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Deferred Taxation under Default Risk

Abstract

In this article, we have used a continuous EBIT-based model to study deferred taxation under default risk. Quite surprisingly, default risk has been disregarded in research on deferred taxation. In order to underline its importance, we first calculated the probability of default, over a given time period, together with the contingent value of tax deferral. We then applied our theoretical model to a sample of 27,749 OECD companies. We showed that, when accounting for both firms with a negative EBIT and firms with a probability of default higher than 50% (over a 10-year period), a relevant percentage of firms were close enough to default. Hence, these taxpayers should not consider deferred taxation in their financial statements, for the sake of prudence. Moreover, under default, the expected present value of deferred taxes was much lower than that obtained in a deterministic context. Hence, if we look at deferred taxes from the Government's point of view, we must consider them as being risk-free loans. However, only a portion are subsequently repaid, due to default. This implies that, when a Government allows accelerated tax depreciation it should be aware of future losses due to default. So far, these estimates have been missing, although techniques do exist and are quite practical.

JEL-Codes: H250, M410.

Keywords: capital structure, contingent claims, corporate taxation and tax depreciation allowances.

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1 Introduction

Book accounting differs from tax accounting in most OECD countries, due to divergence between the purposes of Accounting Principles (e.g., US GAAP and IFRS) for financial reporting and those for tax forms. This could cause a gap between the value of assets and liabilities for book and tax purposes. As we know, this gap can depend on either permanent or temporary differences.[1] For instance, Poterba, Rao and Seidman (2011) find that, in the USA, the book-tax gap is mainly caused by temporary differences generated by the intertemporal mismatch between the carrying amount of assets and liabilities for tax and financial accounting (regarding, for instance, depreciation).

The theoretical implications of deferred taxation for corporations have been extensively studied in the accounting literature. For instance, Sansing (1998) and Guenther and Sansing (2000) demonstrated that deferred taxes have real effects on corporations, whether or not they revert over time. This is because they are directly charged against corporate earnings in the income statement. Mills (2006) highlighted the importance of the book-tax gap and pointed out several implications of deferred taxation for corporate policy. For example, corporations with large deferred tax assets are likely to lobby against tax cuts, whereas corporations with net deferred tax liabilities positions are likely to lobby for tax rate cuts.

Furthermore, the empirical accounting research has demonstrated that in the United States, the aggregate deferred tax balance for the corporate sector is a liability. This has increased over time, reaching about $400 billion by the end of 2004. Its main driver has been the difference between book and tax depreciation.[2] It is worth noting that existing literature has neglected the possible effects of default on deferred taxation. As we know, if a firm defaults, deferred taxes are written off. This means that a Government, that allows generous tax depreciation allowances, faces a potential loss. As will be shown, this social cost may well be relevant.

In order to explain the nexus between tax deferral and default we will use a twofold approach. Firstly, using a stochastic continuous-time model, we will calculate both the probability of default within a given period and the expected value of deferred taxes. Secondly, we will apply our theoretical model to a sample of 27,749 OECD companies. As shown, if we account for both firms with a negative EBIT and those with a probability of default higher than 50% (over a 10-year period), a relevant portion of firms in

[1] Positive temporary differences generate deferred tax liabilities, i.e., taxes to be paid in the future, which increase the total tax liability of a corporation; negative temporary differences generate deferred tax assets, i.e., credits against current taxes, thus reducing the total tax liability of a corporation.

[2] See, e.g., Mills and Plesko (2003), Hanlon and Shevlin (2005) as well as Poterba, Rao and Seidman (2011).

the sample is not far from default. For this reason, these taxpayers should not reckon deferred taxes in their financial statements, for the sake of prudence. Moreover, under default, the expected present value of deferred taxes is much lower than that obtained in a deterministic context. If we look at deferred taxes from the Government's point of view, we must therefore consider them as risk-free loans. However, only a portion is repaid, due to default.

The structure of the article is as follows. Section 2 provides a review of the literature on deferred taxation. Section 3 develops a contingent claim model, which enabled us to calculate both the probability of default and the present value of deferred taxes. Section 4 uses a dataset of European companies to provide a numerical analysis. Section 5 summarizes our findings and discusses their policy implications.

2 Literature Review

In this Section we briefly review the relevant literature on accelerated tax depreciation allowances and its effects on tax deferral. To our knowledge however, nobody has yet analyzed the effects of default risk on the evaluation of the (fair) value of deferred taxes.

The economic effects of deferred taxation on investment choices and corporate tax policy analysis were first analyzed by King (1974), who stressed the fact that law imposes a binding dividend constraint on corporations. Boadway and Bruce (1979) and Boadway (1980) pointed out that dividends should not exceed after-tax profit. Sinn (1987) suggested a slightly modified constraint, according to which dividends cannot exceed after-tax current profit, net of (accelerated) depreciation. Kanniainen and Södersten (1995) proved that the present form of the deferred tax constraint is ultimately defined by the financial reporting rules to which a corporation must adhere.

To the extent that book depreciation equals economic depreciation, the formulation of the deferred tax constraint proposed by Kanniainen and Södersten (1995) is therefore consistent with the temporary differences approach currently required by a country's GAAP. The deferred tax constraint, however, creates liquidity in the firm as it increases cash holdings. In Kanniainen and Södersten (1995), extra liquidity is neither distributed to shareholders (due to the deferred tax constraint), nor is invested in physical capital since it arises at the margin, i.e., when the optimal capital stock has already been accumulated.

Polito (2009) applied Kanniainen and Södersten's (1995) model of the deferred tax constraint to calculate the Effective Tax Rates on domestic investment financed by retained earnings implicit in the 2008 tax codes of five European countries. He showed that forward-looking effective tax rates are negatively biased because traditional models overlook dividend constraints associated with financial tax incentives, such as accelerated depreciation. More recently, Polito (2012) incorporated deferred taxation in Devereux and Griffith (1998, 2003) and showed that the impact of deferred taxation holds whether it is assessed using a framework consistent with the "old view", the "new view" and the "neutral view" of corporate taxation.[3] All these articles applied deterministic frameworks.

The accounting research deals with tax deferral by focusing on several aspects.[4] Some research aims to find whether taxation provides additional information to investors (see, e.g., Chludek, 2011, Dhaliwahl et al. 2004, and Gleason et al., Gleason and Mills, 2002). Other work looks at managers' behavior in terms of tax aggressiveness (see, e.g., Huseynov and Klamm, 2012, as well as Klassen et al., 2016). Moreover, there are many papers that discuss the valuation of tax deferral (see Amir et al., 1997 and 2001) as well as reversal (see Laux, 2013). Finally, Schackelford et al. (2012) point out that empirical research studying the relationship between tax and financial reporting focuses on tax compliance and discusses some issues regarding tax uncertainty.

This strand of literature has had a relevant boost due to the introduction of a new accounting rule (FIN 48) in the USA. The aim of this reform was to standardize the reckoning of uncertain tax benefits and to induce companies to disclose their tax reserve amounts (see Blouin et al.,2007 and Mills et al., 2010). Tax uncertainty arises from the difficulty in applying ambiguous tax laws and anticipating the consequences of a future tax audit (Diller et al., 2016; Mills et al., 2010).[5] Despite this reform, other sources of uncertainty (such as default) have still been neglected.[6] Moreover, with a few exceptions (Sansing, 1998, Guenther and Sansing, 2000, Diller et al., 2017), tax experts have analyzed taxation in a deterministic context. In any case, default risk has been disregarded.[7]

[3] See also Edgerton (2012), who compared the impact of investment tax credits and accelerated tax depreciation. He showed that the former relief has more impact on investment choices than the latter.

[4] For further details see Graham (2005), Hanlon and Heitzman (2010), Graham et al. (2012).

[5] Tax uncertainty combines three aspects: an investment decision, the tax compliance, and the financial reporting matters. Investment decision, tax effects and financial reporting incentives can potentially interact in ways that affect investment decisions (Hanlon et al. 2010). First of all, value, timing and uncertainty of tax payments affect the present value of a project and therefore invest choices. Secondly, through deprecation or expensing, the investment decision will affect pre-tax accounting earnings. Lastly, as pointed out by Edgerton (2012), tax policies that do not affect accounting profit (e.g., accelerated depreciation) are less effective in stimulating investment than those that increase accounting profit (e.g., investment tax credit). Concluding tax uncertainty is harmful for investment (Edmiston, 2004; Scholes et al., 2015).

[6] So far, scholars have mainly focused on the tax compliance uncertainty (Diller et al. 2016; De Simone et al. 2013; Mills et al. 2010).

[7] If we look at the main accounting principles all over the world (US GAAP as well as IAS/IFRS),

3 The model

In order to study deferred taxation under default risk, let us focus on a representative firm which makes an investment I at time 0.[8] Defining the EBIT (Earnings Before Interest and Taxes) at time t as Π_t, we introduce the following:[9]

Assumption 1 *A firm's investment I yields an EBIT Π_t, that evolves according to a geometric Brownian motion (with Π_0), where α is its deterministic growth rate, σ is the instantaneous standard deviation and dz_t is the increment of a standard Wiener process. Moreover, growth rate α cannot exceed the risk-free interest rate r.*[10]

Assumption 2 *Depreciation and amortization (DA) is constant and equal to λI with $\lambda \in (0,1)$. These resources are used to maintain asset I.*

Assumption 3 *At time 0, the firm decides how much to borrow from a perfectly competitive risk-neutral lender. Given r, C is the coupon paid to the lender under non-arbitrage.*

Assumption 4 *Debt is not renegotiable.*

Assumption 5 *If the firm does not meet its debt obligations (towards both the Government and lender), default occurs, namely the firm is expropriated by the lender.*

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we realize that they deal with uncertainty using a "detection free-risk" approach. According to IFRIC, detection risk is "the risk that the tax authority will detect an error or misapplication of the taxation requirement and accordingly, assess additional (less) tax. Detecting risk of 100% means the tax authority will detect all such errors or misapplication" (IFRIC, July 2014). Both US GAAP and IAS/IFRS adopt the detection risk free approach. Indeed, "it shall be presumed that the tax position will be examined by the relevant tax authorities that has full knowledge of all relevant information" (ASC 740-10-25-27) and similarly "an entity shall assume that a taxation authority with the rights to examine amounts reported to it will examine those amounts and have full knowledge of all relevant information making those examination" (IFRIC 23 par. 13). Under the detection risk-free approach, two (quite restrictive and unrealistic) assumptions are used. Firstly, the tax authority have full information about current and future events. Secondly, markets are fully efficient. Also in the GAAP perspective no space is given to default risk.

[8] For simplicity, we assume that a firm cannot postpone its investment decision. For a discussion of this choice see Panteghini (2012), and Panteghini and Vergalli (2016). Moreover, we also assume that economic and book depreciation coincide. In doing so, we can focus on the tax effects of depreciation allowances.

[9] For further details on the EBIT-based models see also Panteghini (2006, 2007, 2012).

[10] Notice that the difference $r - \alpha$ is the well-known convenience yield (see, e.g., McDonald and Siegel, 1985).

Assumption 6 *After default, the Government loses the deferred tax which was originally granted.*

Under Assumption 1, a firm's EBIT evolves as follows:

$$\frac{d\Pi_t}{\Pi_t} = \alpha dt + \sigma dz_t, \text{ with } \Pi_0 > 0. \quad (1)$$

According to Assumption 2, Depreciation and amortization is proportional to the initial investment I and therefore, the EBITDA (Earnings Before Interest, Taxes, Depreciation and Amortization) will be $\Pi_t + \lambda I$. As pointed out, the maintenance cost of the investment is equal to λI. In line with Leland (1994), Assumption 3 states that our representative firm pays a coupon C.[11] For simplicity, we assume that C is not optimally chosen but rather is given exogenously.[12] Moreover, according to assumption 4, debt cannot be renegotiated: this means that the firm's financial policy cannot be reviewed later.[13] Assumption 5 introduces the risk of default, respectively. Given (1), it is assumed that if the firm's EBIT falls to a given threshold level, the firm is expropriated by the lender (assumption 5). Moreover, according to assumption 6, default causes a loss for the Government. This assumption is of course in line with the main accounting principles.[14]

3.1 Taxation

Let us next introduce taxation. We define τ as the tax rate and assume that interest payments are fully deductible. For simplicity, we also assume full loss-offset.[15]

[11] Given this coupon and the risk-free interest rate r, the market value of debt can be calculated in the absence of arbitrage (see Leland, 1994).

[12] For a joint analysis of financial and investment choices with accelerated tax depreciation allowances see Panteghini and Vergalli (2016).

[13] The absence of debt renegotiation simplifies our analysis, although it does not affect the qualitative properties of the model. For a detailed analysis of financial decisions, with costly debt renegotiation, see e.g. Goldstein et al. (2001), and Hennessy and Whited (2005).

[14] For instance, according to the International GAAP 2009, (Generally Accepted Accounting Practice under International Financial Reporting Standards, vol.2., Wiley, West Sussex): "Deferred tax is an accounting model based on the premise that, for financial reporting purposes, the tax effects on transactions should be recognized in the same period as the transactions themselves". Moreover, "[t]he tax authorities cannot demand payment of an entity's deferred tax liability until it forms part of the legal tax liability for a future period; equally, an entity cannot recover its deferred tax assets from the tax authorities until they form a deduction in arriving at the legal tax liability for a future period."This means that, in the event of default, deferred taxes simply vanish and cause no tax liability.

[15] Of course, this simplifying assumption could be relaxed and we intend to leave these extensions for future research.

Here, we assume that tax depreciation allowances follow the straight-line method. According to these assumptions, a tax deduction will be ensured as long as our representative firm continue to produce. Since we want to focus on accelerated tax depreciation, we assumed that the inequality $\lambda_F \geqslant \lambda$ holds.[16]

In this case, deferred tax liabilities are generated whenever tax depreciation exceeds economic depreciation (i.e., $\lambda_F > \lambda$). This means that our firm can postpone a portion of the tax liability, thereby leading to deferred taxes. More precisely, a firm must usually set a provision for the tax rate levied on the difference between tax and economic depreciation, namely, $(\lambda_F - \lambda)$. Thus the term $\tau(\lambda_F - \lambda) I dt$ measures the deferred tax burden in an interval dt.[17]

Since we aim to study the effects of default, we must also take into account the lenders' tax treatment. For simplicity, we disregarded personal taxation and assumed that the lender's pre-default tax burden is nil. This means that, before default, the lender receives C at any time t and pays zero taxes. When however, default takes place, the lender earns Π_t instead of C.

3.2 Default

The calculation of the default threshold level of Π_t depends on the definition of default. In this article, we let a company default when Π_t does not allow to pay both taxes and the coupon. According to this definition, default may be triggered when the firm's EBIT falls to the exogenously given threshold point $\underline{\Pi}^D$. Given these assumptions, the after-tax cash flow will be equal to:

$$\Pi^N(\Pi_t; C) = (1-\tau)(\Pi_t - C) + \tau\lambda_F I. \tag{2}$$

According to Assumption 5, if $\Pi^N(\Pi_t; C) > 0$, the firm continues to operate. When however $\Pi^N(\Pi_t; C)$ goes to zero, default takes place and the firm is expropriated. Solving

[16] Guenther and Sansing (2004) used the declining balance method to measure the benefits of tax depreciation allowances. The quality of results however, does not change. To show this, let us define the tax depreciation rate and the economic depreciation rate under the declining balance method as δ and b, respectively. The present value of the tax benefit due to tax and economic depreciation will then be $\frac{\delta}{r+\delta}$ and $\frac{b}{r+b}$. According to our model we are assuming that $\frac{\delta}{r+\delta} \geq \frac{b}{r+b}$, or equivalently $\delta \geq b$. In our framework we focused on deferred taxation and therefore, we let the inequality $\lambda_F \geq \lambda$ hold. In doing so, we disregarded the effects of reversal, that occurs when tax depreciation is less than economic depreciation.

[17] As pointed out by Polito (2009), if tax is deferred, there is some undistributed cash, which could be re-invested in a risk-free bond. For simplicity, we did not account for this extra-provision and considered this cash as a collateral that reduces the risk of default. As long as the level of risk-free interest rates is low enough, this simplifying assumption does not affect the quality of results.

the equality $\Pi^N(\Pi_t; C) = 0$ for Π_t thus gives the threshold level below which default occurs:[18]

$$\underline{\Pi}^D = C - \frac{\tau}{1-\tau}\lambda_F I. \tag{3}$$

Notice that the inequality $\underline{\Pi}^D < C$ holds. This means that, thanks to tax depreciation allowances, the firm does not necessarily default if Π_t is less than C.[19] It is worth noting that $\underline{\Pi}^D$ is affected by taxation, and in particular, given (3) we can see that:

Proposition 1 *The higher the tax parameters λ_F and τ, the lower the threshold point $\underline{\Pi}^D$ is.*

According to Proposition 1, we can say that when deferred taxation is allowed, there is a reduction in the threshold point $\underline{\Pi}^D$. This means that, for any Π_t, default is delayed. Of course, this result has an important implication, in that accelerated depreciation and other causes of deferred taxation reduce the probability of default for any Π_t and affect the contingent evaluation of future events (see Panteghini and Vergalli, 2016).

Given these results we can therefore calculate the probability of default over a certain period as well as the expected present value of deferred taxes, contingent on the event of default. These computations are necessary for the numerical analysis of Section 4.

3.3 Probability

Let us focus on a given period $[0, T]$ and calculate the probability of default within time T. In other terms, we will have to measure the probability that Π hits $\underline{\Pi}^D$ in the $[0, T]$ period.

It worth noting that, given the inequality $\Pi_0 > \underline{\Pi}^D$, the probability of default is equivalent to the the probability of the geometric Brownian motion Π_t reaching the critical value $\underline{\Pi}^D$ within $[0, T]$. Given the geometric Brownian motion (1), we can write that, at any time t, the value of EBIT is $\Pi_t = \Pi_0 \exp\left[\left(\alpha - \frac{\sigma^2}{2}\right)t + \sigma W_t\right]$. As shown in

[18] In this model, for simplicity, we also assumed that the cutoff level of Π, below which default takes place, was exogenously given. Leland (1994) also analyzed a case where this threshold level was optimally chosen by shareholders.

[19] It is worth noting that this default assumption causes no loss of generality. If, for instance, we assumed that shareholders can decide when to default, the threshold level would be lower. However, the new threshold would be similar to $\underline{\Pi}^D$. For a detailed analysis of default conditions see, e.g., Panteghini (2006, 2007), who shows that the quality of results does not change when an endogenously given threshold point is found.

Appendix A the probability of default over the $[0, T]$ period is therefore equal to:

$$\begin{aligned} P\left[\tau_{\underline{\Pi}^D} \leq T\right] &= P[\xi_T^{(\theta)} \leq \ln\left(\frac{\underline{\Pi}^D}{\Pi_0}\right)] \\ &= \left(\frac{\underline{\Pi}^D}{\Pi_0}\right)^{2\theta} \Phi\left(\frac{\ln \frac{\underline{\Pi}^D}{\Pi_0} + \sigma^2\theta T}{\sigma\sqrt{T}}\right) + \Phi\left(\frac{\ln \frac{\underline{\Pi}^D}{\Pi_0} - \sigma^2\theta T}{\sigma\sqrt{T}}\right) \end{aligned} \quad (4)$$

where $\overline{\alpha} = \left(\alpha - \frac{\sigma^2}{2}\right)$ and $\theta = \frac{\overline{\alpha}}{\sigma^2}$. Apart from the complexity of formula (4), which will be used in our numerical analysis, we can say that, *coeteris paribus*, an increase in the tax parameters τ and λ_F reduces $\underline{\Pi}^D$ and hence, the probability of default.

3.4 The contingent value of deferred taxation

In the absence of default (and of other uncertain events, such as those referring to uncertain tax positions), the benefit due to tax deferral would be higher. If, for instance, the present value of deferred taxation is equal to 1 Euro, under default risk we expect that its "fair" value is less. In this Section we therefore calculate the value of deferred taxation, contingent to the event of default. As shown in Appendix B, this value, denoted by $DT(\Pi; C)$, will be equal to:

$$DT(\Pi; C) = \begin{cases} 0 & \Pi = \underline{\Pi}^D \\ \left[1 - \left(\frac{\Pi}{\underline{\Pi}^D}\right)^{\gamma_2}\right]\Phi & \Pi > \underline{\Pi}^D, \end{cases} \quad (5)$$

where $\Phi \equiv \frac{\tau(\lambda_F - \lambda)I}{r}$ is the present value of the benefit arising from deferred taxation in a deterministic context.[20] As can be seen, if the EBIT reaches the threshold level $\underline{\Pi}^D$, default occurs and the benefits from tax deferral vanish. If Π is higher, the benefit is positive. It is worth noting that $\left(\frac{\Pi}{\underline{\Pi}^D}\right)^{\gamma_2}$ measures the present value of 1 Euro contingent on the event of default (see Panteghini, 2007). Since $\left(\frac{\Pi}{\underline{\Pi}^D}\right)^{\gamma_2} \in (0, 1)$, the present value of the benefit arising from deferred taxation is lower under default risk. As shown in Appendix C we can see that:

Proposition 2 *An increase in both τ and λ_F time reduces $\left(\frac{\Pi}{\underline{\Pi}^D}\right)^{\gamma_2}$, and thus increases the contingent value of 1 Euro of deferred tax.*

[20] Using the notation of footnote 16, under a declining balance method, the deterministic value of the benefit arising from deferred taxation would be $\tau\left(\frac{\delta}{r+\delta} - \frac{b}{r+b}\right) I \geq 0$. Of course, the quality of results would not change.

Proof. See Appendix C. ■

According to Proposition 2, tax parameters affect the contingent evaluation of future events. An increase in both τ and λ_F reduces $\underline{\Pi}^D$. This means that, given Π, default is less likely. Hence, the contingent value of 1 Euro of deferred taxation increases.

4 A numerical analysis

It is worth noting that λ_F affects the probability of default. Furthermore, the higher the parameter λ_F the more valuable deferred taxation is. In order to study both effects we will run a numerical example based on the information contained in the ORBIS dataset.

4.1 Some descriptive statistics

This dataset provides information on approximately 250 million companies across the world and is provided by Bureau van Dijk. Here, we focused on active manufacturing companies belonging to the NACE class, letter C, from 10 to 33. We selected companies operating in OECD countries, with a turnover of over 20 million Euros. By choosing the 2011-2015 period, we obtained a set of 33,791 OECD companies (see Table 1).

Table 1: Descriptive statistics of the ORBIS population (millions of Euros).

Aggregate and %	**2015**	**2014**	**2013**	**2012**	**2011**	**Avg.**
Total Assets (TAs)	21,804	20,167	18,113	18,422	17,756	19,252
Turnover	18,079	17,661	16,439	16,940	16,691	17,162
EBIT	1,368	1,321	1,205	1,160	1,287	1,268
Interest paid	192	183	171	180	182	182
EBIT/TAs	15.0%	16.3%	16.5%	15.6%	18.1%	16.3%
Interest Paid/TAs	2.1%	2.3%	2.3%	2.4%	2.6%	2.3%

Our analysis was conducted exclusively on companies with available data, limited to those with at least three years of data. We therefore eliminated 6,042 firms and obtained a sample of 27,749 companies (see Table 2).

Table 2: Descriptive statistics of our sample (millions of Euros).

Aggregate and %	2015	2014	2013	2012	2011	Avg.
Total Assets (TAs)	19,696	18,346	16,505	16,800	16,261	17,522
Turnover	16,045	15,720	14,609	15,061	14,882	15,263
EBIT	1,183	1,148	1,055	1,003	1,129	1,104
Interest paid	185	177	165	175	177	176
EBIT/TAs	6.0%	6.3%	6.4%	6.0%	6.9%	6.3%
Interest Paid/TAs	0.9%	1.0%	1.0%	1.0%	1.1%	1.0%

Finally, the sample was divided into two groups: the first one contained companies with EBIT>0; the second consisted of companies with negative EBIT. The former (latter) group contains 87.8% (12.2%) of the whole sample. Tables 3 and 4 contain some descriptive statistics of these two groups.

Table 3: Descriptive statistics of the sub-sample with positive EBIT (millions of Euros).

Aggregate and %	2015	2014	2013	2012	2011	Avg.
Total Assets (TAs)	18,510	17,164	15,394	15,623	15,032	16,345
Turnover	14,885	14,531	13,451	13,839	13,595	14,060
EBIT	1,228	1,178	1,087	1,084	1,137	1,143
Interest paid	168	160	149	159	160	159
EBIT/TAs	6.6%	6.9%	7.1%	6.9%	7.6%	7.0%
Interest Paid/TAs	0.9%	0.9%	1.0%	1.0%	1.1%	1.0%

Table 4: Descriptive statistics of the sub-sample with negative EBIT (millions of Euros).

Aggregate and %	2015	2014	2013	2012	2011	Avg.
Total Assets (TAs)	1,186	1,181	1,111	1,176	1,229	1,177
Turnover	1,160	1,189	1,157	1,223	1,288	1,203
EBIT	-45	-29	-31	-80	-8	-39
Interest paid	17	16	16	16	17	16
EBIT/TAs	-3.8%	-2.5%	-2.8%	-6.8%	-0.7%	-3.3%
Interest Paid/TAs	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%

It is worth noting that firms with a systematically negative EBIT are relatively close to default, unless they can find additional resources. For precautionary reasons, therefore, they should not account for deferred taxation. For this reason, hereafter we will mainly focus on the positive-EBIT group of companies.

Table 5 reports the relevant parameter values that will be used for our numerical analysis. As can be seen, some are derived from the ORBIS data; others are chosen according to the following reasons. Since we want to focus on a long-term scenario T is set equal to 10 years.[21] The tax depreciation rate is assumed to range from λ to 1.5λ. The values of Π, C and I are obtained by using the ORBIS dataset. In line with Bilicka and Devereux (2012), we used the same risk-free real interest rate ($r = 5\%$). As regards the drift parameter α, we used two different values: 0 and 0.02. The value of the investment cost I was equal to the sum between the tangible and intangible assets contained in the financial statements. Subsequently, it was normalized to 100, according to the ORBIS data. Finally, in line with Dixit and Pindyck (1994), we assumed that the benchmark volatility is 20%. We also accounted for a more volatile context where the standard deviation was higher (30% and 40%, respectively).

Table 5: Average values used for the numerical simulation.

Variable	**Parameters**	**Value(s)**
Time horizon	T	10 and 15
Tax depreciation allowance	$\lambda_F = \gamma \cdot \lambda$	$\gamma \in \{1, 1.5\} \cdot \lambda$
EBIT	Π	from ORBIS
Coupon	C	from ORBIS
Tax rate	τ	25%
Investment cost	I	Normalized to 100
Growth rate	α	0% and 2%
risk-free real interest rate	r	5%
volatility	σ	20%, 30%, 40%

We then analyzed the probability of default in our sub-sample of firms characterized by a positive EBIT. To do so, we divided the sub-sample into ten deciles, obtained by means of the distribution of interest payments. This allowed us to calculate the number of firms with a default probability higher than 50%.[22] Table 6 shows that the percentage of distressed firms crucially depends on the parameter values. Table 6 also shows that an increase in σ leads to an increase in the probability of default. This also suggests that if a firm's risk is higher than the average one (about $\sigma = 0.2$), the probability of default is higher.

As regards tax depreciation allowances, we have used two values of λ_F: 0.10 and 0.15. In the former case, no deferred taxation is reckoned, since $\lambda_F = \lambda = 0.1$. In

[21] For sake of brevity, we did not add the results obtained with $T = 15$. Data are available upon request. In any case, according to our theoretical framework, we know that the longer the arrival time T the higher the probability of default and the smaller the contingent value of deferred taxation will be.

[22] Of course, the choice of the threshold probability of 50% is fully arbitrary. However, it allows to identify the distribution of firms of firms with high default risk. Obviously, if we raised the threshold level of probability, the number of firms in financial distress would be smaller and vice versa.

the latter case, with $\tau = 25\%$, the yearly deferred tax is $\tau(\lambda_F - \lambda)I = 1.25$. In the absence of default therefore, the present value of deferred taxation will then be equal to $\frac{\tau(\lambda_F-\lambda)I}{r} = \frac{1.25}{0.05} = 25$.

4.2 Results

As shown in Table 6, deciles 9 and 10 contain a relevant number of firms, whose default probability is higher than 50%. If we sum these firms to those with negative EBIT (that is, 3385 companies, as shown in Table 4) we can say that many companies are close to default. This means that a great deal of resources might be lost by the Government. In particular, if $\sigma = 0.2$, the number of firms with negative EBIT and/or with a probability of default higher than 50% is 3977 on a sample of 27,749 firms (i.e., 14.33%). Of course if volatility is higher the number of almost distressed firms rises. If $\sigma = 0.4$ the number of firms with negative EBIT and/or with a probability of default higher than 50% is 4735 (i.e., 17.06% of the sample).

Table 6: Number of firms with probability of default higher than 50%(distribution according to interest payments)

Interest paid		**Probability of default higher than 50%**			
σ	λ_f	**Decile 10**	**Decile 9**	**Decile 8**	**Decile 1-7**
0.2	**0.1**	654 (27%)	211 (9%)	16 (1%)	0 (0%)
0.3	**0.1**	1000 (41%)	329 (14%)	21 (1%)	0 (0%)
0.4	**0.1**	1414 (58%)	593 (24%)	33 (1%)	0 (0%)
0.2	**0.15**	534 (22%)	58 (2%)	0 (0%)	0 (0%)
0.3	**0.15**	809 (33%)	85 (3%)	0 (0%)	0 (0%)
0.4	**0.15**	1213 (50%)	137 (6%)	0 (0%)	0 (0%)

Number of firms for each decile = 2437; for the deciles 1-7 the total sample consists of 17052 firms, T=10 years.

Finally, if we look at the number of distressed firms (i.e., firms with a probability of default higher than 50%), we see that an increase in λ_F has a twofold effect: on the one hand, it increases the amount of deferred taxes (and therefore the risk-free loan ensured by the Government), on the other hand, it reduces the probability of default. This is in line with Proposition 1, according to which an increase in λ_F delays default and at the same time increases the contingent value of deferred taxation. In order to understand this twofold effect, we calculated the contingent value of deferred taxation. As usual, we focused on the sub-sample of firms with a positive EBIT. Again, we analyzed the sample by focusing on the interest payments distribution.

Table 7: The contingent value of deferred taxation for deciles obtained by using the ratio Interest payments/Total assets

	$\lambda_f = 0.15$					
Interest paid	$\alpha = 0.00$			$\alpha = 0.02$		
Decile	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
1	24.99	24.78	23.87	25.00	24.99	24.98
2	24.87	23.79	21.53	24.99	24.97	24.78
3	24.57	22.60	19.59	24.99	24.89	24.36
4	24.36	21.97	18.70	24.99	24.82	24.08
5	23.97	21.03	17.48	24.98	24.67	23.60
6	23.51	20.07	16.35	24.97	24.45	23.04
7	23.26	19.60	15.81	24.95	24.32	22.74
8	23.00	19.16	15.32	24.94	24.20	22.44
9	22.70	18.67	14.80	24.91	24.01	22.09
10	22.25	17.97	14.10	24.86	23.74	21.58
Total	22.85	18.92	15.07	24.92	24.10	22.27

As shown in Table 7, if $\lambda = 0.10 < \lambda_f = 0.15$, the contingent value of deferred taxation not only depends on the ratio between Interest payments and Total assets (the higher this ratio, the lower the value of deferred taxation is), but also on volatility. In particular, this effect has a dramatic impact on the "fair" value of deferred taxation. If, for instance, we look at firms in decile 5, we can see that, when $\sigma = 0.2$ the contingent value is almost 24. This means that volatility has a negligible impact on the "fair value" of deferred taxation. When however volatility increases, the result is quite different and, with $\sigma = 0.4$, the contingent value of deferred taxation decreases by 27%. This effect is less relevant if a firm's EBIT has a positive growth rate. As a result, we can see that the contingent value of deferred taxes depends on α, σ and the starting value of EBIT.

5 Conclusion

In this article, we have analyzed deferred taxation under default risk. Despite the fact that tax deferral has long been analyzed from many points of view, we have not found any work about this specific topic. To fill the gap, we have therefore developed a simple continuous-time model aimed at calculating both the probability of default and the contingent value of deferred taxation. The calculation of the probability of default within a given time period allows us to check whether deferred taxes should be reckoned or not. In our view (and according to many accounting principles), firms with a high probability of default should not reckon deferred taxes because of their low opportunity of recovery. Quite surprisingly, both the existing literature on deferred taxation and the current accounting principles are silent on this point.

The calculation of the contingent ("fair") value of deferred taxes allows us to evaluate the present value of the tax benefit arising from accelerated tax depreciation, conditional on default. Again, this point is missing in existing literature, despite its importance. Since deferred tax allowances are equivalent to a risk-free loan guaranteed by the Government, we have shown how to evaluate the expected loss faced by the Government in the event of default. Our results lead to a twofold implication. The first regards existing accounting principles, that should consider default risk. For precautionary reasons, indeed, firms with a high probability of default should omit deferred tax reckoning. The second implication is about the public effect of default: from a Government's point of view, deferred taxes are a considered as risk-free loans, although a relatively large portion of them are not repaid. For this reason we suggest that a Government willing to apply accelerated depreciation should also account for the expected possible loss due to the default. We have shown that techniques do exist and are quite easy to manage.

A Probability

Let us focus on a stochastic process that follows a geometric Brownian motion. If at time 0 we have $\Pi_0 > 0$ then at time $t > 0$ we will have $\Pi_t = \Pi_0 \exp\left[\left(\alpha - \frac{\sigma^2}{2}\right) t + \sigma W_t\right]$. Let us next introduce a lower bound $\underline{\Pi}^D < \Pi_0$ and calculate the probability to hit $\underline{\Pi}^D$ within time T. To do so we must define the hitting time as follows: $\left\{\tau_{\underline{\Pi}^D} \leq T\right\} = \left\{\Pi^* \leq \underline{\Pi}^D\right\}$ where $\Pi^* = (\Pi_t^*)_{t\geq 0}$ and $\Pi_t^* = \min_{t\epsilon[0,T]}$. Given this notation the probability that Π_t hits $\underline{\Pi}^D$ in the $[0,T]$ period is:

$$P\left[\tau_{\underline{\Pi}^D} \leq T\right] = P\left[\Pi^* \leq \underline{\Pi}^D\right] = P[\xi_T^{(\theta)} \leq \ln\left(\frac{\underline{\Pi}^D}{\Pi_0}\right)], \tag{6}$$

where

$$\xi_t^{(\theta)} = \min_{s\in[0,t]} \left\{\overline{\alpha} t + \sigma W_t\right\}. \tag{7}$$

Using the definitions $\overline{\alpha} \equiv \left(\alpha - \frac{\sigma^2}{2}\right)$ and $\theta \equiv \frac{\overline{\alpha}}{\sigma^2}$, we can rewrite (7) as follows:

$$\xi_t^{(\theta)} = \min_{s\in[0,t]} \left\{\overline{\alpha} t + \sigma W_t\right\} = \min_{s\in[0,t]} \left\{\theta t + W_t\right\}. \tag{8}$$

Let us next use (6) and (8). Following Harrison (1985, pp. 11-14), Sarkar (2000, pp. 222-223) and Cappuccio and Moretto (2001, pp. 8-9) gives:

$$\begin{aligned}
P\left[\tau_{\underline{\Pi}^D} \leq T\right] &= P\left[X_T^* \leq B\right] \\
&= P[\xi_T^{(\theta)} \leq \ln\left(\frac{\underline{\Pi}^D}{\Pi_0}\right)] \\
&= e^{2\theta \ln\left(\frac{\underline{\Pi}^D}{\Pi_0}\right)} \Phi\left(\frac{\ln\left(\frac{\underline{\Pi}^D}{\Pi_0}\right) + \sigma^2\theta T}{\sigma\sqrt{T}}\right) + \Phi\left(\frac{\ln\left(\frac{\underline{\Pi}^D}{\Pi_0}\right) - \sigma^2\theta T}{\sigma\sqrt{T}}\right) \\
&= \left(\frac{\underline{\Pi}^D}{\Pi_0}\right)^{2\theta} \Phi\left(\frac{\ln\left(\frac{\underline{\Pi}^D}{\Pi_0}\right) + \sigma^2\theta T}{\sigma\sqrt{T}}\right) + \Phi\left(\frac{\ln\left(\frac{\underline{\Pi}^D}{\Pi_0}\right) - \sigma^2\theta T}{\sigma\sqrt{T}}\right).
\end{aligned}$$

Formula (4) is thus obtained.

B The derivation of (5)

The value of deferred taxation can be written as:

$$DT\left(\Pi;C\right)=\begin{cases}0 & \Pi=\underline{\Pi}^{D},\\ \left[\tau\left(\lambda_{F}-\lambda\right)I\right]dt+e^{-rdt}\mathbb{E}\left[DT\left(\Pi+d\Pi;C\right)\right] & \Pi>\underline{\Pi}^{D}.\end{cases}\tag{9}$$

$DT\left(\Pi;C\right)$ is given by the solution of the following system of Second Order Differential Equation, and the threshold level $\underline{\Pi}^{D}$ is given by (3). Differentiating (9) and applying Itô's Lemma gives the following non-arbitrage condition:

$$\frac{1}{2}\sigma^{2}\Pi^{2}DT_{\Pi}\left(\Pi;C\right)+\alpha\Pi DT_{\Pi}\left(\Pi;C\right)-rDT\left(\Pi;C\right)=-\tau\left(\lambda_{F}-\lambda\right)I.\tag{10}$$

To solve equation (10), let us apply the following general function:

$$DT\left(\Pi;C\right)=\begin{cases}0 & \Pi=\underline{\Pi}^{D},\\ \Phi+A_{1}\Pi^{\gamma_{1}}+A_{2}\Pi^{\gamma_{2}} & \Pi>\underline{\Pi}^{D},\end{cases}\tag{11}$$

where $\Phi\equiv\frac{\tau(\lambda_{F}-\lambda)I}{r}$ is the present value of the benefit arising from deferred taxation in a deterministic context. To solve (10) we use (11) and apply the following boundary condition (see Dixit and Pindyck, 1994):

$$DT\left(\underline{\Pi}^{D};C\right)=0.\tag{12}$$

Moreover, we assume the absence of financial bubbles. This means that the equality $A_{1}=0$ must hold. Setting $A_{1}=0$, and using (12) gives:

$$\Phi+A_{2}\underline{\Pi}^{D\gamma_{2}}=0,\tag{13}$$

and thus $A_{2}=-\Phi\underline{\Pi}^{D^{-\gamma_{2}}}$. Using these results in (11) gives function (5).

C Comparative statics

Given $\underline{\Pi}^{D}=C-\frac{\tau}{1-\tau}\lambda_{F}I$ and $\gamma_{2}<0$ it is easy to show that:

$$\begin{aligned}\frac{\partial\underline{\Pi}^{D}}{\partial\tau} &= -\frac{1}{1-\tau}\lambda_{F}I-\frac{\tau}{\left(1-\tau\right)^{2}}\lambda_{F}I=\left[-\frac{1-\tau}{\left(1-\tau\right)^{2}}-\frac{\tau}{\left(1-\tau\right)^{2}}\right]\lambda_{F}I\\ &= -\frac{\lambda_{F}I}{\left(1-\tau\right)^{2}}<0,\\ \frac{\partial\underline{\Pi}^{D}}{\partial\lambda_{F}} &= -\frac{\tau}{1-\tau}I<0.\end{aligned}$$

Moreover, we obtain:

$$\frac{d\left[\left(\frac{\Pi}{\underline{\Pi}^D}\right)^{\gamma_2}\right]}{d\tau} = \underbrace{\left[(-\gamma_2)\,\Pi^{\gamma_2}\underline{\Pi}^{D-\gamma_2-1}\right]}_{>0}\frac{\partial\underline{\Pi}^D}{\partial\tau} \text{ with } \frac{\partial\underline{\Pi}^D}{\partial\tau} = -\frac{\lambda_F I}{(1-\tau)^2} < 0,$$

$$\frac{d\left[\left(\frac{\Pi}{\underline{\Pi}^D}\right)^{\gamma_2}\right]}{d\lambda_F} = \underbrace{\left[-\gamma_2\Pi^{\gamma_2}\underline{\Pi}^{D-\gamma_2-1}\right]}_{>0}\frac{\partial\underline{\Pi}^D}{\partial\lambda_F} \text{ with } \frac{\partial\underline{\Pi}^D}{\partial\lambda_F} = -\left(\frac{\tau}{1-\tau}\right)I < 0.$$

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