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Labor Responses, Regulation and Business Churn

Abstract

We develop a model of sluggish firm entry to explain short-run labor responses to technology shocks. We show that the labor response to technology and its persistence depend on the degree of returns to labor and the rate of firm entry. Existing empirical results support our theory based on short-run labor responses across US industries. We derive closed-form transition paths that show the result occurs because labor adjusts instantaneously whilst firms are sluggish, and closed-form eigenvalues show that stricter entry regulation results in slower convergence to steady state.

JEL-Codes: D250, E200, L110, O330.

Keywords: deregulation, dynamic entry, endogenous entry costs.

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The short-run response of labor hours to technology shocks is widely debated in macroeconomics.[1] Empirical studies, such as Chang and Hong 2006, document different labor responses to technology shocks across U.S. manufacturing industries. They show that while some industries exhibit a temporary reduction in employment in response to a permanent increase in technology, many more industries exhibit a short-run increase in both employment and hours per worker. However, the theory underlying these responses is not fully understood. In this paper, we identify a novel mechanism based on dynamic firm entry to explain short-run labor responses and subsequent persistence. Cross-industry data supports our theory. Additionally, we show that persistence of labor responses depends on firm sluggishness which regulation affects through endogenous entry costs.

Our mechanism focuses on endogenous variation in labor *per firm* which occurs when firm creation is sluggish but labor adjusts instantaneously. Endogenous variation in labor per firm is important for aggregate labor responses if the marginal product of labor (MPL) in a firm's production function is non-constant. For example, if a positive technology shock increases hours, but the stock of firms is fixed, hours per firm increase. With short-run increasing MPL, the rise in hours per firm increases MPL, increases wages and increases hours. Subsequent firm entry decreases hours per firm, decreases MPL, decreases wage, and decreases labor to its long-run level.[2] This channel is typically overlooked because either labor per firm is fixed or the MPL is constant so wages do not respond.

We develop a DGE small open economy (SOE) model in continuous time extended to include dynamic firm entry.[3] There is no capital, only labor, and there is an internationally traded bond with world interest rates equal to the household discount rate. Hence the household perfectly smooths utility, so consumption dynamics do not play a role, which allows a closed-form

[1] Cantore, Ferroni, and Leon-Ledesma 2017 provide a recent survey. The classic references are Gali 1999 for positive responses and Christiano, Eichenbaum, and Vigfusson 2003 for negative responses. See also Basu, Fernald, and Kimball 2006; Christiano, Eichenbaum, and Vigfusson 2004; Wang and Wen 2011; Rebei 2014.

[2] With decreasing MPL, the fall in hours per firm from entry, increases MPL, increases wages and increases labor to its long-run level.

[3] Sen and Turnovsky 1990; Mendoza 1991 are early papers in the SOE-RBC literature.

analysis of firm dynamics. Households can invest in new firms by paying an endogenous sunk entry cost. Once operational, firms compete under monopolistic competition and pay a fixed overhead cost each period. The restriction to one state variable (number of firms) keeps eigenvalues tractable, so we can study persistence and short-run versus long-run effects analytically. To model dynamic entry we assume that the entry costs depend on the flow of entry due to a congestion effect caused by red tape (Datta and Dixon 2002). Our model is parsimonious in order to derive general analytic results and qualitatively replicates key stylized facts.[4]

Related Literature: As mentioned at the start, the work of Chang and Hong 2006 provides evidence on the heterogeneity of short-run employment responses to technology shocks. Our work provides a new explanation for their findings based on labor returns to scale, and is broadly supported by their data. In relation to existing theoretical literature, we generalize the firm production function for increasing, decreasing or constant MPL and combine this with dynamic firm entry whilst maintaining tractability.[5] This distills the importance of dynamic firm entry, and contributes to growing evidence that dynamic (sluggish), rather than static, entry is crucial to understand business cycle dynamics. Bilbiie, Ghironi, and Melitz 2012 (BGM) is the seminal work in this literature. They show that dynamic entry and endogenous markups greatly improve RBC moment matching, and their modelling approach has been successfully adopted in quantitative DSGE exercises.[6] Our mechanism to achieve sluggish entry differs as it relies on endogenous sunk costs. This modelling choice pertains to tractable continuous time analysis, and allows us to study how deregulation can increase business churn and thus speed of adjustment following short-run responses. Lewis 2009 provides

[4]Procylical net entry which lags the cycle (Campbell 1998; Bergin, Feng, and Lin 2016); the existence of variable returns to scale in labor (Basu, Fernald, and Kimball 2006); the existence of monopoly power (De Loecker and Eeckhout 2017); procylical average firm scale (capacity utilization) which is contemporaneous with the cycle (Burnside and Eichenbaum 1996); procylical firm profits also contemporaneous with the cycle (Lewis 2009; Bilbiie, Ghironi, and Melitz 2012); countercylical labor share (Young 2004); and procylical measured productivity (Basu and Fernald 2001; Jaimovich and Floetotto 2008).

[5]Barseghyan and DiCecio 2016 study the relationship between returns to scale and entry in a perfectly competitive Hopenhayn model.

[6]Etro and Colciago 2010; Lewis and Poilly 2012; Lewis and Winkler 2017.

evidence on the importance of entry congestion in macroeconomic propagation.[7] Cantore, Ferroni, and Leon-Ledesma 2017 (Fig. 1, p.70) implies that short-run responses have reversed over the past century in the US from decreasing to increasing, and that the deviation now persists for longer. We explain that this could be caused by a decline in business churn.[8] Lastly, we show that entry effects on aggregate output are non-trivial with variable returns to scale in labor (MPL slope). This features is crucial to our understanding of transition, but also adds a new element to analyses of *optimal entry* by Etro and Colciago 2010; Bilbiie, Ghironi, and Melitz 2016 who focus on endogenous markups with constant MPL.

Roadmap: Section 1 outlines the household problem; Section 2 analyzes firm production and entry decisions; Section 3 summarizes equilibrium, solves for steady-state and solves for transition paths; Section 4 analyzes labor responses; Section 5 shows that deregulation speeds-up convergence.

1 Household

There is a small open economy, with a world capital market interest rate r equal to the discount rate ρ of the Ramsey household.[9]

$$r = \rho \tag{1}$$

[7] A number of recent papers have adopted entry adjustment costs (Lewis and Poilly 2012; Bergin and Lin 2012; Loualiche et al. 2014; Berentsen and Waller 2015; Poutineau and Vermandel 2015).

[8] This relates to recent literature on *'declining business dynamism'* (Decker et al. 2018) that links 'declines in the pace of business formation' to slower reallocation of resources.

[9] This so-called knife-edge condition is a widely-discussed model closing device (Turnovsky 2002; Oxborrow and Turnovsky 2017). Under perfect foresight, this will cause steady-state to depend on initial conditions (Uribe and Schmitt-Grohé 2017, Ch 2 & 3), so the deterministic steady-state is history dependent. Schmitt-Grohé and Uribe 2003 analyse techniques to induce stationarity for approximating equilibrium dynamics in stochastic models. Since our model is deterministic, non-stationarity is not an issue (Turnovsky 1997, Ch. 3).

We assume King-Plosser-Rebelo preferences with logarithmic consumption

$$U(C, 1-H) = \ln C - \frac{H^{1+\eta}}{1+\eta} \tag{2}$$

$\eta \in (0, \infty)$ is inverse Frisch elasticity of labor supply to wages.[10] The household earns income from three sources: supplying labor at wage w, receiving interest income from net foreign bonds rB and receiving profit income Π from owning firms. The household treats profit income as a lump sum payment. The household solves:

$$\max \int_0^\infty U(C, H) e^{-\rho t} dt \tag{3}$$

$$\text{subject to} \quad \dot{B} = rB + wH + \Pi - C \tag{4}$$

$$B(0) = B_0 \tag{5}$$

$$\text{where} \quad r = \rho \tag{1}$$

Given KPR preferences the optimal solutions satisfy

$$\dot{\lambda} = 0 \implies \lambda = \bar{\lambda} \tag{6}$$

$$\bar{C} = \frac{1}{\bar{\lambda}} \tag{7}$$

$$H(w, \lambda) = (\lambda w)^{\frac{1}{\eta}}, \quad \eta \in (0, \infty) \tag{8}$$

where we use bar notation for variables that are constant over time. For a given wage, labor supply H is increasing in λ. Frisch elasticity of supply measures the substitution effect of a change in the wage rate on labor supply $H_w \frac{w}{H} = \frac{1}{\eta}$.[11] The perfect capital markets assumption $r = \rho$ (implies constant

[10] We ignore indivisible labor $\eta = 0$. Additive separability $U_{CH} = 0$ is sufficient for our results to hold when there are increasing marginal costs (decreasing returns to labor). But we require KPR preferences for the decreasing and constant marginal cost cases.

[11] See the appendix for full derivation of first-order conditions. We rule out indivisible labor $\eta = 0$ which would imply $C = w$. If $r \neq \rho$ then no interior steady state exists. The trajectory of consumption will then be either increasing $r > \rho$ or decreasing $r < \rho$ through time. There are many discussions of *'closing devices'* (or *'stationarity-inducing devices'*) in the SOE literature, which are necessary because the exogenous world interest rate causes an incomplete market. See Seoane 2015 based on Mendoza 1991. Oxborrow

consumption $\dot{\lambda} = 0$) and additively separable utility $U_{CH} = 0$ simplify dynamics.[12] The result is that the only dynamics in the model will be a result of firm entry, which will affect wage. The advantage is to pinpoint the precise role of firm entry. λ is the marginal utility of consumption: high λ means low consumption and vice versa. Lastly, to ensure the private agent satisfies the intertemporal budget constraint, the transversality condition must hold

$$\lim_{t\to\infty} \lambda B e^{-rt} = 0 \tag{9}$$

Hence the solution to the problem is characterized by two boundary conditions (5), (9) and two ordinary differential equations (ODEs) $\dot{\lambda}, \dot{B}$ that solve to give trajectories $B(t), \lambda(t)\ \forall t$. Subsequently $\lambda(t)$ gives $C(t)$ and in turn $H(t)$ through the static conditions. However before solving we need to characterize the endogenous behaviour of w and Π in general equilibrium according to factor market equilibrium.

2 Firms: Technology, Entry and Exit

The aggregate consumption good C is either imported or produced domestically by a perfectly competitive industry with a constant returns production function using intermediate inputs which are monopolistically supplied. There is a continuum of possible intermediate products, $i \in [0, \infty)$. At instant t, there is a range of active products defined by $N(t) < \infty$ so that $i \in [0, N(t))$ are active and available, whilst $i > N(t)$ are inactive and not produced. Hence total domestic output Y is related to inputs y_i by the following technology

$$Y = N^{\varsigma - \frac{\theta}{\theta-1}} \left[\int_0^N y_i^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{10}$$

and Turnovsky 2017 give overview and close the model using demography.

[12] Additive separability $u_{CH} = 0$ creates the simple relationship between consumption and marginal utility of consumption. The presence of a small open economy and perfect international capital markets $\rho = r$ implies the household can completely smooth its consumption so $\dot{\lambda} = 0 \implies \lambda = \bar{\lambda}$. Therefore together they imply the marginal utility of consumption is unchanging over time.

where $\theta > 1$ is the elasticity of substitution between products. The N^{ς} multiplier captures any variety effect. We assume $\varsigma = 1$ so no variety effect which implies an increase in the range of intermediates does not affect the unit cost function.[13] Treating the unit price of the consumption good as the numeraire, under monopolistic competition the demand for each available product i takes the constant elasticity form

$$y_i = p_i^{-\theta} \frac{Y}{N^{\varsigma}} \tag{11}$$

with corresponding price elasticity of demand $\varepsilon_{py} \equiv \frac{dp_i}{dy_i}\frac{p_i}{y_i}$ given by $\varepsilon_{py} = -\frac{1}{\theta}$. There is a continuum of potential firms, and each firm can produce one product. At time t, firm $i \in [0, N(t))$ has labor demand h_i to supply output y_i using the technology

$$y_i = \begin{cases} Ah_i^{\nu} - \phi, & \text{if } Ah_i^{\nu} > \phi, \\ 0 & \text{else}, \end{cases} \tag{12}$$

where $\nu > 0$ captures labor returns to scale (slope of MPL): $\nu < 1$ decreasing returns; $\nu = 1$ constant returns; $\nu > 1$ increasing returns. $\phi \geq 0$ is a fixed overhead cost denominated in output terms. A is a technology parameter. The fixed cost implies that labor returns to scale ν are not equivalent to

[13] A common case is $\varsigma = 0$ which leads to a variety effect, we want to remove this as it will create an additional mechanism adding to the main result we want to distill. Without removing love of variety, N will enter the labor market equilibrium condition, even with constant returns to scale.

overall returns to scale measured as average cost over marginal cost[14]

$$\frac{AC}{MC} = \nu(1 + s_{\phi}) \tag{14}$$

where $s_{\phi} \equiv \frac{\phi}{y}$ is the fixed cost share in output. The marginal product of labor and its slope are

$$\text{MPL} = \nu \frac{y+\phi}{h}, \tag{15}$$

$$\text{MPL}_h = (\nu - 1)\frac{\text{MPL}}{h}, \tag{16}$$

The MPL is always positive, but can be increasing $\nu > 1$, decreasing $\nu < 1$ or constant $\nu = 1$ in hours, corresponding to increasing, decreasing or constant returns to labor at the firm-level.[15]

[14]The cost function dual of our production function is $\text{TC} = \text{MC}\,\nu(y + \phi)$. This follows because factor prices equal their marginal revenue product, in the case for labor $w = \text{MR} \times \text{MPL}$. An optimizing firm produces where $MR = MC$, hence as labor is the sole input $\text{TC} = wh = \text{MC} \times \text{MPL} \times h = \text{MC}\,\nu(y+\phi)$. Multiply by $\frac{1}{y\,\text{MC}}$ to get AC / MC which captures overall returns to scale. Furthermore, where w is nominal wage, as labor is the only input, total costs are $\text{TC} = wh = w\left(\frac{y+\phi}{A}\right)^{\frac{1}{\nu}}$ so that marginal cost is

$$\text{MC} = \frac{w}{\nu A}\left(\frac{y+\phi}{A}\right)^{\frac{1-\nu}{\nu}} = \frac{\text{TC}}{\nu(y+\phi)} \tag{13}$$

and average cost is $\text{AC} = \frac{\text{TC}}{y}$ which in the U-shaped AC case ($\nu < 1$ and $\phi > 0$) will achieve minimum at firm scale $y^{\text{MES}} = \frac{\nu\phi}{1-\nu}$, the firm's *minimum efficient scale* (MES).

[15]When $\nu < 1$, $\phi > 0$ there is a U-shaped average cost curve with increasing marginal cost. This is compatible with both perfect and imperfect competition. When $\nu = 1, \phi = 0$, there are constant returns to scale: AC = MC. When $\nu = 1, \phi > 0$, there is a constant MC and decreasing AC. When $\nu > 1$ there is decreasing AC and MC. The extent of increasing returns to labor $\nu > 1$ is limited by the degree of imperfect competition. In the two cases with globally increasing returns to scale, equilibrium can only exist with imperfect competition.

2.1 Aggregate Output

Perfect factor markets imply aggregate labor is divided equally across firms $h_i = H/N,\ \forall i \in N$. Under symmetry the aggregate production function is

$$Y(N,H) = Ny = AH^{\nu}N^{(1-\nu)} - N\phi \tag{17}$$

It is homogeneous of degree 1 in inputs H, N which implies

$$Y = Y_N N + Y_H H \tag{18}$$

The intuition corresponds to $Y = Ny$. Output per firm is homogeneous of degree 0 because a change in aggregate labor is offset by a change in number of producers so that labor per firm is unchanged, then output per firm is unchanged, hence aggregate output expands proportionally to the expansion in number of firms. Treating N, H independently, the effect of entry on aggregate output is ambiguous whereas extra labor always raises aggregate output[16]

$$Y_N \equiv \frac{\partial Y}{\partial N} = (1-\nu)Ah^{\nu} - \phi = y - \nu A h^{\nu} = (1-\nu)y - \nu\phi \gtreqless 0 \tag{19}$$

$$Y_H \equiv \frac{dY}{dH} = A\nu(H/N)^{\nu-1} = A\nu h^{\nu-1} = \nu\frac{y+\phi}{h} > 0 \tag{20}$$

When there are increasing returns to labor $\nu > 1$, an additional firm dividing aggregate labor into smaller units can decrease aggregate output as it employs labor less productively than the incumbents did prior to its entry. Aggregate and firm level MPL are equivalent $Y_H = y_h$.

[16] It is important to note the N derivative is partial, as the in general equilibrium the total derivative would recognize that a variation in N implicitly varies H, that is $\frac{dY}{dN} = \frac{\partial Y}{\partial N} + \frac{dY}{dH}\frac{dH}{dN}$. Since N is independent of H then its partial and total derivative are equivalent.

2.2 Profits and Factor Market Equilibrium

Due to imperfect competition, a profit maximizing firm chooses employment to satisfy the factor market equilibrium[17]

$$w = \frac{1}{\mu} Y_H = \frac{\nu}{\mu} A \left(\frac{H}{N} \right)^{\nu - 1} = \frac{\nu}{\mu} \frac{y + \phi}{h} \tag{21}$$

Where $\mu \equiv \frac{\theta}{\theta - 1} \in [1, \infty)$ is the markup, which is 1 with perfect competition when products are perfectly substitutable $\theta \to \infty$, so demand curves are perfectly elastic.[18] The labor demand curve will be increasing, decreasing or constant depending on the shape of the MPL schedule

$$w_H = \frac{1}{\mu} Y_{HH} = (\nu - 1) \frac{w}{H} \gtreqless 0 \iff \nu \gtreqless 1 \tag{22}$$

We assume the degree of increasing returns to labor is bounded above by the degree of monopoly power. This ensures the second-order condition for profit maximization holds.

Lemma 1. *$\nu < \mu$ is a sufficient condition for the second-order profit maximization condition to hold.*

Later we show it is necessary and sufficient for steady-state existence.[19] This restriction implies that for profit maximizing output MR must intersect MC from above (the second order condition for profit maximization). A higher degree of monopoly μ (more differentiated products) implies steeper

[17] The result follows from the profit maximization problem outlined in Appendix A.4. In the increasing returns case $\nu > 1$, the second-order condition for profit maximization is not always satisfied, so we give a necessary condition for this. However, our later condition $\nu < \mu$ is sufficient for this second-order necessary condition to hold.

[18] Labor demand h will vary depending on returns to scale. The relationship captures *'aggregate labor demand'* (Jaimovich 2007), the right-hand side is the marginal revenue product of labor which is the inverse of the markup multiplied by the marginal product of labor. The number of firms affects the relationship through the marginal product of labor since the markup is fixed. With endogenous markups and constant returns to scale, the number of firms also affect the MRPL (also true of LOV). Both can create upward sloping marginal product schedule $dw/dH > 0$.

[19] Hornstein 1993; Devereux, Head, and Lapham 1996 provide similar conditions in instantaneous-entry, zero-profit models with returns to scale.

MR which allows steeper downward sloping MC (higher ν). Horizontal MC only exists if MR is downward sloping, so some monopoly power exists. Increasing marginal costs $\nu < 1$ is compatible with any level of imperfect competition $\mu \in [1, \infty)$ including perfect competition.

Operating profits and output per firm (thus labor per firm) are isomorphic since $\pi = y - \frac{\nu}{\mu}(y + \phi)$ hence

$$\pi = y\left(1 - \frac{\nu}{\mu}\right) - \frac{\nu}{\mu}\phi = Ah^{\nu}\left(1 - \frac{\nu}{\mu}\right) - \phi \tag{23}$$

$$y = \frac{\mu\pi + \nu\phi}{\mu - \nu} \tag{24}$$

$$h = \left(\frac{y + \phi}{A}\right)^{\frac{1}{\nu}} = \left(\frac{\mu(\pi + \phi)}{A\,(\mu - \nu)}\right)^{\frac{1}{\nu}} \tag{25}$$

Operating profits respond proportionally but strictly less than output $0 < \pi_y = 1 - \frac{\nu}{\mu} < 1$. The implication is that economic profits are less volatile than output, and lemma 1 implies that this relationship cannot be negative.

2.3 Labor Market Equilibrium

In labor market equilibrium labor supply (8) equals labor demand (21): $H^{\eta}\bar{C} = \frac{A\nu}{\mu}H^{\nu-1}N^{1-\nu}$.[20] It is useful to write as a function of (N, λ)[21]

$$H(\lambda, N) = \left(N^{1-\nu}\lambda\frac{\nu A}{\mu}\right)^{\frac{1}{1+\eta-\nu}}, \quad 1 + \eta - \nu > 0 \tag{27}$$

[20]If labor is indivisible ($\eta = 0$) then all wage is consumed $\bar{C} = \frac{1}{\mu}A\nu h^{\nu-1}$, so there is no substitution effect. With constant marginal costs $\nu = 1$ then $C = A/(\mu H^{\eta})$ there is only an income effect as wage is fixed. Jaimovich 2007 studies the effect of instantaneous entry on this relationship with both constant returns and indivisible labor, but N affects the relationship through endogenous markups $\mu(N)$ which causes indeterminacy.

[21]If we substitute out $N = H/h = H\left(\frac{A(\mu-\nu)}{\mu(\pi+\phi)}\right)^{\frac{1}{\nu}}$ in (27) we get labor as a function of profits

$$H = \left[\left(\frac{A(\mu - \nu)}{\mu(\pi + \phi)}\right)^{\frac{1-\nu}{\nu}}\frac{\lambda\nu A}{\mu}\right]^{\frac{1}{\eta}} \tag{26}$$

Whether labor increases, decreases or does not respond to a change in profits depends on returns to scale ν.

Lemma 2 (Labor Market Equilibrium Existence). *To ensure that the labor market condition is well-defined $\nu < 1 + \eta$*

The restriction $\nu < 1 + \eta$ implies that the slope of the labor supply curve exceeds the slope of the labor demand curve. The labor supply curve slope is $\frac{dw}{dH} = \frac{\eta w}{H}$, and upward sloping in (H, w) space (or flat with indivisible labor $\eta = 0$). This must be greater than the slope of labor demand (marginal (revenue) product schedule) which is $\frac{dw}{dH} = w_h h_H = \frac{(\nu-1)w}{h} \frac{1}{N} = \frac{(\nu-1)w}{H}$. As noted, demand for labor can be upward sloping if returns to labor are increasing $\nu > 1$.

Proposition 1 (Existence). *Necessary and sufficient condition for existence*

$$\nu < \min [\mu, 1 + \eta] \tag{28}$$

A sufficient condition is that there are increasing marginal costs $\nu < 1$. Where $1 + \eta > 1$ because we rule out indivisible labor $\eta = 0$.

Proof. Combine profit existence Lemma 1 and labor market existence Lemma 2. □

Entry alters employment per firm which, through marginal costs, affects the efficiency of labor and thus the real wage it is paid. With a decreasing MPL, entry increases the real wage and hence labor supply; with increasing MPL the opposite holds.

Proposition 2 (General Equilibrium Labor Behavior). *From the labor market equilibrium condition (27), we can see that labor responses to entry are*

$$H_N > 0 \iff \nu \in (0, 1) \tag{29}$$

$$H_N = 0 \iff \nu = 1 \tag{30}$$

$$H_N < 0 \iff \nu \in (1, \infty) \tag{31}$$

In deriving this result we show that labor elasticity to number of firms

$\varepsilon \equiv H_N \frac{N}{H}$ is constant and bounded

$$\varepsilon = \frac{1-\nu}{1+\eta-\nu} \tag{32}$$

It is bounded by $\frac{-\eta}{1-\nu+\eta} < \varepsilon < 1$. The upper bound occurs with indivisible labor $\eta \to 0$. The lower bound follows from $\nu < 1+\eta$ so that (working right to left) $\frac{-\eta}{1-\nu+\eta} < \frac{1-(1+\eta)}{1-\nu+\eta} < \frac{1-\nu}{1-\nu+\eta} = \varepsilon$. If $\nu = 1$ then $\varepsilon = 0$. If $\nu < 1$ then $0 < \varepsilon < 1$. And if $\nu > 1$ then $-\infty < \varepsilon < 0$.[22]

2.3.1 Total Derivatives: Labor Effect Vs. Business Stealing

In section 2.1 we derived the partial derivatives of aggregate output with respect to labor $Y_H > 0$ and firms $Y_N \gtreqless 0$, assuming H and N were independent. Now that we have determined $H(\lambda, N)$ we can assess total derivatives of output with respect to entry by considering that labor changes endogenously. Understanding this mechanism is important for our results on the effect of entry on aggregate output to be derived later. The main point is that entry has an ambiguous effect on aggregate output if there are decreasing returns $\nu < 1$ so that $\varepsilon > 0$. This is because entry strengthens labor supply which can increase output. Whereas with constant or increasing returns $\nu \geq 1$ an entrant always decreases aggregate output.

$$\frac{dY}{dN} = y + N\frac{dy}{dN} = \varepsilon(1+\eta)Ah^{\nu} - \phi \tag{33}$$

The first equality states that an entrant contributes its own output y but has a *business stealing* (Mankiw and Whinston 1986) effect on the output of all other incumbents. In the appendix we show this business stealing effect is strictly negative $N\frac{dy}{dN} = \nu(y+\phi)(\varepsilon - 1) < 0$. The second equality of (33) emphasizes the role of firm level returns to scale. It states that an entrant has a negative effect by bringing in an extra fixed cost, but it has another positive, negative or zero effect depending on the labor elasticity to entry ε.

The aggregate flow of operating profits given w equals $N\pi$, where π is

[22] See Appendix A.5 for proof.

firm level profit.[23]

$$\pi = Y_N + \left(1 - \frac{1}{\mu}\right) Y_H \frac{H}{N} \tag{34}$$

In terms of profits this can be written $\frac{dY}{dN} = Y_N + Y_H H_N = \pi - \left(1 - \frac{1}{\mu} - \varepsilon\right) Y_H h$ which is useful when we analyze zero-profit steady state.[24] The first term is the partial derivative effect of an entrant (19) which we have explained is ambiguous based on ν, and the second term is the labor response which is also ambiguous based on ν. Since y and π are in a one-one relationship, the business stealing effect can also be interpreted as entrants diminishing profits, from (23) $\frac{d\pi}{dN} = \frac{dy}{dN}\left(1 - \frac{\nu}{\mu}\right) < 0$. In the dynamic analysis we shall use the expression for dividends with $H(\lambda, N)$ substituted out:

$$\pi(\lambda, N) = \left(\frac{A^{1+\eta}(\nu\lambda)^{\nu}}{\mu^{1+\eta} N^{\eta\nu}}\right)^{\frac{1}{1+\eta-\nu}} (\mu - \nu) - \phi \tag{35}$$

2.4 The Entry Decision

What determines the number of firms operating at each instant t? We develop a congestion effects model of firm entry such that at time t there is a flow cost of entry $q(t)$ which increases in net entry $E(t)$.[25]

$$E(t) \equiv \dot{N} \tag{36}$$

$$q(t) = \gamma E(t) \tag{37}$$

The sensitivity to congestion parameter $\gamma \in (0, \infty)$ represents red tape or regulation in firm creation. Filing papers or gaining accreditation makes start-ups more sensitive to flows of entry as regulator's offices become more

[23] The result follows from substituting w (21) and Y (18) out of the aggregate profit expression $N\pi = Y - wH$ such that $N\pi = Y_N N + Y_H H - \frac{Y_H}{\mu} H$, which rearranges to (34).

[24] See Appendix for full derivation and discussion.

[25] Entry and exit are symmetric, with $-q$ being the cost of exit at time t. There are sunk costs to entry and dismantling fees, such as severance payments, to exit. See Das and Das 1997; Datta and Dixon 2002 for further details. Exit and entry symmetry is not essential, exit could require a fixed cost, perhaps zero, as in Das and Das 1997 and Hopenhayn 1992 or evolve endogenously according to productivity Melitz 2003; Hamano and Zanetti 2017.

congested (i.e. a queuing cost). Aggregating across all entry in a period gives a quadratic firm entry adjustment cost function

$$\mathcal{C}(E) \equiv \int_0^E q \, dE = \frac{\gamma}{2} E^2 = \frac{q^2}{2\gamma} \tag{38}$$

$\mathcal{C}(E)$ is a non-negative, convex function of the rate of entry. With zero entry, the aggregate cost and marginal cost of firm creation is zero $\mathcal{C}(0) = \mathcal{C}_E(0) = 0$. The interpretation of modelling the aggregate sunk cost as an adjustment cost is that firm creation and destruction, whether positive (net entry) or negative (net exit), generates resource costs.

The flow of entry in each instant is determined by an *arbitrage condition* that equates the return on bonds (opportunity cost of entry) with the return on setting up a new firm. It is a differential equation in q, which determines the entry flow by (37).[26]

$$\frac{\pi}{q} + \frac{\dot{q}}{q} = r \tag{39}$$

π is given by (34) which will make this a nonlinear differential equation in N.[27] The first left-hand side term is the number of firms per dollar $(1/q)$ times the flow operating profits (dividends) the firm will make if it sets up. The second term reflects the change in the cost of entry. If $\dot{q}/q > 0$, then it means that the cost of entry is increasing, so that there is a capital gain associated with entry at time t if $\dot{q}/q < 0$ it means entry is becoming cheaper, thus discouraging immediate entry. The sunk cost $q(t)$ represents the net present value of incumbency: it is the present value of profits earned if you are an incumbent at time t.[28] This arises since the entrants are indifferent between entering and staying out. When $q < 0$, the present value of profits is negative: in equilibrium this is equal to the cost of exit. In steady state, we

[26]The arbitrage equation can be written in a way directly analogous to the user cost of capital $\pi = q\left(r - \frac{\dot{q}}{q}\right)$ in capital adjustment cost models.

[27]Note that our entry model has the standard models as limiting cases: when $\gamma = 0$, we have instantaneous free entry so that (39) becomes $\pi = 0$ and there are zero profits each instant. If $\gamma \to +\infty$, then changes in N become very costly and N moves little if at all which approximates the case of a fixed number of firms.

[28]This is because of the free-entry assumption that sunk costs equal the net present value of the firm. See Stokey 2008 for a general discussion.

have $E = q = 0$, so that the entry model implies the zero-profit condition. Entry costs only arise on convergence to steady state.

Accounting for entry costs, aggregate profits Π are the operating profits (dividends) of firms less the entry costs paid by the entrants

$$\Pi = N\pi - \gamma\frac{E^2}{2} = NY_N + \left(1 - \frac{1}{\mu}\right) Y_H H - \gamma\frac{E^2}{2} = Y(N,H) - wH - \frac{q^2}{2\gamma} \quad (40)$$

3 Equilibrium, Steady State and Solution

The economic system is five dimensional $\{\lambda, N, q, B, H\}$ with four differential equations and one static equation. The static intratemporal condition (27) implies $H(\lambda, N)$, so the system can be reduced to four differential equations in four unknowns, and since the consumption differential equation implies consumption is constant $\lambda(t) = \bar{\lambda}$, we have three dynamic equations in N, q, B.

$$\dot{\lambda} = 0 \implies \lambda(t) = \bar{\lambda}$$

$$\dot{N}(q) = \frac{q}{\gamma} \quad (41a)$$

$$\dot{q}(N, \bar{\lambda}, q) = rq - \pi(N, H(\bar{\lambda}, N)) \quad (41b)$$

$$\begin{aligned} \dot{B}(B, N, \bar{\lambda}, q) &= rB + wH(\bar{\lambda}, N) + \Pi(N, H(\bar{\lambda}, N), q) - \bar{C}(\bar{\lambda}) - G \\ &= rB + Y(N, H(\bar{\lambda}, N)) - \mathcal{C}(q) - \bar{C}(\bar{\lambda}) - G \end{aligned} \quad (41c)$$

Accompanying the differential equations in system (41) there are three boundary conditions: the household transversality (9); the initial condition on bonds; the initial condition on number of firms. Notably the industry dynamics (N, q) form a two dimensional subsystem of the three dimensional system, with bonds being B determined through (41c) alone. Therefore we shall solve recursively: first solving the industry dynamics subsystem for $N(t), q(t)$, then solve for bonds $B(t)$ based on these solutions.

3.1 Steady-state

Steady state is non-standard because there are three steady state conditions $\dot{N} = \dot{q} = \dot{B} = 0$ but four unknowns $\bar{\lambda}, q, N, B$.[29] In order to get an extra equation to solve this system for steady state, first we find a solution to the dynamic system for its timepaths of $N(t,\bar{\lambda}), q(t,\bar{\lambda}), B(t,\bar{\lambda})$ conditional on knowing one steady-state variable $\bar{\lambda}$. Second we use the limit of the bond solution and transversality to acquire an extra steady state condition, allowing us to solve for steady state. It is this procedure which causes steady state to depend on initial conditions N_0, B_0, so-called path dependency or hysteresis.[30]

We use a tilde to denote a steady state variable. The $\dot{N} = 0$ differential equation immediately implies that steady-state sunk costs are zero, which equivalently implies the net present value of a firm in steady state is zero.

$$\tilde{q} = 0 \tag{42}$$

This leaves two steady-state conditions $\dot{q} = \dot{B} = 0$ in three unknowns $\tilde{N}, \bar{\lambda}, \tilde{B}$. Through the arbitrage condition (41b), zero sunk costs (42) imply operating profits are zero

$$\tilde{\pi} = 0 \tag{43}$$

The zero profit condition determines labor per firm (or aggregate labor as a linear function of number of firms $\tilde{H}(\tilde{N})$)

$$\tilde{h} = \left(\frac{\mu\phi}{A(\mu - \nu)} \right)^{\frac{1}{\nu}} \tag{44}$$

[29] This occurs because the consumption differential equations is always in steady-state ($\dot{\lambda} = 0$) due to perfect consumption smoothing from $r = \rho$ which implies consumption is fixed $\lambda = \bar{\lambda}$, but it does not relate to other variables in the system.

[30] An implication of this feature is that temporary shocks may have permanent effects.

Labor per firm determines output per firm and wage[31]

$$\tilde{y} = \frac{\nu}{\mu - \nu}\phi \tag{45}$$

$$\tilde{w} = \left(\frac{A}{\mu}\right)^{\frac{1}{\nu}} \nu \left(\frac{\phi}{\mu - \nu}\right)^{1-\frac{1}{\nu}} \tag{46}$$

With $\tilde{h}$ and $\tilde{w}$ determined by the free entry arbitrage condition $\tilde{\pi} = 0$, then the labor market equilibrium condition (27) determines the number of firms as a function of the consumption index, and therefore labor as a function of consumption index:

$$\tilde{N}(\bar{\lambda}) = \frac{(\bar{\lambda}\tilde{w})^{\frac{1}{\eta}}}{\tilde{h}} \tag{47}$$

$$\tilde{H}(\bar{\lambda}) = (\bar{\lambda}\tilde{w})^{\frac{1}{\eta}} \tag{48}$$

In order to find $\bar{\lambda}$, we are left with one steady-state condition $\dot{B} = 0$ that we have not used: the output market clearing condition (steady-state bond accumulation equation).

$$G + \bar{C}(\bar{\lambda}) - \tilde{w}\tilde{H}(\bar{\lambda}) - r\tilde{B} = 0 \tag{49}$$

This is an excess demand function for the steady state in terms of the price of marginal utility $\bar{\lambda}$. The first two terms $G + C(\bar{\lambda})$ represent expenditure and are decreasing in $\bar{\lambda}$. The second two terms $wH(\bar{\lambda}) + r\tilde{B}$ represent income and are increasing in $\bar{\lambda}$. By the intermediate value theorem, this implies that there exists a $\bar{\lambda} > 0$ such that the economy is at the steady state equilibrium given $\tilde{B}$ (See Appendix A.8 for proof of existence and uniqueness with endogenous $\tilde{B}(\bar{\lambda})$.).

In this section we partly defined steady-state $\{\tilde{N}, \bar{\lambda}, \tilde{B}\}$ for the primitive variables of the dynamical system $N, \bar{\lambda}, B$, given steady-state bonds $\tilde{B}$. We gave $\tilde{N}(\bar{\lambda})$ analytically in (47), then used (49) to prove a steady-state $\bar{\lambda}$ must exist given $\tilde{B}$. In the next section, we derive solutions for dynamics which

[31] Since zero profits imply $0 = \tilde{y} - \tilde{w}\tilde{h}$ then steady-state wage is equivalent to labor productivity $\tilde{w} = \frac{\tilde{y}}{\tilde{h}}$.

provide an additional steady-state condition $\tilde{B}(\bar{\lambda})$ that teamed with (49) and (47) can solve for $\bar{\lambda}$ by expressing (49) entirely in $\bar{\lambda}$ terms

$$G + \frac{1}{\bar{\lambda}} - \tilde{w}^{1+\frac{1}{\eta}}\bar{\lambda} - r\tilde{B}(\bar{\lambda}) = 0$$

3.2 Linearized system

The analysis of the steady state was conditional on the level of steady state bonds $\tilde{B}$. However to determine $\tilde{B}$ we need to know the path taken to equilibrium. The dynamics of the system will be analyzed by linearizing around the steady state. Where the 3×3 matrix is the Jacobian $\mathbf{J}$, the linearized system is[32]

$$\begin{bmatrix} \dot{N} \\ \dot{q} \\ \dot{B} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\gamma} & 0 \\ \frac{1}{\tilde{N}(\bar{\lambda})}\frac{\nu\eta\phi}{1+\eta-\nu} & r & 0 \\ \tilde{\Omega} & 0 & r \end{bmatrix} \begin{bmatrix} N(t) - \tilde{N} \\ q(t) - \tilde{q} \\ B(t) - \tilde{B} \end{bmatrix} \tag{50}$$

$$\text{where} \quad \tilde{\Omega} = \frac{\nu\phi\mu}{\mu - \nu}\left(\varepsilon - 1 + \frac{1}{\mu}\right) \tag{51}$$

Since the total effect of an entrant on aggregate output is an important mechanism for our analysis we denote it

$$\Omega \equiv \frac{dY}{dN}$$

The ambiguous effect of entry on aggregate output ($\Omega \gtreqless 0$) explored away from steady state in section 2.3.1 is also ambiguous in steady state ($\tilde{\Omega} \gtreqless 0$). It depends on $\left(\varepsilon - 1 + \frac{1}{\mu}\right)$. We discuss this extensively in section 3.2.3. For dynamics it implies that the Jacobian element corresponding to the effect of entry on bond accumulation $\frac{d\dot{B}}{dN}|_{\tilde{}} = \tilde{\Omega}$ is ambiguous.

[32] Detailed derivation in Appendix A.6

3.2.1 Number of Firms and Entry (industry dynamics) Solution

The determinant and trace of the industry dynamics $\{N, q\}$ sub-system $\mathbf{B} \in \mathbb{R}^2$ in (50) are

$$\det(\mathbf{B}) = \Delta = \frac{\frac{d\tilde{\pi}}{dN}}{\gamma} = -\frac{\nu\eta\phi}{\gamma\left(1+\eta-\nu\right)\tilde{N}(\bar{\lambda})} < 0 \tag{52}$$

$$tr(\mathbf{B}) = r \tag{53}$$

$\det(\mathbf{B})$ is negative as $1 + \eta > \nu$ and is increasing in $\bar{\lambda}$.[33] The root to the characteristic polynomial corresponding to the subsystem is

$$\Gamma(\bar{\lambda}) = \frac{r}{2}\left(1 \pm \frac{1}{r}\left[r^2 - 4\Delta(\tilde{N}(\bar{\lambda}))\right]^{\frac{1}{2}}\right) \tag{54}$$

The discriminant (square root term) is positive since the determinant is negative ($\Delta < 0$). This implies two distinct real roots. And since the discriminant exceeds 1, then so does its square root so there will be one positive and one negative root. Hence the system is saddle-path stable, with a negative real root Γ and a positive real root Γ^U. Furthermore the trace is positive so the sum of the eigenvalues is positive implying the positive eigenvalue is larger than the absolute value of the negative eigenvalue. Our focus is the stable root which is negative

$$\Gamma = \frac{1}{2}\left(r - \left[r^2 - 4\Delta\right]^{\frac{1}{2}}\right) \tag{55}$$

Lemma 3. *The stable eigenvalue is increasing in* $\bar{\lambda}$

Proof. See Appendix A.7. □

The solution to the linearized subsytem is

$$N(t) = \tilde{N} + \exp[\Gamma(\bar{\lambda})t](N_0 - \tilde{N}) \tag{56}$$

take derivative to get the net entry rate $E = \dot{N} = \Gamma \exp[\Gamma t](N_0 - \tilde{N})$ and

[33] See Appendix A.7 for proof.

substitute $q = \gamma E$ for the sunk cost solution

$$q(t) = \gamma \Gamma \exp[\Gamma t](N_0 - \tilde{N}) \tag{57}$$

The derivative of the solution is $\dot{q} = \Gamma^2 \gamma \exp(\Gamma t)(N_0 - \tilde{N})$, so the growth (shrinkage) in the cost of entry (firm NPV) is given in absolute terms by the stable eigenvalue

$$\left| \frac{\dot{q}}{q} \right| = \Gamma$$

with the sign being determined by whether profits are positive (firms accumulation) or negative (decumulation).

3.2.2 Bonds Solution

Combining (41c) and (9) provides a condition that the solution for bonds must satisfy in the long run (full derivation Appendix A.3).

$$0 = B_0 + \int_0^\infty e^{-rt} \left[Y - \frac{q^2}{2\gamma} - C - G \right] dt \tag{58}$$

The two terms must cancel out, which has an intuitive interpretation. The first term is the initial position of bond holdings. $B_0 > 0$ implies the country begins as a borrower, $B_0 < 0$ implies it begins as a creditor. The second term represents trade surplus if positive and deficit if negative. Therefore (58) states that if a country begins as a borrower, at some point over the time horizon it must run a trade deficit.

Linearizing the differential equation in bonds gives

$$\dot{B}(t) = \tilde{\Omega} \left[N(t) - \tilde{N} \right] - \frac{\tilde{q}}{\gamma} [q(t) - \tilde{q}] + r \left[B(t) - \tilde{B} \right] \tag{59}$$

where $\tilde{q} = 0$. Then substitute in the $N(\bar{\lambda}, t)$ solution (56) restricts the differential equation to be a linear first-order nonhomogeneous differential equation in $B(t)$

$$\dot{B}(t) = \tilde{\Omega} \left[\exp[\Gamma t](N_0 - \tilde{N}) \right] + r \left[B(t) - \tilde{B} \right] \tag{60}$$

If the economy starts with bonds $B(0) = B_0$ the solution to (60) is

$$B(t) = \tilde{B} + \frac{\tilde{\Omega}}{\Gamma(\bar{\lambda}) - r} \exp[\Gamma(\bar{\lambda})t](N_0 - \tilde{N}) \tag{61}$$

where $\frac{d\dot{B}}{dN}|^{\sim} = \tilde{\Omega}$ implies the effect of entry on aggregate output equals the effect of entry on the flow of bonds evaluated at steady state. $\tilde{\Omega}$ affects how accumulation of firms $N_0 \to \tilde{N}$ so $N_0 - \tilde{N} < 0$ changes stock of bonds $B(t)$. $\tilde{\Omega} > 0$ then entry strengthens home production and increases bond investment, whereas $\tilde{\Omega} < 0$ then entry weakens home production and decreases bond investment. In the Walrasian case $(\mu = 1, \nu < 1)$, $\tilde{\Omega} > 0$ and the accumulation of firms leads to a reduction in bonds. The main mechanism here is that there is a positive effect of N on labor supply and output $(Y_{HN} > 0)$, so that having too few firms means that wages, labor income and home production are below their steady state level. To maintain consumption, this low level of income is compensated by higher than steady state imports, financed by running down bonds. An *increase* in firms per se makes wages higher. However, the number of firms is increasing because it is below the steady-state. The stock of bonds decreases because entry implies that the initial level of N was low in the first place, not because the accumulation of firms lowers income.

However, given $\mu > 1, \nu < 1$, if μ is large enough then bonds will increase as firms are accumulated. This is because the level of profits along the path to equilibrium is large: whilst the number of firms is below equilibrium, the extra profits generated are enough to exceed the adjustment costs and lower wage. In addition, there is a capacity effect, so that productivity is higher whilst the number of firms is below equilibrium (for $\mu > 1$, free-entry leads to excessive number of firms in steady-state). In the case of $\nu \geq 1$, the flow of entry leads to an increase in the stock of bonds: this is because N has a negative effect on wages and profits, so that N below its steady state implies income above the steady state.

3.2.3 Effect of Entry on Aggregate Output

In steady state entry may increase, decrease or have no effect on aggregate output $\tilde{\Omega} \gtreqless 0$. This corresponds to whether entry increases, decreases or has no effect on labor supply, which depends on whether labor is employed with decreasing, increasing or constant returns.

Proposition 3 (Entry and Aggregate Output)**.** *The effect of entry on aggregate output $\tilde{\Omega}$ is ambiguous in steady-state.*

1. *Lack of Entry:* $\tilde{\Omega} > 0 \iff 1 - \nu > \eta(\mu - 1)$
2. *Excess Entry:* $\tilde{\Omega} < 0 \iff 1 - \nu < \eta(\mu - 1)$
3. *Optimal Entry:* $\tilde{\Omega} = 0 \iff 1 - \nu = \eta(\mu - 1)$

For $\nu \geq 1$ there is always excessive entry $\tilde{\Omega} < 0$. For $\nu < 1$ all outcomes are possible.[34]

Next we provide a discussion of the three possible cases.[35] From the proof the outcome depends on whether the negative business stealing effect $-\left(\frac{\mu-1}{\mu}\right) \leq 0,\ \mu \in [1, \infty)$ dominates the labor elasticity to entry effect $\frac{-\eta}{1+\eta-\nu} < \varepsilon < 1$, which may be positive, negative or zero.

Excess Entry $\tilde{\Omega} < 0$: If there are constant $\nu = 1$ or increasing $\nu > 1$ returns to labor, $\varepsilon \leq 0$, then the fall in labor reinforces the negative business stealing effect, so there is unambiguously a negative effect of entrants on aggregate output in steady state. This is a sufficient condition but is not necessary, providing the business stealing effect is large enough it can override even a positive labor elasticity effect that arises with decreasing returns $\nu < 1$.

[34] Optimal entry refers to the number of firms that maximizes steady-state aggregate output, conditional on a markup existing. There is no maximum with perfect competition $\mu = 1$, always a lack of entry due to a positive labor effect and no negative markup (business stealing) effect.

[35] Etro 2009; Etro and Colciago 2010 provide a discussion of 'golden rule' number of firms when there is endogenous imperfect competition, constant returns and love-of-variety. The golden rule number of firms is that which maximizes consumption and therefore output in steady-state. They show that imperfect competition causes excessive entry in steady-state, which our proposition corroborates ($\mu > 1$ and $\nu = 1$ implies $1 - \nu < \eta(\mu - 1)$, so excess entry).

1. Example: Positive labor elasticity effect, dominated by negative business stealing effect $\nu = 0.9,\ \eta = 1$ therefore $\varepsilon = 0.\overline{09}$ with $\mu = 1.15$ business stealing is -0.13.

2. Constant Returns Special Case $\nu = 1$: The labor effect is zero, so only the negative business stealing effect is present. The smaller the markup $\mu \to 1$ the smaller the negative business stealing effect. But it cannot equal 1 due to the existence condition $\nu < \mu$.

With large markups this outcome is likely. With less divisible labor $\eta \to 0$ this outcome is more likely.

Lack of Entry $\tilde{\Omega} > 0$: If there are decreasing returns $\nu < 1$ then $0 < \varepsilon < 1$ and the boost in labor from entry works against the negative business stealing effect, so there can be too little entry if this positive effect dominates the negative business stealing effect. $\varepsilon > 0$, hence $\nu < 1$, is necessary but not sufficient, sufficiency requires it is positive *and* larger than the negative business stealing effect.

1. Example: Positive labor elasticity effect dominates negative business stealing effect $\nu = 0.9,\ \eta = 1$ therefore $\varepsilon = 0.\overline{09}$ with $\mu = 1.05$ business stealing is -0.05.

2. Perfect Competition Special Case $\mu = 1, \nu < 1, \tilde{\Omega} > 0$: There is no negative business stealing effect, and the the existence condition $\nu < \mu$ enforces decreasing returns. Therefore entry always has a positive effect, implying lack of entry in steady state in the Walrasian (perfect competition) economy.

Optimal Entry $\tilde{\Omega} = 0$: A necessary condition is that the ambiguous labor elasticity effect is positive $\varepsilon > 0$, so it can counterbalance the negative business stealing effect. Therefore a necessary condition is decreasing returns $\nu < 1$.

1. Example: $\nu = 0.9,\ \eta = 1,\ \mu = 1.1$

3.3 Steady-state Bonds

The linearized dynamics give an explicit solution for steady state bonds as a function of $\bar{\lambda}$ and the initial conditions N_0, B_0. Evaluate (61) at $t = 0$ implies

$$\tilde{B}(\bar{\lambda}) = B_0 - \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}(N_0 - \tilde{N}(\bar{\lambda})) \tag{62}$$

therefore the steady-state bond condition (62) and steady-state arbitrage condition (47) give the excess demand condition (49) in terms of $\bar{\lambda}$ only

$$\tilde{w}\tilde{H}(\bar{\lambda}) + r\tilde{B}(\bar{\lambda}) - \bar{C}(\bar{\lambda}) - G = 0 \tag{63}$$

We can solve this for the steady-state consumption index $\bar{\lambda}$, which then provides $\tilde{C}(\bar{\lambda}), \tilde{H}(\bar{\lambda}), \tilde{N}(\bar{\lambda}), \tilde{B}(\bar{\lambda})$. We cannot solve (63) analytically since it is highly nonlinear in $\bar{\lambda}$. However we can show analytically that a unique solution exists, and then solve for this numerically. A useful lemma to show uniqueness (and other results) is that the steady-state excess demand function is strictly increasing in inverse consumption, so is decreasing in consumption given N_0 begins within a neighbourhood of $\tilde{N}$.

Lemma 4 (Excess Demand Monotonically Increasing)**.** *The steady-state market-clearing condition is monotonically increasing in* $\bar{\lambda}$

$$\tilde{w}\frac{d\tilde{H}}{d\lambda} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\tilde{C}}{d\bar{\lambda}} > 0 \tag{64}$$

if the following sufficient condition holds

$$\left(\varepsilon - 1 + \frac{1}{\mu}\right)\left(\frac{N_0}{\tilde{N}(\bar{\lambda})} - 1\right) \geq -\left(\frac{\varepsilon - 1}{\Gamma(\bar{\lambda})} + \frac{1}{r\mu}\right)(r - 2\Gamma(\bar{\lambda})) \tag{65}$$

Proof. See appendix A.8. □

The right-hand side of (65) is strictly negative and the left-hand side is ambiguous. This condition is weaker than the simpler sufficient condition $N_0 - \tilde{N}(\bar{\lambda}) \to 0$ which is commonly assumed and ensures the left-hand side is zero.[36] The condition always holds if there is entry $N_0 < \tilde{N}$ and $\varepsilon - 1 + \frac{1}{\mu} < 0$ (i.e. $\tilde{\Omega} < 0$) implying the left-hand side is positive.

Corollary 1 ($\bar{\lambda}$ Uniqueness). *If* (65) *holds then there is a unique* $\bar{\lambda}$ *that solves* (64).

Proof. Lemma 4 shows that given (65) the steady state market clearing condition (excess of income over expenditure) is strictly monotonic in $\bar{\lambda}$. Hence, if a steady-state exists it is a *unique* steady state solution for $\bar{\lambda}$. □

4 Technological Change

4.1 Comparative Statics

An improvement in technology A reduces employment per firm but output per firm (firm scale) (12) is unaffected. Consequently an improvement in technology increases wages[37]

$$\frac{d\tilde{h}}{dA} = -\frac{\tilde{h}}{\nu A} < 0 \tag{66}$$

$$\frac{d\tilde{w}}{dA} = \frac{\tilde{w}}{\nu A} > 0 \tag{67}$$

Therefore in the long run technological progress crowds-out labor at the product-level but output is unaffected (aggregate output will expand as there are more products each requiring less labor). These comparative statics are simple as they only depend on exogenous variables. However, the aggregate endogenous variables $\{\bar{C}, \tilde{N}, \tilde{B}\}$ ((7), (47), (62)), excluding $\tilde{q}$ which is zero, are a function of A directly but also depend on $\bar{\lambda}(A)$. Therefore technology change has a direct (partial) and an indirect (consumption) effect.[38]

[36] See Turnovsky 1997, p.68 (footnote 8) for a justification of this.

[37] An increase in steady-state wages is equivalent to an increase in labor productivity since $\tilde{w} = \frac{\tilde{y}}{\tilde{h}}$.

[38] We call the indirect effect a consumption effect as $\bar{\lambda}(A)$ is inverse consumption by (7).

Proposition 4 (Long-run Effect of Technology). *A permanent improvement in technology:*

$$\frac{d\bar{C}}{dA} > 0 \tag{68}$$

$$\frac{d\tilde{N}}{dA} > 0 \tag{69}$$

$$\text{sgn}\, \frac{d\tilde{B}}{dA} = \text{sgn}\, -\tilde{\Omega} \tag{70}$$

$$\text{sgn}\, \frac{d\tilde{H}}{dA} = \text{sgn}\left[B_0 - \frac{\tilde{\Omega}}{\Gamma - r} N_0\right] \tag{71}$$

$$\frac{d\tilde{Y}}{dA} = \tilde{y}\frac{d\tilde{N}}{dA} > 0 \tag{72}$$

From the steady-state market clearing condition, the implicit function theorem implies that technology unambiguously increases consumption. This rise in consumption (indirect effect) decreases aggregate labor and number of firms, whereas the direct partial effects of increased technology increase labor and number of firms. Overall, the partial effect dominates in the number of firms case, whereas it is ambiguous in the labor case. The increase in the stock of firms implies an increase in aggregate output, and a bond response that depends on the whether there are excessive, insufficient or optimal number of firms. The effect on the labor supply is ambiguous because there is a conflict of income and substitution effects: the higher wage causes a substitution effect for less leisure and more consumption, which increases labor. Whereas the income effect increases leisure and decreases labor. Which effect dominates depends on the level of initial wealth. From (62) $B_0 - \frac{\tilde{\Omega}}{\Gamma - r} N_0$ is the initial value of wealth in terms of bonds.[39] If $\tilde{\Omega} > 0$, that is $\nu < 1$ and μ small enough, then a sufficient condition for employment to increase $\frac{d\tilde{H}}{dA} > 0$ is that bond holdings are non-negative $B_0 \geq 0$. Likewise, if $\tilde{\Omega} < 0$, (for which $\nu \geq 1$ is sufficient) then a sufficient condition for employment to decrease $\frac{d\tilde{H}}{dA} < 0$ is that bond holdings are non-positive $B_0 \leq 0$.

[39] From (62), $-\frac{\tilde{\Omega}}{\Gamma - r} N_0 = \tilde{B} - B_0 - \frac{\tilde{\Omega}}{\Gamma - r}\tilde{N}$ thus the term $-\frac{\tilde{\Omega}}{\Gamma - r} N_0$ is the present value of the bonds that would have been decumulated/accumulated if $\tilde{N} = 0$.

Bonds respond in the opposite direction to the entry effect on output. If technology-induced entry increases GDP, then bonds decrease (less borrowing is necessary). If technology-induced entry decreases GDP, then bonds increase (more borrowing is necessary). Since steady-state bonds only depend on technology through $\tilde{N}$, the bond response follows the number of firms increase:m $\frac{d\tilde{B}}{dA} = \frac{d\tilde{B}}{dN}\frac{d\tilde{N}}{dA}$, and to a first-order approximation sgn $\frac{d\tilde{B}}{dN} \approx \text{sgn} -\tilde{\Omega}$.[40] Similarly the increase in number of firms determines that aggregate output increases as long-run output per firm (firm scale) is constant.

4.2 Comparative Dynamics

From the dynamic solution for number of firms (56), we can see that on impact $t = 0$ of a shock the number of firms is fixed $N(0) = N_0$, whereas entry adjusts $E(0) = \Gamma(N_0 - \tilde{N})$, which affects the stock of firms an instance later. In other words number of firms is a stock (state) variable, and entry is a flow (jump) variable. Thus entry jumps the economy onto its stable manifold instantaneously as the shock hits, subsequently the number of firms responds as the economy evolves along this manifold. Therefore the difference between the impact and long-run effects depend on the effect of entry.

Proposition 5. *On impact of a technology shock hours and wages will increase, decrease or remain constant relative to their long-run level depending on whether labor returns to scale are increasing $\nu > 1$, decreasing $\nu < 1$ or constant $\nu = 1$.*

$$\text{sgn}\left[\frac{dH(0)}{dA} - \frac{dH(\infty)}{dA}\right] = \text{sgn}\left[\nu - 1\right] \tag{73}$$

$$\text{sgn}\left[\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA}\right] = \text{sgn}\left[\nu - 1\right] \tag{74}$$

On impact the labor effect is ambiguous, as in the long run, due to competing substitution and income effects. The reason is also the same (income and substitution effects may clash). However, if we look at the difference

[40] The approximation arises from assuming we begin close to steady-state $N_0 - \tilde{N} \to 0$. From (62) removes the effect of the eigenvalue responding to $\tilde{N}$.

between the impact and long-run effect, this depends on whether there is an increasing or decreasing MPL at the firm level. When $\nu < 1$, on impact there is a negative relationship between the real wage and employment.; when $\nu > 1$ a positive relation; when $\nu = 1$ no relation. We can thus get undershooting of employment ($\nu > 1$) or overshooting ($\nu < 1$) on impact relative to the new long-run level depending on whether entry increases or decreases the marginal product.

Table 1 captures the combination of static (Proposition 4) and dynamic effects (Proposition 5) on labor. Rows capture the static effect that labor might in the long-run increase, decrease or remain constant depending on initial wealth. Columns capture the dynamic effect that labor might initially overshoot, undershoot or perfectly reflect its long-run level.

	$\nu < 1$	$\nu > 1$	$\nu = 1$
$B_0 > \frac{\tilde{\Omega}}{\Gamma - r} N_0$	Increase, Overshoot	Increase, Undershoot	Increase, Constant
$B_0 < \frac{\tilde{\Omega}}{\Gamma - r} N_0$	Decrease, Overshoot	Decrease, Undershoot	Decrease, Constant
$B_0 = \frac{\tilde{\Omega}}{\Gamma - r} N_0$	Constant, Overshoot	Constant, Undershoot	Constant, Constant

Table 1: Conditions for Taxonomy of Labor Dynamics

4.3 Reconciling with Empirical Evidence

In the theoretical model we derived the result that the short-run response of labor depends on whether the marginal product of labor is increasing or decreasing. In most models of entry, such as Bilbiie, Ghironi, and Melitz 2012, there is a constant marginal product of labor, so that there is no short-run impact on labor. Chang and Hong 2006 conduct an SVAR analysis of labor responses to technology shocks across US manufacturing industries. They show that of their 2-digit industry estimates, 14 industries show a positive response (4 significant) while 6 industries show a negative response (1 significant).[41] Additionally they provide estimates of returns to scale using the

[41] *Instruments* and *Non-electornic* are zero at 3 decimal places but positive with greater precision. Statistical significance is at the 10% level. *Misc* are significant with greater precision than reported in Table 2: $\frac{SRR}{SD} = 0.01626/0.0098 = 1.6492 > t^{crit.} = 1.6449$.

methodology of Basu, Fernald, and Kimball 2006 (BFK). The BFK methodology is to run a log-linear regression of output on inputs with a common coefficient γ on capital and employment for each industry, with an additional coefficient β on hours per worker.[42] The coefficient γ is interpreted as returns to scale which is reported by Chang and Hong (Table 5) for their dataset. In terms of our model, in which there is only labor, we can interpret the increasing or decreasing marginal product of labor $\nu \gtreqless 1$ either as the coefficient γ (i.e. interpreting labor input as employment) or as the sum of the coefficients γ and β (i.e. the coefficient on total hours, the product of employment and hours-per-worker). Chang and Hong (Table 5) provide estimates of γ for 20 two-digit industries (ten durables and ten non-durables) plus an estimate of β for durables $\beta^D = 0.17$ and non-durables $\beta^{ND} = 0.76$ (β is assumed constant across industries within each sector). Our theory predicts a positive relationship between labor returns to scale (ν) and short run responses (SRR) of labor to technology shocks that is supported by their evidence. In Table 2 the SRR of labor for 2-digit industries, and standard deviations, are taken directly from Chang and Hong replication files, while the labor returns to scale are proxied by the returns to scale reported in their table 5. Our main result is the levels prediction that short-run responses are positive with increasing returns to labor $\nu > 1$ and negative with decreasing returns to labor $\nu < 1$. The results show that 14 of 20 industries respond the way we would expect,[43] and of the 5 significant (asterisk) responses reported by Chang and Hong all but textile conform to our theory.[44]

Chang and Hong find that there are increasing returns in the majority of industries (14 out of 20) in terms of γ. Estimates of β are both positive: if we combine β with γ, all of the industries have increasing returns so that all of the sectors with a negative or zero short-run impact are inconsistent with our theory: this is 7 industries, meaning 13 are theory consistent. Hence, Chang and Hong's results are broadly supportive of our theoretical result: 13 or 14 of the industries are consistent with our results whether we use γ or

[42]See Basu, Fernald, and Kimball 2006 equation 18, p1424.

[43]This includes *Instruments* which has no short-run response and is the closest estimate to constant returns.

[44]In Appendix A.10 Figure 2 we report the results as a scatter plot.

SIC	Industry	RTS	SRR	SD
23	Apparel	1.24	0.012	0.009
28	Chemicals	1.52	-0.004	0.004
36	Electronic	1.53	-0.009	0.012
34	Fab. Metal	1.29	0.024	0.090
20	Food	0.38	0.001	0.003
25	Furniture	1.18	0.021	0.009*
38	Instruments	0.97	0.000	0.011
31	Leather	0.39	-0.002	0.012
24	Lumber	0.92	-0.028	0.011*
33	Metal	1.29	0.012	0.017
39	Misc	1.41	0.016	0.010*
35	Non-electronic	1.67	0.000	0.013
26	Paper	1.48	0.001	0.008
29	Petrol	0.53	-0.004	0.007
27	Printing	1.49	-0.001	0.008
30	Rubber	1.15	0.022	0.010*
32	Stone	1.36	0.009	0.008
22	Textile	0.86	0.017	0.006*
21	Tobacco	1.08	0.005	0.006
37	Transport	1.12	0.018	0.013

Table 2: Chang and Hong 2006 Results Comparison

$\gamma + \beta$ as our measure of ν.

5 Entry Regulation Shock

We interpret γ in the cost of entry equation (37) as red tape. When red tape increases firm entry costs become more sensitive to the flow of entry. For example, if a resource needed to setup a firm is in inelastic supply, like a government office that provides certificates to enter an industry, then a rise in red tape amplifies congestion. This makes entry more costly, and a firm may wait until a less congested period to attain certification. A 'deregulatory' policy decreases γ.[45] Data reported in Figure 1 indicate that red tape, proxied

[45]We adopt the term deregulatory shock following Bilbiie, Ghironi, and Melitz 2007 and authors who interpret entry costs as influenced by regulation (Blanchard and Giavazzi 2003; Poschke 2010; Barseghyan and DiCecio 2011). Whereas these focus on differences in

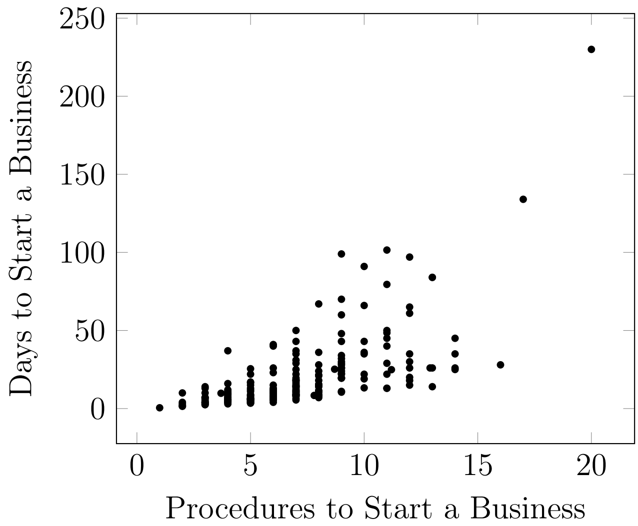

Figure 1: Red Tape and Business Churn

by procedures to start a business, is positively related to the length of time it takes to start a firm which proxies pace of business formation.[46]

Proposition 6. *The economy's speed of adjustment is monotonically decreasing in regulation of business creation.*

The magnitude of the stable root captures the economy's speed of adjustment, as it dictates the speed of adjustment of the sole state variable (number of firms) through the exponential term of (56). Taking the derivative of the stable root, which is negative, with respect to the regulatory parameter gives[47]

$$\Gamma_\gamma = \Gamma_\Delta \Delta_\gamma = \frac{\Delta_\gamma}{(r^2 - 4\Delta)^{\frac{1}{2}}} = \frac{-\Delta}{\gamma(r^2 - 4\Delta)^{\frac{1}{2}}} > 0 \tag{75}$$

The stable root is increasing in the discriminant and the discriminant $\Delta_\gamma =$

fixed exogenous sunk costs and changes in the steady-state stock of operating firms, our interest is endogenous sunk costs and changes in speed of adjustment of firms.

[46] Figure 1 represents 2016 World Bank Doing Business data for 211 countries. Venezuela is the 20 procedures 230 days outlier. New Zealand is the 0.5 days 1 procedure point. Ebell and Haefke 2009 report similar trends in number of procedures and days to start-up for OECD data.

[47] This result is for a given steady-state $\tilde{N}(\bar{\lambda}))$ as γ will also affect $\tilde{N}$ through $\bar{\lambda}$.

$-\frac{\Delta}{\gamma}$ is increasing in the regulatory parameter. Therefore an increase in regulation, increases the the value of the negative root moving it closer to zero and implying slower adjustment. The result implies that economies with less red tape recover faster following a shock.[48] In the context of labor responses to technology shocks, it implies that labor achieves its new steady state faster. The implication that less red tape, helps business churn and aids the dissipation of shocks supports policy work by the IMF and academic literature focused on structural reform in Europe (e.g.di Mauro and Lopez-Garcia 2015).[49]

6 Conclusion

This paper studies the effect of dynamic entry on short-run labor responses to technology shocks. The main insight is that if firm entry is slow to react, then the response of labor to technology shocks will depend on whether labor is employed with decreasing, increasing or constant returns to scale at the firm level. Furthermore the persistence of these deviations will depend on the level of regulation and consequently on the pace of firms' adjustment.

[48] This line of analysis relates to Chatterjee 2005 who focuses on speed of convergence related to capital utilization.

[49] For example, see The Case for Fiscal Policy to Support Structural Reforms (IMF blog, 2017) and Eurozone rebalancing: Are we on the right track for growth? Insights from the CompNet micro-based data (voxEU, Bartelsman, di Mauro, Dorrucci, 2015) on the policy side and Cacciatore, Duval, et al. 2016a; Cacciatore and Fiori 2016; Cacciatore, Duval, et al. 2016b on the academic side.

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A Appendix

A.1 Household Optimization

The Hamiltionian and optimality conditions are

$$\hat{\mathcal{H}}(t) = U(C,H) + \lambda(t)[rB + wH + \Pi - C - G] \tag{76}$$

$$\begin{aligned}
\hat{\mathcal{H}}_C = 0: && U_C(C) - \lambda =& 0 && (77)\\
\hat{\mathcal{H}}_H = 0: && U_H(H) + \lambda w =& 0 && (78)\\
\hat{\mathcal{H}}_B = \rho\lambda - \dot{\lambda}: && \lambda r = \rho\lambda - \dot{\lambda} && (79)\\
\hat{\mathcal{H}}_\lambda = \dot{B}: && \dot{B} = rB + wH + \Pi - C - G && (80)
\end{aligned}$$

The presence of a small open economy and international capital markets $\rho = r$ means that the household can completely smooth its consumption so (79) implies $\dot{\lambda} = 0$. Therefore marginal utility of wealth is unchanging over time. $\lambda = \bar{\lambda}$ combined with additively separable preferences $u_{CH} = 0$ this implies from (77) that consumption is constant and in a one-one relationship with marginal utility of wealth.[50]

$$\bar{C} = C(\bar{\lambda}) \tag{81}$$

This relationship from (77) then implies labor only varies with real wage from (78)

$$H = H(\bar{\lambda}, w) = H(\bar{C}, w) \tag{82}$$

This represents the households labor supply.

[50]We could not make the final step from (77) is $u_{CH} \neq 0$. Imposing additive separability and therefore constant consumption, we simplify analysis of dynamics as C can be treated as fixed.

A.2 General Equilibrium Effect of Entry on Output

There are two ways to think of the effect of an entrant on aggregate output $\frac{dY}{dN}$, and they offer different intuitions. The first begins with $Y = Ny$ and the second begin with $Y = AN^{1-\nu}H^{\nu} - N\phi$.

1. $\frac{dY(N,y(N,H))}{dN} = \frac{d[Ny]}{dN} = y + N\frac{dy}{dN}$ An entrant always causes 'business stealing' from other firms: a fall in output at the firm level or analogously, by (24), a fall in an each incumbents' profits.

$$\frac{dy}{dN} < 0 \tag{83}$$

$$\frac{dy}{dN} = \frac{d\left(AN^{-\nu}H^{\nu} - \phi\right)}{dN} \tag{84}$$

$$= -\nu\frac{(y+\phi)}{N} + \nu\frac{(y+\phi)}{H}\frac{dH}{dN} \tag{85}$$

$$= \nu\frac{(y+\phi)}{N}\left[\varepsilon - 1\right] < 0 \tag{86}$$

$$= Y_H\frac{h}{N}\left[\varepsilon - 1\right] \tag{87}$$

Therefore the aggregate business stealing effect is

$$N\frac{dy}{dN} = \nu(y+\phi)(\varepsilon - 1) \tag{88}$$

This also implies the effect on operating profits is negative and less than proportional

$$\frac{d\pi}{dN} = \left(1 - \frac{\nu}{\mu}\right)\frac{dy}{dN} < 0 \tag{89}$$

At the aggregate level it is not clear whether the negative business stealing effect of an entrant aggregated across all incumbents offsets the positive effect of the new firms' extra output.

$$\frac{dY}{dN} = \frac{d(Ny)}{dN} \tag{90}$$
$$= y + N\frac{dy}{dN} \tag{91}$$
$$= y + \nu A h^{\nu}(\varepsilon - 1) \tag{92}$$
$$= Ah^{\nu}(1 - (1-\varepsilon)\nu) - \phi \tag{93}$$
$$= \frac{(1-\nu)(1+\eta)}{1+\eta-\nu} Ah^{\nu} - \phi \tag{94}$$
$$= \varepsilon(1+\eta)Ah^{\nu} - \phi \tag{95}$$

The final representation makes clear the crucial effect of returns to scale. It reads that an entrant has a negative effect by bringing in an extra fixed cost, but it has another positive negative or zero effect depending on ε.

2. Alternatively use (34), where the first term is the partial derivative effect of an entrant which we have explained is ambiguous based on ν, and the second term is the labor response which is also ambiguous based on ν.

$$\frac{dY(N,H)}{dN} = \frac{d[AN^{1-\nu}H^{\nu} - N\phi]}{dN} = Y_N + Y_H H_N \tag{96}$$
$$= \pi - \left(1 - \frac{1}{\mu}\right) Y_H \frac{H}{N} + Y_H H_N \tag{97}$$
$$= \pi - \left(1 - \frac{1}{\mu} - \varepsilon\right) Y_H h \tag{98}$$

A.3 Bonds

The dynamic equation (41c) is a first-order, linear, nonhomogeneous ordinary differential equation in B. Rewrite in standard form

$$\dot{B} - rB = Y - \frac{q^2}{2\gamma} - C - G \tag{99}$$

Multiply by the integrating factor e^{-rt}

$$e^{-rt}\dot{B} - re^{-rt}B = e^{-rt}\left[Y - \frac{q^2}{2\gamma} - C - G\right] \tag{100}$$

Notice the left-hand side as the result of a product rule differentiation, and use this to help integrate

$$e^{-rt}B = \kappa + \int_0^\infty e^{-rt}\left[Y - \frac{q^2}{2\gamma} - C - G\right] dt \tag{101}$$

To find the constant of integration κ, evaluate at $t = 0$ and use the initial condition $B(0) = B_0$

$$B(0) = \kappa = B_0 \tag{102}$$

Substitute this back in (101), then evaluate at $t \to \infty$. Use the transversality condition (9) which makes the left-hand side zero as $\lambda = \bar{\lambda}$. Therefore

$$0 = B_0 + \int_0^\infty e^{-rt}\left[Y - \frac{q^2}{2\gamma} - C - G\right] dt \tag{58}$$

A.4 Profit Maximization with Variable Returns to Scale

$$\max_h \pi_i = p_i y_i - wh_i \tag{103}$$

$$\text{s.t.} \quad \frac{p_i}{P} = \left(\frac{Y}{N^{\varsigma} y_i}\right)^{\frac{1}{\theta}} \tag{11}$$

$$y_i = Ah_i^{\nu} - \phi \tag{12}$$

$$\pi = \left(\frac{Y}{N^{\varsigma}}\right)^{\frac{1}{\theta}} (Ah^{\nu} - \phi)^{1-\frac{1}{\theta}} - wh \tag{104}$$

$$\pi_h = \left(\frac{Y}{N^{\varsigma}}\right)^{\frac{1}{\theta}} \left(1 - \frac{1}{\theta}\right) (Ah^{\nu} - \phi)^{-\frac{1}{\theta}} \cdot A\nu h^{\nu-1} - w \tag{105}$$

$$\pi_{hh} = -\frac{1}{\theta}\frac{(\pi_h + w)}{y}\frac{(y+\phi)\nu}{h} + \frac{(\pi_h + w)(\nu - 1)}{h} \tag{106}$$

$$= \frac{\pi_h + w}{h}\left[\nu\left(1 - \frac{1}{\theta} - \frac{\phi}{\theta y}\right) - 1\right] \tag{107}$$

$$\pi_{hh} < 0 \quad \iff \quad 1 - \frac{1 + s_{\phi}}{\theta} < \frac{1}{\nu} \tag{108}$$

The second-order condition $\pi_{hh} < 0$ for maximization is always satisfied when $\nu \leq 1$. However with increasing returns $\nu > 1$ it is possible that the term in square brackets is positive, hence there is a necessary and sufficient condition, which can be expressed as

$$1 - \frac{1 + s_{\phi}}{\theta} < \frac{1}{\nu} \tag{109}$$

$$\theta\left(\frac{\theta - 1}{\theta} - \frac{1}{\nu}\right) < s_{\phi} \tag{110}$$

$$\left(\frac{1}{\mu} - \frac{1}{\nu}\right) < \frac{s_{\phi}}{\theta} \tag{111}$$

Throughout the paper we impose that the markup $\frac{\theta}{\theta-1} \equiv \mu$ exceeds returns to scale $\mu > \nu$ (this is necessary for a well-defined steady-state), but it is also a sufficient condition for the second-order condition to hold since $\frac{1}{\mu} - \frac{1}{\nu} < 0 < \frac{\phi}{\theta y}$. Under perfect competition $\theta \to \infty$ so $\mu \to 1$, there must be increasing marginal costs $\nu < 1$ which gives the outcome that with a fixed cost, Walrasian equilibrium only exists with increasing marginal costs, where marginal cost intersect minimum average costs at a firm's *minimum efficient scale.*

To find the profit maximizing ($\pi_h = 0$) outcome, exploit symmetry $Y/N =$

y.

$$\pi_h = \left(\frac{Y}{N^{\varsigma} y}\right)^{\frac{1}{\theta}} \left(1 - \frac{1}{\theta}\right) \cdot A\nu h^{\nu-1} - w = 0 \tag{112}$$

$$\pi_h = N^{\frac{1-\varsigma}{\theta}} \left(1 - \frac{1}{\theta}\right) \cdot A\nu h^{\nu-1} - w = 0 \tag{113}$$

$$\pi_h = N^{\frac{1-\varsigma}{\theta}} \left(\frac{\theta - 1}{\theta}\right) \nu A h^{\nu-1} - w = 0 \tag{114}$$

where we ignore love of variety by assuming $\varsigma = 1$.

A.5 General Equilibrium Labor Behavior

Proof of Proposition 2. Take the derivative of (27)

$$H_N = \frac{1-\nu}{1+\eta-\nu} \frac{H}{N} \tag{115}$$

Therefore, the elasticity follows naturally

$$\varepsilon = H_N \frac{N}{H} = \frac{1-\nu}{1+\eta-\nu} \tag{116}$$

The elasticity is less than 1, it approaches 1 in the indivisible labor limit.

$$\lim_{\eta \to 0} \varepsilon = 1 \tag{117}$$

$$\lim_{\eta \to \infty} \varepsilon = \begin{cases} 0^+ & \nu < 1 \\ 0^- & \nu > 1 \end{cases} \tag{118}$$

□

A.6 Detailed Jacobian

The Jacobian matrix of the 3-dimensional system is as follows (all elements are evaluated at steady state)

$$\mathbf{J} = \left[\left[\begin{array}{ccc} 0 & \frac{d\dot{N}}{dq} & 0 \\ \frac{d\dot{q}}{dN} & \frac{d\dot{q}}{dq} & 0 \\ \frac{d\dot{B}}{dN} & \frac{d\dot{B}}{dq} & \frac{d\dot{B}}{dB} \end{array}\right]\right|^{\tilde{}} = \left[\begin{array}{ccc} 0 & \frac{1}{\gamma} & 0 \\ -\frac{d\tilde{\pi}}{dN} & r & 0 \\ \frac{d\tilde{Y}}{dN} & -\frac{d\tilde{\mathcal{C}}}{dq} & r \end{array}\right] \tag{119}$$

where,

$$\frac{d\tilde{\mathcal{C}}}{dq} = \frac{\tilde{q}}{\gamma} \tag{120}$$

$$\frac{d\tilde{\pi}}{dN} = \frac{\tilde{\pi} + \phi}{\tilde{N}(\bar{\lambda})} \left(\frac{-\eta\nu}{1 + \eta - \nu} \right) \tag{121}$$

$$\frac{d\tilde{Y}}{dN} = A\tilde{h}^{\nu} \left(1 + \nu \left(\frac{1 - \tilde{h}}{\tilde{h}} \right) \right) - \phi \tag{122}$$

where $\tilde{q} = \tilde{\pi} = 0$ (from (42) and (43)) and (44) gives $\tilde{h}$ as a function of exogenous parameters, but $\tilde{N}(\bar{\lambda})$ depends on endogenously determined steady-state consumption index given in (47). Section 2.3.1 and equation (35) help with these derivations, and make clear that both π an Y responses depend on business stealing $\frac{d\tilde{y}}{dN}$.

A.7 Jacobian Results

In the results that follow, the trace, determinant, eigenvalue relationships are useful

$$\Delta = \Gamma\Gamma^U \tag{123}$$

$$r = \Gamma + \Gamma^U \tag{124}$$

$$\Delta = \Gamma(r - \Gamma) \tag{125}$$

$$(r^2 - 4\Delta)^{\frac{1}{2}} = r - 2\Gamma \tag{126}$$

The determinant of the entry subsystem $\det(\mathbf{B}) = \Delta(\tilde{N}(\bar{\lambda}))$ is increasing in $\bar{\lambda}$.

$$\Delta_\lambda = \Delta_N \tilde{N}_\lambda = -\frac{\Delta}{\tilde{N}} \cdot \frac{\tilde{N}}{\eta\bar{\lambda}} = -\frac{\Delta}{\eta\bar{\lambda}} > 0 \tag{127}$$

The stable root is increasing in the determinant

$$\Gamma_\Delta = -\frac{r}{2}\left(\frac{1}{2}\left(1 - \frac{4\Delta}{r^2}\right)^{\frac{-1}{2}} \cdot \frac{-4}{r^2}\right) \tag{128}$$

$$= \frac{1}{(r^2 - 4\Delta)^{\frac{1}{2}}} = \frac{1}{r - 2\Gamma} > 0 \tag{129}$$

and therefore increasing in the number of firms

$$\frac{d\Gamma}{d\tilde{N}} = \Gamma_\Delta \Delta_N = \frac{\Gamma(\Gamma - r)}{r - 2\Gamma}\frac{1}{\tilde{N}} > 0 \tag{130}$$

Therefore the stable root is increasing in $\bar{\lambda}$

$$\Gamma_{\bar{\lambda}} = \Gamma_\Delta \Delta_\lambda = \Gamma_\Delta \Delta_{\tilde{N}} \tilde{N}_\lambda > 0 \tag{131}$$

which proves Lemma 3.

This can be written

$$\Gamma_{\bar{\lambda}} = -\frac{\Delta}{\eta\bar{\lambda}(r^2 - 4\Delta)^{\frac{1}{2}}} = \frac{1}{\eta\bar{\lambda}}\frac{\Gamma(\Gamma - r)}{r - 2\Gamma} > 0$$

A.8 Steady-state Proofs

Proof of Proposition 3.

$$\tilde{\Omega} = \left(\varepsilon - 1 + \frac{1}{\mu}\right) Y_H \tilde{h}$$

$$\tilde{\Omega} = \nu \frac{\phi}{1 - \frac{\nu}{\mu}}\left(\varepsilon - 1 + \frac{1}{\mu}\right) = \frac{\nu\phi\mu}{\mu - \nu}\left(\frac{1}{\mu} - \frac{\eta}{1 + \eta - \nu}\right)$$

$$\operatorname{sgn}\tilde{\Omega} = \operatorname{sgn}\left[\varepsilon - \left(\frac{\mu - 1}{\mu}\right)\right]$$

where $\operatorname{sgn}\varepsilon = \operatorname{sgn}(1 - \nu)$ since $\varepsilon = \frac{1-\nu}{1+\eta-v}$ from (32). □

Repeating the steady-state bond condition here

$$\tilde{B}(\bar{\lambda}, A) = B_0 - \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}(N_0 - \tilde{N}(\bar{\lambda})) \tag{62}$$

The total derivative of steady-state bonds with respect to inverse consumption is

$$\frac{d\tilde{B}}{d\bar{\lambda}} = -\tilde{\Omega}\left(\frac{d\left(\frac{N_0 - \tilde{N}(\bar{\lambda})}{\Gamma(N(\bar{\lambda}))-r}\right)}{d\bar{\lambda}}\right) = \tilde{\Omega}\left[\frac{(\Gamma(\bar{\lambda}) - r)\frac{d\tilde{N}}{d\bar{\lambda}} + [N_0 - \tilde{N}(\bar{\lambda})]\frac{d\Gamma(\tilde{N})}{d\bar{\lambda}}}{(\Gamma(\bar{\lambda}) - r)^2}\right] \tag{132}$$

The response of steady-state bonds to inverse consumption $\bar{\lambda}$ is ambiguous because both $\tilde{\Omega}$ and $[N_0 - \tilde{N}(\bar{\lambda})]$ are ambiguously signed. Since this model is path-dependent (steady-state depends on initial conditions $\tilde{N}(\bar{\lambda}, N_0)$ due to (62)), we cannot evaluate at $N_0 = \tilde{N}$, which removes the changing eigenvalue effect (see Caputo 2005, p. 475-477 for this common approach).[51] Instead we follow Turnovsky 1997, p.68 (footnote 8) and assume this component $[N_0 - \tilde{N}]$ is small, which – to a linear approximation – removes the changing eigenvalue effect.

Lemma 5. *The effect of a change in the consumption index on bonds is*

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\left[\frac{\Gamma}{r - 2\Gamma}\left(\frac{r}{\Gamma} - 3 + \frac{N_0}{\tilde{N}}\right)\right]\frac{\tilde{N}}{\bar{\lambda}\eta} \tag{133}$$

Proof. From (62) a change in consumption index only affects steady-state bonds indirectly through its effect on steady-state stock of firms

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{d\bar{\lambda}} \tag{134}$$

Then steady-state stock of firms affects bonds directly $\frac{\partial \tilde{B}}{\partial \tilde{N}}$ through $\tilde{N}$ and

[51] Attempting this approach here introduces another fixed point problem since changing N_0 to equal $\tilde{N}$ will in turn change $\tilde{N}$ due to path-dependency.

indirectly $\frac{d\tilde{B}}{d\Gamma}\frac{d\Gamma}{d\tilde{N}}$ through the eigenvalue $\Gamma(\tilde{N}(\bar{\lambda}))$:

$$\frac{d\tilde{B}}{d\tilde{N}} = \frac{\partial\tilde{B}}{\partial\tilde{N}} + \frac{d\tilde{B}}{d\Gamma}\frac{d\Gamma}{d\tilde{N}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\left[1 + \left(\frac{N_0 - \tilde{N}(\bar{\lambda})}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\right)\frac{d\Gamma}{d\tilde{N}}\right] \tag{135}$$

Therefore the effect of a change in consumption index on bonds through eigenvalues is an indirect-indirect effect.

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{d\bar{\lambda}} = \left(\frac{\partial\tilde{B}}{\partial\tilde{N}} + \frac{d\tilde{B}}{d\Gamma}\frac{d\Gamma}{d\tilde{N}}\right)\frac{d\tilde{N}}{d\bar{\lambda}} \tag{136}$$

$$= \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\left[1 + \left(\frac{N_0 - \tilde{N}(\bar{\lambda})}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\right)\frac{d\Gamma}{d\tilde{N}}\right]\frac{d\tilde{N}}{d\bar{\lambda}} \tag{137}$$

Using (130) the term in square brackets simplifies

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\left[\frac{\Gamma}{r - 2\Gamma}\left(\frac{r}{\Gamma} - 3 + \frac{N_0}{\tilde{N}}\right)\right]\frac{d\tilde{N}}{d\bar{\lambda}} \tag{138}$$

Therefore substituting in (159) gives (133). □

Corollary 2. *If* $\frac{N_0}{\tilde{N}(\bar{\lambda})} < 3 - \frac{r}{\Gamma}$ *then*

$$\operatorname{sgn}\frac{d\tilde{B}}{d\bar{\lambda}} = -\operatorname{sgn}\tilde{\Omega} \tag{139}$$

Proof. From (133) this result ensures the term in curled parenthesis is negative. □

Hence a sufficient condition is $\frac{N_0}{\tilde{N}} < 3$, which allows for both entry and exit $-\tilde{N} < N_0 - \tilde{N} < 2\tilde{N}$. The economic interpretation is that the initial stock of firms (market size) is greater than zero and less than three times the steady-state stock of firms. This is more general than the (commonly assumed) stronger condition that the initial condition is arbitrarily close to steady state $\frac{N_0}{\tilde{N}} \to 1$. This condition simply ensures we ignore the changing eigenvalue effect.

Corollary 3. *If* $[N_0 - \tilde{N}(\bar{\lambda})] \to 0$ *then*

$$\operatorname{sgn} \frac{d\tilde{B}}{d\bar{\lambda}} = -\operatorname{sgn} \tilde{\Omega} \tag{140}$$

Proof. From (135) as $N_0 - \tilde{N}(\bar{\lambda}) \to 0$

$$\frac{d\tilde{B}}{d\tilde{N}} \approx \frac{\partial \tilde{B}}{\partial \tilde{N}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r} \tag{141}$$

$$\frac{d\tilde{B}}{d\bar{\lambda}} \approx \frac{\partial \tilde{B}}{\partial \tilde{N}} \frac{d\tilde{N}}{d\bar{\lambda}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r} \frac{\tilde{N}}{\bar{\lambda}\eta} \tag{142}$$

□

Lemma 6 (Steady-state Existence)**.** *By the intermediate-value theorem at least one steady-state solution exists.*

Proof of Lemma 6. Split the steady-state excess demand function into two functions: an income function $f(\bar{\lambda}) = \tilde{w}\tilde{H}(\bar{\lambda}) + rB(\bar{\lambda})$ and an expenditure function $g(\bar{\lambda}) = C(\bar{\lambda}) + G$, so we have $f(\bar{\lambda}) - g(\bar{\lambda}) = 0$. Analyze the functions for the limits of $\bar{\lambda}$. Existence follows from the functional forms for $H(\bar{\lambda}, A) = (\bar{\lambda}w)^{\frac{1}{\eta}}$ and $C(\bar{\lambda}) = \frac{1}{\bar{\lambda}}$. Also that $\tilde{B}$ is bounded in (62) since $\tilde{N}$ is bounded as it is proportional to $\tilde{H}$, which lies in $[0, 1]$. $\lim_{\lambda \to 0} H = 0$ and $\lim_{\lambda \to 0} C = \infty$ so expenditure exceeds income. $\lim_{\lambda \to \infty} H = 1$ and $\lim_{\lambda \to \infty} C = 0$, so income exceeds expenditure. Hence for at least one intermediate value of λ (63) is satisfied. □

Proof of Lemma 4. We aim to show

$$\tilde{w}\frac{d\tilde{H}}{d\lambda} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\tilde{C}}{d\bar{\lambda}} > 0 \tag{64}$$

Since $\frac{d\tilde{C}}{d\lambda} < 0$, a sufficient condition is to show that $\tilde{w}\frac{d\tilde{H}}{d\lambda} + r\frac{d\tilde{B}}{d\lambda} > 0$. That is, we show that the positive labor effect always dominates the (potentially) negative bond effect.

$$\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{\tilde{Y}_H}{\mu}\frac{d\tilde{H}}{d\bar{\lambda}} + r\tilde{\Omega}\left[\frac{(\Gamma - r)\frac{d\tilde{N}}{d\lambda} + [N_0 - \tilde{N}]\frac{d\Gamma}{d\lambda}}{(\Gamma - r)^2}\right] \tag{143}$$

Substitute $\tilde{\Omega} = \left(\varepsilon - 1 + \frac{1}{\mu}\right)\tilde{Y_H}\tilde{h}$ and $\frac{d\tilde{N}}{d\bar{\lambda}} = \frac{d\tilde{H}}{d\bar{\lambda}}\frac{1}{\tilde{h}}$

$$= \left[\frac{Y_H}{\mu}\frac{d\tilde{H}}{d\bar{\lambda}}(\Gamma - r) + r\left(\varepsilon - 1 + \frac{1}{\mu}\right)Y_H\frac{d\tilde{H}}{d\bar{\lambda}} + \frac{r\left(\varepsilon - 1 + \frac{1}{\mu}\right)Y_H\tilde{h}(N_0 - \tilde{N})}{\Gamma - r}\frac{d\Gamma}{d\bar{\lambda}}\right]\frac{1}{\Gamma - r} \tag{144}$$

$$= \left[\frac{1}{\mu}(\Gamma - r) + r\left(\varepsilon - 1 + \frac{1}{\mu}\right) + \frac{r\left(\varepsilon - 1 + \frac{1}{\mu}\right)\tilde{h}(N_0 - \tilde{N})}{(\Gamma - r)\frac{d\tilde{H}}{d\bar{\lambda}}}\frac{d\Gamma}{d\bar{\lambda}}\right]\frac{Y_H\frac{d\tilde{H}}{d\bar{\lambda}}}{\Gamma - r} \tag{145}$$

Cancel $\frac{r}{\mu}$ and use that $\frac{d\tilde{H}}{d\bar{\lambda}} = \frac{d\tilde{N}}{d\bar{\lambda}}\tilde{h}$

$$= \left[\frac{1}{\mu}\Gamma + r(\varepsilon - 1) + \frac{r\left(\varepsilon - 1 + \frac{1}{\mu}\right)(N_0 - \tilde{N})}{\Gamma - r}\frac{\frac{d\Gamma}{d\bar{\lambda}}}{\frac{d\tilde{N}}{d\bar{\lambda}}}\right]\frac{Y_H\frac{d\tilde{H}}{d\bar{\lambda}}}{\Gamma - r} \tag{146}$$

Remembering $\varepsilon - 1 < 0$, the first two terms are negative and the third term (the changing eigenvalue term $\frac{d\Gamma}{d\bar{\lambda}}$) is ambiguous. As with signing $\tilde{B}_{\bar{\lambda}}$, a sufficient condition to remove the problematic changing eigenvalue term is $N_0 - \tilde{N} \to 0$. Although a weaker, but messier, sufficient condition is:

$$\left(\varepsilon - 1 + \frac{1}{\mu}\right)\left(\frac{N_0}{\tilde{N}} - 1\right)\frac{\Gamma}{r - 2\Gamma} \leq -\left(\frac{\Gamma}{r\mu} + \varepsilon - 1\right) \tag{147}$$

$$\left(\varepsilon - 1 + \frac{1}{\mu}\right)\left(\frac{N_0}{\tilde{N}} - 1\right) \geq -\left(\frac{\varepsilon - 1}{\Gamma} + \frac{1}{r\mu}\right)(r - 2\Gamma) \tag{148}$$

The right-hand side is negative so this condition always holds if there is entry $N_0 < \tilde{N}$ and $\varepsilon - 1 + \frac{1}{\mu} < 0$ implying $\tilde{\Omega} < 0$. Or if there is exit $N_0 > \tilde{N}$ and and $\varepsilon - 1 + \frac{1}{\mu} > 0$ implying $\tilde{\Omega} > 0$.

□

A.9 Dynamics

Rather than defining steady-state as a function of $\tilde{h}(A), \tilde{w}(A)$ as in (47) and (48), since both depend on A and we are investigating changes in A it is useful substitute out. Repeating $\tilde{B}$, expressing dependence on A, is also useful. A only affects $\tilde{B}$ through $\tilde{N}$, which it affects directly and indirectly: $\tilde{N}(A, \bar{\lambda}(A))$ via (149).

$$\tilde{N}(\bar{\lambda}, A) = \left(\bar{\lambda}\frac{\nu}{\mu}\right)^{\frac{1}{\eta}} A^{\frac{1+\eta}{\nu\eta}} \left(\frac{\mu - \nu}{\mu\phi}\right)^{\frac{1+\eta-\nu}{\nu\eta}} \tag{149}$$

$$\tilde{H}(\bar{\lambda}, A) = \tilde{h}(A)\tilde{N}(\bar{\lambda}, A) = \left(\bar{\lambda}\frac{\nu}{\mu}\right)^{\frac{1}{\eta}} A^{\frac{1}{\nu\eta}} \left(\frac{\mu - \nu}{\mu\phi}\right)^{\frac{1-\nu}{\nu\eta}} \tag{150}$$

$$\tilde{B}(\tilde{N}(A, \bar{\lambda}(A))) = B_0 - \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(A, \bar{\lambda}(A))) - r}(N_0 - \tilde{N}(\tilde{N}(A, \bar{\lambda}(A))) \tag{62}$$

Technology change has a direct (partial) and an indirect (consumption) effect on the core endogenous model variables

$$\frac{dX}{dA} = \frac{\partial X}{\partial A} + \frac{dX}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}, \quad X \in \{\bar{C}, \tilde{N}, \tilde{B}\} \tag{151}$$

The direct (partial) effects of A holding $\bar{\lambda}$ constant are simple to calculate. There is no partial effect on consumption, only an indirect effect.

$$\frac{\partial \bar{C}}{\partial A} = 0 \tag{152}$$

$$\frac{\partial \tilde{N}}{\partial A} = \frac{(1+\eta)\tilde{N}}{\nu\eta A} > 0 \tag{153}$$

$$\frac{\partial \tilde{B}}{\partial A} \approx \frac{\tilde{\Omega}}{\Gamma - r}\frac{\partial \tilde{N}}{\partial A} \gtreqless 0 \implies \operatorname{sgn}\frac{\partial \tilde{B}}{\partial A} = \operatorname{sgn} -\tilde{\Omega} \tag{154}$$

$$\frac{\partial \tilde{H}}{\partial A} = \frac{\tilde{H}}{\nu A\eta} > 0 \tag{155}$$

From the steady state market clearing condition (63), we can use the implicit function theorem to infer that technology decreases the marginal utility of consumption and therfore increase consumption (since through (7) consumption and marginal utility are inversely related).

Proposition 7 (Technology Effect on Steady-state Consumption)**.**

$$\frac{d\bar{\lambda}}{dA} < 0 \tag{156}$$

$$\frac{d\bar{C}}{dA} = \frac{d\bar{C}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} > 0 \tag{157}$$

$$\frac{d\bar{C}}{d\bar{\lambda}} = -\frac{1}{\bar{\lambda}^2} < 0 \tag{158}$$

Therefore an increase in technology increases consumption (decreases marginal utility), which, from (47) and (48), will have an indirect effect of decreasing numbers of firms and labor. This is because consumption crowds out investment in firms.

$$\frac{d\tilde{N}}{d\bar{\lambda}} = \frac{\tilde{N}}{\eta\bar{\lambda}} > 0 \tag{159}$$

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{d\bar{\lambda}} \approx \frac{\tilde{\Omega}}{\Gamma - r}\frac{d\tilde{N}}{d\bar{\lambda}} \implies \operatorname{sgn}\frac{d\tilde{B}}{d\bar{\lambda}} = -\operatorname{sgn}\tilde{\Omega} \tag{160}$$

$$\frac{d\tilde{H}}{d\bar{\lambda}} = \tilde{h}\frac{d\tilde{N}}{d\bar{\lambda}} = \frac{\tilde{H}}{\eta\bar{\lambda}} > 0 \tag{161}$$

Proof of Proposition 7. The total derivative of (63) with respect to technology is

$$\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\left(\frac{\partial\tilde{H}}{\partial A} + \frac{d\tilde{H}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right) + r\left(\frac{\partial\tilde{B}}{\partial A} + \frac{d\tilde{B}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right) - \frac{dC}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} = 0 \tag{162}$$

Therefore

$$\frac{d\bar{\lambda}}{dA} = -\frac{\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\frac{\partial\tilde{H}}{\partial A} + r\frac{\partial\tilde{B}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}} < 0 \tag{163}$$

The denominator is positive under sufficient condition (65) or stronger sufficient condition $N_0 - \tilde{N} \to 0$. Let's focus on the numerator

$$\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\frac{\partial \tilde{H}}{\partial A} + r\frac{\partial \tilde{B}}{\partial A} \tag{164}$$

which appears to be ambiguous. We shall show it is positive implying (163) is negative.

$$\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\frac{\partial \tilde{H}}{\partial A} + r\frac{\partial \tilde{B}}{\partial A} \tag{165}$$

$$= \frac{\tilde{w}}{\nu A}\tilde{H} + \tilde{w}\frac{\tilde{H}}{\nu A\eta} + r\frac{\tilde{\Omega}}{\Gamma - r}\frac{(1+\eta)\tilde{N}}{\nu\eta A} = \frac{1+\eta}{\nu A}\left[\frac{\tilde{w}\tilde{H}}{(1+\eta)} + \frac{\tilde{w}\tilde{H}}{(1+\eta)\eta} + r\frac{\tilde{\Omega}}{\Gamma - r}\frac{\tilde{N}}{\eta}\right] \tag{166}$$

$$= \frac{1+\eta}{\nu A}\left[\frac{\tilde{w}\tilde{H}}{\eta} + r\frac{\tilde{\Omega}}{\Gamma - r}\frac{\tilde{N}}{\eta}\right] = \frac{1+\eta}{\nu A}\left[\frac{\frac{\tilde{Y_H}}{\mu}\tilde{H}}{\eta} + r\frac{\tilde{\Omega}}{\Gamma - r}\frac{\tilde{N}}{\eta}\right] \tag{167}$$

Substitute $\tilde{\Omega} = (\varepsilon - 1 + \frac{1}{\mu})\tilde{Y_H}\frac{\tilde{H}}{\tilde{N}}$

$$= \frac{1+\eta}{\nu A}\left[\frac{\frac{\tilde{Y_H}}{\mu}\tilde{H}}{\eta} + r\frac{(\varepsilon - 1 + \frac{1}{\mu})\tilde{Y_H}\frac{\tilde{H}}{\tilde{N}}}{\Gamma - r}\frac{\tilde{N}}{\eta}\right] = \frac{(1+\eta)\tilde{Y_H}\tilde{H}}{\nu A\eta}\left[\frac{1}{\mu} + r\frac{(\varepsilon - 1 + \frac{1}{\mu})}{\Gamma - r}\right] \tag{168}$$

$$= \frac{(1+\eta)\tilde{Y_H}\tilde{H}}{\nu A\eta}\frac{1}{(\Gamma - r)}\left[\frac{\Gamma}{\mu} + r(\varepsilon - 1)\right] = \frac{(1+\eta)\tilde{N}(\tilde{y} + \phi)}{A\eta}\frac{1}{(\Gamma - r)}\left[\frac{\Gamma}{\mu} + r(\varepsilon - 1)\right] > 0 \tag{169}$$

Using $\frac{\tilde{H}}{\eta\bar{\lambda}} = \frac{d\tilde{H}}{d\bar{\lambda}}$ we can show

$$= \frac{(1+\eta)\bar{\lambda}}{\nu A}\frac{\tilde{Y_H}\frac{d\tilde{H}}{d\bar{\lambda}}}{(\Gamma - r)}\left[\frac{\Gamma}{\mu} + r(\varepsilon - 1)\right] \tag{170}$$

Substitute (146) (ignore changing eigenvalue effect)

$$= \frac{(1+\eta)\bar{\lambda}}{\nu A}\left(\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}\right) > 0 \tag{171}$$

Therefore

$$\frac{d\bar{\lambda}}{dA} = -\frac{\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\frac{\partial \tilde{H}}{\partial A} + r\frac{\partial \tilde{B}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}} = -\frac{(1+\eta)\bar{\lambda}}{\nu A}\left(\frac{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right) < 0 \tag{172}$$

□

Proof of Proposition 4. **Firms**

$$\frac{d\tilde{N}}{dA} = \frac{\partial \tilde{N}}{\partial A} + \frac{d\tilde{N}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} \tag{173}$$

$$= \frac{(1+\eta)}{\nu\eta A}\tilde{N} - \frac{\tilde{N}}{\bar{\lambda}\eta}\left[\frac{(1+\eta)\bar{\lambda}}{\nu A}\left(\frac{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right)\right] \tag{174}$$

$$= \frac{\partial \tilde{N}}{\partial A}\left[1 - \frac{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right] = \frac{\partial \tilde{N}}{\partial A}\left[\frac{-\frac{d\bar{C}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right] > 0 \tag{175}$$

Bonds

$$\frac{d\tilde{B}}{dA} = \frac{\partial \tilde{B}}{\partial A} + \frac{d\tilde{B}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} = \frac{d\tilde{B}}{d\tilde{N}}\frac{\partial \tilde{N}}{\partial A} + \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} \tag{176}$$

$$= \frac{d\tilde{B}}{d\tilde{N}}\left[\frac{\partial \tilde{N}}{\partial A} + \frac{d\tilde{N}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right] = \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{dA} \tag{177}$$

From (135) if $N_0 - \tilde{N} \to 0$ then $\frac{d\tilde{B}}{d\tilde{N}} = \frac{\partial \tilde{B}}{\partial \tilde{N}} + \frac{d\tilde{B}}{d\Gamma}\frac{d\Gamma}{d\tilde{N}} = \frac{\tilde{\Omega}}{\Gamma - r}\left(1 + \frac{N_0 - \tilde{N}}{\Gamma - r}\frac{d\Gamma}{d\tilde{N}}\right) \approx \frac{\tilde{\Omega}}{\Gamma - r}$ thus

$$\frac{d\tilde{B}}{dA} \approx \frac{\tilde{\Omega}}{\Gamma - r}\frac{d\tilde{N}}{dA} \gtreqless 0 \implies \operatorname{sgn}\frac{d\tilde{B}}{dA} = \operatorname{sgn} -\tilde{\Omega} \tag{178}$$

Labor:

$$\frac{d\tilde{H}}{dA} = \frac{\partial \tilde{H}}{\partial A} + \frac{d\tilde{H}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} = \frac{\tilde{H}}{\nu A\eta} + \frac{\tilde{H}}{\nu\bar{\lambda}}\frac{d\bar{\lambda}}{dA} = \frac{\partial \tilde{H}}{\partial A}\left[1 + \frac{\nu A}{\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right] \tag{179}$$

Substitute out (172)

$$= \frac{\partial \tilde{H}}{\partial A}\left(1 - \frac{(1+\eta)\left(\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}\right)}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\right) \tag{180}$$

$$= \frac{\frac{\partial \tilde{H}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\left(-\eta\left(\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}\right) - \frac{d\bar{C}}{d\bar{\lambda}}\right) \tag{181}$$

Substitute out $\frac{d\tilde{H}}{d\bar{\lambda}} = \frac{\tilde{H}}{\bar{\lambda}\eta}$, $\frac{d\tilde{B}}{d\bar{\lambda}} \approx \frac{\tilde{\Omega}}{\Gamma - r}\frac{d\tilde{N}}{d\bar{\lambda}}$ and $\frac{d\bar{C}}{d\bar{\lambda}} = -\frac{1}{\bar{\lambda}^2} = -\frac{\bar{C}}{\bar{\lambda}}$

$$= \frac{\frac{\partial \tilde{H}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\frac{1}{\bar{\lambda}}\left(\bar{C} - \tilde{w}\tilde{H} - r\frac{\tilde{\Omega}}{\Gamma - r}\tilde{N}\right) \tag{182}$$

In steady state $\tilde{C} - \tilde{w}\tilde{H} = r\tilde{B}$

$$\frac{d\tilde{H}}{dA} = \frac{\frac{\partial \tilde{H}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\frac{1}{\bar{\lambda}}\left(r\tilde{B} - r\frac{\tilde{\Omega}}{\Gamma - r}\tilde{N}\right)$$

From (62) $\tilde{B} - \frac{\tilde{\Omega}}{\Gamma - r}\tilde{N} = B_0 - \frac{\tilde{\Omega}}{\Gamma - r}N_0$

$$\frac{d\tilde{H}}{dA} = \frac{\frac{\partial \tilde{H}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\frac{r}{\bar{\lambda}}\left(B_0 - \frac{\tilde{\Omega}}{\Gamma - r}N_0\right)$$

□

Proof of Proposition 5. **Labor**: Totally differentiating $H = H(\bar{\lambda}, N, A)$ keeping N fixed yields.

$$\frac{dH(0)}{dA} = \frac{dH}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} + \frac{\partial H}{\partial A} \tag{183}$$

$$= -\frac{\partial H}{\partial A}\left[\frac{(1+\eta-\nu)(w\frac{dH}{d\bar{\lambda}} + r\frac{dB}{d\bar{\lambda}}) - \nu\frac{dC}{d\bar{\lambda}})}{\nu\left(w\frac{dH}{d\bar{\lambda}} + r\frac{dB}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}\right)}\right] \tag{184}$$

As in the long-run case, the income and substitution effects of a technological improvement work in opposite directions. The difference between the long-run and impact multiplier is accounted for by the effect of entry, so that

$$\frac{dH(0)}{dA} - \frac{dH(\infty)}{dA} = \frac{dH}{dN}\frac{dN}{dA} = \frac{dH}{dN}\left[\frac{\partial N}{\partial A} + \frac{dN}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right] \tag{185}$$

$$= \frac{dH}{dN}\frac{\partial \tilde{N}}{\partial A}\left[\frac{-\frac{d\bar{C}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right] \tag{186}$$

$$\text{sgn}\left[\frac{dH(\infty)}{dA} - \frac{dH(0)}{dA}\right] = \text{sgn}\;\; H_N = \text{sgn}\,[1-\nu]$$

Wages:

$$\frac{dw(0)}{dA} = \frac{1}{\mu}Y_{HH}\frac{dH(0)}{dA} + \frac{w}{A\nu} \tag{187}$$

Hence

$$\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA} = \frac{1}{\mu}Y_{HH}\frac{dH(0)}{dA} \tag{188}$$

$$\text{sgn}\left[\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA}\right] = \text{sgn}\,[\nu - 1] \tag{189}$$

The difference between the long-run and short run wage effect depends on whether an increase in employment increases the MPL ($\nu > 1, Y_{HH} > 0$), or decreases it ($\nu < 1$, $Y_{HH} < 0$). □

A.10 Extra Figures

Figure 2 plots a scatter of the Chang and Hong results from Table 2. Red triangles represent the 14 observations that are consistent with our theory.

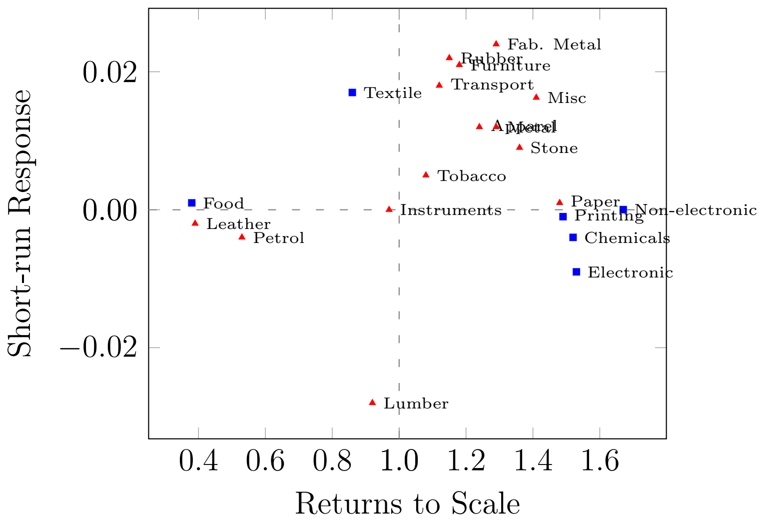

Figure 2: Empirical Evidence