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Abstract

We propose one of the first dynamic models of the optimal transition from fossil fuels to renewables in electricity generation that takes into account the variability and intermittency of renewable energy as well as storage. This work sheds light on the extent to which variability and intermittency constitute a serious obstacle to energy transition and, given these constraints, the value of storage. The results of this model provide useful insight into the complexity of transitioning to a clean energy mix, as well as the role climate policy can play in facilitating both the growth of renewables and storage.

JEL-Codes: Q420, Q540.

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Energy transition with variable and intermittent renewable electricity generation*

1 Introduction

In the short/medium term, renewables cannot be deployed at a large scale to replace coal and other fossil fuels in electricity generation. Indeed, on the one hand they are still more costly on average than fossil fuels, and, on the other hand, they are non-dispatchable and are not continuously available. But in the future the production of electricity has to be decarbonized, and producing energy by renewable sources seems to be the best option to do so,[1] before nuclear fusion becomes eventually available.

The production of electricity by renewable sources has grown rapidly in recent years. At the end of 2017, the estimated renewable energy share in global electricity production is 26.5%, of which 16.4% is hydropower, 5.6% wind power, and 1.9% solar PV (REN21, 2018). Hydropower has limited expansion possibilities, and the combined shares of solar and wind power is still very small. It is a long way to the total elimination of the 73.5 % share of non-renewable electricity that is left.

Contrary to what was the dominant view a few years ago, the cost is not the main obstacle to a larger penetration of renewable sources in the electric mix. The investment cost for wind and solar plants has already been widely reduced, due to technical progress and learning effects in production and installation, and the decrease is expected to continue, until a limit lower bound is reached. For instance, according to the International Energy Agency (2011), solar PV costs have been reduced by 20% for each doubling of the cumulative installed capacity.

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[1] Carbon Capture and Storage (CCS) is another option, but it is still expensive and can only offer a partial solution as potential carbon sinks are of limited capacity. CCS has already been extensively studied. See for instance Lafforgue, Magne and Moreaux (2008). Examining the nuclear option would require to write another, very different paper.

The serious problem is posed by the fact that wind and solar energy are not continuously available: they are both variable and intermittent. Variability is predictable, it follows a natural pattern such as the alternation of day and night, or of seasons. Intermittency is stochastic, as it is due to unpredictable weather events like cloud formation. Sinn (2017), discussing the German energy transition (*die Energiewende)*, stresses that in Germany in 2014 the installed wind power production capacity was 35.92 GW and the average production 5.85 GW, that is 16.3% of the capacity; for solar the corresponding figures are 37.34 GW of capacity, and a production of 3.7 GW, that is 9.9% of the capacity. The need for solutions to handle variability and intermittency is obvious; without solutions, intermittent sources cannot be considered as good substitutes for conventional fossil sources (Joskow, 2011, Laskano *et al.*, 2017). However, at the moment, issues posed by variability and intermittency are not satisfactorily solved. Practical solutions are the backup of renewables by fossil fuel-fired power plants (the German double structure strategy, see Sinn, 2017), the diversification of sources associated to a dense transmission network, storage—from the classic hydro-pumped storage to new solutions like modern batteries, compressed air storage, flywheels etc., and demand side management. Heal (2016) offers an overview of these solutions and possible lines of modelling.

The literature considering the penetration of renewables in the energy mix consists so far in two separate trends.

There is first a very abundant literature of macro-dynamic models à la Hotelling considering renewable energy as a "backstop technology". It consists in an abundant and steady flow available with certainty, at a unit cost higher than the unit extraction cost of fossil energy.[2] In the simplest version of this representation of the energy transition, the unit extraction cost of fossil fuels rises over time by a Hotelling effect, and the switch to clean renewable energy occurs when this cost reaches the cost of the backstop. Useful as it may have been, this representation is not accurate for the actual energy transition problem. First, it overlooks the fact that the relevant cost to be taken into account as far as renewables are concerned is the investment cost, rather than the variable operating cost, which is very low. Second, the standard literature ignores the variable and intermittent nature of renewable sources, which is to our point of view the main obstacle to their penetration in the electric mix.

Another strand of literature is composed of models that are not directly interested in energy transition, but focus on the design of the electric mix when intermittency is taken into account, with or without storage devices. Ambec and Crampes (2012, 2015) are representative of this literature. They study in a static framework the optimal electric mix with intermittent renewable sources, and contrast it with the mix chosen by agents in a decentralized economy where the retailing price of electricity does not vary with its availability. They examine the properties of different public policies and their impacts on renewable penetration in the electric mix: carbon tax, feed-in tariffs, renewable portfolio standards, demand-side

[2]See, for early path-breaking papers, Hoel and Kverndokk (1996), and Tahvonen (1997).

management policies. Helm and Mier (2018) build a dynamic model with a fossil fuel, a variable renewable source and a storage technology. Their aim is to study the optimal subsidies for the renewables and the storage capacity when a carbon tax cannot be put into place. They do not account for intermittency.

A recent survey on the economics of solar electricity (Baker *et al.,* 2013) emphasizes the lack of economic analysis of clean energy provision through renewable sources. We intend to contribute to fill this lack by putting together the two strands of the literature mentioned above, in order to make macro-dynamic models more relevant for the study of the energy transition. Indeed we believe that the energy transition is by essence a dynamic problem, which cannot be fully understood through static models. On the other hand, dynamic models are so far unable to take into account properly some crucial features of renewables, namely investment in capacity and, above all, variability and intermittency, with the need for storage they create. Our aim is to propose a simple and tractable dynamic model incorporating these features, to be able to study the extent to which they actually constitute a serious obstacle to energy transition.

In a first step we tackle the variability issue alone. We build a stylized deterministic dynamic model of the optimal choice of the electric mix (fossil and renewable), where the fossil energy, let us say coal for the purpose of illustration, is abundant but CO_2-emitting, and the renewable energy, let us say solar, is variable and clean. The originality of the model is that electricity produced when the renewable source of energy is available, and electricity produced when it is not, are considered two different goods: day-electricity and night-electricity in the case of solar energy that we use for illustration. These two goods are produced by potentially different technologies: day-electricity can be produced with coal and/or solar, whereas night-electricity can be produced with coal, or by the release of day-electricity that has been stored to that effect. Storage is costly due to the loss of energy during the restoration process. These two goods are also potentially valued differently by consumers, who, at each period of time, derive utility from the consumption of the two goods: consumers may prefer to consume electricity at day, in which case solar is at peak, or at night, in which case solar is off-peak. Considering that there are two different goods allows us to take into account intra-day variability. We consider that coal and solar are available at zero variable costs, in order to focus on the variability and intermittency issues. We also make the assumption that at the beginning of the planning horizon coal-fired power plants already exist so that there is no capacity constraint on the production of electricity by the fossil source, but that the existing solar capacity is small so that investments are to be made in order to build up a sizable capacity.

We solve the centralized program under the constraint of a carbon budget that cannot be exceeded, and derive an optimal succession of regimes. We show that with a low initial solar capacity it is optimal to first use fossil fuels during night and day, then use fossil fuels during night only and finally go for no fossil fuels at all, when the carbon budget is exhausted. The optimal dynamics for the capacity of solar power plants is derived, as well as the optimal amount of electricity stored in time. We show that investment in solar panels is not necessarily monotonous, and that storage begins when fossil fuels have been abandoned at day and the solar capacity is large enough. We characterize the long term of this economy, and especially the determinants of long run solar capacity and storage rate. Finally, we analyze the consequences of a more stringent climate policy, a lower investment cost in solar panels, and improvements in the storage and solar power generation technologies on the optimal investment decisions and energy mix.

In a second step, we introduce intermittency in addition to variability, and study the design of the power system enabling to accommodate it. With intermittency, day-electricity generation by solar power plants becomes uncertain. We consider that there is only partial generation if solar radiations are too weak due for instance to the cloud system, which occurs with a given probability. We exhibit two very different situations. If the cloud problem is not too severe, we show that intermittency does not matter so much, rejoining the empirical result of Gowrisankaran *et al.* (2016). The energy transition follows the same succession of phases as under variability only. On the contrary, if there may be very few sun during the day, the transition is very different as fossil fuels are abandoned later while storage starts earlier. In both cases, ignoring intermittency leads to underinvest in solar capacity in the long run. Finally, we show that renewable electricity generation and storage are complement: absent storage devices, the long run renewable capacity is smaller, and so is electricity consumption.

The structure of the paper is the following. Section 2 sets up the framework, solves the model and studies the sensitivity of the solution to the main parameters in the case where variability only is taken into account. In Section 3, we introduce intermittency, examine under what circumstances it qualitatively changes the previous solution, and wonder whether in the presence of intermittency renewable capacity and storage are substitute or complements. Section 4 concludes.

2 Optimal energy transition with a variable renewable source

As explained above, our solution to handle the variability issue is very simple. It is based on the fact that the concept of variability supposes the existence of a predictable pattern. Consider for the purpose of illustration that the renewable energy we are talking about is solar energy. It can be harnessed at day but not at night, and this is clearly predictable: we have a pattern here.[3] Then, we consider that at each point in time the representative

[3] there is no such general pattern for wind energy, but specific patterns can be found depending on the location.

household wants to consume two different goods, day and night-electricity. They are different because they are not produced with the same technology and also because the consumer may value them differently, since they are not available at the same time.

Energy requirements may be satisfied by fossil sources, let's say coal, at day and night. Coal is abundant[4] and carbon-emitting: the issue with coal extraction and consumption is not scarcity but climate change. To focus on the variability issue, we neglect extraction costs and suppose that a large fossil capacity exists at the beginning of the planning horizon, so there is no need of investing to build new fossil-fired power plants. Climate policy takes the form of a carbon budget that society decides not to exceed, to have a good chance to maintain the temperature increase at an acceptable level -typically 2°C. This carbon budget is consumed when coal is burned. It is also possible to use a renewable source of energy, namely solar, abundant and clean, provided that a production capacity is built but with no variable cost. Costly investment allows to increase solar capacity. Coal and solar are perfect substitutes in electricity generation. There exists a storage technology that allows to store imperfectly electricity from day to night at no monetary cost but with a physical loss.

Our objective is to determine the optimal electric mix, the path of investment in renewable capacity, and the optimal storage policy.

2.1 Objective and constraints

The social planner seeks to maximize the discounted sum of the net surplus of the economy. Instantaneous net surplus is the difference between the utility of consuming day and night-electricity and the cost of the investment in solar capacity. Day-electricity can be produced by coal-fired power plants and/or solar plants. A fraction of solar electricity can be stored to be released at night.[5] In addition to fossil electricity, night-electricity can be produced by the release of solar electricity stored during the day, with a loss.[6]

[4]Coal reserves and, overall, resources are so large that coal, although non-renewable, can safely be considered as not scarce. See for instance IPCC (2014), Chapter 7, Table 7.2.

[5]It does not make sense to store coal electricity since coal-fired power plants can be operated at night as well and there is no capacity constraint.

[6]For instance, according to Yang (2016), the efficiency of pumped hydroelectric storage (defined as the electricity generated divided by the electricity used to pump water) is lower than 60% for old systems, but over 80% for state-of-the-art ones.

The social planner's programme reads:

$$\max_{x_d,x_n,a,I} \int_0^\infty e^{-\rho t} \left[u\left(e_d(t), e_n(t)\right) - C(I(t))\right] dt \tag{1}$$

$$\begin{aligned}
e_d(t) &= x_d(t) + (1-a(t))\overline{\phi} Y(t) \\
e_n(t) &= x_n(t) + k a(t)\overline{\phi} Y(t) \\
\dot{X}(t) &= x_d(t) + x_n(t) \\
\dot{Y}(t) &= I(t) \\
0 \leq a(t) &\leq 1 \\
X(t) &\leq \overline{X} \\
x_d(t) &\geq 0, x_n(t) \geq 0 \\
X_0 \geq 0, &\quad Y_0 \geq 0 \text{ given}
\end{aligned}$$

e_d and e_n are respectively day and night-electricity consumption. $u(e_d, e_n)$ is the instantaneous utility function, supposed to have the standard properties and to be separable in day and night-electricity. This assumption may be justified by a temporal separability argument: temporal separability holds at the intra-day level as it holds across days. Then the marginal utility u_d only depends on e_d, and the marginal utility u_n only on e_n. Moreover, we suppose that marginal utilities become infinite when consumption tends to zero: $\lim_{e_d \to 0} u_d(e_d) = 0$ and $\lim_{e_n \to 0} u_n(e_n) = 0$.

x_d and x_n are fossil-generated electricity consumed respectively at day and night. X is the stock of carbon accumulated into the atmosphere due to fossil fuel combustion. $\overline{X}$ is the carbon budget, that is the ceiling on the atmospheric carbon concentration imposed by climate policy.

Y is solar capacity. $\overline{\phi}$ measures the efficiency of solar electricity generation. I is the investment in solar capacity. $C(I)$ is the investment cost function, increasing and strictly convex to reflect adjustment costs. a is the share of solar electricity produced at day that is stored to be released at night. The efficiency of the storage technology is represented by the parameter $k \in [0,1]$. Conversely, $1-k$ is the leakage rate of this technology. ρ is the discount rate, strictly positive.

2.2 General results

The current value Hamiltonian associated to the social planner's programme (1) reads, dropping the time index:

$$\mathcal{H} = u\left(x_d + (1-a)\overline{\phi} Y, x_n + k a \overline{\phi} Y\right) - C(I) - \lambda\left(x_d + x_n\right) + \mu I$$

where λ is the shadow price of carbon, that is the carbon value, and μ is the shadow price of the solar capacity. Introducing Lagrange multipliers for the different inequality constraints allows us to write the Lagrange function as:

$$\mathcal{L} = \mathcal{H} + \underline{\omega}_a a + \overline{\omega}_a (1-a) + \omega_d x_d + \omega_n x_n + \omega_X \left(\overline{X} - X\right)$$

with obvious notations.

The first order optimality conditions are:

$$u_d = \lambda - \omega_d \tag{2}$$
$$u_n = \lambda - \omega_n \tag{3}$$
$$Y\left(u_d - k u_n\right) = \underline{\omega}_a - \overline{\omega}_a \tag{4}$$
$$C'(I) = \mu \tag{5}$$
$$\dot{\lambda} - \rho\lambda = -\omega_X \tag{6}$$
$$\dot{\mu} - \rho\mu = -(1-a)\overline{\phi} u_d - ka\overline{\phi} u_n \tag{7}$$

where u_d and u_n denote respectively the marginal utilities of consumption of day and night-electricity. The complementarity slackness conditions read:

$$\underline{\omega}_a a = 0, \underline{\omega}_a \geq 0, a \geq 0$$
$$\overline{\omega}_a (1-a) = 0, \overline{\omega}_a \geq 0, 1-a \geq 0$$
$$\omega_d x_d = 0, \omega_d \geq 0, x_d \geq 0$$
$$\omega_n x_n = 0, \omega_n \geq 0, x_n \geq 0$$
$$\omega_X \left(\overline{X} - X\right) = 0, \omega_X \geq 0, \overline{X} - X \geq 0$$

Before the ceiling, $X < \overline{X}$ and $\omega_X = 0$. Then FOC (6) reads $\dot{\lambda}/\lambda = \rho$, i.e.:

$$\lambda(t) = \lambda(0)e^{\rho t} \tag{8}$$

The carbon value follows a Hotelling rule before the ceiling, as long as fossil fuel is used.

Eq. (5), together with $\dot{Y} = I$, yields:

$$\dot{Y}(t) = I(t) = C'^{-1}(\mu(t)) \tag{9}$$

According to the properties of the investment cost function (C' is strictly increasing), this equation shows that investment in solar panels, that is the accumulation of solar capacity, is an increasing function of the shadow price of solar capacity.

Remember that we have supposed that the initial solar capacity is low, which is the most relevant assumption from an empirical point of view. It ensures that fossil fuel is going to be used night and day at the beginning of the planning horizon, and allows us to avoid the multiplication of different cases that do not bring much to our purpose.

Phase (1)

The first phase is then a phase where fossil fuel is used night and day. Then, $\omega_d = \omega_n = 0$. FOC (2), (3), (4) and (7) may be written as:

$$u_d(e_d) = \lambda \tag{10}$$

$$u_n(e_n) = \lambda \tag{11}$$

$$Y(1-k)\lambda = \underline{\omega}_a - \overline{\omega}_a \tag{12}$$

$$\dot{\mu} - \rho\mu = -\left((1-a) + ka\right)\overline{\phi}\lambda \tag{13}$$

The marginal utilities of consumption of day and night-electricity are the same in this phase, and equal to the carbon value. It implies that $e_d = u_d^{-1}(\lambda)$ and $e_n = u_n^{-1}(\lambda)$. We will say that sun is "at peak" if $u_d^{-1}(\lambda) > u_n^{-1}(\lambda)$, meaning that for a given price λ demand is higher at day, when sun shines, than at night, when it does not. Sun will be "off-peak" in the opposite case. Whether sun is at peak or off-peak will prove to have important consequences on the design of the transition to solar energy.

The left-hand side member of Eq. (12) is positive. Hence the case $\underline{\omega}_a = 0$ and $\overline{\omega}_a > 0$, i.e. $a = 1$ (full storage) is excluded. The interior case $\underline{\omega}_a = 0$ and $\overline{\omega}_a = 0$, i.e. $0 < a < 1$, is possible if and only if $Y = 0$. But it does not make sense to store absent any intermittent source of energy. This case is also excluded. The only possibility is thus $\underline{\omega}_a > 0$ and $\overline{\omega}_a = 0$, i.e. $a = 0$. It is never optimal to store electricity when coal is used night and day.

With no storage, Eq. (13) simplifies into:

$$\dot{\mu} - \rho\mu = -\overline{\phi}\lambda \tag{14}$$

Integrating this equation, we obtain:

$$\mu(t) = e^{\rho t}\left(\mu(0) - \overline{\phi}\lambda(0)t\right) \tag{15}$$

Then

$$\dot{\mu}(t) = e^{\rho t}\left(\rho\mu(0) - \overline{\phi}\lambda(0)(1+\rho t)\right)$$

implying that $\dot{\mu}(t)$ is strictly positive if and only if $t < \underline{T}_I$, defined by:

$$\underline{T}_I = \frac{\mu(0)}{\overline{\phi}\lambda(0)} - \frac{1}{\rho} \tag{16}$$

As according to Eq. (9) $\dot{I}(t)$ has the same sign as $\dot{\mu}(t)$, we may conclude that in this first phase investment in solar panels increases during a time interval all the longer since the initial shadow value of solar capacity is high, reflecting a small capacity, the initial carbon value is small, reflecting a lenient climate constraint, and the discount rate is high, reflecting an impatient society. It could be the case that $\underline{T}_I$ is either negative or higher than the date at which this first phase will come to an end (denoted $\underline{T}$). We will show below that $\underline{T}_I > \underline{T}$ is impossible. Hence investment in solar capacity either decreases or is bell-shaped during the first phase.

At the end of this phase, $x_d(\underline{T}) = 0$. Indeed, $x_n = 0$ is impossible: according to the properties of the utility function, it would require $\lambda \to +\infty$. Therefore this first phase is followed by a second phase where fossil fuel has been dropped for the production of day-electricity, and is only used in the production of night-electricity.

Phase (2)

In this phase, solar is used at day, fossil fuel at night, and, by continuity with the previous phase, there is no storage.[7]

FOC (2), (3), (4) and (7) may be written as:

$$u_d(\overline{\phi}Y) \leq \lambda \tag{17}$$
$$u_n(x_n) = \lambda \tag{18}$$
$$u_d(\overline{\phi}Y) \geq k\lambda \tag{19}$$
$$\dot{\mu} - \rho\mu = -\overline{\phi}u_d(\overline{\phi}Y) \tag{20}$$

Eqs. (9) and (20) yield a saddle-path dynamic system in (μ, Y). At the steady state of the system, the shadow price of solar capacity and solar capacity itself are:

$$\mu^* = C'(0), \qquad Y^{**} = \frac{1}{\overline{\phi}} u_d^{-1}\left(\frac{\rho\mu^*}{\overline{\phi}}\right) \tag{21}$$

In the presence of storage possibilities, this steady state where fossil fuel use is strictly positive at night will never be reached because of the carbon budget constraint, which imposes that this phase comes to an end in finite time.[8] As solar capacity cannot decrease, we necessarily have $Y(\underline{T}) < Y^{**}$. Therefore, along the saddle path, investment in solar panels and the shadow price of solar capacity are decreasing, while solar capacity is increasing (see Figure 1).

According to Eqs. (17) and (19), we must have $k\lambda \leq u_d(\overline{\phi}Y) \leq \lambda$, which gives the boundaries

[7] Of course, the continuity argument only holds in $\underline{T}^+$. It may be the case that this phase is reduced to one date, if storage begins immediately. We are going to show that this is not what happens, as soon as $k < 1$.

[8] It would be different absent storage possibilities: then fossil fuel use at day might tend to zero asymptotically and this steady state would be reached.

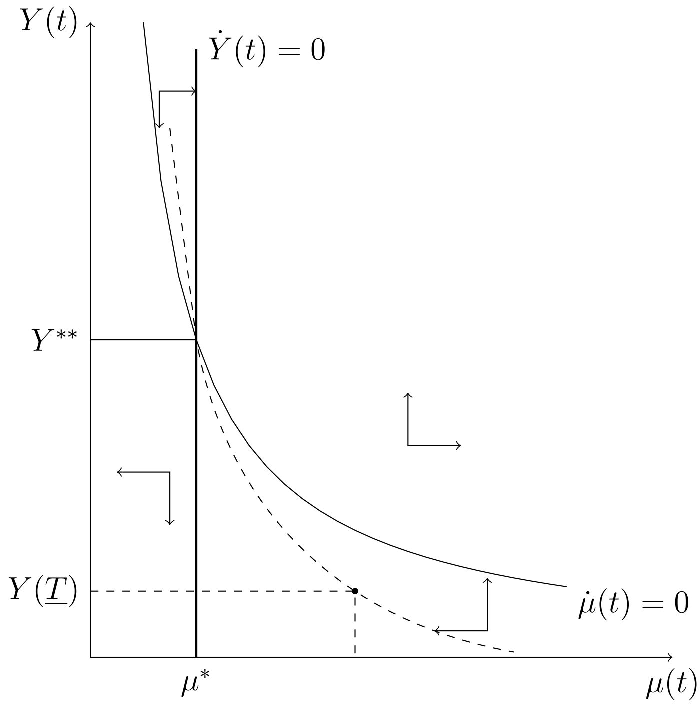

Figure 1: Phase diagram for phase (2)

of this phase: it begins at date $\underline{T}$ characterized by:

$$u_d^{-1}(\lambda(\underline{T})) = \overline{\phi}Y(\underline{T}) \tag{22}$$

and ends at date T_i defined by:

$$u_d^{-1}(k\lambda(T_i)) = \overline{\phi}Y(T_i) \tag{23}$$

As the demand function u_d^{-1} is decreasing, Eqs. (22) and (23) show that as soon as $k < 1$, $T_i > \underline{T}$: this phase is not reduced to a single date. If on the contrary $k = 1$, this phase does not exist.

We finally come back to the shape of the investment function. As $I(t) = C'^{-1}(\mu(t))$ in phases (1) and (2), we have in these two phases $\dot{I}(t) = \dot{\mu}(t)/C''(C'^{-1}(\mu(t)))$. According to Eq. (14) and (20), we have in phase (1) $\dot{\mu}(t) = \rho\mu(t) - \overline{\phi}\lambda(t)$, and in phase (2) $\dot{\mu}(t) = \rho\mu(t) - \overline{\phi}u_d(\overline{\phi}Y(t))$. At the date of the switch between the two phases μ, λ and Y are continuous, and Eq. (22) shows that $\lambda(\underline{T}) = u_d(\overline{\phi}Y(\underline{T}))$. Therefore $\dot{\mu}$ is continuous at $\underline{T}$, which implies that $\dot{I}$ is continuous as well. As in phase (2) the shadow price of solar capacity μ is strictly decreasing over time and so is investment, we conclude that investment is decreasing in phase (1) when t approaches $\underline{T}$. Therefore $\underline{T}_I > \underline{T}$ is impossible.

<u>Phase (3)</u>

In this phase, solar is used at day, fossil fuel at night, and storage is introduced ($0 < a < 1$). FOC (2), (3), (4) and (7) now read:

$$u_d((1-a)\overline{\phi}Y) \leq \lambda \tag{24}$$
$$u_n(x_n + ka\overline{\phi}Y) = \lambda \tag{25}$$
$$u_d((1-a)\overline{\phi}Y) = k\lambda \tag{26}$$
$$\dot{\mu} - \rho\mu = -k\overline{\phi}\lambda \tag{27}$$

Eq. (26) yields:

$$a(t) = 1 - \frac{1}{\overline{\phi}Y(t)} u_d^{-1}(k\lambda(t)) \tag{28}$$

As λ and Y are increasing during this phase, a is increasing as well, from $a(T_i) = 0$ to $a(\overline{T}) > 0$.

Eq. (25) yields:

$$x_n(t) = u_n^{-1}(\lambda(t)) + ku_d^{-1}(k\lambda(t)) - k\overline{\phi}Y(t) \tag{29}$$

This phase ends when $x_n = 0$, that is at date $\overline{T}$ defined by:

$$u_d^{-1}(k\lambda(\overline{T})) + \frac{1}{k}u_n^{-1}(\lambda(\overline{T})) = \overline{\phi}Y(\overline{T}) \tag{30}$$

Using this equation it is possible to compute $a(\overline{T}) = \frac{1}{k\overline{\phi}Y(\overline{T})} u_n^{-1}(\lambda(\overline{T}))$.

A comparison of Eqs. (23) and (30) shows that $\overline{T} > T_i$.[9]

At $\overline{T}$ the carbon budget is exhausted. Indeed, it cannot be optimal not to have exhausted it, since fossil fuel will never be used anymore.

As in phase (1), integrating Eq. (27) yields:

$$\mu(t) = e^{\rho(t-T_i)} \left(\mu(T_i) - k\overline{\phi}\lambda(T_i)(t - T_i)\right) \tag{31}$$

Then

$$\dot{\mu}(t) = e^{\rho(t-T_i)} \left(\rho\mu(T_i) + \rho k\overline{\phi}\lambda(T_i)T_i - k\overline{\phi}\lambda(T_i)(1 + \rho t)\right)$$

implying that $\dot{\mu}(t)$ is strictly positive if and only if $t < \overline{T}_I$, with:

$$\overline{T}_I = T_i + \frac{\mu(T_i)}{k\overline{\phi}\lambda(T_i)} - \frac{1}{\rho} \tag{32}$$

If $\overline{T}_I < T_i$ investment in solar panels is decreasing; it is bell-shaped if $T_i < \overline{T}_I < \overline{T}$, and increasing if $\overline{T} < \overline{T}_I$. With the same continuity argument as for $\underline{T}_I$, it is possible to show

[9]Proof: Suppose that $\overline{T} < T_i$; then $\lambda(\overline{T}) < \lambda(T_i)$, implying that $u_d^{-1}(k\lambda(\overline{T})) > u_d^{-1}(k\lambda(T_i))$, implying in turn, by Eqs. (23) and (30), that $Y(\overline{T}) > Y(T_i)$, a contradiction.

that the only possible case is the first one. Investment in solar panels is decreasing during this phase.

Phase (4)

This is a clean phase where no fossil fuel is used. It begins at date $\overline{T}$. FOC (2), (3), (4) and (7) now read:

$$u_d((1-a)\overline{\phi}Y) \leq \lambda \tag{33}$$
$$u_n(ka\overline{\phi}Y) \leq \lambda \tag{34}$$
$$u_d((1-a)\overline{\phi}Y) = ku_n(ka\overline{\phi}Y) \tag{35}$$
$$\dot{\mu} - \rho\mu = -k\overline{\phi}u_d((1-a)\overline{\phi}Y) \tag{36}$$

Eq. (35) implicitly gives the storage rate $a(t)$ as a function of solar capacity $Y(t)$. The solution exists and is unique as soon as the marginal utilities tend to infinity when consumption tends to zero.[10] Then, Eqs. (9) and (36) yield a saddle-path dynamic system in (μ, Y). At the steady state, the shadow price of solar capacity and solar capacity itself are defined by:

$$\mu^* = C'(0), \qquad (1-a(Y^*))Y^* = \frac{1}{\overline{\phi}}u_d^{-1}\left(\frac{\rho\mu^*}{k\overline{\phi}}\right) \tag{37}$$

Along the saddle path, investment in solar panels and the shadow price of solar capacity are decreasing, while solar capacity is increasing (the argument is the same as in phase (2)).

The model is completed by specifying that the total quantity of fossil fuel burned is equal to the carbon budget:

$$\int_0^{\underline{T}}(x_n(t)+x_d(t))dt + \int_{\underline{T}}^{T_i} x_n(t)dt + \int_{T_i}^{\overline{T}} x_n(t)dt = \overline{X} \tag{38}$$

The global phase diagram is represented on Figure 2, for a given $\lambda(0)$ that reflects the stringency of the climate constraint. The last phase is a saddle path leading to the steady state (μ^*, Y^*). Moving backward from the steady state along the stable branch, date $\overline{T}$ is reached. The relevant path then corresponds to phase (3) where the dynamics of Y and μ are independent and the evolution of μ is driven by λ, until date T_i is reached. From T_i on, the saddle path of phase (2), corresponding to the steady state (μ^*, Y^{**}) is followed until date $\underline{T}$.[11] Before $\underline{T}$ the path followed corresponds to phase (1) where Y and μ are independent, until date 0. We have chosen to represent the case where μ is bell-shaped in this phase. The initial shadow value of solar panels, $\mu(0)$ that we obtain (as a function of $\lambda(0)$) matches the initial condition $Y(0) = Y_0$.

[10] Proof: $u_d((1-a)\overline{\phi}Y)$ is a strictly increasing function of $a \in (0,1)$, from $u_d(\overline{\phi}Y)$ to $+\infty$, and $u_n(ka\overline{\phi}Y)$ is a strictly decreasing function of a, from $+\infty$ to $u_n(k\overline{\phi}Y)$.

[11] Phases (1), (2) and (3) coalesce if there is no loss in the storage technology ($k = 1$).

For a given value of $\lambda(0)$, the joint evolutions of λ and Y trigger the phase switchings, i.e. give dates $\overline{T}$, T_i and $\underline{T}$ (see Eqs. (22), (23) and (30)).

The climate constraint then pins down $\lambda(0)$.

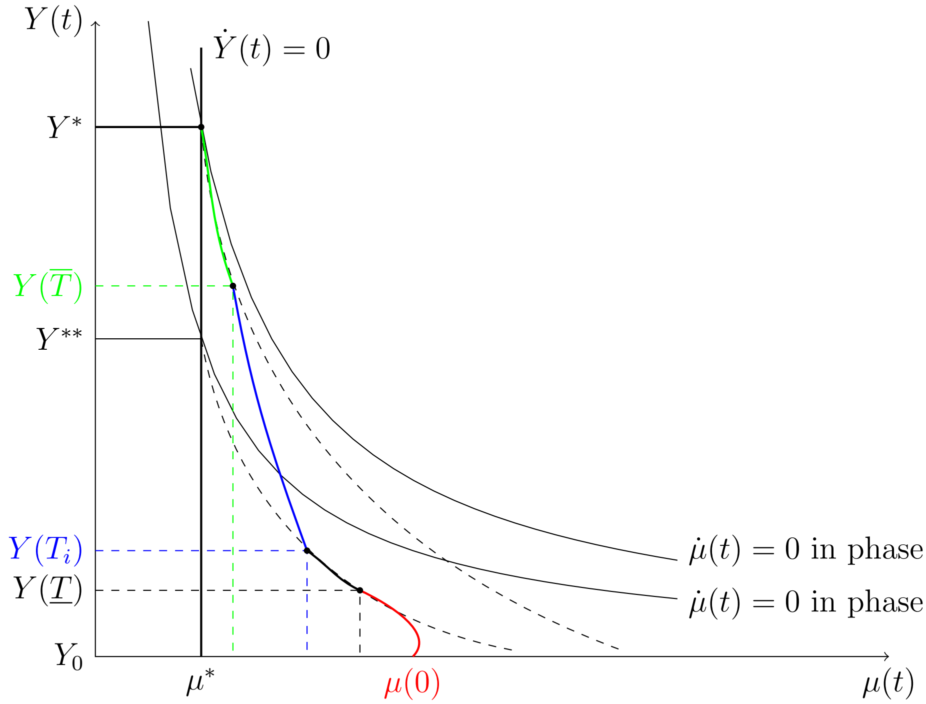

Figure 2: Global phase diagram

The previous results are summarized in the following propositions.

Proposition 1 *In the case where only variability of renewable energy is taken into account and the initial solar capacity is low, the optimal solution consists in 4 phases:*

(1) production of day and night-electricity with fossil fuel-fired power plants complemented at day by solar plants, no storage, investment in solar panels to increase solar capacity (from 0 *to* $\underline{T}$*);*

(2) production of day-electricity with solar plants only, use of fossil fuel-fired power plants at night while proceeding with the building up of solar capacity, no storage (from $\underline{T}$ *to* T_i*);*

(3) production of day-electricity with solar plants only, use of fossil fuel-fired power plants at night while proceeding with the building up of solar capacity, progressive increase of storage (from T_i *to* $\overline{T}$*);*

(4) production of day and night-electricity with solar plants only, storage of electricity at day to be released at night, and investment in solar panels to increase capacity, up to a steady state (from $\overline{T}$ *to* ∞*). This last phase begins when the carbon budget is exhausted.*

Proposition 2 *Investment in solar panels may be always decreasing, or may exhibit a bell shape in the first phase and be decreasing elsewhere, depending on the discount rate, the initial solar capacity, the stringency of climate policy and the efficiency of solar panels.*

Proposition 1 shows that it is always optimal to begin installing solar panels immediately and to use them to complement fossil energy at daytime. There is simultaneous use of fossil fuel and renewable energy from the beginning of the planning horizon.[12] However, it is never optimal to begin storage immediately. Storage would allow saving fossil energy at night, but at the expense of more fossil at day to compensate for the solar electricity stored; it would also cause a physical loss of electricity. Even if the storage technology is available, as it is the case in the model, storage must only begin after fossil has been abandoned at day because the installed solar capacity has become high enough and solar energy accounts for a significant fraction of electricity generation.Then, storing electricity becomes optimal, and storage increases to accompany the penetration of solar in the electric mix. Notice that this timing implies that there is overcapacity of fossil generation during daytime, in the sense that fossil-fuel power plants are maintained for night use only.

The result of Proposition 2 may seem counter-intuitive. It states that whereas solar capacity has to be accumulated from the beginning of the planning horizon, the bulk of the effort of solar capacity building is not necessarily to be made right away. Two forces are at play, explaining the bell shape: discounting, that makes society willing to postpone the effort, and adjustment costs, that makes it willing to smooth the effort over time.

2.3 More results with isoelastic utility and quadratic investment cost

To go further in the characterization of the optimal solution, we proceed by specifying the utility and investment cost functions. These specifications will be used in the remainder of the paper.

For the instantaneous utility function, day and night-electricity consumption are in a first step aggregated with a CES function. Then, in a second step, the utility derived from this consumption bundle is supposed to be isoelastic. Finally, we make the assumption that the intertemporal elasticity of substitution of consumption and the elasticity of substitution between day and night-electricity are the same, and we denote this elasticity σ. Under these assumptions, the instantaneous utility reads:

$$u(e_d, e_n) = \frac{1}{1-\frac{1}{\sigma}} \left(\alpha e_d^{1-\frac{1}{\sigma}} + (1-\alpha) e_n^{1-\frac{1}{\sigma}} \right), \qquad 0 < \alpha < 1, \qquad \sigma > 0, \sigma \neq 1 \tag{39}$$

[12] This is very different from what happens in energy transition models where renewable energy is modeled as a backstop technology.

or

$$u(e_d, e_n) = \alpha \ln e_d + (1-\alpha) \ln e_n, \qquad 0 < \alpha < 1 \tag{40}$$

The investment cost function is classically supposed to be quadratic:

$$C(I) = c_1 I + \frac{c_2}{2} I^2, \qquad c_1, c_2 > 0 \tag{41}$$

We are then able to obtain explicitly from Eqs. (26) and (35) the storage rate in phases (3) and (4):

$$\text{Phase (3)} \qquad a(t) = 1 - \left(\frac{\alpha}{k\lambda(t)}\right)^{\sigma} \frac{1}{\overline{\phi} Y(t)} \tag{42}$$

$$\text{Phase (4)} \qquad a(t) = a^* = a(\overline{T}) = \frac{(1-\alpha)^{\sigma}}{k^{1-\sigma}\alpha^{\sigma} + (1-\alpha)^{\sigma}} \tag{43}$$

The storage rate increases in phase (3), from 0 to its maximal value a^*. It is constant in phase (4) at the level a^*.

Eq. (9) reads:

$$\dot{Y}(t) = \frac{1}{c_2} (\mu(t) - c_1) \tag{44}$$

and the evolution of the shadow value of solar panels in the four phases is (see Eqs. (14), (20), (27), (36) and (51)):

$$\begin{aligned}
&\text{Phase (1)} \qquad \dot{\mu}(t) = \rho\mu(t) - \overline{\phi}\lambda(t) \\
&\text{Phase (2)} \qquad \dot{\mu}(t) = \rho\mu(t) - \alpha\overline{\phi}^{1-\frac{1}{\sigma}} Y(t)^{-\frac{1}{\sigma}} \\
&\text{Phase (3)} \qquad \dot{\mu}(t) = \rho\mu(t) - k\overline{\phi}\lambda(t) \\
&\text{Phase (4)} \qquad \dot{\mu}(t) = \rho\mu(t) - (k\overline{\phi})^{1-\frac{1}{\sigma}} \left(k^{1-\sigma}\alpha^{\sigma} + (1-\alpha)^{\sigma}\right)^{\frac{1}{\sigma}} Y(t)^{-\frac{1}{\sigma}}
\end{aligned} \tag{45}$$

Dates $\underline{T}$, T_i and $\overline{T}$ are given by:

$$\lambda(\underline{T}) = \frac{\alpha}{\left(\overline{\phi} Y(\underline{T})\right)^{\frac{1}{\sigma}}} \tag{46}$$

$$\lambda(T_i) = \frac{\alpha}{k(\overline{\phi} Y(T_i))^{\frac{1}{\sigma}}} \tag{47}$$

$$\lambda(\overline{T}) = \left(\frac{k^{1-\sigma}\alpha^{\sigma} + (1-\alpha)^{\sigma}}{k\overline{\phi} Y(\overline{T})}\right)^{\frac{1}{\sigma}} \tag{48}$$

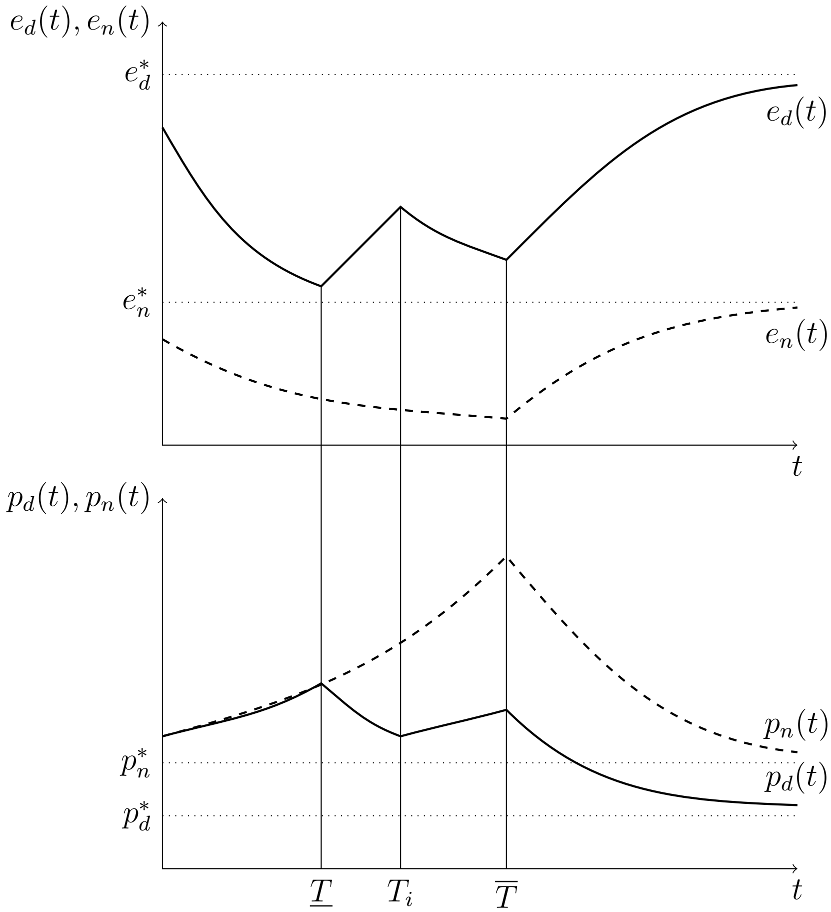

Figure 3: Day and night consumptions and prices, sun at peak

Phase (1)	$x_d(t) = \left(\frac{\alpha}{\lambda(t)}\right)^\sigma - \overline{\phi}Y(t)$ $e_d(t) = \left(\frac{\alpha}{\lambda(t)}\right)^\sigma$ $p_d(t) = \lambda(t)$	$x_n(t) = \left(\frac{1-\alpha}{\lambda(t)}\right)^\sigma$ $e_n(t) = \left(\frac{1-\alpha}{\lambda(t)}\right)^\sigma$ $p_n(t) = \lambda(t)$	$a(t) = 0$
Phase (2)	$x_d(t) = 0$ $e_d(t) = \overline{\phi}Y(t)$ $p_d(t) = \frac{\alpha}{(\overline{\phi}Y(t))^{\frac{1}{\sigma}}}$	$x_n(t) = \left(\frac{1-\alpha}{\lambda(t)}\right)^\sigma$ $e_n(t) = \left(\frac{1-\alpha}{\lambda(t)}\right)^\sigma$ $p_n(t) = \lambda(t)$	$a(t) = 0$
Phase (3)	$x_d(t) = 0$ $e_d(t) = \left(\frac{\alpha}{k\lambda(t)}\right)^\sigma$ $p_d(t) = k\lambda(t)$	$x_n(t) = \frac{k^{1-\sigma}\alpha^\sigma+(1-\alpha)^\sigma}{\lambda(t)^\sigma} - k\overline{\phi}Y(t)$ $e_n(t) = \left(\frac{1-\alpha}{\lambda(t)}\right)^\sigma$ $p_n(t) = \lambda(t)$	$a(t) = 1 - \left(\frac{\alpha}{k\lambda(t)}\right)^\sigma \frac{1}{\overline{\phi}Y(t)}$
Phase (4)	$x_d(t) = 0$ $e_d(t) = (1-a^*)\overline{\phi}Y(t)$ $p_d(t) = \frac{\alpha}{((1-a^*)\overline{\phi}Y(t))^{\frac{1}{\sigma}}}$	$x_n(t) = 0$ $e_n(t) = a^*k\overline{\phi}Y(t)$ $p_n(t) = \frac{1-\alpha}{(a^*k\overline{\phi}Y(t))^{\frac{1}{\sigma}}}$	$a^* = \frac{(1-\alpha)^\sigma}{k^{1-\sigma}\alpha^\sigma+(1-\alpha)^\sigma}$

Table 1: Day and night consumptions, prices and storage rate in the variability case

Finally, fossil fuel use, storage, total electricity consumption and prices[13] in each phase are given in Table 1. Using these results, it is possible to compute the share of solar electricity in the electric mix that triggers storage:

$$\frac{\overline{\phi}Y(T_i)}{e_d(T_i)+e_n(T_i)} = \frac{\overline{\phi}Y(T_i)}{\overline{\phi}Y(T_i)+x_n(T_i)} = \frac{\alpha^\sigma}{\alpha^\sigma+((1-\alpha)k)^\sigma} \tag{49}$$

It is all the higher since the preference for day-electricity is high and the storage technology inefficient.

In phase (1), day and night-electricity consumptions and prices are only driven by the carbon value, that is by climate policy. The reason is that we have assumed away variable costs, of extraction for coal, of production for solar. The only cost that remains is the social cost of carbon. Consumptions decrease whereas the electricity price, the same at day and night, increases over time, following the Hotelling rule. In phase (2), night-electricity consumption is still driven by the carbon value only, and its price increases, but during daytime, electricity consumption, of solar origin, only depends on installed solar capacity. Then day-electricity consumption increases and its price decreases over time. In this phase, day and night-electricity consumptions and prices are decoupled. Things are very different in phase (3). Now, storage creates a link between day and night-electricity consumptions, which are not decoupled anymore. The storage rate increases in this phase from 0 to its maximum value a^*. Due to storage, it is again only the climate constraint that determines electricity consumption and price at each period. The price at day is only a fraction k

[13]"Price" is used here as a shortcut for marginal utility of consumption.

of the price at night. Both increase, following the Hotelling rule. In phase (4), when electricity production is totally carbon-free, electricity consumptions and prices only depend on installed solar capacity, and prices decrease. The storage rate is constant. To sum up, day-electricity consumption is W-shaped, and its price inverted W-shaped, whereas night-electricity consumption first decreases and then increases, while its price follows the opposite pattern (see Figure 3).

We now examine more precisely the long term of this economy, when the transition to clean energy has been achieved.

Proposition 3 *With our specifications, solar capacity at the steady state is:*

$$Y^* = \frac{1}{\rho c_1}, \quad \sigma = 1, \qquad \textit{or} \quad Y^* = \frac{k^{1-\sigma}\alpha^\sigma + (1-\alpha)^\sigma}{(k\bar{\phi})^{1-\sigma}(\rho c_1)^\sigma}, \quad \sigma < 1 \tag{50}$$

The storage rate at the steady state is:

$$a^* = 1-\alpha, \quad \sigma = 1, \qquad \textit{or} \quad a^* = \frac{(1-\alpha)^\sigma}{k^{1-\sigma}\alpha^\sigma + (1-\alpha)^\sigma} \quad \sigma < 1 \tag{51}$$

Day and night-electricity consumptions are respectively $e_d^* = \left(\frac{\alpha\bar{\phi}}{\rho c_1}\right)^\sigma$ *and* $e_n^* = \left(\frac{(1-\alpha)k\bar{\phi}}{\rho c_1}\right)^\sigma$*.*

The price of day-electricity is $p_d^* = \rho c_1/\bar{\phi}$*, and the price of night-electricity is* $p_n^* = p_d^*/k$*.*

Proof. These long term values are easily deduced from Table (1). ■

The long term solar capacity is all the higher since the discount rate and the marginal investment cost are low and, for $\sigma < 1$, since the efficiency of the solar panels and storage technologies are poor. The impatience and cost effects are conform to intuition. The effect of the efficiency of the technology is a compensation effect: more capacity is needed to obtain the same amount of day and night-electricity with less efficient technologies. The role played by the preference parameter is less straightforward. It is necessary to consider simultaneously the effect of a change in this parameter on long run electricity consumption, storage and solar capacity. We see easily that e_d^* is an increasing function of α, e_n^* a decreasing function of α, and $e_d^* > e_n^* \iff \alpha > (1-\alpha)k$, that is if and only if the preference for day-electricity is higher than the preference for night-electricity corrected for the loss due to storage. On the other hand, a^* is always a decreasing function of α. Then, when α increases, less solar capacity is required to ensure that consumption increases at day; but on the other hand more solar capacity may be necessary to ensure that consumption does not decrease too much at night. Overall, solar capacity decreases (resp. increases) if and only if $\alpha >$ (resp. $<$) $(1-\alpha)k$, that is solar capacity decreases if consumption at day is higher than consumption at night. It is in particular the case when sun is at peak ($\alpha > 1/2$).

On the contrary, when sun is off-peak and the storage technology efficient enough, a higher preference for day-electricity leads to increase long run solar capacity.

The link between the long run storage rate and the preference parameter deserves some additional comments. If $\alpha > 1/2$, consumers prefer consuming at day, when there is sun. This means that peak time consumption coincides with the availability of solar electricity. This is obviously the most favorable case for solar penetration, and in this case the storage rate is smaller than $1/(1 + k^{1-\sigma})$. If on the contrary $\alpha < 1/2$, sun is shining at off-peak time, which requires to store in the long run more than half the electricity produced at day. This would correspond to the case that gives rise to the Californian duck documented by CAISO[14] and recently analyzed by Fowlie[15] and Wolfram:[16] in California, there has been more and more solar generation during day in the recent years, while consumption is mainly in the evening, meaning that sun shines off-peak. These dynamics generate a daily net generation (i.e. electricity generation net of electricity consumption) profile that evolves to look like a duck. What we show here is that in addition off-peak sun implies the need of large storage capacities, corresponding to more than half the daily production of electricity.

2.4 Comparative statics

We now perform some comparative statics exercises to complete our analysis of the energy transition under variability of solar generation. Whereas the model is too complex to allow for a full analytical analysis, some results may be obtained by direct inspection of the equations in Table 1. In addition, we will provide some conjectures that will need to be checked numerically.

Let us first examine the effect of a more stringent climate policy.

Our first conjecture is that strengthening climate policy translates in a higher initial carbon value $\lambda(0)$.[17] Then, fossil and total energy consumptions at day and night are smaller, and prices are higher. Fossil is dropped at day earlier.[18] Storage occurs earlier, and the switch to clean energy is brought forward.[19]

[14] "What the duck curve tells us about managing a green grid", `https://www.caiso.com/Documents/FlexibleResourcesHelpRenewables\_FastFacts.pdf`

[15] See "The duck has landed", the Energy Institute Blog, `https://energyathaas.wordpress.com/2016/05/02/the-duck-has-landed/`

[16] See "What's the Point of an Electricity Storage Mandate?" `https://energyathaas.wordpress.com/2013/07/29/whats-the-point-of-an-electricity-storage-mandate/`

[17] Even if we are unable to compute analytically $\lambda(0)$ to make sure that this conjecture is true, the very meaning of the carbon value, that is the shadow price of the climate constraint, indicates that it is the case.

[18] Proof: According to Eq. (46), $Y(\underline{T})^{\frac{1}{\sigma}} e^{\rho \underline{T}} = \alpha/(\overline{\phi}\lambda(0))$; as the left-hand side member of this equation is an increasing function of $\underline{T}$, a larger $\lambda(0)$ translates in a smaller $\overline{T}$.

[19] Proof along the same line as above, using Eqs. (47) and (48).

Our second conjecture is that the shadow value of solar panels $\mu(0)$ increases as well. Indeed, with a smaller carbon budget, solar capacity becomes more valuable. If this conjecture is correct, investment in solar panels is higher and solar capacity is higher in the short/medium run. But long term solar capacity as well as storage are unaffected by climate policy, as soon as there actually exists a climate policy (Proposition 3). We may conclude that climate policy has a timing effect on solar capacity accumulation, but not a permanent effect.

Let us now turn to the effect of a decrease in the investment cost. We have seen (Proposition 3) that it leads to higher long run solar capacity and day and night-electricity consumptions, at lower prices. Contrary to a direct climate policy, the effects of a subsidy to solar are permanent. The relevant question now is: is there a Green Paradox? Answering this question analytically is out of reach because it is not possible to compute $\lambda(0)$. Intuition has little to offer either. The question remains open.

Finally, let us examine the effects of technological improvements. With more efficient solar panels, the abandonment of coal at day is brought forward, as well as the beginning of storage and the switch to clean energy. The long term solar capacity is smaller because of the compensation effect. The effects are similar with a more efficient storage technology, except that the date at which fossil is abandoned at day is unchanged.

3 Accounting for intermittency

3.1 Variability, intermittency and storage

We now account for the fact that solar electricity generation is not only variable but *intermittent* as well, i.e. some of its variations are not predictable. During the day, solar radiations can be fully harnessed if there is sun, but can only be partially harnessed if there are clouds. Very simply, we suppose that during the day the weather is sunny with a probability q and solar panels are then producing electricity at full capacity Y, and that with a probability $(1-q)$ the weather is cloudy and solar panel only produce ϕY electricity with $0 < \phi < 1$.[20] With intermittency, the sequences of storage, solar panel accumulation and electricity consumptions are decided at the beginning of the program, accounting for the fact that weather will be uncertain. e^u denotes electricity consumption when the sun is shining,

[20]To ensure that comparisons can be made between this model and the one with variability only, we impose in the variability only case $\overline{\phi} = q + (1-q)\phi$: the efficiency of solar panel in the variability only case is equal to their expected efficiency in the case where intermittency is taken into account.

while it is noted e^l when there are clouds. The social planner's programme becomes:

$$\max \int_0^\infty e^{-\rho t} \left[qu\left(e_d^u(t), e_n^u(t)\right) + (1-q)u\left(e_d^l(t), e_n^l(t)\right) - C(I(t))\right] dt \tag{52}$$
$$e_d^u(t) = x_d(t) + (1-a(t))Y(t), \quad e_d^l(t) = x_d(t) + (1-a(t))\phi Y(t)$$
$$e_n^u(t) = x_n(t) + ka(t)Y(t), \quad e_n^l(t) = x_n(t) + ka(t)\phi Y(t)$$
$$\dot{X}(t) = x_d(t) + x_n(t)$$
$$\dot{Y}(t) = I(t)$$
$$0 \leq a(t) \leq 1$$
$$X(t) \leq \overline{X}$$
$$x_d(t) \geq 0, x_n \geq 0,$$
$$X_0 \geq 0, \quad Y_0 \geq 0 \text{ given}$$

Obtaining a complete analytical solution of the model is now beyond reach. Nevertheless, we were able to obtain a number of analytical results with an isoelastic utility function and a quadratic investment cost. They are gathered in Proposition 4, Corollary 5 and Proposition 6.

Let us define $\tilde{\phi}$ as the real positive and smaller than 1 root of the following equation:

$$\phi^{1+\frac{1}{\sigma}} - \frac{1-kq}{k(1-q)}\phi^{\frac{1}{\sigma}} - \frac{1-k(1-q)}{kq}\phi + 1 = 0 \tag{53}$$

The characteristics of the optimal solution are then the following.

Proposition 4 *When the intermittency of renewable energy is taken into account and the initial solar capacity is low, there exist two different types of optimal solution:*

(1) For $\phi > \tilde{\phi}$ defined in equation (53), the optimal solution under intermittency and variability exhibits the same succession of phases as under variability only;

(2) For $\phi < \tilde{\phi}$, the optimal solution under intermittency and variability significantly differs from the one under variability only: storage optimally begins before fossil has been abandoned at day.

Proof. See Appendix A. ■

Proposition 4 shows that two very different solutions are possible.

If ϕ is high enough, the optimal solution when intermittency is taken into account is qualitatively the same as when it is not. The planner does not make a big mistake by ignoring intermittency. At the limit, when $\phi \to 1$, we obtain exactly the variability-only solution for $\overline{\phi} = 1$.

If ϕ is small we get a very different solution, characterized by a reluctance of the planner to abandon fossil at day in case of the occurrence of the bad event, which is here actually very bad.

Intuition runs as follows. There are two different methods to satisfy night-electricity demand under a climate constraint that prevents to use as much fossil as desired. The first one is to abandon fossil at day to "save" fossil for night when solar capacity is high enough to ensure that day-electricity needs are satisfied. The second one is to transfer electricity from day to night through storage, at the expense of a loss. With variability only, and with intermittency characterized by a small cloud problem (ϕ is high, corresponding to case (1) in Proposition 4), it is optimal to use the first method first, that is to begin storage after fossil has been abandoned at day, to avoid incurring the loss. With intermittency characterized by a severe cloud problem (ϕ is low, corresponding to case (2) in Proposition 4), the second method is used first, in spite of the loss. Fossil is abandoned at day later, to make sure that, in the case of no or few sun, day-electricity consumption can be satisfied. To compensate for the smaller quantity of fossil left available for night, a part of day-electricity production by solar panels is stored.

It is possible to study more precisely the determinants of the cutoff value $\tilde{\phi}$.

Corollary 5 *$\tilde{\phi}$ is an increasing function of k, meaning that the more efficient the storage technology is, the smaller is the range of ϕs for which intermittency may be safely ignored. $\tilde{\phi}$ is a symmetric and concave function of q, the no-clouds probability, tending to zero when q tends to 0 or 1, and reaching its maximum value for $q = 1/2$.*

Proof. See Appendix A. ■

The interpretation of the first result is the following: the more efficient the storage technology is, the less costly it is to take intermittency into account, therefore the larger the range of ϕs for which the planner will adapt its decisions accordingly. The intuition for the second result is that when q is very close to 0 or to 1, there is actually little uncertainty; then intermittency does not matter a lot, and the range of ϕs for which the planner can ignore it is large.

Finally, the following proposition characterizes the long run of the economy.

Proposition 6 *Solar capacity at the steady state is:*

$$Y_i^* = \frac{\left(q + (1-q)\phi^{1-\frac{1}{\sigma}}\right)^{\sigma} \left((1-\alpha)^{\sigma} + k^{1-\sigma}\alpha^{\sigma}\right)}{k^{1-\sigma}(\rho c_1)^{\sigma}}$$

for $\phi > 0$. When $\sigma = 1$, it is the same as when intermittency is ignored. When $\sigma < 1$, it is higher: the planner ignoring intermittency underinvests in solar capacity in the long run. In the extreme case where $\phi = 0$, it becomes:

$$Y^*_{i,0} = \frac{q^\sigma \left((1-\alpha)^\sigma + k^{1-\sigma}\alpha^\sigma\right)}{k^{1-\sigma}(\rho c_1)^\sigma} < Y^*_i$$

Proof. See Appendix A, and B for the case $\phi = 0$. ■

Taking into account intermittency leads to invest more in solar capacity in the long run ($Y^*_i \geq Y^*$), except in the limit case where there is no sun at all at day with a strictly positive probability. Indeed, we have $Y^*_{i,0} = q^\sigma k^{1-\sigma} Y^* < Y^*$. Then, the optimal long run solar capacity is all the smallest since this probability is large. The intuition supporting this result is that in this case, it is not possible to abandon fossil at day completely –fossil consumption tends to zero asymptotically, and less solar capacity is required.

3.2 Dealing with intermittency without storage

As storage is suspected to play a major role in tackling variability and intermittency, we appraise its importance by comparing the general model with variability, intermittency and storage analyzed above with a model with variability, intermittency but no storage possibilities.

In the case with variability, intermittency but no storage, the social planner program reads:

$$\begin{aligned}
&\max \int_0^\infty e^{-\rho t}\left[qu\left(e^u_d(t), e_n(t)\right) + (1-q)u\left(e^l_d(t), e_n(t)\right) - C(I(t))\right] dt \\
&e^u_d(t) = x_d(t) + Y(t) \\
&e^l_d(t) = x_d(t) + \phi Y(t) \\
&e_n(t) = x_n(t) \\
&\dot{X}(t) = x_d(t) + x_n(t) \\
&\dot{Y}(t) = I(t) \\
&X(t) \leq \overline{X} \\
&x_d(t) \geq 0, x_n(t) \geq 0 \\
&\quad X_0 \geq 0,\ Y_0 \geq 0 \text{ given}
\end{aligned}$$

We show in Appendix C that only two phases appear when $0 < \phi < 1$. In phase (1), coal is used night and day, whereas it is used only during night in phase (2). The equations characterizing these two phases are formally the same as the equations characterizing the phases with fossil night and day, no storage, and fossil night only, no storage in the general model with a storage technology. But now phase (2) lasts forever: it is never possible to abandon fossil completely at night as there exists no mean to transfer daytime solar generation towards the night. Therefore electricity generation is never carbon-free and night consumption (that is equal to fossil fuels use) asymptotically tends towards zero as it is driven by the climate constraint.

Phase (2) is a saddle path leading to a steady state characterized in the following proposition:

Proposition 7 *Absent storage possibilities, solar capacity at the steady state is:*

$$Y_{ns}^* = \left(\frac{\alpha(q + (1-q)\phi^{1-\frac{1}{\sigma}}}{\rho c_1} \right)^{\sigma}$$

We have:

$$\frac{Y_i^*}{Y_{ns}^*} = 1 + \frac{(1-\alpha)^{\sigma}}{k^{1-\sigma}\alpha^{\sigma}} > 1$$

Solar capacity at the steady state is higher with storage: solar electricity generation and storage are complements.

Proof. See Appendix C. ■

Thanks to storage, it is possible to transfer electricity generated at day using solar capacity towards the night. Benefits from solar generation are therefore higher with storage, which explains that long run solar capacity is higher. It is all the higher since the preference for night electricity $(1-\alpha)$ is high and, when $\sigma < 1$, the storage technology inefficient (k small), to compensate for this inefficiency. Notice that having a very inefficient storage technology is drastically different from having no storage technology at all.

The complementarity of solar capacity and storage is not a totally intuitive result. Indeed, without storage, it could have been optimal to invest more in solar to produce more renewable electricity and be able to save coal for the night-electricity generation. In this case, solar capacity and storage would have been substitutes. It may be the case that this effect appears in the short run, but this is impossible to check analytically.

4 Conclusion

This work can be considered as a first step in the study of the energy transition under variability and intermittency of the clean renewable sources. We build a stylized dynamic model allowing to account for important characteristics of the energy transition that have been neglected in the literature so far. In the case where the renewable energy is only variable, we are able to obtain an almost complete analytical resolution of the model — almost in the sense that it is not possible to compute the initial carbon value. The optimal dynamics for the capacity of solar power plants is derived, as well as the optimal amount of electricity stored in time and the timing of the energy transition. In the general case with intermittency, an analytical solution of the model is not possible. Nevertheless, we are able to determine when the solution is very different from the one with variability only, and when it is not, that is when the planner makes a big mistake if he ignores intermittency and when he does not, and to show that solar capacity and storage technology are complements.

The next step should concern numerical simulations, applying the model to real country cases. In addition, a decentralized version of the model would be interesting and challenging as the energy market exhibits several peculiar features, and it would allow designing policy instruments.

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APPENDICES

A Variability and intermittency

The current value Hamiltonian associated to the social planner's programme reads:

$$\begin{aligned}\mathcal{H} = & qu\left(x_d+(1-a)Y, x_n+kaY\right)+(1-q)u\left(x_d+(1-a)\phi Y, x_n+ka\phi Y\right) \\ & -C(I)-\lambda\left(x_d+x_n\right)+\mu I\end{aligned}$$

and the Lagrangian is:

$$\mathcal{L}=\mathcal{H}+\underline{\omega}_a a+\overline{\omega}_a(1-a)+\omega_d x_d+\omega_n x_n+\omega_X\left(\overline{X}-X\right)$$

The first order conditions are:

$$qu_d(x_d+(1-a)Y)+(1-q)u_d(x_d+(1-a)\phi Y)=\lambda-\omega_d \tag{A.1}$$

$$qu_n(x_n+kaY)+(1-q)u_n(x_n+ka\phi Y)=\lambda-\omega_n \tag{A.2}$$

$$\begin{aligned}& qY\left[u_d(x_d+(1-a)Y)-ku_n(x_n+kaY)\right] \\ & \quad +(1-q)\phi Y\left[u_d(x_d+(1-a)\phi Y)-ku_n(x_n+ka\phi Y)\right]=\underline{\omega}_a-\overline{\omega}_a\end{aligned} \tag{A.3}$$

$$C'(I)=\mu \tag{A.4}$$

$$\dot{\lambda}-\rho\lambda=-\omega_X \tag{A.5}$$

$$\begin{aligned}& \dot{\mu}-\rho\mu=-q\left[(1-a)u_d(x_d+(1-a)Y)+kau_n(x_n+kaY)\right] \\ & \quad -(1-q)\phi\left[(1-a)u_d(x_d+(1-a)\phi Y)+kau_n(x_n+ka\phi Y)\right]\end{aligned} \tag{A.6}$$

The complementarity slackness conditions read:

$$\begin{aligned}\underline{\omega}_a a=0, \underline{\omega}_a\geq 0, a\geq 0 \\ \overline{\omega}_a(1-a)=0, \overline{\omega}_a\geq 0, 1-a\geq 0 \\ \omega_d x_d=0, \omega_d\geq 0, x_d\geq 0 \\ \omega_n x_n=0, \omega_n\geq 0, x_n\geq 0 \\ \omega_X\left(\overline{X}-X\right)=0, \omega_X\geq 0, \overline{X}-X\geq 0\end{aligned}$$

Before the ceiling, $X<\overline{X}$ and $\omega_X=0$. Then FOC (A.5) reads $\dot{\lambda}/\lambda=\rho$, i.e.:

$$\lambda(t)=\lambda(0)e^{\rho t} \tag{A.7}$$

Moreover, Eq. (A.4) yields:

$$\dot{Y} = I = C'^{-1}(\mu) \tag{A.8}$$

A.1 Fossil night and day

We have $x_d > 0$, $x_n > 0$, $\omega_d = 0$, $\omega_n = 0$. FOC (A.1) and (A.2) then read:

$$qu_d(x_d + (1-a)Y) + (1-q)u_d(x_d + (1-a)\phi Y) = \lambda \tag{A.9}$$

$$qu_n(x_n + kaY) + (1-q)u_n(x_n + ka\phi Y) = \lambda \tag{A.10}$$

A.1.1 No storage

We have $\underline{\omega}_a > 0$, $\overline{\omega}_a = 0$, $a = 0$. Eqs. (A.9) and (A.10) then read:

$$qu_d(x_d + Y) + (1-q)u_d(x_d + \phi Y) = \lambda \tag{A.11}$$

$$u_n(x_n) = \lambda \tag{A.12}$$

Eq. (A.11) gives implicitly x_d as a function of Y and λ, and Eq. (A.12) yields $x_n = u_n^{-1}(\lambda)$. Eq. (A.6) simplifies into:

$$\dot{\mu} - \rho\mu = -qu_d(x_d + Y) - (1-q)\phi u_d(x_d + \phi Y) \tag{A.13}$$

Eq. (A.3) reads:

$$qY\left[u_d(x_d + Y) - ku_n(x_n)\right] + (1-q)\phi Y\left[u_d(x_d + \phi Y) - ku_n(x_n)\right] \geq 0$$

i.e., using Eq. (A.12):

$$qu_d(x_d + Y) + (1-q)\phi u_d(x_d + \phi Y) \geq k\overline{\phi}\lambda$$

with $\overline{\phi} = q + (1-q)\phi$. Using Eq. (A.11) to replace λ in this last equation we obtain:

$$(1 - k\overline{\phi})qu_d(x_d + Y) \geq (k\overline{\phi} - \phi)(1-q)u_d(x_d + \phi Y) \tag{A.14}$$

Let us now determine the boundaries of this phase. First, with our assumption that initial solar capacity is nil, this phase is necessarily the first one and starts at date 0. Then, at the end of this phase, either $x_d = 0$ ($x_n = 0$ is impossible, since it would require $\lambda \to +\infty$) or Eq. (A.14) is satisfied as an equality, meaning that the next phase will be a phase with positive storage. Therefore the end of this phase is $\underline{T}$ or $\underline{T}_a$ respectively defined by:

$$\lambda(\underline{T}) = qu_d(Y(\underline{T})) + (1-q)u_d(\phi Y(\underline{T})) \text{ and } x_d(\underline{T}) = 0 \tag{A.15}$$

and

$$(1-k\overline{\phi})qu_d(x_d(\underline{T}_a) + Y(\underline{T}_a)) = (k\overline{\phi} - \phi)(1-q)u_d(x_d(\underline{T}_a) + \phi Y(\underline{T}_a))) \tag{A.16}$$

A necessary condition for the existence of $\underline{T}_a$ is:

$$k\overline{\phi} - \phi > 0 \Longleftrightarrow \phi < \frac{1}{1 + \frac{1-k}{qk}} = \tilde{\phi}_0 \tag{A.17}$$

meaning that when ϕ is small enough, reflecting a serious cloud problem, storage must begin before fossil is abandoned at day. See Figure 4.

If this condition is not satisfied the relevant date of end of this phase is $\underline{T}$. It is $\underline{T}$ as well if this condition is satisfied but $\underline{T} < \underline{T}_a$. Comparing directly $\underline{T}$ and $\underline{T}_a$ is out of reach. Nevertheless, we will obtain later on an argument allowing us to determine when the relevant date of end of this phase is $\underline{T}$ and when it is $\underline{T}_a$.

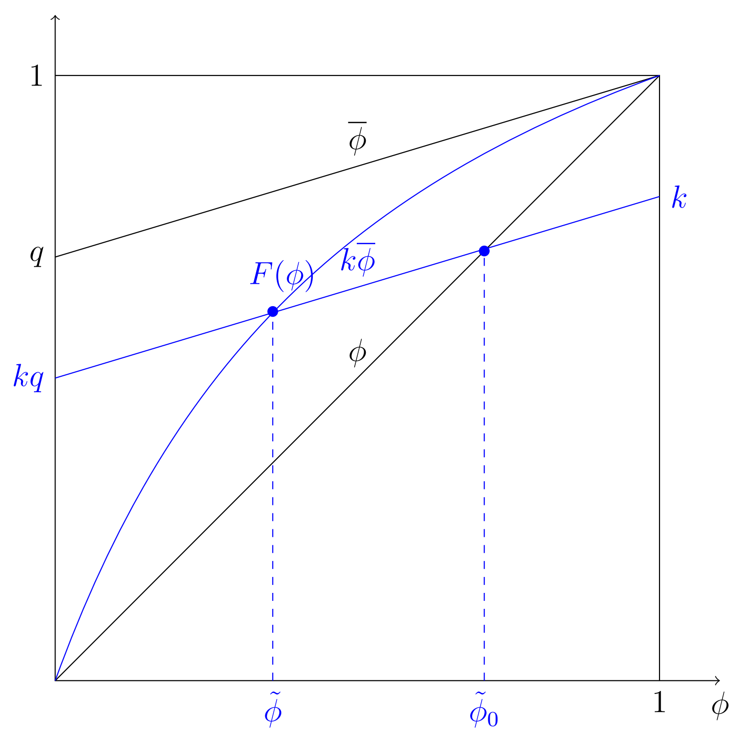

Figure 4: Boundaries of the two possible solutions in the fossil night and day and no storage case

A.1.2 Interior storage

If it exists, this phase starts at $\underline{T}_a$. We have $\omega_d = \omega_n = \underline{\omega}_a = \overline{\omega}_a = 0$, $x_d > 0$, $x_n > 0$ and $0 < a < 1$. FOC (A.3) reads:

$$\begin{aligned} & qY\left[u_d(x_d + (1-a)Y) - ku_n(x_n + kaY)\right] \\ & + (1-q)\phi Y\left[u_d(x_d + (1-a)\phi Y) - ku_n(x_n + ka\phi Y)\right] = 0 \end{aligned} \tag{A.18}$$

Eqs. (A.1), (A.2) and (A.18) allow us to obtain implicitly x_d, x_n and a as functions of Y and λ. Notice that algebraic expressions of these variables are impossible to obtain, even if we specify the utility function as an isoelastic function.

Turning to the boundaries of this phase, we have seen that if it exists it begins with no storage at date $\underline{T}_a$ defined in Eq. (A.16). At the end of this phase (date $\underline{T}_b$) fossil consumption at day stops. From Eq. (A.1), date $\underline{T}_b$ is characterized by:

$$\lambda(\underline{T}_b) = qu_d((1 - a(\underline{T}_b))Y(\underline{T}_b)) + (1-q)u_d((1 - a(\underline{T}_b))\phi Y(\underline{T}_b)) \tag{A.19}$$

A.1.3 Full storage

This case with full storage cannot appear under our assumption that initial solar capacity is nil (note that it is true as well in the variability only case). Indeed, intuitively, full storage is likely to occur if on the one hand the preference for night-electricity is higher that the preference for day electricity that is if agents prefer consuming electricity off-peak, and if on the other hand initial solar capacity is large, so that solar electricity is produced at day in excess of the consumption needs. But it will never be optimal to build solar capacity to obtain this kind of solution at a date other than the initial one, because of the loss of electricity attached to the storage technology.

Relying on these arguments, we exclude in the following the case of full storage.

A.2 Fossil at night only

This or these phases are intermediate. We have here $x_d = 0$, $x_n > 0$, $\omega_d > 0$, $\omega_n = 0$ and the FOC read:

$$u_d((1-a)\underline{\phi}Y) = \lambda - \omega_d \tag{A.20}$$

$$qu_n(x_n + kaY) + (1-q)u_n(x_n + ka\phi Y) = \lambda \tag{A.21}$$

$$\begin{aligned} &qY\left[u_d((1-a)Y) - ku_n(x_n + kaY)\right] \\ &\quad + (1-q)\phi Y\left[u_d((1-a)\phi Y) - ku_n(x_n + ka\phi Y)\right] = \underline{\omega}_a - \overline{\omega}_a \end{aligned} \tag{A.22}$$

$$\begin{aligned} &\dot{\mu} - \rho\mu = -q\left[(1-a)u_d((1-a)Y) + kau_n(x_n + kaY)\right] \\ &\quad - (1-q)\phi\left[(1-a)u_d((1-a)\phi Y) + kau_n(x_n + ka\phi Y)\right] \end{aligned} \tag{A.23}$$

to which we add Eqs. (A.7) and (C).

A.2.1 No storage

In the case of no storage, the FOC become:

$$u_d(\underline{\phi}Y) \leq \lambda \tag{A.24}$$

$$u_n(x_n) = \lambda \tag{A.25}$$

$$qu_d(Y) + (1-q)\phi u_d(\phi Y) \geq k\overline{\phi}\lambda \tag{A.26}$$

$$\dot{\mu} - \rho\mu = -qu_d(Y) - (1-q)\phi u_d(\phi Y) \tag{A.27}$$

Eqs. (C) and (A.27) yield a saddle-path dynamic system in (μ, Y). The steady state values of μ and Y are given by:

$$\mu^* = C'(0) \tag{A.28}$$

$$qu_d(Y_i^{**}) - (1-q)\phi u_d(\phi Y_i^{**}) = \rho C'(0) \tag{A.29}$$

According to (A.24) and (A.26), we must have $qu_d(Y) + (1-q)u_d(\phi Y) \leq \lambda \leq \frac{1}{k\overline{\phi}}[qu_d(Y) + (1-q)\phi u_d(\phi Y)]$. Hence this phase begins at $\underline{T}$ (see Eq. (A.15)), and it ends at T_i defined by:

$$\lambda(T_i) = \frac{1}{k\overline{\phi}}[qu_d(Y(T_i)) + (1-q)\phi u_d(\phi Y(T_i))] \tag{A.30}$$

The existence of this phase requires that $qu_d(Y) + (1-q)u_d(\phi Y) \leq \frac{1}{k\overline{\phi}}[qu_d(Y) + (1-q)\phi u_d(\phi Y)]$, which simplifies into:

$$\frac{qu_d(Y)}{(1-q)u_d(\phi Y)} \geq \frac{k\overline{\phi} - \phi}{1 - k\overline{\phi}}$$

It seems difficult to go further without specifying the utility function. However, with an isoelastic utility function, the terms in Y cancel out and the above condition simplifies into:

$$\frac{q}{1-q}\phi^{\frac{1}{\sigma}} \geq \frac{k\overline{\phi}-\phi}{1-k\overline{\phi}}$$

This condition is equivalent to the following:

$$k\overline{\phi} \leq \frac{(1-q)\phi+q\phi^{\frac{1}{\sigma}}}{(1-q)+q\phi^{\frac{1}{\sigma}}} = F(\phi) \tag{A.31}$$

$F(.)$ is a function of ϕ on the interval $(0,1)$, with $F(0)=0$ and $F(1)=1$. It is increasing: we have $F'(\phi) = (1-q)\left[\frac{1}{\sigma}q\phi^{\frac{1}{\sigma}}\left(\frac{1}{\phi}-1\right)+q\phi^{\frac{1}{\sigma}}+(1-q)\right]/[(1-q)+q\phi^{\frac{1}{\sigma}}]^2 > 0$. It is concave: we easily see that $F(\phi) \geq \phi \quad \forall \phi \in (0,1)$. See Figure 4. Denoting by $\tilde{\phi}$ the value of ϕ for which the condition is satisfied as an equality, it may simply be written as $\phi > \tilde{\phi}$: ϕ must be high enough, meaning that the cloud problem is not too severe. If this condition is not satisfied this phase does not exist. It implies that the phase with fossil night and day and no storage necessarily ends at date $\underline{T}_a$. This solves the problem of the validity of the boundaries $\underline{T}$ and $\underline{T}_a$.

Let us come back to Eq. (A.31) taken as an equality and study how $\tilde{\phi}$ depends on k and q. $\tilde{\phi}$ is solution of:

$$G(\phi) = kq(1-q)\phi^{\frac{1}{\sigma}+1} - q(1-kq)\phi^{\frac{1}{\sigma}} - (1-q)(1-k(1-q))\phi + kq(1-q) = 0$$

We have $G(0)=0$, $G(1)=k-1<0$, and

$$\begin{aligned} G'(\phi) =& \left(\frac{1}{\sigma}+1\right)kq(1-q)\phi^{\frac{1}{\sigma}} - \frac{1}{\sigma}q(1-kq)\phi^{\frac{1}{\sigma}-1} - (1-q)(1-k(1-q)) \\ <&(1-q)\left[\frac{1}{\sigma}q\phi^{\frac{1}{\sigma}-1}(k\phi-1)+kq(\phi^{\frac{1}{\sigma}}-1)-(1-k)\right] < 0 \end{aligned}$$

Therefore, $\tilde{\phi}$ is clearly an increasing function of k: a higher k makes the whole $G(\tilde{\phi})$ curve shift to the right. It is an increasing (resp. decreasing) function of q when $q <$ (resp. $>$)$\frac{1}{2}$: $\frac{\partial G(0)}{\partial q} = k(1-2q) > 0$ for $q < \frac{1}{2}$, < 0 otherwise.

A.2.2 Interior storage

In the case of an interior solution on a, Eq. (A.22) yields:

$$k\left[qu_n(x_n+kaY)+(1-q)\phi u_n(x_n+ka\phi Y)\right] = qu_d((1-a)Y)+(1-q)\phi u_d((1-a)\phi Y) \tag{A.32}$$

Eqs. (A.21) and (A.32) implicitly give a and x_n as functions of Y and λ.

Eq. (A.23) reads:

$$\dot{\mu} - \rho\mu = -\left[qu_d((1-a)Y) + (1-q)\phi u_d((1-a)\phi Y)\right] \tag{A.33}$$

This phase may follow either a phase where fossil fuel is used at night only and there is no storage, or a phase where fossil fuel is used night and day and there is positive storage. In the first case, Eqs. (A.25) and (A.32) show that this phase begins at T_i defined in Eq (A.30). In the second case, Eqs. (A.10) and (A.32) show that this phase begins at $\underline{T}_b$ defined in Eq. (A.19).

The date at which this phase ends is given by the fact that fossil fuel consumption at night becomes nil. Then Eq. (A.32) shows, in the isoelastic case, that $a = \frac{(1-\alpha)^\sigma}{k^{1-\sigma}\alpha^\sigma + (1-\alpha)^\sigma}$ at the end of this phase, and Eq. (A.21) shows that this date is $\overline{T}$ given by:

$$\lambda(\overline{T}) = \left(q + (1-q)\phi^{-\frac{1}{\sigma}}\right)\left(\frac{k^{1-\sigma}\alpha^\sigma + (1-\alpha)^\sigma}{kY(\overline{T})}\right)^{\frac{1}{\sigma}} \tag{A.34}$$

A.3 Fossil at day only

Quite intuitively, this case could occur if initial solar capacity was very large, so that it is optimal to start with full storage at the beginning of the planning horizon, and make storage decrease in time. It cannot occur under our assumption of nil initial solar capacity.

A.4 No fossil

This phase is necessarily the last one, and it always exists. More precisely, it exists as such as soon as $\phi > 0$. For $\phi = 0$ fossil consumption asymptotically decreases towards zero. Moreover, storage is always positive in this phase.

We have $x_d = 0$, $x_n = 0$, $\omega_d > 0$, $\omega_n > 0$, $0 < a < 1$, $\underline{\omega}_a = \overline{\omega}_a = 0$ $\forall t \geq \overline{T}$.

The FOC then read:

$$qu_d((1-a)Y) + (1-q)qu_d((1-a)\phi Y) \leq \lambda \tag{A.35}$$
$$qu_n(kaY) + (1-q)u_n(ka\phi Y) \leq \lambda \tag{A.36}$$
$$qu_d(1-a)Y) + (1-q)\phi u_d((1-a)\phi Y) = k\left(qu_n(kaY) + (1-q)\phi u_n(ka\phi Y)\right) \tag{A.37}$$
$$\dot{\mu} - \rho\mu = -q\left[(1-a)u_d((1-a)Y) + kau_n(kaY)\right] - (1-q)\phi\left[(1-a)u_d((1-a)\phi Y) + kau_n(ka\phi Y)\right] \tag{A.38}$$

to which we add Eqs. (A.7) and (C).

In the isoelastic case, Eq. (A.37) yields:

$$a^* = \frac{(1-\alpha)^\sigma}{k^{1-\sigma}\alpha^\sigma + (1-\alpha)^\sigma} \tag{A.39}$$

There is a constant rate of storage all along this phase, the same as in the variability only model.

In the isoelastic case again, Eq. (A.38) reads, after some computations:

$$\dot{\mu} - \rho\mu = -\left(q + (1-q)\phi^{1-\frac{1}{\sigma}}\right)\left((1-\alpha)^\sigma + k^{1-\sigma}\alpha^\sigma\right)^{\frac{1}{\sigma}} k^{-\frac{1-\sigma}{\sigma}} Y^{-\frac{1}{\sigma}} \tag{A.40}$$

Eqs. (C) and (A.40) yield a saddle-path dynamic system in (μ, Y). The values of μ and Y at the steady state are:

$$\mu^* = c_1 \tag{A.41}$$

$$Y_i^* = \frac{\left(q + (1-q)\phi^{1-\frac{1}{\sigma}}\right)^\sigma \left((1-\alpha)^\sigma + k^{1-\sigma}\alpha^\sigma\right)}{k^{1-\sigma}(\rho c_1)^\sigma} \tag{A.42}$$

The shadow value of solar panels at the steady state is the same whether intermittency is taken into account or not. If $\sigma = 1$ (logarithmic utility function), solar capacity at the steady state is the same as well: $Y_i^* = Y^*$ (see Eq. (50)). If $\sigma < 1$, we have:

$$\frac{Y_i^*}{Y^*} = \frac{(q + (1-q)\phi)^{1-\sigma}}{\left(q + (1-q)\phi^{1-\frac{1}{\sigma}}\right)^{-\sigma}}$$

This ratio is strictly greater than 1 for all $q \in (0,1), \phi \in (0,1)$. Indeed, we have:

$$\left(\frac{Y_i^*}{Y^*}\right)^{-\frac{1}{\sigma}} = \frac{(q + (1-q)\phi)^{1-\frac{1}{\sigma}}}{q + (1-q)\phi^{1-\frac{1}{\sigma}}} \leq 1$$

because the function $x \to x^{1-\frac{1}{\sigma}}$ is convex.

Finally, Eqs. (A.35) and (A.36) imply $\frac{1}{\underline{\phi} Y} \leq \lambda$ and $\frac{1}{k\underline{\phi} Y} \leq \lambda$. The second condition is more stringent than the first one. It shows that this phase begins at date $\overline{T}$, which is also the date at which the ceiling is reached.

A.5 Respect of the carbon budget

The model is completed by specifying that the total quantity of fossil fuel burned cannot exceed the carbon budget. The constraint reads:

$$\overline{X} = \int_0^{\underline{T}} (x_n(t) + x_d(t))dt + \int_{\underline{T}}^{T_i} x_n(t)dt + \int_{T_i}^{\overline{T}} x_n(t)dt \tag{A.43}$$

in the case $\phi > \tilde{\phi}$, and

$$\overline{X} = \int_0^{\underline{T_a}} (x_n(t) + x_d(t))dt + \int_{\underline{T_a}}^{\underline{T_b}} (x_n(t) + x_d(t))dt + \int_{\underline{T_b}}^{\overline{T}} x_n(t)dt \tag{A.44}$$

in the case $\phi < \tilde{\phi}$.

B A polar case: intermittency with no sun at all in the bad case $\phi = 0$

Note that this case is very specific. It does not constitute the limit when ϕ tends to 0 of the $0 < \phi < \tilde{\phi}$ case. It is the only case where fossil fuels use never stops, because there is a chance that solar panels will not be able to produce electricity some days.

B.1 No storage, fossil night and day

With isoelastic utility, Eq. (A.11) reads:

$$q\frac{\alpha}{(x_d + Y)^{\frac{1}{\sigma}}} + (1-q)\frac{\alpha}{x_d^{\frac{1}{\sigma}}} = \lambda \tag{B.1}$$

which gives implicitly x_d as a function of Y and λ. Eq. (A.16) yields:

$$\lambda(\underline{T}_a) = \frac{1}{k}\frac{\alpha}{(x_d(\underline{T}_a) + Y(\underline{T}_a))^{\frac{1}{\sigma}}} \tag{B.2}$$

and Eq. (A.13) reads:

$$\dot{\mu} - \rho\mu = -q\frac{\alpha}{(x_d + Y)^{\frac{1}{\sigma}}} \tag{B.3}$$

B.2 Interior storage, fossil night and day

Eqs. (A.18) and (A.6) respectively read:

$$\frac{\alpha}{(x_d+(1-a)Y)^{\frac{1}{\sigma}}} = k\frac{1-\alpha}{(x_n+kaY)^{\frac{1}{\sigma}}} \tag{B.4}$$

and

$$\dot{\mu}-\rho\mu = -q\left[(1-a)\frac{\alpha}{(x_d+(1-a)Y)^{\frac{1}{\sigma}}} + ka\frac{1-\alpha}{(x_n+kaY)^{\frac{1}{\sigma}}}\right] \tag{B.5}$$

This phase starts with no storage at date $\underline{T}_a$. It never ends: in the long run, x_d and x_n tend asymptotically to zero, and, as shown by Eq. (B.4), a tends asymptotically to a^*.

The asymptotic steady state is characterized by $\mu^* = c_1$ and, according to Eq. (B.5):

$$Y_{i,0}^* = \frac{q^\sigma\left((1-\alpha)^\sigma + k^{1-\sigma}\alpha^\sigma\right)}{k^{1-\sigma}(\rho c_1)^\sigma} \tag{B.6}$$

The long run value of solar panels is the same as when $\phi > 0$, but the long run value of solar capacity is smaller, because solar capacity is less useful when there is a chance that it cannot produce electricity at all and the transition to clean energy lasts forever. Indeed, we have:

$$\frac{Y_i^*}{Y_{i,0}^*} = \left(1+\frac{1-q}{q}\phi^{1-\frac{1}{\sigma}}\right)^\sigma > 1$$

C Variability and intermittency without storage

The current value Hamiltonian associated to the social planner's programme reads:

$$\mathcal{H} = qu\left(x_d+Y,x_n\right)+(1-q)u\left(x_d+\phi Y,x_n\right)-C(I)-\lambda\left(x_d+x_n\right)+\mu I$$

and the Lagrangian is:

$$\mathcal{L} = \mathcal{H}+\underline{\omega}_a a+\omega_d x_d+\omega_n x_n+\omega_X\left(\overline{X}-X\right)$$

The first order conditions are:

$$qu_d(x_d+Y)+(1-q)u_d(x_d+\phi Y)=\lambda-\omega_d \tag{C.1}$$
$$u_n(x_n)=\lambda-\omega_n \tag{C.2}$$
$$C'(I)=\mu \tag{C.3}$$
$$\dot{\lambda}-\rho\lambda=-\omega_X \tag{C.4}$$
$$\dot{\mu}-\rho\mu=-qu_d(x_d+Y)-(1-q)\phi u_d(x_d+\phi Y) \tag{C.5}$$

The complementarity slackness conditions read:

$$\omega_d x_d=0, \omega_d\geq 0, x_d\geq 0$$
$$\omega_n x_n=0, \omega_n\geq 0, x_n\geq 0$$
$$\omega_X\left(\overline{X}-X\right)=0, \omega_X\geq 0, \overline{X}-X\geq 0$$

Before the ceiling, $X<\overline{X}$ and $\omega_X=0$. Then FOC (C.4) reads $\dot{\lambda}/\lambda=\rho$, as in the general model. Eq. (C.3) yields:

$$\dot{Y}=I=C'^{-1}(\mu)$$

also as in the general model.

C.1 Fossil night and day

We have $x_d>0$, $x_n>0$, $\omega_d=0$, $\omega_n=0$. FOC (C.1) and (C.2) then read:

$$qu_d(x_d+Y)+(1-q)u_d(x_d+\phi Y)=\lambda \tag{C.6}$$
$$u_n(x_n)=\lambda \tag{C.7}$$

Eq. (C.6) gives implicitly x_d as a function of Y and λ, and Eq. (C.7) yields $x_n=u_n^{-1}(\lambda)$.

Let us now determine the boundaries of this phase. First, with our assumption that initial solar capacity is nil, this phase is necessarily the first one and starts at date 0. Then, at the end of this phase, fossil stops being used at day. Therefore the end of this phase is date T defined by $x_d(T)=0$. Notice that this equation is formally the same as the equation giving the boundary of the first phase in the case with intermittency and storage (see Eq. (A.15)), but of course the solar capacity is not necessarily the same, so that the dates may be different.

C.2 Fossil at night only

In this second case, the first order conditions reduce to:

$$qu_d(Y) + (1-q)u_d(\phi Y) \leq \lambda \tag{C.8}$$

$$u_n(x_n) = \lambda \tag{C.9}$$

$$\dot{\mu} - \rho\mu = -qu_d(Y) - (1-q)\phi u_d(\phi Y) \tag{C.10}$$

This phase lasts forever: it is not possible to abandon fossil at night absent storage possibilities. It is a saddle-path.

With our specification of the utility and investment cost functions, the steady state is characterized by $\mu^* = c_1$, and:

$$Y^*_{ns} = \left(\frac{\alpha(q + (1-q)\phi^{1-\frac{1}{\sigma}}}{\rho c_1} \right)^{\sigma}$$