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Burkhard Heer, Vito Polito, Michael R. Wickens

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Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
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Population Aging, Social Security and Fiscal Limits

Abstract

We study the sustainability of pension systems using a life-cycle model with distortionary taxation that sets an upper limit to the real value of tax revenues. This limit implies an endogenous threshold dependency ratio, i.e. a point in the cross-section distribution of the population beyond which tax revenues can no longer sustain the planned level of transfers to retirees. We quantify the threshold using a computable life-cycle model calibrated on the United States and fourteen European countries which have dependency ratios among the highest in the world. We examine the effects on the threshold and welfare of a number of policies often advocated to improve the sustainability of pension systems. New tax data on dynamic Laffer effects are provided.

JEL-Codes: E620, H200, H550.

Keywords: dependency ratio, fiscal space, Laffer effects, pensions, fiscal policy sustainability.

Burkhard Heer
University of Augsburg
Universitaetsstrasse 16
Germany – 86159 Augsburg
burkhard.heer@wiwi.uni-augsburg.de

Vito Polito
University of Sheffield
9 Mappin Street
United Kingdom – S1 4DT Sheffield
v.polito@sheffield.ac.uk

Michael R. Wickens
University of York
United Kingdom – York, YO10 5DD
mike.wickens@york.ac.uk

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1 Introduction

Background. Population aging is a major challenge for the public finances of both advanced and developing economies. Longer life expectancy and declining birth rates are causing dependency ratios (the number of retirees as a proportion of the labor force) to rise world-wide. This is generating an increasing burden of taxation on the working population. It raises the issue of whether existing social security nets for older people are sustainable in the longer term and, if not, whether there are policy changes that would make them sustainable and what the welfare cost of this would be. The main reason why existing social security nets for older people may not be sustainable is that they cannot be funded under existing taxation policy. The question that then arises is whether it would be possible to increase tax revenues sufficiently to achieve sustainability.

Focusing mainly on the pension component of social security systems, this paper examines their sustainability using a life-cycle dynamic general equilibrium model with distortionary taxation that takes into account the possibility of an upper limit on the real value of tax revenues raised through direct taxation. The limit exists because tax revenues are subject to dynamic Laffer effects (DLEs) due to the distortionary taxation of the factors of production.

As we are concerned with a generational issue, the sustainability of publicly funded support for older people, we use a life-cycle, multi-period, overlapping generations (OLG) model in the tradition of Auerbach et al. (1983) and Auerbach and Kotlikoff (1987). In contrast, the economic literature on DLEs typically employs models with infinitely-lived agents.

We find that, in an OLG life-cycle model, DLEs imply the existence of an upper bound, or threshold, on the dependency ratio. This threshold identifies a critical point in the cross-section of the age-distribution of the population beyond which tax revenue from direct taxation can no longer sustain the planned level of transfers to retirees. We refer to this as the *threshold dependency ratio*. This is determined by the structure of the economy, the design of fiscal policy and evolves over time due to demographic changes.

We show that the threshold dependency ratio is derived from a subset of the competitive equilibria achievable in an OLG life-cycle economy. This subset includes all competitive equilibria in which the government chooses tax policy to maximize tax revenue. The threshold is then endogenously derived from the government budget constraint. We are interested in characterizing the level of the threshold in a given period and its projection over the medium and long term, with a view of comparing this against existing demographic projections.

Demographic projections possess a significant degree of uncertainty. We exploit this to derive a statistical measure of the *distance* between the projected dependency ratio and the threshold. We use this distance in conjunction with the distribution of stochastic demographic forecasts to measure the probability of an economy reaching the threshold at some point in the future. The distance from the threshold indicates to what extent the government can exploit its ability to raise revenue through direct taxation in order to maintain current levels of publicly funded support for older people. The probability of reaching the threshold indicates how likely a government is to be able to sustain the pension system in the medium and long run. The probability of reaching the threshold also provides a direct comparison of the effects of policy on the sustainability of a pension system, including changes in the retirement age.

The existence of the threshold affects both the beneficiaries of and the contributors to the social security net. Once the dependency ratio reaches the threshold - the distance is then zero - the government can no longer sustain the social security net for older people through an increase in direct taxation. It then faces a choice of either partially reneging on its social security commitments, for example, by reforming the pension system and making people retire later, or of increasing indirect taxation, or possibly reducing other types of public spending.

Quantitative studies on dynamic fiscal policy based on large-scale (life-cycle) simulation models typically focus on the United States. Our analysis breaks new ground by covering, in addition to the United States, fourteen European (EU14) countries. This extension is particularly interesting as the results for these EU14 countries are much more dramatic than those for the United States; their dependency ratios have reached some of the highest values in the world by 2010, and are projected to increase very rapidly by 2100. For each country, we start by quantifying the current size of the fiscal space as measured by the potential increase in tax revenue that could be achieved if tax rates on income from capital and labor were set to maximize tax revenues. This gives an indication of a country's ability to sustain the pension system through increase in direct taxation alone. We then measure the threshold over the period 2010-2100 and use stochastic population forecasts to quantify the distance from the threshold and the probability of reaching the threshold in the medium and long run.

We consider four alternative policy scenarios. The first covers the case of no-policy change (S1-NPC). The remaining three policy scenarios reflect reforms typically advocated for improving the sustainability of existing pension systems (National Research Council, 2012): increasing the consumption tax rate by 5 percentage points (S2-ICT), reducing the replacement ratio of pensions by 10 percentage points (S3-RRR) and increasing the retirement age from 65 to 70 (S4-IRA). We examine the contribution that these policy changes may make in increasing the distance from the threshold and/or reducing the probability of reaching the threshold in the medium and long term. We also rank these reforms based on their welfare effects on the cohorts of individuals alive during 2010-2100.

Quantitative Results. We find that the size of the fiscal space in the United States ranges between 32 and 47 percent in 2010 (depending on whether the public sector is committed to maintain either the level or the replacement ratio of pensions, respectively) and is expected to grow over the period 2010-2100, though not fast. The threshold dependency ratio in the United States in 2010 is about three times larger than the actual dependency ratio (61% vs 22%). If no policy change is implemented, the probability of reaching the threshold is zero in 2050 but about 4% in 2100. Under the policy scenario S2-ICT the probability of reaching the threshold by 2100 declines to about 2%. Under the policy scenarios S3-RRR and S4-IRA the probability of reaching the threshold falls to zero by 2100.

The outlook is very different for the EU14 countries. Compared to the United States, they have, on average, narrower fiscal spaces, more generous pension systems, are older (higher dependency ratios) and are expected to age much faster. On average across the EU14 countries the threshold dependency ratio is only 0.2 times larger than the actual dependency ratio in 2010. If no policy change is implemented, dependency ratios in all EU14 countries are expected to overtake the threshold well before 2100. Under the policy reform scenarios S2-ICT, S3-RRR and S4-IRA, respectively, the number of countries that are expected to exceed the threshold dependency ratio before 2100 reduces to thirteen, eleven and nine. The outlook is worst for Austria, Belgium, France, Greece, Italy, Spain and the three Scandinavian countries. If no change in policy is undertaken, on average, these countries are expected to exceed the threshold dependency ratio by 2030. This date is postponed by 5, 15 and 40 years under the policy reform scenarios S2-ICT, S3-RRR and S4-IRA, respectively. These results highlight how imminent is the need of significant pension system reforms for the public finances of the EU14 countries.

The welfare analysis compares the effects of three alternative changes to policy that would give the same degree of protection, and hence sustainability, to existing pension provision through yielding the same distance from the threshold by 2050. The three policies are a change to the consumption tax rate, to pension contributions and to the retirement age. For the United States we find that of the three policy reforms, the greatest welfare gains are obtained through an increase of the taxation of consumption, as this leads to the largest reduction of the distortionary taxation on income from capital and labor. A similar result for the United States is found by De Nardi et al. (1999), Kotlikof et al. (2007), and Conesa and Garriga (2016). In contrast, we find that this is not necessarily the best policy option for most of the EU14 countries, as increasing the retirement age and/or reducing pension contributions achieve greater welfare gains than increasing the taxation of consumption. These contrasting welfare results reflect differences in tax burdens, demographic structures and discount factors among the EU14 countries.

A by-product of our numerical analysis is the quantification of revenue maximizing tax rates in an OLG life-cycle model. This contributes to the existing quantitative literature on DLEs which is based on infinitely-lived agent models. In particular, our OLG life-cycle model calibrated on the United States highlights four new dimensions of DLEs: (i) how the cross-section distribution of the tax burden changes once the economy moves to the peak of the Laffer hill, (ii) how the measurement of the fiscal space depends on how tax revenue is shared among retirees, (iii) the extent to which population aging impacts on the position and the shape of the Laffer curves and (iv) how uncertainty about demographic projections impacts on DLEs.

We also provide a new data set of revenue-maximizing tax rates on capital and labor for the United States and the EU14 countries based on a life-cycle model. When keeping constant the replacement ratio of pensions, these tax rates are generally in line with those obtained by Trabandt and Uhlig (2011) using infinitely-lived agent models. The OLG life-cycle model, however, gives significantly lower revenue-maximizing tax rates on capital and labor when the level of pension per-capita is kept constant.

Related Literature. As noted previously, the present paper is related to the extensive literature on the implications of aging for the sustainability of social security systems based on multi-period overlapping generation models. See, for example, Auerbach et al.(1983) and Auerbach and Kotlikoff (1987), De Nardi et al. (1999), Fuster et al. (2007), Kotlikoff et al. (2007), Heer and Irmen (2014), Conesa and Garriga (2016), Imrohoroğlu et al (2016).[1] These studies evaluate how aging is likely to increase the tax burden required to fund the social security system over a given period of time and how the resulting welfare cost could be mitigated through various reforms of the social security system, including partial financing with a consumption tax, reduction of social security transfers or increase in the eligibility age. This paper contributes to this literature by providing a measure of the limits faced by tax policy in maintaining the sustainability of pension systems through the threshold dependency ratio and by assessing the probability that an economy will reach a point at which reforms will be inevitable. It also extends the literature by focusing on a multi-country analysis rather than just on a single economy.

Our paper is also related to the growing literature on the implications for public finances and macroeconomic policy of DLEs, typified by the works Davig et al. (2010), Trabandt and Uhlig (2011), Bi (2012), Polito and Wickens (2014, 2015), D'Erasmo et al. (2016).[2] The common denominator among these studies is their use of infinitely-lived agent models. We contribute to this literature by studying DLEs in a life-cycle model and by considering their implications for the sustainability of pension systems.

Two recent works, Holter et al. (2017) and Guner et al. (2016), also consider DLEs in a large-scale model of overlapping generations. Their aim is to quantify how much extra tax revenue can be generated in the United States by increasing the progressivity of the tax system. In these two studies, DLEs impose an upper bound on a government's ability to redistribute resources in the economy. The scope of our paper is different. We are interested in how DLEs contribute to determining the threshold dependency ratio which imposes an upper bound on a government's ability to sustain the pension system in the medium and long terms.

A number of issues concerning particular features of existing pension systems are beyond the scope of this paper. These include normative questions such as why we have the pension systems that we do and whether there is a socially

[1] A related branch of this literature focuses on the macroeconomic effects of reforms of the United States' tax system, see for example Altig et al. (2001), Conesa et al. (2009), Guner et al. (2012), Guner et al (2016).

[2] In these works, DLEs are quantified by calculating competitive equilibria over given grids for the tax rates. Babel and Huggett (2017) illustrate how to predict the top of the Laffer curve directly using the sufficient statistic approach.

optimal level of redistribution from workers to older people.[3] Our analysis is positive, being confined to the financial sustainability of a pension system in the presence of fiscal limits, the policy changes that can be implemented to maintain the social security net for older people and the welfare costs that societies may incur in implementing these changes.

Paper Structure. The paper is organized as follows. Section 2 provides a summary of global demographic trends and motivates our focus on advanced economies. Section 3 describes a stylized life-cycle model suitable for the analysis of dynamic fiscal policy and derives the threshold dependency ratio, the distance from the threshold and the probability of reaching the threshold. We also employ a restricted version of the model to derive a closed-form solution for the threshold and examine its determinants more closely. Section 4 describes the assumptions made for the quantitative model, its calibration and the scope of the numerical analysis. Sections 5 and 6 present the results for the United States and the EU14 countries, respectively. Section 7 concludes. Appendix A describes the numerical algorithm, while Appendix B reports the parameter values calibrated in each country.

2 Demographic Trends

Figure 1 shows the historic and projected evolution of dependency ratios over the period 1950-2100 in four regions: the world, the United States, Europe and fourteen European (EU14) countries.[4] We consider two measures of the dependency ratio endorsed by the United Nations (2015). The first is the Old-Age Dependency Ratio 2 (OADR2), which measures the number of people in the population aged 65 and above as a percentage of those aged between 20 and 64. The second is Old-Age Dependency Ratio 3 (OADR3) which measures the number of people aged 70 and above as a percentage of those aged between 20 and 70.[5] For the period 2015-2100, data for the world and Europe refer to the United Nations (2015) projections under the assumption of a medium fertility scenario, while data for the United States and the EU14 countries are based on the mean forecasts from the Bayesian hierarchical model underpinning the United Nations (2015)'s projections, see Alkema et al. (2011), Raftery et al. (2012), Raftery et al. (2013), Gerland et al. (2014) and United Nations

[3] Diamond (2004) and Diamond and Orszag (2005) present various economic arguments underpinning the existence of social security contributions. Shiller (2005) and Beetsma et al. (2011) survey advantages and disadvantages of individual savings accounts for social insurance. Volume 19, issue 2, of the Journal of Economic Perspectives collects a series of different views on social security and reforms of social security systems.

[4] The United States and Europe cover about 4.4 and 10 percent of world population in 2015. The EU14 countries cover about half of the European population.

[5] Strictly speaking, the relevant indicator for our study is the retirees-to-workers ratio, defined as the number of retirees as a proportion of the labor force. We use the OADR2 because this is the closest proxy available the retiree-to-worker ratio, see National Research Council (2012) and forecasts of retirees-to-workers ratios are not available for the countries covered in our quantitative analysis. The OADR3 is the relevant dependency ratio for our analysis of reforms of the pension system based on increase of the retirement age to 70.

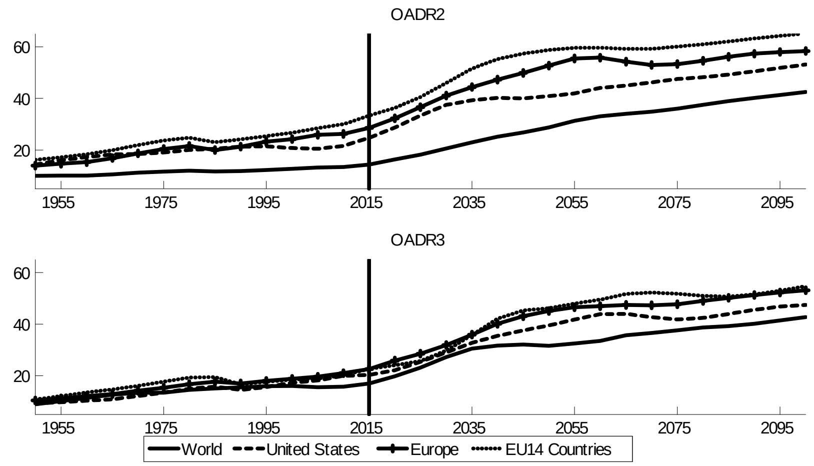

Figure 1: OADR2 and OADR3 in percentage, selected regions, 1950-2100. Source: United Nations (2015).

(2015).[6] The Figure clearly shows that aging is a worldwide phenomenon. It is however more relevant for advanced than developing economies as OADR2 and OADR3 for the United States and Europe are well above the world trend.

Table 1 reports average dependency ratios for the four regions and for each EU14 country in 1950, 2010, 2050 and 2100. The EU14 countries are ranked in descending order according to their OADR2 in 2050 (in bold). The upper part of the table shows that the dependency ratios for the four regions are generally expected to double over the period 2010-2100. There are, however, significant differences in the levels and the (expected) rates of change of dependency ratios across the EU14 countries. Italy, Germany, Portugal, Greece and Sweden have the highest dependency ratios in 2010. Those of Italy, Greece, Germany and Portugal are projected to remain above the average of the EU14 for 2100. Spain, Austria, Ireland and Portugal are projected to have very large increases in their dependency ratios over the 2010-2100 period. Greece and Italy are forecasted to have the highest dependency ratios by 2100. Compared to the United States, the EU14 countries are older and are expected to age more rapidly over the period 2010-2100.

Figure 2 shows the OARD2 in each EU14 country and in the United States over the period 2010-2100. In the figure, data from 2015 onward refer to the mean and the two-standard-error bands from the empirical distribution of the

[6] We thank Hana Sevcikova for providing the data on dependency ratio forecasts in the United States and the EU14 countries.

	OADR2				OADR3			
	1950	2010	2050	2100	1950	2010	2050	2100
World	10	13	29	42	7	10	21	33
US	14	22	41	53	9	16	32	43
Europe	14	26	53	58	9	20	40	47
EU14	16	29	58	65	11	21	45	53
ESP	13	27	**76**	71	9	20	59	59
ITA	14	34	**74**	72	10	25	60	60
PRT	13	31	**72**	75	9	23	56	61
GRE	13	31	**72**	73	9	24	53	60
GER	16	34	**64**	71	10	25	49	58
AUS	17	29	**60**	67	11	20	46	55
NET	14	25	**53**	63	9	18	43	51
BEL	18	29	**52**	60	12	22	40	49
FRA	20	29	**51**	62	13	23	41	51
IRL	21	18	**50**	59	14	13	38	49
FIN	12	29	**50**	63	8	21	39	51
GBR	18	27	**46**	59	12	20	36	48
DNK	16	28	**45**	58	10	19	37	46
SWE	17	31	**45**	56	11	22	36	46

Source: United Nations (2015).

Table 1: OADR2 and OADR3 in percentage, selected regions and dates over 1950-2100.

forecasts for the OADR2.[7] The figure shows that there is significant uncertainty over future demographic trends. The extent of this uncertainty tends to increase with the level of the OADR2. This is shown by the amplitude of the bands which are larger over time and for countries with a higher forecasted value of the dependency ratio. We will exploit the uncertainty surrounding dependency ratio forecasts in constructing our statistical measure of the distance from the threshold and in quantifying the probability of reaching the threshold.

3 The Model

The economy is described by a stylized life-cycle model comprising a large number of overlapping generations of households with a finite life, a representative firm and government. Each household includes one individual who makes consumption, saving and labor supply decisions to maximize lifetime utility. The

[7] See National Research Council (2012) for a review of different methods available for predicting demographic trends. For consistency with the measure of uncertainty in the formulae of the distance from the threshold and the probability of reaching the threshold described in Section 3.5 we base the confidence bands on multiples of the standard errors.

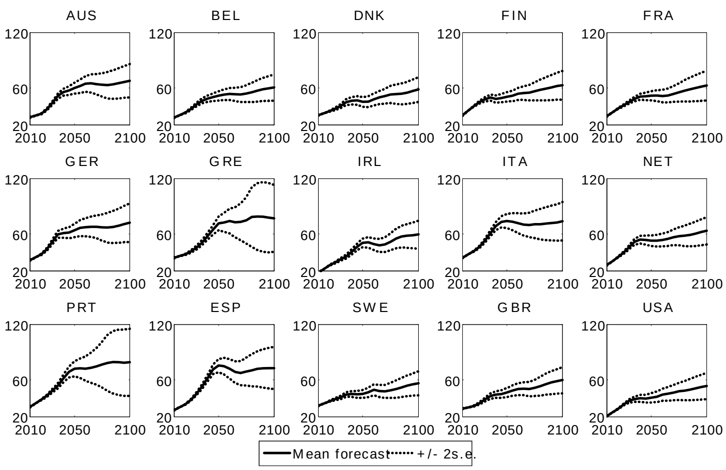

Figure 2: OADR2 in percentage, EU14 countries and USA, 2010-2100. Source: Alkema et al. (2011), Raftery et al. (2012), Raftery et al. (2013), Gerland et al. (2014) and United Nations (2015).

firm uses aggregate capital and labor to maximize profits, while operating a neoclassical production technology. Consumption, income from labor and income from capital are subject to proportional taxes. The government uses tax revenue and issues debt to finance the provision of public consumption goods and the social security system, which includes transfers to all individuals and pension payments.

3.1 Demographics

In each period $t \geq 0$ a new cohort of individuals is born and denoted by its date of birth. Individuals in each cohort live for $J+1$ periods, with $J \geq 1$. In $t = 0$, J cohorts of individuals are already alive, each indexed by their date of birth $(-1, -2, ..., -J)$. We denote by j_t^0 the age of an individual in $t = 0$, so that for any cohort born in $t \geq -J$, $j_t^0 = \max\{-t, 0\}$. The probability of surviving until age j in period $t+j$, conditional on being alive at age $j-1$ in period $t+j-1$ is $\phi_{t,j}$.[8] Without loss of generality, at this stage we assume $\phi_{t,j} = 1$ for $j \in \left(j_t^0, J-1\right)$ and $\phi_{t,J+1} = 0$. The population grows at the rate $n > -1$. The share of individuals of age j in the population, μ_j, is given by $\mu_j = \mu_0 / \left(1+n\right)^j$ for $j \in (1, J)$, with $\sum_{j=0}^{J} \mu_j = 1$.

Individuals work in the first $j_R - 1$ periods of their life and retire from age

[8] Throughout the paper, unless otherwise indicated, the first subscript denotes the date in which an individual is born, whereas the second denotes the age of the individual. Thus the sum of the two subscripts is the current period. Variables with only one time subscript are not age dependent and the subscript denotes the period in which are observed.

j_R onwards, with $j_R \in (2, J)$. The dependency ratio d is thus defined as:

$$d = d\left(\left(\mu_j\right)_{j=0}^{J}, n, j_R\right) = \frac{\mu^R}{\mu^W}, \tag{1}$$

where $\mu^R = \sum_{j=j_R}^{J} \mu_j$ and $\mu^W = \sum_{j=0}^{j_R-1} \mu_j$ denote the shares in the population of retirees and workers, respectively. The dependency ratio is determined by four factors: the maximum life duration J, the distribution of age-j individuals in the population, the growth rate of the population and the retirement age. The first three are affected by population aging, through reductions in birth rates and increases in life expectancy. Given life expectancy, a decline in the birth rate results in a reduction of n that leads to an increase in the number of retirees relative to workers in the population. Given the birth rate, an increase in life expectancy, for example through a reduction in the mortality rate, leads to increase in the dependency ratio, as would a change in μ_j and/or J for any given n. Without loss of generality, we abstract from exogenous changes in the cross-section distribution of the population due, for example, to migration.[9] We treat J, μ_j's and n as exogenous although, in practice, they could be related to the economic environment and policy, and hence be endogenous. Making these three variables endogenous would not affect our qualitative results. The retirement age j_R indicates the age from which individuals start receiving old-age social security contributions. This could be either an endogenous variable chosen by the individual conditional on the minimum retirement age set by the government or a policy parameter, depending on how social security eligibility is regulated in the economy.[10] The stylized life-cycle model of the economy described here is compatible with both these interpretations, since the existence of the threshold and its related statistics (distance and probability) do not depend on the mechanism underlying the choice of j_R. We appraise the effect of variation of j_R in the quantitative analysis.

3.2 Environment

Households. Individuals within each cohort are the same. They are endowed with an initial allocation of assets in the first period of their life, $a_{t,0}$, and do not leave bequests. They are also endowed with one unit of time at each age of their life. This is shared between labor and leisure during the working age. No labor is supplied during retirement. Each unit of time devoted to labor provides $z_j \geq 0$ units of productivity.

Individual preferences depend on consumption and leisure. For any $t \geq -J$,

[9] In the quantitative analysis we account for the impact on any factor influencing the demographic structure of the population, including migration. This is because our measure of the distance from the threshold and the probability of reaching the threshold depend on forecasts of dependency ratios in the medium and long term that account for these factors.

[10] All the studies based on life-cycle models cited in the Introduction assume that the retirement age is exogenous. Fehr et al. (2013) and Kitao (2014), among others, study a large-scale lfe-cycle model with endogenous retirement.

these are ordered by the utility function:

$$U^t = \sum_{j=j_t^0}^{J} \beta^{j-j_t^0} u\left(c_{t,j}, 1 - l_{t,j}\right), \tag{2}$$

where $\beta = (1+\rho)^{-1}$ is the common discount factor, with ρ denoting the discount rate; $c_{t,j}$ and $l_{t,j}$ are the consumption and the labor supply of an individual of age j born in period t, respectively. Utility u is strictly increasing in consumption and leisure, twice continuously differentiable, strictly concave and satisfies the Inada conditions. Individuals have perfect foresight. The budget constraints faced by individuals for $j \in \left(j_t^0, J\right)$ are:

$$q_{t,j} c_{t,j} + a_{t,j+1} = x_{t,j} + tr_{t,j} + \left(1 + r_{t,j}\right) a_{t,j}, \tag{3}$$

in which

$$x_{t,j} = \begin{cases} w_{t,j} z_j l_{t,j} & \text{for } j \in \left(j_t^0, j_R - 1\right) \\ p_{t,j} & \text{for } j \in \left(j_R, J\right) \end{cases}, \tag{4}$$

$$l_{t,j} = 0 \text{ for } j \in \left(j_R, J\right), \tag{5}$$

$$a_{t,J+1} = 0. \tag{6}$$

Further, $q_{t,j} = 1 + \tau_{t,j}^c$, $w_{t,j} = \left(1 - \tau_{t,j}^l\right) \widehat{w}_{t+j}$ and $r_{t,j} = \left(1 - \tau_{t,j}^k\right) \widehat{r}_{t+j}$ are the after-tax prices of consumption, income from labor and income from capital, respectively; $\tau_{t,j}^c$, $\tau_{t,j}^l$ and $\tau_{t,j}^k$ are the corresponding age-dependent tax rates; $\widehat{w}_{t+j}$ and $\widehat{r}_{t+j}$ denote the pre-tax prices of labor and capital; $tr_{t,j}$ are age-related transfers; $p_{t,j}$ is the pension received by retired individuals. Without loss of generality, we do not include explicitly a payroll tax.

For an individual born in t of age j, the solution to the lifetime maximization problem is the sequence of allocations $\left(c_{t,j}, l_{t,j}, a_{t,j+1}\right)_{j=j_t^0}^{J}$ that for any $t \geq -J$ satisfies the necessary and sufficient conditions:

$$u_{c_{t,j}} = q_{t,j} \lambda_{t,j}, \text{ for } j \in \left(j_t^0, J\right), \tag{7}$$

$$u_{1-l_{t,j}} = \lambda_{t,j} w_{t,j}, \text{ for } j \in \left(j_t^0, j_R - 1\right), \tag{8}$$

$$\lambda_{t,j} = \beta \lambda_{t,j+1} \left(1 + r_{t,j+1}\right), \text{ for } j \in \left(j_t^0, J - 1\right), \tag{9}$$

and the constraints in (3) - (6), where $\lambda_{t,j}$ is the Lagrange multiplier associated with an individual's budget constraint.

Firms. In each period $t \geq 0$ there is a single produced good that can be used as private consumption, public consumption or capital. Goods are produced by a neoclassical production function with constant returns to scale, $y_t = f\left(k_t, l_t\right) - \delta k_t$, where y_t and k_t denote per-capita net output and capital, respectively; δ is the rate of physical depreciation and f is monotonically increasing, strictly concave and satisfies the Inada conditions. Factors of production are paid their marginal products. The before-tax prices of capital and labor are:

$$\widehat{r}_t = f_{k_t} - \delta, \tag{10}$$

$$\widehat{w}_t = f_{l_t}, \tag{11}$$

respectively.

Government. The government finances an exogenous sequence of public consumption, transfers and pension payments, $(g_t, tr_t, p_t)_{t=0}^{\infty}$, through revenue from taxation, $(tax_t)_{t=0}^{\infty}$, and by issuing public debt, $(b_t)_{t=1}^{\infty}$ (all variables are in per-capita terms). The sequence of government budget constraints for $t \geq 0$ is given by:

$$g_t + tr_t + p_t + (1 + \widehat{r}_t)\, b_t = tax_t + (1 + n)\, b_{t+1}, \tag{12}$$

where tax revenue in any $t \geq 0$ is given by

$$\begin{aligned} tax_t \quad = \quad & \textstyle\sum_{j=0}^{J} (q_{t-j,j} - 1)\, \mu_j c_{t-j,j} + \sum_{j=0}^{j_R-1} (\widehat{w}_t - w_{t-j,j})\, \mu_j z_j l_{t-j,j} \\ & + \textstyle\sum_{j=0}^{J} (\widehat{r}_t - r_{t-j,j})\, \mu_j a_{t-j,j}. \end{aligned} \tag{13}$$

Without loss of generality, we abstract from a separate social security budget at this stage. Note how the dependency ratio is implicitly accounted for in the constraints faced by fiscal policy through equations (12) and (13), since these depend on the same J, $\left(\mu_j\right)_{j=0}^{J}$, n and j_R that determine d in equation (1). This observation motivates the derivation of the threshold dependency ratio in the next section. Fiscal policy is subject to the solvency condition:

$$\lim_{T \to \infty} \frac{b_T}{\prod_{s=0}^{t} \frac{(1+\widehat{r}_s)}{(1+n)}} = 0. \tag{14}$$

Market-clearing and Feasibility. The equilibrium conditions for per-capita labor, asset holdings and consumption are:

$$l_t \quad = \quad \textstyle\sum_{j=0}^{j_R-1} \mu_j z_j l_{t-j,j}, \tag{15}$$

$$a_t \quad = \quad \textstyle\sum_{j=0}^{J} \mu_j a_{t-j,j} = k_t + b_t, \tag{16}$$

$$c_t \quad = \quad \textstyle\sum_{j=0}^{J} \mu_j c_{t-j,j}, \tag{17}$$

respectively. The per-capita resource constraint requires

$$y_t + (1 - \delta)\, k_t = c_t + g_t + (1 + n)\, k_{t+1}. \tag{18}$$

Transfers and pension payments per-capita are $tr_t = \sum_{j=0}^{J} \mu_j tr_{t-j,j}$ and $p_t = \sum_{j=j_R}^{J} \mu_j p_{t-j,j}$, respectively.

3.3 Competitive Equilibrium and Threshold Dependency Ratio

First we define the set of competitive equilibria. We then show that the dependency ratio can be derived endogenously as the unique number supporting a specific competitive equilibrium. The threshold dependency ratio is then simply a special case.

Definition 1 ***Competitive Equilibrium.*** *Given an initial aggregate endowment of assets* $a_0 = k_0 + b_0$*, a government spending policy* $(g_t, tr_t, p_t)_{t=0}^{\infty}$*, a tax policy* $((q_{t,j}, w_{t,j}, r_{t,j})_{j=j_t^0}^{J})_{t=-J}^{\infty}$*, a borrowing policy* $(b_{t+J+1})_{t=-J}^{\infty}$ *and a dependency ratio* $d = d((\mu_j)_{j=0}^{J}, n, j_R)$*, a competitve equilibrium is a sequence of relative prices* $(\widehat{r}_t, \widehat{w}_t)_{t=0}^{\infty}$ *and individual allocations* $((c_{t,j}, l_{t,j}, a_{t,j+1})_{j=j_t^0}^{J})_{t=-J}^{\infty}$ *such that:*

1. *(a) The sequence of individual allocations satisfies (3) - (9), for* $t \geq -J$*;*
   (b) The sequence of relative prices satisfies (10) and (11), for $t \geq 0$*;*
   (c) The dependency ratio and the sequence of government spending, tax and borrowing policies satisfy (12)- (13), for $t \geq 0$*, and (14);*
   (d) All markets clear, i.e. (15) - (17) hold, for $t \geq 0$*;*
   (e) Feasibility (18) holds, for $t \geq 0$*.*

A competitive equilibrium is computed in two stages. The first consists in determining the sequence of individual allocations (a) and relative prices (b) that describe the private sector's optimal choices. The sequence of individual allocations in (a) is determined taking as given government policy and two of the variables contributing to the determination of the dependency ratio in (1), namely, the maximum life duration J and the age of retirement j_R.[11] In the second stage, government policy (spending, tax and borrowing) in (c) and aggregate variables in (d and e) are determined subject to the constraints set by the private sector choices, the dependency ratio and the government budget constraint. It is at this second stage that all parameters of the dependency ratio enter the computation of the competitve equilibrium through the government budget constraint in (12) - (13) and the market clearing conditions. Crucially, one degree of freedom is missing at this stage, as the dependency ratio and the government policy need to satisfy the sequence of government budget constraints in (12) and (13). As a result, there are many competitive equilibria, each indexed with a different dependency ratio and government policy. This multiplicity implies that for any given fiscal policy, the dependency ratio can be derived as a residual from the solution of the government budget constraint. This, however, would not uniquely identify d, which is a highly nonlinear combination of the demographic parameters J, $(\mu_j)_{j=0}^{J}$, n and j_R. The residual solution of the dependency ratio from the government budget constraint relies on the fact that the government can always choose at least one of the variables in equation (1). As discussed in section 3.1, governments in advanced economies typically set the minimum age of retirement.

To highlight the relation between changes in tax revenue and the dependency ratio, consider a government implementing a new tax policy that delivers a higher level of tax revenue. For this new policy to be supported as a competitive equilibrium the government budget constraint has to be satisfied. To this end, the additional tax revenue could be used to pay for a higher level of

[11] Both J and j_R could either be taken as given or included among the set of choice variables.

transfers to the existing cohort of retirees. It could also be used to maintain the current level of pensions per-capita while sustaining a higher number of beneficiaries of the pension system. In this second case, increases in tax revenue can be associated with higher dependency ratios, while still be compatible with a competitive equilibrium. The threshold dependency ratio is a special case, being the dependency ratio $\overline{d}$ obtained when tax policy is set to maximize tax revenue given government spending and borrowing policy. In other words, it measures the maximum number of retirees per worker that the government could sustain through tax policy alone.

A maximum dependency ratio sustainable through changes in tax policy emerges naturally in a life-cycle model as long as there is an upper bound on tax revenue. This is provided by the DLE. The upper bound can be exploited in conjunction with the government budget constraint to give the threshold dependency ratio, $\overline{d}$.

Definition 1 implies that there is a competitive equilibrium where $d = \overline{d}$. Still $\overline{d}$ is not uniquely determined being a nonlinear combination of the four parameters in equation (1). The computation of the threshold dependency ratio used in this paper takes J, $(\mu_j)_{j=0}^{J}$ and j_R as given in the second stage of the competitive equilibrium calculation, while determining endogenously the growth rate of the population n. The threshold dependency ratio is therefore defined as follows.

Definition 2 *Threshold Dependency Ratio. Given an initial aggregate endowment of assets $a_0 = k_0 + b_0$, a government spending $(g_t, tr_t, p_t)_{t=0}^{\infty}$, consumption tax $((q_{t,j})_{j=j_t^0}^{J})_{t=-J}^{\infty}$ and borrowing $(b_{t+J+1})_{t=-J}^{\infty}$ policy and a set of J, $(\mu_j)_{j=0}^{J}$ and j_R, a threshold dependency ratio is a dependency ratio calculated from (1) for a competitve equilibrium such that:*

1. *(a) The sequence of individual allocations satisfies (3) - (9), for $t \geq -J$;*
   (b) The sequence of relative prices satisfies (10) and (11), for $t \geq 0$;
   (c) The sequence of labor and capital tax policy $((w_{t,j}, r_{t,j})_{j=j_t^0}^{J})_{t=-J}^{\infty}$ maximizes (13);
   (d) The growth rate of the population satisfy (12), for $t \geq 0$, and (14);
   (e) All markets clear, i.e. (15) - (17) hold, for $t \geq 0$;
   (f) Feasibility (18) holds, for $t \geq 0$.

As noted above the threshold dependency ratio can be calculated endogenously as a residual from the government budget constraint. This requires measuring one of the parameters in (1) from the government budget constraint, while fixing all others. We choose to determine n endogenously, as this is numerically simpler to compute.

3.4 An Illustrative Analytic Example

We illustrate the determinants of the threshold dependency ratio, and how this depends on direct and indirect taxation, using a simplified version of the general model in which (i) individuals live for two periods, thus $J = 1$, and $j = 0, 1$; they work in the first period and retire in the second period, hence $j_R = 1$; (ii) labor productivity is normalized to one, $z_0 = 1$; (iii) there is no aggregate saving, $a_{t,j} = 0$ for $j = 0, 1$ and $t \geq 0$; (iv) there is no government consumption, $g_t = 0$; (v) technology and utility are $y_t = \omega l_t$ and $U(c_{t,0}, c_{t+1,1}, l_t) = \ln c_{t,0} + \chi \ln(1 - l_t) + \beta \ln c_{t+1,1}$, respectively, with $\omega \geq 1$ and $\chi \geq 0$.[12]

With these assumptions, the government budget constraint can be written

$$N_{t-1}p_t + N_t tr_t = \tau_t^l w_t l_t N_t + \tau_t^c c_{t,1} N_t + \tau_t^c c_{t,0} N_{t-1}.$$

The dependency ratio can be solved directly from the government budget constraint as

$$\begin{aligned} d &= \frac{N_{t-1}}{N_t} = (1+n)^{-1} \\ &= \frac{\tau_t^l w_t l_t + \tau_t^c c_{t,1} - tr_t}{p_t - \tau_t^c c_{t,0}}. \end{aligned}$$

The dependency ratio, therefore, equals the ratio of net payments to the government by the young working generation to the net income received from the government by the retired and can be derived from the government constraint. A higher dependency ratio requires either an increase in the net payments of those who are working or a fall in the net income received from the state by the retired. The formula shows that this may be achieved by an increase in the rate of taxation of labor or of consumption. There may, however, be a limit to the tax revenues raised through labor taxes due to DLEs. This implies there would be an upper bound to the dependency ratio which we refer to as the threshold dependency ratio and is the subject of this analysis.

To show this consider the optimality conditions for the consumption of workers and retirees and of labor are $c_{t,0} = [(1 - \tau_t^l)\omega + tr_t]/[(1 + \tau_t^c)(1 + \chi)]$, $c_{t+1,1} = p/(1 + \tau_{t+1}^c)$ and $l_t = (1 + \chi)^{-1} - \chi tr_t/[(1 + \chi)(1 - \tau_t^l)\omega]$, respectively. These capture the trade-offs in equilibrium that give rise to DLEs. Consumption during the working age is a normal good. It reduces if tax rates on either consumption or labor increase. It is also positively related to government transfers. The labor supply is negatively related to the tax rate on labor income and transfers. It does not depend on the consumption tax rate. The demand for consumption goods during retirement is entirely exogenous, being (negatively) related to pensions and the rate of the consumption tax. Using these conditions,

[12] Assumption (i) is the principal difference with the general model where the greater number of cohorts prevents an analytic solution and necessitates a numerical solution. Assumption (ii) removes a redundant variable without loss of generality. Assumption (iii) allows to dispense with the taxation of capital for which the trade-offs due to DLEs are hard to sign when labour supply is also endogenous. It also implies that the government runs a balanced budget in every period. Assumptions (iv) and (v) lead to an analytical solution for the supply of labour that depends on the labour income tax rate.

the equilibrium tax revenue is written as:

$$tax_t = \tau_t^l \left[\frac{\omega}{(1+\chi)} - \frac{\chi tr_t}{(1+\chi)\left(1-\tau_t^l\right)} \right] + \frac{\tau_t^c \left[\left(1-\tau_t^l\right)\omega + tr_t + (1+\chi)\, p_t d\right]}{(1+\tau_t^c)(1+\chi)}. \tag{19}$$

The first term on the right side is the revenue from the labor income tax. This reflects the typical dynamic Laffer trade-off. An increase in the rate of the labor income tax decreases the equilibrium supply of labor (income effect) and thus the labor tax base. The second term on the right is the revenue generated by the consumption tax. If consumption is a normal good, as a higher labor income tax rate reduces disposable income, it also reduces the revenue from a consumption tax.

Differentiating total tax revenue with respect to the labor income tax τ_t^l, gives the peak of the Laffer curve as

$$\overline{\tau}_t^l = 1 - \left[\frac{\chi tr_t \left(1+\tau_t^c\right)}{\omega} \right]^{\frac{1}{2}}.$$

Hence, the tax rate on income from labor that maximizes tax revenues depends negatively on the preference parameter, the level of transfers and the consumption tax rate, but positively on productivity. The negative dependence of $\overline{\tau}_t^l$ on the consumption tax rate is due to the fall in consumption brought about by the increase in the labor tax rate. This further compounds the reduction in the total tax base due to the income effect. Thus, the higher is the consumption tax rate, the lower is the labor tax rate at the peak of the Laffer curve.

Differentiation of total tax revenue in equation (19) with respect to the consumption tax rate yields

$$\frac{\partial tax_t}{\partial \tau_t^c} = \frac{1}{\left(1+\tau_t^c\right)^2} \left[\frac{\left(1-\tau_t^l\right)\omega + tr_t + (1+\chi)\, p_t d}{(1+\chi)} \right] > 0.$$

This result can be viewed as the analog, in a life-cycle model, of proposition 3.1 in Trabandt and Uhlig (2011)'s which states that, as long as τ_t^c is finite and does not affect the supply of labor, the government can generate an ever increasing revenue from the taxation of consumption.[13]

After replacing the equilibrium conditions and the solution for $\overline{\tau}_t^l$ in the government budget constraint, the threshold dependency ratio sustainable in equilibrium for any given τ_t^c is given by:

$$\overline{d} = \frac{\left(1+\tau_t^c\right)\omega - 2\omega \left[\frac{\chi tr_t (1+\tau_t^c)}{\omega}\right]^{\frac{1}{2}} - tr_t}{(1+\chi)\, p_t}.$$

The threshold is therefore entirely dependent on the parameters of the economy and the design of fiscal policy. In particular, the threshold is unambiguously

[13] As common in the literature on DLEs, in our quantitative analysis we also take the tax rate on consumption as given. We however evaluate the impact of changes in τ^c on the threshold and its welfare effects.

lower the higher is the level of government expenditure, whether through transfers or pension payments. The derivative of the threshold with respect to the consumption tax rate is

$$\frac{1}{(1+\chi)\,p_t}\left\{\omega - \chi tr_t\left[\frac{\omega}{\chi tr_t\,(1+\tau_t^c)}\right]^{\frac{1}{2}}\right\}$$

which is always positive for any finite value of the consumption tax rate, as long as $\tau_t^c \geq \frac{\chi tr_t}{\omega} - 1$. Hence, while there is an upper bound to the tax revenues generated by labor taxation due to the DLE, tax revenues from consumption are only constrained by technology and preferences.

The level of the threshold dependency ratio is not, however, fixed. Although not explicitly modelled in this simplified version of the general model, which has a fixed time period for working, a later retirement age would, in effect, raise labor input and the level of consumption of the working age group, and hence their net tax payments, and would reduce the net income of the retired through lower total pension payments. Consequently, a later retirement age would raise the threshold dependency ratio.

3.5 Distance and Probability

We are interested in measuring the distance between any forecast of the dependency ratio at some point in the future and the threshold dependency ratio at that point of time and the probability of reaching the threshold, or equivalently exhausting the distance, at some point in the future.

Consider writing the dependency ratio in $t+1$ as

$$d_{t+1} = E_t d_{t+1} + \xi_{t+1}$$

where $E_t d_{t+1}$ is the expected value of the dependency ratio by the end of period $t+1$ conditional on information available in t, and $\xi_{t+1} = \sigma\epsilon_{t+1}$ is the corresponding innovation in period $t+1$, with ϵ_{t+1} being an independent and identically distributed disturbance, $\epsilon_{t+1} \sim i.i.d.\,(0,1)$. The h-period ahead dependency ratio is therefore

$$d_{t+h} = E_t d_{t+h} + \eta_{t+h},$$

where $\eta_{t+h} = \sum_{j=1}^{h} \xi_{t+j}$ is the h-period ahead innovation and $Var\left(\eta_{t+h}\right) = \sigma_{\eta,t+h} = \sum_{j=1}^{h} Var\left(\xi_{t+j}\right) = h\sigma^2$. The forecast error associated with the h-period ahead dependency ratio can be written as

$$\eta_{t+h} = \sigma \textstyle\sum_{j=1}^{h} \epsilon_{t+j} = \sigma u_{t+h}.$$

The probability that the h-period ahead dependency ratio exceeds the threshold dependency ratio $\overline{d}$, $\Pr\left(\overline{d}, t+h\right)$, is therefore written as

$$\begin{aligned}\Pr\left(\overline{d}, t+h\right) &= \Pr\left[\left(\overline{d} - d_{t+h}\right) \leq 0\right] \\ &= \Pr\left[\frac{\overline{d} - E_t d_{t+h}}{\sigma} \leq u_{t+h}\right]. \qquad (20)\end{aligned}$$

In the special case of $\epsilon_t \sim N(0,1)$, then u_{t+h} is also normally distributed. More generally $\Pr\left(\overline{d}, t+h\right)$ can be computed for any stochastic distribution of the expected dependency ratio. We define the *distance from the threshold,* $D\left(\overline{d}, t+h\right)$ as the number of standard deviations that the h-period ahead dependency ratio is from the dependency ratio threshold $\overline{d}$.[14] This is given by:

$$D\left(\overline{d}, t+h\right) = \frac{\overline{d} - E_t d_{t+h}}{\sigma_{\eta,t+h}}. \tag{21}$$

The probability of exceeding the threshold dependency ratio $\Pr\left(\overline{d}, t+h\right)$ is therefore a function of the distance from the threshold $D\left(\overline{d}, t+h\right)$. It decreases as the gap between the threshold and the forecasted dependency ratios widens, and the uncertainty surrounding the dependency ratio forecast decreases. This probability changes over time due to changes in the base year and to new information which affect the forecast of the dependency ratio, its uncertainty and the threshold.

4 Quantitative Analysis

4.1 Assumptions

Demographics. Each period, t, corresponds to five years. Newborns have a real-life age of 20-24 ($j=0$), retire at age 65 ($j_R = 9$) and live up to age 94 ($J = 14$). The survival probability in each age j is non-zero, other than in the last period.

Households. Preferences are described by the expected lifetime utility

$$U^t = \sum_{j=j_t^0}^{J} \beta^{j-j_t^0} \left(\prod_{s=j_t^0}^{j} \phi_{t,s} \right) u(c_{t,j}, l_{t,j}), \tag{22}$$

where $\phi_{t,s}$ denotes the probability of surviving until age s in period $t+s$, conditional on being alive at age $s-1$ in period $t+s-1$. Following Trabandt and Uhlig (2011), the instantaneous utility is specified as:

$$u(c_{t,j}, l_{t,j}) = \frac{1}{1-\eta} \left(c_{t,j}^{1-\eta} \left[1 - \kappa(1-\eta) l_{t,j}^{1+1/\varphi} \right]^{\eta} - 1 \right), \tag{23}$$

where φ is the Frisch elasticity of labor supply, and η is the inverse of the intertemporal elasticity of substitution. We make five modifications to household budget constraint in equation (3). First, labor productivity is assumed to be also time dependent. Thus we set $z_{t,j} = A_t y_j$, where A_t is the time-varying component of labor productivity growing at the constant rate $g_A \geq 0$ and y_j, with $j \in \left(j_t^0, j_R - 1\right)$, is the age-dependent component of labor productivity. Under this specification, pre-tax labor income is given by $\widehat{w}_t A_t y_j l_{t,j}$. Second,

[14] This is the analog of the distance-to-default in corporate finance, which is defined as the number of standard deviations that a firm is away from default.

taxes are age-independent. Third, households contribute to the pension system through a proportional social security tax levied on wage income at the rate τ_t^p. Thus the after-tax labor income is $(1-\tau_t^p-\tau_t^w)\, A_t y_j \widehat{w}_t l_{t,j}$, with $\tau_t^l=\tau_t^p+\tau_t^w$, for $t\geq 0$. A_t denotes labor productivity. Fourth, transfers are age-independent, $tr_{t,j}=tr_t$ for any $t\geq 0$ and $j\in\left(j_t^0,J\right)$. Fifth, pension payments are also age-independent, being set as a constant proportion (replacement ratio) θ of the average labor income in the economy $\widehat{w}_t l_t/\mu^W$, thus $p_t=\theta\left(\widehat{w}_t l_t/\mu^W\right)$ for any $t\geq 0$ and $j\in(j_R,J)$. In addition, newborns are assumed to start their life with no assets as well as leaving no bequests at the end of their life, thus $a_{t,0}=a_{t,J+1}=0$.

In equilibrium the household is indifferent between holding assets in the form of physical capital or government debt, since both yield the same (certain) after-tax return. With a single household living for two periods the proportion of asset holdings would be the same at the household and the aggregate level, but with many periods, the portfolio allocation is indeterminate. Consequently, without loss of generality, we assume that each household holds the two assets in the same fixed proportions.

Firm. Production is described by a Cobb-Douglas function with labor-augmenting technological progress, $y_t=k_t^{\alpha}\left(A_t l_t\right)^{1-\alpha}-\delta k_t$. Under this specification, the balanced-growth rate of the economy is equal to the growth rate of labor productivity, $g_A\geq 0$.

Government. Government expenditures (consumption and transfers) grow at the exogenous balanced-growth rate. Government revenue is augmented to include all accidental bequests from households that do not survive. This is equivalent to assume that the government collects all accidental bequests and redistributes them as lump-sums to households as, for example, in Krueger and Ludwig (2007), Braun and Joines (2015) and Holter et al. (2017). There is a separate balanced-budget for pensions, so that aggregate expenditure on pensions is equal to the aggregate revenue raised through the social security tax: $\tau^p\widehat{w}_t l_t=p_t\mu^R$.

Aggregate equilibrium. All variables, other than labor, are made stationary by expressing them as a proportion of technological progress. The stationary equilibrium is described in more detail in Appendix A.1.

Computation. The main focus of our quantitative analysis is on the sustainability of public pension systems in aging economies. Quantitative studies employing large-scale models with overlapping generations are often concerned with the distributional effects of various forms of macroeconomic policy interventions. These studies therefore account for different forms of heterogeneity among agents (for example, with regard to income shocks, financial wealth distribution, education attainments, health, disability status, sex, marital status and household composition), other than age and productivity.

In our judgment, we can, without any loss of generality, dispense with most of these features when defining the threshold dependency ratio in Section 3. In principle, all these forms of heterogeneity could be included in our quantitative analysis, depending on the availability and comparability of these data for each country. Doing so would, however, add significantly to the computation time required in a multi-country analysis. We estimate that with existing computer technologies and a version of the model including all the forms of heterogeneity described above, the solution of a single equilibrium takes not less than one hour, and iteration of the algorithm over the tax rates grid would take about four weeks. Added to this, the time required to iterate over the 2010-2100 period and for each country. Under our specification, the solution of a single equilibrium takes about one second and the search over the three-dimensional grid for τ^n, τ^k and d takes about two hours for each country, depending on the grid size and the number of years considered. Our specification is an attempt to balance the accuracy of the results with the feasibility of their computation.[15]

4.2 Benchmark Calibration

As in Holter et al. (2017), the parametrization of the model starts with a benchmark calibration that follows as closely as possible Trabandt and Uhlig (2011). In this way, we can better appraise how DLEs in infinitely-lived-agent models change due to the life-cycle structure of the economy and aging.[16]

The benchmark calibration of the country-specific parameters is reported in Table 11 in Appendix B. The second column reports the annual growth rates of the population rate in each country, using the estimates of the United Nations (2015). To be consistent with Trabandt and Uhlig (2011), who use 1995-2007 as the calibration period, we use the average population growth rate prevailing during the years 1990-2010. The 5-year survival probabilities for the 15 different age groups are taken from the United Nations (2015). These data show that survival probabilities have increased over time, and display larger rates of growth for the older age groups. For our benchmark simulation, we use the average survival probabilities during the period 1990-2010.[17] The third column reports the equilibrium dependency ratios implied as residuals by these demographic variables in each country. The fourth column reports our estimate of the discount factor in each country. Trabandt and Uhlig (2011) calibrate discount factors endogenously so that the real interest rate equals 4% on an annual basis. As we are using an OLG model, we recalibrate β in each country to match a rate of interest of 21.9% over a period of 5 years, equivalent to 4% on an annual basis. The two preference parameters, η and φ, are taken from Trabandt and Uhlig (2011). Thus, the intertemporal elasticity of substitution η^{-1} and the Frisch elasticity of labor supply φ are set to 1/2 and 1, respectively, for all countries. Consequently, the values of the parameter κ in the fifth column are calculated endogenously to match the equilibrium average working hours, equal

[15] Appendix A describes the algorithm used for the numerical solution of the model. The GAUSS code implementing the algorithm is available upon request from the authors.

[16] Although this calibration is based on assumptions that are fairly common in quantitative macro models on the effects of taxation, we note that other calibrations have been used in applied work on DLEs. For example, D'Erasmo et al. (2016) use a two-country model with a limited depreciation tax allowance and endogenous capacity utilization in order to better match empirical estimates of the capital income tax base short-run elasticity

[17] For reasons of space we do not report the survival probabilities. These are available upon request from the authors.

to 0.3 across countries. The production parameters in columns 6 and 7 are not affected by the OLG structure of the household sector, and are as in Trabandt and Uhlig (2011). Similarly, the annual economic growth rate is set to 2% in all countries. The remaining columns report country-specific fiscal variables and aggregates. Government consumption-to-GDP and debt-to-GDP (columns 8 and 10 respectively) and the tax rates on labor, capital and consumption (columns 12-14) are taken directly from Trabandt and Uhlig (2011). Thus government transfers-to-GDP ratios in column 9 are determined endogenously to satisfy the government budget constraint in each country. Pension replacement rates in column 11 are computed using data on the gross replacement ratios for pensions from the OECD (2015).[18] The the social security tax rates τ^p in the last column are determined endogenously to close the social security budget constraint in each country. Trabandt and Uhlig (2011) employ effective tax rates on labor income which already include social security contributions. We therefore restrict the tax rate on labor income in column 12 to have the same value of the labor tax rate τ^l used by Trabandt and Uhlig (2011), thus $\tau^l = \tau^w + \tau^p$. We use the hump-shaped age-productivity profiles estimated by Hansen (1993) to measure the labor productivity z_j for the United States. Labor productivity is set equal to 1 for all $j \in (0, J)$ in all other countries.[19]

We employ the benchmark calibration to quantify DLEs and threshold dependency ratios in each country for 2010. The dynamic analysis of the evolution of threshold dependency ratio over the period 2010-2100 is carried out by retaining the benchmak calibration for preferences, production and (most of) the fiscal policy variables, while updating the demographic variables over time. In particular, for the projection of survival probabilities that serve as input into the calculation of the threshold over the period 2010-2100 we continue to use moving averages of 4 periods. For example, the threshold for 2015 is based on average survival probabilities during 1995-2015, the threshold for 2020 is based on average survival probabilities during 2000-2020, and so on. In each period over the projection horizon, tax rates on labor and capital income are set at their Laffer peaks, while the growth rate of the population is computed endogenously as the implied residual from the government budget constraint. The resulting equilibrium dependency ratio is thus the threshold, as in Definition 2. Over the transition period 2010-2100 the social security contribution rate τ^p is adjusted to balance the social security budget.

[18] These are based on the percentage of pre-retirement income for men.

[19] No data is readily available on age-productivity profiles for the majority of European countries. Where there is no information, an alternative would be to use the age-productivity profile estimated from another European country. This would, however, be equally aribtrary. We re-calibrated the labor productivity age-profile in each European country using the labor-productivity estimates calculated for Germany by Heer and Maussner (2009). The impact on the peaks of the Laffer curves and the thresholds is negligible because the revenue loss from those with productivity below 1, namely those aged between 20 and 39 (first four cohorts of workers) is in part offset by the revenue gain from those with productivity above 1, individuals aged between 40 and 64 (last four cohorts of workers). These results for the European countries are not included for reason of space, but are illustrated as an example for the United States in footnote 22.

	2010		2050	
	Benchmark	Laffer	Benchmark	Laffer
	Total Tax Burden			
Workers	86.6	91.5	82.4	89.2
Retirees	13.4	8.5	17.6	10.8
	Tax Burden on Workers			
Labor Tax	64.0	79.5	61.7	79.8
Capital Tax	24.0	16.4	25.9	16.4
Consumption Tax	12.0	4.1	13.4	3.8
	Tax Burden on Retirees			
Labor Tax	0	0	0	0
Capital Tax	73.0	78.2	71.1	78.9
Consumption Tax	27.0	21.8	28.9	21.1

Notes: All numbers are in percentage.

Table 2: Tax Burden distribution across workers and retirees, USA, 2010 and 2050.

5 United States

5.1 Fiscal Space

Table 2 shows the distribution of the tax burden between workers and retirees when tax rates on labor and capital income are set as in the benchmark calibration or at the values that maximize tax revenue, that correspond with the peak of the Laffer hill on tax revenue raised from capital and labor income.[20] The first two columns report the results when the model is calibrated using the demographic of 2010. In the last two columns the model is re-calibrated for the demographic structure in 2050.

Under the benchmark calibration, the largest contribution to tax revenue is made by workers through the taxation of their income from labor (column 1). The government could increase tax revenue by further shifting the tax burden on to workers. This could be accomplished by increasing the taxation of labor income relative to that of capital income and consumption (column 2). As a result of population aging the proportion of workers in the economy falls relative to that of retirees as does the tax burden both from the benchmark and the Laffer calibrations.

Figure 3 shows the cross-section distribution of the tax burden among the cohorts of individuals in the population when the tax rates on income from

[20] For tax revenue maximization, we choose the combination of these two tax rates that maximizes tax revenue as a proportion of GDP, with transfers adjusting to balance the government budget. Alternatively, the budget can be balanced by increasing either government consumption or social security spending. The results are not significantly different from those presented in the main text.

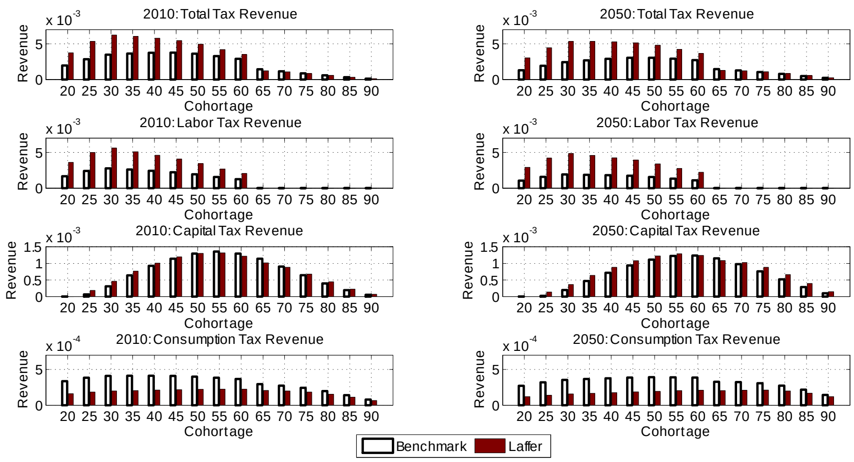

Figure 3: Tax burden distribution across age cohorts under (i) benchmark calibration and at (ii) the peak of the Laffer hill, USA, 2010 and 2050.

labor and capital are set as in the benchmark calibration or at the values that maximize tax revenue in 2010 (left panels) and in 2050 (right panels).

The left panels show that the burden of the labor income tax is slightly higher for the cohorts of those aged between 30 and 50, since labor productivity is hump-shaped in the data for the United States. The burden of the capital income tax falls mainly on the middle-age cohorts (those aged between 40 and 70), as these are the individuals with the largest amount of savings.[21] The burden of the consumption tax is more equally distributed among the cohorts than the capital income tax burden as consumption is more uniformly distributed in the cross-section of individuals than is saving. When taxation is at the peak of the Laffer hill, the skewness in the cross-section distribution increases, due to the higher tax burden on workers aged between 30 and 50. The majority of the increase is generated by the higher revenue from the labor income tax. The taxation of capital is largely unaffected, while the taxation of consumption falls for all generations. However, it falls more for workers due to the negative income effect caused by the higher labor income tax, which reduces consumption. These patterns for 2010 are not significantly altered by the demographic structure of 2050. Both under the benchmark calibration and at the peak of the Laffer hill, total tax revenue declines over time.

Table 3 reports the tax rates on income from labor and capital at the peak of the Laffer hill in 2010, and the implied fiscal space (FS), being measured as the percentage increase of tax revenue when tax rates are at the peak of the Laffer hill relative to the benchmark. We consider two cases: a constant

[21] The cross-section distribution of saving is hump-shaped in the life-cycle model.

replacement ratio (θ) and a constant pension level (p). For convenience we also report the tax rates under the benchmark calibration.[22] The main result is that DLEs in a life-cycle model depend on how tax revenue is shared among retirees. This follows from the peaks of the Laffer curves. They are higher and the size of the fiscal space is larger for the case of constant replacement ratio because pensions fall in absolute value due to the higher tax rates and the government's commitment to maintain a constant replacement ratio. According to life-cycle theory, this will cause workers to increase savings in order to smooth their consumption when retired. This leads to additional capital accumulation that partly offsets the negative effect of a higher tax burden. In contrast, when the pension level is fixed workers no longer need to increase saving. There is then no additional capital accumulation to partly offset the negative effect from a higher tax burden. Additional capital accumulation is a feature specific to a life-cycle model; it is absent in an infinitely-lived agent model, where agents can change their labor supply in every period of their life.

It is useful to relate the results in Table 3 to other studies. First, the tax rate on income from labor could be increased by as much as 115-130%. This increase is larger than the increases in the labor tax rate measured by De Nardi et al. (1999) or by Kotlikoff et al. (2007). This is not surprising as these studies consider the additional tax burden required to sustain a given demographic structure, whereas the peak of the Laffer hill corresponds to the maximum tax burden sustainable for that demographic structure. Second, the rates of the labor income tax at the peak of the Laffer hill reach values similar (60%) to those calculated by Trabandt and Uhlig (2011) using a neoclassical growth model with infinitely-lived agents and by Holter et al. (2017) using an multi-period overlapping generations model. Third, when the government maintains a constant pension level the capital income tax rate is on the "slippery" side of the Laffer hill beyond the peak, as also found by Trabandt and Uhlig (2011). Unless it is explicitly stated, all subsequent results are based on the assumption of a constant replacement ratio.

Table 4 shows how population aging impacts on DLEs. It reports the tax rates on labor and capital income and the fiscal space (FS) at the peak of the Laffer hill in 2050, using the mean forecast, the upper and lower two-standard-error bands for the 2050 dependency ratio.[23] For reference we also include the pension contribution rate that, due to population aging, has to increase to balance the social security budget. We highlight two main results. First, the size of the fiscal space declines the more pessimistic is the demographic projection. Second, for any given demographic forecast, the size of the fiscal space increases over time, from about 47% in 2010 (see Table 3) to 55-66% in

[22] The results in Table 3 are based on the hump-shaped age-productivity profile estimated by Hansen (1993). The age-productivity profile, however, has little impact on the size of the fiscal space. For the case of constant θ, when assuming equal productivity for all agents the revenue-maximizing tax rates become 63.6% and 46.0% for τ^l and τ^k, respectively; τ^p is still equal to 10.8%; additional tax revenues are equal to 46.4%.

[23] The equilibrium growth rate of the population is calculated as an implied residual from either the mean forecast, or the upper and lower two-standard-error bands of the 2050 dependency ratio.

	τ^l	τ^k	τ^p	FS
	Benchmark			
	28	36	11	-
	Laffer			
Constant θ	63.6	47.3	11	46.9
Constant p	60.0	34.5	15.7	31.8

Notes: All numbers are in percentage.

Table 3: Tax rates on income from labor, income from capital and Fiscal Space (FS) at the peak of the Laffer hill, USA, 2010.

	τ^l	τ^k	τ^p	FS
d_{2050}		*Constant* θ		
+2 s.e.	66.1	50.3	16.1	54.8
Mean	64.8	49.7	14.4	60.3
-2 s.e.	64.2	48.5	12.6	66.0
		Constant p		
+2 s.e.	61.2	33.9	21.1	37.6
Mean	60.6	33.9	19.1	43.6
-2 s.e.	60.6	33.3	17.2	49.5

Notes: All numbers in percentage.

Table 4: Tax rates on income from labor, income from capital and Fiscal Space (FS) at the peak of the Laffer hill, USA, 2050.

2050. This is because due to population aging tax revenue under the benchmark calibration declines more rapidly than the maximum tax revenue. Our analysis of the threshold dependency ratio in the next section illustrates to what extent a larger fiscal space is likely to sustain the increasing cost of the pension system in the United States.

Figure 4 displays the Laffer curves for labor and capital income taxes in 2010 (top panel) and 2050 (bottom panels).[24] Laffer curves for the capital income tax are flatter than those for the labor income tax, as also found by Trabandt and Uhlig (2011) using a neoclassical growth model with infinitely-lived agents. This suggests that the slope of the Laffer curve is not qualitatively altered when accounting for the life-cycle structure of individuals in the economy. Comparison of the three panels shows that demographic uncertainty affects the position of the Laffer curve but does not significantly alter their shapes.

In summary, these results highlight four dimensions of DLEs in the a life-cycle model: (i) as an economy moves towards the peak of the Laffer hill the tax burden shifts further towards workers, with the largest increase in tax revenue generated through the labor income tax; (ii) revenue-maximizing tax rates are higher when the government keeps the replacement ratio constant rather than

[24]The lines for the labor (capital) income tax are obtained by varying the labor (capital) income tax rate while keeping τ^k (τ^l) constant at the value that maximizes total tax revenue.

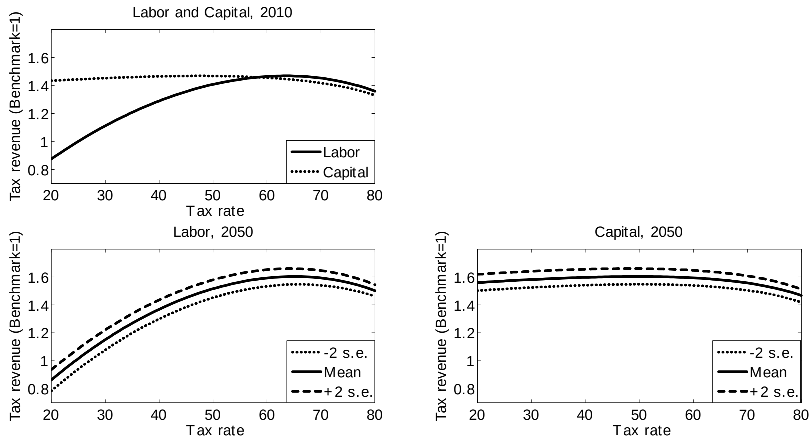

Figure 4: Laffer curves on income from labor and capital, USA, 2010 and 2050.

the level of pensions, as this induces further private sector saving and capital accumulation which partly compensates for the negative effects of an increase in taxation; (iii) population aging affects more the position of the Laffer curves than their shape; and (iv) uncertainty about demographic projections has a significant impact on the predicted position of Laffer curves.

5.2 Threshold Dependency Ratio

We compute a time series of the threshold dependency ratio that is directly comparable with the OADR projections of the United Nations (2015) over the period 2010-2100, as described in section 4.2. Due to population aging, the cost of current pension systems is expected to rise over time. We therefore consider the consequences of four different policy scenarios that are commonly suggested to make pension systems fiscally sustainable, for example De Nardi et al. (1999), Kotlikoff et al. (2007), Nishiyama and Smetters (2007), National Research Council (2012) and Conesa and Garriga (2016). Under the first scenario the government maintains the replacement ratio of pension as in 2010 and finances increases in the cost of pensions over time by raising the social security tax rate τ^p. This quantifies the threshold dependency threshold ratio under a scenario of no policy change, and is referred to as S1-NPC. The second scenario, S2-ICT, considers the effect of increasing the consumption tax rate by 5 percentage points, from 5% to 10% in the case of the United States. The third scenario, S3-RRR, considers the effect of reducing the replacement ratio of pensions by 10 percentage points, from 35.2% to 25.2% in the case of the United States. The fourth scenario, S4-IRA, considers the effect of increasing the retirement age to 70.

Figure 5 shows the evolution of the threshold dependency ratio under the

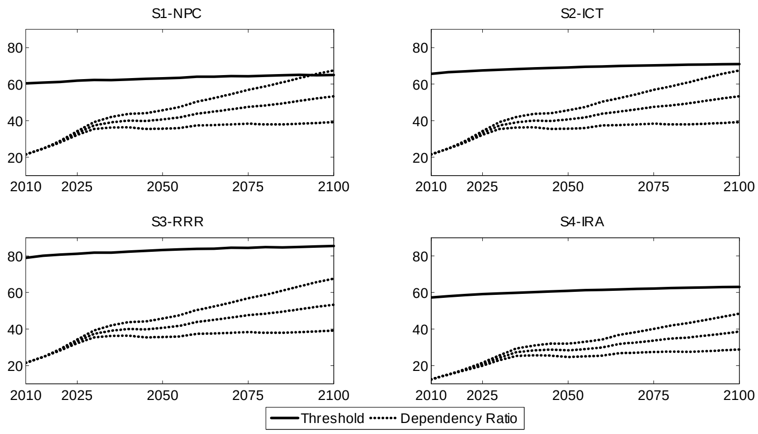

Figure 5: Threshold dependency ratio, USA, 2010-2100.

four policy scenarios over the period 2010-2100. The dotted lines in the panels denote the dependency ratio forecasts (OADR2 for S1-NPC, S2-ICT and S3-RRR; OADR3 for S4-IRA). In 2010, the threshold dependency ratio under S1-NPC is about three times the actual dependency ratio. Doubling the consumption tax rate (S2-ICT) would change this only marginally. The threshold would be about 4 times higher than the actual dependency ratio following either a reduction in the replacement ratio by 10 percentage points (S3-RRR) or an increase in the retirement age to 70 (S4-IRA).

Over the period 2010-2100, threshold dependency ratios increase, but very little. In contrast, the OADR2 and the OADR3 are forecasted to increase very rapidly over the same period of time. Consequently, the gap between the forecasted dependency ratio and the threshold narrows under each policy scenario. The upper two-standard-error band of the forecast of the dependency ratio rises above the threshold under S1-NPC from 2085, but does not reach the threshold under the other three policy scenarios.

Table 5 reports the threshold dependency ratios and the OARD2 and OADR3 forecasts for 2010, 2015, 2050 and 2100. On average, across the four policy scenarios, thresholds increase by about one percentage point between 2010 and 2015, by about three further percentage points until 2050, and by about two more percentage points by 2100. These increases are consistent with the increase in the size of the fiscal space over this period (compare Tables 3 and 4). Under S4-IRA both the threshold dependency ratio and the projected OADR3 are much lower. Therefore, the effects of the four policy scenarios are not comparable when considering the level of the thresholds in isolations, which motivates the use of our measures of distance and probability.

	Actual/Forecasted Dependency Ratio (d)		Threshold Dependency Ratio ($\bar{d}$)			
			Policy Scenarios:			
	OADR2	OADR3	S1-NPC	S2-ICT	S3-RRR	S4-IRA
2010	21.6	14.1	60.4	65.7	79.0	57.3
2015	24.7	15.1	60.9	66.6	80.2	58.0
2050	40.7	28.3	63.3	69.2	83.3	60.9
2100	53.4	38.6	65.0	71.0	85.5	63.0

Notes: All numbers are in percentage.

Table 5: Actual and threshold dependency ratios, USA, selected dates over 2010-2100.

Figure 6 shows the evolution of the distance from the threshold and the probability of reaching the threshold under the four policy scenarios between 2050-2100. The distance declines under each policy scenarios due to the dependency ratios increasing more rapidly than the thresholds over the period 2050-2100, thereby reducing the numerator in equation (21). The standard deviation of the forecast of the dependency ratio also increases, thereby increasing the denominator. The distance increases when moving from S1-NPC to S2-ICT to S3-RRR. This is consistent with the increases in the threshold levels under the three policies reported in Table 5. The distance increases even more under S4-IRA. This implies that increasing the retirement age in the United States to 70 would improve the sustainability of the pension system more than would doubling the taxation of consumption or reducing the replacement ratio by 10 percentage points.

Under S1-NPC, the probability of reaching the threshold is strictly positive from 2085 onwards and reaches about 4.5% by 2100. This suggests that without any change in policy, there is a probability of about 5% that the cost of the pension system will become unsustainable in the United States over the medium and long run. Under S2-ICT, the probability of reaching the threshold is positive from 2090 onward, and reaches around 2.5% by 2100. For the other two policy scenarios, the threshold is reached with probability zero.

5.3 Welfare Analysis

The choice of policy to increase the distance of the economy from the threshold could have significant welfare implications. We therefore compare the implications of changing in 2010 the consumption tax rate, the replacement ratio of pension and the retirement age in order to deliver the same distance from the threshold dependency ratio by 2050. As shown in Figure 6, if the retirement age were to increase to 70 the distance from the threshold dependency ratio would be of about 17.66 standard deviations by 2050. The same distance would also result from an increase in the consumption tax rate to 25.4% or a reduction in

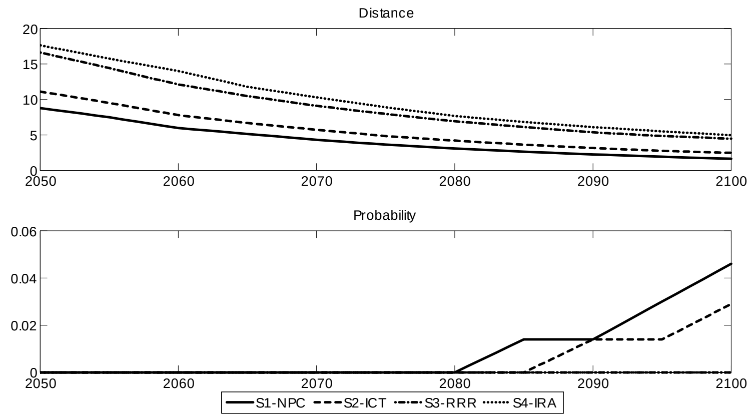

Figure 6: Distance from the threshold dependency ratio and probability of reaching the threshold, USA, 2050-2100.

the replacement ratio to 24.1% We refer to these two adjusted policy scenarios as S2A-ICT and S3A-RRR, respectively.

5.3.1 Newborn Generation 2050

We begin by calculating the average lifetime utility of the newborn in 2050 under S2A-ICT, S3A-RRR and S4-IRA, using the model equilibrium solution for 2050. In each policy scenario, the labor income tax rate and the pension contribution rate are adjusted to balance the general and the social security government budgets, respectively. We then compute the life-cycle profile of consumption and leisure over the 15 and 9 (10 under S4-IRA) lifetime periods. Instantaneous utility in each lifetime period and lifetime utility are then calculated using equations (23) and (22), respectively.

Table 6 presents the values of aggregate consumption, aggregate labor, the tax rate on income from labor and welfare (U) under the three policies for the 2050 newborn generation. The last row reports the percentage consumption equivalent change (Δ) required for welfare under S4-IRA to be the same as under S2A-ICT and S3A-RRR. The results show that the policy reform based on increasing the consumption tax gives higher welfare than the policy based on reducing the replacement ratio or increasing the retirement age. This policy ranking arises from the tax rate on labor, which is lower under S2A-ICT compared to S3A-RRR and S4-IRA. This finding is similar to those of De Nardi et al. (1999), Kotlikof et al. (2007) and Conesa and Garriga (2016). Thus the policy that achieves the lowest distortion on the production factors brings also higher welfare gains. This welfare analysis neglects the (potentially important) welfare effects while the economy is transiting between equilibria from 2010 to 2050. We address this issue in the next sub-section.

	S2A-ICT	S3A-RRR	S4-IRA
c	0.095	0.096	0.097
l	0.230	0.233	0.238
τ^l	0.168	0.287	0.298
U	-102.78	-103.55	-107.07
Δ	+4.17	+3.40	

Notes: U is lifetime utility. Δ is consumption equivalent change w.r.t. S4-IRA.

Table 6: Welfare of generations alive by 2050 under S2A-ICT, S3A-RRR and S4-IRA, USA

5.3.2 Transition experiment

The transition experiment follows a fairly standard protocol in the computational OLG literature, see for example Conesa and Garriga (2016) and Imrohoroğlu et al. (2016). This is based on the following assumptions: between 2010 and 2050 the demographic variables are the same as those forecasted by the United Nations (2015); starting from 2050, the demographic variables remain constant; the government announces the policy change (either an increase in the consumption tax rate as in S2A-ICT, or a reduction of the replacement ratio as in S3A-RRR, or an increase in the retirement age as is S4-IRA) in the second transition period, corresponding to the year 2015, and the policy becomes effective in that year; the policy change is unanticipated and agents adjust their behavior accordingly; government consumption and transfers are fixed throughout the whole transition period, while the labor income tax and the pension contribution rates adjust to balance the general and the social security government budgets, respectively. For comparison, we also consider the transition of the economy when the government finances the higher pension burden by an increase in the taxation of labor without implementing any policy change, as in S1-NPC.[25]

Figure 7 plots the evolution of aggregate capital, aggregate labor, aggregate consumption, and the tax rate on income from labor τ^l during the transition. The results are broadly in line with those of similar transition experiments in the literature. The economy transits to a new equilibrium with the lowest labor supply under S1-NPC, as this yields the highest labor income tax and, consequently, the lowest after-tax income and consumption. S2A-ICT generates higher labor supply due to lowest labor income tax required to finance public spending. The higher level of taxation on consumption under S2A-ICT induces

[25] The transition experiment could be based on a different timing protocol. For example, policy changes could be pre-announced and/or implemented gradually, as in De Nardi et al. (1999) and Kotlikof et al (2007). The choice of policy protocol does not alter the long-run results, but it does impact on the transition dynamic adjustments and the measurement of welfare in the short run.

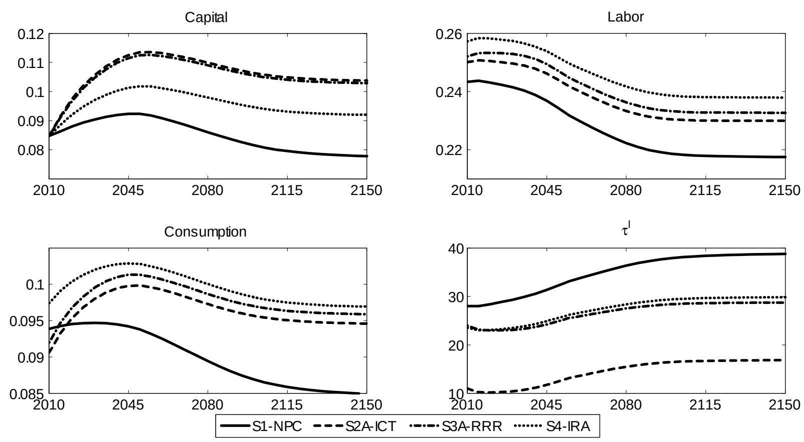

Figure 7: Transition dynamics, USA, 2010-2150.

an increase in savings and a higher level of capital accumulation than the other three policy scenarios. Under S3A-RRR, because pensions are lower, individuals work more as they need extra savings in old age. This increase in the labor supply is also supported by the partial reduction τ^l, due to the reduction in τ^p. As a result, capital is almost as high as in S2A-ICT. Labor supply and consumption are similar to those achieved under S2A-ICT.[26] Under S4-IRA the labor supply is high because individuals work for five extra years. Lifetime income and hence consumption are the highest. Individuals do not save as much under this policy scenario as under S2A-ICT and S3A-RRR. The main finding from these transition dynamics is that all three policy-change scenarios result in a significant fall in the rate of tax on labor income throughout the whole transition period.

Figure 8 shows the cross-section distribution for the consumption-equivalent changes under S2A-ICT, S3A-RRR and S4-IRA relative to S1-NPC for the cohorts alive in 2010 and 2100. Under our policy protocol, none of the cohorts alive in 2010 anticipate the policy change. In contrast, the 2010 policy changes are fully incorporated in the dynamic responses of all those alive in 2100. In addition, the capital stock has not fully adjusted to the new steady state by 2100. Thus, the results in the figure illustrate how policy changes impact on different cohorts depending on whether or not they are fully anticipated and by how much the transition is completed.

The top panel shows that in 2010 the cohorts of younger individuals (those aged less than 40) gain from the unanticipated reforms under policies S2A-ICT,

[26] The relative differences between the transtion dynamics under S2A-ICT and S3A-RRR depend on the relative magnitude of income and substitution effects. The orders may change for different calibrations of the Frisch labor supply elasticity and the intertemporal elasticity of substitution.

S3A-RRR and S4-IRS. As they are already working when the policy change takes place, these cohorts can adjust their labor supply and savings over a relatively long life-time horizon. The middle-aged cohorts (those aged between 40 and 65) face a shorter adjustment horizon. This is less of a problem under scenario S4-IRA, since households can adjust their labor supply at age 65-69. In contrast, middle-aged households under policies S2A-ICT and S3A-RRR are affected much more severely: had they known that they would be able to afford lower consumption in old age (due to higher consumption taxes under policy S2A-ICT or lower pensions under policy S3A-RRR) these cohorts would have adjusted their labor supply when younger. For retirees in 2010, the policy change has less effect due to their lower life expectancy. Figure 8 shows that the welfare losses for older cohorts under all policy-change scenarios are smaller and they decrease with age.[27]

The bottom panel shows that in the long run all age groups make considerable welfare gains from the policy changes made in 2010, although these gains decline gradually with age. This reflects the ability to optimally adjust behavior by 2100 when fully informed beforehand about policy changes. The welfare gains are inversely related to the burden of distortionary taxation on labor income. Raising the tax on consumption as in S2A-ICT generates an increase in welfare of about 33-36% higher than reducing the retirement age as in S4-IRA and, depending on age, about 5-8% higher than lowering the replacement ratio as in S3A-RRR. As in Conesa and Garriga (2016), those who face a lower tax burden gain most from the policy reforms.

In summary, these results show that reforms designed to improve the sustainability of the pension system in the United States can bring welfare gains for all age groups in the long run, but only the young benefit in the short run. The larger benefits in the long run reflect the advantages of having full information about policy changes and therefore being able to respond to them. The greatest welfare gains (for all cohorts in the long run and for the young cohorts in the short run) are obtained through raising consumption taxes.

6 EU14 Countries

In this section we extend our quantitative analysis of the threshold dependency ratio to the EU14 countries. First, we provide a cross-country comparison of the size of their fiscal space and highlight how this is related to differences in the labor tax rate, the replacement ratio, the dependency ratio and aging. We then present our measurement of the threshold (level, distance and probability) in each country under the same four policy scenarios considered for the United States. We conclude our assessment with the welfare analysis. Given the large amount of data involved, we do not report the results for the transition experiments carried out on each individual country, but focus on the 2050 newborn generation. These allow a clear comparison of the effects of policy reforms across

[27] If we had focused instead on remaining life-time utility and disregarded all the periods prior to 2010, welfare losses would have increased with age.

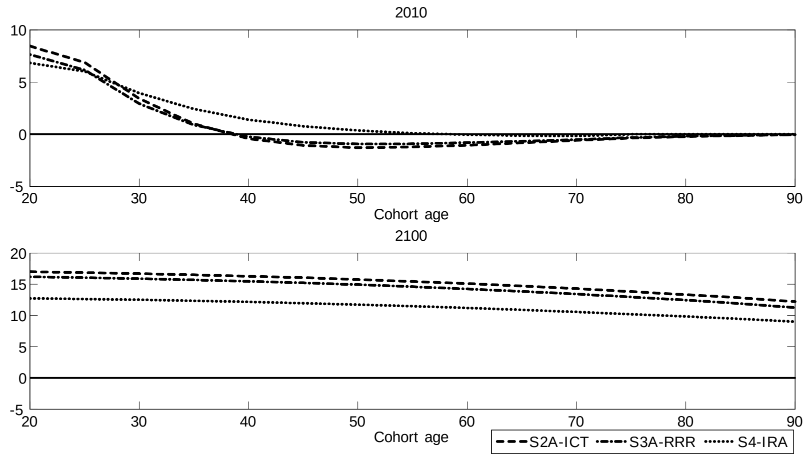

Figure 8: Consumption-equivalent changes across cohorts required for S2A-ICT and S3A-RRR to yield same lifetime utility as under S4-IRA, USA, 2010 and 2100.

countries in the long run, while it is difficult to make any judgment looking at the transition profiles.

6.1 Fiscal Space

Table 7 reports the tax rates on labor income and capital at the peak of the Laffer hill and the size of the fiscal space (FS) for each EU14 country assuming either a constant replacement ratio (θ) or a constant pension level (p) in 2010.[28] For reference we also report tax rates under the benchmark calibration. With a constant pension level, the payroll tax rate τ^p adjusts to balance the social security budget. Numbers in bold indicate revenue-maximizing rates that are lower than the corresponding rates under the benchmark calibration. These highlight instances where current tax rates are higher than those at the peak of the Laffer hill, i.e. on the slippery side of the hill.

The main result emerging from the table is that the revenue-maximizing tax rates and the size of the fiscal space are significantly higher in the case of constant replacement ratio. As for the United States, this is because pensions fall in absolute value as tax rates increase and the government maintains a constant replacement ratio. This induces workers to increase savings in order to smooth their consumption when retired, thereby leading to additional capital accumulation that is absent when the government maintains instead a fixed pension level.

[28] As for the United States, we report our calculations for 2010 so that the results can be compared with those based on a neoclassical model with infinitely-lived agents reported by Trabandt and Uligh (2011).

Country	Benchmark			Laffer						
				Constant θ			Constant p			
	τ^l	τ^k	τ^p	τ^l	τ^k	FS	τ^l	τ^k	τ^p	FS
AUS	50	24	29	62	52	12.4	52	29	30	0.2
BEL	49	42	16	59	47	3.1	53	**34**	16	0.3
DNK	47	51	23	57	**46**	1.5	49	**27**	20	2.3
FIN	49	31	20	60	45	4.6	53	**26**	21	0.3
FRA	46	35	20	59	52	8.4	50	36	21	0.5
GER	41	23	16	60	49	16.6	54	35	19	6.1
GRE	41	16	27	62	52	37.3	52	30	34	7.8
IRL	27	21	9	56	40	23.8	53	29	12	16.1
ITA	47	34	28	64	51	14.3	53	**30**	36	0.9
NET	44	29	32	69	53	31.5	59	**26**	38	6.0
PRT	31	23	28	67	50	68.9	61	25	40	28.9
ESP	36	30	27	64	53	44.0	55	**29**	35	1.0
SWE	56	41	20	60	50	1.0	**50**	**37**	18	1.2
GBR	28	46	7	57	**43**	15.7	54	**36**	9	11.2
EU14	42	32	22	61	49	20	53	30	25	6.0
USA	28	36	11	64	47	47	60	35	16	52

Notes: All numbers are in percentage. EU14 is the arithmetic average.

Table 7: Tax rates on income from labor, income from capital and Fiscal Space (FS) at the peak of the Laffer hill, EU14 countries, 2010.

The results with a constant replacement ratio are generally in line with those of Trabandt and Uhlig (2011). The labor income tax rates for all countries are lower than at the peak of the Laffer hill. For the capital income tax rate we find that only Denmark and Great Britain are on the slippery side of the Laffer hill. The average size of the fiscal space for the EU14 countries in 2010 is 22%, less than half of that of the United States. There are, however, significant cross-country differences. Portugal has the largest fiscal space of about 70%; Belgium and the three Scandinavian countries have the lowest.

The results for a constant pension level show that for the labor income tax, all countries but Sweden are below the peak and hence on the "right" side of the Laffer hill. For the capital income tax rate, we find that 8 of the 14 countries have a tax rate higher than at the peak of the Laffer hill. The average size of the fiscal space with constant pensions is about a quarter of that with constant replacement ratio. Although there are still cross-country differences in the size of the fiscal space, these are less pronounced.

Figure 9 relates the fiscal space of the EU14 countries for a constant replacement ratio to the tax rate on labor income, the replacement ratio of pensions, the dependency ratio (OADR2 in 2010) and aging (change in the OADR2 between 2010 and 2050). The United States is included for comparison. In each panel, the vertical line indicates the average fiscal space and the horizontal line is the average value of the variable on the vertical axis.

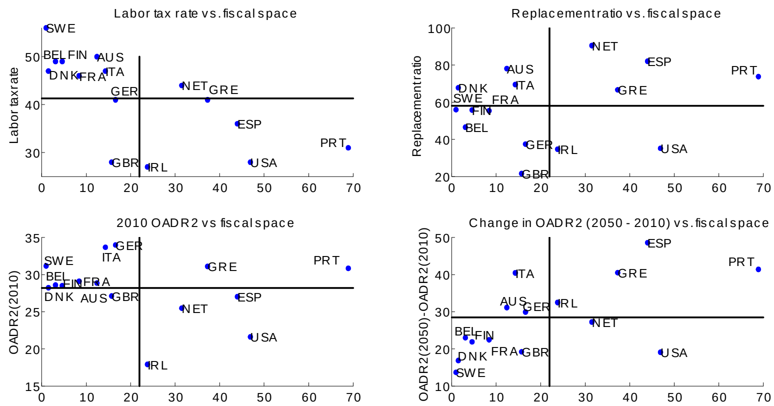

Figure 9: Fiscal space in 2010 (constant replacement ratio) relative to (i) labor income tax rate, (ii) pension replacement ratio, (iii) 2010 OADR2 and (iv) change in the OADR2 over 2010-2050, EU14 countries.

The negative relation between the fiscal space and the labor tax rate in the top-left panel helps to explain the cross-country differences in the sizes of the fiscal spaces reported in Table 7.[29] The top-right panel shows that the four EU14 countries with the largest fiscal spaces (Portugal, Spain, Greece and the Netherlands) also have among the highest replacement ratios. Six countries (Austria, Denmark, Italy, Sweden, Finland and France) have high replacement ratios but relatively small fiscal spaces. The bottom-left panel shows a negative relation between the fiscal space and the age structure of the population in 2010. Eight countries are concentrated in the top-left corner of the panel. They have relatively high dependency ratios and narrow fiscal spaces. The four countries with the largest fiscal space also have relatively high dependency ratios. The bottom-right panel shows a positive relation between the size of the fiscal space and the increase forecasted in the OADR2 between 2010 and 2050. The four countries with the largest fiscal space are located towards the top-right corner. This suggests that countries which in 2010 have a relatively large fiscal space are likely to exhaust it relatively quickly. From this perspective, the fiscal outlooks for Italy, Austria and Germany are the most precarious as they have a relatively narrow fiscal space and are projected to age very rapidly over the period 2010-2050.

In summary, when compared to the United States, the EU14 countries have, on average, narrower fiscal spaces, higher replacement ratios, higher dependency ratios and are expected to age much faster.

[29] Differences in the size of the fiscal space across the EU14 countries is also related to the gap between the tax rate on income from capital under the benchmark calibration and at the peak of the Laffer hill. These are not reported for reason of space.

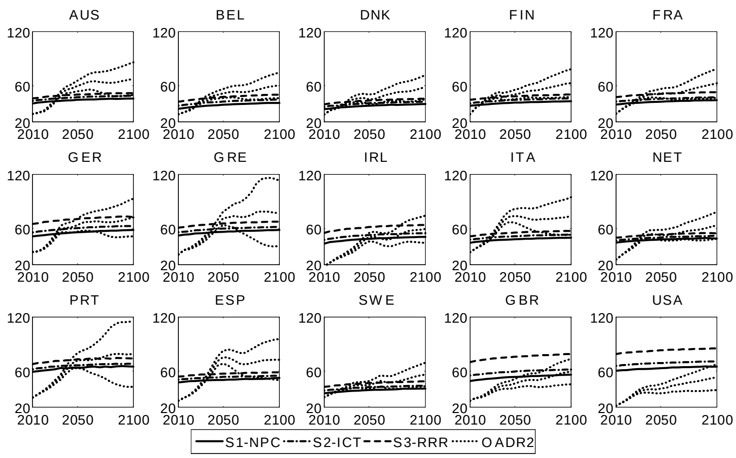

Figure 10: OADR2 and thresholds under S1-NPC, S2-ICT and S3-RRR, EU14 countries, 2010-2100.

6.2 Threshold Dependency Ratio

Figure 10 shows the evolution of the threshold dependency ratio in the EU14 countries over 2010-2100. In each panel, the dotted lines denote the OADR2 forecasts (mean and two-standard-deviation bands); the solid line denotes the threshold dependency ratio under the no-policy-change scenario (S1-NPC); the dashed-dotted and dashed lines denote the threshold obtained when the consumption tax rate is increased by 5 percentage points (S2-ICT) and the replacement ratio is reduced by 10 percentage points (S3-RRR), respectively. Figure 11 shows the evolution of the threshold dependency ratio when the age of retirement is increased from 65 to 70 (S4-IRA) in each EU14 country together with the OADR3 forecasts over the period 2010-2100.

We find that under S1-NPC the majority of the EU14 countries have threshold dependency ratios below the mean forecast of the OARD2 for the largest part of the 2010-2100 period. Under S2-ICT and S3-RRR there is a very modest increase in the threshold in all EU14 countries, other than Great Britain. The outlook appears to improve under S4, though in no country is the threshold above the higher error band of the OADR3.

Table 8 reports our estimate of the year when the OADR2 and OADR3 mean forecast are expected to be higher than the threshold dependency ratio for each EU14 country under the four policy scenarios. Under S1-NPC, the OADR2 is expected to overtake the threshold for all EU14 countries before 2100. Under S2-ICT, S3-RRR and S4-IRA the number of countries overtaking the threshold before 2100 reduces to thirteen, eleven and nine, respectively. The outlook is therefore particularly concerning for this last group of nine,

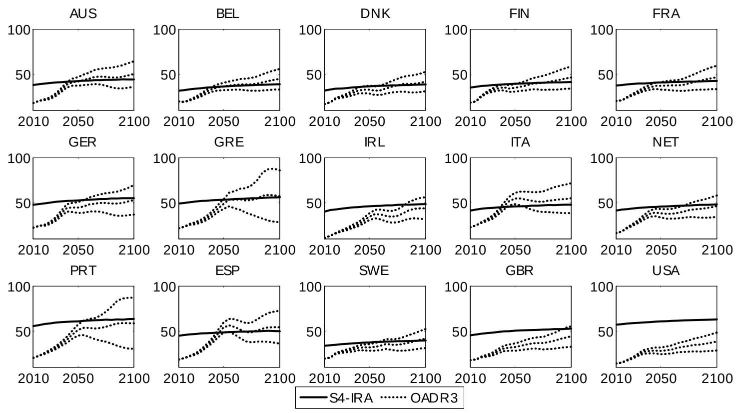

Figure 11: OADR3 and thresholds under S4-IRA, EU14 countries, 2010-2100.

that comprises Austria, Belgium, France, Italy, Greece, Spain and the three Scandinavian countries. On average, under S1-NPC these nine countries are expected to overtake the threshold by 2030. This date is postponed by 5, 15 and 40 years under S2-ICT, S3-RRR and S4-IRA, respectively.

Table 9 reports the distance from the threshold and probability of reaching the threshold in each EU14 country in 2050 under the four policy scenarios. The last two rows report the EU14 average and the United States for reference. Under S1-NPC, for all countries other than Great Britain the distance from the threshold in 2050 is negative implying the dependency ratio exceeds the threshold. The distance is still negative under S2-ICT, though on average smaller than S1-NPC. It becomes positive on average under S3-RRR and S4-IRA. The probability of reaching the threshold by 2050 declines on average from about 90 to about 20 percent when moving from S1-NPC to S4-IRA. In contrast, there is a zero probability of the United States reaching the threshold by 2050 even under the scenario of no change in policy. These results highlight how pressing reforms of the pension system are in the EU14 countries. They also highlight that the extent of these reforms in EU14 countries should be more radical than for the United States.

6.3 Welfare Analysis

Table 10 reports our results on the welfare effects of alternative policy changes that are designed to satisfy the threshold dependency ratio for the EU14 countries based on the lifetime utility of the 2050 newborn generation. The first

	S1-NPC	S2-ICT	S3-RRR	S4-IRA
AUS	2030	2035	2035	2055
BEL	2025	2030	2040	2055
DNK	2025	2030	2035	2080
FIN	2020	2025	2030	2080
FRA	2025	2030	2040	2085
GER	2035	2035	-	-
GRE	2040	2045	2050	2085
IRL	2075	2080	-	-
ITA	2030	2030	2035	2040
NET	2035	2035	2065	-
PRT	2045	2045	2070	-
ESP	2035	2040	2040	2050
SWE	2020	2030	2075	2095
GBR	2090	-	-	-
USA	-	-	-	-

Notes: '-' indicates threshold not reached before 2100.

Table 8: Years when forecasted dependency ratios are estimated to overtake the thresholds under S1-NPC, S2-ICT, S3-RRR and S4-IRA, EU14 countries.

two columns report for each country the targeted distance from the threshold in 2050 and the corresponding probability (these are the same as in the last two columns of Table 9). The next two columns report the tax rate on consumption and the pension replacement ratio required to achieve the targeted distance in each country under S2A-ICT and S3A-RRR, respectively. The last two columns report the percentage consumption change (Δ) required for welfare under S4-IRA to be equal to welfare under S2A-ICT and S3A-RRR, respectively. The results for the United States are also reported in the last row for comparison. Welfare calculations are not provided for Italy and Spain as the distance remains negative for these two countries. We highlight two main results.

First, the increases in the consumption tax rate and/or reduction in the replacement ratio required to achieve the targeted distances are significantly higher than those calculated for the United States. The average consumption tax rate under the benchmark calibration of the twelve countries that have a positive distance is about 21%. This needs to increase on average across these countries to 43%. Similarly the replacement ratio needs to be reduced from 57 to 43%, on average.

Second, the welfare ranking for the three policy reform scenarios is very different across countries. In particular, partial financing using the consumption tax, which is the preferred policy change for the United States, does not yield the higher long-run welfare gains for the majority of EU14 countries. The policy reform S3A-RRR yields higher welfare gains in the long run for Austria, Belgium, Finland, Germany and Great Britain, while S4-IRA yields higher welfare gains for Denmark, France and the Netherlands.

	S1-NPC		S2-ICT		S3-RRR		S4-IRA	
	D	Pr	D	Pr	D	Pr	D	Pr
AUS	-5.22	100	-4.22	100	-3.29	99.9	0.13	1.7
BEL	-5.11	100	-3.68	100	-1.58	95.2	0.01	47.5
DNK	-2.76	99.7	-1.69	94.8	-1.68	70.6	1.56	5.4
FIN	-3.61	100	-2.39	99	-0.81	79.3	2.08	2.3
FRA	-3.68	100	-2.39	99.5	-0.22	56.7	1.72	5.5
GER	-2.81	98.9	-0.96	84.2	1.79	4.9	2.66	0.9
GRE	-4.07	100	-3.09	99.9	-1.57	95.4	1.76	4.8
IRL	-0.85	80.5	1.02	15.2	4.88	0	6.17	0
ITA	-6.79	100	-5.9	100	-4.85	100	-2.86	99.9
NET	-1.71	95.9	-0.96	84.2	0.23	41.3	3.1	0
PRT	-2.06	99.3	-1.4	92.8	0.27	37.9	3.03	0
ESP	-7.07	100	-6.12	100	-5.13	100	-1.99	98.1
SWE	-2.65	99.7	-1.13	87.6	0.98	15.7	3.02	0.2
GBR	2.95	0	5.4	0	12.17	0	8.82	0
EU14	-3.25	91.0	-1.97	82.7	0.09	56.9	2.09	19.0
USA	8.78	0	11.12	0	16.61	0	17.7	0

Notes: All numbers are in percentage.

Table 9: Distance from the threshold and probability of reaching the threshold, EU14 countries, 2050.

To shed light on some of the factors determining these cross-country differences in the welfare ranking, we calculate how the measured consumption compensations correlate with the deep parameters that determine the distance from the threshold in each country.[30]

Several patterns emerge. Policy reforms based on increasing the taxation of consumption tend to yield higher welfare gains in countries with relatively high debt-to-GDP ratios. Under this policy the government can reduce the level of public spending since the equilibrium stock of capital is high, thus leading to a reduction in the cost of servicing public debt. Policy reforms based on a reduction in pension contributions tend to be preferred to those based on higher taxation of consumption in countries where the taxation of income from labor is relatively high, since the consumption tax rate has the effect of further increasing the tax wedge. For the same reason, in countries with relatively high taxation of income from labor, policy reforms based on the reduction of pension contributions tend to be preferred also to those based on increasing the retirement age.

Cross-country differences are also affected by differences in discount factors and the taxation of saving. Under policy S4-IRA there is a reduction in public

[30] We considered tax rates (τ^c, τ^k, τ^l, τ^p), preferences and production parameters (β, κ, α, δ), fiscal expenditure variables and parameters (g/y, tr/y, b/y, θ), the size of the 65-70-year cohort, the OADR2 and OADR3 in 2010, and the change in both of these between 2010 and 2050.

	S4-IRA		S2A-ICT	S3A-RRR	Δ_1	Δ_2
	D	Pr	τ^c	θ		
AUS	0.13	1.7	46	56	1.49	1.78
BEL	0.01	47.5	34	33	-0.23	2.42
DNK	1.56	5.4	55	51	-8.79	-4.15
FIN	2.08	2.3	50	39	-1.36	2.66
FRA	1.72	5.5	39	41	-0.36	-1.18
GER	2.66	0.9	36	25	3.27	4.49
GRE	1.76	4.8	46	47	13.43	7.97
IRL	6.17	0	45	23	1.23	1.20
ITA	-2.86	99.9	41	49		
NET	3.10	0	48	71	-4.45	-8.01
PRT	3.03	0	55	57	1.52	-3.36
ESP	-1.99	98.1	44	60		
SWE	3.02	0.2	40	42	24.2	-1.45
GBR	8.82	0	28	15	0.40	1.77
EU14	2.09	19.0	43	43		
USA	17.7	0	25	25	4.17	3.40

Table 10: Consumption compensations across cohorts required for S2A-ICT and S3A-RRR to yield same equilibrium lifetime utility as under S4-IRA, EU14 countries, 2050.

transfers at age 65-70 (the pension for the cohort that is required to work under S4-IRA). Under policy S3-RRR, there is on average a reduction in transfers at a later point in life for those age 65-95. Different discount factors across countries weight these two policies differently. For any given policy, individuals in countries with high capital tax rates have lower consumption than those living in countries with low capital tax rates in order to build up their savings. Accordingly, for a lower discount rate and capital income tax rate, policy S3-RRR is preferred to policy S4-IRA.

7 Conclusion

The main challenge to the sustainability of state pension systems is population aging. Although a world-wide phenomenon, aging is a particular problem for advanced economies where the ratio of pension recipients to contributors to the pension system - the dependency ratio - are among the highest in the world, and are projected to double over the next 85 years. Moreover, many of these countries are close to the limits of what tax policy can do to relieve the situation. In this paper we provide a theoretical and quantitative analysis of the problem for the United States and for fourteen of the largest European countries over the period 2010-2100.

The paper develops a stylized multi-period overlapping generations model that explicitly accounts for limits to the ability of governments to increase tax revenues through the distortionary taxation of income from capital and labor. This fiscal limit imposes a constraint on pension provision. As a result, under their current pension arrangements governments may find that there is an upper bound to the size of the dependency ratio that they can sustain solely from the revenues from the taxation of income. This threshold to the dependency ratio is obtained as a competitive equilibrium solution to the model. It also allows us to obtain a measure of the distance that an economy is from this threshold and the probability of reaching the threshold at some point in the future. Once this distance falls to zero, reform of the current pension arrangements becomes essential and can no longer be postponed. We consider three possible reforms to pension arrangements: partial financing using consumption taxes, a reduction in pension contributions and an increase in the retirement age.

In all of the countries we study we find that the threshold is increasing over time but not as rapidly as demographic forecasts of the dependency ratio. As a result, the distance from the threshold is found to decline very quickly and the probability of reaching the threshold is increasing.

There are significant differences in thresholds levels, distances and probabilities among these countries. The outlook for most of the European countries is of particular concern. Compared to the United States, all have, on average, smaller fiscal spaces, more generous pension systems, are older and are projected to age much faster. The European countries are therefore found to be much closer to the threshold than the United States in 2010 and are predicted to reach the threshold well before 2050. In contrast, the United States is found to maintain a positive distance until 2100.

The probability of the United States reaching the threshold can be reduced to be close to zero either by increasing consumption taxation by 5 percentage points, or by reducing the replacement ratio of the pension by 10 percentage points, or by increasing the retirement age to 70. In contrast, such policy changes bring only marginal improvements to the pension outlooks for the EU14 countries and only serve to highlight how pressing an issue pension reform is for these EU14 countries.

A further difference is that whereas for the Unites States there is a clear welfare advantage to employing higher consumption taxes to achieve a given distance from the threshold than having a lower replacement ratio or a higher retirement age, there is no such preference ordering for the European countries. Their welfare rankings differ depending on country-specific characteristics, such as the design of the tax system, the current level of public spending, private sector preferences and productivity.

One of the new contributions of this paper is the quantification of dynamic Laffer effects in a multi-period overlapping generations model. The paper shows how, in an OLG life-cycle model, these effects can alter the cross-section distribution of the tax burden depending on how tax revenues are shared among retirees, and how this may change over time due to population aging.

Although the quantitative results presented in this paper are based on a very stylized model of the economy, we believe that they have identified fundamental problems for the provision of public pensions in advanced economies that require urgent attention. A more complete analysis, beyond the scope of this paper, might also take account of several further implications that population aging may have for the macro-economy and for public finances. For example: (i) the political feasibility of extracting the maximum revenue from the taxation of income; (ii) the cost of non-pension-related components of public spending; and (iii) long-term rates of economic growth. We do not, however, anticipate that taking account of these considerations would alter our main finding, namely, that a large number of European countries are likely to reach their threshold dependency ratio within the next 20-30 years.

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A Numerical Algorithm

A.1 Equilibrium

In the following, we describe the derivation of the equilibrium solution for the quantitative model in Section 4. Given the shares of the individuals of age j in the population, μ_j, the population growth rate n, the age of retirement j_R and the survival probabilities $\phi_{t,j}$, the equilibrium is calculated so that (i) individual behavior is consistent with the aggregate behavior of the economy, (ii) firms maximize profits, (iii) households maximize intertemporal utility, (iv) the factor and goods markets clear and (v) the budgets of the government and the social security authority are balanced.

Under the specification in section 4.1, the household chooses the sequence of allocations $(c_{t,j}, l_{t,j}, a_{t,j+1})_{j=j_t^0}^{J}$ that, for any $t \geq -J$, maximizes

$$U^t = \sum_{j=j_t^0}^{J} \beta^{j-j_t^0} \left(\prod_{s=j_t^0}^{j} \phi_{t,s} \right) \frac{1}{1-\eta} \left(c_{t,j}^{1-\eta} \left[1 - \kappa(1-\eta) l_{t,j}^{1+1/\varphi} \right]^{\eta} - 1 \right)$$

subject to

$$(1+\tau_t^c)c_{t,j} + a_{t,j+1} = x_{t,j} + [1 + (1-\tau_t^k)\widehat{r}_t]a_{t,j} + tr_j, \tag{A.1a}$$

where:

$$\begin{aligned} x_{t,j} &= \begin{cases} (1-\tau_t^w - \tau_t^p)\widehat{w}_t y_j A_t l_{t,j} & \text{for } j \in \left(j_t^0, j_R - 1\right) \\ p_{t,j} & \text{for } j \in \left(j_R, J\right) \end{cases}, \\ l_{t,j} &= 0 \text{ for } j \in (j_R, J), \\ a_{t,0} &= a_{t,J+1} = 0, \\ a_{t,j} &= k_{t,j} + b_{t,j}. \end{aligned}$$

The necessary and sufficient conditions for a solution to the consumer's problem in equations (7) - (9) become:

$$\lambda_{t,j}(1+\tau_t^c) = c_{t,j}^{-\eta}[1 - \kappa(1-\eta) l_{t,j}^{1+\frac{1}{\vartheta}}]^{\eta}, \tag{A.1b}$$

for $j \in \left(j_t^0, J\right)$

$$\lambda_{t,j}(1-\tau_t^w - \tau_t^p)\widehat{w}_t y_j A_t = \kappa\eta(1+\frac{1}{\vartheta})c_{t,j}^{1-\eta}[1-\kappa(1-\eta)l_{t,j}^{1+\frac{1}{\vartheta}}]^{\eta-1} l_{t,j}^{\frac{1}{\vartheta}}, \tag{A.1c}$$

for $j \in \left(j_t^0, j_R - 1\right)$

$$\lambda_{t,j} = \beta\phi_{t,j+1}\lambda_{t+1,j+1}[1 + (1-\tau_{t+1}^k)\widehat{r}_{t+1}], \tag{A.1d}$$

for $j \in \left(j_t^0, J\right)$. In equation (A.1b) $l_{t,j} \equiv 0$ for $j \geq j_R$.

To describe the model in terms of stationary variables, we define the following individual stationary variables:

$$\widetilde{c}_{t,j} \equiv \frac{c_{t,j}}{A_t}, \widetilde{a}_{t,j} \equiv \frac{a_{t,j}}{A_t}, \widetilde{k}_{t,j} \equiv \frac{k_{t,j}}{A_t}, \widetilde{b}_{t,j} \equiv \frac{b_{t,j}}{A_t}, \widetilde{tr}_t \equiv \frac{tr_t}{A_t}, \widetilde{p}_t \equiv \frac{p_t}{A_t}, \widetilde{\lambda}_{t,j} \equiv \frac{\lambda_{t,j}}{A_t^{-\sigma}},$$

and the aggregate (per-capita) stationary variables:

$$\widetilde{y}_t \equiv \frac{y_t}{A_t}, \widetilde{k}_t \equiv \frac{k_t}{A_t}, \widetilde{b}_t \equiv \frac{b_t}{A_t}, \widetilde{tax}_t \equiv \frac{tax_t}{A_t}, \widetilde{a}_t = \widetilde{k}_t + \widetilde{b}_t.$$

Thus, factor prices in (10) and (11) become:

$$\widehat{r}_t = \alpha \widetilde{k}_t^{\alpha-1} \widetilde{l}_t^{1-\alpha} - \delta, \tag{A.1e}$$

$$\widehat{w}_t = (1-\alpha) \widetilde{k}_t^{\alpha} \widetilde{l}_t^{-\alpha}. \tag{A.1f}$$

To compute the equilibrium, in any $t \geq 0$, we solve a system of non-linear equations in 28 variables, consisting of the 14 individual asset levels, $\widetilde{a}_{t,j} = \widetilde{k}_{t,j} + \widetilde{b}_{t,j}$, $j \in (0, J)$, with $a_{-j,j} = 0$ for $j \in (0, J)$, the 9 individual labor supplies, $l_{t,j}$, $j \in (0, j_R - 1)$ and the aggregate variables $\widetilde{k}_t$, $\widetilde{l}_t$, $\widetilde{a}_t$, τ^p and $\widetilde{tr}_t$. After replacing factor prices (A.1e) and (A.1f) into (A.1a) - (A.1d), the non-linear equations system consits of 23 equations obtained from the equilibrium conditions (A.1a) - (A.1d) - the 14 Euler conditions and the 9 intra-temporal first-order conditions obtained by combining (A.1a), (A.1b) and (A.1c), expressed in stationary variables - and the following 5 aggregate equilibrium conditions:

$$\begin{aligned}
(1+n)\,\widetilde{a}_{t+1} &= \sum_{j=0}^{J} \mu_j \widetilde{a}_{t-j,j+1}, \\
l_t &= \sum_{j=0}^{j_R-1} \mu_j y_j l_{t-j,j} \\
\widetilde{k}_t &= \widetilde{a}_t - \widetilde{b}_t, \\
\tau^p &= \frac{\mu^R p_t}{\widehat{w}_t l_t}, \\
\widetilde{tr}_t + \widetilde{g}_t &= \widetilde{tax}_t + \widetilde{beq}_t + [(1+g_A)(1+n) - (1+\widehat{r})]\widetilde{b}_t,
\end{aligned}$$

where $\widetilde{tax}_t = \tau^w \widehat{w}_t l_t + \tau^k \widehat{r}_t \widetilde{a}_t + \tau^c \widetilde{c}_t$, with $\widetilde{c}_t = \sum_{j=0}^{J} \mu_j \widetilde{c}_{t-j,j}$, and $(1+n)\,\widetilde{b}_{t+1} = \sum_{j=0}^{J} \mu_j \widetilde{b}_{t-j,j+1}$.Accidental bequests in the stationary equilibrium amount to

$$(1+n)\,\widetilde{beq}_{t+1} = \sum_{j=0}^{J} \mu_j (1 - \phi_{t-j,j+1})[1 + (1-\tau^k)\widehat{r}_{t+1}]\widetilde{a}_{t-j,j+1}.$$

To compute $\widetilde{b}_j$ we used the condition that all agents hold the two assets $\widetilde{k}_j$ and $\widetilde{b}_j$ in the same proportion. All other variables, e.g. individual consumption, factor prices, and aggregate bequests and taxes, can be computed with the help of the 28 endogenous variables. For example, individual consumption levels $\widetilde{c}_j$ for $j \in (0, J)$ are derived from the individual budget constraints; factor prices are computed from the first-order conditions of the firms.

We solve the system of non-linear equations with a modified Newton-Rhapson algorithm, as described in Section 11.5.2 in Heer and Maussner (2009), and applied to a large-scale OLG model, as described in Section 9.1.2 in Heer and Maussner (2009).

The main challenge for the solution is to come up with a good initial value for the individual and aggregate state variables. Therefore, we started from a 9-periods OLG model with exogenous labor where all cohorts are workers. The exogenous labor supply is set equal to 0.3 and the initial value for the aggregate capital stock is set equal to the corresponding value in the Ramsey model. Thereafter, we added one additional cohort of retirees in each step and used the solution of the model in the previous step as an input for the initial value of the next step. Finally, we introduced endogenous labor in the model. During these initial computations, we computed the solution for the individual optimization problem in an inner loop and updated the aggregate capital variables in an outer loop with a dampening iterative scheme, as described in Section 3.9 of Judd (1998), to ensure convergence. For the final calibration and the computation of the equilibria under different tax rates, we applied the modified Newton-Rhapson algorithm to the complete set of the 28 individual and aggregate equilibrium conditions.

A.2 Laffer Curve and Threshold Dependency Ratio

In order to find the revenue-maximizing tax rates (τ^l, τ^k), we employ a nested procedure. In the inner loop, we use a maximization routing (Golden Section search) in order to find the revenue-maximizing capital income tax rate τ^k for given labor income tax rate τ^l. In the outer loop, we use a grid search over τ^l over the interval [20%, 70%] with an accuracy equal to 0.01%. In order to find the threshold dependency ratio in a particular year, we iterate over the dependency ratio starting from a low value. As initial value, we choose the dependency ratio that is associated with the survival probabilities of that year and a population growth rate of 2%. We slowly increase the dependency ratio and compute the revenue maximizing tax combination (τ^l, τ^k) in each step as described in the last paragraph. When we find a tax combination that finances the government expenditures in the particular year, we continue to increase the dependency ratio. We stop the computation, when we cannot find a tax combination (the Laffer peak) that is able to finance government expenditures. In particular, we will stop the computation when the endogenous residual from the government budget in the form of the transfers is below the transfers in the benchmark economy.

A.3 Transition dynamics

We compute the transition dynamics for the United States economy as described in Algorithm 9.2.1 in Heer and Maussner (2009). We first choose a number of transition periods assuming that the transition is complete by 2170. This value for the final year is found with trial and error. However, the transition of the endogenous values is completed well before 2170, as shown in Figure 7. Next, we compute the initial and final equilibrium and project a trajectory for the endogenous values of $\left(\widetilde{k}_t,\widetilde{l}_t,\tau_t^l,\tau_t^p,\widetilde{tr}_t\right)_{2010}^{2170}$. We assume that the economy is in steady state in and prior to 2010. For given path of $\left(\widetilde{k}_t,\widetilde{l}_t,\tau_t^l,\tau_t^p,\widetilde{tr}_t\right)_{2010}^{2170}$, we compute the individual policy functions in each year and aggregate individual labor supply and consumption. Using the first-order conditions and the fiscal budget constraints, we are able to provide a new guess for the path of $\left(\widetilde{k}_t,\widetilde{l}_t,\tau_t^l,\tau_t^p,\widetilde{tr}_t\right)_{2010}^{2170}$. Again, we use the dampening iterative scheme as described in Section 3.9 of Judd (1998) in order to update the sequence $\left(\widetilde{k}_t,\widetilde{l}_t,\tau_t^l,\tau_t^p,\widetilde{tr}_t\right)_{2010}^{2170}$ until convergence (with an accuracy equal to 10^{-5}).

B Benchmark Calibration

	n	d	β	κ	α	δ	g/y	tr/y	b/y	θ	τ^l	τ^k	τ^c	τ^p
AUS	0.3	36.6	0.984	3.99	39	7.1	20	10.2	65	78.1	50	24	20	29
BEL	0.5	33.8	0.970	5.14	39	8.4	24	14.9	107	46.6	49	42	17	16
DNK	0.4	33.4	0.970	3.27	40	9.2	28	15.3	50	67.8	47	51	35	23
FIN	0.4	35.7	0.966	3.96	34	7.0	24	16.4	46	55.8	49	31	27	20
FRA	0.5	36.0	0.976	5.18	41	6.9	27	6.6	60	55.4	46	35	18	20
GER	-0.1	41.4	0.960	5.18	37	6.7	21	9.7	62	37.5	41	23	15	16
GRE	0.1	40.6	0.984	3.36	40	6.1	20	1.7	100	66.7	41	16	15	27
IRL	1.3	25.4	0.951	5.66	36	8.6	19	11.4	43	34.7	27	21	26	9
ITA	0.2	40.0	0.981	5.03	39	7.0	21	7.1	110	69.5	47	34	15	28
NET	0.5	35.1	0.992	5.8	38	7.7	27	-3.6	58	90.5	44	29	19	32
PRT	0.1	38.1	0.974	3.4	39	9.8	23	-6.0	57	73.8	31	23	21	28
ESP	0.7	27.7	0.982	5.17	42	8.5	21	-1.2	54	82.1	36	30	14	27
SWE	0.5	35.9	0.975	2.99	36	4.8	30	14.4	58	56.0	56	41	21	20
GBR	0.6	33.2	0.942	4.36	36	6.4	21	13.6	44	21.6	28	46	16	7
USA	0.95	30.6	0.958	3.62	35	8.3	18	5.2	63	35.2	28	36	5	11

Notes: Parameter values equal for all countries: $\widehat{r} = 4\%$, $\eta = 2$, $\varphi = 1$ and $g_A = 2\%$.
All numbers are in percentage, other than β and κ. The survival probabilities are averages over 1990-2010. Age-productivity z_j equal to 1 in all other countries other than in the USA. Data source is described in the main text.

Table 11: Benchmark calibration