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Short-time Work Subsidies in a Matching Model

Abstract

We consider positive and normative aspects of subsidizing work arrangements where subsidies are paid in time of low demand and reduced working hours so as to stabilize workers' income. In a matching framework such an arrangement increases labor demand. Tightening eligibility to short-time work benefits tends to reduce the wage while the impact on unemployment remains ambiguous. We develop a modified Hosios condition characterizing an efficient combination of labor market tightness and short-time benefit loss rate.

JEL-Codes: E240, H240, J410, J630, J640.

Keywords: short-time work, unemployment insurance, employment subsidies.

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1. Introduction

During the economic crisis after 2008, several countries made extensive use of short-time work, including Japan, Italy and Germany (see Boeri and Bruecker, 2011). Noting the sharp recession, the German government extended the maximum eligibility period for subsidized short-time work up to 24 months. This might have contributed to go through the recession at a comparatively mild increase in open unemployment. Short-time workers accounted for more than 3 per cent of all employed in 2009 in Germany and Italy, and even for more than 5 per cent of all employed in Belgium (Hijzen and Venn, 2011).

This paper analyzes the impacts of subsidizing short-time work on equilibrium wages and unemployment in a matching framework taking the existence of unemployment insurance for granted. Short-time work arises due to partial job destruction. Recovery is in principle possible, but the shock turns out to be permanent for some fraction of cases. The political variable is the short-time benefit loss rate, where the loss terminates the job. Since the government is not able to verify the persistence of the shock, the benefit loss rate is implemented in a non-discriminatory fashion. Both unemployment benefits and subsidies for short-time work are financed by the unemployment insurance. While granting subsidies for short-time work avoids losses of profitable matches and may be less costly than financing unemployment in the short run, its welfare cost consists in keeping up low output jobs.

Increasing the short-time benefit loss rate reduces labor demand as the expected duration of a match is reduced. At the same time, negotiated wages fall at any given labor market tightness since the prospects of unemployed are deteriorated. As a consequence, wages in equilibrium fall under mild condition, while labor market tightness, being correlated with unemployment, may move in either direction. Formulating the normative problem of maximizing output per capita by choosing both the short-time benefit loss rate and labor market tightness subject to the frictions of forming matches yields conditions that can be used to extend the Hosios (1990) condition of achieving efficient labor market tightness.

In practice, short-time work schemes are differentiated across several dimensions. Usually, only full-time workers with a permanent contract are eligible. Working time reduction is usually limited, but sometimes even allowed at a rate of 100 per cent. Maximum duration is always limited, but may vary over the course of the business cycle. The resulting net replacement rate lies above the level for short-term unemployed, and rises in the number of working hours. Typical features of short-time work benefits are given by rate of wage subsidization and benefits. While the average cost to an employer in the first month of short-time work (in 2009) lies below 10% of the normal labor cost in Canada, Switzerland and Spain, it exceeds 30% in France and Japan. Net replacement rates in that year were at least 80% in Luxembourg, Italy and Switzerland, while staying around 60% in Canada, Denmark, Germany, and Spain. Maximum duration of short-time work spells was at 6 months in Denmark and Luxembourg and could be extended up to at least 24 months in Austria, Gemany, Japan, Spain and Switzerland (Hijzen and Venn, 2011). Further details are described by Cahuc and Carcillo (2011).

In the employment contract, short-time work can be understood as part of an insurance arrangement, which might be financed by somewhat lower wages otherwise or some unpaid overtime work in times of high demand. Working time accounts deliver full insurance against fluctuations in normal times and are then preferrered to short-time work arrangements. Layoffs occur only if the flexibility of so-called worksharing is exhausted through downward rigidity in working hours per head (Rosen, 1985). It is not obvious how this arrangement can be improved by subsidies. In the presence of unemployment insurance, subsidizing short-time work might also be associated with reducing benefits for (fully) unemployed. Burdett and Wright (1989) consider a US type unemployment insurance without benefits for short-time work and a European scheme with such benefits according to the degree of working time reduction. They argue that the European scheme induces employers to vary hours per worker more and number of workers less than under a US scheme. While the US scheme implies an overuse of temporary layoffs unless the unemployment insurance is not experience rated, the European scheme tends to distort the hours choice due to subsidization. Relatedly, short-time work cross-subsidizes heavy users of such schemes, reducing aggregate production, where Cahuc and Nevoux (2017) advocate to use experience rated unemployment insurance instead.
Boeri and Bruecker (2011) stress that schemes using stricter eligibility rules (eg only in macroeconomic downturn episodes) and co-financing through experience rating of firms using short-time work – as in Germany – tends to suffer less from moral hazard issues than a pure wage subsidization (similar to the Italian scheme). Fitzroy and Hart (1985) suspect that firms prefer layoffs if short-time work is financed by per-worker taxes instead of wage-related contributions.

The obvious alternative to subsidized short-time work consists in using open unemployment which may accelerate structural change and/or unsubsidized schemes of short-time work. Estimates of Boeri and Bruecker (2011) suggest that there is a substantial deadweight loss associated with using short-time work benefits. Thus, the number of jobs „saved" will fall considerably short of the number of participants in the program. This may be attributed either to windfall gains – subsidies are taken up although the job would be kept anyway- or displacement effects – jobs are kept that are not viable any more (see Hijzen and Martin, 2013). Using data from 23 OECD countries, Hijzen and Martin (2013) estimate the impact of short-time work on aggregate employment at the end of the crisis in 2010 as negligible or even negative. Similarly, Balleer at al. (2016) argue that standard short-time work rules have substantial impacts so as to stabilize employment, while extending eligibility in the recession (discretionary component) mostly subsidizes jobs that would have been kept. Cooper et al. (2017) stress heterogeneity of firms, arguing that short-time work indeed reduces unemployment while at the same time inducing substantial output losses and a fiscal burden.

The contributions closest in spirit are Balleer et al. (2016) and Cooper et al. (2017), both using heterogeneity in output shocks. The latter has an hours restriction as the key policy variable to avoid use of subsidies by low output firms, while in the former the government can directly condition access to short-time work subsidies on output. By contrast, my paper highlights heterogeneity in persistence of the shock rather than in its level, where the government has to rely on a non-discriminating benefit loss rate due to informational asymmetries. The simpler setting also allows analytical comparative statics and a characterization of constrained efficient allocations as novel elements.

The remainder of the paper is organized as follows. Section 2 introduces basic elements of the model, and Section 3 is devoted to labor supply and demand. Section 4 presents the equilibrium and its comparative statics. After defining efficient allocations in the spirit of the Hosios condition in Section 5, Section 6 concludes and indicates directions for futher research.

2. Matching model: Basic elements and unemployment insurance

Basic elements. Consider an adaptation of the standard matching model (Cahuc and Zylberberg, 2004). Let V be the number of vacancies. For simplicity, only the U unemployed search, thus $\theta = V/U$ is labor market tightness. The number of hires is $M(V, U)$ and the matching function, representing matches per vacancy, is defined by $m(\theta) = M(V, U)/V$ with $m' < 0$.

The exit rate from unemployment is given by $M\backslash U = \theta m(\theta)$, which is assumed to increase in tightness, thus $\frac{d\theta m(\theta)}{d\theta} = m(\theta) + \theta m'(\theta) > 0$.

Denote by N the total labor force, which is taken as constant. When L is the number of employed, S is the number of short-time workers, q is the rate of job destruction, and ρ is the benefit loss rate terminating short-time work, we have $\dot{U} = qL + \rho S - \theta m(\theta)U$. The change in the unemployment rate $u = U/N$ is then given by $\dot{u} = \frac{\dot{U}}{N}$. We introduce short-time work via imposing a partial job destruction rate x. Highlighting heterogeneity in short time work an initial share α of jobs is recreated at rate k (for the time being exogenous), while for the share $1-\alpha$ that recreation rate is lower, which for simplicity is set to zero. Denoting $\overline{S}$ as the number of „good" short-time work jobs, it changes according to $\dot{\overline{S}} = \alpha x L - (k+\rho)\overline{S}$. The number of „bad" short-time work jobs changes according to $\dot{\underline{S}} = (1-\alpha)xL - \rho\underline{S}$. Thus, in equilibrium we have $\overline{S} = \frac{\alpha x}{k+\rho}L$ and $\underline{S} = \frac{(1-\alpha)x}{\rho}L$. Accordingly, the equilibrium share of bad short-time work jobs among all short-time work jobs is $\frac{\underline{S}}{S} = \frac{(1-\alpha)/\rho}{\frac{\alpha}{k+\rho}+\frac{1-\alpha}{\rho}} = \frac{1-\alpha}{1-\alpha+\alpha\frac{\rho}{k+\rho}} = \frac{(1-\alpha)(k+\rho)}{\alpha\rho+(1-\alpha)(k+\rho)}$. The change in the total number of short-time workers is determined as $\dot{S} = xL - \rho S - k\overline{S} = xL - \rho S - k\frac{\alpha\rho}{\alpha\rho+(1-\alpha)(k+\rho)}S$. With this specification, the equilibrium share of short-time workers becomes $\frac{S}{L} = \frac{x}{\frac{k\alpha\rho}{\alpha\rho\ (1-\alpha)(k+\rho)}+\rho} = \frac{x[\alpha\rho+(1-\alpha)(k+\rho)]}{k\alpha\rho+\rho[\alpha\rho+(1-\alpha)(k+\rho)]} = \frac{x[\rho+(1-\alpha)k]}{\rho(k+\rho)} = \frac{x[1+(1-\alpha)k/\rho]}{k+\rho}$.
Should the employment structure be in equilibrium, we would have

$$\dot{u} = \frac{\dot{U}}{N} = \left[q\frac{\rho(k+\rho)}{\rho(k+\rho)+x[\rho+(1-\alpha)k)]} + \rho\frac{x[\rho+(1-\alpha)k]}{\rho(k+\rho)+x[\rho+(1-\alpha)k]}\right](1-u) - \theta m(\theta)u. \qquad (1)$$

Accordingly, the related stationary unemployment rate (thus $\dot{u} = 0$) turns out to be

$$u = \frac{q\rho(k+\rho) + \rho x[\rho + (1-\alpha)k]}{q\rho(k+\rho) + \rho x[\rho+(1-\alpha)k] + \theta m(\theta)\big[\rho(k+\rho) + x[\rho+(1-\alpha)k]\big]} \qquad (2)$$
$$= \frac{q(k+\rho) + x[\rho+(1-\alpha)k]}{q(k+\rho) + x[\rho+(1-\alpha)k] + \theta m(\theta)\left[k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]\right]}$$

The last expression describes the so-called Beveridge curve as an inverse relation between the vacancy rate $v = V/N$ and unemployment rate u. The boundary cases when varying the short-time benefit loss rate are

$$\lim_{\rho \to \infty} u = \frac{q + x}{q + x + \theta m(\theta)}$$

and

$$\lim_{\rho \to 0} u = 0$$

The latter result obtains as at a short-time benefit loss rate of zero working short-time in a bad firm becomes an absorbing state.
Lemma 1 describes properties of the equilibrium unemployment rate.

Lemma 1. *The equilibrium unemployment rate* u *decreases in labor market tightness* θ. *At given labor market tightness, a higher short-time benefit loss rate* ρ *increases unemployment.*

Proof. See Appendix A1. ■

Obviously, the higher the labor market tightness, the higher is the exit rate from unemployment, and the lower is the equilibrium unemployment rate. The direct impact of the short-time benefit loss rate – ignoring repercussions via labor market tightness - on unemployment is positive because more people are sent into regular unemployment. Accordingly, a sufficiently strong positive impact on labor market tightness is needed to avoid an increase of the unemployment rate.

Unemployment insurance with short-time work benefits. Consider now an unemployment insurance that also finances short-time work. For simplicity, employed workers pay a lump-sum contribution β financing both unemployment benefits z and short-time work benefits z_s. Accordingly, the budget equation of the unemployment insurance reads

$$\beta(1-u)\frac{k+\rho}{k+\rho+x[1+(1-\alpha)k/\rho]} = zu + z_s(1-u)\frac{x[1+(1-\alpha)k/\rho]}{k+\rho+x[1+(1-\alpha)k/\rho]} \tag{3}$$

Thus, the endogenous contribution β increases in the unemployment rate u:

$$\beta = \frac{u}{1-u} z \frac{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho} + z_s \frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho} \tag{4}$$

The allocation is feasible for unemployment rate and benefit levels remaining sufficiently small. From the definition of equilibrium unemployment rate we obtain

$$\frac{u}{1-u} = \frac{q(k+\rho)+x[\rho+(1-\alpha)k]}{\theta m(\theta)\big[k+\rho+x[1+(1-\alpha)k/\rho]\big]} \tag{5}$$

Thus

$$\begin{aligned}\beta &= \frac{q(k+\rho)+x[\rho+(1-\alpha)k]}{\theta m(\theta)\big[k+\rho+x[1+(1-\alpha)k/\rho]\big]} z \frac{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho} \\ &\quad + z_s \frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho} \\ &= \frac{z\big[q(k+\rho)+x[\rho+(1-\alpha)k]\big]+z_s x\left[1+(1-\alpha)\frac{k}{\rho}\right]\theta m(\theta)}{\theta m(\theta)(k+\rho)} \end{aligned} \tag{6}$$

which gives unemployment insurance contribution β as a function of labor market tightness θ.

For the extreme cases without short-time work programs $\rho \to \infty$ and with unlimited programs $\rho = 0$ we obtain $\lim_{\rho\to\infty} \beta = \frac{z(q+x)}{\theta m(\theta)}$ and $\beta(\rho = 0) = \infty$. The latter again obtains as short-time work becomes an absorbing state. Lemma 2 summarizes the impacts of changes in labor market tightness and varying short-time benefit loss rate.

Lemma 2. *The unemployment insurance contribution falls with increasing labor market tightness θ. At given labor market tightness, the contribution decreases with higher short-time benefit loss rate ρ if and only if* $z_s\left[1+\frac{(1-\alpha)k}{\rho}\left(2+\frac{k}{\rho}\right)\right] > \alpha z k$.

Proof. See Appendix A2. ■

A higher labor market tightness reduces unemployment and thus also the unemployment contribution. The impact of an increase in the short-time benefit loss rate turns out to be ambiguous. It will always decrease the unemployment contribution if ρ is sufficiently small, as reducing „bad“ short-time work then dominates the outcome. The inequality is more likely to hold if the benefit of short-time workers z_s is higher, the benefit of regular unemployed z is lower, or the rate of „good“ short-time work α is smaller, which is all quite intuitive.

3. Firms and workers

Firms. Each firm has one job that can either be vacant or filled. Let output per employed worker be y, where the wage is w and the interest rate is r. With Π_e, $\Pi_{\bar{s}}$, $\Pi_{\underline{s}}$, and Π_v representing expected profit of a filled, partially destroyed „good“, partially destroyed „bad“, and vacant job, respectively, where $\underline{y}$ is reduced output, $\underline{w}$ is the wage paid to a short-time worker, and h is the cost of a vacant job per unit of time, we obtain

$$r\Pi_e = \mathrm{y} - \mathrm{w} + \mathrm{q}(\Pi_v - \Pi_e) + x\big[\alpha(\Pi_{\bar{s}} - \Pi_e) + (1-\alpha)\big(\Pi_{\underline{s}} - \Pi_e\big)\big] \qquad (7)$$

$$r\Pi_{\bar{s}} = \underline{y} - \underline{w} + \rho(\Pi_v - \Pi_{\bar{s}}) + k(\Pi_e - \Pi_{\bar{s}}) \qquad (8a)$$

$$r\Pi_{\underline{s}} = \underline{y} - \underline{w} + \rho\big(\Pi_v - \Pi_{\underline{s}}\big) \qquad (8b)$$

$$r\Pi_v = -h + m(\theta)(\Pi_e - \Pi_v) \qquad (9)$$

The last equation shows that the direct impact of a higher labor market tightness at given $\Pi_e - \Pi_v > 0$ works so as reduce the value of the vacancy, since filling becomes more difficult. With free market entry, we have $\Pi_v = 0$, which yields

$$(r + q + x)\Pi_e = y - w + x\big[\alpha\Pi_{\bar{s}} + (1-\alpha)\Pi_{\underline{s}}\big] \qquad (10)$$

$$(r + \rho + k)\Pi_{\bar{s}} = \underline{y} - \underline{w} + k\Pi_e \qquad (11a)$$

$$\Pi_{\underline{s}} = \frac{\underline{y} - \underline{w}}{r + \rho} \qquad (11b)$$

Thus,

$$\Pi_{\bar{s}} = \frac{\underline{y} - \underline{w}}{r + \rho + k} + \frac{k}{r + \rho + k}\Pi_e \qquad (12)$$

and

$$\Pi_e = \frac{h}{m(\theta)} = (y - w)\frac{r + \rho + k}{(r + q + x)(r + \rho + k) - \alpha k x} + x\left(\underline{y} - \underline{w}\right)\frac{1 + \frac{(1-\alpha)k}{r+\rho}}{(r + q + x)(r + \rho + k) - \alpha k x} \tag{13}$$

Equation (12) shows that expected profit from „good" short-time work falls in the short-time benefit loss rate and converges to zero for $\rho \rightarrow \infty$. The latter property is quite natural as short-time work can then no longer be distinguished from a vacant position.
Equation (13) represents labor demand and relates the wage to the labor market tightness. Its LHS shows that all impacts on profit per regular worker must be reflected in labor market tightness. For the situation without short-time work $(x = k = 0)$, this boils down to the standard condition of the Mortensen-Pissarides framework.

Lemma 3 describes the participation constraint under which the option to use short-time work will be taken up by „good" firms. For „bad" firms the condition boils down to $\underline{y} > \underline{w}$.

Lemma 3 (participation of firm*). A „good" firm will participate in a short-time work program if and only if* $(y - w)k >$-$\left(\underline{y} - \underline{w}\right)(r + q + x + \frac{(1-\alpha)kx}{r+\rho})$.

Proof. See Appendix A3. ■

Small instantaneous losses under short-time work can be tolerated as long as $(y - w)k/(r + q + x + \frac{(1-\alpha)kx}{r+\rho}) >$-$\left(\underline{y} - \underline{w}\right)$. In case of such losses, it becomes more likely that the inequality holds with higher output levels y and $\underline{y}$, lower wages w and $\underline{w}$, lower job destruction rates q and x, and higher job recreation rate k. The acceptable threshold loss decreases with a higher conditional likelihood of becoming a „bad" firm in the future $(1 - \alpha)$.

Lemma 4 describes properties of labor demand.

Lemma 4 (properties of labor demand). *The number of vacancies per unemployed θ decreases with an increasing wage w. Provided that the firm participation constraint in short-time work holds, the number of vacancies per unemployed increases with a higher* *instantaneous net profit $\underline{y} - \underline{w}$ and decreases with a higher short-time benefit loss rate ρ.*

Proof. See Appendix A4. ■

A negative impact of wages on job creation is standard. Regarding the parameters of short-time work, should $\underline{y} - \underline{w}$ increase, short-time work becomes more attractive (if positive) or less severe (if negative). Accordingly, possible future incidences of short-time work will have a more positive impact on the present value of expected profit and vacancies per unemployed rise at any given wage. Increasing the termination rate of short-time benefits ρ reduces the expected length of survival of the employment relationship and therefore diminishes the incentive to create vacancies. This is true also if incidences of short-time work are associated with instantaneous losses because the present value of expected future profits always remains positive as long as the participation constraint holds. Thus, a higher benefit loss rate ρ makes employment less attractive at any given wage, reducing labor market tightness.

Workers. Workers are risk-neutral. Expected return for an employed regular (rV_e), employed „good" short-time ($rV_{\bar{s}}$), employed „bad" short-time ($rV_{\underline{s}}$), and an unemployed (rV_u) worker are respectively given by

$$rV_e = w - \beta + q(V_u - V_e) + x(V_s - V_e), \tag{15}$$

$$rV_{\bar{s}} = \underline{w} + z_s + k(V_e - V_{\bar{s}}) + \rho(V_u - V_{\bar{s}}), \tag{16a}$$

$$rV_{\underline{s}} = \underline{w} + z_s + \rho\left(V_u - V_{\underline{s}}\right), \tag{16b}$$

$$rV_u = z + \theta m(\theta)(V_e - V_u). \tag{17}$$

Wage bargaining divides total surplus $\Omega = V_e - V_u + \Pi_e - \Pi_v$ where the worker receives share $\gamma\Omega = V_e - V_u$. Following the standard approach, the bargaing power parameter $\gamma\epsilon(0,1)$ is a constant. With free entry, we obtain $(1-\gamma)\Omega = \Pi_e$.
Hence

$$\Omega = \frac{h}{(1-\gamma)m(\theta)} = \frac{y-w}{1-\gamma} * \frac{r+\rho+k}{(r+q+x)(r+\rho+k)-\alpha kx} + \frac{\underline{y}-\underline{w}}{1-\gamma} * \frac{x\left(1+\frac{(1-\alpha)k}{r+\rho}\right)}{(r+q+x)(r+\rho+k)-\alpha kx} \tag{18}$$

From equation (18), aggregate surplus of a match decreases in the wage and increases in labor market tightness. As for the profit of the firm, the direct impact of a higher benefit loss rate ρ on the value of a match is negative. Ultimately, the equilibrium profit increases if and only if the equilibrium labor market tightness goes up. Again, all impacts of parameter changes will be reflected in created vacancies per unemployed.

The value of unemployment equation can be written as $rV_u = z + \theta m(\theta)\gamma\Omega$.

The surplus equation is

$$r\Omega = y - \beta - rV_u + q(\Omega_u - \Omega) + x\big(\alpha\Omega_{\bar{s}} + (1-a)\Omega_{\underline{s}} - \Omega\big) \tag{19}$$

where $\Omega_{\bar{s}}$, $\Omega_{\underline{s}}$, and Ω_u denote the surplus under good short-time work, bad short-time work, and a vacant job respectively. With free entry, we have $\Omega_u = 0$. Rearranging terms yields

$$\Omega = \frac{y - \beta - rV_u}{r + q + x} + \frac{x\big(\alpha\Omega_{\bar{s}} + (1-a)\Omega_{\underline{s}}\big)}{r + q + x} \tag{20}$$

The related asset equation on surplus under short-time work are given by

$$r\Omega_{\bar{s}} = \underline{y} + z_s - rV_u + k(\Omega - \Omega_{\bar{s}}) + \rho(\Omega_u - \Omega_{\bar{s}}) \tag{21a}$$

and

$$r\Omega_{\underline{s}} = \underline{y} + z_s - rV_u + \rho\big(\Omega_u - \Omega_{\underline{s}}\big) \tag{21b}$$

Recalling $\Omega_u = 0$, solving for $\Omega_{\bar{s}}$ gives

$$\Omega_{\bar{s}} = \frac{\underline{y} + z_s - rV_u}{r + k + \rho} + \frac{k\Omega}{r + k + \rho} \tag{22a}$$

and

$$\Omega_{\underline{s}} = \frac{\underline{y} + z_s - rV_u}{r + \rho} \tag{22b}$$

Inserting the latter equations (22a) and (22b) into (20) yields

$$(r + q + x)\Omega = y - \beta - rV_u + x\left[\underline{y} + z_s - rV_u\right]\left[\frac{\alpha}{r + k + \rho} + \frac{1 - \alpha}{r + \rho}\right]$$

$$+\frac{xk\alpha}{r+k+\rho}\Omega$$

Isolating Ω gives

$$\Omega = \frac{(r+k+\rho)(y-\beta-rV_u)}{(r+k+\rho)(r+q+x)-\alpha xk} + \frac{x\left(\underline{y}+z_s-rV_u\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]}{(r+k+\rho)(r+q+x)-\alpha xk} \tag{23}$$

The result can be inserted into the asset equation for the value of unemployment:

$$rV_u = z + \frac{\theta m(\theta)\gamma(r+k+\rho)(y-\beta-rV_u)}{(r+k+\rho)(r+q+x)-\alpha xk} + \frac{\theta m(\theta)\gamma x\left(\underline{y}+z_s-rV_u\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]}{(r+k+\rho)(r+q+x)-\alpha x} \tag{24}$$

where for the moment we treat the unemployment contribution β as fixed. Solving for rV_u gives

$$rV_u\left[1+\frac{\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)}{(r+k+\rho)(r+q+x)-\alpha xk}\right]$$
$$= z + \frac{\theta m(\theta)\gamma(r+k+\rho)(y-\beta)}{(r+k+\rho)(r+q+x)-\alpha xk}$$
$$+\frac{\theta m(\theta)\gamma x\left(\underline{y}+z_s\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]}{(r+k+\rho)(r+q+x)-xk}$$

and

$$rV_u = \frac{z[(r+k+\rho)(r+q+x)-\alpha xk]}{N} \tag{25}$$
$$+\theta m(\theta)\gamma\frac{(r+k+\rho)(y-\beta)+x\left(\underline{y}+z_s\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]}{N}$$

with $N \equiv (r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)$.

Plugging in into the wage equation, we obtain the wage curve equation

$$w = \gamma y + (1-\gamma) rV_u = \gamma y + \frac{(1-\gamma)z}{N} \tag{26}$$
$$+ \theta m(\theta)\gamma(1-\gamma)\frac{(r+k+\rho)(y-\beta) + x\left(\underline{y}+z_s\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]}{N}$$

Using the definition of rV_u, a different way to write the wage curve equation is

$$\begin{aligned} w = rV_u + \gamma(y - rV_u) &= z + \theta m(\theta)\gamma\Omega + \gamma(y - z - \theta m(\theta)\gamma\Omega) \\ &= z + \gamma(y-z) + \gamma(1-\gamma)\theta m(\theta)\Omega \end{aligned} \tag{27}$$

Only the last term is a function of labor market tightness where $\theta m(\theta)$ increases in θ. The RHS would increase in θ if Ω is nondecreasing in θ.

For arriving at a positive slope of the wage curve, it is sufficient to demonstrate that rV_u increases in θ. Rewriting rV_u yields

$$rV_u = \frac{z[(r+k+\rho)(r+q+x) - \alpha x k]}{N} \tag{28}$$
$$+ \frac{\theta m(\theta)\gamma\left[(r+k+\rho)(y-\beta) + x\left(\underline{y}+z_s\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]}{N}$$

Note that rV_u is a weighted average of unemployment benefit z, net output $y-\beta$ and short-time income $\underline{y}+z_s$ with weights $\frac{[(r+k+\rho)(r+q+x)-\alpha x k]}{N}$, $[\theta m(\theta)\gamma(r+k+\rho)]/N$ and $\left[\theta m(\theta)\gamma x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]/N\right]$ recalling $N = (r+k+\rho)(r+q+x) - \alpha x k + \theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)$.

Lemma 5. *The negotiated wage as described in the wage curve increases with higher labor market tightness θ. At given labor market tightness, it increases with higher output of short time work $\underline{y}$, and falls with a higher short-time work benefit z_s. It also falls with an increasing short-time benefit loss rate ρ provided that $y-\beta-\underline{y}-z_s$ is sufficiently close to zero and $z_s\left[1+\frac{(1-\alpha)k}{\rho}\left(2+\frac{k}{\rho}\right)\right] \le \alpha z k$.*

Proof. See Appendix A5. ■

The wage curve shifts upward with increasing labor market tightness. Negotiated wages are higher because the position of the unemployed is improved. Following the proof, this would also hold at given unemployment insurance contribution β. Taking into account the endogeneity of β strengthens the result. As higher labor market tightness reduces the unemployment contribution and thus increases the net wage, the value of unemployment rises with higher θ also via the unemployment contribution channel.

While a higher output of short-time workers $\underline{y}$ improves the bargaining position of a worker, a higher benefit of short-time workers z_s does the opposite. This result turns out because an unemployed needs two transitions to benefit - from unemployment to regular work and from regular to short-time work - while he has to finance the subsidy already after the first transition.

A higher short-time benefit loss rate ρ affects negotiated wages via changing the position of an unemployed worker. At given labor market tightness, it reduces the expected length of a future employment relationship. Relative weights of regular employment and unemployment in (28) increase at the expense of the state of short-time work, moving the value of unemployment into opposite directions. Recalling Lemma 2, the unemployment contribution will fall in case of low short-time benefit loss rates, increasing negotiated wages, while the opposite may occur if the loss rates are very high. The net impact is a reduction of the wage if the total replacement rate under short time work is close to unity, when $y - \beta - \underline{y} - z_s$ is close to zero and, at the same time, the unemployment contribution does not decrease – which happens if $z_s \left[1 + \frac{(1-\alpha)k}{\rho}\left(2 + \frac{k}{\rho}\right)\right] \leq \alpha z k$. In that event, the higher likelihood of unemployment in combination with a non-increasing payoff of regular employment determines the outcome.

4. Equilibrium and comparative statics

The labor market equilibrium (w, θ) is defined by the following two equations determining wage w and labor market tightness θ. These are given by the labor demand equation

$$b_1 = (y - w)\frac{r + \rho + k}{(r + q + x)(r + \rho + k) - \alpha k x} + x\left(\underline{y} - \underline{w}\right)\frac{1 + \frac{(1-\alpha)k}{r+\rho}}{(r + q + x)(r + \rho + k) - \alpha k x} - \frac{h}{m(\theta)} = 0 \tag{29}$$

and the wage curve

$$b_2 = \gamma y + (1-\gamma) r V_u - w = \gamma y - w + (1-\gamma) * \left[\frac{z[(r+k+\rho)(r+q+x) - \alpha x k]}{(r+k+\rho)(r+q+x) - \alpha x k + \theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)} \frac{+\theta m(\theta)\gamma\left[(r+k+\rho)(y-\beta) + x\left(\underline{y}+z_s\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]}{(r+k+\rho)(r+q+x) - \alpha x k + \theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)}\right] = 0 \tag{30}$$

where the unemployment insurance contribution β satisfies

$$\beta = \frac{z\left[q(k+\rho) + x[\rho + (1-\alpha)k]\right] + z_s x\left[1 + (1-\alpha)\frac{k}{\rho}\right]\theta m(\theta)}{\theta m(\theta)(k+\rho)} \tag{31}$$

It can easily be demonstrated that any equilibrium of this system is unique. In the following we will assume that existence is ensured.

Lemma 6. *Given existence of the labor market equilibrium (w, θ), it is always unique.*

Proof. See Appendix A6. ∎

Uniqueness of the equilibrium is a consequence of having a positive slope of the wage curve and a negative slope of the labor demand curve.

Proposition 1 summarizes the impacts of changes in the benefit loss rate on the labor market equilibrium.

Proposition 1. *A higher short-time benefit loss rate* ρ *reduces the wage* w *provided that* $y-\beta-\underline{y}-z_s$ *is sufficiently close to zero and* $z_s\left[1+\frac{(1-\alpha)k}{\rho}\left(2+\frac{k}{\rho}\right)\right]\leq \alpha z k$. *Its impact on labor market tightness satisfies*

$$sgn\left[\frac{\partial\theta}{\partial\rho}\right]=-sgn\Bigg[\frac{r+\rho+k}{(r+q+x)(r+\rho+k)-\alpha kx} * \frac{(1-\gamma)x\theta m(\theta)\gamma}{N^2} \tag{32}$$

$$*\Bigg\{\theta m(\theta)\gamma\left[1+\tfrac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\tfrac{(1-\alpha)k}{(r+\rho)^2}\right]\left(y-\beta-\underline{y}-z_s\right)$$

$$-\alpha k(y-\beta-z)-\left(\underline{y}+z_s-z\right)$$

$$*\left[[(r+k+\rho)(r+q+x)-\alpha xk]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]$$

$$+\left[(r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right]$$

$$*\left[(r+k+\rho)\frac{z_s\left[1+(1-\alpha)\frac{k}{\rho}\left(2+\frac{k}{\rho}\right)\right]\theta m(\theta)-\alpha zk}{\theta m(\theta)(k+\rho)^2}\right]\Bigg\}$$

$$+x\Bigg\{\frac{\alpha k(y-w)}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}$$

$$+\frac{\left(\underline{y}-\underline{w}\right)\left[[(r+q+x)(r+\rho+k)-\alpha kx]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}\Bigg]$$

Proof. See Appendix A7. ■

With a higher short-time benefit loss rate, the expected length of employment is reduced, which already tells at the moment of hiring. Recall that we assume participation of the firm assured, implying that the present value of further employment is positive at any time. The labor demand curve shifts downward with a higher short-time benefit loss rate since vacancies are reduced at any given wage. This in turn implies a tendency in favor of a lower wage and a smaller labor market tightness.

The impact of increasing the short-time benefit loss rate on the wage curve works exclusively through affecting the value of unemployment. Weights of both regular employment and unemployment go up. Moreover, the unemployment contribution changes, affecting the payoff of regular employment. Following Lemma 5, the sufficient conditions, repeated here, state that a high replacement rate under short time work in combination with a non-decreasing unemployment contribution will result in lower negotiated wages. In that event, the wage curve shifts down, which in turn is sufficient to have a lower equilibrium wage.

In general, as both the wage curve and the labor demand curve shift down, the impact on labor market tightness is ambiguous. This is refelected in the complex condition given in the second part of Proposition 1. A higher labor market tightness becomes more likely if raising the short-time benefit losss rate increases the unemployment contribution, thus shifting the negotiated wage down at any given labor market tightness.

5. Efficiency

The standard measure in the search-matching framework so as to evaluate the labor market equilibrium is aggregate output minus the vacancy cost. Since the employment rate (including short-time work) is

$$\begin{aligned} 1-u &= \frac{L+S}{L+S+U} \\ &= \frac{\theta m(\theta)[\rho(k+\rho+x)+x(1-\alpha)k]}{q\rho(k+\rho)+\rho x[\rho+(1-\alpha)k]+\theta m(\theta)[\rho(k+\rho+x)+x(1-\alpha)k]} \end{aligned} \tag{33}$$

output per capita taking vacancy cost into account is

$$\begin{aligned} \frac{Y}{N} &= (1-u)*\Bigg[y\frac{\rho(k+\rho)}{[\rho(k+\rho+x)+x(1-\alpha)k]} \\ &\qquad +\underline{y}\frac{\rho x(1+(1-\alpha)k)}{[\rho(k+\rho+x)+x(1-\alpha)k]}\Bigg]-h\theta u \\ &= \frac{\theta\left\{\left[(k+\rho)y+\left[1+(1-\alpha)\frac{k}{\rho}\right]x\underline{y}\right]m(\theta)-h\left[q(k+\rho)+x[\rho+(1-\alpha)k]\right]\right\}}{q(k+\rho)+x[\rho+(1-\alpha)k]+\theta m(\theta)\left[(k+\rho+x)+x(1-\alpha)\frac{k}{\rho}\right]} \end{aligned} \tag{34}$$

For the limiting cases we obtain

$$\frac{Y}{N}(\rho=0)=(1-u)\underline{y}-h\theta u=\ \underline{y} \tag{35}$$

as bad short-time work becomes an absorbing state, and

$$\lim_{\rho\to\infty}\frac{Y}{N}=(1-u)y-h\theta u=\frac{\theta\{ym(\theta)-h(q+x)\}}{q+x+\theta m(\theta)} \tag{36}$$

At given unemployment rate, output per worker is highest without short-time work. Short-time work is associated with benefits due to some instead of zero output as far as unemployment can be reduced. At the same time, losses can occur if the number of vacancies per unemployed shrinks given that short-time workers are not available for other regular jobs.

Suppose now that the government can pick both the benefit loss rate and labor market tightness independently just having to observe frictions as implied by the equation of motion (1). Following an approach as for deriving the Hosios (1990) condition, we need to modify the objective by attaching the a social value to the state of unemployment. For simplicity, and following the standard approach, this value is fixed at the unemployment benefit z. Accordingly, the current value Hamiltonian related to optimization problem of the social planner is

$$\begin{aligned} H=(1-u)&\left[y\frac{k+\rho}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}\right. \\ &\left.+\underline{y}\frac{x(1+(1-\alpha)\frac{k}{\rho})}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}\right]+zu-h\theta u \\ &+\mu\left\{\left[q\frac{k+\rho}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}+\rho\frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}\right](1-u)\right. \\ &\left.-\theta m(\theta)u\right\} \end{aligned} \tag{37}$$

where μ is the costate variable related to the equation of motion. Proposition 2 presents optimality conditions for tightness and the benefit loss rate.

Proposition 2. *Any efficient combination* (θ, ρ) *of labor market tightness and short-time benefit loss rate satisfies*

$$\frac{h}{m(\theta)} = \frac{\left(y - \underline{y}\right)\left[1 + \frac{2(1-\alpha)k}{\rho} + \frac{(1-\alpha)k^2}{\rho^2}\right](1+\eta)}{\alpha k + q + x + (1-\alpha)\left[\frac{2(q+x)k}{\rho} + \frac{k^2[q + x(1-\alpha)]}{\rho^2}\right]}$$

$$= \frac{\left[y\frac{k+\rho}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})} + \underline{y}\frac{x(1+(1-\alpha)\frac{k}{\rho})}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})} - z\right](1+\eta)}{r + q\frac{k+\rho}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]} + \rho\frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]} - \eta\theta m(\theta)}$$

with $\eta = \frac{\theta m'(\theta)}{m(\theta)}$ *where* $\eta\epsilon(-1,0)$.

Proof. See Appendix A8. ■

Solving the optimal control problem yield first-order conditions which boil down to the two conditions given in Proposition 2 that jointly determine optimal tightness and the related optimal short-time benefit loss rate.

Note that the Hosios (1990) condition ensuring efficiency of labor market tightness in equilibrium does not carry over tot he framework with short-time work. Optimal tightness can be compared to equilibrium tightness as follows. As previously shown, inserting the wage curve equation

$$\begin{aligned} w = \gamma y + (1-\gamma)rV_u &= z + \theta m(\theta)\gamma\Omega + \gamma(y - z - \theta m(\theta)\gamma\Omega) \\ &= z + \gamma(y-z) + \gamma(1-\gamma)\theta m(\theta)\Omega \end{aligned} \quad (38)$$

into the labor demand equation

$$\begin{aligned} \frac{h}{m(\theta)} = (y - w)&\frac{r+\rho+k}{(r+q+x)(r+\rho+k) - \alpha kx} \\ &+ \left(\underline{y} - \underline{w}\right)\frac{x\left(1 + \frac{(1-\alpha)k}{r+\rho}\right)}{(r+q+x)(r+\rho+k) - \alpha kx} \end{aligned} \quad (39)$$

and recalling $\Pi_e = \frac{h}{m(\theta)} = (1-\gamma)\Omega$ gives

$$\frac{h}{m(\theta)} = \frac{(y-z)(1-\gamma)(r+\rho+k)+x\left(\underline{y}-\underline{w}\right)}{(r+q+x)(r+\rho+k)-\alpha kx+\gamma\theta m(\theta)(r+\rho+k)} \tag{40}$$

In the standard Mortensen-Pissarides model without short-time work ($\rho \to \infty, x = k = 0$), equilibrium tightness coincides with optimal tightness if and only the worker's bargaining share γ coincides with the absolute of the elasiticity of the matching function, $\gamma = -\eta,$ the Hosios condition (Hosios, 1990). This is no longer the case here.

6. Concluding discussion

Our matching approach draws a balanced point of view of the merits of short-time work rulings. Benefits to be paid by the unemployment insurance are smaller for short-time workers than for unemployed. At the same time, job recreation rates of „good“ short-time work are supposed to be at least as high as job finding rates of regular unemployed. Hence, the direct impact of easier access to short-time work benefits consists in reducing expenditure of unemployment insurance. At the same time, low output jobs may be subsidized for too long. Tightening eligibility rules reduce negotiated wages as the prospects of an unemployed are dampened. Moreover, job offers become less attractive as the expected duration of the job as reduced. Owing to this negative impact on labor demand, lower wages are not neccessarily transferred into higher number of job offers per unemployed and a lower unemployment rate.

It cannot be excluded at the outset that the optimal short-time benefit loss rate is infinite, that is, short-time-work is not allowed. This is the obvious outcome should the share of „bad“ short-time work approach unity. By contrast, using unlimited eligibility is never an optimum as short-time work becomes an absorbing state. Having characterized efficient combinations of short-time benefit loss rate and labor market tightness, the next step will be to determine how the optimal short-time benefit loss rate varies with the parameters of the model, in particular with those characterizing the business cycle. It needs to be checked whether or not the optimal short-time benefit loss rate declines with a higher rate of job destruction.

While the paper explores efficiency issues using output per capita net of vacancy cost into account, alternative welfare measures suggest themselves should some positive approach of explaining implemented policies be pursued. Policy makers may consider either welfare of the unemployed as a Rawlsian measure or welfare of the employed as representing the political majority. In each case several distortions arise. First, the value of the regular employment state is downgraded according to the wage bargaining parameter and the unemployment insurance contribution. Second, probabilities of employment states are biased toward the current state. Finally, the payoff of short-time work is inflated by the short-time work benefit, and the cost of posting vacancies is ignored. In sum, it is to be expected that such a positive approach arrives at a laxer use of short-time work subsidies compared to achieving allocative efficiency as described here.

Appendix

A1: Proof of Lemma 1

From equation (2) we obtain

$$\frac{\partial u}{\partial \theta} * \widetilde{N}^2 = -[m(\theta) + \theta m'(\theta)] \left[k + \rho + x\left[1 + (1-\alpha)\frac{k}{\rho}\right]\right] \\ * \left[q(k+\rho) + x[\rho + (1-\alpha)k]\right] < 0, \qquad (A1)$$

$$\frac{\partial u}{\partial \rho} * \widetilde{N}^2 = \left[q(k+\rho) + x[\rho + (1-\alpha)k] + \theta m(\theta)\left[k + \rho + x + x(1-\alpha)\frac{k}{\rho}\right]\right](q + x) \\ -\left[q(k+\rho) + x[\rho + (1-\alpha)k]\right]\left(q + x + \theta m(\theta)\left[1 - \frac{(1-\alpha)k}{\rho^2}\right]\right) \\ = \theta m(\theta) x\{q\left[1 + (1-\alpha)\frac{k}{\rho} + (1-\alpha)\frac{k(k+\rho)}{x\rho^2}\right] \\ + \left[\alpha k + x + (1-\alpha)\frac{xk}{\rho} + (1-\alpha)\frac{k}{\rho^2}(\rho + (1-\alpha)k)\right]\} > 0 \qquad (A2)$$

with

$$\widetilde{N} = q(k+\rho) + x[\rho + (1-\alpha)k] + \theta m(\theta)\left[k + \rho + x[1 + (1-\alpha)k/\rho]\right]. \qquad (A3)$$

A2: Proof of Lemma 2

The impact of a higher labor market tightness is given by

$$\frac{\partial\beta}{\partial\theta} = -\frac{z\big[q(k+\rho) + x[\rho + (1-\alpha)k]\big][m(\theta) + \theta m'(\theta)]}{(k+\rho)[\theta m(\theta)]^2} < 0 \qquad (A4)$$

Taking the partial derivative with respect to the benefit loss rate yields

$$\frac{\partial\beta}{\partial\rho} = \frac{(k+\rho)\left[z(q+x) - z_s x\theta m(\theta)\frac{(1-\alpha)k}{\rho^2}\right]}{\theta m(\theta)(k+\rho)^2} \qquad (A5)$$

$$-\frac{z\big[q(k+\rho) + x[\rho+(1-\alpha)k]\big] + z_s x\left[1 + (1-\alpha)\frac{k}{\rho}\right]\theta m(\theta)}{\theta m(\theta)(k+\rho)^2}$$

$$= -x\frac{z_s\left[1 + 2(1-\alpha)\frac{k}{\rho} + \frac{(1-\alpha)k^2}{\rho^2}\right]\theta m(\theta) - \alpha z k}{\theta m(\theta)(k+\rho)^2}.$$

A3. Proof of Lemma 3

Noting the expression explaining profit of keeping short-time work, we obtain: Under free entry, short-time work will be used only if $\Pi_{\bar{s}} > 0$, which is equivalent to $-\left(\underline{y} - \underline{w}\right)/k < \Pi_e$. This condition is always satisfied if the instantaneous profit $\underline{y} - \underline{w}$ is nonnegative. Inserting for Π_e and rearranging yields

$$-\left(\underline{y} - \underline{w}\right)[(r+q+x)(r+\rho+k) - \alpha k x] < k(y-w)(r+\rho+k) \qquad (A6)$$

$$+kx\left(\underline{y} - \underline{w}\right)\left[1 + (1-\alpha)\frac{k}{r+\rho}\right]$$

and therefore

$$-\left(\underline{y} - \underline{w}\right)\left[(r+q+x)(r+\rho+k) + (1-\alpha)kx\frac{r+\rho+k}{r+\rho}\right] \qquad (A7)$$

$$< k(y-w)(r+\rho+k)$$

Hence, $\Pi_s > 0$ if and only if $(y - w)k > -\left(\underline{y} - \underline{w}\right)\left(r + q + x + \frac{(1-\alpha)kx}{r+\rho}\right)$.

A4. Proof of Lemma 4

The slope of the curve defined by (13) is negative and equal to

$$\left.\frac{dw}{d\theta}\right|_{d\Pi_e=0} = \frac{hm`(\theta)[(r+q+x)(r+\rho+k)-\alpha kx]}{[m(\theta)]^2(r+\rho+k)} < 0 \qquad (A8)$$

Differentiating the RHS of (13) yields

$$\frac{\partial \dfrac{(y-w)(r+\rho+k)+\left(\underline{y}-\underline{w}\right)x\left[1+\dfrac{(1-\alpha)k}{r+\rho}\right]}{(r+q+x)(r+\rho+k)-\alpha kx}}{\partial\left(\underline{y}-\underline{w}\right)} = \frac{x\left[1+\dfrac{(1-\alpha)k}{r+\rho}\right]}{(r+q+x)(r+\rho+k)-\alpha kx}$$

$$> 0$$

and

$$\frac{\partial \dfrac{(y-w)(r+\rho+k)+\left(\underline{y}-\underline{w}\right)x\left[1+\dfrac{(1-\alpha)k}{r+\rho}\right]}{(r+q+x)(r+\rho+k)-\alpha kx}}{\partial\rho} \qquad (A9)$$

$$= \frac{[(r+q+x)(r+\rho+k)-\alpha kx]\left(y-w-\dfrac{x(1-\alpha)k\left(\underline{y}-\underline{w}\right)}{(r+\rho)^2}\right)}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}$$

$$-\frac{(r+q+x)\left[(y-w)(r+\rho+k)+\left(\underline{y}-\underline{w}\right)x\left[1+\dfrac{(1-\alpha)k}{r+\rho}\right]\right]}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}$$

$$= -\frac{\alpha kx(y-w)}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}$$

$$-\frac{x\left(\underline{y}-\underline{w}\right)\left[[(r+q+x)(r+\rho+k)-\alpha kx]\dfrac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\dfrac{(1-\alpha)k}{r+\rho}\right]\right]}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}$$

Given $x > 0$, the numerator of the RHS in (A9) is always negative.

A5. Proof of Lemma 5

Differentiating the variable part of the wage curve expression (28) gives

$$\frac{\partial rV_u}{\partial\theta} = \frac{\gamma[m(\theta)+\theta m'(\theta)]}{N^2} \tag{A10}$$
$$* \left\{ \left[(r+k+\rho)(r+q+x) - \alpha xk + \theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right]\right.$$
$$* \left[(r+k+\rho)(y-\beta) + x\left(\underline{y}+z_s\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right] - \theta m(\theta)\frac{(r+k+\rho)}{[m(\theta)+\theta m'(\theta)]}\frac{\partial\beta}{\partial\theta}\right]$$
$$-\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\left[z[(r+k+\rho)(r+q+x)-\alpha xk]\right.$$
$$\left.\left. + \theta m(\theta)\gamma\left[(r+k+\rho)(y-\beta) + x\left(\underline{y}+z_s\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]\right]\right\}$$

with

$$N = (r+k+\rho)(r+q+x) - \alpha xk + \theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)$$

Since

$$\frac{\gamma[m(\theta)+\theta m'(\theta)]}{N^2} > 0,$$

and

$$\frac{\partial\beta}{\partial\theta} = -\frac{z\big[q(k+\rho)+x[\rho+(1-\alpha)k]\big][m(\theta)+\theta m'(\theta)]}{[\theta m(\theta)]^2(k+\rho)} < 0$$

we obtain

$$sgn\left[\frac{\partial rV_u}{\partial\theta}\right] = sgn\{[(r+k+\rho)(r+q+x)-\alpha xk]\left[(r+k+\rho)(y-\beta-z)\right.$$
$$\left. + x\left(\underline{y}+z_s-z\right)\right]$$

$$-\left[(r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right]$$
$$*\,\theta m(\theta)\gamma\frac{(r+k+\rho)}{[m(\theta)+\theta m'(\theta)]}\frac{\partial\beta}{\partial\theta}\Bigg\}>0 \qquad (A11)$$

Evaluating the impacts of parameters of short-time-work at given θ, we obtain

$$\frac{\partial rV_u}{\partial \underline{y}}=\frac{\theta m(\theta)\gamma x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]}{N}>0 \qquad (A12)$$

$$\frac{\partial rV_u}{\partial z_s}=\frac{\theta m(\theta)\gamma\left[x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]-(r+k+\rho)\frac{\partial\beta}{\partial z_s}\right]}{N} \qquad (A13)$$

As $\frac{\partial\beta}{\partial z_s}=\frac{x}{k+\rho}\left[1+\frac{(1-\alpha)k}{\rho}\right]$ it turns out that

$$\frac{\partial rV_u}{\partial z_s}=\frac{-\theta m(\theta)\gamma x\left[\frac{r}{k+\rho}+(1-\alpha)k\left[\frac{r+k+\rho}{k+\rho}*\frac{1}{\rho}-\frac{1}{r+\rho}\right]\right]}{N}<0 \qquad (A14)$$

The derivative with respect to the short-time benefit loss rate is

$$\frac{\partial rV_u}{\partial\rho}=\frac{1}{N^2} \qquad (A15)$$
$$*\left\{\left[(r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right]\right.$$
$$*\left[z(r+q+x)+\theta m(\theta)\gamma\left[(y-\beta)-x\left(\underline{y}+z_s\right)\frac{(1-\alpha)k}{(r+\rho)^2}-(r+k+\rho)\frac{\partial\beta}{\partial\rho}\right]\right]$$
$$-\left(r+q+x+\theta m(\theta)\gamma\left[1-\frac{(1-\alpha)xk}{(r+\rho)^2}\right]\right)\left[z[(r+k+\rho)(r+q+x)-\alpha xk]\right.$$
$$\left.\left.+\theta m(\theta)\gamma\left[(r+k+\rho)(y-\beta)+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\left(\underline{y}+z_s\right)\right]\right]\right\}$$

Since $\frac{\partial\beta}{\partial\rho}=-x\frac{z_s\left[1+2(1-\alpha)\frac{k}{\rho}+\frac{(1-\alpha)k^2}{\rho^2}\right]\theta m(\theta)-\alpha zk}{\theta m(\theta)(k+\rho)^2}$ we obtain

$$\frac{\partial rV_u}{\partial\rho}=\frac{\theta m(\theta)\gamma}{N^2}$$

$$
\begin{aligned}
&* \{[(r+k+\rho)(r+q+x)-\alpha xk]\left[(y-\beta-z)-x\left(\underline{y}+z_s-z\right)\frac{(1-\alpha)k}{(r+\rho)^2}\right] \\
&-(r+q+x)\left[(r+k+\rho)(y-\beta-z)+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\left(\underline{y}+z_s-z\right)\right] \\
&-\left[(r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right] \\
&* (r+k+\rho)\frac{\partial\beta}{\partial\rho} \\
&+\theta m(\theta)\gamma x\left[\left[1+\frac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\frac{(1-\alpha)k}{(r+\rho)^2}\right](y-\beta)\right] \\
&-\theta m(\theta)\gamma x\left[\left[1+\frac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\frac{(1-\alpha)k}{(r+\rho)^2}\right]\left(\underline{y}+z_s\right)\right]\Bigg\} \\
&=\frac{\theta m(\theta)\gamma}{N^2} \\
&* \{[-\alpha xk][(y-\beta-z)] \\
&-x\left(\underline{y}+z_s-z\right)\left[[(r+k+\rho)(r+q+x)-\alpha xk]\frac{(1-\alpha)k}{(r+\rho)^2}\right. \\
&\left.+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right] \\
&-\left[(r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right] \\
&* (r+k+\rho)\frac{\partial\beta}{\partial\rho} \\
&+\theta m(\theta)\gamma x\left[1+\frac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\frac{(1-\alpha)k}{(r+\rho)^2}\right]\left(y-\beta-\underline{y}-z_s\right)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{-x\theta m(\theta)\gamma}{N^2} \\
&* \Bigg\{\Bigg[(r+k+\rho)(r+q+x)-\alpha xk \\
&\qquad+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\Bigg]\frac{r+k+\rho}{x}\frac{\partial\beta}{\partial\rho} \\
&+\alpha k(y-\beta-z)-\theta m(\theta)\gamma\left[1+\frac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\frac{(1-\alpha)k}{(r+\rho)^2}\right]\left(y-\beta-\underline{y}-z_s\right)
\end{aligned}
$$

$$+\left[[(r+k+\rho)(r+q+x)-\alpha xk]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]\left(\underline{y}+z_s\right.$$
$$\left.\left.-z\right)\right\}$$

Recalling $\frac{-x\theta m(\theta)\gamma}{N^2}<0$, it follows that

$$sgn\left[\frac{\partial rV_u}{\partial\rho}\right]=sgn\left\{\theta m(\theta)\gamma\left[1+\frac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\frac{(1-\alpha)k}{(r+\rho)^2}\right]\left(y-\beta-\underline{y}-z_s\right)\right.$$
$$-\alpha k(y-\beta-z)-\left(\underline{y}+z_s-z\right)$$
$$*\left[[(r+k+\rho)(r+q+x)-\alpha xk]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]$$
$$+\left[(r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right]$$
$$\left.*\left[(r+k+\rho)\frac{z_s\left[1+(1-\alpha)\frac{k}{\rho}\left(2+\frac{k}{\rho}\right)\right]\theta m(\theta)-\alpha zk}{\theta m(\theta)(k+\rho)^2}\right]\right\}\qquad(A16)$$

Thus, with $y-\beta-\underline{y}-z_s$ being sufficiently close to zero and $z_s\left[1+(1-\alpha)\frac{k}{\rho}\left(2+\frac{k}{\rho}\right)\right]\theta m(\theta)\leq\alpha zk$, we obtain $\left[\frac{\partial r\ u}{\partial\rho}\right]<0$.

A6. Proof of Lemma 6

The labor market equilibrium defined by (29) and (30) taking (31) into account is unique because

$$\left.\frac{dw}{d\theta}\right|_{b_1=0}<0<\left.\frac{dw}{d\theta}\right|_{b_2=0}$$

since

$$\left.\frac{dw}{d\theta}\right|_{b_1=0}=-\frac{\frac{\partial b_1}{\partial\theta}}{\frac{\partial b_1}{\partial w}}=\frac{\frac{hm'(\theta)}{[m(\theta)]^2}}{\frac{r+\rho+k}{(r+q+x)(r+\rho+k)-\alpha kx}}<0\qquad(A17)$$

noting

$$\frac{\partial b_1}{\partial \theta} = \frac{hm'(\theta)}{[m(\theta)]^2} < 0$$

$$\frac{\partial b_1}{\partial w} = -\frac{r+\rho+k}{(r+q+x)(r+\rho+k)-\alpha kx} < 0$$

and

$$\left.\frac{dw}{d\theta}\right|_{b_2=0} = -\frac{\frac{\partial b_2}{\partial \theta}}{\frac{\partial b_2}{\partial w}} < 0 \qquad (A18)$$

as

$$\frac{\partial b_2}{\partial w} = -1 < 0 \qquad (A19)$$

and

$$\begin{aligned}\frac{\partial b_2}{\partial \theta} = \frac{(1-\gamma)\gamma[m(\theta)+\theta m'(\theta)]}{N^2} \qquad (A20)\\ *\{[(r+k+\rho)(r+q+x)-\alpha xk]\left[(r+k+\rho)(y-\beta-z)\right.\\ \left.+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\left(\underline{y}+z_s-z\right)\right]\\ -N(r+k+\rho)\frac{\theta m(\theta)}{m(\theta)+\theta m'(\theta)}\frac{\partial \beta}{\partial \theta}\Big\} > 0\end{aligned}$$

recalling

$$N = (r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right) > 0$$

and

$$\frac{\partial \beta}{\partial \theta} = -\frac{z\big[q(k+\rho)+x[\rho+(1-\alpha)k]\big][m(\theta)+\theta m'(\theta)]}{(k+\rho)[\theta m(\theta)]^2} < 0 \qquad (A21)$$

A7. Proof of Proposition 1

Using the implicit function theorem, the derivatives of w and θ with respect to ρ are given by $\partial w/\partial \rho = -\Delta_{\rho w}/\Delta$ and $\partial\theta/\partial \rho = -\Delta_{\rho\theta}/\Delta$ where Δ is the determinant to the Jacobian related to the system of equations b_1 and b_2 and $\Delta_{\rho w}$ is the related determinant where in the Jacobian the column vector of derivatives with respect to w is replaced by the column vector of derivatives of the system of equations b_1 and b_2 with respect to ρ.

Following Lemma 4, we obtain

$$\frac{\partial b_1}{\partial\rho} = \frac{-\alpha kx(y-w)}{[(r+q+x)(r+\rho+k)-\alpha kx]^2} \tag{A22}$$
$$-\frac{x\left(\underline{y}-\underline{w}\right)\left[[(r+q+x)(r+\rho+k)-\alpha kx]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}$$
$$< 0.$$

Following the proof of Lemma 5, we obtain

$$\frac{\partial b_2}{\partial\rho} = \frac{(1-\gamma)}{N^2} \tag{A23}$$
$$\left\{\left[(r+k+\rho)(r+q+x)-xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right]\right.$$
$$*\left[z(r+q+x)+\theta m(\theta)\gamma\left[(y-\beta)-x\left(\underline{y}+z_s\right)\frac{(1-\alpha)k}{(r+\rho)^2}-(r+k+\rho)\frac{\partial\beta}{\partial\rho}\right]\right]$$
$$-\left[z[(r+k+\rho)(r+q+x)-xk]\right.$$
$$\left.+\theta m(\theta)\gamma\left[(r+k+\rho)(y-\beta)+x\left(\underline{y}+z_s\right)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]\right]$$
$$*\left[(r+q+x)+\theta m(\theta)\gamma-x\left(\underline{y}+z_s\right)\frac{(1-\alpha)k}{(r+\rho)^2}\right]$$
$$=\frac{(1-\gamma)\gamma x\theta m(\theta)}{N^2}$$
$$\left\{\theta m(\theta)\gamma\left[1+\frac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\frac{(1-\alpha)k}{(r+\rho)^2}\right]\left(y-\beta-\underline{y}-z_s\right)\right.$$

$$-\alpha k(y-\beta-z)-\left(\underline{y}+z_s-z\right)$$
$$*\left[\left[(r+k+\rho)(r+q+x)-\alpha xk\right]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]$$
$$+\left[(r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right]$$
$$*\left[(r+k+\rho)\frac{z_s\left[1+(1-\alpha)\frac{k}{\rho}\left(2+\frac{k}{\rho}\right)\right]\theta m(\theta)-\alpha zk}{\theta m(\theta)(k+\rho)^2}\right]\Bigg\}$$

The determinant of the system of equations defining the labor market equilibrium is

$$\Delta=\frac{\partial b_1}{\partial\theta}\frac{\partial b_2}{\partial w}-\frac{\partial b_1}{\partial w}\frac{\partial b_2}{\partial\theta}>0 \qquad (A24)$$

since $\frac{\partial b_1}{\partial\theta}<0$, $\frac{\partial b_2}{\partial w}<0$, $\frac{\partial b_1}{\partial w}<0$, and $\frac{\partial b_2}{\partial\theta}>0$ (see Lemma 6).

Hence, $sgn[\partial w/\partial\rho]=-sgn\left[\Delta_{\rho w}\right]$ and $sgn[\partial\theta/\partial\rho]=-sgn\left[\Delta_{\rho\theta}\right]$. Evaluating these determinants yields

$$\Delta_{\rho w}=\frac{\partial b_1}{\partial\theta}\frac{\partial b_2}{\partial\rho}-\frac{\partial b_1}{\partial\rho}\frac{\partial b_2}{\partial\theta} \qquad (A25)$$
$$=-\frac{hm'(\theta)}{[m(\theta)]^2}*\frac{(1-\gamma)\gamma x\theta m(\theta)}{N^2}$$
$$*\Bigg\{\Bigg[(r+k+\rho)(r+q+x)-\alpha xk$$
$$+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\Bigg]\frac{r+k+\rho}{x}\frac{\partial\beta}{\partial\rho}$$
$$+\alpha k(y-\beta-z)-\theta m(\theta)\gamma\left[1+\frac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\frac{(1-\alpha)k}{(r+\rho)^2}\right]\left(y-\beta-\underline{y}-z_s\right)$$
$$+\left[\left[(r+k+\rho)(r+q+x)-\alpha xk\right]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]\Big(\underline{y}+z_s$$
$$-z\Big)\Bigg\}$$

$$
\begin{aligned}
&+\frac{(1-\gamma)\gamma[m(\theta)+\theta m'(\theta)]}{N_5^2} \\
&*\{[(r+k+\rho)(r+q+x)-\alpha xk]\Bigg[(r+k+\rho)(y-\beta-z) \\
&+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\left(\underline{y}+z_s-z\right)\Bigg] \\
&-N(r+k+\rho)\frac{\theta m(\theta)}{m(\theta)+\theta m'(\theta)}\frac{\partial\beta}{\partial\theta}\Bigg\} \\
&*\left\{\frac{\alpha kx(y-w)}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}\right. \\
&\left.+\frac{x\left(\underline{y}-\underline{w}\right)\left[[(r+q+x)(r+\rho+k)-\alpha kx]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}\right\}
\end{aligned}
$$

Recalling that $m(\theta)+\theta m'(\theta)>0$ by assumption, $\frac{\partial\beta}{\partial\theta}<0$ according to Lemma 2, and $\frac{\partial\beta}{\partial\rho}=-x\frac{z_s\left[1+2(1-\alpha)\frac{k}{\rho}+\frac{(1-\alpha)k^2}{\rho^2}\right]\theta m(\theta)-\alpha zk}{\theta m(\theta)(k+\rho)^2}$, implying $\frac{\partial\beta}{\partial\rho}\geq 0$ iff $z_s\left[1+(1-\alpha)\frac{k}{\rho}\left[2+\frac{k}{\rho}\right]+\frac{(1-\alpha)k^2}{\rho^2}\right]\theta m(\theta)\leq\alpha zk$, we obtain $\Delta_{\rho w}>0$ if $z_s\left[1+(1-\alpha)\frac{k}{\rho}\left[2+\frac{k}{\rho}\right]+\frac{(1-\alpha)k^2}{\rho^2}\right]\theta m(\theta)\leq\alpha zk$ and if at the same time $y-\beta-\underline{y}-z_s$ is sufficiently close to zero.

Regarding the impact on labor market tightness we obtain

$$
\begin{aligned}
\Delta_{\rho\theta}=&\frac{\partial b_1}{\partial\rho}\frac{\partial b_2}{\partial w}-\frac{\partial b_1}{\partial w}\frac{\partial b_2}{\partial\rho} \qquad (A26)\\
&=\frac{r+\rho+k}{(r+q+x)(r+\rho+k)-\alpha kx} \\
&*\frac{(1-\gamma)x\theta m(\theta)\gamma}{N^2} \\
&*\left\{\theta m(\theta)\gamma\left[1+\frac{(1-\alpha)k}{r+\rho}+(r+k+\rho)\frac{(1-\alpha)k}{(r+\rho)^2}\right]\left(y-\beta-\underline{y}-z_s\right)\right. \\
&-\alpha k(y-\beta-z)-\left(\underline{y}+z_s-z\right) \\
&*\left[[(r+k+\rho)(r+q+x)-\alpha xk]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]
\end{aligned}
$$

$$+\left[(r+k+\rho)(r+q+x)-\alpha xk+\theta m(\theta)\gamma\left(r+k+\rho+x\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right)\right]$$
$$*\left[(r+k+\rho)\frac{z_s\left[1+(1-\alpha)\frac{k}{\rho}\left(2+\frac{k}{\rho}\right)\right]\theta m(\theta)-\alpha zk}{\theta m(\theta)(k+\rho)^2}\right]\Bigg\}$$
$$+x\Bigg\{\frac{\alpha k(y-w)}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}$$
$$+\frac{\left(\underline{y}-\underline{w}\right)\left[[(r+q+x)(r+\rho+k)-\alpha kx]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]}{[(r+q+x)(r+\rho+k)-\alpha kx]^2}\Bigg\}$$

Notice that the last numerator term $\alpha k(y-w)+\left(\underline{y}-\underline{w}\right)\left[[(r+q+x)(r+\rho+k)-\alpha kx]\frac{(1-\alpha)k}{(r+\rho)^2}+(r+q+x)\left[1+\frac{(1-\alpha)k}{r+\rho}\right]\right]$ is always positive due to the firm participation constraint and works so as to reduce labor market tightness. However, with small short-time work output (and high short-time work wage) it may be arbitrarily close to zero. If in addition $z_s\left[1+(1-\alpha)\frac{k}{\rho}\left(2+\frac{k}{\rho}\right)\right]\theta m(\theta)\leq\alpha zk$ and $y-\beta-\underline{y}-z_s$ remains close to zero, we obtain $\Delta_{\rho\theta}>0$.

A8. Proof of Proposition 2

As first-order conditions we obtain

$$\frac{\partial H}{\partial\theta}=-hu-\mu u[m(\theta)+\theta m'(\theta)]=0 \qquad (A27)$$

$$\frac{\partial H}{\partial\rho}$$
$$=(1-u)\left\{\frac{\left(k+\rho+x\left[1+\frac{(1-\alpha)k}{\rho}\right]\right)\left(y-\underline{y}\frac{(1-\alpha)xk}{\rho^2}+\mu(q+x)\right)}{\left[k+\rho+x\left[1+\frac{(1-\alpha)k}{\rho}\right]\right]^2}\right. \qquad (A28)$$

$$
-\frac{\left[y(k+\rho)+\underline{y}x\left[1+\frac{(1-\alpha)k}{\rho}\right]+\mu\left[q(k+\rho)+\rho x\left[1+\frac{(1-\alpha)k}{\rho}\right]\right]\right]\left[1-\frac{(1-\alpha)xk}{\rho^2}\right]}{\left[k+\rho+x\left[1+\frac{(1-\alpha)k}{\rho}\right]\right]^2}\Bigg\}
$$

$$
=\frac{x(1-u)}{\left[k+\rho+x\left[1+\frac{(1-\alpha)k}{\rho}\right]\right]^2}
$$

$$
*\begin{bmatrix} \left(y-\underline{y}\right)\left[1+\frac{2(1-\alpha)k}{\rho}+\frac{(1-\alpha)k^2}{\rho^2}\right] \\ +\mu\left(\alpha k+q+x+(1-\alpha)\frac{k}{\rho}\left[2(q+x)+\frac{k[q+x(1-\alpha)]}{\rho}\right]\right)\end{bmatrix}
$$

$$
=0
$$

From the latter equation we obtain

$$
\mu=-\frac{\left(y-\underline{y}\right)\left[1+\frac{2(1-\alpha)k}{\rho}+\frac{(1-\alpha)k^2}{\rho^2}\right]}{\alpha k+q+x+(1-\alpha)\frac{k}{\rho}\left[2(q+x)+\frac{k[q+x(1-\alpha)]}{\rho}\right]} \qquad (A29)
$$

which is a constant. The costate variable is negative as increasing the unemployment rate (state variable) has a negative impact on the objective function. Should the absolute of μ be higher (smaller), then the Hamiltonian is decreasing (increasing) with a higher benefit loss rate ρ. Solving the former equation $(A27)$ for the costate variable yields

$$
\mu=-\frac{h}{m(\theta)+\theta m'(\theta)}=-\frac{h}{m(\theta)(1+\eta)} \qquad (A30)
$$

As $m(\theta)$ is decreasing in θ, the absolute of the costate variable is increasing in labor market tightness given a constant elasticity η. Should an interior optimum (θ,ρ) exist, it has to satisfy $(A33)$ and $(A34)$, thus

$$
\frac{h}{m(\theta)}=\frac{\left(y-\underline{y}\right)\left[1+\frac{2(1-\alpha)k}{\rho}+\frac{(1-\alpha)k^2}{\rho^2}\right](1+\eta)}{\alpha k+q+x+(1-\alpha)\left[\frac{2(q+x)k}{\rho}+\frac{k^2[q+x(1-\alpha)]}{\rho^2}\right]} \qquad (A31)
$$

with $\eta = \frac{\theta m'(\theta)}{m(\theta)}$ where $\eta \epsilon(-1,0)$.

The canonical equation is

$$-\frac{\partial H}{\partial u} = \dot{\mu} - r\mu \qquad (A32)$$

$$= \mu\left[q\frac{k+\rho}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]} + \rho\frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]} + \theta m(\theta)\right]$$

$$+h\theta - z + \left[y\frac{k+\rho}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})} + \underline{y}\frac{x(1+(1-\alpha)\frac{k}{\rho})}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}\right]$$

Given $\dot{\mu} = 0$, solving this for the costate variable yields

$$\mu = -\frac{h\theta + y\frac{k+\rho}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})} + \underline{y}\frac{x(1+(1-\alpha)\frac{k}{\rho})}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})} - z}{r + q\frac{k+\rho}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]} + \rho\frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]} + \theta m(\theta)} \qquad (A33)$$

Finally, the transversality condition is

$$\lim_{t\to\infty}\{\mu(t)u(t)e^{-r}\ \} = 0, \qquad (A34)$$

which would be satisfied for a constant value of the costate variable.

Equating the values for the costate variable derived from the canonical equation and the first-order condition with respect to the control labor market tightness yields

$$\frac{h}{m(\theta)}=\frac{h\theta+\left[y\frac{k+\rho}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}+\underline{y}\frac{x(1+(1-\alpha)\frac{k}{\rho})}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}-z\right]}{r+q\frac{k+\rho}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}+\rho\frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}+\theta m(\theta)}(1+\eta) \qquad (A35)$$

which is identical to the standard formula with a value of unemployment equal to zero in the absence of short-time work.

By collecting terms, this can be reformulated as

$$h\left[r+q\frac{k+\rho}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}+\rho\frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}+\theta m(\theta)\right]$$
$$=h\theta m(\theta)(1+\eta)$$
$$+\left[y\frac{k+\rho}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}+\underline{y}\frac{x(1+(1-\alpha)\frac{k}{\rho})}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}-z\right]m(\theta)(1+\eta)$$

which is equivalent to

$$\frac{h}{m(\theta)}=\frac{\left[y\frac{k+\rho}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}+\underline{y}\frac{x(1+(1-\alpha)\frac{k}{\rho})}{k+\rho+x(1+(1-\alpha)\frac{k}{\rho})}-z\right](1+\eta)}{r+q\frac{k+\rho}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}+\rho\frac{x\left[1+(1-\alpha)\frac{k}{\rho}\right]}{k+\rho+x\left[1+(1-\alpha)\frac{k}{\rho}\right]}-\eta\theta m(\theta)}$$

being the counterpart of characterizing efficient tightness in the Mortensen-Pissarides model.

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