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Growth and the Geography of Knowledge

Abstract

We analyse how spatial disparities in innovation activities, coupled with migration costs, affect economic geography, growth and regional inequality. We provide conditions for existence and uniqueness of a spatial equilibrium, and for the endogenous emergence of industry clusters. Spatial variations in knowledge spillovers lead to spatial concentration of more innovative firms. Migration costs, however, limit the concentration of economic activities in the most productive region. Narrowing the gap in knowledge spillovers across regions raises growth, and reduces regional inequality by making firms more sensitive to wage differentials. The associated change in the spatial concentration of industries has positive welfare effects.

JEL-Codes: O410, O310, L130, J610, R320.

Keywords: growth, economic geography, geographic labour mobility, innovation, knowledge spillovers, regional economics.

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1 Introduction

In this paper we develop a model of industry location, R&D led growth and inter-regional inequality with three distinctive features; an endogenous market structure characterised by oligopolistic firms conducting R&D, spatially constrained knowledge flows and migration costs. We ask how regional disparities in innovation activities, coupled with migration costs, shape the geography of economic activities, the growth rate and inter-regional inequality.

The spatial disparities in technology (knowledge) spillovers encourage the spatial concentration of industries in the most productive region, however, inter- regional migration costs limit the geographical concentration of economic activities. As a result, not all firms necessarily operate in the most productive region. The endogenous market structure allows us to analyse the interaction between R&D, growth and firms' market power, with the implication that the spatial concentration of industries is associated with fewer, but more innovative, firms.[1]

Since the seminal work of Jaffe et al. (1993), regional variations in localized knowledge spillovers have been identified in the empirical literature. These are important factors behind regional variations in innovation performance suggesting that geographic proximity facilitates knowledge flows and that knowledge spillovers are spatially bounded (Audretsch and Feldman (1996); Baldwin and Martin (2004)). Notably, it is the non-codified (tacit) type of knowledge that flows more easily locally than over great distances. This is the type of knowledge that is transferred through person-to-person interaction and is clearly facilitated by geographic proximity and hindered by the costs of people moving. For example Feldman and Lichtenberg (1998) show that the higher the share of non-codified knowledge in total technology the more geographically localized are the benefits from knowledge externalities. In this context, spatial differences in knowledge externalities should act as an agglomeration force, since the proximity of firms and workers make firms more productive.

[1] While the link between market power concentration, innovation and growth is well established in the literature (see, e.g., Aghion and Howitt (1992); Peretto (1996); Aghion et al. (1997); Etro (2009)); in this paper, we add location as an additional variable which allows for the endogenous emergence of regional disparities in innovation clusters.

Yet we observe dispersion. That is, not all innovation activities are clustered in the most productive region. The standard explanation in the new economic geography literature is that transport costs prevent the concentration of R&D activities in one region (Krugman (1991); Fujita et al. (1999)). Emphasizing the role of frictions in movements of goods as the main impediment to agglomeration seems problematic, given the persistence of localization despite the rapid decline in the costs of shipping goods and in communication costs (Glaeser and Kohlhase (2004); Head and Mayer (2004)). One suggestion is that alternative explanations, put forward in the literature, emphasising knowledge diffusion and learning, may be as relevant in explaining clustering of innovation activities (Audretsch and Feldman (2004)). For example, Cohen and Levinthal (1989) argue that firms that invest in R&D also develop the ability to identify, assimilate and exploit knowledge from the environment. As argued by Combes and Duranton (2006) such abilities are usually embodied in workers and diffuse when workers move between jobs. Since relocating is costly for workers, they tend to change jobs within the same local labour market, implying that knowledge diffusion is geographically localised.

More recently, Bloom et al. (2013) using a panel of U.S. firms over the period 1981-2001, estimate that the social return to R&D is two to three times higher than the private return and find evidence in support of the importance of geographic proximity to capture knowledge spillovers. Building on Bloom et al. (2013), and using U.S. firm level accounting data (1980-2000) matched into the U.S. Patent and Trademark Office data, Lychagin et al. (2016) find that both intra- and inter-regional spillovers matter. Furthermore, they find empirical support for the hypothesis that reduced face-to-face knowledge flows account for the weakening of cross-regional spillovers in space.[2]

Drawing from this evidence, the model we propose incorporates disparities in the spatial extent of knowledge externalities so that the region with the larger knowledge spillovers is characterised by higher productivity and a higher population share. Differently from the standard endogenous growth

[2]This evidence also fits with a vast body of literature in the urban and regional economic field, which argues that regional (and urban) units are increasingly relevant for the advancement of a country, as innovation leads increasingly rely on knowledge that tends to remain localised (Glaeser and Gottlieb (2009)).

models featuring a single R&D sector and monopolistic competition (e.g., Romer (1990); Grossman and Helpman (1991)), our model economy features a market structure where each commodity is produced by an endogenous number of firms, and where each firm engages in R&D and competes in Cournot fashion.[3]

We show that economies that start out from a situation where the gap in knowledge externalities between regions is not too large are associated with a unique dispersed equilibrium. In this case, the productivity advantage held by the advanced region is not large enough to compensate for its higher factor price, implying a diminished incentive for firms to set up operations in the region holding the productivity advantage. Workers, on the other hand, have little incentive to move as migration is costly and the wage differential between the advanced and the lagging regions is not high enough.

In this context, a stronger knowledge spillover widens inter-regional income inequality, although the economy's growth rate increases, as higher knowledge spillovers boost overall R&D productivity. Notably, we show that the gains in productivity levels associated with higher spillovers are bigger for the lagging region, since reductions in the productivity gap between regions make firms more sensitive to wage differentials. Intuitively, as R&D becomes more productive more industries choose to operate in the region experiencing productivity gains. Since labour costs are higher in the advanced region, more firms set up in the lagging region when knowledge spillovers increase, than in the advanced region for an equivalent increase in knowledge spillovers there.

Welfare in each region depends on static components related to market imperfection, the relative wage and the industry share, and on dynamic components associated with the rate of growth. These static and dynamic components may move in opposite directions as a result of higher spillovers, since the latter trigger a higher price mark-up but, also, higher long run

[3]High levels of R&D are frequently observed in oligopolistic markets which, arguably, seem to be better suited for innovation (specifically, process innovation or cost reducing R&D). The rationale being that, since the market share of a firm increases as its cost advantage over its rivals increases, less innovative firms will find their markets shrinking and will be forced by competitive pressures to respond to the actions of their rivals. In other words, innovation is essential to the survival of firms and R&D outlays become an integral part of a business (Baumol (2002)).

growth. We show that the growth gains associated with fewer and more productive firms outweigh the static losses associated with a lower degree of competitiveness (higher prices). Furthermore, we show that, under mild parameter restrictions, both regions experience welfare gains as spillovers in the lagging region strengthen. In essence, reducing regional disparities and promoting growth can be mutually compatible.

Throughout the paper we concentrate on spillovers across regions rather than technological gaps (or absorptive capacity). The model, however, naturally extends to economies not lying on the technological frontier.[4]

The paper is closely related to the literature on endogenous growth and endogenous industry location pioneered by Grossman and Helpman (1991). They show that, when knowledge spillovers are global, initial conditions determine the pattern of trade and growth. In their model, however, there is a single R&D sector that innovates and each innovation is used to produce a new variety. Hence, innovation takes place only in the country with the larger stock of knowledge capital. In contrast we show that dispersion of innovation activities can be compatible with regions of different sizes and of, potentially, different knowledge endowments. Building on Grossman and Helpman, Martin and Ottaviano (1999) consider the role of global and local R&D spillovers; they show that geography influences productivity only if spillovers are local and, then, study the effect of lower transport costs on agglomeration and growth. Rather than assume one or the other type of spillover, we take into consideration both the strength and spatial extent of knowledge externalities, and show how regional disparities in innovation-enhancing activities, rather than transport frictions, can affect both industry location and growth.

More importantly, since in our model the location of both firms and workers is endogenous, our analysis can accommodate changing patterns of agglomeration and dispersion. This in itself is a novel framework, as far as we are aware, and complements the emerging empirical literature on tacit knowledge flows and the geography of economic activities.

Notably, we differ from the standard literature by assuming that product variety is exogenous, while the number of firms operating in each sector is

[4]In appendix 8.7 we show that incorporating both spillovers and technological gaps would not alter our qualitative results.

endogenous and, depending on their location, firms obtain different returns. This enables us, among other things, to account for agglomeration without appealing to different initial endowments, or falling transport costs.[5] Furthermore, since the model we propose includes in-house R&D, endogenous firms' market power, knowledge spillovers, wages differing across locations and workers that can move between regions, it accounts for a richer set of equilibria. For instance, we show that a high propensity to move (low migration costs) does not necessarily lead to complete agglomeration. This depends on the spatial distribution of the population, which affects the real wage differential between different locations. If workers do not move, it is firms that move in response to a decrease in the productivity gap between regions, and this brings about income convergence between regions and higher growth.

In our setup, knowledge spillovers directly affect the costs of innovation and, thereby, the equilibrium growth rate, the equilibrium location of industries and their degree of competitiveness. Transport costs do not directly influence the input costs of innovation, however, they do influence income differentials between regions (through their effect on the price index) and, thereby, the equilibrium distribution of population. The latter is a result that resonates with recent spatial economic models incorporating trade costs and labour mobility, where part of the spatial variation in income across regions is explained by variations in trade costs (e.g., Allen and Arkolakis (2014)). Differently from this literature, our focus is on the endogenous emergence of industry clusters, market power concentration, growth and inter-regional inequality. Spatial differences in knowledge externalities (coupled with migration costs) not only ensure uniqueness and existence of a spatial equilibrium but are key for industry location and growth. Crucially, our model is capable of explaining dispersed innovation activity in the presence of low transportation costs.[6]

[5] Tabuchi et al. (2014) also have migration costs acting as the dispersion force. In their paper, however, the distribution of activities is determined by the interplay between labour productivity and migration costs; moreover, in their model, technological progress is exogenous and affects all regions equally. In contrast, we assume that firms' ability to capture knowledge differs across regions, and identify knowledge spillovers as an agglomeration force.

[6] Our work also resonates with recent literature on the dynamic impact of spatial frictions focusing on the interaction between migration restrictions, market size and

Finally, the framework we develop in this paper also makes a theoretical contribution to the literature linking R&D, market concentration, economic integration and endogenous growth (e.g., Peretto (2003); Impulliti and Licandro (2018)). By developing a two location dynamic model with homogenous firms that compete à la Cournot and endogenously choose the extent to which they innovate though R&D, we can combine a number of features (such as imperfect competition, dynamic innovation, spatial frictions) in an extremely tractable way. Thereby, elucidating the mechanism behind the interdependence of the number of firms and the rate of growth, as well as the interaction between technology spillovers, spatial concentration and growth.

The paper is organised as follows. Section 2 describes the model. Section 3 analyses the equilibrium, first conditional on a given population distribution across regions, and then conditional on individuals' location choice. Section 4 analyses the effect of knowledge spillovers on the patterns of agglomeration, or dispersion, of activities and individuals. Section 5 explores the implications for growth and inter-regional income inequality. Section 6 briefly discusses the role of transport costs, and Section 7 concludes. All proofs are presented in the Appendix which also includes Table 1 detailing the notation (parameters and variables) used throughout the paper.

2 The Model

We consider a two-region setup (North and South) where the same disembodied form of technology (e.g., blueprints, intangibles) may be adopted in the whole economy, but one region is better than the other at capturing outside (non-codified) knowledge. As a result, the region with the larger knowledge spillovers experiences lower innovation costs and, thereby, higher productivity. We assume that knowledge is embedded in labour hired and used in R&D activities. Workers are uniformly skilled and are used for both R&D as well as the production of goods. They are perfectly mobile within a region but imperfectly mobile between regions. In what follows,

productivity (e.g., Desmet et al. (2017)). We, in contrast, focus on the determinants of firms' location and innovation in relation to spatial frictions in knowledge spillovers and workers' propensity to move between regions.

we restrict attention to the description of technologies and preferences in the Northern region. Analogous expressions apply to the Southern region. Whenever a distinction is needed variables and/or parameters for the South are denoted with a star, $*$.

Time t is continuous and goes from zero to infinity. The economy as a whole has a constant, exogenous number of identical, infinitely-lived, skilled workers, L^w, each endowed with one unit of labour-time supplied inelastically.

It is assumed that, at date zero, a share η of individuals are born and reside in the North, while the remaining share $\eta^* = 1 - \eta$ is born and reside in the South.

The labour market is perfectly competitive and workers incur a positive non-pecuniary cost of migration. The latter accounts, among other things, for the cost of adapting to a new environment, moving away from friends and family, and similar. As made clear later on, such a migration cost allows a steady-state equilibrium in which individuals stay put while firms, responding to shocks, set up operations in one region or the other.

Preferences and technologies are described next. Individuals derive utility from the consumption of diverse goods with preferences given by

$$U = \int_0^\infty \left[\sum_{j=1}^{N} \alpha_j \log \left(c_{j,t} \right) \right] e^{-\rho t} dt \ , \tag{1}$$

where α_j is a parameter of the taste for variety j, with $\sum_{j=1}^{N} \alpha_j = 1$, $N > 2$ represents the exogenous set of commodities produced in the whole economy, $c_{j,t}$ is the consumption of variety j ($j = 1, ..., N$,) and $\rho > 0$ is the rate of time preference. To simplify we impose $\alpha_j = \alpha = 1/N$. Lifetime utility, expression (1), is slightly different from that used in standard models in the new economic geography literature (NEG). In particular, there is no homogeneous (agricultural) sector good which can be traded at no cost between regions.[7] The budget constraint of an individual residing in the

[7] In NEG models this is the device used to equalise the wage of (unskilled) individuals in both regions. Another point of departure with the standard literature is the use of a Cobb-Douglas felicity function instead of a standard CES form. Such formalisation, however, is made to simplify the analysis. Detailed calculations, available from the authors upon request, show that none of the results we derive hinge on this specification.

North is given by

$$E_t = \sum_{j=1}^{\gamma N} \tau_D p_{j,t} c_{j,t} + \sum_{j=\gamma N+1}^{N} \tau_I p^*_{j,t} \bar{c}_{j,t} \ , \tag{2}$$

where E_t denotes the per-capita level of expenditure, $p_{j,t}$ ($p^*_{j,t}$) is the price of commodity j produced in the North (South), τ_D and τ_I stand for transport (iceberg) costs, and γ represents the (endogenous) share of industries located in the North. Accordingly, γN is the number of commodities produced in the North. The upper-bar indicates the consumption of a good produced in the foreign region (imported good).

The iceberg costs can also be interpreted as capturing the quality of infrastructure within a region, τ_D, or between regions, τ_I (see Martin (1999)). In line with the literature we impose the restriction $1 \leq \tau_D \leq \tau^*_D < \tau_I = \tau^*_I$, that is, transport costs are less costly within a region than between regions, and infrastructure in the North is of better quality than in the South. In each period t, every variety j of commodities ($j = 1, ..., N$) is produced by an endogenous number of identical firms $Q_{j,t} > 1$, each designated by q_j ($q_j = 1, \ldots, Q_{j,t}$), competing"à la Cournot".[8] Both $Q_{j,t}$ and γ are crucial variables of the model as they are related to the extent of firms' market power and the extent of firms' agglomeration, respectively. To keep the analysis simple and in line with empirical evidence, we suppose that in each industry j, every firm q_j engages simultaneously in the production of good j and in R&D (see, Dasgupta and Stiglitz (1980)).

Denoting by $X_{q_j,t}$ the quantity of good produced by firm q_j, the total amount of variety j is $X_{j,t} = \sum_{q_j=1}^{Q_{j,t}} X_{q_j,t}$. The technology of production of every firm is given by

$$X_{q_j,t} = \left(A_{q_j,t}\right)^{\nu} L^X_{q_j,t} \ , \tag{3}$$

where $0 < \nu < 1$ denotes the returns to knowledge, $L^X_{q_j,t}$ is the quantity of labour devoted to the production of a good, and $A_{q_j,t}$ is the stock of (specific) knowledge-capital produced and used by firm q_j. Each firm can improve its productivity over time by engaging in in-house R&D via a process

[8] Notice that there are no segmented markets in this economy. Specifically, we assume a world market for each commodity $j = 1, ..., N$, in which firms cannot price discriminate. Notice also, that we consider a symmetric Cournot equilibrium, which entails industrial specialisation within regions.

of cost reduction driven by the accumulation of firm-specific knowledge-capital (cf. Peretto (1996)). The production function (3) does not include a fixed or sunk cost, only variable costs. However, notice that in the (infinite horizon) open-loop dynamic model we present, firms' R&D expenditures are effectively sunk at every point in time by all active firms. Hence, they are formally equivalent to fixed production or maintenance costs (Spence (1984)).

The number of units of knowledge-capital produced per unit of time by each firm q_j is given by

$$\overset{\bullet}{A}_{q_j,t} = \delta L^A_{q_j,t}\left[\sum\nolimits_{j=1}^{\gamma N}\sum\nolimits_{q_k=1}^{Q_{k,t}} A_{q_k t} + \mu \sum\nolimits_{j=\gamma N+1}^{N}\sum\nolimits_{q_k=1}^{Q_{k,t}} A^*_{q_k t}\right] \ , \qquad (4)$$

where $\delta > 0$, is the R&D productivity parameter. The term $L^A_{q_j,t}$ is the amount of labour devoted to R&D, and μ denotes the degree of inter-regional knowledge spillover. Expression (4) represents the flow of knowledge generated by R&D. The R&D technology (4) above exhibits constant returns to scale in the factor that is accumulated, i.e knowledge. Within each region firms are able to take full advantage of each other's knowledge, also helped by intra-regional perfect mobility of workers; however, outside each region knowledge spillovers are not perfect and are region specific. Formally, this amounts to imposing the restriction: $0 \leq \mu^* < \mu \leq 1$. [9]

To close the model, we set the labour constraint in the North as

$$\eta L^w = L^X_t + L^A_t \ , \qquad (5)$$

where $L^X_t = \sum_{j=1}^{\gamma N}\sum_{q_j=1}^{Q_{j,t}} L^X_{q_j,t}$ and $L^A_t = \sum_{j=1}^{\gamma N}\sum_{q_j=1}^{Q_{j,t}} L^A_{q_j,t}$ denote the *aggregate* quantity of labour employed in the production of differentiated goods and employed in R&D, respectively. In the sequel since the total number of workers is fixed, without loss of generality, we set $L^w = 1$.

[9] It would be possible to also introduce imperfect intra-regional knowledge spillovers. In this case, the technology (4) would read: $\overset{\bullet}{A}_{q_j,t} = \delta L^A_{q_j,t}\left[(1-\lambda) A_{q_j,t} + \lambda \sum_{j=1}^{\gamma N}\sum_{q_k=1}^{Q_{k,t}} A_{q_k t} + \mu \sum_{j=\gamma N+1}^{N}\sum_{q_k=1}^{Q_{k,t}} A^*_{q_k t}\right]$, where λ would stand for the degree of intra-regional knowledge spillover, and verify: $0 \leq \mu^* < \mu < \lambda^* < \lambda \leq 1$. However, imperfect intra-regional knowledge spillovers are not essential for our results and seriously complicate the model. The more general analysis of the model for $0 \leq \mu^* < \mu < \lambda^* < \lambda \leq 1$ is available from the authors upon request.

3 Equilibrium

We proceed in three steps. First, we describe the behaviour of individuals and firms. Second, we derive the equilibrium of the model assuming a given population distribution across regions, $\eta \in [1/2, 1]$, and no migration. Third, we analyse the choice of location of individuals, and study the spatial equilibrium. As regards market structure, we assume Cournot competition with free entry in the goods market (see below), and perfect competition in the labour market. We denote by w_t the price of labour in the North and normalise the price of labour in the South to one, that is $w_t^* = 1$.

3.1 Individuals and Firms

Each individual maximises lifetime utility (1) subject to the budget constraint (2). The solution of this programme is standard. The demand function for consumption good j of an individual living in the North is given by $c_{j,t} = E_t/(N\tau_D p_{j,t})$ if $0 < j \leq \gamma N$ and $\overline{c}_{j,t} = E_t/(N\tau_I p_{j,t}^*)$ if $\gamma N < j \leq N$. The aggregate demand function for variety j, denoted $c_{j,t}^d$, is thus given by,

$$c_{j,t}^d = \frac{1}{N}\left(\frac{\eta E_t}{\tau_D p_{j,t}} + \frac{(1-\eta)\, E_t^*}{\tau_I p_{j,t}}\right) . \tag{6}$$

Firms perform two activities: (i) they produce and sell in an oligopolistic market (competition "à la Cournot") with free entry and exit and, (ii) they generate new pieces of knowledge-capital via their in-house R&D using labour.

The market equilibrium we consider is a symmetric Nash equilibrium in open-loop strategies. Denote by $s_{q_j} = [X_{q_{j,t}}, L^A_{q_{j,t}}, A_{q_{j,t}}]$ for $t \geq 0$ firm q_j's strategy vector. To make the analysis simple, we assume that entry and exit involve zero costs,[10] meaning that the number of firms can freely adjust to its equilibrium level. In equilibrium firms commit to time

[10]Obviously this is a strong assumption. Effectively it is implying that R&D knowledge (as embodied in labour hired and used in R&D activities) is substitutable across firms and varieties. It is made in order to keep the model and its dynamics tractable. Notice though that R&D expenditure forms part of firm's total costs and is determined endogenously in market equilibrium, cf. Peretto (1996), p.897. Prospective entrants are aware that these costs have to be incurred in the post-entry equilibrium.

paths for production, R&D, and labour at time t, with entry and exit determining the number of active firms, Q_j. Therefore, at time t the vector $[Q_j, s_1, ..., s_j, ...s_{Q_j}]$ is an instantaneous equilibrium with free entry and exit if for all firms q_j (and in all sectors, $j = 1, ...N$)

$$\Pi_{q_j,0}[Q_j, s_1, ..., s_j, ...s_{Q_j}] \geq \Pi_{q_j,0}[Q_j, s_1, ..., s_j^{'}, ...s_{Q_j}] \geq 0 \qquad (7)$$

and for $Q_j > 1$

$$\Pi_{q_j,0}[Q_j + 1, s_1, ..., s_j, ...s_{Q_j+1}] \leq 0 \ , \qquad (8)$$

where $[Q_j, s_1, ..., s_j^{'}, ...s_{Q_j}]$ is the strategy vector when firm q_j deviates from its optimal time-paths while all other firms do not. Condition (7) requires that a firm maximises the sum of present values of its net profits while taking as given the behaviour of the other firms, and this value be non-negative. Condition (8) is a standard zero-profit condition. Accordingly, each firm q_j (in the North) maximises

$$\Pi_{q_j,0} = \int_0^{\infty} [p_{j,t} X_{q_j,t} - w_t L_{q_j,t}^X - w_t L_{q_j,t}^A] e^{-\int_0^t r_u du} dt \ , \qquad (9)$$

subject to equations (3) and (6), and taking as given the law of motion of knowledge-capital in the R&D sector (4) and the real interest rate, r_t. After substitution, the current value Hamiltonian becomes

$$CVH_{q_j,t} = \left\{ \begin{array}{c} X_{q_j,t} \left[\Omega_t \left(\sum_{q_j=1}^{Q_{j,t}} X_{q_j,t} \right)^{-1} - w_t \left(A_{q_j,t} \right)^{-\nu} \right] - w_t L_{q_j,t}^A \\ +\xi_t \delta L_{q_j,t}^A \left[\sum_{j=1}^{\gamma N} \sum_{q_k=1}^{Q_{k,t}} A_{q_k t} + \mu \sum_{j=\gamma N+1}^{N} \sum_{q_k=1}^{Q_{k,t}} A_{q_k t}^* \right] \end{array} \right\} ,$$

where

$$\Omega_t \equiv \frac{1}{N} \left[\frac{\eta E_t}{\tau_D} + \frac{(1-\eta) E_t^*}{\tau_I} \right] \ , \qquad (10)$$

is taken as given by each firm. The term ξ_t is the co-state variable associated with (4) and $\sum_{q_j=1}^{Q_{j,t}} X_{q_j,t} = c_{j,t}^d$. In this problem, the choice variables are: $X_{q_j,t}$ (production of commodity j), $L_{q_j,t}^A$ (quantity of labour employed in R&D) and $A_{q_j,t}$ (the path of knowledge-capital). The first order conditions are given by: $\partial CVH_{q_j,t}/\partial X_{q_j,t} = 0$, $\partial CVH_{q_j,t}/\partial L_{q_j,t}^A = 0$ and $\partial CVH_{q_j,t}/\partial A_{q_j,t} = -\dot{\xi}_t + r_t \xi_t$.

The transversality condition is $\lim_{t\to\infty} \xi_t A_{q_j,t} e^{-\int_0^t r_u du} = 0$.

Note that, the only plausible Nash-equilibrium of the model is associated with $\partial CVH_{q_j,t}/\partial L^A_{q_j,t} = 0$, at which the marginal revenue of an extra unit of labour devoted to R&D equals its cost (here w_t) and there is no incentive for firms to deviate from the strategy $L^A_{q_j,t} > 0$. In fact, if $\partial CVH_{q_j,t}/\partial L^A_{q_j,t} < 0$ the marginal revenue of an extra unit of labour devoted to R&D is always negative implying no labour allocated to R&D in equilibrium (i.e., $L^X_t = \sum_{j=1}^{\gamma N} \sum_{q_j=1}^{Q_{j,t}} L^X_{q_j,t} = L$). Such an outcome can be ruled out, as any firm would have an incentive to deviate from the $L^A_{q_j,t} = 0$ strategy and choose $L^A_{q_j,t} > 0$ to produce new pieces of knowledge-capital, thereby improving their productivity and thus profitability vis-a-vis their rivals. On the other hand, if $\partial CVH_{q_j,t}/\partial L^A_{q_j,t} > 0$ the marginal revenue of an extra unit of labour allocated to R&D is always positive implying all labour allocated to R&D in equilibrium (i.e., $L^A_t = \sum_{j=1}^{\gamma N} \sum_{q_j=1}^{Q_{j,t}} L^A_{q_j,t} = L$). The latter can also be ruled out, as it leads to a meaningless solution with no production of (differentiated) goods (cf. Peretto (1996)).

Straightforward computations yield

$$X_{j,t} = \frac{\Omega_t}{w_t \left(A_{q_j,t}\right)^{-\nu}} \left(1 - \frac{X_{q_j,t}}{X_{j,t}}\right) , \tag{11}$$

$$w_t = \xi_t \delta \left(\sum_{j=1}^{\gamma N} \sum_{q_k=1}^{Q_{k,t}} A_{q_k t} + \mu \sum_{j=\gamma N+1}^{N} \sum_{q_k=1}^{Q_{k,t}} A^*_{q_k t} \right) , \tag{12}$$

$$r_t = \frac{w_t \nu X_{q_j,t} \left(A_{q_j,t}\right)^{-\nu-1}}{\xi_t} + \delta L^A_{q_j,t} + \frac{\dot{\xi}_t}{\xi_t} . \tag{13}$$

Equation (11), where $X_{j,t} \equiv \sum_{q_j=1}^{Q_{j,t}} X_{q_j,t}$, is the total production of variety j, implicitly gives the best response of firm q_j (i.e., $X_{q_j,t}$) to the choice of production of good j of the other firms. Note that this condition is used to determine the price level of each variety.

Since $p_{j,t} = \Omega_t \left(c^d_{j,t}\right)^{-1} = \Omega_t \left(X_{j,t}\right)^{-1}$ (see 6), we obtain,

$$p_{j,t} = \frac{w_t \left(A_{q_j,t}\right)^{-\nu}}{\left(1 - X_{q_j,t}/X_{j,t}\right)} . \tag{14}$$

Equation (14) shows that the price of each variety is determined by the product between its marginal cost of production ($w_t \left(A_{q_j,t}\right)^{-\nu}$) and the markup $1/(1-X_{q_j,t}/X_{j,t}) > 1$. Equation (12) is a static condition equating the marginal cost (w_t) and benefit ($\xi_t \delta \left(\sum_{j=1}^{\gamma N} \sum_{q_k=1}^{Q_{k,t}} A_{q_k t} + \mu \sum_{j=\gamma N+1}^{N} \sum_{q_k=1}^{Q_{k,t}} A^*_{q_k t} \right)$) of an additional unit of labour spent in R&D. Finally, equation (13) is a dynamic condition stating that the return (r_t) of a new piece of knowledge-capital depends on three factors: the productivity gains from knowledge accumulation (cost reducing effect of R&D, (first term on the *rhs*)), the future units of knowledge-capital (second term on the *rhs*) and the change in the shadow price of knowledge-capital (third term on the *rhs*). [11]

Using (9), we derive the standard condition $r_t \Pi_{q_j,t} = \overset{\bullet}{\Pi}_{q_j,t} + \pi_{q_j,t}$, where $\pi_{q_j,t} = p_{j,t} X_{q_j,t} - w_t L^X_{q_j,t} - w_t L^A_{q_j,t}$ is the instantaneous profit of a firm. Given that there is free entry and exit, we have $\Pi = \overset{\bullet}{\Pi} \leq 0$. Using (3) and (14), we then obtain,

$$\frac{w_t L^X_{q_j,t}}{\left(1 - X_{q_j,t}/X_{j,t}\right)} - w_t L^X_{q_j,t} - w_t L^A_{q_j,t} \leq 0 \; . \tag{15}$$

This is the zero profit condition for every firm $q_j = 1, ..., Q_j$.

By combining (13), (3) and the zero profit condition above (15) we obtain the rate of return to R&D in the industry (partial) equilibrium, that is

$$r_t = \nu \left(1 - X_{q_j,t}/X_{j,t}\right) \left(\frac{w_t L^A_{q_j,t}}{X_{q_j,t}/X_{j,t}} \right) \frac{1}{\xi_t A_{q_j,t}} + \delta L^A_{q_j,t} + \frac{\overset{\bullet}{\xi}_t}{\xi_t} \; . \tag{16}$$

Noticeably, the cost reducing effect of R&D can be decomposed into the return to R&D earned by the increase in the market share $\nu \left(1 - X_{q_j,t}/X_{j,t}\right)$, and the returns to R&D earned for a given market share $\frac{w_t L^A_{q_j,t}}{X_{q_j,t}/X_{j,t}}$. The latter implies that the incentive to innovate is the incremental profit (*gross-profit effect*), while the former captures the potential gains from an increase in rivals' market shares (*business-stealing effect*). The gross-profit effect is increasing in firms' market power (decreasing in the number of firms) while the business-stealing effect works in the opposite direction and implies that returns to R&D increase with the number of firms. This is in keeping with

[11] Note that, under perfect foresight, a firm can finance its own R&D through debt or equity, as the no arbitrage condition (13)) implies that the rate of return from a risk-less loan must be equal to the cost of financing R&D by borrowing (cf. Peretto (1996), p. 904).

many IO models of R&D and is in line with Peretto (1996).[12]

3.2 Intra-regional equilibrium

Intra-regional equilibrium determines the number of firms in each sector, quantities and prices for given shares of population (η and $1-\eta$). We focus on an *intra*-regional symmetric equilibrium, whereby prices and quantities of goods are identical within a region but, as it will be the case, different between regions. Formally,

Definition 1 *For all sectors j in a given region, an intra-regional symmetric equilibrium is characterised by the number of firms in each sector Q_j, quantities X_{q_j} and prices p_j that are identical for all firms q_j. For the North, we have*

1. $Q_{j,t} = Q_t$ *for all* $j \leq \gamma N$*,*
2. $X_{q_j,t} = X_{j,t}/Q_t = X_t/Q_t$*,* $L^X_{q_j,t} = L^X_{j,t}/Q_t = L^X_t/(\gamma N Q_t)$ *and* $L^A_{q_j,t} = L^A_{j,t}/Q_t = L^A_t/(\gamma N Q_t)$ *for all* q_j *and all* $j \leq \gamma N$*, and*
3. $p_{j,t} = p_t$ *for all* $j \leq \gamma N$*.*

And similarly for the South.

Using the definition above, and combining the labour constraint (5) and the zero-profit condition (15), we obtain aggregate employment in the production of goods and R&D

$$L^X = \eta \left(1 - \frac{1}{Q}\right) , \tag{17}$$

$$L^A = \eta \frac{1}{Q} . \tag{18}$$

That is, the share $1/Q$ of labour force is allocated to the production of R&D and, since $\eta \geq 1/2$ and $\mu > \mu^*$, the share of workers employed in each

[12]In Peretto (1996) the return to R&D is non-monotonic in the number of firms as a result of the tension between the gross profit and the business stealing effects. Consequently, spillovers exert different influences on incentives to undertake R&D depending on which effect dominates. In our model, in contrast, the gross-profit effect always dominates the business stealing effect in (general) equilibrium (see Appendix 8.2). Recent empirical work by Bloom et al. (2013) supports this result.

sector will be larger in the North (See proposition 1 below). Interpreting the number of firms in each sector as a proxy for the degree of markets' competitiveness ($Q = X_j/X_{q_j}$) it follows that: as firms' market share in a given sector increases ($Q \downarrow$), firms allocate more labour to R&D and less to the production of commodities.

To ensure existence, and other results to follow, from now on we impose the following restriction on the returns to knowledge parameter, ν.

Assumption 1 *Let* $\nu > \frac{\rho}{\eta\delta}$.

As shown in Appendix 8.1, Assumption 1 is a necessary condition to ensure a strictly positive long-term growth rate and is easily satisfied for plausible parameter values.

In Appendix 8.1 we also formally demonstrate that a steady-state equilibrium exists, that the adjustment to steady state is instantaneous, and that growth of knowledge-capital is the same in the North and in the South. To simplify we assume that every firm starts with the same endowment of knowledge-capital ($A_{q_j,0} = A^*_{q_j,0}$), though this is not essential for the results.[13] The following proposition summarises the main properties of the steady state.

Proposition 1 *(*STEADY STATE*) For any given population distribution* $\eta \in [1/2, 1]$

(a) There exists a unique steady-state equilibrium solution to which the economy jumps immediately,

(b) The steady state is characterised by a constant (and common to all firms) level of growth of capital-knowledge, $g = g^*$*; and a constant and equal number of firms in each sector,* $Q = Q^*$*, where* $Q > 1 + \frac{1}{\nu}$*,*

(c) $1/2 \leq \eta < \gamma$.

[13] If $A_{q_j,0} \neq A^*_{q_j,0}$, the knowledge gap between firms of different regions would remain constant over time as the economy jumps immediately to the steady state (see Appendix 8.1).

Proof. See Appendix 8.1 ■

Regions grow at the same rate in steady state, but crucially relative productivity levels differ according to a region's ability to make the most of innovation. Also, the number of firms in each sector is the same for both regions in symmetric equilibrium (part b), and the more productive region has a higher share of firms with $\gamma > \eta$ (part c). The latter is supported by ample empirical evidence (Carlino and Kerr (2014)) documenting that R&D activities are more concentrated than employment (the measure of agglomeration economies).

For analytical tractability we focus on the case where N is large ($N \to \infty$); an assumption that is not severely restrictive, as the number of commodities produced in the whole economy, N, is exogenously fixed in our setup.[14] In this case, the equilibrium system expressed in terms of γ and Q (derived in Appendix 8.1) reads as

$$(1-\eta)\left[1+\mu^{*}\frac{\gamma}{(1-\gamma)}\right]=\eta\left[1+\mu\frac{(1-\gamma)}{\gamma}\right] \ , \tag{19}$$

$$\rho = \delta\eta(1+\mu\frac{1-\gamma}{\gamma})(\nu\frac{Q-1}{Q}-\frac{1}{Q}) \ . \tag{20}$$

Equation (19) draws from $g = g^{*}$ (Proposition 1b) together with (4), (5) and (16), while equation (20) draws from $Q = Q^{*}$ (Proposition 1b) together with (4), (12), (17) and (18) and the fact that, at the steady state, $r_t = \rho$. From equation (19) we can express γ as a function of exogenous parameters, which then, through (20), gives Q as function of exogenous parameters.

Starting from the equilibrium share of sectors, γ, few comments are in order. First, there is a differentiated spillover effect according to which the share of industries located in each region is positively related to the region specific spillover (for details see Appendix 8.3). This effect is reinforced by a factor endowment effect (captured by the terms in η in (19)) according to which the share of sectors located in a given region is positively related to its population share: $d\gamma/d\eta > 0$ (readily follows from (19)). Intuitively, a

[14] In Appendix 8.1 it is shown that γ is independent of N and that a sufficient condition to ensure a unique solution for Q is $N > 2$, which we assume from the start. Setting $N \to \infty$ provides us with closed form solutions for the equilibrium system; however, numerical simulations corroborate our results for N ranging from small to very large values.

larger population share creates a larger market for commodities and allows firms to conduct R&D at a greater scale. This is also an agglomerating force, and reminiscent of a home market effect whereby the larger region hosts a more than proportionate share of industries (see Krugman (1980)).

Turning to the equilibrium number of firms, Q, we can establish that $dQ/d\mu < 0$, $dQ/d\mu^* < 0$ and $dQ/d\eta < 0$.[15] Intuitively, when μ^* and/or μ take greater values, the productivity of labour in R&D improves. As a result, firms allocate more labour to R&D (there is an increase in L^A) and less to the manufactured good (there is a decrease in L^X). Due to the fixed labour supply in each region, the reduction in L^X is equivalent to a contraction in production and hence a reduction in the markets' competitiveness. Indeed, the zero profit condition (15) shows that the markup increases to compensate the decrease in L^X. Thereby, the number of firms constituting each industry adjusts downwards, that is some firms are exiting or become inactive and the price of goods increases (see (14)).

Lastly, a higher concentration of people in the North, ceteris paribus, reduces competitiveness ($dQ/d\eta < 0$). In fact, a larger pool of workers allows firms to conduct R&D at a larger scale, which pushes up L^A (relative to L^X) and increases the mark up over marginal costs. The relationship linking labour allocation, population shares and Q follows next.

Up to this point we have seen that the forces affecting the distribution of firms between regions vary in nature: one acts directly through firms' technologies, while the other essentially relies on the population distribution. To establish which force dominates, it is crucial to examine what affects individuals' choice to migrate. We analyse this issue in Section 3.3. Before proceeding, though, we need to evaluate prices and wages.

Turning first to wage determination: note that total production of commodity j must be equal to the number of firms, Q, times quantity produced by each firm, (3); then, using the definition of Ω_t (see (10)) and combining it with (5), (15) and (17), we obtain the wage equation for the North,

$$w = \gamma \left[\frac{E}{\tau_D} + \frac{(1-\eta)}{\eta} \frac{E^*}{\tau_I} \right] \, .$$

[15]See Appendix 8.3 for analytical derivations.

Using the same method, we derive the wage for the South,

$$w^* = (1-\gamma)\left[\frac{\eta}{(1-\eta)}\frac{E}{\tau_I} + \frac{E^*}{\tau_D^*}\right] .$$

Taking into account that the level of expenditure per capita is determined by the wage ($E = w$ and $E^* = w^*$) and $w^* = 1$ (by assumption), the level of wage in the North, w, must adjust so that both conditions are simultaneously satisfied. After straightforward computations, we obtain the following expression

$$\frac{\eta}{(1-\eta)}\left[\frac{\eta}{(1-\eta)}w + \frac{\tau_I}{\tau_D^*}\right] = \frac{\gamma}{1-\gamma}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right] , \qquad (21)$$

implicitly defining w as function of all exogenous variables. Therefore, we establish the following proposition.

Proposition 2 *(*WAGE IN THE NORTH*). For any given $\eta \in [1/2, 1]$, an equilibrium level w exists and is unique; moreover $w > w^* = 1$.*

Proof. See Appendix 8.4. ■

In line with empirical evidence (Head and Mayer (2010); Redding (2011)) we find that the wage differential across regions is positively correlated with the population share in the larger region ($dw/d\eta > 0$).[16] Intuitively, a larger share of workers implies a larger labour supply of labour that, in turn, prompts the location of a greater share of sectors ($d\gamma/d\eta > 0$) and drives up the wage ($dw/d\gamma > 0$). Note, also, that from (14) and $w^* = 1$ it immediately follows that $p/p^* = w$. Finally, transport costs are not crucial to explain the wage/price differential across regions. Indeed, $w > w^*$ for any $1/2 \leq \eta < \gamma$ and no transport costs.[17]

Turning to the general price index, hereafter denoted by P, we obtain[18]

$$P = (\tau_D p)^{\gamma} (\tau_I p^*)^{1-\gamma} < P^* = (\tau_I p)^{\gamma} (\tau_D^* p^*)^{1-\gamma} , \quad \text{for} \quad \eta \geq 1/2 . \qquad (22)$$

[16]Specifically, we obtain $dw/d\eta > 0$, with $\lim_{\eta\to 0} w = \mu\tau_D^*/\tau_I < 1$ and $\lim_{\eta\to 1} w = \tau_I/(\mu^*\tau_D) > 1$. See Appendix 8.4.

[17]The role of transport costs is analysed in depth in Section 6.

[18]The choice of a Fisher price index is made for convenience. Since it is computed as a geometric mean, it will be easier for us to interpret some of the results we derive later, particularly, regarding the migration condition.

In the region that hosts a larger share of industries, a greater proportion of goods are purchased without incurring the high inter-regional transport cost, τ_I. Therefore, individuals residing in the North benefit not only from a higher wage, but also from a lower price index than individuals located in the South; a set of circumstances that should make the Northern region more attractive. As we shall see shortly, this will play a role in the analysis of the migration decision of individuals.

Finally, we close this subsection by writing down the common level of growth of knowledge-capital in the North and South, that is,

$$g \equiv \frac{\overset{\bullet}{A}_{q_j,t}}{A_{q_j,t}} = \delta L^A_{q_j,t} \left[\gamma NQ + \mu \left(1-\gamma\right) NQ \right] \ . \tag{23}$$

Accordingly, growth depends on labour employed in R&D and on the relative production of knowledge (term in squared brackets). In symmetric equilibrium $L^A_{q_j} = L^A / \left(\gamma NQ\right)$, and using (18) into the above we obtain,

$$g = \delta\eta \frac{1}{Q} \left[\frac{\gamma + \mu \left(1-\gamma\right)}{\gamma} \right] \ . \tag{24}$$

Growth is determined by two endogenous variables: the number of firms per industry, Q, and the share of industries located in a given region, γ.

Since in each industry production and R&D compete for labour, a larger number of firms per industry reduces employment in R&D and, therefore, growth. Under Cournot competition this also implies a trade off between firm's market power and growth.

A larger concentration of industries located in the North, γ, also affects growth through the following channels: (i) a direct effect through the relative production of knowledge, whereby the higher the number of industries located in the North the larger the contribution of spillovers to the production of new knowledge, and (ii) an indirect effect through the firm's mark up, as a larger share of sectors in the North triggers a re-allocation of resources between production of varieties and R&D. It should be stressed at this stage that, although instructive, this account is not, as yet, conclusive since both Q and γ are endogenous variables affected by η, μ and μ^*, among other parameters. We shall come back to this point later on in the paper when we carry out the equilibrium growth analysis (see Section 5).

3.3 Spatial equilibrium

Having set out the behaviour of agents, conditional on given shares of individuals in each region (η and $1-\eta$), we now explore the conditions required for a spatial equilibrium. We begin by defining a spatial equilibrium.

Definition 2 *A population share $\hat{\eta} \in (0,1]$ is a spatial equilibrium, if individuals have no incentives to move away from the region in which they are originally located.*

To determine which region individuals choose to reside in, we compare levels of utility in the North and in the South. Recall that individuals incur a non-pecuniary migration cost ($m > 0$) when they move from a region to another. Accordingly, using lifetime utility (1) we derive the following condition,

$$\left| \begin{array}{c} \int_0^\infty \left[\sum_{j=1}^{\gamma N} \log\left(c_{j,t}\right) + \sum_{j=\gamma N+1}^{N} \log\left(\bar{c}_{j,t}\right) \right] e^{-\rho t} dt \\ - \int_0^\infty \left[\sum_{j=1}^{\gamma N} \log\left(\bar{c}^*_{j,t}\right) + \sum_{j=\gamma N+1}^{N} \log\left(c^*_{j,t}\right) \right] e^{-\rho t} dt \end{array} \right| \leq m \ .$$

The above condition states that as long as the differential in utility between North and South (i.e. $\Delta U = U - U^*$) is lower, in absolute value, than the cost of migration, then, individuals born and residing in a given region stay put. Using the property that the economy jumps immediately to the steady state, together with the individual demand functions, and expressions (14) and (22), we can simplify the above to obtain,

$$|\Delta U| = \frac{1}{\rho} \left| \log w \frac{P^*}{P} \right| \leq m \ . \tag{25}$$

Notice that, as $w > 1$ and $P < P^*$, it follows that $\Delta U > 0$ implying that migration never occurs from North to South. Indeed, in terms of utility, workers residing in the North fare better than those residing in the South, since they benefit from a higher wage and a lower price index. For later purposes, using (25) above and (22), the difference in the present value of utility reads as

$$\rho \Delta U = \log w + \log \left(\frac{\tau_I}{\tau_D} \right)^{\gamma} \left(\frac{\tau_D^*}{\tau_I} \right)^{1-\gamma} \ , \tag{26}$$

and, differentiating (26) with respect to η gives,

$$\rho \frac{d\Delta U}{d\eta} = \frac{1}{w}\frac{dw}{d\eta} + \frac{d\gamma}{d\eta} \log\left(\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}\right) > 0 \ . \tag{27}$$

Since $1 \leq \tau_D \leq \tau_D^* < \tau_I$, $d\gamma/d\eta > 0$ and $dw/d\eta > 0$, equation (27) implies that the utility differential is positively correlated with the population share in the North. In other words, a larger population in the North creates more incentives for individuals to migrate from the South to the North.

Depending on the size of m different scenarios emerge. To analyse the possible outcomes, let us define the cut-off value $\underline{m}$ such that $\Delta U - \underline{m} = 0$, that is the level of the migration cost at which individuals are indifferent between staying put or migrating. Then, if migration costs are high, $m > \underline{m}$, and (25) always holds with a strict inequality, that is $\Delta U < m$ when $\Delta U - \underline{m} = 0$, any initial population distribution is a dispersed spatial equilibrium. Next, consider the case where $m \in (0, \underline{m}]$, so that (25) holds with equality. Solving $\Delta U = \underline{m}$ for the population share η potentially gives two possible solutions for the population distribution: one for the case of migration from North to South and the other for migration from South to North. As mentioned above, migration from the North to the South cannot occur, hence this solution can be ruled out. We thus denote by $\underline{\eta}$ the unique possible solution for the population distribution. Since $\Delta U(\underline{\eta}) > 0$ and $d\Delta U/d\eta > 0$, it follows that $\underline{\eta} > 0$. Moreover $\underline{\eta} < 1$ requires us to assume:[19]

Assumption 2 $\left(\frac{\tau_I}{\tau_D}\right)^2 \exp\left[-\rho m\right] < \mu < 1$.

Combining this with the discussion above leads to the following proposition.

Proposition 3 *(*SPATIAL EQUILIBRIA*). Define $\underline{\eta}$ as the population distribution satisfying $\Delta U - \underline{m} = 0$. Then there exists a critical cut-off value for the migration cost, $\underline{m}$, such that*

[19]As shown in Appendix 8.4, $\lim_{\eta \to 1} w = \frac{1}{\mu^*}\frac{\tau_I}{\tau_D}$. So, at the limit $\eta = 1$, the utility differential (25) reads $|\Delta U| = \frac{1}{\rho}\left|\log\left[\frac{1}{\mu^*}\left(\frac{\tau_I}{\tau_D}\right)^2\right]\right|$. Since the term in the square bracket is greater than one, the equation $\Delta U = m$ has a solution for μ^* given by $\mu^* = \left(\frac{\tau_I}{\tau_D}\right)^2 \exp\left[-\rho m\right] < \mu$, as stated in Assumption 2; suitable values of m can be set to ensure that this condition is always satisfied.

(a) For $0 < m \leq \underline{m}$*: Any initial population distribution* $\eta_0 \in (0, \underline{\eta})$ *is a dispersed spatial equilibrium,* $\hat{\eta} = \eta_0$*;*

(b) For $0 < m \leq \underline{m}$*: Any initial population distribution* $\eta_0 \in [\underline{\eta}, 1]$ *is an agglomerated spatial equilibrium,* $\hat{\eta} = 1$*;*

(c) For $m > \underline{m}$*: Any initial population distribution* $\eta_0 \in (0, 1]$ *is a dispersed spatial equilibrium,* $\hat{\eta} = \eta_0$*.*

Proposition 3 establishes that, with migration, different types of spatial equilibria may arise. One is an agglomerated equilibrium: in part (b) individuals and firms locate in the North. In the remaining cases, we obtain a dispersed equilibrium where individuals and firms are located in both regions. Note, in particular, that because the utility differential between North and South is increasing in the share of workers living in the North, it is possible to sustain a dispersed equilibrium for relatively small migration costs provided that η is sufficiently low (part a). In other words, whether low migration costs are associated with complete agglomeration will depend on the spatial distribution of population, which affects the real wage differential between North and South.[20]

Next we link Proposition 3 with Proposition 1 that established conditions of existence and uniqueness for a potential dispersed equilibrium, requiring $\eta \in [1/2, 1]$. Recall that in the context of Proposition 1, individuals were assumed to stay put and migration costs had not been introduced. To proceed, we need to compare $\underline{\eta}$ and $1/2$. In principle, we can have $0 < \underline{\eta} < 1/2$ or $1/2 \leq \underline{\eta} < 1$. Indeed, if $\underline{\eta}$ is close to $1/2$ we can have $\Delta U(\underline{\eta}) > 0$, with $\underline{\eta} < 1/2$. However, this case can be ruled out as $\underline{\eta} < 1/2$ would contradict the condition of existence and uniqueness (Proposition 1). Hence, under Proposition 1, the solution to (25) necessarily implies $1/2 \leq \underline{\eta}$. Accordingly, we establish the following.

Corollary 1 *For* $m \in (0, \underline{m})$ *and* $\eta_0 \in [1/2, \underline{\eta})$*, there exists a unique spatial equilibrium* $\hat{\eta} = \eta_0$ *where individuals and firms are dispersed.*

Figure 1 illustrates both Proposition 3 and the Corollary.

[20]See Desmet et al. (2017) who use a quantitative framework to analyse the impact of migration frictions on local markets through the interaction of population density and productivity.

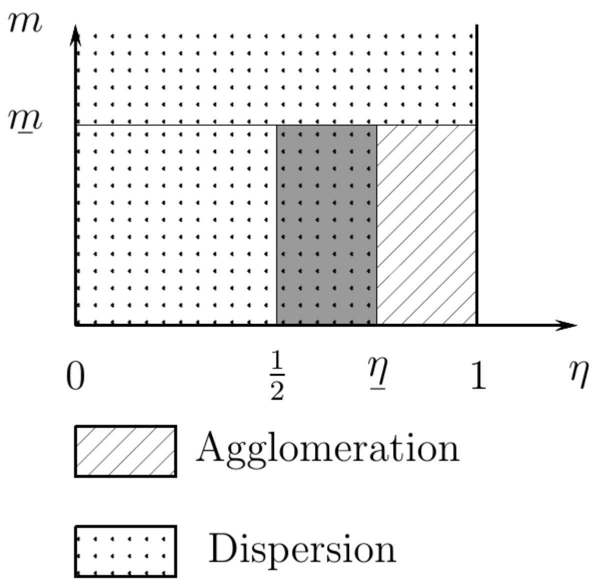

Figure 1: Illustration of Proposition 3 and Corollary 1 (shaded area)

4 Dispersion and Agglomeration

The previous sub-section has highlighted the role played by the non-pecuniary migration cost in establishing the kind of steady state we obtain. Here, we assess whether the inter-regional knowledge spillovers may or may not induce individuals and firms to agglomerate in a single (core) region. We carry out the analysis assuming that, initially, the economy is at a unique dispersed equilibrium (Corollary 1) and study how the knowledge spillovers affect the cut-off level $\underline{\eta}$ and, thereby, either reinforce the dispersed equilibrium or trigger a switch to an agglomerated equilibrium where all workers agglomerate in the North.

To this end, using (26), we define the following function,

$$F(\underline{\eta}) = \frac{1}{\rho}\left[\log w + \gamma \log\left(\frac{\tau_I}{\tau_D}\right) + (1-\gamma)\log\left(\frac{\tau_D^*}{\tau_I}\right)\right] - m = 0 \ .$$

Since μ is set higher than μ^* by default (and it needs to be fulfilled at all times), and both parameters are bounded from above, to study the effect of changes in spillovers we simply look at a decrease in μ^* for a given μ. This is as if spillovers are (in relative terms) more potent in the North.

Thus, applying the implicit function theorem to $F(\bullet)$ above, we obtain,

$$\frac{d\underline{\eta}}{d\mu^*} = -\frac{\frac{1}{w}\frac{\partial w}{\partial \mu^*} + \frac{\partial \gamma}{\partial \mu^*}\log\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}}{\frac{1}{w}\frac{\partial w}{\partial \underline{\eta}} + \frac{\partial \gamma}{\partial \underline{\eta}}\log\left(\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}\right)} > 0 \ .$$

The denominator of $\frac{d\underline{\eta}}{d\mu^*}$ is positive while the numerator is negative and, overall, the effect is positive. Accordingly, a reduction in μ^* (that is, an increase in the knowledge spillovers gap between North and South), shrinks the range of dispersed equilibria $(1/2, \underline{\eta})$. In the case of μ^* decreasing, ceteris paribus, the dispersed equilibrium becomes less likely. In terms of Figure 1, this amounts to a shrinking of the shaded area. A greater gap in knowledge spillovers leads to a greater productivity in the R&D sectors of firms located in the North, thereby the share of industries operating in the North increases. This has also a positive impact on the wage, making the Northern region more attractive. Indeed, by differentiating (26) with respect to μ^* it can be easily checked that the utility differential between North and South expands as μ^*decreases, that is

$$\frac{d\Delta U}{d\mu^*} = \frac{1}{\rho}\left[\frac{1}{w}\frac{dw}{d\mu^*} + \frac{d\gamma}{d\mu^*}\log\left(\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}\right)\right] < 0 \ , \tag{28}$$

making it more desirable for individuals to migrate. The following proposition summarises.

Proposition 4 *(*SPILLOVERS, DISPERSION AND AGGLOMERATION*). For any initial population distribution $\eta_0 \in [1/2, \underline{\eta})$, there exists a cut-off value $\overline{\mu}^*$ such that*

(a) $\hat{\eta} = \eta_0$ is a dispersed spatial equilibrium for all $\overline{\mu}^* < \mu^* < \mu$, and

(b) $\hat{\eta} = 1$ is an agglomerated spatial equilibrium for all $\mu^* < \overline{\mu}^* < \mu$.

The above implies that, ceteris paribus, higher regional dispersion in knowledge spillovers may lead to agglomeration. However, it may be also compatible with a dispersed equilibrium, if differences in spillovers between regions are initially not too large which is the most empirically plausible scenario.[21] In this case, the productivity advantage held by the most advanced

[21] The most recent empirical work, using U.S. firm level accounting data matched into

region is not high enough to compensate for its higher wage triggered by higher demand of labour in the R&D sector. Moreover, labour mobility will not help, as migration is costly and workers will not move. As a result, not all industries will operate in the region with higher spillovers.

5 Growth and inter-regional inequality

To study the implications for growth and inter-regional income inequality, we focus on the case where the economy is at an equilibrium in which not all R&D activities and not all workers are concentrated in the most productive region (Corollary 1). Evidence suggests that this is indeed the most plausible real world scenario.

Using $L_{q_j}^{A} = L_j^A/Q = L^A/(\gamma N Q_t)$, (18) and plugging in equation (23) the value of Q given in Appendix 8.1 (see equation(A.6)), we obtain the equilibrium growth rate,

$$g = \frac{1}{1+\nu}\left\{\nu\frac{\delta\eta}{\gamma}\left[\gamma+\mu\left(1-\gamma\right)\right]-\rho\right\} \quad . \tag{29}$$

Recall that in equilibrium the share of sectors γ depends on η, μ and μ^* (see (19)). Therefore, the equilibrium growth rate depends on the share of population located in the North and on knowledge spillovers. Differentiating the above we obtain[22]

$$\frac{dg}{d\eta} = \frac{\nu}{1+\nu}\left\{\delta\left(1+\mu\frac{1-\gamma}{\gamma}\right)\left(\frac{\mu^*\frac{\gamma}{1-\gamma}-\mu\left(\frac{\eta}{1-\eta}\right)^2\frac{1-\gamma}{\gamma}}{\mu^*\frac{\gamma}{1-\gamma}+\mu\frac{\eta}{1-\eta}\frac{1-\gamma}{\gamma}}\right)\right\} > 0 \, , \tag{30}$$

$$\frac{dg}{d\mu} = \frac{\nu}{1+\nu}\left\{\delta\eta\frac{\mu^*}{\mu^*\frac{\gamma}{1-\gamma}+\mu\frac{\eta}{1-\eta}\frac{1-\gamma}{\gamma}}\right\} > 0 \, , \tag{31}$$

$$\frac{dg}{d\mu^*} = \frac{\nu}{1+\nu}\left\{\delta\eta\frac{\mu}{\mu^*\frac{\gamma}{1-\gamma}+\mu\frac{\eta}{1-\eta}\frac{1-\gamma}{\gamma}}\right\} > 0 \, . \tag{32}$$

Accordingly, we establish the following.

U.S. Patent and Trademark Office data, documents that cross-regional spillovers are reduced in strength and correlated to geographical distance (Lychagin et al. (2016)).

[22] Details on computations in this Section are in Appendix 8.3.4

Proposition 5 *(*GROWTH*). Along the equilibrium path defined by Corollary 1, and for any $\overline{\mu}^* < \mu^* < \mu$*

(a) growth increases with η;

(b) growth increases with μ and μ^*;

(c) higher knowledge spillovers in the South bring higher overall growth, i.e., $\frac{dg}{d\mu^*} > \frac{dg}{d\mu}$.

Intuitively, a larger share of people working in the more productive region encourages growth (part a), and the productivity of R&D increases as knowledge spillovers increase (part b). Notably, this effect is stronger the larger the R&D spillover originating in the South (part c). Recall that, as the R&D sector becomes more productive two things happen: (i) firms allocate a greater amount of labour to R&D and, (ii) more industries choose to operate in the region experiencing productivity gains. Since labour cost is higher in the North ($w > w^*$), more industries set up in the South when μ^* increases than in the North for an equivalent increase in μ. As a result, μ^* has a stronger effect on growth than μ.

From (31) and (32) a related result emerges.

Proposition 6 *$\frac{dg}{d\mu}$ and $\frac{dg}{d\mu^*}$ increase with η.*

This implies that, the growth effect of higher spillovers in the North, or in the South, is amplified the higher is the share of workers in the North (η).

Turning to inter-regional income inequality, in the context of the present model, this is given by the wage gap between North and South. Recall that $w^* = 1$ and w comes from expression (21). The latter, is indirectly affected by μ and μ^* through γ.

Proposition 7 *(*SPILLOVERS AND INTER-REGIONAL INEQUALITY*). Along the equilibrium path defined by Corollary 1, and for any $\overline{\mu}^* < \mu^* < \mu$, inter-regional inequality increases (decreases) with μ (μ^*). The effect is larger the higher the share of workers in the North.*

Proof. See Appendix 8.5. ■

These findings suggest that strengthening knowledge spillovers in the South will lead to higher overall growth and a reduction of regional disparities. This is consistent, for example with empirical research documenting local spatial externalities between university research and high technology innovative activity, and the idea that promoting institutions that facilitate knowledge flows are important in supporting regional development (Jaffe (1989); Acs et al. (1992); Anselin et al. (1997); Kantor and Whalley (2014)).

From a welfare stand point, along the dispersed equilibrium path (and for any $\overline{\mu}^* < \mu^* < \mu$), we have seen that improving knowledge diffusion in the South reduces the welfare gap between North and South (see expression (28) establishing that $\frac{d\Delta U}{d\mu^*} < 0$). The latter, however, may come at the cost of creating regional winners and losers. Interestingly, we find that individuals in both regions are likely to gain from higher knowledge spillover in the South. Computations (relegated in Appendix 8.6), show that the welfare effects of μ^* in the North and in the South amount, respectively, to

$$\frac{dU}{d\mu^*} = \frac{1}{\rho}\left[\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} + \frac{1-\gamma}{w}\frac{dw}{d\mu^*} - \frac{d\gamma}{d\mu^*}\log w + \frac{d\gamma}{d\mu^*}\left(\log\frac{1}{\tau_D} - \log\frac{1}{\tau_I}\right) + \frac{\nu}{\rho}\frac{dg}{d\mu^*}\right] \quad (33)$$

and

$$\frac{dU^*}{d\mu^*} = \frac{1}{\rho}\left[\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} - \frac{\gamma}{w}\frac{dw}{d\mu^*} - \frac{d\gamma}{d\mu^*}\log w + \frac{d\gamma}{d\mu^*}\left(\log\frac{1}{\tau_I} - \log\frac{1}{\tau_D^*}\right) + \frac{\nu}{\rho}\frac{dg}{d\mu^*}\right] . \quad (34)$$

In the expressions above we can identify several welfare effects. The first term in squared brackets captures the degree of competition (static) effect, which is common to both regions and negative in sign. As seen earlier (Section 3.2), when μ^* increases, the number of firms in each sector decreases ($\frac{dQ}{d\mu^*} < 0$), reducing competitiveness and increasing the price of each good produced. The second term in squared brackets captures the change in the relative wage, and is negative for the North and positive for the South ($\frac{dw}{d\mu^*} < 0$); while the third and fourth terms capture the impact of the change in the share of industries operating in the North ($\frac{d\gamma}{d\mu^*} < 0$). The latter, noticeably, affects the relative price index between regions in opposite directions, positively for the South and negatively for the North. Finally the fifth term, common to both regions, captures the long-term growth effect induced by greater knowledge spillovers in R&D. By differentiating (24) we obtain $\frac{dg}{d\mu^*} = -\frac{g}{Q}\frac{dQ}{d\mu^*} - \delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*} > 0$, suggesting that the dynamic welfare gains are twofold: (i) associated to the trade off between firms' market power and growth ($-\frac{dQ}{d\mu^*} > 0$), and (ii) related to the effect of knowledge spillovers on the share of industries operating in the North ($-\frac{d\gamma}{d\mu^*} > 0$). Notice, in particular, that the effect of knowledge spillovers on firms' mark up arises because the number of operating firms decreases as R&D becomes more productive; we term this the dynamic competition effect.

Tedious computations (relegated in Appendix 8.6) show that the dynamic competition effect dominates the static competition effect, i.e. $-\frac{g}{Q}\frac{dQ}{d\mu^*} > \frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*}$. In other words, the growth gains associated with fewer and more productive firms outweigh the static losses associated with a lower degree of competitiveness (higher prices). As a result, individuals in the South unambiguously gain from higher knowledge spillover in their region ($\frac{dU^*}{d\mu^*} > 0$). If, in addition, transport costs are negligible, then individuals in the North also gain from higher knowledge spillovers in the South ($\frac{dU}{d\mu^*} > 0$) under mild restrictions on parameters. The following proposition restates the result.

Proposition 8 *(*WELFARE*). Assume transport costs are negligible ($\tau_D^* = \tau_D = \tau_I \to 1$). Then along the equilibrium path defined by Corollary 1, and for any $\overline{\mu}^* < \mu^* < \mu$, higher knowledge spillovers in the South result in welfare gains for both regions, if $\nu \leq \hat{\nu}$ where $\hat{\nu}$ is implicitly given by the solution of*

$$\frac{\gamma\rho(1-\gamma)(1+\hat{\nu})[\gamma+\mu(1-\gamma)]}{\hat{\nu}(1-\eta)\mu\{\hat{\nu}[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho\}} = 1$$

and γ is at its equilibrium level, given by equation (A.5) *in the appendix.*

Proof. See Appendix 8.6 ■

Since γ is independent of the parameters ν, δ and ρ the conditions ensuring $\nu \leq \hat{\nu}$ are not particularly restrictive; for instance, δ large enough relative to ρ would suffice. Proposition 8 implies that, transport costs aside, the welfare gains associated with the increase in the share of industries operating in the South are likely to outweigh the welfare losses associated with the lower relative wage in the North, implying a net welfare gain for the North too.

6 Transport costs

In this section we briefly mention what role transport costs play in our model. In the economic geography literature it is usually argued that transport costs are crucial in shaping the distribution of activities (Krugman (1991); Fujita et al. (1999)) and in explaining the dynamics between growth and agglomeration (see, e.g, Minerva and Ottaviano (2009)). In these models, which typically assume monopolistic competition and increasing returns to scale, the innovation sector requires goods which incur transport costs, so that industrial concentration, by reducing the input cost of innovation, increases the growth rate.

From direct inspection of (29) one can immediately check that, in our setup, the equilibrium growth rate does not depend on transport costs. This is because inter- and intra-regional transport costs $(\tau_I, \tau_D, \tau_D^*)$ do not directly influence the input costs of innovation and, therefore, play no role in the equilibrium location of industries (nor on their degree of competitiveness).

Transport costs, though, do influence the income differential between regions. Consider, for instance, the effect of a higher inter-regional transport cost. Simple algebra shows that w (see (21)) is increasing in τ_I if the number of industries located in the North is greater than in the South (i.e. $dw/d\tau_I > 0$ as $1/2 \leq \eta < \gamma$). The reason is that an increase in τ_I induces a decrease in individual demands for foreign varieties and, as the Northern region is more populated in terms of firms ($\gamma > 1/2$), such an effect is more pronounced in the South than in the North. As a result, the relative price of varieties increases, which, ultimately, generates a larger wage gap across regions.[23] This also suggests that, ceteris paribus, the lower the transport cost the more likely are dispersed equilibria. Intuitively, when the transport cost τ_I decreases, on the one hand foreign

[23]The same logic applies to a change in τ_D and τ_D^*.

varieties are cheaper and, on the other hand, the wage in the North decreases ($dw/d\tau_I > 0$). Both effects make the Northern region less attractive for individuals in the South.[24]Formally, differentiating (26) with respect to τ_I, we obtain $\frac{d\Delta U}{d\tau_I} = \frac{1}{\rho}\left[\frac{1}{w}\frac{dw}{d\tau_I} + \frac{(2\gamma-1)}{\tau_I}\right] > 0$. That is, utility differential between North and South shrinks as the transport cost decreases, thereby reducing the incentives of individuals to migrate. Furthermore, the cut off value $\underline{\eta}$ is decreasing in the transport cost, which translates into an increase in the set of dispersed equilibria $[1/2, \underline{\eta})$ as τ_I decreases.[25] Graphically, the shaded area in Figure 1 expands.

Finally, due to the price index effect highlighted above, transport costs affect the change in regional welfare associated with an improvement in knowledge spillovers in the South. Namely, transport costs strengthen the positive welfare effect in the South but weaken the potential welfare gain in the North.

7 Concluding Remarks

In this paper, we have constructed a two-region growth model in which firms operate under Cournot competition and innovate through in-house R&D. Drawing from the evidence that both intra and inter-regional spillovers matter, our main aim was to study how regional disparities in innovation enhancing activities, coupled with migration costs, shape the geography of economic activities.

We have shown that disparities in knowledge spillovers between regions lead to spatial concentration of industries, and the latter is associated with fewer, but more innovative firms. Frictions in the movement of workers, on the other hand, limit the geographic concentration of economic activities in the most productive region. In this context, the stronger the knowledge spillovers the larger the economy growth rate, and the wider the inter-

[24]Allen and Arkolakis (2014) and Tabuchi et al. (2014), also obtain that a lower inter-regional transport cost does not necessarily lead to agglomeration, while Martin and Ottaviano (1999) point out that transport costs are inconsequential for growth if spillovers are global rather than local.

[25]This can be easily checked by applying the implicit function theorem to (25) which yields: $\frac{d\underline{\eta}}{d\tau_I} = -\frac{\frac{1}{w}\frac{\partial w}{\partial \tau_I} + \frac{(2\gamma-1)}{\tau_I}}{\frac{1}{w}\frac{\partial w}{\partial \underline{\eta}} + \frac{\partial \gamma}{\partial \underline{\eta}}\log\left(\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}\right)} < 0$.

regional income disparities. By contrast, a weakening of the spatial disparities in knowledge spillovers between the advanced and the lagging region reduces income inequality, while preserving the positive effect on growth. This occurs because reductions in productivity advantages make firms more sensitive to wage differentials, leading to a rise in the share of industries operating in the lagging region. Welfare increases because the growth gains associated with fewer and more innovative firms outweigh the static losses associated with higher mark ups, and because the gains associated with the change in the spatial concentration of industries are likely to outweigh the losses associated with lower relative wages in the advanced region.

We have also considered whether transport costs play any role in shaping the distribution of activities across regions and found that the latter do not influence the equilibrium location of industries. This is consistent with the growing body of literature emphasising frictions in knowledge flows as source of agglomeration economies.

Our analysis is kept simple to present the effects clearly and maintain tractability. One natural extension would be to consider heterogeneous workers and spatial sorting, although recent empirical work by De la Roca and Puga (2017) indicates little sorting by innate abilities.

Symbol	Description
N	Number of industrial sectors, $j = 1, ..., N$
Q_j	Number of firms in each sector $j = 1, ..., N$, each denoted by $q_j = 1, ..., Q_j$
α	Parameter of taste for each good j, $j = 1, ..., N$
ρ	Rate of time preference
η	Share of total population residing in the North
L^w	Total number of individuals/total quantity of labour (North+South)
τ_D	Intra-regional transportation cost
τ_I	Inter-regional transportation cost
μ	Degree of inter-regional knowledge spillover in R&D
E_t	Per-capita level of expenditures
$c_{j,t}$	Consumption of good j, $j = 1, ..., N$
$p_{j,t}$	Price of good j, $j = 1, ..., N$
ν	Returns to knowledge in the R&D sector
δ	Productivity parameter in the R&D sector
$A_{q_k t}$	Quantity of knowledge produced by a Northern firm q_j, $q_j = 1, ..., Q_j$
$X_{q_j,t}$	Quantity of good produced by firm q_j, $q_j = 1, ..., Q_j$
$L^X_{q_j,t}$	Quantity of labour devoted to the production of good q_j, $q_j = 1, ..., Q_j$
$L^A_{q_j,t}$	Quantity of labour devoted to R&D in firm q_j, $q_j = 1, ..., Q_j$
γ	Endogenous share of sectors located in the North
w_t	Wage rate in the Northern region
$\Pi_{q_j,0}$	Present values of expected profits of firm q_j, $q_j = 1, ..., Q_j$
$\pi_{q_j,t}$	Time t profits of firm q_j, $q_j = 1, ..., Q_j$
r_t	Real interest rate
m	Individual migration cost
ξ_t	Co-state variable associated with the R&D technology in every firm q_j, $q_j = 1, ..., Q_j$, $j = 1, ..., N$

Table 1: Notation

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8 Appendix

8.1 Proof of Proposition 1

The proof is structured as follows. First we show that, in a *symmetric* equilibrium, the economy jumps immediately to a steady state where $g = g^*$. Second, that in the symmetric equilibrium, $Q = Q^*$. Finally we prove existence and uniqueness.

8.1.1 Step 1, showing that $g = g^*$

Using (3), (11) and (17), recall that the wage equation in the North is given by $\frac{\eta}{\gamma} w_t = \left(\frac{\eta E_t}{\tau_D} + \frac{(1-\eta) E_t^*}{\tau_I} \right)$ and an equivalent condition is satisfied for the South. Since $E_t^* = w_t^* = 1$ at each instant, and the consumer problem implies that $\overset{\bullet}{E}_t / E_t = \overset{\bullet}{E}_t^* / E_t^* = r_t - \rho$ at each instant, it follows and $E_t = E$ and $r_t = \rho$. Thus, the wage equations in the North and in the South imply that the wage in the North, and the share of sectors locating in the North (South) jump immediately to their steady state values.

From the individual demand functions ($c_t = E_t / (N \tau_D p_t)$ and $\overline{c}_t = E_t / (N \tau_I p_t^*)$ for $0 < j \leq \gamma N$), it follows that $p_t c_t$ and $p_t^* \overline{c}_t$ must be constant. Therefore, at the aggregate level, $p_t c_t^d = p_t X_t$ is also constant. Since every firm in a given sector is identical, then $p_t X_t / Q_t$ ($p_t^* X_t^* / Q_t^*$) must be constant. Therefore, the number of firms Q (Q^*) in each sector $j = 1, ..., N$, jumps immediately to its steady-state value.

To complete the proof, we derive the dynamic equation. From equation (12), we have

$$w = \xi_t \delta \frac{A_{q,t}}{\gamma N Q} \left[\gamma N Q + \mu (1-\gamma) N Q^* \frac{A_{q,t}^*}{A_{q,t}} \right] . \tag{A.1}$$

Differentiating with respect to time yields

$$-\frac{\overset{\bullet}{\xi}_t}{\xi_t} = \frac{\delta \eta}{\gamma N Q^2} \left[\gamma N Q + \mu (1-\gamma) N Q^* \frac{1}{\widehat{A_t}} \right] - \frac{\mu (1-\gamma) N Q^*}{\widehat{A_t} \left[\gamma N Q + \mu (1-\gamma) N Q^* / \widehat{A_t} \right]} \frac{\overset{\bullet}{\widehat{A_t}}}{\widehat{A_t}},$$

where we have denoted $\widehat{A_t} = A_{q_j,t} / A_{q_j,t}^*$. Then, using (13), (12), (17) and

(18),we obtain

$$-\frac{\overset{\bullet}{\xi}_t}{\xi_t} = -\rho + \delta\nu\eta\frac{Q-1}{\gamma NQ^2}\left[\gamma NQ + \mu\left(1-\gamma\right)NQ^*\frac{1}{\widehat{A_t}}\right] + \frac{\delta\eta}{\gamma NQ^2}.$$

Combining the two previous equations yields

$$\frac{\overset{\bullet}{\widehat{A_t}}}{\widehat{A_t}} = \frac{\frac{\delta\eta}{\gamma NQ^2}\left[\gamma NQ + \mu\left(1-\gamma\right)NQ^*\frac{1}{\widehat{A_t}}\right]\left[1-\nu\left(Q-1\right)\right] + \rho - \frac{\delta\eta}{\gamma NQ^2}}{\frac{\mu(1-\gamma)NQ^*}{\widehat{A_t}\left[\gamma NQ+\mu(1-\gamma)NQ^*/\widehat{A_t}\right]}}.$$

Taking a first order Taylor approximation of the dynamic equation around the steady state, we obtain

$$\frac{\overset{\bullet}{\widehat{A_t}}}{\widehat{A_t}} = -\frac{\frac{[1-\nu(Q-1)]\delta\eta}{\gamma Q}\mu\left(1-\gamma\right)\frac{\widehat{A_t}-\widehat{A}}{\widehat{A}^2}}{\frac{\mu(1-\gamma)NQ^*}{\widehat{A_t}\left[\gamma NQ+\mu(1-\gamma)NQ^*/\widehat{A_t}\right]}},$$

where we have dropped the subscript t to indicate the steady-state value of $\widehat{A}$. Direct inspection of this dynamic equation shows that, if $1-\nu(Q-1) < 0$ (i.e., $Q > 1/\nu+1$) the economy jumps immediately to its steady state where $g = g_{A^*}$.

8.1.2 Step 2, showing that $Q = Q^*$

Using the fact that the growth rates are the same in both regions, together with (4), (5) and (15), we obtain

$$0 = \frac{\eta}{\gamma}\left[\gamma NQ + \mu\left(1-\gamma\right)NQ^*\right] - \frac{(1-\eta)}{(1-\gamma)}\left[\left(1-\gamma\right)NQ^* + \mu^*\gamma NQ\right]\ . \quad \text{(A.2)}$$

Using (3), (4), (12), (13), (17) and (18), we obtain

$$\rho = \delta\nu\left[\gamma NQ + \mu\left(1-\gamma\right)NQ^*\right]\frac{\eta\left(Q-1\right)}{\gamma NQ^2} - \delta\left[\left(\gamma NQ-1\right) + \mu\left(1-\gamma\right)NQ^*\right]\frac{\eta}{\gamma NQ^2}\ .$$
(A.3)

Noting that we can derive an equivalent expression for the South, the two equations above imply that $Q = Q^*$.

8.1.3 Step 3, Existence and uniqueness

From equation (A.2), using $Q = Q^*$, we obtain

$$\eta \left[1 + \mu \frac{(1-\gamma)}{\gamma} \right] = (1-\eta) \left[1 + \mu^* \frac{\gamma}{(1-\gamma)} \right] . \tag{A.4}$$

It can be easily checked that the left hand side (hereafter LHS) is strictly decreasing, with $\lim_{\gamma \to 0} LHS = +\infty$ and $\lim_{\gamma \to 1} LHS = \eta$. Similarly, the right hand side (hereafter, RHS) is strictly increasing, with $\lim_{\gamma \to 0} RHS = (1-\eta)$ and $\lim_{\gamma \to 1} RHS = +\infty$. Since $0 \leq \mu^* < \mu \leq 1$, it follows that there exists a *unique* solution for γ verifying $0 \leq \gamma \leq 1$.

Rearranging (A.4) yields the following quadratic equation

$$[(1-\eta)\mu^* - (1-2\eta) - \mu\eta]\gamma^2 + [1 - 2\eta + 2\mu\eta]\gamma - \mu\eta = 0.$$

Simple computations show that the solution is given by

$$\gamma = \frac{-(1-2\eta+2\mu\eta) + \sqrt{[1-2\eta+2\mu\eta]^2 + 4\mu\eta[(1-\eta)\mu^* - (1-2\eta) - \mu\eta]}}{2[(1-\eta)\mu^* - (1-2\eta) - \mu\eta]} , \tag{A.5}$$

where it can easily be verified that $0 < 1/2 \leq \eta < \gamma \leq 1$ for any population distribution verifying $\eta \in [1/2, 1]$.

To compute Q , using $Q = Q^*$ in (20) and re-arranging terms, we obtain the following quadratic equation

$$0 = \{\nu\eta\delta[\gamma + \mu(1-\gamma)] - \gamma\rho\} Q^2 - (\nu+1)[\gamma + \mu(1-\gamma)]\eta\delta Q + \frac{\eta\delta}{N}.$$

There are potentially two solutions that we denote by $Q_1$and Q_2. They verify $Q_1 < Q_2$ and given by

$$Q_1 = \frac{(\nu+1)[\gamma+\mu(1-\gamma)]\eta\delta - \sqrt{[(\nu+1)[\gamma+\mu(1-\gamma)]\eta\delta]^2 - 4\frac{\eta\delta}{N}\{\nu\eta\delta[\gamma+\mu(1-\gamma)]-\gamma\rho\}}}{2\{\nu\eta\delta[\gamma+\mu(1-\gamma)]-\gamma\rho\}},$$

and

$$Q_2 = \frac{(\nu+1)[\gamma+\mu(1-\gamma)]\eta\delta + \sqrt{[(\nu+1)[\gamma+\mu(1-\gamma)]\eta\delta]^2 - 4\frac{\eta\delta}{N}\{\nu\eta\delta[\gamma+\mu(1-\gamma)]-\gamma\rho\}}}{2\{\nu\eta\delta[\gamma+\mu(1-\gamma)]-\gamma\rho\}}.$$

To ensure that the solution for Q is unique, let us show that $Q_1 < 1 < Q_2$, so that Q_1 can be excluded. Under the assumption $\nu\eta\delta\left[\gamma+\mu\left(1-\gamma\right)\right]-\gamma\rho>0$ which needs to be verified to ensure a strictly positive long-term growth rate (see below) and $0<\nu<1$, it can be checked that $Q_1 < 1 < Q_2$ if the following condition is verified

$$\left[\gamma+\mu\left(1-\gamma\right)-\frac{1}{N}\right]\eta\delta+\gamma\rho>0.$$

As $\gamma > 1/2$, the above expression implies that a sufficient condition to ensure that $Q_1 < 1 < Q_2$ is $N > 2$ which we impose from the start.

Note that, for N large enough ($N \to \infty$) the above expression for Q_2 (i.e. the unique solution for Q) simplifies to

$$Q=\frac{\left(1+\nu\right)\left[\gamma+\mu\left(1-\gamma\right)\right]\frac{\delta\eta}{\gamma}}{\nu\left[\gamma+\mu\left(1-\gamma\right)\right]\frac{\delta\eta}{\gamma}-\rho}>1\ . \tag{A.6}$$

Plugging (A.6) in (24), we obtain:

$$g=\frac{\nu\delta\eta\left[\gamma+\mu\left(1-\gamma\right)\right]-\gamma\rho}{\gamma\left(1+\nu\right)}.$$

The above expression clearly shows that $\nu > \frac{\rho}{\delta\eta}$ is a necessary condition for $g > 0$ (cf. Assumption 1), since the RHS is strictly increasing in γ.

8.2 Gross-profit and business-stealing effects

By use of equation (3) and the equilibrium value $L^X_{q_j,t}=\frac{L^X}{\gamma QN}=\frac{\eta}{\gamma QN}\left(1-\frac{1}{Q}\right)$, the business stealing effect ($\nu\left(1-X_{q_j,t}/X_{j,t}\right)$) and the gross profit effect ($\frac{w_t L^A_{q_j,t}}{X_{q_j,t}/X_{j,t}}$) are given by, respectively: $\nu\left(\frac{Q-1}{Q}\right)$ and $w_t\frac{\eta}{\gamma QN}$. Recall that the gross profit effect is increasing in firms' market power (decreasing in the number of firms) while the reverse applies to the business stealing effect. In equilibrium the product of the two effects, hereafter denoted by Γ, is given by (for any $N > 2$)

$$\Gamma=\nu\left(\frac{Q-1}{Q}\right)w_t\frac{\eta}{\gamma QN}.$$

Normalising $\tau_D = \tau_D^* = \tau_I = 1$, the equilibrium wage is $w = \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta} > 1$, which is independent of Q. Differentiating the above with respect to Q, we obtain

$$\frac{d\Gamma}{dQ} = -w\frac{\nu\eta}{\gamma N}\left(\frac{Q-1}{Q^3}\right) < 0,$$

implying that the gross profit effect dominates the business stealing effect.

8.3 Comparative statics

8.3.1 Change in γ with respect to μ , μ^* and η

Applying the implicit function theorem to (A.4), we obtain

$$\begin{aligned}
\frac{d\gamma}{d\mu} &= \frac{\frac{(1-\gamma)}{\gamma}\frac{\eta}{(1-\eta)}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)} + \frac{\mu^*}{(1-\gamma)^2}} > 0 \ , \\
\frac{d\gamma}{d\mu^*} &= \frac{-\frac{\gamma}{(1-\gamma)}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)} + \frac{\mu^*}{(1-\gamma)^2}} < 0 \ , \\
\frac{d\gamma}{d\eta} &= \frac{\left[1+\mu\frac{(1-\gamma)}{\gamma} + 1 + \mu^*\frac{\gamma}{(1-\gamma)}\right]\frac{1}{(1-\eta)}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)} + \frac{\mu^*}{(1-\gamma)^2}} > 0 \ .
\end{aligned}$$

8.3.2 Change in Q with respect to μ, μ^* and η

Note that a change in Q with respect to any parameter x is given by $\frac{dQ}{dx} = \frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{dx}$. From (A.6) it readily follows that $\frac{\partial Q}{\partial \gamma} = \frac{\eta\delta(\nu+1)\rho\mu}{[\nu\eta\delta[\gamma+\mu(1-\gamma)]-\gamma\rho]^2} > 0$ and $\frac{\partial Q}{\partial \mu} = \frac{-(1+\nu)\delta\eta\rho(1-\gamma)\gamma}{\{\delta\nu[\gamma+\mu(1-\gamma)]\eta-\rho\gamma\}^2} < 0$. Accordingly, after replacing $\frac{d\gamma}{d\mu}$ and rearranging terms, we obtain,

$$\frac{dQ}{d\mu} = \frac{\partial Q}{\partial \mu} + \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{d\mu} = \frac{\delta\eta\rho\,(1-\gamma)\,\gamma\,(1+\nu)}{\{\delta\nu\,[\gamma+\mu\,(1-\gamma)]\,\eta-\rho\gamma\}^2}\left\{\frac{\mu\frac{\eta}{(1-\eta)}}{\mu\frac{\eta}{(1-\eta)} + \frac{\gamma^2\mu^*}{(1-\gamma)^2}} - 1\right\} < 0 \ .$$

Similarly,

$$\frac{dQ}{d\mu^*} = \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{d\mu^*} < 0 \ .$$

To compute $dQ/d\eta$, let us recall that $L^A = \eta/Q$ (see (18)). Thus, we have

$$\frac{dQ}{d\eta} = \frac{1 - \frac{\eta}{L^A}\frac{dL^A}{d\eta}}{L^A} \ .$$

Therefore, the sign of $dQ/d\eta$ is the same as the sign of $1 - \frac{\eta}{L^A}\frac{dL^A}{d\eta}$. Using (A.6), we obtain

$$\frac{dL^A}{d\eta}\frac{\eta}{L^A} = \eta\frac{\left[\nu\delta - \left(\frac{\gamma\rho\mu(1-\gamma)^2}{\gamma^2(1-\eta)\mu^* + \eta\mu(1-\gamma)^2}\right)\frac{1}{[\gamma+\mu(1-\gamma)](1-\eta)}\right]}{\left[\nu\eta\delta - \frac{\gamma\rho}{[\gamma+\mu(1-\gamma)]}\right]} \ .$$

Simplifying the above expression, $1 - \frac{\eta}{L^A}\frac{dL^A}{d\eta} < 0$ is equivalent to the following condition

$$\frac{\mu\left(1-\gamma\right)}{\gamma}\frac{\eta}{\left(1-\eta\right)^2} < \frac{\gamma}{\left(1-\gamma\right)}\mu^* + \frac{\eta\mu(1-\gamma)}{(1-\eta)\gamma}.$$

Using the equilibrium condition (19), substitute $\mu^*\frac{\gamma}{1-\gamma} = \frac{\eta}{1-\eta}\left[1 + \mu\frac{(1-\gamma)}{\gamma}\right] - 1$ in the equation above to obtain, after simple manipulations

$$\frac{\mu\left(1-\gamma\right)}{\gamma}\frac{\eta}{\left(1-\eta\right)} < 1 \ ,$$

which is always satisfied since $0 < \mu < 1$ and $\gamma > \eta$. Therefore, $dQ/d\eta < 0$.

8.3.3 Change in L_A with respect to η

In a similar way as before, we can use $L^A = \eta/Q$ (see (18)) and (A.6) to obtain

$$\frac{dL^A}{d\eta} = \frac{1}{\delta\left(\nu+1\right)}\left[\nu\delta - \frac{\rho}{\left[\gamma+\mu\left(1-\gamma\right)\right]}\frac{\mu}{\left[\gamma+\mu\left(1-\gamma\right)\right]}\frac{d\gamma}{d\eta}\right] \ .$$

After substituting for $d\gamma/d\eta > 0$, we obtain

$$\frac{dL^A}{d\eta} = \frac{1}{\delta\left(\nu+1\right)}\left[\nu\delta - \left(\frac{\gamma\rho\mu\left(1-\gamma\right)^2}{\gamma^2(1-\eta)\mu^* + \eta\mu(1-\gamma)^2}\right)\frac{1}{\left[\gamma+\mu\left(1-\gamma\right)\right]\left(1-\eta\right)}\right] \ .$$

From (A.6), we know that $\nu\delta > \frac{\gamma\rho}{\eta[\gamma+\mu(1-\gamma)]}$. Therefore, let us show

$$\left(\frac{\gamma\rho\mu\left(1-\gamma\right)^{2}}{\gamma^{2}(1-\eta)\mu^{*}+\eta\mu(1-\gamma)^{2}}\right)\frac{1}{\left[\gamma+\mu\left(1-\gamma\right)\right]\left(1-\eta\right)} < \frac{\gamma\rho}{\eta\left[\gamma+\mu\left(1-\gamma\right)\right]} \ ,$$

implying that $dL^{A}/d\eta > 0$. Simplifying the previous expression, we obtain the following condition

$$\frac{\eta^{2}\mu(1-\gamma)}{(1-\eta)^{2}\gamma} < \frac{\gamma}{(1-\gamma)}\mu^{*}$$

Using the equilibrium condition (19), substitute $\mu^{*}\frac{\gamma}{1-\gamma} = \frac{\eta}{1-\eta}\left[1+\mu\frac{(1-\gamma)}{\gamma}\right]-1$ into the equation above to obtain the following inequality

$$\frac{\mu\left(1-\gamma\right)}{\gamma}\frac{\eta}{(1-\eta)} < 1 \ ,$$

which is always satisfied as $\gamma > \eta$ and $0 < \mu < 1$.Therefore, $dL^{A}/d\eta > 0$.

8.3.4 Change in g with respect to η

Recall that the growth rate is given by

$$g = \frac{\delta\nu\left[\gamma+\mu\left(1-\gamma\right)\right]\frac{\eta}{\gamma}-\rho}{(1+\nu)} \ .$$

Differentiating g with respect to η yields

$$\frac{dg}{d\eta} = \frac{\delta\nu}{(1+\nu)}\frac{1}{\gamma}\left\{\left[\gamma+\mu\left(1-\gamma\right)\right]-\mu\frac{\eta}{\gamma}\frac{d\gamma}{d\eta}\right\} \ .$$

Let us show that $dg/d\eta > 0$, that is

$$-\mu\frac{\eta}{\gamma}\frac{d\gamma}{d\eta}+\left[\gamma+\mu\left(1-\gamma\right)\right] > 0 \ .$$

Recalling that $\frac{d\gamma}{d\eta} = \frac{1}{(1-\eta)}\frac{\left[1+\mu\frac{1-\gamma}{\gamma}\right]}{\left[\frac{\mu^{*}(1-\eta)}{(1-\gamma)^{2}}+\frac{\mu\eta}{\gamma^{2}}\right]}$, and substituting in the inequality above, we obtain

$$\mu^{*}\frac{\gamma}{1-\gamma}-\mu\frac{\eta^{2}}{(1-\eta)^{2}}\frac{1-\gamma}{\gamma} > 0 \ .$$

Using the equilibrium condition (19), substitute $\mu^* \frac{\gamma}{1-\gamma} = \frac{\eta}{1-\eta}\left[1+\mu\frac{(1-\gamma)}{\gamma}\right] - 1$ into the above expression to obtain

$$1-\mu\frac{1-\gamma}{\gamma}\left(\frac{\eta}{1-\eta}\right) > 0 \ ,$$

which is always verified since $1/2 \leq \eta < \gamma$. Therefore, $dg/d\eta > 0$.

8.4 Wage

8.4.1 Proof that $w > w^* = 1$

Equation (21) readily shows that its LHS is strictly increasing with $\lim_{w\to 0} LHS = \eta\tau_I/\tau_D^* < \lim_{w\to 0} RHS = +\infty$ and $\lim_{w\to+\infty} LHS = +\infty > \lim_{w\to+\infty} RHS = \gamma\eta\tau_I/[(1-\gamma)(1-\eta)\tau_D]$. Therefore, the solution for the level of wage is unique. Moreover, under the assumption $\tau_I/\tau_D > \tau_I/\tau_D^*$ and the property $\gamma > \eta \geq 1/2$, we can check

$$\lim_{w\to 1} LHS = \frac{\eta}{(1-\eta)}\left[\frac{\eta}{(1-\eta)}+\frac{\tau_I}{\tau_D^*}\right] < \lim_{w\to 1} RHS = \frac{\gamma}{1-\gamma}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D}+1\right].$$

i.e., the intersection between LHS and RHS in (21) necessarily occurs at $w > 1$.

8.4.2 Proof that $dw/d\eta > 0$

Manipulating (21), we obtain:

$$\frac{\eta}{1-\eta}w^2+\left(\frac{\tau_I}{\tau_D^*}-\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}\right)w-\left(\frac{1-\eta}{\eta}\right)\frac{\gamma}{1-\gamma} = 0 \ . \qquad \text{(A.7)}$$

Thus, the solution for the wage is given by

$$w = \frac{\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}-\frac{\tau_I}{\tau_D^*}+\left[\left(\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}-\frac{\tau_I}{\tau_D^*}\right)^2+\frac{4\gamma}{1-\gamma}\right]^{1/2}}{2\frac{\eta}{1-\eta}} \ . \qquad \text{(A.8)}$$

Applying the implicit function theorem to (A.7), and substituting w by its value given by (A.8) in the denominator, we obtain

$$\frac{dw}{d\eta} = -\frac{\frac{\partial F}{\partial \eta}}{\left[\left(\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D} - \frac{\tau_I}{\tau_D^*}\right)^2 + \frac{4\gamma}{1-\gamma}\right]^{1/2}} ,$$

where

$$\frac{\partial F}{\partial \eta} = -\frac{w}{(1-\gamma)^2}\frac{\tau_I}{\tau_D}\left[\frac{d\gamma}{d\eta} - \frac{(1-\gamma)^2}{(1-\eta)^2}\frac{\tau_D}{\tau_I}w\right] - \frac{(1-\eta)}{\eta}\frac{1}{(1-\gamma)^2}\left[\frac{d\gamma}{d\eta} - \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta}\right].$$

Note that, if $\frac{d\gamma}{d\eta} > \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta}$ and $\frac{(1-\gamma)^2}{(1-\eta)^2}\frac{\tau_D}{\tau_I}w < \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta}$, then $\frac{\partial F}{\partial \eta}$ would be negative.

Starting from the latter, where we substitute w by its value given by (A.8), after some manipulations, we obtain the following inequality

$$-4\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*} + \frac{4\gamma}{1-\gamma} < 0 ,$$

which is always satisfied as $\tau_I > \tau_D^* > \tau_D$.

Next, let us show that $\frac{d\gamma}{d\eta} > \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta}$. Noting that we can write

$$\frac{d\gamma}{d\eta} = \frac{\left[1 + \mu\frac{(1-\gamma)}{\gamma} + 1 + \mu^*\frac{\gamma}{(1-\gamma)}\right]\frac{1}{(1-\eta)}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)} + \frac{\mu^*}{(1-\gamma)^2}} ,$$

after some computations, we obtain

$$\frac{d\gamma}{d\eta} > \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta} \Leftrightarrow \frac{\eta}{[(1-\eta)\mu^* + \eta]} > \gamma .$$

To show that the above condition is always verified, let us recall that (A.4) is given by

$$(1-\eta)\left[1 + \mu^*\frac{\gamma}{(1-\gamma)}\right] = \eta\left[1 + \mu\frac{(1-\gamma)}{\gamma}\right] .$$

Now let us define

$$\begin{aligned} LHS(\gamma) &= (1-\eta)\left[1 + \mu^*\frac{\gamma}{(1-\gamma)}\right] ; \\ RHS(\gamma) &= \eta\left[1 + \mu\frac{(1-\gamma)}{\gamma}\right] . \end{aligned}$$

We know that $LHS(\gamma)$ is strictly increasing and $RHS(\gamma)$ is strictly decreasing. Moreover, we can easily check that

$$RHS\left(\frac{\eta}{\mu^*(1-\eta)+\eta}\right) < LHS\left(\frac{\eta}{\mu^*(1-\eta)+\eta}\right) .$$

As in equilibrium we must have $RHS(\gamma) = LHS(\gamma)$, this implies that $\gamma < \eta/[(1-\eta)\mu^* + \eta]$. That is, the intersection between $RHS(\gamma)$ and $LHS(\gamma)$ occurs for a value of γ such that: $\gamma < \frac{\eta}{[(1-\eta)\mu^*+\eta]}$. Therefore, $dw/d\eta > 0$.

8.4.3 Showing that $\lim_{\eta\to 0} w$ and $\lim_{\eta\to 1} w$

In this section, we compute the level of wage in the limit cases where η tends to 0 or 1. Using (A.5) we can obtain

$$\lim_{\eta\to 0}\frac{d\gamma}{d\eta} = \mu < 1,$$

and

$$\lim_{\eta\to 1}\frac{d\gamma}{d\eta} = \mu^* < 1.$$

Then, using the above results, along with l'Hôpital's rule applied to (A.8), we obtain

$$\lim_{\eta\to 0} w = \lim_{\eta\to 0}\frac{d\gamma}{d\eta}\left(\frac{\tau_I}{\tau_D^*}\right)^{-1} = \mu\frac{\tau_D^*}{\tau_I} < 1,$$

and

$$\lim_{\eta\to 1} w = \left(\frac{d\gamma}{d\eta}\right)^{-1}\frac{\tau_I}{\tau_D} = \frac{1}{\mu^*}\frac{\tau_I}{\tau_D} > 1.$$

8.4.4 Change in w with respect to μ, μ^*, τ_I, τ_D, τ_D^*

Recall that the wage is implicitly given by (see (21))

$$\frac{\eta}{(1-\eta)}\left[\frac{\eta}{(1-\eta)}w + \frac{\tau_I}{\tau_D^*}\right] = \frac{\gamma}{1-\gamma}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right] .$$

Applying the implicit function theorem to the above, we obtain

$$\frac{dw}{d\mu} = \frac{\frac{1}{(1-\gamma)^2}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right]\frac{d\gamma}{d\mu}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} > 0,$$

$$\frac{dw}{d\mu^*} = \frac{\frac{1}{(1-\gamma)^2}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right]\frac{d\gamma}{d\mu^*}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} < 0,$$

$$\frac{dw}{d\tau_I} = -\frac{\frac{\eta}{(1-\eta)}\left(\frac{1}{\tau_D^*} - \frac{\gamma}{1-\gamma}\frac{1}{\tau_D}\right)}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} > 0,$$

$$\frac{dw}{d\tau_D} = -\frac{\frac{\gamma}{1-\gamma}\frac{\eta}{(1-\eta)}\frac{\tau_I}{(\tau_D)^2}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} < 0,$$

$$\frac{dw}{d\tau_D^*} = \frac{\frac{\eta}{(1-\eta)}\frac{\tau_I}{(\tau_D^*)^2}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} > 0.$$

8.5 Proof of Proposition 7

Apply the implicit function theorem to (21) to obtain $\frac{dw}{d\gamma} = \frac{\frac{1}{(1-\gamma)^2}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right]}{\left(\frac{\eta}{1-\eta}\right)^2 + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} > 0$. Since $\frac{d\gamma}{d\mu} > 0$ and $\frac{d\gamma}{d\mu^*} < 0$, then w is positively correlated with μ and negatively correlated with μ^*. Furthermore, since $\frac{d\gamma}{d\eta} > 0$ it follows that w is also positively correlated with η.

8.6 Welfare

Using (6), (14), $E = w$ and $E^* = w^* = 1$, in steady state, the individual lifetime utility in the North reads

$$\rho U = \log\left[w^{1-\gamma}\frac{\left(A_{q_j,0}\right)^{\nu}(Q-1)}{NQ}\left(\frac{1}{\tau_D}\right)^{\gamma}\left(\frac{1}{\tau_I}\right)^{1-\gamma}\right] + \nu\frac{g}{\rho},$$

Similarly, for the South, we have

$$\rho U^* = \log\left[w^{-\gamma}\frac{\left(A_{q_j,0}\right)^{\nu}(Q-1)}{NQ}\left(\frac{1}{\tau_I}\right)^{\gamma}\left(\frac{1}{\tau_D^*}\right)^{1-\gamma}\right] + \nu\frac{g}{\rho}.$$

Differentiating the above expressions with respect to μ^* yields (33) and (34) in the main text.

8.6.1 Proof that $\frac{dU^*}{d\mu^*} > 0$

$$\frac{dU^*}{d\mu^*} = \frac{1}{\rho}\left\{\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} + \frac{1-\gamma}{w}\frac{dw}{d\mu^*} - \frac{d\gamma}{d\mu^*}\log w + \frac{d\gamma}{d\mu^*}\left(\log\frac{1}{\tau_I} - \log\frac{1}{\tau_D^*}\right) + \frac{\nu}{\rho}\frac{dg}{d\mu^*}\right\},$$

where $1 < \tau_D^* < \tau_I$, $Q = \frac{(1+\nu)[\gamma+\mu(1-\gamma)]\frac{\delta\eta}{\gamma}}{\nu[\gamma+\mu(1-\gamma)]\frac{\delta\eta}{\gamma}-\rho} > 1$, $\frac{dQ}{d\mu^*} = \frac{\partial Q}{\partial\gamma}\frac{d\gamma}{d\mu^*} < 0$, $\frac{dw}{d\mu^*} = \frac{\frac{1}{(1-\gamma)^2}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D}+\frac{1}{w}\right]\frac{d\gamma}{d\mu^*}}{\frac{\eta^2}{(1-\eta)^2}+\frac{\gamma}{1-\gamma}\frac{1}{w^2}} < 0$, $\frac{d\gamma}{d\mu^*} < 0$ and $\frac{dg}{d\mu^*} = -\frac{g}{Q}\frac{dQ}{d\mu^*} - \delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*} > 0$. Therefore, for $\frac{dU^*}{d\mu^*} > 0$ to hold it suffices to show that $\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} - \frac{g}{Q}\frac{dQ}{d\mu^*} > 0$, that is the dynamic competition effect outweighs the static competition effect. Since $\frac{dQ}{d\mu^*} < 0$, we need to show that

$$\frac{1}{Q-1} < \frac{\nu}{\rho}g.$$

Substituting for $\frac{1}{Q-1} = \frac{\nu[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho}{[\gamma+\mu(1-\gamma)]\delta\eta+\gamma\rho}$ and $g = \frac{\nu[\gamma+\mu(1-\gamma)]\frac{\delta\eta}{\gamma}-\rho}{(1+\nu)}$, we obtain

$$\nu > \left(\frac{\rho(1+\nu)\gamma}{\nu[\gamma+\mu(1-\gamma)]\delta\eta-\rho\gamma}\right)\left(\frac{\nu[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho}{[\gamma+\mu(1-\gamma)]\delta\eta+\gamma\rho}\right).$$

Simplifying yields,

$$\nu > \gamma\rho\left(\frac{(1+\nu)}{[\gamma+\mu(1-\gamma)]\delta\eta+\gamma\rho}\right)$$

$$\nu\{[\gamma+\mu(1-\gamma)]\delta\eta+\gamma\rho\} - \nu\gamma\rho > \gamma\rho$$

$$\nu > \frac{\gamma\rho}{[\gamma+\mu(1-\gamma)]\delta\eta}.$$

This is always satisfied under assumption 1.

8.6.2 Proof that $\frac{dU}{d\mu^*} > 0$

Assuming negligible transport costs ($\tau_D = \tau_D^* = \tau_I \to 1$) we have

$$\frac{dU}{d\mu^*} = \frac{1}{\rho}\left[\frac{1/Q^2}{1-1/Q}\frac{\partial Q}{\partial\gamma}\frac{d\gamma}{d\mu^*} + \frac{1-\gamma}{w}\frac{dw}{d\mu^*} - \frac{d\gamma}{d\mu^*}\log w + \frac{\nu}{\rho}\frac{dg}{d\mu^*}\right],$$

where $\frac{dg}{d\mu^*} = -\frac{g}{Q}\frac{dQ}{d\mu^*} - \delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*}$. Under assumption 1 (cf. proof above) we know that $\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} - \frac{g}{Q}\frac{dQ}{d\mu^*} > 0$. Hence for $\frac{dU}{d\mu^*} > 0$ to hold it suffices to show that $\frac{1-\gamma}{w}\frac{dw}{d\mu^*} \leq \log w\frac{d\gamma}{d\mu^*} + \frac{\nu}{\rho}\delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*}$, that is the gains associated with the re-location of sectors outweighs the losses associated with the decrease in the relative wage.

Under negligible transport costs the wage equation (21) implies $w = \frac{\left(\frac{2\gamma-1}{1-\gamma}\right)+\left[\frac{4\gamma^2+1-4\gamma}{(1-\gamma)^2}+\frac{4\gamma}{1-\gamma}\right]^{1/2}}{2\frac{\eta}{1-\eta}} = \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta} > 1$ and $\frac{dw}{d\mu^*} = \frac{1}{(1-\gamma)^2}\frac{1-\eta}{\eta}\frac{d\gamma}{d\mu^*}$, hence we need to show that

$$\frac{1}{\gamma} - 1 + \frac{1-\gamma}{\gamma}\frac{\eta}{1-\eta} - \frac{\nu}{\rho}\delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2} \leq 0,$$

which simplifies to

$$\frac{1-\gamma}{1-\eta} \leq \frac{\nu}{\rho}\delta\eta\frac{1}{Q}\frac{\mu}{\gamma}.$$

Replacing Q and simplifying

$$\frac{1-\gamma}{1-\eta}\frac{(1+\nu)\left[\gamma+\mu\left(1-\gamma\right)\right]}{\nu\left[\gamma+\mu\left(1-\gamma\right)\right]\frac{\delta\eta}{\gamma}-\rho} \leq \frac{\nu}{\rho}\mu \quad \Leftrightarrow \quad \frac{\gamma\rho\left(1-\gamma\right)\left(1+\nu\right)\left[\gamma+\mu\left(1-\gamma\right)\right]}{\nu\left(1-\eta\right)\mu\left\{\nu\left[\gamma+\mu\left(1-\gamma\right)\right]\delta\eta-\gamma\rho\right\}} \leq 1.$$

Denote by $\hat{\nu}$ the solution of the implicit function $\frac{\gamma\rho(1-\gamma)(1+\hat{\nu})[\gamma+\mu(1-\gamma)]}{\hat{\nu}(1-\eta)\mu\{\hat{\nu}[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho\}} = 1$; then $\frac{dU}{d\mu^*} > 0$ for any $\nu \leq \hat{\nu}$.

8.7 Asymmetries in productivity

Here we postulate $\delta > \delta^*$ (i.e., R&D productivity in the North is greater than in the South) and $\mu = \mu^* < 1$. In this case, the condition equalising growth rates ($g = g^*$) reads as

$$\delta\eta\left[1+\frac{\mu\left(1-\gamma\right)}{\gamma}\right] = \delta^*\left(1-\eta\right)\left[1+\frac{\gamma\mu}{\left(1-\gamma\right)}\right].$$

Accordingly, the location decision of sectors depends on R&D productivity.

It can be easily checked that the left hand side (hereafter LHS) is strictly decreasing, with $\lim_{\gamma\to 0} LHS = +\infty$ and $\lim_{\gamma\to 1} LHS = \delta\eta$. Similarly, the right hand side (hereafter, RHS) is strictly increasing, with $\lim_{\gamma\to 0} RHS = \delta^*(1-\eta)$ and $\lim_{\gamma\to 1} RHS = +\infty$. Under the assumption $0 \leq \delta\eta \leq 1$, it follows that there exists a unique solution for γ verifying $0 \leq \gamma \leq 1$. Notice, also, that $\delta\eta < 1$ is not stringent as $\eta \leq 1$ and we can expect δ to be much lower than 1.

Note that when $\eta = 1/2$, the equation above can be written as,

$$\delta^* \mu \left(\frac{\gamma}{1-\gamma} \right)^2 - (\delta - \delta^*) \left(\frac{\gamma}{1-\gamma} \right) - \delta \mu = 0.$$

Solving for $\frac{\gamma}{1-\gamma}$ we obtain,

$$\frac{\gamma}{1-\gamma} = \frac{(\delta - \delta^*) + \sqrt{(\delta - \delta^*)^2 + 4\delta^* \mu (\delta - \delta^*)}}{2\delta^* \mu}.$$

Therefore when $\eta = 1/2, \gamma = X$ where

$$X = \frac{(\delta - \delta^*) + \sqrt{(\delta - \delta^*)^2 + 4\delta^* \mu (\delta - \delta^*)}}{2\delta^* \mu + (\delta - \delta^*) + \sqrt{(\delta - \delta^*)^2 + 4\delta^* \mu (\delta - \delta^*)}}.$$

Furthermore, $X > 1/2$ if, after some simplifications, $(\delta - \delta^*) + \sqrt{(\delta - \delta^*)^2 + 4\delta^* \mu (\delta - \delta^*)} > 2\delta^* \mu$.

Since $\delta > \delta^*$, we can as well show that $(\delta - \delta^*) + \sqrt{(\delta - \delta^*)^2 + 4\delta^* \mu (\delta - \delta^*)} > \delta^* \mu + \delta \mu$. Simplifying, the latter condition becomes $\delta (1 - \mu) - \delta^* (1 - \mu) + \sqrt{(\delta - \delta^*)^2 + 4\delta^* \mu (\delta - \delta^*)} > 0$, which is always satisfied. This proves that $\gamma > 1/2$ when $\eta = 1/2$ as in the case analysed in the main section of the paper.

Finally, by applying the implicit function theorem, it can be easily checked that $\frac{d\gamma}{d\delta^*} < 0$. Therefore, the properties of the model described for μ^* apply to δ^*.