CESifo WORKING PAPERS

6906
2018

February 2018

Disagreement and Optimal Security Design

Juan M. Ortner, Martin C. Schmalz

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editors: Clemens Fuest, Oliver Falck, Jasmin Gröschl
www.cesifo-group.org/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: www.CESifo-group.org/wp

CESifo *Working Paper No. 6906*
Category 13: Behavioural Economics

Disagreement and Optimal Security Design

Abstract

We study optimal security design when the issuer and market participants agree to disagree about the characteristics of the asset to be securitized. We show that pooling assets can be optimal because it mitigates the effects of disagreement between issuer and investors, whereas tranching a cash-flow stream allows the issuer to exploit disagreement between investors. Interestingly, pooling and tranching can be complements. The optimality of debt with or without call provisions can be derived as a special case. In a model with multiple financing rounds, convertible securities naturally emerge to finance highly skewed ventures.

JEL-Codes: G300, G320, D840, D860.

Keywords: disagreement, security design, optimism, overconfidence, pooling, behavioral finance.

Juan M. Ortner
Department of Economics
Boston University
USA - Boston, MA 02215
jortner@bu.edu

Martin C. Schmalz
University of Michigan
Stephen M. Ross School of Business
USA - 48109-1234 Ann Arbor MI
schmalz@umich.edu

May 15, 2017
This paper was previously circulated under the title “Robust Security Design.” For helpful comments and discussions we would like to thank Ulf Axelson (discussant), Nittai Bergman, Tilman Börgers, Markus Brunnermeier, Robin Greenwood, Naveen Khanna, Ernst Maug, Benny Moldovanu, Stefan Nagel, Uday Rajan, Adi Sunderam, Anjan Thakor, Boris Vallée, and especially Denis Gromb for very detailed comments, as well as the audience at the AEA Meetings 2016 in San Francisco, Cambridge Corporate Finance Theory Symposium, and seminar participants at Frankfurt School of Finance and Management, Universität zu Köln, and the University of Michigan. All errors are our own.

1 Introduction

Which security does a firm optimally issue when firm and market participants agree to disagree about the firm's cash-flow distribution? Several earlier papers have employed the assumption of belief disagreement between firm and investors – and in particular of optimistic issuers – to explain capital structure choices, investment decisions, the choice of debt maturities, or the emergence of intermediaries. By contrast, this paper studies how differences in beliefs influence a firm's optimal security design in the sense of Allen and Gale (1988), i.e., in a model that imposes only minimal restrictions on the shape of the contract. We show that disagreement in beliefs can generate various commonly observed financial contracts, including the pooling and tranching of securities. Our model also explains empirical patterns in the dynamics of securities issuance that are more difficult to reconcile with existing theories.

We consider an issuer who owns an asset that will pay uncertain cash-flows at a future date. To raise capital, the issuer designs a security which is backed by the asset's cash-flows. Following DeMarzo and Duffie (1999), we assume the issuer discounts future cash-flows more than the market does. We allow for different types of investors in the market, who may have different beliefs about the asset's cash-flow distribution. Our main assumption is that the issuer is more optimistic than market participants about the asset's cash-flow distribution.[1] The issuer's problem is to design the monotonic securities (one for each type of investor) backed by the underlying asset that maximize her expected payoff, which is is given by the sum of the market prices of the securities she sells and the expected discounted value of retained cash-flows.

Our model generates four sets of results. First, we provide conditions under which it is optimal for the issuer to sell different tranches to the different types of investors. Second,

[1] This assumption is different from assuming difference in the success probability of a project with binary payoff, as has been employed in the previous literature. The difference is meaningful as it leads to a distinction between outside equity and debt, among others.

we show that selling a security backed by a pool of several underlying assets can be strictly preferred to selling individual asset-backed securities. Third, we provide conditions under which pooling and tranching are complements: there are situations in which pooling is strictly optimal only when the issuer expects to tranch the pooled assets at a later stage. Fourth, we show that, in a model with multiple financing rounds, the optimal security can be convertible preferred stock, a security commonly used in venture capital (VC) financing.

The intuition behind the optimality of tranching is simple, and related to Garmaise (2001): when there are differences in beliefs among investors, it is optimal for the issuer to design different securities, targeted to the different investor types, and to retain the most junior tranche.[2]

We then specialize to the case in which effectively all investors in the market share the same beliefs and show that, under standard conditions, the optimal security is debt. The intuition is similar to that driving earlier results on capital structure choice and investment amid disagreement between issuer and market, including De Meza and Southey (1996); Heaton (2002); Hackbarth (2008): the issuer finds it optimal to only sell cash-flows in the left tail of the cash-flow distribution, which the market values relatively more, and to retain the right tail of the distribution, which the issuer values relatively more. Our model also predicts that pre-existing debt may lead the firm to optimally stop selling securities. This prediction contrasts with that of the traditional "pecking order" model (Myers and Majluf, 1984), in which firms never issue equity or do so only as a "last resort" – firms in our model sell equity when investors are confident. The fact that firms issue equity when stock prices and sentiment are high (e.g., Marsh, 1982; Baker and Wurgler, 2002; Erel et al., 2011; McLean and Zhao, 2014; Farre-Mensa, 2015) and when agreement between issuer and market is high (Dittmar and Thakor, 2007) is in line with our model's predictions.

[2]The latter prediction depends on the assumption that the issuer is more optimistic than the market. The prediction would be reversed in a (perhaps trivial) twist on the model that involves allowing some investors to be more optimistic than the issuer.

For our result on pooling, we consider an issuer who owns two underlying assets.[3] We start by assuming that there is a single type of investor in the market. In this setting, we show that an optimistic issuer may strictly prefer to sell a security backed by the pool of assets to selling individual asset-backed securities. Intuitively, while outside investors might be very pessimistic (relative to the issuer) about the probability of an individual asset delivering high profits, they will typically be less pessimistic about the event that at least one of several assets pays off a high return. As a result, an issuer who owns multiple assets may find it strictly optimal to combine them and sell a "senior" security backed by the pool of assets. The following example illustrates the mechanics.

Example 1. Consider first an issuer who owns a single asset, which can either pay a return of 1 or a return of 0. The market believes that the probability of the asset paying off is $\frac{1}{3}$; the issuer believes in an upside probability of $\frac{2}{3}$. The issuer discounts future cash-flows with a factor of 0.6, whereas the market does not discount. The market is therefore willing to pay $\frac{1}{3}$ for the asset. Since the asset is worth $\frac{2}{3} \cdot 0.6 = 0.4$ to the issuer, she retains it.

Consider now an issuer who owns two of these assets with *iid* returns. The issuer's payoff from retaining the two assets is 0.8, which is strictly larger than her payoff from selling two individual securities, each backed by an asset. Suppose instead that the issuer sells a "senior" security backed by the pool of assets that pays 1 if at least one asset pays off and zero otherwise. Investors are willing to pay $1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$ for the security, while the issuer assigns to it a value of $\left(1 - \left(\frac{1}{3}\right)^2\right) \cdot 0.6 = \frac{8}{15} < \frac{5}{9}$. Because the issuer retains a cash-flow of 1 in the event that both assets pay off, her expected payoff from selling this security is $\frac{5}{9} + \left(\frac{2}{3}\right)^2 0.6 \approx 0.822$.

We stress that differences in beliefs between the issuer and the market are crucial for pooling to be optimal in this setting. Indeed, because the issuer discounts future cash-flows more

[3] For simplicity we focus on the case of two assets, although the results extend to the case in which the issuer owns several assets.

than the market, with homogenous beliefs it is always optimal for the issuer to sell the entire firm. As a result, when issuer and market share the same beliefs, the issuer is indifferent between pooling her assets or selling them as separate concerns. Moreover, our model has the feature that the optimality of pooling assets breaks down when the correlation of the underlying assets increases. This mechanism can explain empirically observed dynamics of securitization, discussed below.

Next, we consider the case of an issuer who owns two assets, and who faces different types of investors in the market. We impose assumptions on the primitives such that an issuer who does not pool the assets will find it optimal to sell a single tranche (designed for one type of investors), and under which an issuer who does not tranche (i.e., sells to only one type of investors) will find it optimal not to pool. We show that, in this setting, it can still be strictly optimal for the issuer to pool the assets and sell different tranches. In other words, we show conditions under which pooling and tranching are complements.

Finally, we consider a model with multiple financing rounds and show that convertible securities commonly used in VC financing become optimal (see, e.g., Gompers and Lerner, 2001; Kaplan and Strömberg, 2003, 2004). We assume that the issuer (here: the entrepreneur) is more confident about the project's prospects than the financier (here: the VC).[4] Because the entrepreneur assigns a relatively low probability to states in which performance is bad, she finds it relatively cheap to rescind cash-flows to the VC in such states. At the time of initial contracting, the entrepreneur also secures an option for a future financing round that enables her to expand the project conditional on good interim performance.[5] Given her optimistic beliefs, the entrepreneur assigns a relatively high value to this option. By contrast, the VC

[4]See Cooper et al. (1988); Bernardo and Welch (2001); Moskowitz and Vissing-Jørgensen (2002); Koellinger et al. (2007); Puri and Robinson (2007) for discussions of entrepreneurial optimism.

[5]Our predictions do not rely on assuming that the strike of the refinancing option is determined with certainty at the time of initial contracting. Indeed, the fact that disagreement is reduced by learning about project quality over time is one of the key reasons for using convertible securities in early-stage financing, rather than securities that require a precise agreed-upon valuation of the project at the time of contracting.

finds it cheap to write this option to the entrepreneur, because the VC finds good interim performance relatively unlikely. In addition to the optionality, the model also predicts that the VC obtains a stake of the upside of the project conditional on refinancing and expanding the project at the interim stage. It is the upside part of the contract that allows the VC to break even.

Importantly, the key assumption that leads to the financier securing part of the upside is that both the project's required investment and its upside potential are high – in other words, the payoff profile is highly skewed. Indeed, in practice many safer entrepreneurial ventures are financed with straight (bank) debt, whereas VC financing with convertibles are used only for projects with relatively high investment needs and high potential payoffs (Cochrane, 2005). The model's mechanics are also strongly consistent with practitioners' accounts of the drivers of the use of convertibles in the financing of young firms.[6]

2 Related Literature

The idea that belief disagreement can shape the securities that firms issue goes back at least to Modigliani and Miller (1958), who write "Grounds for preferring one type of financial structure to another still exist within the framework of our model. If the owners of a firm discovered a major investment opportunity which they felt would yield much more than [the market's discount rate], they might well prefer not to finance it via common stock. A better course would be to finance the project initially with debt. Still another possibility might be to [issue] a convertible debenture." (excerpts from p. 292) This paper offers a formal investigation into the role of disagreement in optimal design of securities.

Following a long literature in economics, our model assumes that there is open disagree-

[6] Entrepreneurs seeking VC funding are overoptimistic: "Guy Kawasaki, a venture capitalist, says that when an entrepreneur promises to make $50m in four years he adds one year to the delivery time and divides the revenue by ten." (Economist, 2014). "Convertible[s] remains an attractive way to bridge the [resulting] valuation gap." (http://www.cognitionllp.com/convertible-debt-panacea-or-pain/)

ment between issuer and market, and that there are no informational frictions. Our model's mechanism is thus sharply different from contributions that rationalize particular securities (including debt, convertible debt) and pooling with informational asymmetries.[7] Our model also makes no use of moral hazard as a driver of the optimal security. In particular, the alignment of incentives to exert effort plays no role in generating convertibles in our model. This is in contrast to earlier contributions by Green (1984); Admati and Pfleiderer (1994); Schmidt (2003); Cornelli and Yosha (2003).[8]

Several more papers than those cited above have invoked differences in beliefs to explain stylized facts of entrepreneurship as well as corporate investment, financing, payout and capital structure choices.[9] By contrast, we allow for a much less restricted state space (N states instead of two) and/or contracting space (all monotonic securities, rather than a choice between equity and debt, debt of various maturities, or similar) and study the question which security is optimal under these more general conditions.[10] Our paper also relates to Garmaise (2001), who shows that tranching can be optimal in a model in which there is disagreement among investors and in which the prices of securities are determined through a first price auction and Coval and Thakor (2005), who show that rational actors can arise to intermediate between optimistic entrepreneurs and pessimistic investors, issuing safe debt and retaining a mezzanine tranche of the projects they finance (see also Gennaioli et al., 2013).[11]

[7]See, for instance, Myers and Majluf (1984); Noe (1988); Innes (1990); Nachman and Noe (1990); Gorton and Pennacchi (1990); Stein (1992); Nachman and Noe (1994); Manove and Padilla (1999); DeMarzo (2005); Inderst and Mueller (2006); Axelson (2007). As a result, our predictions are less sensitive to changes in distributional assumptions, as emphasized by Nachman and Noe (1990); Fulghieri et al. (2013). We discuss differences in predictions throughout the paper.

[8]For other models linking moral hazard and security design, see Bergemann and Hege (1998); Winton and Yerramilli (2008); Antic (2014); Hébert (2014).

[9]De Meza and Southey (1996); Boot et al. (2006, 2008); Landier and Thesmar (2009); Malmendier et al. (2011); Boot and Thakor (2011); Bayar et al. (2011); Thakor and Whited (2011); Huang and Thakor (2013); Adam et al. (2014); Bayar et al. (forthcoming). See also Simsek (2013), who studies how differences in beliefs among investors affect asset prices in the presence of collateral constraints.

[10]Yang (2013) shows that limited channel capacity can render debt and pooling optimal; Yang and Zeng (2015) explain the use of convertibles in venture capital financing with endogenous information choice.

[11]Our paper is also related to a literature on corporate financial choices amid an ambiguity-averse pool of investors (e.g. Dicks and Fulghieri, 2015), because ambiguity aversion on behalf of the market collapses

3 Basic Model

3.1 Payoffs, Beliefs, and Objectives

At date $t = 0$, an issuer owns a risky asset yielding state-contingent payoffs at date $t = 1$. For now we treat this as a single asset, but this could be a pool of several assets. There is a finite set of possible states of nature $S = \{1, .., K\}$ at $t = 1$, and the asset pays an amount $X_s \in \mathbb{R}_+$ in state $s \in S$.[12] We assume that $X_s > 0$ for all $s \in S$ and that there exists at least one pair of states $s, s' \in S$ such that $X_s \neq X_{s'}$. Without loss of generality, we order the states so that $X_1 \leq X_2 \leq ... \leq X_K$.

We let π^I be the probability distribution over S that represents the issuer's beliefs, and assume that $\pi^I_s > 0$ for all $s \in S$. Market participants have different beliefs about the cash-flow distribution of the underlying asset than the issuer. In particular, we assume that there are two types of investors in the market, $\tau = t_1, t_2$.[13] The two types of investors differ in their beliefs about the cash-flow distribution of the asset that the issuer owns. For $j = 1, 2$, let π^j be the probability distribution over S representing the beliefs of investors of type τ_i. We assume that the issuer is more optimistic than both types of investors: for $j = 1, 2$, π^I first-order stochastically dominates π^j.

The issuer has to design securities $(F^1, F^2) \in \mathbb{R}^K_+$ backed by the cash-flows $X = (X_s)_{s \in S}$ to sell in the market. Thus, securities (F^1, F^2) must be such that $0 \leq F^1_s + F^2_s \leq X_s$ for all $s \in S$. Following DeMarzo and Duffie (1999) we assume that the issuer discounts retained cash-flows at a rate that is higher than the market rate (which is normalized to

to disagreement between issuer and market. Lee and Rajan (2017) study optimal security design when both issuer *and* market are ambiguity averse.

[12] The assumption that states are finite is for simplicity. Our main results are robust to having a continuum of states.

[13] The assumption that there are two types of investors is for simplicity; all of our results extend to the case with $n \geq 1$ types of investors. In particular, the number of tranches will depend on the number of investors with different beliefs.

1).[14] Thus, the issuer attaches a value of $\delta \sum_{s\in S} \pi_s^I \left(X_s - F_s^1 - F_s^2\right)$ to retained cash-flows, where $\delta \in (0,1)$ is the issuer's discount rate. The payoff of an issuer who sells to the market securities (F^1, F^2) at a prices p^1 and p^2 (including the value of the retained cash-flows) is then given by $p^1 + p^2 + \delta \sum_{s\in S} \pi_s^I \left(X_s - F_s^1 - F_s^2\right)$.

The price that investors of type t_j are willing to pay for security F is $p^j(F) := \sum_s \pi_s^j F_s$. For any security F, let $p(F) = \max\{p^1(F), p^2(F)\}$ be the highest price that market participants are willing to pay for F. Overall, the issuer's payoff from selling securities (F^1, F^2) is

$$U(F^1, F^2) := p(F^1) + p(F^2) + \delta \sum_{s\in S} \pi_s^I \left(X_s - F_s^1 - F_s^2\right). \tag{1}$$

As is standard in the literature of optimal security design (e.g. DeMarzo and Duffie, 1999), we assume the issuer is restricted to sell monotonic securities.[15]

Definition 1. Say that securities F^1 and F^2 are *monotonic* if F_s^1 and F_s^2 are increasing in s and if $X_s - F_s^1 - F_s^2$ is increasing in s.

Let $\mathcal{F}$ be the set of feasible securities

$$\mathcal{F} := \left\{F^1, F^2 \in \mathbb{R}_+^K : 0 \le F_s^1 + F_s^2 \le X_s \forall s \in S \text{ and } F^1 \text{ and } F^2 \text{ are monotonic}\right\}. \tag{2}$$

The issuer's problem is to find the securities $(F^1, F^2) \in \mathcal{F}$ that solve

$$\sup_{(F^1,F^2)\in\mathcal{F}} U\left(F^1, F^2\right). \tag{3}$$

[14] The assumption that the issuer discounts future cash-flows at a higher rate than the market is a metaphor, for example for a situation in which the issuer has some profitable investment opportunity. Also, the assumption will hold if the issuer faces credit constraints or, as in the case of financial entities, minimum-capital requirements.

[15] As is well known, this assumption can be microfounded with a moral hazard problem. To avoid high payments implied by a non-increasing security, the issuer could easily inflate cash-flows, e.g., by borrowing privately, and thus decrease payments to the investor.

3.2 Optimal Security Design with Divergent Beliefs

In this section we present the solution to problem (3). We introduce additional notation before presenting our results. For any $s \in S$, let $A_s := \{s, s+1, ..., K\}$ be the event that the asset yields cash-flows weakly larger than X_s. For all $s \in S$, let $\pi^I(A_s) := \sum_{s' \geq s} \pi^I_{s'}$ and $\pi^j(A_s) := \sum_{s' \geq s} \pi^j_{s'}$ be, respectively, the probability that the issuer and investors of type j assign to event A_s. The assumption that π^I first-order stochastically dominates π^j implies that $\pi^I(A_s) \geq \pi^j(A_s)$ for all $s \in S$ and for $j = 1, 2$.

Lemma 1. *Let (F^1, F^2) be a solution to (3). Then, there exists $(\hat{F}^1, \hat{F}^2) \in \mathcal{F}$ with $U(\tilde{F}^1, \tilde{F}^2) = U(F^1, F^2)$ such that, for $j = 1, 2$, $p(\tilde{F}^j) = p^j(\tilde{F}^j)$.*

By Lemma 1, it is without loss of optimality to consider solutions (F^1, F^2) to (3) such that, for $j = 1, 2$, security F^j is bought by investors of type t_j.

The following result characterizes the optimal security. In what follows, for $j = 1, 2$, we use $-j$ to denote the investors of type $t_i \neq t_j$.

Proposition 1. *The optimal securities (F^1, F^2) satisfy: $F^1_1 + F^2_1 = X_1$, and for $j = 1, 2$ and for all $s \in S \backslash \{1\}$,*

$$F^j_s = \begin{cases} F^j_{s-1} + X_s - X_{s-1} & \text{if } \pi^j(A_s) > \max\left\{\pi^{-j}(A_s), \delta\pi^I(A_s)\right\}, \\ F^j_{s-1} & \text{if } \pi^j(A_s) \leq \max\left\{\pi^{-j}(A_s), \delta\pi^I(A_s)\right\}. \end{cases} \tag{4}$$

The key value that determines the shape of the optimal securities (F^1, F^2) at each state s is the difference between $\delta\pi^M(A_s)$ and $\max\{\pi^1(A_s), \pi^2(A_s)\}$; i.e., the difference between the probability that the issuer and market assign to profits being larger than X_s. If $\delta\pi^M(A_s) < \max\{\pi^1(A_s), \pi^2(A_s)\}$, the optimal securities (F^1, F^2) pay the largest possible amount (subject to monotonicity constraints) in state s; i.e., $F^1_s + F^2_s = F^1_{s-1} + F^2_{s-1} + X_s - X_{s-1}$. In contrast, if $\delta\pi^M(A_s) \geq \max\{\pi^1(A_s), \pi^2(A_s)\}$, the optimal securities pay the least possible amount (again, subject to monotonicity constraints) at state s; i.e., $F_s^1 + F_s^2 = F_{s-1}^1 + F_{s-1}^2$.

The following corollaries immediately follow.

Corollary 1. *Suppose that there exists $s_1, s_2 \in S, s_1 < s_2$, such that*

(i) $\pi^1(A_s) > \max\left\{\pi^2(A_s), \delta\pi^I(A_s)\right\}$ *if and only if* $s \leq s_1$, *and*

(ii) $\pi^2(A_s) > \max\left\{\pi^1(A_s), \delta\pi^I(A_s)\right\}$ *if and only if* $s \in (s_1, s_2]$.

Then, the optimal securities (F^1, F^2) *are* $F_s^1 = \min\{X_s, X_{s_1}\}$ *(i.e.,* F^1 *is debt with face value* X_{s_1}*) and*

$$F_s^2 = \begin{cases} 0 & \textit{if } s \leq s_1, \\ X_s - X_{s_1} & \textit{if } s \in (s_1, s_2], \\ X_{s_2} - X_{s_1} & \textit{if } s > s_2. \end{cases}$$

Under the conditions in Corollary 1, the issuer sells a senior tranche F^1, which is bought by investors of type t_1, and a mezzanine tranche F^2, which is bought by investors of type t_2. Finally, the issuer only retains the most junior cash-flows $X_s - X_{s_{t_2}}$ at states $s > s_s$. The real-world correspondence of the mezzanine tranche may be preferred equity or junior debt.

3.3 Single Investor

A special case of the model is one in which there is effectively a single investor in the market. To formalize this, suppose that π^1 FOSD π^2, so investors of type 1 are relatively more optimistic than investors of type 2. We use the convention that $F_0 = X_0 = 0$ for any security F.

Corollary 2. *Suppose that* π^1 *FOSD* π^2*. Then, the optimal securities* (F^1, F^2) *have* $F_s^2 = 0$

for all s*, and*

$$\forall s \in S, \quad F_s^1 = \begin{cases} F_{s-1}^1 + X_s - X_{s-1} & \text{if } \pi^1(A_s) > \delta\pi^I(A_s), \\ F_{s-1}^1 & \text{if } \pi^1(A_s) \leq \delta\pi^I(A_s). \end{cases} \tag{5}$$

Corollary 2 characterizes the optimal security in the case in which all (relevant) investors share the same beliefs. By Corollary 2, when the ratio $\frac{\pi^1(A_s)}{\delta\pi^I(A_s)}$ is decreasing in s, the optimal security is given by a debt contract with face value $s^* = \min\left\{s \in S : \pi^1(A_s) > \delta\pi^I(A_s)\right\}$. Holding δ fixed, the face value of debt s^* depends on how different the beliefs of the issuer and market are. When the market is extremely pessimistic, the firm issues only risk-free debt. (Once that option is exhausted, it stops issuance altogether, as we show below.) By contrast, the issuer sells the whole firm when the market is optimistic and there is less disagreement.

This prediction is strongly consistent with the timing of securities issuances to meet market sentiment (e.g., Marsh (1982); Baker and Wurgler (2002), and in particular Dittmar and Thakor (2007)). At the same time, our prediction is in stark contrast to several theories of security design based on asymmetric information. Most prominently, the traditional "pecking order" hypothesis holds that firms issue equity only as a "last resort" (e.g., Myers, 1984) – hence, only the worst firms that have run out of other options issue equity. Contrasting that prediction is the empirical evidence, which indicates that firms issue equity also (and indeed predominantly) when not in financial distress (Frank and Goyal, 2003; Fama and French, 2005). The empirical evidence is arguably more consistent with the disagreement prediction that the relative optimism of investors versus firms drives issuance decisions: Farre-Mensa (2015) analyses firms that are hit with negative cash-flow shocks and thus face a need to issue securities (a decrease in δ in our model), and shows that firms whose stock is overvalued issue equity, whereas undervalued firms issue debt. Similar in spirit, Erel et al. (2011) and

McLean and Zhao (2014) find that equity issuance is cyclical and higher amid positive investor sentiment, whereas firms turn to issuing safer securities during market downturns.

Pre-existing Debt

We now briefly consider the problem of an issuer who has senior debt outstanding that is backed by the cash-flows that her asset will generate, and who is considering to issue a new security backed by the remaining cash-flows. For simplicity, we maintain the assumption that there is effectively one single investor type.

Suppose the issuer has debt outstanding with face value $D < X_K$. The issuer's goal is to design a security $F \in \mathbb{R}_+^K$ to sell to the market, with F backed by the remaining cash-flows; i.e., for all s, F satisfies $0 \leq F_s \leq X_s - \min\{X_s, D\}$. As before, we restrict the issuer to design monotonic securities; that is, securities F such that F_s and $X_s - F_s - \min\{X_s, D\}$ are increasing in s. Let $\mathcal{F}_D$ denote the set of feasible securities.[16] The issuer's problem is $\sup_{F \in \mathcal{F}_D} U_D(F)$, where for any $F \in \mathcal{F}_D$,

$$U_D(F) := \sum_{s \in S} \pi_s^1 F_s + \delta \sum_{s \in S} \pi_s^I (X_s - \min\{X_s, D\} - F_s).$$

Let $s_D = \max\{s \in S : X_s \leq D\}$, and note that any security $F \in \mathcal{F}_D$ must be such that $F_s = 0$ for all $s \leq s_D$.

Corollary 3. *Suppose the issuer already has debt outstanding with face value D. Then, the*

[16] That is, $\mathcal{F}_D := \{F \in \mathbb{R}^K : 0 \leq F_s \leq X_s - \min\{X_s, D\} \forall s \in S$ and F_s and $X_s - F_s - \min\{X_s, D\}$ are increasing in $s\}$.

optimal security is described by

$$\forall s \in S, \quad F_s = \begin{cases} 0 & \textit{if } s \leq s_D \\ F_{s-1} + X_s - X_{s-1} & \textit{if } \pi^1(A_s) \geq \delta\pi^I(A_s) \textit{ and } s > s_D, \\ F_{s-1} & \textit{if } \pi^1(A_s) < \delta\pi^I(A_s) \textit{ and } s > s_D. \end{cases} \tag{6}$$

Corollary 3 shows that the firm in our model may stop the issuance of all securities when it becomes over-levered, and is thus similar to the underinvestment result in Heaton (2002). This prediction contrasts with that of informational theories of security design as well as with tradeoff models, in which the firm may start to issue equity instead of debt when it has preexisting debt. The existing evidence supports the prediction made here: Erel et al. (2011) show that low market sentiment can indeed lead firms not only to stop equity issuances but to not access credit markets at all.

4 Pooling

This section shows how an issuer who has more optimistic beliefs than the market can strictly benefit from pooling different assets and designing a security backed by the cash-flows generated by the pool. By exploring a new mechanism that can lead to pooling, this result speaks to a question in security design that has become a central item of the policy discussion in the aftermath of the financial crisis. In particular, the mechanism helps explain the dynamics of securitization (Chernenko et al., 2013; Fuster and Vickery, 2014) in ways that is consistent with the empirical evidence on issuers' relatively optimistic beliefs (Cheng et al., 2014) and the procyclical nature of belief disagreement (see e.g. Chen et al., 2002; Scheinkman and Xiong, 2003; Hong and Stein, 2007). Moreover, we show how pooling and tranching can be complements.

4.1 General Framework

Consider an issuer who owns two assets, X^1 and X^2, with *iid* returns.[17] Let $S = \{1, ..., K\}$ and let $\{X_s\}_{s\in S}$ be the possible cash-flow realizations of asset X^a, $a = 1, 2$. Without loss of generality we assume that $X_1 \leq X_2 \leq ... \leq X_K$.

As in Section 3, we assume that there are two types of investors. Let π^I be the probability distributions over S representing the beliefs of the issuer; let π^1 and π^2 be the probability distributions over S representing, respectively, the beliefs of investors of type t_1 and t_2. The issuer is more optimistic than the market, so π^I first-order stochastically dominates π^1 and π^2. The issuer discounts future profits at rate $\delta < 1$, whereas the market discounts future profits at rate 1.

The timing of events is as follows. First, the issuer decides whether to pool the assets or not. If she pools the asset, she designs securities (F^1, F^2) backed by the asset pool $Y = X^1 + X^2$. For $j = 1, 2$, let $F^j_{s,s'}$ be the money that security F^j pays when asset 1's realized return is X_s and asset 2's realized return is $X_{s'}$. We restrict the issuer to sell securities (F^1, F^2) that satisfy the following monotonicity requirements:

Definition 2. Say that securities F^1 and F^2 backed by asset $Y = X^1 + X^2$ are X^1X^2-*monotonic* if:

(i) for $j = 1, 2$, $F^j_{s,s'}$ is increasing in s and s';

(ii) $X_s + X_{s'} - (F^1_{s,s'} + F^2_{s,s'})$ is increasing in s and s'.

This monotonicity restriction assumes it is difficult for the issuer to manipulate profits

[17] We focus on the case of two assets for simplicity. The results can be extended to the case of $n > 2$ assets.

across assets. For example, the issuer may face legal constraints that make it difficult for her to transfer profits from one asset to another.

Let $\hat{S} = S \times S$, and let $\mathcal{F}_Y$ be the set of feasible securities:

$$\mathcal{F}_Y := \left\{ F^1, F^2 \in \mathbb{R}^{|\hat{S}|} : 0 \leq F^1_{s,s'} + F^2_{s,s'} \leq X_s + X_{s'} \forall (s,s') \in \hat{S} \text{ and } (F^1, F^2) \text{ are } X^1X^2\text{-monotonic} \right\}.$$

Note that the price that market participants of type j are willing to pay for security F is $p^j_Y(F) := \sum_{s \in S} \sum_{s' \in S} \pi^j_s \pi^j_{s'} F_{s,s'}$.

If the issuer does not pool the assets, for each asset $a = 1, 2$, the issuer designs securities $(F^{a,1}, F^{a,2})$ backed by asset X^a. In this case, securities $(F^{a,1}, F^{a,2})$ have to be monotonic with respect to asset X^a, as in Section 3.

Pooled assets

The issuer's profits from pooling the assets and selling securities $(F^1, F^2) \in \mathcal{F}(Y)$ are

$$U_Y(F^1, F^2) = p_Y(F^1) + p_Y(F^2) + \delta \sum_{s \in \tilde{S}} \pi^I_s \pi^I_{s'} (X_s + X_{s'} - F^1_s - F^2_s), \tag{7}$$

where, for any security F, $p_Y(F) = \max_{j=1,2} p^j_Y(F)$ is the highest price that investors are willing to pay for F. The problem of an issuer who pools the asset is then

$$\sup_{(F^1,F^2) \in \mathcal{F}_Y} U_Y(F^1, F^2). \tag{8}$$

Separate assets

For each asset X^a, let $\mathcal{F}_{X^a}$ be the set of securities (F^1, F^2) backed by X^a that are monotonic; i.e., that satisfy the conditions in definition 1. For any $(F^1, F^2) \in \mathcal{F}_{X^a}$, we let $U_{X^a}(F^1, F^2)$ be the profits that the issuer obtains from selling securities (F^1, F^2) (calculated as in Section

3). Then, an issuer who doesn't pool the assets solves the following problem for each asset $a = 1, 2$:

$$\sup_{(F^1,F^2)\in\mathcal{F}_{X^a}} U_{X^a}(F^1, F^2). \tag{9}$$

The solution to this problem is characterized by Proposition 1. We are interested in whether pooling can be preferred to selling the assets as separate concerns.

4.2 Single investor

We start by considering the case in which $\pi^1 = \pi^2$, so there is one type of investor in the market. For notational simplicity, we will assume that the issuer sells the security to investor 1. We start with a simple example:

Example 2. Suppose the issuer has two assets, X^1 and X^2. Each of the assets can produce cash-flows in $\{X_1, X_2\}$, with $X_2 > X_1 > 0$. Let $\pi^I \in (0, 1)$ and $\pi \in (0, 1)$ be, respectively, the probability the issuer and market assigns to the asset yielding cash-flows X_1. The issuer is more optimistic than the market, so $\pi^I < \pi$. We further assume that $\pi > 1 - \delta(1 - \pi^I)$.

Consider first the problem of an issuer who does not pool the assets. By Proposition 1, for each asset X^a, an optimal security F has $F_1 = X_1$ and $F_2 = X_1$ (since $\delta(1-\pi^I) > 1-\pi$). The issuer's profits from selling the securities separately are $2X_1 + 2\delta(1 - \pi^I)(X_2 - X_1)$.

Suppose instead that the issuer pools the two assets and sells a single security backed by the pool. Let $Y = X^1 + X^2$ and $F_Y = \min\{Y, X_1 + X_2\}$. The market-price of security F_Y is $p(F_Y) = \pi^2 2X_1 + (1 - \pi^2)(X_1 + X_2)$, and the issuer's payoff from selling F_Y is $2X_1 + (1 - \pi^2 + \delta(1 - \pi^I)^2)(X_2 - X_1)$. The issuer strictly prefers to pool the assets and sell security F_Y if $\pi < \sqrt{1 - \delta(1 - (\pi^I)^2)}$. Therefore, for $\pi \in \left(1 - \delta(1 - \pi^I), \sqrt{1 - \delta(1 - (\pi^I)^2)}\right)$, pooling is strictly optimal.

Intuitively, the market is relatively less pessimistic about the event that one of the two assets yields a cash-flow of X_2. By pooling the two assets, the issuer is able to design a security that pays off a high return precisely when this event occurs. The example illustrates why changes in belief divergence between issuers and the market should relate to the time-series variation in the issuance of asset-backed securities, as documented by Chernenko et al. (2013).

We now present a general result. Let $\hat{F}$ be the optimal security backed by a single asset X^a. The issuer's profits from selling the two assets separately are then $2U_{X^a}(\hat{F})$. As before, for each $s \in S$ let $A_s = \{s, s+1, ..., K\}$ be the event that an asset pays weakly more than X_s. Recall that π^1 are the beliefs of the investors in the market. We make the following assumption.

Assumption 1. *There exists $k \in S \backslash \{K\}$ such that*

(i) $\pi^1(A_s) \geq \delta \pi^I(A_s)$ *if and only if* $s \leq k$, *and*

(ii) $\frac{\pi^1(A_{k+1})}{\delta \pi^I(A_{k+1})} > \frac{2-\pi^I(A_{k+1})}{2-\pi^1(A_{k+1})}$.

Let

$$\mathcal{F}_Y^1 := \left\{ (F^1, F^2) \in \mathcal{F}_Y : F_s^2 = 0 \text{ for all } s \in S \right\}, \tag{10}$$

denote the set of feasible securities when the issuer pools the assets and designs a single security to investors with beliefs π^1. Note that the seller's profits from pooling in this case are $\sup_{F \in \mathcal{F}_Y^1} U_Y(F)$. Our next result shows that, under Assumption 1, pooling is strictly optimal.

Proposition 2. *Suppose that Assumption 1 holds. Then, pooling is optimal:* $\sup_{F \in \mathcal{F}_Y^1} U_Y(F) > 2U_{X^a}(\hat{F})$.

4.3 Multiple investors

We now consider a setting in which there are two types of investors. We show that belief disagreement among them can make pooling and tranching complements.

We make the following assumptions.

Assumption 2. *(i) There exists $k \in S$ such that $\pi^1(A_{s'}) \geq \pi^2(A_{s'})$ for all $s' \leq k$ (with strict inequality for all $s' \neq 1$) and $\pi^1(A_{s'}) < \pi^2(A_{s'})$ for $s' > k$.*

(ii) $\pi^1(A_{s'}) > \delta\pi^I(A_{s'})$ for all $s' \leq k$ and $\delta\pi^I(A_{s'}) \geq \pi^2(A_{s'}) > \pi^1(A_{s'})$ for all $s' > k$.

Assumption 2(i) implies that the c.d.f.'s of the investors' beliefs cross at exactly one point. When the two types of investors assign the same value to the underlying assets (i.e., when $\sum_s \pi_s^1 X_s^a = \sum \pi_s^2 X_s^a$), Assumption 2(i) implies that π^1 second-order stochastically dominates π^2.

Assumption 2(ii) implies that an issuer who sells the two assets separately finds it optimal to not tranch the assets: for each asset $a = 1, 2$, she will sell a single security F^a targeted to investors with beliefs π^1. Indeed, by Proposition 1, under this assumption the optimal securities (F^1, F^2) when selling assets $(X^a)_{a=1,2}$ separately are given by $F_s^{1,a} = \min\{X_k, X_s\}$ and $F_s^{2,a} = 0$ for all s and for $a = 1, 2$. The issuer's profits from selling the two assets separately are then $2U_{X^a}(F^{1,a}, F^{2,a})$.

Our next result shows that, under further conditions, an issuer who only sells securities designed to investors with beliefs π^1 does not benefit from pooling the assets.

Proposition 3. *Suppose Assumption 2 holds, and that*

$$\pi^1(A_s)(2\pi^1(A_{s'}) - \pi^1(A_s)) < \delta\pi^I(A_s)(2\pi^I(A_{s'}) - \pi^I(A_s)) \tag{11}$$

for all s, s' with $s > k$ or $s' > k$. Then, there are no gains from pooling: $\sup_{F \in \mathcal{F}_Y^1} U_Y(F) = 2U_{X^a}(F^{1,a}, F^{2,a})$.

Our next result shows that, in this environment, the issuer finds it strictly optimal to pool the assets: by doing so, she can profit from selling an additional tranch to consumers of type t_2.

Proposition 4. *Suppose that the conditions in Proposition 3 hold, and that* $\frac{\pi^2(A_{k+1})}{\delta\pi^I(A_{k+1})} > \frac{2-\pi^I(A_{k+1})}{2-\pi^2(A_{k+1})}$. *Then, pooling is strictly optimal:* $\sup_{(F^1,F^2)\in\mathcal{F}_Y} U_Y(F^1,F^2) > 2U_{X^a}(F^{1,a},F^{2,a})$.

The results above show that pooling and tranching may be complements: while neither pooling nor tranching and beneficial on their own, the issuer finds it stricly optimal to pool the assets and then tranche them.

We conclude this section by briefly discussing two extensions. For simplicity, we focus on the case in which there is a single investor. Consider first the case in which the underlying assets' returns are not *iid*. In Appendix B we generalize Example 2 to the case of non-zero correlations. Consistent with the time-series variation in the issuance of asset-backed securities discussed above, pooling remains optimal as long as the correlation between the underlying assets is not too high relative to the disagreement in beliefs.

Second, our model assumes that issuer and market disagree about the return distribution of each of the underlying assets, but agree on the correlation between these assets. We stress that disagreement about the correlation in the assets' return can strengthen the investor's incentives for pooling. To see this, consider again the setting in Example 2. Suppose that the market believes that the two assets are *iid*, while the issuer believes that the two assets are perfectly correlated. Assume again that $\pi > 1-\delta(1-\pi^I)$, so that the optimal security backed by asset X^a has $F_s = X_1$ for $s = 1,2$. The issuer's profits from selling the securities separately are given by $2X_1 + 2\delta(1-\pi^I)(X_2 - X_1)$, while her payoff from selling security $F_Y = \min\{Y, X_1+X_2\}$ now is $2X_1 + (1-\pi^2+\delta(1-\pi^I))(X_2-X_1)$. Pooling is strictly optimal whenever $\pi \in (1-\delta(1-\pi^I), \sqrt{1-\delta(1-\pi^I)})$.

5 Convertibles

This section provides a simple dynamic extension of the disagreement framework. The stylized version of the model presented here introduces the possibility of financing a project in multiple stages and contracts between the issuer (here called an entrepreneur) and investor (the bank or venture capitalist) that can depend on interim performance. Given that information about the characteristics of novel types of entrepreneurial ventures tends to be scarce, we find the assumption that beliefs may differ across market participants particularly realistic in this setting. In what follows, we show that a conversion option can be a natural feature of optimal securities when the issuer and investor's beliefs differ. Aside from differences in beliefs, a key assumption needed for the optimality of convertibles (as opposed to straight debt) is that the investment project requires a relatively large investment and has high upside potential – i.e., a highly skewed payoff profile.[18]

The setup is as follows. The entrepreneur is endowed with an investment opportunity, which requires an initial investment I_0 at time $t = 0$, and offers in period $t = 2$ a risky payoff. There are two states of nature, $\{H, L\}$ (high or low). In the interim period, $t = 1$, a public and contractible signal is observed, which we specify below. In response to the signal, there are two options for the project:

- the project can be left as is, in which case the returns of the investment at state $s \in \{H, L\}$ are X_s, with $X_H > X_L > 0$;
- the project can be expanded by way of an interim investment $I_1 > 0$, in which case the returns of the investment at state $s \in \{H, L\}$ are $K \times X_s$, where $K > 1$.

Let $\pi^E \in (0, 1)$ and $\pi^{VC} \in (0, 1)$ be, respectively, the entrepreneur's and the venture capitalist's initial beliefs that the realized state at $t = 2$ will be H. We assume that $\pi^E > \pi^{VC}$,

[18] To be able to most clearly illustrate the role of belief differences, we abstract away from many frictions that are relevant for VC contracting, like moral hazard, adverse selection, taxes, etc., which may help explain other features of contracts used in VC financing.

so the entrepreneur is more optimistic about the project's outcome than the VC.

The interim signal at time $t = 1$, σ, can take either of two values: $\sigma \in \{h, l\}$. We assume that signals $\sigma = h, l$ are informative about the state of nature: the entrepreneur and the VC believe that

$$P(\sigma = h|s = H) = P(\sigma = l|s = L) = \alpha > \frac{1}{2}. \tag{12}$$

For $\sigma \in \{h, l\}$ and for $i = E, VC$, let $\pi^i(\sigma)$ denote the probability that i assigns to the state being H after observing signal σ; $\pi^i(l) = \frac{(1-\alpha)\pi^i}{(1-\alpha)\pi^i+\alpha(1-\pi^i)} < \pi^i < \frac{\alpha\pi^i}{\alpha\pi^i+(1-\alpha)(1-\pi^i)} = \pi^i(h)$.

The contract, in exchange for which the entrepreneur receives funding from a competitive VC sector, specifies:

(i) an expansion decision $\mathbf{1}(\sigma) \in \{0, 1\}$ to be made at $t = 1$ as a function of the signal σ; $\mathbf{1}(\sigma) = 1$ denotes expanding the firm and $\mathbf{1}(\sigma) = 0$ denotes not expanding the firm; and

(ii) repayments $z = (z_L(\sigma), z_H(\sigma))$ from the entrepreneur to the VC to be made at $t = 2$: for $s \in \{H, L\}$ and $\sigma \in \{h, l\}$, $z_s(\sigma) \times (1 + (K - 1)\mathbf{1}(\sigma))$ is the repayment at state s if signal σ was observed at the interim stage (with $z_s(\sigma) \in [0, X_s]$).

We make the following assumptions:

Assumption 3. *(i) the VC believes that the project is profitable enough to invest I_1 at $t = 1$ only after observing signal $\sigma = h$:*

$$(K - 1)\left(\pi^{VC}(h)X_H + (1 - \pi^{VC}(h))X_L\right) > I_1 > K\left(\pi^{VC}(l)X_H + (1 - \pi^{VC}(l))X_L\right). \tag{13}$$

(ii) the VC believes that the project is profitable but risky:

$$\pi^{VC} X_H + (1 - \pi^{VC}) X_L > I_0 > X_L. \tag{14}$$

The entrepreneur's expected payoff from contract $(z, \mathbf{1})$ is

$$\begin{aligned} U^E(z, \mathbf{1}) \ := \ & \left[(1-\alpha)\pi^E (X_H - z_H(l)) + \alpha(1-\pi^E)(X_L - z_L(l))\right](1 + \mathbf{1}(l)(K-1)) \\ & + \left[\alpha\pi^E (X_H - z_H(h)) + (1-\alpha)(1-\pi^E)(X_L - z_L(h))\right](1 + \mathbf{1}(h)(K-1)) \end{aligned} \tag{15}$$

The VC's payoff from this contract is

$$\begin{aligned} U^{VC}(z, \mathbf{1}) \ := \ & \left[(1-\alpha)\pi^{VC} z_H(l) + \alpha(1-\pi^{VC}) z_L(l)\right](1 + \mathbf{1}(l)(K-1)) \\ & + \left[\alpha\pi^{VC} z_H(h) + (1-\alpha)(1-\pi^{VC}) z_L(h)\right](1 + \mathbf{1}(h)(K-1)) \\ & -\rho_l \mathbf{1}(l) I_1 - \rho_h \mathbf{1}(h) I_1 - I_0, \end{aligned} \tag{16}$$

where ρ_l and ρ_h denote, respectively, the probability that the VC assigns to the signal taking values l and h, respectively, i.e., $\rho_l = (1-\alpha)\pi^{VC} + \alpha(1-\pi^{VC})$ and $\rho_h = \alpha\pi^{VC} + (1-\alpha)(1-\pi^{VC})$. The problem of the entrepreneur is

$$\max_{(z,\mathbf{1})} U^E(z, \mathbf{1}) \ s.t. \tag{17}$$

$$U^{VC}(z, \mathbf{1}) \geq 0, \tag{BE}$$

$$K\left(\pi^{VC}(\sigma) z_H(\sigma) + (1 - \pi^{VC}(\sigma)) z_L(\sigma)\right) \geq I_1 \text{ if } \mathbf{1}(\sigma) = 1. \tag{EC}$$

Constraint (BE) is the VC's break-even condition. Constraint (EC) requires that, if the VC expands the project at $t = 1$, her expected return should cover the investment cost.

Proposition 5. *If Assumption 3 holds, the solution to (17) is such that:*

(i) the project is expanded if and only if $\sigma = h$; i.e., $\mathbf{1}(l) = 0$ and $\mathbf{1}(h) = 1$;

(ii) for $\sigma \in \{l, h\}$, $z_L(\sigma) = X_L$;

(iii) $z_H(l)$ and $z_H(h)$ are such that (BE) holds with equality.

Proposition 5 characterizes the main properties of the solution to (17). Part (i) follows immediately from Assumption 3. Part (ii) follows from the fact that the entrepreneur is relatively more optimistic than the VC, and so the cheapest way to satisfy the VC's break-even condition is to repay the entire cash-flows at the low state. (This feature is reminiscent of the results in the optimality of debt in Section 3.2.)

Proposition 5 does not pin down what the exact payments at state H are.[19] However, under further parametric conditions, convertible preferred stock is an optimal contract:

$$\rho_H K X_L + \rho_l X_L \quad < \quad I_0 + \rho_h I_1 < K\left[(1-\alpha)\pi^{VC} X_H + \alpha(1-\pi^{VC})X_L\right] + \rho_l X_L. \quad (18)$$

The first inequality in equation (18) states that the VC does not break even under a contract that specifies repayments $z_s(\sigma) = X_L$ for $s \in \{L, H\}$ and $\sigma \in \{l, h\}$. The second inequality in equation (18), on the other hand, states that the VC makes a strict profit under a contract that specifies repayments $z_H(l) = z_L(l) = z_L(h) = X_L$ and $z_H(h) = X_H$.

Corollary 4. *Suppose Assumption 3 and (18) hold. Then, the following contract solves (17):*

(i) the project is expanded if and only if $\sigma = h$; i.e., $\mathbf{1}(l) = 0$ and $\mathbf{1}(h) = 1$;

(ii) $z_L(\sigma) = X_L$ for $\sigma \in \{l, h\}$.

(iii) $z_H(l) = X_L$ and $z_H(h) \in (X_L, X_H)$ such that (BE) holds with equality.

[19]Indeed, given the linearity of the entrepreneur and the VC's payoffs, there is a continuum of optimal contracts. Increasing $z_H(l)$ by Δ allows the entrepreneur to reduce $z_H(h)$ by $\frac{1-\alpha}{\alpha}\frac{1}{K}\Delta$ (so that the break even constraint is still satisfied with equality). This change in the contract leaves the entrepreneur indifferent since $-\Delta\pi^E(1-\alpha) + \frac{1-\alpha}{\alpha}\frac{1}{K}\Delta\alpha\pi^E K = 0$.

The optimal contract in Corollary 4 can be implemented by a convertible security that promises $\min\{R, K \times X_L\}$ (where R is the return of the project) to the VC, and gives the VC the following options: (i) after observing interim performance, choose whether or not to invest in expanding the project; and (ii) if the project is expanded, choose whether or not to convert the original security into a fraction $z_H(h)$ of equity after observing profits.

The intuition for the result is simple: because the entrepreneur is relatively optimistic about the project's success probability, she finds it relatively valuable to secure the option to expand the project in the future. The VC, on the other hand, finds it relatively cheap to write that option. Of course, both VC and entrepreneur also find it optimal to leave the cash-flows to the VC in case of failure.

Discussion

The above model illustrates that a disagreement-based theory of security design can explain convertible contracts between entrepreneurs and financiers in a natural way. Optimism may not only be a driving force behind the entrepreneur's venture, but also behind the financing vehicle that helps her realize the project. Aside from its simplicity, an attractive feature of the model presented here is that highly skewed projects (those with high investment needs and high potential payoffs) receive financing with convertible securities as typically used in VC, whereas optimistic entrepreneurs with less ambitious and risky projects optimally finance their ventures with straight debt.[20]

[20]Our results are derived in a simplified framework with only two states of nature; the VC shares in on the upside because the compensation on the downside does not allow her to break her even. In a model with more states, the VC's stake would be capped on the upside – unlike in straight convertible preferred contracts. Kramer and Tran (2016) document that 40% of participating convertible preferred stakes are indeed capped.

6 Conclusion

This paper offers a simple but broadly applicable theory of security design based on the premise that issuer and market openly disagree about the asset's cash-flow distribution. We show that issuing securities backed by a pool of assets (instead of issuing one security for each asset) can be optimal. In addition, when there is disagreement among investors, the issuer optimally sells different tranches to the market. Importantly, the possibility of tranching can make pooling more opportune, and vice versa. This result appears useful for our understanding of the procyclical dynamics of securitization.

Straight or callable debt can arise as special cases of our model, namely when disagreement between investors is negligible but issuers are significantly more optimistic than the market. Moreover, when disagreement between issuers and markets is low, the issuer may choose to sell the entire firm. These features are strongly consistent with existing empirical results on the dynamics of issuances in general, and the dynamics of the debt-equity mix in issuances in particular. Viewing a corporation as a pool of assets through the lense of our model can potentially also help shed light into the dynamics of corporate events such as mergers and splits, the dynamics of the conglomerate discount, and the issuance of mezzanine tranches.

Finally, in a stylized setting with multiple financing rounds, we show that conversion features similar to those observed in venture capital contracts are optimal to finance high-risk projects with right-skewed payoff profiles. Consistent with empirical realities, the same model also predicts that lower-risk projects or projects run by less optimistic entrepreneurs are financed with straight debt.

In sum, we find that disagreement between issuer and market can help explain a variety of real-world security designs that have thus far required multiple distinct models and frictions as explanations. For tractability, our model abstracts away from other frictions that are known to be important for security design, such as moral hazard, adverse selection, taxes, etc. Combining disagreement with these frictions may help researchers explain other features of real-world financial contracts that the present study leaves unaddressed. Testing the new empirical predictions arising from our model is also left for future research.

A Proofs

Proofs of Section 3

Proof of Lemma 1. Let $(F^1, F^2) \in \mathcal{F}$ be a solution to the issuer's problem. Note that the lemma clearly holds if the two securities (F^1, F^2) are bought by different types of investors. If the two securities (F^1, F^2) are bought by investors of type t_i, then the issuer's payoff from selling security (F^1, F^2) is

$$\begin{aligned} U(F^1, F^2) &= p^i(F^1) + p^i(F^2) + \delta \sum_{s \in S} \pi_s^I (X_s - F_s^1 - F_s^2) \\ &= \sum_{s \in S} \pi^i (F_s^1 + F_s^2) + \delta \sum_{s \in S} \pi_s^I (X_s - F_s^1 - F_s^2). \end{aligned}$$

Consider the pair of securities $(\tilde{F}^1, \tilde{F}^2)$ with $\tilde{F}_s^i = F_s^1 + F_s^2$ for all s and $\tilde{F}_s^j = 0$ for all s. Since investors of type t_i buy the two securities F^1, F^2, it must be that $p^i(F^j) \geq p^{-i}(F^j)$ for $j = 1, 2$. Note that $p^i(\tilde{F^i}) = p^i(F^1) + p^i(F^2)$ and $p^{-i}(\tilde{F^i}) = p^{-i}(F^1) + p^{-i}(F^2)$. Hence, $p(\tilde{F}^i) = p^i(\tilde{F}^i)$. Moreover, $p^j(\tilde{F^{-i}}) = 0$ for $j = 1, 2$, and $p(\tilde{F}^{-i}) = p^{-i}(\tilde{F}^{-i})$. Finally, note that

$$\begin{aligned} U(\tilde{F}^1, \tilde{F}^2) &= p^i(\tilde{F^i}) + \delta \sum_{s \in S} \pi_s^I (X_s - F_s^1 - F_s^2) \\ &= \sum_{s \in S} \pi_s^i (F_s^1 + F_S^2) + \delta \sum_{s \in S} \pi_s^I (X_s - F_s^1 - F_s^2) = U(F^1, F^2). \end{aligned}$$

□

Proof of Proposition 1. Fix a pair of securities $(F^1, F^2) \in \mathcal{F}$ such that, for $j = 1, 2$, security

F^j is bought by investors of type t_j. The issuer's payoff from selling this pair of securities is

$$\begin{aligned} U(F^1, F^2) &= \sum_{s=1}^{K} \pi_s^1 F_s^1 + \sum_{s=1}^{K} \pi_s^2 F_s^2 + \delta \sum_{s=1}^{K} \pi_s^I (X_s - F_s^1 - F_s^2) \\ &= F_1^1 + F_1^2 + \sum_{s=2}^{K} \left(\pi^1(A_s) - \delta\pi^I(A_s)\right)\left(F_s^1 - F_{s-1}^1\right) \\ &\quad + \sum_{s=2}^{K} \left(\pi^2(A_s) - \delta\pi^I(A_s)\right)\left(F_s^2 - F_{s-1}^2\right) + \delta \sum_{s=1}^{K} \pi_s^I X_s. \end{aligned} \tag{19}$$

Note that any pair of securities $(F^1, F^2) \in \mathcal{F}$ must be such that: (i) $F_1^1 + F_1^2 \in [0, X_1]$, (ii) for all $s > 1$, $F_s^1 + F_s^2 \in [F_{s-1}^1 + F_{s-1}^2, F_{s-1}^1 + F_{s-1}^2 + X_s - X_{s-1}]$ and (iii) for $i = 1, 2$, $F_s^i \geq F_{s-1}^i$. From equation (19), it is optimal for the issuer to set $F_1^1 + F_1^2 = X_1$. Moreover, for $s \in S \setminus \{1\}$ and $j = 1, 2$, it is optimal to set $F_s^j = F_{s-1}^j + X_s - X_{s-1}$ if $\pi^j(A_s) > \max\left\{\delta\pi^I(A_s), \pi^{-j}(A_s)\right\}$, and to set $F_s^j = F_{s-1}^j$ otherwise.

□

Proof of Corollary 3. The proof uses arguments similar to those in the proof of Proposition 1. For any security $F \in \mathcal{F}_D$, the issuer's payoff is

$$\begin{aligned} U(F) &= \sum_{s=1}^{K} \pi_s^M F_s + \delta \sum_{s=1}^{K} \pi_s^I (X_s - \min\{X_s, D\} - F_s) \\ &= F_1 + \sum_{s=2}^{K} \left(\pi^M(A_s) - \delta\pi^I(A_s)\right)(F_s - F_{s-1}) + \delta \sum_{s=1}^{K} \pi_s^I (X_s - \min\{X_s, D\}), \end{aligned} \tag{20}$$

Note that any security $F \in \mathcal{F}_D$ must be such that $F_s = 0$ and for all $s \leq s_D$, $F_s \in [F_{s-1}, F_{s-1} + X_s - X_{s-1}]$ for all $s > s_D$ and $F_s \geq F_{s-1}$ for all s. Moreover, any security that satisfies these conditions belongs to $\mathcal{F}_D$. Mechanically, any optimal security F must have $F_s = 0$ and for all $s \leq s_D$. From equation (20), for any $s > s_D$ it is optimal to set $F_s = F_{s-1} + X_s - X_{s-1}$ if $\pi^M(A_s) \geq \delta\pi^I(A_s)$, and to set $F_s = F_{s-1}$ if $\pi^M(A_s) < \delta\pi^I(A_s)$.

$\square$

Proofs of Section 4

Proof of Proposition 2. By Proposition 1, under Assumption 1 the optimal security backed by a single asset X^a is $F = \min\{X_s, X_k\}$. Note that selling two individual securities F, each backed by one of the assets, is the same as selling security $\tilde{F} \in \mathcal{F}_Y$ such that

$$\tilde{F}_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_k + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_k & \text{if } s \leq k, s' > k, \\ 2X_k & \text{if } s > k, s' > k. \end{cases} \tag{21}$$

Consider the following alternative security $F \in \mathcal{F}_Y$:

$$F_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_{k+1} + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_{k+1} & \text{if } s \leq k, s' > k, \\ X_k + X_{k+1} & \text{if } s > k, s' > k. \end{cases} \tag{22}$$

Note that, for any beliefs π over S,

$$\begin{aligned} \sum_s \sum_{s'} \pi_s \pi_{s'} (F_{s,s'} - \tilde{F}_{s,s'}) &= \sum_{s=1}^{k} \pi_s \left(\sum_{s'=k+1}^{K} \pi_{s'} (X_{k+1} - X_k) \right) + \sum_{s=k+1}^{K} \pi_s \sum_{s'=1}^{K} \pi_{s'} (X_{k+1} - X_k) \\ &= (2 - \pi(A_{k+1}))\pi(A_{k+1})(X_{k+1} - X_k). \end{aligned} \tag{23}$$

Note then that

$$\begin{aligned}
U_Y(F) - 2U_{X^a}(F) &= U_Y(F) - U_Y(\tilde{F}) \\
&= p_Y(F) - p_Y(\tilde{F}) + \delta \sum_s \sum_{s'} \pi_s^I \pi_{s'}^I (\tilde{F}_{s,s'} - F_{s,s'}) \\
&= \sum_s \sum_{s'} \pi_s^1 \pi_{s'}^1 (F_{s,s} - \tilde{F}_{s,s'}) + \delta \sum_s \sum_{s'} \pi_s^I \pi_{s'}^I (\tilde{F}_{s,s'} - F_{s,s'}) \\
&= \left((2 - \pi^1(A_{k+1}))\pi^1(A_{k+1}) - \delta(2 - \pi^I(A_{k+1}))\pi^I(A_{k+1})\right)(X_{k+1} - X_k) > 0,
\end{aligned}$$

where we used equation (23) and Assumption 1.

□

Proof of Proposition 3. Let F^1 be the optimal security when the issuer pools the two assets and sells a single security to market participants with beliefs π^1. Since the two assets have iid returns, it is without loss to assume that F^1 is symmetric: $F^1_{s,s'} = F^1_{s',s}$ for all $s, s' \in S$.[21] We show that $F^1_{s,s'} = \tilde{F}^1_{s,s'}$ for all $s, s' \in S$, where $\tilde{F}^1$ is the security that the issuer will effective sell if she were to sell two individual securities, each backed by its own asset:

$$\tilde{F}^1_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_k + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_k & \text{if } s \leq k, s' > k, \\ 2X_k & \text{if } s > k, s' > k. \end{cases} \tag{24}$$

We start by showing that $F^1_{s,s'} = \tilde{F}^1_{s,s'}$ for all s, s' such that $s \leq k$ and $s' \leq k$. Towards a contradiction, suppose not, and let $\hat{s}, \hat{s}' \leq k$ be such that $F^1_{\hat{s},\hat{s}'} \neq \tilde{F}^1_{\hat{s},\hat{s}'} = X_{\hat{s}} + X_{\hat{s}'}$. Note that

[21]To see why, suppose the seller finds it optimal to sell a security F^1 that is not symmetric. Let $\hat{F}^1$ be the security such that, for all $s, s' \in S$, $\hat{F}^1_{s,s'} = F^1_{s',s}$. Since the two assets have iid returns, securities F^1 and $\hat{F}^1$ yield the same profits to the issuer. Since F^1 satisfies the monotonicity requirements, so does $\hat{F}^1$. Let G be a security such that, for all s, s', $G_{s,s'} = \frac{1}{2}(F^1_{s,s'} + \hat{F}^1_{s,s'})$. Note that security G is symmetric, satisfies the monotonicity requirements, and give the same profits to the issuer as security F_1.

it must be that $F^1_{\hat{s},\hat{s}'} = X_{\hat{s}} + X_{\hat{s}'} - \epsilon$ for some $\epsilon > 0$. Consider the security $\hat{F}$ such that

$$\hat{F}_{s,s'} = \begin{cases} F^1_{s,s'} & s < \hat{s} \text{ and } s' < \hat{s}' \\ F^1_{s,s'} + \epsilon & \text{otherwise.} \end{cases} \tag{25}$$

Note that $\hat{F}$ satisfies the monitonicity requirements.

For any beliefs π,

$$\begin{aligned} \sum_s \sum_{s'} \pi_s \pi_{s'} (\hat{F}_{s,s'} - F^1_{s,s'}) &= \sum_{s=1}^{\hat{s}-1} \pi_s \sum_{s'=\hat{s}'}^{K} \pi_{s'} \epsilon + \sum_{s=\hat{s}}^{K} \pi_s \sum_{s'=1}^{K} \pi_{s'} \epsilon \\ &= \epsilon \left[(1 - \pi(A_{\hat{s}}))\pi(A_{\hat{s}'}) + \pi(A_{\hat{s}}) \right]. \end{aligned} \tag{26}$$

Note that

$$\begin{aligned} U_Y(\hat{F}) - U_Y(F^1) &= p_Y(\hat{F}) - p_Y(F^1) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \hat{F}_{s,s'}) \\ &= \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} (\hat{F}_{s,s'} - F^1_{s,s'}) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \hat{F}_{s,s'}) \\ &= \epsilon \left[(1 - \pi^1(A_{\hat{s}}))\pi^1(A_{\hat{s}'}) + \pi^1(A_{\hat{s}}) - \delta(1 - \pi^I(A_{\hat{s}}))\pi^I(A_{\hat{s}'}) - \delta\pi^I(A_{\hat{s}}) \right], \end{aligned}$$

where we used equation (26). By Assumption 2, $\pi^1(A_{\hat{s}}) > \delta\pi^I(A_{\hat{s}})$ (since $\hat{s} \le k$). Moreover, since π^I fosd π^1, $\pi^I(A_{\hat{s}}) \ge \pi^1(A_{\hat{s}})$. Since $\pi^1(A_{\hat{s}}) > \delta\pi^I(A_{\hat{s}})$, it follows that $(1 - \pi^1(A_{\hat{s}}))\pi^1(A_{\hat{s}'}) > \delta(1 - \pi^I(A_{\hat{s}}))\pi^I(A_{\hat{s}'})$. Therefore, $U_Y(\hat{F}) > U_Y(F^1)$, a contradiction to the fact that F^1 is optimal. Hence, F^1 is such that $F^1_{s,s'} = X_s + X_{s'}$ for all $s, s' \le k$.

Next we show that $F^1_{s,s'} = \tilde{F}^1_{s,s'}$ for all s, s' with $s > k$ or $s' > k$. Since $F^1_{s,s'} = \tilde{F}^1_{s,s'} = X_s + X_{s'}$ for all s, s' with $s \le k$ and $s' \le k$, by monotonicity it must be that that $F^1_{s,s'} \ge \min\{X_s, X_k\} + \min\{X_{s'}, X_k\} = \tilde{F}^1_{s,s'}$ for all s, s' with $s > k$ or $s' > k$. Towards a contradiction, suppose that there is s, s' with $s > k$ or $s' > k$ such that $F^1_{s,s'} > \tilde{F}^1_{s,s'}$. Let $\hat{s}' := \min\{s' > k : F^1_{s,s'} > \tilde{F}^1_{s,s'}$ for some $s\}$, and let $\hat{s} := \min\{s : F^1_{s,\hat{s}'} > \tilde{F}^1_{s,\hat{s}'}\}$. By monotonicity, it must be that

$F^1_{s,s'} > \tilde{F}^1_{s,s'}$ for all s, s' with $s \geq \hat{s}$ and $s' \geq \hat{s}'$. Moreover, since F^1 and $\tilde{F}^1$ are symmetric, it must also be that $F^1_{s,s'} > \tilde{F}^1_{s,s'}$ for all s, s' with $s \geq \hat{s}'$ and $s' \geq \hat{s}$. Lastly, by symmetry of F^1, it must be that $\hat{s}' \geq \hat{s}$.

Let $\hat{S} = \{s, s' \in S^2 : s \geq \hat{s}$ and $s' \geq \hat{s}'$ or $s \geq \hat{s}'$ and $s' \geq \hat{s}\}$, and $\epsilon = \min_{s,s' \in \hat{S}} F^1_{s,s'} - \tilde{F}^1_{s,s'} > 0$. Let $\hat{F}$ be an alternative security with

$$\hat{F}_{s,s'} = \begin{cases} F^1_{s,s'} & s, s' \notin \hat{S} \\ F^1_{s,s'} - \epsilon & s, s' \in \hat{S} \end{cases} \tag{27}$$

One can check that, since F^1 satisfies the monotonicity requirements, so does $\hat{F}$. Note that, for any beliefs π

$$\begin{aligned} \sum_s \sum_{s'} \pi_s \pi_{s'} (\hat{F}_{s,s'} - F^1_{s,s'}) &= \sum_{s=\hat{s}}^{K} \pi_s \sum_{s'=\hat{s}'}^{K} \pi_{s'} (-\epsilon) + \sum_{s=\hat{s}'}^{K} \pi_s \sum_{s'=\hat{s}}^{\hat{s}'-1} \pi_{s'} (-\epsilon) \\ &= -\epsilon \left[\pi(A_{\hat{s}}) \pi(A_{\hat{s}'}) + \pi(A_{\hat{s}'})(\pi(A_{\hat{s}}) - \pi(A_{\hat{s}'})) \right] \\ &= -\epsilon \left[\pi(A_{\hat{s}'})(2\pi(A_{\hat{s}}) - \pi(A_{\hat{s}'})) \right]. \end{aligned} \tag{28}$$

Note that

$$\begin{aligned} U_Y(\hat{F}) - U_Y(F^1) &= p_Y(\hat{F}) - p_Y(F^1) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \hat{F}_{s,s'}) \\ &= \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} (\hat{F}_{s,s'} - F^1_{s,s'}) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \hat{F}_{s,s'}) \\ &= -\epsilon \left[\pi^1(A_{\hat{s}'})(2\pi^1(A_{\hat{s}}) - \pi^1(A_{\hat{s}'})) - \delta \pi^I(A_{\hat{s}'})(2\pi^I(A_{\hat{s}}) - \pi^I(A_{\hat{s}'})) \right], \end{aligned}$$

where we used equation (28). By the conditions in the Proposition, $\pi^1(A_{\hat{s}'})(2\pi^1(A_{\hat{s}}) - \pi^1(A_{\hat{s}'})) < \delta \pi^I(A_{\hat{s}'})(2\pi^I(A_{\hat{s}}) - \pi^I(A_{\hat{s}'}))$, and so $U_Y(\hat{F}) > U_Y(F^1)$, which contradicts the fact tha F^1 is optimal. Hence, it must be that $F^1_{s,s'} = \tilde{F}^1_{s,s'}$ for all s, s', and so there are no gains from pooling. $\square$

Proof of Proposition 4. By the arguments in the main text, under Assumption 2 the optimal securities $(F^{1,i}, F^{2,i})$ backed by a single asset X^i are $F^{1,i}_s = \min\{X_s, X_k\}$ and $F^{2,i}_s = 0$. Note that selling two securities $F^{1,i}$, each backed by one of the assets, is the same as selling security $\tilde{F}^1 \in \mathcal{F}_Y$ such that

$$\tilde{F}^1_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_k + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_k & \text{if } s \leq k, s' > k, \\ 2X_k & \text{if } s > k, s' > k. \end{cases} \tag{29}$$

Suppose the issuer pools the assets and sells securities $\left(\tilde{F}^1, \tilde{F}^2\right) \in \mathcal{F}_Y$, with

$$\tilde{F}^2_{s,s'} = \begin{cases} 0 & \text{if } s, s' \leq k, \\ X_{k+1} - X_k & \text{if } s > k \text{ or } s' > k. \end{cases} \tag{30}$$

Note that, for any beliefs π,

$$\sum_s \sum_{s'} \pi_s \pi_{s'} \tilde{F}^2_{s,s'} = \pi(A_{k+1})(2 - \pi(A_{k+1}))(X_{k+1} - X_k). \tag{31}$$

The issuer's payoff from selling the two assets as separate concerns, issuing for each asset X^a securities $(F^{1,a}, F^{2,a})$ with $F^{1,a}_s = \min\{X_s, X_k\}$ and $F^{2,a}_s = 0$, is equal to

$$2U_{X^a}(F^{1,a}, F^{2,a}) = \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} \tilde{F}^1_{s,s'} + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (X_s + X_{s'} - \tilde{F}^1_{s,s'}). \tag{32}$$

On the other hand, the payoff that the issuer gets from pooling the assets and selling securities

$\left(\tilde{F}^1, \tilde{F}^2\right) \in \mathcal{F}_Y$ is

$$U_Y\left(\tilde{F}^1, \tilde{F}^2\right) = \sum_s \sum_{s'} \pi_s^1 \pi_{s'}^1 \tilde{F}^1_{s,s'} + \sum_s \sum_{s'} \pi_s^2 \pi_{s'}^2 \tilde{F}^2_{s,s'} + \delta \sum_s \sum_{s'} \pi_s^I \pi_{s'}^I (X_s + X_{s'} - \tilde{F}^1_{s,s'} - \tilde{F}^2_{s,s'}). \tag{33}$$

Comparing (32) and (33), it follows that

$$\begin{aligned} U_Y\left(\tilde{F}^1, \tilde{F}^2\right) - 2U_{X^a}(F^{1,a}, F^{2,a}) &= \sum_s \sum_{s'} \pi_s^2 \pi_{s'}^2 \tilde{F}^2_{s,s'} - \delta \sum_s \sum_{s'} \pi_s^I \pi_{s'}^I \tilde{F}^2_{s,s'} \\ &= (\pi^2(A_{k+1})(2 - \pi^2(A_{k+1})) - \delta \pi^I(A_{k+1})(2 - \pi^I(A_{k+1}))(X_{k+1} - X_k) > 0, \end{aligned}$$

where the second equality follows from (31) and the strict inequality follows from the assumption in the statement of the Proposition.

□

Proofs of Proposition 5 and Corollary 4

Before presenting the proofs of Proposition 5 and Corollary 4, we establish two useful lemmas.

Lemma 2. *Let $(z, \mathbf{1})$ be a solution to (17). If $z_H(\sigma) > 0$ for $\sigma \in \{l, h\}$, then it must be that $z_L(\sigma) = X_L$.*

Proof. Suppose by contradiction that $z_H(\sigma) > 0$ and $z_L(\sigma) < X_L$ for $\sigma \in \{l, h\}$. Suppose first that $\sigma = h$, and consider a contract $(\tilde{z}, \mathbf{1})$ such that $\tilde{z}_s(l) = z_s(l)$ for $s = H, L$, $\tilde{z}_L(h) = z_L(h) + \epsilon$ and $\tilde{z}_H(h) = z_H(h) - \frac{1-\alpha}{\alpha}\frac{1-\pi^{VC}}{\pi^{VC}}\epsilon$, with $\epsilon > 0$. Note that contract $(\tilde{z}, \mathbf{1})$ gives the VC the same expected payoff as contract $(z, \mathbf{1})$. Note further that

$$\begin{aligned} U^E(\tilde{z}, \mathbf{1}) - U^E(z, \mathbf{1}) &= (1 + \mathbf{1}(h)(K-1))\left(\alpha \pi^E \frac{1-\alpha}{\alpha}\frac{1-\pi^{VC}}{\pi^{VC}}\epsilon - (1-\alpha)(1-\pi^E)\epsilon\right) \\ &= (1 + \mathbf{1}(h)(K-1))\frac{(1-\alpha)\epsilon}{\pi^{VC}}\left(\pi^E(1-\pi^{VC}) - \pi^{VC}(1-\pi^E)\right) > 0, \end{aligned}$$

where we used $\pi^E > \pi^{VC}$. This contradicts the assumption that $(z, \mathbf{1})$ is optimal.

Suppose next that $\sigma = l$. Consider a contract $(\tilde{z}, \mathbf{1})$ such that $\tilde{z}_s(h) = z_s(h)$ for $s = H, L$, $\tilde{z}_L(l) = z_L(l) + \epsilon$ and $\tilde{z}_H(l) = z_H(l) - \frac{\alpha}{1-\alpha}\frac{1-\pi^{VC}}{\pi^{VC}}\epsilon$, with $\epsilon > 0$. Note that contract $(\tilde{z}, \mathbf{1})$ gives the VC the same expected payoff as contract $(z, \mathbf{1})$, and

$$\begin{aligned} U^E(\tilde{z}, \mathbf{1}) - U^E(z, \mathbf{1}) &= (1 + \mathbf{1}(l)(K-1))\left((1-\alpha)\pi^E \frac{\alpha}{1-\alpha}\frac{1-\pi^{VC}}{\pi^{VC}}\epsilon - \alpha(1-\pi^E)\epsilon\right) \\ &= (1 + \mathbf{1}(l)(K-1))\frac{\alpha\epsilon}{\pi^{VC}}\left(\pi^E(1-\pi^{VC}) - \pi^{VC}(1-\pi^E)\right) > 0. \end{aligned}$$

Again this contradicts the assumption that $(z, \mathbf{1})$ is optimal. □

Lemma 3. *Let $(z, \mathbf{1})$ be a solution to (17). Under Assumption 3, $z_L(\sigma) = X_L$ for $\sigma \in \{l, h\}$.*

Proof. Let $(z, \mathbf{1})$ be a solution to (17). The conditions in Assumption 3 imply that, in order for the VC to break even, it must be that $z_H(h) > 0$ and/or $z_H(l) > 0$. If both of these quantities are strictly positive, then the result follows from Lemma 2.

Suppose next that $z_H(h) = 0$ and $z_H(l) > 0$. By Lemma 2, $z_L(l) = X_L$. Towards a contradiction, suppose $z_L(h) < X_L$. Let $(\hat{z}, \mathbf{1})$ be an alternative contract with $\hat{z}_L(\sigma) = z_L(\sigma)$ for $\sigma = h, l$, $\hat{z}_H(l) = z_H(l) - \epsilon$ and $\hat{z}_H(l) = z_H(h) + \frac{1-\alpha}{\alpha}\frac{1+\mathbf{1}(l)(K-1)}{1+\mathbf{1}(h)(K-1)}\epsilon$, with $\epsilon > 0$. Contract $(\hat{z}, \mathbf{1})$ gives entrepreneur and VC the same expected payoff as contract $(z, \mathbf{1})$, so it is also an optimal contract. But this contradicts Lemma 2, since $\hat{z}_L(h) = z_L(h) < X_L$ and $\hat{z}_H(h) > 0$. Hence, if $(z, \mathbf{1})$ is an optimal contract with $z_H(h) = 0$ and $z_H(l) > 0$, it must that $z_L(\sigma) = X_L$ for $\sigma \in \{l, h\}$.

Finally, consider the case with $z_H(h) > 0$ and $z_H(l) = 0$. By Lemma 2, $z_L(h) = X_L$. Towards a contradiction, suppose $z_L(l) < X_L$. Let $(\hat{z}, \mathbf{1})$ be an alternative contract with $\hat{z}_L(\sigma) = z_L(\sigma)$ for $\sigma = h, l$, $\hat{z}_H(l) = z_H(l) + \epsilon$ and $\hat{z}_H(l) = z_H(h) - \frac{1-\alpha}{\alpha}\frac{1+\mathbf{1}(l)(K-1)}{1+\mathbf{1}(h)(K-1)}\epsilon$. Again, contract $(\hat{z}, \mathbf{1})$ gives entrepreneur and VC the same expected payoff as contract $(z, \mathbf{1})$, so it is also an optimal contract. But this contradicts Lemma 2, since $\hat{z}_L(l) = z_L(l) < X_L$ and

$\hat{z}_H(l) > 0$. Hence, if $(z, \mathbf{1})$ is an optimal contract with $z_H(h) > 0$ and $z_H(l) = 0$, it must be that $z_L(\sigma) = X_L$ for $\sigma \in \{l, h\}$. $\square$

Proof of Proposition 5. Part (ii) follows from Lemma 3.

We now prove part (i). Note first that, under Assumption 3, any optimal contract $(z, \mathbf{1})$ must be such that $\mathbf{1}(l) = 0$: indeed, under the condition (i) in Assumption 3, there are no feasible repayments z that satisfy constraint (EC) for $\sigma = l$ when $\mathbf{1}(l) = 1$.

We now show that, under an optimal contract, $\mathbf{1}(h) = 1$. Suppose that there exists an optimal contract $(z, \mathbf{1})$ with $\mathbf{1}(h) = 0$. Let $(\tilde{z}, \tilde{\mathbf{1}})$ be an alternative contract with $\tilde{\mathbf{1}}(h) = 1$, $\tilde{\mathbf{1}}(l) = \mathbf{1}(l) = 0$, $\tilde{z}_L(\sigma) = z_L(\sigma) = X_L$, $\tilde{z}_H(l) = z_H(l)$ and

$$
\begin{aligned}
\tilde{z}_H(h) &= \frac{\rho_h I_1 + \alpha\pi^{VC} z_H(h) + (1-\alpha)(1-\pi^{VC}) z_L(h)}{K\alpha\pi^{VC}} - \frac{1-\alpha}{\alpha}\frac{1-\pi^{VC}}{\pi^{VC}} z_L(h) \\
&= \frac{\rho_h I_1 + \alpha\pi^{VC} z_H(h) + (1-\alpha)(1-\pi^{VC}) X_L}{K\alpha\pi^{VC}} - \frac{1-\alpha}{\alpha}\frac{1-\pi^{VC}}{\pi^{VC}} X_L,
\end{aligned}
$$

where the second equality follows since, under any optimal contract, $z_L(h) = X_L$. Note that the VC's expected payoff under contract $(\tilde{z}, \tilde{\mathbf{1}})$ is the same as her expected payoff under contract $(z, \mathbf{1})$. Note further that

$$
\begin{aligned}
U^E(\tilde{z}, \tilde{\mathbf{1}}) - U^E(z, \mathbf{1}) &= K\alpha\pi^E(X_H - \tilde{z}_H(h)) - \alpha\pi^E(X_H - z_H(h)) \\
&= \alpha\pi^E\left[(K-1)\, X_H + (K-1)\,\frac{1-\alpha}{\alpha}\frac{1-\pi^{VC}}{\pi^{VC}} X_L - \frac{\rho_h I_1}{\alpha\pi^{VC}}\right] \\
&= \frac{\pi^E}{\pi^{VC}}\left[(K-1)\,\alpha\pi^{VC} X_H + (K-1)(1-\alpha)(1-\pi^{VC}) X_L - \rho_h I_1\right] > 0,
\end{aligned}
$$

where the strict inequality follows from Assumption 3. Hence, if $(z, \mathbf{1})$ is an optimal contract it must have $\mathbf{1}(h) = 1$. This establishes part (i).

Finally, part (iii) follows since any optimal contract $(z, \mathbf{1})$ must be such that $U^{VC}(z, \mathbf{1}) = 0$. Further, we note that if there exists an optimal contract $(z, \mathbf{1})$ such that $U^{VC}(z, \mathbf{1}) = 0$

and such that (EC) is satisfied with slack, then there exists a continuum of optimal contracts. Indeed, increasing $z_H(l)$ by ϵ allows the entrepreneur to reduce $z_H(h)$ by $\frac{1-\alpha}{\alpha}\frac{1}{K}\epsilon$ while still satisfying the VC's break even condition. This change in the contract leaves the entrepreneur indifferent since $-\epsilon\pi^E(1-\alpha)+\frac{1-\alpha}{\alpha}\frac{1}{K}\epsilon\alpha\pi^E K=0$. □

Proof of Corollary 4. Parts (i) and (ii) follow from Proposition (5). Finally, when the first inequality in (18) holds, the VC must get strictly more than X_L at state $s=H$ when $\sigma=l$ and/or $\sigma=h$ (otherwise the VC does not break even). When the second inequality in (18) holds, there exists $z\in(X_L,X_H)$ such that

$$I_0+\rho_h I_1=K\left[(1-\alpha)\pi^{VC}z+\alpha(1-\pi^{VC})X_L\right]+\rho_l X_L. \tag{34}$$

By equation (34), the VC breaks even under a contract $(z,\mathbf{1})$ with $\mathbf{1}(h)=1$, $\mathbf{1}(l)=0$, $z_L(l)=z_L(h)=z_H(l)=X_L$ and $z_H(h)=z$. Finally, since $I_0>\rho_L X_L$ (by Assumption (3)), equation (34) implies that

$$K\left[(1-\alpha)\pi^{VC}z+\alpha(1-\pi^{VC})X_L\right]>\rho_H I_1, \tag{35}$$

so that (EC) holds. Hence, by Proposition (5), contract $(z,\mathbf{1})$ is optimal. □

B Generalization of the Simple Pooling Example

This appendix extends the example of section 4.2 to allow for non-zero correlation between the assets to be securitized. As in section 4.2, suppose the issuer owns two assets, X^1 and X^2, each of which can generate a return in $\{X_1,X_2\}$ (with $X_1<X_2$). In contrast to section 4.2, suppose that the returns of assets X^1 and X^2 are correlated. Let $sk\in\hat{S}=\{11,12,21,22\}$ denote the event that asset 1's return is X_s and asset 2's return is X_k. The beliefs of the

issuer and market over the set of possible return realizations are, respectively, $\hat{\pi}^I$ and $\hat{\pi}^M$. For $j = I, M$, $\hat{\pi}^j_{sk}$ denotes the probability that j assigns to the event sk. We assume that the assets are symmetric, so that $\hat{\pi}^j_{12} = \hat{\pi}^j_{21}$ for $j = I, M$. The *iid* case of section 4.2 is the special case with $\hat{\pi}^j_{sk} = \pi^j_s \pi^j_k$ for $j = I, M$ and for all $sk \in \hat{S}$.

Suppose first that the issuer sells two individual securities, each backed by an asset. By Proposition 1 an optimal security F has $F_1 = X_1$ and $F_2 \geq F_1$. The price that the market is willing to pay for security F is $p(F) = X_1(\hat{\pi}^M_{11} + \hat{\pi}^M_{12}) + F_2(\hat{\pi}^M_{21} + \hat{\pi}^M_{22})$; the issuer's payoff from selling this security is

$$p(F) + \delta(X_2 - F_2)(\hat{\pi}^M_{21} + \hat{\pi}^M_{22}) = X_1(\hat{\pi}^M_{11} + \hat{\pi}^M_{12}) + F_2(\hat{\pi}^M_{21} + \hat{\pi}^M_{22}) + \delta(X_2 - F_2)(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}). \quad (36)$$

The issuer finds it optimal to set $F_2 = X_1$ if $\delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) > \hat{\pi}^M_{21} + \hat{\pi}^M_{22}$ and $F_2 = X_2$ if $\delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) \leq \hat{\pi}^M_{21} + \hat{\pi}^M_{22}$. In what follows we maintain the assumption that $\delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) > \hat{\pi}^M_{21} + \hat{\pi}^M_{22}$, so that an issuer who sells individual securities F^1 and F^2, each backed respectively by asset X^1 and X^2, finds it optimal to set $F^1_s = F^2_s = X_1$ for $s = 1, 2$.

Suppose next that the issuer pools the two assets and sells a single security backed by cash-flows $Y = X^1 + X^2$. Consider a security $F_Y = \min\{Y, X_1 + X_2\}$. The price that the market is willing to pay for security F_Y is $p(F_Y) = \hat{\pi}^M_{11} 2X_1 + (1 - \hat{\pi}^M_{11})(X_1 + X_2)$, and the issuer's payoff from selling this security is

$$p(F_Y) + \delta\hat{\pi}^I_{22}(X_2 - X_1) = \hat{\pi}^M_{11} 2X_1 + (1 - \hat{\pi}^M_{11})(X_1 + X_2) + \delta\hat{\pi}^I_{22}(X_2 - X_1). \quad (37)$$

Comparing (36) and (37), the issuer strictly prefers selling security F_Y backed by the pool of assets than selling the two individual securities $F^1_s = F^2_s = X_1$ for $s = 1, 2$ if and only if $2\hat{\pi}^M_{21} + \hat{\pi}^M_{22} = 1 - \hat{\pi}^M_{11} > \delta(1 - \pi^I_{11}) = \delta(2\hat{\pi}^I_{21} + \hat{\pi}^I_{22})$. Combining this with $\delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) > \hat{\pi}^M_{21} + \hat{\pi}^M_{22}$, the issuer strictly prefers to pool the assets and sell security F_Y if

$$\hat{\pi}_{11}^{M} \in \left(1 - \delta(\hat{\pi}_{21}^{I} + \hat{\pi}_{22}^{I}) - \hat{\pi}_{21}^{M}, 1 - \delta(2\hat{\pi}_{21}^{I} + \hat{\pi}_{22}^{I})\right). \tag{38}$$

If the issuer and the market both perceive the asset to be perfectly correlated (so that $\hat{\pi}_{21}^{j} = 0$ for $j = 1, 2$), the condition in (38) can never be satisfied, and hence pooling does not obtain.

References

Adam, Tim R., Valentin Burg, Tobias Scheinert, and Daniel Streitz, "Managerial Optimism and Debt Contract Design: The Case of Syndicated Loans," *Working Paper*, 2014.

Admati, Anat R. and Paul Pfleiderer, "Robust Financial Contracting and the Role of Venture Capitalists," *The Journal of Finance*, 1994, *49* (2), 371–402.

Allen, Franklin and Douglas Gale, "Optimal security design," *Review of Financial Studies*, 1988, *1* (3), 229–263.

Antic, Nemanja, "Contracting with unknown technologies," *Working Paper*, 2014.

Axelson, Ulf, "Security design with investor private information," *The Journal of Finance*, 2007, *62* (6), 2587–2632.

Baker, Malcolm and Jeffrey Wurgler, "Market timing and capital structure," *The Journal of Finance*, 2002, *57* (1), 1–32.

Bayar, Onur, Thomas J Chemmanur, and Mark H Liu, "A theory of equity carve-outs and negative stub values under heterogeneous beliefs," *Journal of Financial Economics*, 2011, *100* (3), 616–638.

_ , _ , and _ , "A theory of capital structure, price impact, and long-run stock returns under heterogeneous beliefs," *Journal of Corporate Finance Studies*, forthcoming.

Bergemann, Dirk and Ulrich Hege, "Venture capital financing, moral hazard, and learning," *Journal of Banking & Finance*, 1998, *22* (6), 703–735.

Bernardo, Antonio E and Ivo Welch, "On the evolution of overconfidence and entrepreneurs," *Journal of Economics & Management Strategy*, 2001, *10* (3), 301–330.

Boot, Arnoud WA and Anjan V Thakor, "Managerial autonomy, allocation of control rights, and optimal capital structure," *Review of Financial Studies*, 2011, pp. 3434–3485.

_ , Radhakrishnan Gopalan, and Anjan V Thakor, "The entrepreneur's choice between private and public ownership," *The Journal of Finance*, 2006, *61* (2), 803–836.

_ , _ , and _ , "Market liquidity, investor participation, and managerial autonomy: why do firms go private?," *The Journal of Finance*, 2008, *63* (4), 2013–2059.

Chen, Joseph, Harrison Hong, and Jeremy C Stein, "Breadth of ownership and stock returns," *Journal of Financial Economics*, 2002, *66* (2), 171–205.

Cheng, Ing-Haw, Sahil Raina, and Wei Xiong, "Wall Street and the housing bubble," *The American Economic Review*, 2014, *104* (9), 2797–2829.

Chernenko, Sergey, Sam Hanson, and Adi Sunderam, "The Rise and Fall of Securitization," Technical Report, Working Paper, December 2013.

Cochrane, John H, "The risk and return of venture capital," *Journal of Financial Economics*, 2005, *75* (1), 3–52.

Cooper, Arnold C, Carolyn Y Woo, and William C Dunkelberg, "Entrepreneurs' perceived chances for success," *Journal of Business Venturing*, 1988, *3* (2), 97–108.

Cornelli, Francesca and Oved Yosha, "Stage financing and the role of convertible securities," *The Review of Economic Studies*, 2003, *70* (1), 1–32.

Coval, Joshua D and Anjan V Thakor, "Financial intermediation as a beliefs-bridge between optimists and pessimists," *Journal of Financial Economics*, 2005, *75* (3), 535–569.

DeMarzo, Peter and Darrell Duffie, "A liquidity-based model of security design," *Econometrica*, 1999, *67* (1), 65–99.

DeMarzo, Peter M, "The pooling and tranching of securities: A model of informed intermediation," *Review of Financial Studies*, 2005, *18* (1), 1–35.

Dicks, David and Paolo Fulghieri, "Ambiguity, disagreement, and corporate control," Technical Report, Working Paper, University of Northern Carolina, Chapel Hill 2015.

Dittmar, Amy and Anjan Thakor, "Why do firms issue equity?," *The Journal of Finance*, 2007, *62* (1), 1–54.

Economist, The, "Entrepreneurs anonymous," *http://www.economist.com/news/business/21618816-instead-romanticising-entrepreneurs-people-should-understand-how-hard-their-lives-can*, 2014.

Erel, Isil, Brandon Julio, Woojin Kim, and Michael S Weisbach, "Macroeconomic conditions and capital raising," *Review of Financial Studies*, 2011, pp. 341–376.

Fama, Eugene F and Kenneth R French, "Financing decisions: who issues stock?," *Journal of Financial Economics*, 2005, *76* (3), 549–582.

Farre-Mensa, Joan, "The Benefits of Selective Disclosure: Evidence from Private Firms," *Working Paper*, 2015.

Frank, Murray Z and Vidhan K Goyal, "Testing the pecking order theory of capital structure," *Journal of Financial Economics*, 2003, *67* (2), 217–248.

Fulghieri, Paolo, Diego Garcia, and Dirk Hackbarth, "Asymmetric information and the pecking (dis)order," Technical Report, University of North Carolina at Chapel Hill 2013.

Fuster, Andreas and James Vickery, "Securitization and the fixed-rate mortgage," *Review of Financial Studies*, 2014, p. hhu060.

Garmaise, Mark, "Rational Beliefs and Security Design," *Review of Financial Studies*, 2001, *14* (4), 1183–1213.

Gennaioli, Nicola, Andrei Shleifer, and Robert W Vishny, "A model of shadow banking," *The Journal of Finance*, 2013, *68* (4), 1331–1363.

Gompers, Paul and Josh Lerner, "The venture capital revolution," *Journal of Economic Perspectives*, 2001, pp. 145–168.

Gorton, Gary and George Pennacchi, "Financial intermediaries and liquidity creation," *The Journal of Finance*, 1990, *45* (1), 49–71.

Green, Richard C., "Investment incentives, debt, and warrants," *Journal of Financial Economics*, 1984, *13* (1), 115 – 136.

Hackbarth, Dirk, "Managerial traits and capital structure decisions," *Journal of Financial and Quantitative Analysis*, 2008, *43* (04), 843–881.

Heaton, James B., "Managerial optimist and corporate finance," *Financial Management*, 2002, (33-45).

Hébert, Benjamin, "Moral Hazard and the Optimality of Debt," *Available at SSRN 2185610*, 2014.

Hong, Harrison and Jeremy C Stein, "Disagreement and the stock market," *The Journal of Economic Perspectives*, 2007, *21* (2), 109–128.

Huang, Sheng and Anjan V Thakor, "Investor heterogeneity, investor-management disagreement and share repurchases," *Review of Financial Studies*, 2013, *26* (10), 2453–2491.

Inderst, Roman and Holger M Mueller, "Informed lending and security design," *The Journal of Finance*, 2006, *61* (5), 2137–2162.

Innes, Robert D, "Limited liability and incentive contracting with ex-ante action choices," *Journal of Economic Theory*, 1990, *52* (1), 45–67.

Kaplan, Steven N. and Per Strömberg, "Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts," *Review of Economic Studies*, 2003, *70*, 281–315.

_ **and** _ , "Characteristics, Contracts, and Actions: Evidence from Venture Capitalist Analyses," *The Journal of Finance*, October 2004, *59* (5), 2177–2210.

Koellinger, Philipp, Maria Minniti, and Christian Schade, ""I think I can, I think I can": Overconfidence and entrepreneurial behavior," *Journal of Economic Psychology*, 2007, *28* (4), 502–527.

Kramer, Barry and Khang Tran, "Silicon Valley Venture Capital Survey First Quarter 2016," Technical Report, Fenwick & West LLP, available at https://www.fenwick.com/FenwickDocuments/Silicon-Valley-Venture-Capital-Survey-First-Quarter-2016.pdf 2016.

Landier, Augustin and David Thesmar, "Financial contracting with optimistic entrepreneurs," *Review of Financial Studies*, 2009, *22* (1), 117–150.

Lee, Seokwoo and Uday Rajan, "Robust Security Design," *University of Michigan Working Paper*, 2017.

Malmendier, Ulrike, Geoffrey Tate, and Jon Yan, "Overconfidence and early-life experiences: the effect of managerial traits on corporate financial policies," *The Journal of Finance*, 2011, *66* (5), 1687–1733.

Manove, Michael and A Jorge Padilla, "Banking (conservatively) with optimists," *The RAND Journal of Economics*, 1999, pp. 324–350.

Marsh, Paul, "The choice between equity and debt: An empirical study," *The Journal of Finance*, 1982, *37* (1), 121–144.

McLean, R David and Mengxin Zhao, "The business cycle, investor sentiment, and costly external finance," *The Journal of Finance*, 2014, *69* (3), 1377–1409.

Meza, David De and Clive Southey, "The borrower's curse: optimism, finance and entrepreneurship," *The Economic Journal*, 1996, pp. 375–386.

Modigliani, Franco and Merton H Miller, "The cost of capital, corporation finance and the theory of investment," *The American economic review*, 1958, pp. 261–297.

Moskowitz, Tobias J and Annette Vissing-Jørgensen, "The returns to entrepreneurial investment: A private equity premium puzzle?," *The American Economic Review*, 2002, *92* (4), 745–778.

Myers, Stewart C, "The capital structure puzzle," *The Journal of Finance*, 1984, *39* (3), 574–592.

_ **and Nicholas S Majluf**, "Corporate financing and investment decisions when firms have information that investors do not have," *Journal of Financial Economics*, 1984, *13* (2), 187–221.

Nachman, David C and Thomas H Noe, *Design of securities under asymmetric information*, College of Management, Georgia Inst. of Technol., 1990.

_ **and** _ , "Optimal design of securities under asymmetric information," *Review of Financial Studies*, 1994, *7* (1), 1–44.

Noe, Thomas H, "Capital structure and signaling game equilibria," *Review of Financial Studies*, 1988, *1* (4), 331–355.

Puri, Manju and David T Robinson, "Optimism and economic choice," *Journal of Financial Economics*, 2007, *86* (1), 71–99.

Scheinkman, Jose A. and Wei Xiong, "Overconfidence and Speculative Bubbles," *Journal of Political Economy*, 2003, *111* (6), 1183–1219.

Schmidt, Klaus M., "Convertible Securities and Venture Capital Finance," *The Journal of Finance*, 2003, *58* (3), 1139–1166.

Simsek, Alp, "Belief disagreements and collateral constraints," *Econometrica*, 2013, *81* (1), 1–53.

Stein, Jeremy C., "Convertible bonds as backdoor equity financing," *Journal of Financial Economics*, 1992, *32* (1), 3 – 21.

Thakor, Anjan V and Toni M Whited, "Shareholder-manager disagreement and corporate investment," *Review of Finance*, 2011, *15* (2), 277–300.

Winton, Andrew and Vijay Yerramilli, "Entrepreneurial finance: Banks versus venture capital," *Journal of Financial Economics*, 2008, *88* (1), 51–79.

Yang, Ming, "Optimality of debt under flexible information acquisition," *Available at SSRN 2103971*, 2013.

_ **and Yao Zeng**, "Financing Entrepreneurial Production: Security Design with Flexible Information Acquisition," *Working Paper*, 2015.