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Great Volatility and Great Moderation

Abstract

We investigate the sources of the great changes in GDP volatility observed from 1966 to 2000. We develop a general equilibrium model and calibrate it to US data in order to characterize the contribution of micro level productivity shocks, inter-sectoral linkages and households' behavior to aggregate volatility. Our results show that changes in sectoral volatility played an important role in shaping volatility at the aggregate level. Moreover, asymmetries in the economic structure sometimes had an amplifying, and other times a dampening effect on aggregate volatility. We show that the different impact depends on the time-varying correlation between sectoral volatilities and the relative importance of specific sectors in the economy.

JEL-Codes: E320, E230, D570.

Keywords: business cycle, micro-macro volatility, input-output network.

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1 Introduction

GDP volatility in the US exhibited a high degree of time variation in the second part of the last century. The years ranging from 1960s to early-1980s were characterized by a surge in the volatility of GDP, which was then followed by a sharp decline which lasted untile the mid-1990s (see e.g. McConnell and Perez-Quiros, 2000; Stock and Watson, 2002). We refer to these distinct periods respectively as “great volatility” and “great moderation”. Traditional explanations of GDP fluctuations relied on economy-wide disturbances, such as aggregate productivity shocks (see e.g. Lucas, 1977). The origins of such aggregate shocks are, however, still a matter of debate. A significant branch of the literature, influenced by work of Long and Plosser (1983) and Horvath (1998, 2000), advanced the hypothesis that fluctuations at the macro level may originate from micro level, e.g. sectoral, shocks. Dupor (1999) argues instead against the microeconomic origins of aggregate fluctuations on the basis of a diversification argument: in an economy with a large number of sectors, the aggregate effect of idiosyncratic sectoral shocks should decay at a rate proportional to the square root of the number of sectors. More recently, Gabaix (2011) pointed out that when firms’ size is heterogeneous and distributed according to a Zipf law, aggregate fluctuations may emerge from shocks originated at the micro level: idiosyncratic shocks to a handful of very large firms do not wash out in the aggregate and decay at a much lower rate than predicted by the diversification argument. The work of Di Giovanni et al. (2014) provides empirical evidence for the importance of firm-specific shocks in explaining aggregate fluctuations. Moreover, Acemoglu et al. (2012) show that the diversification argument does not hold in the presence of asymmetric input-output linkages: production complementarities may lead to the amplification of sector-specific shocks and generate aggregate volatility.

Focusing on the origins of the large fall in GDP volatility associated with the “great moderation”, several papers provided empirical evidence on the importance of sectoral sources in explaining aggregate fluctuations. Proposed explanations typically connect changes in aggregate volatility to either changes in the weights of different sectors in the economy or changes in the volatility within sectors. McConnell and Perez-Quiros (2000) relate the decline in GDP volatility occurred around 1984 to a decline in the volatility within the sector of durable goods. Carvalho and Gabaix (2013) link changes in aggregate volatility to changes in the structure of the economy described by the time-variation of sectoral sales over GDP. Moro (2012) explains the "great moderation" by means of a structural change described by an increase in the size of the services sector (less intensive in intermediate inputs) relative to the manufacturing sector (more intensive in intermediate inputs). Moro (2012) also hypothesizes that the time-variation of total factor productivity (TFP) at the sectoral level played an important role in shaping aggregate volatility in the first half the 2000s.[1]

In this paper we consider both a time-varying structure of sectoral weights in the economy and time-varying sectoral TFP volatility, and study how their interaction shaped GDP volatility. We frame our analysis in a model with n sectors using capital, labor and intermediate goods to produce gross output along the lines of Long and Plosser (1983), Jones (2011) and Carvalho and Gabaix (2013). Our theoretical framework links the dynamics of GDP volatility to the following time-varying terms: (*i*) the weights of each sector in the economy defined as "Domar weights", i.e. sectoral gross nominal output over GDP (Domar, 1961), (*ii*) sectoral TFP volatilities, (*iii*) a multiplying factor which depends on labor and capital supply decisions of households in the economy.

We calibrate the model to US data and perform a series of counterfactual exercises in order to isolate (*i*) the impact of time-varying sectoral TFP volatilities on GDP volatility, (*ii*) the amplification of idiosyncratic sectoral productivity shocks due to the presence of production linkages, (*iii*) the role played by the asymmetry of the input-output network.

Our main findings can be summarized as follows. First, sectoral TFP volatilities are an important driver of GDP volatility. However, sectoral TFP volatilities alone are not able to fully describe the behavior of aggregate fluctuations, nor to match the observed level of GDP volatility. Second, the presence of an input-output network amplifies sectoral-level fluctuations. This amplification takes place through two channels: (*i*) the degree of overall intermediate input-intensity of production, (*ii*) the asymmetry of production structure. We find that the impact of the asymmetry on aggregate volatility has greatly changed over time. In particular, from 1970 to 1992 asymmetry had an *amplifying* effect on volatility, with a rather strong impact around 1980. Before

[1] Other explanations for the "great moderation" include better inventory management (Kahn et al., 2002), improvements in financial markets facilitating consumption and investment smoothing (Blanchard and Simon, 2001), changes in the volatility of aggregate shocks and impulse-response propagation mechanisms (Stock and Watson, 2002; Galí and Gambetti, 2009), decline in the variability of the shock specific to the investment account equilibrium condition (Justiniano and Primiceri, 2008), changes in the demographic composition of the workforce (Jaimovich and Siu, 2009), and better policy (Clarida et al., 2000) among others.

1970 and after 1992 the asymmetric structure of the economy had a *dampening* effect on aggregate volatility. The different impacts cannot be explained by different levels of asymmetry alone, but a major role is played by changes over time in the correlation between Domar weights and sectoral TFP volatilities. For example, our findings show that during the “great moderation” the level of asymmetry remained almost unchanged, while the correlation between Domar weights and TFP volatilities became more negative. Finally, we show that the “great volatility” and the subsequent “great moderation” can be largely explained by changes in both the Domar weights and the TFP volatilities of just a few sectors.

Our paper contributes to the literature on the microeconomic origins of aggregate fluctuations. It is most related to Carvalho and Gabaix (2013), who introduce a measure of “fundamental volatility”, i.e. volatility derived only from sectoral shocks, and show that GDP volatility tracks fundamental volatility. In Carvalho and Gabaix (2013), the evolution of fundamental volatility only depends on sectoral weights changing over time, while sectoral TFP volatilities are constant. We build on their theoretical framework and introduce time-varying micro-level TFP volatilities as well as sector-specific shares of production inputs. We decompose changes in fundamental volatility to measure the relative contributions of changes in sectoral TFP volatilities and in the Domar weights, finding that 56% of the increase in fundamental volatility during the “great volatility” period is explained by changes in idiosyncratic volatilities, while 78% of the reduction in fundamental volatility occurred during the “great moderation” is due to changes in sectoral TFP volatilities. Moreover, our results suggest that the time-varying interaction between Domar weights and sectoral TFP volatilities is a key driver of aggregate volatility. Therefore, explanations of the volatility dynamics based only on changes of sectoral weights in the economy or only on changes in sectoral volatilities may overlook the important role played by the interaction between these two factors.

The rest of the paper is organized as follows. In Section 2 we develop the theoretical model underpinning our empirical analysis. In Section 3 we discuss the data used for our analysis and the calibration of the model. In Section 4 we discuss the results of our counterfactual exercises, while in Section 5 we conduct a sectoral-level analysis. Section 6 concludes.

2 Model

In this section we present an analytically tractable model which will be the basis for our empirical analysis. A detailed derivation can be found in Appendix A.

There are n sectors producing intermediate goods. Each intermediate good can be used as input for the production of intermediate goods or it can be aggregated into a single final consumption good. Households consume and supply labor. Each sector i producing intermediate goods uses a Cobb-Douglas technology function given by

$$Q_i = A_i \left(K_i^{\alpha_i} H_i^{(1-\alpha_i)} \right)^{(1-w_i)} \prod_{j=1}^{n} d_{ij}^{w_{ij}} \ , \tag{1}$$

where A_i is an exogenous productivity term uncorrelated across sectors, K_i and H_i represent quantities of capital and labor respectively, while d_{ij} represents the quantity of good j used in the production of sector i. The production technology features constant returns to scale so that $0 < \alpha_i < 1$ and $\sum_j w_{ij} = w_i$.

Instead of specifying a utility function over the n different goods, we follow Jones (2011) and specify a single final good given by a log-linear aggregation of the output of the n sectors

$$Y = \prod_{i=1}^{n} c_i^{\beta_i} \ , \tag{2}$$

where Y denotes the quantity of the final good, c_i is the quantity of each intermediate good used to produce the final good and $\sum_{i=1}^{n} \beta_i = 1$. We set the price of the final good $P = 1$ so that Y denotes both real and nominal aggregate good.

Households in the economy are all equal, and can therefore be represented by a representative household with utility function $u(C, H) = C - H^{1+\frac{1}{\phi}}$, where C is consumption, H is supplied labor and $\phi > 0$ is the Frisch elasticity of labor supply. The competitive equilibrium is the result of the planner's problem, which is to maximize the household's utility subject to the resources constraint:

$$\max u(C, H) = C - H^{1+\frac{1}{\phi}} \tag{3a}$$

$$\text{s.t. } C = Y - rK \tag{3b}$$

where r is the price of capital. The aggregate good production, or GDP level, in the competitive equilibrium is given by

$$\log(Y) = m\gamma'\varepsilon + \Psi \ . \tag{4}$$

Eq. (4) describes (log-)GDP as a weighted sum of sectoral total factor productivities, collected in vector $\varepsilon = (\log(A_1), \ldots, \log(A_n))'$, where the weights contained in vector γ are known in the literature as Domar weights (Domar, 1961). The scalar m is a multiplying factor described below, while the constant Ψ is a convolution of structural parameters which only influences the level of economic activity. Since we are interested in studying only the impact of productivity shocks on GDP, we leave the detailed description Ψ to Appendix A and focus below on the Domar weights γ_i and on the multiplying factor m.

The Domar weights are defined as $\gamma' \equiv \beta'(I - W)^{-1}$. The vector $\beta' = (\beta_1, \ldots, \beta_n)$ contains the exponents, or shares of intermediate goods, in final good production, while matrix W collecting the exponents w_{ij} is the input-output matrix of intermediate good shares. Matrix $\bar{W} = (I - W)^{-1}$ is the Leontief inverse matrix, whose ij-th element describes how much an increase in productivity in sector j raises output in sector i taking into account all direct and indirect effects in the production structure. Multiplying the vector of value-added weights β by the j-th column of matrix $\bar{W}$ yields $\gamma_j = \sum_{i=1}^{n} \beta_i \bar{w}_{ij}$. Therefore, the j-th element of γ' sums the effects of sector j on all the other sector of the economy, weighting by their shares of value-added. In other words, the Domar weights describe the impact of a change in productivity in a certain sector on the overall value-added in the economy.

The multiplying factor is defined as $m \equiv (1 - \Gamma)^{-1}$, where the scalar Γ is given by

$$\Gamma \equiv \gamma' \left(k_s + h_s \frac{\phi}{1 + \phi} \right) \ . \tag{5}$$

The vector k_s contains the production elasticities $(1 - w_i)\alpha_i$ for capital, while the vector h_s contains the production elasticities $(1 - w_i)(1 - \alpha_i)$ for labor.

In this paper we are interested in the impact that changes in sectoral TFPs, denoted as $\Delta\varepsilon = (\Delta \log(A_1), \ldots, \Delta \log(A_n))'$, have on GDP. In particular, given Eq. (4), we have that

$$\Delta \log(Y) = m\gamma'\Delta\varepsilon \ . \tag{6}$$

Denoting the variance of a productivity shock in sector i as $\sigma_i^2 = \text{var}(\Delta\varepsilon_i)$ and using the fact that exogenous productivity terms are uncorrelated across sectors we can write GDP volatility defined as $\sigma_Y^2 = \text{var}(\Delta \log(Y))$ as follows:

$$\sigma_Y^2 = m^2 \sum_i \gamma_i^2 \sigma_i^2 \ . \tag{7}$$

The term $\sum_i \gamma_i^2 \sigma_i^2$ is referred to as fundamental volatility in Carvalho and Gabaix (2013) and granular volatility in Gabaix (2011). It describes volatility arising from idiosyncratic shocks to TFP at the sectoral level, pondered by the Domar weights. Fundamental volatility is multiplied by the term m^2 which describes the contribution of saving and labor supply decisions to aggregate volatility.

Based on Eq. (7), the model that we take to the data in order to explain GDP volatility in each period t is the following:

$$\sigma_{Yt}^2 = m_t^2 \sum_i \gamma_{it}^2 \sigma_{it}^2 \ . \tag{8}$$

The terms m_t, γ_{it} and σ_{it} denote respectively the multiplying factor, the Domar weight and the TFP volatility of sector i in year t, thus computed using data in year t (see Section 3 for details). Eq. (8) is similar to the empirical model implemented in Carvalho and Gabaix (2013) since it links the volatility of GDP to sectoral volatilities and Domar weights, but with the following important differences. First, in defining the multiplier m_t we allow for heterogeneity in capital and labor shares, as well as heterogeneity in the shares of intermediate inputs across sectors. This enables us to be consistent with input-output data, resulting in a time-varying expression for production elasticities, i.e. α_{it} and w_{it}, and thus for the multiplying factor. In the presence of production input shares α and w homogeneous across sectors the multiplier reduces to the expression implemented in Carvalho and Gabaix (2013)

$$m = \frac{1+\phi}{1-\alpha} \ ,$$

which is kept constant over time in their analysis. By considering a time-varying multiplier based on Eq. (5), where production elasticities are varying over time and calculated using input-output data, we take into account the time-varying contribution of labor supply and savings decisions to aggregate volatility. Second, we allow for time-varying sectoral TFP volatilities. This choice is motivated by previous empirical work which linked changes in GDP volatility to changes in sectoral volatility (see e.g. McConnell and Perez-Quiros, 2000; Moro, 2012).[2] This enables us to highlight the impact that changes in the covariance between Domar weights and sectoral TFP volatilities had on changes in GDP volatility.

In what follows we show that *i*) our model calibrated to US data can explain the pattern of aggregate volatility both in *deviation* from its mean and in *level*; *ii*) by means of counterfactual analysis, we are able to disentangle the role played by time-varying sectoral TFP volatilities, asymmetric production linkages and the covariance between γ_i and σ_i in explaining aggregate volatility.

3 Calibration

A detailed description of the data used to construct all measures below is provided in Appendix B. Following Carvalho and Gabaix (2013), we consider both a rolling window and an instantaneous measure of GDP volatility. In order to obtain the first measure, we construct the series of quarterly HP-detrended log real GDP, denoted as $\hat{y}$, and then compute the variance at each quarter q using a centered rolling window of 20 quarters:

$$\sigma^2_{RW,q} = \text{var}(\hat{y}_\tau) \qquad \text{for } \tau \in [q-20, q+20] \ .$$

To obtain volatility in year t, denoted by $\sigma^2_{RW,t}$, we take the average of $\sigma^2_{RW,q}$ over the quarters of year t. To compute the second measure of GDP fluctuations we first estimate an AR(1) model on the quarterly growth rate of real GDP, and then use residuals e_q to obtain the series of annualized instantaneous variance as $4e^2_q$. We then compute the average of $4e^2_q$ over the quarters of year t to obtain

$$\sigma^2_t = \sum_{q=1}^{4} e^2_{t:q} \ ,$$

where $t : q$ denotes the quarter q of year t, and finally obtain the instantaneous measure of GDP volatility $\sigma^2_{IV,t}$ using the HP-trend of σ^2_t. In order to explain GDP volatility, measured both as rolling window (RW) and as instantaneous volatility (IV), we calibrate our model using input-

[2]Carvalho and Gabaix (2013) consider a measure of time-varying sectoral volatilities estimated using a GARCH(1,1) model. They find that this measure does not improve the explanatory power of the model when compared to the case of constant sectoral volatilities. We use instead a non-parametric measure of time-varying sectoral volatilities improving the explanatory power of the model. See Table C.1 in Appendix C for a comparison between these models.

output data from Dale Jorgenson and Associates (see Jorgenson et al., 2005). The dataset contains input-output yearly tables for 88 sectors for the US economy, ranging from 1960 to 2005. In order to make our results comparable to previous literature, we consider the same total of 77 sectors analyzed in Carvalho and Gabaix (2013).

In each year t we compute the shares α_{it} as the ratio between the nominal value of capital and the total nominal value of primary inputs used by sector i in that year. The shares w_{it} are computed as the ratio between the sum of the nominal values of all intermediate inputs used by each sector i in that year t, and the nominal value of gross industry output. The Domar weights γ_{it} are computed as the ratio between nominal output of sector i and nominal GDP in year t. We use the same dataset to compute sectoral changes in TFP $\Delta\varepsilon$ employing the accounting methods outlined in Jorgenson et al. (1987). In order to obtain a time-varying measure of sectoral volatilities we compute σ_{it}^2 as the variance of $\Delta\varepsilon_{it}$ using a 10-years rolling window:[3]

$$\sigma_{it}^2 = \text{var}(\Delta\varepsilon_{i\tau}) \qquad \text{for } \tau \in [t-5, t+5] \; .$$

Given the windows' length, our characterization of aggregate volatility focuses on the years 1966–2000. The only free parameter left in the model is the Frisch elasticity of labor supply ϕ, and we calibrate it in order to match the average level of GDP volatility. In what follows we use σ_{RW}^2 as our reference measure of GDP volatility. Robustness of the results to the alternative measure σ_{IV}^2 are reported in Appendix C. The resulting calibrated value of the Frisch elasticity is $\phi = 0.13$,[4] which is consistent with estimates in the microeconometric literature (MaCurdy, 1981; Altonji, 1986).[5]

[3]The window's length is consistent both with the number of years used in Carvalho and Gabaix (2013) and with the length used to compute GDP volatility.

[4]The minimum distance between the average model-implied volatility and actual average volatility is obtained with $\phi = 0.1243$. The value of the Frisch elasticity that minimizes the mean squared distance between the two measures is $\phi = 0.1363$. We take the average between the two values and set $\phi = 0.13$.

[5]In macroeconomic literature, the typical calibrated value of the Frisch elasticity is between 2 and 4. There exists a large number of contributions investigating the reasons behind the different values found in the microeconometric literature and the macroeconomic calibrations (Keane and Rogerson, 2012, 2015; Peterman, 2016). It is beyond the scope of this paper to take a stand about the actual value of the Frisch elasticity. Therefore, our calibrated value should be interpreted as a parameter scaling the reaction of the household to technology shocks. Using $\phi = 2$, our model would overestimate the level of GDP volatility, as in Carvalho and Gabaix (2013).

3.1 Fit of the model

Fig. 1 displays actual GDP volatility computed using both the rolling-window estimate (RW) the HP-filtered instantaneous volatility (IV) together with GDP volatility implied by the model in Eq. (8). Fig. 1 shows that the calibrated model is able to track the pattern of the *level* of aggregate volatility over time. In particular, despite the fact that the peak in volatility is slightly shifted to the right, the model reproduces both the "great volatility" and the subsequent "great moderation" period.

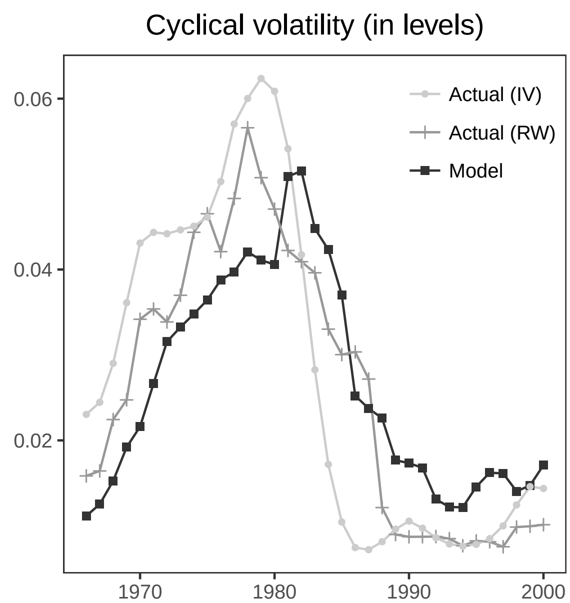


Figure 1: Actual GDP volatility computed using a rolling-window estimate (RW) and HP-filtered instantaneous volatility (IV) together with GDP volatility implied by model (8).

In Table 1 we evaluate the goodness of fit of our model and we compare it with a model with constant sectoral TFPs. The first column of Table 1 shows that the volatility σ^2_{Yt} described in Eq. (8) explains about 77% of actual GDP volatility $\sigma^2_{RW,t}$. This regression confirms the ability of the model to track observed GDP volatility.[6] The second column of Table 1 shows that the volatility $\bar{\sigma}^2_{Yt}$, i.e. aggregate volatility implied by the model with constant sectoral TFP volatilities σ_i, is only able to explain about 57% of actual GDP volatility. This result highlights the importance of time-varying idiosyncratic volatilities to explain the dynamics of aggregate fluctuations. The third column of Table 1 shows that, when both measures σ^2_{Yt} and $\bar{\sigma}^2_{Yt}$ are included in the regression, the

[6] In Appendix C we perform a series of robustness checks. We show that the model retains the ability to reproduce observed GDP volatility also with different measures of time-varying sectoral volatility.

latter is not significant.

Empirical model: $\sigma^2_{RW,t} = a + b\,\sigma^2_{Yt} + c\,\bar{\sigma}^2_{Yt} + \eta_t$

$\hat{a}$	-0.0000 (-0.0001,0.0000)	-0.0003 (-0.0005,-0.0001)	-0.0000 (-0.0002,0.0002)
$\hat{b}$	1.1084 (0.8944,1.3224)	–	1.1767 (0.7281,1.6253)
$\hat{c}$	–	2.5013 (1.7290,3.2737)	-0.2052 (-1.3844,0.9740)
R^2	0.77	0.57	0.77

Table 1: OLS estimates with 95% confidence intervals between brackets. The measure $\bar{\sigma}^2_{Yt}$ is obtained as $\bar{\sigma}^2_{Yt} = m_t^2 \sum_i \gamma_{it}^2 \sigma_i^2$, where σ_i denotes constant sectoral TFP volatility of sector i, i.e. computed over the whole sample, while σ^2_{Yt} is defined in Eq. (8).

In order to stress the importance of sectoral volatilities for aggregate fluctuations, we measure the relative contributions of changes in Domar weights and in sectoral TFP volatilities to changes in fundamental volatility. Focusing on fundamental volatility rather than on the model-implied volatility including the multiplying factor allows us to obtain a simple characterization of such relative contributions. Recalling the definition of fundamental volatility as the combination of sectoral TFP volatilities pondered by the Domar weights, the change in fundamental volatility between two time periods t_1 and t_2 can be written as $\sum_i \gamma^2_{i,t_2}\sigma^2_{i,t_2} - \sum_i \gamma^2_{i,t_1}\sigma^2_{i,t_1}$. We can then decompose this difference to isolate the impacts of changes in γ_i and σ_i relative to the total change in fundamental volatility as

$$\frac{\sum_i \gamma^2_{i,t_1}(\sigma^2_{i,t_2} - \sigma^2_{i,t_1})}{\sum_i \gamma^2_{i,t_2}\sigma^2_{i,t_2} - \sum_i \gamma^2_{i,t_1}\sigma^2_{i,t_1}} + \frac{\sum_i \sigma^2_{i,t_2}(\gamma^2_{i,t_2} - \gamma^2_{i,t_1})}{\sum_i \gamma^2_{i,t_2}\sigma^2_{i,t_2} - \sum_i \gamma^2_{i,t_1}\sigma^2_{i,t_1}} = 1 \ .$$

The first term describes the relative importance of the change in the idiosyncratic volatilities between t_1 and t_2 scaled by the Domar weights in t_1, while the second term describes the relative importance of the change in the Domar weights between t_1 and t_2 scaled by the idiosyncratic volatility in t_2. Both terms have a simple and intuitive interpretation: the first measures the change in volatility that would have occurred if only sectoral TFP had changed and Domar weights had remained constant at their level in t_1 (as a fraction of total change in fundamental volatility), while the second measures the change that would have taken place if the Domar weights had been the only variables to change (as a fraction of total change in fundamental volatility) with sectoral

TFP at their level in t_2. Computing the two terms for the "great volatility" period spanning from 1966 to 1982, we find that 56% of the increase in fundamental volatility is explained by the change in idiosyncratic volatilities, while 44% is explained by the change in the Domar weights. Strikingly, 78% of the reduction in volatility occurred during the "great moderation" between 1983 and 1994 is explained by the sectoral volatility term and only 22% by the change in the Domar weights.

This result shows once again that changes in sectoral TFP volatility have significant impact on the dynamics of aggregate volatility and provides a rationale for the use of time-varying sectoral volatilities. In the next section we perform a series of counterfactual analysis to understand in details the determinants of aggregate volatility.

4 Counterfactual analysis

In this section we perform a series of counterfactual exercises to assess the impact of the different terms in Eq. (8) on aggregate volatility from 1966 to 2000. In particular, the counterfactuals show the impact on observed changes in GDP volatility of *i*) changes in idiosyncratic volatility, *ii*) changes in aggregate intermediate goods intensity in production and *iii*) changes in the distribution of Domar weights, reflecting changes in both the vector of value-added weights β and the input-output network W. Each of these elements played a role both in the upsurge of aggregate volatility during the "great volatility" period, and in the reduction of aggregate volatility during the "great moderation".

In the first counterfactual we compute the level of aggregate volatility emerging from sectoral TFP volatilities σ_{it}^2 in the absence of an input-output network by setting $w_{ij} = 0$ for all sectors i, j. We label this scenario as "No I–O". In this case the vector of Domar weights is simply equal to the vector of exponents in final good production, which we set homogeneous across sectors, i.e., $\gamma_i = \beta_i = 1/n$ for all i. The scalar Γ_t in Eq. (5) which defines the multiplying factor m_t is then given by

$$\Gamma_t = \frac{1}{n} e' \left(k_{st} + h_{st} \frac{\phi}{1+\phi} \right) , \qquad \text{(No I–O)}$$

where $e = (1, \ldots, 1)'$ and the i-th elements of vectors k_{st} and h_{st} are now given respectively by α_{it}

and $(1-\alpha_{it})$. The time-variation of aggregate volatility in this case is described by

$$\sigma_{Yt}^2 = m_t^2 \frac{1}{n^2} \sum_i \sigma_{it}^2 \ , \qquad \text{(No I–O)}$$

and it depends only on sectoral TFP volatilities and production elasticities for capital and labor. Fig. 2(a) reports GDP volatility in the No I–O scenario, where the primary input shares α_i are computed using actual input-output tables. We observe that the simulated No I–O aggregate volatility exhibits a pattern similar to actual GDP volatility (correlation coefficient ≈ 0.65 and significant at 5% level), but sectoral volatilities alone are not able to explain the level of actual aggregate fluctuations. This means that changes in sectoral volatilities alone are an important engine for the time-variation of aggregate volatility, but the network of production linkages plays an important amplification role.

In order to assess the importance of the input-output network for aggregate volatility, we start by considering an hypothetical symmetric structure of production linkages. In this second counterfactual we thus maintain homogeneous exponents in final good production $\beta_i = 1/n$ and we calibrate the total intermediate input shares w_{it} using input-output data. The symmetric input-output network is then obtained by setting $w_{ijt} = w_{it}/n$ for all j. We label this scenario as "Sym I–O". In this case the Domar weights sum up to the observed total nominal value of sales over GDP, but they are homogeneous across sectors. Using actual input-output data to compute total gross nominal output across sectors, we can compute the homogeneous Domar weights as $\gamma_{it} = \frac{1}{n}\sum_i \frac{p_{it}Q_{it}}{Y_t} = \bar{\gamma}_t$ for all i. The scalar Γ_t in Eq. (5) is thus obtained as

$$\Gamma_t = \bar{\gamma}_t e' \left(k_{st} + h_{st} \frac{\phi}{1+\phi} \right) \ , \qquad \text{(Sym I–O)}$$

and the multiplier m_t is computed accordingly. The evolution of GDP volatility is then given by

$$\sigma_{Yt}^2 = m_t^2 \bar{\gamma}_t^2 \sum_i \sigma_{it}^2 \ . \qquad \text{(Sym I–O)}$$

The variation of volatility over time depends therefore on the average level of sales per unit of GDP, which is a measure of the importance of intermediate inputs in production. Fig. 2(a) reports GDP volatility implied by the second counterfactual. By comparing the volatilities obtained in the Sym

I–O and the No I–O scenarios we isolate the amplifying effect of a symmetric input-output network. The mere presence of such a network implies the propagation of idiosyncratic productivity shocks across sectors via production linkages. The greater the importance of the input-output network captured by $\bar{\gamma}_t$, the higher the impact of sectoral shocks on GDP.

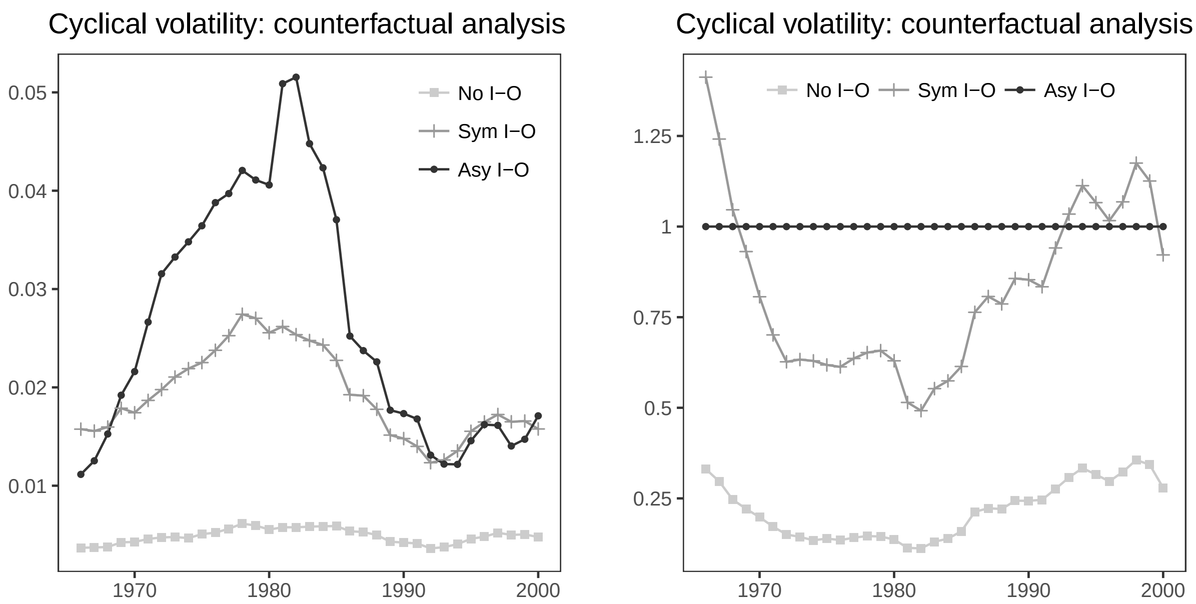
Finally, in the third counterfactual exercise we evaluate the contribution of asymmetries in the vector of value-added weights β and the input-output network W to aggregate volatility. The considered scenario corresponds to the model in Eq. (8) reported below for convenience and labeled as "Asy I–O":

$$\sigma_{Yt}^2 = m_t^2 \sum_i \gamma_{it}^2 \sigma_{it}^2 \,. \qquad \text{(Asy I–O)}$$

In this case the sectoral Domar weights are given by observed gross nominal industry output over GDP. Fig. 2(a) reports model-implied GDP volatility under Asy I–O. The difference between the Sym I–O and the Asy I–O scenarios is that in the latter we introduce asymmetry in the production structure by considering observed Domar weights. This allows us to isolate the impact of asymmetries in the vector of shares in final good production and in the structure of production linkages.

In order to quantitatively assess the contribution of different volatility sources to aggregate fluctuations, we compute the ratios between the volatility obtained in the No I–O and Sym I–O scenarios and the volatility implied by the full model in the Asy I–O scenario. Fig. 2(b) displays the results.

Sectoral TFP volatilities are an important source of GDP fluctuations but, as noted before, the implied level of volatility in the absence of production linkages is too low. In fact, the average ratio between volatilities in the No I–O and the Asy I–O scenario is about 0.22. Introducing a symmetric input-output layer to the model amplifies micro-level volatilities because it allows idiosyncratic shocks to spread through the network. The "great volatility" is partially explained by the increase in the use of intermediate goods in the US. Similarly, the "great moderation" is partially explained by the evolution toward a less intermediate good intensive economy. This last result is in line with the explanation provided in Moro (2012). However, the change in the intensity of intermediate goods in production is not able to fully explain observed changes in aggregate volatility. By comparing the Sym I–O and Asy I–O scenarios it is clear how the non-homogeneous



(a) (b)

Figure 2: **Panel (a):** Model-implied volatilities in the cases of no input-output (No I–O), symmetric (Sym I–O) and asymmetric (Asy I–O) input-output network. **Panel (b):** Ratios between volatilities in the No I–O, Sym I–O, Asy I–O scenarios and volatility in the Asy I–O scenario.

structure of the input-output network had an important amplifying role between 1970 and 1990. Note that the only difference between Sym I–O and Asy I–O counterfactuals is the distribution of Domar weights between sectors. The analysis of the Sym I–O and Asy I–O volatilities ratio reveals that the role played by asymmetries in shaping aggregate volatilities has greatly changed in time. In particular from 1970 to 1992, asymmetry has amplified the volatility (Sym I–O/Asy I–O < 1), especially around 1980. However, before 1970 and after 1992, the asymmetric structure of the economy had a dampening effect on aggregate volatility (Sym I–O/Asy I–O > 1). During the "great moderation", for example, there has been a convergence of aggregate volatility towards the volatility implied by a symmetric input-output network. This convergence could simply be explained by a reduction of the degree of asymmetry in the input-output network over time, or it could be due to a change in the effect that a similarly asymmetric network had over time. In the following section we study the causes of this convergence and investigate further the role played by asymmetry over time.

4.1 The time-varying role of asymmetry

Consider the ratio between volatilities in the Asy I–O and Sym I–O scenarios denoted for simplicity by $\mathrm{vA}_t/\mathrm{vS}_t$:

$$\frac{\mathrm{vA}_t}{\mathrm{vS}_t} = \frac{\mathrm{mA}_t}{\mathrm{mS}_t} \frac{\sum_i \gamma_{it}^2 \sigma_{it}^2}{\bar{\gamma}_t^2 \sum_i \sigma_{it}^2} ,$$

where $\mathrm{mA}_t/\mathrm{mS}_t$ denotes the ratio between the multiplying factors m_t in the Asy I–O and Sym I–O scenarios. The ratio between the volatility with asymmetric and symmetric input-output structures depends on the ratio between the multipliers and on a second term which in turn depends, unsurprisingly, on the distribution of γ_i, and in particular on the product between the vector of Domar weights and the vector of sectoral volatilities in each time period t. To help the intuition, it turns out to be convenient to rewrite the expression above in terms of cross-sectional population moments to obtain

$$\frac{\mathrm{vA}}{\mathrm{vS}} = \frac{\mathrm{mA}}{\mathrm{mS}} \left(\frac{E(\gamma)^2 + \mathrm{var}(\gamma)}{E(\gamma)^2} + \frac{\mathrm{cov}(\gamma^2, \sigma^2)}{E(\gamma)^2 E(\sigma^2)} \right) , \tag{9}$$

where we have suppressed the time subscript t for notational simplicity. Let us define the terms $Asy \equiv (E(\gamma)^2 + \mathrm{var}(\gamma))/E(\gamma)^2$ and $Cov \equiv \mathrm{cov}(\gamma^2, \sigma^2)/(E(\gamma)^2 E(\sigma^2))$. The Asy term describes how the asymmetry of the input–output network evolved over time. When the input-output network is symmetric, i.e. the Domar weights are homogeneous across sectors ($\mathrm{var}(\gamma) = 0$), the term Asy is equal to 1. The Asy term increases with the degree of asymmetry. We remark that the asymmetry in the production network also impacts the ratio of multipliers mA/mS. The Cov term shows that the way in which the covariance between the vector of (squared) Domar weights γ^2 and the vector of sectoral TFP volatilities σ^2 evolves over time has an impact on aggregate fluctuations. Fig. 3 displays the evolution over time of terms mA/mS, Asy, Cov and compares them with changes in the volatility ratio vA/vS.

We notice that the ratio mA/mS oscillated between 0.95 and 1.1, meaning that this term did not play a key role in shaping the time variation of the ratio vA/vS. The level of asymmetry as summarized by the term Asy decreased almost monotonically from the beginning of the sample until 1990 and then pretty much stabilized from 1991 on. We therefore conclude that the evolution of asymmetry alone is not able to explain the different patterns of volatility in the Sym I–O and Asy I–O scenarios. In particular, the dampening effect of the asymmetric structure of the input-output

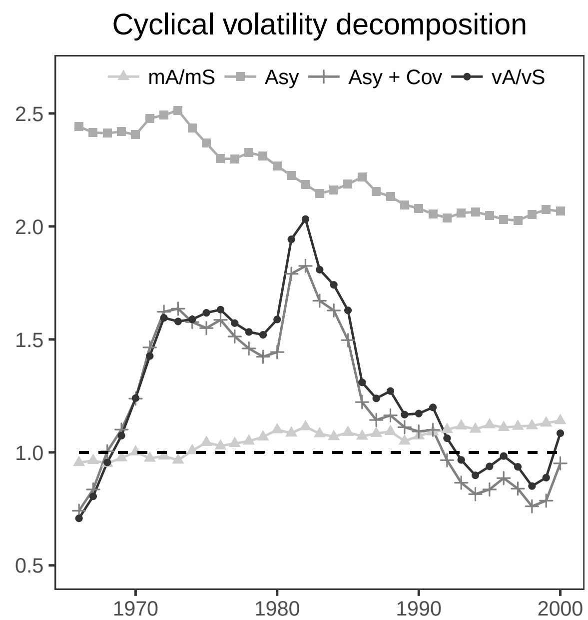


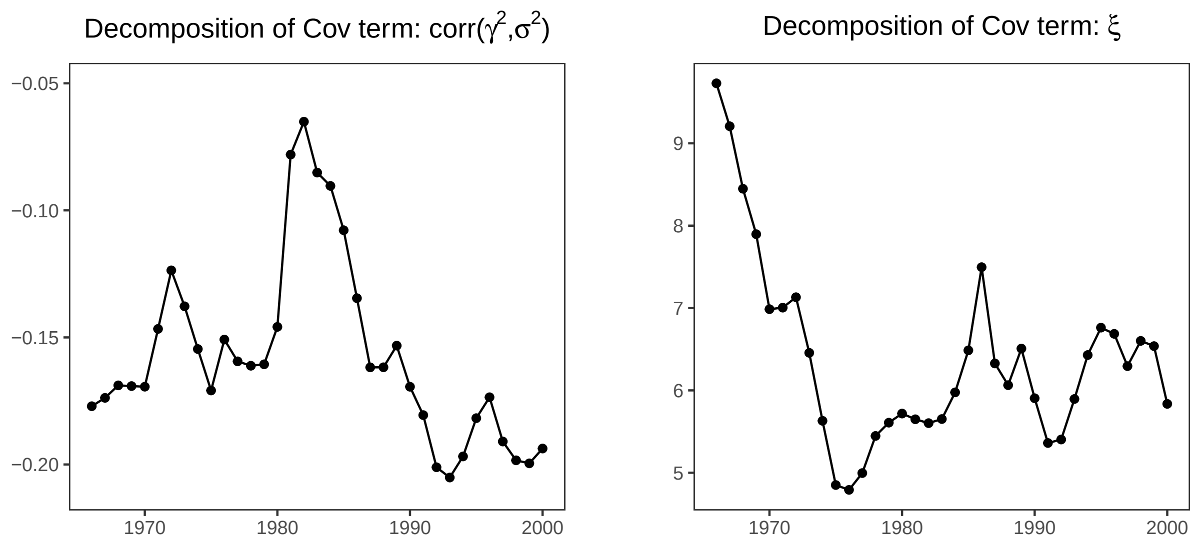
Figure 3: Evolution of terms vA/vS, mA/mS, Asy and Cov over time.

network observed after 1992, or in other words the convergence towards the volatility implied by the model with homogeneous Domar weights, does not seem to be related to a reduction in the degree of asymmetry itself. In fact, what explains the different effect of asymmetry over time is the way in which the Cov term evolved over time. More specifically, the strong amplifying effect of asymmetric production linkages registered around late-1970s/early-1980s is due to a less negative Cov term.[7] Moreover, the dampening effect of asymmetry observed during the "great moderation" is due to a higher Cov term (in absolute value). The importance of the term Cov in explaining aggregate volatility requires a deeper analysis. To ease the interpretation, it is convenient to rewrite the term Cov as:

$$\frac{\text{cov}(\gamma^2, \sigma^2)}{E(\gamma)^2 E(\sigma^2)} = \text{corr}(\gamma^2, \sigma^2)\xi \ ,$$

where $\text{corr}(\gamma^2, \sigma^2)$ is the correlation between the squared Domar weights and the idiosyncratic volatilities in each time period, while $\xi \equiv \frac{\text{std}(\gamma^2)\text{std}(\sigma^2)}{E(\gamma)^2 E(\sigma^2)}$ is a scaling term accounting for dispersion and size of γ^2 and σ^2 in each time period. Fig. 4 shows the evolution over time of $\text{corr}(\gamma^2, \sigma^2)$ and ξ. The time variation of $\text{corr}(\gamma^2, \sigma^2)$ qualitatively matches the pattern of vA/vS. In fact, most of the contribution of the asymmetry in the input-output network to the change of aggregate volatility

[7] Since the term $Asy + Cov$ is always below Asy in Fig. 3, the term Cov is always negative.



(a) (b)

Figure 4: **Panel (a):** Evolution over time of correlation term corr(γ^2, σ^2). **Panel (b):** Evolution over time of scaling term ξ .

can be traced back to changes in the correlation between Domar weights and sectoral volatilities. In particular, asymmetry in the production network played an important role in the "great volatility" because in those decades the correlation between Domar weights and sectoral volatilities increased, becoming less negative. In other words, the increase of aggregate volatility can be explained by the fact that relatively volatile sectors became more "central" in the production network and relatively important sectors became more volatile. On the opposite, during the "great moderation" the correlation decreased and became more negative, i.e. less volatile sectors became relatively important in the production structure and relatively important sectors became less volatile. Our results stress that the interaction between asymmetric production network and heterogeneous idiosyncratic volatilities plays an important role in explaining the dynamics of aggregate fluctuations.

5 Sectoral analysis and aggregate volatility

In this section we investigate which sectors contributed the most to the "great volatility" and to the "great moderation". Defining the index $H_i(t_1, t_2)$ as

$$H_i(t_1, t_2) = \frac{\gamma_{i,t_2}^2 \sigma_{i,t_2}^2 - \gamma_{i,t_1}^2 \sigma_{i,t_1}^2}{\sum_i \gamma_{i,t_2}^2 \sigma_{i,t_2}^2 - \sum_i \gamma_{i,t_1}^2 \sigma_{i,t_1}^2} \ ,$$

we can measure the contribution of a given sector i to the change of aggregate volatility between periods t_1 and t_2. As argued in Section 3.1, focusing on fundamental volatility rather than on the model-implied volatility including the multiplying factor allows us to obtain a simple characterization of such contributions. We divide our sample into two subsamples, namely 1966 – 1982 ("great volatility") and 1983 – 1994 ("great moderation") and compute the values of H_i for each sector i in both subsamples. Fig. 5(a) displays the distribution of $H_i(1966, 1982)$, while Fig. 5(b) plots the distribution of $H_i(1983, 1994)$.

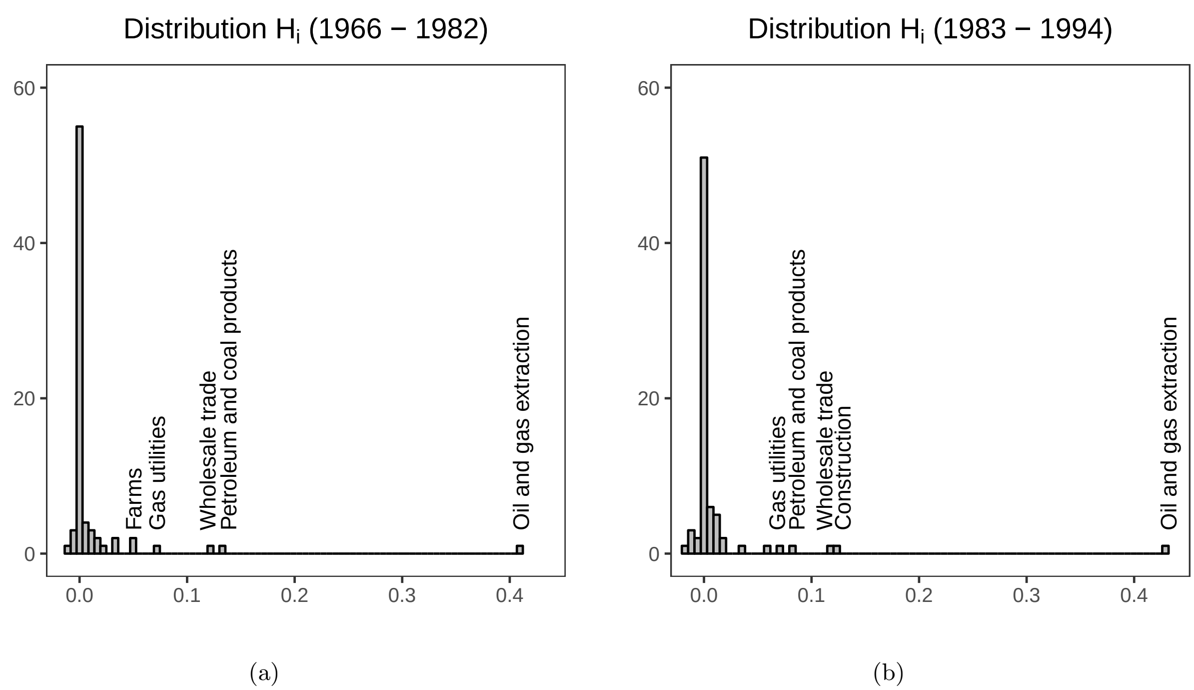


Figure 5: **Panel (a):** Distribution of $H_i(1966, 1982)$. **Panel (b):** Distribution of $H_i(1983, 1994)$. Labels show the 5 most relevant sectors.

The five most important sectors for the "great volatility" period, namely Oil and gas extractions, Petroleum and coal products, Wholesale trade, Gas utilities and Farms, jointly account for about

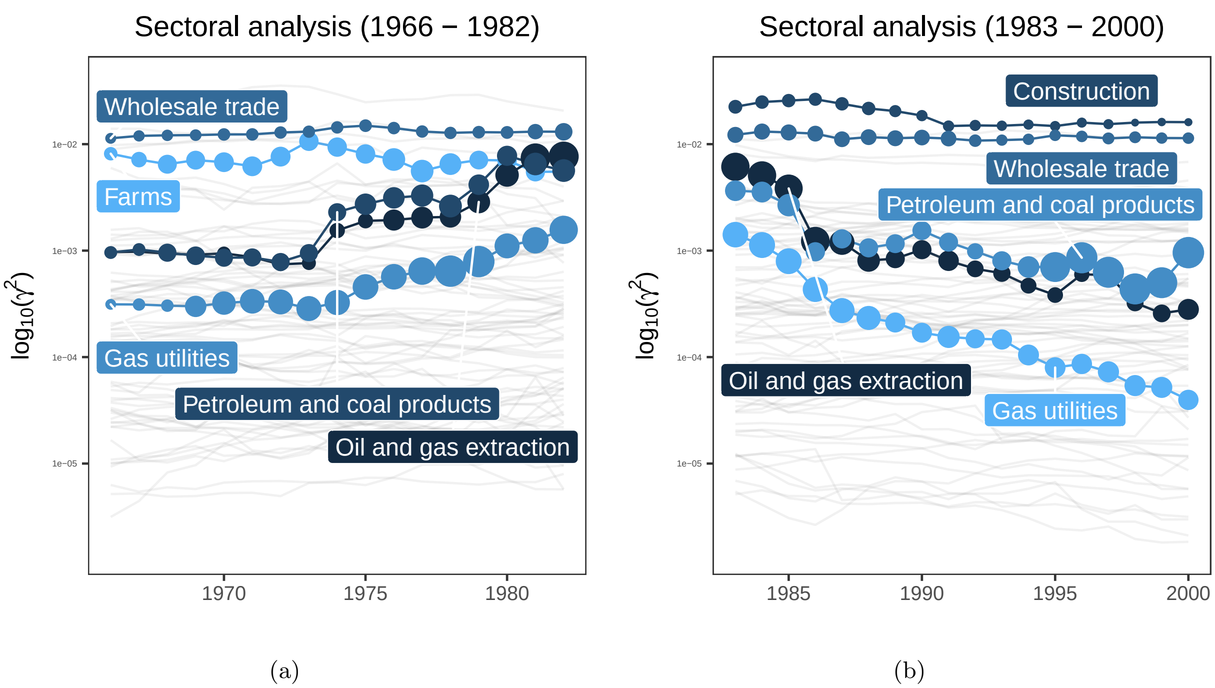


Figure 6: **Panel (a):** Time variation of γ_i^2 (in $\log_{10}$ scale) for first 5 sectors ranked according to $H_i(1966, 1982)$. Darker to lighter color reflects decreasing sector importance according to H_i. Point size is proportional to σ_i^2. **Panel (b):** Time variation of γ_i^2 (in $\log_{10}$ scale) for first 5 sectors ranked according to measure $H_i(1983, 1994)$. Darker to lighter color reflects decreasing sector importance according to H_i. Point size is proportional to σ_i^2.

79% of the increase in aggregate volatility. On the other hand, the five most relevant sectors for the “great moderation”, namely Oil and gas extractions, Construction, Wholesale trade, Petroleum and coal products and Gas utilities are jointly responsible for about 83% of the decline in GDP volatility. Figs. 6(a) and 6(b) display instead the logarithm of the Domar weight associated to each sector over time, with the size of the circles proportional to idiosyncratic volatility. For the sake of readability, we highlighted the five most important sectors according to $H_i(1966, 1982)$ (Fig. 6(a)) and to $H_i(1983, 1994)$ (Fig. 6(b)), while darker to lighter color reflects decreasing sector importance according to H_i.

Fig. 6(a) shows that some very large sectors in the US production structure, such as Farms and Whole trade, experienced an increase in their TFP volatility while their Domar weights remained roughly constant. On the other hand, sectors related to fossil fuels displayed an increase in both their importance in terms of Domar weights and their TFP volatility. These phenomena can explain the increase in the correlation between the vectors of Domar weights and TFP volatilities depicted in Figure 4(a). As shown in Fig. 6(b), the reduction in aggregate volatility leading to the “great moderation” is explained by the fact that sectors related to fossil fuel became less central in the production network and less volatile. On the other hand, the Construction and Wholesale trade sectors became less volatile while their Domar weights remained roughly constant. Finally, we remark that from the second half of 1990s, the volatility of the sector related to Petroleum and coal products largely increased, causing an upward trend in GDP volatility in the final part of the sample.

6 Conclusions

This paper highlights the importance of the interaction between the structure of the production network and sectoral volatilities in determining GDP volatility. We show that sectoral TFP volatilities are important sources of GDP fluctuations. Sectoral volatilities are transmitted at the aggregate level and amplified through the network of production linkages. The amplification effect depends on two factors. First, the intermediate goods intensity of the production system, measured as total gross nominal production over GDP. Second, the asymmetry of the production structure determined by the distribution of value-added weights β and the topology of the input-output network

W. We explain the rise and fall of aggregate volatility as the combination of three factors. First, an increase in the average sectoral volatility, which then decreased starting from mid-1980s (No I–O counterfactual). Second, an increase and a subsequent decrease of the intermediate input intensity of the production structure occurred in the same years (Sym I–O counterfactual). The higher the intermediate input intensity, the higher the amplification effect. Third, the change over time of the impact of asymmetry on the transmission of sectoral shocks (Asy I–O counterfactual). We observed that asymmetry had both an amplifying and a dampening effect on aggregate volatility. In fact, although the literature typically associates asymmetric production networks to stronger amplification of micro-level volatility, we find that the actual effect depends on the correlation between the vectors of Domar weights and sectoral TFP volatilities. We find that this correlation is negative throughout our sample, implying that on average sectors with higher Domar weights are associated to lower volatilities. The peak in aggregate volatility is associated to an higher correlation (less negative) and thus to a stronger amplifying effect of asymmetry. In the first and last part of the sample, correlation is relatively more negative implying that more volatile sectors are more peripheral in the production network, leading therefore to a dampening effect of asymmetry.

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A Detailed model derivation

We first derive the competitive equilibrium of the economy and express total production of the aggregate good as a function of sectoral TFPs, Domar weights and aggregate levels of primary inputs. We then derive the optimal level of capital and labor supply as a solution to the planner's problem. Finally, we compute GDP volatility.

Final good sector: A representative firm produces the final good using the technology:

$$Y = \prod_{i=1}^{n} c_i^{\beta_i} \ ,$$

with $\sum_{i=1}^{n} \beta_i = 1$. The profit maximization of this firm reads as follows:

$$\max_{c_i} \ Y - \sum p_i c_i \ ,$$

where p_i denotes the price of input i and we assume that the price index for the aggregate good is $P = 1$. The first order conditions imply that

$$\beta_i = \frac{p_i c_i}{Y} \ . \tag{A.1}$$

Intermediate goods sectors: Intermediate goods are produced using the following technology:

$$Q_i = A_i \left(K_i^{\alpha_i} H_i^{(1-\alpha_i)} \right)^{(1-w_i)} \prod_{j=1}^{n} d_{ij}^{w_{ij}}$$

where $w_i = \sum_j w_{ij}$. The profit maximization reads as follows:

$$\max \ p_i Q_i - \sum_j p_j d_{ij} - rK_i - wH_i \ ,$$

where r and w denote the prices of capital and labor respectively. The first order conditions are given by

$$
\begin{aligned}
rK_i &= \alpha_i(1-w_i)p_iQ_i && \text{(A.2)} \\
wH_i &= (1-\alpha_i)(1-w_i)p_iQ_i && \text{(A.3)} \\
p_jd_{ij} &= w_{ij}p_iQ_i \; . && \text{(A.4)}
\end{aligned}
$$

We now derive an expression for (log-)GDP as a function in terms of sectoral TFP and Domar weights.

GDP as function of sectoral TFPs and Domar weights: We start by writing the resource constraints

$$
c_j + \sum_{i=1}^{n} d_{ij} = Q_j \; , \tag{A.5}
$$

according to which the quantity of intermediate good j used in the production of the final good plus the quantity of intermediate good j used in the production of intermediate goods equals the total production of intermediate good j. Multiplying both sides of Eq. (A.5) by p_j and substituting Eq. (A.4) to eliminate the terms d_{ij} we get

$$
p_jc_j + \sum_{i=1}^{n} w_{ij}p_iQ_i = p_jQ_j \; .
$$

Using Eq. (A.1) to substitute for prices in the equation above we get

$$
\begin{aligned}
\frac{\beta_jY}{c_j}c_j + \sum_{i=1}^{n} w_{ij}\frac{\beta_iY}{c_i}Q_i &= \frac{\beta_jY}{c_j}Q_j \\
\beta_j + \sum_{i=1}^{n} w_{ij}\frac{\beta_i}{c_i}Q_i &= \frac{\beta_j}{c_j}Q_j \; .
\end{aligned}
$$

Defining $v_j \equiv \beta_jQ_j/c_j$ and matrix W whose ij-th element is given by w_{ij}, we can write the equation above in vector form

$$
\beta + W'v = v \; ,
$$

from which we get an expression for the Domar weights given by

$$v^* = (I - W')^{-1}\beta \equiv \gamma \ . \tag{A.6}$$

In fact, using Eq. (A.1) we obtain $\gamma_j = \beta_j Q_j / c_j = p_j Q_j / Y$. In order to interpret the weights in γ we can write $\gamma' = \beta'(I - W)^{-1}$. Matrix $\bar{W} = (I - W)^{-1}$ is the Leontief inverse matrix, whose ij-th element describes how much an increase in productivity in sector j raises output in sector i taking into account all direct and indirect effects in the production structure. Multiplying this matrix by the vector of value-added weights β yields $\beta'\bar{W} = \sum_{i=1}^{n} \beta_i \bar{w}_{ij}$. Therefore, the j-th element of γ' sums the effects of sector j on all the other sector of the economy, weighting by their shares of value-added. In other words, Domar weights describe the impact of a change in productivity in a certain sector on the overall value-added in the economy.

In order to write production of the aggregate good as a function of the Domar weights we notice that Eq. (A.1) implies that

$$\frac{p_i}{p_j} = \frac{\beta_i}{\beta_j}\frac{c_j}{c_i} \ ,$$

which can be substituted in Eq. (A.4) to get

$$d_{ij} = \frac{\beta_i}{\beta_j}\frac{c_j}{c_i} Q_i w_{ij} \ .$$

Given that $\gamma_j = p_j Q_j / Y = \beta_j Q_j / c_j$, we can write the FOC for the optimal choice of intermediate input above as

$$d_{ij} = \frac{\gamma_i}{\gamma_j} Q_j w_{ij} \ . \tag{A.7}$$

Moreover, using again that $\gamma_j = p_j Q_j / Y = \beta_j Q_j / c_j$, we can write the FOCs for optimal capital and labor choices in Eqs. (A.3)–(A.4) as

$$K_i = \frac{\alpha_i (1 - w_i)\gamma_i Y}{r}$$
$$H_i = \frac{(1 - \alpha_i)(1 - w_i)\gamma_i Y}{w} \ ,$$

from which it follows that

$$\frac{K_i}{\sum_i K_i} = \frac{K_i}{\bar{K}} = \frac{\alpha_i(1-w_i)\gamma_i Y}{\sum_i \alpha_i(1-w_i)\gamma_i Y} \equiv \bar{\Theta}_{K_i} \tag{A.8}$$

$$\frac{H_i}{\sum_i H_i} = \frac{H_i}{\bar{H}} = \frac{(1-\alpha_i)(1-w_i)\gamma_i Y}{\sum_i(1-\alpha_i)(1-w_i)\gamma_i Y} \equiv \bar{\Theta}_{H_i} \ . \tag{A.9}$$

Substituting the expressions for the production inputs derived in Eqs. (A.7)–(A.9) in the intermediate goods production function we obtain

$$Q_i = A_i \left(\left(\Theta_{K_i}\bar{K}\right)^{\alpha_i} \left(\Theta_{H_i}\bar{H}\right)^{1-\alpha_i} \right)^{1-w_i} \prod_{j=1}^{n} \left(\frac{w_{ij} Q_j \gamma_i}{\gamma_j} \right)^{w_{ij}} ,$$

which can be rewritten in logs as

$$\log(Q_i) = \log(A_i) + (1-w_i)\alpha_i \left(\log(\bar{\Theta}_{K_i}) + \log(\bar{K})\right) + (1-w_i)(1-\alpha_i)\left(\log(\bar{\Theta}_{H_i}) + \log(\bar{H})\right) + \sum_j w_{ij}\left(\log(w_{ij}) + \log(Q_j) + \log(\gamma_i) - \log(\gamma_j)\right)$$

Let us define ω_K, ω_H and ω_d as the vectors whose i-th elements are given by

$$[\omega_K]_i = (1-w_i)\alpha_i \log(\bar{\Theta}_{K_i})$$

$$[\omega_H]_i = (1-w_i)(1-\alpha_i)\log(\bar{\Theta}_{H_i})$$

$$[\omega_d]_i = \sum_j w_{ij} \log(w_{ij}\gamma_i/\gamma_j) \ ,$$

and $\omega_q = \omega_K + \omega_H + \omega_d$. Defining the vectors of sectoral capital shares $(1-w_i)\alpha_i$ and labor shares $(1-w_i)(1-\alpha_i)$ respectively as k_s and h_s, we can finally write the expression for the sectoral productions derived above as

$$q = \varepsilon + \omega_q + k_s \log(\bar{K}) + h_s \log(\bar{H}) + Wq \ ,$$

where q is the vector collecting sectoral productions, i.e. $\log(Q_i)$, and ε is the vector collecting sectoral TFPs, i.e. $\log(A_i)$. Solving for q we get

$$q = (I-W)^{-1}\varepsilon + (I-W)^{-1}\left(\omega_q + k_s\log(\bar{K}) + h_s\log(\bar{H})\right) \ . \tag{A.10}$$

After having derived the equilibrium expressions for gross sectoral outputs, we proceed by writing the equilibrium expression of intermediate goods used in the production of the aggregate good. Recalling that $c_i = \beta_i Q_i / \gamma_i$, we can write

$$\log(c_i) = \log(\beta_i) + \log(Q_i) - \log(\gamma_i) \ ,$$

or using vector notation

$$c = \omega_c + q \ , \tag{A.11}$$

where c is the vector collecting sectoral outputs aggregated into final good, i.e. $\log(c_i)$, and ω_c is a vector whose i-th element is defined as $[\omega_c]_i = \log(\beta_i) - \log(\gamma_i)$.

Finally, using the production technology for the final good in logs, i.e. $\log(Y) = \sum_{i=1}^{n} \beta_i \log(c_i) = \beta' c$, and Eq. (A.11) we can write

$$\log(Y) = \beta'(\omega_c + q) \ . \tag{A.12}$$

Substituting in Eq. (A.12) the equilibrium expression for q derived in Eq. (A.10) we obtain

$$\log(Y) = \beta'\omega_c + \beta'(I - W)^{-1}\varepsilon + \beta'(I - W)^{-1}\left(\omega_q + k_s \log(\bar{K}) + h_s \log(\bar{H})\right) \ . \tag{A.13}$$

Defining the scalar $\Xi \equiv \beta'\omega_c + \beta'(I - W)^{-1}\left(\omega_q + k_s \log(\bar{K}) + h_s \log(\bar{H})\right)$ we can write Eq. (A.13) as

$$\log(Y) = \beta'(I - W)^{-1}\varepsilon + \Xi$$

which leads to the expression of (log-)GDP as a function of sectoral TFPs aggregated using the Domar weights

$$\log(Y) = \gamma'\varepsilon + \Xi \ . \tag{A.14}$$

Optimal capital and labor supply: The competitive equilibrium implements the planner's problem defined as

$$\max u(C, H) = C - H^{1+\frac{1}{\phi}}$$

$$\text{s.t. } C = Y - rK$$

where r is the price of capital.

The FOCs for capital and labor can respectively be written as

$$Y\frac{\partial \log(Y)}{\partial K} - r = 0 \tag{A.15}$$
$$Y\frac{\partial \log(Y)}{\partial H} - \frac{1+\phi}{\phi} H^{\frac{1}{\phi}} = 0 \,. \tag{A.16}$$

Using Eq. (A.13) and the fact that ω_c and ω_q do not depend neither on K nor on H, we get

$$\begin{aligned} \frac{\partial \log(Y)}{\partial K} &= \beta'(I-W)^{-1}k_s K^{-1} \\ \frac{\partial \log(Y)}{\partial H} &= \beta'(I-W)^{-1}h_s H^{-1} \,. \end{aligned}$$

Substituting the above expressions in Eqs. (A.15)–(A.16) we get

$$\begin{aligned} K &= \frac{Y\beta'(I-W)^{-1}k_s}{r} \\ H &= \left(Y\beta'(I-W)^{-1}h_s\frac{\phi}{1+\phi}\right)^{\frac{\phi}{1+\phi}} \,. \end{aligned}$$

The optimal capital and labor supply expressions in logs are thus given by

$$\log(K) = \log(Y) + \Gamma_K \tag{A.17}$$
$$\log(H) = \frac{\phi}{1+\phi}\log(Y) + \frac{\phi}{1+\phi}\Gamma_H \,, \tag{A.18}$$

where $\Gamma_K \equiv \log(\beta'(I-W)^{-1}k_s) - \log(r)$ and $\Gamma_H \equiv \log\left(\beta'(I-W)^{-1}h_s\left(\frac{\phi}{1+\phi}\right)\right)$.

Optimal aggregate production and GDP volatility: Substituting Eqs. (A.17)–(A.18) in Eq. (A.13) we get

$$\begin{aligned} \log(Y) &= \gamma'\varepsilon + \beta'\omega_c + \gamma'\left(\omega_q + k_s(\log(Y)+\Gamma_K) + h_s\left(\frac{\phi}{1+\phi}\log(Y) + \frac{\phi}{1+\phi}\Gamma_H\right)\right) \\ \log(Y) &= \gamma'\varepsilon + \beta'\omega_c + \gamma'\left(k_s + h_s\frac{\phi}{1+\phi}\right)\log(Y) + \gamma'\left(\omega_q + k_s\Gamma_K + h_s\frac{\phi}{1+\phi}\Gamma_H\right) \,. \end{aligned}$$

Defining the scalar $\Gamma \equiv \gamma'\left(k_s + h_s\frac{\phi}{1+\phi}\right)$ and collecting terms yields

$$\log(Y) = (1-\Gamma)^{-1}\gamma'\varepsilon + (1-\Gamma)^{-1}\bar{\Psi} \,, \tag{A.19}$$

where $\bar{\Psi} \equiv \beta' \omega_c + \gamma' \left(\omega_q + k_s \Gamma_K + h_s \frac{\phi}{1+\phi} \Gamma_H \right)$. Defining $m \equiv (1 - \Gamma)^{-1}$ and $\Psi \equiv m\bar{\Psi}$ leads to Eq. (4) in the main text.

B Data

We use sectoral data provided by Dale Jorgenson and Associates (see Jorgenson et al., 2005) and downloaded from the on line appendix of Carvalho and Gabaix (2013). Data includes annual input-output tables for 88 US sectors from 1960 to 2005. In this dataset, for each year and each sector we observe the nominal value in US$ of gross output, capital input, labor input, intermediate input of commodities from all 88 sectors and corresponding price indexes. Following Carvalho and Gabaix (2013) we concentrate on private sector industries, thus excluding the following sectors: 8. Uranium, thorium ores; 60. Real Estate- owner occupied; 62. Renting of machinery; 81. Federal gen govt excl. health; 82. Federal govt enterprises; 83. Government Hospitals; 84. Govt other health; 85. S&L education; 86. S&L excl. health,educ.; 87. S&L govt. enterprises; 88. Military.

To compute US GDP volatility we use US Real Gross Domestic Product (GDPC1) retrieved from FRED, Federal Reserve Bank of St. Louis.

C Robustness to alternative measures of volatility

In this appendix we test the robustness of our results by comparing different measures of sectoral and aggregate volatility. We start by defining the following alternative measures of sectoral TFP volatility:

a) *Instantaneous sectoral volatility.* We estimate an AR(1) model on the growth rate of sectoral TFP of each sector i and compute the series of residuals e_{it}. Since $E(e_{it}) = 0$ is known, $\sigma^2_{it} = e^2_{it}$ is an unbiased estimator of variance in year t. We consider the HP-trend of σ^2_{it}, denoted as $\sigma^2_{a,it}$.

b) *10-years rolling window instantaneous sectoral volatility.* The estimation of variance considered in *a*) is based on only one observation. To improve the precision of the volatility estimation we consider also a 10-years rolling window average of the same measure:

$$\sigma^2_{b,it} = \frac{1}{11} \sum_{\tau=t-5}^{t+5} e^2_{i\tau} \ .$$

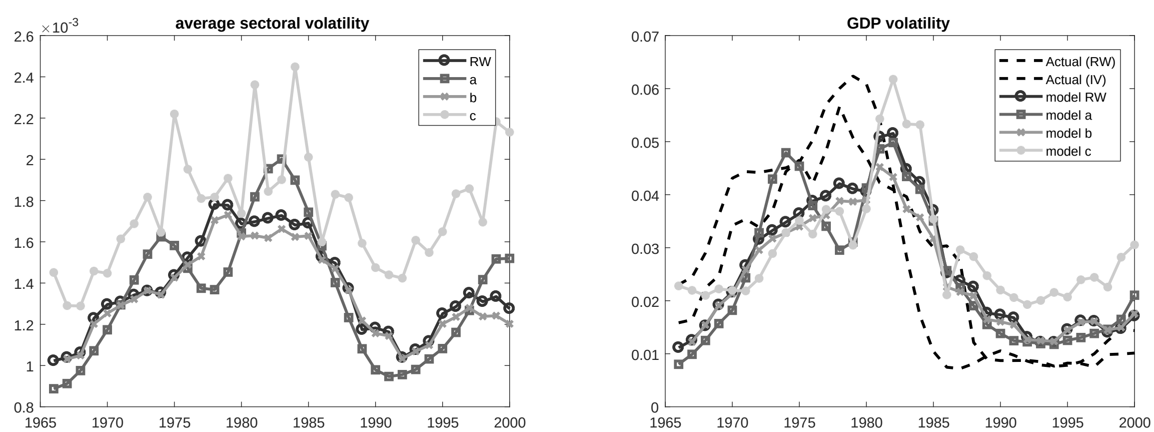
•

c) *Garch(1,1)* We estimate a third measure of sectoral volatility $\sigma^2_{c,it}$ by running a GARCH(1,1) for each sector i. This measure of time-varying sectoral volatility has been used in Carvalho and Gabaix (2013).

GDP volatility corresponding to each of these measures is computed as follows:

$$\sigma^2_{M,Yt} = m^2_t \sum_i \gamma^2_{it} \sigma^2_{M,it} \ ,$$

where $\sigma^2_{M,it}$ denotes sectoral volatility implied by measure $M \in \{RW, a, b, c\}$, where RW refers to our benchmark time-varying volatility measure used in the main text. Panel a) in Fig. C.1 displays



(a) (b)

Figure C.1: **Panel (a):** Average sectoral volatility. **Panel (b):** GDP volatility computed using the different measures of sectoral volatility.

the dynamics of average sectoral volatility. Average sectoral volatilities estimated using measures RW, a and b display similar qualitative pattern: they increase from mid-60s to mid-80s and decrease from mid-1980s to mid-1990s. Although the Garch(1,1) measure displays a high degree of variability, the long-term dynamics follows roughly the dynamics exhibited by the other considered measures.

Panel b) in Fig. C.1 shows the GDP volatility computed using the different measures of sectoral volatility. Observed GDP volatility is represented using both the RW and the IV measure, while the volatility implied by the benchmark model is denoted as "model RW". Regardless of the considered measure, the model reproduces both the "great volatility" and the "great moderation" phenomena. To compare the performance of the model in explaining observed GDP volatility using the different measures of sectoral volatility, we estimate the following regression:

$$\sigma^2_{RW,t} = \alpha + \beta\ \sigma^2_{M,Yt} + \eta_t\ . \tag{C.1}$$

Results of the regressions are shown in Table C.1. The first important result to highlight in Table C.1

Dependent variable: σ^2_{RW}

model:	RW	a	b	c
$\hat{\alpha}$	-0.0000 (-0.0001,0.0000)	-0.0000 (-0.0001,-0.0000)	-0.0001 (-0.0001,0.0001)	0.0000 (-0.0001,0.0001)
$\hat{\beta}$	1.1084 (0.8944,1.3224)	0.9464 (0.7018,1.1910)	1.3821 (1.1533,1.6109)	0.8573 (0.4441,1.2705)
R^2	0.77	0.65	0.83	0.35

Table C.1: Coefficients (95% confidence intervals). OLS estimation of Eq. (C.1).

is that each model yields $\hat{\alpha}$ very close to 0 and $\hat{\beta}$ very close to 1, meaning that all models are capable of representing GDP volatility dynamics. The second important result is that the R^2 statistic is high in all regressions but the regression involving model c, which measures sectoral volatility by estimating a GARCH(1,1). This is due to the very erratic behavior of the resulting sectoral volatility, as evident from the behavior of the its average sectoral volatility displayed in Panel (a) of Fig. C.1. As a further robustness check, we perform the same analysis using instantaneous

Dependent variable: σ^2_{IV}

model:	RW	a	b	c
$\hat{\alpha}$	-0.0000 (-0.0001,0.0001)	0.0000 (-0.0001,0.0002)	-0.0001 (-0.0002,0.0000)	0.0001 (-0.0001,0.0002)
$\hat{\beta}$	1.0892 (0.7016,1.4768)	0.9376 (0.5467,1.3284)	1.4462 (1.0192,1.8732)	0.7319 (0.1608,1.3030)
R^2	0.50	0.42	0.60	0.17

Table C.2: Coefficients (95% confidence intervals). OLS estimation of Eq. (C.1) with $\sigma^2_{IV,t}$ as dependent variable.

volatility, $\sigma^2_{IV,t}$ as dependent variable. Results are shown in Table C.2 and they confirm the ability of the model to match the observed GDP volatility with different measures of sectoral volatility.