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Abstract

Quality and quantity are very common features of production processes. People care about these two features and they tend to be connected. I consider a contest in which the quantity and quality of output are rewarded. The output in the quality contest plays a dual role. It counts in the quality contest but it is also converted into quantity-equivalent output to obtain *total* output in the quantity contest. This latter feature implies that the two contests are interlinked. I find that when the unit cost of producing quality is sufficiently high, then treating quality and quantity as the same has a disincentive effect on the production of quality. In contrast, when the unit cost of producing quality is sufficiently low, treating quality and quantity as the same has no disincentive effect on the production of quality. I also find an equilibrium in which no one exerts effort in the quantity contest. When there is a binding budget constraint on effort, I find that effort in the quantity contest is smaller relative to the unconstrained case but effort in the quality contest *may* remain unchanged.

JEL-Codes: D720.

Keywords: Bayesian Nash equilibrium, contests, quality, quantity.

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Introduction

A contest is a game in which players compete over a prize or set of prizes by making irreversible outlays or expenditures. The seminal works by Tullock (1980) and Lazear and Rosen (1981) have resulted in a vibrant literature on contests (see, for example, Konrad, 2009, and Vojnovic, 2015).

In this paper, I want to study an unexplored contest, what one may refer to as quantity-cum-quality contests. Let me give some examples. Leaders of a country may be ranked by their major or most enduring policies (quality of work) and also by their entire set of policies (quantity of work). In contests for professional recognition or prizes in academia, a scholar's success depends on the quantity (number) of publications and quality of publications (i.e., citations, rank or impact factor of journals, etc). In some universities, internal competitions over research chairs, course reductions, promotion, etc that are evaluated by committees of administrators and members, drawn from different fields, may focus more on quantity of publications than on quality of publications. In contrast, external academic awards, recognition, and reputation within a scholar's field tends to be evaluated by people within his/her field and is more likely to be influenced by the quality of publications. Sports teams are ranked by the quantity (number of competitions won) and also by the quality of competitions won. In European soccer, teams are ranked by the number of domestic league cups won and the number of European club championships won with the UEFA Champions league cup being the most prestigious. In the Olympics, countries and athletes are ranked by the number of gold medals (quality) won and the total number (quantity) of medals (i.e., gold, silver, bronze) won. In fact, whether total medals or only gold medals should be used is a subject of debate.[1] And a gold medal in the 100 meter race for men -- which comes with the accolade of "the world's fastest man" -- may have more weight than a gold medal in an 800 meter race or triple jump.[2] A world or Olympic record will carry even more weight. A musician's or novelist's work is judged by the number of albums or books written and also by the number of hits or bestsellers produced. Departments in a university may be ranked by quality of students (e.g., graduate student placements) and quantity of students (enrollment numbers). In film and television, actors may be ranked by the number of *Academy Awards* (i.e., the Oscars) won and also by the total number of awards won.[3] Some of these rankings need not be formal rankings because there is certainly an informal contest for recognition that human beings, as social beings, care about.

These contests would be trivial if the quantity contest and quality contest were separate. But they are not. By a quantity-cum-quality contest, I mean a contest with the following features: (i) There is a prize for quantity of output and a prize for quality of output, (ii) in the quantity contest, the quality of output is converted into quantity-equivalent output to obtain *total* quantity of output, but not vice-versa, and (iii) the unit cost of effort in the quality contest is higher than the unit cost of effort in quantity contest. I note that the rent-givers or contest-designers of quantity contests need not be the same as the rent-givers or contest-designers of quality contests. I return to this point in section 2.1.

[1] For example, see the February 2018 article in *Slate Magazine* titled "The Great Olympics Debate: Should We Rank Countries by Gold Medals or Total Medal Count."

[2] In a given event (e.g., triple jump), the same performance by an athlete may win him a gold, silver, or bronze medal. But for different events (e.g., if he competes in 100 m and 400 m races), he may allocate different levels of effort and have different aspirations. He may put in more effort to prepare for the Olympic games than he would for the Commonwealth games or National games.

[3] While the effort put into the same movie can win both an *Academy award* and *Golden Globe award*, it is also true that in the same year or during an actor's career, she may put different efforts into different movies depending on her sense of the quality of the script.

In the preceding paragraph, it is the feature in (ii) that makes the quantity and quantity contests interlinked. Otherwise, they will be two separate contests. The feature in (ii) is captured in cases where publication in a prestigious journals is considered to be equivalent to, for example, two or more publications in a less prestigious journal. An Olympic gold medal may be equivalent to three bronze medals.

It is important to note that while output in the quality contest can be translated into equivalent units of output in the quantity contest, output in the quantity contest *cannot* be translated into quality-equivalent output. For example, five or ten books, none of which made it to a best-seller list, may not be equivalent to one book that made it to a best-seller list. At the 2017 meetings of the American Economic Association, Nobel laureates Robert Akerlof and James Heckman bemoaned the overemphasis on publishing in the top five journals,[4] a phenomenon that the Roberto Serrano of Brown University has humorously dubbed *Top5itis*. According to Serrano (2018), "Top5itis is a disease that currently affects the economics discipline. It refers to the obsession of the profession of academic economists with the so-called "top5 journals." Heckman and Moktan (2018) refer to this obsession as the "tyranny of the top five". Whether this obsession is detrimental to the Economics profession is not my focus. My point is that *Top5itis* may explain why many economists will not consider multiple publications in lower-tier journals to be equivalent to one publication in the so-called top-5 journals. In film and television, multiple *Golden Globes* awards may not be seen as equivalent to an *Academy Award* (i.e., the Oscars).

[4]See *Publishing and Promotion in Economics: The Curse of the Top5*, a discussion session at the American Economic Association annual meetings, https://www.aeaweb.org/webcasts/2017/curse.php

It is obvious that in a quantity-cum-quality contest, the players have the opportunity to invest in two dimensions: quality and quantity. There are contests with multiple dimensions of efforts. An example is a contest with sabotage where the players can invest in both productive effort and destructive effort. These contests have been studied in Lazear (1989), Konrad (2000), Chen (2003), Kräkel (2005), Amegashie and Runkel (2007), and Amegashie (2012). There are also multiple dimensions of efforts in contests in which the players can engage in production and appropriation (theft) or in the defense of their properties (e.g., Hirshleifer, 1995; Skapedas and Syropoulous, 1997). Finally, Epstein and Hefeker (2003), Rai and Sarin (2009) and Arbatskaya and Mialon (2010, 2012) have studied contests with multiple dimensions of efforts, none of which is necessarily destructive (sabotage). Contests in which the prize is awarded by a committee and so the players direct their efforts toward multiple committee members also fall in this class of contests (see, for example, Amegashie, 2002; Congleton, 1984).

There is also a literature on selection or entry of participants into contests that is related to the present paper. In Lazear and Rosen (1981), Amegashie and Wu (2004), Azmat and Moller (2009, 2016), Edwin Leuven et al. (2011), Konrad and Kovenock (2012), Damiano, Hao, and Suen (2012), and Morgan, Sisak, and Vardy (2017), contestants can choose from a set of contests to participate in. However, unlike the contest in the present paper, the participants can only compete in one contest. A participant in this paper can compete in two contests.

Iluz and Sela (2018) study a model of two contests and two prizes. Effort in the first contest counts as effort in the second contest. While there are some similarities between this paper and Iluz and Sela (2018), some important differences remain. In their paper, efforts in the two contests take place sequentially while they take place simultaneously in this paper. They assume that effort in the first contest counts as one-for-one effort in the second contest. This is equivalent to $r = 1$ in my model, where r is the rate of transformation from effort in the quality contest to equivalent units in the quantity contest. In my model, I focus on $r \geq 1$ and look at the different effects of r on total efforts in the quality and quantity contests including the incentive effects of treating quality and quantity of output as the same. I also study an extension in which r is a random variable. They have two players while this paper has $N \geq 2$ players. Finally, I also consider budget constraints, incomplete information, and discuss different applications.

What I refer to as quantity contest may be referred to as a low-cost or low-quality contest and what I refer to as quality contest may be referred to as a high-cost or high-quality contest. But because output in the quality contest can be converted into quantity-equivalent output and added to output primarily intended for the quantity contest, I prefer to use quantity contest and quality contest. Serena (2017) has a paper titled "quality contests". It is a model with only one contest. Naturally, his definition of quality contest is different from the definition in this paper. By "quality contest", he means a contest in which the objective is the maximization of the expected effort of the winner.

The paper is organized as follows: the next section presents a model of a quantity-cum-quality contest with complete information and a single prize in each contest. It derives some results and discusses them. Section 3 considers some extensions: budget constraints and incomplete information. Section 4 concludes the paper.

1. A quantity-cum-quality contest

Consider a contest with $N \geq 2$ risk-neutral players each with valuation, $V > 0$, of winning the prize. Player k chooses his effort $e_k \geq 0$, $k = 1, 2, ..., N$. The cost of a unit of effort is $\alpha > 0$. Call this contest a *quantity contest*. The same N players may also participate in another contest, a *quality contest*, in which each player has valuation $W > 0$ of winning the prize. Player k chooses his effort $x_k \geq 0$, $k = 1, 2, ..., N$. The cost of a unit of effort is $\beta > \alpha$.

Suppose that an effort of x_k by player k in the quality contest produces an output of x_k and an effort of e_k by player k in the quantity contest produces an output of e_k.[5] Assume that a unit of output in the quality contest is equivalent to r units of output in the quantity contest, where $r \geq 1$. We may write player k's *total* output in the quantity contest as $e_k + rx_k$. Then, using the Tullock contest success function, player k's probability of winning the quantity contest and quality contest respectively are:

$$p_k = \frac{e_k + rx_k}{e_k + rx_k + \sum_{j \neq k}(e_j + rx_j)} \text{ and } q_k = \frac{x_k}{x_k + \sum_{j \neq k} x_j}, \tag{1}$$

$k = 1, ..., N$.

Because quantity can be directly produced in the quantity contest or produced in the quality contest and then converted to quantity, I shall sometimes use "*direct* production of quantity" or "*directly* produce quantity" (in the quantity contest) to refer to e_k as distinct from rx_k.

This is a simultaneous-move game with complete information. My solution concept is Nash equilibrium.

The payoff of player k who participates in both the quantity and quality contests is:

$$\Pi_k = p_k q_k (V + W) + (1 - p_k) q_k W + (1 - q_k) p_k V + (1 - p_k)(1 - q_k)(0) - \alpha e_k - \beta x_k,$$

$$= \frac{e_k + rx_k}{e_k + rx_k + \sum_{k \neq j}(e_j + rx_j)} V + \frac{x_k}{x_k + \sum_{k \neq j} x_j} W - \alpha e_k - \beta x_k. \tag{2}$$

Note that if $r = 0$, we have two separate or non-connected Tullock contests. We know that in this case, every contestant will exert a positive effort in each contest. Then given $r > 0$,

[5] The production function is linear and deterministic. This is not necessary. A non-linear and stochastic production function can yield the contest success functions in (1). See, for example, Lu and Fu (2012), Jia (2008) and Jia et al. (2013).

an immediate observation is that there is no Nash equilibrium with zero effort by any player in the quality contest. This follows from the fact that effort in the quantity contest does not enter the contest success function of the quality contest and zero effort by any player is not a Nash equilibrium in the quality contest for the same reasons that it is not a Nash equilibrium in a standard Tullock contest. With this in mind, we write the first-order conditions as:

$$\frac{\partial \Pi_k}{\partial e_k} = \left(\frac{1}{e_k + rx_k + \sum_{k \neq j}(e_j + rx_j)} - \frac{e_k + rx_k}{\left(e_k + rx_k + \sum_{k \neq j}(e_j + rx_j)\right)^2}\right) V - \alpha \leq 0,$$

$$e_k \geq 0, e_k \frac{\partial \Pi_k}{\partial e_k} = 0, \tag{3}$$

and

$$\frac{\partial \Pi_k}{\partial x_k} = \left(\frac{1}{e_k + rx_k + \sum_{k \neq j}(e_j + rx_j)} - \frac{e_k + rx_k}{\left(e_k + rx_k + \sum_{k \neq j}(e_j + rx_j)\right)^2}\right) rV + \left(\frac{1}{x_k + \sum_{k \neq j} x_j} - \frac{x_k}{\left(x_k + \sum_{k \neq j} x_j\right)^2}\right) W -$$

$$\beta = 0, \tag{4}$$

Using the first-order conditions and evaluating them at a symmetric Nash equilibrium for each type of contestant, where $e_k = \tilde{e} > 0$ and $x_k = \tilde{x} > 0$ for all k, gives:

$$\frac{\partial \Pi_k}{\partial e_k} = V\left(\frac{1}{N(\tilde{e} + r\tilde{x})} - \frac{\tilde{e} + r\tilde{x}}{[N(\tilde{e} + r\tilde{x})]^2}\right) - \alpha = 0, \tag{5}$$

and

$$\frac{\partial \Pi_k}{\partial x_k} = rV\left(\frac{1}{N(\tilde{e} + r\tilde{x})} - \frac{\tilde{e} + r\tilde{x}}{[N(\tilde{e} + r\tilde{x})]^2}\right) + W\left(\frac{1}{N\tilde{x}} - \frac{\tilde{x}}{[N\tilde{x})]^2}\right) - \beta = 0. \tag{6}$$

Put (5) into (6) to get $\alpha r + W\left(\frac{1}{M\tilde{x}} - \frac{\tilde{x}}{[M\tilde{x})]^2}\right) - \beta = 0$. Then $\tilde{x} = \frac{N-1}{N^2}\frac{W}{\beta - \alpha r}$. Put this into (5) and solve to get $\tilde{e} = \frac{N-1}{N^2}\frac{V}{\alpha} - \frac{N-1}{N^2}\frac{rW}{\beta - \alpha r} = \frac{N-1}{N^2}\left(\frac{V}{\alpha} - \frac{rW}{\beta - \alpha r}\right) > 0$. This gives:

Proposition 1: *Suppose* $\beta - \alpha r > \alpha r\frac{W}{V}$. *Then there exists a Nash equilibrium in which each contestant exerts an effort of* $\tilde{e} = \frac{N-1}{N^2}\left(\frac{V}{\alpha} - \frac{rW}{\beta-\alpha r}\right)$ *in the quantity contest and* $\tilde{x} = \frac{N-1}{N^2}\frac{W}{\beta-\alpha r}$ *in the quality contest.*[6]

Note that given $\beta - \alpha r > \alpha r\frac{W}{V}$, it follows that $\beta > \alpha r$. This also ensures that $\tilde{x} = \frac{N-1}{N^2}\frac{W}{\beta-\alpha r} > 0$. Why is it necessary to have $\beta > \alpha r$? To see this, observe that the cost of x_k units of effort in the quality contest is βx_k. Because this gives $r x_k$ units of output in the quantity contest, it means that the effective per unit cost of these $r x_k$ units of output is $\frac{\beta x_k}{r x_k} = \frac{\beta}{r}$. If $\frac{\beta}{r} \leq \alpha$, then the effective unit cost of producing quantity-equivalent output (using output in the quality contest) is not more than the unit cost, α, of producing in the quantity contest (using *direct* output in the quantity contest). Given that output in the quality contest also counts in the quantity contest, the marginal benefit of x_k is always strictly greater than marginal benefit of e_k. Then, an equilibrium with $e_k > 0$ and $x_k > 0$ cannot exist if $\frac{\beta}{r} \leq \alpha$. Therefore, the equilibrium in proposition 1 requires $\beta > \alpha r$.

In proposition 1, total quantity produced is

$$N\tilde{e} + rN\tilde{x} = N\frac{N-1}{N^2}\left(\frac{V}{\alpha} - \frac{rW}{\beta-\alpha r}\right) + rN\frac{N-1}{N^2}\frac{W}{\beta-\alpha r}.$$

$$= \frac{N-1}{N}\frac{V}{\alpha}.$$

Total quality produced in proposition 1 is:

$$N\tilde{x} = \frac{N-1}{N}\frac{W}{\beta-\alpha r}.$$

This gives:

[6] In appendix A, we show that second-order conditions hold.

Corollary 1: *Suppose $\beta - \alpha r > \alpha r \frac{W}{V}$. Then there exists a Nash equilibrium in which total quantity produced is independent of the rate of transformation from quality to quantity (i.e., r) but total quality is increasing in r. In addition, the quality produced by each contestant is increasing in the prize (i.e., W) in the quality contest but is independent of the prize (i.e., V) in the quantity contest, although there is a link between both contests. In contrast the quantity directly produced in the quantity contest by each contestant is increasing in V but decreasing in W (i.e.,* $\frac{\partial \tilde{e}}{\partial V} > 0$ and $\frac{\partial \tilde{e}}{\partial W} < 0$).

Now towards the construction of an equilibrium with a corner solution, note that it is possible that some players may not *directly* produce output in the quantity contest. This is because output produced in the quality contest can be converted into output in the quantity contest. Accordingly, suppose each contestant *directly* chooses zero effort in the quantity contest and $x_k = x^* > 0$ in the quality contest. Then (6) becomes:

$$\frac{\partial \Pi_k}{\partial x_k} = rV\left(\frac{1}{Nrx^*} - \frac{rx^*}{[Nrx^*]^2}\right) + W\left(\frac{1}{Nx^*} - \frac{x^*}{[(Nx^*)]^2}\right) - \beta = 0. \quad (7)$$

Solving (7) gives $x^* = \frac{N-1}{N^2}\frac{V+W}{\beta}$ for each player. Then evaluating $\frac{\partial \Pi_k}{\partial e_k}$ (i.e., the expression in (3)) at $e^* = 0$ and x^* for all k gives $\frac{\partial \Pi_k}{\partial e_k} = \left(\frac{1}{Nrx^*} - \frac{rx^*}{(Nrx^*)^2}\right)V - \alpha \leq 0$. Substituting $x^* = \frac{N-1}{N^2}\frac{V+W}{\beta}$ and simplifying gives $\alpha r\left(1 + \frac{W}{V}\right) - \beta \geq 0$. Note that $\frac{\partial \Pi_k}{\partial e_k}$ is strictly decreasing in e_k. Therefore, if $\frac{\partial \Pi_k}{\partial e_k} \leq 0$ at $e^* = 0$ and $x^* = \frac{N-1}{N^2}\frac{V+W}{\beta}$, then for any $e_k > 0$, $\frac{\partial \Pi_k}{\partial e_k} < 0$ at $x^* = \frac{N-1}{N^2}\frac{V+W}{\beta}$ and $e_j = 0$ for all $j \neq k$.

This leads to the following proposition:

Proposition 2: *Suppose* $\beta - \alpha r \leq \alpha r \frac{W}{V}$. *Then there is a symmetric Nash equilibrium with* $e_k^* = e^* = 0$ *and* $x_k^* = x^* = \frac{N-1}{N^2}\frac{V+W}{\beta}$ *for all* k *= 1, 2, ... ,* N.[7]

In proposition 2, the total quantity produced is rNx^* and total quality is Nx^*. Noting that x^* is independent of r gives:

Corollary 2: *Suppose* $\beta - \alpha r \leq \alpha r \frac{W}{V}$. *Then there exists a Nash equilibrium in which total quantity is increasing in the rate of transformation from quality to quantity (i.e.,* r*) but total quality is independent of* r. *In addition, the quality produced by each contestant is increasing in the prize (i.e.,* W*) in the quality contest and also in the prize (i.e.,* V*) in the quantity contest.*

2.1 Discussion

Note that the condition $\beta - \alpha r \leq \alpha r \frac{W}{V}$ in proposition 2 can be rewritten as $r \geq \frac{\beta}{\alpha}\left(\frac{V}{V+W}\right)$. Therefore, if r is sufficiently large and/or the ratio of the marginal cost of effort in the quality contest to the marginal cost of effort in the quantity contest (i.e., $\frac{\beta}{\alpha}$) is sufficiently low, no contestant competes directly in the quantity contest.

Suppose quality and quantity are treated as the same in the quantity contest. That is, $r = 1$. For example, this occurs when the system of rewards in a research institution is based on *only* the number of publications, the reputation of an Olympic athlete is based on *only* the number of medals won, an innovator's reputation is based on *only* the number of patents, etc. Corollary 2 implies that if the unit cost of producing quality is sufficiently low

[7]Note that for a *fixed* value of rx_k, the contest success function in the quantity contest, given by p_k, boils down to the contest success function in Amegashie (2006). Therefore, the corner solution for one of the effort levels in proposition 2 is consistent with the analysis in Amegashie (2006).

(i.e., $\beta - \alpha r \leq \alpha r \frac{W}{V}$), then treating quantity and quality as the same does not have an effect on the output of quality in the contest. There is no disincentive effect. The low cost of producing quality implies that contestants do not focus on the direct production of quantity but only focus on producing quality and thereby quantity-equivalent output. Then given that it is optimal to focus on only producing quality (i.e., e = 0 for all contestants), the rate at which quality is converted into quantity is irrelevant because the contest success function in the game, whose arguments now become only efforts in quality, is homogenous of degree zero in quality efforts (i.e., x_k). Therefore, corollary 2 holds for any contest success function that is homogenous of degree zero, a property that is satisfied by the Tullock contest success function (see, for example, Jia et al., 2013).

In contrast, corollary 1 implies that if the unit cost of producing quality is sufficiently high, then treating quality and quantity as the same, has a disincentive effect on the production of quality. Paradoxically, it is institutions with people who have high costs of producing quality that tend to care less about quality. In effect, they focus too much on quantity and do not reward the production of quality.

On the preceding point, one is inclined to argue that when proposition 1 holds (in effect, corollary 1 holds), a contest designer should choose a very high r if he cares about quality or even if his objective function is convex combination of total quality and total quantity. This is correct. But while high r is desirable, it is important to note that r cannot be chosen arbitrarily high. This is because it has an obvious lower bound (i.e., r = 1) and it also has an upper bound. For example, in the Olympics, there is an attempt to get a "reasonable" set of rules for determining the value of a gold medal relative to a silver medal and a bronze.[8] In academia, the quality of journals and research are based on hard data like impact factors and citations. The journal ranked first may be given a reference weight (index) of 100 and all other journals below it are assigned smaller numbers accordingly based on relative impact factors.

The effort in the quality contest gives a contestant the chance of winning a prize of $V + W$ while effort in quantity contest gives a contestant the chance of winning a prize of W. Hence, one may argue that quality and quantity are not treated the same even if $r = 1$. That is, they are only treated as the same in the quantity contest but not in the overall contest. But, as pointed out in the introduction, the groups that award the prizes V and W need not be the same. People who are more concerned about quality are less likely to care about quantity relative to people who are concerned about a person's or an entity's entire body of work. For example, university-wide committees that award prizes in quantity contests by looking at a scholar's entire body of work are not the same as the scholar's peers in his field of study who tend to focus much more on only quality. A certain segment of sports fans and analysts consider teams as great only if they have won the most prestigious trophy in that sport while others consider a host of factors in ranking teams.

In the equilibrium in proposition 1, the prize in the quantity contest has no effect on effort in the quality contest. But in proposition 2, the prize in the quantity contest has a positive effect on effort in the quality contest. The result in proposition 2 accords with intuition while the result in proposition 1 is counter-intuitive. Regarding the result in proposition 1, note that when the

[8]According to an August 2012 article in the *New York Times* that ranked countries based on medals won at the 2008 and 2012 Summer Olympics, the rules used to generate the rankings "... assumed more medals are better, gold is better than silver and silver is better than bronze. Beyond that, the chart accounts for any possible weighting scheme, including counting all medals equally, counting gold medals only, or assigning different points for gold, silver and bronze.": http://archive.nytimes.com/www.nytimes.com/interactive/2012/08/07/sports/olympics/the-best-and-worst-countries-in-the-medal-count.html

prize in the quantity contest increases, a contestant who competes in both the quality and quantity contests can respond by increasing his direct effort in the quantity contest or by increasing his effort in the quality contest (because it will be converted into quantity) or both. It turns out that in proposition 1, the contestants choose to only increase their direct effort in the quantity contest in response to an increase in the prize of winning the quantity contest (i.e., $\frac{\partial \tilde{e}}{\partial V} > 0$). This makes sense given that proposition 1 holds when the unit cost of producing quality is very high.

3. Extensions

I consider two extensions in this section, one at a time: (i) budget-constrained contestants,[9] and (ii) incomplete information about r.

3.1 Budget constraint

Suppose there is a budget $B > 0$ that a contestant can use to allocate his effort between the quantity and quality contests. To be precise, I assume that $e_k + x_k \leq B$. Player k solves:

$$\max_{e_k, x_k} \Pi_k = \frac{e_k + r x_k}{e_k + r x_k + \sum_{k \neq j}\left(e_j + r x_j\right)} V + \frac{x_k}{x_k + \sum_{k \neq j} x_j} W - \alpha e_k - \beta x_k, \tag{8}$$

subject to: $e_k + x_k \leq B$,

$k = 1, 2, \dots, N$.

The interesting case to consider is $e_k + x_k = B$ (i.e., the constraint binds). Put $e_k = B - x_k$ into (8) to get:

$$\max_{x_k} \widetilde{\Pi}_k = \frac{B + (r-1)x_k}{B + (r-1)x_k + \sum_{k \neq j}\left(B + (r-1)x_j\right)} V + \frac{x_k}{x_k + \sum_{k \neq j} x_j} W - (\beta - \alpha) x_k - \alpha B, \tag{8a}$$

[9] I thank Jingfeng Lu for suggesting this extension.

I look for a symmetric Nash equilibrium (e_B^* , x_B^*). A useful observation is that $e_B^* = B$ and $x_B^* = 0$ is not an equilibrium. A player k who deviates to $e_k^d = B - \varepsilon > 0$ and $x_k^d = \varepsilon$, where ε is very small but positive, has a profitable deviation. His total cost of effort will rise marginally by $(\beta - \alpha)\varepsilon$ but his benefit will rise discontinuously from W/N to W in the quality contest. His total effort in the quantity contest will also rise by $(r-1)\varepsilon$. Therefore, $x_B^* > 0$ for $B > 0$.

Recall that the equilibrium in the unconstrained budget case (i.e., proposition 1) is $\tilde{x} = \frac{N-1}{N^2}\frac{W}{\beta-\alpha r}$ and $\tilde{e} = \frac{N-1}{N^2}\left(\frac{V}{\alpha} - r\frac{W}{\beta-\alpha r}\right) > 0$. Therefore, $\tilde{e} + \tilde{x} = \frac{N-1}{N^2}\left(\frac{V}{\alpha} - (r-1)\frac{W}{\beta-\alpha r}\right)$. So for the constrained case to be meaningful, we require

$$e_B^* + x_B^* = B < \frac{N-1}{N^2}\left(\frac{V}{\alpha} - (r-1)\frac{W}{\beta-\alpha r}\right) \equiv \overline{B}(r).$$

The FOC for the problem in (8a) for $x_B^* \in (0, B]$ is:

$$\frac{\partial \tilde{\Pi}_k}{\partial x_k} = \frac{(r-1)V}{B+(r-1)x_B^*}\frac{N-1}{N^2} + \frac{W}{x_B^*}\frac{N-1}{N^2} - (\beta - \alpha) \geq 0. \quad (9)$$

Note that the derivative in (9) is strictly decreasing in x_B^*. Therefore, if the weak inequality in (9) holds at $x_B^* = B$, then it holds for all $x_B^* \in (0, B]$. Then the optimal value is $x_B^* = B$. Putting $x_B^* = B$ into (9) and simplifying, it follows that if

$0 < B \leq \frac{N-1}{(\beta-\alpha)N^2}\left(W + \left(\frac{r-1}{r}\right)V\right) \equiv \underline{B}(r)$, then the Nash equilibrium is $x_B^* = B$ and $e_B^* = 0$,[10]

where we assume that $\underline{B}(r) < \overline{B}(r)$.

Suppose $\underline{B}(r) < B < \overline{B}(r)$. For $x_B^* \in (0, B)$, we require that $\frac{\partial \tilde{\Pi}_k}{\partial x_k} = 0$. This gives a quadratic equation whose only positive root is:

$$x_B^* = \frac{-\left((r-1)(V+W) - \theta B\right) + \sqrt{[(r-1)(V+W) - \theta B]^2 + 4(r-1)\theta W B}}{2\theta(r-1)}, \quad (10)$$

[10]These results are similar to the results in Che and Gale (1997) who first studied a Tullock contest with budget-constrained players.

where $r \neq 1$ and $\theta \equiv \frac{(\beta-\alpha)N^2}{N-1}$.[11] Then $e_B^* = B - x_B^*$.

Suppose (9) holds with strict equality. Then differentiating (9) with respect to B gives $\frac{\partial x_B^*}{\partial B} = -\frac{(r-1)^2 V}{(r-1)V + W\left(\frac{B+(r-1)x_B^*}{x_B^*}\right)^2} < 0$. It follows that $\frac{\partial e_B^*}{\partial B} = 1 - \frac{\partial x_B^*}{\partial B} > 0$.

Now suppose $r = 1$. If $0 < B \leq \frac{N-1}{N^2}\frac{W}{\beta-\alpha} \equiv \underline{B}(1)$, the equilibrium is $x_B^* = B$ and $e_B^* = 0$. And if $\underline{B}(1) \equiv \frac{N-1}{N^2}\frac{W}{\beta-\alpha} < B < \frac{N-1}{N^2}\frac{V}{\alpha} \equiv \overline{B}(1)$. Then $x_B^* \in (0, B)$. Put $r = 1$ into $\frac{\partial \tilde{\Pi}_k}{\partial x_k} = 0$ and solve to get is $x_B^* = \frac{N-1}{N^2}\frac{W}{\beta-\alpha}$. This gives $e_B^* = B - \frac{N-1}{N^2}\frac{W}{\beta-\alpha}$. In either case, effort in the quantity contest is smaller relative to the unconstrained case but effort in the quality contest *may* remain unchanged.

The analysis in this section can be summarized in the following proposition:

Proposition 3: *Suppose* $r > 1$ *and the budget to be allocated between efforts in the quality and quantity contests is binding. If the budget is sufficiently small* (*i.e.,* $0 < B \leq \underline{B}(r)$), *then there exists a Nash equilibrium in which an increase in the budget may lead to an increase in effort in the quality contest but no change in effort in the quantity contest. If the budget is sufficiently big* (*i.e.,* $\underline{B}(r) < B < \overline{B}(r)$), *then there exists a Nash equilibrium in which an increase in the budget leads to a decrease in effort in the quality contest and an increase in effort in the quantity contest.*

[11] The expression in (10) implies that we have an equilibrium in which although the efforts in both contests are positive, the effort in the quality contest is not independent of the prize in this quantity contest. This is different from the result in corollary 1.

3.2 Incomplete information

Suppose the players do not know the value of r. But they know that r is a random variable that is continuously distributed on $[1, \bar{r}]$ with positive density $f(r)$, $\bar{r} > 1$.[12]

Player k's payoff is:

$$\Omega_k = V \int_1^{\bar{r}} \frac{e_k + r x_k}{e_k + r x_k + \sum_{k \neq j}(e_j + r x_j)} f(r) dr + \frac{x_k}{x_k + \sum_{k \neq j} x_j} W - \alpha e_k - \beta x_k, \tag{11}$$

$k = 1, ..., N$.

Evaluating the first-order conditions at a symmetric Bayesian Nash equilibrium, $e_k = \bar{e} \geq 0$ and $x_k = \bar{x} > 0$ for all k, and simplifying gives:

$$\frac{V(N-1)}{N^2} \int_1^{\bar{r}} \frac{1}{\bar{e} + r\bar{x}} f(r) dr - \alpha \leq 0, \tag{12}$$

and

$$\frac{V(N-1)}{N^2} \int_1^{\bar{r}} \frac{r}{\bar{e} + r\bar{x}} f(r) dr + \frac{W(N-1)}{N^2} \frac{1}{\bar{x}} - \beta = 0. \tag{13}$$

For $\bar{e} = 0$ and $\bar{x} > 0$, equation (13) gives $\bar{x} = \frac{(V+W)(N-1)}{\beta N^2}$. Put this and $\bar{e} = 0$ into (12) to get $\frac{\beta V}{V+W} \int_1^{\bar{r}} \frac{1}{r} f(r) dr - \alpha \leq 0$. Hence, if β is sufficiently low, the Bayesian Nash equilibrium is $\bar{e} = 0$ and $\bar{x} = \frac{(V+W)(N-1)}{\beta N^2}$. This was also the equilibrium in the case of complete information (i.e., proposition 2), except that the cut-off value of β was different. In general, when $\bar{e} = 0$, the effort put into the production of quality is independent of the density of r. Again, this is a consequence of the fact that Tullock contest success function is homogenous of degree zero in effort levels. However, the density of r affects the cut-off value of β that ensures that $\bar{e} = 0$ is an equilibrium.

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[12]Note that, in this case, the players have incomplete information about an aspect of the contest that is *not* an attribute of the players and no player has private information. Starting with Hurley and Shogren (1998), there is a literature in which players in a contest have private information about the contest technology or their own attributes (e.g., cost, valuation of the prize, budget constraints, etc). To the best of my knowledge, the most up-to-date summary of this literature is in Ewerhart and Quartieri (2018).

The incomplete-information analogue of proposition 1 where $\bar{e} > 0$ and $\bar{x} > 0$ requires that an equilibrium pair $(\bar{e}, \bar{x})$ satisfies (12) and (13) with strict equality. I could not obtain a solution to this pair of integral equations.[13] Thus, I assume that r is uniformly distributed on [1, 2]. Therefore, $f(r) = 1$ on [1, 2] and $f(r) = 0$ otherwise.

Carrying out the integration in (12) and (13) gives:

$$\frac{V(N-1)}{\bar{x}N^2} ln\left(\frac{\bar{e}+2\bar{x}}{\bar{e}+\bar{x}}\right) - \alpha = 0, \tag{12a}$$

and

$$\frac{V(N-1)}{N^2}\left(\frac{1}{\bar{x}} - \frac{\bar{e}}{\bar{x}^2} ln\left(\frac{\bar{e}+2\bar{x}}{\bar{e}+\bar{x}}\right)\right) + \frac{W(N-1)}{N^2}\frac{1}{\bar{x}} - \beta = 0. \tag{13a}$$

Put (12a) into (13a) to get:

$$\bar{x} = \frac{(N-1)(V+W)}{\beta N^2} - \frac{\alpha}{\beta}\bar{e}. \tag{14}$$

The same step in the complete-information case immediately gave a solution for the equilibrium effort in the quality contest which was independent of the prize, V, in the quantity contest. That is not the case here as (14) clearly shows and so, in equilibrium, the quality produced may not be independent of the prize in the quantity contest. I illustrate this result with the following example: $W = 4, \beta = 4, \alpha = 1,$ and $N = 10.$ I also compute the effort levels, $\tilde{e} > 0$ and $\tilde{x} > 0,$ in proposition 1 (the complete-information case) by setting $r = \int_1^2 f(r)dr = 1.5$, which is the expected value of r in the incomplete-information case. This gives the following table of results:

[13] Rykin (2010) studied a Tullock contest in which each player had *private* information about his cost of effort and the players had only one choice of effort. He made the observation that the first-order conditions, being multi-dimensional nonlinear integral equations, "... can only be solved numerically, and even then finding a solution becomes increasingly difficult and ultimately unfeasible for large N." Using the case of a uniform distribution, as in Fey (2008), Rykin (2010) looked for numerical solution for only small values of N (i.e., $N \leq 4$).

Table 1: $W = 4, \beta = 4, \alpha = 1,$ and $N = 10$

V	$\bar{e}$	$\tilde{e}$	$\bar{x}$	$\tilde{x}$
2.5	0.0207	0.0090	0.1410	0.1440
3.0	0.0638	0.0540	0.1415	0.1440
3.2	0.0812	0.0720	0.1417	0.1440
3.4	0.0987	0.0900	0.1418	0.1440
3.6	0.1162	0.1080	0.1419	0.1440
3.8	0.1338	0.1260	0.1420	0.1440
4.0	0.1514	0.1440	0.1421	0.1440

Clearly, Table 1 shows that, in the case of incomplete information, effort in quality contest is a function of the prize in the quantity contest even if effort in the quality contest is positive. This is different from the result in corollary 1. Furthermore, $\bar{e} > \tilde{e}$ but $\bar{x} < \tilde{x}$.[14]

Start from the complete-information case where r = 1.5. Then consider the case of incomplete information where r is uniformly distributed on [1, 2]. Its expected value is 1.5 and it has a positive variance. Then the brief exercise in this section gives the following result:

Proposition 4: *Suppose there is uncertainty about the rate at which quality is converted to quantity but its expected value is equal to its value in the case of complete information . Then there exists a Bayesian Nash equilibrium in which, relative to the complete-information case, the players, although risk neutral, increase effort in the quantity contest but decrease effort in the quality contest.*

[14] I have also checked that second-order conditions for a maximum hold.

4. Conclusion

In this paper, I have studied a contest in which quality and quantity matter. I labeled it quantity-cum-quality contest. Quality and quantity are very common features of production processes or things that people care about and they tend to be connected. Therefore, one might say that the quantity-cum-quality contest in this paper is ubiquitous.

There are further issues that could be studied in this contest. For example, further analysis of the incomplete-information case is a potentially interesting avenue to explore. Also, the contest success function for the quantity contest, being a combination of direct output in the quantity contest and quantity-equivalent output in the quality contest, could be derived from a model with micro-foundations where the random component in the direct production of quantity may be correlated with or is independent from the random component in the production of quality.

Appendix A: Second-order conditions

Equilibrium in proposition 1

Define $\Pi_{ee} \equiv \frac{\partial^2 \Pi_k}{\partial e_k^2}$, $\Pi_{xx} \equiv \frac{\partial^2 \Pi_k}{\partial x_k^2}$, $\Pi_{ex} \equiv \frac{\partial^2 \Pi_k}{\partial e_k \partial x_k}$,

$T_v \equiv e_k + rx_k + \sum_{k \neq j}(e_j \; + \; rx_j)$, and $T_w \equiv x_k + \sum_{k \neq j} x_j$. Then

$\Pi_{ee} = \frac{V}{(T_v)^2}\left(\frac{e_k + rx_k}{T_v} - 2\right) < 0$, $\Pi_{xx} = \frac{r^2 V}{(T_v)^2}\left(\frac{e_k + rx_k}{T_v} - 2\right) + \frac{W}{(T_w)^2}\left(\frac{x_k}{T_w} - 2\right) < 0$. We get

$\Pi_{ex} = \frac{rV}{(T_v)^2}\left(\frac{e_k + rx_k}{T_v} - 2\right)$. This gives

$\Pi_{ee}\, \Pi_{xx} - (\Pi_{ex})^2 = \frac{VW}{(T_v)^2 (T_w)^2}\left(2 - \frac{e_k + rx_k}{T_v}\right)\left(2 - \frac{x_k}{T_w}\right) > 0.$

Therefore, the second-order conditions for a maximum for a player who competes in both contests and exerts a positive effort in each contest are satisfied. For the other players who compete in only the quantity contest, their problem is identical to the problem of a player in a single Tullock contest. It is well known that second-order conditions hold in this case.

Equilibria in propositions 2, 3, and 4: It is trivial to show that the second-order conditions hold.

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