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Optimal Stabilization in an Emission Permits Market

Abstract

We develop a 2-period emission trading model for a stock pollutant with demand shocks resolving over time. We find precise conditions for efficiency of a stabilization mechanism where cumulative available permits decrease with excess supply in early periods. Our model describes the stabilization rule, and identifies optimal parameters. The market stability mechanism substantially increases welfare, increases the domain of parameter values where (Stabilized) Banking outperforms Prices, and reduces price volatility. Our findings are important for emission trading schemes worldwide, such as California's Global Warming Solutions Act Scoping Plan, the U.S. Regional Greenhouse Gas Initiative, EU-ETS, and China's National ETS, the world's largest carbon market.

JEL-Codes: H230, Q540, Q580.

Keywords: prices, quantities, emission trading, regulatory instruments, pollution, climate change.

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Comments appreciated.

1 Introduction

This paper studies optimal regulation of a stock pollutant under asymmetric information with information updating over time. We build a dynamic model where damages are caused by the stock of accumulated emissions, as in Hoel and Karp (2001, 2002); Newell and Pizer (2003); Fell et al. (2012). Several important global environmental problems are best described as resulting from a stock pollutant – there is, for instance, near-consensus that climate change is caused by the stock of atmospheric CO_2 (Stocker, 2014), and though there is depreciation of atmospheric CO2, the intricate carbon cycle and temperature feedbacks lead to cumulative emissions standing out as the best predictor of expected temperature rise (Allen et al., 2009).[1] Our analysis therefore complements a relatively larger literature on optimal regulation of flow pollutants suited for other emission problems such as local air pollution (e.g. Weitzman, 1974; Roberts and Spence, 1976; Cronshaw and Kruse, 1996; Rubin, 1996; Kling and Rubin, 1997; Yates and Cronshaw, 2001; Newell et al., 2005; Weitzman, 2018; Lintunen and Kuusela, 2018).[2]

The main complexity in analyzing a stock pollutant stems from its dynamic nature, which often renders the formal analysis convoluted or even unsolvable, manifested in a reliance on quantitative methods. We therefore study a 2-period model, enabling us to derive relatively simple analytic results that still capture the profoundly dynamic nature of stock pollutant regulation. Confined to this fairly limited time-frame, we can transcend analysis of the static instruments traditionally considered by, in addition, studying intertemporally integrated policy instruments, such as banking, where firms can save unused permits to be used in the future, or alternatively can give up future emission rights in order to emit more today (Cronshaw and Kruse, 1996; Rubin, 1996; Kling and Rubin, 1997; Yates and Cronshaw, 2001; Weitzman, 2018; Lintunen and Kuusela, 2018). Banking can then be compared with essentially static instruments such as pricing and fixed per-period quantities.

While our first contribution is that we consider a stock-pollutant rather than a flow pollutant, the second contribution is that we add imperfect foresight of future shocks on the side of the firms to the base case of perfect foresight. The intertemporal approach we

[1] If one assumes that atmospheric CO_2 directly causes damages, without considering delayed temperature adjustment (c.f. Newell and Pizer, 2003; Golosov et al., 2014), one (mistakenly) concludes that effects of current emissions fade over time. Such an approximation can lead to quantitatively substantial deviations in calculated optimal policies; see Dietz and Venmans (2017) and (Gerlagh and Liski, 2018, Fig 1) for a discussion.

[2] Although it need not be a universally applicable rule, generally we find flow pollutants to be specific both to a temporal and a geographical location. Conversely, stock pollutants, by being persistent over time, tend to be diffuse and are more spread both temporally and geographically.

follow warrants deliberation along such lines: whilst assuming that firms know perfectly the realization of shocks that occur at the present time may be roughly correct, it is certainly much less credible when we consider shocks yet to be realized in the future. This holds true almost by definition of a shock, but even absent linguistic considerations it is fairly obvious that we know more about the world today than about its fate tomorrow. Nonetheless, this fact has been ignored by most of the papers studying a multi-period setting in the literature (Cronshaw and Kruse, 1996; Rubin, 1996; Kling and Rubin, 1997; Yates and Cronshaw, 2001; Newell et al., 2005; Weitzman, 2018). Notable exceptions are Newell and Pizer (2003), Fell et al. (2012), and Lintunen and Kuusela (2018), who explicitly take into account uncertainty of future shock realizations.

Optimal regulation when the regulator is uncertain about some parameters relevant to the firms has a flavor of mechanism design. That literature searches for regulation rules (contract offers) that implement the socially optimal outcome (Baron and Myerson, 1982). It requires that the regulator is unconstrained by any pre-imposed structure to achieve its goal, and efficient regulation can become increasingly complex (for a more in-depth discussion of mechanism design, see Mas-Colell et al., 1995; Hurwicz and Reiter, 2006). We start at the other end of the spectrum. The literature on emission control focuses on very simple highly structured instruments – prices, quantities, banking – accepting suboptimal ex-post outcomes for the benefit of policy feasibility. We add to the asymmetry in information that is common in the emission permit market literature an alternative environment in which both regulator and regulated firms share uncertainty about future emission permit demand. As we will see, optimal policies in such an environment deviate substantially from the optimal policies in an economy where firms perfectly anticipate future demand shocks.

Our third contribution to the literature is that we add a refinement of the banking instrument, labeled Stabilized Banking to the three most common instruments – Quantities, Prices, and Banking. We introduce an automated quota updating into the basic banking instrument, as an optimal stabilization mechanism. We show that a marginally more complex regulation can achieve substantially higher ex-ante welfare compared to the traditional instruments. We thereby contribute to a literature on banking refinements. Roberts and Spence (1976) combine a quantity policy with price floors and ceilings, which, if properly implemented, is shown to achieve higher efficiency than quantities or prices in isolation. Kling and Rubin (1997) and Newell et al. (2005) propose that the regulator depreciates or tops up banked permits, similar to the financial bank setting its interest rate on loans and deposits. Similarly, Yates and Cronshaw (2001) consider banking with a discount rate for permits; they identify conditions under which the optimal discount rate is non-zero. Newell et al. (2005) discuss adjusting quota in response to the quantity of outstanding permits. The regulator's response can stabilize permit prices over time and mimic price-based policies. Their idea is conceptually close to ours, and their motivation similar. Yet, their model is fundamentally different as they consider a flow pollutant, while we consider a stock pollutant, such as CO_2. Lintunen and Kuusela (2018) search for an optimal Markov rule where the regulator chooses a new emissions cap every period in response to the amount of outstanding allowances banked for future use. They find a relatively simple rule that, as in Roberts and Spence (1976), combines characteristics of both price and quantity instruments. The other papers in this literature generally consider only the traditional instruments of prices, quantities, and banking.

Though highly stylized, our model yields important policy implications for greenhouse gas emissions trading systems (ETSs) worldwide, such as Europe's EU-ETS, California's Global Warming Solutions Act[3], the Regional Greenhouse Gas Initiative (RGGI)[4], and China's National ETS, the world's largest carbon market.[5] Recent developments in Europe underscore its relevance. After a start with volatile and 'low' prices, the European Commission decided early in 2015 to revise the EU-ETS, introducing a 'Market Stability Reserve' (MSR), to be operative as of 2019 (Erbach, 2017). This was motivated by the large surplus – the cumulative gap between planned auctioned allowances and allowances surrendered by emitting firms – of above 2 billion tCO_2 built up since 2009.[6] The MSR

[3]California's Global Warming Solutions Act was created in Assembly Bill 32, this plan constitutes a comprehensive, multi-year program to reduce greenhouse gas (GHG) emissions in California. In its most recent update of 2016, the plan codifies a 2030 GHG emissions reduction target of 40 percent below 1990 levels. The California Air Resources Board is charged with developing a Scoping Plan that describes the approach California will take to achieve this goal.

[4]RGGI is the first mandatory market-based program in the United States to reduce greenhouse gas emissions. It constitutes cooperation between the states of Connecticut, Delaware, Maine, Maryland, Massachusetts, New Hampshire, New York, Rhode Island, and Vermont, whose joint effort aims to cap and reduce CO2 emissions from the power sector.

[5]Launched on 19 December 2017, China's National ETS is now the world's largest carbon market, regulating some 1,700 companies in the power sector that each emitted around 26,000 t/CO_2 in any year over the 2013-2015 period. In its initial phase, China's National ETS covers more than 3 billion tons of CO_2, some 30% of national emissions, making it the largest emission trading system currently in existence. By 2020, China's National ETS is aimed to have achieved 40-45% reductions in carbon intensity compared to 2005 levels; by 2030 CO_2 emissions should peak, with best efforts to achieve the peak earlier. Although for now the system covers only the power sector, over time it is planned to be expanded to also cover the following industries: petrochemical, chemical, building materials, steel, nonferrous metals, paper, and aviation.

[6]To assess the economic importance of this surplus, note that permit prices in the EU are currently about 7€/tCO_2, so that the monetary value of the surplus amounts to some €14 billion. However, these prices are widely perceived as too low. Optimal prices, calculated in e.g. Nordhaus (2014), Golosov et al. (2014), and Gerlagh and Liski (2018), are in the range of $25 - 40$€/tCO_2, so that the value of the surplus

takes allowances out of the market, to bring them back into the market at a later stage when private holdings of unused permits has decreased.[7] The rules set out imply a rate of transfer from the private permits surplus to the MSR that varies between 12 and 24%. In November 2017, a further revision was announced. By 2024, permits in the MSR will be canceled when its level exceeds the amount of auctioned permits in the previous year (Erbach, 2017). Combining the rules, we see the ultimate effect: a positive (negative) demand shock in early years leads to reduced (increased) banking, and this leads to an increase (decrease) of the cumulative amount of allowances available for auctioning. This is a radical deviation from the typical banking principle in which cumulative permits are fixed, and the market only decides on the inter-temporal allocation. Though the MSR proposal is an endogenous policy response to prices perceived as too low, it raises the question whether such a stabilization mechanism is ex ante optimal. Using our formal model, we show that adjusting cumulative quota as an automatic response to early period demand shocks is efficient indeed. Our results pin down a precise formula for the optimal response rate.

The paper is organized as follows. In Section 2 we introduce the model and define ex-post and ex-ante (expected) welfare losses. Section 3 analyzes different policy instruments when firms perfectly observe the current and future cost-structure. The instruments considered are Quantities, Prices, Banking, and Banking with Stabilization. We show that specific parametric settings reduce our model to the one of Weitzman (1974), so that these well-known results can be considered special cases of our more general conclusions. Section 4 relaxes the perfect foresight assumption; now firms can forecast only part of the future cost structure. For both the perfect and partial foresight model, we derive optimal parameters for banking with a stabilization mechanism. Our results suggest that, consistent with current EU-ETS proposals, optimal regulation dampens the effect of emission permit demand shocks in early periods on later periods. Section 5 discusses matters of implementation and concludes.

ranges between €50 and €80 billion. Moreover, Nordhaus (2014) shows that within the context of a limit on the global temperature rise of 2℃ – which the EU has formulated as one of its goals for climate policy – optimal permit prices may be as high as 60€/tCO_2 for emissions today, reaching up to above 200€/tCO_2 in 2050, with prices expressed in 2005 US dollars. These prices would imply the current surplus has a value beyond €100 billion now. Gerlagh and Liski (2018), using non-exponential time preferences, find an optimal permit price of €130 for emissions in 2010, yielding a value of the total surplus of €260 billion.

[7] In some way, the MSR is a form of 'public savings' of allowances. Indeed, it has been argued by many economists that Ricardian equivalence holds: public savings decisions do not affect market permit prices as it crowds out private savings one to one.

2 Model Set Up

2.1 A Stock Pollutant and Complete but Asymmetric Information

Our notation of the descriptive model, welfare analysis, and characterization of policies builds on Weitzman (2018). We consider a two-period world and a representative profit-maximizing firm in the business of producing a homogeneous good with polluting emissions as a negative externality.[8] At every time t, emitting an amount q_t of the pollutant allows the firm to produce a quantity $Y_t(q_t;\theta_t)$ of the good. The parameter θ_t may be thought of as a 'productivity shock', affecting how much can be produced for a given emission input q_t, and is observed by the firm, but unknown to the regulator. Although the regulator cannot discern the actual realization of θ_t, it is common knowledge that $\mathbb{E}[\theta_t]=0$, $\mathbb{E}[\theta_t^2]=\sigma_t^2$, and $\mathbb{E}[\theta_1\theta_2]=\rho\sigma_1\sigma_2$. For the purposes of our study, the variance σ_t^2 provides a natural measure of the amount of uncertainty present. In the remainder of the paper, we will use the terms 'variance' and 'uncertainty' interchangeably.

Emissions damage the environment and potentially the economic system. The context of our study is that of a long-lived pollutant, such as CO_2. We focus on the simplest possible case, where emissions add to a pure stock pollutant, and emission-related damages only enter welfare through cumulative emissions. The rationale for this assumption is that, for e.g. CO_2, most damages appear after the regulation period.[9] Thus, damages enter welfare as a proxy for expected future welfare losses (see Gerlagh and Michielsen, 2015) and are given by $D(q_1+q_2)$.

The problem facing the regulator is finding quantities q_1 and q_2 that maximize

$$Y_1(q_1;\theta_1)+Y_2(q_2;\theta_2)-D(q_1+q_2). \tag{1}$$

In the absence of asymmetric information, the fully knowledgeable regulator can set these quantities directly or else put a price on emissions that will make the profit-maximizing firm produce the same quantities, and these two instruments are perfectly equivalent, see Montgomery (1972). However, as was first shown by Weitzman (1974), this formal equivalence between instruments breaks down once we introduce an informational disparity,

[8]This simplest of possible settings is equivalent to a model with a continuum of competitive profit-maximizing firms in a market without free entry or exit and households buying the good and supplying labor. A micro-foundation of our simple model from such primitives can be found in Appendix B.

[9]Climate change has very persistent dynamics. See Gerlagh and Liski (2018) for an extensive discussion of the time-structure and its implications for climate policies.

captured here by θ_t.

It will serve the analysis to make some restrictive assumptions regarding the forms productivity and damages take. Let optimal quantities and prices in expectations (when $\theta_1 = \theta_2 = 0$) be labeled $q_t = q^*$ and $p_t = p^*$, respectively. We assume linear marginal productivity, of the form:

$$MY_t(q_t) = p^* - 2c(q_t - q^*) + \theta_t. \tag{2}$$

Marginal damages due to emissions are also linear, and given by:

$$MD = p^* + b(q_1 + q_2 - 2q^*). \tag{3}$$

Although possibly unrealistic in several ways, these simplifications allow us to find neat analytic solutions to the questions we wish to address. As any model is an abstraction from reality, so too is ours, and we acknowledge that the world functions in a way much more complicated than would appear from the simple structure imposed in this work. We do, however, believe that our model captures some important mechanisms governing existing emission trading markets and our results to be relevant outside the realm of economic theories.

Before proceeding to the analysis, we introduce some notation. We use an asterisk for the quantities and prices that are optimal in expectations. We use superscripts scenario labels for equilibrium outcomes. We use superscripts SO, Q, P, B, SB, for the social optimum, and the equilibria outcomes with quantities set per period, prices set per period, banking, and banking with quota updating for stabilization, respectively. Moreover, let x^i denote the value of a variable x under policy i. Let $\Delta^i_j x := x^i - x^j$ be the difference in x under scenario i and j, respectively. Finally, let $\Delta^i x := \Delta^i_* x = x^i - x^*$ be the deviation of x under policy i from the ex-ante expected optimal value x^*, and let $\widetilde{\Delta}^i x := \Delta^i_{SO} = x^i - x^{SO}$ denote the difference between the value of x under scenario i and its ex post socially optimal value.

The game has the following stages:

1. The regulator chooses its policy instruments and their levels, which can be either prices or quantities including rules for banking.

2. The firms observe the productivity shocks θ_1 and θ_2.

3. Prices and quantities are chosen, jointly in both periods, consistent with profit

maximization by the firms,

$$-2c\Delta^i q_t + \theta_t = \Delta^i p_t, \tag{4}$$

while the policy rules determine the relation between quantities and prices within and across periods.

The special case of perfectly correlated and equally sized shocks, $\rho = 1$, $\sigma_1 = \sigma_2$, brings us back to the well-known (one-period) model of Weitzman (1974). Throughout the paper, we consider that specific parametric setting to compare our results with those in Weitzman (1974). This will help us develop an intuition for the more general results obtained.

2.2 Ex-post Social Optimum With Complete Information

Since damages are caused by a stock pollutant, the marginal damage of a unit of emissions is the same in period 1 as in period 2. Because in the social optimum, marginal productivity should equal marginal damage, marginal productivity must therefore also be alike in both periods. The optimal quantities q_1^{SO} and q_2^{SO} set by the regulator are thus given by the condition $MY_1 = MY_2 = MD$, that is, matching prices in both periods, $\Delta^{SO}p_1 = \Delta^{SO}p_2$, so that we can omit the price time subscript. This condition is a major deviation from analysis of a flow pollutant and will prove to be of fundamental importance for comparison between instruments. Since prices are equal, we have (4) and

$$b(\Delta^{SO}q_1 + \Delta^{SO}q_2) = \Delta^{SO}p.$$

By solving the above FOCs, we can easily characterize the social optimum:

$$\Delta^{SO}p = \frac{b}{2(b+2c)}(\theta_1 + \theta_2), \tag{5}$$

$$\Delta^{SO}q_1 = \frac{1}{4c}\theta_1 - \frac{b}{4(b+2c)c}\theta_2, \tag{6}$$

$$\Delta^{SO}q_2 = -\frac{b}{4(b+2c)c}\theta_1 + \frac{1}{4c}\theta_2, \tag{7}$$

$$\Delta^{SO}Q = \frac{1}{2(b+2c)}(\theta_1 + \theta_2). \tag{8}$$

In case of constant marginal damages that do not depend on cumulative emissions, $b = 0$, we immediately see that optimal prices do not change, $\Delta^{SO}p = 0$, so that shocks are fully absorbed by changes in emission levels in the same period that the shock occurs. In case of an almost flat marginal productivity, $c \searrow 0$, we can think of a backstop technology that provides an alternative for fossil fuels at constant marginal costs. An unforeseen cost rise of the backstop leads to a (positive) shock in marginal productivity, which is then half absorbed by higher prices, while the other half is absorbed by increased overall emissions. But, more importantly, abatement will move sharply between periods, with a sharp increase (decrease) in emissions in the period with increased (decreased) marginal productivity.

2.3 Welfare Costs of Policies

By definition of the difference under policy i with the social optimum and considering the firms' optimization (4), it is immediate that quantity deviations from the social optimum scale with price deviations:

$$\widetilde{\Delta}^i p_t = 2c\widetilde{\Delta}^i q_t. \tag{9}$$

The welfare loss is then given by:

$$\begin{aligned}
\widetilde{\Delta}^i W &= \mathbb{E}\left[\widetilde{\Delta}^i Y_1 + \widetilde{\Delta}^i Y_2 - \widetilde{\Delta}^i D\right] \\
&= \mathbb{E}\left[\sum_t \widetilde{\Delta}^i q_t \left(p_t^{SO} - c\widetilde{\Delta}^i q_t\right) - \widetilde{\Delta}^i Q \left(p_t^{SO} + \frac{b}{2}\widetilde{\Delta}^i Q\right)\right] \\
&= -\frac{b}{2}\mathbb{E}\left[\left(\widetilde{\Delta}^i Q^2\right)\right] - c\sum_t \mathbb{E}\left[\left(\widetilde{\Delta}^i q_t^2\right)\right] \qquad (10)
\end{aligned}$$

Through adding parameters to our notation, $\widetilde{\Delta}^i W(\sigma_1, \sigma_2, \rho)$, we spell out that the welfare loss of policy i depends on specific parametric values, which facilitates comparison of our results to those in for example Weitzman (1974) ($\sigma_1 = \sigma_2, \rho = 1$) as a means of guiding our intuition.

3 Policies

3.1 Quantities

The first policy we consider fixes quantities in each period:

Definition 1 (Quantities)**.** In both periods, independently, the regulator auctions the ex-ante optimal amount of permits q^*:

$$\Delta^Q q_1 = \Delta^Q q_2 = 0, \tag{11}$$

while prices adjust to reach equilibrium on the emission permits market (4).

We can substitute (6-8) into equation (10), and readily obtain expected welfare losses:

$$-\widetilde{\Delta}^Q W = \frac{1}{8c}\frac{1}{b+c}\left[(b+2c)\left(\sigma_1^2+\sigma_2^2\right) - 2b\rho\sigma_1\sigma_2\right] \tag{12}$$

For the reader's convenience, we relegate this and future derivations to Appendix C.

Inspecting (12), we find the expected welfare losses $-\widetilde{\Delta}^Q W$ to be decreasing in the correlation between shocks, ρ. That is, the expected ex post welfare loss from quantities as a policy instrument are lower for higher correlation between shocks over time. We can understand this as follows. Consider (6); while a positive shock in the first period increases the efficient level of first-period emissions, a positive shock in the second period decreases efficient first-period emissions. Thus, shocks of equal sign in both periods tend to mitigate each other, reducing the welfare loss of fixed quantities. On the other hand, considering (8), we see that a positive correlation increases the welfare loss associated with cumulative emissions. Yet, a careful writing out of welfare losses establishes that the former effect strictly dominates the latter.

3.2 Prices

The second policy we consider fixes permit prices in each period:

Definition 2 (Prices)**.** The regulator fixes permit prices at the ex-ante optimal level $p_1 = p_2 = p^*$:

$$\Delta^P p_1 = \Delta^P p_2 = 0. \tag{13}$$

The firm can buy any number of permits from the regulator at the stated price; quantities adjust to reach equilibrium on the emission permits market (4).

Firms respond to a shock θ_t by adjusting quantities according to $\Delta^P q_t = \theta_t/2c$, see (4). Recall from our previous discussion that, since optimally $MD = MY_1 = MY_2$, in social optimum prices should be equal in both periods, $\Delta^{SO} p_1 = \Delta^{SO} p_2 = \Delta^{SO} p$. Thus the price distortion is the same in both periods: $\widetilde{\Delta}^P p_t = \Delta^P p_t - \Delta^{SO} p = -\Delta^{SO} p$. We now invoke (9) and get our welfare loss measure, expressed in the price gap with the ex post Social

Optimum:

$$\widetilde{\Delta}^i W = -\frac{b}{2}\mathbb{E}\left[\left(\widetilde{\Delta}^i Q\right)^2\right] - \sum_t c\mathbb{E}\left[\left(\widetilde{\Delta}^i q_t\right)^2\right]$$
$$= -\frac{1}{2}\left(\frac{b+c}{c^2}\right)\mathbb{E}\left[\left(\widetilde{\Delta}^i p\right)^2\right]. \tag{14}$$

The above equation holds for all scenarios with constant prices, such as Banking and Stabilized Banking.[10] Considering a Prices policy, we find:

$$-\widetilde{\Delta}^P p = \frac{1}{2}\frac{b}{b+c}(\theta_1+\theta_2) = -\Delta^{SO} p, \tag{15}$$

and combination of (15) with (14) yields:

$$-\widetilde{\Delta}^P W = \frac{1}{8}\frac{b^2}{c^2}\frac{1}{b+c}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2). \tag{16}$$

In contrast to the Quantities policy, we now find that expected welfare losses $\widetilde{\Delta}^P W$ are increasing with the correlation ρ. We can understand the intuition as follows. In social optimum, prices are the same in both periods; this feature is copied in the fixed-prices policy. That is, the fixed-price policy ensures an efficient allocation of emissions over both periods, conditional on cumulative emissions. Consequently, welfare losses brought by the fixed-price policy must come from a distortion with respect to total cumulative emissions. We can thus consider (8) and compare it with the fixed-prices allocation $\Delta^P Q = (\theta_1+\theta_2)/c$. We immediately see that cumulative emissions differ by a constant factor $(b+2c)/c$. That is, the distortion is proportional to the change in the social-optimal (SO) cumulative emissions. As a positive shock in either period tends to increase SO cumulative emissions, the variation in cumulative emissions is minimal when shocks are negatively correlated and maximal when shocks are positively correlated. There are no welfare losses if $\sigma_1 = \sigma_2$ and $\rho = -1$.

We can now compare the Quantities and Prices (policies) in our economy with a stock pollutant under complete but asymmetric information:

Proposition 1. *Quantities outperform Prices, $\widetilde{\Delta}_P^Q W < 0$, iff:*

$$\frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\rho > \frac{1+\frac{b}{2c}-2\left(\frac{b}{2c}\right)^2}{\frac{b}{2c}\left(1+2\frac{b}{2c}\right)} \tag{17}$$

[10] When permits can be banked, equal prices across periods is a result of the profit-maximizing behavior of the firm.

A higher correlation between the two periods, ρ, increases the domain of parameter value ratios b/c for which Quantities outperform Prices.

See Figure 1, which illustrates the proposition by showing iso-welfare lines of Prices and Quantities for perfectly negatively, perfectly positively, and uncorrelated shocks, $\rho \in \{-1, 0, 1\}$.[11] Consider the horizontal line at $b/(b+c) = 2/3$, labeled "$P = Q$ ($\rho = 0$)", which depicts parameter values yielding equal welfare under Quantities and Prices when shocks are uncorrelated over time. Below this line, Prices outperform Quantities. Above, vice versa. The curved lines depict the same condition, but for shocks that are (negatively or positively) correlated over time.

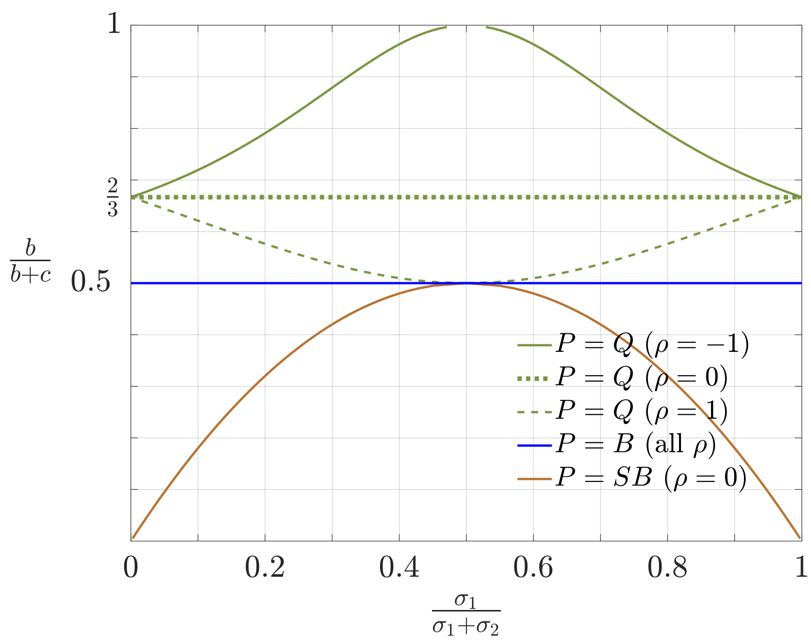

Figure 1: Iso-welfare curves for Prices, Quantities, and (Stabilized) Banking. Below the iso-welfare lines, Prices perform better; above Prices perform worse.

As a corollary to Proposition 1, we list some special cases of our proposition, including the third case that replicates Weitzman (1974):

Corollary 1. *For equally sized negatively correlated shocks, $\sigma_1 = \sigma_2$ and $\rho = -1$, Prices reproduce the Social Optimum and always outperform Quantities. For independent shocks,*

[11] These curves represent cuts (for fixed ρ) of the iso-welfare *planes* visualizing the proposition for any possible ρ. Due to its three dimensions, such a figure is less straightforward to read – for completeness, we nonetheless present it in Figure 2.

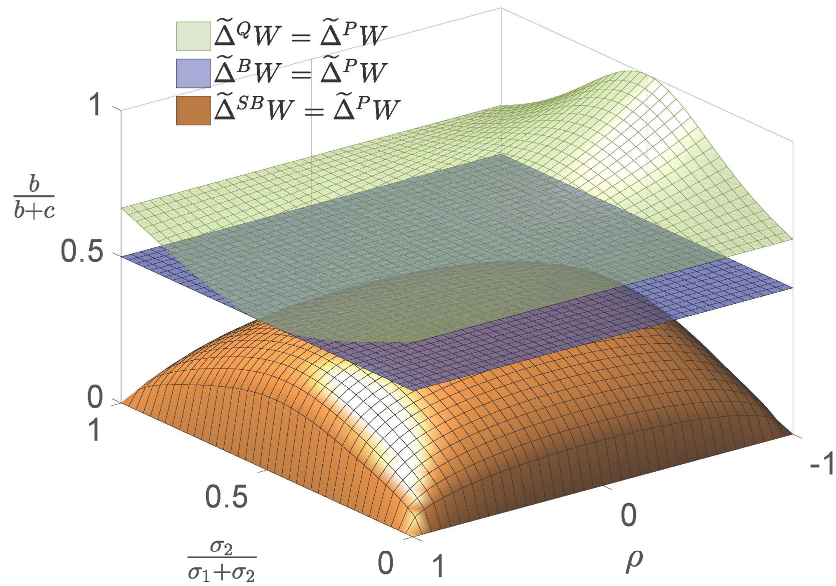

Figure 2: Indifference planes under perfect foresight.

$\rho = 0$, *Quantities outperform Prices if and only if* $b > 2c$. *For* $\sigma_1 = \sigma_2$ *and* $\rho = 1$, *Quantities outperform Prices if and only if* $b > c$.

As the Corollary states, when uncertainty is equally large in both periods and shocks are perfectly correlated, Quantities are preferred to Prices, $\widetilde{\Delta}_P^Q W(\sigma, \sigma, \rho = 1) < 0$, only when $b > c$, which is the canonical result of Weitzman (1974). If the productivity curve is very flat, or the damage curve very steep, fixing prices is a risky endeavor: the slightest error in setting the price right yields a large deviation from socially optimal emissions (as firms will produce up to the point where marginal productivity equals the stipulated price), with all associated critical consequences. Under such conditions, it is thus natural to favor the more conservative instrument of anchoring quantities.

3.3 Banking

Definition 3 (Banking)**.** The regulator auctions the ex-ante optimal amount of permits q^* in both periods. Firms can freely substitute permits between periods. They bank permits not used in the first period, for use in the second period, or borrow permits if first-period demand exceeds supply, subject to:

$$\Delta^B q_1 + \Delta^B q_2 = 0. \tag{18}$$

Equilibrium on the emission permits market implies (4). Profit maximization and free banking and borrowing ensures that permits are allocated so that marginal productivity is equal in both periods:

$$\Delta^B p_1 = \Delta^B p_2.$$

Banking allows firms the flexibility to efficiently distribute their permit use over time subject to the constraint that total emissions are fixed. Note that our model abstracts from the borrowing constraint applied in the EU-ETS. We briefly elaborate on these matters in our discussion section (Section 5).

Combining these observations with the firms' FOCs, (4), we find the change in permit use by period:

$$\Delta^B q_1 = \frac{\theta_1 - \theta_2}{4c} \tag{19}$$

$$\Delta^B q_2 = \frac{\theta_2 - \theta_1}{4c}. \tag{20}$$

From (19) and (20) and using (4), we can solve:

$$\Delta^B p = \frac{\theta_1 + \theta_2}{2} = (b+c)\,\frac{\theta_1 + \theta_2}{2(b+c)}. \tag{21}$$

The socially optimal price response to shocks θ_1 and θ_2 is given by (5). Hence:

$$\widetilde{\Delta}^B p = \Delta^B p - \Delta^{SO} p = \frac{1}{2}\frac{c}{b+c}(\theta_1 + \theta_2). \tag{22}$$

If we compare the deviation from the socially optimal price under a Banking policy to that under a Prices policy, (22) versus (15), we find these to be linearly related:

$$\widetilde{\Delta}^B p = -\frac{c}{b}\widetilde{\Delta}^P p. \tag{23}$$

It is easily seen that:[12]

$$\left|\widetilde{\Delta}^P p\right| > \left|\widetilde{\Delta}^B p\right| \iff b > c. \tag{24}$$

From (14) we know that the welfare loss of Banking and Prices depends on the squared

[12] Not only in expectations, but also ex-post.

deviation of prices from their socially optimal level only. Thus, we conclude immediately that Banking outperforms Prices if and only if $b > c$. In general, using (15) we see that the welfare loss of a Banking policy is given by:

$$-\widetilde{\Delta}^B W = \frac{1}{8}\frac{1}{b+c}(\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2). \tag{25}$$

Like deviations from socially optimal prices, welfare losses under a Banking and a Prices policy are linearly related. This conclusion can be drawn directly by comparing (16) and (25) or, alternatively, by feeding (23) into (14). Specifically, we find:

$$\widetilde{\Delta}^B W = \frac{c^2}{b^2}\widetilde{\Delta}^P W.$$

The following proposition is now immediate:

Proposition 2. *Banking always outperforms Quantities. Banking outperforms Prices if and only if* $b > c$.

The iso-welfare line for Banking compared to either Prices or Quantities is depicted in Figure 1 at $b/(b+c) = 0.5$.

That banking outperforms quantities comes as no surprise; banking allows firms to use their superior information, quantities doesn't. What is, at first, surprising in the result is that the comparison between banking and prices does not depend on the measure of uncertainty in either period, nor the correlation between shocks. On further inspection, this result has a clear background. Both Prices and Banking as policies secure equal prices in both periods, so that the only deviation from the Social Optimum must come from cumulative emissions. Prices as well as Banking prompt a change in aggregate emissions proportional to the cumulative shock, so that only the proportionality between the change in cumulative emissions in equilibrium and in social optimum matters. For both policies, that fraction is a constant, dependent on the ratio b/c.

Our result is complementary to a major conclusion in Weitzman (2018), where Banking is always outperformed by either Prices or Quantities. The different ordering of instruments signifies the importance of carefully considering whether an emission trading market is set up for a stock or a flow pollutant. While Banking is superior for a stock pollutant, it is not for a flow pollutant.

3.4 Banking with stabilization

The standard approach towards banking, as described above, presumes a unity marginal rate of substitution between permits used in period 1 and 2, for both individual firms ($MRS_i = 1$), and at the aggregate level ($MRS_A = 1$). In principle, both rates can be chosen independently, and differently from unity.

The individual firm's marginal rate of substitution, MRS_i, measures the number of permits that an individual firm receives (has to pay back) in the second period for each permit it banks (borrows) in the first period. Commonly, and often implicitly, this rate of conversion is assumed to be unity. Profit maximization by firms then implies that permit prices rise by the interest rate, the Hotelling rule (Cronshaw and Kruse, 1996; Rubin, 1996; Kling and Rubin, 1997; Fell et al., 2012). A few studies consider the firm's marginal rate of substitution MRS_i as a policy instrument, chosen at the discretion of the regulator (Yates and Cronshaw, 2001; Newell et al., 2005). The resulting deviation from Hotelling's rule is part of an efficient policy if marginal damages differ between periods. From our assumption that emissions contribute to a perfect stock pollutant, with marginal damages unconnected to the period of emissions, it follows that equal prices in both periods is an essential characteristic of the ex-post social optimum.[13] We therefore keep $MRS_i = 1$ in our Stabilized Banking analysis.

The aggregate rate of substitution, MRS_A, measures the change in the amount of permits available for use in the second period, at the aggregate level, if one more permit remains unused (banked) in the first period. This aggregate rate of substitution differs from unity if the regulator dynamically adapts the total amount of auctioned permits in the second period, conditional on the amount of used permits in the first period. Indeed, this procedure is part of the EU-ETS revisions proposed in November 2017. Lintunen and Kuusela (2018) study rules for efficient aggregate substitution rates in the context of a flow pollutant. Our analysis extends theirs by considering the stock pollutant-case. We follow the custom that only information on quantities is used. Thus, the regulator can decide to increase or decrease the amount of auctioned permits in the second period, dependent on the number of used permits in the first period. We call this policy Banking with Stabilization, or Stabilized Banking, and abbreviate it as SB. We use $\delta = MRS_A$ for notational convenience, and we will refer to it as the *stabilization rate*.

Definition 4 (Stabilized Banking)**.** The regulator adapts the total amount of auctioned

[13]Extending our model to cover time discounting, it is easily shown that Hotelling's rule would hold in our model. Our base model can be interpreted as the special case where $r = 0$, with r the interest rate, so that the discount factor $1/(1 + r) = 1$ by construction.

permits in the second period based on permits used in the first period for fixed $MRS_A = \delta$:

$$\delta\Delta^{SB}q_1 + \Delta^{SB}q_2 = 0. \tag{26}$$

Profit maximization and free banking and borrowing with $MRS_i = 1$ ensures that firms allocate permits so that marginal productivity is equal in both periods:

$$\Delta^{SB}p_2 = \Delta^{SB}p_1. \tag{27}$$

We can now derive welfare losses:

$$\begin{aligned}-\widetilde{\Delta}^{SB}W = &\frac{1}{2}\frac{b+c}{c^2}\left[\frac{\delta}{1+\delta} - \frac{b}{2(b+c)}\right]^2\sigma_1^2 + \frac{1}{2}\frac{b+c}{c^2}\left[\frac{1}{1+\delta} - \frac{b}{2(b+c)}\right]^2\sigma_2^2 \\ &+ \frac{b+c}{c^2}\left[\frac{\delta}{1+\delta} - \frac{b}{2(b+c)}\right]\left[\frac{1}{1+\delta} - \frac{b}{2(b+c)}\right]\rho\sigma_1\sigma_2. \end{aligned}\tag{28}$$

Note that regular Banking is equivalent to Stabilized Banking with stabilization rate $\delta = 1$ (this is easily verified by plugging $\delta = 1$ into (28) and comparing to (25) or by setting $\delta = 1$ in (26) and comparing to (18)).

From (28) we see that the stabilization rate δ is an important determinant of the welfare loss under Stabilized Banking. This rate is chosen by the regulator and can in principle be any real number. Choosing the stabilization rule optimally reduces welfare losses substantially. To see this, consider the special case where first period marginal productivity is publicly observed (ex-ante known by the regulator), $\theta_1 = 0$, and only second-period productivity is privately observed. That specific case requires a stabilization rate $\delta^* = \frac{b+2c}{b}$, which allows the regulator to perfectly reproduce the outcome of the social optimum, despite uncertainty. That is, $\widetilde{\Delta}^{SB}(0, \sigma_2, \rho, \frac{b+2c}{b}) = 0$. This seems surprising. Yet, by inspection of the Social Optimum, (6) and (7), and substituting $\theta_1 = 0$, we indeed find that:

$$-\frac{\Delta^{SB}q_1}{\Delta^{SB}q_2} = \delta = \frac{b+2c}{b} = -\frac{\Delta^{SO}q_1}{\Delta^{SO}q_2}. \tag{29}$$

Similarly, we have $\delta^* = \frac{b}{b+2c}$ and $\widetilde{\Delta}^{SB}(\sigma_1, 0, \rho, \frac{b}{b+2c}) = 0$ (for $\theta_2 = 0$). We can more generally solve for the optimal stabilization rate δ^* as a function of any feasible vector of parameter values, as stated in the following proposition:

Proposition 3. *Optimal Stabilized Banking equals Banking ($\delta = 1$) when $\sigma_1 = \sigma_2$. In all other cases, Optimal Stabilized Banking strictly outperforms Banking; the optimal* *stabilization rate δ^* is given by:*

$$\delta^* = \frac{b\sigma_1^2 + (b+2c)\sigma_2^2 - 2(b+c)\rho\sigma_1\sigma_2}{(b+2c)\sigma_1^2 + b\sigma_2^2 - 2(b+c)\rho\sigma_1\sigma_2}. \quad (30)$$

The optimal stabilization rate as a function of any possible vector $(\sigma_1, \sigma_2, \rho)$ when $b = c$ (i.e. when Banking and Prices perform equally well in terms of welfare) is illustrated in Figure 3. It has a few outstanding features. First, when the measure of uncertainty in both periods is equal, $\sigma_1 = \sigma_2$, then independently of the correlation between shocks ρ, the optimal stabilization rate δ^* is unity, so that Stabilized Banking and Banking are equivalent.

Second, if the first (second) period has larger measure of uncertainty, then the optimal stabilization dampens its effect on overall emissions through a below-unity (above-unity) exchange rate between the two periods. This result is stated formally in the following corollary:

Corollary 2. *The optimal stabilization rate is less (more) then unity, $\delta^* < 1$ ($\delta^* > 1$), if first-period uncertainty exceeds second-period uncertainty, $\sigma_1 > \sigma_2$ ($\sigma_1 < \sigma_2$).*

Third, the deviation of the optimal stabilization rate from unity increases with a higher correlation between the two shocks.[14] We look at the cut through the surface for fixed $\sigma_2 > \sigma_1$, and we increase the correlation between shocks ρ from -1 to 1. Along this cut, we see that δ^* exceeds unity, and the more so for higher correlations ρ. The intuition is as follows. Assume that shocks are negatively correlated, $\rho < 0$. Recall that the regulator does not observe the realization of shocks, only firms do. Furthermore, recall that, upon observing borrowing of permits, $\widetilde{\Delta}^{SB}q_1 > 0$, the regulator changes cumulative emissions by $(\delta^* - 1)\Delta^{SB}q_1$. Having noticed firms borrow emission permits in the first period, $\widetilde{\Delta}^{SB}q_1 > 0$, and knowing shocks are negatively correlated, the regulator discerns a scenario along the lines of $\theta_2 < 0 < \theta_1$ has realized. Moreover, since $\sigma_2 > \sigma_1$, the regulator concludes that the aggregate shock has been negative, $\theta_1 + \theta_2 < 0$. Next, consider positively correlated shocks, and again assume the regulator observes first-period borrowing: $\widetilde{\Delta}^{SB}q_1 > 0$. Shocks being positively correlated, this means the regulator expects that $\theta_2 > \theta_1$ and, since $\sigma_2 > \sigma_1$, that $\theta_1 < \theta_2 < 0$. Thus, the aggregate shock is now 'more negative' than for negatively correlated shocks. The more negative the aggregate shock, the cheaper it is to cut emissions, and thus second-period emission allowances should be adjusted much more strongly, ie the stabilization rate δ^* should be more different from unity, the more

[14] See also Figure 8 in the appendix.

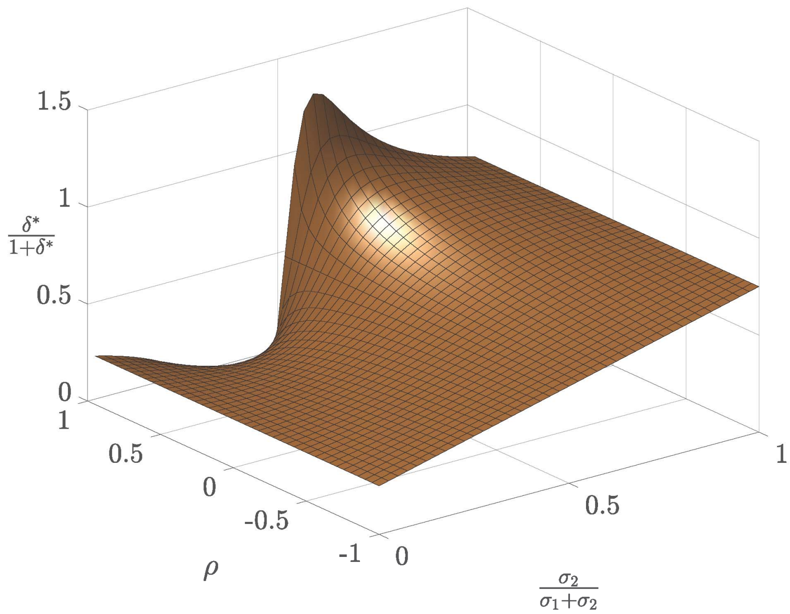

Figure 3: The optimal stabilization rate δ^* as a function of the correlation between shocks over time, ρ, and the relative magnitude of per-period shocks, $\sigma_2/(\sigma_1+\sigma_2)$, when $b = c$, that is, when Prices and Banking do equally well.

negative is the cumulative shock. That is, for a given relative measure of uncertainty, the optimal stabilization rate δ^* deviates more from unity as we increase the correlation ρ.

Having derived the welfare loss under Stabilized Banking for any stabilization rate δ given by (28), as well as the optimal or welfare loss minimizing stabilization rate δ^* given by (30), we can solve for the welfare loss under Stabilized Banking with an optimally chosen stabilization rate for any feasible vector of parameters $(\sigma_1, \sigma_2, b, c, \rho)$. With a slight abuse of notation, we obtain the following remarkably simple expression:

$$-\widetilde{\Delta}^{SB}W(\delta^*;\sigma_1,\sigma_2,b,c,\rho) = \frac{1}{2}\frac{1}{b+c}\frac{(1-\rho^2)\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2}. \tag{31}$$

Consider as a special case the situation where $\sigma_1 = \sigma_2 = \sigma$. This allows us to rewrite (31) as:

$$-\widetilde{\Delta}^{SB}W(\delta^*;\sigma,\sigma,b,c,\rho) = \frac{1}{4}\frac{1}{b+c}(1+\rho)\sigma^2. \tag{32}$$

Figure 4 illustrates the normalized welfare losses under Banking and Stabilized Banking as a function of the correlation between shocks and the relative magnitude of period-specific uncertainty $\sigma_2/(\sigma_1+\sigma_2)$. The welfare loss under Banking always exceeds that under Stabilized Banking, and, moreover, this difference is generally increasing in the correlation between shocks ρ. Consider, for instance, the case when $\sigma_2 = 2\sigma_1$ and shocks are perfectly correlated, $\rho = 1$. The loss (vis-a-vis the social optimum) under Banking is then approximately twice as large as under Stabilized Banking, or stated inversely Stabilized Banking (with an optimal stabilization rate δ) yields welfare gains of more than 50% as compared to regular Banking.

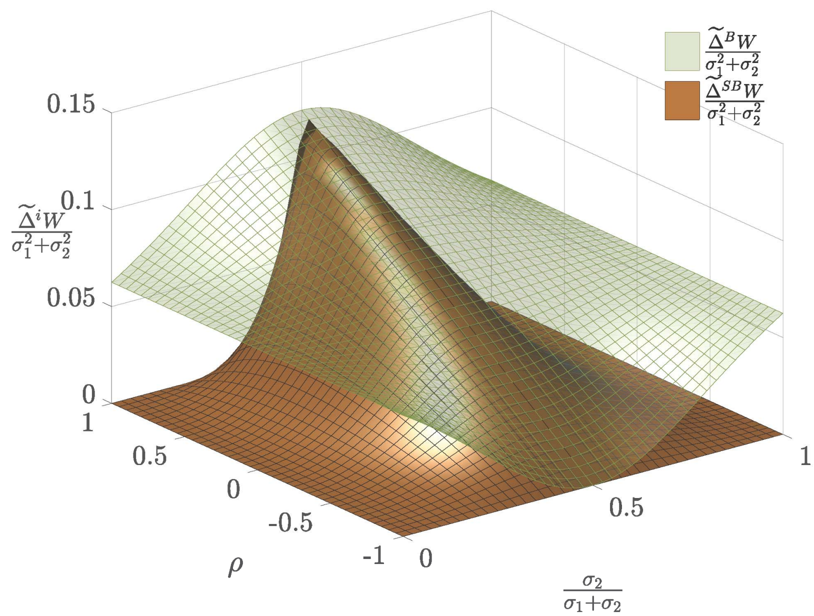

Figure 4: Welfare losses for Banking and Stabilized Banking plotted against the correlation between shocks over time, ρ, and the relative magnitude of per-period shocks, $\sigma_2/(\sigma_1+\sigma_2)$. We assume $b = c$, so that Banking and Prices have the same expected welfare loss.

The next corollary establishes monotonicity of the stabilization rate for the special case of independent or negatively correlated shocks.

Corollary 3. *For independent or negatively correlated shocks, $\rho \leq 0$, the optimal stabilization rate, δ^* increases with the relative size of second-period uncertainty, σ_2/σ_1. The optimal ratio ranges between $b/(b+2c)$ and its inverse $(b+2c)/b$.*

Notably, monotonicity does not hold for positively correlated shocks. The ratio can fall below $b/(b+2c)$, or reach above $(b+2c)/b$:

Corollary 4. *For positively correlated shocks, $\rho > 0$, there is a value for the relative measure of second-period uncertainty, $0 < \sigma_2/\sigma_1 < 1$ such that the optimal ratio satisfies $\delta^* < b/(b+2c)$. Similarly, there exists $1 < \sigma_2/\sigma_1$ such that $\delta^* > (b+2c)/b$.*

What is important to realize is that Figures 3 and 4 are graphical illustration of analytic results, where the only chosen parameter values are $b = c$, which has the advantage that Prices and Banking have the same expected welfare costs. The figures are thus inclusive; they present fundamental features of a general model, and are not mere numerical simulations for a convenient set of parameters.

Finally, we can analytically compare welfare losses under Prices and (optimal) Stabilized Banking. The precise condition for the former to outperform the latter is stated in the following proposition:

Proposition 4. *Stabilized Banking with optimally chosen stabilization rate δ^* outperforms Prices, $\widetilde{\Delta}_P^{SB} W(\delta^*) < 0$, if and only if:*

$$\left[\frac{\sigma_1^2 + \sigma_2^2}{2\sigma_1\sigma_2}\right]^2 > \frac{\rho^2(b^2 - c^2) + c^2}{b^2}. \tag{33}$$

The left-hand side measures the skewness of uncertainty. Stabilization is able to correct for a skewed uncertainty distribution, while Prices performs better, in relative terms, under uniform uncertainty. Whether positive or negative correlated shocks improve the performance of Price or Stabilized Banking depends not only on the correlation between shocks, but this interacts with the relative slopes of benefits and costs of emission reductions.

This concludes our analysis of Quantities, Prices, Banking and Stabilized Banking with an optimal stabilization mechanism. We have found that it is almost always optimal to deviate from the one-to-one aggregate exchange rate between periods. Typically, the optimal stabilization rate is such that it dampens the largest shock. The model so far assumes that firms perfectly observe the shocks to their marginal productivity in the second period before deciding on first-period emissions. This is an assumption that stretches the boundaries of credulity. We therefore relax it in the next section.

4 Imperfect foresight

Now assume that the second-period shocks are not known by the firms when period-one production decisions are made. The stages of the game become:

1. The regulator chooses its policy instruments and their levels, which can be either Prices or Quantities including rules for Banking.

2. The firms observe the first-period productivity shock θ_1.

3. First-period prices and quantities are chosen, that maximize expected production, given policies.

4. The firms observe the second-period shock θ_2.

5. Second-period prices and quantities are chosen, that maximize production, given policies.

For this model, it serves the analysis to write the shock as an AR(1) process. We decompose θ_2 in two parts:

$$\theta_2 = \alpha\theta_1 + \mu, \tag{34}$$

with $-1 \leq \alpha \leq 1$ and μ white noise, so that $\sigma_2^2 = \alpha^2\sigma_1^2 + \sigma_\mu^2$, and $\rho = \alpha\sigma_1/\sigma_2$. For $\alpha > 0$, we can think of θ_t as a persistent technology shock, where the better technology of the first period is carried on to the second, positively affecting productivity in both periods. For $\alpha < 0$, we can think of θ_t as demand shocks in a business cycle framework, where a negative demand shock in the first period is met with counter-cyclical policy by the government, boosting demand in the second period. For realism, we restrict ourselves to $|\alpha| \leq 1$; it appears unlikely for firms to expect shocks of ever-increasing magnitude. We think of a small positive value of α as the most realistic case.

When considering the different policies, we immediately see that Quantities and Prices give identical outcomes to the perfect foresight economy, independent of when the firms observe the shock. With these 'static' policies, knowledge about future conditions does not affect current decisions. The analysis of Section 2 thus also applies to the model with imperfect foresight. We only need to analyze Banking with and without Stabilization. Note, however, that the comparison of Prices versus Banking must be strictly better for Prices under imperfect foresight as compared to perfect foresight. Lengthy derivations in this section are relegated to Appendix D. Deviations from the socially optimal level will be notated shorthand with a $\widehat{\cdot}$.

4.1 Banking

Ex post, we maintain the price-emissions conditions (4) from profit-maximization, and fixed cumulative emissions from the policy rule (18). Yet firms can equalize prices between periods only in expectations, not ex-post:

$$\mathbb{E}[\Delta^B p_2|\theta_1] = \Delta^B p_1, \tag{35}$$

where expected prices depend on expected changes in productivity:

$$2c\mathbb{E}[\Delta^B q_2|\theta_1] + \mathbb{E}[\theta_2|\theta_1] = \mathbb{E}[\Delta^B p_2|\theta_1]. \tag{36}$$

Combining the price-emissions condition (4) and the fixed total emissions (18) with the above equations and using that, by hypothesis, $\mathbb{E}[\theta_2|\theta_1] = \alpha\theta_1$, we derive deviations in permit use in both periods:

$$\Delta^B q_1 = \frac{(1-\alpha)\theta_1}{4c} \tag{37}$$

$$\Delta^B q_2 = \frac{(\alpha-1)\theta_1}{4c}. \tag{38}$$

Equations (37) and (38) have the following interpretation. A profit-maximizing firm equates expected marginal productivity in both periods by smoothing shocks over time. When making production decisions, the firm has only observed the first-period shock. Thus, both first- and second-period emission levels depend on the realization of the first shock and the expectation of the second shock. This expectation depends on the realized first shock, namely $\mathbb{E}[\theta_2|\theta_1] = \alpha\theta_1$. The firm thus chooses emission levels as if smoothing shocks θ_1 and $\alpha\theta_1$. Consider then $\alpha = 1$, i.e. the firm expects shocks of exactly equal magnitude in both periods. In that case, there is nothing to smooth, and hence emission levels should be unaffected by the realization of θ_1. Similarly, consider $\alpha = -1$, i.e. the firm expects a second-period shock which fully 'cancels out' the first period shock. Then, since for any given level of permit use marginal productivity will be higher in the first period as compared to the second, profit maximization will shift permit use very strongly from period 1 to period 2. Equations (37) and (38) show this to be true indeed.

From the per-period emissions given by (37) and (38) we obtain a measure for welfare losses:

$$-\widehat{\Delta}^B W = \frac{1}{8}\frac{(1+\alpha)^2}{b+c}\sigma_1^2 + \frac{1}{8c}\frac{b+2c}{b+c}\sigma_\mu^2 \tag{39}$$

This and subsequent results are equivalently presented in the notation familiar from the analysis of perfect foresight in Appendix D. When comparing the above result to the welfare loss of Banking under perfect foresight, given by (25), it is immediate that:

$$-\widehat{\Delta}^B W = -\widetilde{\Delta}^B W + \frac{1}{8c}\sigma_\mu^2 \geq -\widetilde{\Delta}^B W, \tag{40}$$

with equality if and only if $\sigma_\mu = 0$. That is, the welfare loss of a Banking policy under perfect and imperfect foresight is the same, $\widehat{\Delta} W^B = \widetilde{\Delta} W^B$, exactly when de facto there is no second period uncertainty for firms. In all other cases, imperfect foresight strictly increases welfare costs. This result is formalized in the following proposition:

Proposition 5. *The welfare loss of Banking under imperfect foresight is as least as large as under perfect foresight. Welfare losses are equal if and only if there is no second-period uncertainty for firms ($\sigma_\mu = 0$).*

Next, we compare the welfare outcomes under Banking and Prices and provide conditions under which the former outperforms the latter:

Proposition 6. *Banking outperforms Prices, $\widehat{\Delta}_P^B > 0$, if and only if*

$$\frac{\sigma_\mu^2}{(1+\alpha)^2\sigma_1^2 + \sigma_\mu^2} < \frac{b-c}{c}. \tag{41}$$

Proof. See Appendix D. Q.E.D.

The proposition is visualized in Figure 7, which allows for the immediate conclusion that under imperfect insight, too, Quantities cannot outperform Banking in terms of welfare losses: the indifference plane for Quantities is consistently at the same level or above that for Banking. Thus, a fundamental insight supported by a clear economic intuition in the setting with perfect foresight is carried over to the case where foresight is less than perfect. Therefore, and importantly, even under imperfect predictability of future productivity shocks, our conclusions continue to deviate from the main finding in Weitzman (2018) that Banking is *always* outperformed by either Prices or Quantities. This once again stresses the critical importance of expertly assessing whether the externality under regulation is caused by a stock or a flow pollutant.

4.2 Banking With Stabilization

Under imperfect foresight of future shocks, Banking with Stabilization is defined through the demand equation (4), the stabilization rule (26), constant expected prices (35), and expected efficient allocation of permits (36). Since $\mathbb{E}[\theta_2|\theta_1] = \alpha\theta_1$, we can derive:

$$\Delta^{SB} q_1 = \frac{(1-\alpha)\theta_1}{2c}\frac{1}{1+\delta} \tag{42}$$

$$\Delta^{SB} q_2 = \frac{(\alpha-1)\theta_1}{2c}\frac{\delta}{1+\delta} \tag{43}$$

$$\Delta^{SB} Q = \frac{(1-\alpha)\theta_1}{2c}\frac{1-\delta}{1+\delta}. \tag{44}$$

Through straightforward (but tedious) algebra, we can derive welfare losses:

$$\widehat{\Delta}^{SB} W = \frac{1}{2}\frac{b+c}{c^2}\left[\frac{\delta}{1+\delta} - \frac{b}{2(b+c)}\right]^2 \sigma_1^2 + \frac{1}{2}\frac{b+c}{c^2}\left[\frac{1}{1+\delta} - \frac{b}{2(b+c)}\right]^2 \alpha^2\sigma_1^2 + \frac{b+c}{c^2}\left[\frac{1}{1+\delta} - \frac{b}{2(b+c)}\right]\left[\frac{\delta}{1+\delta} - \frac{b}{2(b+c)}\right]\rho\sigma_1^2 + \frac{b+2c}{b+c}\frac{1}{8c}\sigma_\mu^2. \tag{45}$$

Comparing (45) to the welfare loss of Stabilized Banking under perfect foresight, (28), we see immediately that imperfect foresight increases expected welfare costs:

$$\widehat{\Delta}^{SB} W = \widetilde{\Delta}^{SB} W + \frac{1}{2}\frac{b+c}{c^2}\left[\frac{1}{1+\delta} - \frac{b}{2(b+c)}\right]^2 \sigma_\mu^2 + \frac{b+2c}{b+c}\frac{1}{8c}\sigma_\mu^2 \geq \widetilde{\Delta}^{SB} W \geq \widetilde{\Delta}^{SB} W \tag{46}$$

with equality if and only if $\sigma_\mu = 0$, i.e. if there is no second-period uncertainty for the firms. From (45), we find the optimal δ^* to depend only on the relative size of predictable shocks, α, and the relative slope of benefits and costs b/c, but not on the relative size of unpredictable uncertainty for the regulator nor firms (σ_μ/σ_1):

Proposition 7. *Under imperfect foresight, optimal Stabilized Banking equals Banking ($\delta = 1$) when $\alpha = -1$. In all other cases, Optimal Stabilized Banking strictly outperforms Banking; the optimal stabilization rate δ^* is given by:*

$$\delta^* = \frac{b + (b+2c)\alpha^2 - 2(b+c)\alpha}{(b+2c) + b\alpha^2 - 2(b+c)\alpha}. \tag{47}$$

The optimal stabilization rate δ^* is graphically illustrated in Figure 5. Note, importantly, that under imperfect foresight, the optimal stabilization rate δ^* is at most unity, which a careful writing out of (47) will show. This is a major deviation from the case of perfect foresight, where the optimal stabilization rate δ^* could reach values considerably larger than one.

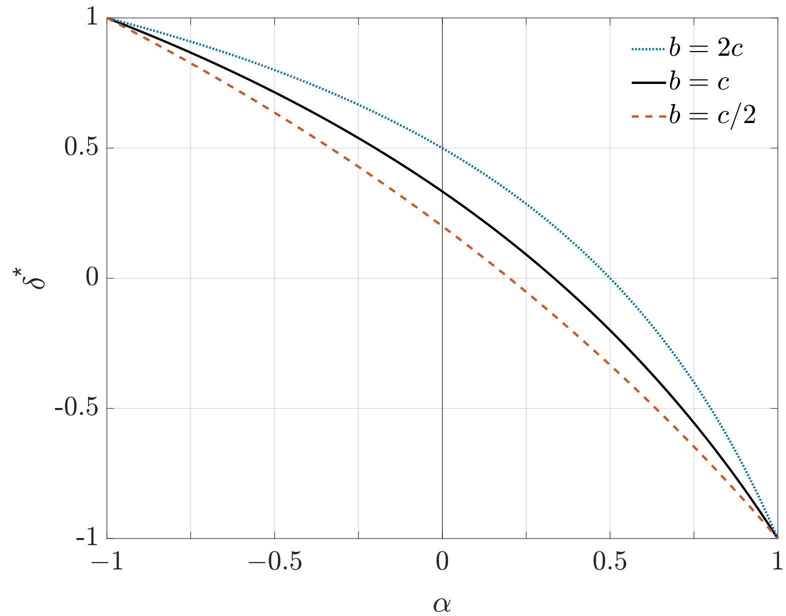

Figure 5: Optimal Stabilization Rate δ^*, for different ratios b/c, dependent on the correlation between shocks α.

Corollary 5. *The optimal stabilization rate δ^* is monotone decreasing in α. For $\alpha = -1$, the optimal stabilization rate δ^* is unity, that is, regular Banking is ex ante optimal. For all other $\alpha \in (-1, 1]$ the optimal stabilization rate is less than unity, $\delta^* < 1$. For $\alpha = 1$, $\delta^* = -1$.*

How do these mathematical observations translated in economists' language? Recall from the definition of Stabilized Banking that $\delta\widehat{\Delta}^{SB}q_1 = -\widehat{\Delta}^{SB}q_2$ and assume, for simplicity, that $\sigma_2 = \sigma_1$. First, let us examine a world in which subsequent shocks are expected to perfectly offset each other, that is, $\alpha = -1$ and thus $\mathbb{E}[\theta_2|\theta_1] = -\theta_1$. This means that an increase in marginal productivity today is met by a decrease in marginal productivity tomorrow. As it is more costly at present to cut emissions, the firms ideally produce more now, cutting emissions in the future, when productivity is low. Since these shocks to productivity are expected to be equal in magnitude but opposite in direction, and combined with unchanged marginal damages, socially optimal total emissions remain unchanged. It is then clearly efficient to allow banking and borrowing of permits on a one-to-one basis, where every extra unit emitted at present translates into exactly one unit forfeited in the future, i.e. to set $\delta = 1$. Next, consider the case where $\alpha = 1$, so that the second period shock is expected to be the exact same magnitude as the first period shock: $\mathbb{E}[\theta_2|\theta_1] = \theta_1$. For a positive productivity shock, it is then optimal to produce more in both periods, since the costs of cutting production have risen in both periods as well. In fact, due to the perfect persistence of shocks and period-independence of marginal damages, this increase of production should be the same in both periods. This can be achieved by setting $\delta = -1$.

Armed with equation (45) – the welfare loss under Stabilized Banking for generic stabilization rate δ – as well as with equation (47) – the optimal stabilization rate δ^* –, we can solve for the welfare loss under Stabilized Banking when $\delta = \delta^*$ is implemented. Almost magically, this yields us a short formula, which provides a decisive and profound insight, of key importance both for theory and policy. For that reason, we state it as Theorem:

Theorem 1. *Under imperfect foresight, Stabilized Banking enables the regulator to mitigate all shocks that are anticipated by the market through setting the stabilization rate δ optimally (at δ^*). That is, shocks anticipated by the regulated firms do not induce welfare losses. Only those shocks which market participants cannot predict cause welfare losses under optimal Stabilized Banking:*

$$-\widehat{\Delta}^{SB}W(\delta^*;\sigma_1,\sigma_2,b,c,\rho) = \frac{1}{8c}\frac{b+2c}{b+c}\sigma_\mu^2. \tag{48}$$

By dynamically adapting future permit allocations in response to information revelation through firms' production choices, the regulator can fully incorporate all knowledge about present and future (productivity) shocks. Thus, any remaining ex post sub-optimality in total permit allowances derives no longer from an asymmetry of information between firms and regulator but solely from the unpredictable element, for both parties, in productivity shocks. Only those shocks which take by surprise both regulated and regulating parties cannot be overcome by setting the optimal δ^*, and therefore are the only factor driving welfare away from its socially optimal level. What is important to note is that all the other instruments considered in this work – Quantities, Prices, and Banking – generally fail to achieve this, save for some exceptional parametric cases.

Our discussion has now brought to light another critical piece of understanding: namely, that the welfare loss under regular Banking when compared to that under optimal Stabilized Banking derives entirely from the failure of the former to wholly incorporate all anticipated productivity shocks into aggregate permit allocations. Indeed, comparing the welfare loss under Stabilized Banking to that under regular Banking, given by (39), we obtain the following:

$$-\widehat{\Delta}^B W = -\widehat{\Delta}^{SB} W + \frac{1}{8}\frac{(1+\alpha)^2}{b+c}\sigma_1^2 \geq -\widehat{\Delta}^{SB} W, \quad (49)$$

which shows that Banking cannot do better than Stabilized Banking in terms of welfare, also when foresight is imperfect, and that the difference between the two is proportional to the measure of cumulative uncertainty $(1+\alpha)^2\sigma_1^2$, multiplied that is, of anticipated productivity shocks, only. Finally, we see that the two instruments are equivalent if and only if $\alpha = -1$. The latter remark in turn corroborates Corollary 5.

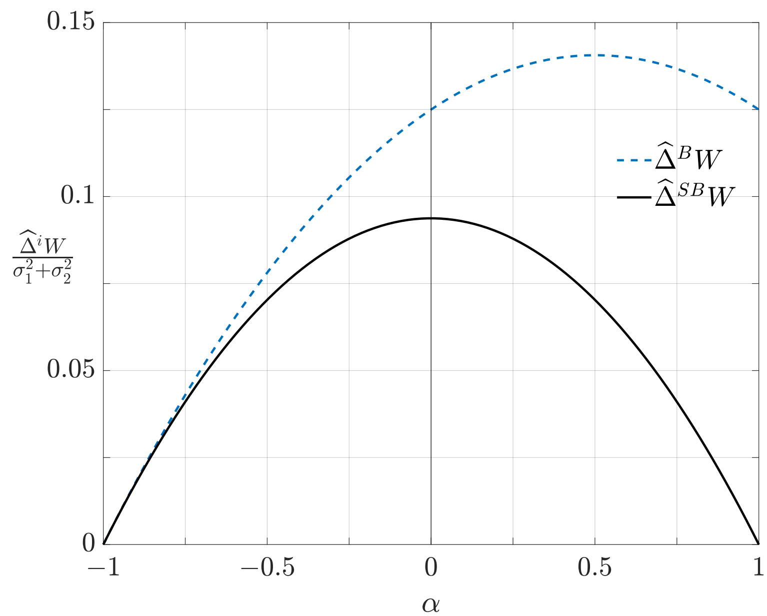

Figure 6: Normalized welfare losses under Banking and Stabilizefd Banking for $\sigma_1 = \sigma_2$ and $b = c$.

Figure 6 illustrates Proposition 7 and equation (49), by plotting (for $b = c$) the normalized welfare losses of regular Banking and Stabilized Banking with optimally chosen stabilization rate $\delta = \delta^*(\alpha)$ when foresight is imperfect and uncertainty is of equal measure in both periods ($\sigma_1 = \sigma_2$). We see that, as was stated in Proposition 7, Stabilized Banking always outperforms regular Banking, except in the extreme cases where $\alpha = -1$, when welfare losses are equal. The graph shows that Stabilized Banking can yield welfare gains of up to $\sim 50\%$ for relevant small but positive values of α. For any given α, the distance between the solid line (depicting normalized welfare losses under optimal Stabilized Banking) and the dashed line (giving normalized welfare losses under regular Banking) is proportional to σ_1^2 and represents the additional welfare losses caused by sub-optimally adjusting aggregate permit availability conditional on the revelation of information through an efficient stabilization mechanism.

It has now become straightforward to compare the welfare losses under Stabilized Banking with those under Prices and to identify conditions under which the either instrument is preferred. We do so formally in the following proposition:

Proposition 8. *Stabilized Banking with optimally chosen stabilization rate δ^* outperforms Prices, $\widehat{\Delta}_P^{SB} W(\delta^*) < 0$, if and only if:*

$$\frac{\sigma_\mu^2}{(1+\alpha)^2\sigma_1^2 + \sigma_\mu^2} < \frac{b^2}{c(b+2c)}. \tag{50}$$

Proposition 8 is illustrated in Figure 7. We observe that, as the future becomes more unpredictable even for firms, $\sigma_\mu \nearrow$, the comparative advantage of different instruments over Prices becomes more similar. Indeed, for $\sigma_\mu/(\sigma_\mu + \sigma_1) \to 1$, that is, when first-period uncertainty is very small compared to added second-period uncertainty, we see that Quantities, Banking, and Stabilized Banking are all favored over Prices if and only if $b < 2c$, whilst the latter instrument is superior with a reversed inequality. Alternatively, when period 2 shocks are perfectly predictable, $\sigma_\mu = 0$, Stabilized Banking takes out all welfare losses whereas the other instruments do not, so that Stabilized Banking outperforms Prices, Quantities, and Banking for all parameter vectors (b, c, α).

5 Discussion and Conclusions

5.1 Implementation

Many real-world emissions trading systems allow banking of unused permits to be used in future periods but at the same time do not allow borrowing of future endowments to support present-day production. In the EU-ETS, for example, this holds by construction of the system: in every regulatory period i a total amount a_i of emissions permits is auctioned (or grandfathered) by the regulator, and emissions in period i cannot exceed the total amount auctioned plus those permits that remain unused from previous periods. Thus, in every period i emissions permits can be banked, but not borrowed. To some extent, this is surprising. The advantage of banking and borrowing over period-specific quantity-setting derives from allowing firms to make use of their superior information regarding the value of emissions permits for production. By (de facto) allowing only banking and not borrowing, the regulator exploits only half of the better information.

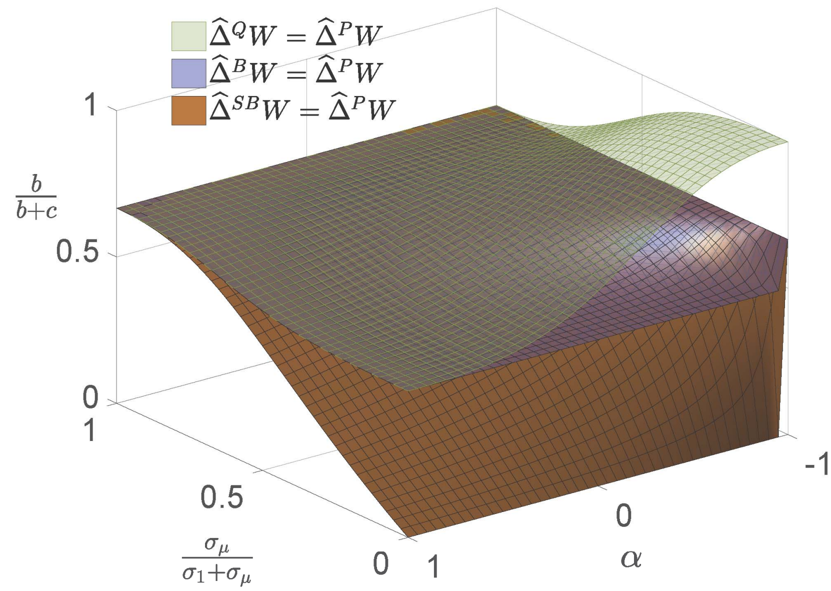

Figure 7: Indifference planes under imperfect foresight.

Consequently, in terms of economic efficency, the system performs worse than when full banking *and* borrowing are allowed. Yet, the inefficiency induced by prohibiting borrowing can be mitigated by increasing the auctioned allowances in early periods. Indeed, this is how the EU-ETS has been implemented.

In the extreme case, for the model developed in this paper, a Banking policy can effectively be implemented by auctioning all permits in the first period, setting $a_1 = 2q^*$ and $a_2 = 0$, and allowing firms to spread use of these permits over time. That is, firms are at once endowed with the emission permits for all periods, and it is left to the market how these permits are allocated intertemporally. The resulting allocation is at least as efficient as the allocation imposed by the regulator if it chose any other auctioned amounts in periods 1 and 2.

Stabilized Banking can achieve substantial welfare gains, and be implemented if in the first period, the minimum amount of cumulative permits is auctioned, $a_1 = (1 + \delta^*)q^*$. We define banking through $s_1 \equiv a_1 - q_1$. The remaining permits are auctioned in the second period, $a_2 = (1 - \delta^*)q_1 \geq 0$. Second-period auctioning decreases with first-period banking: $\partial a_2/\partial s_1 = -(1 - \delta^*)$. The new EU-ETS rules suggest still a modest stabilization mechanism, $0.5 < \delta < 1$, while our findings as presented in Fig 5 suggest a more aggressive stabilization mechanism is efficient; for a positive correlation between shocks, $\alpha > 0$, it is optimal to reduce future auctioning close to one-to-one in response to above-expected current banking.

5.2 Empirical Predictions

The model we developed provides predictions that can be empirically tested when the EU-ETS changes of rules become internalized in the market. The model predicts that exogenous shocks (e.g. weather or oil price shocks) have a lesser effect on price volatility after implementation of the stabilization rule. The high volatility of permit prices in the initial phase has been documented, among others, in Paolella and Taschini (2008); Hintermann (2010); Chesney and Taschini (2012). Our theoretical predictions about price volatility are formally stated and proven in Propositions 9 and 10 for the case of perfect and imperfect foresight, respectively. We show that, in both cases, Stabilized Banking with the stabilization rate δ chosen optimally reduces price volatility of emission permits as compared to regular Banking.

Proposition 9. *With perfect foresight, Stabilized Banking with optimally chosen stabilization rate δ has lower price volatility in both periods compared to Banking. More generally,*

$$\mathbb{E}\left[(\Delta^{SB}p_t)^2\right] < \mathbb{E}\left[(\Delta^{B}p_t)^2\right] \iff (\delta-1)(\sigma_1-\sigma_2) < 0. \tag{51}$$

The RHS condition is always satisfied for $\delta = \delta^*(\rho, \sigma_1\sigma_2)$.

Proof. In Appendix C. Q.E.D.

Proposition 10. *With imperfect foresight, Stabilized Banking with optimally chosen stabilization rate δ has lower price volatility in both periods compared to Banking. More generally,*

$$\mathbb{E}\left[(\Delta^{SB}p_t)^2\right] < \mathbb{E}\left[(\Delta^{B}p_t)^2\right] \iff \delta < 1. \tag{52}$$

The RHS condition is always satisfied for $\delta = \delta^*(\alpha)$.

Proof. In Appendix D. Q.E.D.

5.3 Perfect Versus Imperfect Foresight

Our analysis of optimal policy instruments for a stock pollutant under asymmetric information shows some remarkable differences between scenarios where markets can fully anticipate future shocks, versus those with incomplete private foresight. Comparison of findings in Sections 3 and 4 shows that the comparison between prices and quantities remains unchanged, but banking performs worse, under imperfect foresight, as private firms cannot correctly anticipate future demand shocks. Also, we find that the stabilization rate used for stabilized banking is very different between the two scenarios. Whereas perfect foresight implies a stabilization rate above unity when shocks increase over time, the stabilization rate δ drops below unity, and substantially, with imperfect foresight. What matters, in the latter case, is not the size of future shocks, but the size of the shock that can be correctly forecast. It is natural to assume that shocks increase over time, but that the part of shocks that can be predicted decreases over time. Thus, while perfect foresight suggests a high stabilization rate, imperfect foresight suggests a low stabilization rate. This feature was illustrated in Figures 3 and 5.

A peculiar feature that arises from our model is that a more complex economic model, with imperfectly anticipated future shocks, results in a more simple rule for optimal stabilization. This can be seen from the optimal stabilization rate δ^* as established in equations (30) and (47), respectively. Whilst remarkable upon first sight, there is support for economic intuition. The model evaluates instruments for asymmetric information; heterogeneity between the information sets of firms and the regulator is largest when firms' foresight is perfect and the regulator has to choose its instruments while knowing nothing. Assuming the firms' foresight to be imperfect renders their knowledge set more similar to that of the regulator. We recognize that the complexity of regulation increases with the information advantage of the regulated agents.

5.4 Internationally Connected Permit Markets

Our model under perfect foresight can also be interpreted in terms of spatially (rather them temporally) connected permit markets. Although in that context it is perhaps less natural to consider the amount of permits auctioned in one jurisdiction as independent of that in another jurisdiction, we may nonetheless carry over a fundamental insight insight from our basic model to this alternative interpretation: that it is not, in general, efficient to fix the total amount of auctioned permits exogenously. Rather, it should take into account the uncertainty measures for demand in both jurisdictions, as well as information revealed through inter-jurisdictional trade of permits.

5.5 Conclusions

We built and analyzed a formal model to compare policy instruments for regulating a stock pollutant under uncertainty. Conditions were found under which prices outperform quantities in terms of welfare (Proposition 1) and these were shown to be a generalization of Weitzman (1974). We then compared Prices to Banking (and borrowing) of permits and derived when Banking is preferable (Propositions 2 and 6; Figure 1). Although Banking has been analyzed previously in the context of a flow pollutant Cronshaw and Kruse (1996); Rubin (1996); Kling and Rubin (1997); Yates and Cronshaw (2001); Weitzman (2018); Lintunen and Kuusela (2018), we contribute to the literature by considering a stock pollutant, which is relevant for several important global environmental problems, notably climate change. Our results are radically different from those derived for a flow pollutant.

Having analyzed the policy instruments traditionally considered in the literature, we next introduced a slightly more evolved instrument where periodic permit issues respond automatically to the amount of permits banked in the system, called Stabilized Banking. We showed that Stabilized Banking yields substantial welfare gains compared to the traditional instruments (Propositions 3 and 7), cutting welfare losses by up to 50% as compared to regular Banking (Figures 4 and 6). In November 2017, a modification of EU-ETS rules was proposed. Permits in the Market Stability Reserve (MSR) will be canceled when its level exceeds the amount of auctioned permits in the previous year (Erbach, 2017). That is, the new rules describe a stabilization mechanism akin to our Stabilized Banking policy. Using our model, we can conclude that the ad hoc change of rules indeed appears to be welfare improving, possibly substantially so. Moreover, as an addendum we predict that permit price volatility will be lower under Stabilized Banking (with an optimal stabilization rate) as compared to regular Banking (Propositions 9 and 10). It would be an interesting avenue for future investigation to test this hypothesis empirically once Stabilized Banking has been implemented.

References

Allen, M. R., Frame, D. J., Huntingford, C., Jones, C. D., Lowe, J. A., Meinshausen, M., and Meinshausen, N. (2009). Warming caused by cumulative carbon emissions towards the trillionth tonne. *Nature*, 458(7242):1163–1166.

Baron, D. P. and Myerson, R. B. (1982). Regulating a monopolist with unknown costs. *Econometrica: Journal of the Econometric Society*, pages 911–930.

Chesney, M. and Taschini, L. (2012). The endogenous price dynamics of emission allowances and an application to co2 option pricing. *Applied Mathematical Finance*, 19(5):447–475.

Cronshaw, M. B. and Kruse, J. B. (1996). Regulated firms in pollution permit markets with banking. *Journal of Regulatory Economics*, 9(2):179–189.

Dietz, S. and Venmans, F. (2017). Cumulative carbon emissions and economic policy: in search of general principles. *Grantham Research Institute WP*, 283.

Erbach, G. (2017). Post-2020 reform of the eu emissions trading system. Technical Report PE 614.601, European Parliamentary Research Service.

Fell, H., MacKenzie, I. A., and Pizer, W. A. (2012). Prices versus quantities versus bankable quantities. *Resource and Energy Economics*, 34(4):607–623.

Gerlagh, R. and Liski, M. (2018). Consistent climate policies. *Journal of the European Economic Association*, 16:1–44.

Gerlagh, R. and Michielsen, T. O. (2015). Moving targets: cost-effective climate policy under scientific uncertainty. *Climatic change*, 132(4):519–529.

Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88.

Hintermann, B. (2010). Allowance price drivers in the first phase of the eu ets. *Journal of Environmental Economics and Management*, 59(1):43–56.

Hintermann, B. (2011). Market power, permit allocation and efficiency in emission permit markets. *Environmental and Resource Economics*, 49(3):327–349.

Hintermann, B. (2017). Market power in emission permit markets: theory and evidence from the eu ets. *Environmental and Resource Economics*, 66(1):89–112.

Hoel, M. and Karp, L. (2001). Taxes and quotas for a stock pollutant with multiplicative uncertainty. *Journal of public Economics*, 82(1):91–114.

Hoel, M. and Karp, L. (2002). Taxes versus quotas for a stock pollutant. *Resource and Energy Economics*, 24(4):367–384.

Hurwicz, L. and Reiter, S. (2006). *Designing economic mechanisms*. Cambridge University Press.

Kling, C. and Rubin, J. (1997). Bankable permits for the control of environmental pollution. *Journal of Public Economics*, 64(1):101–115.

Lintunen, J. and Kuusela, O.-P. (2018). Business cycles and emission trading with banking. *European Economic Review*, 101:397–417.

Liski, M. and Montero, J.-P. (2011). Market power in an exhaustible resource market: The case of storable pollution permits. *The Economic Journal*, 121(551):116–144.

Mas-Colell, A., Whinston, M. D., Green, J. R., et al. (1995). *Microeconomic theory*, volume 1. Oxford university press New York.

Montgomery, W. D. (1972). Markets in licenses and efficient pollution control programs. *Journal of economic theory*, 5(3):395–418.

Newell, R., Pizer, W., and Zhang, J. (2005). Managing permit markets to stabilize prices. *Environmental and Resource Economics*, 31(2):133–157.

Newell, R. G. and Pizer, W. A. (2003). Regulating stock externalities under uncertainty. *Journal of Environmental Economics and Management*, 45(2):416–432.

Nordhaus, W. (2014). Estimates of the social cost of carbon: concepts and results from the dice-2013r model and alternative approaches. *Journal of the Association of Environmental and Resource Economists*, 1(1/2):273–312.

Paolella, M. S. and Taschini, L. (2008). An econometric analysis of emission allowance prices. *Journal of Banking & Finance*, 32(10):2022–2032.

Roberts, M. J. and Spence, M. (1976). Effluent charges and licenses under uncertainty. *Journal of Public Economics*, 5(3-4):193–208.

Rubin, J. D. (1996). A model of intertemporal emission trading, banking, and borrowing. *Journal of Environmental Economics and Management*, 31(3):269–286.

Stocker, T. (2014). *Climate change 2013: the physical science basis: Working Group I contribution to the Fifth assessment report of the Intergovernmental Panel on Climate Change*. Cambridge University Press.

Weitzman, M. L. (1974). Prices vs. quantities. *The review of economic studies*, 41(4):477–491.

Weitzman, M. L. (2018). Prices or quantities dominate banking and borrowing (no. 24218). *National Bureau of Economic Research*.

Yates, A. J. and Cronshaw, M. B. (2001). Pollution permit markets with intertemporal trading and asymmetric information. *Journal of Environmental Economics and Management*, 42(1):104–118.

A Onlinde Appendix A: Figures for Comparative Statics

In Figure 8 we present cuts for three different correlations between shocks, $\rho = -1, 0, 0.95$, of Figure 3 in the main text. It clearly illustrates the property that the optimal stabilization rate deviated more from unity the larger is the correlation ρ.

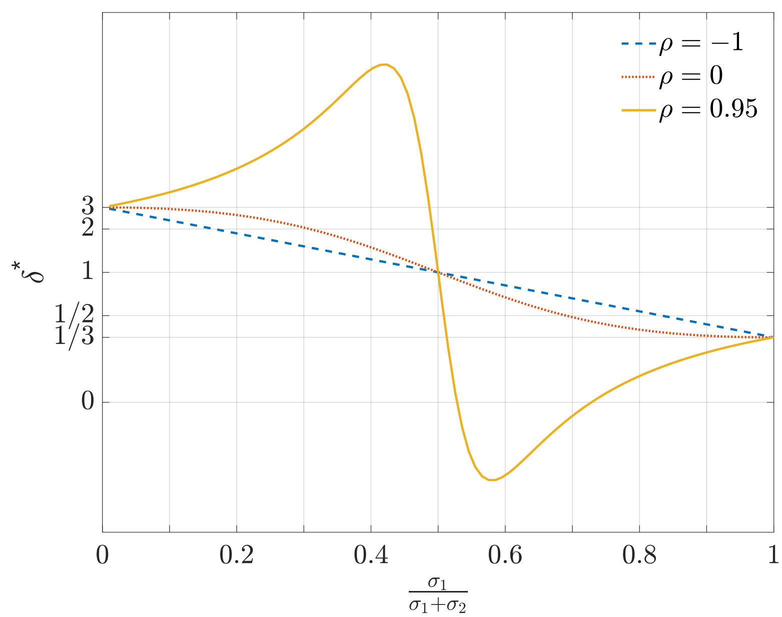

Figure 8: Optimal stabilization rate δ^* when $b = c$ for $\rho \in \{-1, 0, 0.95\}$.

B Online Appendix B: Micro-foundation of the Model

Firms

Consider a continuum of competitive profit-maximizing firms, indexed $i \in [0, 1]$, each producing at every time $t \in \{1, 2, ..., T\}$ quantity Y_{it} of a homogeneous good.[15] Production by the firm causes environmental damages to consumers as an externality, which the firm does not take into account. The production technology $Y : \mathbb{R}^2_+ \to \mathbb{R}_+$ has decreasing returns to scale, with inputs emission permits q_{it} and labor l_{it}. Permit prices and wages are equal for all firms and given by price p_t and w_t, respectively. Marginal productivity of emission permits by firm i at time t is subject to random vertical shocks θ_{it}, while labour productivity is unaffected. Shocks are observed by the firm but not by the regulator. Given these assumptions, firm i solves:

$$\max_{q_{it}, l_{it}} \sum_{t=1}^{T} Y_{it}(q_{it}, l_{it}; \theta_{it}) - p_t q_{it} - w_t l_{it}. \tag{53}$$

We assume symmetry between firms and strictly decreasing returns to scale (positive profits) so that $l_{it} = l_t$, and we label the optimal quantities in expectations $q_t = q^*$ and prices $p_t = p^*$. In particular, we assume linear marginal productivity, of the form:

$$\forall i : \frac{\partial Y_{it}}{\partial q_{it}} = p^* - Tc(q_{it} - q^*) + \theta_{it} \tag{54}$$

Aggregate emissions, shocks, and production at time t are given by:

$$q_t = \int_0^1 q_{it} di \tag{55}$$

$$\theta_t = \int_0^1 \theta_{it} di, \tag{56}$$

$$Y_t(q_t; \theta_t) = \int_0^1 Y_{it}(q_{it}, l_{it}; \theta_{it}) di, \tag{57}$$

with $\mathbb{E}[\theta_t] = 0$ and variance captured by the parameter $\mathbb{E}[\theta_t^2] = \sigma_t^2 \geq 0$. We denote correlations between periods t and $t+s$ by $\mathbb{E}[\theta_t \theta_{t+s}] = \rho^s \sigma_t \sigma_{t+s}$.[16] Because of competitive markets and separation of emission productivity shocks from labor productivity, we can

[15] Environments where firms are not competitive and permit allocations or prices can be manipulated by dominant firms are studied in Liski and Montero (2011), Hintermann (2011), Hintermann (2017).

[16] We do not need more structure, such as the distribution. Only expectations, standard deviations, and correlations enter our results.

reduce equilibrium through the aggregate or representative firm, which faces the problem:

$$\max_{q_t, l_t} \sum_{t=1}^{T} \left[Y_t(q_t; \theta_t) - p_t q_t - w_t l_t \right]. \tag{58}$$

In equilibrium, marginal productivity for emission permits of the representative firm equals prices:

$$p_t = \frac{\partial Y_t}{\partial q_t} = p^* - Tc(q_t - q^*) + \theta_t. \tag{59}$$

Note that for the special cases where $T = 1$ or $\theta_1 = \theta_2 = ... = \theta_T = \theta$, $p_t = p$, the first order condition (59) simplifies to $c(Tq^* - \sum_t q_t) + \theta_t = p - p^*$, which exactly reproduces the model in Weitzman (1974) (with $\sum_t q_t$ aggregate emissions over all periods).

Finally, note that a decreasing returns to scale technology implies positive profits, so that:

$$Y_{it} = p_t q_{it} + w_t l_t + \pi_{it} \tag{60}$$

$$\Longrightarrow Y_t = p_t q_t + w_t l_t + \Pi_t. \tag{61}$$

Households

Households, normalized to size 1, maximize utility, which is derived from consumption C_t and environmental damages resulting from the stock of emissions as determined by the damage function $D : \mathbb{R}_+ \to \mathbb{R}_+$. We focus on the simplest possible case, wherein emission-related damages only enter welfare through cumulative emissions. The rationale for this assumption is that, for long-lived pollutants such as CO_2, most damages appear after the regulation period.[17] Thus, damages enter welfare as a proxy for expected future welfare losses (see Gerlagh and Michielsen, 2015). Moreover, we abstract from discounting between periods.

To defray their consumption, households supply labor inelastically $L_t = 1$ and earn wages w_t in every period t. Households receive profits from the firm and a lump-sum transfer τ_t from the regulator.

[17] Climate change has very persistent dynamics. See Gerlagh and Liski (2018) for an extensive discussion of the time-structure and its implications for climate policies.

Households face the constrained optimization problem given by:

$$\max_{C_t} \left[\sum_{t=1}^{T} C_t\right] - D\left(\sum_{t=1}^{T} q_t\right) \tag{62}$$

$$\text{s.t.} \quad \sum_{t=1}^{T} C_t \leq \sum_{t=1}^{T} \left[w_t + \Pi_t + \tau_t\right]. \tag{63}$$

We assume marginal damages are linear in emissions. Specifically:

$$MD = p^* + b\left[\sum_{t=1}^{T} q_t - Tq^*\right]. \tag{64}$$

Market and Regulator
Since all consumption must be produced and vice versa, we have:

$$Y_t(q_t, l_t; \theta_t) = C_t, \tag{65}$$

for all t. Moreover, the market for labor equates supply and demand, so that

$$l_t = L_t = 1, \tag{66}$$

which in turn determines the wage w_t, for all t.

Households receive a lump-sum amount of money τ_t from the regulator in every period t. From the fact that the regulator collects money only through selling (or auctioning) permits, its budget-balancing constraint implies:

$$\tau_t = p_t q_t. \tag{67}$$

The regulator maximizes the sum of consumer surplus and producer surplus, which equals consumer welfare (62). From the equilibrium in the goods market (65), we see that the welfare-maximizing regulator's objective is given by:

$$\max W = \max_{q_t} \sum_{t=1}^{T} Y_t(q_t; \theta_t) - D\left(\sum_{t=1}^{T} q_t\right) \tag{68}$$

For purposes of tractability, in the main text of the paper we study a two-period model, $T = 2$, which allows to derive neat analytic expressions while still capturing the dynamic nature of a stock pollutant.

C Online Appendix C: Derivations Perfect Foresight

Here we provide the derivations of various results under perfect foresight.

C.1 Quantities

DERIVATION OF (12):

$$
\begin{aligned}
-\widetilde{\Delta}^Q W &= \frac{1}{(2b+2c)^2 4c^2}\left[c\mathbb{E}[((b+2c)\theta_1 - b\theta_2)^2] + c\mathbb{E}[((b+2c)\theta_2 - b\theta_1)^2] + \frac{b}{2}\mathbb{E}[(2c(\theta_1+\theta_2))^2]\right] \\
&= \frac{1}{(2b+2c)^2 4c^2}\left[c\mathbb{E}\left[((b+2c)^2 + b^2 + 2bc)(\theta_1^2+\theta_2^2) + (4bc - 4b(b+2c))\,\theta_1\theta_2\right]\right] \\
&= \frac{1}{(2b+2c)^2 4c^2}\left[c\mathbb{E}\left[(2b+2c)(b+2c)\left(\theta_1^2+\theta_2^2\right) - 2b(2b+2c)\theta_1\theta_2\right]\right] \\
&= \frac{1}{2(2b+2c)2c}\left[\mathbb{E}\left[(b+2c)(\theta_1^2+\theta_2^2) - 2b\theta_1\theta_2\right]\right] \\
&= \frac{1}{8}\frac{1}{(b+c)c}\left[(b+2c)\left(\sigma_1^2+\sigma_2^2\right) - 2b\rho\sigma_1\sigma_2\right] \qquad (69)
\end{aligned}
$$

C.2 Prices

DERIVATION OF (16):

$$
\begin{aligned}
-\widetilde{\Delta}^P W &= c\mathbb{E}\left[\left(-\frac{\theta_1}{2c} + \frac{b+2c}{(2b+2c)2c}\theta_1 - \frac{b}{(2b+2c)c}\theta_2\right)^2 + \left(-\frac{\theta_2}{2c} + \frac{b+2c}{(2b+2c)2c}\theta_2 - \frac{b}{(2b+2c)2c}\theta_1\right)^2\right] \\
&+ \frac{b}{2}\mathbb{E}\left[\left(-\frac{2b}{(2b+2c)2c}(\theta_1+\theta_2)\right)^2\right] \\
&= \frac{1}{16}\frac{1}{(b+c)^2c^2}\left[2cb^2(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2) + 2b^3(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)\right] \\
&= \frac{1}{8}\frac{b^2}{(b+c)c^2}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2). \qquad (70)
\end{aligned}
$$

C.3 Prices versus Quantities

PROOF OF PROPOSITION 1:

Proof.

$$\begin{aligned}
(b+2c)\left(\sigma_1^2+\sigma_2^2\right)-2b\rho\sigma_1\sigma_2 \leq \frac{b^2}{c}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2) \Rightarrow \\
\left(b+2c-\frac{b^2}{c}\right)(\sigma_1^2+\sigma_2^2) \leq \left(\frac{2b^2}{c}+2b\right)\rho\sigma_1\sigma_2 \Rightarrow \\
\left(bc+2c^2-b^2\right) \leq 2\left(b^2+bc\right)\frac{\rho\sigma_1\sigma_2}{(\sigma_1^2+\sigma_2^2)} \Rightarrow \\
\frac{2\rho\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2} \geq \frac{bc+2c^2-b^2}{b(b+c)} \Rightarrow \\
\frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\rho \geq \frac{1+\frac{b}{2c}-2\left(\frac{b}{2c}\right)^2}{\frac{b}{2c}(1+2\frac{b}{2c})}
\end{aligned} \tag{71}$$

Q.E.D.

C.4 Banking

DERIVATION OF (25):

$$\begin{aligned}
-\widetilde{\Delta}^B W &= c\mathbb{E}\left[\left(\frac{\theta_2-\theta_1}{2c}+\frac{b+2c}{(2b+2c)2c}\theta_1-\frac{b}{(2b+2c)2c}\theta_2\right)^2\right] \\
&\quad + c\mathbb{E}\left[\left(\frac{\theta_1-\theta_2}{4c}+\frac{b+2c}{(2b+2c)2c}\theta_2-\frac{b}{(2b+2c)2c}\theta_1\right)^2\right]+\frac{b}{2}\mathbb{E}\left[\left(-\frac{\theta_1+\theta_2}{2b+2c}\right)^2\right] \\
&= \frac{1}{4(b+c)^2}\mathbb{E}\left[\frac{2c}{4}\left(\theta_1+\theta_2\right)^2+\frac{b}{2}\left(\theta_1+\theta_2\right)^2\right] \\
&= \frac{1}{8}\frac{1}{b+c}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2).
\end{aligned} \tag{72}$$

C.5 Banking with Stabilization

DERIVATION OF (28):
After substituting $\widetilde{\Delta}^{SB}p_1=\widetilde{\Delta}^{SB}p_2$ in the firms' FOCs, (4), changes in permit use are given by:

$$\widetilde{\Delta}^{SB}q_1 = \frac{\theta_1-\theta_2}{2c}\frac{1}{1+\delta} \tag{73}$$

$$\widetilde{\Delta}^{SB}q_2 = \frac{\theta_2-\theta_1}{2c}\frac{\delta}{1+\delta} \tag{74}$$

$$\widetilde{\Delta}^{SB}Q = \frac{\theta_1-\theta_2}{2c}\frac{1-\delta}{1+\delta}. \tag{75}$$

Combining (73)-(75) with (5)-(8), it follows that:

$$\widetilde{\Delta}^{SB} q_1 = \widetilde{\Delta}^{SB} q_2 = \left[\frac{\delta}{1+\delta} - \frac{b}{2b+2c}\right]\frac{\theta_1}{2c} + \left[\frac{1}{1+\delta} - \frac{b}{2b+2c}\right]\frac{\theta_2}{2c} \tag{76}$$

and after some algebra we furthermore obtain:

$$\widetilde{\Delta}^{SB} Q = 2\widetilde{\Delta}^{SB} q_1. \tag{77}$$

Consequently, from (10), we derive:

$$-\widetilde{\Delta}^{SB} W = 2(b+c)\mathbb{E}\left[\left(\widetilde{\Delta}^{SB} q_1\right)^2\right] \tag{78}$$

$$= \frac{1}{2}\frac{b+c}{c^2}\left[\frac{\delta}{1+\delta} - \frac{b}{2(b+c)}\right]^2 \sigma_1^2 + \frac{1}{2}\frac{b+c}{c^2}\left[\frac{1}{1+\delta} - \frac{b}{2(b+c)}\right]^2 \sigma_2^2$$

$$+ \frac{b+c}{c^2}\left[\frac{\delta}{1+\delta} - \frac{b}{2(b+c)}\right]\left[\frac{1}{1+\delta} - \frac{b}{2(b+c)}\right]\rho\sigma_1\sigma_2. \tag{79}$$

C.5.1 Optimal Stabilization Rate

DERIVATION OF (30):
For analytical tractability, we define $z \equiv \frac{\delta}{1+\delta}$, mapping $\delta \in \mathbb{R}_+$ onto $[0, 1]$. The variable z is symmetric around $\frac{1}{2}$. Furthermore, we define $a \equiv \frac{b+c}{2c^2}$, and $\gamma \equiv \frac{b}{2(b+c)}$. We rewrite (28) as:

$$a[z-\gamma]^2\sigma_1^2 + a[1-z-\gamma]^2\sigma_2^2 + 2a[z-\gamma][1-z-\gamma]\rho\sigma_1\sigma_2,$$

which can be differentiated with respect to z:

$$\frac{\partial}{\partial z}(28) = 2a[z-\gamma]\sigma_1^2 - 2a[1-z-\gamma]\sigma_2^2 + 2a[[1-z-\gamma]-[z-\gamma]]\rho\sigma_1\sigma_2.$$

The first order condition reads:

$$2z[\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2] = 2\gamma\sigma_1^2 + 2(1-\gamma)\sigma_2^2 - 2\rho\sigma_1\sigma_2. \tag{80}$$

In optimum, z should therefore satisfy:

$$z = \frac{\gamma\sigma_1^2 + (1-\gamma)\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}. \tag{81}$$

Plugging in $\gamma \equiv \frac{b}{2b+2c}$ and $z \equiv \frac{\delta}{1+\delta}$, we find the optimal δ^*:

$$\delta^* = \frac{b\sigma_1^2 + (b+2c)\sigma_2^2 - 2(b+c)\rho\sigma_1\sigma_2}{(b+2c)\sigma_1^2 + b\sigma_2^2 - 2(b+c)\rho\sigma_1\sigma_2}, \tag{82}$$

as given.

PROOF OF PROPOSITION 9:

Proof. From (4), we know that $\Delta^i p_t = \theta_t - 2c\Delta^i q_t$. Hence, if we plug in $\Delta^B q_t$ found in (19) and (20), we obtain:

$$\Delta^B p_1 = \Delta^B p_2 = \frac{\theta_1 + \theta_2}{2}.$$

Thus:

$$\mathbb{E}\left[(\Delta^B p_1)^2\right] = \mathbb{E}\left[(\Delta^B p_2)^2\right] = \frac{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}{4}$$

Next we analyze Stabilized Banking. From (73) and (74), giving us $\Delta^{SB} q_t$, we know:

$$\Delta^{SB} p_1 = \Delta^{SB} p_1 = \frac{\delta}{1+\delta}\theta_1 + \frac{1}{1+\delta}\theta_2.$$

Thus:

$$\mathbb{E}\left[(\Delta^{SB} p_1)^2\right] = \mathbb{E}\left[(\Delta^{SB} p_2)^2\right] = \frac{\sigma_1^2 + \delta^2\sigma_2^2 + 2\rho\delta\sigma_1\sigma_2}{(1+\delta)^2}$$

It is immediate that $\mathbb{E}\left[(\Delta^{SB} p_t)^2\right] = \mathbb{E}\left[(\Delta^B p_t)^2\right]$ if and only if $\delta = 1$. Next:

$$\mathbb{E}\left[(\Delta^B p_t)^2 - (\Delta^{SB} p_t)^2\right] = \frac{\left((1+\delta)^2 - 4\right)\sigma_1^2 + \left((1+\delta)^2 - 4\delta^2\right)\sigma_2^2 + 2\left((1+\delta)^2 - 4\delta\right)\rho\sigma_1\sigma_2}{4(1+\delta)^2}.$$

We showed that for $\delta = 1$, Banking and Stabilized Banking have equal price deviations. After careful manipulation, we find that for $\delta > 1$, $\mathbb{E}\left[(\Delta^{SB} p_t)^2\right] < \mathbb{E}\left[(\Delta^B p_t)^2\right]$ whenever:

$$\sigma_1 < \sigma_2,$$

with the reversed inequality for $\delta < 1$. It follows that:

$$\mathbb{E}\left[(\Delta^{SB} p_t)^2\right] < \mathbb{E}\left[(\Delta^B p_t)^2\right] \iff (\delta - 1)(\sigma_1 - \sigma_2) < 0.$$

Next, as we show in Corollary 2, $\delta^*(\rho, \sigma_1, \sigma_2) \lesseqgtr 1$ if and only if $\sigma_2 \lesseqgtr \sigma_1$, which establishes the result. Q.E.D.

D Online Appendix D: Derivations Imperfect Foresight

Here we provide the derivations of various results under imperfect foresight.

D.1 Banking

DERIVATION OF (39):

Comparison of (37) and (38) with the SO (6)-(8) yields:

$$\widehat{\Delta} q_1 = \frac{1}{4}\frac{1}{(b+c)c}\left[(\alpha(b+c)+c)\theta_1 - b\theta_2\right]$$
$$\widehat{\Delta} q_2 = \frac{1}{4}\frac{1}{(b+c)c}\left[(-\alpha(b+c)+c)\theta_1 + (b+2c)\theta_2\right]$$
$$\widehat{\Delta} Q = \frac{1}{2(b+c)}(\theta_1+\theta_2).$$

Therefore:

$$\begin{aligned}
-\widehat{\Delta}^B W &= c\mathbb{E}\left[\left(\frac{(\alpha(2b+2c)+2c)\theta_1 - 2b\theta_2}{8(b+c)c}\right)^2 + \left(\frac{(\alpha(2b+2c)-2c)\theta_1 - 2(b+2c)\theta_2}{8(b+c)c}\right)^2\right] \\
&+ \frac{b}{2}\mathbb{E}\left[\left(\frac{\theta_1+\theta_2}{2(b+c)}\right)^2\right] \\
&= \frac{1}{8(b+c)^2 c}\left[4c^2(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2) + (1-\rho^2)(2b+2c)^2\sigma_2^2\right] \\
&+ \frac{1}{8(b+c)^2} b(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2) \\
&= \frac{1}{8}\frac{1}{b+c}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2) + \frac{(1-\rho)(1+\rho)}{8c}\sigma_2^2 \qquad (83) \\
&= \frac{1}{8}\frac{(1+\alpha)^2}{b+c}\sigma_1^2 + \frac{1}{8c}\frac{b+2c}{b+c}\sigma_\mu^2 \qquad (84)
\end{aligned}$$

D.2 Banking versus Prices

PROOF OF PROPOSITION 5:

Proof. Banking outperforms prices if and only if $\widehat{\Delta}_P^B W < 0$. First, as noted, expected welfare losses under Prices are equal in perfect and imperfect foresight. From equation

(39) and (16), we have:

$$\begin{aligned}
\widehat{\Delta}_P^B W &= \frac{c^2-b^2}{c^2}[\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2] + \frac{b+c}{c}(1-\rho^2)\sigma_2^2 < 0 \\
&\implies \frac{b+c}{c}(1-\rho^2)\sigma_2^2 < \frac{b^2-c^2}{c^2}[\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2] \\
&\implies (1-\rho^2)\sigma_2^2 < \frac{b-c}{c}[\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2] \\
&\implies \frac{(1-\rho^2)\sigma_2^2}{\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2} < \frac{b-c}{c}
\end{aligned} \tag{85}$$

Using that $\sigma_2^2 = \alpha^2\sigma_1^2 + \sigma_\mu^2$ and $(1-\rho^2)\sigma_2^2 = \sigma_2^2 - \alpha^2\sigma_1^2 = \sigma_\mu^2$, we can rewrite the above condition as:

$$\frac{\sigma_\mu^2}{(1+\alpha)^2\sigma_1^2+\sigma_\mu^2} < \frac{b-c}{c}, \tag{86}$$

as stated in the Proposition. Q.E.D.

D.3 Banking with Stabilization

DERIVATION OF (45):

Simply using the five equations defining the policy and plugging in that $\mathbb{E}[\theta_2|\theta_1] = \alpha\theta_1$, we obtain:

$$\begin{aligned}
\Delta q_1 &= \frac{(1-\alpha)\theta_1}{2c}\frac{1}{1+\delta} \\
\Delta q_2 &= \frac{(\alpha-1)\theta_1}{2c}\frac{\delta}{1+\delta} \\
\Delta Q &= \frac{(1-\alpha)\theta_1}{2c}\frac{1-\delta}{1+\delta},
\end{aligned}$$

so that from equations (6)-(8) it follows:

$$\begin{aligned}
\widehat{\Delta} q_1 = \Delta q_1 - \Delta^{SO} q_1 &= \frac{(1-\alpha)}{2c}\frac{1}{1+\delta}\theta_1 + \frac{b+2c}{(2b+2c)2c}\theta_1 - \frac{b}{(2b+2c)2c}\theta_2 \\
&= \frac{1}{4}\frac{(1-\alpha)(2b+2c)+(1+\delta)(b+2c)}{(1+\delta)(b+c)c}\theta_1 - \frac{1}{4}\frac{(1+\delta)b}{(1+\delta)(b+c)c}\theta_2
\end{aligned}$$

$$\widehat{\Delta}q_2 = \Delta q_2 - \Delta^{SO} q_2 = \frac{(\alpha-1)}{2c}\frac{\delta}{1+\delta}\theta_1 + \frac{b+2c}{(2b+2c)2c}\theta_2 - \frac{b}{(2b+2c)2c}\theta_1$$
$$= \frac{1}{4}\frac{(\alpha-1)(2b+2c)\delta - (1+\delta)b}{(1+\delta)(b+c)c}\theta_1 - \frac{1}{4}\frac{(1+\delta)(b+2c)}{(1+\delta)(b+c)c}\theta_2$$

$$\widehat{\Delta}Q = \Delta Q - \Delta^{SO} Q = \frac{(1-\alpha)}{2c}\frac{1-\delta}{1+\delta}\theta_1 + \frac{1}{2b+2c}(\theta_1+\theta_2)$$
$$= \frac{1}{4}\frac{(1-\alpha)(1-\delta)(2b+2c)+2c(1+\delta)}{(1+\delta)(b+c)c}\theta_1 + \frac{1}{4}\frac{(1+\delta)c}{(1+\delta)(b+c)c}\theta_2$$

Note that Δq_1, Δq_2, and ΔQ are functions of the expected θ_2, given the realization of θ_1. This is given by $\mathbb{E}[\theta_2|\theta_1] = \alpha\theta_1$. Thus, effectively they are functions of θ_1 only. This makes sense: the firm only knows θ_1 and can therefore only operate on the basis of beliefs derived from θ_1. However, Δ^{SO} is a function of the true, real, or realized θ_2, which is given by $\theta_2 = \mathbb{E}[\theta_2|\theta_1] + \mu = \alpha\theta_1 + \mu$. We can plug this into the equations derived above to obtain:

$$\widehat{\Delta}q_1 = \frac{(1-\alpha)(2b+2c)+(1+\delta)(b+2c)}{(1+\delta)(2b+2c)2c}\theta_1 - \frac{(1+\delta)b}{(1+\delta)(2b+2c)2c}\theta_2$$
$$= \frac{(1-\alpha)(2b+2c)+(1+\delta)(b+2c)}{(1+\delta)(2b+2c)2c}\theta_1 - \frac{(1+\delta)b}{(1+\delta)(2b+2c)2c}(\alpha\theta_1+\mu)$$
$$= \frac{(1-\alpha)(2b+2c)+(1+\delta)(1-\alpha)b+(1+\delta)2c}{(1+\delta)(2b+2c)}\frac{\theta_1}{2c} - \frac{b}{2b+2c}\frac{\mu}{2c}$$
$$= \frac{(1-\alpha)(2b+2c)+(1+\delta)(2b+2c)-(1+\delta)(1+\alpha)b}{(1+\delta)(2b+2c)}\frac{\theta_1}{2c} - \frac{b}{2b+2c}\frac{\mu}{2c}$$
$$= \left[\frac{\alpha+\delta}{1+\delta} - \frac{1+\alpha}{2b+2c}b\right]\frac{\theta_1}{2c} - \frac{b}{2b+2c}\frac{\mu}{2c}.$$

$$\widehat{\Delta}q_2 = \frac{(\alpha-1)(2b+2c)\delta-(1+\delta)b}{(1+\delta)(2b+2c)2c}\theta_1 - \frac{(1+\delta)(b+2c)}{(1+\delta)(2b+2c)2c}\theta_2$$
$$= \frac{(\alpha-1)(2b+2c)\delta-(1+\delta)b}{(1+\delta)(2b+2c)2c}\theta_1 - \frac{(1+\delta)(b+2c)}{(1+\delta)(2b+2c)2c}(\alpha\theta_1+\mu)$$
$$= \frac{(\alpha-1)(2b+2c)\delta-(1+\delta)b+\alpha(1+\delta)(b+2c)}{(1+\delta)(2b+2c)}\frac{\theta_1}{2c} + \frac{b+2c}{(2b+2c)2c}\frac{\mu}{2c}$$
$$= \left[\frac{\alpha+\delta}{1+\delta} - \frac{1+\alpha}{2b+2c}b\right]\frac{\theta_1}{2c} + \frac{b+2c}{2b+2c}\frac{\mu}{2c}$$

$$
\begin{aligned}
\widehat{\Delta}Q &= \frac{(1-\alpha)(1-\delta)(2b+2c)+2c(1+\delta)}{(1+\delta)(2b+2c)2c}\theta_1 + \frac{(1+\delta)2c}{(1+\delta)(2b+2c)2c}\theta_2 \\
&= \frac{(1-\alpha)(1-\delta)(2b+2c)+2c(1+\delta)}{(1+\delta)(2b+2c)2c}\theta_1 + \frac{(1+\delta)2c}{(1+\delta)(2b+2c)2c}(\alpha\theta_1+\mu) \\
&= \frac{(1-\alpha)(1-\delta)(2b+2c)+(1+\delta)(1+\alpha)2c}{(1+\delta)(2b+2c)}\frac{\theta_1}{c} + \frac{2c}{2b+2c}\frac{\mu}{2c} \\
&= 2\left[\frac{\alpha+\delta}{1+\delta} - \frac{1+\alpha}{2b+2c}b\right]\frac{\theta_1}{2c} + \frac{2c}{2b+2c}\frac{\mu}{2c}.
\end{aligned}
$$

Before proceeding, note that:

$$
\begin{aligned}
\alpha+\delta &= \frac{\rho\sigma_2+\delta\sigma_1}{\sigma_1} \\
1+\alpha &= \frac{\rho\sigma_2+\sigma_1}{\sigma_1} \\
(\alpha+\delta)^2 &= \frac{\rho^2\sigma_2^2+\sigma_1^2+2\rho\sigma_1\sigma_2}{\sigma_1^2} \\
(1+\alpha)^2 &= \frac{\rho^2\sigma_2^2+\sigma_1^2+2\rho\sigma_1\sigma_2}{\sigma_1^2} \\
(1+\alpha)(\alpha+\delta) &= \frac{\rho^2\sigma_2^2+\delta\sigma_1^2+(1+\delta)\rho\sigma_1\sigma_2}{\sigma_1^2}
\end{aligned}
$$

and that $\mathbb{E}[\theta_1\mu]=0$. Finally, note that $\sigma_\mu^2=(1-\rho^2)\sigma_2^2$. It thus follows that:

$$\begin{aligned}
-\widehat{\Delta}^{SB}W &= c\mathbb{E}\left[\left(\widehat{\Delta}q_1\right)^2\right]+c\mathbb{E}\left[\left(\widehat{\Delta}q_2\right)^2\right]+\frac{b}{2}\mathbb{E}\left[\left(\widehat{\Delta}Q\right)^2\right]\\
&= c\left[\left[\frac{\alpha+\delta}{1+\delta}-\frac{1+\alpha}{2b+2c}b\right]^2\frac{\sigma_1^2}{4c^2}+\frac{b^2}{(2b+2c)^2}\frac{\sigma_\mu^2}{4c^2}\right]+c\left[\left[\frac{\alpha+\delta}{1+\delta}-\frac{1+\alpha}{2b+2c}b\right]^2\frac{\sigma_1^2}{4c^2}+\frac{(b+2c)^2}{(2b+2c)^2}\frac{\sigma_\mu^2}{4c^2}\right]\\
&+\frac{b}{2}\left[\left[\frac{\alpha+\delta}{1+\delta}-\frac{1+\alpha}{2b+2c}b\right]^2\frac{4\sigma_1^2}{4c^2}+\frac{4c^2}{(2b+2c)^2}\frac{\sigma_\mu^2}{4c^2}\right]\\
&= (2b+2c)\left[\frac{\alpha+\delta}{1+\delta}-\frac{1+\alpha}{2b+2c}b\right]^2\frac{\sigma_1^2}{4c^2}+\frac{1}{2}\frac{b^2c+(b+2c)^2c+4bc^2}{4(b+c)^2}\frac{\sigma_\mu^2}{4c^2}\\
&= (b+c)\left[\frac{\alpha+\delta}{1+\delta}-\frac{1+\alpha}{2b+2c}b\right]^2\frac{\sigma_1^2}{2c^2}+\frac{b+2c}{8c}\frac{1}{b+c}\sigma_\mu^2\\
&= \frac{b+c}{2c^2}\left[\frac{(\alpha+\delta)^2}{(1+\delta)^2}+\frac{(1+\alpha)^2b^2}{4(b+c)^2}-2\frac{(\alpha+\delta)(1+\alpha)b}{(1+\delta)(2b+2c)}\right]\sigma_1^2+\frac{b+2c}{8c}\frac{(1-\rho^2)}{b+c}\sigma_2^2\\
&= \frac{b+c}{2c^2}\left[\frac{\rho^2\sigma_2^2+\delta^2\sigma_1^2+2\rho\sigma_1\sigma_2}{(1+\delta)^2}+\frac{\rho^2\sigma_2^2+\sigma_1^2+2\rho\sigma_1\sigma_2}{4(b+c)^2}b^2-2\frac{\rho^2\sigma_2^2+\delta\sigma_1^2+(1+\delta)\rho\sigma_1\sigma_2}{2(1+\delta)(b+c)}b\right]\\
&+\frac{b+2c}{8c}\frac{(1-\rho^2)}{b+c}\sigma_2^2\\
&= \frac{b+c}{2c^2}\left[\frac{\delta}{1+\delta}-\frac{b}{2(b+c)}\right]^2\sigma_1^2+\frac{b+c}{2c^2}\left[\frac{1}{1+\delta}-\frac{b}{2(b+c)}\right]^2\rho^2\sigma_2^2\\
&+2\frac{b+c}{2c^2}\left[\frac{1}{1+\delta}-\frac{b}{2(b+c)}\right]\left[\frac{\delta}{1+\delta}-\frac{b}{2(b+c)}\right]\rho\sigma_1\sigma_2+\frac{b+2c}{8c}\frac{(1-\rho^2)}{b+c}\sigma_2^2
\end{aligned}$$

D.3.1 Optimal Stabilization Rate

PROOF OF PROPOSITION 7:

We can rewrite (45) as:

$$a[z-\gamma]^2\sigma_1^2+a[1-z-\gamma]^2\rho^2\sigma_2^2+2a[z-\gamma][1-z-\gamma]\rho\sigma_1\sigma_2,$$

which we can differentiate with respect to z:

$$\frac{\partial}{\partial z}(45)=2a[z-\gamma]\sigma_1^2-2a[1-z-\gamma]\rho^2\sigma_2^2+2a[[1-z-\gamma]-[z-\gamma]]\rho\sigma_1\sigma_2.$$

The optimal z should therefore satisfy:

$$z = \frac{\gamma\sigma_1^2 + (1-\gamma)\rho^2\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \rho^2\sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{87}$$

$$= \frac{\gamma + (1-\gamma)\alpha^2 - \alpha}{(1-\alpha)^2} \tag{88}$$

Plugging in $\gamma \equiv \frac{b}{2(b+c)}$ and $z \equiv \frac{\delta}{1+\delta}$, using that $\alpha = \rho\frac{\sigma_2}{\sigma_1}$ and dividing through by σ_1^2, we find the optimal stabilization rate δ^*:

$$\delta^* = \frac{b\sigma_1^2 + (b+2c)\rho^2\sigma_2^2 - 2(b+c)\rho\sigma_1\sigma_2}{(b+2c)\sigma_1^2 + b\rho^2\sigma_2^2 - 2(b+c)\rho\sigma_1\sigma_2} \tag{89}$$

$$= \frac{b + (b+2c)\alpha^2 - 2(b+c)\alpha}{(b+2c) + b\alpha^2 - 2(b+c)\alpha}, \tag{90}$$

as given.

DERIVATION OF (48):

$$
\begin{aligned}
\widehat{\Delta}^{SB} W(\delta^*;\cdot) &= \frac{1}{2}\frac{b+c}{c^2}\left[\frac{\delta}{1+\delta}-\frac{b}{2(b+c)}\right]^2\sigma_1^2+\frac{1}{2}\frac{b+c}{c^2}\left[\frac{1}{1+\delta}-\frac{b}{2(b+c)}\right]^2\rho^2\sigma_2^2 \\
&+\frac{b+c}{c^2}\left[\frac{1}{1+\delta}-\frac{b}{2(b+c)}\right]\left[\frac{\delta}{1+\delta}-\frac{b}{2(b+c)}\right]\rho\sigma_1\sigma_2+\frac{b+2c}{b+c}\frac{1-\rho^2}{8c}\sigma_2^2 \\
&= \frac{1}{2}\frac{b+c}{c^2}\left[\frac{1}{2(b+c)}\frac{b\sigma_1^2+(b2+c)\rho^2\sigma_2^2-2(b+c)\rho\sigma_1\sigma_2-b\sigma_1^2-b\rho^2\sigma_2^2+2b\rho\sigma_1\sigma_2}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right]^2\sigma_1^2 \\
&+\frac{1}{2}\frac{b+c}{c^2}\left[\frac{1}{2(b+c)}\frac{(b2+c)\sigma_1^2+b\rho^2\sigma_2^2-2(b+c)\rho\sigma_1\sigma_2-b\sigma_1^2-b\rho^2\sigma_2^2+2b\rho\sigma_1\sigma_2}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right]^2\rho^2\sigma_2^2 \\
&+\frac{1}{2}\frac{b+c}{c^2}\left[\frac{1}{2(b+c)}\frac{b\sigma_1^2+(b2+c)\rho^2\sigma_2^2-2(b+c)\rho\sigma_1\sigma_2-b\sigma_1^2-b\rho^2\sigma_2^2+2b\rho\sigma_1\sigma_2}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right]\times \\
&\left[\frac{1}{2(b+c)}\frac{(b2+c)\sigma_1^2+b\rho^2\sigma_2^2-2(b+c)\rho\sigma_1\sigma_2-b\sigma_1^2-b\rho^2\sigma_2^2+2b\rho\sigma_1\sigma_2}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right]\rho\sigma_1\sigma_2 \\
&+\frac{b+2c}{b+c}\frac{1-\rho^2}{8c}\sigma_2^2 \\
&= \frac{1}{2}\frac{1}{b+c}\left[\frac{\rho\sigma_2^2-\rho\sigma_1\sigma_2}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right]^2\sigma_1^2+\frac{1}{2}\frac{1}{b+c}\left[\frac{\sigma_1^2-\rho\sigma_1\sigma_2}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right]^2\rho^2\sigma_2^2 \\
&+\frac{1}{b+c}\left[\frac{\rho\sigma_2^2-\rho\sigma_1\sigma_2}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right]\left[\frac{\sigma_1^2-\rho\sigma_1\sigma_2}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right]\rho\sigma_1\sigma_2+\frac{b+2c}{b+c}\frac{1-\rho^2}{8c}\sigma_2^2 \\
&= \frac{1}{b+c}\left(\frac{1}{\sigma_1^2+\rho^2\sigma_2^2-2\rho\sigma_1\sigma_2}\right)^2\left[(\rho^4\sigma_1^2\sigma_2^4+\rho^2\sigma_1^4\sigma_2^2-2\rho\sigma_1^3\sigma_2^3)-(\rho^4\sigma_1^2\sigma_2^4+\rho^2\sigma_1^4\sigma_2^2-2\rho\sigma_1^3\sigma_2^3\right. \\
&+\frac{b+2c}{b+c}\frac{1-\rho^2}{8c}\sigma_2^2 \\
&= \frac{b+2c}{b+c}\frac{1-\rho^2}{8c}\sigma_2^2 \\
&= \frac{1}{8c}\frac{b+2c}{b+c}\sigma_\mu^2. \qquad (91)
\end{aligned}
$$

PROOF OF PROPOSITION 10:

Proof. We know that $\theta_1 - 2c\Delta^i q_1 = \widehat{\Delta}^i p_1$. Moreover, we derived in the main text the quantity responses to a shock θ_1, which are given by equations (37) and (42) for Banking and Stabilized Banking, respectively. It is thus easily seen that:

$$
\begin{aligned}
\Delta^B p_1 &= \frac{1+\alpha}{2}\theta_1 \\
\Delta^{SB} p_1 &= \frac{\alpha+\delta}{1+\delta}\theta_1,
\end{aligned}
$$

from which it follows:

$$\mathbb{E}\left[\left(\Delta^{B}p_1\right)^2\right] = \frac{(1+\alpha)^2}{4}\sigma_1^2$$
$$\mathbb{E}\left[\left(\Delta^{SB}p_1\right)^2\right] = \frac{(\delta+\alpha)^2}{(1+\delta)^2}\sigma_1^2.$$

Thus:

$$\mathbb{E}\left[(\Delta^{SB}p_1)^2\right] < \mathbb{E}\left[(\Delta^{B}p_1)^2\right] \iff \delta < 1.$$

From Corollary 5, we know that under imperfect foresight the optimal stabilization rate is less than unity, $\delta^* \leq 1$, for all α, with equality if and only if $\alpha = -1$, which establishes the result for period 1.

For period 2, we have the derived the quantity deviations conditional on first-period shocks. Using that $\delta^i p_2 = \theta_2 - 2c\Delta^i q_2$ and that $\theta_2 = \alpha\theta_1 + \mu$, we can write:

$$\Delta^{B}p_2 = \frac{1+\alpha}{2}\theta_1 + \mu$$
$$\Delta^{SB}p_2 = \frac{\alpha+\delta}{1+\delta}\theta_1 + \mu,$$

, and thus we can derive:

$$\mathbb{E}\left[\left(\Delta^{B}p_2\right)^2\right] = \frac{(1+\alpha)^2}{4}\sigma_1^2 + \sigma_\mu^2$$
$$\mathbb{E}\left[\left(\Delta^{SB}p_2\right)^2\right] = \frac{(\delta+\alpha)^2}{(1+\delta)^2}\sigma_1^2 + \sigma_\mu^2.$$

Thus:

$$\mathbb{E}\left[(\Delta^{SB}p_2)^2\right] < \mathbb{E}\left[(\Delta^{B}p_2)^2\right] \iff \delta < 1.$$

From Corollary 5, we know that under imperfect foresight the optimal stabilization rate is less than unity, $\delta^* \leq 1$, for all α, with equality if and only if $\alpha = -1$. This establishes our result. Q.E.D.

FIGURE 7 IN TERMS OF CORRELATION ρ:

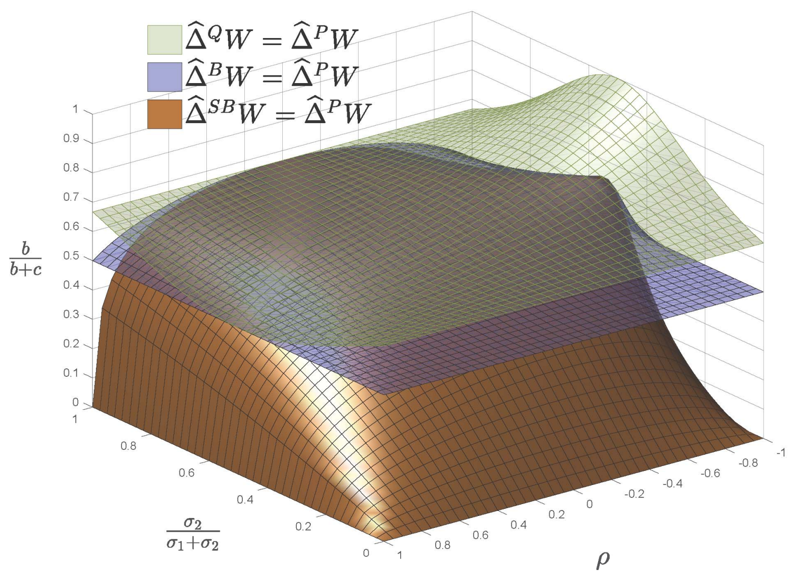

Figure 9: Indifference planes under imperfect foresight.