Guaranteed Renewable Life Insurance Under Demand Uncertainty

Michael Hoy, Afrasiab Mirza, Asha Sadanand

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Abstract

Guaranteed renewability is a prominent feature in many health and life insurance markets. It is well established in the literature that, when there is (only) risk type uncertainty, the optimal GR contract with renewal price set at the actuarially fair price for low risk types provides full insurance against reclassification risk. We develop a model that includes unpredictable (and unobservable) fluctuations in demand for life insurance as well as changes in risk type (observable) over individuals' lifetimes. The presence of demand type heterogeneity leads to the possibility that optimal GR contracts may have a renewal price that is either above or below the actuarially fair price of the lowest risk type in the population. Individuals whose type turns out to be high risk but low demand renew more of their GR insurance than is efficient due to the attractive renewal price. This results in incomplete insurance against re-classification risk. Although a first best efficient contract is not possible in the presence of demand type heterogeneity, the presence of GR contracts nonetheless improves welfare relative to an environment with only spot markets. Our results also apply to a comparison of environments with short versus long term (front loaded) insurance contracts.

JEL-Codes: D800, D860, G220.

Keywords: insurance, guaranteed renewability, re-classification risk, demand uncertainty.

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1   Introduction

Guaranteed renewability, which is a prominent feature in health and life insurance markets, provides an opportunity for individuals to insure against reclassification risk. This works as follows. Consider a set of ex ante identical individuals each of whom purchases an initial ten year term life insurance contract with a view of possibly purchasing a subsequent policy at the end of the term. By the end of the contract period, some insureds may have discovered that their mortality status has changed. If this change is observable to insurers, then the price for a new insurance contract will reflect that change in risk. Individuals recognize ex ante that their risk type may change over time and so prefer to avoid the prospect of premium risk associated with stochastic mortality prospects. Guaranteed renewable (GR) contracts contain a promise to offer a subsequent insurance policy at the expiry date of the first contract at a price that is independent of any changes in mortality risk. The premium for the implicit insurance against reclassification risk is embedded in the first contract (earlier period) through an extra premium assessment – a phenomenon known as front loading. This allows insurers to offer insurance to those individuals who turn out to be higher risk types in the subsequent ten year period at a price below their actuarially fair rate. As long as the amount of front loading is sufficient, the added profit from the first (period) contract compensates for insurers' losses from the second (period) contract. A similar phenomenon may be reflected in short term versus long term insurance contracts (e.g., ten versus twenty years) with longer contracts providing insurance against reclassification risk through front loading in order to keep premiums later in the contract sufficiently low to avoid lapsation by better risks.

In our paper we focus on the implications for GR (and long term) insurance to ameliorate premium risk when individuals face uncertainty over future changes in both mortality risk and insurance needs. We consider a model that has no market frictions or other impediments such as imperfect capital markets or the existence of resettlement or viatical market opportunities (as mentioned above) that can thwart GR insurance to fully protect against

reclassification risk. We develop a two-period model of insurance in which individuals are homogeneous in the first period and hold the same beliefs about the likelihood of becoming a low or high demand type in the second period. An important feature of insurance demand is how it changes over the life cycle. As noted in Hong and Rios-Rull (2012), average demand follows a life cycle pattern that rises from young adulthood to "around age 45 for males and 35 - 40 for females" (based on 1990 data). They also show, in their Figure 1 (p. 3705), that there is substantial variation in demand across individuals at all ages and especially around the peak level of demand. This means that in order to have an ideal amount of coverage for premium risk in the future, one may have to hold more insurance than is optimal early in life (i.e., for the younger part of the life cycle where demand tends to be increasing). This turns out to be a critical factor in determining the extent to which GR can provide insurance against reclassification risk. We allow for the possibility that second period demand may be higher or lower than first period demand for either or both demand types. Each individual's risk type also evolves over time in a similar manner; that is, individuals have the same mortality risk in the first period but their mortality risk diverges in the second period. Moreover, in period 1 individuals hold the same beliefs about the evolution of their risk type for period 2.[1]

There are many potential sources of (evolving) demand type heterogeneity relevant to life insurance. The amount of coverage an individual desires at any point in time is affected by a number of factors, including marital status, income of the insured individual, number of children, earning options for other family members, expenditure requirements for the survivor family should death of the individual occur, the insureds pure (altruistic) preferences, etc.... All of these factors can change over time.[2]. Some of these characteristics are unobservable

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[1]In a similar environment but without differential demand types, Peter, Richter, and Steinorth (2015) consider the implications of individuals learning imperfectly about their risk type over time with this information being private. Fei, Fluet, and Schlesinger (2013) also use a model that features demand uncertainty but that do not include risk type uncertainty nor any dynamic features of insurance demand in our model of GR insurance.

[2]See the survey by Zietz (2003), and particularly tables 2 and 3, for empirical evidence on the effect of personal characteristics on the demand for life insurance. Some of these characteristics would typically change stochastically over a person's lifetime.

to the insurer and others, while observable, typically have idiosyncratic and unobservable implications on individuals' preferences for insurance. We treat demand type as unknown to the insurer. Insureds, on the other hand, learn about their demand preferences and change their valuation of insurance accordingly. This uncertainty in future demand represents a challenge to individuals when deciding how much guaranteed renewable insurance (GR) is appropriate to purchase at a given point in their lifetime, which in turn compromises the ability of guaranteed renewable insurance contracts to protect consumers against reclassification risk. Moreover, given the non-contractible nature of demand risk, the combination of variations in both morality risk and demand risk creates a type of adverse selection problem, as described below.

One important feature of interest regarding the performance of GR insurance is contract lapsation. If the renewal terms are not sufficiently attractive to people who discover they have become relatively low risk, then they will have an incentive to opt out of the first contract at or before the expiry date and not purchase a subsequent contract at the agreed upon price. Moreover, if the renewal price is below the actuarially fair rate for high risk types, which it must be in order to provide protection against reclassification risk, then those with low insurance demand who turn out to be high risk will wish to renew more insurance than is efficient. This means the second period contract will have a disproportionate share of demand from high risk types which creates a stress on the degree of front loading required to make GR insurance financially sustainable.[3]

Our paper contributes to the literature on GR insurance by providing an explicit welfare analysis of a two-period model of decision making based on expected utility preferences which evolve over time.[4] Individuals may find their preference for insurance either rises or falls for the later (second) period under consideration. The various possible demands for second period insurance may not align with first period insurance needs and so the only way for an

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[3]Evidence of this phenomenon in a health insurance market is provided by Hofmann and Browne (2013).
[4]In this paper we focus on life insurance, although the basic principles of GR insurance apply also to health insurance.

individual to hold the optimal amount of GR insurance from the second period perspective may be to overinsure in the first period. This scenario would be expected during (typically earlier) periods of life when future insurance demand tends to increase (on average). We illustrate how these two factors compromise the effectiveness of GR insurance to protect against premium risk. We also see how these patterns influence the structure of premiums of GR insurance; that is, both the degree of front loading and the renewal price.

It has been shown (e.g., Pauly, Kunreuther, and Hirth (1995) and Hendel and Lizzeri (2003)) that, in ideal settings, GR insurance or long term insurance with sufficient front-loading of premiums can fully eliminate reclassification risk.[5] Moreover, Hendel and Lizzeri (2003) show that more front-loading is consistent with increased efficiency as it generates more "commitment" on the part of households to renew their GR. However, in the presence of demand fluctuations this is no longer true and we show that less commitment or front-loading can improve welfare. Alternatively, Frick (1998) demonstrates how capital market imperfections can destroy the potential for GR insurance to provide complete protection against reclassification risk. An imperfect capital market is but one market characteristic that can limit the ability of GR insurance to offer protection against reclassification risk.[6] In different contexts, Polborn, Hoy, and Sadanand (2006) and Daily, Hendel, and Lizzeri (2008) show that if there is uncertainty about future insurance needs as well as risk type and individuals have access to settlement markets where they can sell their previously purchased (long term) insurance coverage, then GR insurance contracts cannot completely eliminate premium risk.

The rest of the paper is organized as follows. The next section presents the basic model while section 3 characterizes the first best (social) optimum as well as the characteristics

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[5]In an early contribution to this literature, Cochrane (1995) developed an interesting alternative approach to protecting individuals from reclassification risk by introducing lump-sum severance payments for individuals whose risk deteriorates at the end of a period of insurance cover. The ideal setting in Hendel and Lizzeri (2003) corresponds to sufficiently low growth in income relative to importance of front loading that individuals are willing to accept sufficient front loading to cover even protection against reclassification risks for relatively good health states. This follows from their results (ii) and (iii) of Proposition 1 and conditions explained in footnote 13, p. 310.

[6]For a more complete discussion, see Peter et al. (2015).

of the allocation when (only) spot insurance markets are available and when GR insurance is also available. The main welfare analysis is provided in section 4 where we show that (i) adding GR insurance to spot markets will improve welfare as long as there is some uncertainty about risk type and (ii) may, under certain conditions, improve welfare when demand type uncertainty is not present despite the absence of other frictions such as capital market imperfections that are used in existing models. We also show that GR insurance cannot generate a first best allocation in the presence of demand type uncertainty. Moreover, contrary to existing models, it turns out that in our model GR insurance may not lead to a first best allocation even when demand type uncertainty is not present. Section 5 presents simulation results which help to further our understanding of how the combination of demand and risk type uncertainty affects the structure of contracts and the welfare effects of GR contracts. Section 6 provides conclusions.

2 Model

We consider an economy populated by a measure one of ex-ante (identical) individuals who buy life insurance and live at most two periods. Each such individual has a family associated with him. In case of the individual'sdeath, we refer to his associated family as the *survivor family* while in any period that he lives we refer to his associated family as the *whole family*. No other members of the family may die. Preferences relate to those of the insurance buyer, albeit he takes his family members well-being into account. For simplicity, we assume he is the only income earner in the family and receives income y_1 at the beginning of period 1. If he survives to period 2, he receives a further y_2 at the beginning of period 2. His risk and demand type evolve over time. Each individual has a probability of death of p, $0 < p < 1$, in the first period of life. If an individual survives the first period, then his probability of death in the second period depends on whether he is a high or low risk type. We describe risk type by index $i \in \{L, H\}$ for low and high risk type, respectively,

with associated probabilities p_L, p_H where $0 < p_L < p_H < 1$. Moreover, we assume all risk types have a higher mortality in period 2 than in period 1 $(0 < p < p_L < p_H < 1)$.[7]

The individuals (and associated families) are homogeneous in all respects in the first period and discover their risk type associated with second period mortality at the beginning of period 2. Insurers also observe individuals' risk type and so there is no asymmetric information in this regard. However, individuals also discover their demand type at the beginning of period 2 which insurers do not observe.[8] In period 1 individuals perceive their prospects about both risk type and demand type development according to the actual population portions of q_i, $i \in \{L, H\}$ for risk type and r_j, $j \in \{l, h\}$ for demand type where $i \in \{L, H\}$ represents low and high risk type while $j \in \{l, h\}$ represents low and high demand type. Risk and demand type are not correlated (i.e., the probability of an individual being risk type i and demand type j is $q_i \cdot r_j$). These differing preferences (demand type) for life insurance in period 2 are reflected in the felicities for death state consumption in period 2 as described below.[9]

So, period 2 decisions depend on both the individual's risk and demand type, characterized by the pair ij, with $i \in \{L, H\}$ and $j \in \{l, h\}$. In cases where confusion may occur, we index the time period and the state (life or death) using superscripts. We refer to the death state by D and the life state by N (i.e., not death). Thus, consumption in period 2 for a person of type ij is represented by C_{ij}^{2D} in the death state and C_{ij}^{2N} in the life (i.e., non-death) state. We write their felicity for consumption in the life state for period t as $u_t(\cdot), t = 1, 2$. Their felicities in the period 2 death state, which depend on demand type j are modelled by the function $v_2(\cdot; \theta_j)$ and $v_2'(x; \theta_h) > v_2'(x; \theta_l)$ for all $x > 0$. The latter captures that high demand types want more insurance than low demand types. The functions u_2 and v_2 satisfy the usual assumptions for risk averters (i.e., $u_2', v_2' > 0$ and $u_2'', v_2'' < 0$).

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[7]See Hendel and Lizzeri (2003).

[8]It is equivalent to alternatively assume demand characteristics are be observable but not contractible.

[9]Note that one could instead introduce demand heterogeneity through different felicities in the life state. This would have similar effects as in our model. Note that such an example may be a liquidity shock associated with the life state of the world.

Individuals have homogeneous preferences in period 1 with their felicity in the life state being $u_1(\cdot)$ and that in the death state being $v_1(\cdot)$, the latter of which is meant to reflect the insurance purchaser's perspective on the survivor family's future utility (including prospects for period 2 and beyond).[10] This can naturally be different from the felicity in the death state of period 2. Similar to the above notation for period 2, consumption in the death and life states of period 1 are C^{1N} and C^{1D}, respectively. Note that since individuals do not know their demand or risk type in period 1, there is no subscript pair ij attached to these consumptions.

Timing of information revelation and taking of decisions is as follows. At the beginning of period 1 individuals decide on the amount of spot insurance to hold for period 1 (L^1), amount of guaranteed renewable insurance (L^{1G}), and an amount of savings, s.[11] $L^1 + L^{1G}$ is the insurance coverage in period 1 and savings is also available to the survivor family should the insured die in period 1. Note that s is not deducted from consumption in the death state as the survivor family gets to use savings from period 1 into the future. The felicity v_1 reflects continuation utility for this survivor family. L^{1G} is the amount of that coverage that could be renewed at a guaranteed (predetermined) rate in the second period should the insured survive to period 2. We let π^1 be the price of first period spot insurance. We assume risk neutral insurers in a competitive environment and having no administrative costs. Insurers can fully commit to long-term contracts. Thus, since coverage from first period spot insurance expires at the end of period 1, competition leads to $\pi^1 = p$ (i.e., actuarially fair insurance).

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[10]This is an indirect utility based on how the family's circumstances will evolve should the income earning insurance buyer die in the first period. The family may be expected to evolve in the sense that a surviving spouse has uncertain prospects of generating income in period 2 (as well as the remainder of period 1) and so on. That is, death felicities should be interpreted as continuation utilities. This simplistic "main breadwinner" sort of model could be transformed to one with two earners and two potential insurance buyers. However, that would lead to a much more complicated model and, we believe, not significantly improved insights.

[11]In an intertemporal model, insurance purchases shift consumption in a current period into any loss state of a future period and therefore creates in some degree consumption smoothing, albeit state contingent consumption smoothing. As shown by Hofmann and Peter (2015), if one omits the savings decision in such a model, the role of insurance (reimbursement for financial losses) becomes contaminated with the motive for income smoothing.

The front-loading of guaranteed renewable insurance allows an individual the option to renew at a price which earns the insurer expected losses. This implies that the unit price of this coverage, π^{1G}, must exceed p, the expected unit cost of providing first period insurance cover. This is explained in greater detail later. At the beginning of period 2 the spot insurance from period 1 expires and individuals learn about their risk type (i) and their demand type (j). Insurers know the risk type of insureds but not their demand type. Each insured then chooses how much guaranteed renewable insurance that was purchased in period 1 (L^{1G}) to renew (L_{ij}^{2G}) at the predetermined (guaranteed) price of π^{2G}. This amount will depend on both risk and demand type with (obviously) $L_{ij}^{2G} \leq L^{1G}$. Importantly, following Hendel and Lizzeri (2003) we assume that individuals cannot commit to long-term contracts. Therefore, they may opt out of their GR contracts and purchase spot insurance (L_{ij}^2) at the risk type specific price $(\pi_i^2 = p_i)$. Note that if $\pi^{2G} > p_L$, low risk types would not renew any of their guaranteed renewable insurance from period 1.[12] Expected utility from the perspective of the beginning of period 1 is

$$EU = pv_1(C^{1D}) + (1-p)u_1(C^{1N}) + (1-p)\sum_i\sum_j q_i r_j \left[p_i v_2(C_{ij}^{2D}; \theta_j) + (1-p_i)u_2(C_{ij}^{2N}) \right], \quad (1)$$

where

$$C^{1N} = y_1 - s - \pi^1 L^1 - \pi^{1G} L^{1G},$$

$$C^{1D} = y_1 + (1 - \pi^1)L^1 + (1 - \pi^{1G})L^{1G},$$

$$C_{ij}^{2N} = y_2 + s - \pi_i^2 L_{ij}^2 - \pi^{2G} L_{ij}^{2G},$$

$$C_{ij}^{2D} = y_2 + s + (1 - \pi_i^2)L_{ij}^2 + (1 - \pi^{2G})L_{ij}^{2G},$$

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[12]We also assume that low risk types renew all of their L^{1G} if $\pi^{2G} = p_L$, the spot price for low risk types in period 2. This is of no consequence since competition means π_L^2 is equal to the low risk type loss probability which means the lapsation-renewal decision has no consequence on insurer profits and hence on π^{1G} or π^{2G}.

with constraints

$$0 \leq L^1, 0 \leq L^{1G}, 0 \leq L^{2G}_{ij} \leq L^{1G}, 0 \leq L^2_{ij}.$$

We now explain more explicitly the timing of events and decisions for the model. This is illustrated in Figures 1 and 2 (see Appendix A). A literal approach to timing would recognize that in each period, which represents say 10 years of life, the main breadwinner earns income throughout the period and could die at any point in the period (i.e., both income generation and mortality are flow variables throughout the period). We simplify the problem by assuming that income is earned at the beginning of the period (before mortality is realized) and that decisions about savings (s) and life insurance purchases (L^1 for spot and L^{1G} for guaranteed renewable in the first period) are also made at the beginning of the period. If death occurs then it does so at the end of period 1 and felicity $v_1(\cdot)$ represents the future stream of expected utility (from the breadwinner's perspective) of the survivor family. In case of death in the first period, C^{1D} is not literally the consumption of the survivor family in period 1 but rather is the income received by the survivor family to use going forward in time (i.e., including period 2 and beyond). This income includes the savings decided upon by the individual as well as life insurance payments. This is why, if the insured lives through period 1, then consumption of the "whole family" for period 1 is C^{1N} (income earned in that period minus savings and the cost of any spot and guaranteed renewable insurance purchased).[13]

If the individual survives period 1, which he does with probability $(1-p)$, then at the beginning of period 2 income y_2 is earned by the main breadwinner and saving from period 1 is also available for consumption. Again, the main breadwinner makes insurance purchasing decisions; i.e., how much spot insurance (L^2_{ij}) and how much of the first period guaranteed renewable insurance that he bought to renew (L^{2G}_{ij}). If he dies, which happens at the end of

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[13]One could, admittedly, quibble with this timing presumption since it requires the same amount of savings to be carried forward in both life and death states of the world, albeit by "different families". We believe, however, that the simplification is worthwhile and the model remains both rich and a reasonable reflection of the decision making environment.

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period 2 (with probability p_i), the survivor family receives C_{ij}^{2D} which includes second period insurance payments (and this generates felicity $v_2(C_{ij}^{2D}; \theta_j)$). If he lives, then the whole family receives C_{ij}^{2N} and this generates felicity $u_2(C_{ij}^{2D})$. These amounts $(C_{ij}^{2D}, C_{ij}^{2N})$ are meant to reflect not just consumption for period 2 but also consumption for an implicit third period and beyond. By not formally including a third period we admittedly omit modeling any further income generation or intertemporal income transfer possibilities (e.g., from period two income for consumption in period three, et cetera). The fact that $C_{ij}^{2D} > C_{ij}^{2N}$ even though the survivor family has fewer individuals can be accounted for by imagining that in a third period (and beyond) the main breadwinner earns income at the beginning of that period which accommodates for higher consumption for the whole family than would be available for the survivor family. This can readily be captured by the state contingent felicities $v_2(\cdot; \theta_j)$ and $u_2(\cdot)$. Moreover, this would also be consistent with the usual rationale for life insurance demand (i.e., loss of income due to death which would be the loss of income that would have been generated by the breadwinner at the beginning of period 3 and beyond). It seems reasonable to leave these issues aside as explicitly including additional periods would unduly complicate the model.

3   Analysis of Possible Market Structures

Before examining the allocations that may be supported with GR contracts, it is instructive to examine two important alternative allocations: the first-best and spot markets only. These are useful to clarify both the value of GR and also its shortcomings.

3.1   Benchmark: First-Best

The first-best allocation is obtained by maximizing ex-ante utility (i.e., from the perspective of individuals in period 1) subject to a set of aggregate resource constraints, one for each

period.[14] These resource constraints simply require that the total consumption across types and states in each period be equal to the total available resources. The first best allocation is the solution to:

$$\max_{C^{1D},C^{1N},s,\{C^{2D}_{ij},C^{2N}_{ij}\}} EU \equiv pv_1\left(C^{1D}\right) + (1-p)u_1\left(C^{1N}\right)$$

$$+ (1-p)\left[\sum_i\sum_j q_ir_j\left(p_iv_2\left(C^{2D}_{ij};\theta_j\right) + (1-p_i)u_2\left(C^{2N}_{ij}\right)\right)\right] \quad \text{s.t.} \quad (2)$$

$$y_1 \geq pC^{1D} + (1-p)(C^{1N} + s), \tag{3}$$

$$y_2 + s \geq \sum_i\sum_j q_ir_j\left[p_iC^{2D}_{ij} + (1-p_i)C^{2N}_{ij}\right]. \tag{4}$$

Proposition 1. *The social optimum is characterized by:*

- *Marginal utilities in all time/state contingent scenarios are equal across all risk and demand type combinations:*

$$v'_1(C^{1D}) = u'_1(C^{1N}) = v'_2(C^{2D}_{ij};\theta_j) = u'_2(C^{2N}_{ij}), \text{ for all } ij \in \{H,L\} \times \{h,l\}. \tag{5}$$

- *Consumption in the life state or death state for each period is independent of risk type.*

- *Second period consumption level for high demand types exceeds that for low demand types (but is independent of risk type, as noted above).*

Proof. See Appendix B.1. □

It follows directly from this proposition that the first-best allocation requires marginal utilities of consumption in each time and state to be equalized. This is an application of the fundamental theorem of risk-bearing. This implies that, for a given demand type

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[14]As there is a measure one of individuals, aggregates are equal to per capita values.

consumption in the period 2 death state is the same for both risk types and likewise for the period 2 life state consumption. However, consumption in the death state is higher for the high demand type than for the low demand type. This is easily established as

$$v_2'(C_{il}^{2D}; \theta_l) = v_2'(C_{ih}^{2D}; \theta_h) < v_2'(C_{il}^{2D}; \theta_h) \Rightarrow C_{ih}^{2D} > C_{il}^{2D}. \tag{6}$$

If it were feasible, one way to decentralize the first-best allocation would be to allow individuals to write contracts at time 1 that offer transfers contingent on their realized demand and risk type at time 2. Such contracts replicate the social planner's ability to effectuate transfers across agents at time 2.[15] Given unobservability of demand type, this is not meant to be a realistic representation of insurance contracts available in actual markets but help illustrate differences with alternative market structures.

3.2   Spot-Markets Only

Now consider the equilibrium choices of individuals when only spot insurance is available in period 2. Determining each individual's optimal consumption requires first solving the second period optimization problem for each individual conditional on risk and demand type, which is known at that point in time, conditional on a given set of first period choices (i.e., for s and L^1). We then use the value functions from the second period optimization problem to determine optimal values for decision variables relating to the first period.

Second period choice problem is, given type ij:

$$Z_{ij}^{spot} = \max_{L_{ij}^2} p_i v_2(C_{ij}^{2D}; \theta_j) + (1 - p_i) u_2(C_{ij}^{2N}), \tag{7}$$

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[15]The first-best may also be decentralized by a tax and transfer scheme that is type contingent.

where

$$C_{ij}^{2N} = y_2 + s - \pi_i^2 L_{ij}^2, \tag{8}$$

$$C_{ij}^{2D} = y_2 + s + (1 - \pi_i^2)L_{ij}^2, \tag{9}$$

which leads to the first order condition:

$$p_i v_2'(C_{ij}^{2D}; \theta_j)(1 - \pi_i) - (1 - p_i)u_2'(C_{ij}^{2N})\pi_i = 0$$

When spot market prices are actuarially fair (i.e., $\pi_i^2 = p_i$), we have

$$v_2'(C_{ij}^{2D}; \theta_j) = u_2'(C_{ij}^{2N}), \tag{10}$$

in other words, ex-post efficiency prevails.

Let Z_{ij}^{spot} be the value function relating to the second period optimization problem. Since no GR insurance is available to purchase in period 1 for potential renewal in period 2, it follows that the only decision variable from period 1 that carries over to period 2 is s. Note that $Z_{ij}^{spot}(s)$ is strictly concave given our assumptions regarding the period 2 felicities, and via the envelope theorem we obtain: $\frac{\partial Z_{ij}^{spot}(s)}{\partial s} = p_i u_2'(C_{ij}^{2N}) + (1 - p_i)v_2'(C_{ij}^{2D}; \theta_j) = u_2'(C_{ij}^{2N})$.

We go back to the first period choice problem to complete the description of the optimal plan. In the first period, households choose savings and spot purchases to maximize expected utility:

$$\max_{s, L^1} EU = p v_1(C^{1D}) + (1 - p)u_1(C^{1N}) + (1 - p)\sum_i \sum_j q_i r_j Z_{ij}^{spot}(s), \tag{11}$$

where

$$C^{1N} = y_1 - s - \pi^1 L^1, \ C^{1D} = y_1 + (1 - \pi^1)L^1. \tag{12}$$

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First order conditions are:

$$\frac{\partial EU}{\partial L^1} = pv_1'(C^{1D})(1 - \pi^1) + (1 - p)u_1'(C^{1N})(-\pi^1) = 0, \tag{13}$$

$$\frac{\partial EU}{\partial s} = (1 - p)u_1'(C^{1N})(-1) + (1 - p)\sum_i \sum_j q_i r_j u_2'(C_{ij}^{2N}) = 0. \tag{14}$$

Competition ensures first period insurance is actuarially fair, $\pi^1 = p$, and so we get

$$v_1'(C^{1D}) = u_1'(C^{1N}), \tag{15}$$

$$u_1'(C^{1N}) = \sum_i \sum_j q_i r_j u_2'(C_{ij}^{2N}). \tag{16}$$

The last equation shows that the optimal savings amount equalizes the marginal utility of consumption in the first period life state to the expected marginal utility of consumption in the second period life state. Thus, marginal utilities will generally not be equal over time, confirming that spot insurance does not insure individuals against re-classification risk.

Proposition 2. *Characterization of Allocation Under Spot Insurance Only*

If the only markets for insurance in both periods is spot insurance, then it follows that:

- Ex-post efficiency (in period 2) prevails; that is, for a given risk type, demand type combination, marginal utility of consumption in the death state is equal to marginal utility in the life state.

- Consumption in the life and death states in period 2 are **not** independent of risk type. Conditional on a given demand type, high risk types have lower consumption in both life and death states of the world than do low risk types. (This follows from the fact that high risk types face a higher price of insurance.)

- The period two consumption level for high demand types of a given risk type is higher than that for low demand types.

Proof. See Appendix B.2.   □

3.3 GR insurance contracts

We now examine the model of primary interest; that is, the one where guaranteed renewable insurance is available. Information assumptions are the same as in the preceding model. In this case, however, in the second period the individuals hold an amount of guaranteed renewable insurance (L^{1G}) that they purchased in the first period. They may renew any amount of this $(L_{ij}^{2G} \leq L^{1G})$ in period 2 at the predetermined price π^{2G}. Individuals may also purchase spot insurance in period 2 (L_{ij}^2) which, since insurers also observe risk type, is priced at the risk type specific actuarially fair price (p_i). Determining each individual's optimal consumption requires first solving the second period optimization problem for each individual conditional on risk and demand type, which is known at that point in time, based on a given set of first period choices (i.e., for s, L^1, and L^{1G}). We then use the value functions from the second period optimization problem, Z_{ij}, to determine optimal values for decision variables relating to the first period.

Second period choice problem is, given type ij:

$$Z_{ij} = \max_{L_{ij}^2, L_{ij}^{2G}} p_i v_2(C_{ij}^{2D}; \theta_j) + (1 - p_i) u_2(C_{ij}^{2N}) \tag{17}$$

where

$$C_{ij}^{2N} = y_2 + s - \pi_i^2 L_{ij}^2 - \pi^{2G} L_{ij}^{2G}, \tag{18}$$

$$C_{ij}^{2D} = y_2 + s + (1 - \pi_i^2) L_{ij}^2 + (1 - \pi^{2G}) L_{ij}^{2G}, \tag{19}$$

$$L_{ij}^{2G} \leq L^{1G}. \tag{20}$$

We denote the multipliers for each type pair's constraint by λ_{ij}. The first order condition

with respect to the choice variables are:

$$L_{ij}^2 : p_i v_2'(C_{ij}^{2D}; \theta_j)(1 - \pi_i^2) - (1 - p_i)u_2'(C_{ij}^{2N})\pi_i^2 = 0, \tag{21}$$

$$L_{ij}^{2G} : p_i v_2'(C_{ij}^{2D}; \theta_j)(1 - \pi^{2G}) - (1 - p_i)u_2'(C_{ij}^{2N})\pi^{2G} - \lambda_{ij} = 0, \tag{22}$$

$$\lambda_{ij}(L^{1G} - L_{ij}^{2G}) = 0. \tag{23}$$

For the scenario with only spot insurance available, the resource constraints are trivial. That is, spot insurance is actuarially fair in each period which means $\pi^1 = p$, since individuals have the same first period mortality risk. In period 2 the spot market price is $\pi_i^2 = p_i$, $i \in \{L, H\}$, since insurers observe risk type. There is an additional resource constraint for GR insurance since front period loading must be sufficient to cover any second period costs associated with any risk types renewing at a rate that is more favourable than their actuarially fair rate (e.g., for $\pi^{2G} < p_i$). The extent to which the first period contract must be front loaded (i.e., the difference $\pi^{1G} - p$) depends on the extent to which the renewal price falls below the actuarial cost of providing risk types with insurance as well as the amount of L^{1G} that is purchased and amounts that will be renewed in equilibrium by risk types of both low and demand type.

Although insureds who turn out to be low demand types but are of high risk type may not renew all of L^{1G}, they have an incentive to renew more than would a low demand type who is also of low risk type since the price is more favourable to them. This means that low demand types who are high risk types typically end up with more second period insurance coverage than their low demand - low risk counterparts.[16] From the characterization of the social optimum, we know this cannot be efficient and so insureds would prefer contracts that are designed so this does not happen. However, once a person knows he is of high risk type, he cannot "resist" renewing more insurance than is necessarily efficient even though, from the ex ante perspective, everyone would like to prevent such an outcome. This "over

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[16]For this to happen depends on both how different is the desired demand of these two types of individual as well as on how much GR insurance L^{1G} they hold entering the second period.

renewal" by $Hl-types$ creates undesirable adverse selection costs which must be covered by a combination of increasing the front loading and/or the second period renewal price compared to what would be required if such inefficient behaviour could be controlled. The following equation explains this additional resource constraint which ensures zero expected profits for insurers offering GR insurance.

$$\pi^{1G}L^{1G} = pL^{1G} + (1-p)\sum_i \sum_j q_i r_j (p_i - \pi^{2G})L_{ij}^{2G}$$

Note that the LHS of this equation represents the total revenue from sales of GR insurance in period 1. The first term of the RHS is the expected cost of insurance claims of GR insurance in period 1 while the second term is the sum of net expected costs of claims from all possible risk and demand types who pay π^{2G} to renew amount L_{ij}^{2G} of their holdings of GR insurance.

The zero-profit condition can also be written as follows:

$$\pi^{1G} = p + (1-p)\sum_i \sum_j q_i r_j (p_i - \pi^{2G})\frac{L_{ij}^{2G}}{L^{1G}}. \tag{24}$$

There are several important points regarding this constraint with some admittedly obvious. Firstly, the amount of front loading per contract, as measured by the difference $\pi^{1G} - p$, is increasing with the (average) fraction of GR insurance holdings from the first period that is in fact renewed in the second period. It is also increasing in the amount of effective subsidy $(p_i - \pi^{2G})$ to each risk type i.[17] An increase in π^{1G} will affect the demand for GR insurance (L^{1G}) and so affect the amount of front loading that is required through the ratio $\frac{L_{ij}^{2G}}{L^{1G}}$. L^{1G} is also naturally a function of π^{2G} since GR insurance is more attractive the lower is its renewal price. This means that the way to control adverse selection problems arising from those who become low demand but high risk is not simply through increasing the renewal price as changing in both prices π^{1G} and π^{2G} affects the desirability of GR insurance in

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[17]Clearly, there will be no market if the renewal price equals or exceeds the actuarially fair cost of insurance of high risk types (i.e., if $\pi^{2G} \geq p_H$).

opposing ways.

To gain further insights into drivers of GR renewals, note that we write the first-order condition for L_{ij}^{2G} as follows:

$$v_2'(C_{ij}^{2D}; \theta_j) - u_2'(C_{ij}^{2N}) = \left(\frac{\pi^{2G} - p_i}{\pi^{2G}(1 - p_i)} \right) v_2'(C_{ij}^{2D}; \theta_j) + \frac{\lambda_{ij}}{\pi^{2G}(1 - p_i)}. \tag{25}$$

Equation (25) is helpful in understanding a number of possible scenarios to be discussed more fully below. Consider, for example, an individual who is both high risk and low demand and so renews some but not all of first period GR ($0 < L_{Hl}^{2G} < L^{1G}$). For such a person the shadow value of L^{1G} is zero ($\lambda_{Hl} = 0$) and so the second term on the RHS of equation (25) is zero. With the renewal price for high risk types being below their actuarially fair rate ($\pi^{2G} < p_H$), the RHS of equation (25) is negative; that is $v_2'(C_{ij}^{2D}; \theta_j) < u_2'(C_{ij}^{2N})$ and so this person ends up in a position of oversinsurance. In this sense the renewal price is mispriced from the ex post (period 2) perspective and there are adverse selection costs created in the renewal market for GR. If an individual renews all of his first period GR, then the shadow value of GR is postive ($\lambda_{Hl} > 0$) and so the second term of the RHS of equation (25) is positive, mitigating the influence of "mispricing" that leads to over-insurance. Because of the existence of spot markets, an individual will never end up with too little insurance from the perspective of second period consumption choice. However, whenever second period spot markets are active, it follows that individuals are not fully protected against reclassification risk since high risk types face a higher spot price.

We write value functions (indirect utilities) from this exercise as $Z_{ij}(L^{1G}, s; \pi^1, \pi_i^2, \pi^{2G})$. Using the envelope theorem, it follows that

$$\frac{\partial Z_{ij}}{\partial L^{1G}} = \lambda_{ij} \text{ for all } i, j, \tag{26}$$

$$\frac{\partial Z_{ij}}{\partial s} = p_i v_2'(C_{ij}^{2D}; \theta_j) + (1 - p_i) u_2'(C_{ij}^{2N}) \text{ for all } i, j. \tag{27}$$

This implies that for types that fail to fully renew their GR in period 2, increasing the

quantity of GR ex-ante has no impact on their welfare.

We now turn our attention to the period 1 optimization problem to complete the description of the optimal plan.

$$\max_{s,L^1} EU = pv_1(C^{1D}) + (1-p)u_1(C^{1N}) + (1-p)\left[\sum_i \sum_j q_i r_j Z_{ij}(\cdot)\right] \tag{28}$$

where

$$C^{1N} = y_1 - s - \pi^1 L^1 - \pi^{1G} L^{1G}, \tag{29}$$

$$C^{1D} = y_1 + (1-\pi^1)L^1 + (1-\pi^{1G})L^{1G}. \tag{30}$$

First order conditions are:

$$\frac{\partial EU}{\partial L^1} = pv_1'(C^{1D})(1-\pi^1) - (1-p)u_1'(C^{1N})\pi^1 = 0, \tag{31}$$

$$\frac{\partial EU}{\partial L^{1G}} = pv_1'(C^{1D})(1-\pi^{1G}) - (1-p)u_1'(C^{1N})\pi^{1G} + (1-p)\sum_i \sum_j q_i r_j \lambda_{ij} = 0, \tag{32}$$

$$\frac{\partial EU}{\partial s} = -(1-p)u_1'(C^{1N}) + (1-p)\sum_i \sum_j q_i r_j \left[p_i v_2'(C_{ij}^{2D};\theta_j) + (1-p_i)u_2'(C_{ij}^{2N})\right] = 0. \tag{33}$$

We can re-write the first-order condition on savings as follows:

$$v_1'(C^{1D}) = u_1'(C^{1N}) = \sum_i \sum_j q_i r_j [p_i v_2'(C_{ij}^{2D};\theta_j) + (1-p_i)u_2'(C_{ij}^{2N})]. \tag{34}$$

This demonstrates that the optimal savings (or borrowing if negative) amount allows households to smooth consumption over time by equalizing marginal utility of consumption in the first period life state to the expected marginal utility of consumption in the second period.

We can also re-write the first-order condition with respect to L^{1G} as follows

$$v_1'(C^{1D}) - u_1'(C^{1N}) = \underbrace{-\frac{(1-p)\sum_i \sum_j q_i r_j \lambda_{ij}}{(1-p)\pi^{1G}}}_{\text{re-classification risk effect}} + \underbrace{\left(\frac{\pi^{1G} - p}{(1-p)\pi^{1G}}\right) v_1'(C^{1D})}_{\text{mispricing effect}}. \tag{35}$$

As for equation (25), there is a mispricing effect illustrated in equation (35) but from the perspective of period 1 insurance purchasing. Due to front loading ($\pi^{1G} > p$), the second term on the RHS being positive implies GR is too expensive for their first period insurance needs and they would purchase too little. However, as long as the first period spot insurance is active it follows that $v_1'(C^{1D}) = u_1'(C^{1N})$. The first term, which is negative if $\lambda_{ij} > 0$ for at least some ij pair, reflects the value of GR in providing protection against reclassification risk (i.e., at least some of $\lambda_{ij} > 0$). However, if expected demand for insurance in the second period is large relative to first period demand, the value for GR insurance in providing protection against reclassification risk can lead to excessive first period insurance coverage; i.e., if all first period insurance needs are more than met by L^{1G} (overinsurance in period 1) we would have $L^1 = 0$ and $v_1'(C^{1D}) < u_1'(C^{1N})$. The important conclusions are summarized in the proposition below:

Proposition 3. *Characterization of Allocation with GR Insurance Available*

If there are markets for both spot and GR insurance, then it follows that

- Ex post efficiency (in period 2) will not generally prevail. In particular, marginal utility in the death state may be less than marginal utility in the life state for high risk types who are also low demand types (over-insurance).

- Consumption in the life and death states in period 2 are **not** necessarily independent of risk type. Conditional on a given demand type, high risk types may have lower consumption in both life and death states of the world than do low risk types. (This follows if second period spot purchases are non-zero due to the fact that high risk types face a higher spot price of insurance.)

- The period two consumption level for high demand types of a given risk type is at least as high as that for low demand types.

4 Welfare analysis of GR contracts

Upon comparing Propositions 1-3, it appears that there are at least as many tendencies towards inefficiency when GR insurance is available compared to the situation in which only spot markets are available. However, the presence of GR insurance allows for individuals who turn out to be high risk types to obtain some insurance coverage at a price below the actuarially fair rate. This ameliorates the inefficiency of reclassification risk (i.e., the pushing apart of consumption levels of any given demand type in both states of period 2 due to risk based pricing in period 2 spot markets). However, GR insurance may also lead to the phenomenon that low demand types who are also high risk types will renew so much of their GR insurance that they end up with greater consumption in the death state of period 2 than that of low demand but low risk types. This reflects a type of ex post inefficiency (see the second statement of Proposition 1).

Proposition 4. *In the presence of both demand and risk heterogeneity, the equilibrium with GR and spot markets is always inefficient relative to first-best. GR contracts can achieve a first best efficient allocation if and only if:*

1. *There is no heterogeneity of demand types. (i.e., demand for insurance is identical across individuals in period 2.)*

2. *The renewal price is sufficiently attractive and thus front-loading sufficiently high that renewing GR is (weakly) preferable to purchasing spot insurance in period 2 for all individuals and there is no lapsation.*

3. *Demand for insurance is non-increasing over-time.*

Proof. See Appendix B.3. □

Two observations regarding the above proposition are important. Firstly, if any demand heterogeneity evolves over individuals' lifetimes, GR insurance cannot achieve a first best efficient allocation of resources. Second, even if there is no demand heterogeneity, it is only possible for GR insurance to achieve first best efficiency if demand for insurance is non-increasing over time. As noted earlier, the average life cycle for insurance purchases has demand increasing while individuals are young (i.e. under 45 for males and under 40 for females). These two factors suggest important limitations for the role of GR for mitigating re-classification risk. As Proposition 5 below and simulations demonstrate, however, introducing GR into a model with only spot markets will improve welfare and possibly substantially so.

Proposition 5. *Making GR insurance available alongside spot markets is always strictly welfare enhancing in the presence of re-classification risk.*

Proof. See Appendix B.4. □

The intuition for the proof is straightforward. Consider starting from a position of only spot market insurance being available with demand for first period insurance being positive (although possibly "small"). Consider substituting a small amount of first period spot insurance with GR which is renewable at a price at least slightly below the second period spot price for high risk types but above the spot price for low risk types. This GR insurance will be renewed only by high risk types. The envelope theorem guarantees that there is no first order effect on welfare. However, there is a transfer of consumption from both risk types in period 1 (due to a small amount of front loading) to high risk types who have a higher marginal utility of consumption in period 2 due to their higher loss probability and so higher price they face in the period 2 market for spot insurance. This transfer represents a first order improvement in welfare.

Note that the above set of steps does not work if there is only demand type heterogeneity. The reason is that, in order to transfer consumption from a lower to a higher marginal utility

state (i.e., from low to high demand types in period 2) requires that the renewal price for GR insurance be less than the actuarially fair price in period 2 for all consumers (i.e., since there is only one risk type). Following the above steps, making a small amount of GR insurance available which is renewable at a price below the actuarially fair price of insurance in period 2 will lead to all consumers (i.e., both low and high demand types) renewing this insurance. This implies a transfer of consumption to from all types to all types. Moreover, the effect of such a scheme will be undone by all consumers by reducing savings in period 1 in order to "re-establish" their optimal choices. Therefore, GR will not always provide a welfare improvement when there is only demand type heterogeneity.

A sufficient condition that will guarantee a welfare improvement in these circumstances is that the demand for insurance in period 2 by low demand types be less than first period demand. In this circumstance, replacing an amount of first period insurance equal to first period spot demand plus a small amount extra with GR insurance that can be renewed at a price marginally below the actuarially fair price for second period insurance (which is the same for all consumers) will transfer consumption from low demand types to high demand types who have higher marginal utility of income in the death state.[18] This transfer will induce a first order increase in welfare. This sufficient condition, however, is not to be taken lightly. Recall that demand for insurance on average is rising for 'young' individuals and so even low demand types may have higher demand in period 2 of our model than in period 1. The proof of this result is available from the authors upon request.

5 Simulations

We develop a set of simulations in order to demonstrate the types of properties of GR insurance when both risk and demand uncertainty persist and to investigate conditions under which availability of GR insurance does significantly better in terms of improving social

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[18]Note that the marginal renewal price decrease for GR insurance is so slight that the low demand types do not renew all of it.

welfare compared to the presence of spot insurance only. We adopt CRRA felicities which are varied according to time and state through use of a multiplicative constant as shown in Table 1. We consider the following parameter combinations (also see Table 2 below):

1. Large differences in demand and risk ex-post.

2. Small differences in demand (decreasing over time) and large differences in risk.

3. Large differences in risk only (increasing demand).

4. Large differences in risk only (decreasing demand)

5. Large differences in demand (decreasing) and small differences in risk.

6. Large differences in demand only and same small increase in risk for all types.

	Period 1	Period 2
Life	$u_1(C^{1N}) = \frac{1}{1-\beta}(C^{1N})^{1-\beta}$	$u_2(C_{ij}^{2N}) = \frac{1}{1-\beta}(C_{ij}^{2N})^{1-\beta}$
Death	$v_1(C^{1D}) = \alpha^D u_1(C^{1D})$	$v_2(C_{ij}^{2D}; \theta_j) = \theta_j \frac{1}{1-\beta}(C_{ij}^{2D})^{1-\beta}$

Table 1: Felicities. Note that $\theta_h > \theta_l > 1$, and we set $\beta = 2$, $\alpha^D = 2$.

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
α^D	2.0	10.0	2.0	10.0	10.0	10.0
p_L	0.1	0.1	0.1	0.1	0.1	0.1
p_H	0.5	0.5	0.5	0.5	0.2	0.1
q_L	0.9	0.9	0.9	0.9	0.5	0.9
q_H	0.1	0.1	0.1	0.1	0.5	0.1
θ_l	1.2	4.0	20.0	5.0	1	1.2
θ_h	20.0	5.0	20.0	5.0	5.0	20.0
r_l	0.5	0.5	0.5	0.5	0.5	0.5
r_h	0.5	0.5	0.5	0.5	0.5	0.5

Table 2: Parameter values for all cases. Note: $p = 0.08$, $y_1 = y_2 = 1$ for all cases.

We compute the first-best optimal allocation for each case and report the results (period-state-contingent consumption levels) in Table 4. We also generate the compensating variation

(CV) for each market outcome for each case.[19] The CV values reflect the extent to which efficiency is compromised relative to the social optimum for each scenario of Spot Markets Only, and GR Insurance available (along with spot insurance). These results are reported in Table 3. This allows us to compare how close to the social optimum each market scenario achieves. Note that the CV values represent loss of efficiency relative to the social optimum and so the lower is the CV value, the better is the market outcome relative to first-best. Note that, given $y_1 = y_2 = 1$, the CV values describe the loss of welfare due to mortality risk in the various market scenarios as a percentage of a person's annual income. Tables 5 and 6 summarize those results and also include relevant information about the GR Insurance contracts (initial price for coverage, π^{1G} and renewal price, π^{2G}).

Regime	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Spot Insurance Only	4.80	1.20	4.44	0.80	0.40	0.98
GR plus spot	3.67	0.01	2.73	0.00	0.32	0.75

Table 3: Comparing Insurance Regime Efficiency (CV loss).

From Table 4 we see that it is optimal for individuals to augment their consumption in the period 1 death state by a factor of approximately 1.4-3.2 times their consumption in the life state. For the high demand types in the various scenarios it would be socially optimal for individuals to augment their consumption in the period 2 death state by a factor of approximately 2.2-4.5 relative to the life state. With spot insurance only available, we find individuals come reasonably close to first period optimal state contingent consumption by purchasing insurance of (roughly) amount 0.2 to 1.8 times their first period income depending on the relative (average) importance and cost of insurance that they "forecast" for period 2.[20] This is not surprising since individuals have homogeneous tastes, income, and mortality risk in period 1.

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[19]This value is computed by subtracting the amount CV from the socially optimal level of each period-state-contingent conumption and solving implicitly by setting the resulting expected utility equal to the expected utility obtained under each market outcome.

[20]In case 3 with risk differences only, all individuals are high demand types and so in that scenario people shift more income from period 1 to period 2 due to a greater expectation of having high need for life insurance (i.e., having a high demand with probability 1).

The allocation under spot insurance differs significantly from the social optimum when risk differences are large (i.e., Cases 1 and 3). In particular, looking at Table 5 we see that consumption in the period 2 death and life states for individuals who are both high demand and high risk types is only about 50% the level of that for individuals who are high demand but low risk types. These pairs of consumption levels are the same (i.e., independent of risk type) in the social optimum. The divergence under spot insurance is due to the associated income effect created by the higher cost of second period life insurance for high risk types. These results highlight the problem of reclassification risk from a welfare perspective.

By examining Table 6, we see that individuals purchase a substantial amount of renewable insurance in period 1, i.e. L^{1G} is approximately 0.6-1.8 times their income, and meet more than half of their first period insurance needs through their GR purchases. Purchasing GR in period 1 not only serves their first period insurance needs but also offers some protection against reclassification risk for period 2 insurance. As a result, for the Cases 1 and 3 (i.e., p_H large), individuals essentially overinsure for period 1 (relative to the social optimum), ending up with consumptions in the period 1 death state of 154 and 174 respectively compared to the social optima of 124 and 113. In Case 1 with both demand and risk heterogeneity, the loss of efficiency is relatively large at 3.67% while in Case 3 with only risk-heterogeneity, it is lower at 2.73%. In both cases there is a loss of efficiency in that individuals hold more than the socially optimal amount of first period insurance in order to provide protection against reclassification risk in period 2. In both cases the choice of L^{1G} exceeds 60 units.

In the case of both risk and demand differences, the high risk but low demand types renew significantly more of their first period GR insurance (61 units) than is socially optimal: that is, Hl types end up with period 2 death state consumption of 163 compared to the socially optimal amount of 88 units. This overconsumption of insurance is due to the fact that high risk - low demand types value the GR insurance more highly than their demand type "warrants" due to the relatively attractive renewal price of 0.082 per unit of insurance (i.e., compared to their actuarially fair price of 0.50).

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It is interesting to dig a little deeper to understand just how GR insurance offers welfare improvements over spot insurance in the various cases of demand differences only, risk differences only, and instances with both types of heterogeneity. Consider the case for demand type differences only. There is, of course, no role of GR insurance to play in reducing reclassification risk (i.e., to smooth consumption across individuals who become different risk types). However, people who become higher demand types end up with a higher (average) marginal utility of consumption in period 2. Therefore, if period 2 insurance purchases are effectively subsidized through GR insurance purchased in period 1 which is then renewable in period 2 at a price lower than the actuarially fair price (for all homogeneous risk types), then consumption that delivers higher marginal utility can be enhanced. This is seen by comparing the outcomes in Case 1 under *Spot Insurance Only* and under *GR Insurance*. Under GR Insurance, L^{1G} is front loaded ($\pi^{1G} = 0.129$ while $p = 0.08$); that is, the price exceeds the actuarial cost of first period coverage. The renewal price of $\pi^{2G} = 0.082$ is less than the actuarial cost of second period insurance even for low risk types, which is $p_L = 0.10$. The result is that under GR insurance, individuals who are high demand types, and so have relatively high marginal utility of consumption, end up with period 2 consumption in the death state of $C_{Lh}^{2D} = 357$ while in the case of Spot Insurance Only, they end up with consumption of only $C_{Lh}^{2D} = 372$. This is a modest move in the direction of the socially optimal allocation of $C_{Lh}^{2D} = 393$. (Note: we use L to index the single risk type in this case.) GR insurance does not provide a perfect solution in that individuals who end up being low demand types have an incentive to renew too much insurance since the renewal price is below the actuarially fair price for them as well. This has a spoiling effect on the market for GR insurance as low demand types end up holding (renewing) too much of their first period GR insurance holding. In fact, under GR insurance these low demand types consume $C_{Ll}^{2D} = 128$ while the socially optimal level is $C_{Ll}^{2D} = 96$. This is a rather different type of adverse selection phenomenon than the customary one in that it can even occur when the population of insureds are all of the same risk type. It is low demand types who are overinsuring rather than high risk types.

We described earlier how spot insurance naturally has no ability to provide protection against reclassification risk and so high risk types end up with significantly lower consumption in the (period 2) death state than do low risk types. These consumption levels are equal in the socially optimal allocation (i.e., full insurance against reclassification risk is socially and ex ante individually optimal). In case 3 (i.e., only risk differences which are "large"), GR insurance provides for substantial welfare gains compared to *Spot Insurance Only*. The first-best allocation is not achieved in this case because to hold full protection against reclassification risk would require holding more GR insurance in period 1 than is optimal. In fact, $L^1 = 0$ as individuals opt for more GR (higher level of L^{1G}) than is efficient from a period 1 perspective in order to get protection against reclassification risk. The result is that first period consumption in the death state is 174 under GR compared to socially efficient level of 113. Also, under GR insurance, the optimal renewal price is below the actuarial cost even for low risk types ($\pi^{2G} = 0.085$ while $\pi_L = 0.10$). Having such a low renewal price enhances the ability of GR to cross-subsidize and provide higher consumption to those with high marginal utility (i.e., individuals who are high risk type). But this also leads to overconsumption by people who are low risk types. (NOTE: In this case individuals are of the same demand type which we denote with the index h.) This is demonstrated by the result in Case 3 that the first-best level of consumption in the period 2 death state is independent of risk type with $C_{Lh}^{2D} = C_{Hh}^{2D} = 358$ while under GR insurance we have $C_{Lh}^{2D} = 373$. High risk types end up with $C_{Hh}^{2D} = 239$ which, although substantially less than the first-best level, is substantially more than under *Spot Insurance Only* ($C_{Hh}^{2D} = 197$). In this example, therefore, GR Insurance provides partial insurance against reclassification risk compared to having only spot insurance available. We also see that there is an income effect that leads low risk types to end up with more consumption in the death state than is first-best efficient (i.e., $C_{Lh}^{2D} = 373$ versus 358.) This income effect arises because both low and high risk types are active on the spot market in this example and this favours low risk types (i.e. there is incomplete insurance against re-classification risk).

Finally, we see that in a scenario with heterogeneity of both demand and risk type (Case 1), GR Insurance may perform rather weakly, although still better than having only spot markets available. Introducing demand type differences (Case 1) into the scenario of only risk type differences (Case 3) leads to a worsening of efficiency from a loss of 2.73% in Case 3 to 3.67% in Case 1. The efficiency loss under *Spot Insurance Only* is even worse in these two cases (4.44% for Case 3, 4.80% for Case 4). The reason GR insurance looses some of its advantage when demand type differences are present is that individuals who turn out to be both high risk but low demand type face a very favourable renewal price ($\pi^{2G} \approx 0.082$ compared to $\pi_H = 0.50$). These individuals end up renewing substantially too much of their GR insurance with the result that their second period death state consumption is $C_{Hl}^{2D} = 162.83$ while the first-best level is $C_{Hl}^{2D} = 95.35$. This is a sort of "normal" adverse selection (i.e., high risk types ending up being over-insured). Note, however, that it is the presence of low demand types who are also high risk types that creates this sort of adverse selection and not simply the presence of high risk types who are assessed the same (renewal) price as are low risk types.

6   Conclusions

We have developed a two period model of life insurance in which individuals face uncertainty over future changes in both mortality risk and insurance needs (bequest motives). In the first period individuals are identical in all respects, including their current insurance needs and beliefs about how their risk and demand type will evolve in the second period. We allow for spot markets in each period as well as guaranteed renewable (or long term insurance) that can be purchased in the first period. In the second period individuals may become either a high or low demand type as well as a high or low risk type. Guaranteed renewable insurance (GR) offers the potential to ameliorate reclassification risk. As in Pauly et al. (1995) and other previous work, we find that when individuals face only future risk

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type uncertainty, GR may allow individuals to fully insure against reclassification risk and achieve a first best efficient allocation. However, if insurance demand is increasing over time, then one can only fully insurance insure against reclassification risk by holding more (GR) insurance in period 1 than is desirable. The result is that a first best efficient allocation is not possible.

Moreover, if there is also uncertainty about future demand, individuals do not know how much GR insurance would be ideal to hold for possible use in period 2. This creates problems for the efficiency of the renewals market. To offer protection against reclassification risk, the renewal price must be less than the actuarially fair price for (period 2) high risk types. Therefore, individuals who turn out to be low demand but high risk type will face a renewal price that is below their actuarially fair price and will renew too much and so end up over-insured. This creates a type of adverse selection problem which leads to inefficiency. As a result, unlike in the model of Pauly et al. (1995), we have shown through simulations that the optimal GR contract may involve a renewal price which exceeds the actuarially fair price for (period 2) low risk types and so leads to lapsation by some insureds. Through a series of propositions, we have shown that, although a first best efficient outcome is not possible when there is both risk and demand type uncertainty, adding GR contracts to spot contracts does improve social welfare (i.e., ex ante utilities).

We demonstrate through use of simulations that, as in Fei et al. (2013), GR insurance can be effective in smoothing consumption across demand types and so can improve welfare even if there is no reclassification risk (i.e., individuals are of homogeneous risk type). The source of the welfare improvement is that first period purchases of GR with favourable ("subsidized") renewal terms allows for shifting second period death-state consumption towards those with higher marginal utility of consumption (i.e., towards high demand types). However, as noted above, in the presence of both demand and risk type it may be that the optimal GR contact involves a renewal price above the actuarially fair price for low risk types who allow their policies to lapse and purchase their second period insurance needs

from the spot market. In this scenario, which is not considered in Fei et al. (2013), high demand types who are also low risk types are excluded from the benefits of the "subsidy". Although Polborn et al. (2006) consider a two period model with both demand and risk type uncertainty, they do not model insurance needs in the first period. Therefore, as with Fei et al. (2013), they do not capture the importance of the interaction of these characteristics with the possibility of increasing demand over time which creates an additional obstacle for GR or long term insurance to improve welfare.

Our model has shown the importance of identifying consumers who have higher marginal utility of consumption. This can be due to taste differences in regards to bequest motive or due to risk type (i.e., high demand types of a given risk type have higher marginal utility in the death state as do high risk types of a given demand type). We also demonstrate the importance of life cycle effects in demand for insurance and the relationship between future insurance demand by different types. There are other reasons for changing preferences over time for insurance, including health shocks or income shocks which may increase or decrease marginal utility in the life state versus death states. Our analysis shows the importance of explicitly modeling such changes when analyzing welfare implications of GR or long term versus short term insurance contracts. Future work should use such explicit modelling strategies as reflective of circumstances both for understanding contracts and for any regulations that may be of interest (e.g., (partially) enforced guaranteed renewability of health insurance contracts as in the Affordable Care Act).

References

Cochrane, John H., 1995, Time-Consistent Health Insurance, *Journal of Political Economy* 103, 445–473.

Daily, G., I. Hendel, and A. Lizzeri, 2008, Does the secondary Life Insurance Market Threaten Dynamic Insurance?, *American Economic Review:Papers and Proceedings* 98, 151–156.

Fei, Wenan, Claude Fluet, and Harris Schlesinger, 2013, Uncertain Bequest Needs and Long-Term Insurance Contracts, *The Journal of Risk and Insurance* 82, 125–148.

Frick, Kevin D., 1998, Consumer Capital market Constraints and Guaranteed Renewable Insurance, *Journal of Risk and Uncertainty* 16, 271–278.

Hendel, Igal, and Alessandro Lizzeri, 2003, The Role of Commitment in Dynamic Contracts: Evidence from Life Insurance, *Quarterly Journal of Economics* 118, 299–328.

Hofmann, Annette, and Mark Browne, 2013, One-Sided Commitment in Dynamic Insurance contracts: Evidence from private Health Insurance in Germany, *Journal of Risk and Uncertainty* 46, 81–112.

Hofmann, Annette, and Richard Peter, 2015, Self-Insurance, Self-Protection, and Saving: On Consumption Smoothing and Risk Management, *Journal of Risk and Insurance* 83, 719–734.

Hong, J.H., and J-V Rios-Rull, 2012, Life Insurance and Household Consumption, *American Economic Review* 102, 3710–3730.

Pauly, Mark V., Howard Kunreuther, and Richard Hirth, 1995, Guaranteed Renewability in Insurance, *Journal of Risk and Uncertainty* 10, 143–156.

Peter, Richard, Andreas Richter, and Petra Steinorth, 2015, Yes, No, Perhaps? Premium Risk and Guaranteed Renewable Insurance Contracts with heterogeneous Incomplete Private Information, *The Journal of Risk and Insurance* 83, 363–385.

Polborn, Mattias K., Michael Hoy, and Asha Sadanand, 2006, Advantageous Effects of Regulatory Adverse Selection in the Life Insurance Market, *The Economic Journal* 83, 327–354.

Zietz, Emily Norman, 2003, An Exmination of the Demand for Life Insurance, *Risk Management and Insurance Review* 6, 159–191.

Appendix A   Figures

Figure 1

$$t = 1 \quad y_1; s, L^{1G}, L^1$$

with branches labeled p to C^{1D} and $(1-p)$ to C^{1N}, and

$$y_2, \{i, j\}; L^{2G}_{ij}, L^2_{ij}$$

Figure 2

end of $t = 2$

branches: r_l, r_h

q_L, q_H

$p_L \to C^{2D}_{Ll}$

$(1 - p_L) \to C^{2N}_{Ll}$

$p_H \to C^{2D}_{Hl}$

$(1 - p_H) \to C^{2N}_{Hl}$

same as above except replace l with h
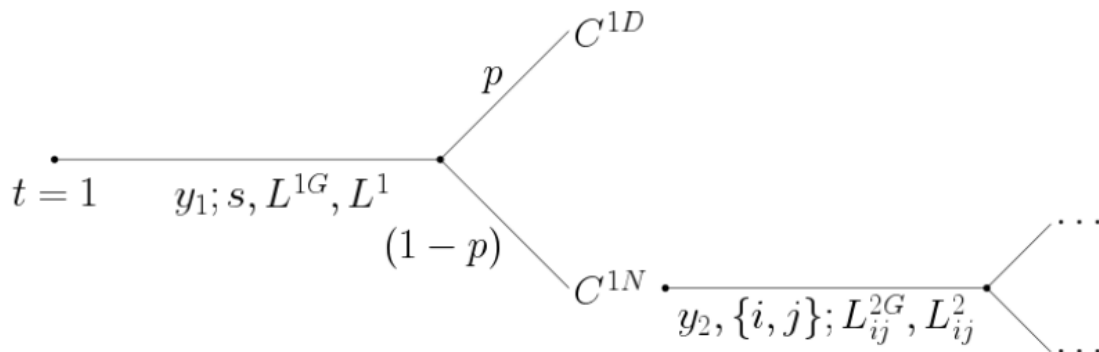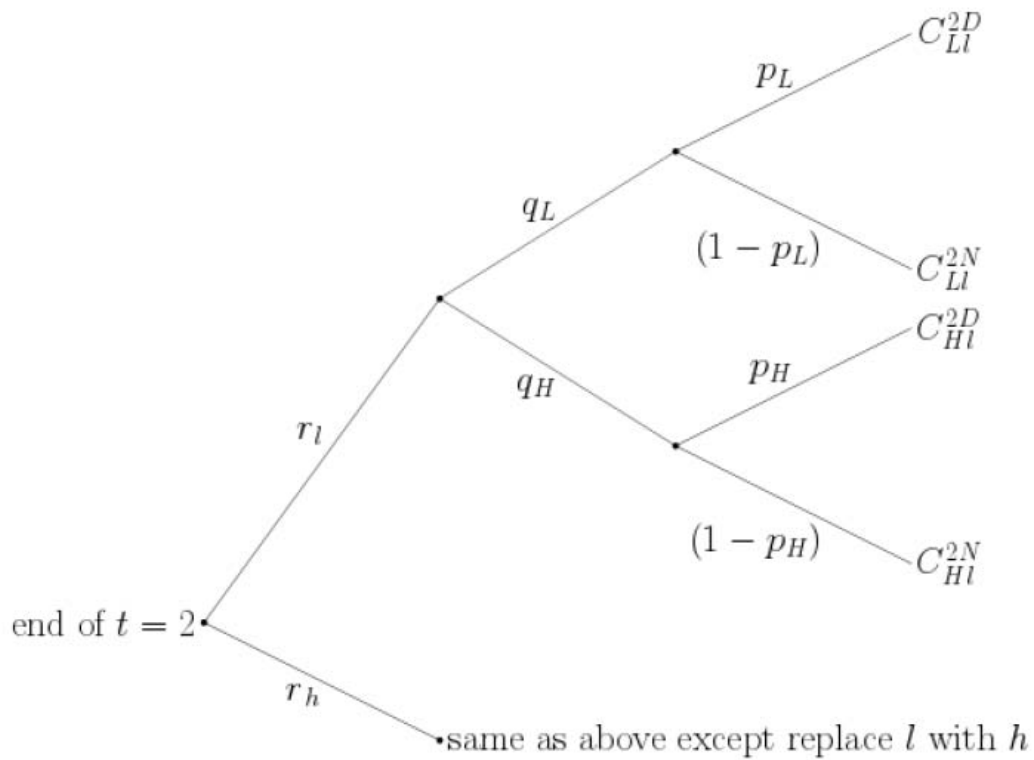
Appendix B   Proofs

B.1   Proof of Proposition 1

Denoting the Lagrange multipliers on the resource constraints in each period by λ_1, and λ_2, the necessary optimality conditions for an interior socially optimal allocation are:

$$C^{1D} : pv'_1(C^{1D}) = p\lambda_1, \tag{36}$$

$$C^{1N} : (1-p)u'_1(C^{1N}) = (1-p)\lambda_1, \tag{37}$$

$$s : (1-p)\lambda_1 = \lambda_2, \tag{38}$$

$$C^{2D}_{ij} : (1-p)q_i r_j p_i v'_2(C^{2D}_{ij}; \theta_j) = q_i r_j p_i \lambda_2, \tag{39}$$

$$C^{2N}_{ij} : (1-p)q_i r_j (1-p_i)u'_2(C^{2N}_{ij}) = q_i r_j (1-p_i)\lambda_2. \tag{40}$$

Combining (36) and (37) we obtain $v'_1(C^{1D}) = u'_1(C^{1N}) = \lambda_1$, and similarly combining (39) and (40) we obtain $v'_2(C^{2D}_{ij}; \theta_j) = u'_2(C^{2N}_{ij}) = \frac{\lambda_2}{1-p}$. Then, using (38), we have

$$v'_1(C^{1D}) = u'_1(C^{1N}) = v'_2(C^{2D}_{ij}; \theta_j) = u'_2(C^{2N}_{ij}) \text{ for all pairs } (i,j) \in \{H, L\} \times \{h, l\}.$$

This implies that, for a given demand type, consumption in the period 2 death state is the same for both risk types and likewise for the period 2 life state consumption. However, consumption in the death state is higher for the high demand type than for the low demand type. This is easily established as

$$v'_2(C^{2D}_{il}; \theta_l) = v'_2(C^{2D}_{ih}; \theta_h) < v'_2(C^{2D}_{il}, \theta_h) \Rightarrow C^{2D}_{ih} > C^{2D}_{il}. \tag{41}$$

Note also that the relationship between the period 2 death state consumption levels according to demand type is independent of risk type.

B.2   Proof of Proposition 2

When only spot markets for insurance are available, the first-order conditions for the households are:

$$L^1 : pv'_1(C^{1D})(1-\pi^1) + (1-p)u'_1(C^{1N})(-\pi^1) = 0,$$

$$s : -u'_1(C^{1N}) + \sum_i \sum_j q_i r_j \left(v'_2(C^{2D}_{ij}; \theta_j)(1+(1-\pi_i^2))\frac{\partial L^2_{ij}}{\partial s} + u'_2(C^{2N}_{ij})\left(1 - \pi_i^2 \frac{\partial L^2_{ij}}{\partial s}\right)\right) = 0,$$

$$L^2_i : p_i v'_2(C^{2D}_{ij}; \theta_j)(1-\pi_i^2) + (1-p_i)u'_2(C^{2N}_{ij})(-\pi_i^2) = 0.$$

Actuarially fair spot insurance contracts require $\pi^1 = p$ and $\pi_i^2 = p_i$. Then, using the first and last conditions above we obtain:

$$v'_1(C^{1D}) = u'_1(C^{1N}),$$

$$v'_2(C^{2D}_{ij}; \theta_j) = u'_2(C^{2N}_{ij}) \text{ for all pairs } (i,j) \in \{H, L\} \times \{h, l\}.$$

That is, marginal utilities are equated across life and death for all types ex-post and also ex-ante.

Now, differentiating the first condition with respect to p_i for a given level of savings, and solving for the change in insurance purchases we obtain:

$$\frac{\partial L_{ij}^2}{\partial p_i} = \frac{(v_2''(C_{ij}^{2D}; \theta_j) - u_2''(C_{ij}^{2L}))L_{ij}^2}{(1 - p_i)v_2''(C_{ij}^{2D}; \theta_j) + p_i u_2''(C_{ij}^{2L})}. \tag{42}$$

For a given demand level θ_j, this is positive whenever returns to consumption diminish at a faster rate in the death state. Given that we assume this, higher risk types buy more insurance. Therefore, high risk types consume less in both the life and death states than low risk types as they also face higher prices.

Finally, differentiating the first-order condition with respect to θ_j for a given level of savings, and solving for the change in insurance purchases we obtain:

$$\frac{\partial L_{ij}^2}{\partial \theta_j} = \frac{-v_2'}{p_i(1 - \pi_i^2)^2 v_2'' + (1 - p_i)(\pi_i^2)^2 u_2''} > 0. \tag{43}$$

This says that for a given risk level, higher demand types buy more insurance and therefore have more consumption that low demand types.

B.3   Proof of Proposition 4

Proof. We will first show that in the presence of fluctuations in demand, the use of GR contracts alone to insure against mortality risk is inefficient so that individuals always have incentives to use spot markets. Then, as long as spot markets are active, we will show that the equilibrium is inefficient as individuals do not receive full insurance against re-classification risk. However, we begin by demonstrating two important results that we will use throughout.

First, it is clear that if demand differences exist then in period 2 at the common price π^{2G}, higher demand types will want to purchase more coverage than low demand types. Formally, suppose that low demand types renew $L_{il}^{2G} \leq L^{1G}$ units of their GR. Then, the difference between their marginal utilities across life and death is always smaller at this amount of coverage than for the high demand types:

$$v_2'\left(y_2 + s + (1 - \pi^{2G})L_{il}^{2G}; \theta_l\right) - u_2'\left(y_2 + s - \pi^{2G}L_{il}^{2G}\right)$$
$$< v_2'\left(y_2 + s + (1 - \pi^{2G})L_{il}^{2G}; \theta_h\right) - u_2'\left(y_2 + s - \pi^{2G}L_{il}^{2G}\right),$$

as $\theta_h > \theta_l$ so high demand types want more coverage.

Second, in period 2 all risk types of a given demand type want to purchase the exact same coverage at a common price if feasible. To see this note that full insurance is obtained for a type ij individual in period 2 by renewing $L_{ij}^{2G} \leq L^{1G}$ units of GR when

$$v_2'\left(y_2 + s + (1 - \pi^{2G})L_{ij}^{2G}; \theta_j\right) = u_2'\left(y_2 + s - \pi^{2G}L_{ij}^{2G}\right).$$

Then clearly L_{ij}^{2G} will differ across demand types but not across risk types as the above equation is independent of p_i.

Now, when renewal of GR contracts are the sole means of obtaining insurance against mortality risk ex post, there are three possible outcomes in period 2:

1. Low demand types fully renew: $\lambda_{il} > 0$. This implies that high demand types also fully renew. However, as they want more insurance than low demand types, they are under-insured and purchase additional spot insurance. Formally, when all types fully renew we have:

$$0 \leq v_2' \left(y_2 + s + (1 - \pi^{2G})L^{1G}; \theta_l\right) - u_2' \left(y_2 + s - \pi^{2G}L^{1G}\right)$$
$$< v_2' \left(y_2 + s + (1 - \pi^{2G})L^{1G}; \theta_h\right) - u_2' \left(y_2 + s - \pi^{2G}L^{1G}\right),$$

   so high demand types have an incentive to purchase spot insurance as they are under-insured when they only renew GR. This case arises when demand for insurance increases over-time for all types.

2. High demand types do not fully renew: $\lambda_{ih} = 0$. This implies that low demand types also do not fully renew their GR contracts as they demand less insurance. However, they purchase too much GR as is cheap. To see this, note that via (25) the foc on L_{il}^{2G} implies:

$$v_2' \left(y_2 + s + (1 - \pi^{2G})L_{il}^{2G}; \theta_l\right) - u_2' \left(y_2 + s - \pi^{2G}L_{il}^{2G}\right) = \frac{\pi^{2G} - p_i}{\pi^{2G}(1 - p_i)} v_2'(C_{ij}^{2D}; \theta_j), \quad (\star)$$

   whenever $\lambda_{il} = 0$. However, note that if there is sufficient front-loading $\pi^{2G} > p_i$ and therefore low demand types are over-insured. This case arises when demand for insurance decreases over-time for all types.

3. High demand types fully renew but low demand types do not. In this case, we again have over-insurance by low demand types for the exact same reason as in the previous case. This case arises when demand is increasing for high demand types but decreasing for low demand types over time.

To see the inefficiencies in the first period note that in Case 1 above, (35) can be combined with the zero-profit condition to yield:

$$(1 - p) \sum_i \sum_j q_i r_j (p_i - \pi^{2G}) \left[v_1'(C^{1D}) - v_2'(C_{ij}^{2D}; \theta_j)\right] < 0,$$

whenever demand is decreasing over time for all types. This implies $v_1'(C^{1D}) - u_1'(C^{1N}) < 0$ or that there is over-insurance ex-ante as excessive amounts of GR is purchased in period 1. Suppose, instead that we are in Case 2, then (35) implies

$$v_1'(C^{1D}) - u_1'(C^{1N}) = \frac{\pi^{1G} - p}{p(1 - \pi^{1G})} u_1'(C^{1N}) > 0,$$

as all the multipliers are zero and $\pi^{1G} > p$ due to front-loading. This implies that individuals are under-insured and have incentives to purchase additional spot coverage ex-ante.

Finally, note that if we are in Case 1 above (e.g. high demand types desire more coverage in period 2), they always fully renew their GR and purchase additional spot insurance. The latter implies that consumptions depend on risk as the additional coverage is purchased

at different prices for different risk types with the same demand. Hence, in equilibrium, individuals are not fully insured against re-classification risk. Now, suppose we are in Case 2 above (e.g. low demand types desire less coverage in period 2), then the amount of GR low demand types renew depends on their risk type. This is obvious by noting that the RHS of equation $(\star)$ depends on p_i. The same is true for Case 3. Hence, individuals do not receive full coverage against re-classification risk with demand fluctuations.

Now, to see the second-part of the result, note that from Proposition 1 first best efficiency requires $v_1'(C^{1D}) = u_1'(C^{1N}) = v_2'(C_{ij}^{2D}; \theta_j) = u_2'(C_{ij}^{2N})$. Since spot prices differ by risk type, this means that all insurance in period 2 must be acquired through renewal of L^{1G} (i.e., $L_{ij}^2 = 0$). This requires that $\pi^{2G} \leq p_L$ in order that all types (including the lowest risk type) at least weakly prefer to renew their holding of GR rather than access the spot market for period 2 insurance needs. Full insurance in period 2 requires the RHS of (25) be zero. Thus, we must have

$$v_2'(C_{ij}^{2D}; \theta_j) - u_2'(C_{ij}^{2N}) = \left(\frac{\pi^{2G} - p_i}{\pi^{2G}(1 - p_i)} \right) v_2'(C_{ij}^{2D}; \theta_j) + \frac{\lambda_{ij}}{\pi^{2G}(1 - p_i)} = 0, \qquad (44)$$

and so

$$\left(\frac{\pi^{2G} - p_i}{\pi^{2G}(1 - p_i)} \right) v_2'(C_{ij}^{2D}; \theta_j) + \frac{\lambda_{ij}}{\pi^{2G}(1 - p_i)} = 0 \qquad (45)$$

which implies that

$$\lambda_{ij} = (p_i - \pi^{2G}) v_2'(C_{ij}^{2D}; \theta_j) \geq 0, \ \forall i, j. \qquad (46)$$

Since $\pi^{2G} \leq p_i$ for all i, it follows that $\lambda_{ij} \geq 0$ with strict inequality applying to all but the lowest risk type. Without loss of generality, we can assume that if $\pi^{2G} = p_L$ then $L - types$ (and hence all types) will renew all of L^{1G}. (For $L - types$, this follows by considering $\pi^{2G} = p_L - \varepsilon$ for $\varepsilon \to 0^+$ and relying on insurance demand being continuous in price. For all other risk types, $\lambda_{ij} > 0$ which implies $L_{ij}^{2G} = L^{1G}$).

No lapsation and no second period spot market activity means

$$C_{ij}^{2D} = y_2 + s + (1 - \pi^{2G}) L^{1G}, C_{ij}^{2N} = y_2 + s - \pi^{2G} L^{1G}. \qquad (47)$$

Therefore, we can write $C_{ij}^{2D} = C^{2D}$ and $C_{ij}^{2N} = C^{2N}$, $\forall \ i, j$. It follows that $v_2'(C^{2D}, \theta_j) = u_2'(C^{2N})$, $\forall j$ which is possible only if θ_j does not vary with j; i.e., $\theta_j = \theta$ for some $\theta > 0$.

We have now shown all conditions for period 2 that are required for first-best efficiency are met. We now need to consider conditions required for the first period allocation to satisfy efficiency, for inter-temporal efficiency to hold, and for the resource constraint to be satisfied.

First best efficiency also requires $v_1'(C^{1D}) = u_1'(C^{1N})$. Except for the possibility of a corner solution, $v_1'(C^{1D}) = u_1'(C^{1N})$ means $L^1 > 0$. Due to the requirements of no lapasation and no second period spot market purchases, this means that demand for insurance in period 1 ($L^1 + L^{1G}$) must exceed (or in the case of $L^1 = 0$ be equal to) demand for insurance in period 2. This confirms requirement 3 of the proposition. We now need to check that the above conditions ensure inter-temporal efficiency and satisfaction of the resource constraint. $v_1'(C^{1D}) = u_1'(C^{1N})$ implies that the RHS of (35) is zero; i.e.,

$$- (1 - p) \sum_i \sum_j q_i r_j \lambda_{ij} + (\pi^{1G} - p) v_1'(C^{1D}) = 0 \qquad (48)$$

which implies

$$(\pi^{1G} - p)v_1'(C^{1D}) = (1-p)\sum_i\sum_j q_i r_j \lambda_{ij}. \tag{49}$$

Using $\lambda_{ij} = (p_i - \pi^{2G})v_2'(C^{2D};\theta)$ gives us

$$(\pi^{1G} - p)v_1'(C^{1D}) = (1-p)v_2'(C^{2D};\theta)\sum_i\sum_j q_i r_j (p_i - \pi^{2G}). \tag{50}$$

Inter-temporal efficiency implies $v_1'(C^{1D}) = v_2'(C^{2D};\theta)$ and so we have

$$\pi^{1G} = p + (1-p)\sum_i\sum_j q_i r_j (p_i - \pi^{2G}). \tag{51}$$

The zero profit condition is

$$\pi^{1G}L^{1G} = pL^{1G} + (1-p)\sum_i\sum_j q_i r_j (p_i - \pi^{2G})L_{ij}. \tag{52}$$

Therefore, no lapsation ($L_{ij}^{2G} = L^{1G}$) implies the above two equations are consistent; that is, the resource constraint is satisfied. This completes the proof. $\qquad\square$

B.4 Proof of Proposition 5

Proof. Consider a spot markets only equilibrium and let $\hat{L}^1$, $\hat{L}_{ij}^2$, $\hat{s}$ denote the corresponding equilibrium values. Recall that as spot markets are active, marginal utilities between life and death are always equated. Suppose now that an ε unit of GR is offered with some front-loading ($\pi^{1G} > p$), and a second period renewal price such that $p_L < \pi^{2G} < p_H$. Such a contract is always feasible by making π^{2G} arbitrarily close to (but less than) p_H and therefore π^{1G} arbitrarily close to (but above) p for any set of model parameters and ε. Moreover, such a contract is fully renewed by high risks as GR is cheaper relative to spot – they will substitute some spot for GR. However, such a contract doesn't affect the behaviour of low risks – they continue to purchase the same amount of spot as before.

Then, $\lambda_{Hj} = (p_H - \pi^{2G})v_2'(y_2 + \hat{s} + (1-p_H)\hat{L}_{Hj}^2;\theta_j)$ and $\lambda_{Lj} = 0$, and the change in welfare from the marginal unit of GR is non-negative if:

$$\lim_{\varepsilon\to 0}\frac{\partial EU}{\partial L^{1G}}\bigg|_{L^{1G}=L_{Hj}^{2G}=\varepsilon}$$
$$= p(1-\pi^{1G})v_1'(y_1+(1-p)\hat{L}_1)-(1-p)\pi^{1G}u_1'(y_1-\hat{s}-p\hat{L}_1)+(1-p)\left(q_H r_h \lambda_{Hh} + q_L r_l \lambda_{Hl}\right) > 0. \tag{53}$$

Note that the zero-profit condition on the GR contract implies:

$$\pi^{1G} - p = (1-p)q_H(p_H - \pi^{2G}), \tag{54}$$

as it is fully renewed by high risks only. Using this expression and the definitions of λ_{ij} we obtain

$$p(1 - \pi^{1G})v_1'(y_1 + (1-p)\hat{L}_1) - (1-p)\pi^{1G}u_1'(y_1 - \hat{s} - p\hat{L}_1) + (1-p)\left(q_H r_h \lambda_{Hh} + q_L r_l \lambda_{Hl}\right)$$
$$= \sum_j q_H r_j (p_H - \pi^{2G}) \left[v_2'(y_2 + \hat{s} + (1-p_H)\hat{L}_{Hj}^2; \theta_j) - v_1'(y_1 + (1-p)\hat{L}_1)\right]$$
$$= q_H(p_H - \pi^{2G}) \sum_j r_j \left[v_2'(y_2 + \hat{s} + (1-p_H)\hat{L}_{Hj}^2; \theta_j) - v_1'(y_1 + (1-p)\hat{L}_1)\right].$$

Finally, using the first-order condition on savings, we have

$$\lim_{\varepsilon \to 0} \left.\frac{\partial EU}{\partial L^{1G}}\right|_{L^{1G}=L^{2G}_{Hj}=\varepsilon}$$
$$= q_H(p_H - \pi^{2G})q_L \sum_j r_j \left[v_2'(y_2 + \hat{s} + (1-p_H)\hat{L}_{Hj}^2; \theta_j) - v_2'(y_2 + \hat{s} + (1-p_L)\hat{L}_{Lj}^2; \theta_j)\right] > 0,$$

$$\tag{55}$$

as whenever there is re-classification risk, high risks have higher marginal utility in the death state than low risks.

$\square$

Appendix C   Simulation Results

Note that all figures in the tables below have been multiplied by 100.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
C^{1D}	124.39	271.42	113.12	269.59	278.73	269.00
C^{1N}	87.96	85.83	79.99	85.25	88.14	85.07
C^{2D}_{Ll}	96.35	171.66	357.72	190.63	88.14	93.18
C^{2N}_{Ll}	87.96	85.83	79.99	85.25	88.14	85.07
C^{2D}_{Lh}	393.35	171.66	357.72	190.63	197.09	380.42
C^{2N}_{Lh}	87.96	85.83	79.99	85.25	88.14	85.07
C^{2D}_{Hl}	96.35	171.66	357.72	190.63	88.14	93.19
C^{2N}_{Hl}	87.96	85.83	79.99	85.25	88.14	85.07
C^{2D}_{Hh}	393.35	171.66	357.72	190.63	197.09	380.42
C^{2N}_{Hh}	87.96	85.83	79.99	85.25	88.14	85.07
Utility	75.47	48.45	144.63	51.34	44.47	86.98

Table 4: Social Optimum: consumptions and expected utility.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
s	12.19	-2.52	20.80	0.42	-3.46	0.77
L^1	23.40	191.12	11.62	183.22	193.66	182.27
L^2_{Ll}	10.61	88.62	311.34	110.47	0.00	9.53
L^2_{Lh}	289.15	107.24	311.34	110.47	106.20	259.71
L^2_{Hl}	10.22	84.77	153.30	76.71	0.00	9.53
L^2_{Hh}	142.38	101.65	153.30	76.71	95.68	259.71
C^{1D}	121.53	275.83	110.69	268.56	278.17	267.69
C^{1N}	85.93	87.23	78.27	84.93	87.97	84.65
C^{2D}_{Ll}	121.74	177.24	401.00	199.84	96.54	109.34
C^{2N}_{Ll}	111.13	88.62	89.67	89.37	96.54	99.82
C^{2D}_{Lh}	372.43	194.00	401.00	199.84	192.13	334.51
C^{2N}_{Lh}	83.28	86.76	89.67	89.37	85.92	74.80
C^{2D}_{Hl}	117.30	169.54	197.45	138.77	96.54	109.34
C^{2N}_{Hl}	107.08	84.77	44.15	62.06	96.54	99.82
C^{2D}_{Hh}	183.38	183.89	197.45	138.77	173.09	334.51
C^{2N}_{Hh}	41.01	82.24	44.15	62.06	77.41	74.80
Utility	63.65	45.46	131.31	49.32	43.47	74.80
CV	4.80	1.20	4.44	0.80	0.40	0.98

Table 5: Spot markets only: equilibrium choices, consumptions and expected utility.

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**	**Case 6**
p^{1G}	12.89	11.62	13.05	15.33	9.05	9.82*
p^{2G}	8.17	10.52	8.51	6.03	15.98	6.65*
s	6.37	-4.43	11.00	-8.39	-4.64	-3.16
L^1	0.00	95.96	0.00	87.31	95.07	0.00
L^{1G}	61.48	94.05	84.79	105.42	99.86	187.62
L^2_{Ll}	0.00	86.88	204.56	0.00	0.00	0.00
L^{2G}_{Ll}	23.43	0.00	84.79	105.42	0.00	34.76
L^2_{Lh}	215.56	105.13	204.56	0.00	104.90	78.15
L^{2G}_{Lh}	61.48	0.00	84.79	105.42	0.00	187.62
L^2_{Hl}	0.00	0.00	100.72	0.00	0.00	0.00
L^{2G}_{Hl}	61.48	94.05	84.79	278.29	13.65	34.76
L^2_{Hh}	106.14	7.32	100.72	0.00	0.00	78.15
L^{2G}_{Hh}	61.48	94.05	84.79	105.42	99.86	187.62
C^{1D}	153.56	271.40	173.72	269.59	278.29	269.20
C^{1N}	85.71	85.82	77.94	85.25	88.00	84.73
C^{2D}_{Ll}	127.89	173.76	372.67	190.67	95.36	129.30
C^{2N}_{Ll}	104.46	86.88	83.33	85.25	95.36	94.53
C^{2D}_{Lh}	356.84	190.19	372.67	190.67	189.77	342.32
C^{2N}_{Lh}	79.79	85.05	83.33	85.25	84.87	76.55
C^{2D}_{Hl}	162.83	179.72	238.93	190.67	106.83	129.30
C^{2N}_{Hl}	101.35	85.67	53.43	85.25	93.18	94.53
C^{2D}_{Hh}	215.90	183.38	238.93	190.67	179.26	342.32
C^{2N}_{Hh}	48.27	82.01	53.43	85.25	79.40	76.55
Utility	66.40	48.42	136.49	51.34	43.68	85.00
CV	3.67	0.01	2.73	0.00	0.32	0.75

Table 6: GR plus spot equilibrium prices, choices, consumptions and expected utility.
*These are not unique, alternatives sets of prices can also deliver the same allocation.
Note: $p = 0.08$ and $p_L = 0.10$ in all cases.