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Abstract

Rising income inequalities are widely debated in public and academic discourse. In this paper, we contribute to this debate by proposing a new family of measures of unfair inequality. To do so, we acknowledge that inequality is not bad per se, but that its underlying sources need to be taken into account. Thereby, this paper is the first to reconcile two prominent fairness principles, namely equality of opportunity and freedom from poverty, into a joint measure of unfair inequality. Two empirical applications provide important new insights on the development of unfair inequality both over time (in the US) and across countries (in Europe). First, unfair inequality shows different time trends and country rankings compared to total inequality. Second, average unfair inequality doubles when complementing the ideal of an equal opportunity society with poverty aversion. Furthermore, we show that an exclusive focus on top incomes may misguide fairness judgments.

JEL-Codes: D310, D630, I320.

Keywords: inequality, equality of opportunity, poverty, fairness, measurement.

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1 Introduction

Rising income inequality in many countries around the world has led to intense debates – both in academia and in the public. However, calls for more redistribution are often countered by pointing out that outcome inequalities are both necessary to incentivize individuals and may comply with the requirements of justice in a market economy.¹ Along those lines, philosophical theories of distributive justice differentiate between fair (justifiable) and unfair (unjustifiable) inequalities. However, previous economic applications of such theories are subject to several criticisms concerning their theoretical scope, their empirical implementation as well as their policy implications (Kanbur and Wagstaff, 2016). In this paper, we address these shortcomings by proposing a new family of measures of unfair inequality that can be implemented empirically. Precisely, we are the first to reconcile two prominent fairness principles, namely equality of opportunity and freedom from poverty, into a joint measure of unfair inequality. Bringing these new measures to data, we provide important new insights on the development of unfair inequality both over time (in the US) and across European countries.

Following the seminal work by Piketty and Saez (2003), the literature on long-run trends in top income shares in the Western world has documented an increase of income inequality since the beginning of the 1980s (Atkinson and Piketty, 2007; Leigh, 2007; Atkinson et al., 2011; Roine and Waldenström, 2015; Guvenen and Kaplan, 2017; Piketty et al., 2018). This literature has directly influenced ensuing public controversies. For example, the Occupy Wall Street movement’s slogan – “*We are the 99%*” – directly follows from research on the income share of the top 1%. Among other interest groups, this movement has fiercely advocated for more redistribution. To the contrary, free-market pundits emphasize that through trickle-down effects everybody benefits from growth among job creators at the top. As a consequence, more redistribution would dis-incentivize those individuals and lead to lower welfare for everybody in the long-run.

While the equity-efficiency trade-off dominates public discourse on inequality, most theories of distributive justice agree that we should not be concerned by inequalities in outcomes per se, but that we should rather focus on the sources of outcome inequalities. To do so, these theories differentiate between fair (justifiable) and unfair (unjustifiable) sources of inequality. Unfair sources of inequality shall be eliminated completely while fair inequalities ought to persist.² According to many of these theories, outcome inequalities are unfair if they are rooted in factors beyond individual control. These factors could not have been influenced by individual choice and therefore people should not be held responsible for life outcomes that follow from them.³ In line with this reasoning, elicited preferences for redistribution show that individuals are more

¹For example, in his influential book Piketty (2014, p. 443) argues for increased redistribution on both efficiency and equity grounds: “*no matter how justified inequalities of wealth may be initially, fortunes can grow and perpetuate themselves beyond all reasonable limits and beyond any possible rational justification in terms of social utility.*” To the contrary, Mankiw, N. Gregory (2013, p. 33) argues against substantial tax hikes since “*such confiscatory tax rates are wrong, even ignoring any incentive effects. By [the just deserts] view, using the force of government to seize such a large share of the fruits of someone else’s labor is unjust, even if the taking is sanctioned by a majority of the citizenry.*”

²In the social choice literature these two intuitions are formally represented by *compensation* and *reward* principles (Fleurbaey, 2008; Fleurbaey and Maniquet, 2011).

³A non-comprehensive list of works emphasizing this distinction includes Rawls (1971), Sen (1980), Dworkin (1981a, 1981b), Arneson (1989), Cohen (1989), Roemer (1993, 1998), and Lippert-Rasmussen (2001).

willing to accept income differences which are due to effort (or laziness) rather than exogenous circumstances (Fong, 2001; Cappelen et al., 2007; Alesina and Giuliano, 2011; Alesina et al., 2017). A prominent field of research in economics that has been inspired by the distinction between exogenous circumstances and individual effort is the literature on intergenerational mobility (see, e.g., Björklund and Jäntti, 1997; Black and Devereux, 2011; Chetty et al., 2014a; Chetty et al., 2014b). Also the vast research on the gender pay gap (see, e.g., Blau and Kahn, 2017; Kleven et al., 2017) and racial disparities (see, e.g., Lang and Lehmann, 2012; Kreisman and Rangel, 2015) are concerned with inequalities that are rooted in factors beyond individual control.

Yet, in spite of its wide acceptance, invoking the notion of individual responsibility alone is insufficient to define fairness (e.g. Konow, 2003; Konow and Schwettmann, 2016). For example, when an outcome is such that it brings deep deprivation to an individual, questions of how it came about seem secondary to the moral imperative of addressing the extremity of the outcome, be it hunger, homelessness, violence or insecurity (Bourguignon et al., 2006).⁴ Hence, while outcome differences based on exogenous circumstances imply violations of fairness, the reverse statement does not hold true. To the contrary, in addition to the responsibility criterion there are many reasons why a given outcome distribution could be considered unfair – one of them being the notion of individual need.

In this paper, we propose the first family of measures for unfair inequality that incorporate the principles of equal opportunities (EOp) and freedom from poverty (FfP) in a co-equal fashion. In line with the previous discussion, we therefore take seriously the idea that life outcomes should be orthogonal to exogenous circumstances (EOp) *and* that everybody should have enough to make ends meet (FfP).

Our paper makes several contributions. First, both EOp and FfP have a vast theoretical and empirical literature. However, attempts to reconcile the two principles are scant. In existing approaches, priority is given to either principle, while the second serves as a mere weighting factor. To the contrary, in our work we treat EOp and FfP as co-equal principles conveying different grounds for compensation. That is, we develop the first inequality measure that detects unfairness emanating from unequal opportunities or poverty even if one of our two guiding principles is satisfied. To do so we build on the norm-based approach towards inequality measurement (Cowell, 1985; Magdalou and Nock, 2011). That is, we first reshape observed income distributions to comply with the principles of EOp and FfP, the satisfaction of both of which is required from a fairness perspective. In a second step, we then measure unfair inequality as the divergence between the observed and the fair income distribution. In fact we show that our proposed measure is easily interpretable and carries desirable properties identified in the respective literature branches on the measurement of EOp and FfP. Standard measures of the

⁴To illustrate this point, Kanbur and Wagstaff (2016) suggest the following thought experiment: Imagine yourself serving on a soup line. The indigents move forwards towards you and you hand out hot soup. But in one case a new piece of information is given to you. You are told that the outcome of the person in front of you was not the result of exogenous circumstances but a lack of effort. Would you then withdraw your soup holding hand because her outcome is morally justifiable according to the responsibility criterion? If not, clearly some other principle is cutting across the power of the equality of opportunity argument. A similar concern is expressed in Fleurbaey (1995a) who argues for the adoption of outcome egalitarianism in spheres of social interest, while maintaining responsibility induced differences in others. Along these lines we would consider the satisfaction of basic needs as indicated by a minimum threshold of income a matter of social interest.

two principles are nested in our approach, i.e. excluding either FfP or EOp from the list of normative principles yields a measure with sensible properties from the perspective of the remaining principle.

Second, we effectively propose a new route to address the downward bias inherent in previous empirical estimates of inequality of opportunity. This downward bias is a result of the partial observability of individual information (Ferreira and Gignoux, 2011) and has evoked controversy with respect to the usefulness of such lower bound estimates (Kanbur and Wagstaff, 2016). While previous works have addressed these informational constraints by the use of better datasets and the exploitation of alternative econometric techniques (see, e.g., Niehues and Peichl, 2014; Hufe et al., 2017), we cushion this concern by acknowledging that judgments on the fairness of an outcome distribution are informed by multiple normative principles.

Third, our measure has important implications for the debate about inequalities and the design of appropriate policy responses. We provide two empirical applications of our measure. First, we calculate unfair inequality in a harmonized dataset of 31 European countries. In our baseline estimations we find that unfair inequality doubles when complementing the concern for unequal opportunities with a concern for freedom from poverty. Second, we measure unfair inequality for the US in the time period from 1969 to 2012. We show that the inequality expansion from the early 1980s up until the beginning 1990s was largely due to the expansion of fair inequality. That is, increases in inequality were only weakly related to factors beyond individual control or based on people falling behind the poverty line. To the contrary, since the beginning of the 1990s the majority share of inequality increases must be considered unfair. This result is equally driven by increasing violations of EOp and FfP. Furthermore, we show that unfairness is nearly unrelated to increases of affluence. Therefore, we argue that the wide-spread focus on top-incomes (Atkinson et al., 2011; Piketty et al., 2018) may miss important aspects of the picture – especially when assessing the development of inequality with respect to aspects of fairness.

Furthermore, our framework may also be fruitfully integrated in models on fair taxation (Fleurbaey and Maniquet, 2006; Weinzierl, 2014; Ooghe and Peichl, 2015; Weinzierl, 2018) that address the equity-efficiency trade-off by reference to specific notions of fairness.

This paper is organized as follows. In section 2, we clarify the underlying normative principles of EOp and FfP. We continue by outlining our approach towards reconciling both principles into one measure of unfair inequalities in section 3. Section 4 describes the data sources for the empirical application, the results of which are presented in section 5. In particular, we provide empirical evidence on the extent of unfair inequalities for a set of 31 European countries. Moreover, we analyze the development of unfair inequality in the US for the time period 1969-2012. Lastly, section 6 concludes.

2 Normative Background

Equality of Opportunity. Equality of opportunity is a popular concept of fairness that is used to evaluate distributions of various outcomes, including health status (Rosa Dias, 2009; Trannoy et al., 2010), education (Ferreira and Gignoux, 2014; Oppedisano and Turati, 2015) or income (Aaberge et al., 2011; Ferreira and Gignoux, 2011). Following the seminal contributions

by Van de gaer (1993), Fleurbaey (1995b), and Roemer (1998), there now is a vivid theoretical and empirical literature that weaves the idea of personal responsibility into inequality research.⁵ For opportunity egalitarians inequalities are ethically acceptable to the extent that they are rooted in factors of individual responsibility. To the contrary, they condemn inequalities that follow from factors beyond individual control. Prominent examples of the latter are, for example, the biological sex, race, the place of birth, or the socioeconomic outcomes of parents. If individual responsibility factors were the sole determinants of the observed outcome distribution, the ideal of EOp would be realized to its full extent.

To operationalize the idea of EOp, the literature draws on the concepts of *circumstances* and *efforts*, where circumstances are those determinants of individual outcomes for which we do not want to hold individuals responsible. Differences in circumstances are typically taken as an indication that people are of a different *type*. To the contrary, efforts belong to the realm of personal responsibility. To the extent that the former rather than the latter are stronger (weaker) determinants of the empirical outcome distribution, society is considered less (more) fair than otherwise. Note that measures of intergenerational mobility (Black and Devereux, 2011; Chetty et al., 2014a; Chetty et al., 2014b), the gender pay gap (Blau and Kahn, 2017; Kleven et al., 2017) or racial disparities (Lang and Lehmann, 2012; Kreisman and Rangel, 2015) can be interpreted as measures of inequality of opportunity with one circumstance variable, only. Theoretically, EOp is underpinned by two fundamental ideas. First, people should be compensated for unequal circumstances. A prominent formulation of this idea is the principle of *ex-ante compensation* which postulates that the value of opportunity sets ought to be equalized across people with differential circumstances. The principle is *ex-ante* in the sense that opportunity sets are evaluated without regard to the individual level of effort exertion. The second idea inherent to the concept of EOp is that people should be appropriately rewarded for differential efforts. While there are again different formulations of this idea, one prominent version is the principle of *utilitarian reward*. Utilitarian reward states that effort should be rewarded so as to maximize the aggregate outcome of people with the same circumstances. This entails that outcome differences between individuals with the same circumstances are a matter of indifference, i.e. that within-type outcome distributions should only be compared by reference to their aggregate means.⁶

A simple way to formulate the equal-opportunity doctrine by reference to ex-ante compensation and utilitarian reward is the following. Consider a set of individuals $N = \{1, 2, \dots, n\}$ and an associated vector of non-negative incomes $Y^e = \{y_1^e, y_2^e, \dots, y_n^e\}$.⁷ Furthermore, assume that a set of circumstances $\Omega = \{C^1, C^2, \dots, C^k\}$ and a scalar of effort, θ , jointly determine the outcome of interest y , i.e. $g : \theta \times \Omega \mapsto \mathbb{R}^+$. Based on the expressions x_i^k of all circumstances $C^k \in \Omega$, we can partition N into a set of mutually exclusive types: $T = \{t_1, t_2, \dots, t_m\}$. Individuals i and j belong to the same type $t \in T$ if $x_i^k = x_j^k \forall C^k \in \Omega$. They belong to different types $t \in T$ if $\exists C^k \in \Omega : x_i^k \neq x_j^k$. From ex-ante compensation we know that the value of opportunity sets

⁵See Roemer and Trannoy (2015), Ferreira and Peragine (2016), and Van de gaer and Ramos (2016) for recent overviews.

⁶For a comprehensive discussion of different compensation and reward principles see the works of Fleurbaey and Peragine (2013) and Van de gaer and Ramos (2016).

⁷Note that superscript e is used to indicate the empirical income distribution. In section 3 we will also introduce a norm income distribution which is indexed by superscript r .

needs to be equalized across these types, while utilitarian reward specifies that opportunity sets need to be compared based on type-specific means. We therefore can construct a counterfactual distribution, where each outcome y_i^e is replaced by the mean realization of the individual's type mean, μ_i^e , to summarize the opportunity structure of N . This counterfactual distribution is indicative of unequal opportunities since any inequality can be traced back to differential circumstances and thus a non-compliance with the principle of ex-ante compensation.

Beyond theoretical reasoning, there is compelling empirical evidence that people indeed disapprove of inequalities that are rooted in factors beyond individual control. For instance, Alesina et al. (2017) use information treatments to show for a set of industrialized countries that policy preferences with respect to taxation and spending on opportunity-equalizing policies are robustly correlated with perceptions of social mobility. The lower social mobility within a society, the more people are willing to remedy existing inequalities by appropriate policy interventions. Faravelli (2007) demonstrates that perceptions of justice tend to more equal distributions when income differences originate from contextual factors that could not have been influenced by individuals. The works of Cappelen et al. (2007), Cappelen et al. (2010), and Krawczyk (2010) confirm that people uphold the equal-opportunity ideal even if it adversely affects their own material interests.

Freedom from Poverty. “Who is poor and who is not?” is one of the fundamental questions of poverty research (Alkire and Foster, 2011). As in the equal-opportunity literature, the insight that poverty cannot be construed in a uni-dimensional fashion – say, by focusing on some definition of household income only – resonates in many works on the measurement of poverty (Aaberge and Brandolini, 2015). Regardless of the specific outcome dimension of interest, the measurement of poverty is implemented through a poverty line y_{\min} which provides the threshold of deprivation, and a poverty index which aggregates the deprivations below the threshold. As such the population N can be partitioned into the groups $P = \{i : y_i^e \leq y_{\min}\}$ and $R = \{i : y_i^e > y_{\min}\}$, where the aggregation index is applied to the incomes of the former group.⁸ Akin to the literature on EOp, the normative concern for poverty operates on a principle of compensation: People should be compensated to the extent that they fall short of the poverty line. Naturally, the appropriate setting of the poverty line is a widely debated issue in the literature (among others Foster, 1998; Decerf, 2017). All else equal, the more lenient the definition of y_{\min} , the larger the group P to which compensation is owed.

There is ample evidence that people are willing to compensate not just on the base of ex-ante opportunity differences but also based on outcome differences among people with identical circumstances. For example, in Cappelen et al. (2013) both stakeholders and impartial spectators tend to equalize outcomes among participants who engaged in risky choices and the differential outcomes of which are the result of luck. Furthermore, the fairness preferences of people appear to be sensitive to individual needs, i.e. the intuition that everybody should have enough to make ends meet irrespective of how the given outcome came about (Konow, 2001; Gaertner and Schwettmann, 2007).

⁸Widely used indexes in the literature are the members of the family of measures proposed by Foster et al. (1984), which nests the headcount ratio as well as the poverty gap ratio.

Reconciling EOp and FfP. While the respective literature branches on equality of opportunity and poverty are immense individually, attempts to reconcile the two concerns are scant. In the current literature, there seem to be at least three ways to think about EOp and FfP in a joint framework. In spite of their merits, however, all three approaches are characterized by important shortcomings when assessing the fairness of empirical distributions with an eye to both EOp and FfP.

First, in analogy to Lefranc et al. (2008, 2009) one could incorporate a concern for poverty into the evaluation of opportunity sets. In fact, Lefranc et al. (2008, 2009) propose to describe opportunity sets by the area under the type-specific generalized Lorenz curve, $\mu_t^e(1 - G_t)$, where G_t is the type-specific Gini coefficient. As a consequence opportunity sets are evaluated by discounting the type-specific average advantage level μ_t^e with the type-specific income risk G_t . To shift the focus from income risk across the entire income distribution to the risk of deprivation, one could also conceive a different weighting of mean-advantage levels by replacing G_t with some measure of poverty P_t .⁹ Ceteris paribus, the value of opportunity sets would then decrease with the type-specific prevalence of poverty. Note, however, that such an approach would not treat EOp and FfP on an equal footing. To the contrary, it gives priority to the measurement of EOp, whereas FfP is of secondary importance and serves as a mere weighting factor. Therefore, such an approach would magnify (contract) the violations EOp in case of positive (negative) correlations between type-specific opportunity disadvantages and poverty prevalence. However, FfP carries no weight once the prioritized EOp principle is fulfilled. For illustration purposes, imagine an equal-opportunity society of two equally sized types in each of which 50% of the population fall below the poverty threshold. In this case, $\mu_1^e(1 - P_1) = \mu_2^e(1 - P_2) = \mu(1 - 0.5)$, i.e. the value of all opportunity sets would equal the population mean. Hence, such a measure would not raise any moral concern in spite of the fact that 50% of the population fall below the poverty threshold.¹⁰

Secondly, one could approach this question from the perspective of poverty measurement. This is the strategy pursued by Brunori et al. (2013), who propose an opportunity sensitive poverty measure that effectively weighs incomes below the poverty line by the value of the individual's opportunity set. While their approach is agnostic towards the specific criterion to evaluate opportunity sets, the ex-ante utilitarian evaluation is one of the available options. In this case, identical incomes below the poverty line receive less attention the more advantageous the average outcome of the individual's type. However, just as in the first approach their measures does not treat EOp and FfP on an equal footing. The measures are constructed as poverty indices and therefore they are compliant with the focus axiom (Sen, 1976), i.e. they are invariant to income changes among the non-poor. As a consequence, they do not detect any unfairness in societies that are free from poverty but that are characterized by severe violations of EOp among people earning an income above the poverty line. Again, the secondary principle – in this case EOp – carries no weight once the prioritized principle – in this case FfP – is

⁹Note that this would require the formulation of some *poverty-averse reward* principle which to the best of our knowledge has not been put forward to this date. For further reference, see also Van de gaer and Ramos (2016) who show how the inequality-averse measure proposed by Lefranc et al. (2008, 2009) can be linked to a combination of ex-ante compensation and inequality-averse reward.

¹⁰Clearly, this example draws on the headcount ratio as the poverty criterion that weighs opportunity sets. Without loss of generality, however, similar examples can be constructed for alternative poverty measures.

realized.

A third approach proposed in the literature is the construction of “opportunity-deprivation profiles” (Ferreira and Peragine, 2016), where members of circumstance types are considered opportunity-deprived if their average outcome falls below a pre-specified deprivation threshold. Effectively, this amounts to applying standard poverty measures to types instead of individuals. Hence, this approach identifies the counterfactual distribution of type-means as its object of analysis. As a consequence it is informative for the identification of particularly opportunity deprived types, however, less so when it comes to evaluating the overall fairness of an empirical income distribution.

In this work we treat EOp and FfP as co-equal principles conveying different grounds for compensation. That is, we develop the first family of measures that is able to detect unfairness emanating from unequal opportunities or poverty even if one of the two guiding principles is satisfied. Its construction will be outlined in the following section.

3 Norm-Based Inequality Measurement

For the purpose of constructing our novel measure of unfair inequality, we rely on the norm-based approach towards inequality measurement (Cowell, 1985; Magdalou and Nock, 2011; Almås et al., 2011). Again consider the society $N = \{1, 2, \dots, n\}$ and the associated vector of non-negative incomes $Y^e = \{y_1^e, y_2^e, \dots, y_n^e\}$. The norm-based approach compares the empirical distribution Y^e to a reference distribution $Y^r = \{y_1^r, y_2^r, \dots, y_n^r\}$.¹¹ For our purposes Y^r can be interpreted as the distribution of fair incomes in a society the construction of which is informed by a set of normative principles. Thus, the population is characterized by a vector of income pairs $Y = \{(y_1^e, y_1^r), (y_2^e, y_2^r), \dots, (y_n^e, y_n^r)\}$. Unfair inequality is now measured by summarizing the differences between Y^e and Y^r . Note that standard measures of outcome inequality, such as the Gini index, can also be understood as norm-based measures, in which the norm vector requires perfect equality among all individuals, i.e. $y_i^r = \frac{1}{N} \sum_i y_i^e \forall i \in N$.

Constructing the Norm Vector. While the empirical distribution Y^e is given in the data, the norm vector Y^r must be constructed by reference to a set of normative requirements. Let’s specify D as the set of all possible income distributions. Consider the following restrictions on D , which jointly define the set of eligible norm vectors for our purposes:

$$D^1 = \left\{ D : \sum_i y_i^r = \sum_i y_i^e \right\}. \quad (\text{Constant Resources})$$

Since we are concerned with the fair distribution of available resources in a given society, we want to avoid *creatio ex nihilo*. Thus, D^1 is the subset of norm distributions for which the total amount of resources matches its empirical counterpart.

We evaluate opportunity sets by reference to the principles of ex-ante compensation and utilitarian reward. Hence, we are infinitely inequality averse with respect to inequalities between circumstance types, while we are indifferent with respect to inequality within types (Fleurbaey and Peragine, 2013). As a consequence, opportunity sets can be compared by the respective

¹¹In the following we will use the terms *norm* and *reference* interchangeably.

type means. The ideal of an equal-opportunity society is realized if and only if there is equality across types with respect to their average outcomes. D^2 is the subset of distributions for which this criterion is satisfied:

$$D^2 = \left\{ D : \mu_t^r = \frac{1}{N_t} \sum_{i \in t} y_i^r = \frac{1}{N} \sum_i y_i^e = \mu \ \forall t \in T \right\}. \quad (\text{EOp})$$

We maintain that people have a claim for a minimal level of subsistence regardless of whether their empirical income is a result of pre-determined circumstance, effort or any other form of luck.¹² Thus, D^3 is the subset of distributions for which this ideal is realized:

$$D^3 = \{ D : y_i^r = y_{\min} \ \forall i \in P \}. \quad (\text{FfP})$$

Note that we can break down this requirement into two parts: $y_i^r = \frac{1}{N_P} \sum_{i \in P} y_i^r = \mu_P^r \ \forall i \in P$ and $\mu_P^r = y_{\min}$. The first component states that we are infinitely inequality averse with respect to inequalities among the poor – they all have an *equal* claim to a certain level of resources. The second component states that we are infinitely inequality averse with respect to the average shortfall of the poor population from the poverty line – they all have an equal claim to nothing less (but also nothing more) than exactly the *subsistence level*.

With **Constant Resources** and **FfP** in place, it is evident that Y^r must be constructed by funneling resources from the individuals of R to the poor population P . In order to avoid overcharging the rich and pushing them below the subsistence threshold, D^4 yields the subset of distributions for which each rich individual indeed maintains an income above the poverty line:

$$D^4 = \{ D : y_i^r > y_{\min} \ \forall i \in R \}. \quad (\text{Financing I})$$

Conditional on the satisfaction of **EOp** and **FfP** we have zero inequality aversion with respect to the share of income that exceeds the poverty line. Hence, D^5 denotes the subset of possible distributions for which within-type inequality of excess income above the poverty line remains unaltered:

$$D^5 = \left\{ D : \forall t \in T, \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \ \forall i, j \in t \cap R \right\}. \quad (\text{Financing II})$$

The intersection of these subsets, $\cap_{s=1}^5 D^s$, yields a singleton which defines our norm distribution. The construction of Y^r can be conceived by way of the following two-step procedure in which the funds for transfers are raised through linear tax rates: The realization of **FfP** requires bottom fill-up until every $i \in P$ realizes an income equal to y_{\min} . In order to avoid pushing the rich below the poverty line (**Financing I**) while maintaining inequality among excess incomes above y_{\min} within types (**Financing II**), it must be the case that the funds to eradicate poverty are raised by a linear tax on the share of income that exceeds the poverty line. With **FfP** satisfied we can now turn to **EOp** in the second step. Clearly, we must leave incomes of the poor intact in order to maintain **FfP**. As a consequence, transfers across types to realize **EOp** are conducted only among the non-poor individuals. Again, the respective taxes and transfers are calculated

¹²See Lefranc et al. (2009) and Lefranc and Trannoy (2017) for the notion of luck within the equality of opportunity framework.

in a linear fashion on excess incomes above the poverty line in order to assure compliance with **Financing I** and **Financing II**.

In general, the construction of the reference distribution can be summarized as follows:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i (\tau^{\text{FFP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FFP}}))], & \text{otherwise.} \end{cases} \quad (\text{EOp+FFP})$$

where $\tilde{y}_i = \left(\frac{y_i^e - y_{\min}}{y_i^e} \right)$, $\tau^{\text{FFP}} = \frac{N_P (y_{\min} - \mu_P^e)}{N_R (\mu_R^e - y_{\min})}$ and $\tau_t^{\text{EOP}} = \frac{\mu_t^e + \frac{N_{P \cap t}}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FFP}} \left(\frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) - \mu}{\mu_t^e + \frac{N_{P \cap t}}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FFP}} \left(\frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) - y_{\min}}$.

We provide a step-by-step derivation of the implicit tax rates and the ensuing norm vector in Appendix A.

Measuring Divergences. Endowed with both Y^e and Y^r , one must decide how to aggregate the discrepancies between both vectors into a scalar measure of unfair inequality. Prominent proposals for appropriate divergence measures include the works by Cowell (1985), Almås et al. (2011), and Magdalou and Nock (2011), each of which can be viewed as a generalization of standard inequality measures. The key difference to standard measures of inequality is that these generalized measures do not decrease (increase) with progressive (regressive) transfers from rich (poor) to poor (rich) but rather with transfers that reduce (increase) the aggregate distance between the empirical and the reference distribution. Hence, transfers from poor to rich can be desirable if the empirical income of the poor exceeds its reference value, while the income of the rich falls short of it. Note that the latter requirement is equivalent to the standard Pigou-Dalton principle of transfers if the reference distribution is equivalent to the sample mean μ .

Our baseline estimates are presented in terms of the measures proposed by Magdalou and Nock (2011). This choice is guided by the intuition that the measures proposed by Cowell (1985) and Magdalou and Nock (2011) nest the mean log deviation, i.e. the generalized entropy measure with $\alpha = 0$, if we set $y_i^r = \mu \forall i \in N$. As such we ensure close proximity to the literature on the measurement of unequal opportunities, in which the use of the mean log deviation is prevalent (among others Ferreira and Gignoux, 2011; Hufe et al., 2017). Robustness checks with respect to the specific choice of the divergence measure, however, including the measures proposed by Cowell (1985) and Almås et al. (2011) are provided in section 5.2.

Magdalou and Nock (2011) propose the following generalization of the general entropy class of inequality measures:¹³

$$D(Y^e || Y^r) = \sum_i [\phi(y_i^e) - \phi(y_i^r) - (y_i^e - y_i^r) \phi'(y_i^r)],$$

where

$$\phi(y) = \begin{cases} -\ln y, & \text{if } \alpha = 0 \\ y \ln y, & \text{if } \alpha = 1 \\ \frac{1}{\alpha(\alpha-1)} y, & \text{otherwise.} \end{cases}$$

As in the family of generalized entropy measures, α is indicative of different value judgments.

¹³We abbreviate this class with *MN* in the following.

The higher α , the higher the valuation of positive divergences of empirical incomes y_i^e from their respective norm incomes y_i^r . To the contrary, the lower α , the more weight is attached to shortfalls from y_i^r . In the baseline we choose $\alpha = 0$. This parameterization yields the mean log deviation if the norm vector was equal to the sample mean.¹⁴ Thus, for our baseline estimates of unfair inequality, we aggregate divergences between the reference and empirical distribution as follows:

$$D(Y^e||Y^r) = \frac{1}{N} \sum_i \left[\ln \frac{y_i^r}{y_i^e} + \frac{y_i^e}{y_i^r} - 1 \right]. \quad (1)$$

Replacing y_i^r with norm distribution **EOp+FfP** we obtain the following baseline measure of unfair inequality:

$$D(Y^e||Y_{\text{EOp+FfP}}^r) = \frac{1}{N} \sum_{i \in P} \left\{ \ln \frac{y_{\min}}{y_i^e} - \left(\frac{y_{\min} - y_i^e}{y_{\min}} \right) \right\} + \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FfP}}))) + \frac{\tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FfP}}))}{1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FfP}}))} \right\}. \quad (\text{Baseline})$$

Properties. To explore the properties of our measure, we first ask the question of how it behaves if we focus on one of our normative principles only. Ideally, excluding either FfP or EOp from our list of normative concerns yields a measure with sensible properties and interpretability from the perspective of opportunity measurement or poverty measurement, respectively.

First, assume our normative evaluation of a given income distribution was blind to FfP. Then, $y_{\min} = 0$, $N_R = N$ and $N_{R \cap t} = N_t$. This leads to $\tau^{\text{FfP}} = 0$ and $\tau_t^{\text{EOp}} = \frac{\mu_t^e - \mu}{\mu_t^e}$. Hence, our norm vector would look as follows:

$$y_i^r = y_i^e \left[1 - \left(\frac{\mu_t^e - \mu}{\mu_t^e} \right) \right] = y_i^e \left[\frac{\mu}{\mu_t^e} \right]. \quad (\text{EOp Only})$$

Using the norm distribution **EOp Only** in equation 1, we see that our measure of unfair inequality collapses to measuring inequality in the distribution of average outcomes by circumstance type. Hence, we obtain the standard ex-ante utilitarian measure of equality of opportunity in which the mean log deviation is applied to a smoothed distribution of type-specific mean incomes:

$$D(Y^e||Y_{\text{EOp}}^r) = \frac{1}{N} \sum_i \ln \frac{\mu}{\mu_t^e}. \quad (2)$$

Reversely, one could ask whether we obtain a poverty measure with sensible properties when focusing on FfP only. Therefore, second, let us assume that our normative evaluation of Y^e was indifferent to EOp. In this case $\mu_t^e = \mu$, $N_{R \cap t} = N_R$, $\mu_{R \cap t}^e = \mu_R$ and hence $\tau_t^{\text{EOp}} = 0$. As a consequence, the norm-vector would be constructed as follows:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i \tau^{\text{FfP}}], & \text{otherwise.} \end{cases} \quad (\text{FfP Only})$$

¹⁴Sensitivity checks with alternative specifications of α are provided in section 5.2.

¹⁵Note that we can scale the measure by $\frac{1}{N}$ to satisfy the *principle of populations* without further adjustment since we have specified the reference distribution to satisfy **Constant Resources** (Magdalou and Nock, 2011).

Using the norm distribution **FfP Only** in equation 1, we obtain the following measure of unfair inequality:¹⁶

$$D(Y^e || Y_{\text{FfP}}^r) = \underbrace{\frac{1}{N} \sum_{i \in P} \ln \frac{y_{\min}^e}{y_i^e}}_{\text{Watts Index}} - \underbrace{\frac{1}{N} \sum_{i \in P} \left(\frac{y_{\min} - y_i^e}{y_{\min}} \right)}_{\text{Poverty Gap}} \quad (3)$$

$$+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i \tau^{\text{FfP}}) + \left(\frac{\tilde{y}_i \tau^{\text{FfP}}}{1 - \tilde{y}_i \tau^{\text{FfP}}} \right) \right\}. \quad (4)$$

As the MN-measures are additively decomposable, we can evaluate unfairness in the truncated distribution of $Y_P^e = \{y_1^e, y_2^e, \dots, y_{\min}^e\}$. Focusing on the contributions of $i \in P$, we see that unfairness in this part of the distribution is characterized by the difference between the Watts index (Zheng, 1993) and the poverty gap ratio. Individually, these are well-known measures of poverty. However, also their combination bears a number of desirable properties that have been identified in the literature on poverty measurement (e.g. Ravallion and Chen, 2003). These include *monotonicity* (as opposed for example to the headcount ratio), the *principle of transfers* (as opposed for example to the poverty gap taken as a stand-alone measure) and *additive decomposability*.¹⁷

We conclude that our measure nests sensible measures of both EOp and FfP. Therefore, it is easily interpretable and carries desirable properties identified in the respective literature on the measurement of either EOp or FfP.

Comparison to Other Approaches. Our critique of previous approaches to reconcile EOp and FfP included the fact that there are cases in which unfair inequality remains undetected once the prioritized principle is satisfied. So how does our composite measure behave once we live in a society free from deprivation or in a society in which individual outcomes cannot be traced to differential circumstances?

First, assume that all individuals earn above the minimum threshold of income. Then, $N_R = N$ and $N_{R \cap t} = N_t$. This leads to $\tau^{\text{FfP}} = 0$ and $\tau_t^{\text{EOp}} = \frac{\mu_t^e - \mu}{\mu_t^e - y_{\min}}$. We would obtain the following norm-vector:

$$y_i^r = y_i^e [1 - \tilde{y}_i \tau_t^{\text{EOp}}]. \quad (\text{Poverty Free})$$

Note the difference to our previous thought experiment, in which we abstracted from FfP altogether. Here our concern for poverty remains intact, which by **Financing I** leads us to calculate opportunity-equalizing transfers across types by reference to the income share that exceeds y_{\min} . After all, we want to avoid pushing non-deprived households into poverty for the sake of realizing EOp. As a consequence, within a given type, the contribution to opportunity equalization across types raises with the proportion of individual income that exceeds y_{\min} . In contrast to standard ex-ante utilitarian measures of EOp, we are not indifferent to any inequality within types and hence obtain a measure that detects more unfairness in a given outcome distribution.

Second, assume that we live in an equal opportunity society without disparities across dif-

¹⁶For the sake of visual clarity we do not spell out the norm income of individuals above the poverty line.

¹⁷Note that it is unsurprising that we do not obtain a measure of poverty that satisfies the *focus axiom*. Our approach frames poverty as a question of inequality and thus imposes requirements on how the funds to eradicate poverty should be raised (**Financing I** and **Financing II**). Therefore, it is not indifferent to transfers between individuals with incomes above the poverty line and thus violates the *focus axiom*.

ferent circumstance types. In such a society assuring FfP remains the only normative ground for policy intervention. In this case, $\mu_t^e = \mu$, $N_{R \cap t} = N_R$, $\mu_{R \cap t}^e = \mu_R$ and hence $\tau_t^{\text{EOP}} = 0$. As a consequence, the norm vector would be constructed exactly as the norm distribution **FfP Only**. Thus, our measure coincides with the measure constructed in our previous thought experiment, in which we abstracted from the goal of EOp altogether:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i \tau^{\text{FfP}}], & \text{otherwise.} \end{cases} \quad (\text{Equal Opportunity})$$

To conclude: In contrast to previous attempts to reconcile EOp and FfP, our measure detects unfairness in a given outcome distribution even if either of our normative principles is perfectly satisfied.

Table 1 summarizes the measures that have been discussed in the previous paragraphs.

Table 1: Measures

Scenario	Description	Measure
EOp+FfP (Baseline)	$D(Y^e Y_{\text{EOp+FfP}}^r)$	$\frac{1}{N} \sum_{i \in P} \left\{ \ln \frac{y_{\min}}{y_i^e} - \left(\frac{y_{\min} - y_i^e}{y_{\min}} \right) \right\}$ $+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))) \right\}$ $+ \frac{1}{N} \sum_{i \in R} \left\{ \frac{\tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))}{1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))} \right\}$
EOp Only	$D(Y^e Y_{\text{EOp}}^r)$	$\frac{1}{N} \sum_i \ln \frac{\mu}{\mu_t^e}$
FfP Only	$D(Y^e Y_{\text{FfP}}^r)$	$\frac{1}{N} \sum_{i \in P} \left\{ \ln \frac{y_{\min}}{y_i^e} - \left(\frac{y_{\min} - y_i^e}{y_{\min}} \right) \right\}$ $+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i \tau^{\text{FfP}}) + \left(\frac{\tilde{y}_i \tau^{\text{FfP}}}{1 - \tilde{y}_i \tau^{\text{FfP}}} \right) \right\}$
Poverty Free	$D(Y^e Y_{\text{EOP+FfP}}^r)$ if $y_i^e > y_{\min} \forall i \in N$	$\frac{1}{N} \sum_i \left\{ \ln (1 - \tilde{y}_i \tau_t^{\text{EOP}}) + \left(\frac{\tilde{y}_i \tau_t^{\text{EOP}}}{1 - \tilde{y}_i \tau_t^{\text{EOP}}} \right) \right\}$
Equal Opportunity	$D(Y^e Y_{\text{EOP+FfP}}^r)$ if $\mu_t^e = \mu \forall t \in T$	$\frac{1}{N} \sum_{i \in P} \left\{ \ln \frac{y_{\min}}{y_i^e} - \left(\frac{y_{\min} - y_i^e}{y_{\min}} \right) \right\}$ $+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i \tau^{\text{FfP}}) + \left(\frac{\tilde{y}_i \tau^{\text{FfP}}}{1 - \tilde{y}_i \tau^{\text{FfP}}} \right) \right\}$

Note: The displayed measures are constructed based on MN with $\alpha = 0$. $\tilde{y}_i = \left(\frac{y_i^e - y_{\min}}{y_{\min}} \right)$, $\tau^{\text{FfP}} = \frac{N_P (y_{\min} - \mu_P^e)}{N_R (\mu_R^e - y_{\min})}$ and $\tau_t^{\text{EOP}} = \frac{\mu_t^e + \frac{N_{P \cap t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FfP}} (\frac{N_{R \cap t} (\mu_{R \cap t}^e - y_{\min}) - \mu}{N_t})}{\mu_t^e + \frac{N_{P \cap t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FfP}} (\frac{N_{R \cap t} (\mu_{R \cap t}^e - y_{\min}) - y_{\min}}{N_t})}{N_t}}$.

4 Data

To illustrate the suggested measure of unfair inequalities, we draw on two data sources. First, we provide cross-country evidence for 31 European countries in 2010 using the EU Statistics on Income and Living Conditions (EU-SILC). Second, we use the Panel Study of Income Dynamics (PSID) to illustrate the development of unfair inequality in the US over the time period

1969-2012. The majority of country-year cells in these datasets are subject to the well-known limitations of survey data as regards the underrepresentation of the very top of the income distribution (Atkinson and Brandolini, 2001). In spite of this limitation, we prefer survey over tax-record data, since the former provide a rather accurate representation of the lower tail of the income distribution and even more importantly provide rich information on individual background characteristics that are necessary to perform inequality of opportunity calculations. Such information is typically missing in tax return data. However, EU-SILC also includes seven “register countries” that strike a balance between both requirements.¹⁸ For these countries, income information from administrative data sources is combined with rich background information from survey questionnaires.

EU-SILC. The 2011 wave of EU-SILC covers 31 European countries.¹⁹ It is a well-researched database for monitoring inequality, poverty and social exclusion in Europe (see for example Atkinson et al. (2017) and the references cited therein), which lends our results to easy comparison with previous works. In particular, we use the 2011 wave as it provides a module on the intergenerational transmission of advantages, which allows us to construct types from a broad range of circumstance variables.²⁰ As is common in survey data, incomes are reported for the year preceding the survey, i.e. 2010 in our case.

We follow standard practices from the literature branches on inequality and poverty measurement in setting up the data. We focus on disposable household income adjusted by the modified OECD-equivalence scale as the outcome of interest.²¹ The few observations with $y_i^c < 0$ are excluded from the analysis. We replace zero incomes by one to avoid sample reductions through logarithmic transformations. To curb the influence of outliers in the lower and the upper part of the income distribution, we winsorize at the 1st and the 99.95th-percentile of the country-specific income distribution. We hold the poverty rate y_{\min} fixed at the so called European *At-Risk-Of-Poverty Rate* which is drawn at 60% of the country-specific median equivalized disposable household income.²² Furthermore, we restrict the sample to working age individuals of 25-59 years. To assure the representativeness of the sample all calculations are performed considering personal cross-sectional sample weights.

¹⁸The “register countries” are Denmark, Finland, Iceland, Netherlands, Norway, Sweden and Slovenia.

¹⁹The sample consists of Austria (AT), Belgium (BE), Bulgaria (BG), Switzerland (CH), Cyprus (CY), Czech Republic (CZ), Germany (DE), Denmark (DK), Estonia (EE), Greece (EL), Spain (ES), Finland (FI), France (FR), Croatia (HR), Hungary (HU), Ireland (IE), Iceland (IS), Italy (IT), Malta (MT), Lithuania (LT), Luxembourg (LU), Latvia (LV), Netherlands (NL), Norway (NO), Poland (PL), Portugal (PT), Romania (RO), Sweden (SE), Slovenia (SI), Slovakia (SK), and Great Britain (UK).

²⁰The 2005 wave also comprises a module on the intergenerational transmission of advantages for a sample of 26 European countries. Results for the 2005 wave are available on request.

²¹This is standard in poverty measurement since the notion of deprivation typically refers to individual well-being as approximated by consumption possibilities. To the contrary, individual gross incomes are often used in the measurement of unequal opportunities. However, usually the differences between inequality of opportunity in gross and disposable incomes are small since existing tax-benefit system typically ignore circumstances used to estimate inequality of opportunity. For example, tax rates usually do not include tags (Immonen et al., 1998; Gelber and Weinzierl, 2016) for gender or the education of parents.

²²The specification of y_{\min} is a normative assumption in itself and a strong driver of our empirical results. Results for alternative poverty lines are provided in section 5.2.

PSID. PSID is a panel study that tracks more than 18,000 individuals from approximately 5,000 families in the United States over time. From its inception in 1968 up until today it provides nationally representative information on the US based non-immigrant population including their incomes, labor market histories, background characteristics and living practices. Starting with annual surveys, it has switched to a biennial rhythm in 1997. Analogously to EU-SILC, it is well researched with respect to aspects of inequality and poverty (e.g. Heathcote et al., 2010; Huggett et al., 2011) which allows us to benchmark our results against existing studies. In terms of data preparation we exactly follow the procedure outlined for EU-SILC. Equivalized disposable household income is calculated based on the cross-national equivalent file (CNEF) of the PSID. We exclude negative incomes, replace zero incomes by one, and winsorize at the 1st and the 99.95th-percentile of the year-specific income distribution. The poverty line is fixed at 60% of the year-specific median equivalized disposable household income and the sample is restricted to the age corridor 25-59. To assure representativeness, all analyses are conducted using the cross-sectional weights provided by the PSID. All incomes refer to the year prior to the survey. Since CNEF is only available until 2013 and the PSID does not contain important circumstance information in waves prior to 1970, we present results for the time period 1969-2012.

Circumstances and Types. For the estimation of inequality of opportunity it is indispensable to partition the population into types. In this work we use four circumstance variables, that are frequently utilized in the empirical literature on equality of opportunity. The first circumstance is the biological sex of the respondent. Second, in EU-SILC we proxy the respondent’s migration background by a binary indicator for whether the respondent lived in her country of birth at time of survey completion. In the US, we replace this circumstance with a racial indicator differentiating among whites and non-whites. Third, we use information on the educational status of the parents. Particularly, we construct types based on whether the highest educated parent of a respondent dropped out of secondary education, attained a secondary school degree or whether the highest educated parent of a respondent attended at least some tertiary education. Lastly, we proxy the occupational status of both parents by grouping them in either elementary occupations, semi-skilled occupations, or top-rank positions.²³ We only retain information on the parent of highest occupational status. As such each of the considered populations is partitioned into a maximum of $2 * 2 * 3 * 3 = 36$ non-overlapping circumstance types.²⁴ As illustrated in Tables B.1 and B.2 some country (year) observations fall short of 36 types. This is due to the fact that some combinations of circumstances appear rather seldom in the data. To give an intuitive example, the combination of the highest educated parent having less than a secondary school degree but occupying a top-rank position in her profession is extremely rare. In order to curb the influence of very small types we only retain those types for which we have a minimum of 20 observations in the respective country-year cell.

²³Details on the harmonization of classifications within and across datasets are provided in Table C.1 in the Appendix.

²⁴A special case is Sweden (SE). The occupation variables are sparsely populated in the Swedish sample. To circumvent sample size reductions that would render the analysis of the Swedish sample infeasible we exclude the occupational variables for this particular country case. Hence, for Sweden we obtain a maximum of $2 * 2 * 3 = 12$ non-overlapping circumstance types.

Descriptive Statistics. Table B.1 shows descriptive statistics for the income distributions in the European sample. In 2010 mean disposable household income was lowest in Romania, Bulgaria and Lithuania ($\mu < 5,000\text{€}$). At the top of the intra-European country distribution we find Luxembourg, Norway and Switzerland the average disposable household incomes of which hover around the mark of 40,000€. ²⁵ To characterize the lower end of the income distributions we draw on three measures of poverty: The headcount ratio (P_0), the poverty gap ratio (P_1) and the Watts index. The headcount ratio is only sensitive to the number of people falling short of y_{\min} , while the poverty gap also measures their average distance to the poverty line. Beyond these two properties, the Watts index additionally varies with inequality among the poor. Indeed country rankings based on these three measures show some variation. For example, Hungary (Rank 17), Luxembourg (Rank 18) rank fairly bad in terms of the headcount ratio, but perform considerably better in terms of the poverty gap (Ranks 11 and 10). To the contrary, Denmark and Iceland rank 7th and 9th in terms of P_0 but only 19th and 16th in terms of P_1 . This indicates that in the first group of countries there is a relatively high mass of people falling short of y_{\min} , yet on average this shortfall is relatively small in comparison to other countries. In the two Nordic countries, however, poverty is not very pervasive, but relatively severe on average for those who actually fall behind the deprivation threshold. Rankings according to the Watts index are largely comparable to the poverty gap ratio.

Going beyond the subset P , we characterize the entire income distribution by means of the Gini index and the mean log deviation. While the Gini index is particularly sensitive to transfers in the middle, the mean log deviation has a normative focus on the lower end of the income distribution. In spite of their different foci, both inequality indices yield rather stable country rankings. According to both measures Norway is the most equal society within our sample. At the upper end of the inequality spectrum we find the Baltic countries of Lithuania and Latvia. The most severe re-ranking occurs for Denmark, that falls back from rank 9 to rank 16 when considering the mean log deviation instead of the Gini index. This suggests that inequality in Denmark is particularly driven by the poor falling short of the population average.

Descriptive statistics for the US are shown in Table B.2. Consistent with other works we observe pronounced increases of both poverty and inequality beginning with the 1980s. These observations are independent of the particular measure we employ. For example, the poverty headcount increased from 11.6% in the first period of observation in 1969 to 12.9% in 1980. This moderate increase is followed by a stronger upwards movement to a level of 16.8% in 1990, 17.3% in 2000 and 20.4% in 2012. Similar upwards movements are observed for the poverty gap and the Watts index. In terms of inequality the Gini index remains stable at a level of around 0.280 in the 1970s. It then increases to 0.339 in 1990 followed by a further surge to a level of 0.371 at the turn of the century in 2000. In the last period of observation it has reached a value of 0.430.

In the following section we turn to the estimates of unfair inequality, i.e. inequality that can be traced back to violations of the normative ideals of EOP and FfP.

²⁵Note that the data is not adjusted for purchasing power parity. This has no bearing on our central results since the statistics of interest are both calculated on the country (year) level and scale invariant.

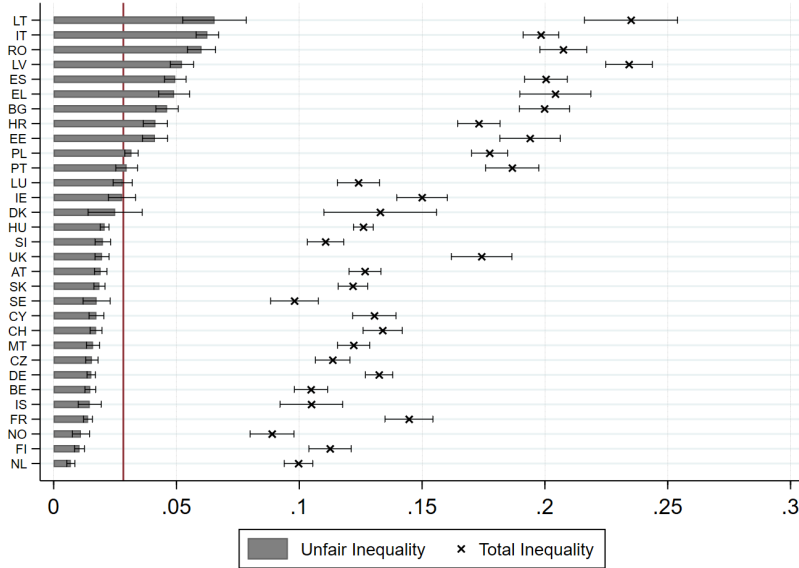
5 Results

5.1 Baseline Results

The gray bars in Figure 1 illustrate unfair inequality for the sample of 31 European countries. For these baseline estimates we use the MN-measure with $\alpha = 0$ to aggregate divergences between the empirical outcome distribution and the norm distribution EOp+FfP . The black crosses indicate total inequality as measured by the mean log deviation.²⁶

We find that on average unfair inequality amounts to 0.029, i.e. that 17.6% of total inequality can be explained by violations of EOp and FfP. Unfair inequality is most prevalent in Lithuania, Italy and Romania, with values of 0.066 (27.9%), 0.063 (31.6%) and 0.060 (29.0%), respectively. From the perspective of our normative stance, income is most fairly distributed in the Netherlands (0.007, 7.0%), Finland (0.011, 9.3%) and Norway (0.011, 12.5%).

Figure 1: Unfair Inequality by Country (Europe)

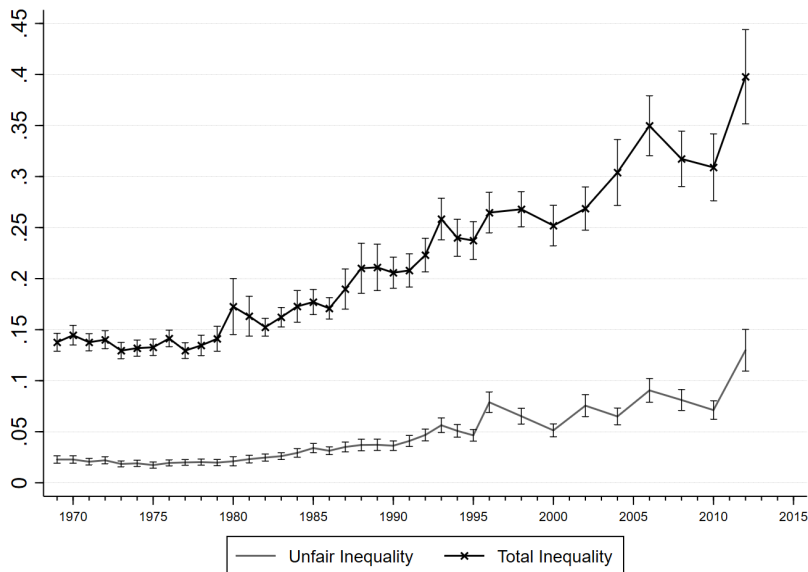


Data: EU-SILC 2011 cross-sectional (rev.5, June 2015).

Note: This figure shows estimates of unfair inequality for a cross-section of European countries in 2010. Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The norm distribution is given by $Y_{\text{EOp+FfP}}^r$. See also Table 1 for information on the calculation of norm distributions. Point estimates are presented in Table F.1. The vertical red line indicates the unweighted cross-country average. The black crosses indicate total inequality as measured by the mean log deviation. 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws.

²⁶In order to ease interpretation, the literature often converts measures of unfair inequality into a percentage term relative to the observed inequality in outcomes. For our measure, this can be done as follows: $\frac{D(Y^e || Y_{\text{EOp+FfP}}^r)}{D(Y^e || \mu)}$. The numerator is our measure of unfair inequality and the denominator is total inequality (i.e. all observed inequality in Y^e is judged to be unfair). Therefore, the relative measure is the ratio of the values indicated by the gray bars and the black crosses in Figure 1. It shows how much of total inequality remains of ethical concern once we specify norm incomes according to EOp and FfP. Clearly, the conversion into a relative measure is not innocuous. Consider two countries with equal levels of $D(Y^e || Y_{\text{EOp+FfP}}^r)$, but unequal levels of total inequality. Due to the division by $D(Y^e || \mu)$ the relative measure would characterize the country with lower total inequality as more unfair than the country with higher total inequality. In view of this misleading property, we refrain from cross-country and longitudinal comparisons based on the relative measure. However, we also report percentage values in brackets in order to facilitate the interpretation of our results. See also Tables D.1 and D.2 in the Appendix.

Figure 2: Unfair Inequality over Time (USA)



Data: PSID public use dataset and PSID-CNEF.

Note: This figure shows estimates of unfair inequality for the US over the period 1969–2013. Unfair inequality is calculated as $D(Y^e||Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The norm distribution is given by $Y_{EOP+FfP}^r$. See also Table 1 for information on the calculation of norm distributions. Point estimates are presented in Table F.2. The black crosses indicate total inequality as measured by the mean log deviation. 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws.

Figure 2 illustrates the evolution of unfair inequality in the US over time. The gray line yields our baseline measure of unfair inequality while the black crosses again are indicative of total inequality as measured by the mean log deviation.

Starting from a level of 0.023 (16.6%) in 1969, unfair inequality attained a level of 0.130 (32.6%) in 2012. The co-movement of total inequality and unfair inequality shows an interesting pattern. Up until 1980 both total inequality and unfair inequality were relatively stable. The 1980s and early 1990s were characterized by strong increases in inequality from 0.173 in 1980 up to 0.237 in 1995. These increases, however, were only weakly related to violations of either EOP or FfP. In this period only 39% of the well-documented rally in inequality levels (Piketty et al., 2018) must be ascribed to increasing unfairness. After 1995 total inequality further increased to 0.398 in 2012. In contrast to the previous period, however, 52% of the increase in inequality can be accounted for by increases in unfairness. Unfair inequality in the US exceeds the corresponding levels of the most unjust European societies, such as Italy and Lithuania, by far in the last periods of observation.

Decomposition. The previous discussion provokes the question to what extent our baseline results are driven by violations of one principle rather than the other. Therefore, in this section we decompose unfair inequality into its EOP and its FfP components. We cannot cleanly allocate all unfair inequality to either FfP or EOP, since they are partly overlapping. To illustrate this point, consider a society where all $i \in P$ belong to the same type $t_p \in T$. By EOP we know that $\mu_{t_p}^r = \mu$ in the norm distribution. Re-scaling all incomes in t_p to realize EOP thus also entails that a fraction of $i \in t_p$ will be lifted out of poverty. In this case, by coincidence an opportunity

egalitarian planner would also be considered poverty-averse. Reversely, assume that $i \in t_p$ is congruent with $i \in P$. Therefore, rescaling all incomes below the poverty line to realize FfP simultaneously lifts the incomes of t_p closer to the population mean. In this sense, aiming for FfP also entails opportunity equalization. Therefore, the upper bound share of unfairness explained by violations of either EOp or FfP is given by how much unfair inequality we can attribute to either principle once we abstract from the other. The construction of these norm distributions were discussed in section 2 and are referenced as scenarios **EOp Only** and **FfP Only** in Table 1. The respective lower bound of how much unfair inequality we can attribute to either principle is given by how much unfair inequality rises once we add this principle to the other. The decomposition of unfair inequality into the two components thus reads as follows:

$$D(Y^e || Y_{\text{EOp+FfP}}^r) = \underbrace{D(Y^e || Y_{\text{EOp+FfP}}^r) - D(Y^e || Y_{\text{EOp}}^r)}_{\text{LB}_{\text{FfP}}} + \underbrace{D(Y^e || Y_{\text{EOp}}^r)}_{\text{UB}_{\text{EOp}}},$$

$$D(Y^e || Y_{\text{EOp+FfP}}^r) = \underbrace{D(Y^e || Y_{\text{EOp+FfP}}^r) - D(Y^e || Y_{\text{FfP}}^r)}_{\text{LB}_{\text{EOp}}} + \underbrace{D(Y^e || Y_{\text{FfP}}^r)}_{\text{UB}_{\text{FfP}}}.$$

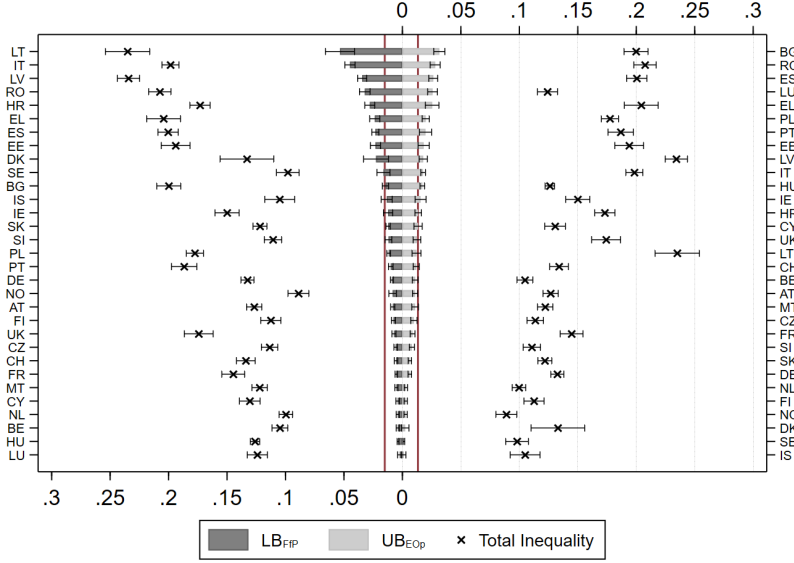
As can be easily inferred from the formulas above, summing upper and lower bounds across principles yields the results presented in Figures 1 and 2. For the following comparisons we will refer to the upper bound measure of EOp and the lower bound measure of FfP, that is we allocate the entirety of the overlap component to EOp. This has the advantage that we can benchmark our results against standard measures of inequality of opportunity.²⁷

As documented in Figures 3 and 4, our baseline results are driven by EOp and FfP in about equal proportions. In the right-hand panel of Figure 3 we show upper-bound unfair inequality attributable to violations of EOp in Europe in 2010. In the left-hand panel we show the corresponding lower bounds for the extent of unfair inequality attributable to FfP. A standard opportunity egalitarian (UB_{EOp}) would detect an average unfairness level of 0.014 (8.4%) in the income distributions of our sample. From her perspective the distributions of Bulgaria (0.032, 15.8%), Romania (0.028, 13.5%) and Spain (0.026, 13.2%) would raise most moral concern. On the other side of the spectrum we find the Nordic countries Iceland, Sweden, Denmark and Norway, in all of which unfair inequality due to EOp is lower than 0.003.

Augmenting the concern for equal opportunities with poverty aversion (LB_{FfP}), unfair inequality on average increases by 0.015 (9.3pp). The strongest increases are observed for Lithuania (0.053, 22.7pp), Italy (0.045, 22.6pp) and Latvia (0.034, 14.7pp). With increases of at most 0.04 points, Luxembourg, Hungary and Belgium experience the lowest upwards correction of unfair inequality when complementing EOp with FfP. As outlined in section 3, our measure of unfair inequalities as originating from FfP is closely related to the Watts index. Therefore, it does not come as a surprise that the upward correction in unfair inequalities coming from FfP is strongly aligned with the country-specific Watts index. The Pearson's correlation coefficient between LB_{FfP} and the Watts index amounts to 0.943 and 0.965 for the EU-SILC and the PSID

²⁷Recall that $D(Y^e || Y_{\text{EOp}}^r)$ collapses to the standard measure of EOp in which the smoothed distribution of type means is evaluated by the mean log deviation (Table 1). A decomposition of unfair inequality into the respective lower bound components of EOp and FfP as well as the overlap component is shown in Tables D.1 and D.2.

Figure 3: Decomposition by Country (Europe)



Data: EU-SILC 2011 cross-sectional (rev.5, June 2015).

Note: This figure shows the decomposition of unfair inequality for a cross-section of European countries in 2010. Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The right-hand panel shows UB_{EOp} – the upper bound estimate of unfair inequality attributable to EOp. The norm distribution for UB_{EOp} is given by Y_{EOp}^r . The left-hand panel shows LB_{FfP} – the lower bound estimate of unfair inequality attributable to FfP. LB_{FfP} is calculated as the difference in the divergence measures for the norms $Y_{EOp+FfP}^r$ and Y_{EOp}^r . See also Table 1 for information on the calculation of norm distributions. The vertical red line indicates the unweighted cross-country average. The black crosses indicate total inequality as measured by the mean log deviation. 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws.

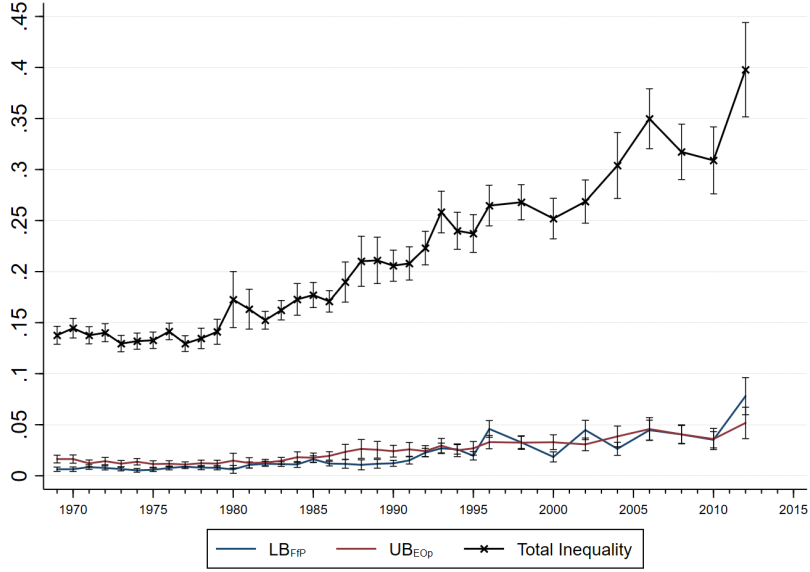
sample, respectively (Figure E.1).

In analogy to the cross-sectional pattern observed in Europe, the development of unfair inequality in the US is attributable to increasing violations of both EOp and FfP (Figure 4). The trends of the series only diverge in the late 1980s when the contribution of FfP slightly decreased and subsequently stagnated at its level until the mid 1990s. Otherwise, the contributions of both sources of unfairness are steadily increasing over the time period of observation. The significant contribution of FfP suggests that an exclusive focus on top income inequality (Piketty et al., 2018) may be misguided – especially when it comes to analyzing inequality patterns with an eye to fairness considerations.

Figure 5 shows the correlation pattern of UB_{EOp} and LB_{FfP} . With values of 0.261 (Europe) and 0.881 (US) the correlations between UB_{EOp} and LB_{FfP} are positive and sizable. This indicates that countries with strong opportunity differences are also characterized by more severe violations of FfP. Furthermore, the US example shows evidence of poverty increases over time being accompanied by rising inequality of opportunity.²⁸

²⁸Furthermore, current research suggests that contemporary increases in poverty may translate into future opportunity shortages for children of affected families (Heckman and Mosso, 2014; Hoynes et al., 2016). Hence, one would expect a positive correlation between current poverty and future inequality of opportunity as well.

Figure 4: Decomposition over Time (USA)



Data: PSID public use dataset and PSID-CNEF.

Note: This figure shows the decomposition of unfair inequality for the US over the period 1969–2013. Unfair inequality is calculated as $D(Y^e||Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The red line shows UB_{EOp} – the upper bound estimate of unfair inequality attributable to EOp. The norm distribution for UB_{EOp} is given by Y_{EOp}^r . The blue line shows LB_{FfP} – the lower bound estimate of unfair inequality attributable to FfP. LB_{FfP} is calculated as the difference in the divergence measures for the norms $Y_{EOp+FfP}^r$ and Y_{EOp}^r . See also Table 1 for information on the calculation of norm distributions. The black crosses indicate total inequality as measured by the mean log deviation. 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws.

5.2 Sensitivity Analysis

We conduct several sensitivity checks including alternations in the specific divergence measure, alternative poverty thresholds and modifications of our normative requirements.

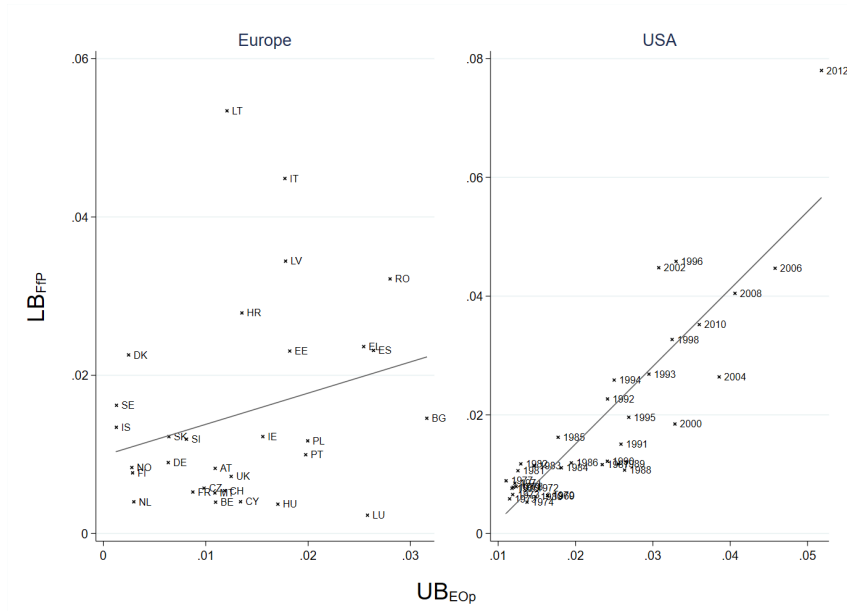
Alternative Divergence Measures. All results presented until this point were based on the divergence measures proposed by Magdalou and Nock (2011) with $\alpha = 0$. In addition to alternations in the weighting parameter α , we present results based on the measures put forward by Cowell (1985) and Almås et al. (2011) in Tables F.1 and F.2.

Within the MN-family, an increase of α induces a shift in the weight put on FfP and EOp, respectively. With increasing α the measure puts less weight on negative divergences of Y^e from Y^r . Since $\forall i \in P, y_i^e \leq y_i^r = y_{\min}$ this translates into decreasing relative importance of FfP. To the contrary, increasing α puts more weight on EOp since this principle induces stronger positive divergences between Y^e from Y^r in the middle and upper parts of the distribution. Just as the measures proposed by Magdalou and Nock (2011), the family of measures put forward by Cowell (1985) is a generalization of the general entropy class of inequality measures and thus varies with the parameter specification of α .²⁹ Therefore, within the Cowell-class the same observations with respect to the parameter α apply. The Cowell-measure and the MN-family coincide exactly for

²⁹To be precise the Cowell-family aggregates divergences as follows:

$$D(Y^e||Y^r) = \sum_i \left[y_i^r \phi \left(\frac{y_i^e}{y_i^r} \right) \right],$$

Figure 5: Correlation of EOp and FfP



Data: EU-SILC 2011 cross-sectional (rev.5, June 2015). PSID public use dataset and PSID-CNEF.
Note: This figure shows the correlation between unfair inequality due to EOp and unfair inequality due to FfP. Correlations are based on UB_{EOp} and LB_{FfP} , respectively. Unfair inequality is calculated as $D(Y^e||Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The norm distribution for UB_{EOp} is given by Y_{EOp}^r . LB_{FfP} is calculated as the difference in the divergence measures for the norms $Y_{EOp+FfP}^r$ and Y_{EOp}^r . See also Table 1 for information on the calculation of norm distributions. The Pearson's correlation coefficient of UB_{EOp} with LB_{FfP} in Europe (USA) is 0.261 (0.881).

$\alpha = 1$. Lastly, we employ the unfairness Gini proposed by Almås et al. (2011),³⁰ which is a generalization of the well-known Gini index. The unfairness Gini tends to put relatively less weight on large negative divergences from the reference distribution, which results in decreases of the importance of FfP as opposed to EOp as explanatory factor for the unfair share of inequality.

In spite of these variations, the measures yield highly comparable results in terms of cross-country and -period comparisons. Tables 2 and 3 show rank-correlations for the different measures and their parameterizations for the European and the US sample, respectively. All correlation coefficients lie above the level of 0.910. Hence, we conclude that our results are robust to alternations in the way in which divergences between Y^e and Y^r are aggregated.

where

$$\phi(z) = \begin{cases} -\ln z, & \text{if } \alpha = 0 \\ z \ln z, & \text{if } \alpha = 1 \\ \frac{1}{\alpha(\alpha-1)} z, & \text{otherwise.} \end{cases}$$

³⁰To be precise the unfairness Gini aggregates divergences as follows:

$$D(Y^e||Y^r) = \frac{2}{N(N-1)\mu} \sum_i i(y_i^r - y_i^e).$$

Table 2: Rank Correlation of Measures by Country (Europe)

Magdalou and Nock			Cowell			Almås et al.
$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	
Magdalou and Nock						
$\alpha = 0$	1.000					
$\alpha = 1$	0.953	1.000				
$\alpha = 2$	0.911	0.982	1.000			
Cowell						
$\alpha = 0$	0.975	0.988	0.963	1.000		
$\alpha = 1$	0.953	1.000	0.982	0.988	1.000	
$\alpha = 2$	0.939	0.994	0.986	0.976	0.994	1.000
Almås et al.						
	0.912	0.970	0.971	0.955	0.970	0.970
						1.000

Data: EU-SILC 2011 cross-sectional (rev.5, June 2015).

Note: The coefficients indicate rank correlation coefficients between the point estimates of unfair inequality as calculated by the different divergence measures. Unfair inequality is calculated as $D(Y^\epsilon || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^ϵ and the norm distribution Y^r . The norm distribution is given by $Y_{EOP+FFP}^r$. See also Table 1 for information on the calculation of norm distributions.

Table 3: Rank Correlation of Measures over Time (USA)

Magdalou and Nock			Cowell			Almås et al.
$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	
Magdalou and Nock						
$\alpha = 0$	1.000					
$\alpha = 1$	0.991	1.000				
$\alpha = 2$	0.961	0.975	1.000			
Cowell						
$\alpha = 0$	0.994	0.998	0.971	1.000		
$\alpha = 1$	0.991	1.000	0.975	0.998	1.000	
$\alpha = 2$	0.986	0.998	0.979	0.994	0.998	1.000
Almås et al.						
	0.972	0.984	0.966	0.978	0.984	0.985
						1.000

Data: PSID public use dataset and PSID-CNEF.

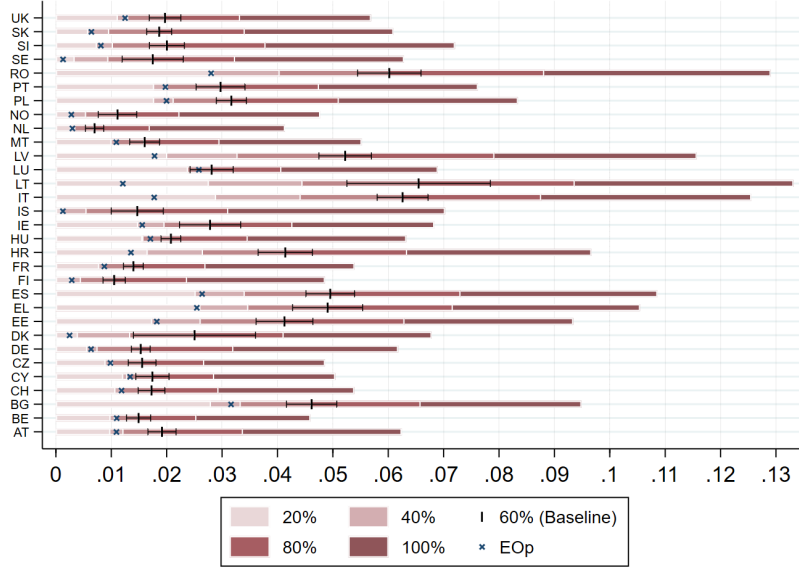
Note: The coefficients indicate rank correlation coefficients between the point estimates of unfair inequality as calculated by the different divergence measures. Unfair inequality is calculated as $D(Y^\epsilon || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^ϵ and the norm distribution Y^r . The norm distribution is given by $Y_{EOP+FFP}^r$. See also Table 1 for information on the calculation of norm distributions.

Alternative Poverty Thresholds. Naturally, what it means “to make ends meet” is contentious both philosophically (e.g. Casal, 2007) and empirically (e.g. Meyer and Sullivan, 2012). It is beyond the ambit of this work to resolve these issues. However, in Figures 6 and 7 we illustrate the sensitivity of our proposed measure to variations in the level of y_{\min} .

In particular, we vary the percentage level of the population median income that is required to trespass the poverty threshold in 20 percentage point steps. By definition at the 100%-mark y_{\min} would equal the population’s median income and 50% of the population would be in P and R , respectively. If we set $y_{\min} = 0$, we would obtain the upper bound measure of EOp. The 60%-mark replicates the baseline results presented in Figures 1 and 2. Intuitively, the unfair share of inequality varies positively with the deprivation threshold: The higher y_{\min} , the stronger our inequality aversion with respect to inequalities within circumstance types.³¹ The

³¹Note that there are some exceptions for very low poverty thresholds. For example, in the Netherlands the standard EOp measure yields higher unfair inequality than the combined measure up until the 20%-poverty mark. Similarly, in the US the standard EOp measure consistently exceeds the combined measure with at a poverty threshold of 20% up until 1995. Intuitively, low levels of poverty require very little re-scaling of incomes below the poverty threshold for FFP to be satisfied. As a consequence, inversely re-scaling incomes by type means to achieve EOp may overshoot the requirements of FFP, especially if differences in type-means are rather pronounced.

Figure 6: Alternative Poverty Thresholds by Country (Europe)



Data: EU-SILC 2011 cross-sectional (rev.5, June 2015).

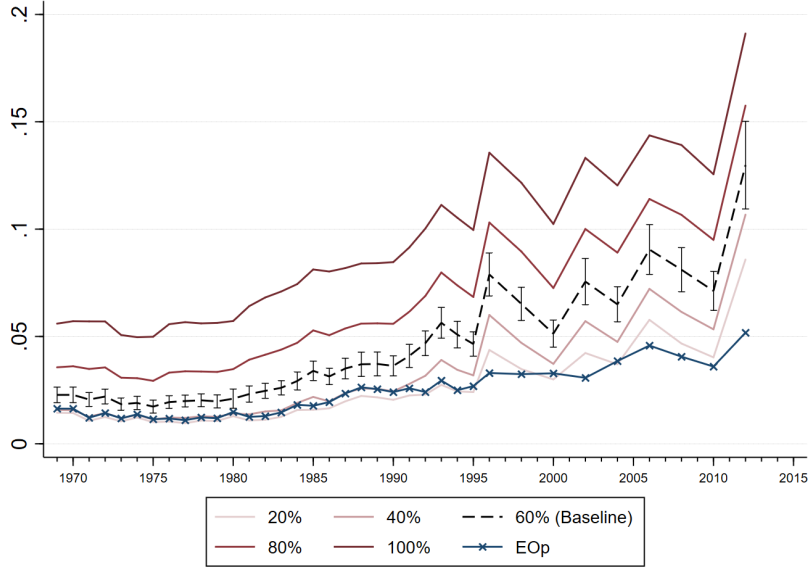
Note: This figure shows unfair inequality based on different poverty thresholds y_{\min} for a cross-section of European countries in 2010. Unfair inequality is calculated as $D(Y^e||Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The norm distribution is given by $Y_{\text{EOP+FfP}}^r$. See also Table 1 for information on the calculation of norm distributions. Alternative poverty thresholds are calculated as percentage of the country-specific equalized disposable household median income. The black dashes indicate our baseline estimates. The blue crosses indicate UB_{EOP} – the upper bound measure of unfair inequality attributable to EOp. 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. For reasons of visual clarity confidence bands are shown for the baseline estimate, only.

susceptibility of our measure of unfair inequality to alternations of the deprivation threshold is remarkably country-specific. For example, in countries in which the lowest percentiles of the income distribution are strong determinants of observed total inequality, even the lowest deprivation threshold (20% of the median) yields strong upward corrections of unfair inequality in comparison to the standard EOp measure. Cases in point are the examples of Lithuania and Italy. To the contrary, in countries in which inequality is a phenomenon of increasing income dispersion in the upper percentiles, introducing FfP with a very stringent poverty threshold has little effect on the unfair share of inequality. Instead, for FfP to provide substantial upward corrections in comparison to standard EOp measures one would have to rely on a rather lenient definition of what it means to be deprived. An example for this latter group of countries is Luxembourg, in which even the upper bound EOp measure (0% poverty threshold) lies within the 95% confidence band of our baseline measure (60% poverty threshold).

Unsurprisingly, given that the fundamental questions of poverty research is “Who is poor and who is not?” (Alkire and Foster, 2011), the specification of the poverty threshold is a strong driver of our empirical results. All alternative poverty scenarios lie outside the 95% confidence bands of our baseline measure. The same holds for all observations in the US sample.

Alternative Norm Distributions. In a last step, we investigate three variations in the way we reconcile FfP and EOp by varying the set of eligible norm distributions as given by the intersection $\cap_{s=1}^5 D^s$ in section 3.

Figure 7: Alternative Poverty Thresholds over Time (USA)



Data: PSID public use dataset and PSID-CNEF.

Note: This figure shows unfair inequality based on different poverty thresholds y_{\min} for the US over the period 1969-2013. Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The norm distribution is given by $Y_{\text{EOP+FfP}}^r$. See also Table 1 for information on the calculation of norm distributions. Alternative poverty thresholds are calculated as percentage of the year-specific equalized disposable household median income. The black dashed line indicates our baseline estimates. The blue crosses indicate UB_{EOp} – the upper bound measure of unfair inequality attributable to EOp. 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. For reasons of visual clarity confidence bands are shown for the baseline estimate, only.

Alternative (a). In our baseline scenario, we constructed the norm vector by first redistributing from R to P to realize FfP. In a second step we redistributed among the non-poor of each type in order to realize EOp. Recall that opportunity equalization was achieved by a linear tax rate on the income shares exceeding y_{\min} in order to leave FfP intact. Thus, $\forall i \in P$ we conceded dominance to the principle of FfP over EOp. To illustrate this, consider the incomes of $i, j \in t$ such that $y_j^e = y_i^e - \epsilon = y_{\min} < y_i^e$ where ϵ is an arbitrarily small positive quantity. Thus, i and j are from the same circumstance type and are close to indistinguishable in terms of their effort exertion $\theta \in \Theta$. In our baseline scenario the corresponding norm incomes would be $y_j^r = y_{\min}$ whereas $y_i^r = y_i^e \left[1 - \left(\frac{y_i^e - y_{\min}}{y_i^e} \right) (\tau^{\text{FfP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FfP}})) \right]$. Since we pin down the norm income of those below the poverty line to equal y_{\min} , their income cannot exceed the poverty line irrespective of whether they come from a very disadvantaged or an advantaged type. If in our example τ_t^{EOp} was negative – as it should for disadvantaged types with $\mu_t < \mu$ – individual i would benefit from opportunity equalization, while j would not. Thus, around the poverty threshold there is a discontinuity in the extent to which the members of one circumstance type benefit from opportunity equalization.

To circumvent this property one has to concede dominance to the principle of EOp over the principle of FfP in the lower end of the distribution. Thus, we define the sets $P^{(a)}$ and $R^{(a)}$ such that $P^{(a)} = \{i : y_i^e (1 - \frac{\mu_t - \mu}{\mu_t}) \leq y_{\min}\}$ and $R^{(a)} = \{i : y_i^e (1 - \frac{\mu_t - \mu}{\mu_t}) > y_{\min}\}$. Note that in contrast to our baseline scenario we now identify the deprived after their incomes had been re-scaled to realize EOp. While **Constant Resources** and **EOp** are unaffected by this re-definition, it entails

changes in the remaining three set restrictions:

$$D^{3(a)} = \left\{ D : y_i^r = y_{\min} \ \forall i \in P^{(a)} \right\}, \quad (\text{FFP (a)})$$

$$D^{4(a)} = \left\{ D : y_i^r \geq y_{\min} \ \forall i \in R^{(a)} \right\}, \quad (\text{Financing I (a)})$$

$$D^{5(a)} = \left\{ D : \forall t \in T, \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \ \forall i, j \in t \cap R^{(a)} \right\}. \quad (\text{Financing II (a)})$$

The intersection $\cap_{s=1}^5 D^{s(a)}$ again yields a singleton:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e (1 - \tau_t^{\text{EOp}(a)}) \leq y_{\min} \\ y_i^e \left[1 - \tau_t^{\text{EOp}(a)} - \tilde{y}_i^{(a)} \tau_t^{\text{FFP}(a)} (1 - \tau_t^{\text{EOp}(a)}) \right], & \text{otherwise.} \end{cases} \quad (\text{Alternative (a)})$$

where $\tilde{y}_i^{(a)} = \left(\frac{y_i^e (1 - \tau_t^{\text{EOp}(a)}) - y_{\min}}{y_i^e (1 - \tau_t^{\text{EOp}(a)})} \right)$, $\tau_t^{\text{FFP}(a)} = \frac{N_{P^{(a)} \cap t} \left[\frac{y_{\min} - \mu_{P^{(a)} \cap t}^e (1 - \tau_t^{\text{EOp}(a)})}{\mu_{R^{(a)} \cap t}^e (1 - \tau_t^{\text{EOp}(a)}) - y_{\min}} \right]}{N_{R^{(a)} \cap t} \left[\frac{\mu_{R^{(a)} \cap t}^e (1 - \tau_t^{\text{EOp}(a)}) - y_{\min}}{\mu_{R^{(a)} \cap t}^e (1 - \tau_t^{\text{EOp}(a)}) - y_{\min}} \right]}$ and $\tau_t^{\text{EOp}(a)} = \frac{\mu_t - \mu}{\mu_t}$.

While such a norm distribution eliminates the discontinuity around the poverty threshold, it imposes another discontinuity across circumstance types. Since tax rates for poverty eradication are now type-specific, the norm income of high income persons in $R^{(a)}$, decreases with the type-specific poverty gap ratio: The more funds are necessary to eradicate intra-type poverty, the higher the linear tax-rate on the non-deprived type members. To illustrate this, consider the incomes of $i \in t_m$ and $j \in t_n$ such that $y_j^e = y_i^e$ and $\mu_m^e = \mu_n^e$. Thus, i and j are from different circumstance types that, however, are on par in terms of the value of their respective opportunity sets. In our baseline scenario the corresponding norm incomes would be $y_j^r = y_i^r = y_i^e \left[1 - \left(\frac{y_i^e - y_{\min}}{y_i^e} \right) (\tau^{\text{FFP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FFP}})) \right]$. Both households would receive the same norm income as they have the same background characteristics ($\mu_m^e = \mu_n^e$) to transform resources into income and share the same responsibility to alleviate deprivation in society ($\tau_m^{\text{FFP}} = \tau_n^{\text{FFP}} = \tau^{\text{FFP}} \ \forall t \in T$). To the contrary, in the alternative scenario as specified by the intersection $\cap_{s=1}^5 D^{s(a)}$, $\tau_t^{\text{FFP}(a)}$ is not constant across types but varies with the average shortfall (surplus) of incomes in $P^{(a)} \cap t$ ($R^{(a)} \cap t$) from the poverty line y_{\min} . The greater the divergence between τ_m^{FFP} and τ_n^{FFP} the greater the divergence between the norm incomes of y_i^r and y_j^r . Thus, instead of interpreting the eradication of poverty as a shared burden across types, the alternative scenario suggests the eradication of poverty to be a type-specific endeavor.

In spite of their different normative implications, the empirical differences between the baseline and this alternative scenario are negligible. In both the European cross-section (Figure 8, Panel 1) and the longitudinal US sample (Figure 9, Panel 1), the alternative scenario yields slightly higher values of unfair inequality. However, in every country-year cell, the point estimate of the alternative specification lies comfortably within the 95% confidence band of our baseline estimate.

Alternative (b). In our baseline scenario, we assume that type-specific opportunity sets should be evaluated by reference to the average incomes of all $i \in t$. However, one may also claim that the requirement of opportunity equalization only operates for households whose basic needs are satisfied. In line with this normative assumption we replace **EOp** by the following

requirement:

$$D^{2(b)} = \left\{ D : \mu_{t \cap R}^r = \frac{1}{N_{t \cap R}} \sum_{i \in t \cap R} y_i^r = \frac{1}{N_R} \sum_{i \in R} y_i^r = \mu_R \quad \forall t \in T \right\}. \quad (\text{EOp (b)})$$

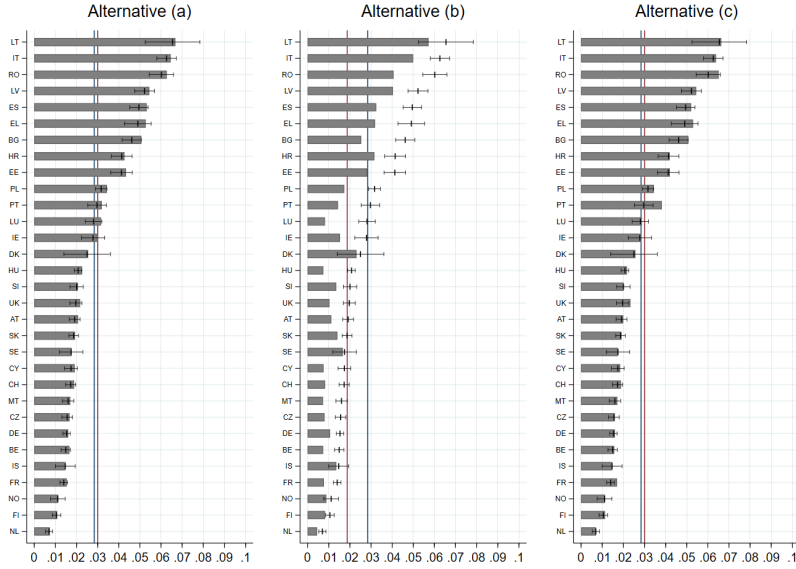
Instead of rating type-specific advantages by the type mean, **EOp (b)** draws on the average excess income above the poverty line to evaluate opportunity sets. Hence, according to this alternative interpretation the norm distribution would be constructed as follows:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i (\tau^{\text{FFP}} + \tau_t^{\text{EOp (b)}} (1 - \tau^{\text{FFP}}))], & \text{otherwise.} \end{cases} \quad (\text{Alternative (b)})$$

where $\tilde{y}_i = \left(\frac{y_i^e - y_{\min}}{y_i^e} \right)$, $\tau^{\text{FFP}} = \frac{N_P (y_{\min} - \mu_P^e)}{N_R (\mu_R^e - y_{\min})}$ and $\tau_t^{\text{EOp (b)}} = \frac{\mu_{R \cap t}^e - \tau^{\text{FFP}} \left(\frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) - \mu_R^e}{\mu_{R \cap t}^e - \tau^{\text{FFP}} \left(\frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) - y_{\min}}$.

In comparison to our baseline measure this alternative interpretation leads to decreases in the extent of inequality that is deemed unfair. Restricting the evaluation of opportunity sets to incomes above the poverty line makes the distribution of type-specific advantages more homogeneous and therefore requires less transfers across types. As a consequence, $D(Y^e || Y_{\text{EOp+FFP}}^r)$ on average decreases by approximately one third in both the European (Figure 8, Panel 2) and the US sample (Figure 9, Panel 2).

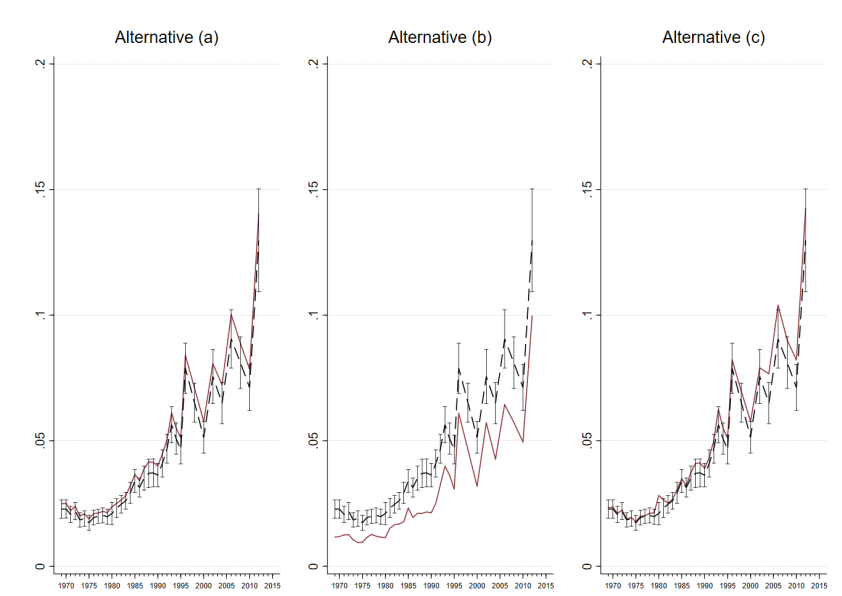
Figure 8: Alternative Norm Distributions by Country (Europe)



Data: EU-SILC 2011 cross-sectional (rev.5, June 2015).

Note: This figure shows unfair inequality based on different normative assumptions in comparison to our baseline estimates for a cross-section of European countries in 2010. Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The gray bars show unfair inequality calculated by reference to the respective alternative scenario indicated in the panel header. The black dashes indicate our baseline estimates. The vertical blue and red lines indicate the unweighted cross-country average for the baseline and the alternative estimate, respectively. 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. For reasons of visual clarity confidence bands are shown for the baseline estimate, only.

Figure 9: Alternative Norm Distributions over Time (USA)



Data: PSID public use dataset and PSID-CNEF.

Note: This figure shows unfair inequality based on different normative assumptions in comparison to our baseline estimates for the US over the period 1969-2013. Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. The red lines show unfair inequality calculated by reference to the respective alternative scenario indicated in the panel header. The black dashed line indicates our baseline estimates. 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. For reasons of visual clarity confidence bands are shown for the baseline estimate, only.

Alternative (c). Note that up to this point we maintained zero inequality aversion within circumstance types conditional on the fact that all $i \in P$ were lifted to the minimal income y_{\min} . However, there may be many arguments for cushioning income disparity beyond our concerns for EOp and FfP. Therefore, in analogy to the poverty threshold y_{\min} , we introduce an affluence threshold y_{\max} and consider any income exceeding this threshold as unfair. Normatively, such a requirement may be defended by reference to concerns about bargaining power and political capture such as in Piketty et al. (2014). As a consequence, we partition $R = \{i : y_i^e > y_{\min}\}$ into a middle class group, $M^{(c)} = \{i : y_{\min} < y_i^e < y_{\max}\}$ and an affluent group, $A^{(c)} = \{i : y_i^e \geq y_{\max}\}$ in order to adjust **Financing II** as follows:

$$D^{5(c)} = \left\{ D : \forall t \in T, \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \quad \forall i, j \in t \cap M^{(c)} \right\}. \quad (\text{Financing II (c)})$$

Financing II (c) reflects the fact that zero inequality aversion with respect to incomes exceeding the poverty threshold only exists for the non-affluent middle class M . On all incomes that exceed the affluence threshold we impose a freedom from affluence (FfA) restriction:

$$D^{6(c)} = \left\{ D : y_i^r = y_{\max} \quad \forall i \in A^{(c)} \right\}. \quad (\text{FfA})$$

By **FfA** there is infinite inequality aversion for the share of income exceeding the affluence threshold. While maintaining all other restrictions as in the baseline scenario, the norm distribution

is now pinned down to a singleton by $\cap_{s=1}^6 D^{s(c)}$:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_{\max}, & \text{if } y_i^e \geq y_{\max} \\ y_i^e [1 - \tilde{y}_i (\tau^{\text{FFP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FFP}}))], & \text{otherwise.} \end{cases} \quad (\text{Alternative (c)})$$

$$\text{where } \tilde{y}_i = \left(\frac{y_i^e - y_{\min}}{y_i^e} \right), \tau_t^{\text{EOp}} = \frac{\mu_t^e + \frac{N_P \cap t}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FFP}(c)} \left(\frac{N_{M(c) \cap t}}{N_t} (\mu_{M(c) \cap t}^e - y_{\min}) \right) - \frac{N_{A(c) \cap t}}{N_t} (\mu_{A(c) \cap t}^e - y_{\max}) - \mu}{\mu_t^e + \frac{N_P \cap t}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FFP}(c)} \left(\frac{N_{M(c) \cap t}}{N_t} (\mu_{M(c) \cap t}^e - y_{\min}) \right) - \frac{N_{A(c) \cap t}}{N_t} (\mu_{A(c) \cap t}^e - y_{\max}) - y_{\min}}$$

$$\text{and } \tau^{\text{FFP}(c)} = \frac{N_P (y_{\min} - \mu_P^e) - N_{A(c)} (\mu_{A(c)}^e - y_{\max})}{N_{M(c)} (\mu_{M(c)}^e - y_{\min})}.$$

Just as in the case of poverty lines, the appropriate setting of affluence thresholds is a contentious and normatively laden issue (Peichl et al., 2010; Atkinson and Brandolini, 2013). For this empirical exercise we set the affluence line to 400% of the country- and year-specific median income. The ensuing results for Europe and the US are shown in the right-hand panels of Figures 8 and 9. Complementing our baseline principles – EOp and FfP – with FfA leads to little changes in the extent of unfair inequality in both samples. As expected, the introduction of FfA leads to upward adjustments in the extent of unfair inequality. However, all estimates lie comfortably within the 95% confidence bands of our baseline estimates. This finding holds although the affluence threshold arguably is very low. On the one hand, it is hardly defensible to apply a marginal tax rate of 100% once a household disposes of four times the resources available to the median household. On the other hand, increasing the affluence threshold would lead to decreases in the extent of inequality aversion within circumstance types and thus lower levels of unfair inequality. Therefore, these results re-emphasize our previous claim that an exclusive focus on high income earners may be misleading – especially when it comes to fairness evaluations of income distribution.

One may be concerned that this result is driven by the under-representation of top incomes in survey data. To address this concern we focus on the seven “register countries” of EU-SILC: Denmark, Finland, Iceland, Netherlands, Norway, Sweden and Slovenia. For these countries, income information is drawn from administrative tax return data. Hence, if our results were driven by aforementioned reporting issues, we should see a much larger increase in unfair inequality for the group of “register countries”. However, this is not what we observe. In comparison to the baseline scenario, the unweighted country average for the entire sample increases from 0.028 to 0.030. The increase of the unweighted country average for the sample of “register countries” from 0.015 to 0.016 is of comparable magnitude. We take this as indication that our results are robust to under-reporting at the top of the income distribution.

6 Conclusions

In this paper we have provided a new measure of unfair inequality that reconciles the ideals of equal opportunities and absence of poverty. In fact, we provide the first work that combines the measurement of equality of opportunity and poverty by treating both as co-equal grounds for compensation. As such, we differ from previous works that have analyzed both principles in a joint framework by prioritizing either FfP or EOp.

While standard measures of EOp can be interpreted as information on the extent of unfair inequalities in a given outcome distribution, concerns have been expressed about them producing very low estimates of unfair inequality due to the partial observability of circumstances (Kanbur and Wagstaff, 2016). In this paper, we propose a new route to address the lower bound nature of current measures of unfair inequality by effectively acknowledging that judgments on the fairness of an outcome distribution are informed by multiple normative principles.

Empirical application of our measures in a standardized dataset of 31 European countries and a longitudinal sample for the US show important implications for current debates on inequality. First, we indeed find sizable upward corrections of unfair inequality when complementing the concern for equal opportunities with poverty aversion. While the extent of this upward correction remains sensitive to the exact specification of the deprivation threshold, our results are largely robust to the use of different inequality indexes and slight modifications of our normative requirements. Second, we show that our unfairness measures are much more sensitive to variations at the bottom of the income distribution than at the top. This suggests to re-shift the recent focus of attention from the upmost parts of the income distribution to the lower percentiles if the issue at stake is unfairness instead of a mere description of aggregate inequality.

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A Construction of Norm Vector

Freedom form Poverty. **FfP** requires bottom fill-up so that $y_i^r = y_{\min} \forall i \in P$. Invoking **Constant Resources**, the funds for bottom fill up must be raised in group R , where by **Financing I** the tax base is given by $y_i^e - y_{\min}$. Again invoking **Constant Resources**, it must be the case that $\sum_P (y_{\min} - y_i^e) = \sum_R \tau_i^{\text{FfP}} (y_i^e - y_{\min})$. Abstracting from **EOp**, there is zero inequality aversion for the share of income exceeding y_{\min} (**Financing II**) and therefore $\tau_i^{\text{FfP}} = \tau_j^{\text{FfP}} = \tau^{\text{FfP}} \forall i, j \in R$. Since τ_i^{FfP} must be linear, $\tau^{\text{FfP}} = \frac{N_P (y_{\min} - \mu_P^e)}{N_R (\mu_R^e - y_{\min})}$. Hence, the only distribution complying with **Constant Resources**, **FfP**, **Financing I** and **Financing II** is given by:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i (\tau^{\text{FfP}})], & \text{otherwise.} \end{cases}, \quad (\text{FfP Only})$$

where $\tilde{y}_i = \left(\frac{y_i^e - y_{\min}}{y_i^e} \right)$ and $\tau^{\text{FfP}} = \frac{N_P (y_{\min} - \mu_P^e)}{N_R (\mu_R^e - y_{\min})}$.

Equality of Opportunity. Let's consider the distribution **FfP Only** as an intermediate distribution that has to be reshaped by means of the tax rate τ_i^{EOp} in order to comply with **EOp**. By **FfP** we know that $\tau_i^{\text{EOp}} = 0 \forall i \in P$. Invoking **Financing I** it must be the case that the tax base for τ_i^{EOp} is $y_i^e [1 - \tilde{y}_i (\tau^{\text{FfP}})] - y_{\min} \forall i \in R$. By **Financing II**, $\tau_i^{\text{EOp}} = \tau_j^{\text{EOp}} = \tau_t^{\text{EOp}} \forall i, j \in R \cap t, \forall t \in T$. Taken together, these provisions specify that **EOp** must be realized by means of transfers among the rich constituents of each type. These transfers are linear within types and calculated on the share of income exceeding the poverty threshold. Therefore:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ \{y_i^e [1 - \tilde{y}_i (\tau^{\text{FfP}})] - y_{\min}\} (1 - \tau_t^{\text{EOp}}) = y_i^e [1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FfP}}))], & \text{otherwise.} \end{cases}$$

To realize **EOp** the following must hold: $\mu_t^r = \frac{1}{N_t} \sum_{i \in t} y_i^r = \frac{1}{N_t} \sum_i y_i^e = \mu, \forall t \in T$. Hence, it must be the case that:

$$\frac{1}{N_t} \left\{ \sum_{i \in t \cap P} y_{\min} + \sum_{i \in t \cap R} y_i^e [1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FfP}}))] \right\} = \mu, \forall t \in T.$$

$$\text{Solving for } \tau_t^{\text{EOp}} \text{ then yields: } \tau_t^{\text{EOp}} = \frac{\left[\mu_t^e + \frac{N_{P \cap t}}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FfP}} \left(\frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) \right] - \mu}{\left[\mu_t^e + \frac{N_{P \cap t}}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FfP}} \left(\frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) \right] - y_{\min}}.$$

B Descriptive Statistics

Table B.1: Descriptive Statistics by Country (Europe)

	N	μ	P ₀	P ₁	Watts Index	Gini	MLD	Types
AT	6,350	25,590 (195)	0.131 (0.006)	0.032 (0.002)	0.043 (0.003)	0.270 (0.003)	0.127 (0.003)	36
BE	5,407	24,116 (182)	0.105 (0.005)	0.024 (0.001)	0.031 (0.002)	0.244 (0.004)	0.105 (0.003)	35
BG	6,931	3,798 (38)	0.163 (0.005)	0.056 (0.002)	0.081 (0.004)	0.332 (0.004)	0.200 (0.005)	30
CH	6,897	42,240 (350)	0.106 (0.005)	0.026 (0.002)	0.034 (0.002)	0.279 (0.004)	0.134 (0.004)	36
CY	4,906	21,142 (219)	0.104 (0.005)	0.025 (0.002)	0.033 (0.002)	0.278 (0.005)	0.131 (0.004)	36
CZ	6,752	9,038 (73)	0.098 (0.005)	0.025 (0.002)	0.033 (0.002)	0.256 (0.004)	0.114 (0.004)	29
DE	12,316	22,394 (133)	0.146 (0.004)	0.037 (0.001)	0.047 (0.002)	0.276 (0.003)	0.132 (0.003)	36
DK	2,532	30,803 (483)	0.111 (0.012)	0.039 (0.006)	0.062 (0.012)	0.258 (0.009)	0.133 (0.012)	32
EE	5,374	7,178 (90)	0.162 (0.006)	0.057 (0.003)	0.085 (0.005)	0.324 (0.005)	0.194 (0.006)	36
EL	6,331	13,458 (166)	0.187 (0.006)	0.060 (0.003)	0.091 (0.005)	0.331 (0.006)	0.204 (0.007)	35
ES	15,360	17,359 (130)	0.188 (0.005)	0.063 (0.002)	0.095 (0.004)	0.329 (0.003)	0.200 (0.004)	36
FI	4,563	25,958 (240)	0.120 (0.006)	0.027 (0.002)	0.035 (0.003)	0.253 (0.005)	0.113 (0.004)	36
FR	11,145	24,572 (197)	0.124 (0.004)	0.027 (0.001)	0.035 (0.002)	0.290 (0.005)	0.145 (0.005)	36
HR	5,947	6,722 (59)	0.167 (0.005)	0.056 (0.002)	0.087 (0.004)	0.302 (0.003)	0.173 (0.004)	36
HU	13,583	5,394 (30)	0.137 (0.003)	0.029 (0.001)	0.037 (0.001)	0.276 (0.002)	0.126 (0.002)	31
IE	3,069	25,386 (349)	0.136 (0.008)	0.034 (0.003)	0.049 (0.005)	0.292 (0.005)	0.150 (0.005)	36
IS	1,579	20,616 (291)	0.122 (0.011)	0.035 (0.004)	0.048 (0.006)	0.241 (0.007)	0.105 (0.007)	34
IT	20,152	18,985 (98)	0.184 (0.004)	0.066 (0.002)	0.115 (0.004)	0.310 (0.002)	0.198 (0.004)	36
LT	5,295	4,810 (62)	0.193 (0.008)	0.073 (0.004)	0.129 (0.010)	0.340 (0.005)	0.235 (0.010)	34
LU	6,871	38,243 (400)	0.140 (0.006)	0.029 (0.002)	0.037 (0.003)	0.272 (0.005)	0.124 (0.004)	36
LV	6,437	5,457 (52)	0.195 (0.005)	0.071 (0.002)	0.111 (0.004)	0.353 (0.004)	0.234 (0.005)	36
MT	4,255	13,408 (128)	0.130 (0.006)	0.028 (0.002)	0.036 (0.002)	0.269 (0.004)	0.122 (0.003)	36
NL	5,513	24,007 (213)	0.092 (0.007)	0.017 (0.002)	0.022 (0.002)	0.245 (0.003)	0.100 (0.003)	36
NO	2,493	40,730 (416)	0.092 (0.008)	0.023 (0.003)	0.032 (0.004)	0.221 (0.005)	0.089 (0.005)	36
PL	14,616	6,233 (44)	0.168 (0.003)	0.047 (0.001)	0.064 (0.002)	0.320 (0.003)	0.177 (0.004)	23
PT	5,923	11,036 (128)	0.153 (0.005)	0.042 (0.002)	0.057 (0.003)	0.332 (0.005)	0.187 (0.006)	33
RO	7,565	2,575 (24)	0.211 (0.006)	0.076 (0.003)	0.116 (0.005)	0.332 (0.004)	0.207 (0.005)	23
SE	2,851	24,755 (219)	0.123 (0.008)	0.031 (0.003)	0.048 (0.006)	0.225 (0.004)	0.098 (0.005)	12
SI	4,870	13,124 (115)	0.143 (0.008)	0.040 (0.003)	0.054 (0.004)	0.248 (0.004)	0.111 (0.004)	36
SK	7,288	7,494 (47)	0.127 (0.005)	0.036 (0.002)	0.050 (0.003)	0.259 (0.003)	0.122 (0.003)	32
UK	6,242	23,320 (263)	0.123 (0.005)	0.031 (0.002)	0.041 (0.002)	0.319 (0.006)	0.174 (0.006)	36

Data: EU-SILC 2011 cross-sectional (rev.5, June 2015).

Note: Standard errors are calculated based on a bootstrap procedure with 500 draws and reported in parentheses.

Table B.2: Descriptive Statistics over Time (USA)

	N	μ	P_0	P_1	Watts Index	Gini	MLD	Types
1969	2,552	4,839 (59)	0.116 (0.007)	0.034 (0.002)	0.045 (0.003)	0.278 (0.005)	0.138 (0.004)	36
1970	2,638	5,224 (67)	0.119 (0.007)	0.035 (0.002)	0.047 (0.003)	0.285 (0.005)	0.145 (0.005)	36
1971	2,758	5,561 (63)	0.121 (0.006)	0.037 (0.002)	0.049 (0.003)	0.277 (0.004)	0.138 (0.004)	36
1972	2,868	6,043 (71)	0.121 (0.006)	0.035 (0.002)	0.048 (0.003)	0.280 (0.005)	0.140 (0.004)	36
1973	2,980	6,567 (73)	0.112 (0.006)	0.032 (0.002)	0.042 (0.003)	0.270 (0.004)	0.129 (0.004)	36
1974	3,071	7,174 (83)	0.108 (0.006)	0.029 (0.002)	0.038 (0.003)	0.274 (0.004)	0.132 (0.004)	36
1975	3,242	7,656 (81)	0.106 (0.006)	0.029 (0.002)	0.039 (0.003)	0.277 (0.004)	0.133 (0.004)	36
1976	3,316	8,501 (95)	0.122 (0.006)	0.034 (0.002)	0.045 (0.003)	0.284 (0.004)	0.141 (0.004)	36
1977	3,509	8,837 (85)	0.123 (0.006)	0.035 (0.002)	0.048 (0.003)	0.269 (0.004)	0.129 (0.004)	36
1978	3,607	9,619 (109)	0.117 (0.006)	0.034 (0.002)	0.046 (0.003)	0.275 (0.005)	0.135 (0.005)	36
1979	3,764	10,440 (136)	0.121 (0.006)	0.034 (0.002)	0.045 (0.003)	0.282 (0.006)	0.141 (0.006)	36
1980	3,835	11,593 (224)	0.129 (0.006)	0.035 (0.002)	0.046 (0.003)	0.308 (0.011)	0.173 (0.014)	36
1981	3,955	12,173 (196)	0.133 (0.006)	0.041 (0.002)	0.056 (0.003)	0.300 (0.009)	0.163 (0.010)	36
1982	4,054	12,800 (137)	0.147 (0.006)	0.042 (0.002)	0.058 (0.004)	0.292 (0.004)	0.152 (0.004)	36
1983	4,119	13,822 (150)	0.149 (0.006)	0.046 (0.002)	0.062 (0.003)	0.301 (0.005)	0.162 (0.005)	36
1984	4,248	14,845 (207)	0.153 (0.006)	0.046 (0.002)	0.064 (0.003)	0.310 (0.007)	0.173 (0.008)	36
1985	4,306	15,568 (187)	0.152 (0.006)	0.051 (0.002)	0.074 (0.004)	0.310 (0.006)	0.177 (0.006)	36
1986	4,371	16,130 (187)	0.157 (0.006)	0.051 (0.002)	0.070 (0.003)	0.309 (0.005)	0.171 (0.005)	36
1987	4,456	18,049 (285)	0.155 (0.006)	0.051 (0.002)	0.072 (0.004)	0.323 (0.008)	0.190 (0.010)	36
1988	4,497	19,352 (331)	0.163 (0.006)	0.052 (0.002)	0.073 (0.004)	0.342 (0.010)	0.210 (0.013)	36
1989	4,549	20,320 (351)	0.157 (0.006)	0.052 (0.002)	0.074 (0.004)	0.342 (0.009)	0.211 (0.012)	36
1990	4,593	20,925 (294)	0.168 (0.006)	0.053 (0.002)	0.074 (0.003)	0.339 (0.007)	0.206 (0.008)	36
1991	4,659	21,365 (309)	0.169 (0.006)	0.056 (0.002)	0.081 (0.004)	0.339 (0.007)	0.208 (0.008)	36
1992	4,553	22,586 (329)	0.172 (0.007)	0.060 (0.003)	0.093 (0.005)	0.346 (0.007)	0.223 (0.008)	36
1993	4,624	22,618 (353)	0.175 (0.007)	0.067 (0.003)	0.106 (0.006)	0.369 (0.008)	0.258 (0.010)	36
1994	4,896	23,118 (331)	0.177 (0.006)	0.064 (0.003)	0.100 (0.005)	0.357 (0.007)	0.240 (0.009)	36
1995	4,898	24,422 (366)	0.175 (0.006)	0.060 (0.003)	0.091 (0.004)	0.360 (0.007)	0.237 (0.009)	36
1996	3,808	25,936 (394)	0.178 (0.007)	0.071 (0.003)	0.131 (0.007)	0.359 (0.007)	0.265 (0.010)	36
1998	4,052	27,774 (416)	0.184 (0.006)	0.071 (0.003)	0.117 (0.006)	0.373 (0.006)	0.268 (0.009)	36
2000	4,027	32,204 (571)	0.173 (0.007)	0.061 (0.003)	0.093 (0.005)	0.371 (0.008)	0.252 (0.010)	36
2002	4,042	31,870 (489)	0.186 (0.007)	0.070 (0.003)	0.127 (0.008)	0.365 (0.007)	0.269 (0.011)	36
2004	4,948	36,390 (810)	0.193 (0.007)	0.072 (0.003)	0.114 (0.005)	0.402 (0.011)	0.304 (0.016)	36
2006	4,861	39,197 (805)	0.185 (0.006)	0.073 (0.003)	0.137 (0.008)	0.419 (0.009)	0.350 (0.015)	36
2008	4,763	40,007 (814)	0.201 (0.007)	0.077 (0.004)	0.134 (0.008)	0.403 (0.009)	0.317 (0.014)	36

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Table B.2 – *Continued from previous page*

	N	μ	P ₀	P ₁	Watts Index	Gini	MLD	Types
2010	4,595	39,732 (883)	0.183 (0.007)	0.071 (0.003)	0.120 (0.006)	0.399 (0.011)	0.309 (0.017)	36
2012	4,406	42,461 (1,142)	0.204 (0.007)	0.088 (0.004)	0.187 (0.013)	0.430 (0.013)	0.398 (0.024)	36

Data: PSID public use dataset and PSID-CNEF.

Note: Standard errors are calculated based on a bootstrap procedure with 500 draws and reported in parentheses.

C Occupational Classification

Table C.1: Occupational Classification

	Census 1970	Census 1990	ISCO-08
High	(1) Professional, Technical and Kindred workers	(1) Managerial and Professional Specialty Occ.	(1) Managers
	(2) Managers, Officials and Proprietors	(2) Technical and Sales Op.	(2) Professionals
	(3) Self-Employed Businessmen	–	(3) Technicians and Associate Professionals
Middle	(4) Clerical and Sales Workers	(3) Administrative Support Occ., Including Clerical	(4) Clerical Support Workers
	(5) Craftsmen, Foremen and Kindred Workers	(5) Precision Production, Craft, and Repair Occ.	(7) Craft and Related Trade Workers
	(6) Operatives and Kindred Workers	(7) Machine Op., Assemblers, and Inspectors	(8) Plant and Machine Op.s and Assemblers
	–	(6) Extractive and Precision Production Occ.	–
Low	(7) Laborers, Service Workers and Farm Laborers	(4) Service, Farming, Forestry, and Fishing Occ.	(6) Skilled Agric., Forestry and Fishery Workers
	(8) Farmers and Farm Managers	(8) Transportation and Material Moving Occ., Handlers, Equipment Cleaners, Helpers, and Laborers	(9) Elementary Occ.
	(9) Miscellaneous (incl. Armed Services, Protective Workers etc.)	(9) Military Occ.	(0) Armed Forces Occ.
	(-) Not in Labor Force	(-) Not in Labor Force	(-) Not in Labor Force

Note: In the PSID, waves 1970-2001 report occupation codes with reference to 1970 Census codes. Waves 2003-2013 report occupation codes with reference to 2000 Census codes. If available on 3-digit level we use the cross-walk routine provided by Autor and Dorn (2013) to standardize codes based on the 1990 Census classification. Few occupational codes available in the PSID are not included in the routine provided by Autor and Dorn (2013). These groups are allocated to their 1990 Census classification analogues by the authors of this paper. We then aggregate all codes to the 1-digit level and apply the classification scheme outlined above. In EU-SILC, the 2005 wave reports occupation codes with reference to the ISCO-88 classification, while the 2011 wave reports occupation codes with reference to the ISCO-08 classification. We apply the cross-walk routine provided by the International Labour Organization to standardize codes based on the ISCO-08 classification. We then aggregate all codes to the 1-digit level and apply the classification scheme outlined above.

D Upper and Lower Bounds of FfP and EOp

Table D.1: Relative Point Estimates by Country (Europe)

	Baseline			Decomposition		
	N	μ	$D(Y^e Y_{\text{EOp+FfP}}^r)$	LB _{EOp}	LB _{FfP}	Overlap
AT	6,350	25,590	15.1%	6.3%	6.5%	2.3%
BE	5,407	24,116	14.2%	7.3%	3.8%	3.1%
BG	6,931	3,798	23.1%	10.4%	7.3%	5.4%
CH	6,897	42,240	12.9%	6.8%	4.1%	2.1%
CY	4,906	21,142	13.3%	7.6%	3.1%	2.6%
CZ	6,752	9,038	13.7%	6.7%	5.1%	2.0%
DE	12,316	22,394	11.6%	3.5%	6.8%	1.2%
DK	2,532	30,803	18.8%	1.4%	17.0%	0.4%
EE	5,374	7,178	21.3%	6.8%	11.9%	2.6%
EL	6,331	13,458	24.0%	8.4%	11.6%	4.0%
ES	15,360	17,359	24.7%	8.6%	11.6%	4.6%
FI	4,563	25,958	9.3%	2.1%	6.8%	0.5%
FR	11,145	24,572	9.7%	4.3%	3.6%	1.7%
HR	5,947	6,722	23.9%	5.7%	16.1%	2.1%
HU	13,583	5,394	16.5%	10.6%	3.0%	2.9%
IE	3,069	25,386	18.6%	8.4%	8.2%	2.0%
IS	1,579	20,616	14.0%	1.1%	12.8%	0.1%
IT	20,152	18,985	31.6%	6.4%	22.6%	2.5%
LT	5,295	4,810	27.9%	3.5%	22.7%	1.6%
LU	6,871	38,243	22.7%	16.1%	1.9%	4.7%
LV	6,437	5,457	22.3%	5.1%	14.7%	2.5%
MT	4,255	13,408	13.1%	7.2%	4.2%	1.7%
NL	5,513	24,007	7.0%	2.5%	4.0%	0.4%
NO	2,493	40,730	12.5%	2.6%	9.4%	0.5%
PL	14,616	6,233	17.8%	8.1%	6.6%	3.1%
PT	5,923	11,036	15.9%	8.3%	5.3%	2.3%
RO	7,565	2,575	29.0%	9.4%	15.5%	4.0%
SE	2,851	24,755	17.8%	0.9%	16.5%	0.4%
SI	4,870	13,124	18.1%	5.9%	10.8%	1.4%
SK	7,288	7,494	15.3%	3.8%	10.1%	1.5%
UK	6,242	23,320	11.3%	5.4%	4.2%	1.8%

Data: EU-SILC 2011 cross-sectional (rev.5, June 2015).

Note: Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. Rows titled $D(Y^e || Y_{\text{EOp+FfP}}^r)$ indicate the baseline measure of total unfair inequality attributable to both EOp and FfP. Rows titled LB_{EOp} indicate the lower bound measure of unfair inequality attributable to EOp. Rows titled LB_{FfP} indicate the lower bound measure of unfair inequality attributable to FfP. Rows titled Overlap indicate the overlap component that can be attributed to either EOp or FfP. All components are divided by total inequality as measured by the MLD to yield relative measures of unfair inequality.

Table D.2: Relative Point Estimates over Time (USA)

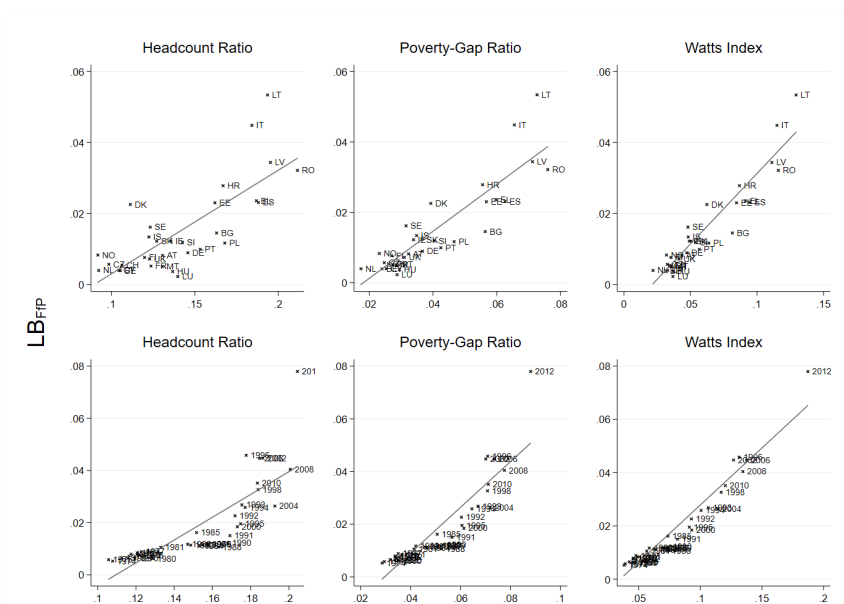
	Baseline			Decomposition		
	N	μ	$D(Y^e Y_{EOP+FIP}^r)$	LB _{EOP}	LB _{FIP}	Overlap
1969	2,552	4,839	16.6%	8.1%	4.7%	3.8%
1970	2,638	5,224	15.8%	7.6%	4.4%	3.8%
1971	2,758	5,561	15.0%	5.9%	6.2%	2.9%
1972	2,868	6,043	15.7%	6.7%	5.5%	3.5%
1973	2,980	6,567	14.3%	6.1%	5.1%	3.0%
1974	3,071	7,174	14.4%	7.2%	4.0%	3.2%
1975	3,242	7,656	13.1%	5.9%	4.4%	2.8%
1976	3,316	8,501	13.7%	5.6%	5.4%	2.8%
1977	3,509	8,837	15.4%	5.6%	6.9%	2.9%
1978	3,607	9,619	15.1%	6.1%	5.9%	3.0%
1979	3,764	10,440	14.0%	5.8%	5.5%	2.6%
1980	3,835	11,593	12.2%	5.5%	3.6%	3.0%
1981	3,955	12,173	14.2%	4.9%	6.5%	2.8%
1982	4,054	12,800	16.2%	5.3%	7.7%	3.1%
1983	4,119	13,822	16.1%	5.7%	7.1%	3.3%
1984	4,248	14,845	16.9%	6.7%	6.4%	3.8%
1985	4,306	15,568	19.2%	6.1%	9.2%	3.9%
1986	4,371	16,130	18.4%	7.0%	7.0%	4.4%
1987	4,456	18,049	18.5%	7.4%	6.1%	5.0%
1988	4,497	19,352	17.6%	7.6%	5.1%	4.9%
1989	4,549	20,320	17.6%	7.4%	5.6%	4.7%
1990	4,593	20,925	17.6%	7.3%	5.9%	4.4%
1991	4,659	21,365	19.7%	7.6%	7.3%	4.8%
1992	4,553	22,586	21.0%	6.4%	10.2%	4.4%
1993	4,624	22,618	21.8%	6.4%	10.4%	5.0%
1994	4,896	23,118	21.2%	6.1%	10.8%	4.3%
1995	4,898	24,422	19.6%	6.7%	8.3%	4.7%
1996	3,808	25,936	29.8%	6.8%	17.3%	5.7%
1998	4,052	27,774	24.3%	7.0%	12.2%	5.1%
2000	4,027	32,204	20.4%	7.7%	7.3%	5.3%
2002	4,042	31,870	28.1%	6.9%	16.7%	4.6%
2004	4,948	36,390	21.4%	7.4%	8.7%	5.3%
2006	4,861	39,197	25.9%	7.5%	12.8%	5.6%
2008	4,763	40,007	25.6%	7.5%	12.8%	5.3%
2010	4,595	39,732	23.0%	7.0%	11.4%	4.6%
2012	4,406	42,461	32.6%	7.6%	19.6%	5.4%

Data: PSID public use dataset and PSID-CNEF.

Note: Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . Divergences are aggregated based on MN with $\alpha = 0$. Rows titled $D(Y^e || Y_{EOP+FIP}^r)$ indicate the baseline measure of total unfair inequality attributable to both EOp and FfP. Rows titled LB_{EOP} indicate the lower bound measure of unfair inequality attributable to EOp. Rows titled LB_{FIP} indicate the lower bound measure of unfair inequality attributable to FfP. Rows titled Overlap indicate the overlap component that can be attributed to either EOp or FfP. All components are divided by total inequality as measured by the MLD to yield relative measures of unfair inequality.

E Correlation of FfP with Poverty Measures

Figure E.1: Correlation of FfP with Poverty Measures



Data: EU-SILC 2011 cross-sectional (rev.5, June 2015). PSID public use dataset and PSID-CNEF.
Note: This figure shows the correlation between unfair inequality due to FfP and different measures of poverty. Correlations are based on LB_{FfP} – the lower bound estimate of unfair inequality attributable to FfP – and the respective poverty measure. Unfair inequality is calculated as $D(Y^E||Y^T)$, which measures the aggregate divergence between the empirical distribution Y^E and the norm distribution Y^T . Divergences are aggregated based on MN with $\alpha = 0$. LB_{FfP} is calculated as the difference in the divergence measures for the norms Y^{E+T} and $Y^{E+T}_{EOp+FfP}$. See also Table 1 for information on the calculation of norm distributions. The Pearson's correlation coefficients of LB_{FfP} with headcount ratio, poverty gap ratio and Watts index in Europe (USA) are 0.764, (0.800), 0.885, (0.888), 0.943, (0.965).

F Divergence Measures

Table F.1: Divergence Measures by Country (Europe)

	Magdalou and Nock			Cowell			Almås et al.
	$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	
AT	0.019 (0.001)	0.016 (0.001)	0.024 (0.003)	0.018 (0.002)	0.016 (0.001)	0.015 (0.001)	0.098 (0.005)
BE	0.015 (0.001)	0.014 (0.001)	0.023 (0.004)	0.016 (0.002)	0.014 (0.001)	0.013 (0.001)	0.090 (0.005)
BG	0.046 (0.002)	0.041 (0.003)	0.072 (0.007)	0.043 (0.003)	0.041 (0.003)	0.041 (0.003)	0.172 (0.006)
CH	0.017 (0.001)	0.016 (0.001)	0.026 (0.004)	0.017 (0.002)	0.016 (0.001)	0.015 (0.002)	0.103 (0.005)
CY	0.017 (0.002)	0.017 (0.002)	0.028 (0.006)	0.017 (0.002)	0.017 (0.002)	0.018 (0.002)	0.098 (0.006)
CZ	0.016 (0.001)	0.014 (0.002)	0.021 (0.005)	0.014 (0.002)	0.014 (0.002)	0.014 (0.002)	0.097 (0.006)
DE	0.015 (0.001)	0.011 (0.001)	0.015 (0.002)	0.012 (0.001)	0.011 (0.001)	0.010 (0.001)	0.083 (0.004)
DK	0.025 (0.006)	0.011 (0.003)	0.012 (0.009)	0.015 (0.003)	0.011 (0.003)	0.009 (0.003)	0.059 (0.008)
EE	0.041 (0.003)	0.028 (0.003)	0.037 (0.005)	0.032 (0.003)	0.028 (0.003)	0.027 (0.003)	0.134 (0.008)
EL	0.049 (0.003)	0.038 (0.004)	0.061 (0.010)	0.041 (0.003)	0.038 (0.004)	0.038 (0.005)	0.141 (0.008)
ES	0.050 (0.002)	0.038 (0.002)	0.060 (0.007)	0.044 (0.002)	0.038 (0.002)	0.035 (0.002)	0.148 (0.005)
FI	0.011 (0.001)	0.007 (0.001)	0.007 (0.002)	0.007 (0.001)	0.007 (0.001)	0.006 (0.001)	0.056 (0.005)
FR	0.014 (0.001)	0.013 (0.001)	0.026 (0.004)	0.014 (0.001)	0.013 (0.001)	0.013 (0.001)	0.089 (0.005)
HR	0.041 (0.003)	0.024 (0.002)	0.027 (0.003)	0.029 (0.002)	0.024 (0.002)	0.023 (0.002)	0.116 (0.005)
HU	0.021 (0.001)	0.022 (0.001)	0.033 (0.002)	0.022 (0.001)	0.022 (0.001)	0.023 (0.001)	0.124 (0.004)
IE	0.028 (0.003)	0.022 (0.003)	0.030 (0.004)	0.024 (0.003)	0.022 (0.003)	0.022 (0.003)	0.121 (0.008)
IS	0.015 (0.002)	0.007 (0.001)	0.006 (0.002)	0.009 (0.002)	0.007 (0.001)	0.006 (0.001)	0.049 (0.007)
IT	0.063 (0.002)	0.034 (0.001)	0.041 (0.003)	0.044 (0.002)	0.034 (0.001)	0.031 (0.001)	0.130 (0.003)
LT	0.065 (0.007)	0.028 (0.003)	0.029 (0.004)	0.039 (0.004)	0.028 (0.003)	0.025 (0.003)	0.115 (0.007)
LU	0.028 (0.002)	0.032 (0.003)	0.053 (0.005)	0.035 (0.003)	0.032 (0.003)	0.031 (0.003)	0.152 (0.006)
LV	0.052 (0.002)	0.031 (0.002)	0.039 (0.004)	0.036 (0.002)	0.031 (0.002)	0.029 (0.002)	0.135 (0.006)
MT	0.016 (0.001)	0.015 (0.002)	0.021 (0.003)	0.015 (0.002)	0.015 (0.002)	0.016 (0.002)	0.093 (0.006)
NL	0.007 (0.001)	0.005 (0.001)	0.007 (0.001)	0.006 (0.001)	0.005 (0.001)	0.005 (0.001)	0.055 (0.005)
NO	0.011 (0.002)	0.007 (0.001)	0.006 (0.001)	0.008 (0.001)	0.007 (0.001)	0.006 (0.001)	0.056 (0.006)
PL	0.032 (0.001)	0.029 (0.002)	0.049 (0.006)	0.030 (0.002)	0.029 (0.002)	0.030 (0.002)	0.136 (0.005)
PT	0.030 (0.002)	0.029 (0.004)	0.049 (0.008)	0.027 (0.003)	0.029 (0.004)	0.034 (0.005)	0.116 (0.008)
RO	0.060 (0.003)	0.043 (0.003)	0.056 (0.005)	0.046 (0.002)	0.043 (0.003)	0.045 (0.003)	0.154 (0.006)
SE	0.017 (0.003)	0.007 (0.001)	0.005 (0.001)	0.011 (0.002)	0.007 (0.001)	0.006 (0.001)	0.037 (0.004)
SI	0.020 (0.002)	0.015 (0.001)	0.017 (0.002)	0.016 (0.001)	0.015 (0.001)	0.014 (0.001)	0.091 (0.005)
SK	0.019 (0.001)	0.012 (0.001)	0.012 (0.001)	0.013 (0.001)	0.012 (0.001)	0.011 (0.001)	0.077 (0.004)
UK	0.020 (0.001)	0.017 (0.002)	0.030 (0.006)	0.018 (0.002)	0.017 (0.002)	0.017 (0.002)	0.109 (0.007)

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Table F.1 – *Continued from previous page*

Magdalou and Nock			Cowell			Almås et al.
$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	

Data: EU-SILC 2011 cross-sectional (rev.5, June 2015).

Note: Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . The norm distribution is given by $Y_{\text{EOP+FFP}}^r$. See also Table 1 for information on the calculation of norm distributions. Divergences are aggregated according to the divergence measure indicated in the column header.

Table F.2: Divergence Measures over Time (USA)

	Magdalou and Nock			Cowell			Almås et al.
	$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	
1969	0.023 (0.002)	0.020 (0.002)	0.026 (0.004)	0.022 (0.002)	0.020 (0.002)	0.018 (0.002)	0.106 (0.007)
1970	0.023 (0.002)	0.020 (0.002)	0.027 (0.004)	0.022 (0.002)	0.020 (0.002)	0.019 (0.002)	0.109 (0.007)
1971	0.021 (0.002)	0.016 (0.002)	0.020 (0.003)	0.018 (0.002)	0.016 (0.002)	0.015 (0.002)	0.096 (0.007)
1972	0.022 (0.002)	0.018 (0.002)	0.024 (0.003)	0.020 (0.002)	0.018 (0.002)	0.017 (0.002)	0.104 (0.007)
1973	0.018 (0.001)	0.015 (0.002)	0.020 (0.003)	0.017 (0.002)	0.015 (0.002)	0.014 (0.002)	0.094 (0.007)
1974	0.019 (0.002)	0.017 (0.002)	0.023 (0.003)	0.018 (0.002)	0.017 (0.002)	0.016 (0.002)	0.100 (0.007)
1975	0.017 (0.002)	0.014 (0.002)	0.020 (0.003)	0.016 (0.002)	0.014 (0.002)	0.014 (0.002)	0.094 (0.007)
1976	0.019 (0.001)	0.015 (0.002)	0.020 (0.003)	0.017 (0.002)	0.015 (0.002)	0.014 (0.002)	0.092 (0.007)
1977	0.020 (0.001)	0.015 (0.001)	0.017 (0.002)	0.017 (0.002)	0.015 (0.001)	0.013 (0.001)	0.088 (0.005)
1978	0.020 (0.002)	0.016 (0.002)	0.021 (0.003)	0.018 (0.002)	0.016 (0.002)	0.014 (0.002)	0.093 (0.007)
1979	0.020 (0.002)	0.016 (0.002)	0.022 (0.005)	0.018 (0.002)	0.016 (0.002)	0.014 (0.002)	0.093 (0.007)
1980	0.021 (0.002)	0.019 (0.005)	0.053 (0.037)	0.021 (0.004)	0.019 (0.005)	0.018 (0.008)	0.095 (0.011)
1981	0.023 (0.002)	0.018 (0.003)	0.031 (0.019)	0.020 (0.003)	0.018 (0.003)	0.016 (0.004)	0.095 (0.009)
1982	0.025 (0.002)	0.018 (0.002)	0.023 (0.003)	0.021 (0.002)	0.018 (0.002)	0.016 (0.002)	0.101 (0.006)
1983	0.026 (0.002)	0.020 (0.002)	0.027 (0.004)	0.023 (0.002)	0.020 (0.002)	0.018 (0.002)	0.107 (0.007)
1984	0.029 (0.002)	0.024 (0.003)	0.041 (0.015)	0.027 (0.003)	0.024 (0.003)	0.022 (0.004)	0.123 (0.009)
1985	0.034 (0.002)	0.024 (0.003)	0.036 (0.007)	0.029 (0.003)	0.024 (0.003)	0.022 (0.003)	0.120 (0.008)
1986	0.031 (0.002)	0.025 (0.002)	0.037 (0.005)	0.029 (0.003)	0.025 (0.002)	0.023 (0.002)	0.124 (0.007)
1987	0.035 (0.002)	0.030 (0.005)	0.064 (0.034)	0.034 (0.004)	0.030 (0.005)	0.028 (0.006)	0.135 (0.010)
1988	0.037 (0.003)	0.033 (0.006)	0.081 (0.046)	0.037 (0.005)	0.033 (0.006)	0.031 (0.008)	0.148 (0.012)
1989	0.037 (0.003)	0.032 (0.005)	0.070 (0.033)	0.036 (0.005)	0.032 (0.005)	0.031 (0.007)	0.147 (0.011)
1990	0.036 (0.002)	0.031 (0.003)	0.052 (0.009)	0.034 (0.003)	0.031 (0.003)	0.029 (0.004)	0.144 (0.009)
1991	0.041 (0.003)	0.033 (0.004)	0.061 (0.015)	0.037 (0.004)	0.033 (0.004)	0.032 (0.005)	0.147 (0.009)
1992	0.047 (0.003)	0.032 (0.003)	0.049 (0.009)	0.039 (0.003)	0.032 (0.003)	0.029 (0.003)	0.141 (0.008)
1993	0.056 (0.004)	0.039 (0.004)	0.069 (0.015)	0.046 (0.004)	0.039 (0.004)	0.036 (0.004)	0.158 (0.010)
1994	0.051 (0.003)	0.035 (0.003)	0.062 (0.013)	0.041 (0.004)	0.035 (0.003)	0.032 (0.004)	0.150 (0.009)
1995	0.046 (0.003)	0.036 (0.004)	0.070 (0.016)	0.041 (0.004)	0.036 (0.004)	0.033 (0.004)	0.152 (0.010)
1996	0.079 (0.005)	0.044 (0.004)	0.070 (0.010)	0.059 (0.004)	0.044 (0.004)	0.039 (0.004)	0.162 (0.008)
1998	0.065 (0.004)	0.043 (0.004)	0.073 (0.010)	0.053 (0.004)	0.043 (0.004)	0.039 (0.004)	0.171 (0.009)
2000	0.051 (0.003)	0.042 (0.004)	0.091 (0.015)	0.048 (0.004)	0.042 (0.004)	0.039 (0.005)	0.174 (0.010)
2002	0.076 (0.005)	0.043 (0.004)	0.071 (0.013)	0.057 (0.004)	0.043 (0.004)	0.038 (0.004)	0.170 (0.008)
2004	0.065 (0.004)	0.049 (0.006)	0.123 (0.046)	0.058 (0.006)	0.049 (0.006)	0.045 (0.006)	0.191 (0.013)
2006	0.091 (0.006)	0.058 (0.006)	0.137 (0.029)	0.073 (0.006)	0.058 (0.006)	0.053 (0.008)	0.203 (0.012)
2008	0.081 (0.005)	0.053 (0.005)	0.107 (0.025)	0.066 (0.005)	0.053 (0.005)	0.048 (0.006)	0.194 (0.011)

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Table F.2 – Continued from previous page

	Magdalou and Nock			Cowell			Almås et al.
	$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	
2010	0.071 (0.005)	0.048 (0.006)	0.108 (0.035)	0.058 (0.006)	0.048 (0.006)	0.044 (0.006)	0.186 (0.013)
2012	0.130 (0.010)	0.069 (0.009)	0.185 (0.062)	0.094 (0.009)	0.069 (0.009)	0.064 (0.011)	0.224 (0.017)

Data: PSID public use dataset and PSID-CNEF.

Note: Unfair inequality is calculated as $D(Y^e || Y^r)$, which measures the aggregate divergence between the empirical distribution Y^e and the norm distribution Y^r . The norm distribution is given by $Y_{EOP+FFP}^r$. See also Table 1 for information on the calculation of norm distributions. Divergences are aggregated according to the divergence measure indicated in the column header.