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Abstract

I study the optimal taxation of robots and labor income. In the model, robots substitute for routine labor and complement non-routine labor. I show that while it is optimal to distort robot adoption, robots may be either taxed or subsidized. The robot tax exploits general-equilibrium effects to compress the wage distribution. Wage compression reduces income-tax distortions of labor supply, thereby raising welfare. In the calibrated model, the optimal robot tax for the US is positive and generates small welfare gains. As the price of robots falls, inequality rises but the robot tax and its welfare impact become negligible.

JEL-Codes: D310, D330, D500, H210, H230, H240, H250, J240, J310, O330.

Keywords: optimal taxation, input taxation, production efficiency, technological change, robots, inequality, general equilibrium, multidimensional heterogeneity.

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1 Introduction

Public concern about the distributional consequences of automation is growing (see e.g. Ford, 2015; Brynjolfsson and McAfee, 2014; Frey et al., 2017). It is feared that the "rise of the robots" is going to disrupt the labor market and will lead to extreme income inequality. These concerns have raised the question how redistributive policy should respond to automation. Some policy makers and opinion leaders have suggested a "tax on robots".[1,2] Is this a good idea? This paper tries to answer that question. I find that while it is generally optimal to distort the use of robots, robots may be either taxed or subsidized. The robot tax exploits general equilibrium effects to compress the wage distribution. Wage compression makes it less distortionary to tax income, which allows for more redistribution overall and raises welfare. If robots primarily substitute for routine labor at medium incomes, a tax on robots decreases wage inequality at the top of the wage distribution, but raises inequality at the bottom. The sign of the robot tax is then theoretically ambiguous. Quantitatively, in the US, the optimal robot tax equals 1.8% if occupations are fixed, but decreases to 0.86% with occupational choice. The welfare impact of introducing a robot tax is small. As the price of robots falls, inequality increases but the robot tax and its welfare impact become negligible.

To reach these conclusions, I first build intuition by studying a stylized model based on Stiglitz (1982). The full model then embeds automation similar to Acemoglu and Restrepo (2018a) and labor market polarization as in Autor and Dorn (2013) in an optimal taxation framework based on Rothschild and Scheuer (2013, 2014). For the quantitative analysis, I calibrate the full model to the US economy, using data from the Current Population Survey (CPS), as well as evidence on the impact of robots on the labor market from Acemoglu and Restrepo (2017).

The stylized model extends Stiglitz (1982) to three occupations: manual non-routine, routine, and cognitive non-routine. It captures that cognitive non-routine workers earn on average higher wages than routine workers, who in turn earn higher wages than manual non-routine workers (see e.g. Acemoglu and Autor, 2011). Workers are fixed-assigned to one of the three occupations. Moreover, while in Stiglitz (1982) output is produced by labor only, I introduce robots as additional production factor. Crucially, robots are more

[1]See e.g. this quote from a Draft report by the Committee on Legal Affairs of the European Parliament.

> *"Bearing in mind the effects that the development and deployment of robotics and AI might have on employment and, consequently, on the viability of the social security systems of the Member States, consideration should be given to the possible need to introduce corporate reporting requirements on the extent and proportion of the contribution of robotics and AI to the economic results of a company for the purpose of taxation and social security contributions;"*

[2]Bill Gates has advocated for a tax on robots. See https://qz.com/911968/bill-gates-the-robot-that-takes-your-job-should-pay-taxes/

complementary to non-routine labor than to routine labor. An increase in the amount of robots therefore lowers demand and wages for routine workers relative to non-routine workers. The robot tax exploits this differential impact of robots on wages. I derive a formula for the optimal robot tax which features elasticities of relative wage rates with respect to robots and *incentive effects*. Under the realistic assumption that income taxes may not be conditioned on occupation, the robot tax is in general not zero, violating production efficiency (see Diamond and Mirrlees, 1971). However, it is theoretically ambiguous whether robots should be taxed or subsidized. This is because the robot tax has counteracting effects on wages at the top and the bottom of the wage distribution.

Ceteris paribus, the robot tax is larger, the more robots raise wage inequality between cognitive non-routine workers and routine workers; it is lower, the more robots compress the wage gap between routine workers and manual non-routine workers. The incentive effects capture how much a change in relative wages affects income-tax distortions of labor supply. In addition, they capture how much the government values redistribution between workers with different incomes. Ceteris paribus: the more a reduction of inequality at the top of the wage distribution lowers income-tax distortions of labor supply, the larger is the optimal tax on robots; moreover, the robot tax is larger, the more the government cares about reducing income-tax distortions at the top. In contrast, the tax on robots is smaller, the more an expansion of inequality at the bottom of the wage distribution worsens income-tax distortions of labor supply, and the more the government cares about these. Optimal marginal income tax rates are adjusted if robots can be taxed. If manual and cognitive non-routine labor are sufficiently complementary, the presence of a robot tax leads to higher marginal tax rates at low incomes, and to lower marginal tax rates at high incomes. The resulting labor-supply responses contribute to wage compression.

What the sign and size of the optimal robot tax should be is ultimately a quantitative question. The stylized model misses two features which are particularly important for a quantitative analysis: continuous wage distributions which overlap occupations, and the possibility of switching occupations. With continuous wages, the impact of robots on inequality can be captured more realistically, while occupational choice is a relevant margin of adjustment to automation (Dauth et al., 2018). The full model incorporates both features, building upon Rothschild and Scheuer (2013). Individuals now differ in three-dimensional ability, based on which they choose labor supply and their occupation. The expression for the optimal robot tax has a similar structure as in the stylized model: elasticities of relative wage rates with respect to robots are again central. In addition, a tax on robots now leads to a reallocation of labor supply within occupations, which affects how much labor supply is distorted by income taxation. In the robot tax formula, this is captured by *effort-reallocation effects*. Also, since a tax on robots drives up wages in routine occupations relative to non-routine occupations, some non-routine workers find it beneficial to switch to routine work. The robot-tax formula captures this by including *occupational-shift effects.* By switching occupations, individuals offset part of the wage compression which can be achieved by taxing robots. As a result, the robot tax becomes a less effective policy instrument.

To assess the optimal policy quantitatively, I calibrate the full model to the US economy. To do so, I use data on wages and occupational choice from the CPS. Moreover, I calibrate the impact of robots on wages based on Acemoglu and Restrepo (2017). I compute optimal policy for two scenarios: one in which occupations are fixed, and one in which occupational switching is possible. Without occupational switching, the optimal robot tax is 1.8%. The welfare gain of introducing a robot tax expressed in dollars per person per year is 21.14$. With occupational switching, the robot tax equals 0.86%, and its welfare impact is reduced to 9.22$. In both scenarios, optimal marginal income tax rates are adjusted if robots can be taxed. With the robot tax, marginal income tax rates are higher at low and medium incomes, and lower at high incomes, thereby exploiting labor-supply responses for wage compression.

Finally, I study the impact of a drop in the price of robots for the two scenarios. In both scenarios, wage inequality ultimately increases as the average wage of routine workers falls, while non-routine workers experience wage gains. Moreover, if possible, individuals switch from routine into non-routine occupations. Although wage inequality increases, the robot tax eventually approaches a value close to zero. Without occupational switching, the welfare gain from taxing robots increases at first as the price of robots falls, but it never exceeds 25$ per person per year. Moreover, in both scenarios the welfare gain of taxing robots eventually becomes negligible. In light of the small welfare gains, I conclude that this paper does not provide a strong case for a tax on robots.

The remainder of the paper is structured as follows: Section 2 discusses the related literature. Section 3 sets up a simplified model with discrete worker types and without occupational choice to build intuition. Section 4 introduces continuous types and occupational choice and characterizes the optimal robot tax in the full model. Section 5 studies the quantitative implications of the model. Section 6 concludes. Proofs and additional material are contained in an Appendix.

2 Related literature

This paper builds upon the framework by Rothschild and Scheuer (2013, 2014) who study optimal non-linear income taxation with multi-dimensional heterogeneity and sectoral choice, thereby extending and generalizing Stiglitz (1982). The modeling of the economy in the quantitative part combines a production technology similar to Acemoglu and Restrepo (2018a) and Autor and Dorn (2013) with a Roy (1951) model of occupational choice. In addition, this paper is related to different strands in the literature which are discussed below.

Optimal taxation and technological change. A growing number of papers investigates the question how taxes should respond to technological change. Most closely related is Guerreiro et al. (2017), who in parallel and independent work also ask whether robots should be taxed. Their model features two discrete types of workers – routine and non-routine – who are assigned tasks. In addition, some tasks are performed by robots. In a model like Stiglitz (1982) in which labor income is taxed non-linearly, they show that it is optimal to tax robots (provided that some tasks are still performed by routine labor). The rationale for taxing robots is the same as in this paper: compressing the wage distribution to reduce income-tax distortions of labor supply. Arguing that such a non-linear tax system can be complex and difficult to implement, Guerreiro et al. (2017) then focus on parametric tax schedules. Under the parametric tax systems, a tax on robots is also optimal. Finally, Guerreiro et al. (2017) introduce occupational choice by assuming that individuals have different preferences for non-routine work.

This paper differs from Guerreiro et al. (2017) in important ways. First, by writing technology as a function of the aggregate amount of robots, this paper arrives at optimal tax expressions which are easily interpretable. For example, they feature the elasticities of relative wage rates with respect to robots. Second, by considering three groups of occupations, I allow for wage polarization. The empirical literature on the labor market effects of technological change has highlighted that routine workers are found in the middle of the income distribution (see e.g. Acemoglu and Autor, 2011). The sign of the robot tax is then theoretically ambiguous. Third, my model features heterogeneity within occupations and thus generates a realistic wage distribution, while the model by Guerreiro et al. (2017) only features two levels of wages in the economy. Fourth, I model occupational choice based on individuals' earnings abilities in different occupations, relating to the literature on employment polarization (see e.g. Acemoglu and Autor, 2011; Autor and Dorn, 2013; Goos et al., 2014; Cortes, 2016).

What the optimal level of the robot tax should be is ultimately a quantitative question. Here, my analysis goes beyond that of Guerreiro et al. (2017) whose numerical examples are mostly illustrative.[3] I calibrate my model based on data for the US economy and match the distribution of incomes and employment. Moreover, I use the empirical evidence on the labor market effects of robots by Acemoglu and Restrepo (2017). Based on the calibrated model, I find an optimal robot tax which is substantially lower than the maximum levels found by Guerreiro et al. (2017).[4] Finally, while Guerreiro et al. (2017) compare welfare across different tax systems, they do not isolate the welfare impact of introducing a robot tax – though this is arguably a relevant number to answer the ques-

[3]For example, in their simulations, routine and non-routine workers initially earn the same wage and make up equal shares of the population.

[4]They find an optimal robot tax of up to 10% in the model with non-linear taxes. In the version in which they augment the parametric tax function by Heathcote et al. (2017) with a lump-sup rebate, the robot tax reaches up to 30%.

tion whether robots should be taxed. Based on the small welfare gains of taxing robots, this paper does not provide a strong case for taxing robots.

Related, Costinot and Werning (2018) ask how tax policy should respond to inequality driven by technology or trade if the set of policy instruments is restricted, such that production efficiency as in Diamond and Mirrlees (1971) is not optimal (see below for more on the relation to production efficiency). As one application, they study the optimal tax on robots, assuming that labor income can be taxed non-linearly, but may not depend on a worker's type. As in this paper, if wages of different workers are differentially affected by robots, taxing robots is optimal to reduce inequality in order to dampen income-tax distortions of labor supply. For a general production technology, Costinot and Werning (2018) derive a sufficient-statistics formula for the optimal robot tax which depends only on elasticities, factor shares, and marginal income tax rates. One important ingredient is the elasticity of wages with respect to robots, which is also central for optimal robot taxation in this paper. Using their formula, Costinot and Werning (2018) find optimal robot taxes of very similar magnitude as this paper.

The sufficient-statistics approach makes the paper by Costinot and Werning (2018) complementary to this paper. It allows to make statements about the optimal robot tax without having to assume a lot of structure on the economy. However, the sufficient-statistics formula is only valid if the economy is already at a policy optimum. In contrast, the more structural approach in this paper does not impose that restriction. Moreover, it allows to analyze counterfactuals, such as the impact of a drop in the robot price on the optimal robot tax. In fact, Costinot and Werning (2018) also assume more structure when they analyze the impact of a drop in the price of robots on the optimal robot tax in a stylized model. They show that despite robots being used more and inequality growing, the optimal robot tax falls. In my fully calibrated quantitative analysis, I find as well that the optimal robot tax may drop as robots get cheaper.

Related, Tsyvinski and Werquin (2018) derive how a given tax system needs to be adjusted to compensate individuals for the distributional effects of, for example, trade or automation. In an application, they use the results from Acemoglu and Restrepo (2017) to investigate how individuals should be compensated for changes in income generated by the increased use of industrial robots. In contrast to this paper, Tsyvinski and Werquin (2018) do not study optimal taxation. Other papers (Gasteiger and Prettner, 2017; Hemous and Olsen, 2018) study the impact of taxing robots, taking a positive – rather than a normative – perspective.

The implications of technological change for tax policy are also analyzed by Ales et al. (2015) who study a model in which individuals are assigned to tasks based on comparative advantage. However, Ales et al. (2015) do not model automation nor do they study robot taxation.

Production efficiency. A tax on robots violates production efficiency. This paper is thus related to the *Production Efficiency Theorem* (Diamond and Mirrlees, 1971) which states that production decisions should not be distorted, provided that the government can tax all production factors – inputs and outputs – linearly and at different rates. In addition, the *Atkinson-Stiglitz Theorem* (Atkinson and Stiglitz, 1976) states that if utility is weakly separable between consumption and leisure and the government can use a non-linear income tax, commodity taxes should not be used for redistribution. Combining the two theorems implies that neither consumption nor production should be distorted for redistributive reasons, provided the government can tax labor income non-linearly and has access to sufficient instruments to tax inputs and outputs. This implication has subsequently been put into perspective by Naito (1999); Saez (2004); Naito (2004); Jacobs (2015); Shourideh and Hosseini (2018); Costinot and Werning (2018) who all study settings which feature fewer tax instruments than required for achieving production efficiency. Similarly, in this paper, the set of tax instruments is too restricted for production efficiency to be optimal. In particular, income taxes may not be conditioned on occupation.[5] In a related setting, Scheuer (2014) studies optimal taxation of labor income and entrepreneurial profits. He shows that when labor income and profits are subject to the same non-linear tax schedule, it is optimal to distort production efficiency in order to compress wages differentially. Production efficiency is restored if labor income and profits can be subject to different tax schedules.[6] This paper focuses on the realistic case in which income taxes are not conditioned on occupation.

Robots and the labor market. A recent empirical literature studies the impact of robots on the labor market.[7] Using data on industrial robots from the International Federation of Robotics (IFR, 2014), Acemoglu and Restrepo (2017) exploit variation in exposure to robots across US commuting zones to identify the causal effect of industrial robots on employment and wages between 1990 and 2007. I use their results to inform the quantitative analysis. Other articles which study the impact of robots on labor markets are Graetz and Michaels (2018) for a panel of 17 countries and Dauth et al. (2018) for Germany. A related literature studies labor market polarization due to technological change (see e.g. Goos et al., 2014; Cortes et al., 2017). Autor and Dorn (2013) investigate the impact of ICT technology on wages and employment in routine and non-routine occupations. In their model, ICT technology substitutes for routine labor and complements

[5] Saez (2004) refers to this as a violation of the labor-types-observability assumption.

[6] Gomes et al. (2018) set up a model in which workers with continuously distributed ability choose both, intensive margin labor supply and occupation, as they do in this paper. They then study optimal occupation-specific non-linear income taxation and show that occupational choice is optimally distorted. They refer to this as a distortion of production efficiency. In Scheuer (2014), occupational choice is distorted in the presence of occupation-specific taxes – however, production is efficient.

[7] Also related are papers which study the impact of robots on the economy theoretically (see Berg et al., 2018; Acemoglu and Restrepo, 2018b).

non-routine labor. To capture the differential impact of robots on routine and non-routine occupations, I model production in a similar way. Moreover, to capture automation, I use elements from Acemoglu and Restrepo (2018a) (see also Guerreiro et al., 2018).

Taxation of capital. Robots are a specific type of capital, which relates this paper to the literature on capital taxation. However, most arguments for taxing capital do not depend on the differential impact of capital on wages. Such arguments are therefore orthogonal to the reason for which robots are taxed in this paper. An exception is Slavík and Yazici (2014) who give a similar argument for taxing equipment capital as this paper does for taxing robots. Due to capital-skill complementarity (Krusell et al., 2000), a tax on equipment capital depresses the skill-premium, thereby reducing income-tax distortions of labor supply. In contrast, structures capital, which is equally complementary to low and high-skilled labor, should not be taxed. In their quantitative analysis for the US economy, they find an optimal tax on equipment capital of almost 40%. Moreover, they find large welfare gains of moving from non-differentiated to differentiated capital taxation. One reason for the different welfare implications is that I study the effect of introducing a robot tax into a system which taxes labor income optimally, whereas Slavík and Yazici (2014) start out from the current US tax system in which this is not the case. Moreover, I allow for robots to polarize the wage distribution, which leads to counteracting effects of the robot tax on the top and the bottom of the wage distribution.[8]

3 Model with discrete types and no occupational choice

To develop intuition, I first discuss a simple model which features discrete types and abstracts from occupational choice. The model extends Stiglitz (1982) to three sectors (or occupations) and features endogenous wages. The model illustrates the key arguments for taxing robots. However, it is too stylized for a quantitative analysis, and by abstracting from occupational choice leaves out an important adjustment margin. In Section 4, I discuss a richer model with continuous types and occupational choice which is amenable to a realistic calibration, and thus suitable to analyze the optimal taxation of robots quantitatively.

[8]In Slavík and Yazici (2014) the returns to capital are taxed, whereas in my model the tax is levied on the stock of robots, which is another reason for the smaller magnitude of taxes in my paper.

3.1 Setup

3.1.1 Workers, occupations and preferences

There are three *types* of workers $i \in \mathcal{I} \equiv \{M, R, C\}$ with corresponding mass f_i. A worker's type corresponds to his *occupation*, where M refers to an occupation which requires manual non-routine labor, R refers to an occupation requiring routine labor, and C denotes a cognitive non-routine occupation. The distinction between routine and non-routine occupations is motivated by the empirical literature which has established that in recent decades technology has substituted for routine work, and has complemented non-routine work (see e.g. Autor et al., 2003). Moreover, the literature on labor market polarization suggests to distinguish between low-skilled and high-skilled non-routine occupations (see e.g. Cortes, 2016; Cortes et al., 2017). Workers derive utility from consumption c and disutility from labor supply ℓ, according to the quasi-linear utility function

$$U(c, \ell) = c - \frac{\ell^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}, \tag{1}$$

where ε is the labor-supply elasticity.

3.1.2 Technology

Denote by $\mathbf{L} \equiv (L_M, L_R, L_C)$ the vector of aggregate labor supplies with $L_i = f_i \ell_i$ for all $i \in \mathcal{I}$. Let B denote *robots*. The final good is produced by a representative firm according to a constant-returns-to-scale production function $Y(\mathbf{L}, B)$. The firm maximizes profits by choosing the amount of total labor of each type $i \in \mathcal{I}$ and the number of robots, taking wages w_i and the price of robots p as given. Normalizing the price of the final good to one, the firm's profit maximization problem is

$$\max_{\mathbf{L}, B} Y(\mathbf{L}, B) - \sum_{i \in \mathcal{I}} w_i L_i - pB. \tag{2}$$

Denote the marginal products of total effective labor as

$$Y_i(\mathbf{L}, B) \equiv \frac{\partial Y(\mathbf{L}, B)}{\partial L_i} \; \forall i \in \mathcal{I}, \tag{3}$$

and the marginal product with respect to robots as

$$Y_B(\mathbf{L}, B) \equiv \frac{\partial Y(\mathbf{L}, B)}{\partial B}. \tag{4}$$

In equilibrium, we then have $w_i(\mathbf{L}, B) = Y_i(\mathbf{L}, B) \;\; \forall i \in \mathcal{I}$ and $p = Y_B(\mathbf{L}, B)$. Unless stated otherwise, I assume throughout that robots are better substitutes for routine work than for non-routine work. More specifically, I make the following assumption

Assumption 1. *A marginal increase in the amount of robots raises the marginal product of non-routine labor relative to routine labor.*

- $\frac{\partial}{\partial B}\left(\frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)}\right) > 0$,
- $\frac{\partial}{\partial B}\left(\frac{Y_C(\mathbf{L},B)}{Y_R(\mathbf{L},B)}\right) > 0$.

Due to constant returns to scale, equilibrium profits are zero. Robots are produced linearly with the final good, according to

$$B\left(x\right) = \frac{1}{q}x, \tag{5}$$

where I denote by x the amount of the final good allocated to the production of robots, and where $1/q$ is the marginal rate of transformation between robots and the consumption good. In the absence of taxes, we then have $p = q$ in equilibrium. Later, when taxes drive a wedge between p and q, I refer to q as the *producer price* of robots and to p as the *user price* of robots.

3.1.3 Government and tax instruments

There is a benevolent government whose objective it is to maximize social welfare

$$\mathcal{W} \equiv f_M\psi_M V_M + f_R\psi_R V_R + f_C\psi_C V_C, \tag{6}$$

where ψ_i is the Pareto weight attached to workers of type i, where the weights satisfy $\sum f_i\psi_i = 1$, and $V_i \equiv U\left(c_i, \ell_i\right)$ are indirect utilities. While the government is aware of the structure of the economy, it cannot observe an individual's occupation. This assumption is satisfied by real-world tax systems which also do not condition taxes on occupation, for example, because enforcement may be difficult. However, the government can observe individual income and consumption, as well as the value of robots purchased by the final goods producer. Accordingly, I assume that the government has access to two tax instruments: a non-linear income tax, and a tax on the value of robots.

Denote by $y_i \equiv w_i\ell_i$ gross labor income earned by an individual of type i. The government levies a non-linear income tax $T(y)$ on gross labor income y. Taking the wage and income tax schedule as given, a worker of type i then maximizes utility (1) by choosing consumption and labor supply subject to a budget constraint:

$$\max_{c_i,\ell_i} U\left(c_i, \ell_i\right) \ s.t. \ c_i \leq w_i\ell_i - T\left(w_i\ell_i\right). \tag{7}$$

The value of robots purchased by the final goods producer is given by qB, on which the government may levy a proportional tax τ, to which I refer as *robot tax*.[9] The user

[9] I focus on a linear tax on robots, since with a non-linear tax and constant returns to scale there

price of robots is then $p = (1+\tau)\,q$. While throughout this paper I refer to τ as a tax on robots, I highlight that τ may be negative, and may thus be a subsidy. The government faces the budget constraint

$$f_M T(y_M) + f_R T(y_R) + f_C T(y_C) + \tau q B = 0, \tag{8}$$

stating that by raising tax revenue with the income tax and the robot tax, it must break even. Introducing an exogenous revenue requirement does not change the analysis.

3.2 Optimal policy

The government chooses tax instruments $T(\cdot)$ and τ such as to maximize social welfare (6) subject to budget constraint (8). To characterize optimal taxes, I follow the conventional approach of first solving for the optimal allocation from a mechanism design problem. Afterward, prices and optimal taxes that decentralize the allocation are determined.

The optimal allocation can be implemented using a linear tax on the value of robots (in conjunction with the optimal non-linear income tax). To see this, note that since firms maximize profits, they equate the marginal return to robots with the price of robots. Under laissez-faire, we thus have

$$Y_B(\mathbf{L}, B) = q. \tag{9}$$

However, given $\mathbf{L}$, the planner might want to distort the choice of robots such that (9) is no longer satisfied. By setting a linear tax τ on the value of robots, profit maximization of the firm leads to

$$Y_B(\mathbf{L}, B) = (1+\tau)\,q. \tag{10}$$

Since $Y_B(\mathbf{L}, B)$ is strictly monotone in B, for each B there exists a unique robot tax τ such that (10) holds. For given optimal $\mathbf{L}$, the optimal robot tax τ thus uniquely implements the optimal B.

In a direct mechanism, workers announce their type i, and then get assigned consumption c_i and labor supply ℓ_i. Here, I consider the equivalent problem in which instead of consumption, the planner allocates indirect utilities V_i and define $c(V_i, \ell_i)$ as the inverse of $U(c_i, \ell_i)$ with respect to its first argument.

The allocation must induce workers to truthfully report their type and thus needs to be incentive compatible. Since there is no heterogeneity of types within occupations, the only way in which workers can imitate one another is by mimicking incomes of workers in other occupations. I assume that the primitives of the model are such that $w_C > w_R > w_M$ is satisfied. Moreover, I limit attention to those cases in which only the downward adjacent

would be incentives for firms to break up into parts until each part faces the same minimum tax burden. With linear taxes, such incentives are absent.

incentive constraints may be binding, while all other incentive constraints are slack. This case is the relevant one for gaining intuition which carries over to the continuous-type model.[10] To induce a cognitive worker to truthfully report his type, the following must hold

$$V_C \geq U\left(c\left(V_R, \ell_R\right), \ell_R \frac{w_R\left(\mathbf{L}, B\right)}{w_C\left(\mathbf{L}, B\right)}\right), \tag{11}$$

where $\ell_R \frac{w_R}{w_C}$ is the amount of labor which a cognitive worker needs to supply to mimic the income of a routine worker. Similarly, a routine worker has to be prevented from mimicking the income of manual workers, and thus

$$V_R \geq U\left(c\left(V_M, \ell_M\right), \ell_M \frac{w_M\left(\mathbf{L}, B\right)}{w_R\left(\mathbf{L}, B\right)}\right). \tag{12}$$

3.2.1 Separation into inner and outer problem

I follow Rothschild and Scheuer (2013, 2014) and separate the mechanism design problem into an *inner problem* and an *outer problem*.[11] In the inner problem, the planner takes the tuple of inputs $(\mathbf{L}, B)$ as given and maximizes welfare $\mathcal{W}(\mathbf{L}, B)$ over $\{V_i, \ell_i\}_{i\in\mathcal{I}}$ subject to constraints (specified below). In the *outer problem*, the planner chooses the vector $\mathbf{L} = (L_M, L_R, L_C)$ and robots B such that $\mathcal{W}(\mathbf{L}, B)$ is maximized. The mechanism design problem can thus be written as

$$\max_{\mathbf{L},B} \mathcal{W}\left(\mathbf{L}, B\right) \equiv \max_{\{V_i,\ell_i\}_{i\in\mathcal{I}}} \quad f_M\psi_M V_M + f_R\psi_R V_R + f_C\psi_C V_C \tag{13}$$

subject to the incentive constraints (11) and (12), the *consistency conditions*

$$f_i\ell_i - L_i = 0 \;\forall i \in \mathcal{I}, \tag{14}$$

and the *resource constraint*

$$\sum_{i\in\mathcal{I}} f_i\ell_i Y_i\left(\mathbf{L}, B\right) + Y_B\left(\mathbf{L}, B\right) B - \sum_{i\in\mathcal{I}} f_i c_i - qB = 0. \tag{15}$$

The consistency conditions (14) restate the definition of aggregate labor supplies. Since the inner and outer problem separate optimization over individual labor supplies and aggregate labor supplies, including the consistency conditions ensures that the labor market clears. The first two terms in the resource constraint (15) sum to total output $Y(\mathbf{L}, B)$. The final term captures that $x = qB$ units of the final good have to be used to

[10] See also the discussion in Stiglitz (1982) regarding downward-binding incentive constraints.

[11] The approach has the advantage of generating expressions for the optimal income tax similar to models without occupations. While I could characterize optimal policy in this simple framework without this separation, the approach will turn out to be useful in the full model. Already applying it here leads to expressions which can be easily compared to those in the full model, as the structure of the problem remains the same.

produce B robots.

3.2.2 Optimal robot tax

I first characterize the optimal robot tax by using that in the outer problem at the optimum $\partial \mathcal{W}(\mathbf{L}, B) / \partial B = 0$, hence a change in robots may not lead to a change in welfare.

Proposition 1. *The optimal tax on robots is characterized by*

$$\tau q B = \varepsilon_{w_C/w_R,B} \; I_{CR} - \varepsilon_{w_M/w_R,B} \; I_{RM} \tag{16}$$

with elasticities of relative wages with respect to the number of robots defined as

$$\varepsilon_{w_C/w_R,B} \equiv \frac{\partial (w_C/w_R)}{\partial B} \frac{B}{w_C/w_R} > 0, \tag{17}$$

$$\varepsilon_{w_M/w_R,B} \equiv \frac{\partial (w_M/w_R)}{\partial B} \frac{B}{w_M/w_R} > 0, \tag{18}$$

and incentive effects

$$I_{CR} \equiv f_C (1 - \psi_C) \left(\ell_R \frac{w_R}{w_C} \right)^{1+\frac{1}{\varepsilon}}, \tag{19}$$

$$I_{RM} \equiv f_M (\psi_M - 1) \left(\ell_M \frac{w_M}{w_R} \right)^{1+\frac{1}{\varepsilon}}. \tag{20}$$

Proof. See Appendix A.1. □

The left-hand side of (16), $\tau q B$, is the tax revenue raised with the robot tax. Ceteris paribus, the robot tax is thus larger in magnitude, the smaller the cost of producing robots, q, and the lower the number of robots, B. At the optimum, robot-tax revenue is equal to the difference in incentive effects I_{CR} and I_{MR}, weighted by the respective elasticity terms, $\varepsilon_{w_C/w_R,B}$ and $\varepsilon_{w_M/w_R,B}$. The elasticity terms capture the percentage increase in wages of non-routine workers relative to the wage of routine workers due to a one-percent increase in the number of robots. By Assumption 1, an increase in robots raises the equilibrium-wage of non-routine workers relative to routine workers. As a consequence, $\varepsilon_{w_C/w_R,B} > 0$ and $\varepsilon_{w_M/w_R,B} > 0$. The incentive effects I_{CR} and I_{RM} capture how incentive constraints (11) and (12) are affected by a marginal increase in robots, and how this, in turn, affects welfare.

I first focus on I_{CR}. Raising the number of robots increases w_C/w_R, and since $w_C > w_R$, wage inequality at the top of the wage distribution rises.[12] Regular welfare weights

[12] There are only three levels of wages, w_M, w_R, w_C. With inequality at the top of the distribution, I refer to the gap between w_C and w_R. Inequality at the bottom of the distribution refers to the gap between w_R and w_M.

decrease with income, leading to $\psi_C < 1$. The government thus attaches a lower-than-average weight to cognitive non-routine workers. In this case, it is desirable to redistribute income from cognitive non-routine workers to workers who earn less. The increase in wage inequality at the top then tightens the incentive constraint (11): cognitive non-routine workers now need to put in less labor than before to imitate the income of a routine worker. This tightening of (11) corresponds to increased income-tax distortions of labor supply, which limits redistribution and lowers welfare.

The robot tax has the *opposite* effect to an increase in B. By increasing the user price of robots p, the equilibrium number of robots falls. As a consequence, w_C/w_R drops, which corresponds to a reduction in wage inequality at the top of the wage distribution; and to a relaxation of incentive constraint (11). Relaxing (11) is welfare improving: it becomes less distortionary to use the income tax to redistribute income from cognitive non-routine workers to other workers. Ceteris paribus, a larger incentive effect I_{CR} calls for a higher tax on robots. If labor supply is more elastic (higher ε) income taxation is more distortionary. As a result, I_{CR} is larger, and so is the optimal robot tax. The weighting of incentive effect I_{CR} by elasticity $\varepsilon_{w_C/w_R,B}$ captures how effective taxing robots is in reducing the wage gap w_C/w_R.

I now turn to the second term on the right-hand side of (16). The incentive effect I_{RM} captures how reducing the wage gap between w_R and w_M affects welfare via the incentive constraint (12). With regular welfare weights we have $\psi_M > 1$, hence the government values redistributing income to manual non-routine workers. In this case, lowering the gap between w_R and w_M relaxes (12), and makes it less distortionary to redistribute income with the income tax, captured by $I_{RM} > 0$. The weighting with elasticity $\varepsilon_{w_M/w_R,B} > 0$ captures how effective the robot tax is in changing the wage gap between w_R and w_M. The minus sign is crucial: a tax on robots increases the wage of routine workers relative to manual non-routine workers. As inequality at the bottom of the wage distribution increases, the incentive constraint (12) tightens. Redistributing to manual non-routine workers with the income tax becomes more distortionary, thereby lowering welfare. This effect, ceteris paribus, calls for a lower tax on robots.

To summarize: it is welfare-maximizing to distort the price of robots to make income redistribution less distortionary – and to thereby violate production efficiency (Diamond and Mirrlees, 1971). A tax on robots decreases wage inequality between cognitive and routine workers, thereby reducing income-tax distortions of labor supply and increasing welfare. At the same time, a tax on robots raises the wage gap between routine and manual workers, which worsens income-tax distortions of labor supply and lowers welfare. Due to these opposing forces, the sign of the robot tax is ambiguous. If the first effect dominates, robots should be taxed, whereas if the second effect is more important, robots should be subsidized.

Ceteris paribus, several factors make it more likely for the optimal robot tax to be positive: if robots increase wage inequality at the top of the distribution more than they reduce inequality at the bottom; if the share of cognitive non-routine workers is large, whereas the share of manual non-routine workers is small; if the wage gap between cognitive and routine workers is small, whereas the wage gap between routine and manual workers is large; finally, if the government attaches relatively little weight ψ_C and ψ_M to cognitive workers and manual workers, respectively. The final point can be restated as the government attaching relatively more weight ψ_R to routine workers. This is intuitive: as routine workers gain relative to non-routine workers when robots are taxed, putting relatively more weight on them calls for a larger tax on robots.

Only in special cases is the tax on robots zero. First, it is zero if the government can condition income taxes on occupation, which restores production efficiency (Diamond and Mirrlees, 1971). Moreover, since in the simple model worker types and occupations coincide, occupation-specific income taxes correspond to individualized lump-sum taxes, leading to the first-best outcome. Similarly, the optimal robot tax is zero if income taxation does not distort labor supply, corresponding to $\varepsilon \to 0$. The robot tax is also zero if the effect of reducing labor-supply distortions at the top of the wage distribution exactly cancels against the effect of raising labor-supply distortions at the bottom. Finally, if in contrast to what I have assumed so far, robots are equally complementary to labor in all occupations, the optimal robot tax is zero, since in this case $\varepsilon_{w_C/w_R} = \varepsilon_{w_R/w_M} = 0$.[13]

The ambiguous sign of the robot tax is in contrast to Guerreiro et al. (2017) who argue that the robot tax should be positive. This is due to Guerreiro et al. (2017) considering only two groups of workers: routine and non-routine. In their model, taxing robots unambiguously relaxes the single binding incentive constraint. My result highlights that aggregating workers into just two groups can be misleading as it masks heterogeneous effects of robots on wages along the income distribution. The empirical literature (see e.g. Autor and Dorn, 2013) finds that routine workers are not found at the very bottom of the income distribution. Instead, those who earn least often perform manual non-routine work which is hard to automate. In this case, taxing robots will widen inequality at the bottom of the wage distribution, thereby worsening income-tax distortions of labor supply. If these effects are taken into account, the sign of the robot tax becomes ambiguous.

3.2.3 Optimal income taxes

I now use the inner problem, taking $\mathbf{L}$ and B as given, to characterize optimal marginal income taxes.

Proposition 2. *Let μ denote the multiplier on the resource constraint (15). Let $\mu\eta_{CR}$ be the multiplier on incentive constraint (11) and $\mu\eta_{RM}$ the multiplier on (12). Define*

[13] Along the same lines, Slavík and Yazici (2014) show that structures capital which is equally complementary to labor in all occupations should not be taxed.

$\mu\xi_i$ as the multiplier on the consistency condition for occupation i in (14). The optimal marginal income tax rates satisfy

$$\frac{T'_M + \frac{\xi_M}{Y_M}}{1 - T'_M} = (\psi_M - 1)\left(1 - \left(\frac{w_M}{w_R}\right)^{1+\frac{1}{\varepsilon}}\right) \tag{21}$$

$$\frac{T'_R + \frac{\xi_R}{Y_R}}{1 - T'_R} = \frac{f_C}{f_R}(1 - \psi_C)\left(1 - \left(\frac{w_R}{w_C}\right)^{1+\frac{1}{\varepsilon}}\right) \tag{22}$$

$$T'_C = -\frac{\xi_C}{Y_C}, \tag{23}$$

with

$$\xi_i = \tilde{\varepsilon}_{w_R/w_C,L_i} I_{CR} + \tilde{\varepsilon}_{w_M/w_R,L_i} I_{RM}, \tag{24}$$

where the semi-elasticities of relative wages with respect to L_j *are defined as*

$$\tilde{\varepsilon}_{w_R/w_C,L_i} \equiv \frac{\partial}{\partial L_i}\left(\frac{w_R}{w_C}\right)\frac{w_C}{w_R} \tag{25}$$

and

$$\tilde{\varepsilon}_{w_M/w_R,L_i} \equiv \frac{\partial}{\partial L_i}\left(\frac{w_M}{w_R}\right)\frac{w_R}{w_M}. \tag{26}$$

Proof. See Appendix A.2 □

First, note that each expression for optimal marginal income tax rates features a correction for general-equilibrium effects, ξ_i/Y_i. Suppose for the moment that general-equilibrium effects are absent. We then have $\xi_i = 0 \; \forall i \in \mathcal{I}$. Moreover, assume that welfare weights satisfy $\psi_M > 1$ and $\psi_C < 1$, as will be the case with a welfarist government which attaches higher weights to individuals who earn lower incomes. As a consequence, marginal tax rates T'_M and T'_R are positive.[14] This is in line with the function of marginal income tax rates: the role of the marginal tax rate at income y is to redistribute income from individuals who earn more than y to individuals earnings equal to, or less than, y. Consider (21): the social marginal value of distributing income from f_R routine workers and f_C cognitive workers to a mass of f_M manual workers is $(\psi_M - 1) = f_M^{-1}\left[f_R(1-\psi_R) + f_C(1-\psi_C)\right]$. Similarly, in (22), the term $f_R^{-1} f_C (1-\psi_C)$ captures the social marginal value of redistributing income from f_C cognitive workers to a mass of f_R routine workers. However, marginal tax rates distort labor supply, which is captured by the terms $1 - (w_M/w_R)^{1+1/\varepsilon}$ and $1 - (w_R/w_M)^{1+1/\varepsilon}$. With $\varepsilon > 0$, both terms are smaller than 1 and thus scale down marginal tax rates, whereas in the absence

[14] Note that there are three discrete income levels in the economy. The income tax function T is thus not differentiable. Marginal tax rates T' are defined as $T'_i \equiv 1 + \frac{U_\ell(c(V_i,\ell_i),\ell_i)}{U_c(c(V_i,\ell_i),\ell_i)}\frac{1}{w_i}$.

of labor supply responses $\varepsilon \to 0$, and both terms tend to 1. Finally, without general-equilibrium effects $T'_C = 0$. This is the famous result of "no distortion at the top" (Sadka, 1976; Seade, 1977). Since there are no individuals who earn more than cognitive workers, setting a positive marginal tax rate has no distributional benefits but would distort labor supply. It is thus optimal to set a marginal tax rate of zero.

Now consider the case with general-equilibrium effects. Under special conditions, it is possible to sign the multiplier terms ξ_M and ξ_C.

Corollary 1. *With general-equilibrium effects, the multiplier terms ξ_M and ξ_C can be signed as follows:*

- *if* $\frac{\partial}{\partial L_M}\left(\frac{w_R}{w_C}\right) < 0$, $\xi_M < 0$,
- *if* $\frac{\partial}{\partial L_C}\left(\frac{w_M}{w_R}\right) > 0$, $\xi_C > 0$.

Proof. See Appendix A.2.4 □

Suppose that manual and cognitive non-routine labor are sufficiently complementary, such that $\tilde{\varepsilon}_{w_R/w_C,L_M} < 0$, and thus $\xi_M < 0$. In this case T'_M is larger than in the absence of general-equilibrium effects. The intuition has again to do with the relaxation of incentive constraints. A higher marginal tax rate T'_M discourages labor supply of manual workers, which increases their wage relative to routine workers, thereby relaxing incentive constraint (11). Moreover, since by assumption $\tilde{\varepsilon}_{w_R/w_C,L_M} < 0$, a reduction in the supply of manual labor also increases the wage of routine workers relative to cognitive workers, which relaxes incentive constraint (11). A similar reasoning applies to the case in which $\tilde{\varepsilon}_{w_M/w_R,L_C} > 0$ and thus $\xi_C > 0$. Now T'_C becomes negative, as in Stiglitz (1982), which encourages labor supply of cognitive workers. As a result, their wage drops relative to routine workers, whereas, by assumption, the wage of manual workers increases relative to routine workers. Again, this overall wage compression relaxes incentive constraints, and is therefore welfare improving. Whether the marginal income tax for routine workers is scaled up or down in the presence of general-equilibrium effects is ambiguous. Consider an increase in T'_R: As a result, labor supply of routine workers falls, raising their wage relative to manual and cognitive workers. This change has opposing effects on incentive constraints: it relaxes (11) but tightens (12). Whether an increase in T'_R is desirable thus depends on which of the two effects is more relevant for welfare.

3.2.4 Effect of robot tax on marginal income taxes

How are marginal income taxes affected by the presence of the robot tax? To answer this question, I first derive expressions for the case in which taxing robots is not possible. To do so, I impose the additional constraint $Y_B(\mathbf{L}, B) = q$, which corresponds to $\tau = 0$.

The expressions in (21), (22) and (23) are not affected by the absence of the robot tax. However, the multipliers on the consistency conditions are now different.

Corollary 2. *In the absence of the robot tax, let $\mu\kappa$ denote the multiplier on the additional constraint $Y_B(\mathbf{L}, B) - q = 0$. Let $\mu\xi_i$ be the multiplier on the consistency condition for occupation i, $\mu\eta_{CR}$ the multiplier on incentive constraint (11) and $\mu\eta_{RM}$ the multiplier on incentive constraint (12). The following condition holds for ξ_i:*

$$\xi_i = \tilde{\varepsilon}_{w_R/w_C, L_i} I_{CR} + \tilde{\varepsilon}_{w_M/w_R, L_i} I_{RM} + \kappa \frac{\partial Y_B(\mathbf{L}, B)}{\partial L_i} \; \forall i, \tag{27}$$

with

$$\kappa \frac{\partial Y_B(\mathbf{L}, B)}{\partial B} B = \varepsilon_{w_C/w_R, B} \; I_{CR} - \varepsilon_{w_M/w_R, B} \; I_{MR}. \tag{28}$$

Without the robot tax, ξ_i is thus adjusted by $\kappa \partial Y_B(\mathbf{L}, B)/\partial L_i$. Note that the right-hand-side of (28) is the same as in the expression for the optimal robot tax (16). Since $\partial Y_B(\mathbf{L}, B)/\partial B < 0$, κ thus has the *opposite* sign of the optimal robot tax which would result if we were not to rule out robot taxation. Suppose that the optimal robot tax would be positive, and thus $\kappa < 0$. I first focus on the unambiguous cases $i \in \{M, C\}$. We then have $\partial Y_B(\mathbf{L}, B)/\partial L_i > 0$ and hence ξ_i is lower in the absence of the robot tax. It thus follows that *with* the robot tax, both T'_M and T'_C are *lower*. Intuitively, a tax on robots lowers the wages of manual and cognitive workers, which induces them to reduce their labor supply. Lower marginal income tax rates encourage labor supply, which partly offset the reduction. Moreover, in the case of cognitive workers, the drop in wages which results from increased labor supply further compresses the wage distribution. Now consider routine workers. The sign of $\partial Y_B(\mathbf{L}, B)/\partial L_R$ is ambiguous, and as a result it is not clear in which direction the marginal income tax for routine workers is adjusted if robots are taxed. Suppose that $\partial Y_B(\mathbf{L}, B)/\partial L_R > 0$, which will be the case if the difference in elasticities of substitution between robots and routine workers on the one hand and robots and cognitive or manual workers on the other hand is not too large. ξ_R is now lower without the robot tax, and thus, T'_R will be lower if robots can be taxed.

4 Continuous types and occupational choice

Having developed intuition, I now extend the model to allow for continuous types as well as for occupational choice. To do so, I build on the framework by Rothschild and Scheuer (2013, 2014).

4.1 Setup

4.1.1 Skill Heterogeneity

There is a unit mass of individuals. Each individual is characterized by three-dimensional skill-vector $\theta \in \Theta \equiv \Theta_M \times \Theta_R \times \Theta_C$, with $\Theta_i \equiv [\underline{\theta}_i, \overline{\theta}_i]$ and $i \in \mathcal{I} \equiv \{M, R, C\}$. As before, each dimension of skill determines an individual's productivity in one of the three occupations: manual non-routine (M), routine (R), and cognitive non-routine (C). Skills are distributed according to a continuous cumulative distribution function $F : \Theta \rightarrow [0, 1]$ with corresponding density f.

4.1.2 Technology

The final good is produced using aggregate effective labor in the three occupations, $\mathbf{L} \equiv (L_M, L_R, L_C)$, and robots B according to a constant-returns-to-scale production function $Y(\mathbf{L}, B)$. As before, robots are better substitutes for routine labor than for non-routine labor, and Assumption 1 holds. Aggregate effective labor is now defined as

$$L_M \equiv \int_{\mathcal{M}} \theta_M \ell(\theta)\, dF(\theta), \quad L_R \equiv \int_{\mathcal{R}} \theta_R \ell(\theta)\, dF(\theta), \quad L_C \equiv \int_{\mathcal{C}} \theta_C \ell(\theta)\, dF(\theta), \tag{29}$$

where $\mathcal{M}$, $\mathcal{R}$ and $\mathcal{C}$ are the sets of individuals θ working in occupations M, R and C, respectively. Robots are produced linearly with the final good, as described in Section 3.1.2. The representative firm maximizes profits

$$\max_{\mathbf{L}, B} Y(\mathbf{L}, B) - \sum_{i \in \mathcal{I}} \omega_i L_i - pB, \tag{30}$$

taking wage rates ω_i and the user price of robots, p, as given. As before, we have $p = (1+\tau)q$, with τ the tax on robots and q the producer price of robots. In contrast to the firm problem in the simple model, wages and wage rates now differ due to heterogeneity within occupations, and w_i has thus been replaced by ω_i. In equilibrium, $\omega_i = Y_i(\mathbf{L}, B)$, and $p = Y_B(\mathbf{L}, B)$.

4.1.3 Preferences and occupational choice

Individuals derive utility from consumption c and disutility from supplying labor ℓ according to the strictly concave utility function $U(c, \ell)$ with $U_c > 0$, $U_\ell < 0$. Let y denote an individual's gross income and w the wage. I assume that U satisfies the standard Spence-Mirrlees single-crossing property (Mirrlees, 1971; Ebert, 1992; Hellwig, 2004), that is, the marginal rate of substitution between income and consumption, $-U_\ell\left(c, \frac{y}{w}\right) / \left(wU_c\left(c, \frac{y}{w}\right)\right)$, decreases in w. Moreover, I assume that the monotonicity condition is satisfied, that is, gross-income needs to increase in w.[15] Individuals choose their occupation according to a Roy (1951) model, such that their wage is maximized and, in equilibrium, given by

$$w_{\mathbf{L},B}\left(\theta\right)=\max\left\{Y_M\left(\mathbf{L},B\right)\theta_M,Y_R\left(\mathbf{L},B\right)\theta_R,Y_C\left(\mathbf{L},B\right)\theta_C\right\}, \tag{31}$$

where the subscript indicates that the wage for individual θ is pinned down by factor inputs $\mathbf{L}$ and B.

4.2 Optimal taxation

4.2.1 Reducing dimensionality

I follow Rothschild and Scheuer (2013) to reduce the dimensionality of the problem: Given factor inputs $\mathbf{L}$ and B, wage rates and sectoral choice are determined, and the three-dimensional heterogeneity in skill can be reduced to one-dimensional heterogeneity in wages. The distribution of skills $F\left(\theta\right)$ corresponds to the wage distribution

$$F_{\mathbf{L},B}\left(w\right)=F\left(\frac{w}{Y_M\left(\mathbf{L},B\right)},\frac{w}{Y_R\left(\mathbf{L},B\right)},\frac{w}{Y_C\left(\mathbf{L},B\right)}\right) \tag{32}$$

with occupational wage densities[16]

$$f^M_{\mathbf{L},B}\left(w\right)=\frac{1}{Y_M\left(\mathbf{L},B\right)}\int_{\underline{\theta}_C}^{w/Y_C(\mathbf{L},B)}\int_{\underline{\theta}_R}^{w/Y_R(\mathbf{L},B)}f\left(\frac{w}{Y_M\left(\mathbf{L},B\right)},\theta_R,\theta_C\right)d\theta_R d\theta_C, \tag{33}$$

$$f^R_{\mathbf{L},B}\left(w\right)=\frac{1}{Y_R\left(\mathbf{L},B\right)}\int_{\underline{\theta}_C}^{w/Y_C(\mathbf{L},B)}\int_{\underline{\theta}_M}^{w/Y_M(\mathbf{L},B)}f\left(\theta_M,\frac{w}{Y_R\left(\mathbf{L},B\right)},\theta_C\right)d\theta_M d\theta_C, \tag{34}$$

$$f^C_{\mathbf{L},B}\left(w\right)=\frac{1}{Y_C\left(\mathbf{L},B\right)}\int_{\underline{\theta}_R}^{w/Y_R(\mathbf{L},B)}\int_{\underline{\theta}_M}^{w/Y_M(\mathbf{L},B)}f\left(\theta_M,\theta_R,\frac{w}{Y_C\left(\mathbf{L},B\right)}\right)d\theta_M d\theta_R. \tag{35}$$

The wage density for occupation i at wage w is thus obtained by evaluating the skill density at that skill θ_i which corresponds to earning w in occupation i, $\theta_i=w/Y_i\left(\mathbf{L},B\right)$, and by integrating over the mass of individuals whose skill is not sufficient to earn more in any occupation other than i. I denote the support of the wage distribution for any given $\mathbf{L},B$ by $\left[\underline{w}_{\mathbf{L},B},\overline{w}_{\mathbf{L},B}\right]$, where $\underline{w}_{\mathbf{L},B}=w_{\mathbf{L},B}\left(\underline{\theta}_M,\underline{\theta}_R,\underline{\theta}_C\right)$ and $\overline{w}_{\mathbf{L},B}=w_{\mathbf{L},B}\left(\overline{\theta}_M,\overline{\theta}_R,\overline{\theta}_C\right)$ are the wages earned by the least and most skilled individuals, respectively. The overall wage density is given by $f\left(w\right)=f^M_{\mathbf{L},B}\left(w\right)+f^R_{\mathbf{L},B}\left(w\right)+f^C_{\mathbf{L},B}\left(w\right)$.

Like in the discrete model, the social planner attaches general cumulative Pareto

[15]With non-linear taxes, first-order conditions are necessary, but generally not sufficient for utility maximization. The single-crossing and monotonicity conditions ensure that second-order conditions for utility maximization hold.

[16]Technically, the expressions are not densities, since they do not integrate to one. Instead, the expressions integrate to the mass of individuals in the respective occupation.

weights to types, which I now denote by $\Psi(\theta)$ with the corresponding density $\psi(\theta)$. Since for given $\mathbf{L}, B$ there is a unique mapping from types to wages, Pareto weights can be written as function of wages, $\Psi_{\mathbf{L},B}(w)$, with occupation-specific densities $\psi^M_{\mathbf{L},B}(w)$, $\psi^R_{\mathbf{L},B}(w)$, and $\psi^C_{\mathbf{L},B}(w)$ such that the overall density is $\psi_{\mathbf{L},B}(w) = \psi^M_{\mathbf{L},B}(w) + \psi^R_{\mathbf{L},B}(w) + \psi^C_{\mathbf{L},B}(w)$. Moreover, once $\mathbf{L}, B$ is fixed, all endogenous variables which depend on an individual's type can be written in terms of wages. This feature allows for a useful separation of the planner problem.

4.2.2 Separation into inner and outer problem

As in Section 3, I separate the planner problem into an *inner problem* which maximizes welfare over labor supply and indirect utilities for given $\mathbf{L}$ and B, and an *outer problem* which maximizes welfare over $\mathbf{L}$ and B.

Inner problem. Denote by $V(\theta)$ the indirect utility of type θ. Social welfare is defined as an integral over weighted indirect utilities $\int_\Theta V(\theta)\, d\Psi(\theta)$, which given $\mathbf{L}, B$ can be written as $\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} V(w)\, d\Psi_{\mathbf{L},B}(w)$. As is common, I maximize social welfare by directly choosing an allocation of indirect utilities and labor supplies, subject to incentive and resource constraints. In addition, the allocation needs to be consistent with $\mathbf{L}$ and B to make sure that the market for robots and labor clears. Labor-market clearing is ensured by consistency conditions. Apart from these consistency conditions, the problem is a standard Mirrlees (1971) problem. I define the inner problem as

$$\mathcal{W}(\mathbf{L}, B) \equiv \max_{V(w),\ell(w)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} V(w)\, d\Psi_{\mathbf{L},B}(w) \tag{36}$$

subject to

$$V'(w) + U_\ell\left(c\left(V(w), \ell(w)\right), \ell(w)\right) \frac{\ell(w)}{w} = 0 \;\forall w \in \left[\underline{w}_{\mathbf{L},B}, \overline{w}_{\mathbf{L},B}\right] \tag{37}$$

$$\frac{1}{Y_M(\mathbf{L}, B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell(w)\, f^M_{\mathbf{L},B}(w)\, dw - L_M = 0 \tag{38}$$

$$\frac{1}{Y_R(\mathbf{L}, B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell(w)\, f^R_{\mathbf{L},B}(w)\, dw - L_R = 0 \tag{39}$$

$$\frac{1}{Y_C(\mathbf{L}, B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell(w)\, f^C_{\mathbf{L},B}(w)\, dw - L_C = 0 \tag{40}$$

$$\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \left(w\ell\left(w\right) - c\left(V\left(w\right), \ell\left(w\right)\right)\right) f_{\mathbf{L},B}\left(w\right) dw + Y_B\left(\mathbf{L}, B\right) B - qB = 0. \tag{41}$$

Here, (37) is the set of incentive constraints, and (38), (39), and (40) are the consistency conditions for occupations M, R and C, respectively. Equation (41) is the resource constraint and the continuous-type equivalent of (15).

Outer problem. In the outer problem, the planner chooses inputs $\mathbf{L}$ and B such that welfare is maximized, that is, he solves $\max_{\mathbf{L},B} \mathcal{W}\left(\mathbf{L}, B\right)$. It is useful that $\mathcal{W}\left(\mathbf{L}, B\right)$ corresponds to the value of the Lagrangian of the inner problem, evaluated at optimal indirect utilities and labor supplies.

4.2.3 Optimal robot tax

I obtain a condition for the optimal robot tax from the outer problem by differentiating the maximized Lagrangian with respect to robots, B. Using (10), I characterize the optimal robot tax as follows.

Proposition 3. *Let μ denote the multiplier on the resource constraint and $\mu\xi_i$ the multiplier on the consistency condition for occupation $i \in \{M, R, C\}$. Let $\mu\eta\left(w\right)$ denote the multiplier on the incentive constraint at wage w. Denote by q^i_E the income share in occupation i. The optimal tax on robots, τ, is characterized by*

$$\begin{aligned}
&\tau q B = \\
&\varepsilon_{Y_C/Y_R,B}\left(\mathbf{L}, B\right)\left(I_C\left(\mathbf{L}, B\right) + \sum_{i\in\mathcal{I}} \xi_i \left(C_{Ci}\left(\mathbf{L}, B\right) + S_{Ci}\left(\mathbf{L}, B\right)\right)\right) \\
&+\varepsilon_{Y_M/Y_R,B}\left(\mathbf{L}, B\right)\left(I_M\left(\mathbf{L}, B\right) + \sum_{i\in\mathcal{I}} \xi_i \left(C_{Mi}\left(\mathbf{L}, B\right) + S_{Mi}\left(\mathbf{L}, B\right)\right)\right),
\end{aligned} \tag{42}$$

where for $i \in \{M, R, C\}$

$$I_i\left(\mathbf{L}, B\right) \equiv \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \frac{\eta\left(w\right)}{U_c\left(w\right)} V'\left(w\right) w \frac{d}{dw}\left(\frac{f^i_{\mathbf{L},B}\left(w\right)}{f_{\mathbf{L},B}\left(w\right)}\right) dw, \tag{43}$$

and for $i, j \in \{M, R, C\}$

$$C_{ij}\left(\mathbf{L}, B\right) \equiv \frac{1}{Y_j\left(\mathbf{L}, B\right)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell'\left(w\right) Cov\left(q^i_{\mathbf{L},B}(\theta), q^j_{\mathbf{L},B}(\theta)|w\right) f_{\mathbf{L},B}\left(w\right) dw, \tag{44}$$

with

$$q^{i}_{\mathbf{L},B}(\theta)=\begin{cases}1, & \textit{if } \theta \textit{ works in } i\\ 0, & \textit{otherwise,}\end{cases}$$

and S_{ij} *as defined in the Appendix. The elasticities of equilibrium wage rates with respect to the amount of robots are defined as*

$$\varepsilon_{Y_M/Y_R,B}(\mathbf{L},B)\equiv\frac{\partial\left(Y_M(\mathbf{L},B)/Y_R(\mathbf{L},B)\right)}{\partial B}\frac{B}{Y_M(\mathbf{L},B)/Y_R(\mathbf{L},B)}>0, \tag{45}$$

and

$$\varepsilon_{Y_C/Y_R,B}(\mathbf{L},B)\equiv\frac{\partial\left(Y_C(\mathbf{L},B)/Y_R(\mathbf{L},B)\right)}{\partial B}\frac{B}{Y_C(\mathbf{L},B)/Y_R(\mathbf{L},B)}>0. \tag{46}$$

Proof. See Appendix B □

The expression in (42) characterizes the optimal tax revenue raised with the robot tax. First, note the similarity between (42) and the corresponding expression in the simplified model, (16). In both cases, the effect of robots on relative wage rates plays a crucial role.[17] Due to Assumption 1, an increase in the number of robots leads to higher wage rates in non-routine occupations relative to routine occupations. As a consequence, the elasticities of relative wage rates $\varepsilon_{Y_M/Y_R,B}(\mathbf{L},B)$ and $\varepsilon_{Y_C/Y_R,B}(\mathbf{L},B)$ are both positive.

As in the simple model, these elasticities multiply the incentive effects I_i. In addition, they multiply terms which emerge due to heterogeneous types and occupational choice. Following Rothschild and Scheuer (2013), I refer to these as *effort-reallocation effects* C_{ij} and *occupational-shift effects* S_{ij}, where $i,j\in\{M,R,C\}$. Incentive effects, effort-reallocation effects and occupational-shift effects ultimately affect welfare for the same reason: a change in relative wage rates leads to a change in the wage distribution, which affects incentive constraints – and thus – income-tax distortions of labor supply. If income-tax distortions are reduced, more income can be redistributed overall, which raises welfare.

It is instructive to think about incentive effects as capturing the first-round welfare impact of a tax on robots on relative wage rates. In response to changed relative wage rates, individuals adjust their behavior, which then has second-round effects on relative wage rates and on welfare.[18] These second-round effects are captured by the effort-reallocation and occupational-shift effects. They originate from the effect of robots on the consistency conditions. Intuitively, reallocation and occupational-shift effects reduce the effectiveness of the robot tax, by counteracting initial wage compression.

Incentive effects. The incentive effects capture how changes in relative wage rates affect tax-distortions of labor supply, and thus welfare. If $I_i(\mathbf{L},B)>0$, an increase

[17] In the simplified model elasticities of relative wages coincide with elasticities of relative wage rates. Due to heterogeneity of wages within occupations, this is no longer the case here.

[18] In the model, equilibrium is determined simultaneously, hence there are no actual first and second rounds.

in the robot tax leads to welfare-improving wage compression. Ceteris paribus, larger incentive effects thus call for a higher tax on robots. To determine the sign of the incentive effect, suppose that the incentive constraint (37) is downward-binding, and thus $\eta(w) \geq 0$. Since indirect utilities $V(w)$ increase in w, the sign of $I_i(\mathbf{L}, B)$ is determined by $\frac{d}{dw}\left(f^i_{\mathbf{L},B}(w)/f_{\mathbf{L},B}(w)\right)$. This term captures how the share of individuals earning wage w in occupation i changes with a marginal increase in w.[19]

Consider I_C. Since workers in cognitive occupations are concentrated at high wages, the term $\frac{d}{dw}\left(f^C_{\mathbf{L},B}(w)/f_{\mathbf{L},B}(w)\right)$ is positive at most w. As a consequence, we find $I_C > 0$. By reducing Y_C/Y_R, a tax on robots thus compresses wages at the top of the wage distribution, which increases welfare. The intuition is similar as in the stylized model. Wage compression at the top of the distribution makes it more costly for cognitive workers to imitate types who earn marginally lower incomes in routine occupations – which locally relaxes incentive constraints. In other words, income-tax distortions of labor supply are locally alleviated. This allows for more redistribution overall, which raises welfare.

Next, consider I_M and suppose that manual non-routine workers are concentrated at low wages, as observed empirically. The term $\frac{d}{dw}\left(f^M_{\mathbf{L},B}(w)/f_{\mathbf{L},B}(w)\right)$ is then negative at most w, leading to $I_M < 0$. The negative sign captures that a tax on robots lowers welfare by locally tightening incentive constraints at the bottom of the wage distribution. As in the stylized model, a tax on robots thus has opposing effects on incentive constraints – and thus labor-supply distortions – at the top and at the bottom of the wage distribution. As a consequence, the sign of the robot tax is again ambiguous.

Effects on the consistency conditions. Both, effort-reallocation and occupational-shift effects capture changes in aggregate labor supplies, which affect welfare via the consistency conditions. The welfare impact of a marginal increase in L_i via the consistency condition is $\mu\xi_i$, with $\mu > 0$. The multiplier $\mu\xi_i$ thereby captures how the change in L_i affects welfare by changing relative wage rates, and as a consequence incentive constraints. Different from the simple model, it is not anymore possible to sign ξ_i analytically.

[19] Note that if $I_i \geq 0$ for some occupation i, it has to hold that there is at least one other occupation $j \neq i$, for which $I_j \leq 0$. To see this, write

$$\begin{aligned} I_i(\mathbf{L}, B) &= \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \frac{\eta(w)}{U_c(w)} V'(w)\, w \frac{d}{dw}\left(\frac{f^i_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}\right) dw \\ &= \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \frac{\eta(w)}{U_c(w)} V'(w)\, w \frac{d}{dw}\left(\frac{f_{\mathbf{L},B}(w) - \sum_{j\neq i} f^j_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}\right) dw \\ &= -\sum_{j\neq i} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \frac{\eta(w)}{U_c(w)} V'(w)\, w \frac{d}{dw}\left(\frac{f^j_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}\right) dw. \end{aligned}$$

Effort-reallocation effects. The *effort-reallocation* effect C_{ij} captures the welfare-relevant impact of a marginal increase in Y_i on aggregate labor supply L_j which arises due to individuals adjusting their labor supply within occupations, while keeping occupational choice fixed.

Recall that the expression for the optimal robot tax is derived from the outer problem, taking as given $\ell(w)$ which is chosen optimally in the inner problem. Still, by affecting relative wage rates, a change in the number of robots has an impact on labor supplies within occupations since individuals move along the schedule $\ell(w)$, leading to a change in the wage density $f(w)$ at w. Instead of deriving changes in L_j by keeping $\ell(w)$ fixed and adjusting the densities, I follow Rothschild and Scheuer (2014) and construct a variation of the $\ell(w)$ schedule which, at each w, neutralizes average changes in $\ell(w)$ across occupations. As a result, the wage density $f(w)$ is unaffected. Moreover, at the margin, the schedule variation has no effect on welfare.

Consider for example C_{CM}. The term captures how an increase in Y_C – ceteris paribus – affects aggregate labor supply L_M due to effort-reallocation. At each w, individuals increase their labor supply in occupation C, whereas labor supply remains unchanged in occupations M and R. As a result, at each w, labor supply increases more than average in C and less than average in M. Intuitively, this negative correlation of changes in labor supplies in C and M at w gives rise to the covariance term in C_{CM}. After neutralizing average changes, L_M decreases, which is captured by $C_{CM} < 0$.

Occupational-shift effects. The set of *occupational-shift effects* $S_{ij}(\mathbf{L}, B)$ capture the welfare-relevant impact of a marginal increase in Y_i on aggregate labor supply L_j due to individuals switching between occupations i and j, while keeping the labor supply schedule $\ell(w)$ fixed and ruling out shifts along $\ell(w)$ within occupations.[20] Since a marginal increase in Y_i lets individuals shift from occupation j to occupation i (with $i \neq j$), aggregate labor supply L_j is reduced. This is captured by $S_{ij} < 0$ for $i \neq j$. In contrast, $S_{ii} > 0$ accounts for the inflow into occupation i due to an increase in Y_i, leading to an increase in L_i.

4.2.4 Marginal income taxes

Turning to optimal marginal income taxes, I note that the only difference with the inner problem in Rothschild and Scheuer (2014) is the presence of the term qB in the resource constraint. In the inner problem this term is kept fixed and does therefore not influence the optimal allocation of indirect utilities and labor supply. As a consequence, the characterization of optimal marginal income taxes is the same as in Rothschild and Scheuer

[20] If, instead, one wanted to compute the total change in income due to individuals switching, one would also have to account for changes in labor supply within occupations. However, these effects have already been taken into account in the effort-reallocation effects.

(2014).

Proposition 4. *Let μ be the multiplier on the resource constraint and denote by $\mu\xi_i$ the multiplier on the consistency condition pertaining to occupation i. Let ε^u be the uncompensated labor-supply elasticity and ε^c the compensated labor-supply elasticity. Given* **L** *and B, optimal marginal tax rates are characterized by*

$$1 - T'(w) = \left(1 - \sum_{i \in \mathcal{I}} \frac{\xi_i}{Y_i(\mathbf{L}, B)} \frac{f^i_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}\right) \left(1 + \frac{\eta(w)}{w f_{\mathbf{L},B}(w)} \frac{1 + \varepsilon^u(w)}{\varepsilon^c(w)}\right)^{-1} \tag{47}$$

with

$$\eta(w) = \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \left(1 - \frac{\psi_{\mathbf{L},B}(z)}{f_{\mathbf{L},B}(z)} \frac{U_c(z)}{\mu}\right) \exp\left(\int_w^z \left(1 - \frac{\varepsilon^u(s)}{\varepsilon^c(s)}\right) \frac{dy(s)}{y(s)}\right) f_{\mathbf{L},B}(z)\, dz. \tag{48}$$

Proof. See Rothschild and Scheuer (2014) □

As highlighted by Rothschild and Scheuer (2014), the formula closely resembles the optimal income tax expression in the standard Mirrlees model. The only difference is the correction term $1 - \sum_{i \in \mathcal{I}} \frac{\xi_i}{Y_i(\mathbf{L},B)} \frac{f^i_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}$, which adjusts retention rates $1 - T'(w)$ to account for general-equilibrium effects. The fact that the expression for optimal marginal income tax rates in my model is the same as in Rothschild and Scheuer (2014) does not mean that the tax on robots does not interact with optimal income taxes. As in the simple model, the presence of a robot tax changes the multipliers ξ_i. Since it not possible to solve for ξ_i, I only study the effect of the robot tax on marginal income taxes as part of the quantitative analysis below.

5 Quantitative analysis

In this section, I study optimal taxes on robots and labor income quantitatively. To do so, I first calibrate the model to the US economy for the existing tax system. While some parameters are set directly from the data, other parameters are set by minimizing the sum of squared distances between model and data moments. The calibrated model is then used for optimal tax analysis.

The theoretical results are derived using a very general definition of robots, expressed in Assumption 1. In the quantitative analysis, I study the taxation of industrial robots.[21] Industrial robots are an important automation technology. Moreover, their impact on the economy has been studied empirically. Acemoglu and Restrepo (2017) analyze the

[21] An industrial robot is defined as "an automatically controlled, reprogrammable, multipurpose manipulator programmable in three or more axes, which can be either fixed in place or mobile for use in industrial automation applications." See https://ifr.org/img/office/Industrial_Robots_2016_Chapter_1_2.pdf

impact of industrial robots on employment and wages in the US. I use their findings to guide the quantitative analysis. In addition to being specific about robots, I make specific functional form assumptions for the distribution of skills, the production function, and preferences.

5.1 Functional forms

5.1.1 Skill-distribution

I follow Heckman and Sedlacek (1990) and Heckman and Honoré (1990) by assuming that skills θ follow a joint log-Normal distribution. Moreover, I normalize the mean to be the zero vector. Log skills are then distributed according to

$$\begin{bmatrix} \ln\theta_M \\ \ln\theta_R \\ \ln\theta_C \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_M^2 & \rho_{MR}\sigma_M\sigma_R & \rho_{MC}\sigma_M\sigma_C \\ \rho_{MR}\sigma_M\sigma_R & \sigma_R^2 & \sigma_{RC}\sigma_R\sigma_C \\ \rho_{MC}\sigma_M\sigma_C & \rho_{RC}\sigma_R\sigma_C & \sigma_C^2 \end{bmatrix} \right), \tag{49}$$

where I denote by σ_i the standard deviation of latent skill for occupation i and by ρ_{ij} the correlation between skills in occupations i and j. The distribution of skills in (49) gives rise to occupational wage densities $f^M_{\mathbf{L},B}$, $f^R_{\mathbf{L},B}$, $f^C_{\mathbf{L},B}$ which I provide in Appendix C.

5.1.2 Production function

Robots are produced linearly as described in Section 3.1.2. The production function for the consumption good combines elements from Acemoglu and Restrepo (2018a) (see also Guerreiro et al., 2017) and Autor and Dorn (2013) to capture the assignment of routine tasks to robots and routine labor, as well as the contribution of manual and cognitive non-routine labor. Acemoglu and Restrepo (2018a) argue that automation differs from factor-augmenting technical change. Under factor-augmenting technical change, the substitution between labor and capital – such as robots – is governed only by the elasticity of substitution. In such models, the scope for a reduction in demand for routine labor is limited. This changes if automation is not only thought of as a change in the amount of capital, but also as a shift of tasks from labor to capital. Such a model of task assignment endogenizes the weight given to capital and labor. As the price of capital drops, the firm not only uses more capital, but also gives it more weight in production. As a result, demand for routine labor falls relative to a model which only features factor-augmenting technical change.

In addition to modeling automation similar to Acemoglu and Restrepo (2018a) and Guerreiro et al. (2017), I build on Autor and Dorn (2013) to capture that robots have a different impact on manual non-routine, routine and cognitive non-routine occupations, as has been documented by Acemoglu and Restrepo (2017) and Dauth et al. (2018). Autor and Dorn (2013) study the role of ICT capital for wage polarization.Their production function has the feature that ICT capital substitutes for routine labor and complements non-routine labor, thereby affecting wages in the three occupations differentially.[22] In Autor and Dorn (2013), production of services and goods takes place in separate sectors. In contrast, in my model, a single consumption good is produced by combining all factors on the production side, according to

$$\begin{aligned} Y(\mathbf{L}, B) \equiv & A \left(\mu_M L_M^{\frac{\gamma-1}{\gamma}} + (1-\mu_M) \right. \\ & \times \left(\mu_C L_C^{\frac{\rho-1}{\rho}} + (1-\mu_C) \right. \\ & \left. \left. \times \left(\mu_B(L_R, B)^{\frac{1}{\sigma}} B^{\frac{\sigma-1}{\sigma}} + (1-\mu_B(L_R, B))^{\frac{1}{\sigma}} L_R^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \end{aligned} \tag{50}$$

with

$$\mu_B(L_R, B) \equiv \frac{B}{L_R + B} \in [0, 1]. \tag{51}$$

This is a nested constant-elasticity-of-substitution (CES) production function with three layers. The innermost layer captures output of routine tasks, where σ is the elasticity of substitution between robots B and routine labor L_R. Different from a standard CES production function, the firm's assignment of tasks to robots and routine labor gives rise to endogenous weight $\mu_B(L_R, B)$ (see Appendix D for the derivations). The second layer combines cognitive non-routine labor L_C and output of routine tasks, where ρ is the substitution elasticity. The outermost layer combines manual non-routine labor L_M with the combined output of routine tasks and cognitive labor, where the elasticity of substitution is γ. The production function implies that in equilibrium the marginal products of robots and routine labor are equalized, that is $Y_B(\mathbf{L}, B) = Y_R(\mathbf{L}, B)$. As a consequence, a marginal increase in the number of robots – for example due to a drop in the price of robots – lowers both, the returns to robots as well as the return to routine labor.

5.1.3 Preferences and labor supply

Preferences over consumption and labor supply are quasi-linear and given by

$$U(c, \ell) = c - \frac{\ell^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}, \tag{52}$$

where c is consumption, ℓ is individual labor supply and ε is the labor-supply elasticity. Let $T'(y)$ denote the marginal income tax rate at income y. Using that preferences are

[22] Autor and Dorn (2013) refer to manual non-routine as 'services' and to cognitive non-routine as 'abstract'.

quasi-linear, optimal labor supply ℓ is then implicitly given by the solution to

$$\ell(w)^{\frac{1}{\varepsilon}} = (1 - T'(y(w)))\, w. \tag{53}$$

In the calibration, I use the parametric tax function proposed by Heathcote et al. (2017) as an approximation to the US income tax schedule, with

$$T(y) = y - \lambda y^{-t}, \tag{54}$$

where t is referred to as the progressivity parameter, while λ can be used to calibrate total tax revenue. Based on (54), marginal tax rates are given by

$$T'(y) = 1 - (1 - t)\, \lambda y^{-t}. \tag{55}$$

Using that $y = \ell w$ and substituting (55) in (53), one can explicitly solve for

$$\ell(w) = \left[(1 - t)\, \lambda w^{1-t}\right]^{\frac{\varepsilon}{1+t\varepsilon}}. \tag{56}$$

5.2 Calibration

5.2.1 Data and calibration targets

The calibration aims to accurately capture the key determinants of the optimal robot tax. To do so, I target moments of the distribution of wages and employment, as well as moments related to the impact of industrial robots on the economy.

Moments for the distribution of wages and employment are based on the CPS Merged Outgoing Rotation Groups (MORG) as prepared by the National Bureau of Economic Research (NBER).[23] I focus on the year 1993, since the effect of robots on the labor market studied by Acemoglu and Restrepo (2017) is also based on data from this year.[24] Occupations are categorized into three groups: manual non-routine, routine, and cognitive non-routine following Acemoglu and Autor (2011).[25] I provide an overview of the occupations contained in the three categories as well as summary statistics in Tables 4 and 5 in the Data Appendix F. For the model to generate a realistic wage and employment distribution, I target the mean, standard deviation and skewness of wages by occupation as well as employment shares.

To capture the impact of robots on the wage distribution, I use results from Acemoglu and Restrepo (2017) who compute the impact of an additional robot per thousand workers

[23] See http://www.nber.org/data/morg.html

[24] The data on industrial robots from the IFR (2014) which Acemoglu and Restrepo (2017) use start in 1993. For labor market outcomes, Acemoglu and Restrepo (2017) use Census Data for 1990 and 2000 and Amercian Community Survey Data for 2007.

[25] Acemoglu and Autor (2011) use the labels ‘abstract’ for cognitive non-routine and ‘services’ for manual non-routine. See Cortes (2016) for the same classification and labels as used in this paper.

on deciles of the wage distribution (Acemoglu and Restrepo, 2017, Figure 13). I use results from their long-differences specification and convert the reported semi-elasticities to elasticities by using that in 1993 there were 0.36 robots per thousand workers in the US (IFR, 2014).[26]

I also target the impact of a change in the price of robots on robot adoption. This is an important moment since a tax on robots affects robot adoption via the same channel: by changing the (user) price of robots. According to data from the IFR (2006), the quality-adjusted price of robots dropped by about 60% between 1993 and 2005 (the latest date for which I have quality-adjusted price data), while the stock of robots roughly tripled. For lack of better evidence, I treat the price change in the data as exogenous, and as the only driver of robot adoption, implying a price-elasticity of robot adoption of 3.33.

Finally, in the calibration I need to take a stance on the role of occupational switching in response to a change in the number of robots. Using cross-sectional data for the US, Acemoglu and Restrepo (2017) do not find evidence for occupational switching. In contrast, using panel data for Germany, Dauth et al. (2018) find that many workers adjust by switching occupation while staying at their original employer. Since the calibration aims to capture the impact of robots in the US, I rule out the possibility of individuals switching occupation. To do so, I assume that individuals choose their wage-maximizing occupation given the initial number of robots, but do not adjust their occupation as the number of robots changes. Still, the results by Dauth et al. (2018) demonstrate that occupational switching can be an important adjustment mechanism. When computing optimal policy, I therefore also explore the role of occupational switching.

Based on evidence reported by Blundell and McCurdy (1999) and Meghir and Phillips (2010), I set the labor-supply elasticity to $\varepsilon = 0.3$. The tax progressivity parameter in (54) is set to $t = 0.181$ as estimated by Heathcote et al. (2017). To calibrate the revenue parameter λ, I target the share of income-tax revenue in GDP which in the US in 1993 was 9.3% (OECD, 2017).[27]

5.2.2 Approach

All parameters which are not directly set from the data are based on minimizing the sum of weighted distances between model and data moments. To compute moments in the model, first, factor market equilibrium is computed for a given set of parameters. Then, model moments are computed given the equilibrium. In the model, factor market equilibrium is determined by the price of robots. In the data, no clear target for this price exists (the price index from the IFR (2006) is only meaningful for relative price changes,

[26] Note that Acemoglu and Restrepo (2017) have multiplied all figures by 100.

[27] When computing optimal policy below, I introduce an exogenous revenue requirement into the resource constraint (41) whose level is set to the level of tax revenue raised by the calibrated tax system.

Table 1: Calibration - Moments

Moment	Model	Data	Source of data moment
Employment share: manual	0.12	0.11	
Employment share: routine	0.57	0.59	Computed based on US CPS
Employment share: cognitive	0.32	0.30	
Income-tax revenue as share of GDP	0.09	0.09	OECD (2017)
Price-elasticity of robot adoption	18.41	3.33	Computed based on IFR (2006)

Note: Model moments are based on the calibrated model. All values are rounded. Matching of the remaining calibration targets is illustrated in Figures 1 and 2, and in Table 3 in the Appendix.

but not for levels). I therefore treat the price of robots as additional parameter (which is equivalent to treating the number of robots as a parameter).

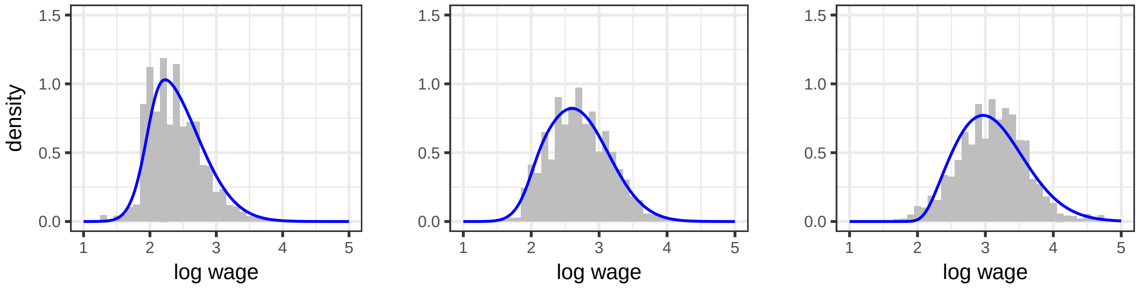
Moments of the distribution of wages and employment are based on the equilibrium which corresponds to the year 1993. In contrast, the elasticities of the wage-distribution-deciles, as well as the price-elasticity of robot adoption are computed based on changes to a new equilibrium which is brought about by a drop in the price of robots. To compute the new equilibrium, I treat the new price of robots as additional parameter, while keeping all other parameters fixed.[28]

5.2.3 Results

Table 2 in the Appendix summarizes the calibrated parameters. I highlight the substitution elasticity σ between routine labor and robots which is calibrated at 4.41, implying a high degree of substitutability. In contrast, cognitive labor is more complementary to the output of routine labor and robots, captured by $\rho = 1.67$, while the substitution elasticity of manual labor and all other factors is calibrated at 2.06. The calibrated production function implies elasticities $\varepsilon_{Y_C/Y_R,B} = 0.17$ and $\varepsilon_{Y_M/Y_R,B} = 0.15$. Assumption 1 is thus satisfied – and robots are better substitutes for routine labor than for non-routine labor. Moreover, the marginal product of cognitive labor increases relatively more with robots than the marginal product of manual labor.

To assess how well the calibration works, I report model and data moments for employment shares, the income-tax share in GDP, and the price-elasticity of robot adoption in Table 1. The performance regarding levels and changes of the wage distribution is illustrated in Figures 1 and 2. As reported in Table 1, employment shares as well as the share of income-tax revenue in GDP are matched well. However, the price-elasticity of robot adoption in the model is much larger than in the data. The inability to match the price-elasticity better is a result of also targeting changes in the wage distribution.

[28]While I describe the calibration as a sequence of steps, it is implemented in a single step as an equality-constrained non-linear minimization problem, in which the equality constraints ensure factor market clearing. The problem is solved using the optimization software Knitro®.



(a) manual non-routine (b) routine (c) cognitive non-routine

Figure 1: Calibration – Densities in the model vs. data

Note: The densities in blue are based on the calibrated model. The histograms in gray are based on US CPS data. The corresponding moments are provided in Table 3 in the Appendix.

The better the model matches the price-elasticity, the worse it does in matching changes in the wage distribution, and vice versa. The calibration is a compromise which, as I argue below, is likely to make taxation of robots more desirable than it would be if both types of targets were matched perfectly. Figure 1 plots the density of wages as implied by the calibration against a histogram of the data. Both line up well, and the model thus generates a realistic wage distribution. The plots also reveal considerable overlap of wage distributions across occupations.[29]

Figure 2 plots the elasticities of wage-distribution-deciles with respect to robots as implied by the calibration against the elasticities obtained from Acemoglu and Restrepo (2017). Qualitatively, the calibration matches the data: an increase in robots reduces deciles at the bottom of the wage distribution and raises deciles at the top. However, quantitatively, the impact of robots on wage deciles in the model is more negative at the bottom and more positive at the top than in the data. An increase in the number of robots thus generates too much inequality. As discussed above, matching changes in the wage distribution better would come at the expense of generating an even larger price-elasticity of robot adoption.

I will now argue that exaggerating the impact of robots on wage inequality and generating a too large price-elasticity of robot adoption makes taxation of robots more desirable than it would be otherwise: First, it is the goal of the robot tax to distort the use of robots, and a high price-elasticity of robot adoption makes the robot tax more distortionary – and as a consequence more desirable. Suppose, to the contrary, that the price-elasticity was zero. In this case, a tax on robots could not achieve any welfare gains. Second, the more inequality robots generate, the more a robot tax can achieve desirable wage compression. As a consequence, the welfare impact of the robot tax in the optimal tax simulations should be viewed as an upper bound. When it comes to the level of the robot tax, the effect of exaggerating the price-elasticity as well as the wage impact of

[29] The moments which correspond to the wage distributions are reported in Table 3 in the Appendix.

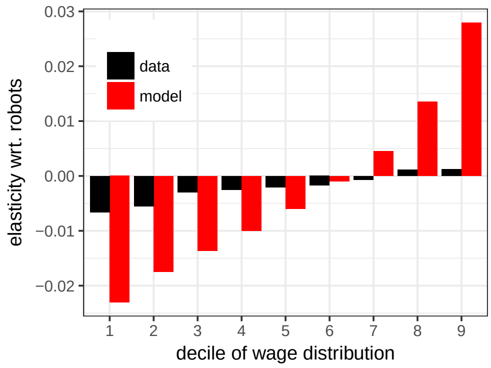


Figure 2: Elasticities of wage-distribution-deciles with respect to a change in robots
Note: Model moments are based on the calibrated model. Data moments are based on Figure 13 in Acemoglu and Restrepo (2017).

robot adoption is less clear. A high price-elasticity suggests a lower robot tax, whereas the generation of more inequality is a force for a higher robot tax.

5.3 Optimal policy

5.3.1 Social Welfare Weights

Before I can compute optimal policy, welfare weights need to be specified. I follow Rothschild and Scheuer (2013) and assume relative social welfare weights according to

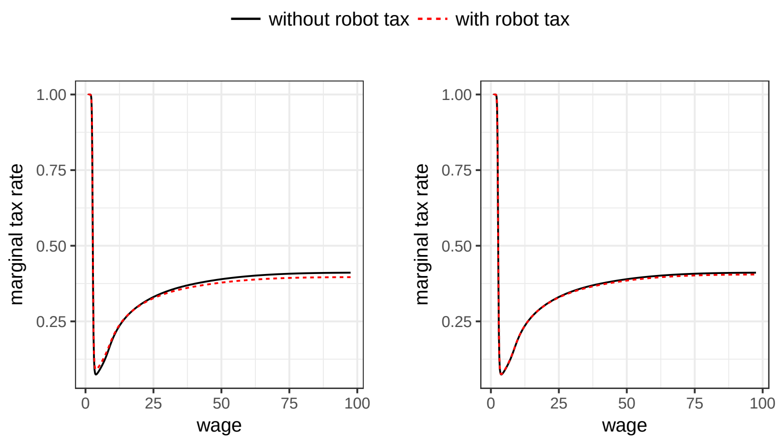
$$\Psi(w) = 1 - (1 - F)^r, \tag{57}$$

where r parametrizes the government's desire to redistribute. Here, $r = 1$ corresponds to utilitarian preferences, which combined with quasi-linear utility would imply no redistribution. As $r \to \infty$, the welfare weights approach that of a Rawlsian social planner. I set $r = 1.2$ such as to generate average marginal tax rates similar to those in the data. In the optimal tax simulations, average marginal tax rates are around 27%. For the US in 1993, the NBER reports an average marginal tax rate of 28%.[30]

5.3.2 Results

I compute optimal policy – non-linear income taxes and the robot tax – for two scenarios: without and with occupational switching.[31] One interpretation is that the scenario without occupational switching captures the short-run, whereas the scenario with occupational switching captures the medium-run. The two scenarios only differ by one factor: whether or not individuals respond to the introduction of a robot tax by switching occupation. The first scenario which rules out occupational switching corresponds to the

[30]See http://users.nber.org/~taxsim/allyup/fixed-ally.html
[31]Optimal policy is computed using the software package GPOPS-II (see Patterson and Rao, 2013).



(a) without occupational switching (b) with occupational switching

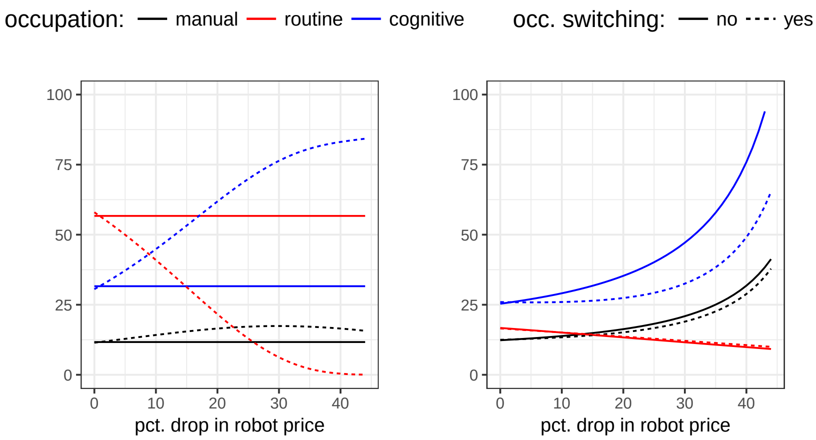
Figure 3: Effect of robot tax on optimal marginal income taxes

Note: The wage corresponds to hourly earnings in 2016-dollars. Results are for moderate redistributive preferences with $r = 1.2$. A wage of 100 corresponds to a value above the 99th percentile of the wage distribution. To obtain the U-shape, a mass point at the bottom has been imposed. Without this mass point, the optimal marginal income tax rate at the bottom is zero.

approach taken in the calibration. To prevent individuals from switching occupation in response to a tax on robots, I proceed as follows: first, equilibrium is computed for a model with occupational choice, but in which it is not possible to tax robots; next, I fix individuals' occupations, but allow for a tax on robots. In the second scenario, individuals always choose their occupation optimally.

For the case without occupational switching, I find an optimal tax on the stock of robots of 1.8%. It is thus optimal to distort the use of robots downward. Next, I compute the consumption-equivalent welfare gain of introducing a robot tax, provided that the income tax is already set optimally. Being able to tax robots is worth 0.04% of GDP, which based on US per capita GDP in 2016 translates into 21.14$ per person per year. In contrast, in the scenario with occupational switching, the optimal robot tax is 0.86%. Intuitively, individuals adjust their occupation in response to changes in wages which are brought about by the robot tax – and thereby partly offset those changes. With occupational switching, the robot tax is thus less effective in compressing wages, and is therefore optimally smaller. Moreover, occupational switching reduces the welfare gains to a share of 0.02% of GDP or 9.22$ per person per year.

Figure 3 plots optimal marginal income tax rates both for the case in which robots can and cannot be taxed. First, note that marginal tax rates follow the common U-shape as for example in Saez (2001). In both scenarios, if robots can be taxed, marginal income tax rates are slightly higher at low-to-medium wages, and slightly lower at high wages. However, the effect is stronger if occupational switching is ruled out. For the simple model in Section 3, I derive that marginal income taxes in the presence of a robot tax are ceteris paribus lower for manual and cognitive workers. The effect on marginal income



(a) Employment shares (b) Average wage

Figure 4: Effect of drop in price of robots on employment and wages

Note: The horizontal axis measures the percentage drop in the (producer) price of robots with respect to the calibrated initial price. All outcomes are computed given the optimal non-linear income tax and the optimal robot tax at each level of the price of robots.

taxes for routine workers is ambiguous. As cognitive workers are concentrated at high wages, the result of lower taxes carries over to the full model. Moreover, in terms of the simple model, lower marginal tax rates for manual workers are overturned by higher marginal taxes for routine workers. Of course, the equivalence with the simple model is imperfect as wage distributions now overlap occupations.

Effect of a drop in the price of robots. Next, I study the effect of productivity increases in robot production. To do so, I ask how the corresponding drop in the (producer) price of robots affects the economy, as well as the optimal tax on robots. I start from the calibrated robot price and then compute optimal policy and other outcomes while gradually lowering the price of robots. Again, I consider two scenarios: one without and one with occupational switching. In the scenario without occupational switching, I fix employment shares at the optimal choice, given the initial price of robots, and given that income is taxed optimally, but ruling out robot taxation. As a result, the optimal robot tax at the initial robot price is 1.8% as above. In the second scenario, individuals choose their occupation optimally at each level of the robot price. The optimal robot tax at the initial robot price is therefore 0.86% like before.

Figure 4 shows the impact of a drop in the price of robots on employment shares and average wages. The horizontal axis measures the drop in the robot price relative to its initial level, hence 0 corresponds to the initial price of robots, while 40 corresponds to a robot price which is 40% lower than its initial level. Without occupational switching, employment shares are constant (Panel 4a). With occupational switching, cheaper robots substitute more and more for routine workers, who then switch to either manual or cognitive work. Eventually, the share of routine occupations goes to zero. If one interprets the employment changes as changes in the cross-section, then they correspond qualitatively to the empirically documented employment polarization. For example, Acemoglu and Autor (2011) report that the employment share of cognitive non-routine workers increased by 4.6 percentage points between 1989 and 2007, whereas the share of manual non-routine workers increased by 3.5 percentage points. In contrast, the share of routine workers dropped by 8.1 percentage points.[32]

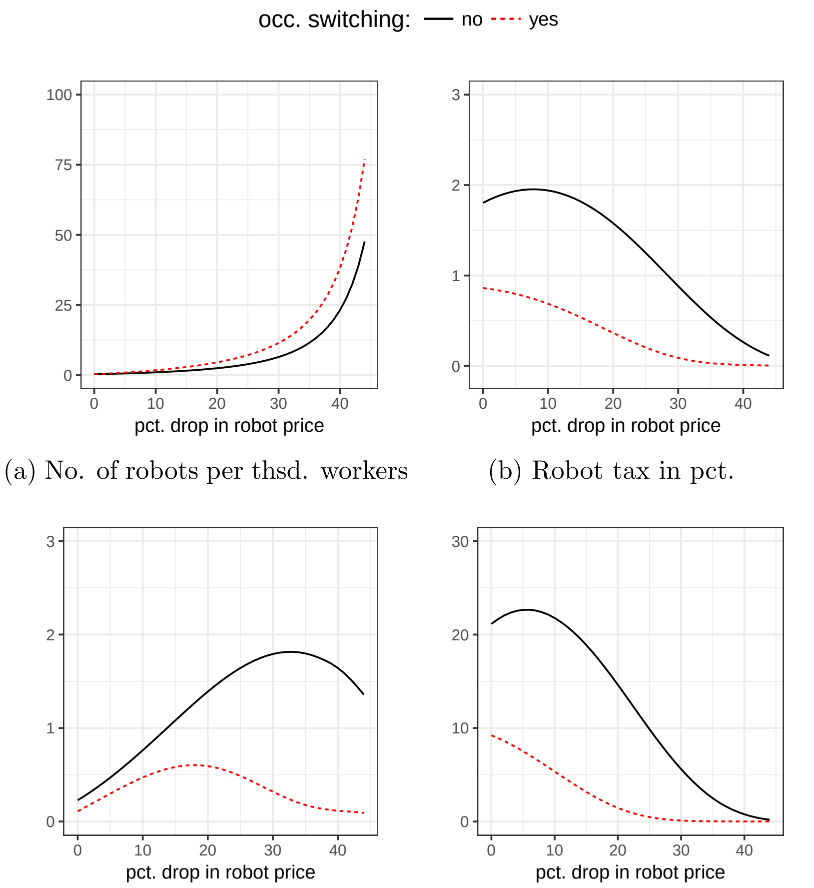
Panel 4b shows that in both scenarios the average wage of routine workers decreases and eventually drops below the initial average wage of manual workers. Average wages for non-routine occupations increase, and more so for cognitive workers. Without occupational switching, average wages are driven by changing wage rates alone, as the composition of skills within occupations remains constant. With occupational switching, the wage impact of cheaper robots is dampened for two reasons: first, due to occupational switching, aggregate labor supply increases in non-routine occupations relative to routine occupations; second, the skill-composition within non-routine occupations worsens, whereas the skill-composition within routine occupations improves.

Figure 5 illustrates how a drop in the price of robots affects the number of robots, as well as optimal policy, robot-tax revenue and welfare. Panel 5a shows a strong increase in the number of robots per thousand workers, from below 1 to more than 50. The increase is stronger if occupational switching is possible. There are two reasons for this difference. First, with occupational switching, the allocation of skills is more efficient. As individuals move into occupations which are more complementary to robots, the return to robot adoption is higher. Second, the robot tax is lower in the scenario with occupational switching.

Panel 5b shows how the optimal robot tax changes as the price of robots falls. If occupational switching is ruled out, the robot tax first increases slightly, but then falls. With occupational switching, the robot tax declines monotonically to approach a level close to zero.

One reason for the falling robot tax in Panel 5b is mechanical: since the number of robots increases faster than their price falls, the value of robots goes up – which according to the optimal robot-tax formula (42) calls for a lower tax on robots for a given level of robot-tax revenue. However, as shown in Panel 5c, robot-tax revenue is not constant. Instead, it is hump-shaped in both scenarios. Since robot-tax revenue represents the left-hand side of (42), the right-hand side of (42) follows the same pattern. Incentive and effort reallocation effects (and occupational shift effects in the scenario with occupational switching) thus first become more important as the price of robots falls, but eventually decline.

[32]Based on Table 3a in Acemoglu and Autor (2011), where I consider professional, managerial and technical workers as cognitive non-routine, service workers as manual non-routine, and clerical, sales, and production workers as well as operators as routine.



(a) No. of robots per thsd. workers

(b) Robot tax in pct.

(c) Robot-tax revenue

(d) Welfare in 2016-dollars per cap.

Figure 5: Effect of drop in price of robots on various outcomes

Note: The horizontal axis measures the percentage drop in the (producer) price of robots with respect to the calibrated initial price. All outcomes are computed given the optimal non-linear income tax and the optimal robot tax at each level of the price of robots. Robot-tax revenue is expressed as share of initial GDP.

Next, I turn to the welfare impact of introducing a robot tax, provided that income is taxed optimally. Panel 5d plots the consumption-equivalent welfare gain of introducing a robot tax expressed in 2016-dollars per capita per year. The pattern mirrors that of the robot tax: without occupational switching, the welfare gain first increases to about 23$, but then declines; with occupational switching, welfare gains are lower to begin with, and monotonically approach a value close to zero.

5.4 Discussion

The quantitative results show that a positive robot tax is optimal. However, its welfare impact is small. Moreover, even though inequality increases as robots become cheaper, this does not warrant larger robot taxes. To the contrary, the robot tax and its welfare impact become negligible. The optimal robot tax is in the same order of magnitude as the tax found by Costinot and Werning (2018) who – using their sufficient-statistics formula – find an optimal robot tax of 2.7% for a labor-supply elasticity of $\varepsilon = 0.3$. Moreover, for a stylized model they show that a drop in the price of robots leads to greater inequality, but may reduce the optimal robot tax. I confirm these results in my fully calibrated model.

Guerreiro et al. (2017) also find that the robot tax eventually decreases as the price of robots falls. In their paper, the optimal robot tax becomes zero once there are no routine workers left. The reason is that the remaining non-routine workers are homogeneously affected by robots. As a result, a tax on robots cannot anymore compress wage differentials, and should therefore no longer be used. In this paper, manual and cognitive non-routine workers are differentially affected by robots, and a tax on robots can thus still achieve desirable wage compression, even if there are no routine workers left. As a consequence, while becoming negligible, the robot tax in this paper does not reach zero.

6 Conclusion

This paper studies the optimal taxation of robots and labor income in a model in which robots substitute for routine labor and complement non-routine labor. Intuition is developed in a stylized model based on Stiglitz (1982), which features intensive-margin labor supply and endogenous wages, but in which types are discrete and occupations are fixed. The full model then introduces continuous wage distributions and occupational choice, building upon Rothschild and Scheuer (2013, 2014).

I find that in general, the optimal robot tax is not zero, thereby violating production efficiency (Diamond and Mirrlees, 1971). The robot tax exploits general-equilibrium effects to compress the wage distribution. As a consequence, income taxation becomes less distortionary – which allows for more redistribution overall, and increases welfare. Since workers in routine occupations are concentrated at medium incomes, the sign of the optimal robot tax is theoretically ambiguous. Taxing robots reduces inequality at high incomes, thereby locally lowering income-tax distortions of labor-supply; but it increases inequality at low incomes, and thus locally worsens labor-supply distortions. If manual and cognitive non-routine labor are sufficiently complementary, the presence of a robot tax leads to optimal marginal income tax rates which are higher at incomes earned by routine workers, and lower at incomes earned by non-routine workers.

To assess the optimal robot tax quantitatively, I calibrate the full model to the US economy. The calibration matches the distribution of wages and employment across manual non-routine, routine, and cognitive non-routine occupations. Moreover, it is informed by the labor-market impact of industrial robots studied by Acemoglu and Restrepo (2017). I compute optimal policy for two scenarios, one in which occupational switching is ruled out, and one in which individuals can switch occupation in response to the introduction of a robot tax. Without occupational switching, the optimal tax on the stock of robots is 1.8% and the consumption-equivalent welfare gain of introducing the tax is 21.14$ per person per year. With occupational switching, the effectiveness of the robot tax for compressing wages is reduced. The optimal robot tax and its welfare impact are then 0.86% and 9.22$. In both scenarios, optimal marginal income taxes are higher at low-to-medium incomes and lower at high incomes if robots can be taxed.

Finally, I study the effect of a drop in the price of robots for the two scenarios. With occupational switching, the share of routine workers approaches zero, as they move into non-routine occupations. Moreover, in both scenarios, wage inequality increases. Nevertheless, the robot tax eventually approaches a value close to zero. The same holds for the welfare gains of taxing robots: they never exceed 25$ per capita and eventually go to (almost) zero.

In light of the small welfare gains from taxing robots, this paper does not provide a strong case for a robot tax. Additional costs cast doubt on the optimality of a robot tax in practice. For example, with a tax on robots come considerable administrative costs as machinery needs to be classified into robots and non-robots. Moreover, I have abstracted from implications which a tax on robots would have in an open economy. As any tax on capital, a tax on robots could impact a firm's location choice with additional implications for inequality and welfare.

Appendix

A Derivations for simple model

A.1 Optimal robot tax

Assuming that only the adjacent downward-binding incentive constraints are relevant, maximized social welfare is given by the Lagrangian

$$\begin{aligned}\mathcal{L} =& f_M \psi_M V_M + f_R \psi_R V_R + f_C \psi_C V_C \\ &+ \mu \eta_{CR} \left(V_C - U \left(c_R \left(V_R, \ell_R \right), \ell_R \frac{Y_R \left(\mathbf{L}, B \right)}{Y_C \left(\mathbf{L}, B \right)} \right) \right) \\ &+ \mu \eta_{RM} \left(V_R - U \left(c_M \left(V_M, \ell_M \right), \ell_M \frac{Y_M \left(\mathbf{L}, B \right)}{Y_R \left(\mathbf{L}, B \right)} \right) \right) \\ &+ \mu \xi_M \left(f_M \ell_M - L_M \right) + \mu \xi_R \left(f_R \ell_R - L_R \right) + \mu \xi_C \left(f_C \ell_C - L_C \right) \\ &+ \mu \left(\sum_{i \in \mathcal{I}} f_i \ell_i Y_i \left(\mathbf{L}, B \right) + Y_B \left(\mathbf{L}, B \right) B - \sum_{i \in \mathcal{I}} f_i c_i - qB \right),\end{aligned} \tag{58}$$

with $\mathcal{I} \equiv \{M, R, C\}$, where μ is the multiplier on the resource constraint, $\mu\eta_{CR}$ is the multiplier on the incentive constraint for cognitive workers, and $\mu\eta_{RM}$ is the multiplier on the incentive constraint for routine workers. Moreover, $\mu\xi_i$ is the multiplier on the consistency condition for occupation i. To find an expression for the optimal robot tax, first write the resource constraint as $Y(\mathbf{L}, B) - \sum_{i \in \mathcal{I}} f_i c_i - qB = 0$. Then differentiate the Lagrangian with respect to B to obtain

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial B} =& - \mu \eta_{CR} U_\ell \left(c_R \left(V_R, \ell_R \right), \ell_R \frac{Y_R \left(\mathbf{L}, B \right)}{Y_C \left(\mathbf{L}, B \right)} \right) \ell_R \frac{\partial}{\partial B} \left(\frac{Y_R \left(\mathbf{L}, B \right)}{Y_C \left(\mathbf{L}, B \right)} \right) \\ &- \mu \eta_{RM} U_\ell \left(c_M \left(V_M, \ell_M \right), \ell_M \frac{Y_M \left(\mathbf{L}, B \right)}{Y_R \left(\mathbf{L}, B \right)} \right) \ell_M \frac{\partial}{\partial B} \left(\frac{Y_M \left(\mathbf{L}, B \right)}{Y_R \left(\mathbf{L}, B \right)} \right) \\ &+ \mu \left(Y_B \left(\mathbf{L}, B \right) - q \right).\end{aligned} \tag{59}$$

Define elasticities of relative wage rates with respect to robots as

$$\varepsilon_{Y_C/Y_R,B} = -\varepsilon_{Y_R/Y_C,B} \equiv \frac{\partial}{\partial B} \left(\frac{Y_R \left(\mathbf{L}, B \right)}{Y_C \left(\mathbf{L}, B \right)} \right) \frac{Y_C \left(\mathbf{L}, B \right)}{Y_R \left(\mathbf{L}, B \right)} B, \tag{60}$$

$$\varepsilon_{Y_M/Y_R,B} \equiv \frac{\partial}{\partial B} \left(\frac{Y_M \left(\mathbf{L}, B \right)}{Y_R \left(\mathbf{L}, B \right)} \right) \frac{Y_R \left(\mathbf{L}, B \right)}{Y_M \left(\mathbf{L}, B \right)} B. \tag{61}$$

Use that

$$Y_B \left(\mathbf{L}, B \right) = p = \left(1 + \tau \right) q \Leftrightarrow Y_B \left(\mathbf{L}, B \right) - q = \tau q, \tag{62}$$

set (59) equal to zero, rearrange and divide by μ to get

$$\begin{aligned}\tau q B = & -\eta_{CR} U_\ell \left(c_R \left(V_R, \ell_R\right), \ell_R \frac{Y_R\left(\mathbf{L}, B\right)}{Y_C\left(\mathbf{L}, B\right)} \right) \ell_R \frac{Y_R\left(\mathbf{L}, B\right)}{Y_C\left(\mathbf{L}, B\right)} \varepsilon_{Y_C/Y_R,B} \\ & + \eta_{RM} U_\ell \left(c_M \left(V_M, \ell_M\right), \ell_M \frac{Y_M\left(\mathbf{L}, B\right)}{Y_R\left(\mathbf{L}, B\right)} \right) \ell_M \frac{Y_M\left(\mathbf{L}, B\right)}{Y_R\left(\mathbf{L}, B\right)} \varepsilon_{Y_M/Y_R,B}.\end{aligned} \tag{63}$$

Now define the *incentive effects* in a similar way as Rothschild and Scheuer (2013) (using that $w_i = Y_i$ and suppressing some arguments)

$$I_{CR} \equiv -\eta_{CR} U_\ell \left(c_R, \ell_R \frac{w_R}{w_C} \right) \ell_R \frac{w_R}{w_C} \tag{64}$$

and

$$I_{RM} \equiv -\eta_{RM} U_\ell \left(c_M, \ell_M \frac{w_M}{w_R} \right) \ell_M \frac{w_M}{w_R}. \tag{65}$$

to write

$$\tau q B = \varepsilon_{Y_C/Y_R,B} I_{CR} - \varepsilon_{Y_M/Y_R,B} I_{RM}. \tag{66}$$

A.1.1 Expressions for incentive effects

To obtain expressions for the incentive effects (64) and (65), one needs to derive expressions for the multipliers η_{CR} and η_{RM}. To do so, differentiate the Lagrangian in (58) with respect to indirect utilities to obtain

$$\frac{\partial \mathcal{L}}{\partial V_M} = f_M \psi_M - \mu \eta_{RM} U_c \left(c_M, \ell_M \frac{w_M}{w_R} \right) \frac{\partial c_M}{\partial V_M} - \mu f_M \frac{\partial c_M}{\partial V_M}. \tag{67}$$

Equating to zero, using that $\partial c_M / \partial V_M = 1/U_c\left(c_M, \ell_M\right)$ and rearranging yields

$$\eta_{RM} = f_M \left(\frac{1}{\mu} \psi_M - \frac{1}{U_c\left(c_M, \ell_M\right)} \right) \frac{U_c\left(c_M, \ell_M\right)}{U_c\left(c_M, \ell_M \frac{w_M}{w_R}\right)}. \tag{68}$$

Analogously, we obtain

$$\frac{\partial \mathcal{L}}{\partial V_R} = f_R \psi_R - \mu \eta_{CR} U_c \left(c_R, \ell_R \frac{w_R}{w_C} \right) \frac{\partial c_R}{\partial V_R} + \mu \eta_{RM} - \mu f_R \frac{\partial c_R}{\partial V_R}, \tag{69}$$

which after equating to zero, substituting for $\partial c_R / \partial V_R = 1/U_c\left(c_R, \ell_R\right)$ and rearranging yields

$$\eta_{RM} = f_R \left(\frac{1}{U_c\left(c_R, \ell_R\right)} - \frac{1}{\mu} \psi_R \right) + \eta_{CR} U_c \left(c_R, \ell_R \frac{w_R}{w_C} \right) \frac{1}{U_c\left(c_R, \ell_R\right)}. \tag{70}$$

Finally, we have

$$\frac{\partial \mathcal{L}}{\partial V_C} = f_C \psi_C + \mu \eta_{CR} - \mu f_C \frac{\partial c_C}{\partial V_C}, \tag{71}$$

which after equating to zero, using $\partial c_C/\partial V_C = 1/U_c\left(c_C, \ell_C\right)$ and rearranging becomes

$$\eta_{CR} = f_C\left(\frac{1}{U_c\left(c_C, \ell_C\right)} - \frac{1}{\mu}\psi_C\right). \tag{72}$$

With quasi-linear utility as in (1) we have $\mu = 1$ and $U_c = 1$. As a result, one obtains

$$\eta_{CR} = f_C\left(1 - \psi_C\right), \tag{73}$$

and

$$\eta_{RM} = f_M\left(\psi_M - 1\right) = f_R\left(1 - \psi_R\right) + \eta_{CR} = f_R\left(1 - \psi_R\right) + f_C\left(1 - \psi_C\right). \tag{74}$$

Moreover, quasi-linear utility leads to $U_\ell = -\ell^{\frac{1}{\varepsilon}}$ such that the incentive effects are

$$I_{CR} = f_C\left(1 - \psi_C\right)\left(\ell_R\frac{w_R}{w_C}\right)^{1+\frac{1}{\varepsilon}}, \tag{75}$$

and

$$I_{RM} = f_M\left(\psi_M - 1\right)\left(\ell_M\frac{w_M}{w_R}\right)^{1+\frac{1}{\varepsilon}}. \tag{76}$$

A.2 Optimal income tax

To derive expressions for the optimal marginal income tax rates, I differentiate the Lagrangian (58) with respect to individual labor supplies ℓ_i .

A.2.1 Expression for T'_M

First consider $\partial\mathcal{L}/\partial\ell_M$, using that $Y_i = w_i$ for $i \in \mathcal{I}$ and suppressing some arguments

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\ell_M} = &- \mu\eta_{RM}\left[U_c\left(c\left(V_M, \ell_M\right), \ell_M\frac{w_M}{w_R}\right)\frac{\partial c\left(V_M, \ell_M\right)}{\partial\ell_M}\right.\\ &\left.+ U_\ell\left(c\left(V_M, \ell_M\right), \ell_M\frac{w_M}{w_R}\right)\frac{w_M}{w_R}\right] + \mu\xi_M f_M + \mu f_M\left(w_M - \frac{\partial c}{\partial\ell_M}\right).\end{aligned} \tag{77}$$

Use that

$$\frac{\partial c}{\partial\ell_i} = -\frac{U_\ell\left(c\left(V_i, \ell_i\right), \ell_i\right)}{U_c\left(c\left(V_i, \ell_i\right), \ell_i\right)} = w_i\left(1 - T'_i\right), \tag{78}$$

where the last step is based on the definition of marginal tax rates. Substituting in (77), I obtain

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \ell_M} = & -\mu \eta_{RM} \Bigg[U_c \left(c\left(V_M, \ell_M\right), \ell_M \frac{w_M}{w_R} \right) w_M \left(1 - T'_M\right) \\ & + U_\ell \left(c\left(V_M, \ell_M\right), \ell_M \frac{w_M}{w_R} \right) \frac{w_M}{w_R} \Bigg] + \mu \xi_M f_M + \mu f_M \left(w_M - w_M \left(1 - T'_M\right)\right)\end{aligned} \tag{79}$$

Setting equal to zero, dividing by μ and collecting terms yields

$$\begin{aligned}\frac{T'_M + \frac{\xi_M}{Y_M}}{1 - T'_M} = & \frac{1}{f_M} \eta_{RM} \Bigg[U_c \left(c\left(V_M, \ell_M\right), \ell_M \frac{w_M}{w_R} \right) \\ & - \frac{U_c\left(c\left(V_M, \ell_M\right), \ell_M\right)}{U_\ell\left(c\left(V_M, \ell_M\right), \ell_M\right)} U_\ell \left(c\left(V_M, \ell_M\right), \ell_M \frac{w_M}{w_R} \right) \frac{w_M}{w_R} \Bigg].\end{aligned} \tag{80}$$

With quasi-linear utility the expression becomes

$$\frac{T'_M + \frac{\xi_M}{Y_M}}{1 - T'_M} = \left(\psi_M - 1\right) \left(1 - \left(\frac{w_M}{w_R} \right)^{1 + \frac{1}{\varepsilon}} \right). \tag{81}$$

A.2.2 Expression for T'_R

The first-order condition with respect to ℓ_R is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \ell_R} = & -\mu \eta_{CR} \left[U_c \left(c\left(V_R, \ell_R\right), \ell_R \frac{w_R}{w_C} \right) \frac{\partial c}{\partial \ell_R} + U_\ell \left(c\left(V_R, \ell_R\right), \ell_R \frac{w_R}{w_C} \right) \frac{w_R}{w_C} \right] \\ & + \mu \xi_R f_R + \mu f_R \left(w_R - \frac{\partial c}{\partial \ell_R} \right).\end{aligned} \tag{82}$$

I thus obtain

$$\begin{aligned}\frac{T'_R + \frac{\xi_R}{Y_R}}{1 - T'_R} = & \frac{1}{f_R} \eta_{CR} \Bigg[U_c \left(c\left(V_R, \ell_R\right), \ell_R \frac{w_R}{w_C} \right) \\ & - \frac{U_c\left(c\left(V_R, \ell_R\right), \ell_R\right)}{U_\ell\left(c\left(V_R, \ell_R\right), \ell_R\right)} U_\ell \left(c\left(V_R, \ell_R\right), \ell_R \frac{w_R}{w_C} \right) \frac{w_R}{w_C} \Bigg],\end{aligned} \tag{83}$$

which with quasi-linear utility translates into

$$\frac{T'_R + \frac{\xi_R}{Y_R}}{1 - T'_R} = \frac{f_C}{f_R} \left(1 - \psi_C\right) \left(1 - \left(\frac{w_R}{w_C} \right)^{1 + \frac{1}{\varepsilon}} \right). \tag{84}$$

A.2.3 Expression for T'_C

Finally, the first-order condition with respect to ℓ_C is

$$\frac{\partial \mathcal{L}}{\partial \ell_c} = \mu \xi_C f_C + \mu f_C \left(w_C - \frac{\partial c}{\partial \ell_C} \right). \tag{85}$$

Setting equal to zero, using (78) and rearranging leads to

$$T'_C = -\frac{\xi_C}{Y_C}. \tag{86}$$

A.2.4 Expressions for multipliers ξ_i

To derive expressions for the multipliers ξ_i, differentiate the Lagrangian (58) with respect to aggregate effective labor supplies L_i to obtain

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial L_i} = & -\mu \eta_{CR} U_\ell \left(c\left(V_R, \ell_R\right), \ell_R \frac{w_R}{w_C} \right) \ell_R \frac{\partial}{\partial L_i} \left(\frac{w_R}{w_C} \right) \\ & -\mu \eta_{RM} U_\ell \left(c\left(V_M, \ell_M\right), \ell_M \frac{w_M}{w_R} \right) \ell_M \frac{\partial}{\partial L_i} \left(\frac{w_M}{w_R} \right) \\ & -\mu \xi_i + \mu \left(\sum_{j \in \mathcal{I}} f_j \ell_j \frac{\partial Y_j}{\partial L_i} + \frac{\partial Y_B}{\partial L_i} B \right). \end{aligned} \tag{87}$$

Substitute $f_j \ell_j = L_j$. By Euler's Theorem, the effect on the resource constraint is zero. Using that at the optimum, a change in L_i has no effect on welfare and rearranging, I obtain

$$\begin{aligned} \xi_i = & -\eta_{CR} U_\ell \left(c\left(V_R, \ell_R\right), \ell_R \frac{w_R}{w_C} \right) \ell_R \frac{\partial}{\partial L_i} \left(\frac{w_R}{w_C} \right) \\ & -\eta_{RM} U_\ell \left(c\left(V_M, \ell_M\right), \ell_M \frac{w_M}{w_R} \right) \ell_M \frac{\partial}{\partial L_i} \left(\frac{w_M}{w_R} \right). \end{aligned} \tag{88}$$

Using the definitions of the incentive effects (75) and (76), we arrive at

$$\begin{aligned} \xi_i = & I_{CR} \frac{\partial}{\partial L_i} \left(\frac{w_R}{w_C} \right) \frac{w_C}{w_R} \\ & + I_{RM} \frac{\partial}{\partial L_i} \left(\frac{w_M}{w_R} \right) \frac{w_R}{w_M}. \end{aligned} \tag{89}$$

Now define the semi-elasticities of relative wages with respect to L_i as

$$\tilde{\varepsilon}_{w_R/w_C, L_i} \equiv \frac{\partial}{\partial L_i} \left(\frac{w_R}{w_C} \right) \frac{w_C}{w_R} \tag{90}$$

and

$$\tilde{\varepsilon}_{w_M/w_R,L_i} \equiv \frac{\partial}{\partial L_i}\left(\frac{w_M}{w_R}\right)\frac{w_R}{w_M} \tag{91}$$

to write

$$\xi_i = \tilde{\varepsilon}_{w_R/w_C,L_i} I_{CR} + \tilde{\varepsilon}_{w_M/w_R,L_i} I_{RM}. \tag{92}$$

Signing the multipliers. The sign of the multiplier is determined by the terms $\frac{\partial}{\partial L_i}\left(\frac{w_R}{w_C}\right)$ and $\frac{\partial}{\partial L_i}\left(\frac{w_M}{w_R}\right)$. The sign of ξ_i is unambiguous if both of these terms have the same sign. Consider ξ_M. We have $\frac{\partial}{\partial L_M}\left(\frac{w_M}{w_R}\right) < 0$. A sufficient condition for $\xi_M < 0$ is thus $\frac{\partial}{\partial L_M}\left(\frac{w_R}{w_C}\right) < 0$. Now consider ξ_C. We have $\frac{\partial}{\partial L_C}\left(\frac{w_R}{w_C}\right) > 0$, and hence $\frac{\partial}{\partial L_C}\left(\frac{w_M}{w_R}\right) > 0$ is a sufficient condition for $\xi_C > 0$. Finally, since $\frac{\partial}{\partial L_R}\left(\frac{w_R}{w_C}\right) < 0$, and $\frac{\partial}{\partial L_R}\left(\frac{w_M}{w_R}\right) > 0$ the sign of ξ_R is ambiguous and depends on the magnitudes of the different terms in (92).

B Optimal tax on robots with continuous types and occupational choice

In order to derive an expression for the optimal tax on robots, I use that at the optimum, a marginal change in robots B, has no first-order welfare effect. The welfare effect of a marginal change in B corresponds to differentiating the Lagrangian of the inner problem, evaluated at the optimal allocation, with respect to B. The Lagrangian of the inner problem is given by

$$\begin{aligned}
\mathcal{L} = &\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} V\left(c\left(w\right),\ell\left(w\right)\right) d\Psi\left(w\right) \\
&+ \mu \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \eta\left(w\right) U_\ell\left(c\left(V\left(w\right),\ell\left(w\right)\right),\ell\left(w\right)\right)\frac{\ell\left(w\right)}{w} dw \\
&+ \mu\xi_M\left(L_M - \frac{1}{Y_M\left(\mathbf{L},B\right)}\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell\left(w\right) f^M_{\mathbf{L},B}\left(w\right) dw\right) \\
&+ \mu\xi_R\left(L_R - \frac{1}{Y_R\left(\mathbf{L},B\right)}\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell\left(w\right) f^R_{\mathbf{L},B}\left(w\right) dw\right) \\
&+ \mu\xi_C\left(L_C - \frac{1}{Y_C\left(\mathbf{L},B\right)}\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell\left(w\right) f^C_{\mathbf{L},B}\left(w\right) dw\right) \\
&+ \mu\left(\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell\left(w\right) f\left(w\right) dw + Y_B\left(\mathbf{L},B\right) B - \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} c\left(V\left(w\right),\ell\left(w\right)\right) f\left(w\right) dw - qB\right).
\end{aligned} \tag{93}$$

The impact of a marginal change in B can be decomposed into four effects. First, there is a *direct effect* on the resource constraint. The other three effects result from the impact which a change in B has on wages: The second effect is the *direct* result from a change in wages, leading to changes in $\ell(w)$ and $V(w)$. The third effect is the *indirect* result of changing wages. As wages change, individuals move along the schedules $\ell(w)$ and $V(w)$. Finally, as relative wage rates change, some individuals *switch occupation*, which has an effect on the consistency conditions.

Instead of computing the effects by holding the schedules $\ell(w)$ and $V(w)$ fixed, using the envelope theorem, I follow Rothschild and Scheuer (2014) and construct variations in the schedules $\ell(w)$ and $V(w)$ which simplify the derivations. The idea is as follows: Instead of having to take into account that changes in B alter the wage densities, the adjustment of $\ell(w)$ and $V(w)$ is chosen such that it offsets, at each w, changes which otherwise would require adjusting the densities. Denote the adjusted schedules by $\tilde{\ell}(w)$ and $\tilde{V}(w)$.

Schedule variations. In what follows, I first derive $\tilde{\ell}(w)$ and $\tilde{V}(w)$ which requires some preparation: Indicate occupations by $i \in \mathcal{I} \equiv \{M, R, C\}$ and denote by

$$\beta_B^i(\mathbf{L}, B) \equiv \frac{\partial Y_i(\mathbf{L}, B)}{\partial B} \frac{1}{Y_i(\mathbf{L}, B)}, \tag{94}$$

the semi-elasticity of the skill-price $Y_i(\mathbf{L}, B)$ with respect to B. I define the indicator $q_{\mathbf{L},B}^i(\theta)$ such that

$$q_{\mathbf{L},B}^i(\theta) = \begin{cases} 1, & \text{if } \theta \text{ works in } i \\ 0, & \text{otherwise.} \end{cases} \tag{95}$$

Using that wages are given by

$$w_{\mathbf{L},B}(\theta) = \max\{Y_M(\mathbf{L}, B)\theta_M, Y_R(\mathbf{L}, B)\theta_R, Y_C(\mathbf{L}, B)\theta_C\}, \tag{96}$$

the wage of individual θ can thus be written as

$$w_{\mathbf{L},B}(\theta) = \sum_{i\in\mathcal{I}} q_{\mathbf{L},B}^i(\theta)\,\theta Y_i(\mathbf{L}, B). \tag{97}$$

The semi-elasticity of wages with respect to B for individual θ is thus

$$\begin{aligned} \frac{\partial w_{\mathbf{L},B}(\theta)}{\partial B}\frac{1}{w_{\mathbf{L},B}(\theta)} &= \sum_{i\in\mathcal{I}} q_{\mathbf{L},B}^i(\theta)\frac{\partial Y_i(\mathbf{L}, B)}{\partial B}\frac{1}{Y_i(\mathbf{L}, B)} \\ &= \sum_{i\in\mathcal{I}} q_{\mathbf{L},B}^i(\theta)\,\beta_B^i(\mathbf{L}, B). \end{aligned} \tag{98}$$

Like Rothschild and Scheuer (2014), I now construct $\tilde{\ell}(w)$ and $\tilde{V}(w)$ such that at each w, changes in average labor supply and indirect utility which come from individuals shifting along $\ell(w)$ and $V(w)$ are offset. I focus on deriving $\tilde{\ell}(w)$. The derivations for $\tilde{V}(w)$ are analogue. First, consider an individual θ. A change dB, which leads to a change in $w_{\mathbf{L},B}(\theta)$, causes θ to adjust labor supply by

$$\ell'(w)\frac{\partial w_{\mathbf{L},B}(\theta)}{\partial B}dB = \ell'(w)\,w\sum_{i\in\mathcal{I}} q^i_{\mathbf{L},B}(\theta)\,\beta^i_B(\mathbf{L},B)\,dB, \tag{99}$$

where I use (98). Now, note that the same wage w can be earned by different individuals if wage distributions overlap across occupations. In order to compute the average adjustment in labor supply, we need to compute the expected adjustment over all types θ earning w, that is

$$\ell'(w)\,w\sum_{i\in\mathcal{I}}\mathbb{E}\left[q^i_{\mathbf{L},B}(\theta)\,|w\right]\beta^i_B(\mathbf{L},B)\,dB = \ell'(w)\,w\sum_{i\in\mathcal{I}}\frac{f^i_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}\beta^i_B(\mathbf{L},B)\,dB, \tag{100}$$

where I use that $\mathbb{E}\left[q^i_{\mathbf{L},B}(\theta|w)\right]$ corresponds to the share of individuals who earn w in occupation i, that is $f^i(w)/f(w)$. I now obtain the adjusted schedule $\tilde{\ell}(w)$ by subtracting the change in $\ell(w)$, induced by a change dB, from $\ell(w)$, that is

$$\tilde{\ell}(w) \equiv \ell(w) - \ell'(w)\,w\delta^B_{\mathbf{L},B}(w)\,dB,$$

where I define

$$\delta^B_{\mathbf{L},B}(w) \equiv \sum_{i\in\mathcal{I}}\frac{f^i_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}\beta^i_B(\mathbf{L},B). \tag{101}$$

The adjusted schedule $\tilde{V}(w)$ is

$$\tilde{V}(w) = V(w) - V'(w)\,w\delta^B_{\mathbf{L},B}(w)\,dB. \tag{102}$$

Since $l(w)$ and $V(w)$ are chosen optimally, by the envelope theorem, a marginal adjustment to $\tilde{\ell}(w)$ and $\tilde{V}(w)$ brought about by a marginal change dB has no first-order effect on welfare. I now consider the effect of a change dB on the different parts of the Lagrangian. The objective is not affected by a change dB.[33]

[33] An exception is the case in which the planner redistributes based on non-welfarist principles. For example, the planner might favor redistribution to certain occupations based on criteria other than the distribution of indirect utilities. In this case, effects on the objective need to be taken into account, to which Rothschild and Scheuer (2013, 2014) refer to as *redistributive effects*.

Incentive constraint effect. The incentive constraint is as in Rothschild and Scheuer (2014), who show that the schedule modification to $\tilde{\ell}(w)$ and $\tilde{V}(w)$ changes

$$V'(w) - U_\ell(c(w), \ell(w))\,\ell(w)/w \tag{103}$$

by $-V'(w)\,w\ d\delta^i_{\mathbf{L},B}(w)/dw\ dB$. Integrating over all wages then leads to the following effect on the incentive constraint

$$\begin{aligned} &-\sum_{i\in\mathcal{I}}\beta^i_B(\mathbf{L},B)\,\mu\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}}\frac{\eta(w)}{U_c(w)}V'(w)\,w\frac{d}{dw}\left(\frac{f^i_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}\right)dw\ dB \\ =&-\mu\sum_{i\in\mathcal{I}}\beta^i_B(\mathbf{L},B)\,I_i(\mathbf{L},B)\ dB, \end{aligned} \tag{104}$$

with the *incentive effect*

$$I_i(\mathbf{L},B)\equiv\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}}\frac{\eta(w)}{U_c(w)}V'(w)\,w\frac{d}{dw}\left(\frac{f^i_{\mathbf{L},B}(w)}{f_{\mathbf{L},B}(w)}\right)dw. \tag{105}$$

Resource constraint effect. The expression in (93) pertaining to the resource constraint is

$$\mu\left(\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell(w)\,f(w)\,dw-\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} c(V(w),\ell(w))\,f(w)\,dw+Y_B(\mathbf{L},B)\,B-qB\right). \tag{106}$$

First, a change in B has a direct effect on $Y_B(\mathbf{L},B)\,B-qB$, given by

$$\frac{\partial Y_B(\mathbf{L},B)}{\partial B}B+Y_B(\mathbf{L},B)-q. \tag{107}$$

Second, there is a direct effect on w in the first integrand, leading to

$$\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}}\delta^B_{\mathbf{L},B}(w)\,w\ell(w)\,f(w)\,dw. \tag{108}$$

Next, there are direct effects on $\ell(w)$ and $V(w)$, and thus on $c(V(w),\ell(w))$. However, these effects are exactly canceled out by varying the schedules to $\tilde{\ell}(w)$ and $\tilde{V}(w)$ to offset the indirect effect. Use $\delta^B_{\mathbf{L},B}(w) = \sum_i \beta^i_B \frac{f^i_{\mathbf{L},B}(w)}{f^i_{\mathbf{L},B}(w)}$ to rewrite (108) as

$$\begin{aligned} &\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \delta^i_B(w)\, w\ell(w)\, f(w)\, dw \\ =&\sum_{i\in\mathcal{I}} \beta^i_B(\mathbf{L},B) \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell(w)\, f^i_{\mathbf{L},B}(w)\, dw \\ =&\sum_{i\in\mathcal{I}} \frac{\partial Y_i(\mathbf{L},B)}{\partial B}\frac{1}{Y_i(\mathbf{L},B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w\ell(w)\, f^i_{\mathbf{L},B}(w)\, dw \\ =&\sum_{i\in\mathcal{I}} \frac{\partial Y_i(\mathbf{L},B)}{\partial B} L_i. \end{aligned} \tag{109}$$

Now use that due to linear homogeneity of the production function,

$$\sum_{i\in\mathcal{I}} \frac{\partial Y_i(\mathbf{L},B)}{\partial B} L_i + \frac{\partial Y_B(\mathbf{L},B)}{\partial B} B = 0, \tag{110}$$

hence adding (107) and (108) and multiplying by μ yields the resource constraint effect

$$\mu\left(Y_B(\mathbf{L},B) - q\right). \tag{111}$$

Consistency condition effects. Next, I turn to the effects on the consistency conditions. There are effort-reallocation effects and occupational-shift effects. Consider the consistency condition for M. The derivations for the other consistency conditions are analogue.

Effort-reallocation effect. First, rather than writing the condition in terms of wages, write it in terms of types θ as

$$L_M - \int_\Theta \theta_M \ell_M(\theta)\, dF(\theta). \tag{112}$$

Now use that in the Roy model individuals fully specialize and write

$$\ell_M(\theta) = \ell(w)\, q^M_{\mathbf{L},B}(\theta), \tag{113}$$

where $w = Y_M(\mathbf{L},B)\,\theta_M$. The integrand can then be written as

$$\theta_M q^M_{\mathbf{L},B}(\theta)\, \ell(w). \tag{114}$$

A change in B affects the expression via three channels: First, there is a direct effect on $\ell(w)$ as a change in B affects wages. Second, there is an indirect effect, as due to a change in wages, individuals move along the schedule $\ell(w)$. Third, a change in B affects relative wage rates across sectors, and thus occupational choice, captured by $q^M_{\mathbf{L},B}(\theta)$. Here, I focus on the first two effects. The third effect will be discussed as occupational-shift effect below.

For a single individual θ, the direct effect changes (114) by

$$\begin{aligned}&\theta_M q^M_{\mathbf{L},B}(\theta)\,\ell'(w)\,w\sum_{i\in\mathcal{I}} q^i_{\mathbf{L},B}(\theta)\,\beta^i_B(\mathbf{L},B)\,dB\\=&\frac{1}{Y_M(\mathbf{L},B)}\sum_{i\in\mathcal{I}}\beta^i_B(\mathbf{L},B)\,\ell'(w)\,w^2 q^M_{\mathbf{L},B}(\theta)\,q^i_{\mathbf{L},B}(\theta)\,dB,\end{aligned} \tag{115}$$

where the second step substituted $\theta_M = \frac{w}{Y_M(\mathbf{L},B)}$ and rearranged. To compute the effect at $w = Y_M(\mathbf{L},B)\,\theta_M$, one needs to take the expectation over all individuals θ who earn w, leading to

$$\frac{1}{Y_M(\mathbf{L},B)}\sum_{i\in\mathcal{I}}\beta^i_B(\mathbf{L},B)\,\ell'(w)\,w^2\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,q^i_{\mathbf{L},B}(\theta)\,|w\right]dB. \tag{116}$$

To offset the indirect effect on (114), change the schedule $\ell(w)$ to $\tilde{\ell}(w)$ by subtracting

$$\ell'(w)\,w\sum_{i\in\mathcal{I}}\mathbb{E}\left[q^i_{\mathbf{L},B}(\theta)\,|w\right]\beta^i_B(\mathbf{L},B)\,dB \tag{117}$$

from $\ell(w)$, which changes (114) by

$$-\theta_M q^M_{\mathbf{L},B}(\theta)\,\ell'(w)\,w\sum_{j\in\mathcal{I}}\mathbb{E}\left[q^i_{\mathbf{L},B}(\theta)\,|w\right]\beta^i_B(\mathbf{L},B)\,dB. \tag{118}$$

Again, this expression is for a single individual. To compute the effect at w, take the expectation over all θ earning w, which by the law of iterated expectations yields

$$\begin{aligned}&-\theta_M\ell'(w)\,w\sum_{j\in\mathcal{I}}\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\mathbb{E}\left[q^i_{\mathbf{L},B}(\theta)\,|w\right]\beta^i_B(\mathbf{L},B)\,dB\\=&-\frac{1}{Y_M(\mathbf{L},B)}\sum_{i\in\mathcal{I}}\beta^i_B(\mathbf{L},B)\,\ell'(w)\,w^2\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\mathbb{E}\left[q^i_{\mathbf{L},B}(\theta)\,|w\right]dB.\end{aligned} \tag{119}$$

Combine (116) and (119) to arrive at the change in (114) due to the direct and indirect effect

$$\begin{aligned}&\frac{1}{Y_M(\mathbf{L},B)}\sum_{i\in\mathcal{I}}\beta^i_B(\mathbf{L},B)\,\ell'(w)\,w^2\times\\&\left(\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,q^i_{\mathbf{L},B}(\theta)\,|w\right]-\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\mathbb{E}\left[q^i_{\mathbf{L},B}(\theta)\,|w\right]\right)dB\\=&\frac{1}{Y_M(\mathbf{L},B)}\sum_{i\in\mathcal{I}}\beta^i_B(\mathbf{L},B)\,\ell'(w)\,w^2\mathrm{Cov}\left[q^i_{\mathbf{L},B}(\theta)\,,q^M_{\mathbf{L},B}(\theta)\,|w\right]dB.\end{aligned} \tag{120}$$

Integrate over wages to obtain

$$\sum_{i\in\mathcal{I}} \beta_B^i(\mathbf{L},B)\frac{1}{Y_M(\mathbf{L},B)}\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \ell'(w)\,w^2\text{Cov}\left[q^i_{\mathbf{L},B}(\theta),q^M_{\mathbf{L},B}(\theta)\,|w\right]f(w)\,dw\;dB, \tag{121}$$

and define

$$C_{iM} \equiv \frac{1}{Y_M(\mathbf{L},B)}\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \ell'(w)\,w^2\text{Cov}\left[q^i_{\mathbf{L},B}(\theta),q^M_{\mathbf{L},B}(\theta)\,|w\right]f(w)\,dw, \tag{122}$$

and generally for $j\in\mathcal{I}$

$$C_{ij} \equiv \frac{1}{Y_j(\mathbf{L},B)}\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} \ell'(w)\,w^2\text{Cov}\left[q^i_{\mathbf{L},B}(\theta),q^j_{\mathbf{L},B}(\theta)\,|w\right]f(w)\,dw. \tag{123}$$

The effort-reallocation effect for the consistency condition which corresponds to occupation $j\in\mathcal{I}$ is then

$$-\mu\xi_j\sum_{i\in\mathcal{I}}\beta_B^i(\mathbf{L},B)\,C_{ij}\;dB. \tag{124}$$

No effort-reallocation effect if wage distributions do not overlap. Now suppose that wage distributions do not overlap sectors, that is, $q^i_{\mathbf{L},B}(\theta)\,|w=0$ for $i\neq M$. The expression in (120) then becomes

$$\begin{aligned}&\frac{1}{Y_M(\mathbf{L},B)}\beta_B^M(\mathbf{L},B)\,\ell'(w)\,w^2\times\\&\left(\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,q^M_{\mathbf{L},B}(\theta)\,|w\right]-\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\right)dB.\end{aligned} \tag{125}$$

Now use that $q^M_{\mathbf{L},B}(\theta)\,q^M_{\mathbf{L},B}(\theta)=q^M_{\mathbf{L},B}(\theta)$. Moreover, with no overlap of distributions all individuals who earn $w=Y_M(\mathbf{L},B)\,\theta_M$ are in occupation M, and thus $q^M_{\mathbf{L},B}(\theta)\,|w=1$. As a result, the term in parenthesis becomes

$$\begin{aligned}&\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,q^M_{\mathbf{L},B}(\theta)\,|w\right]-\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\\=&\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]-\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\mathbb{E}\left[q^M_{\mathbf{L},B}(\theta)\,|w\right]\\=&1-1,\end{aligned} \tag{126}$$

and hence there is no effort-reallocation effect if wage distributions do not overlap across occupations.

Occupational-shift effect. To derive the impact of occupational change on the consistency conditions, I focus on the effect on the condition for occupation M. The derivations for the other consistency conditions are analogue. Instead of writing the consistency conditions in terms of types θ, now write them again in terms of wages. The condition for occupation M is thus

$$L_M - \int_{\underline{w}_{\mathbf{L},B}}^{\bar{w}_{\mathbf{L},B}} \frac{1}{Y_M(\mathbf{L},B)} w\ell(w) f(w)\, dw = 0. \tag{127}$$

Focus on the effect on $\frac{1}{Y_M(\mathbf{L},B)} y(w)$, with income $y(w) \equiv w\ell(w)$. I first derive how income $y(w)$ earned in occupation M changes due to occupational shifts in response to an increase in B. A change in B alters wage rates $Y_i(\mathbf{L},B)$, which in turn affect occupational choice. Write the impact of a marginal increase in B on wage rate $Y_i(\mathbf{L},B)$ as $Y_i(\mathbf{L},B)\,\beta_B^i(\mathbf{L},B)$. Now, first consider how a marginal increase in $Y_R(\mathbf{L},B)$ affects occupational choice, and thus income $y(w)$ earned in occupation M. As $Y_R(\mathbf{L},B)$ increases, individuals are going to shift from occupation M to R. Since I consider a marginal change in Y_R, I focus on those individuals who are indifferent between M and R, which implies that they earn the same wage in both occupations, and thus

$$\theta_M Y_M(\mathbf{L},B) = \theta_R Y_R(\mathbf{L},B) \Leftrightarrow \theta_R = \theta_M \frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)}. \tag{128}$$

Moreover, individuals need to be better off working in occupation M than working in occupation C, thus

$$\theta_M Y_M(\mathbf{L},B) \geq \theta_C Y_C(\mathbf{L},B) \Leftrightarrow \theta_C \leq \theta_M \frac{Y_M(\mathbf{L},B)}{Y_C(\mathbf{L},B)}. \tag{129}$$

Having characterized the affected individuals, I next, consider how a change in relative prices due to a change in Y_R, ΔY_R, affects conditions (128) and (129). We get

$$\theta_R^* = \theta_M \frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)} + \theta_M \frac{\partial}{\partial Y_R}\left(\frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)}\right)\Delta Y_R, \tag{130}$$

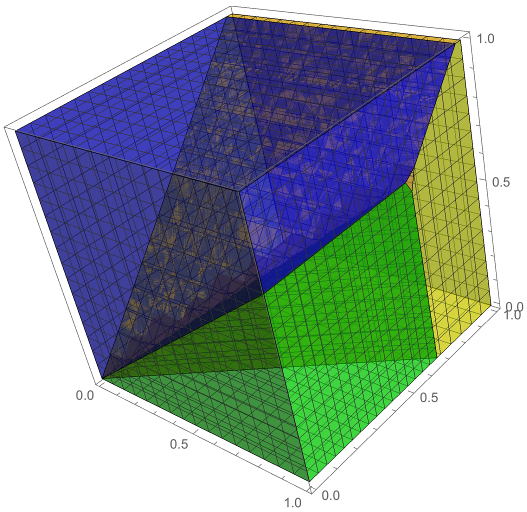
while (129) is not affected.

At a given point $(\theta_M, \theta_C, \theta_R)$, geometrically, the height of the polyhedron of individuals changing from occupation M to occupation R due to an increase in Y_R is given by (see Figure 6 for an illustration)

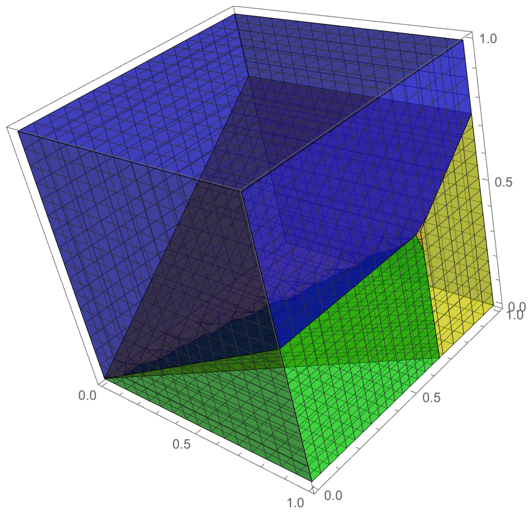
$$\theta_R^* - \theta_R = \theta_M \frac{\partial}{\partial Y_R}\left(\frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)}\right)\Delta Y_R. \tag{131}$$

The density at this point is

$$f(\theta_M, \theta_R, \theta_C) = f\left(\theta_M, \theta_M \frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)}, \theta_C\right). \tag{132}$$



(a) Before increase in Y_R



(b) After increase in Y_R

Figure 6: Illustration of occupational shifting due to increase in Y_R

Note: The axes correspond to the three dimensions of skill, θ_M, θ_R, θ_C. The green volume corresponds to the mass of manual workers, the yellow volume to the mass of cognitive workers, and the blue volume to the mass of routine workers. As Y_R increases, manual and cognitive workers move into routine occupations.

In order to compute the mass of individuals who switch from occupation M to R at a given θ_M with $\theta_R = \theta_M \frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)}$, we first need to integrate over all values θ_C for which individuals do not work in occupation C. This range of values is given by $[\underline{\theta}_C, \theta_M \frac{Y_M(\mathbf{L},B)}{Y_C(\mathbf{L},B)})$. Integrating over the density yields

$$\int_{\underline{\theta}_C}^{\theta_M \frac{Y_M(\mathbf{L},B)}{Y_C(\mathbf{L},B)}} f\left(\theta_M, \theta_M \frac{Y_M\left(\mathbf{L}, B\right)}{Y_R\left(\mathbf{L}, B\right)}, \theta_C\right) d\theta_C, \tag{133}$$

which at a given θ_M corresponds to a slice of the surface of indifference between sectors M and R. To arrive at the first expression for the mass of switchers, we need to multiply this slice of the surface by the height of the polyhedron of switchers, $\theta_M^* - \theta_M$, leading to

$$\theta_M \frac{\partial}{\partial Y_R}\left(\frac{Y_M\left(\mathbf{L}, B\right)}{Y_R\left(\mathbf{L}, B\right)}\right) \Delta Y_R \times \int_{\underline{\theta}_C}^{\theta_M \frac{Y_M(\mathbf{L},B)}{Y_C(\mathbf{L},B)}} f\left(\theta_M, \theta_M \frac{Y_M\left(\mathbf{L}, B\right)}{Y_R\left(\mathbf{L}, B\right)}, \theta_C\right) d\theta_C. \tag{134}$$

In order to compute the income moving from occupation M to occupation R, due to ΔY_R, write income as $\theta_M Y_M\left(\mathbf{L}, B\right) \ell\left(\theta_M Y_M\left(\mathbf{L}, B\right)\right)$, multiply by the mass of switchers at θ_M with $\theta_R = \theta_M \frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)}$ and integrate over θ_M, leading to

$$\begin{aligned} &\int_{\underline{\theta}_M}^{\overline{\theta}_M} \theta_M Y_M\left(\mathbf{L}, B\right) \ell\left(\theta_M Y_M\left(\mathbf{L}, B\right)\right) \times \\ &\theta_M \frac{\partial}{\partial Y_R}\left(\frac{Y_M\left(\mathbf{L}, B\right)}{Y_R\left(\mathbf{L}, B\right)}\right) \Delta Y_R \times \int_{\underline{\theta}_C}^{\theta_M \frac{Y_M(\mathbf{L},B)}{Y_C(\mathbf{L},B)}} f\left(\theta_M, \theta_M \frac{Y_M\left(\mathbf{L}, B\right)}{Y_R\left(\mathbf{L}, B\right)}, \theta_C\right) d\theta_C \, d\theta_M. \end{aligned} \tag{135}$$

Next, apply the change of variables $w = \theta_M Y_M(\mathbf{L}, B)$, which implies $d\theta_M = dw \frac{1}{Y_M(\mathbf{L},B)}$, to obtain

$$\begin{aligned} &\frac{1}{Y_M(\mathbf{L}, B)^2} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \\ &\frac{\partial}{\partial Y_R}\left(\frac{Y_M(\mathbf{L}, B)}{Y_R(\mathbf{L}, B)}\right) \Delta Y_R \times \int_{\underline{\theta}_C}^{\frac{w}{Y_C(\mathbf{L},B)}} f\left(\frac{w}{Y_M(\mathbf{L}, B)}, \frac{w}{Y_R(\mathbf{L}, B)}, \theta_C\right) d\theta_C \; dw. \end{aligned} \tag{136}$$

Now use that $\Delta Y_R = Y_R(\mathbf{L}, B)\, \beta_B^R(\mathbf{L}, B)\, \Delta B$ to get

$$\begin{aligned} &\beta_B^R(\mathbf{L}, B) \frac{Y_R(\mathbf{L}, B)}{Y_M(\mathbf{L}, B)^2} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \\ &\frac{\partial}{\partial Y_R}\left(\frac{Y_M(\mathbf{L}, B)}{Y_R(\mathbf{L}, B)}\right) \times \int_{\underline{\theta}_C}^{\frac{w}{Y_C(\mathbf{L},B)}} f\left(\frac{w}{Y_M(\mathbf{L}, B)}, \frac{w}{Y_R(\mathbf{L}, B)}, \theta_C\right) d\theta_C \; dw \Delta B. \end{aligned} \tag{137}$$

Use that $\frac{\partial}{\partial Y_R}\left(\frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)}\right) = -\frac{Y_M(\mathbf{L},B)}{Y_R(\mathbf{L},B)^2}$ to write

$$\begin{aligned} &-\beta_B^R(\mathbf{L}, B) \frac{1}{Y_M(\mathbf{L}, B)\, Y_R(\mathbf{L}, B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \\ &\int_{\underline{\theta}_C}^{\frac{w}{Y_C(\mathbf{L},B)}} f\left(\frac{w}{Y_M(\mathbf{L}, B)}, \frac{w}{Y_R(\mathbf{L}, B)}, \theta_C\right) d\theta_C \; dw \Delta B. \end{aligned} \tag{138}$$

Finally, use that in order to obtain the effect on $\frac{1}{Y_M(\mathbf{L},B)} y(w)$ I have to divide by $Y_M(\mathbf{L}, B)$, leading to

$$\begin{aligned} &-\beta_B^R(\mathbf{L}, B) \frac{1}{Y_M(\mathbf{L}, B)^2\, Y_R(\mathbf{L}, B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \\ &\int_{\underline{\theta}_C}^{\frac{w}{Y_C(\mathbf{L},B)}} f\left(\frac{w}{Y_M(\mathbf{L}, B)}, \frac{w}{Y_R(\mathbf{L}, B)}, \theta_C\right) d\theta_C \; dw \Delta B. \end{aligned} \tag{139}$$

Now define

$$\begin{aligned} S_{RM}(\mathbf{L}, B) \equiv &-\frac{1}{Y_M(\mathbf{L}, B)^2\, Y_R(\mathbf{L}, B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \\ &\int_{\underline{\theta}_C}^{\frac{w}{Y_C(\mathbf{L},B)}} f\left(\frac{w}{Y_M(\mathbf{L}, B)}, \frac{w}{Y_R(\mathbf{L}, B)}, \theta_C\right) d\theta_C \; dw. \end{aligned} \tag{140}$$

In an analogue way, derive

$$S_{CM}(\mathbf{L},B) \equiv -\frac{1}{Y_M(\mathbf{L},B)^2 Y_C(\mathbf{L},B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \int_{\underline{\theta}_R}^{\frac{w}{Y_R(\mathbf{L},B)}} f\left(\frac{w}{Y_M(\mathbf{L},B)}, \theta_R, \frac{w}{Y_C(\mathbf{L},B)}\right) d\theta_R \, dw. \tag{141}$$

Before providing expressions for the other terms, I repeat (part of) a Lemma from Rothschild and Scheuer (2014):[34]

Lemma 1. *With $\mathcal{I} \equiv \{M, R, C\}$, $\sum_{i\in\mathcal{I}} = C_{ij}(\mathbf{L},B) = \sum_{i\in\mathcal{I}} S_{ij}(\mathbf{L},B) = 0$ for all $j \in \mathcal{I}$.*

By Lemma 1

$$S_{MM}(\mathbf{L},B) = -S_{RM}(\mathbf{L},B) - S_{CM}(\mathbf{L},B). \tag{142}$$

This is intuitive: the inflow into occupation M is equal to the flows from occupations R and C into M.

Similarly, derive

$$S_{MR}(\mathbf{L},B) \equiv -\frac{1}{Y_M(\mathbf{L},B) Y_R(\mathbf{L},B)^2} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \int_{\underline{\theta}_C}^{\frac{w}{Y_C(\mathbf{L},B)}} f\left(\frac{w}{Y_M(\mathbf{L},B)}, \frac{w}{Y_R(\mathbf{L},B)}, \theta_C\right) d\theta_C \, dw, \tag{143}$$

$$S_{CR}(\mathbf{L},B) \equiv -\frac{1}{Y_R(\mathbf{L},B)^2 Y_C(\mathbf{L},B)} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \int_{\underline{\theta}_M}^{\frac{w}{Y_M(\mathbf{L},B)}} f\left(\theta_M, \frac{w}{Y_R(\mathbf{L},B)}, \frac{w}{Y_C(\mathbf{L},B)}\right) d\theta_M \, dw, \tag{144}$$

and by Lemma 1

$$S_{RR}(\mathbf{L},B) = -S_{MR}(\mathbf{L},B) - S_{CR}(\mathbf{L},B). \tag{145}$$

Finally, derive

$$S_{MC}(\mathbf{L},B) \equiv -\frac{1}{Y_M(\mathbf{L},B) Y_C(\mathbf{L},B)^2} \int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2 \ell(w) \times \int_{\underline{\theta}_R}^{\frac{w}{Y_R(\mathbf{L},B)}} f\left(\frac{w}{Y_M(\mathbf{L},B)}, \theta_R, \frac{w}{Y_C(\mathbf{L},B)}\right) d\theta_R \, dw, \tag{146}$$

[34] The corresponding Lemma in Rothschild and Scheuer (2014) is (the second part of) Lemma 6.

$$S_{RC}(\mathbf{L},B) \equiv -\frac{1}{Y_R(\mathbf{L},B)\,Y_C(\mathbf{L},B)^2}\int_{\underline{w}_{\mathbf{L},B}}^{\overline{w}_{\mathbf{L},B}} w^2\ell(w)\times \int_{\underline{\theta}_M}^{\frac{w}{Y_M(\mathbf{L},B)}} f\left(\theta_M,\frac{w}{Y_R(\mathbf{L},B)},\frac{w}{Y_C(\mathbf{L},B)}\right) d\theta_M\ dw, \tag{147}$$

and

$$S_{CC}(\mathbf{L},B) = -S_{MC}(\mathbf{L},B) - S_{RC}(\mathbf{L},B). \tag{148}$$

Having derived all occupational-shift effects, it remains to combine them. The occupational-shift effect which corresponds to the consistency condition for occupation j is

$$-\mu\xi_j \sum_{i\in\mathcal{I}} \beta_B^i(\mathbf{L},B)\,S_{ij}\ dB. \tag{149}$$

Putting everything together. Combining the terms derived above, we get

$$\frac{\partial\mathcal{L}(\mathbf{L},B)}{\partial B} = \mu\left[Y_B(\mathbf{L},B) - q - \sum_{i\in\mathcal{I}}\beta_B^i(\mathbf{L},B)\left(I_i(\mathbf{L},B) + \sum_{j\in\mathcal{I}}\xi_j\left(C_{ij}(\mathbf{L},B)+S_{ij}(\mathbf{L},B)\right)\right)\right]. \tag{150}$$

Now use that

$$Y_B(\mathbf{L},B) = (1+\tau)\,q \Leftrightarrow Y_B(\mathbf{L},B) - q = \tau q. \tag{151}$$

Since at the optimum $\partial\mathcal{L}(\mathbf{L},B)/\partial B = 0$, we get

$$\tau q = \sum_{i\in\mathcal{I}}\beta_B^i(\mathbf{L},B)\left(I_i(\mathbf{L},B) + \sum_{j\in\mathcal{I}}\xi_j\left(C_{ij}(\mathbf{L},B)+S_{ij}(\mathbf{L},B)\right)\right). \tag{152}$$

To further rewrite the expression, first focus on

$$\begin{aligned}
&\beta_B^R(\mathbf{L},B)\left(I_R(\mathbf{L},B) + \sum_{j\in\mathcal{I}}\xi_j\left(C_{Rj}(\mathbf{L},B)+S_{Rj}(\mathbf{L},B)\right)\right)\\
&= -\beta_B^R(\mathbf{L},B)\big[I_M(\mathbf{L},B) + I_C(\mathbf{L},B)\\
&\quad -\xi_M\left(C_{RM}(\mathbf{L},B)+S_{RM}(\mathbf{L},B)\right)\\
&\quad -\xi_R\left(C_{RR}(\mathbf{L},B)+S_{RR}(\mathbf{L},B)\right)\\
&\quad -\xi_C\left(C_{RC}(\mathbf{L},B)+S_{RC}(\mathbf{L},B)\right)\big]\\
&= -\beta_B^R(\mathbf{L},B)\big[I_M(\mathbf{L},B) + I_C(\mathbf{L},B)\\
&\quad +\xi_M\left(C_{MM}(\mathbf{L},B)+C_{CM}(\mathbf{L},B)+S_{MM}(\mathbf{L},B)+S_{CM}(\mathbf{L},B)\right)\\
&\quad +\xi_R\left(C_{MR}(\mathbf{L},B)+C_{CR}(\mathbf{L},B)+S_{MR}(\mathbf{L},B)+S_{CR}(\mathbf{L},B)\right)\\
&\quad +\xi_C\left(C_{MC}(\mathbf{L},B)+C_{CC}(\mathbf{L},B)+S_{MC}(\mathbf{L},B)+S_{CC}(\mathbf{L},B)\right)\big],
\end{aligned} \tag{153}$$

where the first step uses $\sum_{i\in\mathcal{I}} I_i = \sum_{i\in\mathcal{I}} R_i = 0$ and the second step uses Lemma 1. Substituting (153) for the respective expression in (152) and collecting terms yields

$$\begin{aligned}\tau q =& \left(\beta_B^C(\mathbf{L},B) - \beta_B^R(\mathbf{L},B)\right)\left(I_C(\mathbf{L},B) + \sum_{j\in\mathcal{I}} \xi_j \left(C_{Cj}(\mathbf{L},B) + S_{Cj}(\mathbf{L},B)\right)\right) \\ &+ \left(\beta_B^M(\mathbf{L},B) - \beta_B^R(\mathbf{L},B)\right)\left(I_M(\mathbf{L},B) + \sum_{j\in\mathcal{I}} \xi_j \left(C_{Mj}(\mathbf{L},B) + S_{Mj}(\mathbf{L},B)\right)\right).\end{aligned} \tag{154}$$

Finally, use that

$$\begin{aligned}\varepsilon_{Y_C/Y_R,B}(\mathbf{L},B) \equiv& \frac{\partial\left(Y_C(\mathbf{L},B)/Y_R(\mathbf{L},B)\right)}{\partial B}\frac{B}{Y_C(\mathbf{L},B)/Y_R(\mathbf{L},B)} \\ =& B\left(\beta_B^C(\mathbf{L},B) - \beta_B^R(\mathbf{L},B)\right),\end{aligned} \tag{155}$$

and

$$\begin{aligned}\varepsilon_{Y_M/Y_R,B}(\mathbf{L},B) \equiv& \frac{\partial\left(Y_M(\mathbf{L},B)/Y_R(\mathbf{L},B)\right)}{\partial B}\frac{B}{Y_M(\mathbf{L},B)/Y_R(\mathbf{L},B)} \\ =& B\left(\beta_B^M(\mathbf{L},B) - \beta_B^R(\mathbf{L},B)\right),\end{aligned} \tag{156}$$

to arrive at

$$\begin{aligned}\tau q B =& \varepsilon_{Y_C/Y_R,B}(\mathbf{L},B)\left(I_C(\mathbf{L},B) + \sum_{j\in\mathcal{I}} \xi_j \left(C_{Cj}(\mathbf{L},B) + S_{Cj}(\mathbf{L},B)\right)\right) \\ &+ \varepsilon_{Y_M/Y_R,B}(\mathbf{L},B)\left(I_M(\mathbf{L},B) + \sum_{j\in\mathcal{I}} \xi_j \left(C_{Mj}(\mathbf{L},B) + S_{Mj}(\mathbf{L},B)\right)\right).\end{aligned} \tag{157}$$

C Wage densities

The wage densities by occupation are given by[35]

$$f_{\mathbf{L},B}^M(w) \equiv \frac{1}{\sigma_M}\phi\left(\frac{\ln w - Y_M}{\sigma_M}\right)\Phi\left(\frac{\frac{\ln w - Y_R}{\sigma_R} - \rho_{MR}\frac{\ln w - Y_M}{\sigma_M}}{\sqrt{1-\rho_{MR}^2}}, \frac{\frac{\ln w - Y_C}{\sigma_c} - \rho_{MC}\frac{\ln w - Y_M}{\sigma_M}}{\sqrt{1-\rho_{MC}^2}}; \rho_{RC,M}\right), \tag{158}$$

$$f_{\mathbf{L},B}^R(w) \equiv \frac{1}{\sigma_R}\phi\left(\frac{\ln w - Y_R}{\sigma_R}\right)\Phi\left(\frac{\frac{\ln w - Y_M}{\sigma_M} - \rho_{MR}\frac{\ln w - Y_R}{\sigma_R}}{\sqrt{1-\rho_{MR}^2}}, \frac{\frac{\ln w - Y_C}{\sigma_C} - \rho_{RC}\frac{\ln w - Y_R}{\sigma_R}}{\sqrt{1-\rho_{RC}^2}}; \rho_{MC,R}\right), \tag{159}$$

$$f_{\mathbf{L},B}^C(w) \equiv \frac{1}{\sigma_C}\phi\left(\frac{\ln w - Y_C}{\sigma_C}\right)\Phi\left(\frac{\frac{\ln w - Y_M}{\sigma_M} - \rho_{MC}\frac{\ln w - Y_C}{\sigma_C}}{\sqrt{1-\rho_{MC}^2}}, \frac{\frac{\ln w - Y_R}{\sigma_R} - \rho_{RC}\frac{\ln w - Y_C}{\sigma_C}}{\sqrt{1-\rho_{RC}^2}}; \rho_{MR,C}\right). \tag{160}$$

[35] Technically, the expressions are not densities, since they do not integrate to one. Instead, the expressions integrate to the mass of individuals in the respective occupation.

Here, ϕ is the standard Normal density and Φ is the CDF of a bivariate standard Normal distribution with covariance $\rho_{ab,c}$. Following Bi and Mukherjea (2010), we have

$$\rho_{RC,M} \equiv \frac{\rho_{RC} - \rho_{MR}\rho_{MC}}{\sqrt{1-\rho_{MR}^2}\sqrt{1-\rho_{MC}^2}}, \ \rho_{MC,R} \equiv \frac{\rho_{MC} - \rho_{MR}\rho_{RC}}{\sqrt{1-\rho_{MR}^2}\sqrt{1-\rho_{RC}^2}}, \ \rho_{MR,C} \equiv \frac{\rho_{MR} - \rho_{MC}\rho_{RC}}{\sqrt{1-\rho_{MC}^2}\sqrt{1-\rho_{RC}^2}}.$$

D Production of routine tasks

Consider a continuum of tasks $i \in [0,1]$ which can be either produced linearly by robots or by routine labor. Denote by $b(i)$ output of task i produced by robots, and by $l(i)$ output of task i produced by routine labor. Since all tasks are symmetric, assume without loss of generality that the production of tasks $i \in [0, \mu_B]$ is assigned to robots, whereas tasks $i \in (\mu_B, 1]$ are assigned to routine labor. The combined output of robots and routine labor is assumed to satisfy

$$\left(\int_0^{\mu_B} b(i)^{\frac{\sigma-1}{\sigma}} di + \int_{\mu_B}^1 l(i)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}. \tag{161}$$

Since tasks are produced linearly with robots and routine labor, we have

$$B = \int_0^{\mu_B} b(i)di, \ L_R = \int_{\mu_B}^1 l(i)di.$$

Since all tasks produced by robots cost the same to produce, it follows that $b(i) = b = \frac{B}{\mu_B}$. Similarly, we have $l(i) = l = \frac{L_R}{1-\mu_B}$. Moreover, since the firm is technologically indifferent between producing tasks with robots or routine labor, it follows that μ_B is chosen such that the price of producing a task with routine labor or robots is equalized. As a consequence, $b = l$, and thus

$$\frac{L_R}{1-\mu_B} = \frac{B}{\mu_B} \Rightarrow \mu_B(L_R, B) \equiv \frac{B}{L_R + B}.$$

We can thus rewrite (161) as $\left(\mu_B(L_R,B)^{\frac{1}{\sigma}} B^{\frac{\sigma-1}{\sigma}} + (1-\mu_B(L_R,B))^{\frac{1}{\sigma}} L_R^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$.

E Tables for calibration

Table 2: Calibration - Parameters

Parameter	Description	Value
	Labor supply and taxes	
ε	Labor supply elasticity (Blundell and McCurdy, 1999; Meghir and Phillips, 2010)	0.3
t	Tax progressivity (Heathcote et al., 2017)	0.181
λ	Revenue parameter	1.83
	Skill distribution	
σ_M	Standard deviation: manual	0.67
σ_R	Standard deviation: routine	0.53
σ_C	Standard deviation: cognitive	0.75
ρ_{MR}	Correlation: manual vs. routine	-0.61
ρ_{MC}	Correlation: manual vs. cognitive	0.12
ρ_{RC}	Correlation: routine vs cognitive	0.69
	Production function	
γ	Elasticity of substitution: manual labor vs. all other factors	2.06
ρ	Elasticity of substitution: cognitive labor vs. routine labor and robots	1.67
σ	Elasticity of substitution: routine labor vs. robots	4.41
A	Productivity shifter	28.46
μ_M	Share parameter: manual labor	0.10
μ_C	Share parameter: cognitive labor	0.43
q_0	Producer price of robots at time 0	13.33
q_1	Producer price of robots at time 1	12.49

Note: Parameters ε and t are set directly. All other parameters are calibrated to match the following moments: mean, standard deviation, and skewness of wages by occupation, employment shares, elasticities of wage-distribution-deciles with respect to a change in robots, the price-elasticity of robot adoption, and the share of income-tax revenue in GDP.

Table 3: Calibration - Moments of wage distribution

Moment	Occupation	Model	Data
Mean of wages	manual	12.51	11.89
	routine	16.42	16.91
	cognitive	25.87	25.36
St. dev. of wages	manual	6.19	6.15
	routine	8.67	9.31
	cognitive	16.02	14.24
Skewness of wages	manual	2.47	3.47
	routine	2.01	2.99
	cognitive	2.55	2.26

Note: Data moments are based on the CPS for 1993. Wages are in 2016-dollars.

F Data Appendix

I obtain data on wages and occupational choice from the Current Population Survey (CPS) Merged Outgoing Rotation Groups (MORG) as prepared by the National Bureau of Economic Research (NBER).[36] To make results comparable with Acemoglu and Restrepo (2017), I focus on data from 1993 which is the first in year in which data on industrial robots is available for the US.

F.1 Sample

Selection of the sample follows Acemoglu and Autor (2011). I include individuals aged 16 to 64 whose usual weekly hours worked exceed 35. Hourly wages are obtained by dividing weakly earnings by usual hours worked. All wages are converted into 2016 dollar values using the personal consumption expenditures chain-type price index.[37] The highest earnings in the CPS are top-coded. I therefore windsorize earnings by multiplying top-coded earnings by 1.5. Like Acemoglu and Autor (2011), I exclude those individuals who earn less than 50% of the 1982 minimum wage (3.35$) converted to 2016-dollars. Self-employed individuals are excluded, as are individuals whose occupation does not have an `occ1990dd` classification. Like Acemoglu and Autor (2011), I exclude individuals employed by the military as well as agricultural occupations. As will be discussed in Section F.2, I also exclude the following occupations: Police, detectives and private investigators, Fire fighting, prevention and inspection, Other law enforcement: sheriffs, bailiffs, correctional institution officers. Observations are weighted by CPS sample weights.

F.2 Classifying occupations

I classify occupations into three categories: manual non-routine, manual routine and cognitive. To do so, I proceed in several steps.

Two-digit classification as in Acemoglu and Autor (2011). First, I apply the classification from David Dorn.[38] Next, I follow Acemoglu and Autor (2011) and group occupations into the following categories: Managers, Professionals, Technicians, Sales, Office and admin, Personal care and personal services, Protective service, Food prep, buildings and grounds, cleaning, Agriculture, Production, craft and repair, Operators, fabricators and laborers. Autor and Dorn (2013) highlight that protective services is a heterogeneous category with wages in police, firefighters and other law enforcement occupations being substantially higher than in other protective services occupations. I

[36] See http://www.nber.org/data/morg.html

[37] I obtain the price index from https://fred.stlouisfed.org/series/DPCERG3A086NBEA

[38] See http://www.ddorn.net/data.htm

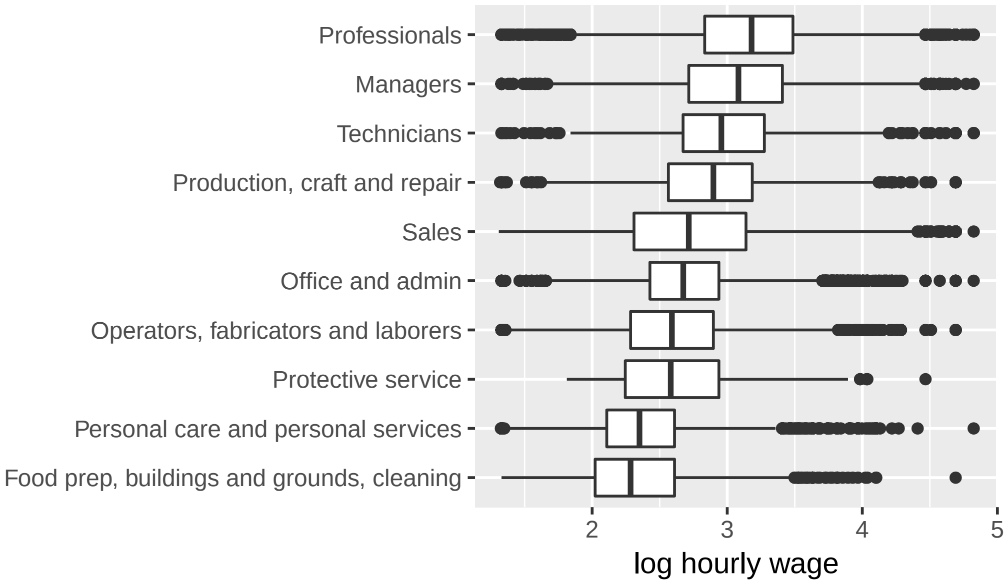


Figure 7: Distribution of log wages for 2-digit occupations

Note: NBER CPS Merged Outgoing Rotation Groups. The sample is explained in Section F.1. Data is for the year 1993. Wages are in 2016-Dollars.

therefore exclude the former occupations from the analysis. Figure 7 shows a box-plot of log wages for the different recoded occupations, ordered by median wage.

Classification into three categories. I base my classification of occupations on Acemoglu and Autor (2011). The classification is described in Table 4. Summary statistics are given in Table 5.

Table 4: Occupation classification based on Acemoglu and Autor (2011)

Three-group classification	Contained occupations
Cognitive non-routine	Managers
	Professionals
	Technicians
Routine	Sales
	Office and admin
	Production, craft and repair
	Operators, fabricators and laborers
Manual non-routine	Protective service
	Food prep, buildings and grounds, cleaning
	Personal care and personal services

Table 5: Summary Statistics for CPS Sample

	man. non-rout.	rout.	cogn. non-rout.	all
Employment share in pct	10.98	58.68	30.34	100
Wage in 2016-$				
Mean	11.89	16.91	25.36	18.98
St. dev.	6.15	9.31	14.24	11.74
Schooling Shares in pct				
Less than high school	24.73	13.63	1.3	11.11
High school	44.78	46.15	13.53	36.1
Some college	19.31	21.72	14.31	19.21
College	10.37	16.91	46.42	25.15
More than college	0.82	1.58	24.45	8.44

Note: Based on data from the NBER CPS Merged Outgoing Rotation Groups. The sample is explained in Section F.1. The classification into three categories is explained in Section F.2. Data is for the year 1993. Wages are in 2016-Dollars.

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