CESifo WORKING PAPERS

6965
2018

March 2018

Intersectoral Markup Divergence

Kristian Behrens, Sergey Kichko, Philip Ushchev

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editors: Clemens Fuest, Oliver Falck, Jasmin Gröschl
www.cesifo-group.org/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: www.CESifo-group.org/wp

CESifo *Working Paper No. 6965*
Category 8: Trade Policy

Intersectoral Markup Divergence

Abstract

We develop a general equilibrium model of monopolistic competition with a traded and a non-traded sector. Using a broad class of homothetic preferences—that generate variable markups, display a simple behavior of their elasticity of substitution, and nest the ces as a limiting case—we show that trade liberalization: (i) reduces domestic markups and increases imported markups in the traded sector; (ii) increases markups in the non-traded sector; and (iii) increases firm sizes in both sectors. Thus, while domestic and export markups in the traded sector converge across countries, markups diverge across sectors within countries. The negative welfare effects of higher markups and less consumption diversity in the non-traded sector dampen the positive welfare effects of lower markups and greater diversity in the traded sector.

JEL-Codes: F120, F150.

Keywords: monopolistic competition, variable markups, trade liberalization, non-traded goods, markup divergence.

Kristian Behrens
Department of Economics
University of Québec at Montréal
ESG-UQAM
Montréal / Québec / Canada
behrens.kristian@uqam.ca

Sergey Kichko
National Research University
Higher School of Economics
Saint Petersburg / Russian Federation
skichko@hse.ru
sergey.kichko@gmail.com

Philip Ushchev
National Research University
Higher School of Economics
Saint Petersburg / Russian Federation
ph.ushchev@gmail.com

March 20, 2018

1 Introduction

Having been out of fashion for a long time, monopolistic competition models with variable elasticity of substitution (VES) have attracted more attention in recent years. They have been used to study, among others, the selection effects of freer trade (Melitz and Ottaviano, 2008), the efficiency of the market outcome (Dhingra and Morrow, 2018), the welfare consequences of trade liberalization (Behrens and Murata, 2012), the interdependence between prices and incomes (Simonovska, 2015), and how preferences affect the market outcome in general (Zhelobodko *et al.*, 2012). One key reason for this renewed interest is that micro-level evidence on markups and firm sizes is by and large inconsistent with the constant elasticity of substitution (CES) model. While VES models help us to make sense of the micro-level data, they have largely abstracted from intersectoral issues. There are indeed few general equilibrium models with multiple VES sectors.[1] Our paper aims to fill that gap by developing a tractable multisector general equilibrium framework with VES preferences that allows us to study the market outcome under trade liberalization.

Monopolistic competition models with VES and multiple sectors are important for at least three reasons. First, empirical evidence shows that markups are variable and change systematically with market structure and trade liberalization (Syverson, 2007; De Loecker *et al.*, 2012). Second, industrial policies and trade liberalization affect the distribution of resources across heterogeneous sectors and firms. In the presence of variable markups, this has consequences for the composition of welfare gains (Feenstra and Weinstein, 2017) and the allocative efficiency of the market outcome (Holmes *et al.*, 2014; Dhingra and Morrow, 2018).[2] Third, distortions also arise between sectors (Epifani and Gancia, 2011; Behrens *et al.*, 2016), so that taking into account both the intra- and the intersectoral allocation of resources is important.

The absence of multisector general equilibrium VES models in the literature may be explained by the fact that those models are difficult to handle. First, there are only few specifications that remain analytically tractable, even with a single sector. Second, when extended to multiple sectors, the intersectoral allocation and, especially, the sectoral budget shares are difficult

[1]While multisector CES models abound in the literature (see, e.g., Bernard *et al.*, 2007; and Costinot and Rodriguez-Clare, 2014, for a state-of-the-art survey), multisector VES models are notably absent. The vast majority of multisector monopolistic competition models use either only one VES sector and a competitive outside sector (Zhelobodko *et al.*, 2012), or quasi-linear preferences (Melitz and Ottaviano, 2008) thus giving those models a 'partial equilibrium flavor'. To our knowledge, Behrens *et al.* (2016) is one of the few papers that develops a full general equilibrium framework with heterogeneous firms and multiple VES sectors to analyze intra- and intersectoral distortions in the economy. See also d'Aspremont and Dos Santos Ferreira (2015) for a closed-economy multisector model with oligopolistic competition.

[2]Analyzing optimality and changes in allocative efficiency in the presence of variable markups has become an important topic in theoretical studies (Arkolakis *et al.*, 2018; Dhingra and Morrow, 2018; Holmes *et al.*, 2014; Behrens *et al.*, 2016). On the empirical side, using Chilean data for 1995-2007, Weinberger (2015) shows that productivity gains are not always accompanied by allocative efficiency gains. Edmond *et al.* (2015) draw on Taiwanese firm-level data to show that trade liberalization leads to a reduction of distortions driven by variable markups by approximately 66%, provided that dominant firms are exposed to tougher competition when trade gets freer.

to pin down. Even with the workhorse Cobb-Douglas upper-tier utility function, the budget shares vary in a non-trivial way with the whole equilibrium allocation when the lower-tier utility is non-homothetic. These difficulties largely explain why the 'Cobb-Douglas with nested CES' specification has been adopted by the vast majority of the extant multisector general equilibrium literature with imperfect competition.

In this paper, we focus on homothetic preferences, which implies that price indices are well defined. In addition, we want preferences to have variable elasticity of substitution, with the latter displaying a 'simple' behavior. Such preferences are necessarily non-additive, which makes the analysis a priori more involved. We provide, however, a simple characterization of Kimball's (1995) preferences by clarifying what are the fundamental properties of the elasticity of substitution for that type of preferences. We show, in particular, that the elasticity of substitution depends solely on the individual consumption level 'scaled' by a quantity index, thus implying that those preferences mimic the behavior of additive preferences. They also nest CES preferences as a special case.[3] We can therefore choose instances of those preferences that are arbitrarily close to the CES, which allows us to derive many results by continuity in a 'small neighborhood' of the CES. We refer to those preferences as 'ε-CES preferences'. They make the analysis of the interactions between sectors fairly tractable while allowing for empirically relevant features like variable markups and varying firm sizes. In addition, they make it straightforward to bridge our results with the existing findings obtained within CES-based settings.

We apply this approach to develop an analytically tractable two-sector VES trade model where one sector produces traded manufactured goods while the other produces non-traded services. Both industries produce a continuum of horizontally differentiated varieties and have variable markups. Having non-traded goods makes the analysis both more tractable and realistic. It is well-documented that the share of non-traded goods in modern economies is high. It varies substantially across countries but averages around 45% (Lombardo and Ravenna, 2012). Furthermore, in the international macro context, variations in the relative prices of non-traded goods across countries are one of the common explanations for real exchange rate volatility (Burstein *et al.*, 2005 and 2006). Thus, the presence of a non-traded sector might have a significant impact on trade patterns. Observe further that the presence of a non-traded sector automatically implies that trade liberalization affects sectors differentially—markups change in sector-specific ways. We thus believe it is important to look more closely at models that feature both non-traded goods and variable markups.[4]

[3]Since CES preferences are the workhorse in the international trade literature (e.g., Krugman, 1980; Eaton and Kortum, 2002; Melitz, 2003), staying close to those preferences does not require us to go into completely new modeling directions. Moreover, departing from the CES within this setup does not render intensive-margin and competitive effects 'fragile', as found by Bertoletti and Epifani (2014) for additive preferences.

[4]The link between the elasticity of substitution and the variation in purchasing power parity across countries is also widely studied. For instance, non-proportional pricing-to-market is one of the explanations for the large deviations from relative purchasing power parity (Atkenson and Burstein, 2008). Corsettia and Dedola (2005) investigate the impact of exchange rate shocks on firms' pricing decisions for domestic and foreign markets by

Our key results may be summarized as follows. First, while trade liberalization promotes the convergence of markups across countries, it leads to the *divergence of markups across sectors* within countries. Our model hence allows us to rationalize the observation that markups for traded manufactured goods have decreased in the wake of the Single Market Program of the European Union in the 1990s, whereas markups for non-traded services have increased (see Badinger, 2007). Although anti-competitive practices have been put forward as a possible explanation for those changes, our model suggests that simple market-driven general equilibrium effects may be enough to generate that outcome. Second, as markups fall and product diversity expands in the traded sector, markups rise and product diversity shrinks in the non-traded sector. The reason for these opposing changes is the intersectoral reallocation of resources triggered by the liberalization of trade in one sector. The negative welfare effects of higher markups and less consumption diversity in the non-traded sector dampen the positive welfare effects of lower markups and greater diversity in the traded sector. To gauge the overall welfare effect, we provide comparisons of welfare gains for our setting with variable markups and the benchmark case of CES preferences. We show that the overall welfare gains are slightly large in the ε-CES case than under the CES preferences, i.e., variable markups are an additional source of gains.

It is important to stress that our welfare results are fully driven by general equilibrium effects. While trade liberalization has a direct impact on the traded sector, freer trade affects the non-traded sector only indirectly through changes in the traded sector. This highlights the importance of working with multisector models to investigate the positive and normative effects of trade liberalization under monopolistic competition.

The remainder of the paper is organized as follows. Section 2 introduces and characterizes our preference structure and describes the trade model. Sections 3 and 4 study the market outcomes in the non-traded and in the traded sectors, respectively. Section 4 introduces our concept of ε-CES preferences. Section 5 explores the consequences of trade liberalization for sectoral markups, firm sizes, and the intersectoral allocation. Section 6 establishes our welfare results and provides numerical illustrations to show how far beyond the CES we can go for our key qualitative results to remain unchanged. Finally, Section 7 concludes.

2 The model

We develop a two-sector general equilibrium model of monopolistic competition with variable elasticity of substitution and costly trade between two countries: home, H, and foreign, F. For simplicity, countries are assumed to have identical technologies and the same population, L.[5]

employing a model with non-tradables and country-specific price elasticities. We deviate from this literature by studying price formation in traded and non-traded sectors under the presence of an external shock in the traded sector, i.e., trade liberalization.

[5]We relax the assumption of equal population size in the Appendix D and show that our key results extend to the case of asymmetric population sizes. Introducing technological differences between sectors does not add

There are two sectors in each country. The first one produces a continuum of horizontally differentiated varieties of a traded good ('manufacturing'), whereas the second one produces a continuum of varieties of a non-traded good ('services'). Labor is the only production factor.

2.1 Preferences

Since we are interested in the behavior of markups, we cannot work with standard CES preferences. Yet, we want to work with preferences for which we can define price indices, i.e., we would like our preferences to be homothetic. As shown by Parenti *et al.* (2017), no additive preferences of the form $U \equiv \int_0^N \mathrm{v}(x_i)\mathrm{d}i$ satisfy jointly the two properties of variable markups and homotheticity. Hence, we need to work with non-additive preferences. The difficulty with non-additive preferences is that the markups (or, alternatively, the elasticity of substitution) are no longer simple functions of solely the quantity x_i, as they are in the additive case.[6]

To deal with that difficulty, we will work with a class of preferences that: (i) are homothetic; (ii) have a 'simple' behavior of their elasticities of substitution; and (iii) display variable markups. In other words, we seek for a class of homothetic preferences mimicking the behavior of additive preferences (see equation (1) below for a mathematical formulation). We show that preferences described by Kimball's flexible aggregator (Kimball, 1995) satisfy these properties. Since they are closely related to additive preferences, they display a similar behavior. In particular, the elasticity of substitution—and hence the profit-maximizing markups—are linked to the relative risk aversion of Kimball's aggregator, whereas they are linked to the relative risk aversion of the lower-tier utility function $\mathrm{v}(\cdot)$ in the additive case. Given their conceptual similarity, we can make use of the numerous tools developed previously in the literature to analyze the equilibrium properties in that type of model. This sheds new light on some fundamental properties of these preferences and provides also a new characterization for them.

Let $\overline{\sigma}(x_i, x_j, \mathbf{x})$ denote the elasticity of substitution between varieties i and j, where $\mathbf{x}$ denotes the whole consumption profile. To combine the three desirable properties mentioned above—the existence of a price index, a simple behavior of the elasticity of substitution, and variable markups—we focus on homothetic preferences u for which there exists a function $s(x, u)$ such that

$$\overline{\sigma}(x, x, \mathbf{x}) = s(x, u(\mathbf{x})) \tag{1}$$

anything substantial to the analysis but makes the algebra more complicated.

[6]As shown by Zhelobodko *et al.* (2012), under symmetric additive preferences the elasticity of substitution between varieties i and j, $\overline{\sigma}(x_i, x_j, \mathbf{x})$, is independent of the remaining consumption pattern $\mathbf{x}$, given that both varieties are consumed in equal quantities. Put differently, if $x_i = x_j = x$, we have $\overline{\sigma}(x, x, \mathbf{x}) = \sigma(x)$. Thus, additive preferences have a simple behavior with respect to the elasticity of substitution. Since the substitutability across varieties is a key feature of the demand side in models of imperfect competition, the aforementioned property explains, at least partly, why additive preferences are so popular. Examples include, among others: Krugman (1979, 1980); Eaton and Kortum (2002); Melitz (2003); Behrens and Murata (2007); Zhelobodko *et al.* (2012); and Simonovska (2015).

at a symmetric consumption pattern where $x_i = x_j$. In expression (1), $u(\mathbf{x})$ is a consumption index, which is increasing, strictly quasi-concave, and positive homogeneous of degree 1 in $\mathbf{x}$. Moreover, observe that at a symmetric consumption pattern, given by $\mathbf{x} = x\,\mathbf{1}_{[0,N]}$, where $\mathbf{1}_{[0,N]}$ stands for the indicator of $[0, N]$ and where N is the mass of available varieties, we have

$$u(\mathbf{x}) = x\nu(N), \quad \text{so that} \quad s(x, u(\mathbf{x})) = s(x, x\nu(N)), \tag{2}$$

where $\nu(N)$ is defined as $\nu(N) \equiv u\big(\mathbf{1}_{[0,N]}\big)$.

As shown by Parenti *et al.* (2017), when preferences are homothetic, the elasticity of substitution $\overline{\sigma}$ depends solely on N at a symmetric consumption pattern. Combining this result with (2) shows that $s(x, u)$ is positive homogeneous of degree 0. Hence, expression (1) boils down to

$$\overline{\sigma}(x, x, \mathbf{x}) = \sigma(x/u(\mathbf{x})). \tag{3}$$

The intuition behind expression (3) is as follows. If, for example, $\sigma(\cdot)$ is a decreasing function, then a higher consumption level of both varieties makes them worse substitutes, while an increase in the overall level of consumption captured by $u(\mathbf{x})$ does the opposite. However, a *proportional change in the consumption of all varieties* leaves the degree of substitutability between any two of them unchanged.

The following proposition provides an alternative characterization of the preferences satisfying (3).

Proposition 1. (Kimball preferences) *A symmetric homothetic preference relationship satisfies (3) if and only if the utility function u is described by*

$$\int_0^N \theta\left(\frac{x_i}{u(\mathbf{x})}\right) \mathrm{d}i = 1, \tag{4}$$

where $\theta(\cdot)$ is an arbitrary non-negative, increasing, concave, and twice continuously differentiable function. Moreover, $\sigma(x/u(\mathbf{x}))$ is given by

$$\sigma(x/u(\mathbf{x})) = \frac{1}{r_\theta(x/u(\mathbf{x}))}, \tag{5}$$

where $r_\theta(\cdot)$ is the relative risk aversion of $\theta(\cdot)$:

$$r_\theta(x/u(\mathbf{x})) \equiv -\frac{[x/u(\mathbf{x})]\theta''(x/u(\mathbf{x}))}{\theta'(x/u(\mathbf{x}))}. \tag{6}$$

Proof. See Appendix A.1. □

To the best of our knowledge, Kimball (1995) was the first to use (4) to represent a production function for a fixed range of varieties.[7] What is hence new in Proposition 1? Firstly, it provides a new mathematical characterization of the class of preferences described by Kimball's flexible aggregator. Secondly, it shows that Kimball-type preferences are very close to additive preferences, in a sense of a simple behavior of their elasticity of substitution. For these preferences, the whole relevant information on the demand side to understand the market outcome is captured by $1/r_\theta(\cdot)$, i.e., the elasticity of substitution, which depends on $x/u(\mathbf{x})$ only. In other words, Kimball preferences (4) mimic the property of additive preferences where the whole relevant information is captured by the relative risk aversion of the lower-tier utility function $r_{\mathrm{v}}(x) \equiv -x\mathrm{v}''(x)/\mathrm{v}'(x)$. This leads to a substantial simplification of the analysis and allows us to derive analytical results on the impacts of trade liberalization. Last, observe that the CES utility with elasticity of substitution σ can be obtained as a special case of (4) by setting $\theta(z) \equiv z^{(\sigma-1)/\sigma}$. This will prove useful later since we can work with preferences satisfying (4), which are arbitrarily close to CES preferences yet are homothetic and have variable markups. These ε-CES preferences allow us to derive many results invoking continuity arguments within the class of Kimball preferences. We return to this point in detail in Section 4.2.

2.2 Consumers

Each country hosts L consumers, who have identical preferences given by

$$\mathcal{U}^k = U(u(\mathbf{x}^{kk}, \mathbf{x}^{lk}), v(\mathbf{y}^k)), \quad k, l \in \{H, F\}, \quad k \neq l, \tag{7}$$

where U is an upper-tier utility; u and v are lower-tier utilities for the consumption of, respectively, traded and non-traded goods; and $\mathbf{x}^{kk}$, $\mathbf{x}^{lk}$, and $\mathbf{y}^k$ are vectors of individual consumption of, respectively, locally produced varieties of the traded good in country k, imported varieties of the traded good in country k, and varieties of the non-traded good produced and consumed in k.

We assume that U, u, and v are strictly increasing, strictly quasi-concave, and homothetic. We also assume that the lower-tier utilities are Kimball type, as defined in Section 2.1, i.e., there exist increasing and concave functions θ and ψ, such that for any $\mathbf{x}^{kk}$, $\mathbf{x}^{lk}$, and $\mathbf{y}^k$, the lower-tier

[7] Dotsey and King (2005) use the same approach to model preferences in a macroeconomic setting, having again a fixed range of varieties, while Barde (2008) does the same in the setting of a economic geography model with a variable range of varieties. Arkolakis *et al.* (2018) use a more general demand system, which encompasses non-homothetic additive preferences and homothetic preferences whose ideal price index is described by Kimball's (1995) flexible aggregator. They focus mainly on measuring gains from trade under pro-competitive effects mostly using empirical estimates and numerical simulations. We, instead, work with a narrower class of demand systems, which allows us to provide an analytical characterization of the equilibrium and to derive a number of comparative static results.

utilities u and v satisfy

$$\int_0^{N^k} \theta\left(\frac{x_i^{kk}}{u^k}\right) \mathrm{d}i + \int_0^{N^l} \theta\left(\frac{x_j^{lk}}{u^k}\right) \mathrm{d}j = 1, \quad \int_0^{M^k} \psi\left(\frac{y_i^k}{v^k}\right) \mathrm{d}i = 1, \quad k,\, l \in \{H,\, F\}, \tag{8}$$

where $u^k \equiv u(\mathbf{x}^{kk},\, \mathbf{x}^{lk})$, $v^k \equiv v(\mathbf{y}^k)$; N^k and N^l are the masses of firms in the traded sector in countries k and l, respectively; and M^k is the mass of firms in the non-traded sector in country k. Note that N^k, N^l, and M^k, while treated parametrically by consumers, will be endogenously determined in equilibrium.

Normalizing the wage to one by choice of numéraire—and recalling that countries are symmetric and thus have the same equilibrium wage—consumers maximize utility (7) subject ot their budget constraint $\int_0^{N^k} p_i^{kk} x_i^{kk} \mathrm{d}i + \int_0^{N^l} p_i^{lk} x_i^{lk} \mathrm{d}i + \int_0^{M^k} \wp_i^k y_i^k \mathrm{d}i = 1$, where p_i^{kk} and p_i^{lk} are the prices of domestic and imported traded varieties in country k; and $\wp_i^k$ are the prices of non-traded varieties.

Let P^k and $\mathcal{P}^k$ denote the price indices in the traded and in the non-traded sectors in country k, respectively. Let also $\alpha(P^k/\mathcal{P}^k)$ denote the share of expenditure for traded goods, which depends solely on the ratio of the traded and the non-traded sectoral price indices since preferences are homothetic. The functional form of $\alpha(\cdot)$ is determined by the upper-tier utility. The value of $\alpha(P^k/\mathcal{P}^k)$ is taken as given by the consumers, although it changes with the price aggregates in the two sectors.

The inverse demands can be derived as follows. Applying standard two-stage budgeting techniques, note first that the consumers' subproblem for the consumption of the traded good is given by:

$$\begin{aligned} \max_{u^k,\, \mathbf{x}^{kk},\, \mathbf{x}^{lk}} \quad & u^k \\ \text{s.t.} \quad & \int_0^{N^k} p_i^{kk} x_i^{kk} \mathrm{d}i + \int_0^{N^l} p_i^{lk} x_i^{lk} \mathrm{d}i = \alpha\left(\frac{P}{\mathcal{P}}\right) \\ & \int_0^{N^k} \theta\left(\frac{x_i^{kk}}{u^k}\right) \mathrm{d}i + \int_0^{N^l} \theta\left(\frac{x_j^{lk}}{u^k}\right) \mathrm{d}j = 1, \end{aligned} \tag{9}$$

where we make use of the representation of u. Setting $\mathbf{z}^{kk} \equiv \mathbf{x}^{kk}/u^k$ and $\mathbf{z}^{lk} \equiv \mathbf{x}^{lk}/u^k$ for notational convenience, we reformulate the constraints as

$$\int_0^{N^k} p_i^{kk} z_i^{kk} \mathrm{d}i + \int_0^{N^l} p_i^{lk} z_i^{lk} \mathrm{d}i = \frac{\alpha(P/\mathcal{P})}{u^k}, \quad \text{and} \quad \int_0^{N^k} \theta\left(z_i^{kk}\right) \mathrm{d}i + \int_0^{N^l} \theta\left(z_i^{lk}\right) \mathrm{d}j = 1.$$

Since maximizing u^k is equivalent to minimizing $1/u^k$, we obtain the following equivalent reformulation of (9) as an expenditure minimization problem:

$$\min_{\mathbf{z}} \quad \int_0^{N^k} p_i^{kk} z_i^{kk} \mathrm{d}i + \int_0^{N^l} p_i^{lk} z_i^{lk} \mathrm{d}i \quad \text{s.t.} \quad \int_0^{N^k} \theta\left(z_i^{kk}\right) \mathrm{d}i + \int_0^{N^l} \theta\left(z_i^{lk}\right) \mathrm{d}j = 1. \tag{10}$$

The first-order conditions of (10) are given by

$$\frac{\theta'(z_i^{kk})}{\mu^k} = p_i^{kk}, \quad \frac{\theta'(z_j^{lk})}{\mu^k} = p_j^{kl}, \tag{11}$$

where μ^k is a sectoral market aggregate that involves the Lagrange multiplier of the budget constraint and the marginal utilities of traded varieties. Using the subproblem for the non-traded sector, we analogously obtain the inverse demands for the non-traded goods as follows:

$$\frac{\psi'\left(y_i^k/v\right)}{\lambda^k} = \wp_i^k, \tag{12}$$

where λ^k is a sectoral market aggregate for the non-traded good.

Note from (11) and (12) that the first-order conditions for Kimball preferences are very similar to those for additive preferences. To see this similarity, it suffices to replace θ' with v$'$ and the 'scaled quantities' z with normal quantities x.

2.3 Firms

Firms in both sectors incur a constant fixed cost, f, and a constant marginal cost, c, both paid in terms of labor. The costs for shipping goods between countries in the traded sector are of the standard iceberg form: $\tau \geq 1$ units of the good have to be dispatched for one unit to arrive. We now turn to the profits of firms in country k. The profits of firm i in the traded sector and of firm j in the non-traded sector are given by

$$\Pi_i^k = (p_i^{kk} - c)Lx_i^{kk} + (p_i^{kl} - c\tau)Lx_i^{kl} - f \quad \text{and} \quad \Pi_j^k = (\wp_j^k - c)Ly_j^k - f,$$

respectively. Because countries are symmetric, we naturally focus on a symmetric outcome. In what follows, we use the notation, where d denotes domestic and m denotes import values of variables.[8] As before, we also define $z^d \equiv x^d/u^d$, $z^m \equiv x^m/u^m$, and $z^n \equiv y/v$. Because we work with a continuum of monopolistically competitive firms, no single firm has any impact on the market aggregates μ and λ. Combining this with the definition (6) of r_θ and the inverse demands (11), we find that r_θ is the inverse demand elasticity. Hence, applying the standard monopoly

[8]More formally, $x^d \equiv x^{HH} = x^{FF}$, $x^m \equiv x^{FH} = x^{HF}$, $y \equiv y^H = y^F$, $p^d \equiv p^{HH} = p^{FF}$, $p^m \equiv p^{FH} = p^{HF}$, $\wp \equiv \wp^H = \wp^F$, $\lambda \equiv \lambda^H = \lambda^F$, $\mu \equiv \mu^H = \mu^F$, $N \equiv N^H = N^F$, $M \equiv M^H = M^F$, $u \equiv u^H = u^F$, $v \equiv v^H = v^F$.

pricing rule yields

$$p^d = \frac{c}{1 - r_\theta(z^d)}, \quad p^m = \frac{c\tau}{1 - r_\theta(z^m)}, \quad \wp = \frac{c}{1 - r_\psi(z^n)}, \tag{13}$$

where $r_\theta(z^d)$ is the markup for domestically produced varieties of the traded good; $r_\theta(z^m)$ is the markup for imported varieties; and $r_\psi(z^n)$ is the markup for non-traded varieties, where we replace u by v in (6).[9] The second-order conditions are relegated to Appendix B.

Expressions (13) imply that, unlike in the standard CES model of international trade, markups vary with the mass of firms. The impact of entry on the markups is channeled through changes in the relative consumptions z^d, z^m, and z^n. Intuitively, per variety relative consumption should decrease as the product range expands, because of love for variety. As consumers have more consumption choices, they diversify their consumption bundles so that the individual consumption levels of each variety decrease. This is not the end of the story, however. Love for variety also implies that an increase in the mass of varieties makes consumers better off, i.e., increases the lower-tier utilities u and v, thereby magnifying the reduction of relative consumptions. Note that the former effect has been studied in Zhelobodko *et al.* (2012), while the latter effect cannot be captured by a model with additive preferences. What happens to the markups as more firms enter? As implied by (13), the answer to this question is fully determined by the behavior of the functions $r_\theta(\cdot)$ and $r_\psi(\cdot)$. We will focus mostly on the case where both functions are increasing: this entails a pro-competitive effect, i.e., entry of firms increases competition, thus leading to a fall in the markups. Empirical evidence points to the existence of pro-competitive effects (e.g., De Loecker *et al.*, 2012).[10] An alternative justification for this assumption can be derived from the conventional wisdom prevailing in industrial organization: the more firms operate in the market, the less differentiated their products are. Because $\sigma = 1/r_\theta$ by Proposition 1, $r_\theta(\cdot)$ and $r_\psi(\cdot)$ can be viewed as measures of product differentiation in, respectively, the traded and the non-traded sectors. These measures naturally change with entry in the different industries. In particular, if they are increasing functions, an increase in the quantity indices, u and v, or, conversely, lower quantity indices, make varieties closer substitutes. Combining (11) and (13), we readily obtain

$$\theta'(z^d)\left[1 - r_\theta(z^d)\right] = \mu c, \quad \theta'(z^m)\left[1 - r_\theta(z^m)\right] = \mu c\tau, \quad \theta'(z^n)\left[1 - r_\psi(z^n)\right] = \lambda c. \tag{14}$$

Conditions (14), which equate marginal revenue and marginal cost, allow us to pin down the quantities z^d, z^m, and z^n as functions of the market aggregates μ and λ only.

[9]The analogy with the first-order conditions in the additive case is again very clear from (13). See, e.g., Zhelodobko *et al.* (2012).

[10]In addition, this assumption guarantees that the second-order conditions for profit maximization hold. See Appendix B for details.

3 Equilibrium in the non-traded sector

We now study how the equilibrium in the non-traded sector changes with trade liberalization, as measured by a decrease in trade costs τ. Using (8), (13), and the sectoral budget constraint $M\wp y = 1 - \alpha$, for a given mass M of non-traded varieties the symmetric equilibrium conditions can be expressed as follows:

$$\mathcal{P}v = M\frac{cy}{1 - r_{\psi}\left(z^{n}\right)} = 1 - \alpha\left(\frac{P}{\mathcal{P}}\right), \tag{15}$$

and

$$M\psi(z^{n}) = 1. \tag{16}$$

Solving (16) for z^n yields

$$z^{n} = \psi^{-1}\left(1/M\right). \tag{17}$$

Since ψ is an increasing function, the relationship (17) implies that the relative consumption of each variety of the non-traded good decreases with an expanding range of varieties in the non-traded sector. Using (13) and (17), we find that the equilibrium markup for a variety of the non-traded good is given by $r_{\psi}\left[\psi^{-1}\left(1/M\right)\right]$. Hence, the markups for non-traded varieties decrease in response to entry if and only if $r_{\psi}(\cdot)$ is increasing or, equivalently, if the elasticity of substitution $\sigma(\cdot)$ is decreasing.

We next analyze how the price index $\mathcal{P}$ varies with entry. Dividing (15) by v, using $z^{n} \equiv y/v$ and (17), we can express the price index as a function of the mass of firms:

$$\mathcal{P} = \frac{cM\psi^{-1}(1/M)}{1 - r_{\psi}\big(\psi^{-1}(1/M)\big)}. \tag{18}$$

As can be seen from (18), if $r_{\psi}(\cdot)$ is increasing, additional entry leads to a fall in prices. In other words, in that case more firms means tougher competition.[11] The total expenditure for the varieties of the non-traded good is given by

$$E(M,\, P) = L\left[1 - \alpha(P/\mathcal{P})\right], \tag{19}$$

where we use the result that the price index in the non-traded sector depends on M only. By (18) and (19), E increases in response to an expansion of product diversity because consumers value variety. Furthermore, E naturally decreases when traded goods become cheaper due to the substitution effect between traded and non-traded goods.[12] This effect will be key in what

[11]The numerator in (18) is always decreasing in M, for the elasticity of $\psi^{-1}(\cdot)$ exceeds 1 by concavity of $\psi(\cdot)$. Actually, $r_{\psi}(\cdot)$ could also be 'moderately decreasing' without changing that result. We henceforth consider mostly the case where it is increasing, as this seems to be empirically more relevant.

[12]Manufactured goods and services are arguably substitutes rather than complements, at least for final goods.

follows to understand how trade liberalization affects the economy as a whole and the intersectoral allocation in particular.

Finally, (19) and the expressions for markups imply that the equilibrium firm size, $q_n = E/(M\wp)$, and operating profit, π_n, in the non-traded sector are given by

$$q_n(M,\, P/\mathcal{P}) = \frac{L}{cM}\left[1 - \alpha(P/\mathcal{P})\right]\left[1 - r_\psi\big(\psi^{-1}(1/M)\big)\right], \tag{20}$$

and

$$\pi_n(M,\, P/\mathcal{P}) = \frac{L}{M}\left[1 - \alpha(P/\mathcal{P})\right] r_\psi\big(\psi^{-1}(1/M)\big), \tag{21}$$

respectively. These expressions will be helpful to analyze how firms' sizes change in response to trade liberalization. As implied by (20), an increasing $r_\psi(\cdot)$ is sufficient (but not necessary) for entry to reduce firms' sizes.

We now determine the equilibrium mass, M, of firms. To this end, we assume that there is free-entry so that profits are zero in (21). The zero-profit condition is given by

$$\frac{L}{M}\big[1 - \alpha(P/\mathcal{P})\big] r_\psi\big(\psi^{-1}(1/M)\big) = F. \tag{22}$$

Condition (22) allows us to pin down the mass of firms in the non-traded sector as a function of the price index, P, of traded varieties. We can show the following result:

Proposition 2. (Traded prices and the mass of non-traded firms) *Assume that $r_\psi(\cdot)$ is increasing and that the budget share of non-traded goods, $\alpha(\cdot)$, decreases not too fast. Then there exists a unique symmetric free-entry equilibrium in the non-traded sector. Furthermore, a reduction in the price index, P, for the traded goods leads to less firms and higher markups in the non-traded sector.*

Proof. When $r'_\psi(\cdot) > 0$, and $\alpha(P/\mathcal{P})$ decreases not too fast, then the operating profit (21) is a decreasing function of M. In this case, (22) has a unique solution $M^*(P)$. Plugging M^* into equations (19) and (20) and using the definition of the markups then determines a unique symmetric free-entry equilibrium. Furthermore, because $\alpha(\cdot)$ is a decreasing function, a reduction in P implies an upward shift of the locus described by the left-hand side of (22). Hence, cheaper traded goods lead to fewer firms in the non-traded sector. □

What is the economic meaning of the assumptions underlying Proposition 2? As discussed above, the first assumption, $r'_\psi > 0$, is a necessary and sufficient condition for entry to generate pro-competitive effects. We believe that this is the empirically plausible case. Moreover, this condition has a purely demand side-related interpretation: a higher consumption index is equivalent to more product differentiation. The second assumption, namely that $\alpha(\cdot)$ is a slowly

For example, public transportation and cars are clearly substitutes. While we acknowledge that complementarities do exist, we believe that substitutability is overall the more plausible assumption.

decreasing function, means that traded and non-traded goods are relatively poor substitutes for consumers. If we think about the former as including mostly manufactured goods, whereas the latter consist mostly of services, this seems to be a fairly natural assumption to make. In what follows, we take the assumptions that $r'_{\psi}(\cdot) > 0$ and that $\alpha(\cdot)$ decreases not too fast as our benchmark.[13]

To illustrate the latter by means of a simple example, consider the CES upper-tier utility:

$$\mathcal{U} = \left[\beta u^{(\sigma-1)/\sigma} + (1-\beta) v^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \quad 0 < \beta < 1 < \sigma.$$

In that case, we have

$$\alpha\left(P/\mathcal{P}\right) = \frac{\left(P/\mathcal{P}\right)^{1-\sigma}}{\left[(1-\beta)/\beta\right]^{\sigma} + \left(P/\mathcal{P}\right)^{1-\sigma}}. \tag{23}$$

From equation (23), we see that when $\sigma \to 1$ (i.e., goods become independent), the slope of $\alpha(\cdot)$ is less and less steep. Hence, the operating profit (21) is more likely to decrease in M since traded and non-traded goods are poor substitutes.

Given the price index P for traded goods, equations (18) and (22) uniquely pin down the equilibrium price index, $\mathcal{P}^*(P)$, and the equilibrium mass of firms, $M^*(P)$. More precisely, we can solve (22) for M and plug the solution into (18). We then obtain a decreasing function $\mathcal{P} = \mathcal{P}^*(P)$. This allows us to express the equilibrium expenditure share for non-traded goods as a function of P only: $a(P) \equiv \alpha(P/\mathcal{P}^*(P))$. Since the expenditure share is the only channel through which the traded sector affects the non-traded sector, *the impact of a fall in trade costs on the non-traded sector is fully captured by changes in the price index in the traded sector.*

How does a change in P affect the consumers' expenditure shares? Two effects are at work. First, there is the standard substitution effect between traded and non-traded goods, which increases $\alpha(\cdot)$ when traded goods become cheaper. Second, there is the income effect that arises because an increase in $\alpha(\cdot)$ leads to more demand for traded goods, thus enticing more firms to enter the traded sector. This makes competition tougher in that sector and leads to an additional fall in P. Since both effects work in the same direction, $a(P)$ unambiguously decreases with P. In other words, if trade liberalization reduces the price index P for traded goods, expenditures on non-traded goods always decrease. The key finding in Proposition 2 is that cheaper traded goods lead to fewer firms in the non-traded sector. This, in turn, makes competition in that sector less tough (because $r_{\psi}(\cdot)$ is increasing function), which allows non-traded firms to charge higher markups. In Section 5.1, we show that trade liberalization reduces the price index for traded goods under fairly plausible assumptions, thereby reducing competition in the non-traded sector.

[13] If α decreases "too steeply", we may loose uniqueness. However, for the existence of a stable free-entry equilibrium, it suffices (though it is not necessary) that both α and r_{ψ} are bounded away from 1. All the comparative statics results remain valid for stable equilibria.

4 Equilibrium in the traded sector

We now analyze the equilibrium in the traded sector. We proceed in two steps. First, we look at the impacts of freer trade on the traded sector for the general case of Kimall preferences. We derive sufficient conditions for the price index in the traded sector to fall with trade liberalization. This comparative static result is crucial to analyze the effects of freer trade in Section 5.1. Second, we introduce 'ε-CES preferences', which are a special—but relevant—case of Kimball preferences, to derive sharper results. We show that these preferences can get arbitrarily close to CES preference so that the equilibrium displays comparative static properties similar to those prevailing under the CES (see Proposition 5 below). This result is important for the subsequent analysis since it allows us to derive many properties of equilibrium by continuity in the vicinity of CES preferences for which results are relatively easy to compute.

4.1 General case

A symmetric equilibrium in the traded sector satisfies the following four equilibrium conditions:

(i) zero profit condition:

$$x^d \frac{r_\theta\left(z^d\right)}{1-r_\theta\left(z^d\right)} + \tau x^m \frac{r_\theta\left(z^m\right)}{1-r_\theta\left(z^m\right)} = \frac{F}{cL}; \tag{24}$$

(ii) Kimball preference representation:

$$N\left[\theta\left(z^d\right)+\theta\left(z^m\right)\right] = 1; \tag{25}$$

(iii) profit maximization:

$$\frac{\theta'\left(z^d\right)\left(1-r_\theta\left(z^d\right)\right)}{\theta'\left(z^m\right)\left(1-r_\theta\left(z^m\right)\right)} = \frac{1}{\tau}; \tag{26}$$

(iv) sectoral budget constraint:

$$Pu = a(P) = Nu\left[\frac{cz^d}{1-r_\theta\left(z^d\right)} + \frac{c\tau z^m}{1-r_\theta\left(z^m\right)}\right]. \tag{27}$$

We first solve (25)–(26) for the relative consumption levels, z^d and z^m. Given the mass of firms, N, in each country, the locus in (25) is downward-sloping in (z^d, z^m)-space, because $\theta(\cdot)$ is an increasing function. The slope of the locus in (26) is positive, which comes from the second-order condition for profit maximization (that condition is given by $r_{\theta'} < 2$; see Appendix B). Consequently, the two curves have a unique intersection denoted by $\left(\overline{z}^d(N,\tau),\, \overline{z}^m(N,\tau)\right)$. How does that intersection vary with the mass of firms, N, and trade costs, τ? When N increases, the locus in (25) shifts downwards, while the locus in (26) remains unchanged. Thus, both $\overline{z}^d$

and $\overline{z}^m$ decrease in the mass of firms. An intuitive explanation for this result is love for variety: additional entry leads consumers to spread their budget over a larger range of varieties. Higher trade costs, τ, lead to a downward shift of the locus in (26), while the locus in (25) remains unchanged. Thus, $\overline{z}^d$ increases with trade costs, while $\overline{z}^m$ decreases with trade cost. Intuitively, freer trade shifts relative demands from domestically produced varieties towards imported ones.

Observe that domestic markups $r_\theta^d(\overline{z}^d)$ and foreign markups $r_\theta^m(\overline{z}^m)$ depend on the mass of firms, N, and trade costs, τ, via the relative consumption levels $\overline{z}^d$ and $\overline{z}^m$. Entry drives both markups downward if and only if $r_\theta(\cdot)$ is an increasing function. In other words, whether the effect of entry is pro- or anti-competitive is fully determined by the nature of consumers' variety-loving behavior. Note that for a given mass of firms, trade liberalization always shifts domestic and foreign markups in opposite directions. More precisely, a decrease in τ drives domestic markups downwards—and foreign markups upwards—if and only if $r_\theta(\cdot)$ is an increasing function. Otherwise, the result is reversed. In the empirically plausible case with pro-competitive effects, domestic markups and foreign markups converge as trade becomes freer: in the limit, when trade is costless, domestic and export markups are equalized.

We can summarize the foregoing results in the following proposition:

Proposition 3. **(Equilibrium in the traded sector)** *Assume that $r_\theta'(z) > 0$. Then: (i) there exists a unique symmetric equilibrium for any given N; (ii) the equilibrium domestic and foreign markups $r_\theta^d\left(\overline{z}^d\right)$ and $r_\theta^m\left(\overline{z}^m\right)$ both decrease with N; and (iii) the domestic markups $r_\theta^d\left(\overline{z}^d\right)$ decrease with τ, whereas the foreign markups $r_\theta^m\left(\overline{z}^m\right)$ increase with τ.*

Proof. In the text. □

How does the price index P vary with the mass of firms N and trade cost τ? To answer that question, we first rewrite (27) as follows:

$$P = \overline{P}(N,\,\tau) \equiv cN\left[\frac{\overline{z}^d(N,\,\tau)}{1 - r_\theta(\overline{z}^d(N,\,\tau))} + \frac{\tau\overline{z}^m(N,\,\tau)}{1 - r_\theta(\overline{z}^m(N,\,\tau))}\right],$$

and use the definition of homothetic preferences to get $u = \overline{u}(N,\,\tau) \equiv a\left(\overline{P}(N,\,\tau)\right)/\overline{P}(N,\,\tau)$. We still need to pin down N, which is endogenously determined by free entry. To this end, we use the zero profit condition (24), which now takes the following form:

$$\overline{u}(N,\,\tau)\left[\overline{z}^d(N,\,\tau)\frac{r_\theta(\overline{z}^d(N,\,\tau))}{1 - r_\theta(\overline{z}^d(N,\,\tau))} + \tau\overline{z}^m(N,\,\tau)\frac{r_\theta(\overline{z}^m(N,\,\tau))}{1 - r_\theta(\overline{z}^m(N,\,\tau))}\right] = \frac{F}{cL}. \tag{28}$$

How the left-hand side of (28), the traded good producers' operating profits $\pi(N,\,\tau)$, varies with N is generally ambiguous, so that multiple equilibria may a priori exist. Let N^* denote a solution to (28). In what follows, we focus on stable equilibria only, i.e., those where $\partial\pi/\partial N < 0$

at $N = N^*$.[14]

The impact of a change in τ on firms' profits is twofold: (i) trade liberalization reduces costs, which leads to rising profits; and (ii) trade liberalization shifts the price index $\overline{P}(N, \tau)$, which may result in tougher competition and hence in lower profits. In any case, (28) implies that at any stable equilibrium trade liberalization leads to an increase in the equilibrium mass of firms N^* if and only if the former effect dominates the latter, i.e., when $\partial\pi/\partial\tau < 0$. If, in addition, $\overline{P}(N, \tau)$ decreases in N and increases in τ, then trade liberalization also drives down the equilibrium value of the price index $P^* \equiv \overline{P}(N^*, \tau)$. Thus, we have the following result.

Proposition 4. (Decreasing price index) *A sufficient condition for* $\mathrm{d}P^*/\mathrm{d}\tau > 0$ *is given by: (i)* $\partial\overline{P}/\partial N < 0$*; and (ii)* $\mathrm{d}N^*/\mathrm{d}\tau < 0$*.*

Proof. In the text. □

Under what conditions on the model's primitives are we sure that $\partial\overline{P}/\partial N < 0$ and $\mathrm{d}N^*/\mathrm{d}\tau < 0$? As we show in the next section, these two properties always hold in the CES model. Hence, they should intuitively also hold for preferences that are 'sufficiently close' to the CES in a sense that we need to make precise. We hence now introduce the concept of ε-CES preferences and show that an ε-CES equilibrium exists. When ε is small enough, that equilibrium has the same qualitative properties than the CES equilibrium, safe that markups are variable. The latter property is very useful to investigate the behavior of models with Kimball preferences that are, however, close to CES preferences and inherit many of their properties by continuity.

4.2 ε-CES preferences

As discussed in Section 2.1, Kimball preferences include the CES as a special case. Given that there is a continuum of Kimball-type preferences, we can choose preferences that are 'arbitrarily close' to the CES within that class. Let

$$\theta(z) = z^{\rho}\exp\big(\varphi(z)\big), \quad \rho \equiv \frac{\sigma}{\sigma - 1} \in (0, 1), \tag{29}$$

where $\varphi(\cdot)$ is 'sufficiently small' in a sense that $||\varphi(\cdot)|| < \varepsilon$. Intuitively, if $\varphi(\cdot)$ is close to zero then (29) will be close to CES preferences.[15] We hence call these preferences ε-CES preferences. We relegate the technical details to the appendix and describe in Appendix A.2 conditions which ensure these preferences are 'close' to standard CES preferences. We also derive the equilibrium conditions of the model for that case. We can prove the following result.

Proposition 5. (Existence and continuity of ε-*ces* equilibrium) *For ε-CES preferences with ε 'sufficiently small', an equilibrium: (i) always exists; and (ii) is a small perturbation of the CES equilibrium.*

[14]By doing so, we do not rule out the possibility of multiple stable equilibria. We will do so in the next subsection, where we focus on a specific instance of preferences that lead to a unique stable equilibrium.

[15]Consider the case where $\varphi(\cdot) \equiv 0$, then (29) boils down exactly to CES preferences.

Proof. See Appendix A.3. □

The key result for the subsequent analysis is the continuity property of ε-CES equilibria with respect to the CES, as stated by claim (ii) of Proposition 5. This property explains why our model largely retains the tractability of the CES case while allowing for departures from it along several empirically relevant dimensions—variable markups and firm sizes. As shown in the next section, it provides a powerful tool for comparative statics analyses to reveal, e.g., the impacts of trade liberalization under non-CES Kimball preferences.

5 The impacts of trade liberalization

We are now equipped to investigate how trade liberalization, i.e., a decrease in τ, influences the equilibrium in the traded and in the non-traded sectors. First, we look at how trade liberalization directly affects the traded sector. We then take into account the indirect effects of trade liberalization on the non-traded sector. This ordering is natural since, as shown before, the effects in the non-traded sector stem entirely from changes in the price index of the traded sector.

5.1 Traded sector

We show in Appendix A.4 that when preferences are CES, the following comparative statics hold:

$$\frac{\mathrm{d}(z^d)^*}{\mathrm{d}\tau} > 0 > \frac{\mathrm{d}(z^m)^*}{\mathrm{d}\tau}, \quad \frac{\mathrm{d}P^*}{\mathrm{d}\tau} > 0, \quad \text{and} \quad \frac{\mathrm{d}N^*}{\mathrm{d}\tau} < 0. \tag{30}$$

Moreover, intuition suggests that (30) must still hold by continuity for the ε-CES case when ε is small enough. Appendix A.4 provides a formal proof relying on Proposition 5. These comparative static results are important for assessing how markups and firm sizes in the traded sector react to trade liberalization.

Markups. We first look at how domestic and export markups in the traded sector change in response to trade liberalization. Recall that in the CES case, markups are invariant to changes in the mass of firms and to changes in trade costs. This is, however, no longer the case under ε-CES preferences, even when ε is small. Put differently, even small departures from the CES lead to variable markups, and we can investigate their behavior in response to freer trade.[16]

In what follows, we focus on small perturbations, φ, in (29) that generate $r'_\theta(z) > 0$. In that

[16] It is worth noting that the method of establishing comparative statics "by continuity" like in Appendix A.3 does not work with markups, for under the CES we have $\partial r_\theta^{d*}/\partial\tau = \partial r_\theta^{m*}/\partial\tau = 0$. Hence, we can say nothing using continuity arguments.

case, $\mathrm{d}(z^d)^*/\mathrm{d}\tau > 0 > \mathrm{d}(z^m)^*/\mathrm{d}\tau$ directly implies

$$\frac{\mathrm{d}(r_\theta^d)^*}{\mathrm{d}\tau} > 0 > \frac{\mathrm{d}(r_\theta^m)^*}{\mathrm{d}\tau}. \tag{31}$$

In words, trade liberalization leads to a reduction in markups for locally produced varieties, and to an increase in the markups of imported varieties. Since domestic markups exceed export markups, this means that freer trade leads to the *convergence of markups for traded goods across countries*. Yet, as we show in the next subsection, it also leads to a *divergence of markups across industries within countries*.

Firm size. We now study how trade liberalization affects firm size, $q \equiv L(x^d + \tau x^m)$. In the CES case, it is well known that trade liberalization does not affect firm size but leads to a decrease in domestic sales, x^d, and an increase in exports, τx^m. Slightly perturbing the CES preferences reveals additional effects of trade liberalization on firm size. To see this, we combine (A-6) and (A-8) in Appendix A.2 to obtain:

$$q\frac{r_\theta(z^d)}{1 - r_\theta(z^d)} - L\tau x^m \left[\frac{r_\theta(z^d)}{1 - r_\theta(z^d)} - \frac{r_\theta(z^m)}{1 - r_\theta(z^m)}\right] = \frac{F}{c}. \tag{32}$$

As we have shown before, a reduction in trade costs τ leads to: (i) an increase in output for the foreign market, τx^m, by continuity with the CES case; and (ii) an increase in the markups of imported varieties, $r_\theta(z^m)$, and a decrease in the markup of domestic varieties, $r_\theta(z^d)$. Note also that, because we are close to the CES, the latter effect is of second order compared to the former effect. More generally, all new effects that arise under ε-CES preferences are of second order and thus are negligible compared to the first-order effects for sufficiently small values of ε. Furthermore, as shown in Appendix A.2., we have $z^m < z^d$, so that the expression in square brackets in (32) is positive. To sum up, the second term on the left-hand side of (32) increases with trade liberalization, because the positive effect of trade liberalization on output (τx^m) dominates the negative effect on markups. This implies that the first term must also increase for the zero-profit condition to hold. Since domestic markups, $r_\theta(z^d)$, decrease with a fall in trade costs, we can conclude that firm output q must hence increase with trade liberalization.[17]

Size of the manufacturing sector. We now investigate how trade liberalization affects the relative size of the traded sector, given by $N^*(cq^* + F)/L$. Note firstly that the labor force of individual firms, $cq^* + F$, increases, yet that this is a second-order effect under ε-CES preferences (because firm size is constant under the CES). Note secondly that the mass of firms in the traded sector N^* increases because of changes in $a(P)$. This constitutes the first-order effect for changes

[17] Evidence of increasing firm sizes in the export sector following trade liberalization is provided by, e.g., Levinsohn (1999).

Table 1: Trade openness and the share of manufacturing employment.

	Dependent variable: log(manufacturing share)					
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
ln(openness)	-0.054	-0.024	0.001			
	(0.034)	(0.026)	(0.024)			
ln(openness, imports)				-0.229^a	-0.208^a	-0.192^a
				(0.033)	(0.033)	(0.029)
ln(openness, exports)				0.157^a	0.168^a	0.172^a
				(0.027)	(0.027)	(0.027)
ln(value-added per employee)		-0.072^a			-0.057^a	
		(0.021)			(0.019)	
ln(output per employee)			-0.104^a			-0.097^a
			(0.025)			(0.023)
p-value: ln(openness, imports) + ln(openness, exports) $= 0$				0.024	0.138	0.389
Observations	2,147	1,988	2,003	2,147	1,988	2,003
R-squared	0.952	0.955	0.958	0.956	0.958	0.961

Notes: Unbalanced panel of 145 countries between 1980 and 2004. See Appendix C for additional information on the data. All specifications include country and year fixed effects. Huber-White robust standard errors are reported in parentheses. Significance levels: ^{a}: $p < 0.01$, ^{b}: $p < 0.05$, ^{c}: $p < 0.1$.

in the size of the traded sector. If we assume that $a(P)$ is 'almost flat', then the amount of labor allocated to the traded sector increases only a little in the wake of trade liberalization (see the discussion in Section 3). In other words, the model predicts that the traded sector expands slightly as trade gets freer. This prediction may seem counterfactual since the traded sector—which is mostly manufacturing—has decreased as a share of employment in all developed countries over the last 40 years, despite increasing trade liberalization. One reason for that change is the increase of manufacturing productivity via the substitution of capital for labor. This effect is absent from our model where firm-level productivity is invariant. Columns (1)–(3) in Table 1 report empirical evidence consistent with those observations. As can be seen from the table, the impact of trade openness—measured by the ratio of imports plus exports over GDP—on the share of manufacturing employment in the economy is very small and insignificant, while productivity gains are associated with falling shares of manufacturing employment. In our model, trade costs are symmetric so that trade liberalization increases both import competition and export opportunities. When breaking down trade openness in terms of imports and exports in the data, we see that a 1% increase in both imports and exports has a negative and statistically significant effect on the share of manufacturing employment in specification (4). This result seems to run against the expansion of manufacturing with increasing trade liberalization as predicted by the model. However, the negative effect disappears once we control for productivitiy in regressions (5) and (6), i.e., a symmetric trade expansion has an insignificant impact on manufacturing employment.[18]

The following proposition summarizes our key results concerning the impacts of trade liberalization on firms in the traded sector.

Proposition. (Effects of trade liberalization on the traded sector) *Consider the case of ε-CES preference with $r'_\theta > 0$. Then there exists $\overline{\varepsilon}$ such that for every $0 < \varepsilon < \overline{\varepsilon}$, trade liberalization leads to: (i) lower markups for domestically produced traded varieties; (ii) higher markups for imported traded varieties; (iii) more firms in the traded sector; (iv) larger firms, as measured by their total output, in the traded sector; and (v) more labor allocated to the traded sector.*

Proof. In the text. □

Observe that the same qualitative properties also hold in other models of monopolistic competition, such as those by Krugman (1979), Behrens and Murata (2007, 2012), Zhelobodko *et al.* (2012), and Kichko *et al.* (2014). The novelty of our results is to bring together multiple sectors and costly trade in general equilibrium with variable elasticity of substitution for a rich class of homothetic preferences. We further show in Section 6 that the results of Proposition 5.1 may still hold outside of a 'small neighborhood', i.e., when we move farther away from the CES case while keeping preferences as given by (4).

5.2 Non-traded sector

We now turn to the non-traded sector. As shown in Section 3, the impacts of trade costs on the non-traded sector are fully captured by the changes in the price index associated with the traded sector. Since we have established the impact of trade costs on the price index in the foregoing subsection, the analysis is now straightforward.

Markups. Because trade liberalization makes competition in the traded sector tougher, as shown in Section 5.1, the price index for traded goods falls as trade gets freer. Furthermore, when traded goods get relatively cheaper, the expenditure share on non-traded goods falls. As a consequence, if $r'_\psi > 0$ and $\alpha(\cdot)$ is a slowly decreasing function, a decrease in P reduces the equilibrium mass of firms, M^*, in the non-traded sector. This, in turn, leads to an increase in both the price index and the markups in that sector, as shown in Section 3. It is worth pointing out here that this effect is entirely driven by the general equilibrium nature of the model, which leads to

[18] These results are illustrative only and we do not claim that they have any causal interpretation. For example, the productivity changes may pick up trade liberalization effects as emphasized in the heterogeneous firms literature following Melitz (2003). When firms are heterogeneous, the intersectoral allocation effects may well be different. For example, Trefler (2004) documents that the Canada-US free trade agreement led to a decrease of Canadian manufacturing employment by 5%, despite substantial productivity gains and plant expansions for the more productive firms. Levinsohn (1999) finds only little between-sector reallocations using episodes of trade liberalization in Chile.

a reallocation of expenditure across industries in the wake of trade liberalization. There is no 'anti-competitive behavior' of firms, although such behavioral reactions of firms to trade liberalization have often been put forward to explain the increase in markups in service industries in the wake of deeper European integration (see Badinger, 2007). To sum up, with trade liberalization, the *markups for domestically produced traded and non-traded goods move in opposite directions*, and this is simply a market reaction to a change in relative prices and the associated changes in expenditures, entry and, ultimately, toughness of competition.

Firm size. Recall that the size of a firm in the non-traded sector is given by (20), i.e.,

$$q_n(M,\, P) = \frac{M}{cL}[1 - a(P)]\left[1 - r_\psi[\psi^{-1}(1/M)\right].$$

Under our assumption that $a(P)$ decreases slowly with the price index, and since changes in r_ψ are of second-order magnitude, the change in q_n is fully captured by changes in the mass of firms M. Because M^* increases with τ, so does the denominator in (20), and hence the whole fraction decreases.

The following proposition summarizes our key results concerning the impacts of trade liberalization on firms in the non-traded sector.

Proposition 6. (Effects of trade liberalization on the non-traded sector) *Assume an ε-CES lower-tier utility in the traded sector and $r'_\psi > 0$ in the non-traded sector. Then trade liberalization leads to: (i) an increase in the markups for non-traded varieties; (ii) an increase in the price index of non-traded varieties; (iii) fewer firms in the non-traded sector; and (iv) larger firm sizes in the non-traded sector.*

Proof. In the text. □

We next investigate how 'far' we can move away from CES preferences while having the same qualitative results and study the welfare effects of trade liberalization. As we have shown

in Propositions 6 and 5.1, a decrease in τ increases market power and reduces product diversity in the non-traded sector, whereas the opposite holds in the traded sector. Hence, the welfare effects of trade liberalization are a priori ambiguous.[19]

6 Numerical illustrations and welfare effects

We now investigate how 'far' we can deviate from the CES model. Answering this question is important for three reasons. First, we know that the results of the preceding sections hold in

[19] Behrens *et al.* (2018) show that with an unspecified upper-tier utility function and CES lower-tier utilities, positive price shocks to one sector never translate into aggregate welfare losses. Thus, this result also holds by continuity under ε-CES preferences. However, it is a priori unclear whether gains from trade are higher or lower under ε-CES preferences compare to standard CES preferences.

a 'small neighborhood' of the CES. Yet, we have no good (numerical) idea what we mean by a 'small neighborhood'. If our results really hold only for very small deviations from the CES, their relevance may be limited. Second, in the CES case, markups are fixed. It is, therefore, of interest to see whether variable markups increase or decrease the welfare changes due to trade liberalization and intersectoral reallocations as compared to the CES case. To put it bluntly, we want to know how variable the variable markups can be so that our main findings still remain valid.

In what follows, we consider the following parametrization. We let $\theta(z) = z^\rho - \varepsilon_\theta z$ and $\psi(z) = z^\rho - \varepsilon_\psi z$, where $\varepsilon_\theta, \varepsilon_\psi \geq 0$ are parameters that capture deviations from the CES in the traded and in the non-traded sectors, respectively. Clearly, $\varepsilon_\theta = 0$ corresponds to the CES case in the traded sector. If, on the contrary, $\varepsilon_\theta > 0$, then $r_\theta(\cdot)$ is increasing, which implies a pro-competitive effect. We also choose upper-tier CES preferences, so that (23) holds. In what follows, we consider the following parameter values: $\sigma = 1.2$, $\gamma = 3.33$, $L = 10$, $c = 1$, $F = 0.4$, $\beta = 1/3$ and $\varepsilon_\psi = 0.1$. We hold these parameters fixed and consider different values of ε_θ and τ. The objective is to investigate how trade liberalization affects key variables as we progressively move away from the CES case by increasing ε_θ. To this end, we move τ from 1 to 2 to generate the different variables for a fixed value of ε_θ. We then run simple linear regressions of the log of the main endogenous variables on the log of trade costs to estimate the elasticity of the variables with respect to trade costs for a given value of ε_θ.[20] We repeat that procedure for values of ε_θ between 0 and 0.07. The value of $\varepsilon_\theta = 0.07$ corresponds to the threshold value beyond which the number of firms in the traded sector is no longer increasing as τ decreases, i.e., the qualitative properties of the model change.[21] Clearly, the important point to notice is that for all $\varepsilon_\theta \in (0, 0.07]$, the model behaves 'as if' it were CES (i.e., the mass of firms increases and the price index decrease in the traded sector with trade liberalization), yet has variable markups and variable firm sizes.

As can be seen from Table 2, the price index for traded goods falls more quickly with trade liberalization in the ε-CES case than in the CES case, even though markups for imported goods increase.[22] The reason is that the decrease in markups for domestically produced varieties offsets the slower increase in the mass of firms than in the CES case. This leads to more substantial welfare gains in the traded sector in the ε-CES case than in the CES case. Although the welfare losses in the non-traded sector are slightly larger in the ε-CES case than in the CES case, the overall effect on welfare is stronger in the former than in the latter. In other words, a 1% decrease in trade costs leads to a 0.24% increase in welfare under our ε-CES preferences, but only to a 0.23% increase

[20]We view these regressions as a simple device that allows us to obtain local approximations of the comparative static properties of the model. The error term in the regressions comes from the linear approximation of a non-linear relationship. It has no structural interpretation other than corresponding to the approximation error of the non-linear relationships.

[21]Since the qualitative properties change, we may view this as a case where we moved out of the neighborhood in which ε-CES preferences inherit the properties of CES preferences.

[22]Yilmazkuday (2015) reports an elasticity of U.S. import markups of 0.16%.

Table 2: Numerical illustration of elasticities

elasticity with respect to τ	CES ($\varepsilon_\theta = 0$)	ε-CES ($\varepsilon_\theta = 0.07$)
markups for tradable goods (domestic)	0.0000	0.0057
markups (imports)	0.0000	−0.0084
mass of firms in the tradable sector	−0.0159	−0.0085
size of manufacturing sector	−0.0156	−0.0164
price index (tradable goods)	0.3118	0.3204
utility u	−0.3276	−0.3368
utility v	0.0763	0.0782
utility U	−0.2324	−0.2383

Notes: Each line reports the coefficients of a log-log regression of an endogenous variable on trade costs τ. We move τ from 1 to 2 with step size 0.01 to generate 100 observations. All coefficients are significant, with p-values that are virtually zero. The R^2 exceeds 0.85 in all cases, thus showing that the linear approximations work very well.

in welfare under CES preferences.[23] This result is different from the one in Arkolakis *et al.* (2018), where variable markups reduce the gains from trade. In a one-sector economy with variable markups and heterogeneous firms, distortions due to trade liberalization stem from increasing within-industry misallocation toward more productive firms that charge higher markups. By contrast, in our two-sector framework, the main source of distortion is the interaction between variable markups in the two sectors, where trade liberalization exacerbates the distortion in the non-liberalized sector.[24]

7 Conclusion

We have developed a tractable model with a traded and a non-traded sector in each country. Both sectors are monopolistically competitive and consumers have non-CES preferences. We show that Kimball's (1995) flexible aggregator leads to preferences that are homothetic and display variable elasticity of substitution. Furthermore, the elasticity of substitution is a function of the ratio of consumption to utility, which allows for a simple behavior. More precisely, Kimball preferences behave in a similar way than standard additive preferences, which simplifies the analysis significantly. They also encompass the CES as a special case, which allows us to derive a number of results for non-CES preferences in a 'small neighborhood' of the CES.

[23]The numbers that we provide are illustrative only. It is well known that they are sensitive to a monotonically increasing transformation of the utility function. However, the ranking of the effects would not be reversed by such a transformation.

[24]See Behrens *et al.* (2016) for a model with both inter- and intra-sectoral misallocations across heterogeneous firms and industries.

Our key result relates to the differential behavior of markups in the wake of trade liberalization. We show that while trade liberalization promotes the convergence of markups for traded goods across countries, it leads to the divergence of markups across sectors—traded and non-traded—within each country. These effects are solely driven by the general equilibrium nature of the model, which leads to a reallocation of expenditure across industries in the wake of trade liberalization. There is no 'anti-competitive behavior' of firms, although such behavioral reactions of firms to trade liberalization have often been put forward to explain the increase in markups in service industries on the way of deeper European integration (Badinger, 2007). We also show that firm sizes increase in both sectors, irrespective of the direction of change in markups.

Since markups fall and product diversity expands in the traded sector, whereas markups rise and product diversity shrinks in the non-traded sector, the welfare impacts of trade integration are ambiguous. In particular, positive effects in the liberalized sector are mitigated by the negative effects in the non-traded sector. The explanation for that result is that freer trade in one sector leads to a reallocation of resources towards that sector. Consequently, there are less firms operating in the other sector, which increases their market power and allows them to raise markups. The overall effect is ambiguous and most likely depends on modeling choices. What our finding suggests is that these modeling choices are not innocuous.

Acknowledgements. We are grateful to Svetlana Demidova, Michal Fabinger, Gabriel Felbermayr, Oleg Itskhoki, Sebastian Krautheim, Andrea Lamorgese, Anders Laugesen, Petros Millionis, Yasusada Murata, Peter Neary, Alexander Tarasov, Jacques-Francois Thisse, and participants at our workshop in Saint Petersburg, the 2015 SAET conference in Cambridge, 2015 NARSC mettings in Portland, 2015 iCare in Perm, 2014 FIW Conference in Vienna, 2015 DIEW in Aarhus, 2015 SMYE in Gent, 2015 APET conference in Luxemburg, 2015 Econometric Society World Congress in Montreal, 2015 EEA congress in Mannheim, 2015 ERSA congress in Lisbon, 2015 EARIE conference in Munich, and the 2015 DEGIT conference in Geneva for valuable comments and suggestions. We thank Darya Vertkina for help with the data. The study has been funded by the Russian Academic Excellence Project '5-100'. Behrens gratefully acknowledges financial support from the CRC Program of the Social Sciences and Humanities Research Council (SSHRC) of Canada for the funding of the *Canada Research Chair in Regional Impacts of Globalization*. Kichko gratefully acknowledges the RFBR within the research project № 18-010-01002. All remaining errors are ours.

References

[1] Arkolakis, Costas, Arnaud Costinot, Dave Donaldson, and Andres Rodriguez-Clare. 2018. "The elusive pro-competitive effects of trade." Forthcoming, *Review of Economic Studies*.

[2] Atkeson, Andrew, and Ariel Burstein. 2008. "Pricing-to-market, trade costs, and international relative prices." *American Economic Review* 98(**5**): 1998–2031.

[3] Badinger, Harald. 2007. “Has the EU’s single market programme fostered competition? Testing for a decrease in mark-up ratios in EU industries.” *Oxford Bulletin of Economics and Statistics* 69**(4)**: 497–519.

[4] Barde, Sylvain. 2008. “A generalised variable elasticity of substitution model of new economic geography.” *Observatoire Francais des Conjonctures Economiques*, Discussion Paper #2008–33, Paris, France.

[5] Behrens, Kristian, Sergey Kichko, Philip Ushchev. 2018. “Intersectoral linkages: Good shocks, bad outcomes?” In progress.

[6] Behrens, Kristian, Giordano Mion, Yasusada Murata, and Jens Suedekum. 2016. “Distorted monopolistic competition.” CEPR Discussion Papers, DP11642.

[7] Behrens, Kristian, and Yasusada Murata. 2012. “Trade, competition, and efficiency.” *Journal of International Economics* 87**(1)**: 1–17.

[8] Behrens, Kristian, and Yasusada Murata. 2007. “General equilibrium models of monopolistic competition: A new approach.” *Journal of Economic Theory* 136**(1)**: 776–787.

[9] Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott. 2007. “Comparative advantage and heterogeneous firms.” *Review of Economic Studies* 74**(1)**: 31-66.

[10] Bertoletti, Paolo, and Paolo Epifani 2014. “Monopolistic competition: CES redux?” *Journal of International Economics* 93(2): 227 – 238.

[11] Brander, James, and Paul R. Krugman. 1983. “A ‘reciprocal dumping’ model of international trade.” *Journal of International Economics* 15**(3-4)**: 313–321.

[12] Burstein, Ariel, Martin Eichenbaum, and Sergio Rebelo. 2005. "Large devaluations and the real exchange rate." *Journal of Political Economy* 113(**4**): 742–784.

[13] Burstein, Ariel, Martin Eichenbaum, and Sergio Rebelo. 2006. "The importance of nontradable goods’ prices in cyclical real exchange rate fluctuations." *Japan and the World Economy,* 18(**3**): 247–253.

[14] Corsetti, Giancarlo, and Luca Dedola. 2005. "A macroeconomic model of international price discrimination." *Journal of International Economics* 67**(1):** 129–155.

[15] Costinot, Arnaud, and Andres Rodriguez-Clare. 2014. “Trade theory with numbers: Quantifying the consequences of globalization.” In Handbook of International Economics (ed. Gopinath, Gita, Elhanan Helpman, and Kenneth Rogoff), volume 4, pp. 197-262. Elsiver.

[16] d'Aspremont, Claude, and Rodolphe Dos Santos Ferreira. 2015. "Oligopolistic vs. monopolistic competition: Do intersectoral effects matter?" CORE *Discussion Paper* #2015/2, Center for Operations Research and Econometrics, Belgium.

[17] De Loecker, Jan, Pinelopi K. Goldberg, Amit K. Khandelwal, and Nina Pavcnik. 2012. "Prices, markups, and trade reforms." NBER Working Paper #17925, National Bureau for Economic Research.

[18] Dhingra, Swati, and John Morrow. 2018. "Monopolistic competition and optimum product diversity under firm heterogeneity." Forthcoming, *Journal of Political Economy*.

[19] Dotsey, Michael, and Robert G. King. 2005. "Implications of state-dependent pricing for dynamic macroeconomic models." *Journal of Monetary Economics* 52**(1)**: 213–242.

[20] Eaton, Jonathan, and Samuel Kortum. 2002. "Technology, geography, and trade." *Econometrica* 70**(5)**: 1741–1779.

[21] Edmond, Chris, Virgiliu Midrigan, Daniel Yi Xu. 2015. "Competition, markups, and the gains from international trade." *American Economic Review* 105**(10)**: 3183–3221.

[22] Epifani, Paolo, and Gino Gancia. 2011. "Trade, markup heterogeneity and misallocations." *Journal of International Economics* 83**(1)**: 1–13.

[23] Feenstra, Robert C., and David E. Weinstein. 2017. "Globalization, Markups, and the U.S. Price Level." *Journal of Political Economy* 125(**4**): 1041–1074.

[24] Holmes, Thomas J., Wen-Tai Hsu, and Sanghoon Lee. 2014. "Allocative efficiency, mark-ups, and the welfare gains from trade." *Journal of International Economics* 94**(2)**: 195–206.

[25] Kichko, Sergey, Sergey Kokovin, and Evgeny Zhelobodko. 2014. "Trade patterns and export pricing under non-CES preferences." *Journal of International Economics* 94**(1)**: 129–142.

[26] Kimball, Miles S. 1995. "The quantitative analytics of the basic neomonetarist model." *Journal of Money, Credit and Banking* 27**(4)**: 1241–1277.

[27] Krugman, Paul R. 1979. "Increasing returns, monopolistic competition, and international trade." *Journal of International Economics* 9**(4)**: 469–479.

[28] Krugman, Paul R. 1980. "Scale economies, product differentiation, and the pattern of trade." *American Economic Review* 70**(5)**: 950–959.

[29] Levinsohn, James. 1999. "Employment responses to international liberalization in Chile." *Journal of International Economics* 47**(2)**: 321–344.

[30] Lipsey, Richard G., and Kelvin Lancaster. 1956. "The general theory of second best." *Review of Economic Studies* 24**(1)**: 11–32.

[31] Lombardo, Giovanni, and Federico Ravenna. 2012. "The size of the tradable and non-tradable sectors: Evidence from input–output tables for 25 countries." *Economics letters* 116(**3**): 558–561.

[32] Melitz, Marc J. 2003. "The impact of trade on intra-industry reallocations and aggregate industry productivity." *Econometrica* 71**(6)**: 1695–1725.

[33] Melitz, Marc J., and Gianmarco IP Ottaviano. 2008. "Market size, trade, and productivity." *Review of Economic Studies* 75**(1)**: 295–316.

[34] Parenti, Mathieu, Philip Ushchev, and Jacques-Francois Thisse. 2017. "Toward a theory of monopolistic competition." *Journal of Economic Theory,* Vol. 167**(1)**: 86-115.

[35] Simonovska, Ina. 2015. "Income differences and prices of tradables: Insights from an online retailer." *Review of Economic Studies* 82(4): 1612–1656.

[36] Syverson, Chad. 2007. "Prices, spatial competition and heterogeneous producers: An empirical test." *Journal of Industrial Economics* 55(2): 197–222.

[37] Trefler, Daniel. 2004. "The long and the short of the Canada-U.S. free trade agreement." *American Economic Review* 94**(4)**: 870–895.

[38] Weinberger, Ariel. 2015. "Markups and misallocation with trade and heterogeneous firms." In progress, University of Oklahoma.

[39] Yilmazkuday, Hakan. 2015. "Pass-through of trade costs to U.S. import prices." *Review of World Economics* 151**(4)**: 609–633.

[40] Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti, and Jacques-Francois Thisse. 2012. "Monopolistic competition: Beyond the constant elasticity of substitution." *Econometrica* 80**(6)**: 2765–2784.

Appendix

A. Proofs of Propositions

A.1. Proof of Proposition 1 Given $\theta(z)$, the proof of the "if" part essentially boils down to choosing a $\sigma(z)$ which satisfies

$$\frac{1}{\sigma(z)} = r_\theta(z) \equiv -\frac{z\theta''(z)}{\theta'(z)}. \tag{A-1}$$

Conversely, to prove the "only if" part, we choose $\theta(z)$ satisfying (A-1) for a given $\sigma(z)$.

Assume that (4) holds. Then, since the inverse demands are given by (11), the elasticity η_i of the inverse demand for variety i is given by

$$\eta_i = r_\theta\left(\frac{x_i}{u(\mathbf{x})}\right), \tag{A-2}$$

where $r_\theta(\cdot)$ is defined by (A-1). As shown by Parenti *et al.* (2017), for any symmetric preferences defined over a continuum of goods, the following equality holds:

$$\overline{\sigma}(x_i, x_j, \mathbf{x})\big|_{x_i=x_j} = \frac{1}{\eta_i}. \tag{A-3}$$

Combining (A-2) with (A-3) and setting $\sigma(z) \equiv 1/r_\theta(z)$, we obtain (3). This proves the "if" part of Proposition 1.

To prove the "only if" part, assume that (3) holds for a given $\sigma(z)$. It is straightforward to verify that implicitly additive preferences with $\theta(\cdot)$ defined by

$$\theta(z) \equiv \int_0^z \exp\left(-\int_1^\varsigma \frac{\mathrm{d}\xi}{\xi\sigma(\xi)}\right)\mathrm{d}\zeta$$

also satisfy (3) for the same $\sigma(z)$. Since a preference relationship is uniquely determined by its elasticity of substitution, this completes the proof. □

A.2. ε-CES preferences. We formally define ε-CES preferences and derive the equilibrium conditions of the model. Let

$$\theta(z) = z^\rho \exp\bigl(\varphi(z)\bigr), \quad \rho \equiv \frac{\sigma}{\sigma - 1} \in (0, 1), \tag{A-4}$$

where $\varphi \in C^3(\mathbb{R}_+)$ is a thrice continuously differentiable function that is sufficiently small in a sense that $||\varphi||_{C^3} < \varepsilon$. In the foregoing expression, $||\cdot||_{C^3}$ is the standard norm in the space of thrice continuously differentiable functions, given by $||\varphi||_{C^3} \equiv ||\varphi||_C + ||\varphi'||_C + ||\varphi''||_C + ||\varphi'''||_C$, with $||\cdot||_C$ being the norm of uniform convergence defined as $||\varphi||_C \equiv \sup_{z\geq 0}|\varphi(z)|$. To use

(A-4) when solving the equilibrium conditions of the model, we need the expressions for $\theta'(\cdot)$ and $r_\theta(\cdot)$, which are given by

$$\theta'(z) = z^{\rho-1} \exp\left[\varphi(z)\right]\left[\rho + z\varphi'(z)\right], \tag{A-5}$$

and

$$r_\theta(z) \equiv -\frac{\theta''(z)z}{\theta'(z)} = 1 - \rho - \xi(z), \tag{A-6}$$

respectively. The residual term, $\xi(\cdot)$, is given by

$$\xi(z) \equiv \frac{z\varphi'(z)}{\rho + z\varphi'(z)}\left[1 + \rho + z\varphi'(z)\right] + \frac{z^2\varphi''(z)}{\rho + z\varphi'(z)}. \tag{A-7}$$

To guarantee that $||\xi||_C \to 0$ as $||\varphi||_{C^3} \to 0$, we need that $z\varphi'(z)$ and $z^2\varphi''(z)$ uniformly converge to zero. These conditions make sure that we can choose $\varphi(\cdot)$ so that (A-4) becomes arbitrarily close to the standard CES case. This will allow us to derive part of our analytical results by continuity in the neighborhood of CES preferences.[25]

Plugging the expressions (A-5)–(A-6) into the equilibrium conditions (24)–(27) yields

$$\begin{aligned}
z^d\frac{1-\rho-\xi^d}{\rho+\xi^d} + \tau z^m\frac{1-\rho-\xi^m}{\rho+\xi^m} &= \frac{F}{cL}\frac{P}{a(P)}, && \text{(A-8)}\\
N\left[(z^d)^\rho \exp(\varphi^d) + (z^m)^\rho \exp(\varphi^m)\right] &= 1, && \text{(A-9)}\\
\frac{\left(z^d\right)^{\rho-1}\exp(\varphi^d)(\rho+\xi^d)\left[\rho + z^d\varphi'(z^d)\right]}{(z^m)^{\rho-1}\exp(\varphi^m)(\rho+\xi^m)\left[\rho + z^m\varphi'(z^m)\right]} &= \frac{1}{\tau}, && \text{(A-10)}\\
cN\left(\frac{z^d}{\rho+\xi^d} + \tau\frac{z^m}{\rho+\xi^m}\right) &= P, && \text{(A-11)}
\end{aligned}$$

where we set $\varphi^k \equiv \varphi(z^k)$ and $\xi^k \equiv \xi(z^k)$ for $k \in \{d, m\}$ for notational convenience. Clearly, $\xi^k \to 0$ when $||\varphi||_{C^3} \to 0$. In the limiting case $\varphi^d = \varphi^m = 0$ and $\xi^d = \xi^m = 0$, so that equations (A-8)–(A-11) boil down to the standard equilibrium conditions under CES preferences. Hence, equations (A-8)–(A-11) may be intuitively viewed as a system in which there are small shocks to the CES equilibrium conditions. However, our approach is not ad hoc, for we explicitly model variable markups, showing where the shocks stem from.

Plugging (A-11) into (A-8), we obtain

$$(1-\rho)\frac{P}{cN} - \zeta = \frac{F}{cL}\frac{P}{a(P)}, \tag{A-12}$$

where $\zeta(\xi^d, \xi^m) \equiv \frac{z^d\xi^d}{\rho+\xi^d} + \tau\frac{z^m\xi^m}{\rho+\xi^m}$ is a residual term. To make sure that this residual term can be

[25]Using numerical illustrations, we also show in Section 6 that these results extend to larger 'neighborhoods' of the CES.

made arbitrarily small, we restrict ourselves to perturbations $\varphi \in \mathcal{F}$, where $\mathcal{F}$ is defined as

$$\mathcal{F} \equiv \left\{ \varphi \in C^3(\mathbb{R}_+) \mid \lim_{z\to\infty} z^2\varphi'(z) = \lim_{z\to\infty} z^3\varphi''(z) = 0 \right\}. \tag{A-13}$$

The second restriction in (A-13) guarantees that the residual terms ξ^d and ξ^m become uniformly small as $||\varphi||_{C^3} \to 0$. Solving the first equation in (A-12) for N, we obtain

$$N = (1-\rho)\frac{La(P)}{F + cL\frac{a(P)}{P}\zeta}. \tag{A-14}$$

Solving (A-9)–(A-10) for z^d and z^m yields

$$\overline{z}^m(N,\tau) = \left[N\left(\tau^{\rho/(1-\rho)}\mathcal{A}^\rho \exp(\varphi^d) + \exp(\varphi^m)\right)\right]^{-1/\rho}, \tag{A-15}$$

$$\overline{z}^d(N,\tau) = \tau^{1/(1-\rho)}\mathcal{A}\overline{z}^m(N,\tau), \tag{A-16}$$

where

$$\mathcal{A}(\varphi^d,\varphi^m,\xi^d,\xi^m) \equiv \left[\frac{\exp(\varphi^d)(\rho+\xi^d)\left(\rho + z^d\varphi'(z^d)\right)}{\exp(\varphi^m)(\rho+\xi^m)\left(\rho+z^m\varphi'(z^m)\right)}\right]^{1/(\rho-1)}.$$

Note that $\mathcal{A} \to 1$ as $||\varphi||_{C^3} \to 0$. Hence, $z^m < z^d$ if we are sufficiently close to CES preferences—since $\mathcal{A}$ is very close to 1 while $\tau^{1/(1-\rho)}$ is strictly greater than 1. Finally, plugging (A-15)–(A-16) into (A-11) yields the expression for the price index:

$$P = cN^{-(1-\rho)/\rho}\left[\mathcal{A}^\rho \exp(\varphi^d) + \tau^{-\rho/(1-\rho)}\exp(\varphi^m)\right]^{-1/\rho}\left(\frac{\mathcal{A}}{\rho+\xi^d} + \frac{\tau^{-\rho/(1-\rho)}}{\rho+\xi^m}\right). \tag{A-17}$$

A.3. Proof of Proposition 5 Denote by $\boldsymbol{\varepsilon} \equiv (\varphi^d, \varphi^m, \xi^d, \xi^m, \mathcal{A}, \zeta)$ the vector of 'shocks' entering (A-14) and (A-17). We first show that equations (A-14) and (A-17) have a unique solution (P^*_{CES}, N^*_{CES}) when $\boldsymbol{\varepsilon} = \mathbf{0}$.

To see this, observe that when $\boldsymbol{\varepsilon} = \mathbf{0}$, equations (A-14) and (A-17) boil down to

$$N = (1-\rho)\frac{L}{F}a(P), \tag{A-18}$$

and

$$P = N^{1/(1-\sigma)}\left(\frac{c\sigma}{\sigma-1}\right)\left(1+\tau^{1-\sigma}\right)^{1/(1-\sigma)}, \tag{A-19}$$

which are the standard expressions for the CES case. Plugging (A-19) into (A-18) yields an equation which uniquely pins down the equilibrium number of firms, N^*_{CES}, provided that $a(\cdot)$ is a sufficiently slowly decreasing function (which we assume, as explained before). Substituting N^*_{CES} back into (A-19), we then obtain the unique equilibrium value of the price index for traded varieties, P^*_{CES}.

It then follows from the implicit function theorem that there exists an open set $V \subseteq \mathbb{R}^6$, such that: (i) $\mathbf{0} \in V$; (ii) (A-14) and (A-17) have a solution $(P^*(\boldsymbol{\varepsilon}),\, N^*(\boldsymbol{\varepsilon}))$ for any $\boldsymbol{\varepsilon} \in V$; and (iii) this solution converges to $(P^*_{CES},\, N^*_{CES})$ as $\boldsymbol{\varepsilon} \to \mathbf{0}$. □

A.4. Expressions for comparative statics The implicit function theorem implies that, for a small enough vector $\boldsymbol{\varepsilon}$, the solution $\left(z^{d*},\, z^{m*},\, P^*,\, N^*\right)$ of equations (A-8)–(A-11) is twice continuously differentiable in $(\boldsymbol{\varepsilon},\, \tau)$. This implies then that $\partial z^{d*}/\partial \tau$, $\partial z^{m*}/\partial \tau$, $\partial P^*/\partial \tau$, and $\partial N^*/\partial \tau$ are all continuous in $\boldsymbol{\varepsilon}$. We may thus conclude that if, for example, $\partial z^{d*}/\partial \tau > 0$ in the CES case, this result will be preserved by continuity for the ε-CES case when $\boldsymbol{\varepsilon}$ is small. Thus, it suffices to conduct comparative statics of $\left(z^{d*},\, z^{m*},\, P^*,\, N^*\right)$ for the CES case, and all results for the ε-CES case are then obtained by continuity.

The expressions for the CES case are given in Appendix A.2. Denote the right-hand side of (A-19) by $\overline{P}(N,\, \tau)$. We have

$$\frac{\partial \overline{P}}{\partial N} < 0 \quad \text{and} \quad \frac{\partial \overline{P}}{\partial \tau} > 0. \tag{A-20}$$

Totally differentiating (A-18) with respect to τ yields

$$\frac{\mathrm{d}N^*}{\mathrm{d}\tau} = (1-\rho)\frac{L}{F} a'\left[\overline{P}(N^*, \tau)\right] \left(\frac{\partial \overline{P}}{\partial N}\frac{\mathrm{d}N^*}{\mathrm{d}\tau} + \frac{\partial \overline{P}}{\partial \tau}\right),$$

so that

$$\frac{\mathrm{d}N^*}{\mathrm{d}\tau} = \frac{(1-\rho)La'\left[\overline{P}(N^*, \tau)\right]}{F - (1-\rho)La'\left(\overline{P}(N^*, \tau)\right)\frac{\partial \overline{P}}{\partial N}} \frac{\partial \overline{P}}{\partial \tau}. \tag{A-21}$$

Since we assume that $a'(\cdot)$ is small enough in absolute value, (A-21) immediately implies

$$\frac{\mathrm{d}N^*}{\mathrm{d}\tau} < 0. \tag{A-22}$$

It is worth pointing out, however, that when $a'(\cdot)$ is sufficiently small, $\mathrm{d}N^*/\mathrm{d}\tau$ is also small in absolute value.

Turning to the relative consumptions $\overline{z}^d(N,\, \tau)$ and $\overline{z}^m(N,\, \tau)$, they are determined from expressions (A-8)–(A-9) and are given by

$$\overline{z}^m(N,\, \tau) = \left[N\left(1 + \tau^{\rho/(1-\rho)}\right)\right]^{-\rho}, \quad \text{and} \quad \overline{z}^d(N,\, \tau) = \tau^{1/(1-\rho)}\left[N\left(1 + \tau^{\rho/(1-\rho)}\right)\right]^{-\rho}.$$

Plugging $N = N^*$ into the foregoing expression and totally differentiating it with respect to τ yields

$$\frac{\mathrm{d}(z^d)^*}{\mathrm{d}\tau} = \left(\frac{\partial \overline{z}^d}{\partial N}\frac{\mathrm{d}N^*}{\mathrm{d}\tau} + \frac{\partial \overline{z}^d}{\partial \tau}\right)\bigg|_{N=N^*}, \qquad \frac{\mathrm{d}(z^m)^*}{\mathrm{d}\tau} = \left(\frac{\partial \overline{z}^m}{\partial N}\frac{\mathrm{d}N^*}{\mathrm{d}\tau} + \frac{\partial \overline{z}^m}{\partial \tau}\right)\bigg|_{N=N^*}. \tag{A-23}$$

Because, as discussed above, the magnitude of $\mathrm{d}N^*/\mathrm{d}\tau$ is small, (A-23) implies

$$\frac{\mathrm{d}(z^d)^*}{\mathrm{d}\tau} > 0 > \frac{\mathrm{d}(z^m)^*}{\mathrm{d}\tau}.$$

We next study how the price index P^* varies with trade costs τ. Totally differentiating $P^* = \overline{P}(N^*, \tau)$ with respect to τ yields

$$\frac{\mathrm{d}P^*}{\mathrm{d}\tau} = \left(\frac{\partial \overline{P}}{\partial \tau} + \frac{\partial \overline{P}}{\partial N} \cdot \frac{\mathrm{d}N^*}{\mathrm{d}\tau}\right)\bigg|_{N=N^*}.$$

Combining this expression with (A-20) and (A-22), we finally obtain

$$\frac{\mathrm{d}P^*}{\mathrm{d}\tau} > 0.$$

Appendix B: Second-order conditions

Using the inverse demands, we find that the profit functions of firm i in the traded sector and of firm j in the non-traded sector in country $k \in \{H, F\}$ are given by

$$\pi_i^k(x_i^{kk}, x_i^{kl}) = \left[\frac{1}{\mu^k}\theta'\left(\frac{x_i^{kk}}{u^k}\right) - c\right] L x_i^{kk} + \left[\frac{1}{\mu^l}\theta'\left(\frac{x_i^{kl}}{u^l}\right) - c\tau\right] L x_i^{kl} - f, \tag{B-1}$$

$$\pi_j^k(y_j^k) = \left[\frac{1}{\lambda^k}\psi'\left(\frac{y_j^k}{v^k}\right) - c\right] L y_j^k - f, \tag{B-2}$$

where $l \neq k$. Because each firm is negligible, μ^k, λ^k, u^k and v^k, $k \in \{H, F\}$ are treated by each firm parametrically. Hence, the second-order conditions are given by

$$\frac{\partial^2 \pi_i^k}{\partial \left(x_i^{kk}\right)^2} < 0, \qquad \frac{\partial^2 \pi_i^k}{\partial \left(x_i^{kl}\right)^2} < 0, \tag{B-3}$$

and

$$\frac{\partial^2 \pi_j^k}{\partial \left(y_j^k\right)^2} < 0. \tag{B-4}$$

Differentiating (B-1) twice with respect to x_i^{kk} and x_i^{kl} yields

$$\frac{\partial^2 \pi_i^k}{\partial \left(x_i^{kk}\right)^2} = \frac{L}{\mu^k u^k}\left[\theta'''\left(\frac{x_i^{kk}}{u^k}\right)\frac{x_i^{kk}}{u^k} + 2\theta''\left(\frac{x_i^{kk}}{u^k}\right)\right],$$

$$\frac{\partial^2 \pi_i^k}{\partial \left(x_i^{kl}\right)^2} = \frac{L}{\mu^l u^l}\left[\theta'''\left(\frac{x_i^{kl}}{u^l}\right)\frac{x_i^{kl}}{u^l} + 2\theta''\left(\frac{x_i^{kl}}{u^l}\right)\right],$$

while differentiating (B-2) twice with respect to y_j^k results in

$$\frac{\partial^2 \pi_j^k}{\partial \left(y_j^k\right)^2} = \frac{L}{\lambda^k v^k} \left[\psi''' \left(\frac{y_j^k}{v^k} \right) \frac{y_j^k}{v^k} + 2\psi'' \left(\frac{y_j^k}{v^k} \right) \right].$$

Combining this with (B-3) and setting $z_i^{kk} \equiv x_i^{kk}/u^k$, $z_i^{kl} \equiv x_i^{kl}/u^l$, we obtain that the second-order conditions for firm i in the traded sector boil down to

$$z_i^{kk} \theta''' \left(z_i^{kk} \right) + 2\theta'' \left(z_i^{kk} \right) < 0, \tag{B-5}$$

$$z_i^{kl} \theta''' \left(z_i^{kl} \right) + 2\theta'' \left(z_i^{kl} \right) < 0. \tag{B-6}$$

Furthermore, using (B-4) and setting $z_j^k \equiv y_j^k/v^k$ yields the following second-order condition for firm j in the non-traded sector:

$$z_j^k \psi''' \left(z_j^k \right) + 2\psi'' \left(z_j^k \right) < 0. \tag{B-7}$$

Finally, it is easy to check that (B-5)–(B-6) hold if and only if $r_{\theta'}(z) \equiv -z\theta'''(x)/\theta'' < 2$ for all $z \geq 0$, while (B-7) holds if and only if $r_{\psi'}(z) \equiv -z\psi'''(x)/\psi'' < 2$ for all $z \geq 0$.

Appendix C: Data

In this appendix, we briefly present the data sources and details concerning the regressions summarized in Table 1. The trade and productivity data is taken from the CEPII database (files Prod_cepii8004.dta and Trade_cepii8004.dta). See Mayer, Paillacar, and Zignago ("TradeProd. The CEPII Trade, Production and Bilateral Protection Database: Explanatory Notes", 2008) for details. For each country and year, we compute the value added per employee and output per employee for overall manufacturing (ISIC 300) from the productivity database. We compute the manufacturing share as the ratio of total manufacturing employment to total employment, where the latter is taken from the Penn World Tables 8.0. Openness is computed as the ratio of total imports plus exports (from the CEPII trade database) to GDP (from the Penn World Tables). Import and export openness are computed as the ratio of either imports or exports over GDP. We drop all country-year pairs for which one of the variables is missing, which results in an unbalanced panel of 145 countries over the years 1980–2004.

Appendix D: Asymmetric countries.

In this appendix, we study the impacts of trade liberalization on markups when the home country has a larger population than the foreign country, i.e., $L^H > L^F$. The objective is to show that our key qualitative results do not hinge on the assumption of symmetric countries.

Let $w = w^H/w^F$ denote the relative wage of country H. We start by studying the equilibrium in the traded sector for the case of Cobb-Douglas upper-tier with CES lower-tier preferences. In that case, $a(P) \equiv a$ is a constant. The equilibrium conditions in the case with asymmetric countries are as follows:

(i) Free entry:

$$L^H x^{HH} + \tau L^F x^{HF} = \frac{F}{c}(\sigma - 1) \quad \text{and} \quad \tau L^H x^{FH} + L^F x^{FF} = \frac{F}{c}(\sigma - 1). \tag{B-8}$$

(ii) Profit maximization:

$$\left(\frac{x^{HH}}{x^{FH}}\right)^{-1/\sigma} = \frac{w}{\tau} \quad \text{and} \quad \left(\frac{x^{FF}}{x^{HF}}\right)^{-1/\sigma} = \frac{1}{\tau w}. \tag{B-9}$$

(iii) Sectoral budget constraints:

$$w\frac{c\sigma}{\sigma - 1}x^{HH}N^H + \tau\frac{c\sigma}{\sigma - 1}x^{FH}N^F = wa \quad \text{and} \quad w\tau\frac{c\sigma}{\sigma - 1}x^{HF}N^H + \frac{c\sigma}{\sigma - 1}x^{FF}N^F = a. \tag{B-10}$$

(iv) Trade balance condition:

$$p^{HF}x^{HF}N^H L^F = p^{FH}x^{FH}N^F L^H \quad \Rightarrow \quad w = \frac{L^H}{L^F}\frac{x^{FH}}{x^{HF}}\frac{N^F}{N^H}. \tag{B-11}$$

Using (B-9), we directly get:

$$x^{FH} = \left(\frac{w}{\tau}\right)^{\sigma} x^{HH} \quad \text{and} \quad x^{HF} = \left(\frac{1}{\tau w}\right)^{\sigma} x^{FF}. \tag{B-12}$$

Plugging this expression into (B-8), we obtain:

$$L^H x^{HH} + \tau^{1-\sigma}w^{-\sigma}L^F x^{FF} = \frac{F}{c}(\sigma - 1) \quad \text{and} \quad \tau^{1-\sigma}w^{\sigma}L^H x^{HH} + L^F x^{FF} = \frac{F}{c}(\sigma - 1).$$

We can combine those two expressions to obtain:

$$L^H x^{HH} = \frac{F(\sigma - 1)}{c}\frac{1 - \tau^{1-\sigma}w^{-\sigma}}{1 - \tau^{2(1-\sigma)}} \quad \text{and} \quad L^F x^{FF} = \frac{F(\sigma - 1)}{c}\frac{1 - \tau^{1-\sigma}w^{\sigma}}{1 - \tau^{2(1-\sigma)}}.$$

Therefore the two domestic and the two import demands are give as follows:

$$\begin{aligned} x^{HH} &= \frac{1}{1 - \tau^{2(1-\sigma)}}\frac{F(\sigma - 1)}{cL^H}\left(1 - \tau^{1-\sigma}w^{-\sigma}\right), \quad x^{FH} = \frac{\tau^{-\sigma}}{1 - \tau^{2(1-\sigma)}}\frac{F(\sigma - 1)}{cL^H}\left(w^{\sigma} - \tau^{1-\sigma}\right) \\ x^{FF} &= \frac{1}{1 - \tau^{2(1-\sigma)}}\frac{F(\sigma - 1)}{cL^F}\left(1 - \tau^{1-\sigma}w^{\sigma}\right), \quad x^{HF} = \frac{\tau^{-\sigma}}{1 - \tau^{2(1-\sigma)}}\frac{F(\sigma - 1)}{cL^F}\left(w^{-\sigma} - \tau^{1-\sigma}\right). \end{aligned}$$

Substituting these expressions into the trade balance condition (iv), we get

$$w = \frac{w^{\sigma} - \tau^{1-\sigma}}{w^{-\sigma} - \tau^{1-\sigma}} \frac{N^F}{N^H}. \tag{B-13}$$

Solving (B-13) for N^F yields

$$N^F = w \frac{w^{-\sigma} - \tau^{1-\sigma}}{w^{\sigma} - \tau^{1-\sigma}} N^H, \tag{B-14}$$

which we can substitute into (B-10) to get:

$$\left(x^{HH} + \tau x^{FH} \frac{w^{-\sigma} - \tau^{1-\sigma}}{w^{\sigma} - \tau^{1-\sigma}} \right) N^H = a \frac{\sigma - 1}{\sigma c}, \quad \text{and} \quad \left(\tau x^{HF} + x^{FF} \frac{w^{-\sigma} - \tau^{1-\sigma}}{w^{\sigma} - \tau^{1-\sigma}} \right) N^H = \frac{a}{w} \frac{\sigma - 1}{\sigma c}. \tag{B-15}$$

Dividing (B-15) for H by the same expression for H and using the expressions of the domestic and import demands, x_{rs} and x_{rr} for $r = H, F, r \neq s$ derived previously, we get:

$$w = \frac{L^F}{L^H} \frac{\left(w^{\sigma} - \tau^{1-\sigma}\right)\left(1 - \tau^{1-\sigma} w^{-\sigma}\right) + \phi \left(w^{-\sigma} - \tau^{1-\sigma}\right)\left(w^{\sigma} - \tau^{1-\sigma}\right)}{\phi \left(w^{\sigma} - \tau^{1-\sigma}\right)\left(w^{-\sigma} - \tau^{1-\sigma}\right) + \left(w^{-\gamma} - \tau^{1-\sigma}\right)\left(1 - \tau^{1-\sigma} w^{\sigma}\right)}$$

which can be simplified to yield:

$$w^{\sigma} + \frac{L^H}{L^F} \left(\tau^{1-\sigma} w - w^{1-\sigma} \right) = \tau^{1-\sigma}. \tag{B-16}$$

It is readily verified that the wage equation (B-16) has a unique solution $w^* \left(L^H/L^F, \tau \right)$, which increases both in L^H/L^F and in τ. These are standard results in this type of model. Rewriting (B-16) as

$$w^{\sigma-1} \frac{w^{\sigma} - \tau^{1-\sigma}}{1 - \tau^{1-\sigma} w^{\sigma}} = \frac{L^H}{L^F},$$

and combining this with (B-14) yields $N^F/L^F = N^H/L^H$. Using (B-15), we then get

$$N^H = \frac{aL^H}{F\sigma} \frac{\left(1 - \tau^{2(1-\sigma)}\right)\left(w^{\sigma} - \tau^{1-\sigma}\right)}{\left(w^{\sigma} - \tau^{1-\sigma}\right)\left(1 - \tau^{1-\sigma} w^{-\sigma}\right) + \tau^{1-\sigma}\left(w^{-\sigma} - \tau^{1-\sigma}\right)\left(w^{\sigma} - \tau^{1-\sigma}\right)},$$

and hence

$$N^H = \frac{aL^H}{F\sigma} \quad \text{and} \quad N^F = \frac{aL^F}{F\sigma}. \tag{B-17}$$

This is the standard result that under Cobb-Douglas and CES preferences the masses of firms in both countries do not change with trade liberalization.

The price indices for the traded good in both countries decrease with a reduction in τ. By continuity, this holds for ε-Cobb-Douglas over ε-CES preferences for sufficiently small values of ε. Here, we mean by 'ε-Cobb-Douglas' a specification in which the expenditure share $a(P)$ is a 'sufficiently slowly' decreasing function of P. Consequently, the markups in the non-traded sector increase in both countries, just like in the case of symmetric countries. The behavior of markups in traded sector are captured by $z^{kl} \equiv x^{kl}/u^l$ variables, which are now origin-destination specific (for $k, l = H, F$ and $k \neq l$):

$$z^{kk} = \frac{x^{kk}}{\left[N^k \left(x^{kk}\right)^{\frac{\sigma-1}{\sigma}} + N^l \left(x^{lk}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}$$

$$z^{kl} = \frac{x^{kl}}{\left[N^k \left(x^{kl}\right)^{\frac{\sigma-1}{\sigma}} + N^l \left(x^{ll}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}},$$

Plugging the expressions for the quantities x_{kk}, x_{lk}, x_{kl} and x_{ll}, as well as (B-17) into the foregoing expressions, some longer yet standard algebra yields:[26]

$$z^{HH} = \frac{1}{\left(\frac{a}{F\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[L^H + L^F \left(\frac{w}{\tau}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}, \tag{B-18}$$

$$z^{HF} = \frac{1}{\left(\frac{a}{F\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[L^H + L^F \left(\tau w\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}, \tag{B-19}$$

$$z^{FH} = \frac{\left(\frac{w}{\tau}\right)^{\sigma}}{\left(\frac{a}{F\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[L^H + L^F \left(\frac{w}{\tau}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}, \tag{B-20}$$

$$z^{FF} = \frac{\left(\tau w\right)^{\sigma}}{\left(\frac{a}{F\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[L^H + L^F \left(\tau w\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}. \tag{B-21}$$

With Cobb-Douglas and CES preferences, $a(P) \equiv a$ is an constant. This remains, by continuity, approximately true with ε-Cobb-Douglas over ε-CES preferences: changes in a due to a reduction in τ are negligible compared to the first-order change in trade costs τ and the relative wage w. Therefore, it immediately follows from the foregoing expressions that

$$\frac{dz^{HF}}{d\tau} < 0, \quad \frac{dz^{FF}}{d\tau} > 0.$$

In words, z^{HF} increases with trade liberalization, whereas z^{FF} decreases. When the lower-tier ε-CES utility satisfies $r'_\theta > 0$, this implies that domestic (foreign) markups in the smaller country decrease (increase) with trade liberalization.

The behavior of z^{HH} and z^{FH} depends solely on how w/τ varies with τ. To study its behavior with respect to τ, we rewrite (B-16) as follows:

$$\left(\frac{w}{\tau}\right)^{\sigma} \tau^{2\sigma-1} + \tau \frac{L^H}{L^F}\frac{w}{\tau} = 1 + \frac{L^H}{L^F}\left(\frac{w}{\tau}\right)^{1-\sigma}. \tag{B-22}$$

[26] The details of the computations are available upon request.

The left-hand side of (B-22) increases with w/τ, while the right-hand side decreases. Hence, the unique solution of (B-22) is given by the intersection of the two curves, one of which is upward sloping while the other is downward sloping. Moreover, an increase in τ shifts upwards the upward-sloping curve and does not change the downward-sloping curve (for any given w/τ). Therefore, w/τ unambiguously decreases with τ. Consequently, under trade liberalization, z^{HH} decreases while z^{FH} increases. We conclude that trade liberalization reduces domestic markups and increases foreign markups in the larger country. We summarize these results in the following proposition:

Proposition 7. *Assume the preferences are ε-Cobb-Douglas and ε-CES, with $r'_\theta > 0$ and $r'_\psi > 0$. Then there exists $\bar{\varepsilon}$ such that for any $0 < \varepsilon < \bar{\varepsilon}$, trade liberalization leads to: (i) lower markups for the traded good in the domestic markets; (ii) higher markups for the traded good in the foreign markets; (iii) higher markups in the non-traded sector; and (iv) a smaller wage differential between the two countries.*

Proof. In the text. □

Markup comparison and dumping. We next turn to the comparision of markups and show that there is 'reciprocal dumping' in the model. From (B-18)-(B-21), one can directly note that $z^{kk} > z^{kl}$ for $k, l = H, F$ and $r \neq s$. We now show that we also have $z^{HH} < z^{FF}$. To see this, note that

$$z^{HH} = \frac{1}{\left(\frac{a}{F\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[L^H + L^F \left(\frac{w}{\tau}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}} < \frac{(\tau w)^\sigma}{\left(\frac{a}{F\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[L^H + L^F (\tau w)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}} = z^{FF}$$

implies equivalently that $L^F w^{2(\sigma-1)} + (L^H - L^F)\tau^{\sigma-1} w^{\sigma-1} - L^H > 0$. This inequality holds for $w \geq 1$, which establishes the result.

We next compare z^{HF} and z^{FH} to show that $z^{FH} < z^{HF}$. To see this, note that

$$z^{HF} = \frac{1}{\left(\frac{a}{F\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[L^H + L^F (\tau w)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}} > \frac{\left(\frac{w}{\tau}\right)^\sigma}{\left(\frac{a}{F\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[L^H + L^F \left(\frac{w}{\tau}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}} = z^{FH}$$

implies equivalently that $L^F w^{2(\sigma-1)} + (L^H - L^F)\tau^{1-\sigma} w^{\sigma-1} - L^H < 0$. Using (B-16) we then obtain

$$\tau^{1-\sigma} = \frac{\frac{L^H}{L^F} w^{1-\sigma} - w^\sigma}{\frac{L^H}{L^F} w - 1}.$$

Note that $(L^H/L^F)w^{1-\sigma} - w^\sigma > 0$, or equivalently, $w^{2\sigma-1} < L^H/L^F$. Plugging this expression into the inequality, we get:

$$L^F w^{2(\sigma-1)} + (L^H - L^F)\frac{\frac{L^H}{L^F} w^{1-\sigma} - w^\sigma}{\frac{L^H}{L^F} w - 1} w^{\sigma-1} - L^H < 0$$

which, after some algebra, reduces to

$$\frac{w^{2\sigma-1}}{w} < \left(\frac{L^H}{L^F}\right)^2.$$

This inequality always holds since $w > 1$, $\frac{L^H}{L^F} > 1$ and $w^{2\sigma-1} < \frac{L^H}{L^F}$. Consequently, when $r'_\theta > 0$, the ordering of markups is unambigously the following:

$$r_\theta^{FF} > r_\theta^{HH} > r_\theta^{HF} > r_\theta^{FH}.$$

These results show that, similar to Kichko *et al.* (2014), firms in both countries practice reciprocal dumping when markets show pro-competitive effects.