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Price Rigidity and the Origins of Aggregate Fluctuations

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Abstract

We document a novel role of heterogeneity in price rigidity: It strongly amplifies the capacity of idiosyncratic shocks to drive aggregate fluctuations. Heterogeneity in price rigidity also completely changes the identity of sectors from which fluctuations originate. We show these results both theoretically and empirically through the lens of a multi-sector model featuring heterogeneous GDP shares, input-output linkages, and idiosyncratic productivity shocks. Quantitatively, we calibrate our model to 341 sectors and find sectoral productivity shocks can give rise to aggregate fluctuations that are half as large as those arising from an aggregate productivity shock. Heterogeneous price rigidity amplifies the aggregate fluctuations by a factor of more than 2 relative to a flexible-price or homogeneous sticky price economy. Hence, idiosyncratic shocks and heterogeneous price rigidity can account for large parts of aggregate fluctuations and there is hope we will not "forever remain ignorant of the fundamental causes of economic fluctuations" (Cochrane (1994)).

JEL-Codes: E310, E320, O400.

Keywords: input-output linkages, nominal price rigidity, idiosyncratic shocks.

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I   Introduction

The role of price rigidity for macroeconomics has been at the center of heated discussions for decades. Are prices sticky, and if so, does it matter? Proponents typically focus on the transmission of nominal shocks, such as monetary policy shocks: If prices do not adjust instantly, monetary policy shocks may have large and persistent real effects. Opponents typically argue prices are flexible or price rigidity – even if it exists – does not imply real effects of nominal shocks (see Head et al. (2012)).

We argue in this paper price rigidity plays a distinct, equally important role for idiosyncratic real shocks. Heterogeneity in price rigidity can interact with other heterogeneous features of the economy such that it crucially affects the propagation properties of the economy. In particular, this interaction can give rise to a much higher capacity of idiosyncratic shocks to drive aggregate fluctuations relative to an economy with flexible prices or homogeneous price stickiness across sectors.

We document this role of price rigidity through the lens of a multi-sector model featuring heterogeneous GDP shares, input-output linkages across sectors, and idiosyncratic sectoral productivity shocks as the source of business cycles.[1] When we calibrate the model to the U.S. for 341 sectors, sectoral TFP shocks can give rise to aggregate fluctuations that are approximately half as large as those arising from an aggregate TFP shock. Heterogeneous price rigidity amplifies the potency of idiosyncratic shocks to generate aggregate fluctuations by a factor of 2.6, relative to the case of flexible prices, and by a factor of 2.2 relative to homogeneously rigid prices.[2] Heterogeneity in price rigidity also completely changes the sectoral origin of aggregate fluctuations: from "Retail Trading," "Real Estate," and "Wholesale Trading" with flexible prices to "Monetary Authorities and Depository Credit Intermediation," "Petroleum Refineries," and "Oil and Gas Extraction" when prices are rigid. Hence, the literature on price rigidity may have for decades disregarded the important interaction of heterogeneity in nominal price rigidity with other heterogeneous, real features of the economy for the potency and identity of sectors driving aggregate fluctuations based on sectoral shocks.

Heterogeneity in price rigidity matters not only because it can substantially augment the potency of real features of the economy to propagate idiosyncratic shocks but also because it can also distort the price response of sectors relative to each other. Consider the example of a

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[1]We model price rigidity à la Calvo (1983), monetary policy following a Taylor-type rule, and use data from the Bureau of Economic Analysis (BEA) and the micro-data underlying the Producer Price Index (PPI) at the Bureau of Labor Statistics (BLS) for the calibration.

[2]Gabaix (2011) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) show in models with flexible prices that fat tails of sectoral size or network centrality may lead to idiosyncratic shocks generating aggregate fluctuations.

two-sector economy with no interlinkages, and with a larger sector with a sales weight of 51% and a smaller sector with a weight of 49%. If the sectors receive one-time productivity shocks of sizes +1% and -1%, the economy will experience a drop in the price level and an economic boom. However, if the larger sector cannot adjust prices at all while the smaller sector can, the economy will experience a recession. The cycle flips sign.

In a nutshell, we can learn about the presence and importance of price rigidity not only from the response of the economy to aggregate shocks, but also by studying aggregate fluctuations that originate from sectoral productivity shocks. The presence of heterogeneity in real features of the economy matters for the aggregate effect of idiosyncratic shocks, but the interaction with heterogeneity in price rigidity is what can substantially increase the power of sectoral shocks to drive aggregate fluctuations. We provide formal intuition in a simplified version of our multi-sector model, assuming firms set their prices before observing shocks, according to a sector-specific Poisson process, and facing exogenous nominal demand. These assumptions allow us to analytically characterize the response of aggregate output and prices up to first-order. At the same time, we nest the results for flexible-price economies in Gabaix (2011) and Acemoglu et al. (2012) on the importance of the size distribution and network linkages as special cases.

In the analytical model, the cross-sectional dispersion of sectoral multipliers determines how much aggregate volatility sectoral shocks can generate. Our key finding is that the interaction of the heterogeneous nominal and real features of the economy determine this dispersion: Intuitively, if prices are fully flexible, a sector's multiplier is large if the sector has a *physically* large output share, and/ or when it is a *physically* large supplier of intermediate inputs or a large supplier to large suppliers of intermediate inputs. However, with price rigidity, a physically large or central sector's multiplier becomes *effectively* larger if (i) its prices are more flexible than the average sector's prices; or (ii) sectors that buy intermediate inputs from it have more flexible prices than the average sector's prices; or (iii) sectors that buy from intermediate input-demanding sectors, have more flexible prices than the average sector's prices.

This intuition of *effective* importance implies sectoral shocks may generate large aggregate volatility even when all sectors have equal physical GDP shares and input-output linkages – as long as high cross-sectional dispersion of price rigidity exists. We provide general conditions for price rigidity to amplify or reduce the potency of heterogeneous real features of the economy – embodied in our setup by heterogeneous GDP shares and network centrality – to generate aggregate fluctuations.

The interaction between heterogeneous price rigidity and real features of the economy also directly speaks to the tension between the diversification argument of Lucas (1977) and the

granular origin of aggregate fluctuations (see Gabaix (2011) and Acemoglu et al. (2012)). As the number of sectors, K, increases, aggregate fluctuations may die out at a rate of $\sqrt{K}$ – the diversification argument of Lucas (1977) – or the rate of decay can be smaller if fat tails exists in the relevant real features of the economy, such as sector size or network centrality.

We provide three novel results in this context. First, fat tails in sectoral price rigidity by itself cannot generate a rate of decay slower than the central limit theorem implies, because price rigidity is bounded: prices cannot get more flexible than fully flexible. Second, heterogeneity in price rigidity is also irrelevant for the rate of decay when it is independent of heterogeneous sectoral features such as size or network centrality – the joint distribution inherits the Pareto properties of the more fat-tailed distribution. Third, however, the distributions of friction-adjusted size or network centrality may be more or less fat-tailed than their frictionless counterparts when we allow for correlations between price rigidities and real features. In this case, the diversification argument of Lucas (1977) might even apply in an economy with price rigidities – whereas the granular hypothesis of Gabaix (2011) and Acemoglu et al. (2012) applies in the frictionless counterpart. Of course, price rigidity may also make the granular hypothesis stronger. We provide one theoretical example for each of these cases.

Our analysis also highlights heterogeneity of price rigidity is economically important even in cases when it is irrelevant for the rate of decay. On the one hand, it can still be quantitatively important in a cross-sectional sense for aggregate volatility from sectoral shocks if the number of sectors is finite. On the other hand, even in the absence of any such quantitative importance, heterogeneity in price rigidity matters. It can dramatically change the effective aggregate importance of each real, heterogeneous feature of the economy. The non-trivial implication for policy-makers is that it may no longer be sufficient to identify the importance of sectors for aggregate fluctuations according to their GDP shares or their centrality in the production network. The interest of the recent granularity literature such as di Giovanni, Levchenko, and Mejean (2018) or Gaubert and Itskhoki (2018) to identify where aggregate fluctuations originate adds further emphasis to this observation.

Ultimately, whether our theoretical insights are quantitatively important is an empirical question. We show they are. As a benchmark, we start by separately establishing the importance of sectoral heterogeneity of price rigidity, and the two real features of the economy. Price rigidity by itself gives rise to a quantitatively relevant "frictional" origin of aggregate fluctuations when sectors differ only in their Calvo parameters: The multiplier of sectoral shocks on GDP volatility is 15.7% of what an aggregate productivity shock would generate.

We can consider economies with flexible prices as a benchmark. Then the multiplier is

5.4% when all sectors are perfectly symmetric, 11.3% when steady-state sectoral GDP shares match U.S. data, 8% when steady-state input-output linkages match U.S. input-output tables, and 16.9% when both match U.S. data.[3] Similar magnitudes arise in case of the GDP deflator. Hence, price rigidity by itself quantitatively generates similar aggregate effects as heterogeneous GDP shares and input-output linkages *combined*.

The interaction between heterogeneous price rigidity with the heterogeneous real features of the economy dramatically amplifies the power of idiosyncratic shocks to drive aggregate fluctuation, confirming our main theoretical finding. The GDP multiplier is 16.9% when prices are flexible and steady-state GDP shares and input-output linkages both match U.S. data, but increases to 44.2% of what an aggregate productivity shock would generate when Calvo parameters match industry-average frequencies of price changes in U.S. data. Sectoral shocks have a 2.6 times larger effect on GDP volatility in an economy with heterogeneous sticky prices relative to an economy with flexible prices across all sectors, and 2.2 times relative to an economy with homogeneous price rigidity. Moreover, the multiplier is 8 times larger than in an economy with perfect symmetry and flexible prices. Similar results emerge for the GDP deflator. Finally, the interaction of nominal and real heterogeneities implies quantitatively, idiosyncratic shocks can have a large impact on aggregate fluctuations compared to an aggregate productivity shock.

Our second key result is the large change in the identity of the most important sectors from which aggregate fluctuations originate. Under fully flexible prices, the sector with the largest multiplier is "Retail Trade" followed by "Real Estate," "Wholesale Trade," with minor roles of "Monetary Authority and Depository Credit Intermediation" and "Telecommunications." Combined, they account for 74% of the total multiplier of sectoral shocks on GDP volatility. Under heterogeneous, rigid prices, the sector with the largest multiplier is "Monetary Authority and Depository Credit Intermediation," followed by "Petroleum Refineries," "Oil and Gas Extraction," "Auto Manufacturing," with a minor role of "Wholesale Trade". Their combined multiplier is 93% of the total multiplier. Thus, price rigidity not only changes the identity of the most important sectors, it also increases the overall importance of the top-5 sectors.

Our results are robust to changes in several assumptions. Theoretically, we analyze variations in the elasticity of labor supply, sectorally segmented labor markets, and monetary policy. Quantitative robustness checks cover variations in the elasticity of labor supply, the elasticity of substitution within and across sectors, and alternative monetary policy rules, including passive monetary policy, price-level targeting, a Taylor-type rule, and a binding zero lower bound. We also calibrate the volatility of sectoral productivity shocks to estimates based on

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[3]This joint effect is remarkably similar to the results of Gabaix (2011) for total sales, which encompasses sales as final goods (GDP) and as intermediate inputs.

the NBER manufacturing dataset, instead of imposing unit volatilities. All our results continue to hold.

We also believe the simple forms of price rigidity we assume are informative for more elaborate, endogenous forms of price setting. Endogenous price rigidity is computationally unsolvable at the level of disaggregation we require for quantitative assessment. Yet although the Calvo friction must be seen as a first-order approximation, we argue it provides all the relevant intuition, because endogenous pricing will not change our mechanism qualitatively: it is the *effective* importance, and not the *physical* importance of a sector that determines the propagation of shocks. This effective importance will now be tied to the determinants of endogenous price adjustment such as menu costs, elasticities, or shock volatility. We leave a formal analysis for future research.

Next we discuss our connection with various strands of literature. Section II presents the model and section III solves its closed-form version and provides all theoretical results. Section IV presents the data and section V the calibrations. Section VI provides quantitative and empirical results. Section VII concludes. The Appendix collects proofs, lengthy derivations, tables, and figures, while additional material is relegated to an online Appendix.

A. Literature Review

Connecting heterogeneous real features of the economy and idiosyncratic shocks as drivers of aggregate fluctuations is one central ingredient of our framework. Long and Plosser (1983) pioneer this microeconomic origin of aggregate fluctuations, and Horvath (1998, 2000) push this literature forward. Dupor (1999) follows Lucas (1977) and argues microeconomic shocks matter only due to poor disaggregation. Gabaix (2011) and Acemoglu et al. (2012) provide a frictionless benchmark for the power of heterogeneous real features of the economy to propagate idiosyncratic shocks to the aggregate, and re-establish the importance of microeconomic shocks by respectively invoking the empirical firm-size distribution and the sectoral network structure of the U.S. economy. Barrot and Sauvagnat (2016), Acemoglu et al. (2016), and Carvalho et al. (2016) provide empirical evidence, and Carvalho (2014) synthesizes this literature.

We introduce heterogeneity in price rigidity as a new focus to the above analyses, recurring to the long-standing question about the role of price rigidity for aggregate fluctuations. In fact, the distortionary role of frictions, and price rigidity in particular, is at the core of the business-cycle literature that conceptualizes aggregate shocks as the driver of aggregate fluctuations, including the New Keynesian literature. This literature has too many contributions to name (see Gali (2015) for a recent textbook treatment). However, to the best of our knowledge, our

paper is the first to study analytically the distortionary role of heterogeneous pricing frictions when aggregate fluctuations have microeconomic origins, and to interact heterogeneous price rigidity with heterogeneous real features of the economy. The disaggregated details of our empirical analysis are also a central novel feature our paper, in particular the ability to measure price rigidity directly in the data at a highly disaggregated level, and to study the quantitative importance of our proposed mechanism.

That said, a few recent theoretical papers that study the microeconomic origin of aggregate fluctuations include frictions in their analyses. Bigio and La'O (2017) study the aggregate effects of the tightening of financial frictions in a production network. Baqaee (2018) shows entry and exit of firms coupled with CES preferences may amplify the aggregate effect of microeconomic shocks. Baqaee and Farhi (2018) decompose the effect of shocks into "direct" and "allocative efficiency" effects due to reduced-form wedges, with a focus on aggregate TFP. These papers ask different questions, but we share with them our finding that the Hulten theorem does not apply in economies with frictions: Total sales of firms/ sectors are not sufficient statistics for their effect on GDP.

In relation to these papers, our work differs in several important dimensions. First, we study pricing as a measurable friction. This approach allows us to pursue quantitative assessments, explore dynamic effects of the friction, and to quantify the implications of a number of relevant model ingredients. Then, our focus also includes the effect on price stability, which is central for the mandate of many central banks around the world, but is absent from this literature entirely. Finally, pricing frictions are a very special friction: Prices are key and first-order to the transmission of shocks. While other frictions may also affect how production adjusts to shocks, that reaction always depends on the ability of prices to pass through the relevant information.

On the empirical side, most closely related to our work are the empirical analyses in Bouakez, Cardia, and Ruge-Murcia (2014) and Carvalho and Lee (2011) that also feature heterogeneity in price rigidity in a New Keynesian setting. Although the focus in these papers is different, some aspects are related to our empirical analysis. The main related result in Bouakez et al. (2014) is the role of sectoral productivity shocks in a 30-sector economy in explaining aggregate variables, but the quantitative importance is modest compared to monetary policy and other shocks. Carvalho and Lee (2011) show the New Keynesian model is consistent with the findings in Boivin, Giannoni, and Mihov (2009) that firms' prices respond slowly to aggregate shocks and quickly to idiosyncratic shocks.

Our model also relates to other previous work studying pricing frictions in production networks. Basu (1995) shows nominal price rigidity introduces misallocation resulting in nominal

demand shocks looking like aggregate productivity shocks. Basu and De Leo (2016) study optimal monetary policy in a model of durable and non-durable goods in the presence of price rigidity and input-output linkages. Nakamura and Steinsson (2010), Midrigan (2011), and Alvarez, Le Bihan, and Lippi (2016), among many others, study monetary non-neutrality in models of endogenous price changes. As we argue above, computational burden makes using such approaches infeasible at highly disaggregated level. In a companion paper (Pasten, Schoenle, and Weber (2016)), we use our model to study the effect of disaggregation on monetary non-neutrality. We generally build on some of this work, relax assumptions on the production structure of the economy, and answer a different set of questions.

II   Model

Our multi-sector model has households supplying labor and demanding goods for final consumption, firms operating under oligopolistic competition producing varieties of goods using labor and intermediate inputs, and a monetary authority. The economy features heterogeneity in two real dimensions: Sectors are heterogeneous in the amount of final goods they produce, and in their input-output linkages. We allow the frequency of price changes to be heterogeneous, which nests cases of homogeneous price rigidity and full price flexibility.

A.   Households

The representative household solves

$$\max_{\{C_t, L_{kt}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \sum_{k=1}^{K} g_k \frac{L_{kt}^{1+\varphi}}{1 + \varphi} \right),$$

subject to

$$\sum_{k=1}^{K} W_{kt} L_{kt} + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t = P_t^c C_t$$

$$\sum_{k=1}^{K} L_{kt} \leq 1,$$

where C_t and P_t^c are aggregate consumption and aggregate prices, respectively. L_{kt} and W_{kt} are labor employed and wages paid in sector $k = 1, ..., K$. Households own firms and receive net income, Π_{kt}, as dividends. Bonds, B_{t-1}, pay a nominal gross interest rate of I_{t-1}. Total labor supply is normalized to 1.

Household demand of final goods, C_t, and goods produced in sector k, C_{kt}, are

$$C_t \equiv \left[\sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

$$C_{kt} \equiv \left[n_k^{-1/\theta} \int_{\Im_k} C_{jkt}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{2}$$

A continuum of goods indexed by $j \in [0,1]$ exists with total measure 1. Each good belongs to one of the K sectors in the economy. Mathematically, the set of goods is partitioned into K subsets $\{\Im_k\}_{k=1}^{K}$ with associated measures $\{n_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} n_k = 1$.[4] We allow the elasticity of substitution across sectors η to differ from the elasticity of substitution within sectors θ.

The first real heterogeneous ingredient of our model is the vector of weights $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]$ in equation (1). This vector summarizes heterogeneity in sectoral GDP in steady state. These weights show up in households' sectoral demand

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{P_t^c} \right)^{-\eta} C_t. \tag{3}$$

All prices are identical in steady state, so $\omega_{ck} \equiv \frac{C_k}{C}$, where variables without a time subscript are steady-state quantities. In our economy, C_t represents the total production of final goods, that is, GDP. The vector Ω_c represents steady-state sectoral GDP shares satisfying $\Omega_c' \iota = 1$, where ι denotes a column vector of 1s. Away from the steady state, sectoral GDP shares depend on the gap between sectoral prices, P_{kt}, and the aggregate price index, P_t^c

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{4}$$

We can interpret P_t^c as the GDP deflator. Household demand for goods within a sector is given by

$$C_{jkt} = \frac{1}{n_k} \left(\frac{P_{jkt}}{P_{kt}} \right)^{-\theta} C_{kt} \text{ for } k = 1, ..., K. \tag{5}$$

Goods within a sector share sectoral consumption equally in steady state. Away from steady state, the gap betwen a firm's price, P_{jkt}, and the sectoral price, P_{kt}, distorts the demand for goods within a sector

$$P_{kt} = \left[\frac{1}{n_k} \int_{\Im_k} P_{jkt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{6}$$

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[4]The sectoral subindex is redundant, but it clarifies exposition. We can interpret n_k as the sectoral share in gross output.

The household first-order conditions determine labor supply and the Euler equation

$$\frac{W_{kt}}{P_t^c} = g_k L_{kt}^\varphi C_t^\sigma \text{ for all } k, j, \tag{7}$$

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} I_t \frac{P_t^c}{P_{t+1}^c} \right]. \tag{8}$$

We implicitly assume sectoral segmentation of labor markets, so labor supply in equation (7) holds for a sector-specific wage $\{W_{kt}\}_{k=1}^K$. We choose the parameters $\{g_k\}_{k=1}^K$ to ensure a symmetric steady state across all firms.

B. Firms

A continuum of monopolistically competitive firms exists, each producing a single good. To facilitate exposition, firms are indexed by the good $j \in [0, 1]$ they produce and the sector, $k = 1, ...K$, to which they belong. The production function is

$$Y_{jkt} = e^{a_{kt}} L_{jkt}^{1-\delta} Z_{jkt}^\delta, \tag{9}$$

where a_{kt} is an i.i.d. productivity shock to sector k with $\mathbb{E}[a_{kt}] = 0$ and $\mathbb{V}[a_{kt}] = v^2$ for all k, L_{jkt} is labor, and Z_{jkt} is an aggregator of intermediate inputs

$$Z_{jkt} \equiv \left[\sum_{k'=1}^K \omega_{kk'}^{\frac{1}{\eta}} Z_{jk} \left(k' \right)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{10}$$

$Z_{kjt}(r)$ is the amount of intermediate inputs firm jk demands from sector r in period t.

The second real, heterogeneous ingredient of our model is heterogeneity in input-output weights $\{\omega_{kk'}\}_{k,k'}$. We denote these weights in matrix notation as Ω, satisfying $\Omega \iota = \iota$. The demand of firm jk for goods produced in sector k' is given by

$$Z_{jkt}\left(k' \right) = \omega_{kk'} \left(\frac{P_{k't}}{P_t^k} \right)^{-\eta} Z_{jkt}. \tag{11}$$

We can interpret $\omega_{kk'}$ as the steady-state share of goods from sector k' in the intermediate input use of sector k, which determines the input-output linkages across sectors in steady state. Away from the steady state, the gap between the price of goods in sector k' and the aggregate price

relevant for a firm in sector k, P_t^k, distorts input-output linkages

$$P_t^k = \left[\sum_{k'=1}^{K} \omega_{kk'} P_{k't}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad \text{for } k = 1, ..., K. \tag{12}$$

P_t^k uses the sector-specific steady-state input-output linkages to aggregate sectoral prices.

The aggregator $Z_{jk}(k')$ gives the demand of firm jk for goods in sector k'

$$Z_{jk}\left(k'\right) \equiv \left[n_{k'}^{-1/\theta} \int_{\Im_{k'}} Z_{jkt}\left(j', k'\right)^{1-\frac{1}{\theta}} dj' \right]^{\frac{\theta}{\theta-1}}. \tag{13}$$

Firm jk's demand for an arbitrary good j' from sector k' is

$$Z_{jkt}\left(j', k'\right) = \frac{1}{n_{k'}} \left(\frac{P_{j''k't}}{P_{k't}} \right)^{-\theta} Z_{jk}\left(k'\right). \tag{14}$$

In steady state, all firms within a sector share the intermediate input demand of other sectors equally. Away from steady state, the gap between a firm's price and the respective sectoral price index (see equation (6)) distorts the firm's share in the production of intermediate goods. Our economy has $K + 1$ different aggregate prices, one for the household sector and one for each of the K sectors. By contrast, households and all sectors face unique sectoral prices.

The third key ingredient of our model is sectoral heterogeneity in price rigidity. For quantitative purposes, we model price rigidity à la Calvo with parameters $\{\alpha_k\}_{k=1}^{K}$ such that the pricing problem of firm jk is

$$\max_{P_{jkt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^{\tau} \left[P_{jkt} Y_{jkt+s} - MC_{kt+s} Y_{jkt+s} \right].$$

Marginal costs are $MC_{kt} = \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta} e^{-a_{kt}} W_{kt}^{1-\delta} \left(P_t^k \right)^{\delta}$ after we impose the optimal mix of labor and intermediate inputs

$$\delta W_{kt} L_{jkt} = (1 - \delta) P_t^k Z_{jkt}, \tag{15}$$

and $Q_{t,t+s}$ is the stochastic discount factor between periods t and $t + s$.

We assume the elasticities of substitution across and within sectors are the same for households and all firms. This assumption shuts down price discrimination across different

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customers, and firms choose a single price P_{kt}^*

$$\sum_{\tau=0}^{\infty} Q_{t,t+\tau} \alpha_k^s Y_{jkt+\tau} \left[P_{kt}^* - \frac{\theta}{\theta-1} MC_{kt+\tau} \right] = 0, \tag{16}$$

where $Y_{jkt+\tau}$ is the total production of firm jk in period $t + \tau$.

We define idiosyncratic shocks $\{a_{kt}\}_{k=1}^{K}$ at the sectoral level. It follows the optimal price, P_{kt}^*, is the same for all firms in a given sector. Thus, aggregating all prices within sector yields

$$P_{kt} = \left[(1 - \alpha_k) P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{17}$$

C.   Monetary policy, equilibrium conditions, and definitions

Throughout the paper, we show results for several alternative monetary rules, always abstracting from monetary policy shocks. Most of our analytical results assume monetary policy targets the steady-state level of nominal aggregate demand $C_t P_t^c$ or the GDP deflator P_t^c. In our quantitative exercises, we also explore results assuming a Taylor rule of the form

$$I_t = \frac{1}{\beta} \left(\frac{P_t^c}{P_{t-1}^c} \right)^{\phi_\pi} \left(\frac{C_t}{C} \right)^{\phi_y}. \tag{18}$$

Monetary policy reacts to inflation, P_t^c / P_{t-1}^c, with elasticity ϕ_π and deviations of GDP from the steady-state level, C_t / C, with elasticity ϕ_y .

We also explore quantitative results for a case in which the nominal interest rate is at zero for 50 periods due to an aggregate preference shock.

Bonds are in zero net supply, $B_t = 0$, labor markets clear, and goods markets clear such that

$$Y_{jkt} = C_{jkt} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k't}(j,k) \, dj', \tag{19}$$

implying a wedge between gross output, Y_t, and GDP, C_t.

III   Theoretical Results in a Simplified Model

We derive closed-form results in a simplified version of our model. Our goal is to illustrate how the interaction of heterogeneous price rigidity and heterogeneous real features of the economy – viewed through the lens of sectoral size and input-output differences – can affect the propagation of idiosyncratic shocks.

Given the focus of the paper, we study log-linear deviations from steady-state GDP. The

Online Appendix contains the steady-state solution, the full log-linear system, and solves for the equilibrium. We discuss at the end of this section implications of a variety of simplifying assumptions.

A. Simplified Setup

All variables in lower cases denote log-linear deviations from steady state. We make the following simplifying assumptions:

(i) Households have log utility, $\sigma = 1$, and linear disutility of labor, $\varphi = 0$. Thus,

$$w_{kt} = p_t^c + c_t; \tag{20}$$

that is, the labor market is integrated, labor supply is fixed, and nominal wages are proportional to nominal GDP.

(ii) Monetary policy targets steady state nominal GDP, so

$$p_t^c + c_t = 0; \tag{21}$$

and wages remain invariant to shocks.

(iii) We replace Calvo price stickiness by a simple form of price rigidity: all prices are flexible, but with probability λ_k, a firm in sector k has to set its price before observing shocks. Thus,

$$P_{jkt} = \begin{cases} \mathbb{E}_{t-1}\left[P_{jkt}^*\right] & \text{with probability } \lambda_k, \\ P_{jkt}^* & \text{with probability } 1 - \lambda_k, \end{cases} \tag{22}$$

where $\mathbb{E}_{t-1}$ is the expectation operator conditional on the $t-1$ information set. This price-setting technology is a simple way to capture the key ingredient in our model: sectoral heterogeneity of the responsiveness of prices to idiosyncratic shocks.

Solution We show in the Online Appendix that under assumptions (i), (ii), and (iii), GDP solves

$$c_t = \chi' a_t, \tag{23}$$

where

$$\chi \equiv (\mathbb{I} - \Lambda)\left[\mathbb{I} - \delta\Omega'\left(\mathbb{I} - \Lambda\right)\right]^{-1}\Omega_c. \tag{24}$$

Λ is a diagonal matrix with price-rigidity probabilities $[\lambda_1, ..., \lambda_K]$ as entries, Ω_c and Ω, respectively, are steady-state sectoral GDP shares and input-output linkages, and δ is the

intermediate inputs share.

Similarly, our specification of monetary policy implies the GDP deflator is pinned down by

$$p_t^c = -c_t = -\chi' a_t. \tag{25}$$

A linear combination of sectoral shocks describes the log-deviation of GDP and the GDP deflator from steady state up to a first-order approximation. Thus, aggregate volatility of output and prices are

$$v_c = v_p = v \sqrt{\sum_{k=1}^{K} \chi_k^2} = \|\chi\|_2 \, v, \tag{26}$$

because all sectoral shocks have the same volatility; that is, $\mathbb{V}[a_{kt}] = v^2$ for all k. $\|\chi\|_2$ denotes the Euclidean norm of χ. For the rest of the paper, we refer to χ and $\|\chi\|_2$, respectively, as the *vector of sectoral multipliers* and the *total multiplier* of sectoral shocks on the volatility of GDP and the GDP deflator.

Below, we study the effect of the sectoral distribution of price rigidity on aggregate fluctuations from two perspectives: first, in a cross-sectional sense for a given finite number of sectors K; and second, with respect to distributional properties as the economy becomes increasingly more disaggregated, $K \to \infty$. We use these two perspectives to study in two separate sections the interaction between price rigidity and two real features of the economy embodied in equation (24): steady-state sectoral GDP shares Ω_c and steady-state input-output linkages Ω.

Finally, in the analysis that follows, we extensively use the following definition:

Definition 1 *A given random variable X follows a **power-law distribution with shape parameter** β when $\Pr(X > x) = (x/x_0)^{-\beta}$ for $x \geq x_0$ and $\beta > 0$.*

B. Price Rigidity and Sectoral GDP

This subsection demonstrates how heterogeneity in price rigidity interacts with heterogeneity in the first real feature of our economy – sectoral size – in propagating idiosyncratic shocks abstracting from input-output linkages across sectors ($\delta = 0$). We first establish as a benchmark the case of interactions with homogeneous price rigidity across sectors. Then, we contrast results with the case of heterogeneous price rigidity. Our analysis also allows us to relate to the flexible-price results in Gabaix (2011), who establishes conditions on the distribution of firms' size such that idiosyncratic shocks survive disaggregation.

Absent input-output linkages ($\delta = 0$), the vector of sectoral multipliers χ in equation (24)

solves

$$\chi = (\mathbb{I} - \Lambda)\, \Omega_c, \tag{27}$$

or, simply, $\chi_k = (1 - \lambda_k)\, \omega_{ck}$ for all k where $\omega_{ck} \equiv C_k / C$. This expression immediately implies that steady-state sectoral GDP shares ω_{ck} fully determine sectoral multipliers only when prices are fully flexible ($\lambda_k = 0$). In general, sectoral multipliers also depend on the sectoral distribution of price rigidity. Sales alone are no longer a sufficient statistic for the importance of sectors for aggregate volatility of output and prices. This interaction result breaks the Hulten (1978) theorem that sales are a sufficient statistic for the sectoral contribution to aggregate fluctuations.

B.1   Homogeneous Price Rigidity and Sectoral GDP

The following lemma presents our first, cross-sectional result when prices are homogeneously rigid across sectors.

Lemma 1 *When $\delta = 0$ and $\lambda_k = \lambda$ for all k,*

$$v_c = v_p = \frac{(1 - \lambda)\, v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}(C_k) + \overline{C}_k^2},$$

where $\overline{C}_k$ and $\mathbb{V}(\cdot)$ are the sample mean and sample variance of $\{C_k\}_{k=1}^{K}$.[5]

This lemma follows from equation (26) for $\delta = 0$ and serves as a benchmark for the case of full heterogeneity in price rigidity, nesting the analysis in Gabaix (2011). GDP volatility and price stability depend on K and the dispersion of sectoral size, here $\mathbb{V}(C_k)$, for a given number of sectors. Equal price rigidity across sectors only enters as a scale effect through the $1 - \lambda$ term.

In most of the analysis, we abstract from this scale effect for two reasons. First, we show below that active monetary policy offsets this scale effect for GDP fluctuations and even completely eliminates it in the case of price-level targeting. Therefore, we view it as an artifact of the passive monetary policy we assume here to facilitate exposition of our core idea. Second, because we focus on the effect of heterogeneous price rigidity on the capacity of sectoral shocks to become drivers of aggregate fluctuations, we are interested in keeping constant the effect of price rigidity per se on the response of GDP and the GDP deflator to aggregate productivity shocks. This scale effect also exists for aggregate productivity shocks, in which case, $v_c = v_p = (1 - \lambda)\, v$.

We now show results for the rate of decay of the total multiplier of sectoral shocks on aggregate volatility of output and prices as the economy becomes increasingly more disaggregated ($K \to \infty$). Again, we assume the degree of price rigidity is homogeneous across sectors.

---

[5]We define $\mathbb{V}(X_k)$ of a sequence $\{X_k\}_{k=1}^{K}$ as $\mathbb{V}(X_k) \equiv \frac{1}{K} \sum_{k=1}^{K} \left(X_k - \overline{X}\right)^2$. The definition of the sample mean is standard.

Proposition 1 *If $\delta = 0$, $\lambda_k = \lambda$ for all k, and $\{C_k\}_{k=1}^K$ follows a power-law distribution with shape parameter $\beta_c \geq 1$, then*

$$v_c = v_p \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c,1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} v & \text{for } \beta_c = 1, \end{cases}$$

where u_0 is a random variable independent of K and v.

Proof. See Online Appendix. ∎

Proposition 1 shows homogeneous price rigidity plays no role for the rate of decay in output and price volatility, except for the scale effect of Lemma 1. The intuition for this result becomes clear when we compare it to the fully flexible price result in Gabaix (2011): When the size distribution of sectors is Pareto fat-tailed with $\beta_c < 2$, aggregate volatility converges to zero at a rate slower than predicted by the central limit theorem, $K^{1/2}$. The rate of decay becomes slower as $\beta_c \to 1$. Intuitively, when the size distribution is Pareto fat-tailed, few sectors remain disproportionately large at any level of disaggregation. However, a constant rate of price rigidity does not change the fatness of the tails – the distribution is only multiplied by a constant.

B.2 Heterogeneous Price Rigidity and Sectoral GDP

We next consider the effect of heterogeneity in price rigidity. First, we study the cross-sectional interaction effect of heterogeneous price rigidity with heterogeneity in sector size.

Lemma 2 *When $\delta = 0$ and price rigidity is heterogeneous across sectors,*

$$v_c = v_p = \frac{v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left((1-\lambda_k)\,C_k\right) + \left[\left(1-\overline{\lambda}\right)\overline{C}_k - \mathbb{COV}\left(\lambda_k, C_k\right)\right]^2},$$

where $\overline{\lambda}$ is the sample mean of $\{\lambda_k\}_{k=1}^K$ and $\mathbb{COV}\left(\cdot\right)$ is the sample covariance of $\{\lambda_k\}_{k=1}^K$ and $\{C_k\}_{k=1}^K$.[6]

Lemma 2 states the volatility of output and prices depends on the sectoral dispersion of the convolved variable $(1-\lambda_k)C_k$, as well as the covariance between sectoral price rigidity and sectoral GDP. This lemma points to the key insight of the paper: Heterogeneity of price rigidity has the power to increase or decrease aggregate volatility of output and prices relative to the case of homogeneous price rigidity in Lemma 1. A scale effect implicitly also exists in Lemma 2 but now depends on the average price rigidity in the economy, $\overline{\lambda}$, rather than λ as in Lemma 1.

---

[6]We define $\mathbb{COV}\left(X_k, Q_k\right)$ of sequences $\{X_k\}_{k=1}^K$ and $\{Q_k\}_{k=1}^K$ as $\mathbb{COV}\left(X_k, Q_k\right) \equiv \frac{1}{K}\sum_{k=1}^K \left(X_k - \overline{X}\right)\left(Q_k - \overline{Q}\right)$.

Depending on the cross-sectional dispersion of price rigidity, sectoral shocks may even generate sizable aggregate volatility when all sectors have equal size. To illustrate this possible implication, consider $C_k = C/K$ for all k, so that Lemma 2 implies

$$v_c = v_p = \frac{v}{K^{1/2}} \sqrt{\mathbb{V}\left(1 - \lambda_k\right) + \left(1 - \overline{\lambda}\right)^2}.$$

This result highlights the potential of heterogeneous price rigidity across sectors to become a "frictional" force that increases the propagation of sectoral productivity shocks on GDP and price volatility at a finite degree of disaggregation. However, heterogeneity in price rigidity does not have the power to affect the rate of decay itself – absent heterogeneity in another real feature – as the next proposition points out.

Proposition 2 *If $\delta = 0$, $C_k/C = 1/K$, and the distribution of λ_k satisfies*

$$\Pr\left[1 - \lambda_k > y\right] = \frac{y^{-\beta_\lambda} - 1}{y_0^{-\beta_\lambda} - 1} \text{ for } y \in [y_0, 1], \beta_\lambda > 0,$$

then $v_c = v_p \sim v/K^{1/2}$.

Proof. See Online Appendix. ∎

This proposition shows that if all sectors have equal size (here, GDP shares), the central limit theorem governs the rate at which shocks die out, independent of the distribution of price rigidity. This result is due to the boundedness of price rigidity. If λ_k followed a Pareto distribution with support unbounded below, the rate of decay would depend on the shape parameter β_λ, exactly as in Proposition 1. However, the bound of λ_k at 0 implies the second moment of the distribution exists for any shape parameter.

The next proposition shows a similar result when GDP shares and price rigidity follow independent Pareto distributions.

Proposition 3 *If $\delta = 0$, λ_k and C_k are independently distributed, the distribution of λ_k satisfies*

$$\Pr\left[1 - \lambda_k > y\right] = \frac{y^{-\beta_\lambda} - 1}{y_0^{-\beta_\lambda} - 1} \text{ for } y \in [y_0, 1], \beta_\lambda > 0,$$

and C_k follows a power-law distribution with shape parameter $\beta_c \geq 1$, then

$$v_c = v_p \sim \begin{cases} \frac{u_0}{K^{\min\{1 - 1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} & \text{for } \beta_c = 1, \end{cases}$$

where u_0 is a random variable independent of K and v.

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Proof. See Online Appendix. ∎

As in Proposition 2, this result is due to the lower bound on the support of price rigidity, which implies the convolution $(1 - \lambda_k) C_k$ follows a Pareto distribution with relevant shape parameter β_c, the shape parameter of the sectoral GDP Pareto distribution.

Yet two remarks about the economic importance of heterogeneous price rigidity are in order. First, at any degree of disaggregation, heterogeneity of price rigidity through its interaction with sector size changes the identity of sectors from which aggregate fluctuations originate. This observation is important given the emphasis of the recent granularity literature that aims at identifying the microeconomic origin of aggregate fluctuations, for example, for stabilization purposes. If shocks are idiosyncratic, re-weighting sectoral shocks of potentially opposite signs can easily change the sign of business cycles relative to a frictionless economy. The intuition is that a large (small) sector can become effectively small (large) if it has rigid (flexible) prices; that is, price rigidity reduces (increases) its *effective* size through the convolution of $(1 - \lambda_k) C_k$. Second, propositions 2 and 3 are consistent with Lemma 2 because they are silent on the degree of disaggregation for which they apply. This distinction is important for our later quantitative analysis where heterogeneity of price rigidity will generate relevant aggregate volatility from sectoral productivity shocks when all sectors have equal size.

Finally, consider the case when price rigidity and GDP shares are not independently distributed.

Proposition 4 *If* $\delta = 0$, $\{(1 - \lambda_k) C_k\}_{k=1}^{K}$ *follows a power-law distribution with shape parameter* $\beta_{\lambda c} \geq 1$, *then*

$$v_c = v_p \sim \begin{cases} \frac{u_0}{K^{\min\{1 - 1/\beta_{\lambda c}, 1/2\}}} v & \text{for } \beta_{\lambda c} > 1 \\ \frac{u_0}{\log K} v & \text{for } \beta_{\lambda c} = 1, \end{cases}$$

where u_0 *is a random variable independent of* K *and* v.

This proposition requires no explicit proof because it is identical to Proposition 1. It simply states that, in the general case, if the convolution $(1 - \lambda_k) C_k$ follows a Pareto distribution, the rate of decay as $K \to \infty$ depends on the shape parameter of the joint distribution, $\beta_{\lambda c}$. This result holds despite the bounded support of price rigidity.

For illustration, consider the following example. Assume C_k follows a Pareto distribution with shape parameter β_c, and price rigidity is given by

$$\lambda_k = \max \left\{ 0, 1 - \phi C_k^{\mu} \right\}$$

for $\mu \in (-1, 0)$ and $\phi > 0$. This relationship implies larger sectors have more rigid prices. In this case, $(1 - \lambda_k) C_k$ is Pareto distributed with shape parameter $\beta_{\lambda c} = \beta_c / (1 + \mu)$. Hence, price rigidity accelerates the rate of decay relative to a frictionless economy or one with homogeneous price rigidity. The rate of decay might even be slower than $\sqrt{K}$ in an economy with flexible prices, whereas the central limit theorem applies in an economy with rigid prices.

An illustrative, slightly more complicated example of the converse holds when we assume $\mu \in (0, 1)$ and the same functional relationship between λ_k and C_k as above. Now, any sector with GDP higher than $\phi^{-1/\mu}$ has fully flexible prices. In other words, price rigidity and sectoral GDP become independent for sectors larger than $\phi^{-1/\mu}$. As a result, $(1 - \lambda_k) C_k$ follows a Pareto distribution with shape parameter $\beta_{\lambda c} = \beta_c / (1 + \mu)$ for $C_k < \phi^{-1/\mu}$ but β_c for $C_k \geq \phi^{-1/\mu}$. Following the same steps as before, one can show the rate of decay depends on $\beta_c / (1 + \mu)$ for $K < K^*$ and on β_c for $K \geq K^*$ with $K^* \equiv \left(x_0 \phi^{1/\mu} \right)^{-\beta_c}$. Intuitively, this condition means when the number of sectors is large enough, $K > K^*$, sectors with fully flexible prices dominate the upper tail of the sectoral GDP distribution. Then, the rate of decay is the same as in a frictionless economy.

This result is weaker than when $\mu \in (-1, 0)$, but this example is still relevant if the cut-off number of sectors, K^*, is large. This happens for a level of disaggregation where fully flexible sectors do not dominate the upper tail of the sectoral GDP distribution. Indeed, in our quantitative analysis with 341 sectors, no single sector exists with fully flexible prices.[7]

C. Price Rigidity and Input-Output Linkages

We now demonstrate how the interaction of heterogeneity in price rigidity and our second real feature of the economy – heterogeneity in input-output linkages – interact in propagating idiosyncratic shocks. To show this point, we assume a positive intermediate input share, $\delta \in (0, 1)$, but shut down the heterogeneity of sectoral GDP shares; that is, $C_k = C/K$.

The vector of multipliers χ in equation (24) now solves

$$\chi \equiv \frac{1}{K} (\mathbb{I} - \Lambda) \left[\mathbb{I} - \delta \Omega' (\mathbb{I} - \Lambda) \right]^{-1} \iota. \tag{28}$$

This expression shows a non-trivial interaction exists between price rigidity and input-

---

[7]Although we have assumed examples with a deterministic relationship between sectoral GDP and price rigidity for expositional convenience, the arguments extend to a stochastic relationship. Similarly, a stochastic relationship between price rigidity and sectoral GDP implies price rigidity has an effect on the identity of the most important sectors for GDP volatility.

output linkages across sectors in general.[8] To study this interaction, we use a second-order approximation of the vector of multipliers, following Acemoglu et al. (2012)

$$\chi \simeq \frac{1}{K} \left(\mathbb{I} - \Lambda\right) \left[\mathbb{I} + \delta\Omega'\left(\mathbb{I} - \Lambda\right) + \delta^2 \left[\Omega'\left(\mathbb{I} - \Lambda\right)\right]^2\right] \iota. \tag{29}$$

C.1 Homogeneous Price Rigidity and Input-Output Linkages

The following lemma presents our first cross-sectional result when price rigidity is homogeneous across sectors.

Lemma 3 *If $\delta \in (0, 1)$, $\Omega_c = \frac{1}{K}\iota$, and $\lambda_k = \lambda$ for all k, then*

$$v_c = v_p \geq \frac{(1-\lambda)\,v}{K^{1/2}} \sqrt{\kappa + \delta'^2\mathbb{V}\left(d_k\right) + 2\delta'^3\mathbb{COV}\left(d_k, q_k\right) + \delta'^4\mathbb{V}\left(q_k\right)}, \tag{30}$$

where $\kappa \equiv 1 + 2\delta' + 3\delta'^2 + 2\delta'^3 + \delta'^4$, $\delta' \equiv \delta\left(1 - \lambda\right)$, $\mathbb{V}\left(\cdot\right)$ and $\mathbb{COV}\left(\cdot\right)$ are the sample variance and covariance statistics across sectors, and $\{d_k\}_{k=1}^K$ and $\{q_k\}_{k=1}^K$ are the outdegrees *and* second-order outdegrees, *respectively, defined for all $k = 1, ..., K$ as*

$$
\begin{aligned}
d_k &\equiv \sum_{k'=1}^K \omega_{k'k}, \\
q_k &\equiv \sum_{k'=1}^K d_{k'}\omega_{k'k}.
\end{aligned}
$$

Lemma 3 follows from equation (26), $d = \Omega'\iota$ and $q = \Omega'^2\iota$. The inequality sign holds because the exact solution for χ is strictly larger than the approximation. "Outdegrees" and "second-order outdegrees" measure the centrality of sectors in the production network, as in Acemoglu et al. (2012): d_k measures the importance of sectors as suppliers of intermediate inputs, whereas q_k measures the importance of sectors as suppliers of large suppliers of intermediate inputs. Upstream effects through demand of intermediate inputs do not play any role here due to our focus on GDP and the GDP deflator, and our assumptions that imply unresponsiveness of wages to shocks. We discuss below what happens when we relax these assumptions.

This lemma establishes two important insights: First, for a finite level of disaggregation, aggregate volatility due to sectoral shocks is higher if the production network is more asymmetric, that is, if a higher cross-sectional dispersion of outdegrees and second-order outdegrees exists.

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[8]This expression nests the solution for the "influence vector" in Acemoglu et al. (2012) when prices are fully flexible, that is, $\lambda_k = 0$ for all $k = 1, ..., K$. The only difference here is $\chi'\iota = 1/\left(1 - \delta\right)$, because Acemoglu et al. (2012) normalize the scale of shocks such that the sum of the influence vector equals 1.

Homogeneous price rigidity across sectors has, similar to Lemma 1, a scale effect on aggregate volatility. Second, price rigidity adds the following new insight: Price rigidity penalizes more strongly the quantitative effect of heterogeneity in second-order outdegrees than in first-order outdegrees. This result is important because the flexible-price analysis of Acemoglu et al. (2012) stresses second-order outdegrees contribute more to aggregate fluctuations from idiosyncratic shocks than first-order outdegrees. In general, inter-connections of order τ are penalized by a factor $(1 - \lambda)^{\tau}$.

The next proposition shows results for the rate of decay of aggregate volatility as $K \to \infty$, still under the benchmark assumption of homogeneous price rigidity.

Proposition 5 *If $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, $\Omega_c = \frac{1}{K}\iota$, the distribution of outdegrees $\{d_k\}$, second-order outdegrees $\{q_k\}$, and the product of outdegrees $\{z_k = d_k q_k\}$ follow power-law distributions with respective shape parameters $\beta_d, \beta_q, \beta_z > 1$ such that $\beta_z \geq \frac{1}{2}\min\{\beta_d, \beta_q\}$, then*

$$
v_c = v_p \geq \begin{cases} \frac{u_3}{K^{1/2}} v & \text{for } \min\{\beta_d, \beta_q\} \geq 2, \\ \frac{u_3}{K^{1-1/\min\{\beta_d, \beta_q\}}} v & \text{for } \min\{\beta_d, \beta_q\} \in (1,2), \end{cases}
$$

where u_3 is a random variable independent of K and v.

Proof. See Online Appendix. ∎

Proposition 5 shows homogeneous sectoral price rigidity has no impact on the rate of decay. The intuition from the previous section continues to hold: multiplying the distributions of a real feature with a constant does not change the fatness of the tails. Therefore, the rate of decay only depends on the shape parameters of measures of network centrality, d and q, as in the flexible-price case of Acemoglu et al. (2012). The fattest tail among these distributions determines the rate of decay of aggregate volatility of output and prices.[9]

C.2 Heterogeneous Price Rigidity and Input-Output Linkages

Next, we study the interaction of heterogeneous price rigidity with heterogeneous input-output linkages. Again, we start with aggregate volatility for a finite level of disaggregation.

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[9] Acemoglu et al. (2012) document in the U.S. data that $\beta_d \approx 1.4$ and $\beta_q \approx 1.2$. We find very similar numbers. Thus, we abstract from the case $\min\{\beta_d, \beta_q\} = 1$ in Proposition 5.

Lemma 4 If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, and price rigidity is heterogeneous across sectors, then

$$v_c = v_p \geq \frac{v}{K^{1/2}}\left[\begin{array}{c}\left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[\widetilde{\kappa} + \delta^2 \mathbb{V}\left(\widetilde{d}_k\right) + 2\delta'^3 \mathbb{COV}\left(\widetilde{d}_k, \widetilde{q}_k\right) + \delta'^4 \mathbb{V}\left(\widetilde{q}_k\right)\right] \\ -\left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[2\delta^2\left(1+\widetilde{\delta}+\widetilde{\delta}^2\right)\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) + \delta^4 \mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right)^2\right] \\ + \mathbb{COV}\left((1-\lambda_k)^2, \left(1+\delta\widetilde{d}_k + \delta^2 \widetilde{q}_k\right)^2\right)\end{array}\right]^{\frac{1}{2}},$$

(31)

where $\widetilde{\kappa} \equiv 1 + 2\widetilde{\delta} + 3\widetilde{\delta} + 2\widetilde{\delta} + \widetilde{\delta}$, $\widetilde{\delta} \equiv \delta\left(1-\overline{\lambda}\right)$, $\overline{\lambda}$ is the sample mean of $\{\lambda_k\}_{k=1}^{K}$, $\mathbb{V}(\cdot)$ and $\mathbb{COV}(\cdot)$ are the sample variance and covariance statistics across sectors, and $\left\{\widetilde{d}_k\right\}_{k=1}^{K}$ and $\{\widetilde{q}_k\}_{k=1}^{K}$ are the **modified outdegrees** and **modified second-order outdegrees**, respectively, defined for all $k = 1, ..., K$ as

$$\widetilde{d}_k \equiv \left(1-\overline{\lambda}\right)\sum_{k'=1}^{K}\left(\frac{1-\lambda_{k'}}{1-\overline{\lambda}}\right)\omega_{k'k},$$

$$\widetilde{q}_k \equiv \left(1-\overline{\lambda}\right)\sum_{k'=1}^{K}\left(\frac{1-\lambda_{k'}}{1-\overline{\lambda}}\right)\widetilde{d}_{k'}\omega_{k'k}.$$

Lemma 4 embodies the central result in this section, derived directly from equation (26) with $\widetilde{d} = \Omega'\left(\mathbb{I} - \Lambda\right)\iota$ and $\widetilde{q} = \left[\Omega'\left(\mathbb{I} - \Lambda\right)\right]^2 \iota$. What matters for aggregate volatility is now the *effective* centrality of sectors in the production network – after adjusting nodes by their degree of price rigidity. In particular, $\widetilde{d}_k$ measures the importance of sectors as large suppliers of sectors with highly flexible prices. Similarly, $\widetilde{q}_k$ measures the importance of sectors as large suppliers of highly flexible sectors that are large suppliers of highly flexible sectors. A key implication of this result is that – depending on the cross-sectoral distribution of price rigidity – $\widetilde{d}$ and $\widetilde{q}$ may be heterogeneous across sectors even when the production network is perfectly symmetric. This result reinforces the importance of the interaction of the nominal and real heterogenous features of the economy and perfectly resembles the result above when we focus on sectoral GDP.

As in the case of heterogeneity in size, we again find a scale effect that depends on the average degree of price rigidity in the economy. On top of this scale effect, the first line on the right-hand side of equation (31) for heterogeneous price rigidity is similar to the one in equation (30) for homogeneous price rigidity in Lemma 3, with two differences. First, by Jensen's inequality,

$$\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2 \geq \left(1-\overline{\lambda}\right)^2.$$

Thus, price rigidity mutes aggregate volatility by less if price rigidity is heterogeneous across

sectors relative to an economy with $\lambda_k = \overline{\lambda}$ for all k.

Second, we now compute key statistics using modified outdegrees, that is, $\widetilde{d}$ and $\widetilde{q}$ instead of d and q. To see the implications, note

$$
\begin{aligned}
\widetilde{d}_k &= \left(1-\overline{\lambda}\right) d_k - K\mathbb{COV}\left(\lambda_{k'}, \omega_{k'k}\right), \\
\widetilde{q}_k &= \left(1-\overline{\lambda}\right)^2 q_k - K\mathbb{COV}\left(\lambda_{k'}, \widetilde{d}_k \omega_{k'k}\right) - \left(1-\overline{\lambda}\right) \sum_{k'=1}^K \omega_{k'k}\mathbb{COV}\left(\lambda_s, \widetilde{d}_s \omega_{sk'}\right).
\end{aligned}
$$

The dispersion of $\widetilde{d}$ is higher than the dispersion of $\left(1-\overline{\lambda}\right) d$ when $\mathbb{COV}\left(\lambda_{k'}, \omega_{k'k}\right)$ is more dispersed across sectors and when it is negatively correlated with d. In words, the dispersion of $\widetilde{d}$ is high when the intermediate input demand of the most flexible sectors is highly unequal across supplying sectors, and when large intermediate input-supplying sectors are also large suppliers to flexible sectors. Similarly, the dispersion of $\widetilde{q}$ is higher than the dispersion of $\left(1-\overline{\lambda}\right)^2 q$ when $\mathbb{COV}\left(\lambda_{k'}, \widetilde{d}_k \omega_{k'k}\right)$ is more dispersed and is negatively correlated with q.

The second and third lines on the right-hand side of the lower bound in equation (31) capture new effects. In particular, aggregate volatility is higher when $\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) < 0$, that is, if sectors with high modified outdegrees, $\widetilde{d}_k$, are the most flexible sectors (second line), and if Jensen's inequality effect is stronger (third line).

Analyzing the effect of the heterogeneous price rigidity on the rate of decay as $K \to \infty$ is more intricate than in the case with no intermediate inputs ($\delta = 0$).

Proposition 6 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, price rigidity is heterogeneous across sectors, the distribution of modified outdegrees $\left\{\widetilde{d}_k\right\}$, modified second-order outdegrees $\left\{\widetilde{q}_k\right\}$, and the product $\left\{z_k = \widetilde{d}_k \widetilde{q}_k\right\}$ follow power-law distributions with respective shape parameter $\widetilde{\beta}_d, \widetilde{\beta}_q, \widetilde{\beta}_z > 1$ such that $\widetilde{\beta}_z \geq \frac{1}{2} \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}$, then*

$$
v_c = v_p \geq \begin{cases} \dfrac{u_4}{K^{1/2}} v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \geq 2, \\[2ex] \dfrac{u_4}{K^{1-1/\min\{\widetilde{\beta}_d, \widetilde{\beta}_q\}}} v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \in (1,2), \end{cases}
$$

where u_4 is a random variable independent of K and v.

This result resembles Proposition 4, but in the context of production networks. Although heterogeneous price rigidity interacts with heterogeneous input-output linkages in a more complicated way than with heterogeneity in size, the fundamental intuition is very similar to that in the previous section: If sectors with the most rigid (flexible) prices are also the most central in the *effective*, price-rigidity-adjusted production network such that $\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} >$

$(<) \min \{\beta_d, \beta_q\}$, then volatility may decay at a rate faster (slower) than when price rigidity is homogeneous across sectors or is independent of network centrality. Just as for the interaction with sectoral size, heterogeneity in price rigidity does have an effect for a finite degree of disaggregation, as stressed in Lemma 4. This result holds regardless of whether or not price rigidity affects the rate of decay. Moreover, akin to the interaction with sectoral size, price rigidity distorts the identity of sectors from which aggregate fluctuations originate when idiosyncratic shocks drive aggregate volatility through the network. Re-weighting sectoral shocks of potentially opposite signs can easily change the sign of business cycles. What matters is the *effective* not the physical network structure.

D. Relaxing Simplifying Assumptions

We now discuss the implications of relaxing several simplifying assumptions.

Elastic labor supply Our first sensitivity analysis allows labor supply to respond elastically to shocks, with the inverse-Frisch elasticity $\varphi > 0$. This possibility opens up new channels for GDP shares, input-output linkages, and price rigidity to affect the propagation of sectoral shocks. However, the qualitative interaction between heterogeneous price rigidity and the heterogeneous real channels remains unaffected. We explore the additional feedback mechanisms below. First, labor supply and demand now jointly determine wages such that

$$w_{kt} = c_t + p_t^c + \varphi l_{kt} \tag{32}$$

becomes the log-linear counterpart to equation (7). Thus, with monetary policy targeting $c_t + p_t^c = 0$, it no longer holds that sectoral productivity shocks have no effect on wages.[10] Because the labor market is sectorally segmented, wages may vary across sectors. The production function implies the log-linear deviation in labor demand is

$$l_{kt}^d = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right). \tag{33}$$

Conditioning on sectors' gross output y_{kt}, productivity shocks have a negative direct effect on labor demand of the shocked sector. But shocks also have an indirect negative effect in all sectors as prices respond to the shock through sector-specific aggregate prices of intermediate inputs, p_t^k, which depend on the input-output linkages across sectors.

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[10]The Online Appendix contains details of the derivations.

In turn, sectoral gross output y_{kt} depends on prices according to

$$y_{kt} = y_t - \eta \left(p_{kt} - \left[(1 - \psi) \, p_t^c + \psi \widetilde{p}_t \right] \right), \tag{34}$$

such that, conditioning on total gross output y_t, a productivity shock in sector k increases its own demand to the extent that its price p_{kt} responds to the shock. This increase in demand is a force pushing wages up in the shocked sector, in opposition to all others forces described so far. But the shock in sector k decreases demand (and thus wages) in all other sectors, depending on the overall responses of prices and GDP shares captured by the GDP deflator, p_t^c, and the share of sectors in the production of intermediate inputs, captured by $\widetilde{p}_t$

$$\widetilde{p}_t = \sum_{k'=1}^{K} \zeta_{k'} p_{k't}, \tag{35}$$

where $\{\zeta_k\}_{k=1}^{K}$ are the steady-state shares of sectors in total production of intermediate inputs

$$\zeta_k \equiv \sum_{k'=1}^{K} n_{k'} \omega_{k'k}, \tag{36}$$

and $\{n_k\}_{k=1}^{\infty}$ are the steady-state shares of sectors in aggregate gross output

$$n_k = (1 - \psi) \, \omega_{ck} + \psi \zeta_k \text{ for all } k = 1, ..., K. \tag{37}$$

$\psi \equiv Z/Y$ is the fraction of total gross output used as intermediate input in steady state.

Finally, the response of total gross output y_t to shocks depends on the response of GDP, c_t, and production of intermediate inputs, z_t, according to

$$y_t = (1 - \psi) \, c_t + \psi z_t, \tag{38}$$

such that z_t solves

$$z_t = (1 + \Gamma_c) \, c_t + \Gamma_p \left(p_t^c - \widetilde{p}_t \right) - \Gamma_a \sum_{k'=1}^{K} n_{k'} a_{k't} \tag{39}$$

and $\Gamma_c \equiv \frac{(1-\delta)(\sigma+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}$. The total production of intermediate inputs z_t depends positively on the production of final goods c_t and the relative price of final goods and intermediate inputs, $p_t^c - \widetilde{p}_t$, and depends negatively on the economy-wide change in productivity, $\sum_{k'=1}^{K} n_{k'} a_{k't}$.

Summing up, equation (23) still gives the solution for c_t, and combined with exogenous

nominal demand, the GDP deflator, p_t^c, but the vector of multipliers χ is now

$$\chi \equiv (\mathbb{I} - \Lambda) \left[\gamma_1 \mathbb{I} + \gamma_2 \aleph \iota' \right] \left[\mathbb{I} - \varphi \left[\gamma_3 \iota \Omega_c' + \gamma_4 \iota \vartheta' - \gamma_5 \iota' \right] (\mathbb{I} - \Lambda) - \gamma_6 \Omega' (\mathbb{I} - \Lambda) \right]^{-1} \Omega_c, \qquad (40)$$

with $\gamma_1 \equiv \frac{1+\varphi}{1+\delta\varphi}$, $\gamma_2 \equiv \frac{\psi(1-\delta)\Gamma_a}{1+\delta\varphi}$, $\gamma_3 \equiv \frac{(1-\delta)[(1-\psi)\eta-1]}{1+\delta\varphi}$, $\gamma_4 \equiv \frac{\psi(1-\delta)(\eta-\Gamma_p)}{1+\delta\varphi}$, $\gamma_5 \equiv \frac{\gamma_2}{\Gamma_a}$, $\gamma_6 \equiv \delta\gamma_1$, $\aleph \equiv (n_1, ..., n_K)'$, and $\vartheta = (\zeta_1, ..., \zeta_K)'$.

Relative to the solution for χ in equation (28), multipliers take a richer functional form, capturing all new channels elastic labor demand introduces on the propagation of sectoral shocks. But the effect of price rigidity still operates through its interaction with sectoral GDP shares and input-output linkages, just like before.

To explore the quantitative effect of elastic labor supply, consider the case of no input-output linkages ($\delta = 0$), in which case

$$\chi = (\mathbb{I} - \Theta) \, \Omega_c,$$

where Θ is a diagonal matrix with entries

$$\frac{1 - \lambda_k}{1 + \varphi\eta(1-\lambda_k)} \left[1 - \varphi(\eta-1) \sum_{k'=1}^{K} \frac{\omega_{ck'}(1-\lambda_{k'})}{1 + \varphi\eta(1-\lambda_{k'})} \right]^{-1},$$

for $k = 1, ..., K$ in its diagonal. Note $\Theta = \Lambda$ when $\varphi = 0$. According to equation (26), the inverse-Frisch elasticity φ has two opposite effects on the capacity of price rigidity to generate aggregate volatility from sectoral productivity shocks. On the one hand, if sector k has more flexible prices, its demand responds by more to its own productivity shocks, so wages in the shocked sector respond by more. This effect is captured by the denominator of the term outside the brackets. On the other hand, the response of prices in the shocked sector has an effect on demand of other sectors. This effect is captured by the term in brackets, which, because it is common to all sectors, enters as part of a scale effect. Thus, in the absence no input-output linkages ($\delta = 0$), more elastic labor supply reduces the quantitative importance of price rigidity to generate fluctuations. However, because both effects operate through sectoral demand, the effect of φ depends on the elasticity of substitution across sectors, η. Quantitatively, empirical estimates suggest η is small (see Atalay (2017) and Feenstra, Luck, Obstfeld, and Russ (2018)).

With input-output linkages ($\delta > 0$), another force exists that reduces the importance of price rigidity: $\varphi \left[\gamma_3 \iota \Omega_c' + \gamma_4 \iota \vartheta' - \gamma_5 \iota' \right] (\mathbb{I} - \Lambda)$ in equation (40). This term collects the common effect of price rigidity on sectoral multipliers that reduce the additional cross-sectoral dispersion of multipliers that price rigidity generates.

Active Monetary Policy The specification of monetary policy crucially affects the response of aggregate output and prices. Because our exogenous nominal demand specification mainly aims at providing intuition, we perform a detailed analysis of various monetary policy specification in the calibration section. Here, we focus on the ability of active monetary policy to eliminate the scale effect emphasized in lemmas 1 and 3.

Consider the general solution for log-linear deviations of GDP for an arbitrary monetary policy assuming the monetary policy instrument is money supply

$$c_t = \left(1 - \chi'\iota\right) m_t + \chi'a_t, \tag{41}$$

where m_t is the log-linear deviation of money supply from steady state. This expression is valid for any assumptions on the inverse-Frisch elasticity, $\varphi \geq 0$, and the joint distribution of sectoral GDP, input-output linkages, and price rigidity.

We can directly see from equation (41) that monetary policy does not interact with the effect of heterogeneous price rigidity for the aggregate implication of sectoral shocks; it only introduces a scale effect regardless of whether shocks are aggregate or sector-specific. As the negative sign in the first term indicates, this effect partially offsets the level effect due to the average level of price rigidity in the economy, which we established in lemmas 1 and 3.

More concretely, consider an extreme version of a price-level targeting such that monetary policy stabilizes the GDP deflator, by setting $p_t^c = 0$. In this case,

$$c_t = \frac{\chi'a_t}{\chi'\iota}. \tag{42}$$

Price-level targeting reproduces the flexible-price response of the economy to aggregate productivity shocks for any joint distribution of sectoral GDP, input-output linkages, and price rigidity. As a result, this monetary policy rule perfectly offsets the scale effect stressed in lemmas 1 and 3, so the multiplier of aggregate productivity shocks on GDP volatility is 1. Clearly, this specification also has an effect on aggregate price stability. Under alternative, inefficient monetary policy rules, the scale effect is not completely offset, because the joint distribution of sectoral GDP, input-output linkages, and price rigidity does affect the response of c_t to aggregate productivity shocks. At the same time, the interactions of heterogeneity in nominal and real features will continue to affect the GDP deflator.

Pricing Friction Our analytical model uses a simplified form of price-setting. However, modeling price rigidities à la Calvo does not change our analytical results. We study the exact

quantitative effects in our subsequent calibrations. Calvo pricing adds serial correlation in the response of prices even when shocks are i.i.d.

$$p_{kt} = (1 + \beta + \xi_k)^{-1} \left[\xi_k mc_{kt} + \beta \mathbb{E}\left[p_{kt+1} \right] + p_{kt-1} \right] \text{ for } k = 1, ..., K, \tag{43}$$

where $\xi_k \equiv (1 - \alpha_k)(1 - \beta\alpha_k)/\alpha_k$.

The exact monetary policy specification will determine how much of the serial correlation the Calvo friction introduces transmits to the response of output. Parallel to our analytical results above, we adjust the definitions of the multipliers to capture possible serial correlation for our calibrations in the next section. Using a general representation of the solution for c_t,

$$c_t = \sum_{\tau=0}^{\infty} \sum_{k=1}^{K} \rho_{k\tau} a_{kt-\tau}, \tag{44}$$

we redefine multipliers as

$$\chi_k \equiv \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}, \tag{45}$$

such that $v_c = \|\chi\|_2 v$ still holds. We compare the multiplier of sectoral shocks to the multiplier of aggregate shocks, which now is given by $\sqrt{\sum_{\tau=0}^{\infty} \left(\sum_{k=1}^{K} \rho_{k\tau} \right)^2}$. For the monetary policy rule that targets the GDP deflator ($p_t^c = 0$), this comparison does not add any information, because the multiplier of aggregate productivity shocks on GDP volatility is 1, exactly as in our analytical model with static price rigidity. This result validates the use of our analytical model to shed light on the interaction of price rigidity with sectoral GDP and input-output linkages to generate aggregate fluctuations even when the pricing friction has dynamic effects as with Calvo.

Endogenous Price Adjustment   In our setup, price rigidity is exogenous, and thus the degree of price rigidity and the process of sectoral shocks are unrelated. With an endogenous pricing friction, a link could exist between price flexibility and the volatility of shocks, or other moments of the distribution of shocks, or model elements that pin down the sS bands. However, how this concern would change our results is not clear.

First, the key *theoretical* mechanism of our model would qualitatively remain unchanged: Again, the effective not the physical importance of a sector would determine the propagation of shocks. The effective importance would now only be differently defined: It would depend on the interaction of the real features with the primitives that pin down the width of the sS bands, such as menu costs, elasticities, or shock volatilities.

Second, this theoretical possibility thus comes with several degrees of freedom. A priori in which direction – if any – it would change our subsequent quantitative results is unclear. We cannot quantify such effects without implementing a state-dependent pricing model. Empirically, no strong relationship exists between volatility of shocks and the frequency of price changes for manufacturing sectors. A regression of the frequency of price changes on the volatility of productivity shocks results in an R^2 of less than 2%, which shows these shocks explain very little of the variation in the frequency of price changes in our data.

Third, such an analysis would face major computational challenges. Our level of disaggregation makes endogenous pricing models such as menu-cost models computationally infeasible because the intermediate input price index represents a different state variable for each sector. Therefore, we leave a formal analysis of the implications of endogenous price rigidity for future research, and our results should be seen as a first-order approximation.

Independent of these considerations, differences in pass-through of shocks – however micro-founded – will change the effective real structure of the economy.

IV Data

This section describes the data we use to construct the input-output linkages, and sectoral GDP, and the micro-pricing data we use to construct measures of price stickiness at the sectoral level.

A. Input-Output Linkages and Sectoral Consumption Shares

The Bureau of Economic Analysis (BEA) produces input-output tables detailing the dollar flows between all producers and purchasers in the U.S. Producers include all industrial and service sectors, as well as household production. Purchasers include industrial sectors, households, and government entities. The BEA constructs the input-output tables using Census data that are collected every five years. The BEA publishes input-output tables every five years beginning in 1982 and ending the most recent tables in 2016. The input-output tables are based on NAICS industry codes. Prior to 1997, the input-output tables were based on SIC codes.

The input-output tables consist of two basic national-accounting tables: a "make" table and a "use" table. The make table shows the production of commodities by industry. Rows present industries, and columns present the commodities each industry produces. Looking across columns for a given row, we see all the commodities a given industry produces. The sum of the entries comprises industry output. Looking across rows for a given column, we see all industries producing a given commodity. The sum of the entries is the output of a commodity. The use

table contains the uses of commodities by intermediate and final users. The rows in the use table contain the commodities, and the columns show the industries and final users that utilize them. The sum of the entries in a row is the output of that commodity. The columns document the products each industry uses as inputs and the three components of value added: compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. The sum of the entries in a column is industry output.

We utilize the input-output tables for 2002 to create an industry network of trade flows. The BEA defines industries at two levels of aggregation: detailed and summary accounts. We use the detailed levels of aggregation to create industry-by-industry trade flows. The BEA also provides the data to calibrate sectoral GDP shares.

The BEA provides concordance tables between NAICS codes and input-output industry codes. We follow the BEA's input-output classifications with minor modifications to create our industry classifications. We account for duplicates when NAICS codes are not as detailed as input-output codes. In some cases, an identical set of NAICS codes defines different input-output industry codes. We aggregate industries with overlapping NAICS codes to remove duplicates.

We combine the make and use tables to construct an industry-by-industry matrix that details how much of an industry's inputs other industries produce. We use the make table ($MAKE$) to determine the share of each commodity each industry k produces. We define the market share ($SHARE$) of industry k's production of commodities as

$$SHARE = MAKE \odot (\mathbb{I} - MAKE)^{-1}_{k,k'}.$$

We multiply the share and use tables (USE) to calculate the dollar amount industry k' sells to industry k. We label this matrix revenue share ($REVSHARE$), which is a supplier industry-by-consumer industry matrix,

$$REVSHARE = SHARE \times USE.$$

We then use the revenue-share matrix to calculate the percentage of industry k' inputs purchased from industry k, and label the resulting matrix $SUPPSHARE$

$$SUPPSHARE = REVSHARE \odot \left((\mathbb{I} - MAKE)^{-1}_{k,k'} \right)'. \tag{46}$$

The input-share matrix in this equation is an industry-by-industry matrix and therefore

consistently maps into our model.[11]

B. Frequencies of Price Adjustments

We use the confidential microdata underlying the producer price data (PPI) from the BLS to calculate the frequency of price adjustment at the industry level.[12] The PPI measures changes in prices from the perspective of producers, and tracks prices of all goods-producing industries, such as mining, manufacturing, and gas and electricity, as well as the service sector. The BLS started sampling prices for the service sector in 2005. The PPI covers about 75% of the service-sector output. Our sample ranges from 2005 to 2011.

The BLS applies a three-stage procedure to determine the sample of goods. First, to construct the universe of all establishments in the U.S., the BLS compiles a list of all firms filing with the Unemployment Insurance system. In the second and third stages, the BLS probabilistically selects sample establishments and goods based on either the total value of shipments or the number of employees. The BLS collects prices from about 25,000 establishments for approximately 100,000 individual items on a monthly basis. The BLS defines PPI prices as "net revenue accruing to a specified producing establishment from a specified kind of buyer for a specified product shipped under specified transaction terms on a specified day of the month." Prices are collected via a survey that is emailed or faxed to participating establishments. Individual establishments remain in the sample for an average of seven years until a new sample is selected to account for changes in the industry structure.

We calculate the frequency of price changes at the goods level, FPA, as the ratio of the number of price changes to the number of sample months. For example, if an observed price path is \$10 for two months and then \$15 for another three months, one price change occurs during five months, and the frequency is 1/5. We aggregate goods-based frequencies to the BEA industry classification.

The overall mean monthly frequency of price adjustment is 16.78%, which implies an average duration, $-1/\log(1 - FPA)$, of 5.44 months. Substantial heterogeneity is present in the frequency across sectors, ranging from as low as 2.74% for the semiconductor manufacturing sector (duration of 35.96 months) to 96.47% for dairy production (duration of 0.30 months).

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[11]Ozdagli and Weber (2016) follow a similar approach.

[12]The data have been used before in Nakamura and Steinsson (2008), Goldberg and Hellerstein (2011), Bhattarai and Schoenle (2014), Gorodnichenko and Weber (2016), Gilchrist, Schoenle, Sim, and Zakrajšek (2017), Weber (2015), and D'Acunto, Liu, Pflueger, and Weber (2016), among others.

V Calibration

We calibrate the steady-state input-output linkages of our model, Ω, to the U.S. input-output tables in 2002. The same 2002 BEA data allow us to calibrate steady-state sectoral GDP shares, Ω_c. The Calvo parameters match the industry-average frequency of price adjustments between 2005 and 2011, using the micro data underlying the PPI from the BLS, for the same classification of industries the BEA uses.[13] After we merge the input-output and the frequency-of-price-adjustment data, we end up with 341 industries, which we refer as "sectors."

The 2002 BEA data have 407 unique sectors. We lose some of them for three reasons. First, some sectors almost exclusively produce final goods, so the BLS data do not contain enough observations to compute reliable average frequencies of price adjustment. Second, the goods some sectors produce do not trade in a formal market, so the BLS has no prices to record. Examples of missing sectors are (with input-output industry codes in parentheses) "Military armored vehicle, tank, and tank component manufacturing" (336992) or "Religious organizations" (813100). Third, the BEA data for some sectors are not available at the six-digit level.

All our calibration exercises are at the monthly frequency, so the discount factor is $\beta = 0.9975$ (3% annual risk-free interest rate). The elasticity of substitution across sectors is $\eta = 0.5$. No good estimates of this elasticity are available at the level of disaggregation we require. Atalay (2017) reports an average elasticity of substitution of 0.15 for 30 industries; presumably a higher level of disaggregation implies a higher elasticity of substitution across sectors. As a robustness check, we also report results for $\eta = 1$. We set the elasticity of substitution within sectors to $\theta = 6$ in our baseline calibration, which implies a markup of 20%. We also report results for $\theta = 11$, implying a 10% markup. We set $\delta = 0.5$ so the intermediate-input share in steady state is $\delta * (\theta - 1)/\theta = 0.42$, which matches the 2002 BEA data.

We report several versions of our model depending on assumptions for the labor market and monetary policy.

MODEL1 has linear disutility of labor, $\varphi = 0$, and monetary policy targeting a constant price level. This model is the closest parametrization of our full-blown New Keynesian model to the simplified model we study in section III, with a monetary rule that eliminates the scale effect of price rigidity on GDP volatility from productivity shocks, either sectoral or aggregate.

MODEL2 is identical to **MODEL1**, but we set the inverse Frisch elasticity to $\varphi = 2$. This model allows for elastic labor supply and sectoral segmentation of the labor market. The analysis in section III shows these features introduce new effects of sectoral GDP and input-output

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[13]We interpret the frequencies of price adjustments as the probability a sector can adjust prices after the shock.

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linkages, and their interaction with price rigidity.

MODEL3 is an intermediate case in which $\varphi = 0$, and a passive monetary policy targets constant nominal GDP at steady-state level. This case allows us to isolate the level effect in the tractable analytical setup with inelastic labor supply by comparing results to **MODEL1**. **MODEL4** assumes the Taylor rule specified in section II with parameters $\phi_c = 0.33/12 = 0.0275$ and $\phi_\pi = 1.34$. **MODEL5** assumes a preference shock that implies the zero-lower-bound constraint on nominal interest rate binds for 50 months and the Taylor rule for **MODEL4** governs monetary policy afterwards. The Online Appendix also reports results for $\eta = 1$ and $\theta = 11$.

VI   Quantitative Results

This section provides quantitative evidence for the importance of the interaction of heterogeneity in price rigidity with heterogeneity in real features of the economy for propagating idiosyncratic shocks. We analyze this interaction through the lens of heterogeneity in sectoral GDP shares and input-output linkages. The main focus is on the size of the volatility of aggregate GDP from idiosyncratic shocks, but we also study the importance for price stability in a separate subsection when we consider alternative monetary policy rules that do not perfectly stabilize the aggregate price level. Finally, we also analyze the distortionary role price rigidities have on the identity of the most important sectors for aggregate volatility. This latter angle is especially important for policy-makers.

To highlight the effects at work in our model, we go through three cases on the nominal side: a flexible price case that serves as a benchmark for price rigidity per se, a homogeneously rigid price case as well as a case of heterogeneous price rigidity. A comparison between the latter two shows the precise role of heterogeneity in price rigidity. On the real side, we assume all sectors are equally large or connected, or fully heterogeneous as in the data.

A.   Multipliers

We base our analysis on two variations of our model, which we label MODEL1 and MODEL2. Both of these model variations assume Calvo price stickiness and monetary policy targeting a constant GDP deflator. The models differ in assuming fixed and elastic labor supply, respectively. We focus on the magnitude of the total multiplier $\|\chi\|_2$ in Table 1. This total multiplier maps the volatility of sectoral shocks into aggregate volatility.

A.1 Multipliers: Flexible Prices

This subsection shows variations in sector size and input-output linkages can substantially propagate idiosyncratic shocks even when prices are fully flexible. The total multiplier can be as large as 1/5 of the multiplier aggregate productivity shocks generate.

To arrive at these results, we first abstract from price rigidity in MODEL 1, which has fixed labor supply. We report this case in panel A, column 1 of Table 1. Line 1 shows the total multiplier is only 5.42% when all sectors have equal GDP shares and equal input-output linkages in steady state. Consistent with section III, the total multiplier in this case equals $K^{-1/2}$ for $K = 341$. Line 2 isolates the role of heterogeneous sectoral GDP shares by shutting down intermediate input use ($\delta = 0$) and calibrating Ω_c to U.S. data. The multiplier is now 20.47%, which is 1/5 of the volatility an aggregate productivity shock generates, and four times larger than the effect of idiosyncratic shocks in an economy with perfect symmetry. This multiplier falls to 11.26% in line 3 when we allow for input-output linkages ($\delta = 0.5$) but assume these linkages are perfectly symmetric. The drop happens because the symmetry in input-output linkages across sectors reduces the cross-sectional dispersion of sectoral multipliers heterogeneous GDP shares introduce.

Next, we show heterogeneous sectoral input-output linkages alone can also propagate idiosyncratic shocks but are a less powerful mechanism than heterogeneity in the sectoral size. We impose equal steady-state GDP shares across sectors in line 4 but calibrate the input-output matrix, Ω, to the actual, heterogeneous U.S. input-output linkages. The multiplier is now 8.01%. Comparing the size of the multiplier with the multiplier in line 2, we see heterogeneous input-output linkages are quantitatively less powerful than heterogeneity in sectoral size as a source of aggregate GDP fluctuations.

When we put both heterogeneities together, we find idiosyncratic shocks can be a substantial driver of aggregate fluctuations. Line 5 reports the multiplier when we calibrate both GDP shares, Ω_c, and input-output linkages, Ω, to U.S. data. The total multiplier is now 16.88%, indicating GDP shares and input-output linkages interact and jointly affect the capacity of sectoral shocks to generate aggregate fluctuations. This multiplier is remarkably close to the one in Gabaix (2011), who uses a measure of firms' total sales. This similarity arises because total sales encompass sales as final goods (i.e., GDP) and as intermediate inputs (i.e., input-output linkages).

Finally, results are almost identical when we relax the assumption of inelastic labor supply (MODEL 2). Panel B of Table 1 summarizes the details.

A.2 Multipliers: Homogeneous Sticky Prices

We now study the role of price rigidity that is homogeneous across sectors for the importance of idiosyncratic shocks as a driver of aggregate fluctiations. Specifically, we calibrate the sectoral Calvo parameters to the average frequency of price changes using PPI micro data. As the theory section predicts, almost all results are identical to the flexible-price case. Column 2 of Table 1 reports the details.

Section III shows price rigidity introduces a scale effect dampening the total multiplier, but this scale effect is completely offset when monetary policy targets a constant price level just as we do in Table 1. Indeed, the total multipliers in column 1 – under perfectly flexible prices – and column 2 – under homogeneously sticky prices – are identical when sectors are perfectly symmetric (line 1) or when labor is the only input in production (line 2). In both cases, the only effect of homogeneous price rigidity is the introduction of the scale effect.

Second, when steady-state GDP shares are heterogeneous across sectors but the production network is symmetric, homogeneous price rigidity increases the total multiplier relative to the case of frictionless prices. Comparing columns (1) and (2) in line 3 shows this increase. The total multiplier increases from 11.26% in column 1 to 20.2% in column 2. This result follows because symmetric input-output linkages reduce the cross-sectional dispersion of multipliers, but the sectoral weight of this reduction is the frequency of price changes, which is 100% for flexible prices but only 16.78% in the case of homogeneous price rigidity.

Asymmetry in the production network introduces another, important effect of homogeneous price rigidity, dampening the total multiplier of sectoral shocks. As our theory section predicts, average price rigidity in the economy enters as a penalty on the quantitative importance of heterogeneity in network centrality. In particular, the higher the order of centrality, the stronger this penalty is. We can see this dampening effect when comparing line 4 across columns (1) and (2): The total multiplier falls from 8.01% when prices are flexible to 5.42% when sectoral prices are equally sticky. This effect is new and important relative to the literature on input-output linkages such as Acemoglu et al. (2012). Quantitatively, homogeneous price rigidity makes the pure network source of aggregate fluctuations irrelevant.

Next, we allow for these two opposite forces jointly. Calibrating our model to steady-state GDP shares and input-output linkages shows the overall effect of homogeneous price rigidity increases the total multiplier. The total multiplier is 20.35% with homogeneous price rigidity (line 5, column 2), whereas it is 16.88% with flexible prices (line 5, column 1).

Once again, results are almost identical for MODEL 2 (panel B), implying elastic labor supply plays a minor quantitative role when all sectoral prices are equally sticky.

A.3 Multipliers: Heterogeneous Sticky Prices

This subsection presents our main empirical result: Heterogeneity in price rigidity is a powerful ingredient to propagate idiosyncratic shocks to the aggregate level. This result holds true both for heterogeneity in price rigidity by itself, as well as for the interaction with our two heterogeneous real features of the economy. Column 3 of Table 1 presents these main results.

First, heterogeneity in price rigidity alone becomes a quantitatively important source of aggregate fluctuations: Focusing on line 1 (equal GDP and input-output linkages across sectors in steady state), the total multiplier goes from 5.42% when prices are fully flexible in column 1 to 15.65% in column 3 when Calvo parameters match the sectoral frequency of price changes in U.S. data. This magnitude of the multiplier is similar to the one obtained in a flex-price economy for the joint effect of heterogeneity in sectoral GDP and input-output linkages. Despite the fact that the theoretical results show price rigidity by itself does not affect the rate of decay of shocks, heterogeneity in price rigidity is quantitatively important for the idiosyncratic origin of aggregate fluctuations and represents a "frictional" origin of aggregate fluctuations that is conceptually different from heterogeneity in sector size or network centrality.

Second, an interaction exists between heterogeneity in size and heterogeneity in price rigidity. In line 2 (no intermediate inputs, $\delta = 0$), the total multiplier goes from 20.47% when prices are fully flexible in column 1 to 51.53% when price rigidity is heterogeneous across sectors in column 3. This change represents a 2.5-fold increase relative to the flex-prices economy and a 10-fold increase relative to the perfectly symmetric case where the multiplier is 5.42%. In line 3 ($\delta = 0.5$ and symmetric input-output linkages), the effect of heterogeneous price stickiness is even stronger: The total multiplier goes from 11.26% in column 1 to 50.32% in column 3. This increase is due to the combined effect of two forces: the average rigidity dampening the effect of symmetric input-output linkages on reducing cross-sectional dispersion in multipliers and the contribution of the heterogeneity in price stickiness to increasing such dispersion.

Third, a strong interaction arises between heterogeneity in price rigidity and the potency of input-output linkages for generating aggregate fluctuations: In line 4, the total multiplier goes from 8.01% when prices are flexible (column 1) to 16.35% when price stickiness is heterogeneous (column 3). This increase is large once we take into account the effect of homogeneous price rigidity on reducing the quantitative importance of input-output linkages (column 2).

Overall, when our calibration matches jointly the sector-size distribution, input-output linkages, and the heterogeneity in price rigidity in the U.S. economy, we find the total multiplier that maps volatility of sectoral productivity shocks into GDP volatility is 44.24% (line 5, column 3). This multiplier is 2.6 times larger than the multiplier in an economy with flexible prices

(16.88%, line 5, column 1) and 8.2 times larger relative to the perfectly symmetric case (5.42%, line 1, columns (1) or(2)).

When we allow for elastic labor demand, all of our results are still qualitatively and quantitatively valid. Additionally, some minor differences exist with respect to MODEL 1 (panel A). As our theory section predicts, relaxing the assumption of inelastic labor supply reduces the quantitative effect of price rigidity via two channels. First, shocks are negatively correlated with the response of wages in the shocked sector. We can see the quantitative effect of this channel by comparing the total multiplier in line 2, column 3 in panel A (51.53%) and panel B (41.57%). Second, elastic labor supply creates aggregate forces from price rigidity that reduce the cross-sectional dispersion of sectoral multipliers when intermediate inputs are used for production. We can see this channel by comparing column 3 in panels A and B in all cases when $\delta = 0.5$ (all lines but line 2).

A.4 Multipliers: Robustness

Our results continue to hold for alternative monetary policy rules and other parameter variations. Table 2 presents results for the inelastic labor-supply version of our model with a monetary policy rule that targets constant steady-state nominal GDP, for a Taylor rule, and for the case when the zero lower bound on the nominal interest rate is binding for 50 periods (and later monetary policy responds according to a Taylor rule.) We label these variations MODEL 3, MODEL 4, and MODEL 5, respectively.

The main difference now is that under these alternative rules, the scale effect of average price rigidity on GDP volatility is not offset by monetary policy. However, this difference is of only minor quantitative importance. All our results described above for Table 1 continue to hold. Magnitudes of multipliers are slightly smaller when monetary policy targets steady-state nominal GDP, as well as for a Taylor rule. When the zero lower bound is binding, they are very close to the baseline results reported in panel A of Table 1.

Results are also robust to further parameter variations: First, given these monetary policy rules, the online appendix reports our results for elastic labor supply (see Table A.1). The dampening effect of elastic labor supply remains similar in magnitude to our baseline specification. Second, regarding sensitivity to other parameters, a low elasticity of substitution within sectors partially driving our findings might be a concern.[14] Table A.2 in the Online Appendix shows our findings for MODEL 1 and MODEL 2 in Panels A and B when we increase the within-sector elasticity of substitution, θ, from a baseline value of 6 to 11. This increase

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[14]We thank Susanto Basu for raising this point.

reduces the markup from 20% to 10%. Results for MODEL 1 remain identical to Table 1. The reason for the similarity is that θ only plays a role when wages respond to labor demand. For MODEL 2, multipliers are somewhat lower than in Table 1, but all results remain qualitatively the same and quantitatively relevant. The same applies when the elasticity of substitution η increases from 0.5 to 1 (Panel C and D).

A final robustness check concerns the volatility of sectoral shocks. In our analysis, we are interested in multipliers that represent elasticities, that is, responses to unit shocks. Although these multipliers allow us to answer the questions we are after, a quantitative variation that calibrates the volatility of sectoral shocks is equally interesting. Therefore, we also use the CES-NBER manufacturing dataset to estimate sectoral volatility of productivity shocks based on AR(1) processes. When we use these estimated volatilities to give differently large one-standard-deviation shocks to sectors, all our results hold. Table A.3 in the appendix presents the results.

B.   Multipliers: Price Stability

Price stability is a policy goal of central banks around the world. Our results show the interaction of heterogeneous price rigidity with our two heterogeneous real features of the economy can have an important impact on the propagation of idiosyncratic shocks into aggregate price stability. We find the impact of idiosyncratic shocks on the volatility of the GDP deflator can be similarly large as in the case of output volatility.

We find a large effect of heterogeneity in price rigidity and its interactions with sectoral size and input-output linkages on aggregate price stability when we consider the alternative monetary policy rules discussed above. Trivially, if the policy-maker targets a constant price level, no effect arises by definition. Therefore, the results we present in Table 3 derive from a constant nominal GDP target, and our Taylor-rule specification. All multipliers in Table 3 are multipliers of idiosyncratic shocks into GDP deflator volatility, relative to the multiplier of an aggregate productivity shock.

First, a qualitative impact similar to that in Table 1 is evident. As our analytical model showed, the intuition for the effects on price stability is the same as for the volatility of aggregate output given the monetary policy specification in the analytical model. The only difference under the monetary policy rules we analyze here is the extent of fluctuations in aggregate prices versus in aggregate GDP. They no longer necessarily fluctuate one for one as is the case under the assumptions of the analytical model.

Second, the quantitative impact on price stability is similar in magnitude to the impact

on GDP volatility. Although it is slightly muted, the effect in some cases more than triples when we allow for heterogeneous price rigidity. For example, in Panel A with constant nominal GDP target, GDP deflator volatility increases from 5.42% under flexible prices to 12.36% under fully heterogeneous prices when sectoral size and linkages are homogeneous (line 1). However, when sectoral size is heterogeneous and linkages are homogeneous (line 3), the multiplier goes from 11.26% under flexible prices to 17% under homogeneous price rigidity to 33.73% under heterogeneous price rigidity. When we allow both real features to be fully heterogeneous (line 5), GDP deflator volatility increases from 16.88% to 31.78% of what an aggregate productivity shock would generate.

If we instead consider a Taylor rule as our monetary policy specification (Panel B), our results continue to hold qualitatively and quantitatively. Heterogeneity in price rigidity interacts with heterogeneity in the real features of the economy in important ways. The propagation of idiosyncratic shocks into CPI volatility is large relative to what an aggregate productivity shock generates.

C. **Distorted Idiosyncratic Origin of Fluctuations**

We now turn to another central result of our quantitative analysis: Sectoral heterogeneity in price rigidity dramatically changes the identity of sectors from which aggregate fluctuations originate. It also increases granularity in the sense that a few sectors account for a larger fraction of the total multiplier.

Table 4 shows the contribution of the five most important sectors for aggregate fluctuations. We report sectoral shares relative to the total multiplier $\|\chi\|_2$ in MODEL1 under different assumptions on steady-state sectoral GDP shares, steady-state input-output linkages, and Calvo parameters. We also report the overall contribution of the five most important sectors; the overall contribution of all sectors is 1 by construction.[15]

We first focus on price rigidity alone. Column 1 shows results when we calibrate Calvo parameters to the sectoral frequency of price changes in the U.S. but impose equal steady-state GDP shares and input-output linkages across sectors. The five sectors with the largest multipliers are those with the most flexible prices, which are mostly commodities and farming products: "Dairy cattle and milk production" (112120), "Alumina refining and primary aluminum production" (33131A), "Primary smelting and refining of copper" (331411), "Oil and gas extraction" (211000), and "Poultry and egg production" (1121A0). Their contributions to the total multiplier range from 13.74% to 7.35%, and together they account for about half of

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[15]Results for MODEL 2 are very similar, so we do not report them here.

the GDP volatility from sectoral shocks (their overall contribution is 48.6%). By contrast, if all prices were perfectly flexible (or equally sticky), each sector would have a contribution of 0.29% (341^{-1}), so the overall contribution of any five sectors would only be 1.47%. Heterogeneity in price rigidity has a quantitatively strong effect on aggregate GDP volatility.

We then focus on the interaction between price rigidity and the sectoral GDP shares of granularity. Columns (2) and (3) in Table 4 assume steady-state sectoral GDP shares in the model matching U.S. data, but symmetric steady-state input-output linkages. Column 2 assumes flexible prices, whereas column 3 matches the sectoral frequency of price changes. The identity and contribution of the sectors with the largest multiplier changes across calibrations: with flexible prices, the most important sector is "Retail trade" (4A0000), followed by "Wholesale trade" (420000), "Real estate" (531000), "Offices of physicians, dentists and other health practitioners" (621A00), and "Hospitals" (622000). With sticky prices instead, the most important sectors become "Oil and gas extraction" (211000), "Monetary authorities and depository credit intermediation" (52A000), "Petroleum refineries" (324110), "Primary smelting and refining of copper" (331411) and "Wholesale trade" (420000). With flexible prices, "Retail trade" (4A0000) accounts for 32.84% of the total multiplier; with sticky prices, it only accounts for 2.22%. In turn, "Oil and gas extraction" (211000) contributes with 0.15% when prices are flexible but with 30% when prices are as sticky as in U.S. data. The overall contribution of the five most important sectors is 70.14% with flexible prices and 94.1% with sticky prices. Thus, heterogeneity in price rigidity changes the sectors with the largest multiplier and it increases the *effective* granularity of the top-5 sectors.

Next, we turn to the interaction between price rigidity and input-output linkages. Columns (4) and (5) of Table 4 assume equal steady-state sectoral GDP across sectors and steady-state input-output linkages matching 2002 U.S. Input-Output tables. Column 4 assumes flexible prices, whereas column 5 also matches sectoral frequencies of price adjustment. Again, the identity of the five sectors with the largest multiplier changes dramatically: With flexible prices, they are "Wholesale trade" (420000), "Real estate" (531000), "Electric power generation, transmission, and distribution" (221100), "Monetary authorities and depository credit intermediation" (52A000), and "Retail trade" (4A0000). With sticky prices, they are "Dairy cattle and milk production" (112120), "Electric power generation, transmission, and distribution" (211000), "Primary smelting and refining of copper" (331411), "Cattle ranching and farming" (1121A0), and "Sugar cane miles and refining" (31131A). The overall contribution of the top-5 sectors increases from 43.24% with flexible prices to 52.48% with heterogeneous sticky prices. The contribution of "Wholesale trade" (420000) falls from 24.58% with flexible

prices to 0.21% with sticky prices. In turn, "Dairy cattle and milk production" (112120) increases its contribution from 0.12% to 12.67% when prices are flexible or sticky, respectively.

Finally, we look at the interaction of price rigidity with both our heterogeneous real features of the economy combined. Columns (6) and (7) in Table 4 report results for flexible prices and when the model matches the empirical industry-average frequency of price changes. The key message is once again the same: With flexible prices, the five sectors with the largest multipliers are "Retail trade" (4A0000), "Real estate" (531000), "Wholesale trade" (420000), "Monetary authorities and depository credit intermediation" (52A000), and "Telecommunications" (517000). Their combined contribution to the total multiplier is 73.66%. With sticky prices, the most important sector by far is "Monetary authorities and depository credit intermediation" (52A000), whose contribution increases from 6.46% with flexible prices to 25.3% with prices sticky as in the data. This sector mainly refers to "businesses engaged in accepting deposits and in lending funds from these deposits." The other sectors in the top-5 with sticky prices are "Petroleum refineries" (324110), "Oil and gas extraction" (211000), "Auto manufacturing" (336111), and "Wholesale trade" (420000). The contribution of "Retail trade" (4A0000), the most important sector with flexible prices, falls from 22.49% with flexible prices to 2.46% with sticky prices. The overall contribution of the five most important sectors is 73.66% with flexible prices and 93.33% with rigid prices.

The discussion has so far focused on the five most important sectors. But the distorting nature of heterogeneous price stickiness is a more general phenomenon. Figure 1 is a scatter plot of the sectoral rank in the contribution of sectors to the total multiplier. Along both axes, the steady-state sectoral GDP shares and input-output linkages in the model are calibrated to match 2002 U.S. data; the x-axis assumes homogeneously rigid prices, whereas the y-axis matches Calvo parameters to the empirical frequency of price changes. If heterogeneity in price rigidity had no effect, we would see no change in ranks and a 45-degree line in the figure. However, although a positive correlation exists in both rankings, the figure shows high dispersion and some dramatic changes in rank, underlining the message from the five most important sectors alone. Heterogeneity in price rigidity has a dramatic effect on the identity of sectors that drive aggregate fluctuations.

VII Concluding Remarks

Are prices sticky and does it matter? We typically answer those questions empirically by looking at moments in micro pricing data and the response of real variables to identified nominal shocks, or quantitatively by studying whether nominal shocks are an important driver of business cycles.

In the paper, we show another, novel, dimension along which price rigidity has first-order importance for macroeconomic research: Heterogeneity in price stickiness across sectors interacts with other heterogeneous real features of the economy and becomes a powerful amplification mechanism for idiosyncratic technology shocks to drive aggregate fluctuations.

We show these results theoretically and quantitatively in a calibrated 341-sector New Keynesian model with intermediate inputs and heterogeneity in sector size, sector input-output linkages, and output-price stickiness.

Heterogeneity in price rigidity has first-order effects: It generates a frictional origin of aggregate fluctuations, it amplifies the effect of idiosyncratic shocks due to the distributions of sectoral GDP and the input-output structure of the economy, and it changes the identity and relative importance of sectors from which fluctuations originate. Importantly, sector sales are no longer a sufficient statistic for sectoral contributions to aggregate fluctuations, as in Hulten (1978). Interestingly, in our most realistic calibration, which allows for heterogeneous price rigidities across sectors matched to U.S. data, we find "Monetary authorities and depository credit intermediation" to be the most important sector for aggregate fluctuations.

To date, the implications of heterogeneous price rigidity, and frictions in general, and its interactions with heterogeneous real features of the economy remain largely unexplored. Our analysis suggests heterogeneity in price rigidity has direct and important implications for the modeling and understanding of aggregate fluctuations originating from idiosyncratic shocks. The interaction also has important implications for the conduct of monetary policy. A central bank that aims to stabilize sectoral prices of "big" or "central" sectors might make systematic policy mistakes if it does not take into account the "frictional" origin of aggregate fluctuations. Although beyond the scope of this paper, future work could explore the design of optimal monetary policy in our heterogeneous production economy.

References

Acemoglu, D., U. Akcigit, and W. Kerr (2016). Networks and the macroeconomy: An empirical exploration. *NBER Macroannual 30*(1), 273–335.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The network origins of aggregate fluctuations. *Econometrica 80*(5), 1977–2016.

Alvarez, F., H. Le Bihan, and F. Lippi (2016). The real effects of monetary shocks in sticky price models: A sufficient statistic approach. *The American Economic Review 106*(10), 2817–2851.

Atalay, E. (2017). How important are sectoral shocks? *American Economic Journal: Macroeconomics 9*(4), 254–80.

Baqaee, D. R. (2018). Cascading failures in production networks. *Unpublished Manuscript, LSE*.

Barrot, J.-N. and J. Sauvagnat (2016). Input specificity and the propagation of idiosyncratic shocks in production networks. *The Quarterly Journal of Economics 131*(3), 1543–1592.

Basu, S. (1995). Intermediate goods and business cycles: Implications for productivity and welfare. *The American Economic Review 85*(3), 512–531.

Basu, S. and P. De Leo (2016, March). Should Central Banks Target Investment Prices? Boston College Working Papers in Economics 910, Boston College Department of Economics.

Bhattarai, S. and R. Schoenle (2014). Multiproduct firms and price-setting: Theory and evidence from U.S. producer prices. *Journal of Monetary Economics 66*, 178–192.

Bigio, S. and J. La'O (2017). Unpublished manuscript, columbia university.

Boivin, J., M. P. Giannoni, and I. Mihov (2009). Sticky prices and monetary policy: Evidence from disaggregated U.S. data. *The American Economic Review 99*(1), 350–384.

Bouakez, H., E. Cardia, and F. Ruge-Murcia (2014). Sectoral price rigidity and aggregate dynamics. *European Economic Review 65*, 1–22.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398.

Carvalho, C. and J. W. Lee (2011). Sectoral price facts in a sticky-price model. *Unpublished Manuscript, PUC-Rio*.

Carvalho, V. M. (2014). From micro to macro via production networks. *The Journal of Economic Perspectives 28*(4), 23–47.

Carvalho, V. M., M. Nirei, Y. U. Saito, and A. Tahbaz-Salehi (2016). Supply chain disruptions: Evidence from the great east Japan earthquake. *Unpublished Manuscript*.

Cochrane, J. H. (1994). Shocks. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 41, pp. 295–364. Elsevier.

D'Acunto, F., R. Liu, C. E. Pflueger, and M. Weber (2016). Flexible prices and leverage. *Unpublished Manuscript, University of Chicago*.

di Giovanni, J., A. A. Levchenko, and I. Mejean (2018, January). The micro origins of international business-cycle comovement. *American Economic Review 108*(1), 82–108.

Dupor, B. (1999). Aggregation and irrelevance in multi-sector models. *Journal of Monetary Economics 43*(2), 391–409.

Durrett, R. (2013). *Probability: theory and examples* (4th ed.). Cambridge; New York: Cambridge University Press.

Feenstra, R. C., P. Luck, M. Obstfeld, and K. N. Russ (2018). In search of the armington elasticity. *Review of Economics and Statistics 100*(1), 135–150.

Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica 79*(3), 733–772.

Gali, J. (2015). *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

Gaubert, C. and O. Itskhoki (2018). Granular comparative advantage. Technical report, National Bureau of Economic Research.

Gilchrist, S., R. Schoenle, J. Sim, and E. Zakrajšek (2017, March). Inflation dynamics during the financial crisis. *American Economic Review 107*(3), 785–823.

Goldberg, P. P. and R. Hellerstein (2011). How rigid are producer prices? *FRB of New York Staff Report*, 1–55.

Gorodnichenko, Y. and M. Weber (2016). Are sticky prices costly? Evidence from the stock market. *American Economic Review 106*(1), 165–199.

Head, A., L. Q. Liu, G. Menzio, and R. Wright (2012). Sticky prices: A new monetarist approach. *Journal of the European Economic Association 10*(5), 939–973.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *The Review of Economic Studies 45*(3), 511–518.

Long, J. B. and C. Plosser (1983). Real business cycles. *The Journal of Political Economy 91*(1), 39–69.

Lucas, R. E. (1977). Understanding business cycles. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 5, pp. 7–29. Elsevier.

Midrigan, V. (2011). Menu costs, multiproduct firms, and aggregate fluctuations. *Econometrica 79*(4), 1139–1180.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics 123*(4), 1415–1464.

Nakamura, E. and J. Steinsson (2010). Monetary non-neutrality in a multisector menu cost model. *Quarterly Journal of Economics 125*(3), 961–1013.

Ozdagli, A. and M. Weber (2016). Monetary policy through production networks: Evidence from the stock market. *Unpublished Manuscript, University of Chicago*.

Pasten, E., R. Schoenle, and M. Weber (2016). Production networks and the propagation of monetary policy shocks. *Unpublished manuscript*.

Weber, M. (2015). Nominal rigidities and asset pricing. *Unpublished manuscript, University of Chicago Booth School of Business*.

Figure 1: **Ranking of Sectors: Heterogeneous versus Homogeneous Price Stickiness**
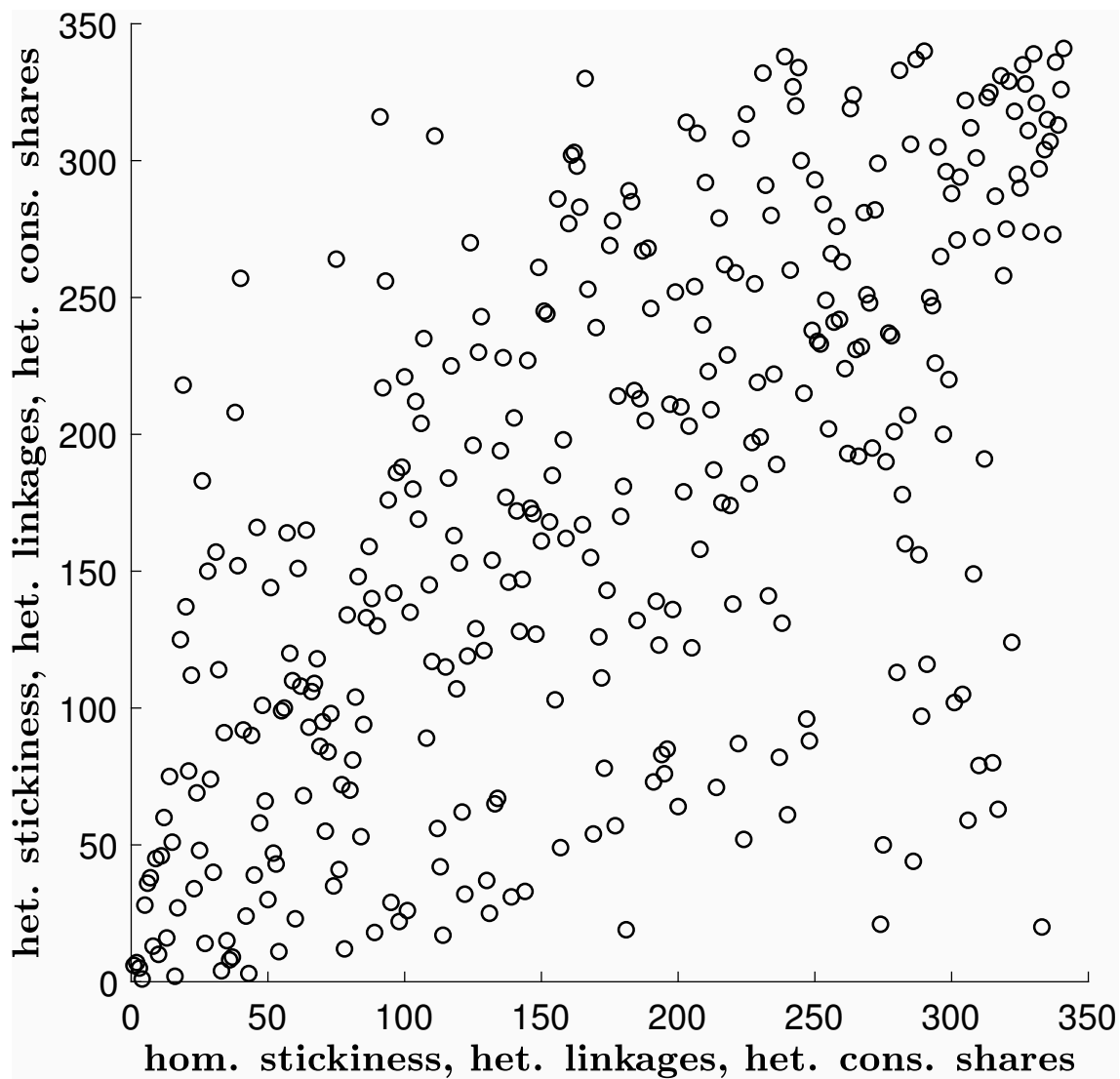
This figure plots the ranking of sectors for aggregate fluctuations originating from sectoral shocks for an economy with heterogeneous price stickiness across sectors (y-axis) and an economy with identical price stickiness for all sectors (x-axis). We assume heterogeneous GDP shares and input-output linkages calibrated to the U.S. in both cases.

Table 1: **Multipliers of Sectoral Shocks into Aggregate Volatility**

This table reports multipliers of sectoral productivity shocks on GDP volatility. Multipliers are defined as the Euclidean norm of vector χ with elements $\chi_k = \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}$, relative to the multiplier of an aggregate productivity shock which equals 1 in our baseline. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS.

			Flexible Prices	Homogeneous Calvo	Heterogeneous Calvo
			(1)	(2)	(3)
			Panel A: MODEL1		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	15.65%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	51.53%
(3)	het Ω_c	hom Ω	11.26%	20.20%	50.32%
(4)	hom Ω_c	het Ω	8.01%	5.42%	16.35%
(5)	het Ω_c	het Ω	16.88%	20.35%	44.24%
			Panel B: MODEL2		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	10.44%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	41.57%
(3)	het Ω_c	hom Ω	10.97%	17.41%	32.13%
(4)	hom Ω_c	het Ω	8.18%	5.73%	11.38%
(5)	het Ω_c	het Ω	16.79%	19.11%	29.27%

Table 2: **Multipliers of Sectoral Shocks into Aggregate Volatility**

This table reports multipliers of sectoral productivity shocks on GDP volatility. Multipliers are defined as the Euclidean norm of vector χ with elements $\chi_k = \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}$, relative to the multiplier of an aggregate productivity shock which equals 1 in our baseline. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS.

			Flexible Prices	Homogeneous Calvo	Heterogeneous Calvo
			(1)	(2)	(3)
			Panel A: MODEL3		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	12.36%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	37.18%
(3)	het Ω_c	hom Ω	11.26%	17.00%	33.73%
(4)	hom Ω_c	het Ω	8.01%	6.09%	13.16%
(5)	het Ω_c	het Ω	16.88%	18.96%	31.78%
			Panel B: MODEL4		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	15.26%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	46.11%
(3)	het Ω_c	hom Ω	11.26%	18.51%	44.82%
(4)	hom Ω_c	het Ω	8.01%	6.00%	15.87%
(5)	het Ω_c	het Ω	16.88%	19.60%	39.81%
			Panel C: MODEL5		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	16.53%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	53.36%
(3)	het Ω_c	hom Ω	20.47%	20.33%	52.22%
(4)	hom Ω_c	het Ω	5.42%	5.42%	16.87%
(5)	het Ω_c	het Ω	20.47%	19.76%	45.26%

This table reports multipliers of sectoral productivity shocks into GDP deflator volatility. Multipliers are defined as the Euclidean norm of vector χ with elements $\chi_k = \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}$, relative to the multiplier of an aggregate productivity shock on GDP deflator volatility, $\sqrt{\sum_{\tau=0}^{\infty} \left(\sum_{k=1}^{K} \rho_{k\tau} \right)^2}$. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS. We consider two monetary policy rules: a constant nominal GDP target in Panel A and a Taylor rule in Panel B.

			Flexible Prices	Homogeneous Calvo	Heterogeneous Calvo
			(1)	(2)	(3)
			Panel A: Constant Nominal GDP Target		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	12.36%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	37.18%
(3)	het Ω_c	hom Ω	11.26%	17.00%	33.73%
(4)	hom Ω_c	het Ω	8.01%	6.09%	13.16%
(5)	het Ω_c	het Ω	16.88%	18.96%	31.78%
			Panel B: Taylor Rule		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	9.97%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	28.75%
(3)	het Ω_c	hom Ω	11.26%	13.74%	25.64%
(4)	hom Ω_c	het Ω	8.01%	6.94%	10.87%
(5)	het Ω_c	het Ω	16.88%	17.70%	25.77%

Table 4: **Contribution of Sectors to Multiplier of Sectoral Shocks on GDP Volatility**

This table reports the contribution of five most important sectors to the multiplier of sectoral shocks on GDP volatility for MODEL1 and the identity of sectors in parentheses. The different columns represent calibrations which match the frequency of price adjustments (λ), the distribution of consumption shares (Ω_c), or the actual input-output matrix (Ω). Sectors are as follows. 1121A0: Cattle ranching and farming; 112120: Dairy cattle and milk production; 211000: Oil and gas extraction; 221100: Electric power generation, transmission, and distribution; 324110: Petroleum refineries; 33131A: Alumina refining and primary aluminum production; 331411: Primary smelting and refining of copper; 336111: Automobile manufacturing; 4A0000: Retail trade; 420000: Wholesale trade; 517000: Telecommunications; 52A000: Monetary authorities and depository credit intermediation; 531000: Real estate; 621A00: Offices of physicians, dentists, and other health practitioners; 622000: Hospitals

	λ (1)		Ω_c (2)		λ, Ω_c (3)		Ω (4)		λ, Ω (5)		Ω_c, Ω (6)		$\lambda, \Omega_c, \Omega$ (7)	
	13.74%	112120	32.84%	4A0000	30.01%	211000	24.58%	420000	12.67%	112120	22.49%	4A0000	25.30%	52A000
	9.53%	31131A	14.25%	420000	21.15%	52A000	8.30%	531000	11.88%	211000	22.29%	531000	20.38%	324110
	9.50%	331411	10.76%	531000	17.71%	324110	3.73%	221100	10.76%	331411	16.76%	420000	19.84%	211000
	8.43%	211000	6.62%	621A00	16.97%	336111	3.58%	52A000	8.76%	1121A0	6.46%	52A000	18.48%	336111
	7.35%	1121A0	5.67%	622000	8.25%	420000	3.06%	4A0000	8.41%	31131A	5.65%	517000	9.33%	420000
Top 5 Contribution	48.55%		70.14%		94.10%		43.24%		52.48%		73.66%		93.33%	

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Online Appendix:
Price Rigidities and the Granular Origins of Aggregate Fluctuations

Ernesto Pasten, Raphael Schoenle, and Michael Weber

Not for Publication

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I Steady-State Solution and Log-linear System

A. Steady-State Solution

Without loss of generality, set $a_k = 0$. We show below conditions for the existence of a symmetric steady state across firms in which

$$W_k = W, \ Y_{jk} = Y, \ L_{jk} = L, \ Z_{jk} = Z, \ P_{jk} = P \ \text{ for all } j, k.$$

Symmetry in prices across all firms implies

$$P^c = P^k = P_k = P$$

such that, from equations (1), (2), (10), and (13) in the main body of the paper,

$$
\begin{aligned}
C_k &= \omega_{ck} C, \\
C_{jk} &= \frac{1}{n_k} C_k, \\
Z_{jk}\left(k'\right) &= \omega_{kk'} Z, \\
Z_{jk}\left(j', k'\right) &= \frac{1}{n_{k'}} Z_{jk}\left(k'\right).
\end{aligned}
$$

The vector $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]'$ represents steady-state sectoral shares in value-added C, $\Omega = \{\omega_{kk'}\}_{k,k'=1}^{K}$ is the matrix of input-output linkages across sectors, and $\aleph \equiv [n_1, ..., n_K]'$ is the vector of steady-state sectoral shares in gross output Y.

It also holds that

$$C = \sum_{k=1}^{K} \int_{\Im_k} C_{jk} dj,$$

$$Z_{jk} = \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk} \left(j', k'\right) dj' = Z.$$

From Walras' law in equation (19) and symmetry across firms, it follows

$$Y = C + Z. \tag{A.1}$$

Walras' law also implies for all j, k

$$Y_{jk} = C_{jk} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k'} \left(j, k\right) dj',$$

$$Y = \frac{\omega_{ck}}{n_k} C + \frac{1}{n_k} \left(\sum_{k'=1}^{K} n_{k'} \omega_{k'k}\right) Z,$$

so $\aleph$ satisfies

$$n_k = \left(1 - \psi\right) \omega_{ck} + \psi \sum_{k'=1}^{K} n_{k'} \omega_{k'k},$$

$$\aleph = \left(1 - \psi\right) \left[I - \psi \Omega'\right]^{-1} \Omega_c,$$

for $\psi \equiv \frac{Z}{Y}$. Note by construction $\aleph' \iota = 1$, which must hold given the total measure of firms is 1.

Steady-state labor supply from equation (7) is

$$\frac{W_k}{P} = g_k L_k^{\varphi} C^{\sigma}.$$

In a symmetric steady state, $L_k = n_k L$, so this steady state exists if $g_k = n_k^{-\varphi}$ such that $W_k = W$ for all k. Thus, steady-state labor supply is given by

$$\frac{W}{P} = L^{\varphi} C^{\sigma}. \tag{A.2}$$

Households' budget constraint, firms' profits, production function, efficiency of production

(from equation (15)) and optimal prices in steady state are, respectively,

$$CP = WL + \Pi \tag{A.3}$$

$$\Pi = PY - WL - PZ \tag{A.4}$$

$$Y = L^{1-\delta} Z^{\delta} \tag{A.5}$$

$$\delta WL = (1 - \delta) PZ \tag{A.6}$$

$$sP = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^{\delta} \tag{A.7}$$

for $\xi \equiv \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta}$.

Equation (A.7) solves

$$\frac{W}{P} = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{1-\delta}}. \tag{A.8}$$

This latter result together with equations (A.5), (A.6), and (A.7) solves

$$\frac{\Pi}{P} = \frac{1}{\theta} Y.$$

Plugging the previous result in equation (A.4) and using equation (A.1) yields

$$
\begin{aligned}
C &= \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right] Y \\
Z &= \delta \left(\frac{\theta - 1}{\theta} \right) Y,
\end{aligned}
\tag{A.9}
$$

such that $\psi \equiv \delta \left(\frac{\theta - 1}{\theta} \right)$.

This result and equation (A.7) gives

$$L = \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\delta}{1-\delta}} Y,$$

where Y from before together with equations (A.2), (A.9) and (A.8) solves

$$Y = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{(1-\delta)(\sigma+\varphi)}} \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{\frac{\delta\varphi}{(1-\delta)(\sigma+\varphi)}} \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\sigma}{\sigma+\varphi}}.$$

B. Log-linear System

B.1   Aggregation

Aggregate and sectoral consumption which we interpret as value-added, given by equations (1) and (2), are

$$
\begin{aligned}
c_t &= \sum_{k=1}^{K} \omega_{ck} c_{kt}, \\
c_{kt} &= \frac{1}{n_k} \int_{\Im_k} c_{jkt} dj.
\end{aligned}
\tag{A.10}
$$

Aggregate and sectoral production of intermediate inputs are

$$
\begin{aligned}
z_t &= \sum_{k=1}^{K} n_k z_{kt}, \\
z_{kt} &= \frac{1}{n_k} \int_{\Im_k} z_{jkt} dj,
\end{aligned}
\tag{A.11}
$$

where equations (10) and (13) imply that $z_{jk} = \sum_{r=1}^{K} \omega_{kr} z_{jk}(r)$ and $z_{jk}(r) = \frac{1}{n_r} \int_{\Im_r} z_{jk}(j', r) dj'$.

Sectoral and aggregate prices are (equations (4), (6), and (12)),

$$
\begin{aligned}
p_{kt} &= \int_{\Im_k} p_{jk} dj \text{ for } k = 1, ..., K \\
p_t^c &= \sum_{k=1}^{K} \omega_{ck} p_{kt}, \\
p_t^k &= \sum_{k'=1}^{K} \omega_{kk'} p_{k't}.
\end{aligned}
$$

Aggregation of labor is

$$
\begin{aligned}
l_t &= \sum_{k=1}^{K} l_{kt}, \\
l_{kt} &= \int_{\Im_k} l_{jkt} dj.
\end{aligned}
\tag{A.12}
$$

4

B.2   Demand

Households' demands for goods in equations (3) and (5) for all $k = 1, ..., K$ become

$$
\begin{aligned}
c_{kt} - c_t &= \eta \left(p_t^c - p_{kt} \right), \\
c_{jkt} - c_{kt} &= \theta \left(p_{kt} - p_{jkt} \right).
\end{aligned}
\tag{A.13}
$$

In turn, firm jk's demands for goods in equation (11) and (14) for all $k, r = 1, ..., K,$

$$
\begin{aligned}
z_{jkt} \left(k' \right) - z_{jkt} &= \eta \left(p_t^k - p_{k't} \right), \\
z_{jkt} \left(j', k' \right) - z_{jkt} \left(k' \right) &= \theta \left(p_{k't} - p_{j'k't} \right).
\end{aligned}
\tag{A.14}
$$

Firms' gross output satisfies Walras' law,

$$
y_{jkt} = (1 - \psi) c_{jkt} + \psi \sum_{k'=1}^{K} \int_{\Im_{k'}} z_{j'k't} (j, k) \, dj'.
\tag{A.15}
$$

Total gross output follows from the aggregation of equations (19),

$$
y_t = (1 - \psi) c_t + \psi z_t.
\tag{A.16}
$$

B.3   IS and Labor Supply

The household Euler equation in equation (8) becomes

$$
c_t = \mathbb{E}_t \left[c_{t+1} \right] - \sigma^{-1} \left\{ i_t - \left(\mathbb{E}_t \left[p_{t+1}^c \right] - p_t \right) \right\}.
$$

The labor supply condition in equation (7) is

$$
w_{kt} - p_t^c = \varphi l_{kt} + \sigma c_t.
\tag{A.17}
$$

B.4   Firms

Production function:

$$
y_{jkt} = a_{kt} + (1 - \delta) l_{jkt} + \delta z_{jkt}
\tag{A.18}
$$

Efficiency condition:

$$w_{kt} - p_t^k = z_{jkt} - l_{jkt} \tag{A.19}$$

Marginal costs:

$$mc_{kt} = (1 - \delta) w_{kt} + \delta p_t^k - a_{kt} \tag{A.20}$$

Optimal reset price:

$$p_{kt}^* = (1 - \alpha_k \beta) mc_{kt} + \alpha_k \beta \mathbb{E}_t \left[p_{kt+1}^* \right]$$

Sectoral prices:

$$p_{kt} = (1 - \alpha_k) p_{kt}^* + \alpha_k p_{kt-1}$$

B.5 Taylor Rule:

$$i_t = \phi_\pi \left(p_t^c - p_{t-1}^c \right) + \phi_c c_t$$

II Solution of Key Equations in Section III

A. Solution of Equation (28)

Setting $\sigma = 1$ and $\varphi = 0$ in equation (A.17) yields

$$w_{kt} = c_t + p_t^c = 0,$$

where the equality follows from the assumed monetary policy rule, so equation (A.20) becomes

$$mc_{kt} = \delta p_t^k - a_{kt}.$$

Here, sectoral prices for all $k = 1, ..., K$ are governed by

$$
\begin{aligned}
p_{kt} &= (1 - \lambda_k) \, mc_{kt} \\
&= \delta (1 - \lambda_k) \, p_t^k - (1 - \lambda_k) \, a_{kt},
\end{aligned}
$$

which in matrix form solves

$$p_t = - \left[\mathbb{I} - \delta \left(\mathbb{I} - \Lambda \right) \Omega \right]^{-1} \left(\mathbb{I} - \Lambda \right) a_t.$$

$p_t \equiv [p_{1t}, ..., p_{Kt}]'$ is the vector of sectoral prices, Λ is a diagonal matrix with the vector $[\lambda_1, ..., \lambda_K]'$ on its diagonal, Ω is the matrix of input-output linkages, and $a_t \equiv [a_{1t}, ..., a_{Kt}]'$ is the vector of realizations of sectoral technology shocks.

The monetary policy rule implies $c_t = -p_t^c$, so

$$c_t = \left(\mathbb{I} - \Lambda' \right) \left[\mathbb{I} - \delta \left(\mathbb{I} - \Lambda' \right) \Omega' \right]^{-1} \Omega'_c a_t.$$

Solution of Equation (40)

Setting $\sigma = 1$ and $\varphi > 0$ in (A.17) yields

$$w_{kt} = \varphi l_{kt}^s + c_t + p_t^c = \varphi l_{kt}^d,$$

which follows from the assumed monetary policy rule.

Labor demand is obtained from the production function in equation (A.18), the efficiency

condition for production in equation (A.19), and the aggregation of labor in equation (A.12)

$$l_{kt}^d = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right).$$

y_{kt} follows from equations (A.10), (A.11), (A.13), (A.14), and (A.15)

$$y_{kt} = y_t - \eta \left(p_{kt} - \left[(1-\psi) p_t^c + \psi \sum_{k=1}^K n_k p_t^k \right] \right),$$

where

$$\widetilde{p}_t \equiv \sum_{k=1}^K n_k p_t^k = \sum_{k=1}^K \zeta_k p_{kt},$$

with $\zeta_k \equiv \sum_{k'=1}^K n_{k'} \omega_{k'k}$.

To solve for y_t, we use equations (A.11), (A.12), (A.16) and $y_t = \sum_{k=1}^K \int_{\Im_k} y_{jkt} dj$ to get

$$y_t = c_t + \psi \left[\Gamma_c c_t - \Gamma_p \left(\widetilde{p}_t - p_t^c \right) - \Gamma_a \sum_{k=1}^K n_k a_{kt} \right],$$

where $\Gamma_c \equiv \frac{(1-\delta)(1+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}, \ \Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}$.

Putting together all these equations, sectoral wages solve

$$w_{kt} = \frac{\varphi}{1+\delta\varphi} \left[\begin{array}{c} (1+\psi\Gamma_c) c_t - a_{kt} - \psi\Gamma_a \sum_{k'=1}^K n_{k'} a_{k't} \\ [(1-\psi)\eta + \psi\Gamma_p] p_t^c + \psi (\eta - \Gamma_p) \widetilde{p}_t + \delta p_t^k - \eta p_{kt} \end{array} \right].$$

With this expression, the solution to equation (40) follows the same steps as the solution to equation (28).

III Proofs

Most proofs below are extensions of the arguments in Gabaix (2011), Proposition 2, which in turn rely heavily on Levy's Theorem (as in Theorem 3.7.2 in Durrett (2013) on p. 138).

Theorem 5 (Levy's Theorem) *Suppose $X_1, ..., X_K$ are i.i.d. with a distribution that satisfies*

(i) $\lim_{x \to \infty} \Pr[X_1 > x] / \Pr[|X_1| > x] = \theta \in (0, 1)$

(ii) $\Pr[|X_1| > x] = x^{-\zeta} L(x)$ *with $\zeta < 2$ and $L(x)$ satisfies $\lim_{x \to \infty} L(tx) / L(x) = 1$.*

Let $S_K = \sum_{k=1}^{K} X_k$,

$$a_K = \inf\{x : \Pr[|X_1| > x] \leq 1/K\} \ \ and \ \ b_K = K\mathbb{E}\left[X_1 \mathbf{1}_{|X_1| \leq a_K}\right], \tag{A.21}$$

As $K \to \infty$, $(S_K - b_K)/a_K \xrightarrow{d} u$, where u has a nondegenerated distribution.

A. Proof of Proposition 1

In the following proofs, we go through three cases: first, when both first and second moments exist, second, when only the first moment exists, and third, when neither first nor second moments exist.

Generally, when there are no intermediate inputs ($\delta = 0$) and price rigidity is homogeneous across sectors ($\lambda_k = \lambda$ for all k),

$$\|\chi\|_2 = \frac{1 - \lambda}{K^{1/2}\overline{C}_k} \sqrt{\frac{1}{K} \sum_{k=1}^{K} C_k^2}.$$

Given the power-law distribution of C_k, the first and second moments of C_k exist when $\beta_c > 2$, so

$$K^{1/2} \|\chi\|_2 \longrightarrow \frac{\sqrt{\mathbb{E}\left[C_k^2\right]}}{\mathbb{E}\left[C_k\right]}.$$

In contrast, when $\beta_c \in (1, 2)$, only the first moment exists. In such cases, by the Levy's theorem,

$$K^{-2/\beta_c} \sum_{k=1}^{K} C_k^2 \xrightarrow{d} u_0^2,$$

where u_0^2 is a random variable following a Levy's distribution with exponent $\beta_c/2$ since $\Pr\left[C_k^2 > x\right] = x_0^{\beta} x^{-\beta_c/2}$.

Thus,

$$K^{1-1/\beta_c} \|\chi\|_2 \xrightarrow{d} \frac{u_0}{\mathbb{E}\left[C_k\right]}.$$

When $\beta_c = 1$, the first and second moments of C_k do not exist. For the first moment, by Levy's theorem,

$$\left(\overline{C}_k - \log K\right) \xrightarrow{d} g,$$

where g is a random variable following a Levy distribution.

The second moment is equivalent to the result above and hence

$$(\log K) \|\chi\|_2 \xrightarrow{d} u'.$$

B. Proof of Proposition 2

Given the distribution of $(1 - \lambda_k)$, $\mathbb{E}[(1 - \lambda_k)^2]$ exists, so the central limit theorem applies.

C. Proof of Proposition 3

Let λ_k and C_k be two independent random variables distributed as specified in the Proposition, the counter-cumulative distribution of $z_k = (1 - \lambda_k) C_k$ is given by

$$f_Z\left(z\right) = \int_z^{z/y_0} f_{C_k}\left(z/y\right) f_{1-\lambda_k}\left(y\right) dy,$$

which follows a Pareto distribution with shape parameter $\beta_{\lambda c}$. The proof of the Proposition then follows the proof of Proposition 1 for

$$\|\chi\|_2 = \frac{1}{K^{1/2}\overline{C}_k} \sqrt{\frac{1}{K} z_k^2}. \tag{A.22}$$

D. Proof of Proposition 4

Analogous to the proof of Proposition 1.

E. Proof of Proposition 5

When $\delta \in (0, 1)$, $\lambda_k = \lambda$ for all k, and $\Omega_c = \frac{1}{K}\iota$, we know

$$
\begin{aligned}
\|\chi\|_2 \;\geq\;& \frac{1-\lambda}{K}\sqrt{\sum_{k=1}^{K}[1+\delta'd_k+\delta'^2 q_k]^2} \\
\geq\;& (1-\lambda)\sqrt{\frac{1+2\delta'+2\delta'^2}{K}+\frac{\delta'^2}{K^2}\sum_{k=1}^{K}\left[d_k^2+2\delta'd_k q_k+\delta'^2 q_k^2\right]}.
\end{aligned}
$$

Following the same argument as in Proposition 2,

$$
K^{-2/\beta_d}\sum_{k=1}^{K}d_k^2 \;\longrightarrow\; u_d^2,
$$

$$
K^{-2/\beta_q}\sum_{k=1}^{K}q_k^2 \;\longrightarrow\; u_q^2,
$$

$$
K^{-1/\beta_z}\sum_{k=1}^{K}d_k q_k \;\longrightarrow\; u_z^2,
$$

where u_d^2, u_q^2 and u_z^2 are random variables. Thus, if $\beta_z \geq 2\min\{\beta_d, \beta_q\}$,

$$
v_c \geq \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}}v
$$

where u_3^2 is a random variable.

F. Proof of Proposition 6

This proposition follows from the dispersion of price stickiness across sectors and hence, the same steps as in the discussion of sectoral GDP.

Table A.1: **Multipliers of Sectoral Shocks into Aggregate Volatility**

This table reports multipliers of sectoral productivity shocks on GDP volatility. Multipliers are defined as the Euclidean norm of vector χ with elements $\chi_k = \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}$. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS.

			Flexible Prices	Homogeneous Calvo	Heterogeneous Calvo
			(1)	(2)	(3)
			Panel A: MODEL6		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	8.88%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	34.57%
(3)	het Ω_c	hom Ω	10.97%	15.60%	24.65%
(4)	hom Ω_c	het Ω	8.18%	6.31%	10.05%
(5)	het Ω_c	het Ω	16.79%	18.39%	24.65%
			Panel B: MODEL7		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	10.23%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	38.96%
(3)	het Ω_c	hom Ω	10.97%	16.26%	29.51%
(4)	hom Ω_c	het Ω	8.18%	6.20%	11.14%
(5)	het Ω_c	het Ω	16.79%	18.65%	27.51%
			Panel C: MODEL8		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	10.93%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	42.79%
(3)	het Ω_c	hom Ω	17.65%	17.53%	33.11%
(4)	hom Ω_c	het Ω	5.69%	5.68%	11.51%
(5)	het Ω_c	het Ω	19.21%	18.55%	28.69%

Table A.2: **Multipliers of Sectoral Shocks into Aggregate Volatility**

This table reports multipliers of sectoral productivity shocks on GDP volatility. Multipliers are defined as the Euclidean norm of vector χ with elements $\chi_k = \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}$. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS.

			Flexible Prices	Homogeneous Calvo	Heterogeneous Calvo
			(1)	(2)	(3)
			$\theta = \mathbf{11}$		
			Panel A: MODEL1		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	15.65%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	51.53%
(3)	het Ω_c	hom Ω	11.26%	20.20%	50.32%
(4)	hom Ω_c	het Ω	8.01%	5.42%	16.35%
(5)	het Ω_c	het Ω	16.88%	20.35%	44.24%
			Panel B: MODEL2		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	7.87%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	26.22%
(3)	het Ω_c	hom Ω	11.26%	15.13%	23.61%
(4)	hom Ω_c	het Ω	8.01%	6.35%	9.06%
(5)	het Ω_c	het Ω	16.88%	18.19%	23.82%
			$\eta = \mathbf{1}$		
			Panel C: MODEL1		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	15.65%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	51.53%
(3)	het Ω_c	hom Ω	11.26%	20.20%	50.32%
(4)	hom Ω_c	het Ω	8.01%	5.42%	16.35%
(5)	het Ω_c	het Ω	16.88%	20.35%	44.24%
			Panel D: MODEL2		
(1)	hom Ω_c	hom Ω	5.42%	5.42%	8.30%
(2)	het Ω_c	$\delta = 0$	20.47%	20.47%	26.22%
(3)	het Ω_c	hom Ω	11.26%	15.85%	25.44%
(4)	hom Ω_c	het Ω	8.01%	6.12%	9.31%
(5)	het Ω_c	het Ω	16.88%	18.47%	25.00%

Table A.3: **Multipliers of Sectoral Shocks into Aggregate Volatility: Calibrated Shocks**

This table reports multipliers of sectoral productivity shocks on GDP volatility. Multipliers are defined as the Euclidean norm of vector χ with elements $\chi_k = \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}$. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables, sector size from the BEA and the frequencies of price adjustment from the BLS. The size of the shocks is calibrated to sectoral TFP shocks from the CES manufacturing database.

			Flexible Prices	Homogeneous Calvo	Heterogeneous Calvo
			(1)	(2)	(3)
(1)	hom Ω_c	hom Ω	7.73%	7.73%	19.71%
(2)	het Ω_c	$\delta = 0$	15.44%	15.44%	38.29%
(3)	het Ω_c	hom Ω	9.97%	13.12%	36.90%
(4)	hom Ω_c	het Ω	9.80%	8.28%	19.74%
(5)	het Ω_c	het Ω	11.82%	13.49%	36.94%