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Abstract

Empirical studies have uncovered an inverted-U relationship between product-market competition and innovation. This is inconsistent with the original Schumpeterian Model, where greater competition reduces the profitability of innovation. We show that the model can predict the inverted-U if the innovators' talent is heterogenous, and privately observable. With competition low and profitability high, talented innovators are credit constrained, since others are eager to mimic them. As competition increases, the mimickers become less eager, and talented innovators can invest more. This generates the increasing part of the relationship. With competition high, talented innovators are unconstrained, and the relationship is decreasing.

JEL-Codes: O380, E600, G380.

Keywords: innovation, competition, Schumpeterian Model of Growth, asymmetric information.

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1 Introduction

Empirical studies have uncovered an inverted-U relationship between product-market competition and innovation (e.g. Aghion et al. (2005)). This is inconsistent with the original Schumpeterian model (Aghion and Howitt (1992)), where stronger competition always reduces the incentives to innovate, because it reduces the post-innovation rents (the *Schumpeterian effect*). To address this inconsistency, the more recent literature has departed from the original setting where innovation is carried out by outsiders, and adopted models where innovation is carried out by insiders. In those models, at low levels of competition, greater competition may reduce the pre-innovation rents more than it reduces the post-innovation rents, thus increasing the incentives to innovate ("escape competition effect").

In this paper, we show that a minimal extension to the original Schumpeterian model can predict the inverted-U relationship between competition and innovation, even in the original setting with innovation by outsiders. We start from a standard version of the model with overlapping generations and a fringe of competitive producers (as in Aghion and Howitt (2009), pp. 130-32 and 90-91), and add two ingredients: heterogeneously talented innovators, and asymmetrically observable talent. Specifically, innovators can be talented or untalented, the former having a higher probability of innovating for given investment; and those types are only observable to the innovators themselves.

We construct a separating equilibrium in which the talented innovators signal themselves to lenders by contributing their entire wage in equity, and by limiting the amount they borrow.[1] At low levels of competition, when post-innovation rents are high, the talented agents would like to invest a lot. However, they cannot borrow enough at favourable conditions, since the untalented innovators are eager to mimic them (given the high profitability of

[1]The general idea of "skin in the game as a screening device" has emerged repeatedly in the academic Finance literature. Applications to the field of entrepreneurship - and its financing - go all the way back to Leland and Pyle (1977); more recent contributions include Kaplan and Stromberg (2004), Skeie (2007), and Conti A. and Rothaermel (2013), among others. DeMarzo and Duffie (1999) and DeMarzo (2005) provide examples of this principle in the context of security design.

innovation). They then invest less than what would be optimal. As competition increases and post-innovation rents decrease, the untalented innovators become less eager to mimic. As this happens, the amount that the talented innovators can borrow at favourable conditions actually increases, leading them to invest *more*. This explains the increasing part of the curve. We call this effect the *selection effect*, because it leads to a higher weight of the talented innovators in overall investment. At high levels of competition, when post-innovation rents are low, the talented innovators would like to invest only a modest amount. Moreover, they can borrow a lot at favourable conditions, since the untalented agents are not eager to mimic them. They then invest their optimal amount, which by the Schumpeterian effect is decreasing in the strength of competition. This generates the decreasing part of the curve.

In further analysis, we show that our separating equilibrium exists as long as the wage is neither too high (or else the talented innovators would not need to borrow) nor too low (or else they would be so much in need of borrowing, that they would opt for being pooled with the untalented innovators). Moreover, we argue that, in this parameter sub-space, our equilibrium outcome is the only one that can "reasonably" realise in a Perfect Bayesian Equilibrium that survives a standard refinement procedure.

This paper relates to several strands of literature. First, it contributes to Schumpeterian growth theory (see Aghion et al. (2014), for a survey). Second, it relates to the burgeoning literature on the macroeconomic implications of financial frictions (see Brunnermeier et al. (2013)), and more specifically the branch analyzing their effects on countries' economic development (see Levine (2005)). Third, our paper proposes a novel explanation for the "inverted-U" relationship between competition and innovation, which has been observed and tested using data from the UK, Japan, the Netherlands, and Switzerland, among other countries.[2]

[2]We refer to Aghion et al. (2005), Michiyuki and Shunsuke (2013), Polder and Veldhuizen (2012) , and Peneder and Woerter (2014), respectively. Looking at U.S. industry-level data, Hashmi (2013) finds a negative relationship between competition and innovation, instead of the "inverted-U" pattern. In his (yet unpublished) Master's Thesis, Astakhov (2015) presents "strong evidence" of the "inverted-U" pattern holding in a "firm-level dataset of publicly traded US companies."

Starting from Aghion et al. (2005) - arguably the seminal study in the contemporary incarnation of this field of research - all of the formal models that have been developed to explain this phenomenon focus squarely on issues of industry organization and dynamics, leaving virtually no role for financial factors.[3] Conversely, our model puts asymmetric information in financial markets at its front and center. This simple, realistic addition to an otherwise standard model allows us to generate the "inverted-U" pattern through an intuitive mechanism.

Finally, there is a small number of papers analyzing Schumpeterian growth models, in which financial features gain center stage. These articles differ from ours in their assumptions (usually and most importantly, the nature of the financial frictions they consider), as well as their subject matters and applications. For example, Diallo and Koch (Forthcoming) investigate the relationship between economic growth and bank concentration; Malamud and Zucchi (2016) study corporate cash management when firms face exogenous financing costs; and Sunaga (2017) extends the standard model to deal with moral hazard in financial markets and monitoring by intermediaries.[4]

Bryce Campodonico et al. (2016) and Plehn-Dujowich (2009) develop Schumpeterian growth models with adverse selection in financing, but use them to study optimal tax policy and to quantify the reduction in the rate of growth stemming from the presence of financial frictions, respectively. Finally, Ates and Saffie (2013) study a general equilibrium endogenous growth model in which financial intermediaries screen the quality of projects from a heterogeneous population of entrepreneurs. None of these papers concerns itself with the relationship between an industry's degree of competition and its R&D outcomes, which is the main focus of the present essay.

The paper is organised as follows. In Section 2 we present the baseline

[3]Models that predict - or are consistent with - the "inverted-U" relationship include: Chernyshev (2016), Mukoyama (2003),Rauch (2008), and Scott (2009). None of them deals with financial market frictions.

[4]In a related contribution, Chiu et al. (2017) develop a model in which "entrepreneurs" get new ideas randomly and without paying any R&D costs, but search frictions and the presence of financial intermediaries influence the process of technological tranfer. That is, their focus is on the allocation of these new blueprints to the agents who have the most talent for developing them and bringing them to market.

model. Section 3 introduces imperfect information in financial markets, and derives the inverted-U relationship between competition and innovation. Section 4 discusses the existence and robustness of our equilibrium. Finally, Section 5 concludes by discussing some empirical and policy implications.

2 Baseline model

The baseline model is a standard Schumpeterian model with overlapping generations and a fringe of competitive producers (as in Aghion and Howitt (2009), pp. 130-32 and 90-91), which we generalise to allow for heterogeneous talent of innovators. A final good is produced competitively using labour and a continuum of intermediate goods, according to

$$Y_t = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} X_{it}^{\alpha}, \tag{2.1}$$

where X_{it} is input of the latest version of intermediate i, and A_{it} is its productivity. Each intermediate is produced and sold by a monopolist, who can produce one unit of the intermediate at the cost of one unit of the final good. However, in each industry, there is also a fringe of competitive firms that can produce the intermediate at cost of $1/\kappa_i$ units of the final good per unit produced. The parameter $\kappa_i \in [\alpha, 1]$ measures the strength of competition faced by the monopolist. As will be clear below, $\kappa_i = \alpha$ denotes the case of no competition, while $\kappa_i = 1$ denotes the case of perfect competition. For simplicity, all industries have the same initial level of productivity, $A_{t-1} \equiv \int_0^1 A_{it-1} di$.[5]

Agents live for two periods, are risk neutral, and have a discount factor equal to one. There are two equally sized cohorts alive in each period, the young and the old. The young work in the final good sector, where they earn a wage. After this, one of them per industry (the “innovator ”) tries to invent

[5]This assumption, also made by Aghion and Howitt (2009), simplifies the model by ruling out that credit constraints are weaker in industries where productivity, and thus the size of investment, is lower. We could have alternatively assumed that workers are immobile across industries, since then both private wealth and investment would be scaled by industry productivity.

a new version of the intermediate which is $\gamma > 1$ times more productive than the previous version. If successful, she invest in the production of the new version, becoming the monopolist in the next period. If unsuccessful, a young agent is chosen at random to produce the previous version, and to become the monopolist. As for the old agents, there is one of them in each industry who is the current monopolist, while all others are all idle consumers.

There are borrowers and lenders in this model. Borrowers include young agents undertaking an investment - be it innovation or production - which they cannot fund through the wage they have earned. The lenders are all young agents. While production is a risk-free activity, innovators only pay back if they are successful. Thus, the financing of innovation is the only interesting part of the financial market. We assume that the maximum supply of credit (the total wage bill) is greater than demand, so that the risk-free interest rate is zero.

A monopolist faces iso-elastic demand $P_{it} = \alpha \left(A_{t-1}L/X_{it}\right)^{1-\alpha}$, given which her optimal price is $1/\alpha$.[6] However, facing competition from the fringe, the monopolist is forced to charge $1/\kappa_i \leq 1/\alpha$ instead. Plugging back in the demand function, we find optimal X_{it}, which can then be multiplied by profit per unit, $\left(1-\kappa_i\right)/\kappa_i$, to find total profits. Normalised by initial productivity, these are

$$\pi\left(\kappa_i\right) = \frac{1-\kappa_i}{\kappa_i}\left(\kappa_i\alpha\right)^{\frac{1}{1-\alpha}} L$$

in an industry that has not innovated, and $\gamma\pi\left(\kappa_i\right)$ in an industry that has. It is easy to show that $\pi\left(\kappa_i\right)$ is decreasing in $\kappa_i \in (\alpha, 1]$: intuitively, the stronger is competition from the fringe, the lower are the monopolist's profits.

Substituting optimal X_{it} in the production function, differentiating with respect to L, and dividing by A_{t-1}, we find the normalised wage,

$$w = (1-\alpha)\left(\kappa\alpha\right)^{\frac{\alpha}{1-\alpha}}, \tag{2.2}$$

[6] This demand function can be found by taking the first derivative of 2.1 with respect to X_{it}.

where $\kappa \equiv \int_0^1 \kappa_i di$ is average level of competition in the economy.

Innovators can be of two types, a high type (H) and a low type (L). If an innovator of type $J \in \{H, L\}$ invests a normalised amount z in research, she is successful with probability $a^J \mu(z)$, where μ is an increasing and concave function satisfying standard conditions, and $a^H > a^L$. In each generation, there is an equal share of high types and low types.

For now, we assume that an innovator's type is perfectly observable to everyone. Then, competing lenders demand an interest rate $1/[a^J \mu(z)]$ from type J, given which the innovator's net present value is

$$\begin{aligned} npv_i^J(z) &= a^J \mu(z) \left[\gamma \pi(\kappa_i) - \frac{1}{a^J \mu(z)} (z - e) \right] - e \\ &= a^J \mu(z) \gamma \pi(\kappa_i) - z, \end{aligned}$$

where e is her normalised equity contribution. With perfect information, the innovator's net present value does not depend on her choice of financing, since the expected cost of both equity and external financing is equal to the risk-free interest rate.

Let $\hat{z}_i^J$ denote optimal, perfect information investment by type J in industry i. This is implicitly defined by condition

$$a^J \mu'\left(\hat{z}_i^J\right) \gamma \pi(\kappa_i) = 1. \tag{2.3}$$

Clearly, at any given level of competition, the high types invest more than the low types, $\hat{z}_i^H > \hat{z}_i^L$. Furthermore, for any type, and for any two industries i and j such that $\kappa_i > \kappa_j$, investment (and thus the probability innovation) is lower in the more competitive industry, $\hat{z}_i^J < \hat{z}_j^J$. This is the standard *Schumpeterian effect* of competition on innovation: by reducing the reward from innovation, stronger competition reduces investment, and thus innovation. With perfect information, only the downward-sloping portion of the observed inverted-U relationship between competition and innovation fits the predictions of the standard Schumpeterian model.

Figure 3.1 illustrates. The $npv_i^J(z)$ functions are represented by thin solid lines (the thick and dashed lines should be ignored for now). Investment choices under perfect information are represented by solid circles. The only parameter that differs across the three panels is κ_i, which increases from top to bottom. By the Schumpeterian effect, a higher κ_i results in an inward rotation of the $npv_i^J(z)$ functions, and in lower investment.

3 Asymmetric information in financial markets

We now assume that the innovator's type is the innovator's private information. Lenders must then determine the interest rate based solely on the subset of information which is observable, that is the size of the proposed investment (z) and equity contribution (e). We here focus on a specific separating equilibrium, and on the parameter range where it exists. In Section 4, we first identify this parameter range, and we then argue that, in that range, the equilibrium we have identified has an attractive feature: namely, its outcome is one of only two that can realise in a Perfect Bayesian Equilibrium (PBE) that survives a standard refinement criterion. We describe the equilibrium intuitively, and relegate the formal derivations to the Online Appendix.

We focus on the first panel of Figure 3.1. It is easy to show that the low types must always invest $\hat{z}_i^L$ at a separating equilibrium, contributing any $e \leq \hat{z}_i^L$ in equity.[7] However, the high types may now be forced to invest less than $\hat{z}_i^H$. To see why, suppose that lenders believed that those who contribute w in equity, and invest $\hat{z}_i^H$, are high types. Clearly, this would allow the high types to borrow $\hat{z}_i^H - w$ at their perfect information rate, given which they would always choose to invest $\hat{z}_i^H$. However, for this to be an equilibrium, the low types must not want to mimic the high types. But by contributing w in equity, and borrowing $z - w$ at the high types' perfect information rate, the

[7] To see this, suppose the low types invested $z \neq \hat{z}_i^L$. Since this is a separating equilibrium, the low types would have to be asked an expected interest rate equal to the risk-free rate, and their payoff would have to be $npv_i^L(z)$. But, by choosing $\hat{z}_i^L$, they could have not been asked a higher expected rate (in equilibrium), and they would have thus obtained at least $npv_i^L(\hat{z}_i^L) > npv_i^L(z^L)$. It follows that z is not the low types' optimal choice, a contradiction.

low types would receive net present value

$$\begin{aligned}\widetilde{npv}_i^L(z) &\equiv a^L\mu(z)\left[\gamma\pi(\kappa_i) - \frac{1}{a^H\mu(z)}(z-w)\right] - w \\ &= a^L\mu(z)\gamma\pi(\kappa_i) - \frac{a^L}{a^H}(z-w) - w,\end{aligned}$$

that is the dashed line in Figure 3.1. By mimicking the high types, the low types can now pay less than the risk-free interest rate on external borrowing (in expectations). Given this low interest rate, it may well be optimal for the low type to invest $\hat{z}_i^H > \hat{z}_i^L$, like the high types, even though successful mimicking also requires contributing w in equity. Indeed, since $\widetilde{npv}_i^L(\hat{z}_i^H) > npv_i^L(\hat{z}_i^L)$, the low types do want to mimic the high types, and the situation that we are describing is not an equilibrium.

Suppose then that lenders believed that those who contribute w in equity are high types if they invest up to z_i^{sep}, while they are high types and low types with equal probability if they invest more. Additionally, suppose that they believed that anyone contributing less than w in equity is a low type. Then, the high types' net present value is represented by the broken, solid line, whose function for the case $z > z_i^{sep}$ is

$$\begin{aligned}\widetilde{npv}_i^H(z) &\equiv a^H\mu(z)\left[\gamma\pi(\kappa_i) - \frac{1}{a\mu(z)}(z-w)\right] - w \\ &= a^H\mu(z)\gamma\pi(\kappa_i) - \frac{a^H}{a}(z-w) - w, \qquad (3.1)\end{aligned}$$

where $a = (a^H + a^L)/2$. Since the expected cost of external financing is more than the risk-free interest rate for $z > z_i^{sep}$, the high types now choose to contribute w in equity, and invest z_i^{sep}. Note that, now, the low types do not necessarily want to mimic the high types, since the latter's high equity contribution and low external financing decisions make mimicking less attractive. By choosing a moderate investment profile, the high types are able to signal themselves as talented innovators to the uninformed lenders.

What we have just described is a PBE, since all innovators invest optimally given the lenders' beliefs, and beliefs are correct in equilibrium. Investment

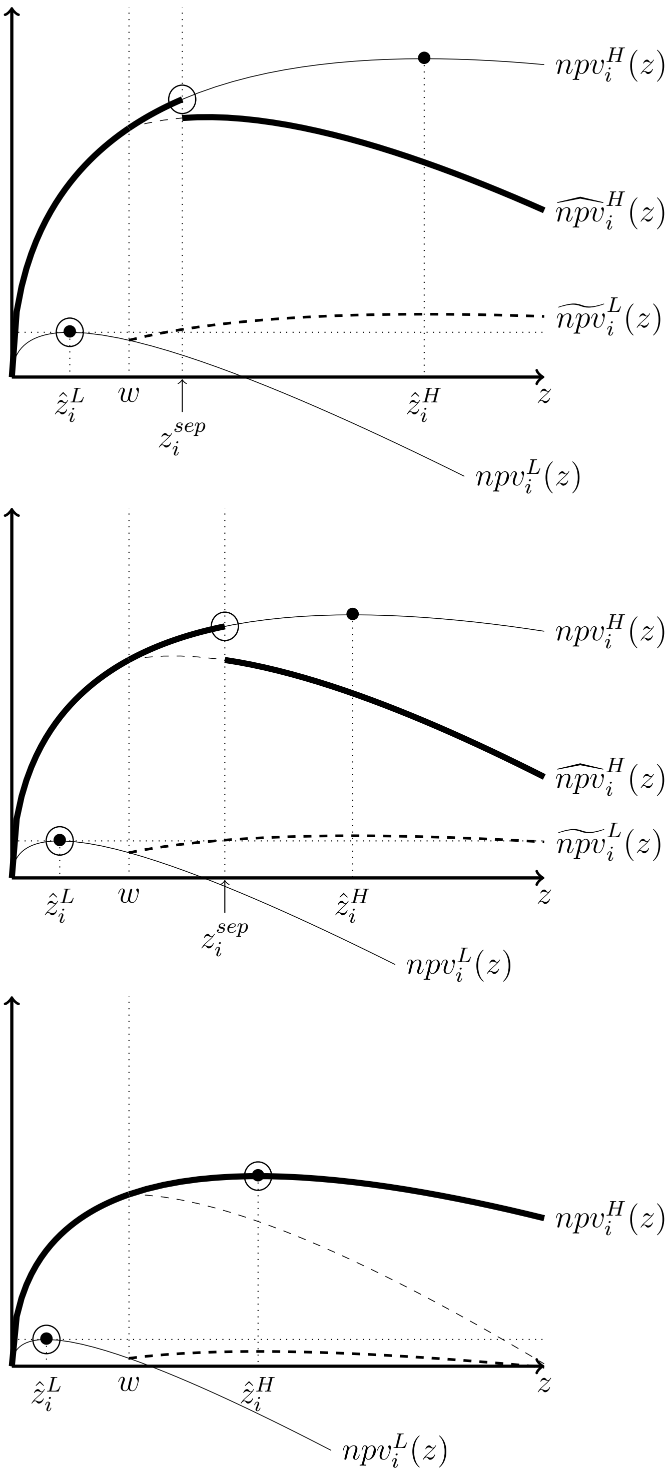

Figure 3.1: Illustration of the separating equilibrium. The three panels only differ by the size of κ_i, which increases from top to bottom.

choices at this separating equilibrium are represented by empty circles in Figure 3.1.

We now study the comparative statics of the equilibrium, by increasing κ_i to the level in the second panel of Figure 3.1. As discussed above, $npv_i^L(z)$ and $npv_i^H(z)$ rotate inwards, and investment by the low types $(\hat{z}_i^L)$ decreases by the Schumpeterian effect. However - and this is the central result of the paper - investment by the high types (z_i^{sep}) *always increases*. To make sense of this result, note that in both the first and second panel of Figure 3.1, the high types' investment decisions are not driven by incentives, but rather by credit constraints. Then, the Schumpeterian effect does not apply, and what matters is the effect of competition on credit constraints. But stronger competition reduces the reward from innovating, and thus the low types' incentives to mimic the high types. This relaxes the credit constraint faced by the high types, allowing them to invest more. To see this in more detail, recall that $z = z_i^{sep}$ and $e = w$ is the high types' investment profile that makes a genuine and a mimicking low type equally well off. But a fall in $\pi(\kappa_i)$ penalises the mimicker more than the genuine agent, since the former has a higher probability of innovating. To restore equality of payoffs, the high types' resort to external borrowing (and hence z_i^{sep}) must increase.

In other words, stronger competition, by creating a tougher operating environment, leads to a better selection of innovators, in the sense that a greater share of available funds is allocated to the high types. We call this the *selection effect* of competition on innovation.

Now suppose that κ_i increases even further, to the level in the third panel of Figure 3.1. Here competition is so strong, and optimal investment so low, that the high types are not credit constrained any more. Then, the Schumpeterian effect kicks back in, and any further increase in competition *decreases* investment by the high types.

This discussion suggests that, at the equilibrium described above, comparing across industries where the innovator is of a high type, one finds an inverted-U relationship between the strength of competition and innovation. This result is formally stated in

Proposition 1. *Consider any two industries i and j where the innovator is a high type, and such that competition is stronger in j than in i, $\kappa_i < \kappa_j$. At the separating equilibrium described above, there exists $\hat{\kappa} \in (\alpha, 1)$ such that, if $\alpha \leq \kappa_i < \kappa_j \leq \hat{\kappa}$, industry j has a higher probability of innovating than industry i, while if $\hat{\kappa} \leq \kappa_i < \kappa_j \leq 1$, industry j has a lower probability of innovating than industry i.*

Proof. Note that $\widetilde{npv}_i^L(z)$ is concave, and reaches a maximum at $\hat{z}_i^H$. Let

$$z_i^{sep} = \min \arg \left\{ a^L \mu(z_i^{sep}) \gamma \pi(\kappa_i) - \frac{a^L}{a^H}(z_i^{sep} - w) - w = a^L \mu\left(\hat{z}_i^L\right) \gamma \pi(\kappa_i) - \hat{z}_i^L \right\}. \tag{3.2}$$

or, if such z does not exist, then $z_i^{sep} = \hat{z}_i^H$. There are two possible cases: $z_i^{sep} < \hat{z}_i^H$ or $z_i^{sep} = \hat{z}_i^H$. Suppose $z_i^{sep} < \hat{z}_i^H$, and consider an increase in κ_i. The total differential of the equation in curly brackets in (3.2) is

$$a^L \mu'(z_i^{sep}) \gamma \pi(\kappa_i)\, dz_i^{sep} + a^L \mu(z_i^{sep}) \gamma \pi'(\kappa_i)\, d\kappa_i - \frac{a^L}{a^H} dz_i^{sep} = a^L \mu\left(\hat{z}_i^L\right) \gamma \pi'(\kappa_i)\, d\kappa_i, \tag{3.3}$$

which can be re-arranged into

$$\frac{dz_i^{sep}}{d\kappa_i} = a^H \frac{\mu\left(\hat{z}_i^L\right) - \mu\left(z_i^{sep}\right)}{a^H \mu'(z_i^{sep}) \gamma \pi(\kappa_i) - 1} \gamma \pi'(\kappa_i) > 0.$$

Since z_i^{sep} is continuously increasing in κ_i, while $\hat{z}_i^H$ is continuously decreasing and $0 \leftarrow \hat{z}_i^H$ as $\kappa_i \rightarrow 1$, there exists $\hat{\kappa} \in (\alpha, 1)$ such that, for $\kappa_i < \hat{\kappa}$, it is $z_i^{sep} < \hat{z}_i^H$, while for $\kappa_i \geq \hat{\kappa}$ it is $z_i^{sep} = \hat{z}_i^H$. In the latter range, it is $dz_i^{sep}/d\kappa_i = d\hat{z}_i^H/d\kappa_i < 0$. The result follows immediately. Note that $\hat{\kappa}$ must be the same across industries, since κ_i is the only parameter that varies across industries. □

Proposition 1 finds an inverted-U relationship between competition and innovation over the entire range of κ_i. In term of our earlier discussion, the region $\alpha \leq \kappa_i < \hat{\kappa}$ is where the high types invest z_i^{sep}, while the region $\hat{\kappa} \leq$

$\kappa_i \leq 1$ is where they invest $\hat{z}_i^H$. The threshold $\hat{\kappa}$ is defined as the unique level of competition such that $z_i^{sep} = \hat{z}_i^H$.

One shortcoming of Proposition 1 is that it only finds an inverted-U relationship between competition and innovation across industries where the innovator is of a high type, while the relationship is decreasing across all other industries. This begs the question of what sort of relationship the model predicts across *all* industries. On the one hand, it is immediate that the relationship will be negative in the region $\hat{\kappa} \leq \kappa_i \leq 1$, since both $\hat{z}_i^H$ and $\hat{z}_i^L$ fall by the Schumpeterian effect. On the other, we are now going to show that a positive relationship is predicted in the region $\alpha \leq \kappa_i < \hat{\kappa}$, for κ_i close enough to $\hat{\kappa}$. So, at least in a subset of $[\alpha, 1]$, the model continues to predict an inverted-U relationship between competition and innovation.

To illustrate this, let μ_i denote the *ex-ante* probability of innovating in industry i,

$$\mu_i = \frac{1}{2} a^H \mu\left(z_i^{sep}\right) + \frac{1}{2} a^L \mu\left(\hat{z}_i^L\right).$$

Then, the following holds:

Proposition 2. *Consider any two industries i and j such that competition is stronger in j than in i, $\kappa_i < \kappa_j$. At the separating equilibrium described above, there exists $\widetilde{\kappa} \in (\alpha, \hat{\kappa})$ such that, if $\widetilde{\kappa} < \kappa_i < \kappa_j \leq \hat{\kappa}$, industry j has a higher ex-ante probability of innovating than industry i, while for $\hat{\kappa}_i \leq \kappa_i < \kappa_j \leq 1$, industry j has a lower ex-ante probability of innovating than industry i.*

Proof. Suppose $z_i^{sep} < \hat{z}_i^H$. It is

$$\frac{d\mu_i}{d\kappa_i} = \frac{1}{2} a^H \mu'\left(z_i^{sep}\right) \frac{dz_i^{sep}}{d\kappa_i} + \frac{1}{2} a^L \mu'\left(\hat{z}_i^L\right) \frac{\hat{z}_i^L}{d\kappa_i}.$$

The total derivative $dz_i^{sep}/d\kappa_i$ was derived in (3.2), while $d\hat{z}_i^L/d\kappa_i$ can be found by taking the total differential of (2.3) and re-arranging,

$$\mu''(\hat{z}_i^L)\gamma\pi\left(\kappa_i\right)d\hat{z}_i^L + \mu'(\hat{z}_i^L)\gamma\pi'\left(\kappa_i\right)d\kappa_i = 0$$

$$\frac{d\hat{z}_i^L}{d\kappa_i} = -\frac{\mu'(\hat{z}_i^L)\pi'\left(\kappa_i\right)}{\mu''(\hat{z}_i^L)\pi\left(\kappa_i\right)} < 0.$$

Replacing $dz_i^{sep}/d\kappa_i$ and $d\hat{z}_i^L/d\kappa_i$ into the expression for $d\mu_i/d\kappa_i$, imposing $d\mu_i/d\kappa_i > 0$, and re-arranging we obtain

$$\frac{1}{\alpha^H\mu'\left(z_i^{sep}\right)\gamma\pi\left(\kappa_i\right)-1} > \frac{a^L\left[\mu'\left(\hat{z}_i^H\right)\right]^2}{\left[-\mu''\left(\hat{z}_i^L\right)\right]\gamma\pi\left(\kappa_i\right)\left[a^H\right]^2\mu'\left(z_i^{sep}\right)\left[\mu\left(z_i^{sep}\right)-\mu\left(\hat{z}_i^L\right)\right]}.$$

As $\kappa_i \to \hat{\kappa}$, it is $z_i^{sep} \to \hat{z}_i^H$. As this happens, the LHS of the last inequality approaches infinity, while the RHS remains finite. Then, there exists $\widetilde{\kappa} \in (\alpha, \hat{\kappa})$ such that, for $\kappa_i \in (\widetilde{\kappa}, \hat{\kappa})$, it is $d\mu_i/d\kappa_i > 0$, while for $\kappa_i > \hat{\kappa}$ it is $d\mu_i/d\kappa_i < 0$. The result follows immediately. Note that $\widetilde{\kappa}$ must be the same across industries, since κ_i is the only parameter that varies across industries. □

To make sense of the upward sloping portion of the curve, recall that this is driven by industries where the innovator is of a high type: in those industries, z_i^{sep} must increase as κ_i increases, to restore equality of payoffs between a genuine and a mimicking low type (given that the latter suffers more from a fall in profits). But as κ_i approaches $\hat{\kappa}$ and z_{sep} approaches $\hat{z}_i^H$, which is the maximum of the mimicker's net present value function, the gain to the mimicker from an increase in z_i^{sep} monotonically decreases to zero. It follows that, as κ_i approaches $\hat{\kappa}$, the increase in z_i^{sep} that follows from an increase in κ_i must grow unboundedly, as greater and greater increases are required to compensate the mimicker. In contrast, in industries where the innovator is of a low type, the decrease in $\hat{z}_i^L$ is always finite. In other words, as κ_i approaches $\hat{\kappa}$, the selection effect must always be stronger than the Schumpeterian effect.

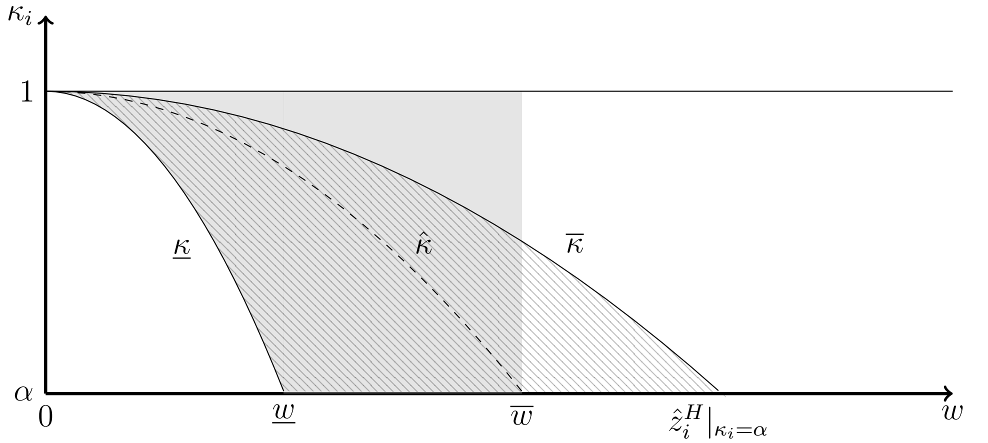

Figure 4.1: the striped area is where the separating equilibrium exists; the grey, shaded area is where the model can predict an inverted-U.

4 Discussion

In this section, we first identify the parameter sub-space where the separating equilibrium exists. Subsequently, we challenge the robustness of our results to a standard equilibrium refinement procedure.

4.1 Existence of the separating equilibrium

Figure 4.1 represents the $(\kappa_i, \kappa, \alpha)$ parameter space, by plotting κ_i on the vertical axis and $w = (1-\alpha)(\kappa\alpha)^{\frac{\alpha}{1-\alpha}}$ on the horizontal axis. Our comparative statics in this paper has consisted of increasing κ_i, for given w. However, we have tacitly focused on a central case ($\underline{w} < w < \overline{w}$ in the figure), while the remaining cases must also be considered.

The term $\hat{z}_i^H|_{\kappa_i=\alpha}$ represents optimal investment by the high types when the monopolist faces effectively no competition (it can charge price $1/\alpha$). It is the highest amount that the high types may ever want to invest. Then, if $w \geq \hat{z}_i^H|_{\kappa_i=\alpha}$, the high types can always finance their optimal investment purely out of equity contributions.[8] The last statement must also be true if

[8]This case must be considered, as there always exist admissible values of the other parameters of the model, γ and a^J, and admissible forms of the function $\mu(.)$, such that

$0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$ and κ_i is high enough, since a high κ_i pushes $\hat{z}_i^H$ down to zero, so that it is $w \geq \hat{z}_i^H$. This second case is represented by the area $\kappa_i \geq \overline{\kappa}$ in the figure, where $\overline{\kappa}$ is the unique value of κ_i such that $w = \hat{z}_i^H$, and is intuitively decreasing in w. In both cases, the separating equilibrium does not exist, if anything because the high types would never contribute w in equity. We show in the Online Appendix that, in a PBE, innovators always invest $\hat{z}_i^J$ in this area.

Consider next the area $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$, $\kappa < \overline{\kappa}$. The separating equilibrium must also not exist if both w and κ_i are very low, that is in the area $0 < w < \underline{w}$, $\alpha \leq \kappa_i < \underline{\kappa}$ in Figure 4.1, (the threshold $\overline{w}$ and $\underline{\kappa}$ are derived in the Online Appendix). To see why, note that $\hat{z}_i^H$ is much greater than w in this case. It follows that $\hat{z}_i^H$ must also be much greater than z_i^{sep}, or else the high types would be leveraging a lot at the separating equilibrium, and the low types would want to mimic them. In other words, there must be a large discrepancy between the high types' optimal investment, and the maximum they can invest by borrowing at their fair rate. But then the high types will prefer to pay an adverse selection premium, borrow more, and invest more. This point can be illustrated using the first panel of Figure 3.1: if w was very low, the maximum of the high types' net present value would not be z_i^{sep}, but rather a local maximum to the right of it.

In summary, the separating equilibrium does not exist outside of the striped area in Figure 4.1. We show in the Online Appendix that, in the striped area, it always exists. Note that this area does not perfectly overlap with the area where the model can predict an inverted-U relationship between competition and innovation in industries where the innovators is of a high type (the shaded area). This is for two reasons. First, in the area $w \geq \overline{w}$, $\kappa_i < \overline{\kappa}$, even if the separating equilibrium exists, the threshold $\hat{\kappa}$ does not, so that Propositions 1 and 2 do not hold. Intuitively, at such high wages, the high types can always invest $\hat{z}_i^H$ at the separating equilibrium, so that only the downward sloping portion of the relationship obtains. Second, in the area $0 < w < \overline{w}$, consider industries i and j such that $\hat{\kappa} \leq \kappa_i < \kappa_j \leq 1$. Suppose further that

$\hat{z}_i^H|_{\kappa_i=\alpha} \leq w$.

$\kappa_i < \overline{\kappa} \leq \kappa_j \leq 1$, or $\overline{\kappa} \leq \kappa_i < \kappa_j \leq 1$. While strictly speaking Propositions 1 and 2 do not apply to industry j, or i and j, since these industries cannot be at the separating equilibrium, their equilibrium investment must still be $\hat{z}_i^H$ and $\hat{z}_i^L$. So, it must still be true that innovation is higher in industry i than in industry j, and the logic of Proposition 1 and 2 carries through.

4.2 Equilibrium refinement

We refine beliefs using a standard, dominance-based criterion (see Mas-Colell et al. (1995), p. 469). Let action $a = (z, e)$ be *dominated* for type J, if there exists another action a' that gives them a strictly higher payoff, for any belief that the lenders might have in equilibrium. The refinement criterion requires that if an action is dominated for one type, but not for the other, then lenders must attach zero probability to the event that the former type undertakes that action (see the Online Appendix for details). We investigate the set of all possible PBE that survive this refinement in the area of existence of our separating equilibrium (the striped area in Figure 4.1).

This analysis leads to two main results. First, the refinement exactly dictates the beliefs that must be associated with certain actions. Most importantly, lenders must believe that only the high types would take actions of the type $\left(z \in [\hat{z}_i, z_i^{sep}), w\right)$ and $\left(z \in [\overline{z}_i^{sep}, \check{z}_i), w\right)$, where $\overline{z}_i^{sep} \geq \hat{z}_i^H$ denotes the second point at which the mimicker's payoff, $\widetilde{npv}_i^L(z)$, cuts through the payoff of the genuine low types,[9] and $\hat{z}_i \in [w, z_i^{sep})$ and $\check{z}_i > \overline{z}_i^{sep}$. This is because these actions are dominated for the low types - any action $\left(\hat{z}_i^L, e \leq \hat{z}_i^L\right)$ gives them a higher payoff, no matter what lenders believe in equilibrium - but not for the high types. The beliefs in our separating equilibrium must be changed slightly for the equilibrium to survive the refinement, however the equilibrium outcome does not change.[10]

[9] The existence of such point can be gauged from the top panel of Figure 3.1. The function $\widetilde{npv}_i^L(z)$ is a parabola reaching its maximum at $\hat{z}_i^H$. It must then cut through the horizontal line passing through $npv_i^L\left(\hat{z}_i^L\right)$ twice, to the left and to the right of $\hat{z}_i^H$.

[10] Beliefs must be changed in the following way. First, lenders must believe that those taking actions of the type $\left(z \in [\overline{z}_i^{sep}, \check{z}_i), w\right)$ must be high types. Second, they must believe the same for those taking actions of the type $\left(z \in (\hat{z}_i, z_i^{sep}), w - \epsilon\right)$ or $\left(z \in (\overline{z}_i^{sep}, \check{z}_i), w - \eta\right)$, where ϵ and η are small enough numbers. No other belief must be changed. Since to invest

Second, the above-described requirement on beliefs implies that the PBE *must* be a separating equilibrium in which the high types contribute w in equity, and invest either z_i^{sep} or $\overline{z}_i^{sep}$. Intuitively, these beliefs make it suboptimal for the high types to take any other action in a separating equilibrium. They also rule out the existence of a pooling equilibrium, for the same reason why our separating equilibrium exists: given the possibility to invest z_i^{sep} and be identified as high types, the high types prefer this to another action that would pool them together with the low types, even if that other action would allow them to invest more. Of course, this logic only works inside the area of existence of the separating equilibrium, where w (and thus z_i^{sep}) is high enough.

To invest $\overline{z}_i^{sep}$ gives the high types exactly the same payoff as to invest z_i^{sep}. Furthermore, the two thresholds behave in an exactly symmetric fashion. Then, $\overline{z}_i^{sep}$ is *decreasing* in κ_i. It follows that the main result of the paper needs to be qualified, since across industries where the innovator is of a high type, and invests $\overline{z}_i^{sep}$, the model still predicts a monotonic, decreasing relationship between competition and innovation. Of course, such a relationship is not due to the Schumpeterian effect, but to the effect of competition on credit constraints.

We think that, on balance, these results are good news for our theory. Most crucially, our key equilibrium outcome, z_i^{sep}, is one of only two which may realise in a refined PBE. And, while the existence of $\overline{z}_i^{sep}$ as an alternative equilibrium outcome makes it in principle harder for the model to predict an inverted-U, one may reasonably question whether such outcome will ever be observed. After all, while z_i^{sep} and $\overline{z}_i^{sep}$ give exactly the same payoffs to both lenders and borrowers, z_i^{sep} always implies a lower debt, and thus a lower expected size of default. If there was any additional cost from default, which increased with the size of the default, then z_i^{sep} would always be the preferred choice.

$\overline{z}_i^{sep}$ gives the high types exactly the same payoff as to invest z_i^{sep}, it is easy to see that the outcome of the equilibrium does not change.

5 Conclusions

We have shown that the observed inverted-U relationship between competition and innovation can be consistent with the original Schumpeterian model, if one allows for heterogeneous, and imperfectly observable, talent of innovators. When competition is low and innovation is profitable, investment by outsiders is likely to be governed by credit constraints. Then, an increase in competition may lead to a positive selection effect, increasing the rate of innovation. When competition is high, however, the low profitability of innovation makes it less likely for credit constraints to be important, and a decreasing relationship is to be expected.

Our main insight is that the Schumpeterian model can predict an inverted-U relationship between competition and innovation, even when innovation is carried out by outsiders. This is a context where, clearly, the escape competition effect, as described in this paper's introduction, would not apply. Our model has two additional empirical predictions. First, the positive relationship between competition and innovation should be more pronounced in industries where credit constraints are more prevalent. Second, the average level of credit constraints in these industries should be decreasing in the strength of competition. Given the plausibility or our assumptions and equilibrium, we believe these predictions provide a promising lead to future empirical research.

One key policy implication of our work is that, at least for low levels of competition, fostering competition is a substitute for reducing asymmetric information in financial markets. Since the government is unlikely to develop an informational advantage over private investors in the market for innovation, its efforts should focus on fostering competition. The alternative explanation of how an inverted-U between innovation and competition relationship occurs, by Aghion et al. (2005), is based on the dynamics of step by step innovation, and relies on the varying incentives of innovators based on how far advanced they are relative to others. These dynamics are also unlikely to be structurally affected by government policy. Hence, both explanations drive toward a similar conclusion: policy should foster competition up to a point, and in particular in industries that exhibit certain properties. However, there is a clear advantage for policy to focus on asymmetric information rather than differences in technological advancement. Differences in technological advancement are practically unobservable and must rely on unsatisfactory proxies such as patenting effort. Asymmetric information, on the other hand, leads to clear volatiliy in innovation outcomes in industries as a whole. By measuring whether that volatility become attenuated as a result of its policy efforts, the government can have a reasonable sense of whether its policy efforts are working.

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Appendix

Section A1 formally derives the equilibrium discussed in the main text, while Section A2 discusses its robustness.

A1. Derivations

This section is organised as follows. We begin, in Theorem 1, by showing that, if $\hat{z}_i^H|_{\kappa_i=\alpha} \leq w$, the two types must invest $\hat{z}_i^H$ and $\hat{z}_i^L$ in any Perfect Bayesian Equilibrium (PBE). Based on this result, Theorems 2-3 focus on the case $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$.

Theorem 2 defines the threshold $\overline{\kappa}$, it establishes its properties as a function of w, and it then shows that, if $\overline{\kappa} \leq \kappa_i \leq 1$, the two types must again invest $\hat{z}_i^H$ and $\hat{z}_i^L$ in any PBE. Based on this result, the theorem further restricts the focus to the case case $\alpha \leq \kappa < \overline{\kappa}$.

Theorem 3 begins by formally defining the separating equilibrium described in Section 3 (points a-d). Subsequently (points 1-3), it defines the threshold $\underline{\kappa}$, and shows that the separating equilibrium exists if and only if $\underline{\kappa} \leq \kappa_i < \overline{\kappa}$. Second, it shows that if $\overline{w} \leq w < \hat{z}_i^H|_{\kappa_i=\alpha}$, it is always $z_i^{sep} = \hat{z}_i^H$ in the area where the separating equilibrium exists.

Theorem 1. *If $\hat{z}_i^H|_{\kappa_i=\alpha} \leq w$, in any Perfect Bayesian Equilibrium (PBE), the two types invest respectively $\hat{z}_i^H$ and $\hat{z}_i^L$, any combination of equity and external financing being possible.*

Proof. Since the opportunity cost of equity financing is zero, the high types are always able to invest $\hat{z}_i^H$ using only personal wealth, and the minimum rate they can be offered on external financing is $1/\left[a^H\mu(z)\right]$, the high types would never select a research effort different from $\hat{z}_i^H$. Furthermore, they would never take on external financing at a rate greater than $1/\left[a^H\mu(z)\right]$. This last fact implies that a pooling equilibrium does not exist. As shown in footnote 7, at any separating equilibrium, the low types must select $\hat{z}_i^L$. Then, there only exists a separating equilibrium in which the two types invest $\hat{z}_i^H$ and $\hat{z}_i^L$ respectively. If an innovator borrows any money at such equilibrium, this must be at a rate $1/\left[a^H\mu\left(\hat{z}_i^H\right)\right]$ for the high types and $1/[a^L\mu\left(\hat{z}_i^L\right)]$ for the low types. Then, the innovator is indifferent as to the amount borrowed, and it is possible to construct an equilibrium with any combination of equity and external financing. □

Theorem 2. *If* $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$*, let*

$$\overline{\kappa} \equiv \arg\left[\hat{z}_i^H = w\right],$$

a threshold that continuously decreases from 1 *to* α *as* w *increases from* 0 *to* $\hat{z}_i^H|_{\kappa_i=\alpha}$*. Then, if* $\overline{\kappa} \leq \kappa_i \leq 1$*, in any PBE, the two types invest respectively* $\hat{z}_i^H$ *and* $\hat{z}_i^L$*, any combination of equity and external financing being possible.*

Proof. The properties of $\overline{\kappa}$ as a function of w follow from the fact that $\hat{z}_i^H$ is equal to $\hat{z}_i^H|_{\kappa_i=\alpha}$ for $\kappa_i = \alpha$, is continuously decreasing in κ, and is equal to 0 for for $\kappa_i = 1$. Then, for $w = 0$, it must be $\overline{\kappa} = 1$; $\overline{\kappa}$ must be continuously decreasing in w; and for $w = \hat{z}_i^H|_{\kappa_i=\alpha}$, it must be $\overline{\kappa} = \alpha$. The rest of the theorem can be shown in the same way as Theorem 1. □

Theorem 3. *If* $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$ *and* $\alpha \leq \kappa_i < \overline{\kappa}$*, consider the following situation:*

a. *Lenders believe that those who contribute* w *in equity and invest* $z \in (w, z_i^{sep}]$ *are high types, where* z_i^{sep} *is the minimum* $z > w$ *such that*

$$\widetilde{npv}_i^L(z) = npv_i^L(\hat{z}_i^L), \tag{5.1}$$

or, if such z *does not exist, then* $z_i^{sep} = \hat{z}_i^H$*. They also believe that those who contribute* w *in equity and invest* $z > z_i^{sep}$ *are high and low types with equal probability. Finally, they believe that everybody else are low types.*

b. *Lenders offer rate* $1/\left[a^H\mu(z)\right]$ *to the first group, rate* $1/\left[a\mu(z)\right]$ *to the second, and rate* $1/[a^L\mu(z)]$ *to the third.*

c. *The low types invest* $\hat{z}_i^L$ *(any combination of equity and external financing being possible).*

d. *The high types invest* z_i^{sep} *(contributing* w *in equity).*

Then, there exists $\underline{w}$ *and* $\overline{w}$*, with* $0 < \underline{w} < \overline{w} < \hat{z}_i^H|_{\kappa_i=\alpha}$*, such that:*

1. *If* $\overline{w} \leq w < \hat{z}_i^H|_{\kappa_i=\alpha}$*, situation a-d is a PBE. It is* $z_i^{sep} = \hat{z}_i^H$*.*

2. *If* $\underline{w} \leq w < \overline{w}$*, situation a-d is a PBE. There exists* $\hat{\kappa} \in (\alpha, \overline{\kappa})$ *such that it is* $z_i^{sep} < \hat{z}_i^H$ *for* $\kappa_i \in [\alpha, \hat{\kappa})$*, and* $z_i^{sep} = \hat{z}_i^H$ *for* $\kappa_i \in [\hat{\kappa}, \overline{\kappa})$*.*

3. *If* $0 < w < \underline{w}$*, point 2 is still true, except that there exists* $\underline{\kappa} \in (\alpha, \hat{\kappa})$ *such that situation a-d is not a PBE if* $\kappa_i \in [\alpha, \underline{\kappa})$*.*

Proof. **I. (Preliminary step). Situation a-d is a PBE if and only if**

$$npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z) \quad \forall z > z_i^{sep}. \tag{5.2}$$

To show this, we proceed in two sub-steps. **I.i.** *If condition 5.2 does not hold, then situation a-d is not a PBE.* This follows from the fact that the high types have a profitable deviation, since they can contribute w in equity and invest some $z > z_i^{sep}$, and obtain a higher payoff. **I.ii.** *If condition (5.2) holds, situation a-d is a PBE.* This follows from the fact that the following three facts hold true. First, for every action that borrowers could play, the lenders' action is optimal given their beliefs. Second, for actions that borrowers play in equilibrium, the lenders' beliefs are correct. Third, borrowers do not have a profitable deviation. To see the last point, let $\succsim^J$ represent type J's preferences, and let $\left(z^J, e^J\right)$ represent type J's investment profile (where e^J denotes the innovator's equity contribution). Consider first the high types. Their equilibrium action, (z_i^{sep}, w), gives payoff $npv_i^H(z_i^{sep})$. We want to show that $(z_i^{sep}, w) \succsim^H (z, e)$ for any feasible (z, e). This follows from the fact that, if

$z < z_i^{sep}$, the high types can at best obtain payoff $npv_i^H(z)$. But $z < z_i^{sep} \leq \hat{z}_i^H$ implies $npv_i^H(z) < npv_i^H(z_i^{sep})$. If $z = z^{sep}$, the only way in which (z, e) may differ from (z_i^{sep}, w) is if $e < w$. But by deviating in this way, the high types are identified as low types, and receive payoff $npv_i^H(z_i^{sep}) - \left(a^H/a^L - 1\right)[z - e] < npv_i^H(z_i^{sep})$. Finally, if $z > z^{sep}$, the high types can at best obtain payoff $\widehat{npv}_i^H(z)$, but it is $\widehat{npv}_i^H(z) \leq npv_i^H(z_i^{sep})$ by condition (5.2). Next, consider the low types. Their equilibrium action, $(\hat{z}_i^L, e^H)$, where $e^H \in \left[0, \hat{z}_i^L\right]$, gives payoff $npv_i^L(\hat{z}_i^L)$. We want to show that $(\hat{z}_i^L, e^H) \succsim^L (z, e)$ for any feasible (z, e). This follows from the fact that, if $e < w$, or if $z \leq w$, or if both conditions hold, the low types obtain payoff $npv_i^L(z) \leq npv_i^L(\hat{z}_i^L)$. If $e = w$, and $z \in (w, z_i^{sep}]$, the low types obtain payoff $\widetilde{npv}_i^L(z)$, and, by definition of z_i^{sep}, $\widetilde{npv}_i^L(z) < npv_i^L(\hat{z}_i^L)$. If $e = w$, and $z > z_i^{sep}$, the low types receive payoff $\widehat{npv}_i^L(z)$. But condition (5.2) must hold for z. Multiplying both sides of it by a^L/a^H, we obtain

$$a^L \mu(\hat{z}_i^{sep}) \gamma \pi\left(\kappa_i\right) - \frac{a^L}{a^H} \hat{z}_i^{sep} \geq a^L \mu(z) \gamma \pi - \frac{a^L}{a}[z - w] - \frac{a^L}{a^H} w,$$

which subtracting $\left[1 - \left(a^L/a^H\right)\right] w$ from both sides becomes $\widetilde{npv}_i^L\left(\hat{z}_i^{sep}\right) \geq \widehat{npv}_i^L(z)$, or, by the definition of z_i^{sep}, $npv_i^L\left(\hat{z}_i^L\right) \geq \widehat{npv}_i^L(z)$.

II. (Preliminary step). There exist $\underline{w}$ and $\overline{w}$, with $\hat{z}_i^L|_{\kappa_i = \alpha} < \underline{w} < \overline{w} < \hat{z}_i^H|_{\kappa_i = \alpha}$, such that, if $\kappa_i = \alpha$, it is $z_i^{sep} < \hat{z}_i^H$ if $w < \overline{w}$, $z_i^{sep} = \hat{z}_i^H$ otherwise; and condition (5.2) holds if and only if $w \geq \underline{w}$. We show these two points in two separate sub-steps. **II.i.** *There exists $\overline{w}$, with $\hat{z}_i^L|_{\kappa_i = \alpha} < \overline{w} < \hat{z}_i^H|_{\kappa_i = \alpha}$, such that, if $\kappa_i = \alpha$, it is $z_i^{sep} < \hat{z}_i^H$ if $w < \overline{w}$, $z_i^{sep} = \hat{z}_i^H$ otherwise.* Suppose $\kappa_i = \alpha$. Recall the definition of z_i^{sep}, provided at part a) of the Theorem. Note that the function $\widetilde{npv}_i^L(z)$ is decreasing in w. Given $w < \hat{z}_i^H|_{\kappa_i = \alpha}$ and $\kappa_i = \alpha$, by Lemma 2, it is $w < \hat{z}_i^H$. Then, the function $\widetilde{npv}_i^L(z)$ (which is only defined for $z > w$), is concave, reaches a maximum at $\hat{z}_i^H > w$, and turns negative for z large enough. As for $npv_i^L(\hat{z}_i^L)$, it is positive and constant in both w and z. It is easy to see that, if $w = \hat{z}_i^L$, it is $\widetilde{npv}^L(\hat{z}_i^L) = npv_i^L(\hat{z}_i^L)$, implying $\widetilde{npv}^L(\hat{z}_i^H) > npv_i^L(\hat{z}_i^L)$. Furthermore, for $w \to \hat{z}_i^H$, it is $\widetilde{npv}^L(\hat{z}_i^H) \to npv_i^L(\hat{z}_i^H) < npv_i^L(\hat{z}_i^L)$. Then, there exists $\overline{w}$, with $\hat{z}_i^L|_{\kappa_i=\alpha} < \overline{w} < \hat{z}_i^H|_{\kappa_i=\alpha}$, such that, if $w < \overline{w}$, equation (5.1) admits two solutions z_i^{sep} and $\overline{z}_i^{sep}$, with $0 < z_i^{sep} < \hat{z}_i^H < \overline{z}_i^{sep} < \infty$; if $w = \overline{w}$, it admits only one solution $z_i^{sep} = \hat{z}_i^H$; and if $w > \overline{w}$, it admits no solutions (which, by definition, still implies $z_i^{sep} = \hat{z}_i^H$). It is also the case that it is $z_i^{sep} = \hat{z}_i^L$ for $w = \hat{z}_i^L$, and $z_i^{sep} > w$ and increasing in w for $w \in \left[\hat{z}_i^L, \overline{w}\right)$. **II.ii.** *There exists $\underline{w}$, with $\hat{z}_i^L|_{\kappa_i=\alpha} < \underline{w} < \overline{w}$, such that, if $\kappa_i = \alpha$, condition (5.2) holds if and only if $w \geq \underline{w}$.* Suppose $\kappa_i = \alpha$. The function $\widehat{npv}_i^H(z)$ is concave and maximum for $\hat{z}_i^{pool} = \arg\left[a\mu'(z)\gamma\pi(\kappa_i) = 1\right]$, and $\hat{z}_i^{pool} \in \left(\hat{z}_i^L, \hat{z}_i^H\right)$. Then, from results in Step II.i, there exists $\hat{w} \in \left(\hat{z}_i^L, \overline{w}\right)$ such that $z_i^{sep} \geq \hat{z}_i^{pool}$ iff $w \geq \hat{w}$. In such a case, a sufficient condition for (5.2) to hold is $npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z_i^{sep})$, which is always true. If $w < \hat{w}$, a necessary and sufficient condition for (5.2) to hold is

$$npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(\hat{z}_i^{pool}). \tag{5.3}$$

There exists $\underline{w}$, with $\hat{z}_i^L|_{\kappa_i=\alpha} < \underline{w} < \hat{w}$ such that (5.3) holds iff $w \in [\underline{w}, \hat{w})$. This can be shown in two steps. First, note that expression $npv_i^H(z_i^{sep}) - \widehat{npv}_i^H(\hat{z}_i^{pool})$ is continuously increasing in w for $w \in (0, \overline{w})$. To see this, start from condition $\widetilde{npv}^L(z_i^{sep}) = npv_i^L(\hat{z}_i^L)$. Multiplying both sides by a^H/a^L and re-arranging, this can re-written as

$$a^H\mu(z_i^{sep})\gamma\pi(\kappa_i) - z_i^{sep} = a^H\mu(\hat{z}_i^L)\gamma\pi(\kappa_i) - \frac{a^H}{a^L}\hat{z}_i^L + \frac{a^H - a^L}{a^H}w, \tag{5.4}$$

where the LHS is equal to $npv_i^H(z_i^{sep})$. Then, expression $npv_i^H(z_i^{sep}) - \widehat{npv}_i^H(\hat{z}_i^{pool})$ can be written as

$$a^H\mu(\hat{z}_i^L)\gamma\pi(\kappa_i) - \frac{a^H}{a^L}\hat{z}_i^L + \frac{a^H - a^L}{a^L}w - \left[a^H\mu(\hat{z}_i^{pool})\gamma\pi(\kappa_i) - \frac{a^H}{a}\hat{z}_i^{pool} + \frac{a^H - a}{a}w\right], \tag{5.5}$$

which is increasing in w (note that $\hat{z}_i^{pool}$ does not depend on w). Second, note that expression $npv_i^H(z_i^{sep}) - \widehat{npv}_i^H(\hat{z}_i^{pool})$ is negative for $w = \hat{z}_i^L$, positive for $w = \hat{w}$. The latter follows from earlier discussion; to see the former, recall that, by Step II.i, it is $z_i^{sep} = \hat{z}_i^L$ for $w = \hat{z}_i^L$. Then, it is

$$\widehat{npv}_i^H(\hat{z}_i^{pool}) > \widehat{npv}_i^H(z_i^{sep}) = \widehat{npv}_i^H(w) = npv_i^H(w) = npv_i^H(z_i^{sep}).$$

III. Point 2 in the Theorem. Suppose $\underline{w} \leq w < \overline{w}$. **III.i.** *If* $\kappa_i \in [\alpha, \overline{\kappa})$, *situation a-d constitutes a PBE.* From Lemma 2, it is $\overline{\kappa} \in (\alpha, 1)$. If $\kappa_i = \alpha$, by Step II, condition (5.2) holds. But the condition also holds for $\kappa_i \in (\alpha, \overline{\kappa})$, which by Step I proves the result. To see this, consider two cases. First, if $z_i^{sep} \geq \hat{z}_i^{pool}$ for $\kappa_i = \alpha$, then such inequality also holds for $\kappa_i \in (\alpha, \overline{\kappa})$. This is because, $\hat{z}_i^{pool}$ is decreasing in κ, while z_i^{sep} is either increasing or equal to $\hat{z}_i^H > \hat{z}_i^{pool}$. But $z_i^{sep} > \hat{z}_i^{pool}$ implies that a sufficient condition for (5.2) to hold is $npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z_i^{sep})$, which is always true. Second, if $z_i^{sep} < \hat{z}_i^{pool}$ for $\kappa_i = \alpha$, there exists $\check{\kappa}$ such that this inequality also holds for $\kappa_i \in (\alpha, \check{\kappa})$, while it is $z_i^{sep} \geq \hat{z}_i^{pool}$ for $\kappa_i \in (\check{\kappa}, \overline{\kappa})$. This follows from the fact that z_i^{pool} is decreasing in κ_i, while z_i^{sep} is increasing and reaches $\hat{z}_i^H > \hat{z}_i^{pool}$ for some $\kappa_i < \overline{\kappa}$. In the first region, that condition (5.2) follows from the fact that it does so for $\kappa_i = \alpha$, and expression (5.5) is increasing in κ_i. In the second region, it follows from the fact that a sufficient condition for (5.2) to hold is $npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z_i^{sep})$, which is always true. **III.ii.** *If* $\kappa_i \in [\alpha, \overline{\kappa})$, *there exists* $\hat{\kappa} \in (\alpha, \overline{\kappa})$ *such that it is* $z_i^{sep} < \hat{z}_i^H$ *for* $\kappa_i \in [\alpha, \hat{\kappa})$, *and* $z_i^{sep} = \hat{z}_i^H$ *for* $\kappa_i \in [\hat{\kappa}, \overline{\kappa})$. Given $\kappa_i < \overline{\kappa}$, by Lemma 2, it is $w < \hat{z}_i^H$. The function $\widetilde{npv}_i^L(z)$ (which is only defined for $z > w$) is concave in z, reaches a maximum at $\hat{z}_i^H > w$, and turns negative for z large enough. At the same time, given $w < \overline{w}$ and $w \geq \underline{w} > \hat{z}_i^L$, by step II.i, if $\kappa_i = \alpha$, equation (5.1) admits two solutions z_i^{sep} and $\overline{z}_i^{sep}$, with $0 < w < z_i^{sep} < \hat{z}_i^H < \overline{z}_i^{sep} < \infty$. But note that $\hat{z}_i^H$ is decreasing in κ_i and, as shown in the proof to Proposition 1, z_i^{sep} is increasing and $\widetilde{npv}_L(z_i^{sep}) - npv_L\left(\hat{z}_i^L\right)$ is decreasing in κ_i (this can be seen by re-arranging equation 3.3). Furthermore, for $\kappa \to \overline{\kappa}$, it is $w \leftarrow \hat{z}_i^H$, which by a result in step II.i implies that $\widetilde{npv}_L(z_i^{sep}) - npv_L\left(\hat{z}_i^L\right)$ converges to $\widetilde{npv}_L\left(\hat{z}_i^H\right) - npv_L\left(\hat{z}_i^L\right) < 0$. The result follows.

IV. Point 1 in the Theorem. Suppose $\overline{w} \leq w < \hat{z}_i^H|_{\kappa_i=\alpha}$. Step III.i still holds, with the simplification that, given $w > \overline{w} > \hat{w}$, by a result in Step II.ii, for $\kappa_i = \alpha$, we only need to consider the case $z_i^{sep} > \hat{z}_i^{pool}$. Step III.iii also still holds. Finally, given $w \geq \overline{w}$, by Step II, if $\kappa_i = \alpha$, it is $z_i^{sep} = \hat{z}_i^H$. Furthermore, by step III.i, $npv_i^L\left(\hat{z}_i^L\right) - \widetilde{npv}_i^L\left(\hat{z}_i^H\right)$ is decreasing in κ_i. It follows that it is $z_i^{sep} = \hat{z}_i^H$ for all $\kappa_i \in [\alpha, \overline{\kappa})$.

V. Point 3 in the Theorem. Suppose $0 < w < \underline{w}$. Steps III.ii and III.iii still hold. By Step II, if $\kappa_i = \alpha$, condition (5.2) does not hold. Furthermore, given $w < \underline{w} < \hat{w}$, by a result in Step II.ii, if $\kappa_i = \alpha$, it is $z_i^{sep} < \hat{z}_i^{pool}$. There exists $\check{\kappa} \in (\alpha, \hat{\kappa})$ such that the last inequality also holds for $\kappa_i \in (\alpha, \check{\kappa})$, while it is $z_i^{sep} \geq \hat{z}_i^{pool}$ for $\kappa_i \in (\check{\kappa}, \overline{\kappa})$. This follows from the fact that z_i^{pool} is decreasing in κ, while z_i^{sep} is increasing and equal to $\hat{z}_i^H > \hat{z}_i^{pool}$ for $\kappa_i = \hat{\kappa}$. There then exists $\underline{\kappa} \in (\alpha, \check{\kappa})$ such that condition (5.2) does not hold for $\kappa_i \in [\alpha, \underline{\kappa})$, while it holds for $\kappa_i \geq \underline{\kappa}$. This follows from the fact that the condition does not hold for $\kappa_i = \alpha$, that expression (5.5) is increasing in κ_i, and that condition (5.2) holds for $z_i^{sep} \geq \hat{z}_i^{pool}$. It follows that, by Step I, situation a-d is not a PBE if $\kappa_i \in [\alpha, \underline{\kappa})$. Otherwise, Step III.i still applies, replacing α with $\underline{\kappa}$ everywhere. □

A2. Equilibrium refinement

This section follows closely the discussion in Mas-Colell et al. (1995), p. 469. We use to the second (and second-weakest) form of domination-based refinement discussed in the textbook (Eq. 13.AA.2). Let $J \in \overline{J} = \{H, J\}$ denote the type of the innovator. Let $a \in A = \{(z, e) : z \geq 0, 0 \leq e \leq z\}$ denote the choice of investment and equity contribution made by the innovator. Let $\pi\left(J|a\right)$ denote the probability that lenders assign to the innovator being of type J, conditional on observing action $a \in A$, and let $r \in R = \{r : r \geq 1\}$ be the interest rate that they require. Let $u\left(a, r, J\right)$ denote the expected payoff to an innovator of type J.

We will say that action a is strictly dominated for type J if there exists another action a' with

$$\min_{r \in [1/[a^H \mu(z')], 1/[a^L \mu(z')]]} u\left(a', r, J\right) > \max_{r \in [1/[a^H \mu(z)], 1/[a^L \mu(z)]]} u\left(a, r, J\right). \tag{5.6}$$

Define the set $\overline{J}^{*}(a) \subseteq \overline{J}$ as

$$\overline{J}^{*}(a) = \{J : \text{there is no } a' \in A \text{ satisfying } (5.6)\}.$$

Our definition of a PBE with reasonable beliefs is as follows:

Definition 1. *A PBE has reasonable beliefs if for all $a \in A$ with $\overline{J}^{*}(a) \neq \varnothing$, $\mu(J|a) > 0$ only if $J \in \overline{J}^{*}(a)$.*

In other words, if an action is dominated for type J, and for type J only, then beliefs are said to be reasonable if and only if lenders attach a zero probability to the event that someone taking action a is of type J.

We are now ready to present our refinement result:

Theorem 4. *If $0 < w < \hat{z}_i^H|_{\kappa_i=\alpha}$ and $\underline{\kappa} < \kappa_i < \overline{\kappa}$, let z_i^{sep} and $\overline{z}_i^{sep}$ be the minimum and maximum $z > w$ such that*

$$\widetilde{npv}_i^L(z) = npv_i^L(\hat{z}_i^L), \tag{5.7}$$

or, if such z is unique or does not exist, then $z_i^{sep} = \overline{z}_i^{sep} = \hat{z}_i^H$. Then, any PBE that has reasonable beliefs in the sense of Definition 1 is a separating equilibrium where the low types invest $\hat{z}_i^L$ (any contribution of equity and external financing being possible), and the high types invest either z_i^{sep} or $\overline{z}_i^{sep}$ (contributing w in equity).

Proof. There exist $\hat{z}_i \in [w, z_i^{sep})$ and $\check{z}_i > \overline{z}_i^{sep}$ such that, at any PBE that has reasonable beliefs in the sense of Definition 1, for any $a = (z, w)$ such that $z \in (\hat{z}_i, z_i^{sep}) \cup (\overline{z}_i^{sep}, \check{z}_i)$, lenders must believe $\mu(L|a) = 0$. To see this, note that there exists $a' = \left(\hat{z}_i^L, e\right)$, with $e \leq \hat{z}_i^L$, such that

$$\min_{r \in \left[1/\left[a^H \mu\left(\hat{z}_i^L\right)\right], 1/\left[a^L \mu\left(\hat{z}_i^L\right)\right]\right]} u(a', r, L) = npv_i^L\left(\hat{z}_i^L\right) > \tag{5.8}$$

$$\widetilde{npv}_i^L(z_i) = \max_{r \in [1/[a^H \mu(z_i)], 1/[a^L \mu(z_i)]]} u(a, r, L), \tag{5.9}$$

where the inequality follows from the definition of z_i^{sep} and $\overline{z}_i^{sep}$.

A PBE that has reasonable beliefs in the sense of Definition 1 cannot be a pooling equilibrium. To see this, proceed by contradiction. Suppose the PBE was a pooling equilibrium, and let $\left(z_i^{pool}, e_i^{pool}\right)$ be the action taken by both types in equilibrium. Distinguish two cases. If $z_i^{pool} > z_i^{sep}$, then the payoff to the high types would be

$$\begin{aligned} a^H \mu\left(z_i^{pool}\right)\gamma\pi\left(\kappa_i\right) - \frac{a^H}{a}\left(z_i^{pool} - e_i^{pool}\right) - e_i^{pool} &\leq a^H \mu\left(z_i^{pool}\right)\gamma\pi\left(\kappa_i\right) - \frac{a^H}{a}\left(z_i^{pool} - w\right) - w \\ &= \widehat{npv}_i^H(z_i^{pool}) \\ &< npv_i^H(z_i^{sep} - \epsilon), \end{aligned}$$

where ϵ is a small enough number. The last inequality follows from Theorem 3 and from continuity: since situation a-d is a PBE in this parameter subspace, it must be $npv_i^H(z_i^{sep}) \geq \widehat{npv}_i^H(z)$, and thus $npv_i^H(z_i^{sep} - \epsilon) \geq \widehat{npv}_i^H(z)$, $\forall z > z_i^{sep}$ (where the strict inequality follows from the fact that we have assumed $\kappa > \underline{\kappa}$ instead of $\kappa \geq \underline{\kappa}$). So, the high types could increase their payoff by choosing $(z_i^{sep} - \epsilon, w)$. If $z_i^{pool} = z_i^{sep}$, then the payoff to the high types would be

$$a^H \mu\left(z_i^{pool}\right)\gamma\pi\left(\kappa_i\right) - \frac{a^H}{a}\left(z_i^{pool} - e_i^{pool}\right) - e_i^{pool} < npv_i^H(z_i^{sep} - \epsilon),$$

where the inequality follows from continuity, given ϵ is low enough. Again, the high types could increase their payoff by choosing $(z_i^{sep} - \epsilon, w)$. Finally, if $0 \leq z_i^{pool} < z_i^{sep}$, then the payoff to the high types would be

$$\begin{aligned} a^H \mu\left(z_i^{pool}\right)\gamma\pi\left(\kappa_i\right) - \frac{a^H}{a}\left(z_i^{pool} - e_i^{pool}\right) - e_i^{pool} &\leq a^H \mu\left(z_i^{pool}\right)\gamma\pi\left(\kappa_i\right) - z_i^{pool} \\ &= npv_i^H(z_i^{pool}) \\ &< npv_i^H(z_i^{sep} - \epsilon), \end{aligned}$$

where the second inequality follows from the fact that $0 \leq z_i^{pool} < z_i^{sep} - \epsilon \leq \hat{z}_i^H$ for ϵ low enough. Once again, the high types could increase their payoff by choosing $(z_i^{sep} - \epsilon, w)$.

Let $a = (z, e)$ be such that either $z \in \{z_i^{sep}, \overline{z}_i^{sep}\}$ and $e < w$, or $z \in$

$(z_i^{sep}, \overline{z}_i^{sep})$ and $e_i \leq w$. Then, at any separating equilibrium, it must be $\pi(H|a) < 1$. To see this, proceed by contradiction. Suppose it was $\pi(H|a) = 1$. Then, the low types could take action a, obtaining payoff $\widetilde{npv}_i^L(z) + \frac{a^H - a^L}{a^H}(w - e) > npv_i^L(\hat{z}_i^L)$. But since, by footnote 7, the low types must be taking action $(\hat{z}_i^L, e_i^L)$ (with $e_i^L \leq \hat{z}_i^L$) in a separating equilibrium, obtaining payoff $npv_i^L(\hat{z}_i^L)$, they would have a profitable deviation, contradicting the notion that this is a PBE.

The Theorem now follows. To see this, note that it was shown in footnote 7 that, in a separating equilibrium, the low types must be taking action $(\hat{z}_i^L, e)$, with $e \leq \hat{z}_i^L$. As for the high types, they could not take an action (z, e) such that either $z \in \{z_i^{sep}, \overline{z}_i^{sep}\}$ and $e < w$, or $z \in (z_i^{sep}, \overline{z}_i^{sep})$ and $e \leq w$, since if they did, by step III, the lenders' beliefs would be incorrect. At the same time, they could not take an action such that $z < z_i^{sep}$, since their payoff would at best be $npv_i^H(z)$, and it would always be possible to find $\epsilon > 0$ small enough so that $z < z_i^{sep} - \epsilon$. Since $npv_i^H(z) < npv_i^H(z_i^{sep} - \epsilon)$, the high types could then increase their payoff by choosing $(z_i^{sep} - \epsilon, w)$. Finally, by a symmetric logic, the high types could not be choosing an action such that $z > \overline{z}_i^{sep}$. It follows that the high types must either take action (z_i^{sep}, w) in a PBE, or action $(\overline{z}_i^{sep}, w)$. □