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Reyer Gerlagh, Roweno J.R.K. Heijmans

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Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
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Regulating Stock Externalities

Abstract

We develop a dynamic regulation game for a stock externality under asymmetric information and future market uncertainty. Within this framework, regulation is characterized as the implementation of a welfare-maximization program conditional on informational constraints. We identify the most general executable programs and find these yield simple and intuitive time-consistent policy rules that implement the stochastic first best as long as a future market exists. We apply our theory to carbon dioxide emissions trading schemes and find substantial welfare gains are possible, compared to current practices.

JEL-Codes: H230, Q540, Q580.

Keywords: asymmetric information, regulatory instruments, policy updating, emission trading, pollution, climate change.

Reyer Gerlagh
Department of Economics
Tilburg University
Tilburg / The Netherlands
r.gerlagh@uvt.nl

Roweno J.R.K. Heijmans
Department of Economics
Tilburg University
Tilburg / The Netherlands
roweno.heijmans@gmail.com

The most recent version of our paper can be found here:
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It is the product of an elaborate series of refinements of CESifo Working Paper 6950, Optimal Stabilization in an Emission Permits Market, which was split into two different analyses. The first and current deals with trade of stock pollutants over time. The second considers trade of stock pollutant emissions across regions - a manuscript can be found here. Comments appreciated. For the many constructive and helpful comments received, we thank Eric van Damme, Corrado Di Maria, Simon Dietz, Sjak Smulders, Inge van den Bijgaart, Thomas Michielsen, Andy Yates, Mauricio Rodriguez Acosta, and seminar participants at Tilburg University, CPB Netherlands Bureau for Economic Policy Analysis, VU Amsterdam, the SURED conference 2018, the 6th World Congress of Environmental and Resource Economists in Gothenburg, the EEA-ESEM congress in Cologne, CESifo area conference on energy and climate 2018, all of whom we hereby exculpate from any remaining errors or omissions.

1 Introduction

When a group of agents produces a good, and when the total amount produced over time has some (un)favorable consequence for society at large, we speak of a stock externality. The question arises how under circumstances of uncertainty such an externality is ideally regulated. We consider two faces of uncertainty. First, there is uncertainty emanating from *asymmetric information*, i.e. the simple fact that firms know better what modifying production costs to them than does the regulator (Montero, 2008). Unfortunately, for a dynamic problem this aspect is just the first hurdle. Firms may well know more than a regulator, their superior knowledge too can only go so far, if not about today than at least about tomorrow. The second face of uncertainty therefore derives form *imperfect foresight*, i.e. our future is partly unpredictable. The current work's quest is thus for regulation that both minimizes the costs of asymmetric information and optimally adapts to new information about expected future market conditions. We abstract from more fundamental uncertainties and risks related to the externality value, which do not resolve before the settlement of allocations.[1]

An inquiry into dynamic regulation is by no means a matter of groundbreaking novelty (*c.f.* Kydland and Prescott, 1977). Considerable theory and practice has been developed to unravel optimal regulation. The key realization in this respect is that markets are inhabited by rational agents who anticipate regulatory changes. Taking the structure of our economy with rational agents, as well as the dynamics of our externality, we maximize expected social welfare subject to informational constraints. The set of optimal allocations over realized uncertain outcomes constructs relations between variables. These relations define our regulation rules. It will turn out that, conveniently, very simple rules emerge.

Optimal regulation when the regulator is uncertain about some parameters relevant to private parties has a flavor of mechanism design. That literature searches for regulation rules, contracts offered to individual agents, that implement the socially optimal outcome (Baron and Myerson, 1982). The focus of our project is different, as we study regulation at an *aggregate* level through open market operations. Specifically, we do not design individual contract schedules, but regulation rules for the aggregate market variables. Developing a constructive approach towards regulating stock externalities for open-market operations is our first contribution. Our results will exhibit some remarkable similarities with those derived for standard mechanism design problems, notably the revelation principle.

The key insight we develop is our second contribution: a stock externality integrates

[1] This also is the principal reason that uncertainty about the externality value plays no role in the early prices versus quantities literature, see Weitzman (1974)

markets over time in such a way that the regulator can distill both current *and* expected future market conditions immediately. Importantly, we show that as long as a future market exists, even within fairly restrictive classes of instruments – i.e. purely quota- or tax-based – all information can be extracted and exploited in such a way that it implements the stochastic first best. Whereas typical responsive regulatory proposals always remain one step behind in adapting policies to observations, we find clean and simple analytic solutions keeping regulation perfectly in step with private information. More involved hybrid instruments – such as quota with price floors and ceilings (Roberts and Spence, 1976) – are not needed for this result, nor are complicated auctions (Ausubel and Cramton, 2004; Montero, 2008). We can rely on simple and therefore easily implementable instruments. This is good news for policymakers. Within each regulatory class, we identify the optimal rule that extracts all information and exploits it immediately. Our rules are called Stabilized Banking (quota-based) and Market Pricing (tax-based). Policies that implement these rules are then provided.

Our problem is of a generic structure; we seek regulation of a market where long-run average or cumulative outcomes carry social costs or benefits. Examples are knowledge produced by academia, resistance to antibiotics, arms races, persistent environmental contamination, and ozone depletion caused by CFCs. We shall illustrate our problem within yet another framework of great practical importance: climate change as caused by the stock of atmospheric CO_2 built up over the course of time. It is by now widely accepted that we are in the soup unless CO_2 emissions are abated, yet how much exactly is a topic that continues to divide opinions. One hefty source of disagreement will be the focus of attention here: the aggregate costs of abatement. While some fear that any emission reduction threatens thousands of jobs, others foresee that the more or less intensive use of natural resources has no significant bearing on neither economic growth nor employment. The emphasis on abatement costs is allegorical; other types of uncertainty resolved before the closing of markets, and relevant for welfare and thus regulation could be used in their stead. Many previous works on stock pollutants employ quantitative methods in response to the dynamic complexities posed by the climate problem. We manage to fully exploit the market integration, inherent to the stock externality, and derive simple analytic policies, by which we suggest some way forward to the intricate problem of regulating CO_2.

To advance the goal of optimally regulating a stock externality, we must properly understand the nature and causes of its underlying problem. For carbon dioxide, climate change is the externality. It is fairly undisputed that the stock of globally emitted atmospheric CO_2, built up over time and not disappearing before centuries (Archer et al., 2009), is the foremost driver of global warming and ocean acidification (Allen et al., 2009; Stocker, 2014). Of course, other greenhouse gases are also known to heat up our planet. For simplicity, we shall aim at CO_2, which is the most important gas by wide margin. Because it is the global *stock* of CO_2 that matters, studies of *flow* externalities will not be particularly helpful for our application, though many such studies present their results as equally applicable to climate policies.[2] Stock pollutants were previously considered by Hoel and Karp (2001); Yates (2002); Williams (2002); Hoel and Karp (2002); Newell and Pizer (2003); Fell et al. (2012) and, most similar to the current work, Kollenberg and Taschini (2016); Karp and Traeger (2017).

Our work also contributes to an influential literature to which Weitzman (1974) paved the way in his celebrated 'Prices vs. Quantities'. Works in the latter tradition fix a collection of policy instruments and identify under which circumstances one instrument is favored over the others (*c.f.* Hoel and Karp, 2001; Newell and Pizer, 2003; Fell et al., 2012; Ambec and Coria, 2013; Meunier et al., 2017; Weitzman, 2018; Burtraw et al., 2018). This literature frequently constructs policy refinements. Roberts and Spence (1976) combine a quantity policy with price floors and ceilings. Kling and Rubin (1997), Newell et al. (2005), and Pizer and Prest (2016) propose that the regulator depreciates or tops up banked – that is, unused and saved for future use – allowances, similar to the financial bank setting its interest rate on loans and deposits. Similarly, Yates and Cronshaw (2001) consider banking with a discount rate for allowances. Newell et al. (2005) and Lintunen and Kuusela (2018) discuss adjusting quota in response to the quantity of outstanding allowances. Finally, Karp and Traeger (2017, 2018), in a work closest to ours, study a cap on emissions that changes in response to aggregate private information inferred from price signals in the market. The ideas in this literature are conceptually close to ours, and their motivation similar. Yet most consider a flow pollutant, while we consider a stock pollutant, such as CO_2, and most importantly, our methodology sets us apart. The approach in the literature has been to construct intuitive policy rules and then to optimize its parameters. Practice in emission trading schemes are based on the recommendations. Instead, our policies are endogenously derived from primitives such as market fundamentals and the externality problem. As we construct regulatory rules under the minium possible informational constraint, those rules that we uncover are strictly welfare superior in their

[2]Major flow externality problems include loud noise or NO_X emissions and acid rain. From first principles, the level of flow pollution in one period does not directly affect the optimal level of flow pollution in another period. Any regulatory connection between periods comes through the economics side, e.g. through slowly resolving uncertainty. In contrast, optimal stock pollution is only defined when considered jointly over all periods.

class. Our third contribution is thus practical: we provide a policy improvement for greenhouse gas emission trading schemes worldwide.

The main results of this paper are stated in three theorems. We begin with a two-period model. Theorems 1 and 2 show that our two rules admit welfare losses depend on (i.e. scale solely with) those market conditions that neither market-participants nor regulator can foresee. In particular, a striking features is that no welfare loss stems from the information asymmetry between the regulated firms and regulator in the first period. Finally, Theorem 3 shows that both instruments approach the operational first-best, the theoretically maximal level of welfare, when increasing the number of inter-temporal trading opportunities for firms.

Considerable preparation will be needed to arrive at these results. We first of all develop the basic machinery in Section 2, where we introduce the model and characterize regulation. Section 3 defines policy benchmarks and ex-post and ex-ante (expected) welfare losses. In Section 4, we introduce and analyze various regulatory policies. Section 5 discusses and concludes.

2 Model Set Up

2.1 Production, Damages, Welfare

Consider a two-period world and a representative profit-maximizing firm in the business of producing a homogeneous good with polluting emissions as a negative externality.[3] At every time t, emitting an amount $\widetilde{q}_t$ of the pollutant allows the firm to produce a quantity $Y_t(\widetilde{q}_t; \theta_t)$ of the good. The parameter θ_t captures the uncertainty and is known to the firms but unknown to the regulator; it can be interpreted as market conditions that capture how valuable a given input of emission $\widetilde{q}_t$ are to producers. Although the regulator cannot discern the actual realization of θ_t, it is common knowledge that $\mathbb{E}[\theta_t] = 0$, $\mathbb{E}[\theta_t^2] = \sigma_t^2$, and $\mathbb{E}[\theta_1\theta_2] = \rho\sigma_1\sigma_2$. In a market with free and competitive trade of allowances, an equilibrium price will emerge, denoted $\widetilde{p}_t$. For the purposes of our study, the variance σ_t^2 provides a natural measure of the amount of uncertainty present. In the remainder of the paper, we will use the terms *variance* and *uncertainty* interchangeably.

Total production, through emissions, damages the environment and the economic system. Emissions add to a pure stock pollutant, so damages only enter welfare through

[3]This simplest of possible settings is equivalent to a model with a continuum of competitive profit-maximizing firms in a market without free entry or exit and households buying the good and supplying labor. A micro-foundation of our simple model from such primitives can be found in Appendix A.

cumulative emissions. This is reasonable since for e.g. CO_2, most damages appear after the regulation period anyway.[4] Thus, damages enter welfare as a proxy for expected future welfare losses, given by $D(\widetilde{q}_1 + \widetilde{q}_2)$.

Some would argue that there is depreciation of atmospheric CO_2 and so a simple aggregate of emissions over time does not suffice to describe environmental damages. However, the intricate natural sciences of climate change lead to cumulative emissions *without* depreciation standing out as the best predictor of temperature rise (Allen et al., 2009).

The problem facing the regulator is to find policies such that outcomes $\widetilde{q}_1$ and $\widetilde{q}_2$ maximize welfare:

$$W(\widetilde{q}_1, \widetilde{q}_2; \theta_1, \theta_2) = Y_1(\widetilde{q}_1; \theta_1) + Y_2(\widetilde{q}_2; \theta_2) - D(\widetilde{q}_1 + \widetilde{q}_2). \tag{1}$$

The timing of regulation and equilibrium follows the stages:

1. The regulator chooses the policy instrument.
2. Firms observe first-period market conditions θ_1.
3. First-period market outcomes are realized.
4. Firms observe second-period market conditions θ_2.
5. Second-period market outcomes are realized.
6. Damages due to the stock of emissions $\widetilde{Q} = \sum_t \widetilde{q}_t$ are realized.

With complete information, the fully knowledgeable regulator can set these quantities $\widetilde{q}_1, \widetilde{q}_2$ directly or else charge a price on emissions that will make the profit-maximizing firm produce the same quantities, and these two instruments are perfectly equivalent, see Montgomery (1972). However, as was first shown by Weitzman (1974), this formal equivalence between instruments breaks down once we introduce an informational disparity, captured here by θ_t.

[4] Climate change has very persistent dynamics. See Gerlagh and Liski (2018b) for an extensive discussion of the time-structure and its implications for climate policies; Gerlagh and Liski (2018a) show that the risk of a future climate catastrophe can decarbonize the economy before evidence about climate damages becomes observable.

2.2 Characterizing Regulation

Armed with a definition of welfare, we characterize regulation as an implementation of the solution to:

$$\max_{x_1} \mathbb{E}_{t_1} \max_{x_2} \mathbb{E}_{t_2} W(\widetilde{q}_1(x_1), \widetilde{q}_1(x_2); \theta_1, \theta_2), \tag{2}$$

subject to market equilibrium conditions captured through $\widetilde{q}_1(x_1), \widetilde{q}_1(x_2)$, with policy variables x_t, and where subscripts $0 \leq t_1 \leq t_2 \leq 2$ of the expectations operators indicate the point in time at which the expectation is evaluated. It is immediately clear that, theoretically, the highest welfare or Social Optimum is obtained when decisions are taken only after all uncertainty has been resolved; this is denoted by $t_1 = t_2 = 2$. Given market conditions and full information, setting quantities q_t or pricing emissions p_t are equivalent as policy variables x_t. We characterize the Social Optimum in shorthand as $(x, t_1, t_2) = (q, 2, 2) = (p, 2, 2)$. Since production decisions cannot be undone in retroaction, the Social Optimum is not implementable. Nonetheless, the Social Optimum provides a clear benchmark against which other policies can be compared.

Another possibility would be to compare regulation with what is in the very best of cases achievable, which is a policy that responds to information as soon as it becomes available to market participants: $(x_1, x_2, t_1, t_2) = (q_1, q_2, 1, 2)$. We call this the symmetric-information Optimal Response. Again, equivalence of instruments imply $OR = (q, 1, 2) = (p, 1, 2)$. Because of the theory appeal yet practical infeasibility of the regulator deciding allocations after observing θ_2, i.e. $t_2 = 2$, these two policies are not part of our regulation set but will constitute the benchmark:

Definition 1 (Class of Benchmarks)**.**

$$\mathcal{B} := \{(x, 1, 2), (x, 2, 2)\}.$$

where $x \in \{q, p\}$, to denote the equivalence between regulated quantities and prices.

As will become clear throughout the exposition, regulatory policies that admit as timing of decisions either $(t_1, t_2) = (0, 0)$ or $(t_1, t_2) = (0, 1)$ already exist both in the literature as well as in practice and therefore only little time will be spent on discussing these.[5] Our main exercise will instead be to fill the gap that so far has either not been noticed or else for other reasons remained void and ignored, namely policies with timing $(t_1, t_2) = (1, 1)$.

[5] Karp and Traeger (2018) is an example of an extensive study of $(q, 0, 1)$ policies, where quota are updated in each subsequent regulation period, given the market signals received in the current period.

Inspired both by the existing literature and political reality, we confine our focus to two distinct classes of instruments: Quantity-based instruments $\mathcal{Q}$ contain all regulations that in some fashion fixate the amount or quantity of an externality directly, $x_t = q$. The most famous example of regulations from this class is presumably a quota.

Definition 2 (Class of Quantity-based instruments)**.**

$$\mathcal{Q} := \{(q, t_1, t_2) \mid 0 \leq t_1 \leq t_2 \leq 1\}$$

The other class of instruments we consider will be called Price-based, $\mathcal{P}$. These are a logical counterpart; The most famous example from this class is the Pigouvian tax.

Definition 3 (Class of Price-based instruments)**.**

$$\mathcal{P} := \{(p, t_1, t_2) \mid 0 \leq t_1 \leq t_2 \leq 1\}$$

Before proceeding to the analysis, we introduce some notation. We use an asterisk for the quantities and prices that are optimal in expectations. We use superscripts scenario labels for equilibrium outcomes. We use superscripts SO, OR, Q, P, B, SB, DP, and MP for the social optimum, the optimal response, and the equilibria outcomes with quantities set per period, prices set per period, banking, 'stabilized banking', 'dynamic prices', and dynamically updated 'market prices', respectively. Moreover, let $\widetilde{x}^i$ denote the value of a variable x under policy i. Let $x^i := \widetilde{x}^i - x^*$ be the deviation of x under policy i from the ex-ante expected optimal value x^*, and let $\Delta^i x := x^i - x^{SO}$ denote the difference between the value of x under scenario i and its ex post socially optimal value. Finally, let $\succ$ denote the (total) welfare ordering of instruments; that is, $X \succ Y$ ($X \succeq Y$) if and only if instrument X is strictly (weakly) welfare superior to instrument Y. When $X \succeq Y$ and $Y \succeq X$, we write $X \approx Y$.

3 Benchmarking

We describe the Social Optimum, which is used as reference to define welfare losses of various policies. We also construct the Optimal Response when the regulator has the same information as all market participants (i.e. there is no problem of asymmetric information). The Social Optimum and Optimal Response provide useful benchmarks $\mathcal{B}$ for the policies we discuss in the next section. To further closed-form solutions, we make some restrictive assumptions regarding functional forms.

3.1 Functional Forms

The descriptive model builds on Weitzman (2018). Let optimal quantities and prices in expectations (when $\theta_1 = \theta_2 = 0$) be labeled $\widetilde{q}_t = q^*$ and $\widetilde{p}_t = p^*$, respectively. The ex-ante optimal stock is then $Q^* = 2q^*$. We assume linear marginal productivity (strictly concave productivity), of the form:

$$MY_t(q_t) = p^* - 2c(\widetilde{q}_t - q^*) + \theta_t. \tag{3}$$

Marginal damages due to emissions are also linear (damages strictly convex), and given by:

$$MD = p^* + b(\widetilde{q}_1 + \widetilde{q}_2 - Q^*). \tag{4}$$

Given competitive markets for emission allowances, prices satisfy:

$$-2cq_t + \theta_t = p_t, \tag{5}$$

For the purposes of this model, it serves the analysis to write the market conditions θ_t as an AR(1) process. We decompose θ_2 in two parts:

$$\theta_2 = \alpha\theta_1 + \mu, \tag{6}$$

with commonly known $\alpha \in [-1, 1]$ and μ white noise, so that $\sigma_2^2 = \alpha^2\sigma_1^2 + \sigma_\mu^2$, and $\rho = \alpha\sigma_1/\sigma_2$. The stochastic variable μ describes that part of the second-period market conditions not known by the firms when period-one production decisions are made. For $\alpha > 0$, we can think of θ_t as a persistent technology shock, where the better technology of the first period is carried on to the second, positively affecting productivity in both periods. For $\alpha < 0$, we can think of θ_t as demand shocks in a business cycle framework, where a negative demand shock in the first period is met with counter-cyclical policy by the government, boosting demand in the second period. For realism, we restrict ourselves to $|\alpha| \leq 1$; it appears unlikely for firms to expect shocks of ever-increasing magnitude. We think of a small positive value of α as the most realistic case.

3.2 Social Optimum

The touchstone for welfare-performance of instruments will be the symmetric information, perfect foresight equilibrium, called the Social Optimum, abbreviated *SO*. No policy, however fanciful, can achieve higher welfare than this theoretical ideal, making it a natural point of departure for our analysis. The Social Optimum solves the following (hypothetical) program:

$$\max_{q_1,q_2} W(q_1, q_2; \theta_1, \theta_2). \tag{7}$$

Since damages are caused by the stock $\widetilde{q}_1+\widetilde{q}_2$, marginal damages of a unit of emissions is the same in period 1 as in period 2. Because in the Social Optimum, marginal productivity should equal marginal damage, marginal productivity must therefore also be the same in both periods. The optimal quantities q_1^{SO} and q_2^{SO} set by the regulator are thus given by the condition $MY_1 = MY_2 = MD$, that is, matching prices in both periods, $p_1^{SO} = p_2^{SO}$, so that we can omit the price time subscript. This condition is a major deviation from analysis of a flow pollutant and will prove to be of fundamental importance for comparison between instruments. Since prices are equal, we have

$$b(q_1^{SO} + q_2^{SO}) = p^{SO} \tag{8}$$

$$-2cq_t^{SO} + \theta_t = p^{SO} \tag{9}$$

By solving the above first order conditions, we can easily characterize the dependence of the social optimum on market conditions θ_t (in Appendix B). Intuitively, the optimal allowance price and emission stock both increase in the market value of emissions in either period. Moreover, socially optimal emissions increase in periods where emissions are of higher value, and ceteris paribus decrease in the other. In case of constant marginal damages that do not depend on cumulative emissions, $b = 0$, we immediately see that optimal prices do not change, $p^{SO} = 0$, so that market conditions are fully absorbed by changes in emission levels in the same period these conditions realize. In case of an almost flat marginal productivity, $c \searrow 0$, we can think of a backstop technology that provides an alternative for fossil fuels at constant marginal costs. An unforeseen cost rise of the backstop leads to a (positive) shock in marginal productivity, which is then half absorbed by higher prices, while the other half is absorbed by increased overall emissions. But, more importantly, abatement will move sharply between periods, with a sharp increase (decrease) in emissions in the period with increased (decreased) marginal productivity.

The result that taxes and quota are perfectly equivalent under full and symmetric information (Montgomery, 1972) can in our notation be condensely stated as:

$$SO = (q, 2, 2) = (p, 2, 2) \in \mathcal{B}.$$

3.3 Welfare Costs of Policies

By definition of the difference under policy i with the social optimum and considering the firms' optimization (5), it is immediate that quantity deviations under asymmetric information and imperfect foresight from the social optimum scale with price deviations:

$$\Delta p_t^i = 2c\Delta q_t^i. \tag{10}$$

The welfare loss is then given by:

$$\begin{aligned} -\Delta W^i &= \mathbb{E}\left[Y_1^i + Y_2^i - D^i\right] \\ &= \mathbb{E}\left[\sum_t q_t^i\left(p_t^{SO} - cq_t^i\right) - Q^i\left(p_t^{SO} + \frac{b}{2}Q^i\right)\right] \\ &= -\frac{b}{2}\mathbb{E}\left[\left(\Delta Q^i\right)^2\right] - c\sum_t \mathbb{E}\left[\left(\Delta q_t^i\right)^2\right] \end{aligned} \tag{11}$$

Through adding parameters to our notation, $W^i(\sigma_1, \sigma_2, \alpha)$, we spell out that the welfare loss of policy i depends on specific parametric values, which facilitates comparison of our results to those in for example Weitzman (1974) ($\sigma_1 = \sigma_2, \rho = 1$) as a means of guiding our intuition.

3.4 Optimal Response

As benchmark to which we compare the instruments, we consider the optimal response for a regulator who has the same information as the firms, and who can set prices or quantities unconstrained by instrument choice. Different from the ex-post Social Optimum described above, in the first period, the second-period market has not been observed and first-period quantities q_1 cannot depend on μ. This framework thus features symmetric information, but imperfect foresight; we call it the symmetric-information optimal response, abbreviated OR:

$$\max_{q_1} \mathbb{E}_1\left[\max_{q_2} W(q_1, q_2; \theta_1, \theta_2)\right]. \tag{12}$$

where the subscript '1' after the expectations sign indicates that θ_1 is observed, but not θ_2 when the planner decides on q_1. The symmetric-information optimal response is defined by two (first-order) conditions:

$$p_1^{OR} = b(q_1^{OR} + \mathbb{E}_1[q_2^{OR}]) \tag{13}$$
$$p_2^{OR} = b(q_1^{OR} + q_2^{OR}) \tag{14}$$

It is straightforward to see that these first-order conditions, together with the demand equations (5) fully characterize the Optimal Response prices and quantities (in Appendix B).

In the Optimal Response Equilibrium, the regulator can fully incorporate all knowledge about present market conditions, allowing full mitigation of the first-period outcome. Indeed, the dependency of (q_1, q_2, Q) with respect to θ_1 is the same in the Social Optimum and Optimal Response allocations. What is lost, compared to the social optimum, is the backwards adjustment of first-period quantities. Because first-period decisions cannot be undone, first-period quantities do not adjust with unforeseen second-period shocks. In terms of prices, the Optimal Responses satisfies $p_1 = \mathbb{E}[p_2|\theta_1]$, whereas the social optimum achieves $p_1 = p_2$. This explains the remaining welfare loss, given by:

$$-\Delta W^{OR} = \frac{1}{8c}\frac{b^2}{(b+c)(b+2c)}\sigma_\mu^2 \tag{15}$$

As we would expect welfare losses in an Optimal Response Equilibrium depend only on the unpredictable shock μ; the regulator by definition responds optimally to the observed market condition θ_1.

The Optimal Response is in our notation characterized by:

$$OR = (q, 1, 2) = (p, 1, 2) \in \mathcal{B}.$$

4 Policies

An overview of the policies studied or derived in this work can be found in Table 1. For reference, we define the two baseline static policies. We do so in conformity with the literature, such as Newell and Pizer (2003). Specifically, a Quantities policy, $Q = (q, 0, 0) \in \mathcal{Q}$, fixes the ex-ante optimal quota for both periods ahead of opening the market, that is, before market-conditions θ_1, θ_2 are observed:

$$\max_{q_1, q_2} \quad \mathbb{E}_0 W(q_1, q_2; \theta_1, \theta_2) \tag{16}$$
$$\text{s.t.} \quad p_t = \theta_t - 2cq_t, \tag{5}$$

Table 1: Overview of policy instruments in this manuscript

Instrument Type	(t_1, t_2)	Quantity-based ($x = q$)	Price-based ($x = p$)
Static	(0,0)	Quantities	Prices
Dynamic	(0,1)	Banking	Dynamic Prices
Optimal Dynamic	(1,1)	*Stabilized Banking*	*Market Pricing*

Note: following the definitional tradition initiated by Weitzman (1974), Quantities will refer to fixed Quota, Prices to a Pigouvian tax. Instruments in *italics* are new to the literature.

A Prices policy solves the mirror-image program, setting expected optimal prices for both periods: $P = (p, 0, 0) \in \mathcal{P}$

$$\max_{p_1, p_2} \quad \mathbb{E}_0 W(q_1, q_2; \theta_1, \theta_2) \tag{17}$$

$$\text{s.t.} \quad p_t = \theta_t - 2cq_t, \tag{5}$$

Appendix B.3 provides a straightforward generalization of Weitzman (1974)'s celebrated result on the relative merits of Prices versus Quantitites in this dynamic setting.

Considering damages, the period in which a pollutant is emitted is immaterial, as damages derive from aggregate emissions only. Consequently, for any given total stock of emissions, e.g. a fixed *carbon budget* $q_1 + q_2$, the sole relevant inter-temporal element remaining pertains to production. It is in the latter, too, that firms possess information about strictly superior to the regulator's. It requires no leap of faith (Kierkegaard, 1846) to see that Dynamic instruments always outperform Static instruments in terms of welfare. This conclusion, however, is diametrically opposed to that obtained when studying a flow pollutant, as in Weitzman (2018). We will not dwell further into the analysis of static policies, something already done lucidly by Newell and Pizer (2003). Instead, we now look into dynamic instruments.

4.1 Banking and Borrowing

Free intertemporal trade of allowances improves welfare. The argument is strongly reminiscent of that in favor of market-based instruments in a static framework. For any total cap on emissions, after observing first-period market conditions θ_1, firms possess superior knowledge about the marginal value of allowances. Consequently, firms are best left to themselves in deciding how to allocate emissions, subject to the cap, over time. In certain subsets of the literature, this policy is known as Banking (and Borrowing)[6] (e.g. Cronshaw and Kruse, 1996; Weitzman, 2018):

Definition 4 (Banking)**.** The regulator allocates the ex-ante optimal amount of allowances q^* in both periods. Firms can freely substitute allowances between periods:

$$q_1 + q_2 = Q. \tag{18}$$

where cumulative emissions Q is chosen optimally. Equilibrium on the emission allowances market implies (5). Free substitution of allowances between periods implies expected marginal productivity is constant over time:

$$\mathbb{E}_1 p_2 = p_1. \tag{19}$$

The price-taking behavior sets us apart from studies on strategic dynamic markets, studied for instance by Liski and Montero (2011) (though note that these authors have a positive approach and do not search for the efficient allocation of permits). Since aggregate emissions (and thus damages) are fixed under both Quantities and Banking but only the latter equalizes expected marginal productivity over time, the inter-temporal allocation of allowances is always weakly more efficient under Banking. It is therefore immediate that $B \succ Q$.

In many ways, Banking is an intuitive policy, of which its widespread use by emission trading systems worldwide is a silent witness. Compared to static types of regulation such as Quantities, it also without doubt is a smart choice for regulating stock pollutants. To get a fuller understanding of the instrument, let us have a look at the program it solves:

$$\max_{Q} \mathbb{E}_0 \Big[\max_{q_1, q_2} \quad \mathbb{E}_1 W(q_1, q_2; \theta_1, \theta_2) \Big] \tag{20}$$

$$\text{s.t.} \quad q_1 + q_2 = Q \tag{18}$$

$$\theta_t - 2cq_t = p_t \tag{5}$$

Equation (18) represents the condition of a fixed cumulative cap on emissions at the ex-ante optimal level. The first order conditions yield (19). This equality of (expected) prices is the crux of most arguments favoring Banking over Quantities. It is a result of the free intertemporal substitution of permits by firms, or what we might call private banking.

[6] If the majority of allowances is auctioned in the first-period, borrowing as outcome becomes unlikely.

Banking is clearly a Quantity-based instruments, which the following characterization, with a slight abuse of notation, makes clear:

$$B = ((Q, q), 0, 1) \in \mathcal{Q}.$$

From the maximization problem (20) and using the general formula for welfare losses of an instrument (11) we obtain a measure for the performance of Banking:

$$-\Delta W^B = \frac{1}{8}\frac{(1+\alpha)^2}{b+c}\sigma_1^2 + \frac{1}{8c}\frac{b+2c}{b+c}\sigma_\mu^2. \tag{21}$$

Importantly, *any* policy that allows individual firms to freely substitute allowances between periods admits equal expected prices (19). It is thus *not* a consequence of the constraint at the *aggregate* level, (18). This insight casts some doubts on the intuitive appeal of Banking. If any instrument permitting perfect individual substitutability of allowances achieves expected price-equalization (19), it does not seem necessary to impose a fixed aggregate quota via (18). In fact, seen through this lens it appears rather arbitrary. Being an unnecessarily restrictive and seemingly ad hoc constraint on policymaking, we will see it relaxed.

To receive some intuition why Banking can be improved upon, we note that realized market conditions affect the ex-post the socially optimal level of *aggregate* emissions. Consequently, unless shocks cancel out perfectly over time, the social optimum cannot be implemented by keeping the stock fixed. This begs the question if Banking can be further refined to what one might call adaptive or an *Optimal* Dynamic Policy, that is, a policy taking into account also the effect on optimal total emissions caused by unobserved market conditions. As we will see, this is possible indeed. Using the fact that firms maximize profits and hold rational expectation about the future, the regulator is able to *infer*, given the policy implemented, all relevant market conditions. Having de facto observed these, the regulator can thus adapt second-period quantities such that, anticipating the regulator's doing so, firms will respond by producing the socially optimal amount of emissions (absent unpredictable shocks μ) in both periods. Deriving this result is not a trivial exercise, however, and before its formal statement, we will need to build some further machinery.

4.2 Stabilized Banking

If we have a look at the regulator's program for Banking (20), we see that the fixed aggregate quota is represented by the constraint (18): $q_1^B + q_2^B = Q^B$ (superscripts added for clarity).

Its incorporation implies one is willing to bind the hands of the regulator by reducing the degrees of freedom available for policy-making. Both mathematically and economically, this is not necessary: it is perfectly possible to solve a program where the relation between q_1 and q_2 is *endogenous* to the program, coming out of a first-order condition, rather than exogenously set. Our exercise will therefore be to remove the constraint. We first propose an ex-ante implementable instrument called Stabilized Banking. Next we show this is the unique instrument that maximizes program (20) without the Banking constraint (18). However, before we introduce the new instrument, a discussion on substitutability will prove illuminating.

The standard approach towards intertemporal trade presumes a unity marginal rate of substitution between allowances used in different periods, both for individual firms and for the aggregate market. In principle, both rates can be chosen independently.

The *individual* marginal rate of substitution (MRS_I) is the ratio at which individual firms can exchange allowances between periods. Commonly, and often implicitly, it is assumed to be one. Profit maximization then implies prices grow with the interest rate, i.e. Hotelling's rule. (Extending our model to cover time discounting, it is easily shown that Hotelling's rule would hold in our model. Our base model can be interpreted as the special case where $r = 0$, with r the interest rate, so that the discount factor $1/(1 + r) = 1$ by construction.) Some authors consider the individual trading ratio as a parameter of policy (Yates and Cronshaw, 2001; Newell et al., 2005; Pizer and Prest, 2016). This is efficient regulation if marginal damages differ by period.

When regulating a pure stock pollutant, one-to-one individual substitution follows from first principles: for a given stock, damages are independent of the period of emission. Consequently, the optimal allocation of allowances equates the marginal value of *emissions* over time, which implies the $MRS_I = 1$. It is here that the study of a stock pollutants differs fundamentally from that of a flow pollutant; for the latter, socially efficient policies require a variable MRS_I, because damages are not linked inter-temporally (Pizer and Prest, 2016). This discrepancy points at the importance of clearly considering the type of pollutant one intends to inquire.

The *aggregate* marginal rate of substitution (MRS_A) measures changes in the total amount of allowances allocated in the second period in response to emissions in the first. Lintunen and Kuusela (2018) study rules for efficient aggregate substitution rates in the context of a flow pollutant. Our analysis extends theirs by considering stock pollutants. We follow the custom that only information on quantities is used. The policy that manipulates the aggregate rate of substitution is called Stabilized Banking. We use $\delta = MRS_A$ for notational convenience, and we will refer to it as the *stabilization rate*. It is defined as follows:

Definition 5 (Stabilized Banking)**.** The regulator adapts second-period allowance allocations to emissions in the first period for fixed $MRS_A = \delta \in \mathbb{R}$:

$$\delta q_1 + q_2 = 0, \tag{22}$$

where δ is chosen to minimize welfare losses. Allowance demand (5) and free individual intertemporal substitution with $MRS_I = 1$ ensures equal expected marginal productivity in both periods:

$$\mathbb{E}_1 p_2 = p_1. \tag{19}$$

It is easily verified that this instrument uniquely implements the solution to the following program:

$$\max_{q_1,q_2} \quad \mathbb{E}_1 W(q_1, q_2; \theta_1, \theta_2) \tag{23}$$
$$\text{s.t.} \quad p_t = \theta_t - 2cq_t, \tag{5}$$

which has first-order conditions (19) and

$$p_1 = b(q_1 + q_2). \tag{24}$$

Importantly, the above properties establish that Stabilized Banking is free from commitment problems. The selected amount of allowances auctioned in the second period is consistent with all information available at the start of the second period, as indicated by the subscript 1 to the expectations symbol above. This can also be seen from the property that second-period prices equal the marginal social damages in expectations, $\mathbb{E}_1 p_2 = b(q_1 + q_2)$.[7]

In short-hand we characterize Stabilized Banking (23) as:

$$SB = (q, 1, 1) \in \mathcal{Q}.$$

Due to the additional degree of freedom, this program has two first-order optimality

[7] Just to be sure, this property is indeed violated for the Banking policy, meaning that the regulator would like to adjust the allocation rules in the second period, if possible. Indeed, political discussions about changing the rules have been observed for e.g. the EU-ETS when private banking turned out to be larger than anticipated.

conditions, compared to just one for Banking (20). Combining them, we can derive:

$$\delta^* := -\frac{q_2}{q_1} = \frac{b - \alpha(b + 2c)}{(b + 2c) - \alpha b}. \tag{25}$$

It follows:

Proposition 1. *Stabilized Banking strictly outperforms Banking for all $\alpha > -1$; the optimal stabilization rate δ^* is given by* (25)*.*

Proofs of this and subsequent results will be given in Appendix B. The optimal stabilization rate δ^* is graphically illustrated in Figure 1. Importantly, $\delta^* \leq 1$, which a careful writing out of (25) will demonstrate. The following corollaries are immediate:

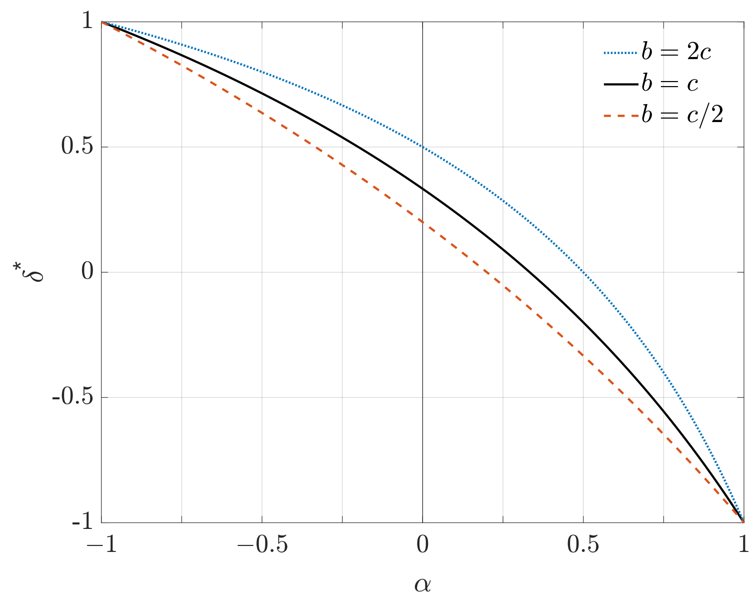

Figure 1: Optimal Stabilization Rate δ^*, for different ratios b/c, dependent on the correlation between shocks α.

Corollary 1. *The optimal stabilization rate δ^* is monotone decreasing in α. For $\alpha = -1$, the optimal stabilization rate δ^* is unity, that is, regular Banking is ex-ante optimal. For all other $\alpha \in (-1, 1]$ the optimal stabilization rate is less than unity, $\delta^* < 1$. For $\alpha = 1$, $\delta^* = -1$.*

Corollary 2. *The optimal stabilization rate δ^* tends to unity as damages due to emissions become increasingly steep:*

$$\lim_{\frac{c}{b} \to 0} \delta^* = 1$$

We now state our first main results as a Theorem:

Theorem 1. *Stabilized Banking is time consistent and strictly welfare superior among the class of quantity-setting instruments $\mathcal{Q}$. It enables the regulator to incorporate all market conditions that private parties can foresee through setting the stabilization rate δ at (25). Only those conditions that neither regulated nor regulating parties can foresee cause welfare losses:*

$$-\Delta W^{SB} = \frac{1}{8c}\frac{b+2c}{b+c}\sigma_\mu^2. \tag{26}$$

By dynamically adapting future allowance allocations in response to information revelation through firms' production choices, the regulator can fully incorporate all knowledge about present and future market conditions. Any remaining ex post sub-optimality in total production derives no longer from an asymmetry of information between firms and regulator but solely from the unpredictability, for both parties, of the future. Only those market conditions which take by surprise both regulated and regulating parties drive welfare away from its socially optimal level. What is important to note is that the other quantity instruments – Quantities and Banking – generally fail to achieve this save for some exceptional parametric cases.

Our discussion has now brought to light another critical piece of understanding: namely, that the welfare loss under regular Banking when compared to that under optimal Stabilized Banking derives entirely from the failure of the former to wholly incorporate all anticipated market conditions into aggregate allowance allocations. Indeed, comparing the welfare loss under Stabilized Banking to that under regular Banking, given by (21), we obtain the following:

$$\Delta W^B - \Delta W^{SB} = \frac{1}{8}\frac{(1+\alpha)^2}{b+c}\sigma_1^2, \tag{27}$$

which shows that Banking cannot do better than Stabilized Banking in terms of welfare and that the difference between the two is proportional to the measure of cumulative uncertainty $(1+\alpha)^2\sigma_1^2$, multiplied that is, of anticipated market conditions, only. Finally, we see that the two instruments are equivalent if and only if $\alpha = -1$. The latter observation in turn corroborates Corollary 1.

Thus, Stabilized Banking accomplishes a seemingly daunting task: to solve the problems caused by asymmetric information. Upon deeper thought, this is clearly not surprising per se, as it resembles the infamous *revelation principle* which shows that in general there exist menus of contracts such that regulated agents will find it in their best interest to truthfully reveal their types (Myerson, 1979). What is, however, surprising is the simplicity and elegance of the optimal *menu* offered when implementing a Stabilized Banking policy: all the regulator needs to do is establish as a pre-specified rule the optimal stabilization rate δ^*. The market does the rest. Those familiar with the literature on mechanism design will be aware of the notorious complexity optimal mechanisms can at times assume, rendering their real-world applicability and political feasibility almost nil. Not so for Stabilized Banking: only one stabilization rate has to be chosen and the system is ready to go; no confusion or overt complications are involved.

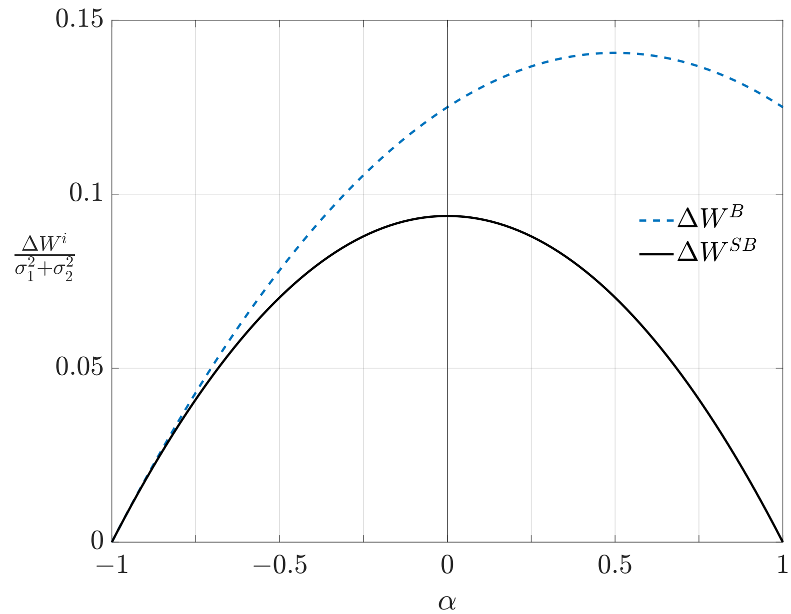

Figure 2: Normalized welfare losses under Banking and Stabilized Banking for $\sigma_1 = \sigma_2$ and $b = c$.

Figure 2 illustrates Proposition 1 and equation (27), by plotting the normalized welfare losses of regular Banking and Stabilized Banking when uncertainty is of equal measure in both periods ($\sigma_1 = \sigma_2$). We see that, as was stated in Proposition 1, Stabilized Banking always outperforms regular Banking, except in the extreme cases where $\alpha = -1$, when welfare losses are equal. The graph shows that Stabilized Banking can yield welfare gains of up to $\sim 50\%$ for relevant small but positive values of α. For any given α, the distance between the solid line (depicting normalized welfare losses under optimal Stabilized Banking) and the dashed line (giving normalized welfare losses under regular Banking) is proportional to σ_1^2 and represents the additional welfare losses caused by sub-optimally adjusting aggregate allowance availability conditional on the revelation of information through an efficient stabilization mechanism.

The Stabilized Banking instrument yields analytical predictions that can be empirically. Our model predicts that exogenous shocks (e.g. weather or oil price shocks) have a lesser effect on price volatility after implementation of the stabilization rule. The high volatility of allowance prices in the initial phases of for instance EU-ETS has been documented, among others, in Paolella and Taschini (2008); Hintermann (2010); Chesney and Taschini (2012). Using our model, we show that Stabilized Banking reduces the price volatility of emission allowances as compared to regular Banking.

Proposition 2. *Stabilized Banking reduces price volatility in both periods compared to Banking,*

$$\mathbb{E}\left[\left(p_t^{SB}\right)^2\right] \leq \mathbb{E}\left[\left(p_t^{B}\right)^2\right], \tag{28}$$

with strict inequality for $\alpha > -1$*.*

Finally, we can compare Stabilized Banking to the Optimal Response equilibrium in terms of welfare losses. The corollary marks that, even though Stabilized Banking strongly outperforms Banking, Stabilized Banking cannot solve the second-period asymmetric information problem; these still carry some welfare costs, compared to the ideal Optimal Response.

Corollary 3. *Welfare losses under a Stabilized Banking policy always exceed those in the Optimal Response Equilibrium:*

$$\frac{\Delta W^{OR}}{\Delta W^{SB}} = \left[\frac{b}{b+2c}\right]^2 < 1. \tag{29}$$

4.3 Dynamic Prices

Our next main objective is to derive an optimal dynamic version of the static Prices policy. As things stand right now, this may be too much of a leap. In the sequence of quota-instruments, after all, the order of exposition was Quantities – Banking – Stabilized Banking, ie static – dynamic – optimal dynamic. It seems reasonable to follow the same procedure for tax-instruments. To that end, we first develop the dynamic version of a Prices policy, the mirror-image of quota-based Banking.

This policy is not a mere finger-exercise in tax-regulation. Like Banking, it has some intuitive appeal, which is as follows. Suppose we have decided the source of a certain externality should be regulated by means of a (pigouvian) tax, which we set at a level we believe is optimal. Nonetheless, over the course of time, it may turn out still too much of the externality is caused, or maybe too little, despite the tax. A logical conclusion for us to draw would be that the initial tax was not optimal after all, that is, given the private information of regulated firms. Thus, next period we adjust the tax in response. Framed this way our policy, labeled Dynamic Prices, sounds pretty much like many a political discussion in many a country. The formal definition follows here:

Definition 6 (Dynamic Prices)**.** The regulator fixes the first-period tax at its ex-ante optimal level

$$p_1 = 0, \tag{30}$$

with emissions given by market demand (5). The second-period tax adapts in response to emissions in the first period,

$$p_2 = \lambda q_1, \tag{31}$$

with λ the *updating rule*. Second-period emissions follow from market demand (5).

Clearly, for $\lambda = 0$ this yields the static Prices instrument. Adopting the convention that Dynamic Prices refers to the instrument employing the optimal updating rule λ^*, we have $DP \succ P$.

It can easily be shown that Dynamic Prices as defined above is the unique policy that solves the following program:

$$\max_{p_1} \quad \mathbb{E}_0 \left[\max_{p_2} \mathbb{E}_1 W(q_1, q_2; \theta_1, \theta_2) \right] \tag{32}$$

$$\text{s.t.} \quad p_t = \theta_t - 2cq_t. \tag{5}$$

From the program (32) it follows Dynamic Prices is a Price-based instrument with the following characterization:

$$DP = (p, 0, 1) \in \mathcal{P}. \tag{33}$$

The optimal updating rule λ^* is determined by the first-order condition to this program:

$$\lambda^* = 2(1+\alpha)\frac{bc}{b+2c}. \tag{34}$$

From this, welfare losses are immediate:

$$-\Delta W^{DP} = \frac{1}{8}\frac{b^2}{(b+2c)c}\frac{1}{b+c}(1+\alpha)^2\sigma_1^2 + \frac{1}{8}\frac{b^2}{c^2}\frac{1}{b+c}\sigma_\mu^2. \tag{35}$$

Corollary 4. *Dynamic Prices outperforms Banking, $DP \succ B$, if and only if $b < 2c$.*

4.4 Market Pricing

Our final task in the exposition of policy instruments is to introduce the optimal dynamic version of a Prices or tax regime, labeled Market Pricing. Though Stabilized Banking is conceptually new, it has some flavor still of trading ratios over time, on which there exists some literature. Market Pricing, on the other hand, is entirely novel, not only in detail but even in kind. Effectively, it is an extension of the Dynamic Prices instruments developed above, with the additional feature that intertemporal trades of allowances are allowed. These market transactions are observed, and prices in future periods can be adapted in response, raising efficiency. It is immediately obvious that $MP \succ DP \succ P$. We will show (Corollary 6) that the comparison between Stabilized Banking and Market Pricing is the famous result in Weitzman (1974). We now define Market Pricing:

Definition 7 (Market Pricing)**.** The regulator auctions a fixed amount of allowances in the first period, where demand determines the price, and auctions allowances in the second-period for fixed price, given by the updating rule with fixed η:

$$p_2 = \eta q_1, \tag{36}$$

where η is chosen to minimize welfare losses. Allowance demand (5) and free intertemporal substitution of allowances ensures equal expected marginal productivity in both periods:

$$\mathbb{E}_1 p_2 = p_1. \tag{19}$$

Thus, the price set for the second period will be made dependent on the emissions observed in the first period. This approach, like Stabilized Banking, leads to the same effect that all first-period information is perfectly incorporated. However, this time the second period is regulated via a price-instrument. It is straightforward that the condition for Market Pricing just conceived versus the Stabilized Banking will become the classic Weitzman condition b versus $2c$. If the Market Pricing is extended over multiple periods, the price setting applies only to the last period.

The essential innovation here has again been to relax a constraint on the optimization problem, thereby handing one more degree of freedom to the regulator, who can then use it to increase welfare. The precise program is as follows:

$$\max_{p_1,p_2} \quad \mathbb{E}_1 W(q_1, q_2; \theta_1, \theta_2) \tag{37}$$

$$\text{s.t.} \quad p_t = \theta_t - 2cq_t. \tag{5}$$

Market Pricing are thus a Price-based instrument,

$$MP = (p, 1, 1) \in \mathcal{P}.$$

This resulting first-order conditions to program (37) are given by:

$$p_1 = p_2 \tag{38}$$

$$p_1 = b\mathbb{E}_1 \left[(q_1 + q_2)\right], \tag{39}$$

with the immediate consequence:

$$p_2 = \eta^* q_1, \tag{40}$$

where

$$\eta^* = 2(1+\alpha)\frac{bc}{b + 2c - \alpha b}. \tag{41}$$

This optimal price response rate η^* is linearly homogeneous with (b, c), continuous and strictly increasing in α. Only for $\alpha = -1$ is the optimal price response rate η^* zero. For $\alpha = 1$, $\eta^* = 2b$. These observations imply the following proposition:

Proposition 3. *Market Pricing outperforms Dynamic Prices, $MP \succ DP$, for all $\alpha > -1$.*

There is an important conceptual difference between the stabilization rate δ for Stabilized Banking and the price response rate η for Market Pricing, making a meaningful direct comparison between the two impossible. The former transforms emissions into emissions and is therefore unit-less. The latter transforms emissions into prices and thus has unit "$price/(emissions)^2$", such as €/tCO_2^2 or \$/$GtCO_2^2$. This is the same unit of measurement as for slope parameters b and c. Since welfare losses in an unregulated market are caused by the firm's neglect of the environmental damages its activities entail, the regulator's task is to somehow make firms incorporate these damages when deciding on production. Consequently, on top of having the same unit of measurement we would in fact expect the optimal price response rate η^* to *scale* with b and c, the slopes of the marginal damage and productivity curve. The normalized optimal price response rate η^*/b is graphically illustrated in Figure 3.

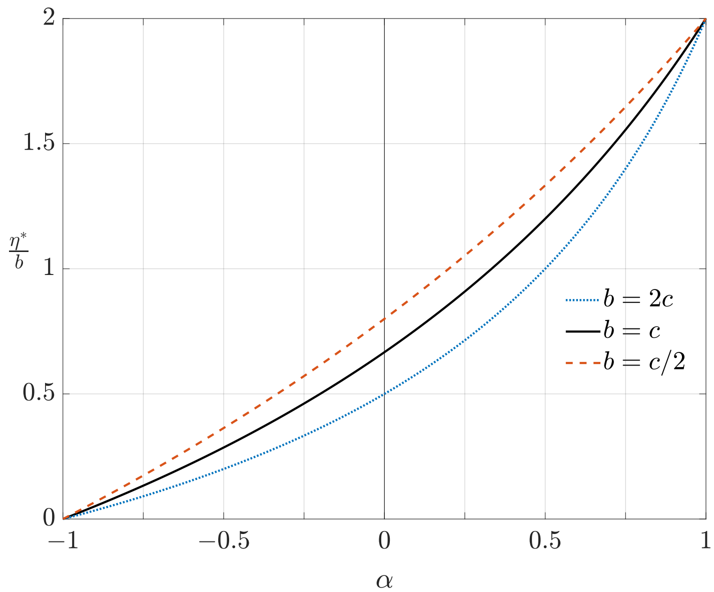

Figure 3: Optimal price response rate η^*, normalized with respect to marginal damage curve slope b.

Our next theorem summarizes the welfare properties of Market Pricing:

Theorem 2. *Market Pricing is time consistent strictly welfare superior among the class of of price-setting instruments $\mathcal{P}$. The instrument enables incorporation of all conditions anticipated by the market through setting the stabilization rate η at* (41)*. Only those market conditions which neither regulated nor regulating parties can predict cause welfare losses:*

$$-\Delta W^{MP} = \frac{1}{8}\frac{b^2}{c^2}\frac{1}{b+c}\sigma_\mu^2. \tag{42}$$

Comparing Market Pricing to the Optimal Response, we see a striking similarity to Stabilized Banking. All three allocations fully absorb first-period market conditions (θ_1) and therefore implement equal first-period price and quantities: $p_1^{SB} = p_1^{MP} = p_1^{OR}$; $q_1^{SB} = q_1^{MP} = q_1^{OR}$. But Market Pricing fixes second-period prices to match expected marginal damages: $p_1 = p_2 = b\mathbb{E}_1[(q_1 + q_2)]$. Fixing prices in the second-period adds to the welfare costs compared to the Optimal Response: $MP \prec OR$.

Corollary 5. *Welfare losses under a Market Pricing policy always exceed those in the Optimal Response Equilibrium:*

$$\frac{\Delta W^{OR}}{\Delta W^{MP}} = \frac{c}{b+2c} \le \frac{1}{2}. \tag{43}$$

Similar to Stabilized Banking, we see that Market Pricing cannot resolve the problem of second-period asymmetric information. By repeated trading, both Stabilized Banking and Market Pricing can absorb all asymmetric information, up to the last period. This is a deep insight that we formalize as Theorem 3 in the next subsection. A comparison of Market Pricing with Stabilized Banking replicates the standard Quanties versus Prices condition:

Corollary 6. *Market Pricing outperforms Stabilized Banking, $MP \succ SB$, if and only if $b < 2c$.*

4.5 A Finer Grid of Trades

We can extend our model to more than two periods; the most relevant case is one where a given piece of time is cut in increasingly smaller pieces. Increasing the number of periods to $N > 2$, we also increase the number of market operations that can be used for regulation and information collection, thus effectively using each trading opportunity as an instrument. This reminds of the result in Roberts and Spence (1976), who show that one can approximate the environmental marginal damage supply curve arbitrarily closely by stapling an increasing number of specific quantity and price instruments. A similar result can be obtained in the pollutant stock context. Given our assumption of imperfect foresight, however, we cannot aim for the ex-post social optimum. Instead, the aim is directed at the symmetric-information optimal response allocation. Recall that the optimal response is defined through $p_t^{OR} = b\mathbb{E}_t Q^{OR}$ for $t = 1, ...N-1$, and $p_N^{OR} = bQ^{OR}$ for the last period. Market Pricing satisfies the same conditions for the first $N-1$ periods, but implements $p_N^{MP} = p_{N-1}^{MP}$ in the last. Stabilized Banking also implements the same allocation in the first $N-1$ periods but fixes q_N^{SB} to implement $\mathbb{E}_{N-1}p_N^{SB} = bQ^{SB}$. That is, both Stabilized Banking and Market Pricing are 'in step' with current information and only deviate from the optimal response allocation in the last period, and welfare losses become vanishingly small when the last period becomes sufficiently short.

Theorem 3. *Let N denote the number of periods. Let ΔW_N^i denote the expected welfare loss under policy i with N regulatory periods. Both Stabilized Banking and Market Pricing approach the symmetric-information optimal response welfare level for an increasingly fine grid of trades:*

$$\lim_{N\to\infty} \Delta W_N^{SB} = \lim_{N\to\infty} \Delta W_N^{MP} = \lim_{N\to\infty} \Delta W_N^{OR} \tag{44}$$

Stated in more condensed notation: $\lim_{N\to\infty} SB_N \approx MP_N \approx OR_N$.

5 Discussion and Conclusions

5.1 Coverage of quantity and price based instruments

We characterize regulation rules as the implementation of expected welfare maximization

$$\max_{x_1} \quad \mathbb{E}_{t_1}\left[\max_{x_2}\mathbb{E}_{t_2}W(q_1,q_2;\theta_1,\theta_2)\right] \tag{2}$$

$$\text{s.t.} \quad p_t = \theta_t - 2cq_t. \tag{5}$$

where $0 \le t_1 \le t_2 \le 2$, is the timing at which policy variables x_t (quantities or prices) are set; we collapse the two expectations signs into one if $t_1 = t_2$, and leave out the expectations sign if $t_2 = 2$. Using notation (x, t_1, t_2) to specify a policy, it is immmediate that $(q, 1, 1) \succ (q, 0, 1) \succ (q, 0, 0)$, and the same for price policies.

In hindsight, we see that the rows in the overview Table 1 move from Static Instruments $(x, t_1, t_2) = (x, 0, 0)$, to Dynamic Instruments $(x, t_1, t_2) = (x, 0, 1)$, to conclude with Optimal Dynamic Instruments $(x, t_1, t_2) = (x, 1, 1)$. Indeed, both the Quantities (Prices) policy sets quantities q_t (prices p_t) before realization of θ_1. Stabilized Banking and Market Pricing set their variables after observing θ_1 but before the realization of θ_2. The Dynamic Instruments require a more subtle discussion. While Dynamic Prices indeed implements $(x, t_1, t_2) = (p, 0, 1)$, Banking is not a pure $(q, 0, 1)$ policy. Reflection reveals that a pure $(q, 0, 1)$ policy will not deviate from the static Quantities policy, $(q, 0, 0) \sim (q, 0, 1)$.[8] To meaningfully adapt q_2 after receiving information θ_1, one has to add a degree of freedom by setting ex-ante cumulative emissions Q rather than q_1.

We can also include Optimal Response in the characterization: $(x, t_1, t_2) = ((q, p), 1, 2)$, for each period t, quantity and price are set after realization of θ_t. The Social Optimum has $(x, t_1, t_2) = ((q, p), 2, 2)$: quantities and prices for both periods are set after realization of θ_2. Table 2 presents the overview of the various rules along these lines and the implied FOCs. The table also reveals that the demand equation (5) always sets the complementary variables (prices for Quantities, quantities for Prices) in their own period.

The table allows us to quickly check whether we have overlooked some policies. We have no quantities nor price policies for $(x, 0, 2)$, but indeed, such timing would make an unnatural combination. Thus, we have covered all relevant combinations (x, t_1, t_2). Our Stabilized Banking and Market Pricing fill a gap in regulation space that was not noted before.

[8] Dynamic Quantities, with updating of q_t after observing the previous market outcome θ_{t-1} through p_{t-1}, will deviate from Static Quantities in a multi-period model.

Table 2: Timing of endogenous variables under different policies

	before	θ_1	between	θ_2	after	FOC1	FOC2
Q	q_1, q_2	–	p_1	–	p_2	$\mathbb{E}_0 p_1 = bQ$	$\mathbb{E}_0 p_2 = bQ$
B	Q	–	p_1, q_1, q_2	–	p_2	$\mathbb{E}_0 p_1 = bQ$	$\mathbb{E}_1 p_2 = bQ$
SB		–	p_1, q_1, q_2	–	p_2	$p_1 = bQ$	$\mathbb{E}_1 p_2 = bQ$
P	p_1, p_2	–	q_1	–	q_2	$p_1 = b\mathbb{E}_0 Q$	$p_2 = b\mathbb{E}_0 Q$
DP	p_1	–	q_1, p_2	–	q_2	$p_1 = b\mathbb{E}_0 Q$	$p_2 = b\mathbb{E}_1 Q$
MP		–	p_1, p_2, q_1	–	q_2	$p_1 = b\mathbb{E}_1 Q$	$p_2 = b\mathbb{E}_1 Q$
OR		–	p_1, q_1	–	p_2, q_2	$p_1 = b\mathbb{E}_1 Q$	$p_2 = bQ$
SO		–		–	p_1, p_2, q_1, q_2	$p_1 = bQ$	$p_2 = bQ$

From top to bottom: Quantities, Banking, Stabilized Banking, Prices, Dynamic Prices, Market Pricing, Optimal Response, Social Optimum.

5.2 EU-ETS: History and Recent Transformation

After a start with volatile and low prices, the European Commission decided early in 2015 to revise the EU-ETS, introducing a Market Stability Reserve (MSR), operative as of 2019 (Erbach, 2017). This was motivated by the large cumulative gap between planned auctioned allowances and allowances surrendered by emitting firms of above 2 billion tCO_2 built up since 2009.[9] The MSR takes allowances out of the market, to bring them back at a later stage when private holdings of unused allowances has decreased. The rules set out imply a rate of transfer from the private allowances surplus to the MSR that varies between 12 and 24%. In November 2017, a further revision, discussed in Perino (2018), was announced. It was adopted by the EU Parliament in February 2018. By 2024 allowances in the MSR will be canceled (not returned to the market) when its level exceeds the amount of auctioned allowances in the previous year (Erbach, 2017). Combining the rules, we see the ultimate effect: a positive demand shock in early years leads to reduced banking, leading to an increase of the cumulative amount of allowances available for auctioning.

The new rules for the MSR incorporate an essential element of our Stabilized Banking policy, dynamic updating of future auctions in response to banked allowances. This is a

[9] To assess the economic importance of this surplus, note that allowance prices in the EU are currently about 20€/tCO_2, so that the monetary value of the surplus amounts to some €40 billion.

radical deviation from traditional policies. Though the MSR proposal is an endogenous policy response to particular problems of the EU-ETS, our analysis suggests general academic relevance. Using our formal SB model, we show that adjusting cumulative quota as an automatic response to early period demand shocks is efficient indeed, both before and after information revelation. Our results pin down a precise formula for the optimal response rate.

5.3 Implementation

Many real-world emissions trading systems allow banking of unused allowances to be used in future periods but at the same time do not allow borrowing of future endowments to support present-day production. In the present EU-ETS, for example, this holds by construction of the system: in every regulatory period i a total amount a_i of emissions allowances is auctioned (or grandfathered) by the regulator, and emissions in period i cannot exceed the total amount auctioned plus those allowances that remain unused from previous periods. Thus, in every period i emissions allowances can be banked, but not borrowed. To some extent, this is surprising. The advantage of banking and borrowing over period-specific quantity-setting derives from allowing firms to make use of their superior information regarding the value of emissions allowances for production. By allowing only unidirectional intertemporal trade, the regulator exploits only half of the better information. Consequently, in terms of economic efficiency, the system performs worse than when full dynamic trading is allowed. Note, however, that the inefficiency induced by prohibiting borrowing can be mitigated by increasing the auctioned allowances in early periods. Indeed, this is how the EU-ETS has been implemented.

Stabilized Banking can effectively be implemented by auctioning the minimum amount of cumulative allowances in the first period: $a_1 = (1 + \delta^*)q^*$. We define banking through $s_1 \equiv a_1 - q_1$. The remaining allowances are auctioned in the second period, $a_2 = (1-\delta^*)q_1 \geq 0$. Second-period auctioning decreases with first-period banking: $\partial a_2/\partial s_1 = -(1 - \delta^*)$. The new EU-ETS rules suggest still a modest stabilization mechanism, $0.5 < \delta < 1$, while our findings as presented in Fig 1 suggest a more aggressive stabilization mechanism to be efficient; for a positive correlation between shocks, $\alpha > 0$, it is optimal to reduce future auctioning nearly one-to-one in response to above-expected current banking.

Market Pricing are considerably less 'natural' an instrument from the perspective of implementation, particularly so when the number of periods expands beyond two. In practice, it would require a Stabilized Banking policy in all but the last period, and prices would have to be set for the last period only, based on cumulative banking according to the pre-announced response rate η^*. This apparent similarity to Stabilized Banking but with increased complexity is a feat making the policy more difficult to understand and implement as well as politically less appealing. Supported by Theorem 3, showing that both instruments approach the same welfare level upon increasing the number of regulatory periods, we believe a strong case can be made for favoring Stabilized Banking over Market Pricing when considering real-world applications.

Finally, we note that our analytical framework is inconclusive as to how allowances are ideally allocated (or grandfathered) across sources. The reader interested in matters of this nature is referred to Böhringer and Lange (2005) or Smith and Yates (2003), where explicit characterizations of optimal allocation schemes are provided.

5.4 Conclusions

We built and analyzed a formal model optimal regulation of stock externalities under uncertainty is characterized as the implementation of a (constrained) welfare maximization problem. The most general executable such program yielded two regulation rules, Stabilized Banking and Market Pricing, each of which is strictly welfare superior among all instruments in the class of quantity- and price-based regulations, respectively. Welfare gains compared to traditional, existing instruments were shown to be substantial. Both rules converge to the Optimal Response equilibrium, the hypothetical level of welfare attained when a regulator learns information as soon as it becomes available to private parties (hence a situation in which only future uncertainty, no asymmetric information, remains) as the number of regulator periods is increased by cutting time into smaller pieces. As our definition of regulation includes implementability, we also suggest policies that implement our rules.

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A Online Appendix: Micro-foundation of the Model

Firms

Consider a continuum of competitive profit-maximizing firms, indexed $i \in [0, 1]$, each producing at every time $t \in \{1, 2, ..., T\}$ quantity Y_{it} of a homogeneous good.[10] Production by the firm causes environmental damages to consumers as an externality, which the firm does not take into account. The production technology $Y : \mathbb{R}^2_+ \to \mathbb{R}_+$ has decreasing returns to scale, with inputs emission allowances q_{it} and labor l_{it}. Permit prices and wages are equal for all firms and given by price p_t and w_t, respectively. Marginal productivity of emission allowances by firm i at time t is subject to random vertical shocks θ_{it}, while labour productivity is unaffected. Shocks are observed by the firm but not by the regulator. Given these assumptions, firm i solves:

$$\max_{\widetilde{q}_{it}, l_{it}} \sum_{t=1}^{T} Y_{it}(\widetilde{q}_{it}, l_{it}; \theta_{it}) - p_t \widetilde{q}_{it} - w_t l_{it}. \tag{45}$$

We assume symmetry between firms and strictly decreasing returns to scale (positive profits) so that $l_{it} = l_t$, and we label the optimal quantities in expectations $q_t = q^*$ and prices $p_t = p^*$. In particular, we assume linear marginal productivity, of the form:

$$\forall i : \frac{\partial Y_{it}}{\partial \widetilde{q}_{it}} = p^* - Tc(\widetilde{q}_{it} - q^*) + \theta_{it} \tag{46}$$

[10]Environments where firms are not competitive and allowance allocations or prices can be manipulated by dominant firms are studied in Liski and Montero (2011), Hintermann (2011), Hintermann (2017).

Aggregate emissions, shocks, and production at time t are given by:

$$\widetilde{q}_t = \int_0^1 \widetilde{q}_{it} di \tag{47}$$

$$\theta_t = \int_0^1 \theta_{it} di, \tag{48}$$

$$Y_t(\widetilde{q}_t; \theta_t) = \int_0^1 Y_{it}(\widetilde{q}_{it}, l_{it}; \theta_{it}) di, \tag{49}$$

with $\mathbb{E}[\theta_t] = 0$ and variance captured by the parameter $\mathbb{E}[\theta_t^2] = \sigma_t^2 \geq 0$. We denote correlations between periods t and $t+s$ by $\mathbb{E}[\theta_t \theta_{t+s}] = \rho^s \sigma_t \sigma_{t+s}$.[11] Because of competitive markets and separation of emission productivity shocks from labor productivity, we can reduce equilibrium through the aggregate or representative firm, which faces the problem:

$$\max_{\widetilde{q}_t, l_t} \sum_{t=1}^{T} \left[Y_t(\widetilde{q}_t; \theta_t) - p_t \widetilde{q}_t - w_t l_t \right]. \tag{50}$$

In equilibrium, marginal productivity for emission allowances of the representative firm equals prices:

$$\widetilde{p}_t = \frac{\partial Y_t}{\partial \widetilde{q}_t} = p^* - Tc(\widetilde{q}_t - q^*) + \theta_t. \tag{51}$$

Note that for the special cases where $N = 1$ or $\theta_1 = \theta_2 = ... = \theta_N = \theta$, $\widetilde{p}_t = p$, the first order condition (51) simplifies to $c(Tq^* - \sum_t \widetilde{q}_t) + \theta_t = p - p^*$, which exactly reproduces the model in Weitzman (1974) (with $\sum_t \widetilde{q}_t$ aggregate emissions over all periods).

Finally, note that a decreasing returns to scale technology implies positive profits, so that:

$$Y_{it} = \widetilde{p}_t \widetilde{q}_{it} + w_t l_t + \pi_{it} \tag{52}$$

$$\Longrightarrow Y_t = \widetilde{p}_t \widetilde{q}_t + w_t l_t + \Pi_t. \tag{53}$$

Households

Households, normalized to size 1, maximize utility, which is derived from consumption C_t and environmental damages resulting from the stock of emissions as determined by

[11] We do not need more structure, such as the distribution. Only expectations, standard deviations, and correlations enter our results.

the damage function $D : \mathbb{R}_+ \to \mathbb{R}_+$. We focus on the simplest possible case, wherein emission-related damages only enter welfare through cumulative emissions. The rationale for this assumption is that, for long-lived pollutants such as CO_2, most damages appear after the regulation period.[12] Thus, damages enter welfare as a proxy for expected future welfare losses (see Gerlagh and Michielsen, 2015). Moreover, we abstract from discounting between periods.

To defray their consumption, households supply labor inelastically $L_t = 1$ and earn wages w_t in every period t. Households receive profits from the firm and a lump-sum transfer τ_t from the regulator.

Households face the constrained optimization problem given by:

$$\max_{C_t} \left[\sum_{t=1}^{T} C_t\right] - D\left(\sum_{t=1}^{T} \widetilde{q}_t\right) \tag{54}$$

$$\text{s.t.} \quad \sum_{t=1}^{T} C_t \leq \sum_{t=1}^{T} \left[w_t + \Pi_t + \tau_t\right]. \tag{55}$$

We assume marginal damages are linear in emissions. Specifically:

$$MD = p^* + b\left[\sum_{t=1}^{T} \widetilde{q}_t - Tq^*\right]. \tag{56}$$

Market and Regulator

Since all consumption must be produced and vice versa, we have:

$$Y_t(\widetilde{q}_t, l_t; \theta_t) = C_t, \tag{57}$$

for all t. Moreover, the market for labor equates supply and demand, so that

$$l_t = L_t = 1, \tag{58}$$

which in turn determines the wage w_t, for all t.

Households receive a lump-sum amount of money τ_t from the regulator in every period t. From the fact that the regulator collects money only through selling (or auctioning)

[12] Climate change has very persistent dynamics. See Gerlagh and Liski (2018b) for an extensive discussion of the time-structure and its implications for climate policies.

allowances, its budget-balancing constraint implies:

$$\tau_t = \widetilde{p}_t \widetilde{q}_t. \tag{59}$$

The regulator maximizes the sum of consumer surplus and producer surplus, which equals consumer welfare (54). From the equilibrium in the goods market (57), we see that the welfare-maximizing regulator's objective is given by:

$$\max W = \max_{\widetilde{q}_t} \sum_{t=1}^{T} Y_t(\widetilde{q}_t; \theta_t) - D\left(\sum_{t=1}^{T} \widetilde{q}_t\right) \tag{60}$$

For purposes of tractability, in the main text of the paper we study a two-period model, $N = 2$, which allows to derive neat analytic expressions while still capturing the dynamic nature of a stock pollutant.

B Online Appendix: Derivations Imperfect Foresight

Here we provide the derivations of various results under imperfect foresight.

B.1 Social Optimum

CHARACTERIZATION OF THE SOCIAL OPTIMUM:

$$p^{SO} = \frac{b}{b+c}\frac{\theta_1+\theta_2}{2}, \tag{61}$$

$$q_1^{SO} = \frac{1}{4c}\frac{b+2c}{b+c}\theta_1 - \frac{1}{4c}\frac{b}{b+c}\theta_2, \tag{62}$$

$$q_2^{SO} = \frac{1}{4c}\frac{b+2c}{b+c}\theta_2 - \frac{1}{4c}\frac{b}{b+c}\theta_1, \tag{63}$$

$$Q^{SO} = \frac{1}{b+c}\frac{\theta_1+\theta_2}{2}. \tag{64}$$

B.2 Optimal Response

CHARACTERIZATION OF OPTIMAL RESPONSE EQUILIBRIUM:

$$p_1^{OR} = \frac{b}{b+c}\frac{(1+\alpha)}{2}\theta_1, \tag{65}$$

$$p_2^{OR} = \frac{b}{b+c}\frac{(1+\alpha)\theta_1}{2} + \frac{b}{b+2c}\mu, \tag{66}$$

$$q_1^{OR} = \frac{1}{4c}\frac{(b+2c)-\alpha b}{b+c}\theta_1, \tag{67}$$

$$q_2^{OR} = \frac{1}{4c}\frac{\alpha(b+2c)-b}{b+c}\theta_1 + \frac{1}{b+2c}\mu, \tag{68}$$

$$Q^{OR} = \frac{1}{b+c}\frac{(1+\alpha)}{2}\theta_1 + \frac{1}{b+2c}\mu. \tag{69}$$

DERIVATION OF (15):

Quantity deviations are given by:

$$\Delta^{OR}q_1 = \frac{1}{4c}\frac{b}{b+c}\mu \tag{70}$$

$$\Delta^{OR}q_2 = -\frac{1}{4c}\frac{b}{b+c}\frac{b}{b+2c}\mu \tag{71}$$

$$\Delta^{OR}Q = \frac{1}{2}\frac{b}{b+c}\frac{1}{b+2c}\mu. \tag{72}$$

Therefore:

$$\Delta^{OR}W = c\left[\frac{1}{4c}\frac{b}{b+c}\frac{b+2c}{b+2c}\right]^2\sigma_\mu^2 + c\left[\frac{1}{4c}\frac{b}{b+c}\frac{b+2c}{b}\right]^2\sigma_\mu^2 \tag{73}$$

$$+\frac{b}{2}\left[\frac{1}{2c}\frac{b}{b+c}\frac{c}{b+2c}\right]^2\sigma_\mu^2 \tag{74}$$

$$= \frac{1}{16c}\frac{b^2}{(b+c)^2}\frac{1}{(b+2c)^2}[(b+2c)^2+b^2]\sigma_\mu^2 + \frac{1}{8c^2}\frac{b^2}{(b+c)^2}\frac{bc^2}{(b+2c)^2}\sigma_\mu^2 \tag{75}$$

$$= \frac{1}{16c}\frac{b^2}{(b+c)^2}\frac{1}{(b+2c)^2}[2b^2+4c^2+6bc]\sigma_\mu^2 \tag{76}$$

$$= \frac{1}{8c}\frac{b}{b+c}\frac{b}{b+2c}\sigma_\mu^2. \tag{77}$$

B.3 Prices versus Quantities

Corollary 7. *Quantities outperform Prices, $Q \succ P$, iff:*

$$\frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\rho > \frac{1+\frac{b}{2c}-2\left(\frac{b}{2c}\right)^2}{\frac{b}{2c}\left(1+2\frac{b}{2c}\right)}$$

A higher correlation between the two periods, ρ, increases the domain of parameter value ratios b/c for which Quantities outperform Prices. For equally sized negatively correlated shocks, $\sigma_1 = \sigma_2$ and $\rho = -1$, Prices reproduce the Social Optimum and always outperform Quantities. For independent shocks, $\rho = 0$, Quantities outperform Prices if and only if $b > 2c$. For $\sigma_1 = \sigma_2$ and $\rho = 1$, Quantities outperform Prices if and only if $b > c$.

Proof. From the definitions of the policies it follows that welfare losses are given by:

$$-\Delta W^Q = \frac{1}{8}\frac{1}{(b+c)c}\left[(b+2c)\left(\sigma_1^2+\sigma_2^2\right) - 2b\rho\sigma_1\sigma_2\right], \tag{78}$$

for Quantities and:

$$-\Delta W^P = \frac{1}{8}\frac{b^2}{(b+c)c^2}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2), \tag{79}$$

for Prices. The comparison is then as follows:

$$(b+2c)\left(\sigma_1^2+\sigma_2^2\right) - 2b\rho\sigma_1\sigma_2 \leq \frac{b^2}{c}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2) \Rightarrow \tag{80}$$

$$\left(b+2c-\frac{b^2}{c}\right)(\sigma_1^2+\sigma_2^2) \leq \left(\frac{2b^2}{c}+2b\right)\rho\sigma_1\sigma_2 \Rightarrow \tag{81}$$

$$\left(bc+2c^2-b^2\right) \leq 2\left(b^2+bc\right)\frac{\rho\sigma_1\sigma_2}{(\sigma_1^2+\sigma_2^2)} \Rightarrow \tag{82}$$

$$\frac{2\rho\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2} \geq \frac{bc+2c^2-b^2}{b(b+c)} \Rightarrow \tag{83}$$

$$\frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\rho \geq \frac{1+\frac{b}{2c}-2\left(\frac{b}{2c}\right)^2}{\frac{b}{2c}(1+2\frac{b}{2c})}, \tag{84}$$

as stated. Q.E.D.

B.4 Banking and Borrowing

Combining the allowance demand (5) and the fixed total emissions (18) with the policy program (20) we derive deviations in allowance use by period:

$$q_1^B = \frac{(1-\alpha)\theta_1}{4c} \tag{85}$$

$$q_2^B = \frac{(\alpha-1)\theta_1}{4c}. \tag{86}$$

Equations (85) and (86) have a clear interpretation. A profit-maximizing firm equates expected marginal productivity in both periods by smoothing shocks over time. When making production decisions, the firm has only observed the first-period shock. Thus, both first- and second-period emission levels depend on the realization of the first shock and the expectation of the second shock. This expectation depends on the realized first shock, namely $\mathbb{E}[\theta_2|\theta_1] = \alpha\theta_1$. The firm thus chooses emission levels as if smoothing shocks θ_1 and $\alpha\theta_1$. Consider then $\alpha = 1$, i.e. the firm expects shocks of exactly equal magnitude in both periods. In that case, there is nothing to smooth, and hence emission levels should be unaffected by the realization of θ_1. Similarly, consider $\alpha = -1$, i.e. the firm expects a second-period shock which fully cancels out the first period shock. Then, since for any given level of allowance use marginal productivity will be higher in the first period as compared to the second, profit maximization will shift allowance use very strongly from period 1 to period 2. The periodic deviations derived in equations (85) and (86) show this to be true indeed.

DERIVATION OF (21):

$$\Delta^B q_1 = \frac{1}{4c}\frac{1}{b+c}[b\mu - (1+\alpha)c\theta_1] \tag{87}$$

$$\Delta^B q_2 = \frac{1}{4c}\frac{1}{b+c}[-(b+2c)\mu - (1+\alpha)c\theta_1] \tag{88}$$

$$\Delta^B Q = \frac{1}{2c}\frac{1}{b+c}[-c\mu - (1+\alpha)c\theta_1]. \tag{89}$$

Therefore:

$$-\Delta^B W = \frac{1}{8}\frac{(1+\alpha)^2}{b+c}\sigma_1^2 + \frac{1}{8c}\frac{b+2c}{b+c}\sigma_\mu^2 \tag{90}$$

B.5 Dynamic Prices

$$q_1^{DP} = \frac{\theta_1}{2c} \tag{91}$$
$$q_2^{DP} = \frac{1}{2c}\left[\frac{2\alpha c - \lambda}{2c}\theta_1 + \mu\right] \tag{92}$$
$$\Delta^{DP} q_1 = \frac{1}{4c}\frac{b}{b+c}\left((1+\alpha)\theta_1 + \mu\right) \tag{93}$$
$$\Delta^{DP} q_2 = \frac{1}{4c}\frac{b}{b+c}\left(\left[1+\alpha - \frac{b+c}{b}\frac{\lambda}{c}\right]\theta_1 + \mu\right) \tag{94}$$
$$\Delta^{DP} Q = \frac{1}{2c}\frac{b}{b+c}\left(\left[1+\alpha - \frac{b+c}{2b}\frac{\lambda}{c}\right]\theta_1 + \mu\right) \tag{95}$$
$$\mathbb{E}\left[(\Delta^{DP} Q)^2\right] = \frac{1}{4c^2}\left(\frac{b}{b+c}\right)^2\left\{\left(1+\alpha - \frac{b+c}{2b}\frac{\lambda}{c}\right)^2\sigma_1^2 + \sigma_\mu^2\right\} \tag{96}$$
$$\mathbb{E}\left[(\Delta^{DP} q_1)^2\right] = \frac{1}{16c^2}\left(\frac{b}{b+c}\right)^2\left\{(1+\alpha)^2\sigma_1^2 + \sigma_\mu^2\right\} \tag{97}$$
$$\mathbb{E}\left[(\Delta^{DP} q_2)^2\right] = \frac{1}{16c^2}\left(\frac{b}{b+c}\right)^2\left\{\left(1+\alpha - \frac{b+c}{b}\frac{\lambda}{c}\right)^2\sigma_1^2 + \sigma_\mu^2\right\} \tag{98}$$

We have the welfare loss function, given by (11). We can plug in the above equations, defining $\gamma = \lambda/c$ and obtain the optimal parameter λ^*:

$$\left(\frac{b+2c}{2b}\right)\gamma^* = 1+\alpha \tag{99}$$
$$\implies \lambda^* = 2(1+\alpha)\frac{bc}{b+2c} \tag{100}$$

DERIVATION OF (35):

Substituting this λ^* into the above equations, we derive:

$$\Delta^{DP} W = \frac{1}{8}\frac{b^2}{(b+2c)c}\frac{1}{b+c}(1+\alpha)^2\sigma_1^2 + \frac{1}{8}\frac{b^2}{c^2}\frac{1}{b+c}\sigma_\mu^2.$$

B.6 Stabilized Banking

DERIVATION OF (25):

From the first-order conditions, and using demand (5), it follows:

$$\theta_1 - 2cq_1 = bq_1 + bq_2 \tag{101}$$

$$\theta_1 - 2cq_1 = \alpha\theta_1 - 2cq_2. \tag{102}$$

It follows immediately that:

$$\frac{q_2}{q_1} = \delta^* = \frac{b - \alpha(b+2c)}{(b+2c) - \alpha b}, \tag{103}$$

as given.

DERIVATION OF (26):

Clearly, welfare losses in the optimum are:

$$-\Delta^{SB} W(\delta^*; \sigma_1, \sigma_2, b, c, \rho) = \frac{1}{8c}\frac{b+2c}{b+c}\sigma_\mu^2. \tag{104}$$

PROOF OF PROPOSITION 2:

Proof. We know that $\theta_1 - 2cq_1^i = p_1^i$. Moreover, we derived in the main text the quantity responses to a shock θ_1, which are given by equations (85) and (**??**) for Banking and Stabilized Banking, respectively. It is thus easily seen that:

$$p_1^B = \frac{1+\alpha}{2}\theta_1$$
$$p_1^{SB} = \frac{\alpha+\delta}{1+\delta}\theta_1,$$

from which it follows:

$$\mathbb{E}\left[\left(p_1^B\right)^2\right] = \frac{(1+\alpha)^2}{4}\sigma_1^2$$
$$\mathbb{E}\left[\left(p_1^{SB}\right)^2\right] = \frac{(\delta+\alpha)^2}{(1+\delta)^2}\sigma_1^2.$$

Thus:

$$\mathbb{E}\left[(p_1^{SB})^2\right] < \mathbb{E}\left[(p_2^{SB})^2\right] \iff \delta < 1.$$

From Corollary 1, we know that under imperfect foresight the optimal stabilization rate is less than unity, $\delta^* \leq 1$, for all α, with equality if and only if $\alpha = -1$, which establishes the result for period 1.

For period 2, we have the derived the quantity deviations conditional on first-period shocks. Using that $p_2^i = \theta_2 - 2cq_2^i$ and that $\theta_2 = \alpha\theta_1 + \mu$, we can write:

$$p_2^B = \frac{1+\alpha}{2}\theta_1 + \mu$$
$$p_2^{SB} = \frac{\alpha+\delta}{1+\delta}\theta_1 + \mu,$$

and thus we can derive:

$$\mathbb{E}\left[\left(p_2^B\right)^2\right] = \frac{(1+\alpha)^2}{4}\sigma_1^2 + \sigma_\mu^2$$
$$\mathbb{E}\left[\left(p_2^{SB}\right)^2\right] = \frac{(\delta+\alpha)^2}{(1+\delta)^2}\sigma_1^2 + \sigma_\mu^2.$$

Thus:

$$\mathbb{E}\left[(p_2^{SB})^2\right] < \mathbb{E}\left[(p_2^B)^2\right] \iff \delta < 1.$$

From Corollary 1, we know that the optimal stabilization rate is less than unity, $\delta^* \leq 1$, for all α, with equality if and only if $\alpha = -1$. This establishes our result. Q.E.D.

B.7 Market Pricing

We find:

$$q_1^{MP} = \frac{\theta_1}{2c+\eta} \tag{105}$$
$$q_2^{MP} = \frac{(2c+\eta)\theta_2 - \eta\theta_1}{2c+\eta}\frac{1}{2c} \tag{106}$$

Next, define $\gamma = 2c/(2c+\eta)$. We solve:

$$\Delta^{MP} q_1 = \frac{1}{4c}\frac{1}{b+c}\left(\left[2(\gamma-1)(b+c) + (1+\alpha)b\right]\theta_1 + b\mu\right) \tag{107}$$
$$\Delta^{MP} q_2 = \frac{1}{4c}\frac{1}{b+c}\left(\left[2(\gamma-1)(b+c) + (1+\alpha)b\right]\theta_1 + b\mu\right) \tag{108}$$
$$\Delta^{MP} Q = 2\Delta^{MP} q_1 = 2\Delta^{MP} q_2 \tag{109}$$

Clearly, the optimal γ^* satisfies $2(\gamma-1)(b+c)+(1+\alpha)b=0$. This implies:

$$2\frac{\eta^*}{\eta^*+2c}(b+c)=(1+\alpha)b \tag{110}$$

$$\eta^*=2(1+\alpha)\frac{bc}{b+2c-\alpha b}. \tag{111}$$

PROOF OF THEOREM (3):

Proof. The expected welfare losses of a policy (11) are straightforwardly extended from 2 to N trading periods:

$$-\Delta W^i=-\frac{b}{2}\mathbb{E}\left[\left(\Delta^i Q\right)^2\right]-cN\sum_{n=1}^{N}\mathbb{E}\left[\left(\Delta^i q_n\right)^2\right]. \tag{112}$$

We will now show that for all $n<N$: $q_n^{SB}=q_n^{MP}=q_n^{OR}$, and since $q_N=O(N^{-1})$, it follows immediately that $\lim_{N\to\infty} SB_N\approx MP_N\approx OR_N$.

Consider a time window of unit length, $t\in[0,1]$, divided in N periods of $\varepsilon=1/N$ length, so that the n^{th} period ($n\in 1,...,N$) covers the interval $[(n-1)\varepsilon,n\varepsilon]$.

We assume a stochastic process θ_t such that for any grid of N periods, the demand shock at period $1<n\le N$, anticipated at period $1\le m<n$, is well defined:

$$\theta_n=\mathbb{E}_m[\theta_n]+\mu_m^n \tag{113}$$

with $\mathbb{E}_m[\theta_n]$ a linear function of $(\theta_1,\theta_2,...,\theta_m)$, and $\mu_m^n\sim N(0,\sigma_m^n)$ orthogonal on past shocks, and $\sigma_{N-1}^N\to\sigma^*$ for $N\to\infty$. We do not need additional structure for the shocks, but assume that for $N\to\infty$ it converges to a well defined continuous stochastic process.

Demand for allowances follows a stochastic process, expressed through marginal productivity equals pricing,

$$p_n=\theta_n-Ncq_n \tag{114}$$

Note that, given the stochastic process, we have $q_N=O(N^{-1})$. We define Q_n as cumulative past emission

$$Q_n=\sum_{m=1}^{n}q_m \tag{115}$$

and $Q_N=Q$. Both the optimal response, market prices, and stabelized banking satisfy rational expectations. For any $1 \leq m < n \leq N$:

$$p_m = \mathbb{E}_m[p_n] \tag{116}$$

We rewrite (114) in expectations as

$$\mathbb{E}_m[p_n] = \mathbb{E}_m[\theta_n] - Nc\mathbb{E}_m[q_n] \tag{117}$$

Through substutition of (114) and (116) at the left-hand side, we can express the equation fully in (expected) quantities:

$$\mathbb{E}_m[q_n] = q_m + \frac{\mathbb{E}_m[\theta_n] - \theta_m}{Nc} \tag{118}$$

which we sum over all future periods, to arrive at

$$\mathbb{E}_m[q_N] = q_{m-1} + q_m + \sum_{n=m+1}^{N} \mathbb{E}_m[q_n] \tag{119}$$

$$= q_{m-1} + (N+1-m)q_m + \sum_{n=m+1}^{N} \frac{\mathbb{E}_m[\theta_n] - \theta_m}{Nc} \tag{120}$$

That is, through rational price expectations, current and expected cumulative supply are connected to each other. We now compare three instruments. The optimal response satisfies the condition that in the last period, marginal productivity equals marginal damages:

$$p_N^{OR} = bQ_N^{OR} \tag{121}$$

Market Pricing auctions prices in the last period at a fixed price

$$p_{N-1}^{MP} = p_N^{MP} = b\mathbb{E}_{N-1}[Q_N^{MP}] \tag{122}$$

while stabilized banking fixes cumulative allowances, over the entire period, in the pre-last period at the expected optimal level:

$$p_{N-1}^{SB} = bQ_N^{SB} \tag{123}$$

All three equilibria, in expectations, imply for $1 \leq m < N$:

$$p_m = b\mathbb{E}_m[Q_N] \tag{124}$$

We combine this with equation (120) to determine quantities in period $1 \leq m < N$:

$$[b(N+1-m) + Nc]q_m = -\theta_m + b \sum_{n=m+1}^{N} \frac{\mathbb{E}_m[\theta_n] - \theta_m}{Nc} \tag{125}$$

which also uniquely identifies p_m through (114), the same for Optimal Response, Stabilized Banking, and Market Pricing. Q.E.D.