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Klaus Adam, Henning Weber

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Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
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Optimal Trend Inflation

Abstract

Sticky price models featuring heterogeneous firms and systematic firm-level productivity trends deliver radically different predictions for the optimal inflation rate than their popular homogenous-firm counterparts: (1) the optimal steady-state inflation rate generically differs from zero and (2) inflation optimally responds to productivity disturbances. We show this by aggregating a heterogenous-firm model with sticky prices in closed form. Using firm-level data from the U.S. Census Bureau, we estimate the historically optimal inflation path for the U.S. economy. In the year 1977, the optimal inflation rate stood at 1.5%, but subsequently declined to around 1.0% in the year 2015. Inflation rates up to twice these numbers can be rationalized if one considers product demand elasticities more in line with the trade literature or if one considers firms that (partially) index prices to lagged inflation rates.

JEL-Codes: E520, E310, E320.

Keywords: optimal inflation rate, sticky prices, firm heterogeneity.

Klaus Adam
University of Mannheim
Mannheim / Germany
adam@uni-mannheim.de

Henning Weber
Deutsche Bundesbank
Frankfurt am Main / Germany
henning.weber@bundesbank.de

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1 Introduction

This paper introduces heterogeneous firms and empirically plausible firm-level productivity trends into an otherwise standard sticky-price economy. It shows that some of the fundamental implications of canonical sticky-price models with homogeneous firms fail to survive within such a generalized setup. The optimal steady-state inflation rate generically differs from zero and inflation optimally responds to productivity disturbances, unlike in settings with homogeneous firms. Moreover, the paper documents that the predictions of the homogeneous firm model turn out to be non-robust in the sense that they are discontinously affected by the presence of firm heterogeneity. We thus present an example in which microeconomic heterogeneity matters for macroeconomic policy prescriptions, an issue that has attracted renewed interest (Ahn et al. (2017), Kaplan and Violante (2014)).

Due to the technical difficulties associated with aggregating heterogeneous-firm models, it is standard in the sticky-price literature to abstract from all firm-level heterogeneity beyond that generated by price adjustment frictions themselves. As is well known, price adjustment frictions then tightly anchor the optimal steady-state inflation rate at zero, see Woodford (2003).[1] As we show, this rather robust but somewhat puzzling implication of standard sticky-price models arises precisely because of the homogeneity assumption. Homogeneity implies that the productivity of price-adjusting firms equals that of non-adjusting firms. With economic efficiency requiring relative prices to reflect relative productivities, it calls for price-adjusting firms to charge the same price as charged on average by non-adjusting firms, i.e., it calls for zero inflation.[2]

The present paper extends the basic sticky-price setup by introducing firm heterogeneity and systematic firm-level productivity trends. Such firm-level trends are clearly present in micro data, but are routinely abstracted from in the sticky-price literature. New firms, for example, tend to be initially small, i.e., tend to be initially unproductive when compared to existing firms.[3] Some of the young firms become more productive over time and grow, others become unproductive and exit the economy. We show how such life-cycle related productivity dynamics cause the average productivity of price-adjusting firms to generally differ from the average productive of non-adjusting firms. Economic efficiency then requires that adjusting firms set on average different prices than existing firms, which causes inflation or deflation to be optimal in steady state. We show this by aggregating the non-linear sticky price model with heterogeneous firms in closed form and by deriving analytical expressions for the optimal inflation rate.

[1] Section 2 discusses a range of extensions of the basic framework considered in the literature and their implications for the optimal inflation rate.

[2] Yun (2005) shows, using a setting with homogeneous firms, that if initial prices do not reflect initial productivities, the optimal inflation rate can display deterministic transitory deviations from zero.

[3] This does not rule out that new firms are in age-adjusted terms more productive than old firms. Our setup will allow for this possibility.

The heterogeneous firm model that we present is formulated in abstract terms and allows for a variety of economic interpretations through which firm heterogeneity arises. One interpretation is - as alluded to above - that heterogeneity arises from firm entry and exit and the associated life-cycle dynamics of firm productivity. This is also the interpretation that we shall consider in our empirical analysis. Yet, as explained in the main text, the model can equally be interpreted as one in which heterogeneity arises from product substitution or product quality improvements.

To fix ideas, consider a sticky-price model with Calvo type or menu-cost type price adjustment frictions in which a measure $\delta \geq 0$ of randomly chosen firms becomes unproductive and exits the economy each period. Exiting firms are replaced by a measure δ of young new firms. Our setup then features three systematic productivity trends, each of which has different implications for the optimal inflation rate. First, there is a common trend in total factor productivity (TFP), which affects all firms equally. The common TFP trend captures general-purpose innovations that are adopted by all firms simultaneously. As in a standard homogeneous-firm model, it does not affect the optimal inflation rate. Second, there is an experience trend in firm-level TFP, which determines how firms accumulate experience with age. The experience trend may capture productivity gains from learning-by-doing or other forms of experience accumulation. As we show, this productivity trend generates a force towards positive inflation rates. Third, there is a cohort productivity trend, which determines the productivity level of newly entering firms. This trend captures the fact that new firms tend to bring new technologies into the economy that are not (yet) used by other firms.[4] The cohort trend will be a force towards making deflation optimal.

Taken together, the optimal steady-state inflation rate in our setting depends on the strength of the experience trend relative to the strength of the cohort trend, whenever there is some positive firm turnover ($\delta > 0$). The optimal steady-state inflation rate is itself independent of the firm turnover rate, as long as $\delta > 0$. Yet, in the absence of firm turnover ($\delta = 0$), the optimal steady-state inflation rate collapses to zero, i.e., to the optimal inflation rate of a homogeneous firm model. It is in this sense, that the inflation predictions of the homogeneous firm model turn out to be non-robust.

To obtain economic intuition for these findings, consider two polar settings. The first setting abstracts from the presence of a cohort trend and considers a setting where the only trend is that firms accumulate experience over time.[5] If an old firm becomes unproductive and exits the economy, the new firm that replaces it will not have accumulated any experience yet. The new firm will thus be less productive than the remaining set of

[4] Newly entering firms are endowed with the cohort productivity level, in addition to the common TFP component, and then gradually accumulate experience over time.

[5] As mentioned before, we can abstract from the common TFP trend, as it does not affect the optimal inflation rate.

old firms.[6] From a welfare standpoint, the optimal price of new firms should therefore exceed the average price of existing firms, so as to accurately reflect relative productivities. Achieving this requires either that new firms choose higher prices or that old firms reduce prices, or a combination thereof.

In the presence of sticky prices, price reductions by old firms are costly in welfare terms. In time-dependent price adjustment models, they lead to inefficient price dispersion due to asynchronous price adjustment; in state-dependent pricing models, they require firms to pay adjustment costs. Therefore, it is optimal to implement the efficient relative price exclusively by having new firms charge higher prices, while all other firms hold their prices steady. Clearly, this implies that the aggregate inflation rate must be positive in the steady state.

Now consider the second polar setting, in which there is no experience effect and the only trend is a positive cohort trend. New firms are then more productive than the existing set of old firms, thus optimally charge lower prices than existing firms. This makes negative rates of inflation optimal.[7]

We also determine in closed form the optimal dynamic response of the inflation rate following shocks to experience and cohort productivity. We show that such shocks have fairly persistent effects on the optimal inflation rate, especially in settings in which δ is positive but close to zero.

To estimate the optimal inflation rate for the United States, we devise a model-consistent estimation approach, which is based on firm-level information from the Business Dynamics Statistics (BDS) of the U.S. Census Bureau. The BDS is based on the Longitudinal Business Database (LBD) and covers all private sector establishments in the United States from the year 1977 onwards. We rely on firm-level information because aggregate information fails to identify the inflation-relevant cohort and experience trends. This is so because inflation-neutral TFP trends mask the underlying inflation-relevant firm-level trends at the aggregate level. The firm-level information allows us to estimate the historically optimal inflation path for the U.S. economy in a model-consistent way and for a setting where the actual inflation rates implemented by the Federal Reserve may have been suboptimal. Our estimation shows that the optimal U.S. inflation rate was strictly positive throughout the years 1977-2015. In our benchmark estimation, it ranges between 1.5% in 1977 and a temporary low of around 0.85% during the Great Recession; the most recent estimate for the year 2015 is 1%.

Inflation rates up to twice these numbers can be rationalized if one is willing to assume lower demand elasticities, in line with estimates in the trade and industrial organization literatures, or if one consider settings in which sticky-price firms index partially prices to

[6] The new firm will be in age-adjusted terms more productive than all old firms, once one allows for a positive cohort trend.

[7] Due to price-setting frictions, it is again not optimal that old firms adjust prices.

past inflation rates. Optimal inflation rates above 1% are large by the standards of the sticky-price literature, e.g., Diercks (2017).

The remainder of the paper is structured as follows. Section 2 discusses the related literature. Section 3 presents our heterogeneous-firm model with sticky prices and section 4 presents a special case that illustrates our main result in rigorous but simple terms. Section 5 analytically aggregates the general model, and section 6 shows that the flexible-price equilibrium in this model is first best when a Pigouvian output subsidy corrects firms' monopoly power. The main optimal inflation result is presented in closed-form in section 7. Section 8 discusses the optimal steady-state inflation rate. It shows how the optimal inflation rate jumps discontinuously when moving from a standard sticky-price economy ($\delta = 0$) to one including firm turnover ($\delta > 0$). Section 9 determines the utility costs of implementing suboptimal inflation. Section 10 presents our estimation approach and our empirical results. Section 11 discusses the robustness of our findings towards various extensions. A conclusion briefly summarizes. Proofs and technical material are relegated to a series of appendices.

2 Related Literature

Few papers discuss the relationship between the optimal inflation rate and productivity trends. All of them focus on aggregate or sectoral productivity trends and find that the optimal inflation rate is (slightly) negative. Amano et al. (2009) consider an economy with aggregate productivity growth and sticky wages and prices. They show how monetary policy affects wage and price mark-ups and that this can make it optimal to implement deflation, so as to reduce wage mark-ups. Wolman (2011) considers a two-sector sticky-price economy with sectoral productivity trends. Despite the absence of monetary frictions, the optimal inflation rate is either negative or close to zero in his setting.

Golosov and Lucas (2007) and Nakamura and Steinsson (2010) consider sticky-price setups with heterogeneous firms and study monetary non-neutrality within these setups. They do not consider the issue of the optimal inflation rate. Firms in their settings are subject to random idiosyncratic productivity shocks. This differs from the present setup which features idiosyncratic shocks that give rise to systematic productivity adjustments (as implied by the cohort and experience trends). The idiosyncratic nature of productivity shocks in Golosov and Lucas (2007) and Nakamura and Steinsson (2010) causes firms with very positive or very negative idiosyncratic productivity shocks to adjust prices. The productivity of price-adjusting firms is thus on average similar to the productivity of non-adjusting firms, suggesting zero inflation to be optimal.

The present paper is also related to a large literature studying the determinants of optimal inflation, most of which finds that the optimal inflation rate is either negative or close to zero. None of these papers makes a connection between the optimal inflation rate and firm-level productivity dynamics.

In classic work, Kahn, King and Wolman (2003) consider a homogeneous-firm model and explore the trade-off between price adjustment frictions, which call for price stability, and monetary frictions, which call for a Friedman-type deflation. They demonstrate how a slight rate of deflation is optimal in such frameworks. In a comprehensive survey, Schmitt-Grohé and Uribe (2010) document the robustness of these findings to a large number of natural extensions. They show that taxation motives, including the presence of untaxed income, foreign demand for domestic currency (Schmitt-Grohé and Uribe (2012a)), as well as a potential quality bias in measured inflation rates (Schmitt-Grohé and Uribe (2012b)), are all unable to rationalize significantly positive rates of inflation.

Adam and Billi (2006, 2007) and Coibion, Gorodnichenko and Wieland (2012) explicitly incorporate a lower bound on nominal interest rates into sticky-price economies. They find that fully optimal monetary policy is consistent with close to zero average rates of inflation. While zero lower bound episodes make it optimal to promise inflation in the future, these promises should only be made conditionally on being at the lower bound, which happens rather infrequently; see Eggertsson and Woodford (2003) for an early exposition.

A number of papers find positive average rates of inflation to be optimal in the presence of downward nominal wage rigidities. Kim and Ruge-Murcia (2009) argue that such rigidities generate optimal inflation rates of approximately 0.35% in a model featuring aggregate shocks only. Looking at a setting with idiosyncratic shocks, Benigno and Ricci (2011) also find a positive steady-state inflation rate to be optimal.[8] Carlsson and Westermark (2016) consider a setting with nominal wage rigidities and search and matching frictions in the labor market. They show how a standard U.S. calibration of the model implies failure of the Hosios condition and justifies an annual inflation rate of about 1.2%. Schmitt-Grohé and Uribe (2013) analyze the case for temporarily elevated inflation in the Euro Area due to the presence of downward rigidity of nominal wages.

Brunnermeier and Sannikov (2016) show that the optimal inflation rate can also be positive in a model without nominal rigidities. They present a model with undiversifiable idiosyncratic capital income risk in which the optimal inflation rate increases with the amount of idiosyncratic risk.

There is also a literature studying endogenous firm entry decisions in homogeneous firm economies, focusing on the effect of inflation on firm entry (Bergin and Corsetti (2008), Bilbiie et al. (2008)). Bilbiie et al. (2014) document that the welfare optimal inflation rate is positive whenever the benefit of additional varieties to consumers falls short of

[8] Since positive inflation has no welfare costs in their setup, they do not quantify the optimal inflation rate.

the market incentives for creating these varieties. Inflation then reduces the value of creating varieties and brings firm entry closer to its efficient (lower) level. The present paper abstracts from endogenous firm entry decisions and thus from the implication of monetary policy for the entry margin. Instead, it considers a setting with heterogeneous firms in which entry and exit is driven by exogenous productivity dynamics.

Part of the sticky price literature incorporates trend inflation via exogenous inflation trends (Ascari and Sbordone (2014), Cogley and Sbordone (2008)). Trend inflation in these setups results from a central bank pursuing an exogenous and potentially time-varying inflation target. The present paper is concerned with determining the optimal inflation rate and how it relates to microeconomic fundamentals.

3 Economic Model

We consider a cashless economy with nominal rigidities and monopolistically competitive firms. The model is entirely standard, except for the more detailed modeling of firm-level productivity and price adjustment dynamics. Specifically, we augment the standard sticky-price setup by idiosyncratic firm-level productivity adjustments that arrive in conjunction with a price adjustment opportunity. This gives rise to a setting with heterogeneous firm-level productivities in which the productivity of price-adjusting firms is not necessarily equal to that of non-adjusting firms.

For simplicity, we derive our results within a time-dependent price adjustment model à la Calvo (1983). As we argue in section 11.1, our main result in proposition 2 below remains unaltered if we look instead at a setting where price adjustment frictions take the form of menu costs. The next section introduces our generalized firm setup in abstract terms. Section 3.2 provides alternative economic interpretations of the setup.

3.1 Technology, Prices and Price Adjustment Opportunities

Each period $t = 0, 1, \ldots$ there is a unit mass of monopolistically competitive firms indexed by $j \in [0, 1]$. Each firm j produces output Y_{jt}, which enters as an input into the production of an aggregate consumption and investment good Y_t according to

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $1 < \theta < \infty$ denotes the price elasticity of product demand. Let P_{jt} denote the price charged by firm j in period t. Firms can adjust prices with probability $1 - \alpha$ each period ($0 \leq \alpha < 1$). The arrival of a Calvo price adjustment opportunity is thereby idiosyncratic and independent of all other exogenous random variables in the economy.

We augment this standard setting by a second price adjustment opportunity that arrives with probability $\delta \geq 0$ each period. This second adjustment opportunity is idiosyncratic across firms, but arrives in conjunction with a firm-level productivity change, as described in detail below. In particular, let $\delta_{jt} \in \{0,1\}$ denote the idiosyncratic i.i.d. random variable governing this second price and productivity adjustment and let $\delta_{jt} = 1$ indicate the arrival of such an adjustment event for firm j in period t ($\Pr(\delta_{jt} = 1) = \delta$). We shall informally refer to the event $\delta_{jt} = 1$ as *the occurrence of a δ-shock*. We introduce such δ-shocks in abstract form below and discuss alternative economic interpretations in section 3.2.

Letting K_{jt} and L_{jt} denote the amount of capital and labor used by firm j, respectively, firm output Y_{jt} is given by

$$Y_{jt} = A_t Z_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right), \tag{2}$$

where A_t captures common productivity, Z_{jt} firm-specific productivity, and $F_t \geq 0$ the potential presence of fixed costs for operating the firm. We have $\phi \geq 1$ and to be consistent with balanced growth, we assume

$$F_t = f \cdot (\Gamma_t^e)^{1-\frac{1}{\phi}} \tag{3}$$

for some $f \geq 0$, where Γ_t^e captures the growth trend in the balanced growth path, as defined in equation (27) below.[9] Common productivity evolves according to

$$A_t = a_t A_{t-1},$$

firm-specific productivity according to

$$Z_{jt} = \begin{cases} g_t Z_{jt-1} & \text{if } \delta_{jt} = 0 \\ Q_t & \text{if } \delta_{jt} = 1, \end{cases} \tag{4}$$

where Q_t is given by

$$Q_t = q_t Q_{t-1}. \tag{5}$$

We also assume that $a_t = a\epsilon_t^a$, $q_t = q\epsilon_t^q$, and $g_t = g\epsilon_t^g$ with $\epsilon_t^a, \epsilon_t^q, \epsilon_t^g > 0$ being stationary shocks with an arbitrary contemporaneous and intertemporal covariance structure, satisfying $E[\epsilon_t^a] = E[\epsilon_t^q] = E[\epsilon_t^g] = 1$. To obtain a well-defined steady state and to insure that relative prices in the flexible-price economy remain bounded, we assume throughout the paper

$$(1-\delta)\,(g/q)^{\theta-1} < 1. \tag{6}$$

Productivity dynamics in the previous setting feature three trends: (1) the common growth trend a_t; (2) the experience growth trend g_t, which applies in the absence of δ-shocks; and (3) the productivity growth trend q_t, which determines the effects of δ-shocks

[9] In the absence of aggregate technology growth, the formulation of fixed costs in equation (3) corresponds to that used in Melitz (2003).

on technology. Each of these three growth trends has a different implication for the optimal inflation rate.

To understand the productivity dynamics implied by the previous setup, consider first the special case with $\delta = 0$. In the absence of idiosyncratic δ-shocks to firm technology, all firms experience the same productivity growth rate $a_t g_t$. Such a setting with homogeneous productivity growth across all firms is the one routinely considered in the sticky-price literature.[10]

Next, consider the case $\delta > 0$ and let s_{jt} denote the number of periods that have elapsed since firm j last experienced a δ-shock (i.e., $\delta_{j,t-s_{jt}} = 1$ and $\delta_{j,\widetilde{t}} = 0$ for $\widetilde{t} = t - s_{jt} + 1, ..., t$). Firm-specific productivity Z_{jt} in equation (4) can then be written as

$$Z_{jt} = G_{jt} Q_{t-s_{jt}},$$

where

$$G_{jt} = \begin{cases} 1 & \text{for } s_{jt} = 0 \\ g_t G_{jt-1} & \text{otherwise,} \end{cases}$$

and where Q_t follows equation (5). This alternative formulation illustrates that all firms hit by a δ-shock in t upgrade idiosyncratic productivity to $Z_{jt} = Q_t$, so that Q_t can be interpreted as capturing a "cohort effect" of productivity dynamics, where cohorts are determined by the arrival time of the last δ-shock. Following a δ-shock, the firm experiences productivity gains, as described by the process G_{jt}, as long as no further δ-shocks arrive. Since the productivity gains G_{jt} are lost with the arrival of the next δ-shock, one can interpret the process G_{jt} as capturing "experience" or "learning-by-doing effects" associated with the cohort production technology $Q_{t-s_{jt}}$. Following a δ-shock in period t, our specification thereby implies that firm productivity increases (temporarily decreases) if Q_t has been growing faster (slower) than G_{jt} since the time of arrival of the last δ-shock prior to period t. Note, however, that as long as Q_t displays a positive growth trend ($q_t > 0$), firms always become more productive over time in experience-adjusted terms, even if Q_t grows slower than G_{jt}. Indeed, in a setting with $\delta > 0$, the long-term growth rate of firms' productivity is determined by the process $a_t q_t$, as the experience growth rate g_t generates - due to the occasional reset - only temporary level effects for firm productivity.

As usual, we define the aggregate price level as

$$P_t \equiv \left(\int_0^1 P_{jt}^{1-\theta} \mathrm{dj} \right)^{\frac{1}{1-\theta}}. \tag{7}$$

[10]For the case $\delta = 0$, our setting still allows for a non-degenerate initial distribution of firm productivities. Typically, this initial distribution is also assumed to be degenerate in the sticky-price literature. As we show below, the additional assumption of a degenerate initial distribution is not key for the conclusion that zero inflation is optimal, as long as initial prices reflect initial productivities, see Yun (2005) for a discussion of this and related issues in a homogeneous firm setting.

Cost minimization in the production of final output Y_t implies

$$Y_{jt} = (P_{jt}/P_t)^{-\theta} Y_t. \tag{8}$$

which shows that the price level can be expressed as an expenditure-weighted average of the prices in the different expenditure categories, in line with the practice at statistical agencies:

$$P_t = \int_0^1 \left(\frac{Y_{jt}}{Y_t}\right) P_{jt} \text{ dj}, \tag{9}$$

Note that the price index contains in any period t all the goods available during this period. This is clearly an idealized notion of how price indices are actually computed by statistical agencies. We discuss in section 11.3 how results are altered if the product basket underlying the price index is biased towards 'older' products.

Using the price index in equation (9), we define the gross inflation rate as

$$\Pi_t \equiv P_t/P_{t-1}.$$

To the extent that δ-shocks capture product substitutions, this assumes that products that become unavailable are replaced by a new products in the subsequent product basket, so that the inflation rate is computed using the price level associated with the old product basket in the earlier period and the price level associated with the new basket in the subsequent period. This assumption is in line with the sampling procedures typically pursued by the BLS and other statistical offices. The section "Item replacement and quality adjustment" in chapter 17 of the BLS Handbook of Methods (BLS (2015)), for instance, describes how the changeover of discontinued product versions is handled. If a data collector cannot find anymore a product version that was previously contained in the basket, the collector replaces it with a new version. The price of the old version enters the previous price index and the price of the new version enters the current price index.[11]

3.2 Alternative Interpretations of the Firm Setup

The previous section defined δ-shocks ($\delta_{jt} = 1$) as an idiosyncratic change in firm-level productivity that is associated with a price adjustment opportunity. This section presents three alternative economic interpretations of δ-shocks that highlight alternative economic sources of firm heterogeneity and that explain why productivity changes may plausibly be associated with price flexibility at the firm level.

[11] The BLS also seeks to adjust for quality differences across versions. Armknecht et al. (1996) shows that about 3% of products are discontinued each month. Their table 9.2 shows that more than 50% of the replacement versions fall into the category "direct comparisons", for which no quality adjustment is made; for the remaining replacements there is either a direct quality adjustment or quality adjustment is imputed via different methods. As will become clear in section 3.2, our setup is consistent with statistical agencies making such quality adjustments.

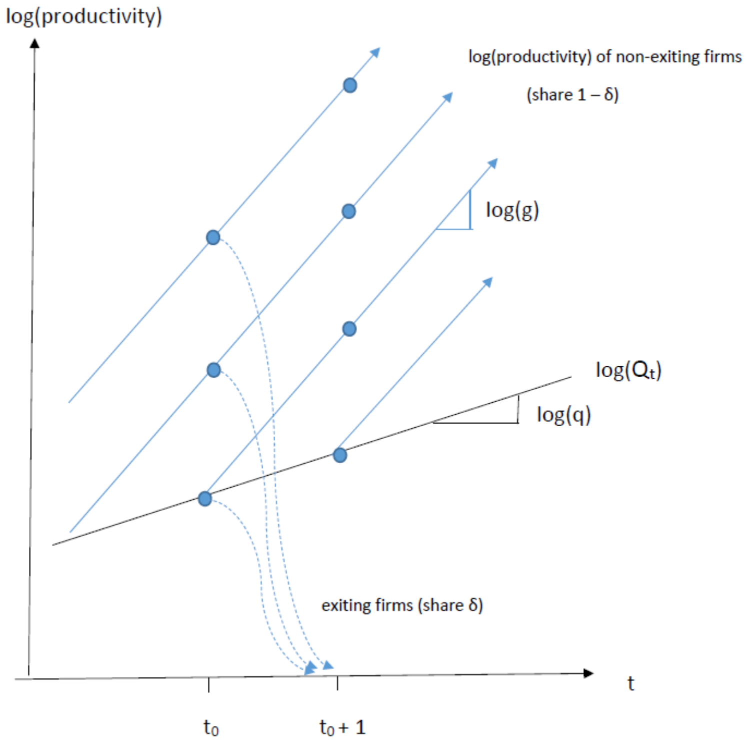

Figure 1: Productivity dynamics in a setting with firm entry and exit

Firm entry and exit. It is possible to interpret δ-shocks as a firm exit and entry event. Indeed, this is the interpretation that we adopt in our empirical application of the model in section 10. Specifically, the event $\delta_{jt} = 1$ can be interpreted as an event in which firm j becomes permanently unproductive and thus exits the economy. Each exiting firm is then replaced by a newly entering firm to which we assign for simplicity the same firm index j. The variable Q_t then captures the productivity level of the cohort of firms that enters in period t, and G_{jt} captures the experience accumulated over the lifetime of firm j. The assumption that firms' prices are flexible following a δ-shock should then be interpreted as newly entering firms being able to freely choose the price of their product. It is worth noting that firm entry and exit rates are high in the United States, see figure 3 in Decker et al. (2014).

Figure 1 illustrates the firm-level productivity dynamics for the empirically plausible setting in which the cohort trend is positive ($q > 0$), but less strong than the experience trend ($g > q$). To simplify the exposition, the figure depicts the deterministic dynamics and abstracts from the common TFP trend a, which does not affect the distribution of relative productivities across firms. The line labeled $\log(Q_t)$ in the figure indicates the cohort trend and captures the productivity of newly entering firms at each point in time. The lines starting at the cohort trend line capture the productivity dynamics of the entering cohorts over time. Since $g > q$, the productivity of existing firms grows faster than the productivity of new entrants, so that existing firms are initially more productive and thus larger than newly entering firms. In experience-adjusted terms, however, newly entering firms are the most productive firms in the economy. The downward-pointing dashed arrows indicate the productivity losses of exiting firms that have been hit by a δ-shock. For simplicity, the figure assumes that their productivity permanently drops to zero. As should be clear from the figure, the entry and exit dynamics imply an exponential distribution for firm age. Coad (2010) shows that such an age distribution is empirically plausible and how it generates, together with (productivity) growth shocks, a Pareto distribution for firm size, in line with the observed firm size distribution.

Product substitution. The event $\delta_{jt} = 1$ can also be interpreted as an event in which the product previously produced by firm j is no longer demanded by consumers. Firm j reacts to this by introducing a new product, which - for simplicity - is assigned the same product index j. The variable Q_t then captures the productivity level associated with products that are newly introduced in t and G_{jt} captures experience accumulation in producing the new product. Product substitutions, e.g., in the form of new product versions or models, take place rather frequently in the data and are also prevalent in the CPI baskets of statistical agencies (see section III.C in Nakamura and Steinsson (2008) for evidence on the rate of product substitution in the U.S. CPI). Evidence provided in Moulton and Moses (1997), Bils (2009) and Melser and Syed (2016) furthermore shows that the prices of new products are typically higher than those that they replace, even after accounting for quality improvements.[12] It thus appears reasonable to assume price flexibility for new products (see also Nakamura and Steinsson (2012)).

Quality improvements. Let Q_{jt} denote the quality of the product produced by firm j in period t. Defining $Q_{jt} = Q_{t-s_{jt}}$, the event $\delta_{jt} = 1$ captures the situation in which firm j upgrades the quality of its product from level $Q_{t-1-s_{j,t-1}}$ to level Q_t. Let aggregate output produced with intermediate inputs of different quality be given by

$$Y_t = \left(\int_0^1 \left(Q_{jt} \widetilde{Y}_{jt} \right)^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}},$$

and let firm j's output of quality level Q_{jt} be given by

$$\widetilde{Y}_{jt} = A_t G_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

where G_{jt} now captures experience effects associated with producing quality Q_{jt}. Finally, let $\widetilde{P}_{jt}$ denote the price of a unit of good j of quality level Q_{jt}. Assuming that statistical

[12] Evidence provided in Bils (2009) shows that inflation for durables ex computers over the period 1988-2006 averaged 2.5% per year, but when including only matched items, the inflation rate was -3.7% per year.

agencies perfectly adjust the price level for quality changes over time, we have

$$P_t = \left(\int_0^1 \left(\frac{\widetilde{P}_{jt}}{Q_{jt}} \right)^{1-\theta} \mathrm{dj} \right)^{\frac{1}{1-\theta}}.$$

As is easily verified, this setup with quality improvements is mathematically identical to the one with productivity changes spelled out in the previous section.[13] Again, it appears natural to assume that firms can flexibly price goods with improved quality features.

3.3 Optimal Price Setting

Firms choose prices, capital and hours worked to maximize profits. While price adjustment is subject to adjustment frictions, factor inputs can be chosen flexibly. Letting W_t denote the nominal wage and r_t the real rental rate of capital, firm j chooses the factor input mix so as to minimize production costs $K_{jt}P_t r_t + L_{jt}W_t$ subject to the constraints imposed by the production function (2). Let

$$I_{jt} \equiv F_t + Y_{jt}/(A_t Q_{t-s_{jt}} G_{jt})$$

denote the units of factor inputs ($K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}}$) required to produce Y_{jt} units of output. As appendix A.1 shows, cost minimization implies that the marginal costs of I_{jt} are given by

$$MC_t = \left(\frac{W_t}{1/\phi} \right)^{\frac{1}{\phi}} \left(\frac{P_t r_t}{1 - 1/\phi} \right)^{1-\frac{1}{\phi}}. \tag{10}$$

Now consider a firm that either experienced a δ-shock or a Calvo shock in period t and that can freely choose its price. Let α denote the Calvo probability that the firm has to keep its previous price ($0 \leq \alpha < 1$), the firm will not be able to reoptimize its price with probability $\alpha(1-\delta)$ at any future date, i.e., whenever it undergoes neither a δ- shock nor a Calvo shock.[14] The price-setting problem of a firm that can optimize its price in period t is thus given by

$$\begin{aligned} \max_{P_{jt}} E_t \sum_{i=0}^{\infty} (\alpha(1-\delta))^i \frac{\Omega_{t,t+i}}{P_{t+i}} \left[(1+\tau) P_{jt+i} Y_{jt+i} - MC_{t+i} I_{jt+i} \right] \qquad (11) \\ s.t. \quad I_{jt+i} = F_{t+i} + Y_{jt+i}/A_{t+i} Q_{t-s_{jt}} G_{jt+i}, \\ Y_{jt+i} = \left(P_{jt+i}/P_{t+i} \right)^{-\theta} Y_{t+i}, \\ P_{jt+i+1} = \Xi_{t+i,t+i+1} P_{jt+i}. \end{aligned}$$

[13]The quality-adjusted price $\widetilde{P}_{jt}/Q_{jt}$ and the quality-adjusted quantity $\widetilde{Y}_{jt}Q_{jt}$ then correspond to the price P_{jt} and quantity Y_{jt}, respectively, in the previous section.

[14]In any period, the firm can adjust its price with probability δ due to the occurrence of a δ-shock and with probability $(1-\alpha)(1-\delta)$ due to the occurrence of a Calvo price adjustment shock.

where τ denotes a sales tax/subsidy and $\Omega_{t,t+i}$ denotes the representative household's discount factor between periods t and $t+i$. The first constraint captures the firm's technology, the second constraint captures the demand function faced by the firm, as implied by equation (1), and the last constraint captures how the firm's price is indexed over time (if at all) in periods in which prices are not reset optimally. We consider general price indexation schemes and allow $\Xi_{t+i,t+i+1}$ to be a function of aggregate variables up to period $t+i$.[15] In the absence of indexation, we have $\Xi_{t+i,t+i+1}=1$ for all $i \geq 0$.

Appendix A.2 shows that the optimal price $P_{jt}^{\star}$ can be expressed as

$$\frac{P_{jt}^{\star}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)=\left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t}, \tag{12}$$

where the variables N_t and D_t are independent of the firm index j and evolve recursively according to

$$N_t=\frac{MC_t}{P_tA_tQ_t}+\alpha(1-\delta)E_t\left[\Omega_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\Xi_{t,t+1}\right)^{-\theta}\left(\frac{P_{t+1}}{P_t}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \tag{13}$$

$$D_t=1+\alpha(1-\delta)E_t\left[\Omega_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\Xi_{t,t+1}\right)^{1-\theta}\left(\frac{P_{t+1}}{P_t}\right)^{\theta-1}D_{t+1}\right]. \tag{14}$$

Equation (12) shows that the optimal reset price of a firm depends only on how its own productivity ($A_tQ_{t-s_{jt}}G_{jt}$) relates to the productivity of a firm hit by a δ-shock in period t (A_tQ_t), as well as on the aggregate variables (N_t, D_t). It is precisely this feature which permits aggregation of the model in closed form. Equation (12) furthermore shows that more productive firms optimally choose lower prices. With homogeneous firms, relative productivity is always equal to one ($Q_{t-s_{jt}}G_{jt}/Q_t = 1$) and equations (12)-(14) then reduce to the ones capturing the optimal price in a standard homogeneous-firm model with sticky prices.

3.4 Household Problem

There is a representative household with balanced growth consistent preferences given by

$$E_0\sum_{t=0}^{\infty}\beta^t\xi_t\left(\frac{\left[C_tV(L_t)\right]^{1-\sigma}-1}{1-\sigma}\right), \tag{15}$$

where C_t denotes private consumption of the aggregate good, L_t labor supply, ξ_t a preference shock with $E[\xi_t]=1$ and $\beta\in(0,1)$ the discount factor. We assume $\sigma>0$ and that

[15] We only require that price indexation is such that the price-setting problem remains well defined, that price indexation does not give rise to multiplicities of the optimal inflation rate and that $\Xi_{t,t+1}=1$ in a steady state without inflation. For instance, when indexing occurs with respect to lagged inflation according to $\Xi_{t,t+1}=(\Pi_t)^{\kappa}$ with $\kappa\geq 0$, we rule out $\kappa>1$ to avoid non-existence of optimal plans and rule out $\kappa=1$ to avoid multiplicities of the steady-state inflation rate.

$V(\cdot)$ is such that period utility is strictly concave in (C_t, L_t) and that Inada conditions are satisfied. The household faces the flow budget constraint

$$C_t + K_{t+1} + \frac{B_t}{P_t} = (r_t + 1 - d)K_t + \frac{W_t}{P_t}L_t + \int_0^1 \frac{\Theta_{jt}}{P_t}\,\mathrm{dj} + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) - T_t,$$

where K_{t+1} denotes the capital stock, B_t nominal government bond holdings, i_{t-1} the nominal interest rate, W_t the nominal wage rate, r_t the real rental rate of capital, d the depreciation rate of capital, Θ_{jt} nominal profits from ownership of firm j, and T_t lump sum taxes. Household borrowing is subject to a no-Ponzi scheme constraint. The first-order conditions characterizing optimal household behavior are entirely standard and are derived in appendix A.3. To insure existence of a well-defined balanced growth path, we assume throughout the paper that

$$\beta < (aq)^{\phi\sigma}.$$

3.5 Government

To close the model, we consider a government which faces the budget constraint

$$\frac{B_t}{P_t} = \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + \tau \int_0^1 \left(\frac{P_{jt}}{P_t}\right) Y_{jt}\,\mathrm{dj} - T_t,$$

where τ denotes a sales subsidy, which will be used to correct for the monopolistic distortions in product markets. The government levies lump sum taxes T_t, so as to implement a bounded state-contingent path for government debt B_t/P_t.[16] Since we consider a cashless limit economy, there are no seigniorage revenues, even though the central bank controls the nominal interest rate. We furthermore assume that monetary policy is not constrained by a lower bound on nominal interest rates. The equilibrium concept is standard and defined in appendix A.5.

4 The Optimal Inflation Rate for a Special Case

We now illustrate the paper's main result using a special setting without economic disturbances, without capital in production ($\phi = 1$), and without price-indexation ($\Xi_{t,t+1} = 1$ for all t). Additionally, we consider the special household preferences[17]

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t - L_t)$$

[16] The household's transversality condition will then automatically be satisfied in equilibrium.

[17] These are obtained from equation (15) for $V(L) = e^{-L}$ and for the limiting case $\sigma \to 1$.

and a strictly positive rate of δ-shocks ($\delta > 0$). The household's first order conditions for consumption and hours worked imply

$$W_t = C_t P_t \qquad \text{for all } t. \tag{16}$$

In a balanced growth path with constant hours worked, aggregate consumption grows at rate[18]

$$\frac{C_{t+1}}{C_t} = aq \qquad \text{for all } t, \tag{17}$$

where a is the productivity growth rate common to all firms and q the cohort productivity growth rate. The experience growth rate (g) does not contribute to long-run growth because accumulated experience is lost once firms are hit by a δ-shock.

Next, consider the firm side of the economy. A necessary condition for optimality in the firm sector is - as in any standard sticky-price model without firm level trends - that firms do not want to change prices over time.[19] For each cohort of firms, production technology progresses at the common TFP growth rate (a) and at the experience trend growth rate (g). For constant prices to be optimal at the firm level, nominal wages must grow at the rate ag, so as to offset the efficiency gains on the real side. This fact together with equation (16) implies

$$\frac{C_{t+1}P_{t+1}}{C_t P_t} = ag \qquad \text{for all } t,$$

which together with equation (17) shows that the optimal inflation rate must satisfy

$$\frac{P_{t+1}}{P_t} = \frac{g}{q}. \tag{18}$$

For this inflation rate to be sufficient for achieving optimality, one only has to insure that the *level* of prices that firms charge in the first period following a δ-shock is the correct one, in the sense that they do not involve a mark-up over production costs. Wages are then equal to the marginal product of labor. As in the standard New Keynesian model with homogeneous firms, this can be achieved with the help of a Pigouvian output subsidy that corrects firms' monopoly power. The optimal inflation rate (18) then achieves full efficiency for the economy: product prices are equal to marginal production costs and the wage equals the marginal product of labor.[20]

The special case presented above highlights a number of aspects that will generalize to the fully fledged model: (1) The Calvo assumption does not drive the optimal inflation

[18] All firms experience the TFP growth rate a. In addition, all firms will eventually be hit by a δ-shock and thus experience the latest cohort productivity level Q_t, where Q_t latter grows at the rate q. Aggregate labor productivity thus grows at the rate aq.

[19] Otherwise, the presence of asynchronous price adjustment opportunities leads to price differences between firms that - on technological grounds - should charge identical prices.

[20] In the absence of the optimal output subsidy, the inflation rate in equation (18) still achieves productive efficiency.

result: since firm-level prices never need to change under the optimal inflation rate, the same inflation rate would be optimal in a setting with menu cost frictions. (2) The experience growth rate (g) and the cohort growth rate (q) have opposite effects on the optimal inflation rate because g affects the growth rate of technology in a cohort of firms, but not the aggregate growth rate, while the reverse is true for q. (3) The flexibility of prices following a δ-shock is key for achieving efficiency.

The remaining part of the paper shows that once one appropriately adjusts for the presence of price indexing, the steady state result (18) survives in more general environments. We furthermore characterize the optimal inflation policy outside of the steady state, where the relationship between marginal costs and consumption is considerably more complex. Achieving efficiency then requires that the optimal inflation rate fluctuates around its steady state value.

5 Analytical Aggregation with Heterogeneous Firms

This section outlines the main steps that allow us to aggregate the model in closed form. In a first step, we derive a recursive representation describing the evolution of the aggregate price level P_t over time. In a second step, we derive a closed-form expression for the aggregate production function. In a last step, we show how to appropriately detrend aggregate variables, so as to render them stationary.

Evolution of the aggregate price level. Let $P^{\star}_{t-s,t-k}$ denote the optimal price of a firm that last experienced a δ-shock in $t-s$ and that has last reset its price in $t-k$ ($s \geq k \geq 0$). In period t, this firm's price is equal to $\Xi_{t-k,t} P^{\star}_{t-s,t-k}$, where $\Xi_{t-k,t} = \prod_{j=1}^{k} \Xi_{t-k+j-1,t-k+j}$ captures the cumulative effect of price indexation (with $\Xi_{t-k,t} \equiv 1$ in the absence of price indexation). Let $\Lambda_t(s)$ denote the weighted average price in period t of the cohort of firms that last experienced a δ-shock in period $t-s$, where all prices are raised to the power of $1-\theta$, i.e.,

$$\Lambda_t(s) = (1-\alpha) \sum_{k=0}^{s-1} \alpha^k (\Xi_{t-k,t} P^{\star}_{t-s,t-k})^{1-\theta} + \alpha^s (\Xi_{t-s,t} P^{\star}_{t-s,t-s})^{1-\theta}. \tag{19}$$

There are α^s firms that have not had a chance to optimally reset prices since receiving the δ-shock and $(1-\alpha)\alpha^k$ firms that have last adjusted $k < s$ periods ago. From equation (7) it follows that one can use the cohort average prices $\Lambda_t(s)$ to express the aggregate price level as

$$P_t^{1-\theta} = \sum_{s=0}^{\infty} (1-\delta)^s \delta \Lambda_t(s), \tag{20}$$

where δ is the mass of firms that experience a δ-shock each period and $(1-\delta)^s$ is the share of those firms that have not undergone another δ-shock for s periods.

To express the evolution of P_t in a recursive form, consider the optimal price $P^\star_{t-s,t}$ of a firm that sustained a δ-shock $s > 0$ periods ago, but can adjust the price in t due to the occurrence of a Calvo shock. Also, consider the price $P^\star_{t,t}$ of a firm where a δ-shock occurs in period t. The optimal price setting equation (12) then implies

$$P^\star_{t,t} = P^\star_{t-s,t} \left(\frac{g_t \times \cdots \times g_{t-s+1}}{q_t \times \cdots \times q_{t-s+1}} \right). \tag{21}$$

The previous equation shows that a stronger cohort productivity trend (higher values for q) causes the firm that experiences a δ-shock in period t to choose lower prices relative to firms that experienced δ-shocks further in the past, as a stronger cohort trend makes this firm relatively more productive. Conversely, a stronger experience effect (higher values for g) increases the optimal relative price of the firm that underwent a δ-shock in t. The net effect depends on the relative strength of the cohort versus the experience effect.

Appendix A.4 shows how to combine equations (19), (20), and (21) to obtain a recursive representation for the evolution of the aggregate price level given by

$$P_t^{1-\theta} = \delta (P^\star_{t,t})^{1-\theta} + (1-\alpha)(1-\delta) \frac{(p_t^e)^{\theta-1} - \delta}{1-\delta} (P^\star_{t,t})^{1-\theta} + \alpha(1-\delta)(\Xi_{t-1,t} P_{t-1})^{1-\theta}, \tag{22}$$

where p_t^e summarizes the history of shocks to cohort and experience productivity and evolves recursively according to

$$(p_t^e)^{\theta-1} = \delta + (1-\delta) \left(p_{t-1}^e g_t / q_t \right)^{\theta-1}. \tag{23}$$

The last term on the r.h.s. of equation (22) captures the price-level effects from the share $\alpha(1-\delta)$ of firms that experienced neither a Calvo shock nor a δ-shock. These firms keep their old price (P_{t-1} on average), adjusted for possible effects of price indexation, as captured by the indexation term $\Xi_{t-1,t}$. The first term on the r.h.s. of equation (22) captures the price effects of the mass δ of firms that experienced a δ-shock in period t; these firms optimally charge price $P^\star_{t,t}$. The second term captures the average price of firms that experienced a Calvo shock in period t; their share is $(1-\alpha)(1-\delta)$ and they set a price that on average differs from the price charged by firms hit by a δ-shock, depending on the value of p_t^e. This latter aspect in equation (22) is the key difference relative to the standard model without firm heterogeneity in productivity. A stronger experience trend (a higher value for g_t), for instance, increases $(p_t^e)^{\theta-1}$, and - ceteris paribus - causes firms hit by a Calvo shock to choose a lower value for the optimal reset price. A stronger cohort trend (a higher value for q_t) has the opposite effect. Overall, the interesting new feature is that price dynamics now depend on the productivity trends.

In a setting where all firms have identical productivity trends, e.g., where the cohort effect is as strong as the experience effect ($q_t = g_t$ for all t), equation (23) implies that p_t^e converges to one, causing the price level to eventually evolve according to

$$P_t^{1-\theta} = [\delta + (1-\alpha)(1-\delta)] (P^\star_{t,t})^{1-\theta} + \alpha(1-\delta)(\Xi_{t-1,t} P_{t-1})^{1-\theta},$$

which is independent of productivity developments at the firm level. If in addition there are no δ-shocks ($\delta = 0$), the previous equation simplifies further to

$$P_t^{1-\theta} = (1-\alpha)(P_{t,t}^{\star})^{1-\theta} + \alpha(\Xi_{t-1,t}P_{t-1})^{1-\theta},$$

which describes the evolution of the aggregate price level in the standard Calvo model with homogeneous firms.

Aggregate production function. In appendix A.6 we show that aggregate output Y_t can be written as

$$Y_t = \frac{A_t Q_t}{\Delta_t}\left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t\right), \tag{24}$$

where K_t denotes the aggregate capital stock, L_t aggregate hours worked and

$$\Delta_t = \int_0^1 \left(\frac{Q_t}{G_{jt}Q_{t-s_{jt}}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} \mathrm{dj} \tag{25}$$

evolves recursively according to

$$\Delta_t = \left[\delta + (1-\alpha)(1-\delta)\frac{(p_t^e)^{\theta-1} - \delta}{1-\delta}\right]\left(\frac{P_{t,t}^{\star}}{P_t}\right)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1}. \tag{26}$$

TFP in the aggregate production function (24) is a function of the TFP of the latest cohort hit by the δ-shock, A_tQ_t, and of the adjustment factor Δ_t. The latter is defined in equation (25) and captures a firm's productivity relative to that of the latest cohort, $Q_t/\left(Q_{t-s_{jt}}G_{jt}\right)$, and weights this relative productivity with the firm's production share $(P_{jt}/P_t)^{-\theta}$. Equations (24) and (25) thus show how relative price distortions may lead to aggregate output losses by negatively affecting aggregate technology, e.g., by allocating more demand to relatively inefficient firms. The evolution of the adjustment factor over time is described by equation (26) and depends on firm-level productivity trends - amongst other ways - through the variable p_t^e. In the limit with homogeneous firm trends (i.e., $q_t = g_t$), p_t^e converges to one and the evolution of Δ_t becomes independent of productivity realizations. If - in addition - there are no δ-shocks ($\delta = 0$), then equation (26) simplifies further to

$$\Delta_t = (1-\alpha)\left(\frac{P_{t,t}^{\star}}{P_t}\right)^{-\theta} + \alpha\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1},$$

which is the equation capturing the distortions from price dispersion within a standard homogeneous-firm model.

Balanced Growth Path. One can obtain stationary aggregate variables by rescaling them by the aggregate growth trend

$$\Gamma_t^e = (A_tQ_t/\Delta_t^e)^{\phi}, \tag{27}$$

where Δ_t^e denotes the efficient adjustment factor chosen by the planner, defined in equation (31) below. Specifically, the rescaled output $y_t = Y_t/\Gamma_t^e$ and the rescaled capital stock $k_t = K_t/\Gamma_t^e$ are now stationary and the aggregate production function (24) can be written as

$$y_t = \left(\frac{\Delta_t^e}{\Delta_t}\right)\left(k_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - f\right). \tag{28}$$

In the deterministic balanced growth path, the (gross) trend growth rate $\gamma_t^e = \Gamma_t^e/\Gamma_{t-1}^e$ is constant and equal to $(aq)^\phi$ and hours worked are constant whenever monetary policy implements a constant inflation rate. Appendices A.7 and A.8 write all model equations using stationary variables only and appendix A.9 determines the resulting deterministic steady state.

6 Efficiency of the Flexible-Price Equilibrium

This section derives the efficient allocation and the conditions under which the flexible-price equilibrium is efficient. Appendix B shows that the efficient consumption, hours and capital allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ solves

$$\max_{\{C_t,L_t,K_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{[C_t V(L_t)]^{1-\sigma} - 1}{1-\sigma}\right) \tag{29}$$

$$s.t. \quad C_t + K_{t+1} = (1-d)K_t + \frac{A_t Q_t}{\Delta_t^e}\left((K_t)^{1-\frac{1}{\phi}}(L_t)^{\frac{1}{\phi}} - F_t\right), \tag{30}$$

where

$$\Delta_t^e \equiv \left(\int_0^1 \left(\frac{Q_t}{G_{jt} Q_{t-s_{jt}}}\right)^{1-\theta} \mathrm{dj}\right)^{\frac{1}{1-\theta}}, \tag{31}$$

which evolves according to

$$(\Delta_t^e)^{1-\theta} = \delta + (1-\delta)\left(\Delta_{t-1}^e q_t/g_t\right)^{1-\theta}. \tag{32}$$

Constraint (30) is the economy's resource constraint, when expressing aggregate output using the aggregate production function (24). The efficient productivity adjustment factor Δ_t^e showing up in the planner's production function is defined in equation (31); its recursive evolution is described by equation (32). The first-order conditions of problem (29)-(30) shown in appendix B are necessary and sufficient conditions characterizing the efficient allocation.

Decentralizing the efficient allocation requires that firms' prices, which enter Δ_t and thus in the aggregate production function (24), satisfy certain conditions. In particular, equations (25) and (31) imply that $\Delta_t = \Delta_t^e$ is achieved if prices satisfy

$$\frac{P_{jt}}{P_t} = \frac{1}{\Delta_t^e} \frac{Q_t}{G_{jt} Q_{t-s_{jt}}}. \tag{33}$$

The previous equation requires relative prices to accurately reflect relative productivities. Furthermore, as in models without firm heterogeneity, one has to eliminate firms' monopoly power by a Pigouvian subsidy to obtain efficiency of the market allocation. We thus impose the following condition:

Condition 1 *The sales subsidy corrects firms' market power, i.e.,* $\frac{\theta}{\theta-1}\frac{1}{1+\tau}=1$.

Appendix C then proves the following result:

Proposition 1 *The flexible-price equilibrium* ($\alpha = 0$) *is efficient if condition 1 holds.*

The proof of the proposition shows that condition (33) holds under flexible prices, so that one achieves $\Delta_t = \Delta_t^e$ and thereby productive efficiency. In the presence of the assumed sales subsidy, consumer decisions are also undistorted, which means that the values of consumption, hours worked and capital in the flexible-price equilibrium are identical to the values that these variables assume in the efficient allocation.

7 Optimal Inflation with Sticky Prices: Main Result

This section determines the optimal inflation rate for an economy with sticky prices ($\alpha > 0$). It derives the optimal rate of inflation for the nonlinear stochastic economy with heterogeneous firms in closed form and shows how inflation optimally depends on the productivity growth rates a_t, q_t and g_t. As it turns out, the optimal inflation rate implements the efficient allocation (the flexible-price benchmark) and is independent of a_t.

To establish our main result in the most straightforward manner, we impose an assumption on initial conditions, in particular on how firms' initial prices and initial productivities are related. Similar conditions are imposed in sticky-price models with homogeneous firms, where it is routinely assumed that there is either no or a sufficiently small dispersion of initial prices, so that relative prices reflect relative productivities. We impose:

Condition 2 *Initial prices in* $t = -1$ *reflect firms' relative productivities, i.e.,*

$$P_{j,-1} \propto \frac{1}{Q_{-1-s_{j,-1}}G_{j,-1}} \qquad \textit{for all } j \in [0,1].$$

We discuss the effects of relaxing this condition below. The following proposition states our main result:

Proposition 2 *Suppose conditions 1 and 2 hold. The equilibrium allocation in the sticky-price economy is efficient if monetary policy implements the gross inflation rate*

$$\Pi_t^\star = \Xi_{t-1,t}^\star \left(\frac{1-\delta\left(\Delta_t^e\right)^{\theta-1}}{1-\delta} \right)^{\frac{1}{\theta-1}} \qquad \textit{for all } t \geq 0, \tag{34}$$

where $\Xi_{t-1,t}^\star$ *captures price indexation between periods* $t-1$ *and* t ($\Xi_{t-1,t}^\star \equiv 1$ *in the absence of indexation) and* Δ_t^e *is defined in equation (31) and evolves according to equation (32).*

The proof of proposition 2, which is contained in appendix D, establishes that with the optimal inflation rate, firms choose relative prices as in the flexible-price equilibrium. This result is established by showing that (1) firms hit by a δ-shock choose the same optimal relative price as in the flexible price economy, and that (2) firms hit by a Calvo shock optimally choose not to adjust their price, which avoids the emergence of price dispersion between otherwise identical firms. This, together with the fact that (3) initial prices reflect initial productivities, ensures that all relative prices are identical to those in the flexible-price equilibrium. Under the assumed output subsidy, it then follows that household allocations are also identical to the flexible-price equilibrium, which has been shown to be efficient (proposition 1).

It may be surprising that a single policy instrument, namely the aggregate inflation rate, can insure that the following two relative prices are optimal: the relative price between firms hit by δ-shocks and sticky-price firms, and the relative price between firms with a Calvo adjustment opportunity and sticky-price firms. To understand this result, note that the optimal price setting equation (12), which holds independently of the reason for why firms i and j can adjust prices, implies

$$\frac{P_{jt}^\star}{P_{it}^\star} = \frac{A_t Q_{t-s_{it}} G_{it}}{A_t Q_{t-s_{jt}} G_{jt}}.$$

This shows that all price-adjusting firms set prices such that relative prices amongst themselves reflect inversely relative productivities. Moreover, this holds true independently of the value of the inflation rate.[21] Therefore, the inflation rate can be used to insure that relative prices between the set of price-adjusting firms and the set of non-adjusting firms are optimal, thereby achieving efficient relative prices amongst all firms.

In the absence of price indexation ($\Xi_{t-1,t}^\star \equiv 1$), the optimal inflation rate $\Pi_t^\star$ is only a function of the variable Δ_t^e, which summarizes the distribution of relative productivities across firms, see equation (31). Since these relative productivities are independent of the common TFP growth rate a_t, it follows that the optimal inflation rate does not depend

[21] The fact that adjusting firms choose optimal relative prices amongst themselves is also a feature of homogeneous firm models. What we show here is that this generalizes to our setting with heterogeneous firms.

on the realizations of a_t. In contrast, the cohort productivity growth rate q_t and the experience growth rate g_t do affect Δ_t^e, see equation (32), albeit in opposite directions: a stronger cohort productivity growth rate q_t decreases the optimal inflation rate, while a stronger experience growth rate g_t increases the optimal inflation rate.

For the special case in which all firms have identical productivity trends ($\delta = 0$ or $g_t = q_t$ for all t) or even identical productivities ($\Delta_t^e = 1$), the optimal gross inflation rate is equal to one in the absence of price indexation, as in a standard homogeneous-firm model. Perfect price stability is then optimal at all times.

To understand the economic logic underlying equation (34), we use equation (32) to obtain the following alternative expression for the optimal inflation rate:

$$\Pi_t^\star = \Xi_{t-1,t}^\star \frac{a_t g_t \frac{A_{t-1}Q_{t-1}}{\Delta_{t-1}^e}}{\frac{A_t Q_t}{\Delta_t^e}}. \tag{35}$$

The denominator in the previous expression ($A_t Q_t / \Delta_t^e$) is the average productivity of all firms in the efficient allocation, see equation (24). The numerator is the average productivity of all firms except those that received a δ-shock in period t: their average productivity was $A_{t-1}Q_{t-1}/\Delta_{t-1}^e$ in period $t-1$ and has grown since by the aggregate TFP growth rate a_t and the experience growth rate g_t. The productivity of all firms is in turn a weighted sum of the productivity of firms with a δ-shock and firms without δ-shocks:

$$\frac{A_t Q_t}{\Delta_t^e} = \left(\delta \left(A_t Q_t\right)^{\theta-1} + (1-\delta)\left(a_t g_t \frac{A_{t-1}Q_{t-1}}{\Delta_{t-1}^e}\right)^{\theta-1}\right)^{\frac{1}{\theta-1}},$$

where $A_t Q_t$ denotes the productivity of firms with a δ-shock. The optimal inflation rate can thus be expressed as

$$\Pi_t^\star = \Xi_{t-1,t}^\star \frac{1}{\left(\delta \left(rp_t\right)^{\theta-1} + (1-\delta)\right)^{\frac{1}{\theta-1}}}, \tag{36}$$

where rp_t denotes the relative relative productivity between firms with and without a δ-shock and is given by

$$rp_t \equiv \frac{A_t Q_t}{a_t g_t \frac{A_{t-1}Q_{t-1}}{\Delta_{t-1}^e}}.$$

In the absence of price indexation ($\Xi_{t-1,t}^\star \equiv 1$), we thus have $\Pi_t^* = 1$ whenever this measure of relative productivity equals one, which is the case in a homogeneous firm model. If δ-shock firms are less productive ($rp_t < 1$), we have $\Pi_t^* > 1$. Conversely, if δ-shock firms are more productive ($rp_t > 1$), we have $\Pi_t^* < 1$.

Price indexation by non-adjusting firms ($\Xi_{t-1,t}^\star \neq 1$), say because of indexation to the lagged inflation rate, introduces additional components into the optimal aggregate inflation rate. In particular, it requires that price-adjusting firms, i.e., firms hit by either a δ-shock or a Calvo shock, also adjust their price by the indexation component. This way prices continue to accurately reflect relative productivities at all times. This explains why indexation affects the optimal inflation rate one-for-one in equations (34)-(36).

Although proposition 2 assumes that firms' initial prices accurately reflect the initial relative productivities, the initial productivity distribution itself is unrestricted. We conjecture that for a setting where condition 2 fails to hold, one would obtain additional transitory and deterministic components to the optimal inflation rate, as in the homogeneous firm setting studied by Yun (2005). The inflation rate stated in proposition 2 would then become optimal only asymptotically.

Interestingly, it follows from the proof of proposition 2 that the inflation rate (34) continues to ensure productive efficiency (but not full efficiency) in settings where condition 1 fails to hold. From the theory of optimal taxation it then follows that it remains optimal to implement the inflation rate (34), as it is suboptimal to distort intermediate production as long as (distortionary) taxes on final goods are available.

8 The Optimal Steady-State Inflation Rate

This section discusses the optimal steady-state inflation rate implied by the model. To simplify the discussion, we initially abstract from price indexation.

Proposition 2 makes it clear that in the case in which the productivity of all firms grows at the same rate ($\delta = 0$), which includes as a special case the setting with homogeneous firms, we obtain $\Pi_t^\star = 1$. For $\delta = 0$, the optimal (gross) steady-state inflation rate is thus trivially equal to one, independently of the values assumed by (a, g, q).

For the case $\delta > 0$, the optimal steady-state inflation rate jumps discontinuously away from $\Pi_t^\star = 1$, with the optimal steady-state inflation rate being itself independent of the value of $\delta > 0$.[22] The following lemma summarizes this result:

Lemma 1 *Suppose conditions 1 and 2 hold, there are no economic disturbances, there is no price indexation ($\Xi_{t-1,t}^\star \equiv 1$) and $\delta > 0$. The optimal inflation rate then satisfies*

$$\lim_{t\to\infty} \Pi_t^\star = g/q. \tag{37}$$

Proof. From equations (6) and (32) it follows that $\lim_{t\to\infty} (\Delta_t^e)^{\theta-1} = [1-(1-\delta)\,(g/q)^{\theta-1}]/\delta$. It then follows from proposition 2 that $\lim_{t\to\infty} \Pi_t^\star = g/q$. ■

Since we allow for arbitrary initial productivity distributions, the absence of shocks does not necessarily imply that the optimal inflation rate is constant from the beginning.

[22] Note that the efficient allocation also discontinuously jumps when moving from $\delta = 0$ to $\delta > 0$, as in the former case efficient aggregate growth is equal to $(ag)^\phi$ and in the latter case it is equal to $(aq)^\phi$ in steady state. Appendix E shows that the discontinuity of the optimal steady-state inflation rate is not due to the discontinuity of the associated aggregate allocation.

This only happens asymptotically, once the productivity distribution converges to its stationary distribution (in detrended terms).[23] The lemma provides the inflation rate that is asymptotically optimal as this stationary distribution is reached.[24]

Surprisingly, the optimal long-run inflation rate is completely independent of the intensity of δ-shocks. To understand the source of this invariance, consider a setting where δ-shocks capture firm exit and entry events and where $g > q$, so that newly entering firms are less productive than the set of continuing old firms. A higher value for δ implies that more young and relatively unproductive firms are amongst the set of price-setting firms. This calls - ceteris paribus - for a higher inflation rate. Yet, the productivity distribution of continuing old firms is not invariant to changes in δ: a higher δ also implies more firm turnover and thus less experience accumulation. Continuing old firms thus tend to be less productive relative to new entrants, which calls for lower inflation rates. In net terms, these two effects exactly cancel each other.

On empirical grounds, it appears plausible to assume $g > q$ so that the optimal steady-state inflation rate from lemma 1 is positive. For the case where δ-shocks capture firm turnover, one obtains $g > q$ from the fact that young firms are small relative to old firms, see also section 10 below. Likewise, interpreting δ-shocks as representing product substitution shocks, $g > q$ implies that new products are more expensive than the average product and that their relative price is falling over the product life cycle. Both of these facts are in line with evidence provided by Melser and Syed (2016). Thus, while the theoretical setup allows the optimal steady-state inflation rate to be potentially negative, these empirical considerations suggest positive inflation to be optimal in steady state.

Interestingly, aggregate productivity dynamics turn out not to be informative about the optimal inflation rate. The aggregate steady-state growth is equal to $(aq)^{\phi}$ and is driven by a factor that affects the optimal inflation rate (q) and a factor that does not affect it (a). Moreover, the experience effect (g) has no aggregate growth rate implications, but affects the optimal inflation rate. Determining the optimal inflation rate thus requires either studying the firm-level productivity trends g and q or the relative productivities of old versus new firms, see equations (35) and (36). We shall come back to this issue in the empirical section 10.

Finally, we discuss the effects of price indexation on the optimal steady state inflation rate. For $\delta > 0$ the optimal long-run inflation rate is then given by $\Xi^{\star}(g/q)$. For the case where prices are indexed to lagged inflation according to $\Xi^{\star}_{t-1,t} = \left(\Pi^{\star}_{t-1}\right)^{\kappa}$ for some $\kappa \in [0, 1)$, we thus obtain

$$\lim_{t\to\infty} \Pi^{\star}_t = (g/q)^{\frac{1}{1-\kappa}} .$$

[23]When $\delta = 0$, the initial distribution remains unchanged (in detrended terms).

[24]The transitional dynamics can easily be derived from proposition 2 using the initial productivity distribution and equation (32).

Standard forms of price indexation thus amplify the divergence of the optimal gross inflation rate from one.

9 The Welfare Costs of Strict Price Stability

This section shows that suboptimally implementing strict price stability, as suggested by sticky-price models with homogeneous firms, gives rise to strictly positive welfare costs whenever $g \neq q$. We derive this fact first analytically for a special case, as this allows considering the limit $\delta \to 0$. In a second step, we illustrate the source of the welfare losses and their magnitude numerically.

The following proposition shows that - as long as $g \neq q$ - there is a strictly positive welfare loss that is bounded away from zero when implementing strict price stability; this holds true even for the limit $\delta \to 0$. The proof of the proposition is contained in appendix F.

Proposition 3 *Suppose conditions 1 and 2 hold, there are no economic disturbances, $\delta > 0$, fixed costs of production are zero ($f = 0$), there is no price indexation ($\Xi^{\star}_{t-1,t} \equiv 1$), and the disutility of work is given by*

$$V(L) = 1 - \psi L^{\nu},$$

with $\nu > 1$ and $\psi > 0$. Assume $g/q > \alpha(1-\delta)$, so that a well-defined steady state with strict price stability exists.

Consider the limit $\beta(\gamma^{e})^{1-\sigma} \to 1$ and a policy implementing the optimal inflation rate $\Pi^{\star}_t$ from proposition 2, which satisfies $\lim_{t\to\infty} \Pi^{\star}_t = \Pi^{\star} = g/q$. Let $c(\Pi^{\star})$ and $L(\Pi^{\star})$ denote the limit outcomes for $t \to \infty$ for consumption and hours worked, respectively, under this policy. Similarly, let $c(1)$ and $L(1)$ denote the limit outcomes under the alternative policy of implementing strict price stability. Then,

$$L(1) = L(\Pi^{\star})$$

and

$$\frac{c(1)}{c(\Pi^{\star})} = \left(\frac{1-\alpha(1-\delta)(g/q)^{\theta-1}}{1-\alpha(1-\delta)}\right)^{\frac{\phi\theta}{\theta-1}} \left(\frac{1-\alpha(1-\delta)\,(g/q)^{-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\phi} \leq 1. \tag{38}$$

For $g \neq q$ the previous inequality is strict and $\lim_{\delta\to 0} c(1)/c(\Pi^{\star}) < 1$.

We now illustrate the nature of the relative price distortions that are generated by a suboptimal rate of inflation and how they give rise to welfare losses. Panel A in figure 2 reports the mean cohort price (relative to the price of all firms), depicted on the y-axis, as a function of the cohort age in quarters (x-axis). It does so once for a setting where monetary policy implements the optimal inflation rate, which for illustration is assumed

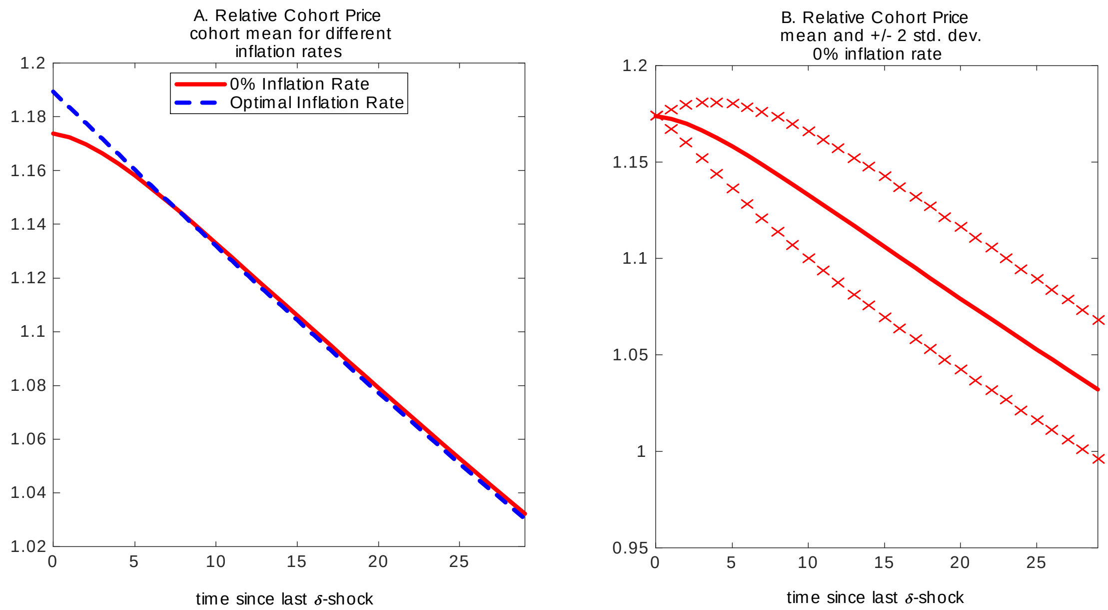

Figure 2: Relative prices and inflation

to be 2% per year (in net terms), and once where monetary policy pursues strict price stability.[25] Panel A shows that young cohorts charge a higher (relative) price and that this price decreases over the lifetime of the cohort. Under the optimal inflation rate, the decline happens at a constant rate.[26] Under strict price stability, however, firms anticipate that their relative prices will not necessarily fall, due to Calvo price stickiness. This causes them to initially "front load" prices, i.e., in an environment with strict price stability young cohorts charge initially lower prices than under the optimal inflation rate. Over time, some firms in the cohort will get the opportunity to lower their prices in response to Calvo shocks, but the average relative price of the cohort will eventually be slightly higher than under the optimal inflation rate. Beyond these distortions in average cohort prices, the suboptimal inflation rate also generates price distortions within a cohort of firms. This is illustrated in panel B of figure 2. Panel B reports the mean cohort price and the two standard deviation bands of the cross-sectional price distribution within the cohort, assuming monetary policy pursues strict price stability. It shows that suboptimal inflation not only gives rise to distortions in the mean price but also to substantial amounts of price dispersion within the cohort. In contrast, price dispersion at the cohort level is zero under the optimal inflation rate.

Figure 3 reports the steady-state value for the ratio Δ_t^e/Δ_t (y-axis) as a function

[25] Figure 2 is computed using $g = 1.02^{0.25}, q = 1, \alpha = 0.75, \delta = 0.035, \theta = 3.8$ and assumes that the initial productivity distribution is equal to the stationary distribution (in detrended terms).

[26] The figure assumes that no shocks hit the economy.

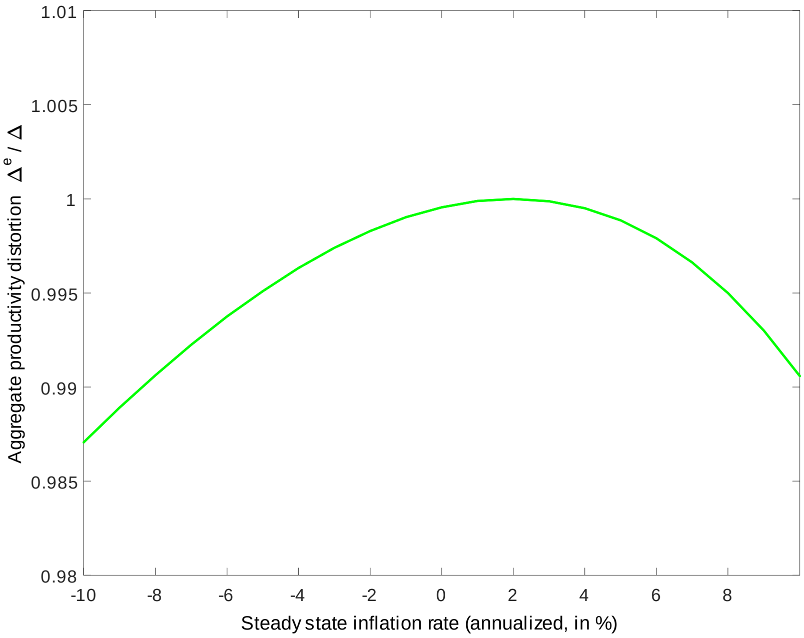

Figure 3: Aggregate productivity for different inflation rates (optimal rate is 2%)

of the implemented steady-state inflation rate (x-axis), when the optimal inflation rate is 2% per year.[27] The aggregate production function (24) shows that one can interpret Δ_t^e/Δ_t as a measure of the aggregate productivity distortion that is implied by the relative price distortions associated with suboptimal inflation rates.[28] The figure shows that a 10 percentage point shortfall of the inflation rate below its optimal value of 2% is associated with an aggregate productivity loss equal to about 1%. In the process, the productivity losses arise rather nonlinearly: a shortfall of inflation of 2 percentage points below its optimal value is associated with an aggregate productivity loss of just 0.05%. Furthermore, inflation losses are asymmetric, with above-optimal inflation leading to relatively larger losses. For instance, increasing inflation 8 percentage points above its optimal value generates a productivity loss of 0.94%, while decreasing inflation by the same amount below its optimal value leads to a productivity losses of only 0.37%.

10 The Optimal Inflation Rate for the United States

Using the theory developed in the preceding sections, we now quantify the optimal inflation rate for the United States. In doing so, we interpret δ-shocks as events in which production establishments are closed down and replaced by new establishments, in line

[27] The figure is based on the same parameterization as figure 2.

[28] Appendix F shows that $c(1)/c(\Pi^\star) = (\Delta^e/\Delta)^\phi$.

with the "firm entry and exit" interpretation spelled out in section 3.2. This interpretation has the advantage that one can use readily available establishment-level information to estimate the optimal inflation rate.

An alternative approach for estimating the optimal inflation rate could make use of even more disaggregate product-level information, which amounts to interpreting δ-shocks as events in which old products (old quality levels) are replaced by new products (new quality levels). Our interpretation of δ-shocks as establishment entry and exit shocks does not preclude that on top of the considered establishment dynamics, additional product substitution dynamics are present at the establishment level. For instance, it would be relatively straightforward to enrich our framework by adding a Poisson shock, which causes establishments to occasionally replace their current product by a new product. These additional product substitutions at the establishment level could also be associated with additional idiosyncratic adjustments to establishment productivity (or product quality).[29] Provided the establishment can again freely choose the prices of newly introduced products, all our theoretical results about the optimal inflation rate remain unchanged. The empirical strategy for estimating the optimal inflation rate presented below would equally remain unaffected. In light of this, we feel comfortable with our interpretation of δ-shocks as establishment entry and exit events.

Let $\overline{L}_t$ denote the average employment per establishment in period t and let $\overline{L}_t^c$ denote the average employment of continuing establishments, i.e., all establishments except the ones that newly entered in period t. The following proposition shows how the optimal inflation rate $\Pi_t^\star$ can be inferred from these employment measures:

Proposition 4 *Suppose conditions 1 and 2 hold and that there are no fixed costs in production ($f = 0$). Suppose monetary policy implements the potentially suboptimal inflation rate Π_t and firms' prices are indexed according to $\Xi_{t-1,t}$ ($\Xi_{t-1,t} \equiv 1$ in the absence of price indexation). The optimal inflation rate net of price indexation $\Pi_t^\star/\Xi_{t-1,t}^\star$ ($\Xi_{t-1,t}^\star \equiv 1$ in the absence of price indexation) then satisfies*

$$\left(\frac{\Delta_t}{\Delta_t^e}\right)^{-1}\left(\frac{1-\alpha(1-\delta)\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta-1}}{1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}} = \frac{1-(1-\delta)\overline{L}_t^c/\overline{L}_t}{1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}} \quad \text{for } t \geq 0, \tag{39}$$

[29] For example, equation (2) could be generalized to

$$Y_{jt} = A_t Z_{jt} \widetilde{Z}_{jt}\left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t\right),$$

where $\widetilde{Z}_{jt}$ is the productivity component that is specific to the product currently produced by establishment j, with $\widetilde{Z}_{jt} = \widetilde{Z}_{jt-1}$ whenever the product is not substituted and $\widetilde{Z}_{jt}$ being an iid draw from some stationary distribution with $E[1/\widetilde{Z}_{jt}] = 1$, whenever there is a Poisson event indicating a product substitution.

where Δ_t/Δ_t^e *evolves recursively according to*

$$\frac{\Delta_t}{\Delta_t^e} = \left[1 - \alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}\right]\left(\frac{1-\alpha(1-\delta)\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta-1}}{1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}$$
$$+ \alpha(1-\delta)\left(\frac{\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta}}{\Pi_t^\star/\Xi_{t-1,t}^\star}\right)\frac{\Delta_{t-1}}{\Delta_{t-1}^e}, \tag{40}$$

with $\Delta_{-1}/\Delta_{-1}^e = 1$.

The proof of proposition 4 can be found in appendix G. Proposition 4 shows how one can determine the optimal inflation path $\Pi_t^\star$ for a sticky-price economy that is subject to stochastic disturbances and in which monetary policy implements potentially suboptimal inflation rates. In particular, equations (39) and (40) allow to infer $\Pi_t^\star/\Xi_{t-1,t}^\star$ for all $t \geq 0$, given values for the parameters (α, δ, θ), the price indexation rule Ξ, the observed actual inflation rate Π_t and the employment ratio $\overline{L}_t^c/\overline{L}_t$.

An instructive special case of proposition 4 arises when monetary policy is optimal ($\Pi_t/\Xi_{t-1,t} = \Pi_t^\star/\Xi_{t-1,t}^\star$ for all t). We then have $\Delta_t = \Delta_t^e$ for all t and equation (39) directly yields

$$\Pi_t^\star = \Xi_{t-1,t}^\star\left(\frac{\overline{L}_t^c}{\overline{L}_t}\right)^{\frac{1}{\theta-1}}. \tag{41}$$

For this special case, the optimal inflation rate is then only a function of price indexation ($\Xi_{t-1,t}^\star$), the ratio of average employment of continuing establishments ($\overline{L}_t^c$) over the average employment of all establishments ($\overline{L}_t$), and the demand elasticity parameter θ. This is closely related to our theoretical result in equation (35), which shows that the optimal inflation rate depends on price indexation ($\Xi_{t-1,t}^\star$) and the ratio of average productivity of continuing firms ($a_t g_t A_{t-1} Q_{t-1}/\Delta_{t-1}^e$) over the average productivity of all firms ($A_t Q_t/\Delta_t^e$). The special case with optimal monetary policy thus illustrates how our estimation approach uses establishment employment information together with information about the demand elasticity θ to determine establishment productivity.[30] In fact, the employment ratio in equation (41) raised to the power $1/(\theta-1)$ is equal to the productivity ratio showing up in equation (35). This holds true because productivity differences translate into price differences, which in turn translate into employment differences, as determined by the elasticity of product demand θ.

[30] It may seem desirable to directly estimate firm-level productivities. It is, however, difficult to measure physical productivity at the firm or establishment level because output prices are typically not observed at this level of observation. As is explained in Foster, Haltiwanger and Syverson (2008), the productivity literature usually measures revenue productivity instead of physical productivity at the firm level, which deflates firm-level output with some industry-level price index. In our setting, firms' revenue productivity is completely unrelated to their physical productivity in the absence of fixed costs of production. For the few industries for which physical and revenue productivities can both be observed, the two productivity measures can be rather different; see Foster, Haltiwanger and Syverson (2008).

For the more general case with suboptimal monetary policy, the expression on the left-hand side of equation (39) starts to generally differ from one. In fact, as a result of inefficient price dispersion, aggregate productivity ($A_t Q_t/\Delta_t$) will fall short of the value it assumes in the efficient allocation ($A_t Q_t/\Delta_t^e$), so that $1/\Delta_t \leq 1/\Delta_t^e$. The evolution of the productivity wedge Δ_t/Δ_t^e under suboptimal policies is captured by equation (40).

10.1 Baseline Estimation

We estimate the optimal inflation rate using the result derived in proposition 4 and the baseline parameters reported in table 1. The parameters refer to a sticky-price model at annual frequency because the employment ratio $\overline{L}_t^c/\overline{L}_t$ is observed at annual frequency. The price stickiness parameter satisfies $\alpha \approx (0.55)^4$, where 0.55 is the baseline value chosen in the quarterly sticky-price model of Coibion, Gorodnichenko and Wieland (2012).[31] The probability for δ-shocks is set to 11.5% per year, which is the midpoint between the average establishment birth rate (12.4%) and the average establishment exit rate (10.7%) over the period 1977 to 2015 reported in the Business Dynamics Statistics (BDS) of the U.S. Census Bureau. We set the elasticity of product demand θ equal to 7, which is the baseline value considered in Coibion, Gorodnichenko and Wieland (2012) and Gorodnichenko and Weber (2016). It implies a steady state mark-up of around 17% over production costs. In general, we consider price indexation rules that index prices to lagged inflation according to

$$\Xi_{t-1,t} = (\Pi_{t-1})^{\kappa},$$

for some $\kappa \in [0, 1)$. For our baseline estimation, we consider a setting without price indexation.

Parameter		Assigned value
Price stickiness	α	0.0915
δ-shock probability	δ	11.5%
Demand elasticity	θ	7
Price indexation	κ	0

Table 1: Baseline parameters (annual model)

We determine the employment ratio $\overline{L}_t^c/\overline{L}_t$ using information from the BDS. The BDS is based on the Longitudinal Business Database (LBD) and provides information on the

[31] The implied price duration is nevertheless larger than in the quarterly model of Coibion, Gorodnichenko and Wieland (2012). Section 10.2 shows that our quantitative results are robust to making prices less sticky.

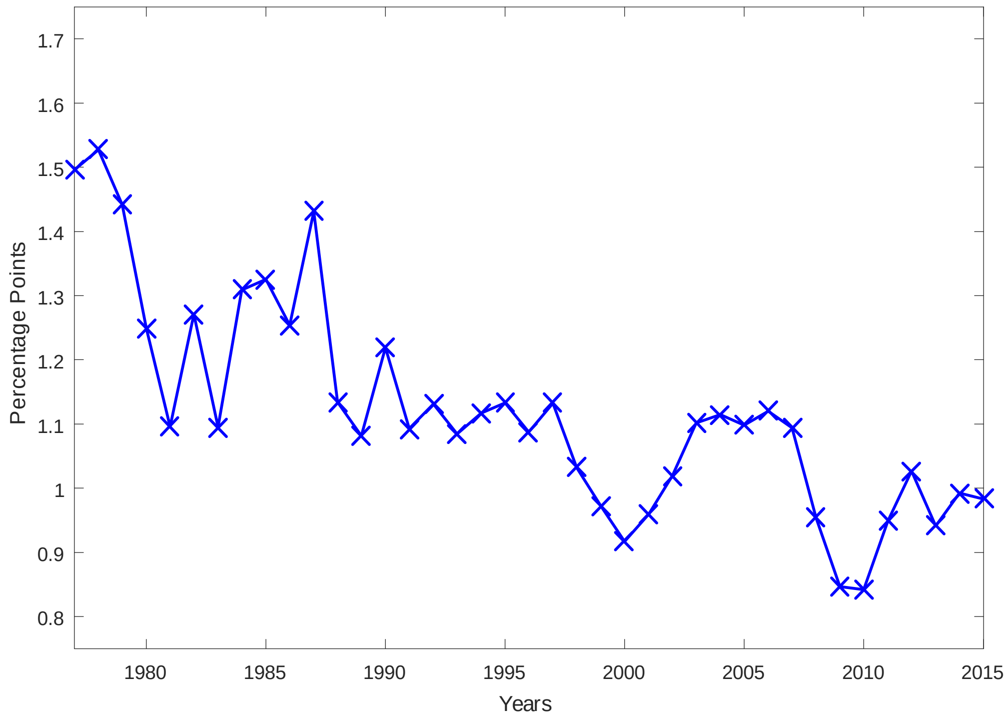

Figure 4: Optimal U.S. inflation $\Pi_t^{\star}$, baseline estimation

number of establishments and establishment employment by establishment age. The database covers the universe of private establishments in the United States for the years 1977-2015. We compute $\overline{L}_t$ by dividing the reported "economy-wide employment" in any year t by the "economy-wide number of establishments" of the year. Similarly, we compute the average employment of continuing establishments $\overline{L}_t^c$ by first subtracting "jobs created by establishment birth over the last 12 months" from "economy-wide employment" and then dividing the result by the number of continuing establishments, which is equal to the "economy-wide number of establishments" minus the number of "establishments born during the last 12 month". We use the GDP deflator provided by the U.S. Bureau of Economic Analysis (GDPDEF_PC1) as our measure for the actual inflation rate. Results are virtually identical when using other measures of actual inflation.

Figure 4 depicts the optimal inflation rate implied by proposition 4 and our baseline calibration. The estimated optimal inflation rate is positive throughout the sample period. This is in line with the empirical observation that older establishments employ (on average) more workers, which means that they are more productive in our framework ($g_t/q_t > 1$). The sample mean of the optimal inflation rate is equal to 1.1% per year, which is a relatively large number within the sticky-price literature.[32] Coibion, Gorod-

[32] See figure 1 in Diercks (2017), which depicts the optimal average inflation rate found in 100 quanti-

nichenko and Wieland (2012), for instance, find similarly high optimal average inflation rate (of around 1.5%) using a model featuring a zero lower bound on nominal interest rates and a monetary authority that follows a Taylor rule.[33]

As is evident from Figure 4, the optimal inflation rate displays a slight downward trend over time. Especially from 1988 onwards, the optimal inflation rate seems to have dropped. While the average over the period 1977-1987 was 1.3%, the average over the remaining sample period was 1.0%. According to the model, this drop in the estimated optimal inflation rate implies that either the experience growth rate g_t has weakened and/or that the cohort growth rate q_t has accelerated. Both of these effects cause older establishments to become smaller relative to new establishments ($\overline{L}_t^c/\overline{L}_t$ drops). Independent evidence on firm employment growth provided in figure 11 in Pugsley, Sedlacek and Sterk (2017) shows indeed that the employment growth rate of cohorts that entered after the year 1987 has slowed significantly, which is consistent with the slight downward trend in the optimal inflation rate showing up in figure 4.

10.2 Robustness of the Empirical Results

We now explore the robustness of our baseline estimation from the previous section. We consider the effects of choosing alternative parameter values for $(\alpha, \delta, \theta, \kappa)$, as well as the effects of positive fixed costs in production ($f \geq 0$).

A first set of results is presented in figure 5. Panel A in the figure depicts the optimal inflation estimate when assuming flexible prices ($\alpha = 0$) and shows that the benchmark results remain virtually unchanged. Panel B depicts the effects of assuming alternative establishment turnover rates. As is well known, the turnover rate has significantly dropped over the considered sample period. The value of $\delta = 0.15$ corresponds to the average establishment entry and exit rate at the beginning of the sample period, while the lower value of $\delta = 0.095$ is the one observed at the end of the sample period. The figure shows that alternative turnover rates within this range do not generate quantitatively significant effects.

Panel C in figure 5 shows the effects of assuming a lower value for the demand elasticity θ. While macroeconomists tend to use high demand elasticities based on the observation that pure profits tend to be low, see Basu and Fernald (1997), the trade and industrial

tative optimal monetary policy studies.

[33]The authors shows that for fully optimal monetary policy, as considered in the present paper, the average optimal inflation rate falls to 0.2% per year in their setting. The large drop in average inflation arises because Taylor rules are severely suboptimal in the vicinty of the strong non-linearities induced by a lower bound constraint. The associated welfare costs can then be reduced by increasing the intercept term of the Taylor rule. In our setup, Taylor rules with optimal intercepts are approximately optimal and lead to a very similar average inflation rate as in a setting with fully optimal monetary policy. This is so because the welfare costs of inflation are relatively symmetric around the optimum, see figure 3.

organization literature usually estimates lower demand elasticity values.[34] The value of $\theta = 3.8$ considered in figure 5 is taken from Bilbiie et al. (2012) and Bernard et al. (2003) and is based on a calibration that fits U.S. plant and macro trade data. The figure shows that the optimal inflation rate then approximately doubles. This increase is driven by the fact that with a lower demand elasticity, any given employment ratio $\overline{L}_t^c/\overline{L}_t$ observed in the data must be associated with larger establishment-level productivity differences (a larger gap between g_t and q_t).

Panel D in figure 5 considers a setting in which prices of non-optimizing firms are indexed to lagged inflation. We choose $\kappa = 1/2$, which Coibion, Gorodnichenko and Wieland (2012) consider to be an upper bound on the range of plausible values for this parameter. Again, this causes the optimal inflation rate to approximately double compared to the baseline estimate.

Since proposition 4 assumes that there are no fixed costs in production ($f = 0$), we also consider the effects of allowing for positive fixed costs. Appendix H shows that the presence of fixed costs tends to slightly increase the estimated optimal inflation rates, but the effect is quantitatively small.

11 Robustness of the Theoretical Results

This section considers various extensions and alternative model setups. Section 11.1 shows that our main finding about the optimal inflation rate (proposition 2) continues to apply in a setting where price adjustment frictions take the form of menu costs. Section 11.2 discusses the effects of a non-constant δ-shock hazard rate, while section 11.3 analyzes the case with a price index which oversamples old products.

11.1 Menu Cost Frictions

While our results are illustrated using time-dependent price-setting frictions, our theoretical finding from proposition 2 extends to a setting in which firms optimally decide to pay a fixed cost to adjust their price. Since the optimal inflation rate in proposition 2 replicates the flexible-price allocation, firms that do not experience a δ-shock have - independently of the nature of their price setting frictions - no incentives to adjust their prices. Since the flexible price allocation is efficient, see proposition 1, monetary policy also has no incentive to deviate from the flexible-price allocation. Both observations together imply that the optimal inflation rate does not depend on whether price-setting frictions are state or time dependent.[35]

[34] These lower elasticities are consistent with pure profits being low if there are fixed costs of production.

[35] Obviously, this requires that, in a setting with menu cost frictions, δ-shocks lead either to these menu cost not having to be paid at all or always having to be paid.

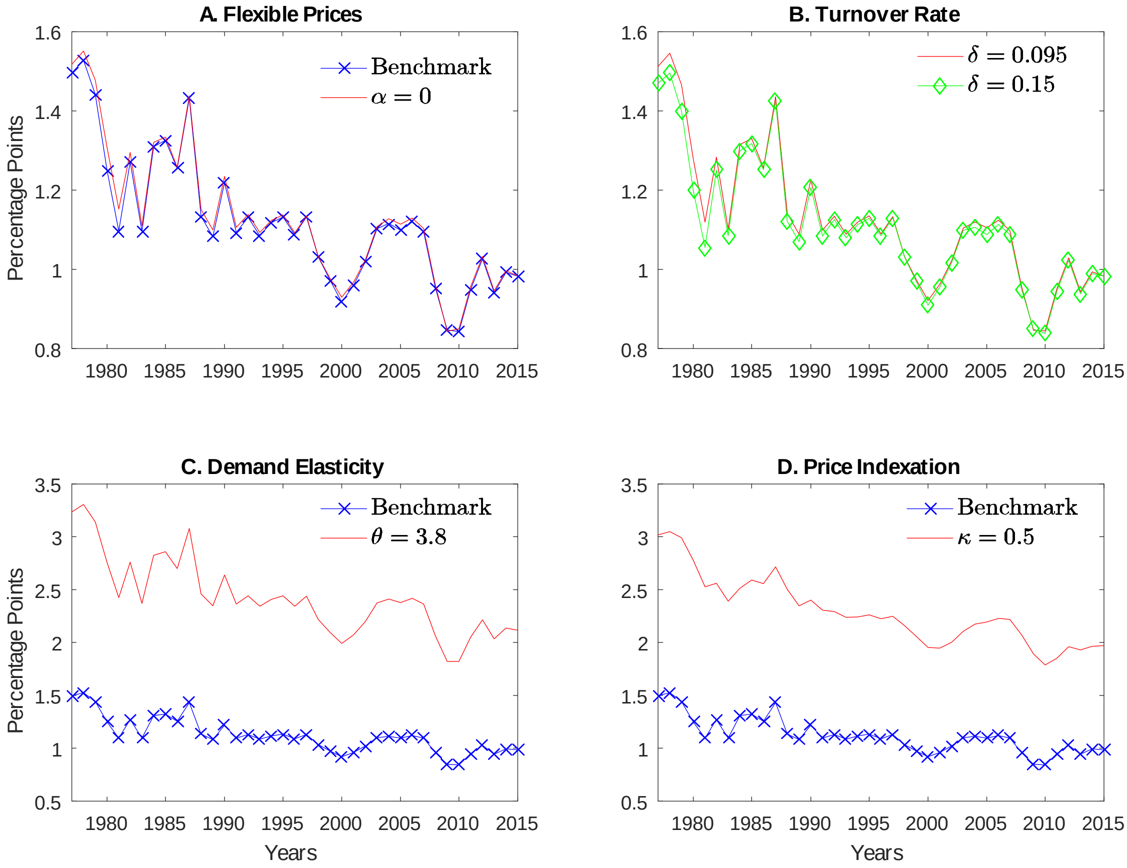

Figure 5: Optimal inflation for the United States, alternative parameter assumptions

11.2 Non-Constant δ-Shock Probability

Throughout the paper we assume that firms face a constant probability for receiving a δ-shock. While analytically convenient, it may on empirical grounds be attractive to consider either increasing or decreasing hazard rate specifications. A decreasing hazard, for instance, can capture the fact that young firms might face higher exit rates. Conversely, increasing hazards may capture the fact that products become increasingly likely to be substituted as they age. To assess how our baseline results are affected by non-constant hazard rates, we consider a setting where firms receive a δ-shock with probability δ_1 in the first period after having received a δ-shock but with probability δ subsequently. We allow for increasing hazard rates ($\delta_1 < \delta$) and decreasing hazard rates ($\delta_1 > \delta$). The following proposition states our main finding:

Proposition 5 *Suppose conditions 1 and 2 hold. Let $\delta_1 \in [0, 1)$ denote the probability of receiving a δ-shock for firms that received a δ-shock in the previous period and let $\delta \in (0, 1)$ denote the probability of receiving a δ-shock for all other firms. The optimal inflation rate is then given by*

$$\Pi_t^\star = \Xi_{t-1,t}^\star \frac{a_t g_t \frac{A_{t-1}Q_{t-1}}{\Delta_{t-1}^e}}{\frac{A_t Q_t}{\Delta_t^e}}, \tag{42}$$

where Δ_t^e evolves according to

$$(\Delta_t^e)^{1-\theta} = \delta_0 - \delta_0(\delta_1 - \delta)\left(q_t/g_t\right)^{1-\theta} + (1-\delta)\left(\Delta_{t-1}^e q_t/g_t\right)^{1-\theta}, \tag{43}$$

with $\delta_0 \equiv \delta/(1 - \delta_1 + \delta)$ denoting the total mass of firms receiving δ-shocks in a given period. In the absence of economic disturbances and price indexation ($\Xi_{t-1,t}^\star \equiv 1$), we have

$$\lim_{t\to\infty} \Pi_t^\star = g/q.$$

The proof of the proposition is contained in appendix I. Equation (42) shows that the optimal inflation rate is determined by the ratio of the average productivity of firms without a δ-shock ($a_t g_t A_{t-1}Q_{t-1}/\Delta_{t-1}^e$) over the average productivity of all firms ($A_t Q_t/\Delta_t^e$), adjusted for the possible presence of price indexation ($\Xi_{t-1,t}^\star$). This is identical to the setting with a constant hazard rate, see the discussion following our main result in proposition 2. The only difference relative to the case with a constant hazard rate is that the recursive equation describing the evolution of Δ_t^e generalizes from equation (32) to equation (43). The optimal steady state inflation rate remains nevertheless unaffected by non-constant hazard rates.

11.3 Bias Towards Older Goods in the Measured Price Index

Throughout the paper, we consider a price index capturing all products available at any given point in time. This idealized price index differs from how statistical agencies tend to compute price indices in practice. In particular, once products become unavailable, the statistical agencies tend to replace them by products that have been around for some time. This generates a bias towards older goods in the basket underlying actual price indices.[36] Since inflation/deflation is optimal in the present setup because of the entry of new products (or firms or qualities), this raises the question whether oversampling of older products drives a wedge between the optimal inflation rate $\Pi_t^\star$ based on the ideal price index and the optimal inflation rate for a price index biased in favor of older goods. To assess this question, we consider an inflation rate Π_t^N that is based on a price index containing only products of "age" $N \geq 1$ or higher. The following proposition summarizes our main result:

Proposition 6 *Suppose conditions 1 and 2 hold and the measured price index features an age bias in the sense that it includes only products that received their last δ-shock at least $N \geq 1$ periods ago. Let Π_t^N denote the inflation rate associated with this price index. The equilibrium allocation in the sticky-price economy is efficient if monetary policy implements the gross inflation rate*

$$\Pi_t^{N\star} = \frac{\Xi_{t-1,t}^\star}{\Xi_{t-N-1,t-N}^\star}\Pi_{t-N}^\star \text{ for all } t \geq N,$$

where $\Pi_t^\star$ denotes the optimal inflation rate for the ideal price index, as stated in proposition 2, and $\Xi^\star$ captures price indexation ($\Xi^\star = 1$ in the absence of indexation).

The proof is contained in appendix J. The proposition shows that in the absence of price indexation ($\Xi_{t-1,t}^\star = 1$ for all t), the optimal inflation rate for an age-biased price index equals the N-period lagged optimal inflation rate for the ideal price index. This is so because it takes N periods for products to enter the statistical agency's product basket.[37] Naturally, the presence of price indexation adds some additional time shifters ($\Xi_{t-1,t}^\star/\Xi_{t-N-1,t-N}^\star$). It follows from the proposition that the optimal steady-state inflation rate is not affected by the presence of an age bias. In the absence of shocks and in the absence of price indexation, we have $\lim_{t\to\infty}\Pi_t^{N\star} = \lim_{t\to\infty}\Pi_t^\star = g/q$.[38]

12 Conclusions

This paper shows how firm-level productivity trends affect the inflation rate that is optimal for the aggregate economy and that the effect of firm-level trends on the optimal

[36] See the discussion on product substitutions in section III.C in Nakamura and Steinsson (2008).

[37] As a result, the optimal inflation rate $\Pi_t^{N\star}$ for the initial periods $t = 0, ..., N-1$ is a function of the initial distribution of prices prevailing at $t = -1$ under optimal monetary policy.

[38] More generally, for the case with price indexation we have $\lim_{t\to\infty}\Pi_t^{N\star}/\Xi_{t-1,t}^\star = \lim_{t\to\infty}\Pi_t^\star/\Xi_{t-N-1,t-N}^\star$. If price indexation does not depend on the inflation measure targeted by the central bank, then we have again $\lim_{t\to\infty}\Pi_t^{N\star} = \lim_{t\to\infty}\Pi_t^\star$.

inflation rate is quantitatively large by the standards of the sticky price literature. Indeed, a puzzling feature of this literature is the considerable gap between the inflation rates it suggests to be optimal, which are typically close to zero or even negative, and the significantly positive inflation targets pursued by leading central banks around the globe. The present paper thus contributes to bridging this gap by providing theoretical underpinnings for current central bank practice.

The paper also opens a range of interesting avenues for further research. In light of the present finding, it appears of interest to understand firm-level productivity trends, including their changes over time, in greater detail and across a larger set of economies. This would allow assessing to what extent the observed downward trend in global inflation rates is fundamentally justified.

It also appears interesting to incorporate firm-level trends into richer sticky-price setups that feature characteristics that the present analysis abstracted from, for instance, a set of different economic sectors and associated input-output linkages. Quantitatively studying such richer frameworks would allow assessing whether or not these characteristics amplify or dampen the effects of firm-level trends on the optimal inflation rate. Finally, it appears to be of interest to incorporate firm-level trends into linear-quadratic formulations of optimal monetary policy problems, so as to study how the optimal policy response to cost-push disturbances is affected by the presence of firm-level trends.

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Appendix

A Derivation of the Sticky-Price Economy

A.1 Cost Minimization Problem of Firms

The cost minimization problem of firm j,

$$\min_{K_{jt},L_{jt}} K_{jt}r_t + L_{jt}W_t/P_t \quad s.t. \quad Y_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

yields the first-order conditions

$$0 = r_t + \left(1 - \frac{1}{\phi}\right) \lambda_t A_t Q_{t-s_{jt}} G_{jt} \left(\frac{L_{jt}}{K_{jt}}\right)^{\frac{1}{\phi}}$$
$$0 = W_t/P_t + \frac{1}{\phi} \lambda_t A_t Q_{t-s_{jt}} G_{jt} \left(\frac{L_{jt}}{K_{jt}}\right)^{\frac{1}{\phi}-1},$$

where λ_t denotes the Lagrange multiplier. The first-order conditions imply that the optimal capital labor ratio is the same for all $j \in [0, 1]$, i.e.,

$$\frac{K_{jt}}{L_{jt}} = \frac{W_t}{P_t r_t}(\phi - 1).$$

Plugging the optimal capital labor ratio into the technology of firm j and solving for the factor inputs yields the factor demand functions

$$L_{jt} = \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}-1} I_{jt} \tag{44}$$

$$K_{jt} = \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}} I_{jt}. \tag{45}$$

Firm j demands these amounts of labor and capital, respectively, to combine them to I_{jt}, which yields Y_{jt} units of output. Accordingly, the firm's cost function to produce I_{jt} is

$$MC_t I_{jt} = W_t \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}-1} I_{jt} + P_t r_t \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}} I_{jt}, \tag{46}$$

where MC_t denotes nominal marginal (or average) costs. This equation can be rearranged to obtain equation (10) in the main text.

A.2 Price-Setting Problem of Firms

The price-setting problem of the firm j, see equation (11), implies that the optimal product price is given by

$$P_{jt}^{\star} = \left(\frac{\theta}{\theta - 1}\frac{1}{1+\tau}\right) \frac{E_t \sum_{i=0}^{\infty}(\alpha(1-\delta))^i \Omega_{t,t+i} Y_{t+i} \left(\Xi_{t,t+i}/P_{t+i}\right)^{-\theta} \frac{MC_{t+i}/P_{t+i}}{A_{t+i}Q_{t-s_{jt}}G_{jt+i}}}{E_t \sum_{i=0}^{\infty}(\alpha(1-\delta))^i \Omega_{t,t+i} Y_{t+i} \left(\Xi_{t,t+i}/P_{t+i}\right)^{1-\theta}}. \tag{47}$$

Rewriting this equation yields

$$\frac{P_{jt}^{\star}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)$$
$$=\left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{-\theta}\frac{MC_{t+i}}{P_{t+i}A_{t+i}Q_{t+i}}\frac{Q_{t+i}/Q_t}{G_{jt+i}/G_{jt}}}{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{1-\theta}}. \quad (48)$$

The multi-period growth rate of the cohort effect relative to the experience effect corresponds to

$$\frac{Q_{t+i}/Q_t}{G_{jt+i}/G_{jt}}=\frac{q_{t+i}\times\cdots\times q_{t+1}}{g_{t+i}\times\cdots\times g_{t+1}},$$

for $i>0$, and equals unity for $i=0$. Hence, this growth rate is independent of the index j, because when going forward in time, firms are subject to the same experience effect. Thus, we can rewrite the equation (48) according to

$$\frac{P_{jt}^{\star}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)=\left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t},$$

where the numerator N_t is given by

$$N_t=E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{-\theta}\frac{MC_{t+i}}{P_{t+i}A_{t+i}Q_{t+i}}\left(\frac{q_{t+i}\times\cdots\times q_{t+1}}{g_{t+i}\times\cdots\times g_{t+1}}\right).$$

The numerator evolves recursively as shown by equation (13). The denominator D_t also evolves recursively, and jointly this yields the recursive pricing equations (12)-(14).

A.3 First-Order Conditions to the Household Problem

The first-order conditions that belong to the household problem comprise the household's budget constraint, a no-Ponzi scheme condition, the transversality condition, and the following equations:

$$\frac{W_t}{P_t}=-\frac{U_{Lt}}{U_{Ct}}$$
$$\Omega_{t,t+1}=\beta\frac{\xi_{t+1}}{\xi_t}\frac{U_{Ct+1}}{U_{Ct}}$$
$$1=E_t\left[\Omega_{t,t+1}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right]$$
$$1=E_t\left[\Omega_{t,t+1}(r_{t+1}+1-d)\right].$$

Here, we denote by $U(.)$ the period utility function. Our assumption that $U(C_t,L_t)=([C_tV(L_t)]^{1-\sigma}-1)/(1-\sigma)$ implies

$$U_{Ct}=C_t^{-\sigma}V(L_t)^{1-\sigma}$$
$$U_{Lt}=C_t^{1-\sigma}V(L_t)^{-\sigma}V_{Lt},$$

where $U_{Ct}=\partial U(C_t,L_t)/\partial C_t$ and $V_{Lt}=\partial V(L_t)/\partial L_t$.

A.4 Recursive Evolution of the Price Level

Plugging the weighted average price of a cohort, equation (19), into the price level, equation (20), yields that

$$P_t^{1-\theta} = \delta(\Xi_{t,t}P_{t,t}^{\star})^{1-\theta} + \sum_{s=1}^{\infty}(1-\delta)^s\delta\left[(1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P_{t-s,t-k}^{\star})^{1-\theta} + \alpha^s(\Xi_{t-s,t}P_{t-s,t-s}^{\star})^{1-\theta}\right].$$

Telescoping the sums yields:

$$\begin{aligned}
P_t^{1-\theta} &= \delta(\Xi_{t,t}P_{t,t}^{\star})^{1-\theta} \\
&\quad + \delta(1-\delta)^1\left[(1-\alpha)(\Xi_{t,t}P_{t-1,t}^{\star})^{1-\theta} + \alpha(\Xi_{t-1,t}P_{t-1,t-1}^{\star})^{1-\theta}\right] \\
&\quad + \delta(1-\delta)^2\left[(1-\alpha)(\Xi_{t,t}P_{t-2,t}^{\star})^{1-\theta} + (1-\alpha)\alpha(\Xi_{t-1,t}P_{t-2,t-1}^{\star})^{1-\theta} + \alpha^2(\Xi_{t-2,t}P_{t-2,t-2}^{\star})^{1-\theta}\right] \\
&\quad + \dots .
\end{aligned}$$

Collecting optimal prices that were set at the same date in square brackets yields:

$$\begin{aligned}
&P_t^{1-\theta} = \\
&\delta\Xi_{t,t}^{1-\theta}\left[(P_{t,t}^{\star})^{1-\theta} + (1-\alpha)(1-\delta)\left\{(P_{t-1,t}^{\star})^{1-\theta} + (1-\delta)(P_{t-2,t}^{\star})^{1-\theta} + (1-\delta)^2(P_{t-3,t}^{\star})^{1-\theta} + \dots\right\}\right] \\
&+ [\alpha(1-\delta)]\delta\Xi_{t-1,t}^{1-\theta}\left[(P_{t-1,t-1}^{\star})^{1-\theta} + (1-\alpha)(1-\delta)\left\{(P_{t-2,t-1}^{\star})^{1-\theta} + (1-\delta)(P_{t-3,t-1}^{\star})^{1-\theta} + \dots\right\}\right] \\
&+ \dots .
\end{aligned}$$

Using equation (21) and the definition of p_t^e in equation (23), we can replace the terms in curly brackets in the previous equation by p_t^e. This yields

$$\begin{aligned}
P_t^{1-\theta} &= \delta(\Xi_{t,t}P_{t,t}^{\star})^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_t^e)^{\theta-1}}{\delta} - 1\right\}\right] \\
&\quad + [\alpha(1-\delta)]^1\delta(\Xi_{t-1,t}P_{t-1,t-1}^{\star})^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_{t-1}^e)^{\theta-1}}{\delta} - 1\right\}\right] \\
&\quad + [\alpha(1-\delta)]^2\delta(\Xi_{t-2,t}P_{t-2,t-2}^{\star})^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_{t-2}^e)^{\theta-1}}{\delta} - 1\right\}\right] + \dots .
\end{aligned}$$

Rearranging the previous equation yields

$$\begin{aligned}
P_t^{1-\theta} &= (\Xi_{t,t}P_{t,t}^{\star})^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right] \\
&\quad + \alpha(1-\delta)(\Xi_{t-1,t})^{1-\theta}\left\{(\Xi_{t-1,t-1}P_{t-1,t-1}^{\star})^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right]\right. \\
&\quad \left. + \alpha(1-\delta)(\Xi_{t-2,t-1}P_{t-2,t-2}^{\star})^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \dots\right\}.
\end{aligned}$$

The term in curly brackets in the previous equation corresponds to $P_{t-1}^{1-\theta}$, which yields the price level equation (22) in the main text.

A.5 Equilibrium Definition

We are now in a position to define the market equilibrium:

Definition 1 *An equilibrium is a state-contingent path for* $\{(P_{jt}, L_{jt}, K_{jt})$ *for* $j \in [0,1]$*,* $W_t, r_t, i_t, C_t, K_{t+1}, L_t, B_t, T_t\}_{t=0}^{\infty}$ *such that*

1. *the firms' choices* $\{P_{jt}, L_{jt}, K_{jt}\}_{t=0}^{\infty}$ *maximize profits for all* $j \in [0,1]$*, given the price adjustment frictions,*

2. *the household's choices* $\{C_t, K_{t+1}, L_t, B_t\}_{t=0}^{\infty}$ *maximize expected household utility,*

3. *the government flow budget constraint holds each period, and*

4. *the markets for capital, labor, final and intermediate goods and government bonds clear,*

given the initial values $B_{-1}(1+i_{-1}), K_0, P_{j,-1}$*, and* $A_{-1}Q_{-1-s_{j,-1}}G_{j,-1}$*, with* $j \in [0,1]$*.*

A.6 Aggregate Technology and Aggregate Productivity

To derive the aggregate technology, we combine firms' technology to produce the differentiated product in equation (2) with product demand $Y_{jt}/Y_t = (P_{jt}/P_t)^{-\theta}$ to obtain

$$\frac{Y_t}{A_t Q_t}\left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} = \left(\frac{K_{jt}}{L_{jt}}\right)^{1-\frac{1}{\phi}} L_{jt} - F_t.$$

Integrating over all firms with $j \in [0,1]$, using labor market clearing, $L_t = \int_0^1 L_{jt}\ \mathrm{dj}$, and the fact that optimizing firms maintain the same (and hence the aggregate) capital labor ratio yields

$$\frac{Y_t}{A_t Q_t}\int_0^1 \left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} \mathrm{dj} = K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t.$$

Rearranging this equation and defining the (inverse) endogenous component of aggregate productivity as in equation (25) in the main text yields the aggregate technology (24).

To derive the recursive representation of Δ_t shown in equation (26), we rewrite equation (25) according to

$$\frac{\Delta_t}{P_t^{\theta}} = \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}}\right)(P_{jt})^{-\theta}\ \mathrm{dj},$$

using the processes describing the evolution of Q_t and G_{jt}. As for the price level, we proceed with the aggregation in two steps. First, we aggregate the optimal prices of all firms operating within a particular cohort. Second, we aggregate all cohorts in the economy. To this end, we rewrite Δ_t/P_t^{θ} in the previous equation according to

$$\frac{\Delta_t}{P_t^{\theta}} = \sum_{s=0}^{\infty}(1-\delta)^s \delta \widehat{\Lambda}_t(s), \tag{49}$$

using

$$\widehat{\Lambda}_t(s) = \begin{cases} \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right) \left[(1-\alpha)\sum_{k=0}^{s-1} \alpha^k (\Xi_{t-k,t} P^\star_{t-s,t-k})^{-\theta} + \alpha^s (\Xi_{t-s,t} P^\star_{t-s,t-s})^{-\theta}\right] & \text{if} \quad s \geq 1 \ , \\ (\Xi_{t,t} P^\star_{t,t})^{-\theta} & \text{if} \quad s = 0 \ . \end{cases}$$

Substituting out for $\widehat{\Lambda}_t(s)$ in equation (49) yields

$$\begin{aligned} \frac{\Delta_t}{P_t^\theta} &= \delta (\Xi_{t,t} P^\star_{t,t})^{-\theta} \\ &+ \delta \sum_{s=1}^{\infty} (1-\delta)^s \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right) \left[(1-\alpha)\sum_{k=0}^{s-1} \alpha^k (\Xi_{t-k,t} P^\star_{t-s,t-k})^{-\theta} + \alpha^s (\Xi_{t-s,t} P^\star_{t-s,t-s})^{-\theta}\right] . \end{aligned}$$

We rearrange the previous equation following corresponding steps to those in appendix A.4. This yields

$$\begin{aligned} \frac{\Delta_t}{P_t^\theta} &= (\Xi_{t,t} P^\star_{t,t})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right] \\ &+ \alpha(1-\delta) \left(\frac{q_t}{g_t}\right) (\Xi_{t-1,t} P^\star_{t-1,t-1})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right] \\ &+ [\alpha(1-\delta)]^2 \left(\frac{q_t q_{t-1}}{g_t g_{t-1}}\right) (\Xi_{t-2,t} P^\star_{t-2,t-2})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \dots \ . \end{aligned}$$

We rearrange the previous equation further to obtain that

$$\begin{aligned} \frac{\Delta_t}{P_t^\theta} &= (\Xi_{t,t} P^\star_{t,t})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right] \\ &+ \alpha(1-\delta) \left(\frac{q_t}{g_t}\right) (\Xi_{t-1,t})^{-\theta} \Big\{ (P^\star_{t-1,t-1})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right] \\ &+ \alpha(1-\delta) \left(\frac{q_{t-1}}{g_{t-1}}\right) (\Xi_{t-2,t-1} P^\star_{t-2,t-2})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \dots \Big\} . \end{aligned}$$

The term in curly brackets in the previous equation is equal to $\Delta_{t-1}/P_{t-1}^\theta$, which yields

$$\frac{\Delta_t}{P_t^\theta} = \left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right] (\Xi_{t,t} P^\star_{t,t})^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)(\Xi_{t-1,t})^{-\theta} \frac{\Delta_{t-1}}{P_{t-1}^\theta} .$$

Multiplying the previous equation by P_t^θ yields equation (26) in the main text.

A.7 Consolidated Budget Constraint

Consolidating the household's and the government's budget constraints shown in the main text yields

$$C_t + K_{t+1} = (1-d)K_t + r_t K_t + \frac{W_t}{P_t} L_t + \frac{\int_0^1 \Theta_{jt} \, \mathrm{dj}}{P_t} - \tau \left(\frac{\int_0^1 P_{jt} Y_{jt} \mathrm{dj}}{P_t}\right) . \tag{50}$$

To compute aggregate firm profits denoted by $\int_0^1 \Theta_{jt}$ dj, we use marginal costs in equation (46) and combine them with the factor demands for L_{jt} and K_{jt}, equations (44) and (45), which yields that $MC_t I_{jt} = W_t L_{jt} + P_t r_t K_{jt}$. We use this equation and product demand $Y_{jt}/Y_t = (P_{jt}/P_t)^{-\theta}$ to rewrite aggregate firm profits according to

$$\begin{aligned}\int_0^1 \Theta_{jt} \text{ dj} &= (1+\tau)\int_0^1 P_{jt}Y_{jt} \text{ dj} - \int_0^1 MC_t I_{jt} \text{ dj} \\ &= (1+\tau)\int_0^1 P_{jt}Y_{jt} \text{ dj} - \int_0^1 (W_t L_{jt} + P_t r_t K_{jt}) \text{ dj} \\ &= (1+\tau)P_t Y_t - W_t L_t - P_t r_t K_t,\end{aligned}$$

with $L_t = \int_0^1 L_{jt}$ dj and $K_t = \int_0^1 K_{jt}$ dj. Thus, the consolidated budget constraint (50) reduces to

$$K_{t+1} = (1-d)K_t + Y_t - C_t.$$

Dividing the previous equation by trend growth Γ_t^e yields

$$\gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t,$$

where $\gamma_t^e = \Gamma_t^e/\Gamma_{t-1}^e$ denotes the gross trend growth rate.

A.8 Transformed Sticky-Price Economy

We define $p_t^\star = P_{t,t}^\star/P_t$ and $mc_t = MC_t/(P_t(\Gamma_t^e)^{1/\phi})$ and $w_t = W_t/(P_t\Gamma_t^e)$ and $c_t = C_t/\Gamma_t^e$. We also use that $p_t^e = 1/\Delta_t^e$, which follows from the equations (23) and (32). This yields the following equations that describe the transformed sticky-price economy.

$$1 = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{1-\theta} + \alpha(1-\delta)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta-1} \tag{51}$$

$$\Delta_t = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1} \tag{52}$$

$$p_t^\star = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t} \tag{53}$$

$$N_t = \frac{mc_t}{\Delta_t^e} + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\gamma_{t+1}^e\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \tag{54}$$

$$D_t = 1 + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\gamma_{t+1}^e\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta-1}D_{t+1}\right] \tag{55}$$

$$mc_t = \left(\frac{w_t}{1/\phi}\right)^{\frac{1}{\phi}}\left(\frac{r_t}{1-1/\phi}\right)^{1-\frac{1}{\phi}} \tag{56}$$

$$r_t k_t = (\phi-1)w_t L_t \tag{57}$$

$$y_t = \left(\frac{\Delta_t^e}{\Delta_t}\right)\left(k_t^{1-\frac{1}{\phi}}L_t^{\frac{1}{\phi}} - f\right) \tag{58}$$

$$\gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t \tag{59}$$

$$\gamma_t^e = (a_t q_t \Delta_{t-1}^e/\Delta_t^e)^{\phi} \tag{60}$$

$$(\Delta_t^e)^{1-\theta} = \delta + (1-\delta)\left(\Delta_{t-1}^e q_t/g_t\right)^{1-\theta} \tag{61}$$

$$w_t = -c_t\left(\frac{V_{Lt}}{V(L_t)}\right) \tag{62}$$

$$1 = E_t\left[\Omega_{t,t+1}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right] \tag{63}$$

$$1 = E_t\left[\Omega_{t,t+1}(r_{t+1}+1-d)\right] \tag{64}$$

$$\Omega_{t,t+1} = \beta\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{\gamma_{t+1}^e c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{V(L_{t+1})}{V(L_t)}\right)^{1-\sigma} \tag{65}$$

After adding a description of monetary policy and a price indexation rule, these seventeen equations determine the paths of the seventeen variables $i_t, \Pi_t, y_t, c_t, k_t, L_t, r_t, w_t, mc_t, \gamma_t^e, \Delta_t, \Delta_t^e, p_t^\star, \Xi_{t-1,t}, N_t, D_t, \Omega_{t-1,t}$ given the four exogenous shocks q_t, g_t, a_t, ξ_t.

A.9 Steady State in the Transformed Sticky-Price Economy

We consider a steady state in the transformed sticky-price economy, in which g and q are constant and the government maintains a constant inflation rate Π, which also implies a constant rate of price indexation Ξ.

To solve for the model variables in this steady state, we first solve for the ratio Δ/Δ^e as a function of model parameters and the inflation rate Π only. To this end, we derive an expression for $p^\star$ as a function of Δ using the equations (51) and (52). Both equations can be rearranged to obtain, respectively,

$$(1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}) = \left[\alpha\delta + (1-\alpha)(\Delta^e)^{1-\theta}\right](p^\star)^{1-\theta} \tag{66}$$

$$\Delta\left(1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}\right) = \left[\alpha\delta + (1-\alpha)(\Delta^e)^{1-\theta}\right](p^\star)^{-\theta}. \tag{67}$$

Dividing the equation (66) by the equation (67) yields

$$p^\star = \Delta^{-1}\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\right). \tag{68}$$

We substitute this expression for $p^\star$ into the equation (67), which yields

$$\left(\frac{\Delta}{\Delta^e}\right)^{1-\theta} = \frac{\alpha\delta(\Delta^e)^{\theta-1}+1-\alpha}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\right)^{-\theta}.$$

We use equation (61) to substitute for $(\Delta^e)^{\theta-1}$ on the right hand side of the previous equation and rearrange the result to obtain

$$\frac{\Delta(\Pi)}{\Delta^e} = \left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}\left(\frac{1-\alpha(1-\delta)(g/q)^{\theta-1}}{1-\alpha(1-\delta)\left(\Pi/\Xi\right)^{\theta}\left(g/q\right)^{-1}}\right), \tag{69}$$

where we have indicated that $\Delta(\Pi)$ depends on the steady-state inflation rate Π. For later use, we define the relative price distortion as

$$\rho(\Pi) = \frac{\Delta^e}{\Delta(\Pi)}. \tag{70}$$

Combining the pricing equations (53) to (55) yields

$$\frac{1}{mc} = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\left(\frac{1}{p^\star\Delta^e}\right)\left(\frac{1-\alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}]\left(\Pi/\Xi\right)^{\theta-1}}{1-\alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}]\left(\Pi/\Xi\right)^{\theta}\left(g/q\right)^{-1}}\right).$$

Using the expression for $p^\star$ in equation (68) to substitute for $p^\star$ in the previous equation and the solution for $\Delta(\Pi)/\Delta^e$ in equation (69), we thus obtain a solution for $1/mc$. Again for later use, we denote the average markup by $\mu = 1/mc$ and thus obtain the solution

$$\mu(\Pi) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{1}{\theta-1}}\left(\frac{1-\alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}]\left(\Pi/\Xi\right)^{\theta-1}}{1-\alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}]\left(\Pi/\Xi\right)^{\theta}\left(g/q\right)^{-1}}\right). \tag{71}$$

Again, we indicate here that $\mu(\Pi)$ depends on the steady-state inflation rate.

Now, we rewrite marginal costs in equation (56) as

$$mc = \left(\frac{w}{r}(\phi-1)\right)^{\frac{1}{\phi}}\left(\frac{r}{1-1/\phi}\right),$$

and use equation (57) to obtain $mc = \left(\frac{k}{L}\right)^{\frac{1}{\phi}}\left(\frac{r}{1-1/\phi}\right)$ or

$$r = \mu(\Pi)^{-1}\left(1-\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{-\frac{1}{\phi}}, \tag{72}$$

after also using $\mu = 1/mc$. Analogous steps for the wage rate also imply

$$w = \mu(\Pi)^{-1} \left(\frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}. \tag{73}$$

Furthermore, the aggregate technology (58), the aggregate resource constraint (59) and the household's optimality conditions (62) to (65) imply the following four equations:

$$\begin{aligned}
y &= \rho(\Pi) \left(\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} L - f \right) \\
w &= c \left(-\frac{V_L}{V(L)} \right) \\
r &= \frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d \\
y &= c + (\gamma^e - 1 + d)k,
\end{aligned}$$

where we have used $\rho(\Pi) = \Delta^e/\Delta(\Pi)$. To simplify these four equations further, we use the equations (72) and (73) to substitute out for w and r. Then, we express all the remaining variables relative to hours worked, which yields the following four equations:

$$\frac{y}{L} = \rho(\Pi) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} \tag{74}$$

$$\frac{c}{L} = \mu(\Pi)^{-1} \left(\frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(-\frac{V(L)}{LV_L}\right) \tag{75}$$

$$\frac{k}{L} = \mu(\Pi)^{-1} \left(1 - \frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-1} \tag{76}$$

$$\frac{y}{L} = \frac{c}{L} + (\gamma^e - 1 + d)\frac{k}{L}. \tag{77}$$

We now show that these four equations determine the four variables y, c, L, k, given a steady-state inflation rate Π and assuming that the ratio of fixed costs over output, f/y, is a calibrated parameter.

First, we solve for hours worked as a function of Π by substituting the equations (74) to (76) into equation (77). This yields

$$\mu(\Pi)\rho(\Pi) \left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} = \left(\frac{1}{\phi}\right) \left(-\frac{V(L)}{LV_L}\right) + \left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right) \left(1 - \frac{1}{\phi}\right),$$

or

$$\begin{aligned}
\left(-\frac{V(L)}{LV_L}\right) &= \phi\mu(\Pi)\rho(\Pi) \left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} - (\phi - 1) \left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right) \\
&= \mathcal{L}(\Pi),
\end{aligned}$$

where $\mathcal{L}(\Pi)$ abbreviates the right-hand-side term, which is a function of the steady-state inflation rate. The previous equation provides an implicit solution for L. We obtain an explicit solution for L, if we assume a functional form for $V(L)$. Using that $V(L) = 1 - \psi L^\nu$, with $\nu > 1$ and $\psi > 0$ yields

$$-\frac{V(L)}{LV_L} = \frac{1 - \psi L^\nu}{\psi \nu L^\nu}$$

and hence

$$L(\Pi) = \left(\frac{1}{\psi + \psi \nu \mathcal{L}(\Pi)} \right)^{1/\nu}, \tag{78}$$

where we have indicated that in general, steady-state hours worked L depend on the steady-state inflation rate Π through $\mathcal{L}(\Pi)$. Recall that in order to compute $\mathcal{L}(\Pi)$, the equations (69), (70) and (71) are required. The solutions for k, c, and y can be recursively computed from the equations (74) to (76). These solutions are

$$k(\Pi) = \mu(\Pi)^{-\phi} \left(1 - \frac{1}{\phi}\right)^{\phi} \left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d \right)^{-\phi} L \tag{79}$$

$$c(\Pi) = \mu(\Pi)^{-1} \left(\frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(-\frac{V(L)}{V_L} \right) \tag{80}$$

$$y(\Pi) = c + (\gamma^e - 1 + d)k. \tag{81}$$

Again, we indicate that these solutions depend on the steady-state inflation rate.

B Planner Problem and Its Solution

The planner allocates resources across firms and time by maximizing expected discounted household utility subject to firms' technologies and feasibility constraints. The planner problem can be solved in two steps. The first step determines the allocation of given amounts of capital and labor between heterogenous firms at date t. The second step determines the allocation of aggregate capital, consumption and labor over time. Endogenous variables in the planner solution are indicated by superscript e.

B.1 Intratemporal Planner Problem

The intratemporal problem corresponds to

$$\max_{L^e_{jt}, K^e_{jt}} \left(\int_0^1 (Y^e_{jt})^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}} \quad s.t. \quad Y^e_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left((K^e_{jt})^{1-\frac{1}{\phi}} (L^e_{jt})^{\frac{1}{\phi}} - F_t \right),$$

and given L^e_t and K^e_t, with $L^e_t = \int_0^1 L^e_{jt}\,\mathrm{dj}$ and $K^e_t = \int_0^1 K^e_{jt}\,\mathrm{dj}$. Optimality conditions yield $K^e_{jt}/L^e_{jt} = K^e_t/L^e_t$ and hence that all firms maintain the same capital labor ratio. Thus, the problem can be recast in terms of the optimal mix of input factors, $I^e_{jt} = (K^e_{jt})^{1-1/\phi}(L^e_{jt})^{1/\phi}$:

$$\max_{I^e_{jt}} \left(\int_0^1 \left[A_t Q_{t-s_{jt}} G_{jt} \left(I^e_{jt} - F_t \right) \right]^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}} \quad s.t. \quad I^e_t = \int_0^1 I^e_{jt} \ \mathrm{dj},$$

with $I^e_t = (K^e_t)^{1-1/\phi}(L^e_t)^{1/\phi}$ being given. Equating the first-order conditions to this problem for two different firms j and k to each other yields the condition

$$Z_{jt} \left[Z_{jt} \left(I^e_{jt} - F_t \right) \right]^{-\frac{1}{\theta}} = Z_{kt} \left[Z_{kt} \left(I^e_{kt} - F_t \right) \right]^{-\frac{1}{\theta}},$$

where $Z_{jt} = Q_{t-s_{jt}} G_{jt}$ denotes productivity of the firm j at date t. Rearranging this condition yields $I^e_{jt} - F_t = (Z_{jt}/Z_{kt})^{\theta-1} (I^e_{kt} - F_t)$, and aggregating this equation over all j's yields

$$I^e_{kt} - F_t = \frac{(G_{kt} Q_{t-s_{kt}}/Q_t)^{\theta-1}}{\int_0^1 (G_{jt} Q_{t-s_{jt}}/Q_t)^{\theta-1} \ \mathrm{dj}} (I^e_t - F_t). \tag{82}$$

Thus, the optimal input mix of the firm k net of fixed costs is proportional to the optimal aggregate input mix net of fixed costs, and the factor of proportionality corresponds to the (weighed) productivity of the firm k relative to the (weighed) aggregate productivity in the economy. Thus, equation (82) shows that the productivity distribution determines the efficient allocation of the optimal input mix across firms.

To obtain the aggregate technology in the planner economy, we combine equation (82) with equation (2) and the Dixit-Stiglitz aggregator (1). This yields

$$Y^e_t = \left(\int_0^1 \left[A_t Q_{t-s_{jt}} G_{jt} \left(\frac{(Q_{t-s_{jt}} G_{jt})^{\theta-1}}{\int_0^1 (Q_{t-s_{jt}} G_{jt})^{\theta-1} \ \mathrm{dj}} (I^e_t - F_t) \right) \right]^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}}.$$

Simplifying this equation yields the aggregate technology in the planner economy,

$$Y^e_t = \frac{A_t Q_t}{\Delta^e_t} \left((K^e_t)^{1-\frac{1}{\phi}} (L^e_t)^{\frac{1}{\phi}} - F_t \right), \tag{83}$$

where the efficient productivity adjustment factor is defined as

$$1/\Delta^e_t = \left(\int_0^1 \left(G_{jt} Q_{t-s_{jt}}/Q_t \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}} \tag{84}$$

and evolves recursively. To see this, rewrite equation (84) as

Assuming that the initial productivity distribution at $t = -1$ is consistent with the assumed productivity process we have

$$\begin{aligned}
(1/\Delta^e_t)^{\theta-1} &= \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}} \right)^{1-\theta} \mathrm{dj} \\
&= \delta \left\{ 1 + \sum_{s=1}^{\infty} (1-\delta)^s \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}} \right)^{1-\theta} \right\} \\
&= \delta \left\{ 1 + (1-\delta) \left(\frac{q_t}{g_t} \right)^{1-\theta} + (1-\delta)^2 \left(\frac{q_t q_{t-1}}{g_t g_{t-1}} \right)^{1-\theta} + \ldots \right\} \\
&= (p^e_t)^{\theta-1}.
\end{aligned}$$

The last step follows from backward-iterating equation (23) and implies that the efficient productivity adjustment factor equals the relative price of firms hit by a δ-shock in period t in the economy with flexible prices,

$$1/\Delta_t^e = p_t^e. \tag{85}$$

It follows also from equation (23) that Δ_t^e evolves recursively as shown in equation (32). The intratemporal planner allocation then consists of equation (82), which determines the efficient allocation of the optimal input mix across firms, and equations (83) and (32), which describe the aggregate consequences of the efficient allocation at the firm level.

B.2 Intertemporal Planner Problem

The intertemporal allocation maximizes expected discounted household utility subject to the intertemporal feasibility condition,

$$\max_{\{C_t^e, L_t^e, K_{t+1}^e\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \xi_t U(C_t^e, L_t^e) \quad s.t. \tag{86}$$

$$C_t^e + K_{t+1}^e = (1-d)K_t^e + \frac{A_t Q_t}{\Delta_t^e}\left((K_t^e)^{1-\frac{1}{\phi}}(L_t^e)^{\frac{1}{\phi}} - F_t\right), \tag{87}$$

with $U(.)$ denoting the period utility function and Δ_t^e given by equation (32). The first order conditions to this problem comprise the feasibility constraint and

$$Y_{Lt}^e = -\frac{U_{Lt}^e}{U_{Ct}^e}, \tag{88}$$

$$1 = \beta E_t\left[\frac{\xi_{t+1}}{\xi_t}\frac{U_{Ct+1}^e}{U_{Ct}^e}\left(Y_{Kt+1}^e + 1 - d\right)\right], \tag{89}$$

denoting by Y_{Kt}^e the marginal product of capital and by Y_{Lt}^e the marginal product of labor. Thus, the planner allocation for aggregate variables is characterized by the aggregate technology, equation (83), the efficient adjustment factor, equation (32), the feasibility condition, equation (87), and the two first-order conditions (88) and (89).

C Proof of Proposition 1

To show that condition (33) holds under flexible prices, we divide equation (22) by $P_t^{1-\theta}$ and impose $\alpha = 0$ to find out that the optimal relative price $p_t^\star$ of firms experiencing a δ-shock in period t is equal to p_t^e. This and the equations (53) to (55) determining the optimal relative price of firms experiencing a δ-shock in t imply with $\alpha = 0$ that

$$p_t^e = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{mc_t}{\Delta_t^e}.$$

Combining the previous equation with the equation (85) yields

$$1 = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) mc_t, \tag{90}$$

which shows that real detrended marginal costs are constant in the economy with flexible prices. From equation (12) it follows that the optimal relative price of the firm j in the flexible-price model satisfies

$$\frac{P^\star_{jt}}{P_t} (G_{jt} Q_{t-s_{jt}} / Q_t) = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{mc_t}{\Delta^e_t}.$$

Combining the previous equation with equation (90), we obtain condition (33) in the main text. Plugging this condition into equation (25) shows that the flexible-price equilibrium implements $\Delta_t = \Delta^e_t$. Thus, the aggregate production function in equation (24) in the flexible-price equilibrium is given by

$$Y_t = \frac{A_t Q_t}{\Delta^e_t} \left((K_t)^{1-\frac{1}{\phi}} (L_t)^{\frac{1}{\phi}} - F_t \right), \tag{91}$$

with $F_t = f \cdot (\Gamma^e_t)^{1-1/\phi}$ and $\Gamma^e_t = (A_t Q_t / \Delta^e_t)^\phi$, and the resource constraint (derived in Appendix A.7) is given by

$$K_{t+1} = (1 - d) K_t + Y_t - C_t. \tag{92}$$

The two equations (91) and (92) are the same constraints faced by the planner under efficient allocation. Combined with the fact that the household decisions in the flexible price economy are undistorted in the presence of the corrective sales subsidy, it follows that the allocation of aggregate consumption, capital, labor, and output in the flexible-price equilibrium is identical to efficient allocation.

D Proof of Proposition 2

Establishing (1): First, we show that firms hit by a δ-shock in period t in the sticky-price economy choose the same optimal relative price as in the flexible-price economy. Let superscript e denote allocations and prices in the flexible-price economy, which we have shown reproduces the efficient allocation. Under flexible prices ($\alpha = 0$) and given condition 1, the optimal relative price implied by equation (12) for firms with a δ-shock in period t is given by

$$p^e_t = \frac{(P^\star_{t,t})^e}{P^e_t} = \frac{MC^e_t}{P^e_t A_t Q_t}.$$

Under sticky prices ($\alpha > 0$) and the efficient allocation, combining this equation with equation (13) implies

$$\frac{N_t}{p^e_t} = 1 + \alpha(1 - \delta) E_t \left[\Omega^e_{t,t+1} \frac{Y^e_{t+1}}{Y^e_t} \left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}} \right)^\theta \left(\frac{q_{t+1}}{g_{t+1}} \right) \left(\frac{p^e_{t+1}}{p^e_t} \right) \left(\frac{N_{t+1}}{p^e_{t+1}} \right) \right]. \tag{93}$$

Furthermore, equation (14) implies

$$D_t = 1 + \alpha(1-\delta)E_t \left[\Omega^e_{t,t+1} \frac{Y^e_{t+1}}{Y^e_t} \left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}} \right)^{\theta-1} D_{t+1} \right]. \tag{94}$$

Firms hit by a δ-shock in period t in the sticky-price economy choose the same optimal relative price as firms receiving a δ-shock in period t in the flexible-price economy, i.e., $P^\star_{t,t}/P_t = N_t/D_t = p^e_t$ or equivalently $N_t/p^e_t = D_t$, if it holds that

$$\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}} \right) \left(\frac{q_{t+1}}{g_{t+1}} \right) \left(\frac{p^e_{t+1}}{p^e_t} \right) = 1, \tag{95}$$

which follows from comparing the equations (93) and (94). To show that equation (95) holds under the optimal inflation rate stated in proposition 2, we lag this equation by one period and rearrange it to obtain

$$\left(\frac{\Pi_t}{\Xi_{t-1,t}} \right) p^e_t = p^e_{t-1} \frac{g_t}{q_t}.$$

Combining this equation with equation (23) implies that the optimal inflation rate as defined in equation (34) satisfies equation (95).

Establishing (2): To show that, under the optimal inflation rate, firms that are subject to a Calvo shock in period t and hence can adjust their price do not find it optimal to change their price, we need to establish that

$$P^\star_{t-k,t} = \Xi^\star_{t-k,t} P^\star_{t-k,t-k}, \tag{96}$$

for all $k > 0$. Dividing this equation by the (optimal) aggregate price level $P^\star_{t-k}$ and using the result from step (1), i.e., $P^\star_{t,t}/P^\star_t = p^e_t$, we obtain

$$\frac{P^\star_{t-k,t}}{P^\star_{t-k}} = \Xi^\star_{t-k,t} \left(\frac{P^\star_{t-k,t-k}}{P^\star_{t-k}} \right) = \Xi^\star_{t-k,t} p^e_{t-k}.$$

Using equation (21), we can rewrite the previous equation as

$$\frac{P^\star_{t,t}}{P^\star_t} \left(\frac{q_t \times \cdots \times q_{t-k+1}}{g_t \times \cdots \times g_{t-k+1}} \right) \frac{P^\star_t}{P^\star_{t-k}} = \Xi^\star_{t-k,t} p^e_{t-k}.$$

Again using $P^\star_{t,t}/P^\star_t = p^e_t$ and that $\Xi_{t-k,t} = \prod_{j=1}^k \Xi_{t-k+j-1,t-k+j}$ further delivers

$$\left(\frac{p^e_t}{p^e_{t-k}} \right) \left(\frac{q_t \times \cdots \times q_{t-k+1}}{g_t \times \cdots \times g_{t-k+1}} \right) \left(\frac{\Pi^\star_t}{\Xi^\star_{t-1,t}} \times \cdots \times \frac{\Pi^\star_{t+1-k}}{\Xi^\star_{t-k,t+1-k}} \right) = 1.$$

Rewriting the previous equation as

$$\left(\frac{\Pi^\star_t}{\Xi^\star_{t-1,t}} \frac{q_t}{g_t} \frac{p^e_t}{p^e_{t-1}} \right) \times \left(\frac{\Pi^\star_{t-1}}{\Xi^\star_{t-2,t-1}} \frac{q_{t-1}}{g_{t-1}} \frac{p^e_{t-1}}{p^e_{t-2}} \right) \times \cdots \times \left(\frac{\Pi^\star_{t+1-k}}{\Xi^\star_{t-k,t+1-k}} \frac{q_{t+1-k}}{g_{t+1-k}} \frac{p^e_{t+1-k}}{p^e_{t-k}} \right) = 1$$

shows that each term in parenthesis is equal to unity under the optimal inflation rate, which follows from equation (95). This establishes that firms that can adjust their price maintain the indexed price as given by equation (96).

Establishing (3): We can establish the fact that the condition 2 causes initial prices to reflect initial relative productivities as follows. The pricing equations (12)-(14) imply under flexible prices and no markup distortion that

$$\frac{P_{jt}^{\star}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)=\frac{MC_t}{P_t A_t Q_t}.$$

For a firm hit by a δ-shock in period t, this equation yields

$$p_t^e=\frac{MC_t}{P_t A_t Q_t}.$$

Combining both previous equations yields

$$\frac{P_{jt}^{\star}}{P_t}=\left(\frac{Q_t}{Q_{t-s_{jt}}G_{jt}}\right)p_t^e.$$

Plugging this equation into the aggregate price level, $P_t^{1-\theta}=\int_0^1 P_{jt}^{1-\theta}\,\mathrm{dj}$, yields

$$1=\int_0^1\left(\frac{Q_t}{Q_{t-s_{jt}}G_{jt}}\right)^{1-\theta}(p_t^e)^{1-\theta}\mathrm{dj}.$$

Rewriting this equation and using $p_t^e=1/\Delta_t^e$ yields equation (31) for $t=-1$.

E Discontinuity of the Optimal Inflation Rate

This appendix compares the optimal inflation rate in an economy with δ-shocks ($\delta>0$) to the economy in the absence of such shocks ($\delta=0$). We refer to the first economy as the δ-economy and to the latter as the 0-economy. Comparing these two economies is not as straightforward as it might initially appear: even if both economies are subject to the same fundamental shocks (a_t, q_t, g_t, ξ_t), the efficient allocation displays a discontinuity when considering the limit $\delta\to 0$. The discontinuity arises because aggregate productivity growth in the δ-economy is driven by $a_t q_t$, while it is driven by $a_t g_t$ in the 0-economy.

To properly deal with this issue, we construct a δ-economy whose efficient aggregate allocation (consumption, hours, capital) is identical to the efficient aggregate allocation in the 0-economy.[39] We then compare the optimal inflation rates in these two economies and show that the optimal inflation rate for the δ-economy differs from the optimal inflation rate for the 0-economy, even for the limit $\delta\to 0$.

[39] The two economies do of course differ in their underlying firm-level dynamics.

Let a_t^δ, q_t^δ, g_t^δ denote the productivity disturbances in the δ-economy and let $A_{-1}^\delta G_{j,-1}^\delta Q_{-1-s_{j,-1}}^\delta$ for $j \in [0,1]$ denote the initial distribution of firm productivities. This, together with the process $\{\delta_{jt}\}_{t=0}^\infty$ for all $j \in [0,1]$, determines the entire state-contingent values for A_t^δ, Q_t^δ, G_{jt}^δ, and $Q_{t-s_{jt}}^\delta$ for all $j \in [0,1]$ and all $t \geq 0$.

Next, consider the 0-economy and suppose it starts with the same initial capital stock as the δ-economy. For the 0-economy, we normalize $Q_{t-s_{jt}}^0 \equiv 1$ for all $j \in [0,1]$ and all t and then set the initial firm productivity distribution in the 0-economy equal to that in the δ-economy by choosing the initial conditions

$$A_{-1}^0 = A_{-1}^\delta,$$
$$G_{j,-1}^0 = G_{j,-1}^\delta Q_{-1-s_{j,-1}}^\delta.$$

Finally, let the process for common TFP in the 0-economy be given by

$$A_t^0 = A_t^\delta \left(\int_0^1 \left(Q_{t-s_{jt}}^\delta G_{jt}^\delta \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}} \left(\int_0^1 \left(G_{jt}^0 \right)^{\theta-1} \mathrm{dj} \right)^{\frac{-1}{\theta-1}},$$

where G_{jt}^0 is generated by an arbitrary process g_t^0, e.g., $g_t^0 = g_t^\delta$. In this setting, it is easily verified that aggregate productivity associated with the efficient allocation, defined as

$$A_t Q_t / \Delta_t^e = A_t Q_t \left(\int_0^1 \left(G_{jt} Q_{t-s_{jt}} / Q_t \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}},$$

is the same in the δ-economy and the 0-economy.[40] We then have the following result:

Proposition 7 *Under the assumptions stated in this section, the efficient allocations in the two economies, the δ-economy and the 0-economy, satisfy*

$$C_t^\delta = C_t^0, L_t^\delta = L_t^0, K_t^\delta = K_t^0$$

for all $t \geq 0$ and all possible realizations of the disturbances.

Proof. Since $A_t^\delta Q_t^\delta / \Delta_t^{e,\delta} = A_t^0 Q_t^0 / \Delta_t^{e,0}$ for all t, it follows from the planner's problem (29)-(30) and the fact that the initial capital stock is identical that both economies share the same efficient allocation. ■

The following proposition shows that (generically) the optimal inflation rate discontinuously jumps when moving from the 0-economy to the δ-economy, even if both economies are identical in terms of their efficient aggregate dynamics:[41]

[40] The fact that $A_t Q_t / \Delta_t^e$ is equal to aggregate productivity in the efficient allocation follows from equations (30) and (31).

[41] Recall that the optimal inflation rates implement the efficient aggregate allocations in these economies.

Lemma 2 *Under the assumptions stated in this section and provided conditions 1 and 2 hold, the optimal inflation rate in the* 0*-economy is* $\Pi_t^{\star,0} = 1$ *for all* t*. The optimal inflation rate in the* δ*-economy is given by equation (34); in particular, for* $g_t^\delta = g$ *and* $q_t^\delta = q$*, and in the absence of price indexation, the optimal rate of inflation in the* δ*-economy satisfies* $\lim_{t\to\infty} \Pi_t^{\star,\delta} = g/q$*.*

Proof. The results directly follow from proposition 2 and lemma 1. ■

The previous result illustrates the fragility of the optimality of strict price stability in standard sticky-price models, once non-trivial firm-level productivity trends are taken into account. Moreover, in combination with proposition 7, the result shows that two economies that can be identical in terms of their aggregate efficient allocations may require different inflation rates for implementing these allocations.

F Proof of Proposition 3

Under the assumptions stated in the proposition, it is straightforward to show that the relative price distortion $\rho(\Pi)$ and the markup distortion $\mu(\Pi)$, which are defined in equations (69), (70) and (71), are inversely proportional to each other,

$$\mu(\Pi) = 1/\rho(\Pi).$$

As a result, the solution of L determined in equation (78) in appendix A.9 simplifies to

$$L = \left(\frac{1}{\psi(1+\nu)}\right)^{1/\nu},$$

because $\mathcal{L}(\Pi) = 1$ and, therefore, L no longer depends on the steady-state inflation rate Π. This result implies that $L(1) = L(\Pi^\star)$, as stated in proposition 3.

In this case, the solutions for capital and consumption, equations (79) and (80), imply

$$k(\Pi) = \rho(\Pi)^\phi \left(1 - \frac{1}{\phi}\right)^\phi (\gamma^e - 1 + d)^{-\phi} L,$$
$$c(\Pi) = \rho(\Pi)^\phi \left(\frac{1}{\phi}\right)\left(1 - \frac{1}{\phi}\right)^{\phi-1} (\gamma^e - 1 + d)^{1-\phi} \left(-\frac{V(L)}{V_L}\right),$$

where we explicitly indicate that steady-state capital and consumption depend on Π.

Comparing steady-state consumption for the policy implementing the optimal inflation rate $\Pi^\star$ and the alternative policy implementing strict price stability in economies without price indexation yields

$$\frac{c(1)}{c(\Pi^\star)} = \left(\frac{\rho(1)}{\rho(\Pi^\star)}\right)^\phi.$$

Equations (69) and (70) imply that the relative price distortion $\rho(\Pi^\star) = 1$. This yields

$$
\begin{aligned}
\frac{c(1)}{c(\Pi^\star)} &= \rho(1)^\phi, \\
&= \left(\frac{\Delta^e}{\Delta(1)}\right)^\phi \\
&= \left(\frac{1-\alpha(1-\delta)(g/q)^{\theta-1}}{1-\alpha(1-\delta)}\right)^{\frac{\phi\theta}{\theta-1}} \left(\frac{1-\alpha(1-\delta)\,(g/q)^{-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^\phi,
\end{aligned}
$$

which is the expression in proposition 3.

To show that $c(1)/c(\Pi^\star) \leq 1$, note that $c(1)/c(\Pi^\star) = 1$, if $g = q$ and hence $\Pi^\star = 1$. To show that the inequality holds strictly, $c(1)/c(\Pi^\star) < 1$, for $g \neq q$, we take the derivative of $c(1)/c(\Pi^\star)$ with respect to g/q. This yields

$$
\frac{\partial}{\partial(g/q)}\left(\frac{c(1)}{c(\Pi^\star)}\right) = \left[\frac{c(1)}{c(\Pi^\star)}\right]\left[\frac{\alpha(1-\delta)\phi}{(g/q)^2}\right] \frac{1-(g/q)^\theta}{\left[1-\alpha(1-\delta)\,(g/q)^{-1}\right]\left[1-\alpha(1-\delta)(g/q)^{\theta-1}\right]}.
$$

Terms in square brackets are positive, because we have assumed that $(1-\delta)(g/q)^{\theta-1} < 1$ (see equation (6)), $\alpha < 1$, and $g/q > \alpha(1-\delta)$. Therefore, the derivative is strictly positive if $1-(g/q)^\theta > 0$ and thus $g/q < 1$. The derivative is strictly negative if $1-(g/q)^\theta < 0$ and thus $g/q > 1$. The derivative is zero if $g/q = 1$.

G Proof of Proposition 4

We start by deriving equation (39) in the proposition. Average employment per firm $\overline{L}_t$ can be written as

$$
\overline{L}_t = \delta \overline{L}_t^\star + (1-\delta)\overline{L}_t^c, \tag{97}
$$

where $\overline{L}_t^\star$ denotes average employment of the firms that received a δ-shock in period t and $\overline{L}_t^c$ average employment of the remaining firms. Equation (2) and equation (24), respectively, imply

$$
\begin{aligned}
\frac{Y_{jt}}{A_t Q_{t-s_{jt}} G_{jt}} + F_t &= (K_{jt}/L_{jt})^{1-\frac{1}{\phi}} L_{jt} \\
\frac{Y_t \Delta_t}{A_t Q_t} + F_t &= (K_t/L_t)^{1-\frac{1}{\phi}} \overline{L}_t,
\end{aligned}
$$

where we used the fact that due to there being a unit mass of firms, we have $L_t = \overline{L}_t$. Taking the ratio of the two previous equations and using the fact that each firm's capital-labor ratio is equal to the aggregate capital-labor ratio, we get

$$
\frac{L_{jt}}{\overline{L}_t} = \left(\frac{1}{1+F_t\frac{A_t Q_t}{Y_t \Delta_t}}\right)\left(\frac{Y_{jt}}{A_t Q_{t-s_{jt}} G_{jt}}\frac{A_t Q_t}{Y_t \Delta_t} + F_t \frac{A_t Q_t}{Y_t \Delta_t}\right).
$$

Using $F_t = f \cdot (\Gamma_t^e)^{1-\frac{1}{\phi}}$ from equation (3), the definition of detrended output $y_t = Y_t/\Gamma_t^e$, and $\Gamma_t^e = (A_t Q_t/\Delta_t^e)^\phi$ from equation (27), the previous equation can be expressed as

$$\frac{L_{jt}}{\overline{L}_t} = \left(1 + \frac{f}{y_t \Delta_t/\Delta_t^e}\right)^{-1} \left(\frac{Y_{jt}}{Y_t \Delta_t}\left(\frac{Q_t}{Q_{t-s_{jt}} G_{jt}}\right) + \frac{f}{y_t \Delta_t/\Delta_t^e}\right).$$

Using the product demand function (8) to substitute Y_{jt}/Y_t, we get

$$\frac{L_{jt}}{\overline{L}_t} = \left(1 + \frac{f\Delta_t^e}{y_t \Delta_t}\right)^{-1} \left(\frac{f\Delta_t^e}{y_t \Delta_t} + \frac{1}{\Delta_t}\left(\frac{Q_t}{Q_{t-s_{jt}} G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta}\right).$$

Firms that receive a δ-shock at date t can charge the optimal price, i.e., $P_{jt}/P_t = P_{t,t}^\star/P_t = p_t^\star$. For these firms, the previous equation implies

$$\frac{\overline{L}_t^\star}{\overline{L}_t} = \left(1 + \frac{f\Delta_t^e}{y_t \Delta_t}\right)^{-1} \left[\frac{f\Delta_t^e}{y_t \Delta_t} + \frac{1}{\Delta_t}(p_t^\star)^{-\theta}\right],$$

where we used the fact that firms that receive a δ-shock are identical, so that on the left-hand side of the previous equation, we can write average employment of these firms in the numerator. Using equation (97) to substitute for $\overline{L}_t^\star/\overline{L}_t$ in the previous equation yields

$$\left(1 + \frac{f\Delta_t^e}{y_t \Delta_t}\right)(1 - (1-\delta)\overline{L}_t^c/\overline{L}_t) - \delta\frac{f\Delta_t^e}{y_t \Delta_t} = \left(\frac{\Delta_t^e}{\Delta_t}\right)\left[\delta(\Delta_t^e)^{\theta-1}\right](\Delta_t^e p_t^\star)^{-\theta}.$$

Equation (34) implies $\delta\,(\Delta_t^e)^{\theta-1} = 1 - (1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}$. This allows us to rewrite the previous equation as

$$(\Delta_t^e p_t^\star)^{-\theta} = \left(\frac{\Delta_t}{\Delta_t^e}\right)\left(\frac{1-(1-\delta)\left[\frac{\overline{L}_t^c}{\overline{L}_t} + \frac{f\Delta_t^e}{y_t\Delta_t}\left(\frac{\overline{L}_t^c}{\overline{L}_t} - 1\right)\right]}{1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}}\right). \tag{98}$$

From equation (51) we obtain

$$1 - \alpha(1-\delta)\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta-1} = \left[\alpha\delta(\Delta_t^e)^{\theta-1} + (1-\alpha)\right](\Delta_t^e p_t^\star)^{1-\theta}.$$

Using again $\delta\,(\Delta_t^e)^{\theta-1} = 1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}$ allows us to rewrite the previous equation as

$$(p_t^\star \Delta_t^e)^{-\theta} = \left(\frac{1-\alpha(1-\delta)\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta-1}}{1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}. \tag{99}$$

Equating the right-hand sides of equation (98) and equation (99) delivers equation (39) in the proposition for the special case with $f = 0$.

We next derive equation (40) in the proposition. From equation (52) we have

$$\Delta_t = \left[\alpha\delta(\Delta_t^e)^{\theta-1} + (1-\alpha)\right]\Delta_t^e\,(p_t^\star\Delta_t^e)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^\theta \Delta_{t-1}.$$

Equation (34) implies $\delta\left(\Delta_t^e\right)^{\theta-1} = 1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}$. Substituting this into the previous equation and dividing by Δ_t^e delivers

$$\frac{\Delta_t}{\Delta_t^e} = \left[1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}\right](p_t^\star\Delta_t^e)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\frac{\Delta_{t-1}^e}{\Delta_t^e}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^\theta \frac{\Delta_{t-1}}{\Delta_{t-1}^e}.$$

Using $\frac{\Pi_t^\star}{\Xi_{t-1,t}^\star} = \frac{g_t}{\Delta_{t-1}^e}\frac{\Delta_t^e}{q_t}$ from equation (35) delivers

$$\frac{\Delta_t}{\Delta_t^e} = \left[1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}\right](p_t^\star\Delta_t^e)^{-\theta} + \alpha(1-\delta)\left(\frac{\left(\Pi_t/\Xi_{t-1,t}\right)^\theta}{\Pi_t^\star/\Xi_{t-1,t}^\star}\right)\frac{\Delta_{t-1}}{\Delta_{t-1}^e}.$$

Using equation (99) to substitute $(p_t^\star\Delta_t^e)^{-\theta}$ in the previous equation delivers equation (40) in the proposition.

H Robustness of Results to Positive Fixed Costs

From the proof of proposition 4 in appendix G, which covers the general case with non-negative fixed costs $f \geq 0$, it follows that equation (40) continues to hold for $f \geq 0$. From equations (98) and (99) it follows that equation (39) generalizes to

$$\left(\frac{\Delta_t}{\Delta_t^e}\right)^{-1}\left(\frac{1-\alpha(1-\delta)\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta-1}}{1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}} = \left(\frac{1-(1-\delta)\left[\frac{\overline{L}_t^c}{\overline{L}_t}+\frac{f\Delta_t^e}{y_t\Delta_t}\left(\frac{\overline{L}_t^c}{\overline{L}_t}-1\right)\right]}{1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}}\right). \tag{100}$$

Using equations (40) and (100), we then evaluate the sensitivity of the optimal inflation estimate in steady state ($y_t = y$) for different fixed cost, using the baseline parameters from table 1. We thereby set $\overline{L}_t^c/\overline{L}_t = 1.0703$, which is the sample mean of this ratio in the data and $\Pi_t/\Xi_{t-1,t} = 1.031$, which is equal to the sample mean of GDP deflator over the considered sample period, i.e., we assume no price indexation ($\Xi_{t-1,t} \equiv 1$). The steady state value of Δ_t^e/Δ_t then follows from (100). We consider fixed costs in a range up to 10% of total (detrended) output, $f/y \in [0, 0.1]$, where $f/y = 0$ is the case considered in the main text. Figure 6 shows that the estimated optimal inflation rate is quite insensitive to assuming alternative fixed costs values: over the considered range of fixed costs, the optimal inflation rate increases, but the maximal effect on the optimal inflation rate is small and around 0.1%. This continues to be true for reasonably sized output fluctuations ($y_t \gtrless y$).

I Proof to Proposition 5

We start by deriving the optimal inflation rate (42) and the recursive equation (43). In the absence of price rigidities, firms choose at all times their price such that their relative

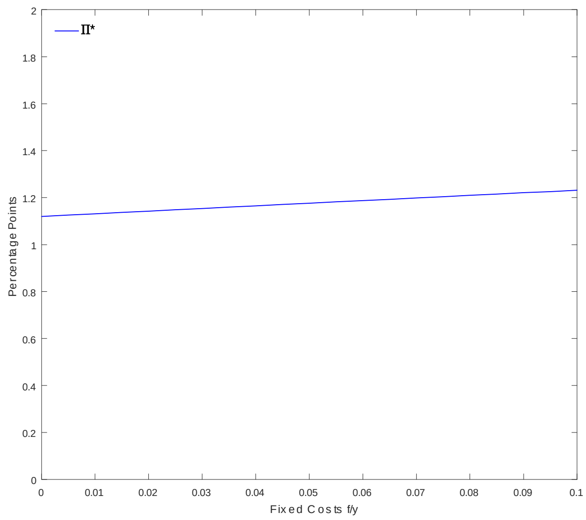

Figure 6: Robustness of optimal inflation estimates towards positive fixed costs.

price is inversely proportional to their relative productivity. This follows from the equation (33), which determines the optimal relative price in the absence of price rigidities and is reproduced here for convenience:

$$\frac{P_{jt}}{P_t} = \frac{1}{\Delta_t^e} \frac{Q_t}{G_{jt} Q_{t-s_{jt}}}. \tag{101}$$

Condition 2 implies that the previous equation holds also for $t = -1$.

We now show that the optimal relative price (101) can also be achieved by firm j in an economy *with* price setting frictions and non-constant δ-shock intensities under the optimal inflation rate stated in the proposition. This is so because absent δ-shocks, the optimal inflation rate insures that the firm's nominal price either remains constant (when there is no price indexation) or evolves over time in line with the price indexation rule, while equation (101) continues to hold. Taking growth rates of equation (101) and imposing $P_{jt} = \Xi_{t-1,t} P_{j,t-1}$, which holds in the absence of δ-shocks, delivers[42]

$$\frac{\Pi_t^\star}{\Xi_{t-1,t}^\star} = \frac{\Delta_t^e}{\Delta_{t-1}^e} \frac{g_t}{q_t}.$$

The previous equation implies equation (42).

To derive equation (43), we can rewrite the definition of Δ_t^e in equation (31) according

[42]In the presence of δ-shocks, prices are flexible so that equation (101) can easily be achieved.

to[43]

$$
\begin{aligned}
(\Delta_t^e)^{1-\theta} &= \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}}\right)^{1-\theta} \mathrm{dj} \\
&= \delta_0 + \delta_0(1-\delta_1)\sum_{s=1}^{\infty}(1-\delta)^{s-1}\left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right)^{1-\theta} \\
&= \delta_0 + \delta_0(1-\delta_1)\left(q_t/g_t\right)^{1-\theta} \\
&+ (1-\delta)\left(q_t/g_t\right)^{1-\theta}\left\{\delta_0(1-\delta_1)\sum_{s=1}^{\infty}(1-\delta)^{s-1}\left(\frac{q_{t-1} \times \cdots \times q_{t-s}}{g_{t-1} \times \cdots \times g_{t-s}}\right)^{1-\theta}\right\},
\end{aligned}
$$

where the term in parenthesis is equal to $(\Delta_{t-1}^e)^{1-\theta} - \delta_0$. This delivers equation (43) in the proposition.

In the absence of economic disturbances, equation (43) implies that Δ_t^e converges to

$$
\Delta^e = \left(\frac{\delta}{1-\delta_1+\delta}\right)^{\frac{1}{1-\theta}} \left(\frac{1-(\delta_1-\delta)(g/q)^{\theta-1}}{1-(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{1}{1-\theta}}.
$$

The steady state result in the proposition then follows from equation (42) and the assumption of no price indexation $\left(\Xi_{t-1,t}^{\star} \equiv 1\right)$.

J Proof to Proposition 6

For simplicity, we shall refer to P_t^N, which contains only products of age N or higher, as the measured price level and to $\Pi_t^N = P_t^N/P_{t-1}^N$ as the measured inflation rate. As before, we let P_t denote the ideal price level (using all products) and Π_t the ideal inflation rate. The proof proceeds in two steps. In a first step, we derive the measured inflation rate $\Pi_t^{N\star}$ in a setting where monetary policy implements $\Pi_t^\star$ from proposition 2 for the ideal inflation rate. In a second step, we show that if monetary policy implements $\Pi_t^{N\star}$ for the measured inflation rate, then this policy implements the same relative product prices as in the case where monetary policy implements $\Pi_t^\star$ for the ideal rate.

Step 1: In analogy to equation (20), which defines the ideal price level, the measured price level is defined as

$$
(P_t^N)^{1-\theta} = \delta\sum_{s=0}^{\infty}(1-\delta)^s\Lambda_t(s+N), \tag{102}
$$

where the weighted average cohort price $\Lambda_t(\cdot)$ is defined in equation (19). From proposition 2 it follows that under the optimal inflation rate $\Pi_t^\star$, firms with a Calvo shock do not find it optimal to adjust their price, so that we have for $s \geq k \geq 0$

$$
P_{t-s,t-k}^{\star} = \Xi_{t-s,t-k}^{\star} P_{t-s,t-s}^{\star}.
$$

[43] The following derivations assume that the initial productivity distribution at $t=-1$ is consistent with the assumed productivity process.

Using this result to rewrite equation (19) shows that the weighted average cohort price under the optimal inflation rate $\Pi_t^\star$ is

$$\Lambda_t(s) = (\Xi_{t-s,t}^\star P_{t-s,t-s}^\star)^{1-\theta}. \tag{103}$$

The previous equation implies

$$\begin{aligned}\Lambda_t(s+N) &= \left(\frac{\Xi_{t-(N+s),t}^\star}{\Xi_{t-(N+s),t-N}^\star}\right)^{1-\theta} \Lambda_{t-N}(s) \\ &= (\Xi_{t-N,t}^\star)^{1-\theta}\Lambda_{t-N}(s).\end{aligned}$$

Substituting this into equation (102) yields

$$(P_t^{N\star})^{1-\theta} = (\Xi_{t-N,t}^\star)^{1-\theta}\left[\delta\sum_{s=0}^{\infty}(1-\delta)^s\Lambda_{t-N}(s)\right],$$

where the expression in brackets is the ideal price level defined in equation (20) shifted N periods into the past. For a policy that implements the optimal inflation rate from proposition 2 for the ideal inflation measure, we thus have

$$P_t^{N\star} = \Xi_{t-N,t}^\star P_{t-N}^\star. \tag{104}$$

From the previous equation we get that measured inflation is then given by

$$\Pi_t^{N\star} = \frac{\Xi_{t-1,t}^\star}{\Xi_{t-N-1,t-N}^\star}\Pi_{t-N}^\star,$$

which is the inflation rate stated in the proposition.

Step 2: Using equation (103) to rearrange equation (102) delivers

$$\begin{aligned}(P_t^{N\star})^{1-\theta} &= \delta\sum_{s=0}^{\infty}(1-\delta)^s(\Xi_{t-(s+N),t}^\star P_{t-(s+N),t-(s+N)}^\star)^{1-\theta} \\ &= \delta(\Xi_{t-N,t}^\star P_{t-N,t-N}^\star)^{1-\theta} + (1-\delta)(\Xi_{t-1,t}^\star P_{t-1}^{N\star})^{1-\theta}.\end{aligned}$$

Dividing the previous equation by $(P_t^{N\star})^{1-\theta}$ and using equation (104) one obtains

$$\Pi_t^{N\star}/\Xi_{t-1,t}^\star = \left(\frac{1-\delta\left(P_{t-N,t-N}^\star/P_{t-N}^\star\right)^{1-\theta}}{1-\delta}\right)^{\frac{1}{\theta-1}}. \tag{105}$$

The previous equation shows how the relative price of firms with a δ-shock $(P_{t-N,t-N}^\star/P_{t-N}^\star)$ is determined so as to be consistent with $\Pi_t^{N\star}$. When monetary policy targets $\Pi_t^{N\star} = \frac{\Xi_{t-1,t}^\star}{\Xi_{t-N-1,t-N}^\star}\Pi_{t-N}^\star$, as assumed, then equation (105) coincides with equation (34) shifted back by N periods. Since equation (33) implies $1/\Delta_t^e = P_{t,t}^\star/P_t^\star$, this shows that monetary policy implements the same relative prices as a policy that implements $\Pi_t^\star$ from proposition 2 for the ideal inflation measure.