CESifo WORKING PAPERS

7331
2018

Original Version: November 2018
This Version: January 2022

Taxes Versus Quantities Reassessed

Larry Karp, Christian Traeger

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editors: Clemens Fuest, Oliver Falck, Jasmin Gröschl
www.cesifo-group.org/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: www.CESifo-group.org/wp

CESifo *Working Paper No. 7331*
Category 10: Energy and Climate Economics

Taxes Versus Quantities Reassessed

Abstract

The ongoing debate concerning the ranking of taxes versus cap and trade for climate policy begins with Weitzman's (1974) seminal slope-based criterion and concludes that taxes dominate quotas. We challenge this conclusion and the intuition behind it. Because technology shocks and pollution stocks are both persistent, a technology shock alters the intercepts of both the marginal damage and abatement cost curves. The ratio of these two intercept shifts is as important as the ratio of slopes in ranking policies. Technology innovations diffuse gradually, strengthening the importance of the ratio of intercept shifts. For plausible parameter combinations, quotas can dominate taxes.

JEL-Codes: Q000, Q500, H200, D800.

Keywords: policy instruments, pollution, climate change, taxes, quantities, regulation, uncertainty, cap and trade, technology.

Larry Karp
Department of Agricultural and Resource Economics / UC Berkeley
Berkeley / CA / USA
karp@berkeley.edu

Christian Traeger
Department of Economics
University of Oslo
Oslo / Norway
christian.traeger@econ.uio.no

January 27, 2022
We are grateful for comments by Lint Barrage, Inge van Den Bijgaart, Reyer Gerlagh, Christian Gollier, Rolf Golombek, Bard Harstad, Michael Hoel, Fortunat Joos, Eric Naevdal, Armon Rezai, Hiroaki Sakamoto, Robert Stavins, Jean Tirole, Martin Weitzman, and seminar participants at the EAERE 2017, SITE 2017, CESifo area network conference 2017, 2018 Workshops on Climate Change in Bologna and Copenhagen, SURED 2018, and EEA 2018. Christian Traeger gratefully acknowledges financial support by the Norwegian Research Council.

1 Introduction

Following the Paris Climate Agreement, 88 countries considered implementing either a tax or a cap and trade system to regulate greenhouse gas emissions. Currently, 45 national jurisdictions regulate over 20% of global carbon emissions by an emission tax or an emission trading scheme (World Bank 2021*b*). At current prices, the European Emissions Trading System alone has an approximate annual market value of over 150 billion USD. Both taxes and cap (quotas) and trade are inefficient policy instruments when we face uncertain technological progress in abatement costs and macroeconomic shocks. Our paper provides new intuition for ranking taxes and cap and trade and challenges the consensus view that taxes dominate quotas in the climate context.

Weitzman's (1974) model of flow pollutants provides the basis for current intuition about the welfare ranking of taxes and quotas. Under a tax, uncertainty about marginal abatement costs creates uncertainty about emissions, and therefore about damages. Under a binding quota, the randomness creates cost uncertainty. Weitzman (1974) shows that taxes create a smaller deadweight loss than quotas if and only if the slope of marginal damages is less than the slope of marginal abatement costs. His paper is among the most widely taught in environmental economics, with over 4000 citations.

Most pollutants, and all climate-related pollutants, have some persistence or cumulative impact; carbon dioxide's effective half-life exceeds a century. The literature has long recognized that for stock pollutants, the relevant marginal damage curve involves the discounted stream of future marginal damages arising from the changes in future pollution stocks caused by current emissions. When no confusion arises, in the stock pollutant context we refer to this discounted stream simply as "marginal damages"; when there is a possibility of ambiguity, we refer to it as the social cost of carbon (SCC).

To a great extent, the literature has transferred Weitzman's logic, developed for a flow pollutant, to the context of stock pollutants, merely substituting the slope of flow marginal damages with the slope of the discounted stream of marginal damages. It is generally agreed that this slope is very small. This consensus view on relative *slopes* (which we do not challenge), together with Weitzman's logic, has led to the conclusion that taxes dominate quotas for controlling climate change.[1] We show that Weitzman's logic fails for climate change, and we derive a ranking criterion applicable to greenhouse gases. A calibration for carbon dioxide illustrates the quantitative importance of the difference.

We focus on the cost uncertainty arising from shocks to technology (deviations from a trend). Section 3 explains this modeling choice. The mechanism we describe is symmetric with respect to positive and negative shocks, so we use the example of a shock that unexpectedly lowers marginal abatement costs in the current period. The optimal level of pollution responds to a change in abatement costs. Equilibrium abatement does not respond to this change under a binding quota, and the response is excessive under a tax. In both cases, the deviation between the equilibrium and the optimal response creates a deadweight loss. Weitzman shows that for a flow pollutant the deadweight loss is larger under the quota if the environmental damages are less sensitive than the costs to a change in emissions: taxes welfare dominate quotas if the marginal damage curve is flatter than the marginal abatement costs curve. For climate change, there is wide agreement that the slope of marginal damages is smaller than the slope of marginal costs.

However, because technology is persistent, the technological innovation also reduces future costs. The costs shocks are firms' private information in the current period, but the regulator learns them in the next period by observing industry behavior. The shock-induced reduction in future abatement costs reduces future emissions,[2] thereby lowering the stock trajectory and lowering

[1] Nordhaus (2008) writes "A major result from environmental economics is that the relative efficiency of price and quantity regulation depends upon the nature – and more precisely the degree of nonlinearity – of costs and benefits (see Weitzman 1974)." Wood & Jotzo (2011) write "It is generally thought that [Weitzman's logic holds] ...with climate change for the comparison between price and quantity instruments." Weitzman (2018) writes "For example in the case of CO2, since the marginal benefit curve within a regulatory period is very flat [...] the theory strongly advises a fixed price as the optimal regulatory instrument."

[2] The same reasoning holds even under business as usual emissions without a regulator; a clean innovation will reduce future emissions.

the stream of future marginal damages: the SCC. As a result, the current shock shifts the marginal damage curve (SCC) in the same direction as the marginal abatement cost curve. Thus, the persistence of *both* the technology and the pollution stock cause the marginal cost and the SCC to be perfectly positively correlated. This perfect correlation arises endogenously from the very nature of the stock pollutant problem. This mechanism is absent if either the pollutant or the cost shock are not persistent. Ignoring the correlation exaggerates the optimal emission response to a clean or dirty technology shock and favors taxes.

We derive a new, simple ranking criterion for stock pollutants. Like Weitzman's criterion for flow pollutant, the policy ranking depends on the relative slopes of the marginal abatement cost curve and the marginal damage curve, provided we understand that the marginal damages involve the discounted cost stream (which we show to be more convex than the corresponding flow damages). In addition, the ranking for stock pollutants depends on the relative shifts in intercepts. This "intercept effect" is a first order effect for ranking taxes versus quotas. For plausible parametrizations, "intercept shifts are as important as slopes". Therefore, the accepted view that the slope of the SCC is very small does not imply that taxes dominate quotas.

Our focus on technology shocks brings into play a previously missing consideration: technology diffuses gradually. Gradual diffusion means that (for example) a good cost shock may shift the current industry marginal abatement costs down by only a small amount, because only a small fraction of the industry adopts the technology. However, future marginal costs fall more substantially with high future adoption. The larger fall in future costs reduces future emissions further, thereby leading to a more substantial fall in the SCC. That is, gradual diffusion makes the shock-induced shift in SCC larger relative to the shift in the current marginal abatement costs. This effect favors quotas because it further reduces the optimal adjustment of abatement. We use the model of technology diffusion introduced in Karp & Traeger (2021). There, the speed of technology diffusion affects the characteristics of two first best policies, which we refer to as the smart cap and the smart tax. In the current paper, the speed of diffusion affects the ranking of the two most important second-best policies, the tax and cap and trade.

Our calibration of the model uses estimates of marginal abatement costs and marginal damages based on Nordhaus & Sztorc (2013) and models climate change based on the "transient climate response to cumulative carbon emissions" (TCRE). The TCRE model posits a linear relation between cumulative emissions and temperature change and is extensively discussed in the IPCC (2013). Recent applications in economics include Anderson et al. (2014), Brock & Xepapadeas (2017), Dietz & Venmans (2018), and Dietz et al. (2021). The model avoids the exaggerated lag in warming generated by the DICE model (Dietz et al. 2021). Our calibration mostly follows Karp & Traeger (2021), adding additional scenarios. The full model is transparent, and produces a policy ranking that depends on only a few parameters. There is substantial consensus about (or at least familiarity with) all of these parameters except for the rate of technological diffusion, for which we conduct sensitivity studies. For some parameter sets we reproduce the conventional view that taxes dominate quotas, but for equally plausible parameters, the ranking favors quotas.

A large literature extends the Weitzman-style analysis in the context of flow pollutants. Stavins (1996) compares taxes and quotas when cost and damage shocks are correlated.[3] Montero (2002) considers policy ranking under incomplete enforcement, and Shinkuma & Sugeta (2016) considers the ranking with endogenous firm entry. Pizer & Prest (2020) compare taxes to quotas for a flow pollutant with intertemporal permit banking and borrowing.[4] Requate

[3]Stavins discusses reasons why damages and abatement costs can be (arbitrarily) correlated for exogenous reasons. We point out that a perfectly positive correlation results endogenously from the very nature of the greenhouse gas abatement problem. This perfect correlation allows us to derive a simple ranking criterion based on slope and intercept shift.

[4]Section 5 of Pizer & Prest (2020) discusses climate economics using a model in which the SCC is a realization of an exogenous autocorrelated random variable. This assumption means that neither cumulative nor current emissions affect either current or future marginal damages, eliding a key feature of climate change. Under their assumed zero slope between the SCC and the stock of carbon, taxes would always dominate quotas in a model focused on cost uncertainty. They create a contest between the policies by introducing uncertainty about the policy-maker's response to the realization of the SCC. In constrast, we treat the SCC as an endogenous function of both cumulative emissions and current technology.

& Unold (2003) show how the incentives to adopt technology vary with the instrument choice, and Perino & Requate (2012) show how policy stringency alters these incentives. Recent reviews of taxes and quotas that discuss both stock and flow pollutants include Hepburn (2006), Aldy et al. (2010), Goulder & Schein (2013), Newbery (2018), and Stavins (2020).

A smaller literature compares policies for a stock pollutant. Pizer (1999), Kelly (2005), Fischer & Springborn (2011), and Heutel (2012) use simulations for this comparison; the latter two papers focus on the effect of business cycles. A separate literature extends Weitzman's linear-quadratic asymmetric information model to produce analytic comparisons for a stock pollutant. In this setting, there is an important difference between open loop and feedback policies. In the former, a regulator at t chooses the sequence of current and future policy levels conditional on information available at t. With feedback policies, a regulator at t chooses the current policy level and understands that future policy levels will be conditioned on information that becomes available in the future. Hoel & Karp (2002) assume serially uncorrelated shocks, thereby ruling out our results. Newell & Pizer (2003) consider serially correlated cost shocks, but only in an open loop setting. They show that positively correlated cost shocks increase stock volatility under taxes, favoring quotas.[5] Karp & Zhang (2005) compare the policy ranking across the open loop and feedback settings, and find that positive serial correlation of cost shocks favors quotas under feedback policies.[6] However, they do not explain the mechanism or include our model of gradual diffusion, and they do not discuss technology.

[5]Under open loop taxes, positively serially correlated shocks produce positively serially correlated levels of emissions. These raise the volatility of the pollution stock and increase the deadweight loss arising from stock uncertainty. In contrast, under open loop quotas, the stock trajectory is deterministic. Serial correlation does not have a similar impact on the deadweight loss arising from abatement cost uncertainty, because abatement costs depend only on a period's shock realization, not on its history. This intuition breaks down under feedback policies, where the stock trajectory is stochastic under both taxes and quotas. By conditioning future policies on historic shock realizations, policy makers eliminate the cumulative deviation between the realized and the optimal stock levels.

[6]Karp & Zhang (2006) show that anticipated learning about climate-related damages favors taxes, but the effect is small. Karp & Zhang (2012) study policy ranking with endogenous investment in abatement capital. Neither paper includes persistent technology.

This literature shows that increasing the regulator's ability to respond to information, either by moving from the open loop to the feedback setting, or by reducing the time step between policy adjustments within the feedback setting, *both favor taxes*. For a long-lived problem such as climate change, a policymaker understands that future policies adjust to new information. Therefore, we consider only the feedback setting. However, we leave the time step as a model parameter, and our application assumes that policy adjusts every five years. Our results do not rely on rapid adjustment of policies.

None of the papers that formally rank taxes and quotas for stock pollutants include our conceptual insight, which explains why Weitzman's reasoning does not carry over to stock pollutants with persistent shocks. This insight can be conveyed in a figure nearly as simple as Weitzman's graph, and therefore can be taught at the undergraduate level (Figure 1 below). The earlier literature also misses our other contributions: We provide a simple and intuitive ranking criterion which shows that the optimal policy choice depends as much on the ratios of intercept shifts as on ratios of slopes of marginal abatement costs and damages. We provide empirical evidence that the case for using taxes instead of cap and trade as a climate policy is weaker than previously thought. Finally, we show that quotas can even be first best, despite an almost flat marginal damage curve, if the intercept effect is strong enough.

2 One-period graphical analysis

Weitzman's static model for a flow pollutant produces a simple criterion for ranking a tax and quota. A variation of this one-period model reveals a fundamental difference between the settings where damages depend on the flow of pollution or the stock of pollution. The criterion for ranking policies in the stock-related case is only slightly more complicated than in the flow-related case, and it closely relates to the formula we develop for the dynamic model.

2.1 Review of standard model

In the classic prices versus quantities setting, marginal damages increase linearly in emissions: $MD = a + b\,E$. The slope parameter b characterizes the convexity of damages. Similarly, the classical setting assumes that marginal benefits from emissions are linear. An optimizing firm emits to the point where the marginal benefits of emissions equal the marginal abatement costs. We write these marginal costs as a function of emissions (instead of abatement): $MAC = \theta - f\,E$. The slope parameter f captures the concavity of the benefits from emitting or, equivalently, the convexity of the abatement cost.[7] The upper left panel in Figure 1 depicts the MD curve as the increasing solid line and shows the expected abatement cost curve as the decreasing solid line.

The parameter θ is private information, known to the firm but not to the policy maker. The planner knows only the expected value of θ. A risk neutral planner sets $\mathbb{E}(MAC) = MD$, equating the marginal damage curve and the expectation of the marginal abatement cost curve.[8] With taxes, the policy fixes the emissions price at the green (horizontal) line in Figure 1. In a quantity setting, the policy caps the emissions at the red (vertical) line.

The dashed lines in Figure 1 shows the realized marginal abatement cost curve for a shock that reduces marginal abatement costs. The results are symmetric with respect to positive and negative shocks, so we illustrate only $\theta < 0$. The top left panel shows the tax and the quota equilibria for a flow

[7] We emphasize that the marginal benefits from emissions are equal to the marginal abatement costs. Let the absolute benefits of emissions be $B(E) = \theta\,E - \frac{f}{2}\,E^2$. Abatement is the difference between business as usual and actual emissions: $A = E^{BAU} - E$. Business as usual emission are industry's optimal emissions in the absence of policy. Firms' first order condition for unregulated emission optimization yields $E^{BAU} = \frac{\theta}{f}$. Thus, the absolute abatement costs are $AC(A) = B(E^{BAU}) - B(E) = \theta E^{BAU} - \frac{f}{2} {E^{BAU}}^2 - \theta\,E + \frac{f}{2}\,E^2 = \frac{1}{2}\frac{\theta^2}{f} - \theta\,E + \frac{f}{2}\,E^2$ resulting in the marginal abatement costs $MAC(A) = (-\theta + f\,E)\frac{dE}{dA} = \theta - f\,E$. Thus, f indeed describes both the concavity of emission benefits and the convexity of abatement costs.

[8] The common assumption that the intercept but not the slope is private information is key to the simplicity of both Weitzman's and our result. Hoel & Karp (2001) rank the two policies in a model with stock pollutants, when a serially uncorrelated shock affects the slope. The resulting criterion for policy ranking is not closely related to the criterion where the shock affects the intercept of marginal cost.

pollutant. The deadweight loss under the tax is the light green triangle and the deadweight loss under a quota is the heavy red triangle. The deadweight loss is smaller under the tax than under the quota because the MAC curve is steeper than the MD curve in this figure. Here, taxes dominate quotas.

2.2 Modification for a stock pollutant

Weitzman's (1974) setting assumes that θ does not shift the MD curve. In a dynamic setting, stock pollutants and abatement technologies evolve over time. The policy maker regulates pollution without knowing the current or future abatement costs. Technology shocks are persistent, so a shock changes both current and future marginal costs. An unexpectedly successful innovation causes both current and future abatement costs to be lower than anticipated; a disappointing result in technology causes both current and future abatement costs to be higher than anticipated. The regulator who learns the current shock in the next period, from having observed industry behavior in the current period, can adjust policy. The policy adjustment alters future levels of the stock pollutant, shifting the marginal damage curve.

The lower left panel in Figure 1 illustrates the consequences of this insight. The solid MD curve now represents the expected discounted stream of future marginal damages arising from current emissions' impact on the pollution stock: the social cost of carbon in the climate context. The two dashed curves show MAC and MD conditional on Δ, the shift in marginal abatement cost. The parameter φ is the ratio of the shift in the intercept of MD to the shift in the MAC curve. Our graphical analysis takes the slope of the MD curve and the ratio of shifts in intercepts (φ) as exogenous, and also assumes that these are the same under taxes and quotas, for any realization of the cost shock, Δ. Our genuinely dynamic model (Section 3) recognizes that both the slope of the MD curve and ratio of intercept shifts are endogenous functions of the model parameters. However, these two functions are the same under taxes and quotas, and they are independent of the cost shock. This invariance is important, because without it the static model in this section would shed

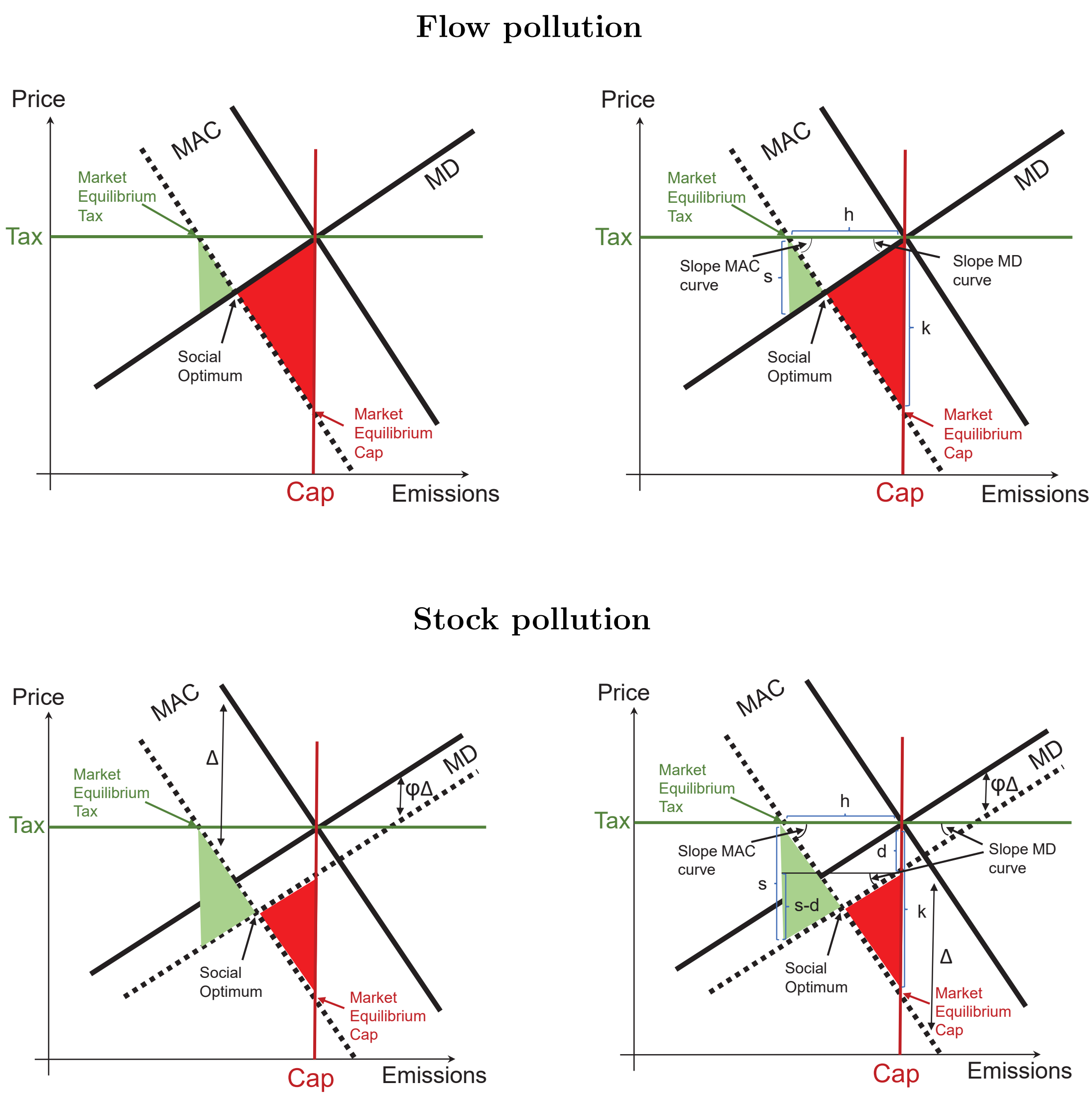


Figure 1: Illustration of Weitzman (1974) insights for a flow pollutant (top panels) and a quasi-static illustration of the changes for a stock pollutant (bottom panels). The light green (left) triangle characterizes the deadweight loss under a tax, whereas the red (right) triangle characterizes the deadweight loss under a quota. The black solid lines represent expectations, and their dashed counterparts represent realizations. The panels on the right add labels of relevant distances and slopes for our graph-based quantitative illustration of taxes versus quotas.

little light on the dynamic model that we use to study a stock pollutant. We therefore explain the basis for this invariance at the end of Section 2.3.

For our example in the Figure, an innovation lowers, by Δ, the marginal abatement cost from the solid line to the dashed line. Because technology is persistent, this reduction in marginal abatement costs makes future emission reductions cheaper, and reduces future emissions (both optimal and Business-as-Usual, BAU).[9] The resulting reduction in the future trajectory of pollution stocks lowers the marginal damage from releasing an additional unit of the pollutant today. As a consequence, the MD curve also shifts by $\varphi\Delta$.

Comparing the top and the bottom left panels in Figure 1, it is apparent that the downward shift of the MD curve increases the deadweight loss of the tax and reduces the deadweight loss of the quota, thus favoring quotas. The right panels in Figure 1 enrich the left panels by adding labels to some slopes and segment lengths. Using these labels, a familiar geometric argument establishes Weitzman's result that taxes dominate quotas for a flow pollutant if and only if the slope of the marginal damage curve is less than the slope of marginal abatement cost curve. An only slightly more complicated geometric argument (Appendix B) shows that taxes dominate quotas for a stock pollutant if and only if

$$\frac{b}{f} \equiv \frac{m^{MD}}{m^{MAC}} < 1 - 2\varphi. \qquad (1)$$

The ranking of taxes versus quantities now depends on both the ratio of the slopes of the two curves and on the ratio of their shifts, φ. The figure shows that a shock induces a greater than optimal emissions adjustment under a tax, and a less than optimal adjustment under a quota. The deadweight loss is monotonic in the deviation between the equilibrium adjustment and the socially optimal adjustment: taxes dominate quotas if and only if the deviation is greater under quotas than under taxes. A shift of the MD curve does not alter the equilibrium emissions adjustment under taxes or quotas, but it reduces the socially optimal adjustment, moving it closer to the equilibrium

[9] We assume throughout that future policy is set optimally. However, even in the absence of any regulation, a persistent shift in the marginal abatement cost changes the trajectory of future BAU emissions, causing a shift in the SCC. The magnitude but not the direction of the shift depends on our assumption that future policy is set optimally.

under quotas (no adjustment). Therefore, a positive value of φ always lowers the deadweight loss under quotas and raises the deadweight loss under taxes.

2.3 Discussion of the graphical model

A footnote in Weitzman (1974), elaborated by Stavins (1996), considers the case where θ and a shock that shifts the MD curve are correlated. Stavins gives the example where a sunny day increases ultraviolet radiation, increasing ozone production, raising ozone abatement costs. If the sunny day causes people to spend more hours outdoors, marginal damages from ozone (respiratory stress) also increase. Here, the correlation between the shocks affecting marginal abatement costs and damages is a primitive, taking any value in $[-1, 1]$.

In contrast, the correlation between marginal abatement costs and damages with a stock pollutant is endogenous, arising from the future response of emissions to a current cost shock. Moreover, correlation is perfect (correlation coefficient of unity). Identifying the correlation here is trivial, but the key to the ranking is φ, the ratio of the shift in the marginal damage curve per unit shift in the marginal abatement cost curve. Given that the mechanisms for the correlation under flow and stock pollutants are entirely different, it is perhaps not surprising that it took a quarter of a century after Stavins' paper to make the link between the two problems.

Our graphical treatment takes the two components of the ranking criterion, the ratio of slopes and the ratio of intercept shifts (φ) as exogenous. These are, of course, endogenous objects in the dynamic setting. Our graphical treatment also assumes that these two ratios do not depend on whether the regulator uses a tax or quota. This invariance is a result, not an assumption, in the dynamic model in Section 3. This fact is important because it means that our graphical treatment accurately reflects the forces at work in the genuinely dynamic model. The dynamic model makes it possible to calculate the values of the two ratios and thereby rank the tax and quota for a stock pollutant.

The invariance is due to three implications of the linear-quadratic model with additive shocks. (i) The value function is quadratic in the pollution stock and the technology shock. This fact means that the realization of the SCC is a linear function of the next-period pollution stock and the technology shock. (ii) The coefficients of the linear and quadratic terms in the value functions under optimal taxes and quotas are identical.[10] This fact means that the intercept (a function of the linear coefficients of the value function) and the slopes (equal to the quadratic coefficients of the value function) of the SCC are the same under the two policies. Therefore, conditional on the next-period stock, the SCC is the same under taxes and quotas. (iii) The *expected* level of emissions, and therefore the expected next-period pollution stock, is the same under the optimal tax and quota. This fact, together with the linearity (in pollution stock) and the equivalence (under taxes and quotas) of coefficients, means that the SCC in the genuinely dynamic model is invariant to the choice of policy – exactly as our static model assumes.

3 The dynamic model

Two sources of asymmetric information cause the non-equivalence of taxes and quotas in the dynamic setting. First, asymmetry arises because technology-related costs are private information when firms choose emissions. Second, asymmetry arises because emissions decisions occur more frequently than policy updates, unless the regulator can condition the policy instrument on the arriving public information.

Our analysis focuses on the asymmetric information resulting from technological innovation, which we consider most relevant for three reasons. First, many technological innovations are genuinely private or unverifiable information at the time firms choose emissions. Second, it is hard to condition policy on technological innovation. Third, technological innovations are persistent, and therefore affect all future periods. The impact of technology shocks is therefore not easily mitigated by intertemporal arbitrage. Macro-economic shocks, in contrast, usually have a low to moderate serial correlation. Intertem-

[10] In contrast, the intercepts of the two value functions differ. The policy ranking depends on a comparison of those intercepts.

poral arbitrage through banking and borrowing of certificates, or through the European Emission Trading System's market stability reserve, reduce market disturbances from such shocks. Moreover, conditioning carbon policy on macroeconomic indicators can eliminate much of the costs arising from macroeconomic shocks (Ellerman & Wing 2003, Jotzo & Pezzey 2007, Newell & Pizer 2008, Doda 2016, Burtraw et al. 2020). (Appendix C.1).

Policy makers will acquire new information during the many decades that climate policy remains relevant. They will adapt regulation following unexpected changes in the cost of renewable energy generation. Even if current policy makers do not intend to adapt policy in the future, they cannot commit their successors to ignore new information for long periods of time. Pizer & Prest (2020) note that "most real-world regulations are updated over time in response to new information". We therefore study a model where the policy maker in each period conditions current regulation on current information, and understands that future policy makers will do the same: the policy rule is feedback, not "open loop with revision". We note that the feedback policy can be implemented by announcing state-contingent policy rules (functions of lagged policy and the equilibrium market response) at the beginning of the planning horizon (Appendix C.4).

3.1 Description of the model

The model contains two state variables: a pollution stock and a technology level. The equation of motion for the pollution stock is

$$S_{t+1} = \delta S_t + E_t,$$

where E_t are emissions. The classic stock pollution model interprets S_t as the pollutant's concentration and $1 - \delta \geq 0$ as the pollutant's decay rate. For our climate change application, we use the fact that atmospheric temperature increase is approximately proportional to cumulative historic emissions and we interpret S_t as temperature (and therefore set $\delta = 1$; see Section 4 for details).

The stock S_t causes annual damages of $\frac{b}{2}S_t^2$.[11] The exogenous parameter b equals the slope of the marginal *flow* damage curve. Both the (discounted stream of) the marginal damage from releasing another unit of emissions and its dependence on technology shocks are endogenous to the model, not an exogenous input as in Section 2.

The abatement technology consists of a deterministic trend and a stochastic deviation θ_t from this trend. This deviation is a highly persistent stochastic process under iid shocks $\varepsilon_t \sim iid\,(0, \sigma^2)$. These shocks represent technological innovations departing from the trend. The equation of motion for θ is

$$\theta_t = \rho\theta_{t-1} + \varepsilon_t, \text{with } \rho > 0 \text{ and } \mathbb{E}_t(\varepsilon_t) = 0.$$

The policy maker knows θ_{t-1} but not ε_t when choosing the policy for period t; firms know both θ_{t-1} and ε_t in period t. This asymmetry provides the dynamic analogue of Weitzman's (1974) asymmetric information.

The speed of technological diffusion plays a critical – and a novel – role in ranking the policies. A large literature documents the fact that many new technologies diffuse gradually through the economy (Rogers 2003). The standard approach uses an $S-$shaped function to describe the relation between the time since a new technology was introduced and the fraction of firms that have adopted it. We use a simpler model, introduced in Karp & Traeger (2021), in which the representative firm adopts only the fraction $\alpha \in (0, 1]$ of the latest technological innovation during the current period, and adopts the remaining fraction in the next period. This model captures gradual diffusion without the need of an additional state variable.[12] We define $\hat{\theta}_t \equiv \rho\theta_{t-1} + \alpha\varepsilon_t$ as the stochastic component of the *adopted* technology. We let h_t denote the deterministic trend of adopted technology; this trend is important for our calibration but it does not appear in the formula for policy ranking. Our

[11] The absence of a linear damage term results from defining S_t as the deviation of the stock from the harm-minimizing level. By adapting the definition of S_t to the model calibration, we avoid the need to include the linear term; Section 4 uses this procedure.

[12] Even in a model with gradual diffusion, the most important characteristic of the diffusion process for the policy ranking would be the amount of technology adopted during the current policy period relative to the long-term future. It is precisely this characteristic of technology diffusion that we capture by α.

formulation complements an AR(1) model for *innovated technology* (θ) by an ARMA(1,1) structure for *adopted technology* ($\hat{\theta}$).[13]

The persistence of innovated technology implies a high autoregressive coefficient ρ, and thus high serial correlation for adopted technology. A lower value of α further increases this serial correlation because a given level of technology adoption today results in a larger future adoption.[14] Higher serial correlation of adopted technology implies that a cost shock today has a stronger impact on both BAU and optimal future levels of emissions. A shock that reduces future abatement costs lowers future emissions, thereby lowering future carbon stocks. That reduction lowers marginal damages, and causes a downwards shift of the SCC in Figure 1. That shift partially offsets the welfare loss resulting from a cost shock under a quota. A shock that increases abatement costs similarly shifts the SCC up, again partially offsetting the welfare loss under a quota. Therefore, a larger ρ and a smaller α favor quotas.

The firms' emission benefits are $(h_t + \hat{\theta}_t)E_t - \frac{f}{2}E_t^2$, where f is the slope of the marginal abatement cost curve. A higher value of $\hat{\theta}_t$ corresponds to a larger marginal benefit from emitting: a larger marginal abatement cost. A better-than-expected technological innovation therefore corresponds to a *negative* realization of the shock ε.

We use superscripts Q and T for the quota and tax policy scenarios. Under a binding **quota**, the regulator chooses the actual emissions level E_t^Q and has the *expected flow net benefit* (using $\mathbb{E}_t\, \alpha\varepsilon_t = 0$)

$$(h_t + \rho\theta_{t-1})\, E_t^Q - \frac{f}{2}\left(E_t^Q\right)^2 - \frac{b}{2}S_t^2.$$

Under a **tax** τ_t the *firm's payoff* is $(h_t + \hat{\theta}_t)E - \frac{f}{2}E_t^2 - \tau_t E_t$, implying the first order condition $h_t + \hat{\theta}_t - fE_t = \tau_t$. This first order condition results in the firm's decision rule

$$E_t^T = e_t^T + \alpha\frac{\varepsilon_t}{f} \quad \text{with} \quad e_t^T \equiv \frac{h_t + \rho\theta_{t-1} - \tau_t}{f} \quad \left(= \mathbb{E}\, E_t^T\right).$$

[13]We have $\hat{\theta}_t = \rho\ (\rho\theta_{t-2} + \alpha\varepsilon_{t-1} + (1-\alpha)\varepsilon_{t-1}) + \alpha\varepsilon_t = \rho\hat{\theta}_{t-1} + \rho(1-\alpha)\varepsilon_{t-1} + \alpha\varepsilon_t$.

[14] Appendix C.7 shows that $corr(\hat{\theta}_t, \hat{\theta}_{t+j}) = \rho^j \left(\rho^2 \frac{\rho^{2j}-1}{\rho^2-1} + \alpha^2\right)^{-0.5}$ and confirms that this function decreases in α and increases in ρ.

It is convenient to model the tax-setting regulator as choosing *expected emissions* e_t^T, which is equivalent to setting the tax $\tau_t = h_t + \rho\theta_{t-1} - f\, e_t^T$. The tax payment is a pure transfer and does not enter the regulator's payoff function. The tax-setting regulator's *expected flow net benefit* from emissions is

$$(h_t + \rho\theta_{t-1})\, e_t^T - \frac{f}{2}\left(e_t^T\right)^2 + \frac{\alpha^2}{2f}\sigma^2 - \frac{b}{2}S_t^2.$$

For both problems, the regulator wants to maximize the expectation of the present discounted stream of net benefit flows, defined as the benefit of emissions minus the stock-related damage. She balances the persistent costs from pollution with the transitory benefits from emitting. The discount factor is β. At the end of period t the regulator learns the value of θ_t by observing the permit price induced by the quota or the level of emissions induced by the tax. Thus, the regulator knows θ_t when choosing the policy level at $t+1$. The pollution stock is public information.

3.2 Policy ranking

The marginal pollution damage, the Social Cost of Carbon (SCC) in the climate setting, is linear in the state variables, i.e. it is of the form

$$SCC_t = \beta\,\mathbb{E}\left(\chi_{t+1} + \lambda S_{t+1} + \mu\theta_t\right),$$

Equations (12) and (13) in Appendix B provide the formulae (functions of model primitives) for λ, the derivative of the SCC with respect to the carbon stock, and μ, the derivative of the SCC with respect to the technology realization. The appendix shows that both are positive constants. An increase in the stock of carbon increases the SCC, and (because $\rho > 0$) a higher cost of abatement (larger θ_{t-1}) shifts up the graph of the SCC as a function of carbon. Equation (14) in Appendix B provides the formula for χ_t. We need this expression to calculate the optimal tax, but not for the policy ranking. The time dependence of χ_t reflects the SCC's response to the technology trend, h_t. The functions χ_t, λ and μ are the same under the optimal tax, the optimal quota, and in the full information (first best) setting where the planner observers ε_t. The levels of emissions are the same in the three settings if and only if the shock equals its expected value, $\varepsilon_t = 0$.

We denote by $r \equiv \frac{b}{f}$ the ratio of the slopes of the marginal flow damage and the marginal abatement cost. This slope describes the relative convexity of the *flow* damage and the abatement cost functions. For a flow pollutant, taxes dominate quotas if and only if $r < 1$. In the case of carbon dioxide, r is tiny, about $r = 5.2\ 10^{-5}$ for our baseline calibration (Section 4). For the case of a stock pollutant, the intertemporally aggregated marginal damages, the SCC, replace the flow marginal damages. Accordingly, we define the ratio $R \equiv \frac{\lambda}{f}$, which relates the convexity of stock damages to that of abatement costs.[15] Lemma 1 gives the relation between these two slopes.

Lemma 1 *Under both taxes and quotas, the slope of the SCC with respect to the stock of carbon, relative to the slope of marginal abatement cost is*

$$R \equiv \frac{\lambda}{f} = \frac{1}{2\beta}\left(-\left(1-\beta\delta^2\right)+\beta r+\sqrt{\left(1-\beta\delta^2-\beta r\right)^2+4\beta r}\right). \qquad (2)$$

Unsurprisingly, the relation between the flow ratio r and the stock ratio R depends on the discount factor β and the persistence of the pollutant δ. Figure 2 graphs $\frac{R}{r}$ as a function of the flow pollution ratio r, using a one-year time step (the length of a period) and two alternative annual discount factors. (We set $\delta = 1$, the value corresponding to the TCRE model described in Section 4.) The heavy dots in Figure 2 show $\frac{R}{r}$ at $r = 5.2\ 10^{-5}$, our baseline value, for a 1.5% and a 0.5% annual discount rate. Aggregate damages are *more convex* than flow damages: the SCC is much steeper in emissions than is the flow

[15] It is instructive to consider the time step and units explicitly when defining R. Appendix C.1 uses a parameter ϕ to denote the time step, enabling a simple scaling of the period's length. There, we define $R \equiv \frac{\lambda}{f}\phi$. R relates the slope of the SCC curve, $\frac{\partial SCC}{\partial S_t}$, to the slope of the MAC curve, $\frac{\partial MAC}{\partial E_t \phi}$. Here, $E_t\phi$ is the amount of emissions over the course of the period, equal to the annual emissions flow times the number of years in a period: we have to compare the cost of the marginal unit change of atmospheric carbon with a unit change of abatement over the course of the period (rather than with the annual *flow*). The parameter ϕ carries the unit time, and $R \equiv \frac{\partial SCC}{\partial S_t} / \frac{\partial MAC}{\partial E_t \phi} = \frac{\lambda}{f}\phi$ is unit free. In the main text, we set $\phi = 1$ (rather than "1 year") for ease of notation. This choice picks units in which years are normalized to unity.

marginal damage curve.[16]

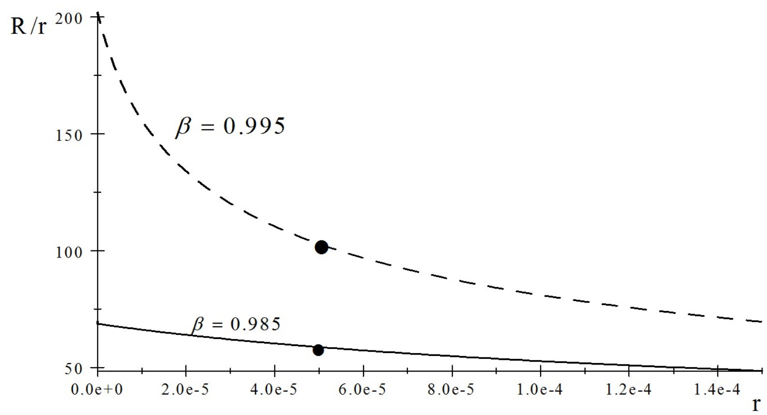


Figure 2: The ratio of $\frac{R}{r}$ for $\delta = \rho = 1$, a one-year time step, and two annual discount factors. The heavy dots identify our baseline value $r = \frac{b}{f} = 5.2\ 10^{-5}$.

The following proposition provides two equivalent characterizations of the criterion for ranking taxes and quotas for a stock pollutant.

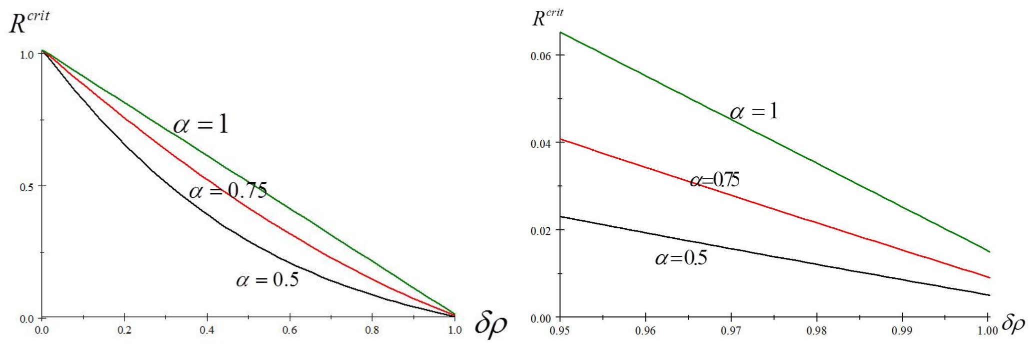
Proposition 1 *Taxes dominate quotas if and only if*

$$R < \frac{1}{\beta} - \frac{2\mu}{\alpha} \qquad \Leftrightarrow \qquad R < R^{crit} \equiv -\frac{1}{2}\kappa_1 + \frac{1}{2}\sqrt{\kappa_1^2 + 4\kappa_0} \tag{3}$$

$$\textit{with } \kappa_1 \equiv \frac{\delta\rho\,(2-\alpha)}{\alpha} \textit{ and } \kappa_0 \equiv \frac{1-\beta\delta\rho}{\beta^2}.$$

For flow pollutants, taxes dominate quotas if and only if $r < 1$. The first condition in Proposition 1 shows that: (i) the relevant slope in the ranking criterion for a stock pollutant is R instead of r; (ii) a higher value of μ favors

[16]With a flow pollutant, E, the slope of the marginal damage of an additional unit of emissions is b. With a stock pollutant, S, the marginal damage of an additional unit of emissions is $\lambda\frac{\partial S_{t+1}}{\partial E_t} = \lambda$.



(a) (b)

Figure 3: Panel (a) graphs R^{crit} as a function of $\delta\rho$ for three values of α, using a one-year time step and an annual discount rate of 1.5%. Panel (b) shows the same graph for values of $\delta\rho$ close to 1, the relevant case for climate change.

quotas; and (iii) slow technology adoption (small α) favors quotas.[17] The endogenous μ is the shadow value of the interaction term $\theta_t S_t$. It measures the responsiveness of the social cost of marginal emissions (SCC) to the technology realization. The right hand side of the equivalence (3) expresses the ranking criterion in terms of the fundamental model parameters. We note that the ratio R and the critical level R^{crit} respond differently to parameters: R depends on all parameters except α, whereas R^{crit} depends on all parameters except r.

Figure 3 graphs R^{crit} as a function of the "joint persistence", $\delta\rho$, of the stock pollutant and technology for three values of α, with an annual time step and the discount factor $\beta = 0.985$. With little or no pollution or technology persistence, $\delta\rho \approx 0$ and the left panel shows that the critical value is close to unity, as in the static criterion. However, for climate change δ is close to 1; and with persistent technology so is ρ. For $\delta\rho \approx 1$ the right panel of Figure 3 shows that the critical value remains bounded away from 0. In the climate change context, quotas might dominate taxes not only when r is tiny, but even if R is close to 0. Section 4 further explores this possibility.

We provide intuition for our results using the case of a *flow pollutant*, where a technology innovation (a negative value of ε) lowers both the socially

[17] Equation (13) in the appendix provides the formula for μ in terms of the the model's fundamentals. Importantly, μ is independent of α.

optimal emission level and marginal abatement cost. Under taxes, firms face constant abatement prices; here the emission quantity overreacts to a cost shock, compared to the socially optimal response. This quantity fluctuation's impact on expected damages is the dominating contribution to the deadweight loss under a tax. By Jensen's inequality, the convexity of the damage function determines the magnitude of the deadweight loss. Under quotas, emissions are constant, but the firms' equilibrium marginal abatement costs overreact compared to the social optimum. This exaggerated abatement cost fluctuation is the dominating contribution to the deadweight loss under quotas, and by Jensen's inequality the convexity of the abatement cost function determines its magnitude. If abatement costs are more convex than damages, the deadweight loss is larger under a quotas. Lemma 1 shows that the damage convexity is greater for the *stock pollution* than for the flow pollution ($R >> r$).

Proposition 1 states that, for a stock pollutant, a greater sensitivity of the SCC to technology (higher μ) and a slower technology diffusion (smaller α) *strengthen the case for quotas*. We start by providing the intuition for the case of immediate technology diffusion ($\alpha = 1$). As discussed in the preceding paragraph, the dominating contribution to the deadweight loss under quantity regulation of flow pollutants is the overreaction of the equilibrium abatement price relative to the socially optimal response. For a stock pollutant, a persistent technological innovation today implies lower future emissions, resulting in a lower future pollution stock.[18] Consequently, a technological innovation *reduces the marginal damages* (the SCC) resulting from an additional emission unit today. This reduction in marginal damages amplifies the socially optimal price fluctuation resulting from the innovation's reduction of marginal abatement costs. Thus, the socially optimal price fluctuation is larger in the stock pollution setting than in the flow pollution setting: the shifts in marginal costs and marginal damages resulting from the innovation are perfectly correlated, differing only in magnitude. A part of what would be an "overreaction" of

[18]In line with the empirical findings for most sectors, our functional forms imply that there is no rebound effect strong enough to increase aggregate emissions in response to an emissions-saving innovation

emission prices under quantity regulation of a flow pollutant becomes a socially optimal variation under a stock pollutant.

Proposition 1 shows that a higher value of μ favors quotas. The endogenous value μ measures the responsiveness of the social cost of marginal emissions to technology realization. It is the derivative of the SCC with respect to the technology level. If the socially optimal abatement cost responds more sharply to innovation (μ large), then the socially optimal response approaches the "overreaction" of equilibrium marginal abatement costs under a quantity regulation, reducing the deadweight loss of a quota. The graphical analysis in Section 2 reflects this intuition. When the technological innovation shifts the marginal damage curve for a stock pollutant (lower panels of Figure 1), it amplifies the optimal price fluctuations in response to the innovation, relative to the case of the flow pollutant (upper panels of Figure 1). Indeed, for $\alpha = 1$, the left side of the policy-ranking equivalence (3) (dynamic model) reproduces the left side of the graph-based equivalence (1) that we derived in the quasi-static setting. The dynamic model introduces the additional discount factor only because we assume that today's emissions contribute to tomorrow's stock and damages, whereas the quasi-static analog treated the damage as instantaneous.

The main difference between the stock pollution extension in Section 2.2 and the dynamic model is that both R and μ are endogenous in equation (3), whereas Section 2.2 simply assumed some slope ratio of marginal damages over marginal abatement costs and merely argued for the existence of some shift, φ, of the marginal damage curve. In addition, the extension in Section 2.2 cannot capture the fact that technology diffusion takes more than one period ($\alpha < 1$). Before continuing the discussion of technology diffusion and the underlying intuition we pose one more question. Can the "overreaction" of marginal abatement costs from the flow pollution perspective become a socially optimal fluctuation for a stock pollutant?

Proposition 2 *Assume that* $b, f, \beta, \rho, \delta > 0$ *and that* $\beta\delta\rho < 1$.

(i) There exists $\alpha^* \in (0, 1)$ *such that the quota is first best.*

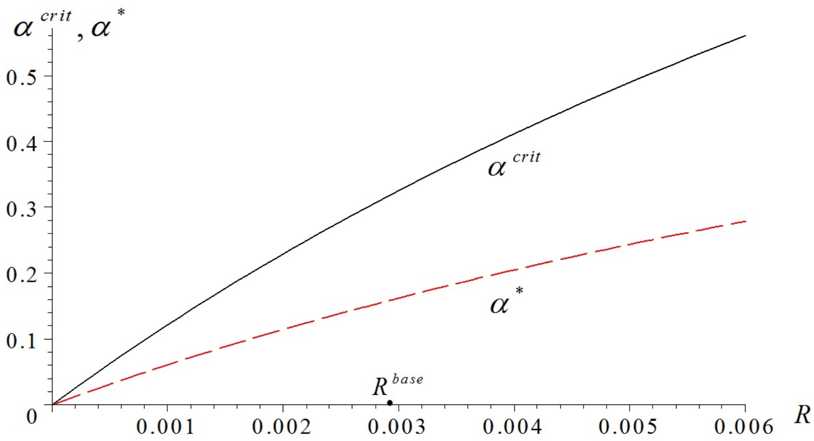


Figure 4: Quotas dominate taxes for $\alpha < \alpha^{crit}$, the solid graph. The quota is first best for $\alpha = \alpha^*$, the red dashed graph. The graphs use an annual time step and baseline values $\beta = 0.985$, $\delta = \delta = 1$. R^{base} marks the slope ratio in our baseline calibration.

(ii) A reduction in α favors quotas, and there exists $\alpha^{crit} \in (\alpha^*, 1)$ such that quotas dominate taxes for all $\alpha < \alpha^{crit}$.

The proposition shows that for any model calibration with convex damages and abatement costs there exists a technology adoption rate α for which quotas dominate taxes. For sufficiently slow technology diffusion, quotas are not only preferred to taxes, but the cap and trade system achieves the first best emission allocation even if the slope of the marginal damages curve is arbitrarily small (but positive) and the slope of the marginal cost curve is arbitrarily large. The proposition also implies that this situation can arise only under partial technology diffusion ($\alpha < 1$). Figure 4 graphs α^* as a function of R, the ratio of stock damage convexity to abatement cost convexity. It also graphs the critical diffusion level α^{crit}, below which quotas dominate taxes,

To understand the role of technology diffusion, note that under partial diffusion today's technology shock provides information not only about today's technology adoption but also about subsequent adoption. We noted above (and in Footnote 14) that partial diffusion increases the correlation between current and future adopted technology. As a result, a given level of adoption today signals even more future adoption. The socially optimal level of marginal abatement cost responds to innovation, anticipating both present and future adoption. In contrast, the marginal abatement cost under quantity regulation responds only to the presently adopted part of the innovation. As a consequence, partial diffusion increases fluctuations of the socially optimal price of emissions relative to the fluctuations arising under quantity regulation. Given that quantity regulation generally suffers from an overreaction of the emissions price, partial diffusion reduces the welfare loss under a quota.

Appendix C.6 notes that for $\alpha < \alpha^*$ the socially optimal emission price fluctuations are even stronger than the fluctuations under a quota. Moreover, in this case, a technological innovation reduces marginal abatement costs but increases socially optimal current emissions: the current innovation strongly reduces future abatement costs (and thus emissions) but only slightly reduces current abatement costs, making it optimal to emit more today in anticipation of the high reductions of future abatement costs.

3.3 A welfare measure

Although the expected emissions trajectories are the same under optimal taxes and quotas, asymmetric information causes welfare to differ under the two policies. The difference in welfare is proportional to σ^2, the variance of the cost shock. Lacking a good estimate of σ^2, we instead provide a measure of the *relative* welfare gain that is independent of σ^2.[19]

To this end, we consider the first-best (full information) setting, where the regulator observes the current shock before choosing the current policy; there, the tax and the quota are equivalent. This problem is still stochastic, because the regulator knows only the expectation of future emissions, not their realization. However, full information eliminates asymmetric information between firms and the regulator. We define the relative welfare gain in moving

[19] We know of only two studies, both over a decade old, that use values of σ^2 to quantify welfare changes in the linear-quadratic setting. However, the models in those papers differ so much from ours that we cannot sensibly adapt their estimates to our model.

from quotas to taxes as a ratio: the loss due to moving from full to asymmetric information conditional on using quotas, relative to the loss due to moving from full to asymmetric information conditional on using taxes:

$$G(\alpha) = \frac{V\,(\text{full info}) - V\,(\text{asym. info, quota})}{V\,(\text{full info}) - V\,(\text{asym. info, tax})}, \tag{4}$$

where $V\,(\cdot)$ denotes the present discounted stream of benefits, conditional on the policy scenario. The function G is non-negative, and it is greater than 1 if and only if taxes dominate quotas. For $G\ = 1$ (where $\alpha = \alpha^{crit}$) the tax and quota produce the same level of welfare.

Proposition 3 *The relative welfare gain of using taxes instead of quotas is*

$$G = \left(\frac{(\alpha - \beta\mu)}{\beta\,(\phi\alpha\lambda + f\mu)} f \right)^2. \tag{5}$$

The function $G(\alpha)$ is decreasing for $\alpha < \alpha^*$ and increasing for $\alpha > \alpha^*$; it reaches its minimum at $\alpha = \alpha^*$, where $G(\alpha^*) = 0$. There, the quota is first best: asymmetric information creates no loss under quotas, although there remains a loss under taxes

4 The climate application

This section quantifies our results above, using the calibration in Karp & Traeger (2021). Costs and damages are based on Nordhaus & Sztorc (2013), and climate dynamics are based on the somewhat better-performing TCRE model (IPCC 2013, Dietz & Venmans 2018). Appendix A graphs the temperature impulse response of this TCRE-based climate model and compares it to a middle-of-the road climate model as well as the 2016 DICE model. Here, we give a brief summary of the calibration; C.5 provides details.

The "transient climate response to cumulative carbon emissions" (TCRE) makes global warming a linear function of the cumulative past emissions (*not* the carbon concentration in the atmosphere). Emissions gradually leave the atmosphere, but they have a cumulative effect on temperature. These two non-linear effects almost cancel each other, resulting in an almost-linear relation between cumulative emissions and the temperature. With the TCRE model, we (i) track cumulative historic emissions so that $\delta = 1$ and (ii) associate the stock variable S_t with temperature. Our baseline uses the IPCC's (2021) best guess transient climate response to cumulative carbon emissions of $TCRE = 1.65 \frac{^oC}{TtC}$; this value coincides with the midpoint of the IPCC's (2013) interval estimate of the transient climate response of $[0.8, 2.5]$ in $\frac{^oC}{TtC}$, which we will relate to for our "greater climate response" scenario, and coincides with the point estimate of the IPCC's (2021).

We use a five-year policy period and assume an annual discount rate of 1.5%. We set $\rho = 1$ because our model describes the role of technological progress that persistently alters abatement costs. Our other baseline calibration follows DICE in assuming that flow damages are zero at the pre-industrial temperature ($T = 0$) and that damages at $T = 2$ equal approximately 1% of world output. Global world output is 130 trillion USD in 2020. Our baseline also adopts Karp & Traeger's (2021) estimate of the DICE's abatement cost slope as $f = 2.5\,10^{-9} \frac{USD}{tCO_2^2}$. We require the estimate of the technology level $h_{2020} = 101 \frac{USD}{tCO_2}$ only to calculate the social costs of carbon for our calibration. The calibration assumes that this intercept falls exogenously by 1% per year.

We also consider three alternatives.[20] The scenario "greater climate response" (greater CR) uses all of the baseline assumptions except that it sets the TCRE to the upper bound of the IPCC's (2013) estimated range, $TCRE = 2.5$. The scenario "greater damage convexity" (greater DC) uses the baseline parameters except that it assumes that a 1 degree temperature anomaly creates zero damage, but a three degree anomaly creates damages equal to 5% of world output. The final scenario assumes both greater damage convexity and greater climate response, combining the changes of the second and third scenarios.[21]

[20]The scenario "greater damage convexity" corresponds to the "concerned" scenario in Karp & Traeger (2021). The other two scenarios are newly calibrated for the current study.

[21]The value of b varies across the scenarios. We have $b = 1.32\,10^{-13}$ (baseline); $b = 3.02\,10^{-13}$ (greater climate response); $b = 6.58\,10^{-13}$ (greater damage convexity); and $b =$

Karp & Traeger (2021) regress carbon emissions on green patents, producing an estimate of $\alpha \approx 0.3$ for a five-year time step. This estimate is consistent with the widely accepted view that technology diffuses with a lag. Here we compare taxes and quotas for a range of α.

Our calibration implies 2020 BAU emissions of 40 $GtCO_2$, slightly higher than estimated emissions, and thus consistent with the current weak climate policy. Optimal 2020 emissions range from $22 - 29$ $GtCO_2$, over the four scenarios, implying reductions of $27 - 45\%$ relative to BAU. The optimal taxes range from $27 - 43$ $\frac{USD}{tCO_2}$. These values are independent of α.

Table 1 identifies the optimal policy instrument, Quota or Tax, under the four scenarios, for three values of α. The row α^{crit} shows the critical value of α, below which quotas dominate taxes. In the baseline, taxes dominate quotas if adoption happens reasonably fast. In the scenarios with greater climate response or greater damage convexity, quotas dominate taxes even if only half of the innovation is adopted within the regulation period. In the scenario with both greater climate response and greater damage convexity, quotas always dominate taxes. The row α^* shows the value of α at which the quota produces the full information first-best outcome.

The lower section of the table reduces the pure rate of time preference from 1.5% to 0.5%, following the median response of Drupp et al.'s (2018) survey. Under such an increased attention to future damages, quotas always dominate taxes. The volatility of emissions under a tax gains in weight relative to the volatility of firms' abatement costs under a quota. Finally, Table 1 presents the expected optimal carbon price in the different scenarios. Our baseline's 2020 price of $27\frac{USD}{tCO_2}$ equals Dietz et al.'s (2021) price of $27\frac{USD}{tCO_2}$ deriving from an enhanced version of the DICE model excluding certain nonlinear temperature-carbon cycle feedbacks and lies slightly below the average 2020 price of the EU ETS of $30\frac{USD}{tCO_2}$ (World Bank 2021*a*). The other scenarios yield higher expected SCCs. Maybe surprisingly, the greater damage convexity reduces the SCC from $55\frac{USD}{tCO_2}$ to $49\frac{USD}{tCO_2}$ under a low pure rate of time preference. The scenario with higher damage convexity reduces damages

$1.51\,10^{-12}$ (both greater climate response and damage convexity).

	baseline	greater climate response	greater damage convexity	greater climate response & damage convexity
p.r.t.p. 1.5%				
$\alpha = 1$:	Tax	Tax	Tax	Quota
$\alpha = 0.5$	Tax	Quota	Quota	Quota
$\alpha = 0.25$	Quota	Quota	Quota	Quota
α^{crit}	0.3	0.53	0.8	>1
α^*	0.15	0.26	0.38	0.52
SCC in USD/tCO_2	27	40	29	42
p.r.t.p. 0.5%				
for any $\alpha \leq 1$	Quota	Quota	Quota	Quota
SCC in USD/tCO_2	55	69	49	60

Table 1: The table presents the optimal policy choice, Quota or Tax, for two different pure rates of time preference (prtp) and three different choices for the share of technological innovation adopted within the regulation period ($\alpha = 1, 0.5, 0.25$). The table also presents α^{crit}, below which quotas dominate taxes, and α^* where a quota produces the full-information first best outcome (no deadweight loss).

at low temperature and increases damages at high temperatures. Under the reduced time preference, optimally regulated temperatures are sufficiently low such that the expected SCC falls relative to the baseline.

Figure 5 provides another perspective, showing graphs of the function G, defined in Proposition 3. Quotas dominate taxes if and only if $G < 1$. The graphs intersect the horizontal line $G = 1$ (where the two policies produce the same level of welfare) at $\alpha = \alpha^{crit}$ and they reach their minimum $G = 0$ (where there is no deadweight loss under quotas) at $\alpha = \alpha^*$.

Compared to previous results, our estimates are more favorable to quotas for three reasons. First, our model of gradual adoption of technology favors quotas when $\alpha < 1$. The gradual adoption of technology and the resulting gradual revelation of otherwise-hidden information reduces one of the major disadvantages of quotas: the concern that aggregate emissions respond too

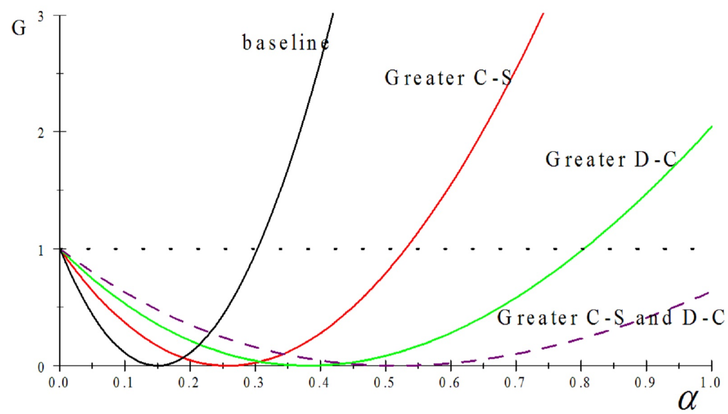


Figure 5: $G(\alpha)$ equals the ratio of the gain in moving from quotas to full information, relative to the gain in moving from taxes to full information. Quotas dominate taxes if and only if $G < 1$, as occurs for $\alpha < \alpha^{crit}$. The curves reach their minimum at $\alpha = \alpha^*$, where $G = 0$; at this value of α, the quota is first-best.

slowly to firms' information.

Second, the TCRE model implies that the state variable does not decay: $\delta = 1$. Simplified models merely relying on the atmospheric carbon stock as the state variable have lower persistence factors, reflecting the removal of the atmospheric carbon. The temperature response even in Nordhaus's (2017) DICE model is too sluggish, performing worse than the TCRE model's immediate persistent response (Mattauch et al. 2020, Dietz et al. 2021). Policy ranking can be sensitive to small changes in δ and β, parameters that determine the future costs of current actions.

Third, we emphasize technology rather than the other more transitory shocks that affect firms' emissions decisions. Thus, we set $\rho = 1$; ρ would be smaller if the shock were an amalgam of both persistent and transitory shocks. Again, our rationale for this focus is that the policy can be conditioned on the more transitory shocks, because these are publicly observed when policy is implemented, although not when policy rules are chosen.

5 Conclusions

A widely used (static) criterion for ranking price-based and quantity-based regulation does not carry over to the dynamic setting where current shocks affect future abatement costs, thereby affecting future regulation. We considered a setting with asymmetric information between the regulator and firms arising from technological change. The policy maker regulates an externality but does not observe current innovations. The standard ranking criterion incorporates the effect of innovations on firms' cost structure. Our criterion recognizes that the current technology innovation also alters future abatement costs and abatement levels, changing the stock trajectory. Both the persistent impact of shocks and a delayed technology diffusion favor quantity regulation.

Our discussion focuses on pollution control to mitigate climate change, where Weitzman's (1974) static ranking criterion is often informally applied. However, contrary to the assumptions of Weitzman's model, all regulated greenhouse gases are persistent and the major greenhouse gas, carbon dioxide, persists for centuries. We emphasize that moving from flow to stock damages substantially increases damage convexity, i.e., the slope of the damage curve. We cannot judge the slope of the cumulative damage curve (the Social Cost of Carbon) based on the (generally very flat) annual damage curve.

Our main contribution is a simple criterion for ranking prices versus quantities for stock externalities under asymmetric information. This criterion depends on both the ratio of the slopes and on the ratio of the shock-induced shifts in the intercepts of the marginal damage and abatement cost curves. The ratio of slopes is a familiar component, but the ratio of shifts in intercepts is novel, and is equally important in determining the ranking. Our graphical derivation furthers the intuition and produces an approximate ranking criterion. Our dynamic model formalizes the ranking criterion. There, we recognize that slope and shift parameters are endogenous. These conceptual changes in the ranking criterion result from the persistence of technology and its (potentially) gradual diffusion.

Our empirical application shows that the conceptual correction of the ranking criterion substantially weakens the case for price regulation in climate change mitigation. We presented several reasonable calibrations for which cap and trade (quantity regulation) dominates taxes (price regulation). We selected our dynamic model to permit general analytic insight, restricting it to two state variables. As a result, the model remains a simple and stylized description of the complex assessment of climate change, even though we calibrate carefully to the integrated assessment literature and climate data. Our quantitative results do not imply that quotas necessarily dominate taxes in controlling carbon dioxide, but they demonstrate that our conceptual correction of the common ranking criterion has serious policy implications.

Technological uncertainty, which lies at the heart of Weitzman's (1974) asymmetric information problem, means that the regulator does not learn firms' current costs even after many observations. In the pollution context, the long-lasting impact of current shocks on future abatement costs alters future emissions, changing social damages because these depend on cumulative emissions. Similar problems arise wherever asymmetric information is important and a regulator's objective depends on cumulative regulated actions.

References

Aldy, J. E., Krupnick, A. J., Newell, R. G., Parry, I. W. & Pizer, W. A. (2010), 'Designing climate mitigation policy', *Journal of Economic Literature* **48**(4), 903–934.

Anderson, E., Brock, W., Hansen, L. P. & Sanstad, A. H. (2014), 'Robust analytical and computational exploration of coupled economic-climate models with carbon-climate response', *RDCEP Working Paper No.13-05* .

Brock, W. & Xepapadeas, A. (2017), 'Climate change policy under polar amplification', *European Economic Review* **99**, 93 – 112. Combating Climate Change. Lessons from Macroeconomics, Political Economy and Public Finance.

Burtraw, D., Holt, C., Palmer, K. & Shobe, W. (2020), 'Quantities and prices: Price-responsive allocance supply in environmental markets', *RFF Working Paper 20-17* .

Dietz, S., van der Ploeg, F., Rezai, A. & Venmans, F. (2021), 'Are economists getting climate dynamics right and does it matter?', *Journal of the Association of Environmental and Resource Economics* **8**(5).

Dietz, S. & Venmans, F. (2018), 'Cumulative carbon emissions and economic policy: in search of general principles', *Grantham Working Paper 283* .

Doda, B. (2016), 'How to price carbon in good times... and bad!', *Climate change, 2016-01* **7**(1), 135–144.

Drupp, M., Freeman, M., Groom, B. & Nesje, F. (2018), 'Discounting disentangled', *American Economic Journal: Economic Policy* **4**(4), 109 – 34.

Ellerman, A. D. & Wing, I. S. (2003), 'Absolute versus intensity-based emission caps', *Climate Policy* **3**, S7–S20.

Fischer, C. & Springborn, M. (2011), 'Emissions targets and the real business cycle: Intensity targets versus caps or taxes', *Journal of Environmental Economics and Management* **62**, 352–366.

Goulder, L. & Schein, A. (2013), 'Carbon taxes versus cap and trade: A critical review', *Climate Change Economics* **04**(03), 1350010/1–28.

Hepburn, C. (2006), 'Regulation by prices, quantities, or both: a review of instrument choice', *Oxford Review of Economic Policy* **22**(2), 226–247.

Heutel, G. (2012), 'How should environmental policy respond to business cycles? optimal policy under persistent productivity shocks', *Review of Economic Dynamics* **15**, 244–264.

Hoel, M. & Karp, L. (2001), 'Taxes and quotas for a stock pollutant with multiplicative uncertainty', *Journal of Public Economics* **82**, 91–114.

Hoel, M. & Karp, L. (2002), 'Taxes versus quotas for a stock pollutant', *Resource and Energy Economics* **24**, 367 – 384.

IPCC (2013), Climate change 2013 - the physical science basis - contribution of working group 1 to the ipcc - summary for policymakers, Technical report, IPCC.

IPCC (2021), *Climate Change 2021: The Physical Science Basis*, Cambridge University Press, in Press, Cambridge.

Jotzo, F. & Pezzey, J. C. (2007), 'Optimal intensity targets for greenhouse gas emissions trading under uncertainty', *Environmental and Resource Economics* **38**, 259–284.

Karp, L. & Traeger, C. (2021), 'Smart cap', *CESifo Working Paper 8971* . First Version: 2016.

Karp, L. & Zhang, J. (2005), 'Regulation of stock externalities with correlated abatement costs', *Environmental and Resource Economics* **32**, 273–299.

Karp, L. & Zhang, J. (2006), 'Regulation with anticipated learning about environmental damage', *Journal of Environmental Economics and Management* **51**, 259–280.

Karp, L. & Zhang, J. (2012), 'Taxes versus quantities for a stock pollutant with endogenous abatement costs and asymmetric information', *Economic Theory* **49**, 371–409.

Kelly, D. L. (2005), 'Price and quantity regulation in general equilibrium', *Journal of Economic Theory* **125**(1), 36 – 60.

Mattauch, L., Matthews, H. D., Millar, R., Rezai, A., Solomon, S. & Venmas, F. (2020), 'Comment on: Steering the climate system - using inertia to lower the cost of policy', *American Economic Review* **114**, 1231–1237.

Montero, J. (2002), 'Prices versus quantities with incomplete enforcement', *Journal of Publicl Economics* **85**, 435–54.

Newbery, D. (2018), 'Policies for decarbonizing carbon in a liberalized power sector', *Kiel Institute for the World Economy Economics Discussion Papers No 2018-29* .

Newell, R. G. & Pizer, W. A. (2008), 'Indexed regulation', *Journal of Environmental Economics and Management* **56**, 221–233.

Newell, R. & Pizer, W. (2003), 'Regulating stock externalitiues under uncertainty', *Journal of Environmental Economics and Management* **45**, 416–32.

Nordhaus, W. D. (2008), *A Question of Balance: Weighing the Options on Global Warming Policies*, Yale University Press, New Haven.
URL: *http://nordhaus.econ.yale.edu/DICE2007.htm*

Nordhaus, W. D. (2017), 'Revisiting the social cost of carbon', *Proceedings of the National Academy of Sciences* **114**(7), 1518–1523.

Nordhaus, W. & Sztorc, P. (2013), *DICE2013R:Introduction and User's Manual*, dicemodel.net.

Perino, G. & Requate, T. (2012), 'Does more stringent environmental regulation induce or reduce technology adoption?', *Journal of Environmental Economics and Management* **64**(3), 456–467.

Pizer, W. A. (1999), 'The optimal choice of climate change policy in the presence of uncertainty', *Resource and Energy Economics* **21**, 255–287.

Pizer, W. & Prest, B. (2020), 'Prices versus quantities with policy updating', *Journal of the Association of Environmental and Resource Economics* **7**, 483 518.

Requate, T. & Unold, W. (2003), 'Environmental policy incentives to adopt advanced abatement technology: Will the true ranking please stand up?', *European Economic Review* **47**, 125–146.

Rogers, E. (2003), *Diffusion of Innovations, 5th ed*, Simon and Schuster.

Shinkuma, T. & Sugeta, H. (2016), 'Taxes versus emissions trade scheme in the long run', *Journal of Environmental Economics and Management* **75**, 12–24.

Stavins, R. (1996), 'Correlated uncertainty and policy instrument choice', *Journal of Environmental Economics and Management* **30**(2), 218–232.

Stavins, R. N. (2020), 'The future of u.s. carbon-pricing policy', *Environmental and Energy Policy and the Economy* **1**, 8–64.

Traeger, C. P. (2021), 'ACE – Analytic Climate Economy', *American Economic Journal: Economic Policy* . conditional accept.

Weitzman, M. (1974), 'Prices versus quantities', *Review of Economic Studies* **41**(4), 477–91.

Weitzman, M. (2018), 'Prices or quantities dominate banking and borrowing', *NBER 24218* .

Wood, P. J. & Jotzo, F. (2011), 'Price floors for emissions trading', *Energy Policy* **39**, 1746–1753.

World Bank (2021*a*), 'Carbon Price EU ETS 2020'. data retrieved from the World Bank's Carbon Pricing Dashboard, https://carbonpricingdashboard.worldbank.org.

World Bank (2021*b*), *State and Trends of Carbon Pricing 2021*, The World Bank, Washington, DC.

A Climate Dynamics

This section discusses our model's implied climate dynamics. Figure 6 shows the temperature response over the coming 150 years to 100Gt of carbon dioxide emitted today. The experiment fixes the background concentration at today's

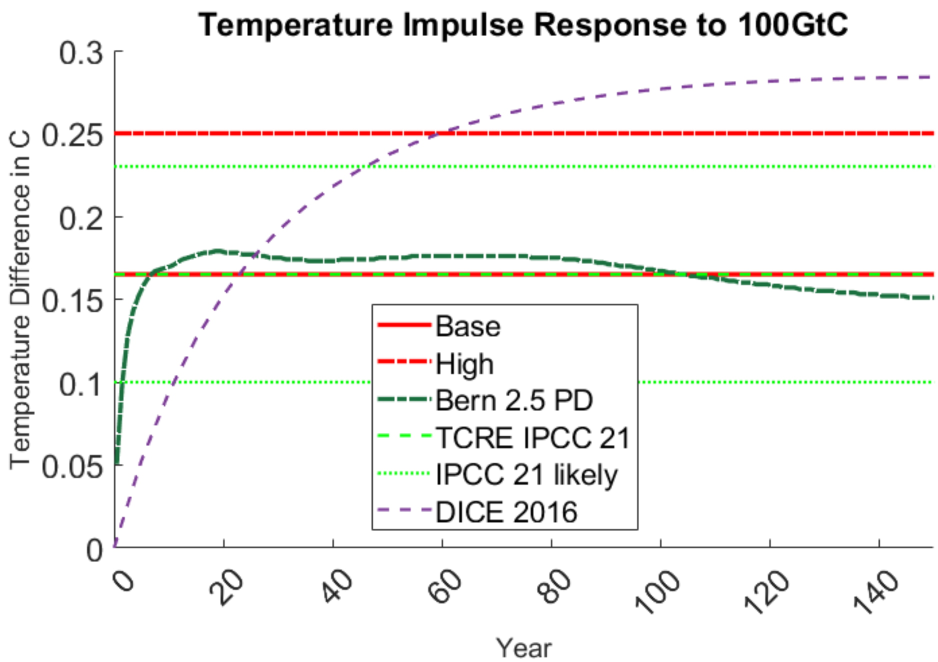


Figure 6: The figure shows the temperatures impulse response to carbon dioxide emissions. Our base scenario coincides with the IPCC's (2021) best guess for the transient climate response to cumulative carbon emissions, which is a good approximation to more sophisticate climate change models like the Bern 2.5 model.

atmospheric carbon dioxide concentration.[22] Our base scenario, taken from Karp & Traeger (2021), matches the IPCC's (2021) best guess for the transient climate response to cumulative carbon emissions, a 1.65C warming for a 1000Gt emission pulse and, thus, one tenth of this response to a 100 Gt impulse. The thick green line labeled "Bern 2.5 PD" presents the results of the Bern 2.5 model, a detailed middle-of-the-road climate change model used in previous iterations of the IPCC reports to calculate global warming potentials. It shows that the actual temperature response is not expected to be a truly flat line, but that the TCRE's constant temperature response is a good

[22]The TCRE model and the DICE model's response is independent of the background concentration, which matters for the Bern 2.5 model where "PD" abbreviates "present day". We are grateful to Fortunat Joos who provided the Bern 2.5 results. Similar data is presented in Traeger (2021) and Karp & Traeger (2021)

approximation to sophisticated climate change models. The graph also depicts the IPCC's (2021) 67% likelihood interval (green dotted). Our "greater climate response" scenario (high, red dash-dotted) lies just outside of this interval, representing the upper bound of the IPCC's (2013) interval estimate for the TCRE. Finally, the graph compares these models to the impulse response of Nordhaus's (2017) DICE 2016 model. As emphasized by previous authors (Mattauch et al. 2020, Dietz et al. 2021), the DICE model's response to carbon emissions is too sluggish, and the medium run temperature response of the DICE 2016 model overshoots the best guess even more than our "high" scenario.

B Appendix: Proofs

Derivation of equation (1): Using the upper right panel of Figure 1, it is straightforward to establish **Weitzman's result** that a tax dominates a quota for a flow pollutant if and only if the MAC curve is steeper than the MD curve. We use the lower right panel in Figure 1 to confirm inequality (1). The ranking criterion depends on both the responsiveness φ of the MD curve to a shift of the MAC and on the relative slopes. We use three geometrical relations from the graph. First, we relate the deadweight loss under the quota to the relative shift φ. Using the relation $\frac{d}{d+k} = \varphi$, we have

$$d = \frac{\varphi\, k}{1-\varphi} \quad \text{or} \quad \frac{d}{k} = \frac{\varphi}{1-\varphi}. \tag{6}$$

The light green and the red triangles representing the deadweight loss in the two settings are similar (same angles), and we compare them based on their sides s and k. By the definition of the slope, $h\, m^{MAC} = k+d \Rightarrow \frac{h\ m^{MAC}}{k} = 1+\frac{d}{k}$. Using equation (6) to replace the fraction $\frac{d}{k}$ delivers

$$\frac{h\ m^{MAC}}{k} = \frac{1}{1-\varphi}. \tag{7}$$

Similarly, we observe that $h\ m^{MD} = s - d \Rightarrow \frac{h\ m^{MD}}{s} = 1 - \frac{d}{s}$. Using equation (6) to replace d, we obtain

$$\frac{h\ m^{MD}}{s} = 1 - \frac{\varphi}{1-\varphi}\frac{k}{s}. \tag{8}$$

Dividing equation (8) by equation (7) and solving for $\frac{s}{k}$ delivers

$$\frac{s}{k} = \frac{1}{1-\varphi}\left(\frac{m^{MD}}{m^{MAC}} + \varphi\right).$$

Taxes dominate quotas if and only if the deadweight loss of the tax is smaller than the deadweight loss of a quota, i.e., $\frac{s}{k} < 1$, leading to equation 1.

Notation for proofs of dynamic model: We take advantage of the linear-quadratic structure to unify the separate the problems when the regulator uses taxes or quotas or in the first best (full information) setting. To this end, we introduce the indicator function $\Phi = 1$ under a tax and $\Phi = 0$ under a quota. Using $x_t \in \{e^T, e^Q\}$ to denote the regulator's control under tax and quantity regulation, respectively, the regulator's problem, for $i \in \{T, Q\}$, is

$$\max \mathbf{E}_t \sum_{\tau=0}^{\infty} \beta \left[\left(h_{t+\tau} + \rho\theta_{t+\tau-1}\right) x_{t+\tau} - \tfrac{f}{2}\left(x_{t+\tau}\right) + \Phi \tfrac{\alpha^2}{2f}\sigma^2 - \tfrac{b}{2} S_{t+\tau}^2 \right] \phi$$

$$\text{subject to } S_{t+\tau+1} = \delta S_{t+\tau} + \phi x_{t+\tau} + \Phi\phi\alpha\tfrac{\varepsilon_t}{f} \text{ and } \theta_t = \rho\theta_{t-1} + \varepsilon_t.$$

The term $\Phi\frac{\alpha^2}{2f}\sigma^2$ in the payoff arises from taking expectations, in each period, of the shock for that period, ε_t. Here we use the assumption that these shocks are iid with mean zero. The problem formulated using x and Φ is the "generic problem" because it subsumes the problems under both taxes and quotas.

Because the problem has two state variables, it is convenient to use matrix notation. We define the state vector as $Y_t = (S_t, \theta_{t-1})'$ and we define:

$$Q = \begin{pmatrix} -b & 0 \\ 0 & 0 \end{pmatrix}, \qquad A = \begin{pmatrix} \delta & 0 \\ 0 & \rho \end{pmatrix}, \qquad W = \begin{pmatrix} 0 & \rho \end{pmatrix}, \tag{9}$$

$$B = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \qquad C = \begin{pmatrix} \Phi\phi\frac{\alpha}{f} \\ 1 \end{pmatrix}.$$

The net flow payoff and equation of motion for the generic problem are:

$$\left[h_t x_t - \tfrac{1}{2} f x_t^2 + \tfrac{1}{2} Y_t' Q Y_t + W Y_t x_t + \Phi \tfrac{\alpha^2}{2f}\sigma^2\right]\phi \text{ and}$$

$$Y_{t+1} = AY_t + Bx_t + C\varepsilon_t.$$

Proof of Lemma 1. The dynamic programming equation for the generic problem is:

$$J_t^i\left(Y_t\right) = \operatorname*{Max}_{x_t}\left[h_t x_t - \frac{1}{2} f x_t^2 + \frac{1}{2} Y_t' Q Y_t + W Y_t x_t + \Phi\frac{\alpha^2}{2f}\sigma^2\right]\phi + \beta \mathbf{E}_t J_{t+1}^i\left(Y_{t+1}\right). \tag{10}$$

The subscript t in J_t takes into account that the value function depends explicitly on calendar time due to the intercept of marginal costs, h_t.

The value function for the LQ problem, for $i \in \{T\,Q\}$, is linear-quadratic: $J_t^i\left(Y_t\right) = V_{0,t}^i + V_{1t}' Y_t + \frac{1}{2} Y_t' V_2 Y_t$. The terms V_{1t} and V_2 are the same under taxes and quotas; only the term $V_{0,t}^i$ differs. The terms V_{1t} and V_{0t}^i inherit the time-dependence of h_t, but V_2 is constant. Denote $v_{1,t}$ as the first element of the column matrix V_{1t}, and define $\chi_t = -\beta v_{1,t}$, the intercept of the graph of the present value of the social cost of carbon. V_2 is a symmetric matrix:

$$V_2 = -\begin{bmatrix} \lambda & \mu \\ \mu & \nu \end{bmatrix}. \tag{11}$$

We write the difference in the payoff under taxes and under quotas as

$$\Delta_t \equiv V_{0,t}^T - V_{0,t}^Q.$$

Online Appendix C.2 provides the details of the following steps:

1) We substitute the equations of motion into the right side of the DPE, equation (10), and take expectations.

2) We use the first order condition for x_t to obtain the linear control rule, $x_t = Z_{0t} + ZY_t$. The coefficients of the control rule, Z_{0t} and Z, are the same under taxes and quotas, a consequence of the "Principle of Certainty Equivalence"; Z is a constant row vector and Z_{0t} is a time-varying scalar.

3) We substitute the optimal control rule back into the right side of the DPE to obtain the maximized DPE.

4) Equating coefficients of the terms that are quadratic in Y_t and independent of Y_t (on the two sides of the DPE) we obtain, respectively, an algebraic Riccati equation for V_2 and a difference equation for V_{0t}^i.

This algorithm produces formulae for the endogenous parameters λ and μ. Using the definition $\varpi \equiv f\left(1 - \beta\delta^2 - \beta\frac{b}{f}\phi^2\right)$, λ and μ satisfy

$$\lambda = \frac{1}{2\beta\phi}\left(-\varpi + \sqrt{\varpi^2 + 4\beta\phi^2 bf}\right) > 0 \tag{12}$$

$$\mu = \frac{\lambda}{f}\left(\frac{\phi\beta\delta\rho}{(1-\beta\delta\rho) + \beta\phi\frac{\lambda}{f}}\right). \tag{13}$$

From inspection of equation (12), $\lambda > 0$, so the numerator of the right side of equation (13) is positive. Therefore, μ has the same sign as ρ, which in our setting is positive, because the shock describes a technological innovation.[23].

We define $r \equiv \frac{b}{f}$, the ratio of the slopes of marginal damages and marginal benefit (equal to marginal abatement cost) and $R \equiv \frac{\lambda}{f}\phi$, the ratio of the slope of the SCC and the marginal flow benefit. The flexible time step ϕ enters the definition of R because we are interested in the ratio of the costs from an additional unit of emissions in the atmosphere λ and the benefits of emitting one more unit of emissions over the course of a period. If the period is not a year, then the benefit from *one* unit of emissions is $\frac{f}{\phi}$ rather than f. The

[23]The units of λ are $\frac{USD}{GtCO_2{}^2}$: The units of ϖ coincide with those of f and $\frac{1}{\phi}$ eliminates the time unit in f. The value function parameter μ is unit-free.

parameter f measures the benefit from increasing the annual emission flow by one unit (so ϕ times the unit increase over the course of a period). Dividing both sides of equation (12) by f establishes Lemma 1.

■

The algorithm described above also produces the formula for χ_t, the intercept of the SCC_t (Online Appendix C.2) Using the definitions

$$M = \frac{\beta\delta f}{f+\beta\phi\lambda} \text{ and } N = -\frac{\beta\delta\phi\lambda}{f+\beta\phi\lambda}.$$

we express χ_t as

$$\chi_t = -N\sum_{j=0}^{\infty} M^j h_{t+j}. \tag{14}$$

If h falls at a constant rate (as in our climate application) we can express this infinite sum as a function of the current level h_t and the model parameters.

Proof of Proposition 1. Step 4 in the algorithm described in the proof of Lemma 1 also produces the difference equation for $V^i_{0,t}$:

$$V^i_{0,t} = \left(h_t Z_{0t} - \tfrac{1}{2} f\left(Z_{0t}\right)^2 + \Phi\tfrac{\alpha^2}{2f}\sigma^2\right)\phi +$$

$$\beta\left(V^i_{0,t+1} + V'_{1t+1}BZ_{0t} + \tfrac{1}{2}\left(BZ_{0t}\right)' V_2\left(BZ_{0t}\right) + \tfrac{1}{2}C'V_2C\sigma^2\right).$$

(Online Appendix C.2 derives this relation; see equation 29.) We define $\Delta_t \equiv V^T_{0,t} - V^Q_{0,t}$, the difference in payoff under taxes and quotas. Using the fact that Z_{0t}, V_{t+1}, and V_2 are the same under taxes and quotas, and the definitions of Φ and C, we obtain the difference equation

$$\Delta_t = V^T_{0,t} - V^Q_{0,t} = \tfrac{\alpha^2}{2f}\sigma^2\phi + \beta\Delta_{t+1}$$

$$-\tfrac{1}{2}\beta\sigma^2\left[\begin{pmatrix}\phi\frac{\alpha}{f} & 1\end{pmatrix}\begin{bmatrix}\lambda & \mu\\ \mu & \nu\end{bmatrix}\begin{pmatrix}\phi\frac{\alpha}{f}\\ 1\end{pmatrix} - \begin{pmatrix}0 & 1\end{pmatrix}\begin{bmatrix}\lambda & \mu\\ \mu & \nu\end{bmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}\right] \Rightarrow$$

$$\Delta_t = \beta\Delta_{t+1} + \tfrac{\alpha^2}{2f}\sigma^2\phi - \tfrac{1}{2}\beta\sigma^2\phi\alpha\tfrac{\phi\alpha\lambda+2\mu f}{f^2} = \beta\Delta_{t+1} + \tfrac{\alpha\phi}{2f}\sigma^2\left(\alpha - \beta\tfrac{\phi\alpha\lambda+2\mu f}{f}\right).$$

The last line follows from carrying out the matrix multiplication and then simplifying. The steady state of this equation is the constant

$$\Delta = \frac{1}{1-\beta}\frac{\alpha\phi}{2f}\sigma^2\left(\alpha - \beta\frac{\phi\alpha\lambda + 2\mu f}{f}\right). \tag{15}$$

Using the definition $R \equiv \frac{\lambda}{f}\phi$ we have

$$\Delta = \frac{1}{1-\beta}\frac{\alpha\phi}{2f}\sigma^2\left(\alpha - \beta\left(2\mu + \alpha R\right)\right).$$

This equation implies that taxes dominate quotas if and only if

$$\alpha - \beta\left(2\mu + \alpha R\right) > 0. \tag{16}$$

Rearranging this inequality establishes the equivalence (3). The explicit dependence on the time step ϕ has dropped out; ϕ matters only through the scaling of the discount factor β and the decay factor δ.

Using inequality (16), the definition of R, and equation (13) we have

$$\alpha - \beta\left(2\frac{\beta\delta\rho R}{(1-\beta\delta\rho) + \beta R} + \alpha R\right) > 0.$$

Multiplying by the positive denominator, this inequality is equivalent to

$$\begin{aligned}
&\alpha\left(\left(1-\beta\delta\rho\right) + \beta R\right) - \beta\left(2\left(\beta\delta\rho R\right) + \alpha R\left(\left(1-\beta\delta\rho\right) + \beta R\right)\right) > 0\\
\Leftrightarrow & R^2 + \frac{1}{\alpha}\delta\rho\left(2-\alpha\right)R - \frac{(1-\beta\delta\rho)}{\beta^2} < 0\\
\Leftrightarrow & R^2 + \kappa_1 R - \kappa_0 < 0,
\end{aligned}$$

where we use the definitions $\kappa_1 \equiv \frac{\delta\rho(2-\alpha)}{\alpha} > 0$ and $\kappa_0 \equiv \frac{1-\beta\delta\rho}{\beta^2} > 0$.

The quadratic expression $R^2 + \kappa_1 R - \kappa_0$ is negative at $R = 0$ and remains negative for R smaller than the positive root of the quadratic, defined as R^{crit} in the proposition. Hence the inequality is satisfied for $R \in [0, R^{crit})$. ■

Proof of Proposition 2. (i) In the first best (full information) world the regulator observes the technology shock in each period before choosing the level of emissions. Here, the regulator conditions emissions on S_t, θ_{t-1} and ε_t. Under asymmetric information and quotas, the regulator chooses emissions conditioned on S_t, θ_{t-1} and $E\varepsilon_t = 0$: under the quota, emissions do not depend on ε_t. Thus, the quota might be first best only if the first best level of emissions does not depend on ε_t.

We use properties of the linear quadratic problem to show that the independence of the first best level of emissions and ε_t is sufficient, not merely necessary, for the quota to be first best. By the Principle of Certainty Equivalence for the linear quadratic problem, the coefficients of the linear and quadratic parts of the value function, V_{1t} and V_2, are the same under taxes and quotas in the scenario with asymmetric information and also in the first best scenario. Thus, the parameters χ_t, λ, and μ are the same across the three scenarios.

The first best level of emissions equates the realized MAC and the present value of the social cost of carbon:

$$\rho\theta_{t-1} + \alpha\varepsilon_t - fE_t^{FB} = \beta\left(\chi_t + \lambda\left(\delta S_t + E_t^{FB}\right) + \mu\left(\rho\theta_{t-1} + \varepsilon_t\right)\right), \tag{17}$$

where E_t^{FB} denotes the first best level of emissions. An innovation ε_t causes the MAC curve to shift up by $\alpha\varepsilon_t$, and the present value of the SCC to shift up by $\beta\mu\varepsilon_t$. We obtain the first order condition for the quota under asymmetric information by replacing ε_t with $E\varepsilon_t = 0$ and by replacing E_t^{FB} with E_t^Q (the quota) in equation (17). The fact that χ_t, λ, and μ are the same in the first best world and under quotas (and taxes) implies that the quota is first best if and only if the first best level of emissions does not depend on ε_t. From equation (17) this necessary and sufficient condition is equivalent to $\alpha = \beta\mu$.

Thus, to establish part (i) of the Proposition we need only establish that there exist an $\alpha \in (0, 1]$ that satisfies $\alpha = \beta\mu$. We have already established (for $\rho > 0$, our maintained assumption) that $\mu > 0$. To complete the proof we need only confirm that $\beta\mu \leq 1$. Using the definitions of μ and R, we have

$$\beta\mu \leq 1 \Leftrightarrow \beta^2 R \tfrac{\delta\rho}{(1-\beta\delta\rho)+\beta R} \leq 1 \Leftrightarrow$$
$$R\beta\left(\beta\delta\rho - 1\right) \leq \left(1 - \beta\delta\rho\right). \tag{18}$$

Because $\beta\delta\rho$ is bounded away from 1 and $R > 0$, the last inequality is always satisfied. Therefore, there exists $\alpha \in (0, 1]$ that satisfies $\alpha = \beta\mu$.

(ii) To show that a reduction in α favors quotas, we note that R^{crit} is a differentiable function of α. Using the chain rule and the definitions of κ_1 and κ_0, we obtain

$$\frac{dR^{crit}}{d\alpha} = -\frac{1}{2}\frac{\kappa_1 - \sqrt{\kappa_1^2 + 4\kappa_0}}{\sqrt{\kappa_1^2 + 4\kappa_0}} 2\delta \frac{\rho}{\alpha^2} > 0. \tag{19}$$

Therefore, a reduction in α lowers the critical value R^{crit}, above which quotas dominate taxes.

To establish the second part of Part (ii), we note from Part (i) that for $\alpha = \beta\mu$ the quota is first best. Under the tax (using $E^T = e^T + \alpha\frac{\varepsilon_t}{f}$), we have

$$\frac{dE^T}{d\varepsilon_t} = \frac{\alpha}{f} > \frac{\alpha - \beta\mu}{f + \beta\lambda} = \frac{dE^{FB}}{d\varepsilon_t}, \tag{20}$$

where the second equality uses the first order condition (17) and the inequality uses $\lambda > 0$ and $\mu > 0$. This inequality means that emissions under the tax are always more responsive to a shock, compared to the first best level of emissions. Therefore, the tax can never support the first best level of emissions; quotas strictly dominate taxes for $\alpha = \beta\mu$, where the quota is first best. This fact and inequality (19) imply that quotas strictly dominate taxes for $\alpha \leq \alpha^* = \beta\mu$. The fact that this dominance is strict means that there exists $\alpha^{crit} > \alpha^*$ for which quotas strictly dominate taxes when $\alpha < \alpha^{crit}$. ■

C Extended Appendix: For online publication

The first section of this appendix discusses the role of marcoeconomic shocks and the time step. The next section collects the details summarized by the algorithm in the proof of Lemma 1. We then provide the proof of Proposition 3. The fourth section discusses discuss implementation of the feedback policies, and the fifth explains our calibration of the climate model. The sixth section provides heuristic arguments for Propositions 1 and 2. The final section derives the formula for the correlation in footnote 13.

C.1 Discussion of model

A comment on macroeconomic shocks: The intercept of the firms' marginal benefit of emissions, $h_t + \rho\theta_{t-1} + \alpha\varepsilon_t$, incorporates exogenous changes via h_t. We treat the function h_t as deterministic, but nothing changes if we add to h_t an iid shock, uncorrelated with the technology shock, ε_t.

If h_t is serially correlated, e.g., if it depends on macroeconomic variables such as the business cycle, we would need to add additional state variables to the model. For example, suppose that a serially correlated macro shock m_t affects the intercept of marginal abatement cost, with $m_t = \rho^m m_{t-1} + \epsilon_t^m$. The regulator knows m_{t-1} when she announces the policy at the beginning of period t, so m_{t-1} is a component of the state variable at time t; m_t is public knowledge when firms choose their emissions. Instead of choosing a level of the tax or the quota, the regulator can choose a rule that is conditioned on m_t. Provided that ϵ_t and ϵ_t^m are uncorrelated, this generalization does not change the ranking criterion. Our paper therefore focuses on the technology-related shock, which remains private information until the regulator observes the emissions response to a tax, or the price of a quota. Hereafter, we ignore the macro-related shock.

Time step: We choose the unit of time to be one year and we use the parameter ϕ to represent the time step. Thus, if the time step is one decade, $\phi = 10$. This parameter does not appear in the formulae used in the text, because there we set $\phi = 1$. The parameter enables us to calibrate the model to a particular time step – we chose one year – and then change the time step of the model without changing f or b.

For example, if the firms' benefit, during one year, of emitting at the annual rate of x_t is $(h_t + \rho\theta_{t-1} + \alpha\varepsilon_t)\, x_t - \frac{f}{2}x_t$, then their benefit of emitting at the same annual rate over a decade is $\left[(h_t + \rho\theta_{t-1} + \alpha\varepsilon_t)\, x_t - \frac{f}{2}x_t\right] 10$. This formulation ignores discounting during a period. However, including intra-period discounting merely introduces a constant factor multiplying each period payoff, without changing the optimization problem or the policy ranking. Similarly, if the stock during a period is S_t, annual damages equal $\frac{b}{2}S_t^2$ and damages during a decade equal $\frac{b}{2}S_t^2 10$. This formulation again ignores intra-period discounting, and additionally assumes that the stock is constant during the period. In the climate context, the stock changes little during a year or a decade, so the assumption of a constant intra-period stock is unimportant. It would be easy to drop this assumption at the cost of slightly more complicated notation. The length of the time step also affects the numerical value of the discount factor β and the decay factor δ. Our numerical evaluation takes this (exponential) scaling into account. In contrast, α denotes the adoption during the current policy period and is therefore independent of the time step.

A comment on macroeconomic shocks: The intercept of the firms' marginal benefit of emissions, $h_t + \rho\theta_{t-1} + \alpha\varepsilon_t$, incorporates exogenous changes via h_t. We treat the function h_t as deterministic, but nothing changes if we add to h_t an iid shock, uncorrelated with the technology shock, ε_t.

If h_t is serially correlated, e.g., if it depends on macroeconomic variables such as the business cycle, we would need to add additional state variables to the model. For example, suppose that a serially correlated macro shock m_t affects the intercept of marginal abatement cost, with $m_t = \rho^m m_{t-1} + \epsilon_t^m$. The regulator knows m_{t-1} when she announces the policy at the beginning of period t, so m_{t-1} is a component of the state variable at time t; m_t is public knowledge when firms choose their emissions. Instead of choosing a level of the tax or the quota, the regulator can choose a rule that is conditioned on

m_t. Provided that ϵ_t and ϵ_t^m are uncorrelated, this generalization does not change the ranking criterion. Our paper therefore focuses on the technology-related shock, which remains private information until the regulator observes the emissions response to a tax, or the price of a quota. Hereafter, we ignore the macro-related shock.

Time step: We choose the unit of time to be one year and we use the parameter ϕ to represent the time step. Thus, if the time step is one decade, $\phi = 10$. This parameter does not appear in the formulae used in the text, because there we set $\phi = 1$. The parameter enables us to calibrate the model to a particular time step – we chose one year – and then change the time step of the model without changing f or b.

For example, if the firms' benefit, during one year, of emitting at the annual rate of x_t is $(h_t + \rho\theta_{t-1} + \alpha\varepsilon_t)\, x_t - \frac{f}{2}x_t$, then their benefit of emitting at the same annual rate over a decade is $\left[(h_t + \rho\theta_{t-1} + \alpha\varepsilon_t)\, x_t - \frac{f}{2}x_t\right] 10$. This formulation ignores discounting during a period. However, including intra-period discounting merely introduces a constant factor multiplying each period payoff, without changing the optimization problem or the policy ranking. Similarly, if the stock during a period is S_t, annual damages equal $\frac{b}{2}S_t^2$ and damages during a decade equal $\frac{b}{2}S_t^2 10$. This formulation again ignores intra-period discounting, and additionally assumes that the stock is constant during the period. In the climate context, the stock changes little during a year or a decade, so the assumption of a constant intra-period stock is unimportant. It would be easy to drop this assumption at the cost of slightly more complicated notation. The length of the time step also affects the numerical value of the discount factor β and the decay factor δ. Our numerical evaluation takes this (exponential) scaling into account. In contrast, α denotes the adoption during the current policy period and is therefore independent of the time step.

C.2 Material for Lemma 1

The equations of motion are

$$Y_{t+1} = AY_t + Bx_t + C\varepsilon_t$$

and the value function is

$$J_t(Y_t) = V_{0,t}^i + V_{1t}'Y_t + \frac{1}{2}Y_t'V_2Y_t.$$

The right side of the DPE, equation 10, is

$$\left(h_t x_t - \tfrac{1}{2}fx_t^2 + \tfrac{1}{2}Y_t'QY_t + WY_t x_t + \Phi\tfrac{\alpha^2}{2f}\sigma^2\right)\phi + \beta\mathbf{E}_t J_{t+1}(Y_{t+1})$$

$$= \left(h_t x_t - \tfrac{1}{2}fx_t^2 + \tfrac{1}{2}Y_t'QY_t + WY_t x_t + \Phi\tfrac{\alpha^2}{2f}\sigma^2\right)\phi$$

$$+\beta\mathbf{E}_t\left(V_{0,t+1}^i + V_{1t+1}'Y_{t+1} + \tfrac{1}{2}Y_{t+1}'V_2Y_{t+1}\right).$$

Substituting in the equations of motion, we write the right side of the DPE as

$$\left(h_t x_t - \tfrac{1}{2}fx_t^2 + \tfrac{1}{2}Y_t'QY_t + WY_t x_t + \Phi\tfrac{\alpha^2}{2f}\sigma^2\right)\phi +$$

$$\beta\mathbf{E}_t[V_{0,t+1}^i + V_{1t+1}'(AY_t + Bx_t + C\varepsilon_t)$$

$$+\tfrac{1}{2}(AY_t + Bx_t + C\varepsilon_t)'V_2(AY_t + Bx_t + C\varepsilon_t)].$$

Taking expectations gives

$$\left(h_t x_t - \tfrac{1}{2}fx_t^2 + \tfrac{1}{2}Y_t'QY_t + WY_t x_t + \Phi\tfrac{\alpha^2}{2f}\sigma^2\right)\phi +$$

$$\beta[V_{0,t+1}^i + V_{1t+1}'(AY_t + Bx_t) + \tag{21}$$

$$\tfrac{1}{2}(AY_t + Bx_t)'V_2(AY_t + Bx_t) + \tfrac{\sigma^2}{2}C'V_2C].$$

The first order condition is

$$(h_t - fx_t + WY_t)\,\phi + \beta\left(V'_{1t+1}B + B'V_2Bx_t + B'V_2AY_t\right) = 0 \Rightarrow$$

$$h_t\phi + \beta V'_{1t+1}B + (W\phi + \beta B'V_2A)\,Y_t = (f\phi - \beta B'V_2B)\,x_t = 0,$$

which implies the control rule

$$x_t = \frac{1}{f\phi - \beta\,(B'V_2B)}\left(h_t\phi + \beta V'_{1t+1}B + (W\phi + \beta B'V_2A)\,Y_t\right)$$

or

$$\begin{gathered} x_t = Z_{0t} + ZY_t \text{ with} \\ Z_{0t} = \tfrac{h_t\phi + \beta V'_{1t+1}B}{f\phi - \beta(B'V_2B)} \text{ and } Z = \tfrac{W\phi + \beta B'V_2A}{f\phi - \beta(B'V_2B)}. \end{gathered} \tag{22}$$

Substituting the control rule into the expectation of the right side of the DPE (expression 21) gives the maximized right side of the DPE:

$$\begin{gathered} [h_t\,(Z_{0t} + ZY_t) - \tfrac{1}{2}f\,(Z_{0t} + Y'_tZ')\,(Z_{0t} + ZY_t) \\ + \tfrac{1}{2}Y'_tQY_t + WY_t\,(Z_{0t} + ZY_t) + \Phi\tfrac{\alpha^2}{2f}\sigma^2]\phi + \\ \beta\left(V^i_{0,t+1} + V'_{1t+1}\,(AY_t + B\,(Z_{0t} + ZY_t))\right) \end{gathered} \tag{23}$$

$$+\beta\left(\tfrac{1}{2}\,(AY_t + B\,(Z_{0t} + ZY_t))'\,V_2\,(AY_t + B\,(Z_{0t} + ZY_t)) + \tfrac{1}{2}C'V_2C\sigma^2\right)$$

The terms that are quadratic in Y in expression (23) are

$$\frac{1}{2}Y'_t\left[(Q - fZ'Z + 2W'Z)\,\phi + \beta\,(A + BZ)'\,V_2\,(A + BZ)\right]Y_t \tag{24}$$

Here we use the fact that $WY_t = Y'_tW'$ (because both are scalars) so

$$WY_t\,(ZY_t) = Y'_tW'ZY_t.$$

Now we use the fact that for any matrix H, $Y'HY = Y'H'Y$ (because $Y'HY$ is a scalar). Therefore $Y'HY = \frac{1}{2}Y'(H + H')Y$. We can write any quadratic form as a symmetric quadratic. Using this fact, write

$$Y_t'W'ZY_t = \frac{1}{2}Y_t'(W'Z + Z'W)Y_t.$$

Using this result we write the quadratic part of the right side of the maximized DPE, expression 24, as

$$\frac{1}{2}Y_t'\left[(Q - fZ'Z + W'Z + Z'W)\phi + \beta(A + BZ)'V_2(A + BZ)\right]Y_t$$

Equating coefficients of the quadratic terms on the left and right sides of the maximized DPE gives

$$V_2 = \left[(Q - fZ'Z + W'Z + Z'W)\phi + \beta(A + BZ)'V_2(A + BZ)\right] \tag{25}$$

We simplify the right side of equation (25) using the definitions of Z (equation 22) and V (equation 11) and the matrices defined in equation (9) to write

$$Z = \left(\begin{array}{cc} -\frac{\delta\beta\lambda}{f+\beta\lambda\phi}, & \frac{\rho-\beta\mu\rho}{f+\beta\lambda\phi}\end{array}\right). \tag{26}$$

With this result, performing the matrix manipulation on the right side of equation (25), and using equation (11), gives a recursive system of equations in λ, μ, and ν:

$$-\left[\begin{array}{cc}\lambda & \mu\\ \mu & \nu\end{array}\right] =$$

$$\left[\begin{array}{cc} -\frac{1}{f+\beta\lambda\phi}\left(f\beta\lambda\delta^2 + b\beta\lambda\phi^2 + bf\phi\right) & -\beta\delta\frac{\rho}{f+\beta\lambda\phi}\left(f\mu + \lambda\phi\right)\\ -\beta\delta\frac{\rho}{f+\beta\lambda\phi}\left(f\mu + \lambda\phi\right) & -\frac{\rho^2}{f+\beta\lambda\phi}\left(-\phi\beta^2\mu^2 + \lambda\nu\phi\beta^2 + 2\phi\beta\mu + f\nu\beta - \phi\right)\end{array}\right].$$

The equation for λ is

$$\lambda = \tfrac{1}{f+\beta\lambda\phi}\left(f\beta\lambda\delta^2 + b\beta\lambda\phi^2 + bf\phi\right) \Rightarrow$$

$$\beta\phi\lambda^2 + \left(f - f\beta\delta^2 - b\beta\phi^2\right)\lambda - bf\phi = 0$$
or
$$\beta\phi\lambda^2 + \varpi\lambda - bf\phi = 0.$$

The last line uses the definition $\varpi \equiv f\left(1 - \beta\delta^2 - \beta\frac{b}{f}\phi^2\right)$. The positive root of this quadratic is

$$\lambda = \frac{1}{2\beta\phi}\left(-\varpi + \sqrt{\varpi^2 + 4\beta\phi^2 bf}\right). \tag{27}$$

We know that the correct root is positive (so $-\lambda < 0$), because the negative root implies that the payoff grows arbitrarily large and positive as the stock of carbon becomes large. However, large carbon stocks result in high damages and a negative payoff.

The equation for μ is

$$\mu = \beta\delta\frac{\rho}{f+\beta\lambda\phi}\left(f\mu + \lambda\phi\right) \Rightarrow \mu = \beta\delta\rho\frac{\lambda}{f}\frac{\phi}{1 - \beta\delta\rho + \beta\frac{\lambda}{f}\phi}. \tag{28}$$

Collecting the terms in expression (23) that are independent of Y_t and equating these to $V^i_{0,t}$ produces the difference equation

$$\begin{gathered} V^i_{0,t} = \left(h_t Z_{0t} - \tfrac{1}{2}f\left(Z_{0t}\right)^2 + \Phi\tfrac{\alpha^2}{2f}\sigma^2\right)\phi + \\ \beta\left(V^i_{0,t+1} + V'_{1t+1}\left(BZ_{0t}\right) + \tfrac{1}{2}\left(BZ_{0t}\right)' V_2\left(BZ_{0t}\right) + \tfrac{1}{2}C'V_2C\sigma^2\right). \end{gathered} \tag{29}$$

We also report the formula for the coefficient of the term in the value function that is linear in S. We need this coefficient, denoted v_{1t}, to calculate the coefficient of the SCC, χ_t (Equation 14). Equating coefficients that are linear in S in the left and right side of the maximzed DPE gives a difference equation for ν_{1t}:

$$\nu_{1t} = M\nu_{1,t+1} + Nh_t, \tag{30}$$

with

$$M = \frac{\beta\delta f}{f + \beta\phi\lambda} \text{ and } N = -\frac{\beta\delta\phi\lambda}{f + \beta\phi\lambda}.$$

Using the forward operator $Fx_t = x_{t+1}$ we write the difference equation (30) as

$$(1 - MF)\, v_{1t} = Nh_t \Rightarrow v_{1t} = \frac{N}{1 - MF} h_t = N \sum_{j=0}^{\infty} (M)^j\, h_{t+j}. \tag{31}$$

If h falls by $x\%$ per year, then $h_{t+j} = \left(1 - \frac{x}{100}\right)^{j\phi} h_t$. Substituting this expression into equation (31) gives

$$v_{1t} = Nh_t \sum_{j=0}^{\infty} \left(M \left(1 - \frac{x}{100}\right)^{\phi} \right)^j = \frac{Nh_t}{1 - M\left(1 - \frac{x}{100}\right)^{\phi}}. \tag{32}$$

Our calibration uses $x = 1$ (a 1% annual decrease in BAU emissions) and $h_t = 95$. The function χ_t in the expression for SCC_t equals $-\nu_{1t}$.

C.3 Proof of Proposition 3

Proof of Proposition 3. We want to compare payoffs (the value functions) under three policy scenarios: full information versus asymmetric information with either taxes or quotas. In all scenarios, the value functions at time t are linear-quadratic functions of S_t and θ_{t-1}. The expected levels of emissions are the same in all scenarios, a reflection of the Principal of Certainty Equivalence for the linear-quadratic model with additive shocks. The actual levels differ only because the shocks have different effects on emissions in the different policy scenarios. As a consequence, the value functions in the different scenarios are identical, except for the coefficients of σ^2. Those coefficients do not depend on the realizations of S_t and θ_{t-1}. Therefore, to obtain the welfare gain in moving from a tax or quota to the full information scenario, we need only find and then compare the coefficients of σ^2 in the different value functions. We used this approach in proving Proposition 2, where we calculated Δ, the welfare gain from using taxes instead of quotas under asymmetric information.

Tedious but straightforward calculations establish that the welfare gain in moving from quotas under asymmetric information to the full information scenario, denoted Δ^{FQ} (F for "full info" and Q for "quota") is

$$\Delta^{FQ} = \frac{1}{2(1-\beta)} \frac{(\alpha - \beta\mu)^2}{f + \beta\lambda\phi} \phi\sigma^2. \tag{33}$$

We obtain the welfare gain in moving from taxes under asymmetric information to the full information scenario, denoted Δ^{FT}, as

$$\Delta^{FT} = \Delta^{FQ} - \Delta = \frac{\beta^2}{2(1-\beta)} \frac{(\phi\alpha\lambda + f\mu)^2}{(f + \beta\lambda\phi) f^2} \phi\sigma^2. \tag{34}$$

Using definition 4 we have

$$G = \frac{\frac{1}{2(1-\beta)} \frac{(\alpha-\beta\mu)^2}{f+\beta\lambda\phi} \phi\sigma^2}{\frac{\beta^2}{2(1-\beta)} \frac{(\phi\alpha\lambda+f\mu)^2}{(f+\beta\lambda\phi) f^2} \phi\sigma^2} = \left(\frac{(\alpha - \beta\mu)}{\beta(\phi\alpha\lambda + f\mu)} f \right)^2,$$

establishing the proposition. ■

C.4 Implementing the feedback policies

The feedback policies can be implemented by announcing, at the beginning of the regulatory period, state-contingent policies to be followed in every period. Here we simplify the exposition by ignoring macro shocks (Appendix A). We use taxes to illustrate the procedure. We assume that at the beginning of the problem, $t = 0$, the regulator knows the lagged technology shock, θ_{-1}. Later we discuss this assumption.

Equation (22) gives the control rule for the general problem. Under taxes, we can think of the regulator choosing expected emissions, so here (using this control rule) $x_t \equiv e_t^T = Z_{0t} + Z_1 S_t + Z_2 \theta_{t-1}$, where Z_1 and Z_2 are the first and second element of the vector Z defined in equation (26). We do not have a closed form expression for the intercept, Z_{0t}. However, this value can be obtained numerically by solving the difference equation for V_{1t}. We obtain that difference equation by equating the terms that are linear in Y in the maximized DPE.

Using the representative firm's first order condition, we have $e_t^T = \frac{h_t + \rho\theta_{t-1} - \tau_t}{f}$, which implies that the optimal tax satisfies

$$\tau_t = h_t - fZ_{0t} + (\rho - fZ_2)\theta_{t-1} - fZ_1 S_t. \tag{35}$$

This control rule for the optimal tax (but not the level of the optimal tax) can be announced at $t = 0$. We can alternatively write the optimal tax rule in period t as a function of lagged emissions, stock, and tax. Equation (35) evaluated at $t-1$ gives $\theta_{t-2} = L_a(S_{t-1}, \tau_{t-1})$, where $L_a(S_{t-1}, \tau_{t-1})$ is a linear function of its two arguments. (We suppress the time-dependent argument.) Using the firm's first order condition, $h_{t-1} + \hat{\theta}_{t-1} - fE_{t-1} = \tau_{t-1}$, and the definition of $\hat{\theta}_{t-1}$, we obtain an expression for the shock $\varepsilon_{t-1} = L_b(\tau_{t-1}, \theta_{t-2}, E_{t-1})$, where $L_b(\tau_{t-1}, \theta_{t-2}, E_{t-1})$ is a different linear function. We can then write $\theta_{t-1} = L_c(S_{t-1}, \tau_{t-1}, E_{t-1}) \equiv \rho L_a(S_{t-1}, \tau_{t-1}) + L_b(\tau_{t-1}, L_a(S_{t-1}, \tau_{t-1}), E_{t-1)})$, where L_c is a third linear function. Replacing θ_{t-1} in equation (35) with $L_c(S_{t-1}, \tau_{t-1}, E_{t-1})$, we have the current tax rule as a (time-dependent) linear function of the lagged emissions, stock, and tax.

Our assumption that at the beginning of the problem, $t = 0$, the regulator knows the lagged technology shock, θ_{-1} is important when $\alpha < 1$ but unimportant when $\alpha = 1$. It is worth recognizing this fact, because it shows that moving from the AR(1) process ($\alpha = 1$) to the ARMA(1,1) process ($\alpha < 1$) is not "free". For $\alpha = 1$ the optimal tax in period 0 requires only the expectation of θ_{-1}. Given this tax and the level of emissions in period 0 the regulator recovers θ_0, independent of the prior expectation of θ_{-1}. In each period there is a single piece of information, the current level of emissions, and a single unknown, ε_t. In contrast, in the ARMA(1,1) model, we have $\hat{\theta}_t = \rho\hat{\theta}_{t-1} + \rho(1-\alpha)\varepsilon_{t-1} + \alpha\varepsilon_t$ (footnote 13). There are potentially two unknowns here, but still a single piece of information (the equilibrium response to the tax). If, contrary to our assumption, the regulator did not know θ_{-1}, subsequent observations of emissions would enable the regulator to update her beliefs about its value, e.g. by using the Kalman filter, but not to learn its value. That approach would be much more complicated and would require knowing the prior probability distribution of θ_{-1}. We avoid this difficulty when the regulator knows θ_{-1}.

C.5 Calibration of the climate model

This appendix describes how we estimate f and explains the mapping from our four sets of calibration assumptions to the four values of the parameter b. Given $b, f, \alpha, \rho, \beta, \delta$, we use the formulae in Section 3.2 to rank policies. We present the slope parameters b and f for a one-year time period and set the parameter $\phi = 5$ to scale to our five-year policy period; see Appendix A for an explanation of the scaling parameter ϕ.

We use the DICE model in Nordhaus & Sztorc (2013) to estimate abatement cost associated with different levels of abatement. Assuming that $h(t)$, the intercept of marginal benefit (= marginal cost), falls at 1% per year, we project these constructed time series on to a linear marginal cost curve to estimate $h(0) = 101.0108 \left(\frac{US\$}{tCO_2}\right)$ and $f = 2.5468 \times 10^{-9} \left(\frac{US\$}{(tCO_2)^2}\right)$.

The TCRE model posits a linear relation between the change in temperature relative to the pre-industrial level, $T\,(^oC)$, and cumulative emissions since the preindustrial era, $X\,(TtC)$. This relation is $T = \tau X$, with τ the "TCRE parameter". Most of our carbon units are tCO_2. With 10^{12} tons in a teraton and $\frac{44}{12}$ units of CO_2 per unit of carbon, $1\,(TtC) = 10^{12}\left(\frac{44}{12}\right)(tCO_2)$.

We now explain our estimation of b. Defining $\bar{S}_t\,(tCO_2)$ as cumulative emissions since the pre-industrial period, we have

$$T_t = \tau X_t = \frac{\tau}{10^{12}\left(\frac{44}{12}\right)}\bar{S}_t. \tag{36}$$

Estimating that the current temperature is about 1^oC above the preindustrial level implies $1 = \frac{\tau}{10^{12}\left(\frac{44}{12}\right)}\bar{S}_0$, or $\bar{S}_0 = \frac{10^{12}\left(\frac{44}{12}\right)}{\tau}$. IPCC (2013) gives a range $(0.8^oC, 2.5^oC)$ for τ with a midpoint 1.65.

The absence of a linear term in our flow damage function, $\frac{b}{2}S_t^2$ means that we need to define S_t as the deviation between the actual and the damage-minimizing stock. Define $T^{\min}$ as the temperature at which damage is minimized, and $S^{\min}$ as the corresponding level of cumulative emissions: $T^{\min} = \frac{\tau}{10^{12}\left(\frac{44}{12}\right)} S^{\min}$. If damages are minimized at the preindustrial temperature, then $T^{\min} = S^{\min} = 0$. However, we need to allow for other possibilities in order to use the quadratic damage function (without a linear term) to flexibly approximate damages. Thus, we define our state variable as $S_t = \bar{S}_t - S^{\min} = \frac{10^{12}\left(\frac{44}{12}\right)}{\tau}\left(T_t - T^{\min}\right)$. Our state variable is an affine function of the temperature anomaly. It is a linear function if $T^{\min} = 0$.

We measure damages (and other payoffs) in US$. Define Y ($\$US$) as the estimate of current annual gross world product (GWP). We set $Y = 130 \times 10^{12}$ (US$ 130 trillion).

Our **baseline** calibration assumes:

(i) Damages are zero at the preindustrial temperature ($T^{\min} = S^{\min} = 0$), implying $S_0 = \bar{S}_0 = \frac{10^{12}\left(\frac{44}{12}\right)}{\tau}$.

(ii) The TCRE parameter is $\tau = 1.65$, implying $\bar{S}_0 = S_0 = \frac{10^{12}\left(\frac{44}{12}\right)}{1.65} \approx 2.2222 \times 10^{12}$.

(iii) A two degree temperature increase reduces GWP, Y ($UD\$$), by 1%.

A $2^o C$ increase corresponds to $S_t = 2\bar{S}_0 = 2\frac{10^{12}\left(\frac{44}{12}\right)}{1.65}$. Assumptions (i), (ii) and (iii) imply $\frac{b}{2}\left(2\frac{10^{12}\left(\frac{44}{12}\right)}{\tau}\right)^2 = 0.01Y$, or $b = 2\frac{0.01Y}{\left(2\frac{10^{12}\left(\frac{44}{12}\right)}{\tau}\right)^2}$. Our estimates $Y = 130 \times 10^{12}$ and $\tau = 1.65$ imply $b = 1.31\,625 \times 10^{-13}$.

Our **greater climate-sensitivity** scenario maintains assumptions (i) and (iii) but replaces assumption (ii) with

(ii') The TCRE parameter is $\tau = 2.5$, implying $\bar{S}_0 = S_0 = \frac{10^{12}\left(\frac{44}{12}\right)}{2.5} = 1.46 \times 10^{12}$.

With assumptions (i), (ii'), and (iii) we have $b = 2\frac{0.01Y}{\left(2\frac{10^{12}\left(\frac{44}{12}\right)}{\tau}\right)^2} = 3.\,021\,694\,214$ $876\,033\,06 \times 10^{-13}$.

Our **greater cost-convexity** scenario maintains assumption (ii), but it replaces assumption (i) and (iii) with

(i') Damages are zero at $T = 1$ ($S^{\min} = \bar{S}_0 = \frac{1}{\tau}10^{12}\left(\frac{44}{12}\right)$), implying $S_t = \bar{S}_t - \bar{S}_0$.

(iii') A 3^oC increase reduces GWP by 5%.

A 3^oC increase corresponds to $3 = \frac{\tau}{10^{12}\left(\frac{44}{12}\right)}\bar{S}_t$, or $\bar{S}_t = \frac{3}{\tau}10^{12}\left(\frac{44}{12}\right)$. Therefore, a 3^oC corresponds to the value of our state variable $S_t = \bar{S}_t - \bar{S}_0 = \frac{2}{\tau}10^{12}\left(\frac{44}{12}\right)$. If this value of the state variable produces a 5% loss in income, then $0.05Y = \frac{b}{2}\left(\frac{2}{\tau}10^{12}\left(\frac{44}{12}\right)\right)^2$, so $b = \frac{2(0.05Y)}{\left(\frac{2}{\tau}10^{12}\left(\frac{44}{12}\right)\right)^2}$. With $\tau = 1.65$ our estimate is $b = 6.58\,125 \times 10^{-13}$.

Our **greater climate sensitivity and cost convexity** scenario uses assumptions (i'), (ii'), and (iii'). Here, with $\tau = 2.5$, the calibration equation is $b = \frac{2(0.05Y)}{\left(\frac{2}{\tau}10^{12}\left(\frac{44}{12}\right)\right)^2} = 1.510\,847\,107\,438\,016\,53 \times 10^{-12}$.

C.6 Heuristic arguments

Equations (17) and (20) rely on only the Principle of Certainty Equivalence for the linear quadratic problem. They do not require formulae for χ_t, λ and μ, and therefore provide the basis for a hueristic argument. As noted in the proof of Proposition 2, the fact that χ_t, λ and μ are the same in the three scenarios (full information and asymmetric information under a tax or a quota) imply that emissions in the three scenarios differ only if $\varepsilon_t \neq 0$, i.e. when the shock does not equal its expected value. For $\varepsilon_t = 0$ we have $E_t^{FB} = E_t^Q = E_t^T$. Emissions under the tax and in the first best scenario are linear in ε_t, with derivatives given in equation (20); of course the quota is independent of ε_t. Thus, for $\alpha \neq \alpha^*$,

$$\frac{dE^T}{d\varepsilon_t} = \frac{\alpha}{f} > \frac{\alpha - \beta\mu}{f + \beta\lambda} = \frac{dE^{FB}}{d\varepsilon_t} > \frac{dE^Q}{d\varepsilon_t} = 0. \qquad (37)$$

The welfare cost of deviating from the first best level of emissions is quadratic in the deviation and symmetric around a zero deviation. The deviation between actual and first best emissions under the tax is

$$\left(\frac{\alpha}{f} - \frac{\alpha - \beta\mu}{f + \beta\lambda}\right)\varepsilon_t,$$

and the deviation between actual and first best emissions under the quota is

$$-\left(\frac{\alpha - \beta\mu}{f + \beta\lambda}\right)\varepsilon_t.$$

We need to consider two cases: $\alpha - \beta\mu > 0$ and $\alpha - \beta\mu < 0$; for $\alpha = \beta\mu$ we know from Proposition 2 that quotas dominate. For $\alpha - \beta\mu > 0$ the absolute value of the deviation under the quota exceeds the absolute value of the deviation under taxes for all $\varepsilon_t \neq 0$ (so taxes dominate quotas) if and only if

$$\left(\frac{\alpha - \beta\mu}{f + \beta\lambda}\right) > \left(\frac{\alpha}{f} - \frac{\alpha - \beta\mu}{f + \beta\lambda}\right).$$

Rearranging this inequality and using the definition of R produces the first equality in Proposition 1. For $\alpha - \beta\mu < 0$ the absolute value of the deviation under the quota exceeds the absolute value of the deviation under taxes (so taxes dominate quotas) if and only if

$$-\left(\frac{\alpha - \beta\mu}{f + \beta\lambda}\right) > \left(\frac{\alpha}{f} - \frac{\alpha - \beta\mu}{f + \beta\lambda}\right) \Rightarrow 0 > \frac{\alpha}{f}.$$

This inequality is never satisfied, so for $\alpha - \beta\mu < 0$ quotas dominate taxes, as established in the proof of Proposition 2.

Note that for $\alpha - \beta\mu < 0$ a larger current innovation (higher abatement costs) reduces the current first best level of emissions. When α is small, a positive current innovation means that abatement costs rise by a small amount, but expected abatement costs for future periods are expected to rise by a large amount. In this case, it is optimal to reduce current emissions (relative to the case where $\varepsilon_t = 0$) in anticipation of high future emissions.

C.7 Intertemporal correlation of adopted technology

This appendix confirms our claim that the intertermporal correlation of adopted technology levels increases in ρ and falls with α.

Using $\theta_t = \rho\theta_{t-1} + \varepsilon_t$ and $\hat{\theta}_t = \rho\theta_{t-1} + \alpha\varepsilon_t$ and solving for θ_{t+j} produces

$$\theta_{t+j} = \rho^{j+1}\theta_{t-1} + \sum_{s=0}^{j} \rho^s \varepsilon_{t+j-s} \tag{38}$$

$$\hat{\theta}_{t+j} = \rho\left(\rho^j\theta_{t-1} + \sum_{s=0}^{j-1} \rho^s \varepsilon_{t+j-1-s}\right) + \alpha\varepsilon_{t+j} \Rightarrow \tag{39}$$

$$var_t\left(\hat{\theta}_{t+j}\right) = \rho^2 \sum_{s=0}^{j-1} \rho^{2s}\sigma^2 + \alpha^2\sigma^2 \text{ and } cov_t\left(\hat{\theta}_t, \hat{\theta}_{t+j}\right) = \alpha\rho^j\sigma^2 \tag{40}$$

Using the formula for covariance, we have

$$corr_t\left(\hat{\theta}_t, \hat{\theta}_{t+j}\right) = \frac{\alpha\rho^j\sigma^2}{\sqrt{\left(\rho^2 \sum_{s=0}^{j-1} \rho^{2s}\sigma^2 + \alpha^2\sigma^2\right)(\alpha^2\sigma^2)}} \tag{41}$$

For $0 \le \rho < 1$, we use $\sum_{s=0}^{j-1} \rho^{2s} = \frac{\rho^{2j}-1}{\rho^2-1}$ to simplify the denominator of the previous expression produces

$$corr_t\left(\hat{\theta}_t, \hat{\theta}_{t+j}\right) = \frac{\alpha\rho^j\sigma^2}{\sqrt{\left(\rho^2 \frac{\rho^{2j}-1}{\rho^2-1}\sigma^2 + \alpha^2\sigma^2\right)\alpha^2\sigma^2}} = \frac{\rho^j}{\sqrt{\left(\rho^2 \frac{\rho^{2j}-1}{\rho^2-1} + \alpha^2\right)}}. \tag{42}$$

By inspection, the correlation is a decreasing function of α. To confirm that the correlation is an increasing function of ρ (for $0 \le \rho < 1$) we denote the correlation by C and use

$$\frac{dC}{d\rho} = \frac{\rho^{j-1}}{\left(\alpha^2+\rho^2\frac{1-\rho^{2j}}{1-\rho^2}\right)^{\frac{3}{2}}(\rho^2-1)^2}\left(j\left(\rho-1\right)^2\left(\rho+1\right)^2\alpha^2+\rho^2 T\right)$$

with

$$T\left(\rho, j\right) \equiv \left(j\left(1-\rho^2\right)-\left(1-\rho^{2j}\right)\right).$$

The first factor in $\frac{dC}{d\rho}$ (the fraction) is positive by inspection, so to confirm that $\frac{dC}{d\rho} > 0$ it is necesary and sufficient to establish that the second factor is positive. The coefficient of α^2 is positive, so to establish that the second factor is positive it is sufficient to show that $T \geq 0$. By inspection, $T\left(\rho, 1\right) = 0$, so we need consider only the case $j > 1$. We have $T\left(1, j\right) = 0$ and

$$\frac{dT}{d\rho} = -2j\rho\left(1-\rho^{2(j-1)}\right) < 0$$

for $j > 1$. Therefore, when $j > 1$, $T > 0$ for $0 \leq \rho < 1$. Thus, we have confirmed that for $0 \leq \rho < 1$, $\frac{dC}{d\rho} > 0$.