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# Optimal Pollution Control in a Mixed Oligopoly with Research Spillovers

## Abstract

We study optimal pollution abatement under a mixed oligopoly game when firms engage in emissions-reducing R&D that is imperfectly appropriable. The regulator uses a tax to curb emissions. Results show that in a mixed oligopoly, the public firm has positive emissions reduction in equilibrium; however, emissions reductions of the private firm could be positive or zero. Under certain conditions, the optimal pollution tax is positive; otherwise, the tax reverts to a subsidy. Comparing mixed and private duopolies, privatization leads to reductions in R&D and output, but to an increase in overall emissions, so privatization tends to make the environment worse.

JEL-Codes: D430, D620, O330, Q550.

Keywords: mixed oligopoly, R&D, pollution, spillovers, taxation, subsidy.

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## 1. Introduction

Emissions of carbon dioxide and nitrogen oxide have increased with economic growth all over the world since the Industrial Revolution, irrespective of the economic system in place. Among others, the consumption of petroleum and coal, which are the main sources of environmental pollution, has rapidly expanded with an improvement in the worldwide standard of living since World War II. Emissions of carbon dioxide cause global warming by the greenhouse effect, and emissions of nitrogen oxide cause air pollution via exhausts from, for example, automobiles, boilers, factories, and thermal power stations, etc. Consequently, the deterioration of earth's environment has been steady, with a rise in temperatures, a rapid reduction in glaciers, and a rise in the sea level. Greenhouse gas emissions are now a serious environmental problem.

In recent decades, there has been policy focus on the environment with policymakers devising various instruments to alter economic agents' behaviors to control environmental degradation. In order to overcome this problem, governments employ various policies such as direct controls and auctioned marketable emissions permits, emissions subsidies, and emissions taxes to control pollution (see, for example, Downing and White (1986), Milliman and Prince (1989), Jung, Krutilla and Boyd (1996), and Requate and Unold (2003)). With pollution continuing to be a global problem, the success of these policy initiatives is debatable.

Researchers have also devoted considerable efforts to recommend and evaluate various initiatives by using various theoretical and empirical approaches. Among theoretical approaches, multi-stage game models (see, for example, Downing and White (1986), Milliman and Prince (1989), Ouchida and Goto (2014, 2016), Poyago-Theotoky (2007, 2010), Youssef and Dinar (2011)) and other methodologies have been used.[1] In these papers, competitive or oligopolistic profit-maximizing firms are assumed. The following papers, on the other hand, extend conventional models to an analytical framework including nonprofit-maximizing (public) firms. There are, for example, Bárcena-Ruiz and Garzon (2006), Gil-Molto and Varvarigos (2014), Kato (2011), and Pal and Saha (2014) addressing the problem of emissions abatement in mixed oligopoly models, with public firms and private firms. Among others, Bárcena-Ruiz and Garzon (2006) employ two-stage game

[1] Fischer and Newell (2008) also assess six policy options for reducing carbon dioxide emissions and promoting renewable energy in the electricity production sector and evaluate the relative policy performance from perspectives such as economic surplus, emissions reduction, renewable energy production and R&D by using a numerical method.

models of a mixed oligopoly and compare market performance in the mixed and private oligopolies, showing that environmental taxes are lower in the mixed oligopoly than in the private oligopoly and that the decision of the government whether to privatize a public firm interacts with the environmental policy. Gil-Molto and Varvarigos (2014), on the other hand, demonstrate that the comparative-static results are affected by the number of firms when firms simultaneously determine outputs and emissions (abatement) under a mixed oligopoly and a private oligopoly. A common feature of Bárcena-Ruiz and Garzon (2006), Gil-Molto and Varvarigos (2014), Kato (2011), and Pal and Saha (2014) approaches is that public and private firms choose output and emissions (abatement) levels in the same stage. Therefore, the roles of emissions abatement technology and the relationship between its technology and emissions taxes are not sufficiently discussed. They do not, for example, refer to spillovers in the development of pollution abatement technology. We take R&D spillovers into consideration.

We examine the interaction and behavior of firms in a mixed oligopoly and the effect of their R&D efforts by separating the timing of decision-making of firms on output and emissions (abatement) into two. The regulator (government) implements a tax on emissions of pollution, and firms can counter these payments by engaging in environmental R&D efforts to abate emissions. Hence, we use a three-stage, mixed oligopoly model, where a public firm and a private firm non-cooperatively determine their R&D to control pollution emissions to maximize social welfare and profits in the first stage, the regulator sets an emissions tax rate (or emissions subsidy rate) to maximize social welfare in the second stage, and firms simultaneously determine outputs to maximize social welfare and profits without cooperation in the final stage, respectively (see, for example, Requate (2005) for a survey of the related literature). We assume externalities for R&D returns as well as for pollution emissions.[2] Thus, our consideration, while still based in a stylized setting, seems more realistic, besides adding to the extant literature.

With regard to papers with the three-stage game model with environmental R&D, there are Poyago-Theotoky (2007) and Youssef and Dinar (2011).[3] These analyses shed light on the behavior of conventional oligopolies (not mixed

[2] Bárcena-Ruiz and Garzon (2006), Gil-Molto and Varvarigos (2014), and Kato (2011) do not take spillovers in pollution abatement technology into consideration. In addition, Youssef and Dinar (2011) assume that there are no R&D externalities between firms.

[3] Besides, Katsoulacos and Xepapadeas (1996) consider the tax-R&D subsidy policy mix, where the government implements in stage one of a three-stage game private duopoly model.

oligopolies). In Youssef and Dinar (2011), a regulator chooses both emissions taxes and R&D subsidies in the first-stage, and it is shown that R&D investment is taxed, not subsidized, when the marginal damage cost of pollution is sufficiently high. Poyago-Theotoky (2007) examine and compare two organization regimes of competitive environmental R&D and an environmental R&D cartel (ERC), and show that environmental R&D, which firms choose in the first stage, is higher in the ERC than in non-ERC for small damages and for large damages when R&D is efficient. In the context of the literature, we consider the optimal tax policy of the government and environmental R&D efforts and outputs of a public firm and a private firm in the presence of R&D spillovers.

Mixed oligopolies are prevalent in many nations where governments partially privatize some sectors and allow private firms to enter markets for power generation, telecommunications, postal and banking services, railway, mining, university education, military production, etc. In such markets, both public and private firms coexist, albeit with differing objectives.

We shed light on government's pollution abatement policy, and firms' environmental R&D behavior and the effect of their R&D on emissions taxes, outputs, and emissions in a mixed duopoly. For our analysis, we invoke Poyago-Theotoky's (2007) model in order to investigate the behavior of the government and a mixed duopoly when their productive activities pollute the environment.[4] Specifically a three-stage game model is employed in order to consider the behavior of the mixed duopoly under pollution control and environmental R&D and seek a sub-game perfect Nash equilibrium. Further, we examine the effects of privatization of a public firm by comparing the results derived under two regimes of a private duopoly and a mixed duopoly and investigate whether different ownership forms affect industry performance and pollution emissions. Consequently, differences in equilibria of the mixed and private oligopolies will be clarified.

We show that a public firm in a mixed duopoly could perform environmental R&D, but a private firm may or may not, i.e., there could exist a corner solution in the environmental R&D stage. This apparently differs from results in a three-stage game model of private duopoly and in two-stage game models of mixed oligopoly. The outputs of the public and private firms depend on total R&D efforts

[4] There is a difference between the production cost functions of Poyago-Theotoky (2007) and our paper: Her cost function is linear in output, but ours is quadratic. As is well known, the final good market gravitates to a monopoly of a welfare-maximizing (public) firm when the production cost functions of firms in a mixed oligopoly are linear.

and are the same. It is, furthermore, obtained that an emissions tax could be negative (i.e., the optimal policy of the government is to subsidize). It is shown that the comparisons of emissions taxes, outputs, and total emissions before and after privatization depend just on the comparison of environmental R&Ds before and after: Privatization causes less environmental R&Ds and less outputs (consumer surplus) and more net emissions in comparison with non-privatization, so that it worsens the environment. Privatization as a policy to improve the environment is not desirable. This revelation is obviously important and could use additional validation, both theoretically and empirically.

The remainder of this paper is organized as follows. In the next section we present a three-stage game model of mixed duopoly and investigate the output and environmental R&D behavior of a public firm and a private firm and the emissions tax choice of the government. In Section 3 we consider the effects of privatization of a public firm by comparing results in a mixed duopoly and a privatized duopoly, and the final section concludes.

## 2. Model and analysis of a mixed duopoly

In the mixed duopoly model we consider, there are a public firm and a private firm producing a homogeneous product. We start with a linear inverse demand function for simplicity: $p = a - Q,\ a > 0$, where $p$ is the output price, $Q = q_0 + q_1$ is the total output, and $a$ is the size of the market. The public firm is denoted by subscript 0, and the private firm by subscript 1.[5] Production costs for both firms are quadratic and symmetric, and denoted by the production cost function of firm $i$: $c(q_i) = q_i^2/2,\ i = 0, 1$. The production processes emit pollution ($e_i$). In particular, each unit of output generates one unit of pollution. However, firms can reduce emissions by performing abatement activities via research and development (R&D). This might be via an environmentally-friendly process innovation. For example, the environmental R&D expenditure of firm $i$ on emissions reduction of $z_i,\ i = 0, 1$, is $\gamma z_i^2/2,\ \gamma > 0$, where $\gamma$ stands for the degree of efficiency of environmental R&D technology.[6] R&D expenditures are increasing and convex in emissions abatement. Firms have the same R&D

[5] The assumptions of duopoly and a linear demand function are made for analytical tractability. Such assumptions are routinely made in the related literature (see, for example, Bárcena-Ruiz and Garzon (2006), Chiou and Hu (2001), Kato (2011), and Poyago-Theotoky (2007)).

[6] Bárcena-Ruiz and Garzon (2006) assume that the degree of efficiency of environmental R&D technology is unity.

technology. The marginal costs of environmental R&D are $\gamma z_i$. We assume that there are externalities of environmental R&D, i.e., spillovers in R&D results. The degree of inappropriability from environmental R&D effort of each firm is given by $\beta$, $0 \le \beta \le 1$.[7] The rival firm is thus able to costlessly unravel some (or all) knowledge and methods related to emissions reduction. Net (pollution) emissions of firm $i$ then follow the following function, given reductions from own environmental R&D and benefits (spillovers) from the rival's R&D: $e_i(q_i,\ z_i) = q_i - (z_i + \beta z_j),\ i \ne j$. The term, $z_i + \beta z_j$, is the effective R&D (or pollution abatement) for firm $i$ (Poyago-Theotoky (2007)). It is now assumed that $e_i(q_i, z_i) > 0,\ i \ne j$.[8] Net pollution emissions of the industry are given by $E = e_i + e_j$, which is the damage to society. The extent of social damage (costs) is captured by a quadratic environmental damage function of *E,* based on emissions from both firms, such as $D(E) = dE^2/2,\ 1/2 < d$, like Poyago-Theotoky (2007).[9] Parameter $d$ in the damage function $D(E)$ can be interpreted as the extent of social seriousness of environmental damage generated by pollution. The marginal environmental damage is given by $MED = dE > 0$.

The social planner (government) commits to environmental tax policy to control emissions, such that $t =$ tax (subsidy) rate per unit of pollution emissions. The emissions tax is introduced to correct pollution externality. Environmental R&D spending on pollution abatement enables firms to reduce tax payments. Emissions taxes have the effect of an intervention in the market as well. With this basic setup, we proceed with the formal model.

### 2A. Partial privatization of the market

In the mixed duopoly of quantity-setting Cournot type, firms have different objectives, with the public firm maximizing social welfare and the private firm maximizing profits, each choosing its environmental R&D and output. The government also plays a positive role in abating pollution via a tax/subsidy policy.

The sequence of decisions can be envisioned in three steps: In the first stage

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[7] Youssef and Dinar (2011) assume that there are no spillovers on R&D performance between private firms. Fischer and Newell (2008) assume the R&D spillover rate to be 0.5 in their numerical analysis. On the other hand, Milliman and Prince (1989) take the diffusion of the new technology across firms into consideration and evaluate environmental policy instruments. Their diffusion corresponds to full spillovers, i.e., a research joint venture.

[8] A similar assumption is employed by, for example, Chiou and Hu (2001), and Katsoulacos and Xepapadeas (1996). According to the behavioral principle of firms, the assumption that net emissions are positive is reasonable.

[9] Bárcena-Ruiz and Garzon (2006) assume $d = 1$.

both the public firm and the private firm simultaneously choose their environmental R&D (or pollution abatement) effort levels without abatement cooperation, in the second stage the government sets a tax or a subsidy to reduce emissions generated by production activities, and in the final stage they simultaneously choose their outputs without collusion. With this recognition, the game model is solved by backward recursion.

The profit function of firm $i$ is given by:

$$\pi_i = (a - q_i - q_j)q_i - \frac{1}{2}q_i^2 - t(q_i - z_i - \beta z_j) - \frac{\gamma}{2}z_i^2, \quad i,\ j = 0,\ 1,\ i \neq j.$$

The profit function includes revenues minus production costs, taxes on net emissions, and environmental research (R&D) expenditures. The social welfare function of the public firm is given by:

$$W = \frac{1}{2}Q^2 + \pi_0 + \pi_1 - \frac{1}{2}dE^2 + t(q_0 - z_0 - \beta z_1) + t(q_1 - z_1 - \beta z_0)$$
$$= \frac{2aQ - Q^2 - (q_0^2 + q_1^2) - \gamma(z_0^2 + z_1^2) - dE^2}{2},$$

where $Q = q_0 + q_1$. The social welfare function is the sum of consumer surplus, profits, emissions tax revenues, minus the environmental damage. The public firm internalizes pollution emissions, but need not pay an emissions tax, unlike the private firm.

I. Output choice - stage three

As the public firm and the private firm engage in Cournot-type quantity competition in the third stage, the first-order condition for social welfare maximization of the public firm is:

$$\frac{dW}{dq_0} = a - Q - q_0 - d(e_0 + e_1) = 0 \qquad (1)$$

The public firm chooses output such that output price equals the sum of marginal costs of both production and environmental damage. An emissions tax never influences the marginal costs. On the other hand, the first-order condition for profit maximization of the private firm is:

$$\frac{d\pi_1}{dq_1} = a - q_0 - 3q_1 - t = 0 \qquad (2)$$

The second-order conditions for both firms are satisfied, and the equilibrium of the output market is locally stable. An increased (declined) emissions tax increases (decreases) the marginal costs of the private firm, consequently reducing (increasing) its output. From the first-order conditions (1) and (2), given

$t$ and $z$, we have the respective outputs:

$$q_0 = \frac{(2-d)a + (1+d)t + 3d(1+\beta)z}{5+2d} \quad \text{and} \quad q_1 = \frac{(1+d)a - (2+d)t - d(1+\beta)z}{5+2d}, \tag{3}$$

where $z = z_0 + z_1$ is total pollution abatement. Given different underlying objectives, the firms have somewhat different output responses to changes in parameters. For instance, with higher emissions taxes, the public firm increases its output, but the private firm does not; and with greater total pollution abatement, the public firm increases its output, but the private firm does not. The effects of spillovers on outputs of both firms are reverse as well. Finally, increased emissions taxes lead to a reduction in total output (consequently, total emissions).

II. Tax choice - stage two

In the second stage, the government chooses an emissions tax in order to maximize social welfare. The government now internalizes pollution emissions like the public firm. The first-order condition for social welfare maximization of the government is (using (3)):

$$\frac{dW}{dt} = \frac{\partial W}{\partial q_0}\frac{dq_0}{dt} + \frac{\partial W}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial W}{\partial t} = -\frac{(2+d)}{(5+2d)^2}\left[(1-2d)a + 4d(1+\beta)z + (3+2d)t\right] = 0, \tag{4}$$

where $\partial W / \partial q_0 = 0$ and $\partial W / \partial t = 0$. Now, the second-order condition is satisfied. So, it follows from (4) that, given $z$, the government sets an emissions tax rate such as:

$$t = \frac{(2d-1)a - 4d(1+\beta)z}{3+2d} \tag{5}$$

The government may have an incentive to increase output through an emissions subsidy even if the social assessment of the pollution damage is less severe.
It, on the other hand, follows from (5) that $dt/dz = -4d(1+\beta)/(3+2d) < 0$. The government could adopt a policy to increase output such as a reduction in emissions taxes or a rise in emission subsidies, because an increase in environmental R&D effort decreases the amount of social damage due to pollution. This shows that the mixed oligopoly incurs the same effect as the private oligopoly (see, for example, Poyago-Theotoky (2007)). Whether the government levies an emissions tax or provides an emissions subsidy depends significantly on the degree of total pollution abatement due to environmental R&D efforts, the seriousness of environmental damage, and R&D spillovers. Meanwhile, the efficiency of R&D technology has no direct impact on the emissions tax.

When substituting (5) into (3), we get the outputs of the public and private firms:

$$q_i = \frac{a + d(1+\beta)z}{3+2d}, \quad i = 0,\ 1 \tag{6}$$

The output levels also depend on the degree of total pollution abatement due to environmental R&D efforts, the seriousness of environmental damage, and R&D spillovers. Particularly, environmental R&D leads to the same increase in both firms' outputs, $dq_i / dz = d(1+\beta)/(3+2d) > 0$, because an increase in its R&D causes emissions taxes (subsidies) to reduce (raise). We find the following lemma:

**Lemma 1.** The public and private firms produce the same outputs for any total pollution abatement.[10]

The outputs of the public and private firms become the same when R&D is determined endogenously. Although this latter result is normal or standard for a private oligopoly with the same production technology, it is not necessarily general that firms with different objectives (as in a mixed oligopoly considered here) produce the same outputs.[11]

III. R&D choice: stage one

In the first stage, firms choose their pollution abatement levels to maximize social welfare and profits by engaging in R&D, respectively. Now, when we employ Lemma 1, the social welfare and profit functions of the public and private firms are reduced to, respectively:

$$W = \frac{1}{2}\left[4aq_0 - 4q_0^2 - 2q_0^2 - \gamma(z_0^2 + z_1^2) - d[2q_0 - (1+\beta)z]^2\right]$$

$$\pi_1 = (a - 2q_1)q_1 - \frac{1}{2}q_1^2 - t(q_1 - z_1 - \beta z_0) - \frac{\gamma}{2}z_1^2$$

When we take (1), (2), and (4) into consideration, the first-order conditions for social welfare and profit maximization are given by, respectively:

$$\frac{dW}{dz_0} = 2a\frac{dq_0}{dz_0} - 6q_0\frac{dq_0}{dz_0} - \gamma z_0 - d[2q_0 - (1+\beta)z][2\frac{dq_0}{dz_0} - (1+\beta)] \tag{7}$$

[10] Bárcena-Ruiz and Garzon (2006) show that the output of a public firm is different from the output of each private firm in a mixed oligopoly by using a two-stage game model and also that the public firm may not produce output, depending on the number of private firms. On the other hand, Kato (2011) mentions that a public firm produces more than a private firm by taking more abatement efforts.

[11] For example, White (1996) obtains the similar result in terms of outputs of public and private firms in mixed oligopolies, but Gil-Molto et al. (2011), and Haruna and Goel (2017) obtain different results from White (1996).

$$= \frac{2ad(1+\beta) - [\gamma(3+2d) + 3d(1+\beta)^2]z_0 - 3d(1+\beta)^2 z_1}{3+2d} = 0$$

$$\frac{d\pi_1}{dz_1} = (a - t - 5q_1)\frac{dq_1}{dz_1} + (-q_1 + z_1 + \beta z_0)\frac{dt}{dz_1} + t - \gamma z_1$$

$$= \{a[3d(1+\beta) + (2d-1)(3+2d)] - d(12+5d)(1+\beta)^2 z_0 - [-3d^2(1+\beta)^2 + 8d(3+2d)(1+\beta) + \gamma(3+2d)^2]z_1\} / (3+2d)^2 = 0 \quad (8)$$

The second-order conditions for maximization of both firms are satisfied. On the abatement plane, $(z_0, z_1)$, the environmental R&D reaction curves of the firms both slope downward, and the R&D equilibrium is given by their intersection and is locally stable. We note from (7) and (8) that the environmental R&D of the public (private) firm is a strategic substitute for the private (public) firm for any spillover rates.[12]

When we solve the first-order conditions (7) and (8), the pollution abatements of firms are derived as:

$$z_0 = \frac{a[2d\gamma(3+2d)(1+\beta) + d(3+7d-3d\beta)(1+\beta)^2]}{\gamma^2(3+2d)^2 + 8d\gamma(3+2d)(1+\beta) + 3d\gamma(3+d)(1+\beta)^2 + 12d^2(1-\beta)(1+\beta)^3} > 0 \quad (9)$$

$$z_1 = \frac{a[\gamma(2d-1)(3+2d) + 3d\gamma(1+\beta) + 3d(2d-1)(1+\beta)^2 - 5d^2(1+\beta)^3]}{\gamma^2(3+2d)^2 + 8d\gamma(3+2d)(1+\beta) + 3d\gamma(3+d)(1+\beta)^2 + 12d^2(1-\beta)(1+\beta)^3} \quad (10)$$

The public firm undertakes environmental R&D, i.e., $z_0 > 0$, for any $\gamma$, $d$, and $\beta$, while the private firm may or may not: The sign of (10) is dependent on its numerator, since the denominator is positive. Thus, the environmental R&D levels of firms are different, $z_0 \neq z_1$. The positive response of the public firm to pollution abatement is due to social welfare maximization being its objective.

We turn to the private firm's emissions abatement behavior. It follows (10) that[13]

$$z_1 > 0 \text{ for } \gamma > \frac{d[5d^2(1+\beta)^3 - 3(2d-1)](1+\beta)^2}{(2d-1)(3+2d) + 3d(1+\beta)} = \overline{\gamma}(d, \beta),$$

$$z_1 \leq 0 \text{ for } \gamma \leq \frac{d[5d^2(1+\beta)^3 - 3(2d-1)](1+\beta)^2}{(2d-1)(3+2d) + 3d(1+\beta)}.$$

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[12] These results are obviously different from results obtained from the analysis of a mixed duopoly without environmental R&D. For example, Haruna and Goel (2017) show that whether R&D of a public firm is a strategic substitute or complement for a private firm is dependent on the degree of R&D spillovers.

[13] Although Poyago-Theotoky (2007) mentions in the private duopoly that the assumption of $d$ >1/2 is sufficient to ensure positive R&D efforts of private firms, its positive one of a private firm in a mixed duopoly is not necessarily ensured.

We call $\bar{\gamma}(d, \beta)$ a critical efficiency value of environmental R&D technology, which is increasing in spillover rates. Since a public firm behaves more aggressively than a private firm in the R&D game and the environmental R&D of the public firm is a strategic substitute for the private firm, the private firm may have no incentive to make R&D investment.

It is not easy to know whether the private firm undertakes environmental R&D. Let us specify the area in which $z_1$ is positive by a numerical example. In Figure 1 the area in which $z_1$ becomes either positive or negative is demonstrated. The horizontal and vertical axes depict $d \in [1/2, 10]$ and $\gamma \in [0, 10]$, respectively. Five borderlines $(\gamma = \bar{\gamma}(d, \beta))$ corresponding to spillover parameter $\beta = 0$, 0.3, 0.5, 0.7, and 1 are illustrated. Environmental R&D effort of the private firm becomes positive (negative) in the area on the upper (lower) or left (right) side of the borderlines.[14] The area is reduced with an increase in the spillover rate. Especially, in the absence of spillovers, $z_1$ becomes positive, namely, the interior solution in the R&D stage holds, in almost all area, while in full spillovers $(\beta = 1)$ it becomes negative, specifically, the corner solution holds, for the efficiency of environmental R&D technology in a fairly large area.[15] As shown, an increase in R&D spillovers leads to greater incentive for the private firm to free ride on R&D of the public firm. We note that whether an interior or a corner R&D solution holds in the first state depends significantly on the degree of research spillovers. However, the efficiency of R&D technology and the extent of social damage have fairly weak impacts on R&D behavior of the private firm.

Insert Figure 1 around here>>>

Then, it follows that the equilibrium environmental R&D effort of the public firm is given by (9), i.e., $z_0^*$, in the sub-game perfect Nash equilibrium (SPNE) and that of the private firm is given by (10) and becomes positive, i.e., $z_1^* > 0$, for $\gamma > \bar{\gamma}(d, \beta)$ and zero, i.e., $z_1^* = 0$, for $\gamma \le \bar{\gamma}(d, \beta)$ in the SPNE. Now we have

$$z^{**} = z_0^* + z_1^* = \frac{a[\gamma(2d-1)(3+2d) + d\gamma(9+4d)(1+\beta) + d^2(13-3\beta)(1+\beta)^2 - 5d^2(1+\beta)^3]}{\gamma^2(3+2d)^2 + 8d\gamma(3+2d)(1+\beta) + 3d\gamma(3+d)(1+\beta)^2 + 12d^2(1-\beta)(1+\beta)^3}$$

and

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[14] Figure 1 and the remaining figures are plotted using Mathematica.

[15] We have the area of $z_1 < 0$ for $\gamma \le 3.5$.

$$z^* = z_0^* = \frac{a[2d\gamma(3+2d)(1+\beta) + d(3+7d-3d\beta)(1+\beta)^2]}{\gamma^2(3+2d)^2 + 8d\gamma(3+2d)(1+\beta) + 3d\gamma(3+d)(1+\beta)^2 + 12d^2(1-\beta)(1+\beta)^3}.$$

Then we obtain the following proposition.

**Proposition 1.**

(i) If the efficiency of environmental R&D technology is larger than its critical value, i.e., $\gamma > \bar{\gamma}(d,\ \beta)$, then both firms undertake environmental R&D to abate emissions, i.e., $(z_0, z_1) = (z_0^*, z_1^*) > 0$, so that total pollution abatement becomes $z^{**} = z_0^* + z_1^*$, where ** stands for the values of $(z_0^*,\ z_1^*)$ in the interior solution of environmental R&Ds;

(ii) if the efficiency of environmental R&D technology is less than or equal to its critical value, i.e., $\gamma \le \bar{\gamma}(d,\ \beta)$, then the public firm only undertakes environmental R&D to abate emissions, i.e., $(z_0, z_1) = (z_0^*, 0)$, so that total pollution abatement becomes $z^* = z_0^*$, where superscript * stands for the value of $(z_0^*, 0)$ in the corner solution of environmental R&Ds.

Now we have $z^{**} > z^*$ with respect to total pollution abatement. Whether the solution of environmental R&D efforts is an interior or a corner solution is dependent on the degree of spillovers, the efficiency of the R&D technology, and social seriousness of environmental damage. On the other hand, it is difficult to compare environmental R&D efforts of the public and private firms. When we resort to a numerical method to compare them, we have $z_0^* > z_1^*$ for $d \in [1/2,\ 10]$ and $\gamma \in [0,\ 10]$: In particular, environmental R&D effort is larger in the public firm than in the private firm. Besides, we have $E^{**} < E^*$ from (6) and the definition of net emissions.

Let us turn to the equilibrium values of an emissions tax and outputs in the second and third stages, successively. First of all, when substituting (9) and (10) into (5), we have the following emissions taxes in the R&D interior and corner solutions of the SPNE, respectively:

for $\gamma > \bar{\gamma}(d,\ \beta)$

$$t^{**} = \frac{a[\gamma^2(2d-1)(3+2d)^2 + 4d\gamma(2d-1)(3+2d)(1+\beta) - d\gamma(9+21d+10d^2)(1+\beta)^2 - 4d^2(3+7d-3\beta+3d\beta)(1+\beta)^3 + 20d^3(1+\beta)^4]}{(3+2d)A} \tag{11}$$

and, for $\gamma \le \bar{\gamma}(d,\ \beta)$

$$t^* = \frac{a[\gamma^2(2d-1)(3+2d)^2 + 8d\gamma(2d-1)(3+2d)(1+\beta) - d\gamma(9+9d+10d^2)(1+\beta)^2 - 4d^2(6+d+3d\beta-3\beta)(1+\beta)^3]}{(3+2d)A}, \tag{12}$$

where $A$ denotes the denominator of (9). The signs of $t^{**}$ and $t^*$ depend on the numerators of (11) and (12), respectively. When the amount of total pollution abatement is large, the government employs lower emissions taxes since the incentive for it to decrease pollution emissions becomes less. Now it follows from Proposition 1 and (5) that $t^{**} < t^*$ since $z^{**} > z^*$. With lower emissions under greater total pollution abatement, the government reduces emissions taxes or raises emissions subsidies. The possibility that the emissions tax can be zero or negative, i.e., an emissions subsidy, is much higher when the public firm and the private firm conduct R&D than when only the public firm does. The following proposition holds:

**Proposition 2.** (1) In the R&D interior equilibrium, the optimal environmental policy of the government is (i) an emissions tax ($t^{**} > 0$) for

$$\gamma^2(2d-1)(3+2d)^2 + 4d\gamma(2d-1)(3+2d)(1+\beta) + 20d^3(1+\beta)^4 > d\gamma(9+21d+10d^2)(1+\beta)^2 + 4d^2(3+7d-3\beta+3d\beta)(1+\beta)^3$$

(ii) laissez-faire ($t^{**} = 0$) for

$$\gamma^2(2d-1)(3+2d)^2 + 4d\gamma(2d-1)(3+2d)(1+\beta) + 20d^3(1+\beta)^4 = d\gamma(9+21d+10d^2)(1+\beta)^2 + 4d^2(3+7d-3\beta+3d\beta)(1+\beta)^3$$

and (iii) an emissions subsidy ($t^{**} < 0$) for[16]

$$\gamma^2(2d-1)(3+2d)^2 + 4d\gamma(2d-1)(3+2d)(1+\beta) + 20d^3(1+\beta)^4 < d\gamma(9+21d+10d^2)(1+\beta)^2 + 4d^2(3+7d-3\beta+3d\beta)(1+\beta)^3$$

(2) In the R&D corner equilibrium, the optimal environmental policy is (i) an emissions tax ($t^* > 0$) for

$$\gamma^2(2d-1)(3+2d)^2 + 8d\gamma(2d-1)(3+2d)(1+\beta) > d\gamma(9+9d+10d^2)(1+\beta)^2 + 4d^2(6+d+3d\beta-3\beta)(1+\beta)^3$$

(ii) laissez-faire ($t^* = 0$) for

[16] Bárcena-Ruiz and Garzon (2006) and Kato (2011) derive a different result that the optimal policy of the government is emissions taxes in mixed oligopolies.

$$\gamma^2(2d-1)(3+2d)^2+8d\gamma(2d-1)(3+2d)(1+\beta)=$$
$$d\gamma(9+9d+10d^2)(1+\beta)^2+4d^2(6+d+3d\beta-3\beta)(1+\beta)^3$$

and (iii) an emissions subsidy ($t^*<0$) for

$$\gamma^2(2d-1)(3+2d)^2+8d\gamma(2d-1)(3+2d)(1+\beta)<$$
$$d\gamma(9+9d+10d^2)(1+\beta)^2+4d^2(6+d+3d\beta-3\beta)(1+\beta)^3$$

We turn to the policy of the government in the R&D corner solution. In Figure 2, five borderlines between $t^*>0$ and $t^*<0$ corresponding to the degrees of spillovers, i.e., $\beta=0,\ 0.3,\ 0.5,\ 0.7,$ and $1$, are illustrated on the plane of the horizontal axis, $d\in[1/2,\ 10]$, and the vertical axis, $\gamma\in[0,\ 10]$.[17] The emissions tax becomes positive in the upward area of the borderlines, but it becomes negative, i.e. an emissions subsidy, in their downward area. The borderlines are shifted upward by increased spillovers. That is, increased spillovers reduce the area where the governmental policy is emissions taxes. This makes sense when one thinks of spillovers disseminating the rewards of R&D (and thereby promoting emissions abatement, which reduces the need for intervention via emissions taxes).

Insert Figure 2 around here>>>

Now it is difficult to illustrate the borderlines between $t^{**}>0$ and $t^{**}<0$ in the R&D interior solution case.[18] In this case, we can deduce results in the R&D interior solution from the results in the R&D corner solution. Specifically, since $z^{**}>z^*$, the area where the optimal tax policy for the case of $t^{**}$ is emissions taxes is reduced in comparison with the case of $t^*$.

Next, when substituting (9) and (10) into (6), we obtain the following SPNE outputs of the public and private firms in the R&D interior and corner solutions: for $\gamma>\overline{\gamma}(d,\ \beta)$

$$q_i^{**}=a[\gamma^2(3+2d)^2+d\gamma(3+2d)(7+2d)(1+\beta)+d\gamma(9+12d+4d^2)(1+\beta)^2+$$
$$d^2(12+13d-12\beta-3d\beta)(1+\beta)^3-5d^3(1+\beta)^4]/(3+2d)A$$

and, for $\gamma\le\overline{\gamma}(d,\beta)$

$$q_i^*=\frac{a[\gamma^2(3+2d)^2+8d\gamma(3+2d)(1+\beta)+d\gamma(9+9d+4d^2)(1+\beta)^2+d^2(15+7d-12\beta-3d\beta)(1+\beta)^3]}{(3+2d)A}$$

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[17] When we illustrate figures, parameter $a$, representing the market size, is excluded.

[18] We are unable to illustrate a figure by separating $z^{**}$ with $z_1^*>0$ from $z^{**}$ with $z_1^*<0$.

We find from (6) that $q_i^{**} > q_i^{*}$, $i = 0, 1$, as $z^{**} > z^{*}$.

It is meaningful to conduct comparative statics to consider the effects of both the efficiency of environmental R&D technology and related spillovers. First of all, let us take the effect of the efficiency of R&D technology. In the R&D interior solution case, when its efficiency improves, i.e., parameter $\gamma$ declines, both firms increase environmental R&D efforts (because the interior equilibrium solution is shifted outward on the R&D plane of $(z_0, z_1)$ as the R&D reaction curve of the public firm turns counterclockwise and that of the private firm turns clockwise, since the marginal R&D costs of both firms decline).[19] Consequently, total pollution abatement is increased. This increased pollution abatement then leads to reduced tax rates (or raised subsidy rates), and to an increase in each firm's output. On the other hand, in the R&D corner solution case, improved R&D technology leads to an increase in the emissions abatement of the public firm. Hence, the same effects on taxes and the outputs of the public and private firms as in the interior solution hold. Secondly, with increased spillovers, the R&D reaction curve of the public firm turns counterclockwise and that of the private firm shifts down at least for $d \geq 1$, so that the interior solution of R&D is shifted right/downward. Consequently, the public firm increases environmental R&D effort, but the private firm reduces it. It is, therefore, ambiguous whether total pollution abatement $(z^{**})$ is increased or not by such increased spillovers. In contrast, increased spillover rates lead to increased total pollution abatement in the R&D corner solution case. This increase causes a decline (a rise) in the emissions tax (subsidy) and hence an increase in the outputs of the public and private firms.[20]

Consequently, the SPNEs with the R&D interior solution and with the R&D corner solution are intermingled at each spillover rate. However, in the reasonable domain of $(d, \gamma)$, we have SPNE with the former solution, $(z^{**}, t^{**}, q^{**})$, at a high percentage in the absence of spillovers, while we have SPNE with the latter solution, $(z^{*}, t^{*}, q^{*})$, at high percentage in the ERJV ($\beta = 1$).

Now, consumer surplus is given as $CS^{**} = (Q^{**})^2 / 2$ or $CS^{*} = (Q^{*})^2 / 2$. Consumer surplus is greater in the R&D interior solution than in the R&D corner solution, i.e., $CS^{**} > CS^{*}$, since $z^{**} > z^{*}$. Profits of the private firm and welfare in the SPNE are given, respectively, by

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[19] In the R&D corner solution, this reduction causes the public firm to make an environmental R&D effort.

[20] It follows from (5) that $dt / d\beta = \partial t / \partial \beta + (\partial t / \partial z)(\partial z / \partial \beta)$, where $\partial t / \partial \beta < 0$ and $\partial t / \partial z < 0$.

$$\pi_1^{**} = 3(q_1^{**})^2/2 + t^{**}(z_1^{**} + \beta z_0^{**}) - \gamma(z_1^{**})^2/2$$
$$\pi_1^{*} = 3(q_1^{*})^2/2 + t^{*}(\beta z_0^{*}),$$
and
$$W^{**} = \frac{4[aq_0^{**} - (q_0^{**})^2] - 2(q_0^{**})^2 - \gamma[(z_0^{**})^2 + (z_1^{**})^2] - d[2q_0^{**} - d(1+\beta)z^{**}]}{2}$$
$$W^{*} = \frac{4[aq_0^{*} - (q_0^{*})^2] - 2(q_0^{*})^2 - \gamma(z_0^{*})^2 - d[2q_0^{*} - d(1+\beta)z^{*}]}{2}.$$

Having dealt with mixed duopoly, we consider the case of the traditional duopoly that occurs when the public firm is privatized. This happens when governments take privatization policy to improve efficiency by increasing competitiveness or to raise funds.

## 3. Privatization and comparisons

We proceed to consider the effect of privatization of a public firm on market performance. Then the public firm 0 is privatized. We examine how the environmental R&D and output of the firm, emissions taxes, and net emissions are influenced by privatization.

First, we deal with the maximization problem of private firms in the third stage. The first-order condition for profit maximization of firm $i(=0,\ 1)$ is given as follows:

$$\frac{\partial \pi_i}{\partial q_i} = a - t - 3q_i - q_j = 0, \quad i,\ j = 0,\ 1,\ i \neq j \qquad (13)$$

When assuming the symmetry, $q_i = q_j$, we have the output of firm $i$, $q_i = (a-t)/4$. Then the welfare function is reduced to

$$W = \frac{4aq_i - 6(q_i)^2 - \gamma(z_i^2 + z_j^2) - d(E)^2}{2},$$

where $E = 2q_i - (1+\beta)z$. The first-order condition for welfare maximization of the government in the second stage is given by

$$\frac{dW}{dt} = \frac{-a + 2ad - 4d(1+\beta)z - (3+2d)t}{8} = 0.$$

Hence we get the emissions taxes for any $z$:

$$t = \frac{(2d-1)a - 4d(1+\beta)z}{3+2d} \qquad (14)$$

Equation (14) is the same as (5) in the mixed duopoly. It follows from (14) that $dt/dz = -4d(1+\beta)/(3+2d) < 0$. When substituting (14) into the output, $q_i = (a-t)/4$, we have the following output of the private firm for any $z$:

$$q_i = \frac{a + d(1+\beta)z}{3+2d}, \quad i = 0,\ 1 \tag{15}$$

Equation (15) is the same as (6). We notice from (5), (6), (14), and (15) that the emissions taxes and outputs of both firms in the mixed duopoly are the same as in the private duopoly, for given $z$. Thus the following lemma holds.

**Lemma 2**. The emissions taxes and firm outputs in mixed and private duopolies are the same for any $z$, and their levels finally depend on total pollution abatements in each duopoly.[21]

Next, we derive the first-order condition for profit maximization of firm $i$ in the first stage. Differentiating the profit function with respect to the environmental R&D effort of firm $i$ and arranging it by using (13), (14), (15), and the symmetry assumption yields

$$\frac{d\pi_i}{dz_i} = \frac{\partial \pi_i}{\partial q_i}\frac{\partial q_i}{\partial t}\frac{dt}{dz_i} + \frac{\partial \pi_i}{\partial q_j}\frac{\partial q_j}{\partial t}\frac{dt}{dz_i} + \frac{\partial \pi_i}{\partial t}\frac{dt}{dz_i} + \frac{\partial \pi_i}{\partial z_i}, \qquad i = 0,\ 1,$$

$$= \frac{a[3d(1+\beta) + (3+2d)(2d-1)] - [2d(1+\beta)[(6+d)(1+\beta) + 4(3+2d)] + \gamma(3+2d)^2]z_i}{(3+2d)^2} = 0. \tag{16}$$

Finally, we obtain the environmental R&D effort of firm $i$ in the SPNE:

$$z_i^+ = \frac{a[3d(1+\beta) + (3+2d)(2d-1)]}{2d(1+\beta)[(6+d)(1+\beta) + 4(3+2d)] + \gamma(3+2d)^2} > 0, \qquad i = 0,\ 1, \tag{17}$$

where superscript + stands for the equilibrium value in the private duopoly. The private firm always performs environmental R&D in the first stage to reduce emissions as long as $d \geq 1/2$, different from the private firm in a mixed duopoly. Increased spillover rates lead to increased environmental R&D efforts of the firms and an improvement in R&D technology leads to increased ones.

Substituting total pollution abatement $z^+ = 2z_i^+$ into (14), we have the emissions tax (subsidy) in the SPNE:

$$t^+ = \frac{a[2d(d-2)(1+\beta)^2 + \gamma(3+2d)(2d-1)]}{2d(1+\beta)[(6+d)(1+\beta) + 4(3+2d)] + \gamma(3+2d)^2} \tag{18}$$

Given $d \geq 2$, the government adopts emissions taxes $(t^+ > 0)$, while whether it levies taxes on emissions for $1/2 < d < 2$ is ambiguous as follows: Given $1/2 < d < 2$, the environmental policy of the government is (i) an emissions tax $(t^+ > 0)$ for $\gamma > 2d(2-d)(1+\beta)^2 / (3+2d)(2d-1)$; (ii) laissez-faire $(t^+ = 0)$ for

[21] Bárcena-Ruiz and Garzon (2006) show that the outputs of firms in both a mixed oligopoly and a private oligopoly are different.

$\gamma = 2d(2-d)(1+\beta)^2/(3+2d)(2d-1)$; and (iii) an emissions subsidy $(t^+ < 0)$ for $\gamma < 2d(2-d)(1+\beta)^2/(3+2d)(2d-1)$. This result is summarized as follows:

**Proposition 3.** (1) Given $1/2 < d < 2$, the optimal environmental policy of the government is (i) an emissions tax for $\gamma > 2d(2-d)(1+\beta)^2/(3+2d)(2d-1)$, (ii) laissez-faire for $\gamma = 2d(2-d)(1+\beta)^2/(3+2d)(2d-1)$, and (iii) an emissions subsidy for $\gamma < 2d(2-d)(1+\beta)^2/(3+2d)(2d-1)$. (3) Given $d \geq 2$, the optimal environmental policy is an emissions tax.

In Figure 3 five curves of $t^+ = 0$ corresponding to $\beta = 0,\ 0.3,\ 0.5,\ 0.7,$ and $1$ are illustrated on the plane of the horizontal and vertical axes of $d \in [1/2, 2]$ and $\gamma \in [0,\ 10]$ in order to specify the area of emissions taxes. Emissions taxes are negative, i.e., $t^+ < 0$, in the left and lower area of each curve, while they are positive, i.e., $t^+ > 0$, in its right and upper area. The areas where emissions taxes are positive are reduced as spillovers rates increase to 1, but its range is not so large. These results show that the policy range to subsidize for firms is very limited, particularly in the absence of R&D spillovers.

Insert Figure 3 around here>>>

Similarly, from (15) and (17) we obtain the outputs of the private firms in the SPNE:

$$q_i^+ = \frac{a[2d(1+\beta)[2(1+\beta)+(3+2d)]+\gamma(3+2d)]}{2d(1+\beta)[(6+d)(1+\beta)+4(3+2d)]+\gamma(3+2d)^2}, \qquad i = 0,\ 1$$

Now an increase in the efficiency of R&D technology leads to increased environmental R&D efforts $(dz_i^+/d\gamma > 0)$. It follows from this result, and (18) that an increase in its efficiency, further, leads to decreased emissions taxes and increased outputs $(dt^+/d\gamma < 0$ and $dq_i^+/d\gamma > 0)$. These results are the same as in the mixed duopoly. On the other hand, we cannot get a definite effect of an increase in spillovers on the environmental R&D effort. However, there is every possibility that an increase in spillovers leads to a decrease in the environmental R&D efforts of the firms: Its increase will then cause emissions taxes to increase, but outputs to decrease.[22] This is the inverse of the effect of the efficiency of environmental R&D technology.

[22] Given $\gamma < [6d(6+d)(1+\beta)^2 + 4(6+d)(3+2d)(2d-1)(1+\beta) + 8(3+2d)^2(2d-1)]/3(3+2d)^2$, then we have $dz_i^+/d\beta < 0$.

The profits of the firm $i$ $(=0,\ 1)$ and welfare in the SPNE are reduced to, respectively,

$$\pi_i^+ = p^+ q_i^+ - \frac{1}{2}(q_i^+)^2 - t^+(q_i^+ - z_i^+ - \beta z_j^+) - \frac{\gamma}{2}(z_i^+)^2, \quad i,\ j = 0,\ 1,\ i \neq j,$$

and

$$W^+ = \frac{2aQ^+ - (Q^+)^2 - [(q_i^+)^2 + (q_j^+)^2] - \gamma[(z_i^+)^2 + (z_j^+)^2] - d(E^+)^2}{2},$$

where $p^+ = a - Q^+$ and $E^+ = q_0^+ + q_1^+ - (1+\beta)z^+$.

We examine the effects of privatization. First, let us make a comparison of pollution abatements (environmental R&D efforts) before and after privatization of a public firm, i.e., $z_0^*$ and $z_0^+$. It is difficult to mathematically compare them, but we are able to compare them by the assistance of a numerical example. According to the numerical example, $z_0^* > z_0^+$ holds for all spillover rates on the plane of the horizontal axis $d \in [1/2,\ 10]$ and the vertical axis $\gamma \in [0,\ 10]$.[23] For reference, let us make their comparison under $\beta = 0.5$. The vertical axis on the three dimensions measures the magnitude of $z_0^* - z_0^+$. The figure illustrates that $z_0^* - z_0^+$ is positive for all range, i.e., a non-privatized firm undertakes more pollution abatement than its privatized counterpart. This is because an increase in environmental R&D leads to larger an increase in welfare for a public firm than an increase in profit for a private firm. A difference between their R&D efforts gets large around small $\gamma$, as shown. It seems that the degree of the difference is reduced as R&D technology advances.

Insert Figure 4 around here>>>

Let us turn to a comparison of emissions taxes, outputs of firms, and net emissions in the mixed and private duopolies. From (5), (6), (14), (15), and the definition of the net emissions, the following relationships for the emissions taxes and outputs in the mixed duopoly with an R&D interior solution and in the private duopoly are established

$$t^{**} - t^+ = -\frac{4d(1+\beta)}{3+2d}(z^{**} - z^+) \tag{19}$$

$$q_i^{**} - q_i^+ = \frac{d(1+\beta)}{3+2d}(z^{**} - z^+) \tag{20}$$

[23] This is verified graphically using Mathematica.

$$E^{**} - E^{+} = -\frac{3(1+\beta)(z^{**} - z^{+})}{3+2d}, \tag{21}$$

where $E^{**} = e_0(q_0^{**}, z_0^{**}) + e_1(q_1^{**}, z_1^{**})$ and $E^{+} = e_0(q_0^{+}, z_0^{+}) + e_1(q_1^{+}, z_1^{+})$. In addition, the same relationships also hold for the emissions taxes, the outputs and net emissions, and total pollution abatement in the mixed duopoly with the R&D corner solution and in the private duopoly. Then it follows from (19), (20), and (21) that the following proposition holds immediately:

**Proposition 4.** (1) In the mixed duopoly with the R&D interior solution,[24]

$t^{**} \le (>)\, t^{+}$, $q_i^{**} \ge (<)\, q_i^{+}$, and $E^{**} \le (>)\, E^{+}$ as $z^{**} \ge (<)\, z^{+}$;

(2) In the mixed duopoly with the R&D corner solution,

$t^{*} \le (>)\, t^{+}$, $q_i^{*} \ge (<)\, q_i^{+}$, and $E^{*} \le (>)\, E^{+}$ as $z^{*} \ge (<)\, z^{+}$

This result shows that in the end, the relationships of emissions taxes, outputs, and net emissions under a mixed duopoly and a private duopoly come down to total pollution abatement.

Now it is difficult to compare total pollution abatement in the mixed duopoly with an R&D interior solution with that in the private duopoly. We take the relatively easy mixed duopoly case of the R&D corner solution, i.e., $z_1^{*} = 0$, but even in this case it is difficult to mathematically specify even the relationship of $z^{*}$ and $z^{+}$ for any $d$, $\gamma$, and $\beta$, where $z^{*} = z_0^{*}$ and $z^{+} = z_0^{+} + z_1^{+}$. Therefore, we can resort to a numerical method to compare both environmental R&D efforts by a figure. We obtain that $z^{*} > z^{+}$ for $d\,(\in [1/2,\ 10])$, $\gamma\,(\in [0,\ 10])$, and $\beta = 0,\ 0.3,\ 0.5,\ 0.7,\ 1$: The figure depicts that total pollution abatements obviously are larger in the mixed duopoly with the R&D corner solution than in the private duopoly. Figure 5, for example, illustrates a comparison of the R&Ds under $\beta = 0.5$, i.e., $z^{*} - z^{+} > 0$, where the magnitude of $z^{*} - z^{+}$ measures on the vertical axis on the three dimensions.

Insert Figure 5 around here>>>

It follows from Proposition 4 (2) that $t^{*} < t^{+}$ as $z^{*} > z^{+}$. Emissions taxes (subsidies) are lower (larger) in the mixed duopoly with the R&D corner solution than in the private duopoly. When invoking the result, we find from Proposition 4

[24] Bárcena-Ruiz and Garzon (2006) derive the result that the government levies lower emissions taxes in a mixed oligopoly than in a private oligopoly. But, their result does not always hold when firms take environmental R&D efforts and there are R&D spillovers.

(2) that the output of firm $i$ is larger in the mixed duopoly with the R&D corner solution than in the private duopoly, i.e., $q_i^* > q_i^+$. Furthermore, we get $E^* < E^+$ as $z^* > z^+$.

We turn to each of the comparisons of emissions taxes, outputs, and net emissions in the mixed duopolies with the R&D interior and corner solutions and in the private duopoly. When reminding $z^{**} > z^*$ as to total pollution abatement in the mixed duopoly, we have the following relationships among environmental R&D efforts, emissions taxes, outputs and net emissions:

$$t^{**} < t^* < t^+,\ q_i^{**} > q_i^* > q_i^+,\ Q^{**} > Q^* > Q^+, \text{ and } E^+ > E^* > E^{**} \text{ for } z^{**} > z^* > z^+ \qquad (22)$$

It follows from the outcome that with respect to consumer surplus we have $CS^{**} > CS^* > CS^+$ for $z^{**} > z^* > z^+$. In other words, consumer surplus is larger in the mixed duopoly than in the private duopoly. After all, privatization leads to small environmental R&D efforts and small industrial output, providing small consumer surplus. Privatization, however, makes the environment worse as a result of non-aggressive R&D activities, irrespective of whether there is an interior solution or a corner one in the first stage equilibrium of the SPNE.[25] It seems that environmental R&D leads to greater effects of reducing emissions under non-privatization (publication) than under privatization. In addition, environmental damage to society is less under non-privatization than under privatization, i.e., $D(E^{**}) < D(E^*) < D(E^+)$. When taking account of the results as to consumer surplus and net emissions, we find that privatization may not be beneficial as an industrial policy. The concluding section follows.

## 4. Conclusions

This paper examines optimal pollution abatement under mixed oligopoly when firms engage in emissions-reducing R&D that is imperfectly appropriable. Mixed oligopolies are common in instances when governments partially privatize public sector undertakings or governments enter private markets to increase competition or offer alternatives. The regulator uses a tax to curb emissions. Firms' behavior under partial privatization is compared to the case of full privatization (i.e., private competition). While some scholars in the literature have considered similar aspects (e.g., Poyago-Theotoky (2007)), our treatment of the issues involved in more general in some respects. At a broader level, this research may be seen in the context of some of the issues of technological change

[25] Gil-Molto and Varvarigos (2014) obtain the same result as ours. Bárcena-Ruiz and Garzon (2006), however, demonstrate that total emissions are greater in the mixed oligopoly than in the private oligopoly. In addition, Beladi and Chao (2006) show the case in which privatization worsens the environment.

and environmental policy noted by Jaffe et al. (2002) and others (e.g., Goel and Hsieh (2006)).

Results show that in a mixed oligopoly, the public firm engages in positive emissions reduction in equilibrium; however, the emissions reductions of the private firm could be positive, negative or zero. In contrast, Bárcena-Ruiz and Garzon (2006), Gil-Molto and Varvarigos (2014), and Kato (2011) show that a private firm undertakes emissions abatement in a mixed oligopoly. Under full privatization, the optimal pollution tax is positive when marginal emissions control expenditures rise sharply or marginal pollution damage is high; otherwise, the tax reverts to a subsidy.

Comparing mixed and private duopolies, privatization leads to reductions in environmental R&D and output, but to an increase in overall emissions. Thus, privatization tends to make the environment worse. This finding underscores the policy challenges of designing effective pollution control in the presence of research and environmental spillovers and call into question blanket recommendations that unequivocally espouse the virtues of privatization. The environmental effects and policy design becomes even more challenging when emissions are durable with impacts that linger over time (Goel and Hsieh (2004)). The other policy take from the results is that pollution taxes may not always be the optimal policy to curb emissions and that subsidies might be the recommended path in some cases. The overall results add to the ongoing theoretical efforts to get a better handle on effect measures to combat environmental degradation. Obviously, these efforts would benefit from some empirical verification that is often constrained by the availability of appropriate data.

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Figure 1. Environmental R&D of the private firm

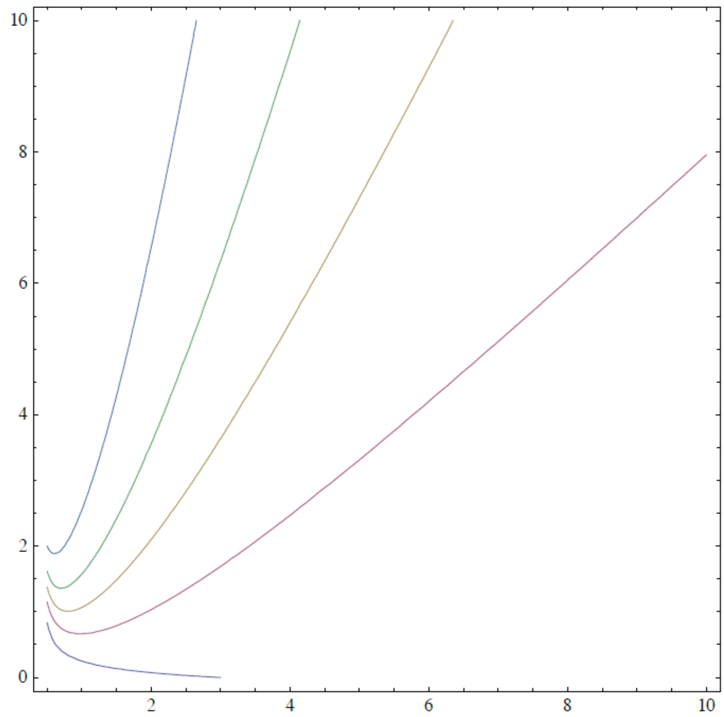


Figure 2. Optimal tax policy in the corner solution

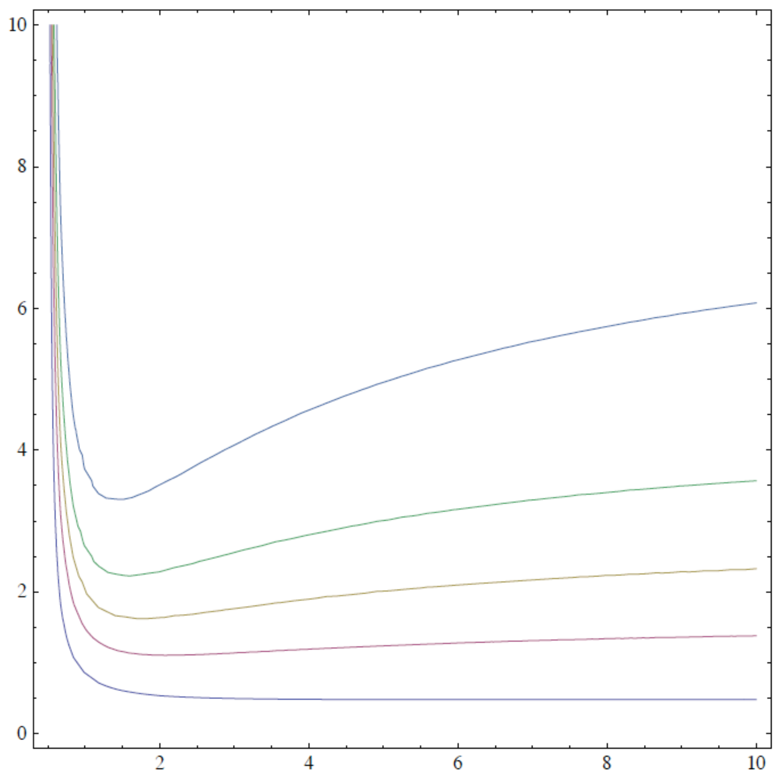

Figure 3. Optimal tax policy in the private duopoly

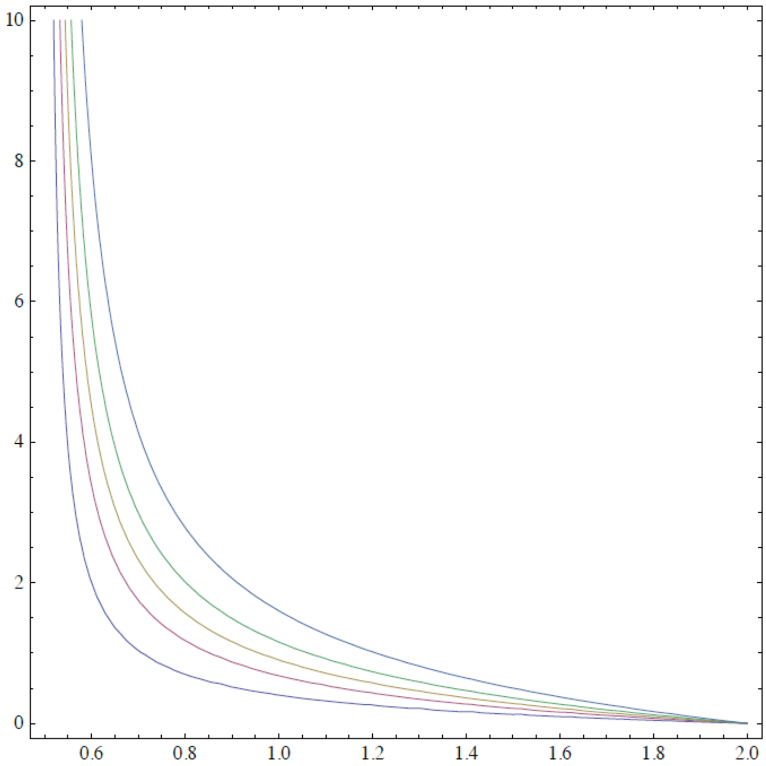


Figure 4. Comparison of environmental R&Ds of non-privatized and privatized firms under $\beta = 0.5$

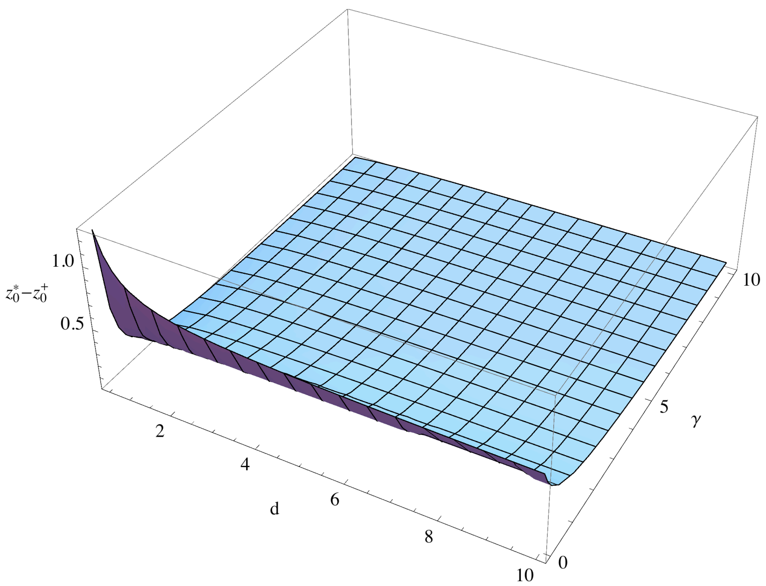

Figure 5. Comparison of environmental R&Ds under a mixed duopoly with an R&D corner solution and a private duopoly under $\beta = 0.5$