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Abstract

Two non-transitive theories to model decision making under risk are regret theory (Loomes and Sugden, 1982, 1987) and salience theory (Bordalo, Gennaioli, and Shleifer, 2012). While the psychological underpinning of these two approaches is different, the models share the assumption that within-state comparisons of outcomes across choice options are a key determinant of choice behavior. We investigate the overlap between these theories and show that original regret theory (Loomes and Sugden, 1982) is a special case of salience theory (Bordalo, Gennaioli, and Shleifer, 2012), which itself is a special case of generalized regret theory (Loomes and Sugden, 1987).

JEL-Codes: D810, D910.

Keywords: choice under risk, regret theory, salience theory.

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1. Introduction

The analysis of decision making under risk is a core topic in microeconomics. The neoclassical workhorse model is *expected utility theory* (EUT), which was first introduced by Bernoulli (1738) and obtained an axiomatic foundation by von Neumann and Morgenstern (1947). Relatively early on, EUT was criticized—most prominently by Allais (1953)—for its failure to predict a significant share of observed individual choices and, thus, also average behavior in a variety of choice situations. The most prominent non-EUT alternative theory for decision making under risk is *prospect theory* (Kahneman and Tversky, 1979), which rests on the three building blocks of probability weighting, reference dependence, and loss aversion. While being able to rationalize many choice anomalies under risk, prospect theory—and in particular probability weighting—was criticized to rely on a set of rather complex and ad hoc assumptions that lack a solid psychological foundation.[1]

With *salience theory*, Bordalo, Gennaioli, and Shleifer (2012) propose a theory for choice under risk that bases the probability weighting process on the psychologically motivated mechanism of salience. Specifically, salience theory posits that a decision maker's attention in a pairwise choice situation is unknowingly drawn to those states of the world in which the respective payoff combination of the two feasible choice options stand out (i.e., are salient). This directional influence on attention is hypothesized to lead to the decision maker placing disproportionally much (little) weight on those states of the world in which the outcomes of the two choice options are perceived as very different (rather similar); i.e., the decision maker over-weighs (under-weighs) the occurrence probability of states with salient (non-salient) outcome combinations. Whether a given state's outcome combination is salient or not is assumed to be determined by the interplay of contrast and level effects, which accounts for the idea that the human perceptional apparatus is attuned to detect differences rather than absolute values and to perceive changes on a log scale (Weber's law).

Salience theory's core assumption that within-state comparisons of outcomes are a key determinant of choice behavior is shared by *regret theory* (Loomes and Sugden, 1982, 1987)—another non-EUT theory for choice under risk.[2] Regarding pairwise choice, regret theory posits that, after uncertainty about the true state of the world has been resolved, the experienced utility derived from receiving the chosen alternative's outcome in that state depends not only on this outcome alone, but also on the outcome that the other

[1] For instance, Loomes and Sugden (1982, p. 817) note that prospect theory as proposed by (Kahneman and Tversky, 1979) requires "a decision weight function which overweights small probabilities, underweights large probabilities, involves subcertainty, subproportionality and subadditivity, and which is discontinuous at both ends". Criticisms of this kind lead to revised and adapted versions of prospect theory (Tversky and Kahneman, 1992; Schmidt, Starmer, and Sugden, 2008), which are surveyed in Wakker (2010).

[2] A similar model was proposed by Bell (1982) and Fishburn (1982).

choice option, which was not chosen, would have yielded in the realized state of the world. If the decision maker had done better (worse) by choosing differently, she suffers from regret (rejoices). The anticipation of these ex post feelings of regret and rejoicing, which arise from within-state comparison of outcomes across choice options, is hypothesized to be factored into ex ante decision making. Specifically, it is presumed that the decision maker has a desire ex ante to avoid ex post feelings of regret. Notably, in contrast to salience theory, regret theory assumes that the decision maker perceives the occurrence probabilities of the different states of the world correctly and without any distortion.

While the respective psychological motivation underlying regret and salience theory is different, both theories have a significant overlap when it comes to their explanatory potential (Loomes and Sugden, 1982, 1983; Bordalo, Gennaioli, and Shleifer, 2012). Both regret theory and salience theory can explain the common consequence effect and the common ratio effect, which go back to Allais (1953).[3] Furthermore, both theories can rationalize the reflection effect identified by Kahneman and Tversky (1979), the preference reversal phenomenon documented by Grether and Plott (1979), as well as the inherent instability of risk attitudes as reflected in the simultaneous preference for gambling and insurance or in the so-called *four-fold pattern* in Tversky and Kahneman (1992). Given this overlap in rationalizable choice patterns, we believe it to be desirable to aim for a thorough analytical comparison of these two theories.

The rest of the paper is structured as follows: After reviewing the related literature in Section 2, we explain the choice situation, regret theory, and salience theory in Section 3. Section 4 and Section 5 contain our main results: While "generalized" regret theory (Loomes and Sugden, 1987) contains salience theory as a special case, salience theory itself contains "original" regret theory (Loomes and Sugden, 1982) as a special case. Section 6 concludes. In Appendix A, we relate our more general salience model to smooth and rank-based salience theory as introduced by Bordalo, Gennaioli, and Shleifer (2012) and we briefly compare the existing approaches that extend regret theory and salience theory beyond pairwise choice.

2. Related Literature

Our paper adds to the extant literature that compares existing theories for choice under risk and uncertainty. An early attempt to distill similarities in how existing theories explain the (common consequence) Allais paradox is made by Sugden (1986), who compares generalized EUT (Machina, 1982), the ratio form (Chew and MacCrimmon, 1979), disappointment theory (Bell, 1985; Loomes and Sugden, 1986), and *regret theory.* Ex-

[3] Both theories predict these effects to be driven by juxtaposition effects rather than by probability effects.

cept for disappointment theory, these theories share the feature that indifference lines in the Machina triangle are "fanning-out". Furthermore, except for regret theory, the aforementioned theories relax the independence axiom from EUT; regret theory, in contrast, relaxes the transitivity axiom. Loomes and Sugden (1987) compare generalized regret theory with skew-symmetric bilinear utility theory (Fishburn, 1982, 1984) and show that for stochastically independent choice options the latter theory is a special case of the former theory. More recently, Masatlioglu and Raymond (2016) show that the intersection of rank-dependent utility (Quiggin, 1982) and quadratic utility (Machina, 1982; Chew, Epstein, Segal, et al., 1991) coincides with expectation-based loss aversion (Kőszegi and Rabin, 2007). Finally, as shown by Gollier (2018), behavior that is seemingly rooted in rank-dependence of preferences can also derive from the aversion to risk of regret such that both rank-dependent utility and regret theory are compatible with a preference for skewed risks in consumption.

As in most of the aforementioned contributions, our comparison of salience theory (Bordalo, Gennaioli, and Shleifer, 2012) with original regret theory (Loomes and Sugden, 1982) and generalized regret theory (Loomes and Sugden, 1987) is based on the utility representation of the theories in question. An alternative way to compare theories is to compare their axiomatic foundations. The first axiomatization of generalized regret theory goes back to Sugden (1993), whereas original regret theory was recently axiomatized by Diecidue and Somasundaram (2017). Notably, both these approaches consider binary preference relations over acts à la Savage (1954). Lanzani (2019) derives a novel axiomatic foundation for general intransitive models under which correlation is relevant and, within this framework, provides a complete set of axioms necessary to derive salience theory. Contrary to the contributions by Sugden (1993) and Diecidue and Somasundaram (2017), the framework in Lanzani (2019) is not based on Savage acts, but builds upon an approach introduced by Fishburn (1991) for multi-attribute choice. As a consequence the axioms provided by Lanzani (2019) cannot be compared directly with the axioms provided by Sugden (1993) and Diecidue and Somasundaram (2017). Importantly, while also showing (albeit indirectly) that salience theory is encompassed by generalized regret theory, Lanzani (2019) neither provides a comprehensive account of how the properties of the two theories relate to each other nor attempts any comparison of salience theory with original regret theory.

The outcomes of a given risky choice option in the different possible states of the world resemble the values of different attributes of an option in riskless multi-attribute choice. Prominent theories for multi-attribute choice under which the preference relation depends only on utility differences within an attribute are Suppes and Winet (1955) and Kőszegi and Szeidl (2013). Our finding that original regret theory is a special case of salience theory for choice under risk suggests that the aforementioned theories for multi-attribute choice are special cases of multi-attribute salience theory (Bordalo, Gennaioli, and Shleifer, 2013), which is highly related to salience theory for choice under risk. While recent comparisons of different theories for multi-attribute choice are provided by Ellis and Masatlioglu (2019) and Landry and Webb (2019), this particular relationship is yet unexplored.

3. Two Context-Dependent Theories for Choice under Risk

3.1. The Choice Situation

Consider a decision maker who faces the choice between two risky choice options L^x and L^y. These choice options, which are acts in the sense of Savage (1954), can be described based on the finite state space $\mathcal{S} = \{1, \dots, S\}$, where the occurrence probability of state s is $p_s \in (0,1)$. The S different states of the world are mutually exclusive such that $\sum_{s=1}^{S} p_s = 1$. Let $P = (p_1, \dots, p_S)$ denote the vector of occurrence probabilities. Act L^i $(i = x, y)$ assigns to each state of the world $s \in S$ a monetary consequence (i.e., an increment or a decrement of the decision maker's wealth). Hence, $L^x = (x_1, \dots, x_S) \in \mathbb{R}^S$ and $L^y = (y_1, \dots, y_S) \in \mathbb{R}^S$. For ease of exposition, we focus on monetary consequences and assume that the decision maker's preferences over *pure consequences* are monotonic: If the decision maker faces an exogenously imposed change of her initial wealth position in which she has no say at all (i.e., there is no decision to make), then the decision maker weakly prefers the exogenous change to be the amount x rather than the amount y if and only if $x \geq y$.[4] We summarize this pairwise choice situation as $\langle \mathcal{S}, P, L^x, L^y \rangle$.

3.2. Regret Theory

Suppose the decision maker chooses act L^x rather than act L^y. If state $s \in \mathcal{S}$ is realized, she obtains outcome x_s. She knows she would have received y_s if she had chosen differently, namely act L^y. According to regret theory, given the choice of act L^x, in state s the decision maker therefore has a composite experience based on obtaining x_s and missing out on y_s. The utility associated with this composite experience based on obtaining x_s and missing out on y_s is denoted by $M(x_s, y_s)$, where $M : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$.

Generalized regret theory (Loomes and Sugden, 1987) postulates that the value assigned to choosing act L^x instead of act L^y is given by

$$V^{RT}(L^x | \langle L^x, L^y \rangle) = \sum_{s=1}^{S} p_s \; M(x_s, y_s). \tag{1}$$

[4] Our findings can be extended to non-monetary outcomes, which requires the existence of a complete, reflexive, and transitive preference ordering on the set of pure consequences.

Thus, the preference relation is described by

$$L^x \succeq L^y \iff \sum_{s=1}^{S} p_s \; \Psi(x_s, y_s) \geq 0, \tag{2}$$

where the bi-variate function $\Psi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is defined as $\Psi(x,y) \equiv M(x,y) - M(y,x)$. Note that $\Psi(\cdot,\cdot)$ is skew-symmetric by construction; i.e., $\Psi(x,y) = -\Psi(y,x)$ for all $x, y \in \mathbb{R}$, which implies $\Psi(x,x) = 0$.

Original regret theory (Loomes and Sugden, 1982) posits that $\Psi(\cdot,\cdot)$ takes the form $\Psi(x,y) \equiv Q(c(x) - c(y))$. Here, the strictly increasing function $c : \mathbb{R} \to \mathbb{R}$ denotes "choiceless utility" and thus $c(x)$ reflects the purely hedonic pleasure experienced from obtaining x without having made a choice that led to obtaining x. Notably, regarding the effect of choiceless utility on choice behavior, only within-state differences of choiceless utilities matter.

Definition 1 (Regret Theory)**.**

(i) The decision maker acts in accordance with generalized regret theory if there is a skew-symmetric function $\Psi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that for any pairwise choice situation $\langle S, P, L^x, L^y \rangle$ the following holds:

$$L^x \succeq L^y \iff \sum_{s=1}^{S} p_s \; \Psi(x_s, y_s) \geq 0. \tag{3}$$

(ii) The decision maker acts in accordance with original regret theory if there is a continuous and strictly increasing function $c : \mathbb{R} \to \mathbb{R}$ and a continuous function $Q : \mathbb{R} \to \mathbb{R}$ such that for any pairwise choice situation $\langle S, P, L^x, L^y \rangle$ the following holds:

$$L^x \succeq L^y \iff \sum_{s=1}^{S} p_s \; Q(c(x_s) - c(y_s)) \geq 0. \tag{4}$$

In order to be consistent with the underlying psychological foundation and to explain certain EUT anomalies, Loomes and Sugden (1987) require the function $\Psi(\cdot,\cdot)$ from generalized regret theory to display the following properties:

(OPC) *Ordering of pure consequences*: For all $x, y \in \mathbb{R}$, $x \geq y$ if and only if $\Psi(x,y) \geq 0$.

(I) *Increasingness:* For all $x, y, z \in \mathbb{R}$, $x \lesseqqgtr y$ if and only if $\Psi(x,z) \lesseqqgtr \Psi(y,z)$.

(C) *Convexity:* For all $x, y, z \in \mathbb{R}$, if $z < y < x$, then $\Psi(x,z) > \Psi(x,y) + \Psi(y,z)$. [5]

[5] As noted by Loomes, Starmer, and Sugden (1991, footnote #6), when considering monetary outcomes, (OPC) and (C) together imply (I). To facilitate comparability with the original contributions, we state the full set of properties as listed in Loomes and Sugden (1987).

Property (OPC) states that obtaining x instead of y is better than obtaining y instead of x if and only if $x \geq y$. According to (I), if x is (weakly) larger than y, then the net advantage of obtaining x instead of z is (weakly) larger than the net advantage of obtaining y instead of z. Thus, if (OPC) and (I) hold, then $\Psi(\cdot,\cdot)$ is strictly increasing in its first argument and, by skew-symmetry, strictly decreasing in its second argument. The behaviorally most important property is (C), which is also often called regret aversion. According to (C), the decision maker is averse to particularly large post-decisional regrets.

For original regret theory with function $Q(\cdot)$ the properties (I) and (C) are replaced by the following conditions:

(I′) *Increasingness:* For all $\Delta \in \mathbb{R}$, $Q(\Delta)$ is strictly increasing.

(C′) *Convexity:* For all $\Delta \in \mathbb{R}_{>0}$, $Q(\Delta)$ is strictly convex.

Property (OPC) is satisfied because $c(\cdot)$ and $Q(\cdot)$ are strictly increasing functions. Notably, (C′) is slightly stricter than (C).

3.3. Salience Theory

The key idea of salience theory is that the decision maker's attention is invariably drawn to states with salient payoff combinations and that this directional influence on attention blurs the perception of the objective occurrence probabilities of the different states of the world. More specifically, it is hypothesized that the objective occurrence probability of a state with a very salient (non-salient) payoff combination is inflated (deflated). When evaluating act L^x, the salience of a state s with payoff combination (x_s, y_s) is denoted by $\sigma(x_s, y_s)$, where the continuous function $\sigma : \mathbb{R} \times \mathbb{R} \to \mathcal{R}_\sigma \subseteq \mathbb{R}$ is the so-called salience function.[6] The function $\sigma(\cdot,\cdot)$ is assumed to be symmetric and to assign the same minimum salience value to all states in which the payoff under L^x coincides with the payoff under L^y; i.e., $\sigma(x,y) = \sigma(y,x)$ for all $x, y \in \mathbb{R}$ and $\sigma(z,z) = \sigma(z',z') < \sigma(x,y)$ for all $x, y, z, z' \in \mathbb{R}$ with $x \neq y$. In the following, $\sigma(\cdot,\cdot)$ is said to satisfy *symmetry* if the former condition holds and *minimal salience* if the latter condition holds.

When evaluating act L^x, the decision weight attached to state s with outcome x_s under act L^x and outcome y_s under act L^y is given by

$$q_s(L^x, L^y) = \frac{f(\sigma(x_s, y_s))}{\sum_{r=1}^{S} f(\sigma(x_r, y_r)) p_r}\, p_s, \tag{5}$$

where $f(\cdot) : \mathcal{R}_\sigma \to \mathbb{R}_{>0}$ is a strictly increasing function. Essentially, the idea is that the objective occurrence probability of state s is inflated (deflated) if the (f-transformed) salience

[6] If $\mathcal{R}_\sigma \subset \mathbb{R}$, then $\sigma(\cdot,\cdot)$ is bounded (as assumed by Bordalo, Gennaioli, and Shleifer (2012)).

of outcome combination (x_s, y_s) is higher (lower) than the probability-weighted average (f-transformed) salience of all possible outcome combinations $(x_1, y_1), \dots, (x_S, y_S)$.

The value that the decision maker attaches to act L^x then is given by

$$V^{ST}(L^x|\langle L^x, L^y\rangle) = \sum_{s=1}^{S} q_s(L^x, L^y)\ v(x_s), \tag{6}$$

where $v : \mathbb{R} \to \mathbb{R}$ is a strictly increasing value function with $v(0) = 0$. Hence, the preference relation is described by

$$L^x \succeq L^y \iff \sum_{s=1}^{S} [q_s(L^x, L^y)\ v(x_s) - q_s(L^y, L^x)\ v(y_s)] \geq 0. \tag{7}$$

Symmetry of the salience function implies that $q_s(L^x, L^y) = q_s(L^y, L^x)$; i.e., the decision weight attached to state s does not depend on which act the decision maker evaluates.

Definition 2 (Salience Theory)**.** *The decision maker acts in accordance with salience theory if there is a strictly increasing function $v : \mathbb{R} \to \mathbb{R}$ with $v(0) = 0$, a strictly increasing function $f : \mathcal{R}_\sigma \to \mathbb{R}_{>0}$, and a continuous function $\sigma : \mathbb{R}\times\mathbb{R} \to \mathcal{R}_\sigma$ that satisfies symmetry and minimal salience such that for any pairwise choice situation $\langle S, P, L^x, L^y\rangle$ the following holds:*

$$L^x \succeq L^y \iff \sum_{s=1}^{S} p_s\ f(\sigma(x_s, y_s))[v(x_s) - v(y_s)] \geq 0. \tag{8}$$

Notably, Definition 2 encompasses the "smooth" salience specification in Bordalo, Gennaioli, and Shleifer (2012) with $f(\sigma) = \delta^{-\sigma}$, where $\delta \in (0, 1)$.

According to Bordalo, Gennaioli, and Shleifer (2012) the salience function $\sigma(\cdot, \cdot)$ displays the following properties:

(O) *Ordering:* For all $x, y, x', y' \in \mathbb{R}$,
if $[\min\{x, y\}, \max\{x, y\}] \subset [\min\{x', y'\}, \max\{x', y'\}]$, then $\sigma(x, y) < \sigma(x', y')$.

(DS) *Diminishing sensitivity:* For all $x, y \in \mathbb{R}_{>0}$ with $x \neq y$, $\sigma(x+\varepsilon, y+\varepsilon) < \sigma(x, y)$ and $\sigma(-(x + \varepsilon), -(y + \varepsilon)) < \sigma(-x, -y)$ for all $\varepsilon > 0$.[7]

(R) *Reflection:* For all $x, y, x', y' \in \mathbb{R}_{>0}$, $\sigma(x, y) < \sigma(x', y')$ if and only if $\sigma(-x, -y) < \sigma(-x', -y')$.

[7] Bordalo, Gennaioli, and Shleifer (2012) assume (O), (DS) and (R) to be satisfied simultaneously and define (DS) only for positive consequences because the corresponding shape of $\sigma(\cdot, \cdot)$ for negative consequences is implied by (R). As we will invoke (DS) without invoking (R), we explicitly include negative consequences into our definition of (DS) to ensure that diminishing sensitivity applies.

The properties (O) and (DS) reflect contrast and level effects, respectively. According to (O), states with higher payoff differences across choice options receive more attention; i.e., the occurrence probabilities of states with a high payoff variation compared to the average outcome in the respective state are inflated. According to (DS), the difference in payoffs across choice options in a given state has a less pronounced effect on the salience of that state the larger the distance of the payoffs to the neutral reference point of zero. As noted by Bordalo, Gennaioli, and Shleifer (2012, p.1250), "[t]he key properties driving our explanation of anomalies are ordering and diminishing sensitivity". Property (R) only plays a role in the explanation of the reflection effect.

4. Generalized Regret Theory and Salience Theory

Theorem 1 below establishes that salience theory is a special case of generalized regret theory in the following sense: If a preference relation allows for a representation by salience theory that satisfies (O), then the same preference relation also allows for a representation by generalized regret theory that satisfies (OPC), (I) and (C). In order to compare the two theories, let the function $\Psi^{ST} : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$\Psi^{ST}(x, y) \equiv f(\sigma(x, y))[v(x) - v(y)] \tag{9}$$

with $v(\cdot)$, $f(\cdot)$ and $\sigma(\cdot, \cdot)$ satisfying the properties listed in Definition 2.

Theorem 1. *Suppose the decision maker behaves according to salience theory with functions $v(\cdot)$, $f(\cdot)$, and $\sigma(\cdot, \cdot)$ where $\sigma(\cdot, \cdot)$ satisfies property (O). Then the decision maker behaves according to generalized regret theory with function $\Psi(\cdot, \cdot) = \Psi^{ST}(\cdot, \cdot)$ where $\Psi(\cdot, \cdot)$ is skew-symmetric and satisfies properties (OPC), (I), and (C).*

Proof of Theorem 1. (i) $\Psi^{ST}(\cdot, \cdot)$ *is skew-symmetric.*

By symmetry of $\sigma(\cdot, \cdot)$,

$$\begin{aligned} \Psi^{ST}(x, y) &= f(\sigma(x, y))[v(x) - v(y)] \\ &= -f(\sigma(y, x))[v(y) - v(x)] = -\Psi^{ST}(y, x). \end{aligned}$$

(ii) $\Psi^{ST}(\cdot, \cdot)$ *satisfies (OPC).*

By the properties of $v(\cdot)$ and $f(\cdot)$,

$$\Psi^{ST}(x, y) \geq 0 \iff f(\sigma(x, y))[v(x) - v(y)] \geq 0 \iff x \geq y.$$

(iii) $\Psi^{ST}(\cdot, \cdot)$ *satisfies (I).*

Regarding the "$\Longrightarrow$" direction, note that

$$\Psi^{ST}(x, z) \gtreqless \Psi^{ST}(y, z) \iff f(\sigma(x, z))[v(x) - v(z)] \gtreqless f(\sigma(y, z))[v(y) - v(z)] \tag{10}$$

for all $x, y, z \in \mathbb{R}$.

If $x = y$, then $\Psi^{ST}(x,z) = \Psi^{ST}(y,z)$ holds trivially.

If $y < x$, we distinguish three cases. First, $z \leq y$. Then $0 \leq v(y)-v(z) < v(x)-v(z)$ because $v(\cdot)$ is strictly increasing. Furthermore, by (O), $\sigma(y,z) < \sigma(x,z)$. As $f(\cdot)$ is strictly positive and strictly increasing, we have $0 < f(\sigma(y,z)) < f(\sigma(x,z))$, such that (10) implies $\Psi^{ST}(x,z) > \Psi^{ST}(y,z)$. Second, $y < z \leq x$. Then $v(y) - v(z) < 0 \leq v(x) - v(z)$ because $v(\cdot)$ is strictly increasing. Furthermore, as $f(\cdot)$ is strictly positive and strictly increasing, $f(\sigma(y,z)) > 0$ and $f(\sigma(x,z)) > 0$, such that (10) implies $\Psi^{ST}(x,z) > \Psi^{ST}(y,z)$. Third, $x < z$. Then $v(y) - v(z) < v(x) - v(z) < 0$ because $v(\cdot)$ is strictly increasing. Furthermore, by (O), $\sigma(x,z) < \sigma(y,z)$. As $f(\cdot)$ is strictly positive and strictly increasing, $0 < f(\sigma(x,z)) < f(\sigma(y,z))$, such that (10) implies $\Psi^{ST}(x,z) > \Psi^{ST}(y,z)$.

If $x < y$, then analogous reasoning reveals that $\Psi^{ST}(x,z) < \Psi^{ST}(y,z)$.

The "$\Longleftarrow$" direction follows by contraposition.

(vi) $\Psi^{ST}(\cdot,\cdot)$ *satisfies (C).*
For any $z < y < x$,

$$\Psi^{ST}(x,z) > \Psi^{ST}(x,y) + \Psi^{ST}(y,z), \tag{11}$$

is equivalent to

$$f(\sigma(x,z))[v(x) - v(z)] > f(\sigma(x,y))[v(x) - v(y)] + f(\sigma(y,z))[v(y) - v(z)]. \tag{12}$$

By (O), together with $f(\cdot)$ being strictly increasing, $f(\sigma(x,z)) > \max\{f(\sigma(x,y)), f(\sigma(y,z))\}$, which implies that (12) holds. Hence, $\Psi^{ST}(\cdot,\cdot)$ satisfying (C) is an implication of $\sigma(\cdot,\cdot)$ satisfying property (O).

□

According to Theorem 1, for any specific functions $v(\cdot)$, $f(\cdot)$, and $\sigma(\cdot,\cdot)$ that satisfies (O), there is a skew-symmetric function $\Psi(\cdot,\cdot)$ that satisfies (OPC), (I) and (C) and that predicts exactly the same behavior as the corresponding generalized regret theory representation in any pairwise choice problem.[8] To establish Theorem 1, only one of the properties that according to Bordalo, Gennaioli, and Shleifer (2012) might be imposed on the salience function $\sigma(\cdot,\cdot)$ is needed—the ordering property. With the convexity property being the behaviorally defining property of generalized regret theory, it is the

[8] For example, a specification of salience theory could prescribe a function $v(\cdot)$ that satisfies the properties of a "typical" value function as stated in Bowman, Minehart, and Rabin (1999), the salience function $\sigma(x,y) = |x-y|/(|x|+|y|+\theta)$ with $\theta > 0$, as proposed by Bordalo, Gennaioli, and Shleifer (2012), and function $f(\sigma) = \delta^{-\sigma}$, which corresponds to smooth salience theory.

ordering property that makes the salience theory representation a generalized regret theory representation and, thus, brings salience theory outside of the expected utility framework.[9]

While following trivially from Theorem 1, it is worthwhile to highlight that salience theory is a special case of generalized regret theory even if the salience function satisfies the entire set of assumptions listed in Bordalo, Gennaioli, and Shleifer (2012).

A yet open question is whether generalized regret theory is also encompassed by salience theory. Given that salience theory imposes diminishing sensitivity on the salience function, one might jump to the conclusion that the answer is "no". The curvature of the value function $v(\cdot)$, however, might neutralize the properties of the salience function $\sigma(\cdot,\cdot)$ such that the bi-variate function $\Psi^{ST}(\cdot,\cdot)$ not necessarily has to display "diminishing sensitivity". Nevertheless, when restricting $v(\cdot)$ to be a typical S-shaped value function that is weakly concave over gains and weakly convex over losses, diminishing sensitivity of the value function and of the salience function work in the same direction such that $\Psi^{ST}(x+\varepsilon, y+\varepsilon) < \Psi^{ST}(x,y)$ for all $0 < y < x$ and $\varepsilon > 0$. As generalized regret theory imposes no such restriction on the bi-variate function $\Psi(\cdot,\cdot)$, in this case salience theory does not encompass generalized regret theory.

5. Original Regret Theory and Salience Theory

According to Bleichrodt and Wakker (2015) the applied theoretical literature on regret theory often relies on the more tractable specification of original regret theory under which $\Psi(x,y) = Q(c(x) - c(y))$. This regret theory specification not only has more structure and therefore more predictive power (i.e., it is easier to falsify) than generalized regret theory, but also disentangles "standard" choiceless utility, as embodied by function $c(\cdot)$, from "behavioral" regret utility, as embodied by function $Q(\cdot)$. The purpose of this section is to compare salience theory with original regret theory.

5.1. Original Regret Theory is a Special Case of Salience Theory

As stated formally in the following Theorem 2, any preference relation allowing for a representation by original regret theory that satisfies (I$'$) and (C$'$) also allows for a representation by salience theory that satisfies (O). Thus, while salience theory is a special case of generalized regret theory (Theorem 1), original regret theory is a special case of salience theory.

Theorem 2. *Suppose the decision maker behaves according to original regret theory with functions $c(\cdot)$ and $Q(\cdot)$ where $Q(\cdot)$ satisfies properties (I$'$) and (C$'$).*

[9] See also Lanzani (2019).

(i) *Then there exist functions $v(\cdot)$, $f(\cdot)$ and $\sigma(\cdot,\cdot)$ such that the decision maker acts in accordance with salience theory with $\sigma(\cdot,\cdot)$ satisfying property (O).*

(ii) *If $c(\cdot)$ is strictly concave on $\mathbb{R}_{>0}$ and strictly convex on $\mathbb{R}_{<0}$, then there exist functions $v(\cdot)$, $f(\cdot)$ and $\sigma(\cdot,\cdot)$ such that the decision maker acts in accordance with salience theory with $\sigma(\cdot,\cdot)$ satisfying properties (O) and (DS).*

(iii) *If $c(\cdot)$ satisfies $c(0) = 0$, is strictly concave on $\mathbb{R}_{>0}$, and, for all $x \in \mathbb{R}_{<0}$, satisfies $c(x) = -\lambda c(-x)$ with $\lambda > 0$, then there exist functions $v(\cdot)$, $f(\cdot)$ and $\sigma(\cdot,\cdot)$ such that the decision maker acts in accordance with salience theory with $\sigma(\cdot,\cdot)$ satisfying properties (O), (DS) and (R).*[10]

Proof of Theorem 2. As a preliminary observation, note that $Q(\cdot)$ being strictly increasing and continuous implies that $Q(\cdot)$ is differentiable almost everywhere such that $\lim_{\Delta\searrow 0} \frac{Q(\Delta)}{\Delta} = \lim_{\Delta\searrow 0} Q'(\Delta)$ exists and is strictly positive.

Now, we prove each part of the statement in turn.

(i) Define $\Delta_{x,y} := c(x) - c(y)$ and consider the following specification of salience theory:

$$v(x) \equiv c(x) - c(0), \quad \forall x \in \mathbb{R},$$

$$\sigma(x, y) \equiv |v(x) - v(y)| = |\Delta_{x,y}|, \quad \forall (x, y) \in \mathbb{R} \times \mathbb{R},$$

and

$$f(\sigma(x, y)) = f(|\Delta_{x,y}|) \equiv \begin{cases} \frac{Q(\Delta_{x,y})}{\Delta_{x,y}} & \text{if } \Delta_{x,y} \in \mathbb{R}_{>0}, \\ \lim_{\Delta\searrow 0} Q'(\Delta) & \text{if } \Delta_{x,y} = 0. \end{cases}$$

First, $v(\cdot)$ is strictly increasing by $c(\cdot)$ being strictly increasing. Furthermore, by construction, $v(0) = 0$.

Second, $\sigma(\cdot,\cdot)$ is continuous by $c(\cdot)$ and the absolute-value function $|\cdot|$ being continuous. Furthermore, $\sigma(\cdot,\cdot)$ is symmetric: $\sigma(x, y) = |v(x) - v(y)| = |v(y) - v(x)| = \sigma(y, x)$. In addition, for all x,y,z,$z' \in \mathbb{R}$ with $x \neq y$, we have $\sigma(z, z) = \sigma(z', z') = 0 < \sigma(x, y)$. Finally, as $\sigma(x, y) = |v(x) - v(y)| = \max\{v(x), v(y)\} - \min\{v(x), v(y)\}$, we have $\sigma(x, y) < \sigma(x', y')$ for all $x, y, x', y' \in \mathbb{R}$, with $[\min\{x, y\}, \max\{x, y\}] \subset [\min\{x', y'\}, \max\{x', y'\}]$ because $v(\cdot)$ is strictly increasing; i.e., $\sigma(\cdot,\cdot)$ satisfies (O).

Third, with $Q(0) = 0$ and $Q(\cdot)$ being continuous, strictly increasing by (I$'$), and strictly convex by (C$'$), $\frac{Q(\Delta)}{\Delta}$ is strictly positive and strictly increasing for all $\Delta \in \mathbb{R}_{>0}$. Hence, as $f(\cdot)$ is continuous on $\mathbb{R}_{\geq 0}$ by construction, $f(\cdot)$ is strictly positive and strictly increasing for all $\sigma \in \mathcal{R}_\sigma = \mathbb{R}_{\geq 0}$.

[10]Strictly spoken, $c(0) = 0$ is implied by continuity of $c(\cdot)$ together with $c(\cdot)$ being a "scaled" skew-symmetric function with scaling factor $\lambda > 0$.

(ii) It remains to show that $\sigma(\cdot,\cdot)$ satisfies (DS).

If $c(\cdot)$ is strictly concave on $\mathbb{R}_{>0}$, then $c(x) - c(y) > c(x+\varepsilon) - c(y+\varepsilon) > 0$ for all $0 < y < x$ and $\varepsilon > 0$. Hence, with $v(\cdot) = c(\cdot) - c(0)$, $|v(x) - v(y)| > |v(x+\varepsilon) - v(y+\varepsilon)|$ and, thus, $\sigma(x,y) > \sigma(x+\varepsilon, y+\varepsilon)$ for all $0 < y < x$ and $\varepsilon > 0$.

Analogously, $c(\cdot)$ being strictly convex on $\mathbb{R}_{<0}$ implies $\sigma(-x,-y) > \sigma(-(x+\varepsilon), -(y+\varepsilon))$ for all $0 < y < x$ and $\varepsilon > 0$.

(iii) It remains to show that $\sigma(\cdot,\cdot)$ satisfies (R).

If $c(z) = -\lambda c(-z)$ for all $z \in \mathbb{R}_{<0}$, where $\lambda \in \mathbb{R}_{>0}$, then $c(0) = 0$ and, as $c(\cdot)$ is strictly increasing, the following holds for all $x, y, x', y' \in \mathbb{R}_{>0}$:

$$
\begin{aligned}
&\sigma(x,y) < \sigma(x',y') \\
&\iff |c(x) - c(y)| < |c(x') - c(y')| \\
&\iff c(\max\{x,y\}) - c(\min\{x,y\}) < c(\max\{x',y'\}) - c(\min\{x',y'\}) \\
&\iff -\frac{1}{\lambda}c(-\max\{x,y\}) - \left[-\frac{1}{\lambda}c(-\min\{x,y\})\right] \\
&\qquad < -\frac{1}{\lambda}c(-\max\{x',y'\}) - \left[-\frac{1}{\lambda}c(-\min\{x',y'\})\right] \\
&\iff -c(\min\{-x,-y\}) + c(\max\{-x,-y\})] \\
&\qquad < -c(\min\{-x',-y'\}) + c(\max\{-x',-y'\})] \\
&\iff |c(-x) - c(-y)| < |c(-x') - c(-y')| \\
&\iff \sigma(-x,-y) < \sigma(-x',-y').
\end{aligned}
$$

□

While for any original regret theory representation there exists a behaviorally equivalent salience theory representation that satisfies the ordering property, this salience theory representation not necessarily satisfies the other properties that Bordalo, Gennaioli, and Shleifer (2012) impose on the salience function. The second key property of salience theory, diminishing sensitivity, is satisfied if the choiceless utility function is S-shaped; i.e., if $c(\cdot)$ is a typical value function. If, in addition, the choiceless utility function is scaled skew-symmetric, then also reflection, holds. Notably, part (iii) of Theorem 2 allows for a value function that is steeper for losses than for gains.

5.2. Salience Theory is not a Special Case of Original Regret Theory

With Theorem 2, we established original regret theory to be a special case of salience theory. Next, by means of an example, we show that salience theory is not encompassed by original regret theory.

Suppose that a decision maker's preference relation can be represented by salience theory with functions

$$v(x) = x, \qquad f(\sigma) = \delta^{-\sigma}, \qquad \sigma(x, y) = \frac{|x - y|}{|x| + |y| + \theta},$$

where $0 < \delta < 1$ and $0 < \theta$; i.e., we have a specification of smooth salience theory with a linear value function and $\sigma(\cdot, \cdot)$ corresponds to the leading example for a salience function in Bordalo, Gennaioli, and Shleifer (2012). Defining $\Delta = x - y$, for $x \geq y$ we obtain

$$\Psi^{ST}(x, y) = \delta^{-\left(\frac{|x-y|}{|x|+|y|+\theta}\right)}(x - y) = \begin{cases} \delta^{-\frac{\Delta}{\Delta+\theta}}\Delta & \text{if} \quad y < 0 < x, \\ \delta^{-\frac{\Delta}{2y+\Delta+\theta}}\Delta & \text{if} \quad 0 \leq y < x. \end{cases} \tag{13}$$

If there were an original regret theory representation of the same preference relation, then there would exist functions $c(\cdot)$ and $Q(\cdot)$ such that $\Psi^{ST}(x, y) = Q(c(x) - c(y))$ for all $x, y \in \mathbb{R}$. For any $y < 0 < x$ and $0 < \varepsilon < |y|$, $\Psi^{ST}(x, y) = \Psi^{ST}(x + \varepsilon, y + \varepsilon)$. If $\Psi^{ST}(x, y) = Q(c(x) - c(y))$ with $Q(\cdot)$ being strictly increasing, then we must have $c(x) - c(y) = c(x + \varepsilon) - c(y + \varepsilon)$, which implies that $c(\cdot)$ is linear. For any $0 < y < x$ and $0 < \varepsilon$, we have $\Psi^{ST}(x, y) > \Psi^{ST}(x + \varepsilon, y + \varepsilon)$. If $\Psi^{ST}(x, y) = Q(c(x) - c(y))$ with $Q(\cdot)$ being strictly increasing, then we must have $c(x) - c(y) > c(x + \varepsilon) - c(y + \varepsilon)$, which implies that $c(\cdot)$ is strictly concave—a contradiction.

Hence, for this particular salience specification there do not exist functions $c(\cdot)$ and $Q(\cdot)$ such that $\Psi^{ST}(x, y) = Q(c(x) - c(y))$ in any pairwise choice situation $\langle S, P, L^x, L^y \rangle$; i.e., it is impossible to find a specification of original regret theory that represents the same preference relation.

5.3. Two Specific Functional Forms

The prominent specification of original regret theory in Bleichrodt, Cillo, and Diecidue (2010) is based on a power regret function of the form

$$Q(\Delta) = \begin{cases} \Delta^r & \text{if } \Delta \geq 0, \\ -(-\Delta)^r & \text{if } \Delta < 0. \end{cases} \tag{14}$$

The parameter $r \in \mathbb{R}_{>0}$ determines the curvature of the function $Q(\cdot)$ and, thus, the decision maker's degree of regret aversion. Clearly, for $Q(\cdot)$ to satisfy (C$'$), we must have $r > 1$.

As an application of Theorem 2, we now identify the specification of salience theory that represents the same preference relation as the above specification of original regret theory.

Proposition 1. *A decision maker acts in accordance with original regret theory with functions $c(\cdot)$ and $Q(\cdot)$ where $Q(\cdot)$ satisfies* (14) *with $r > 1$ if and only if the decision maker acts in accordance with salience theory with functions $v(\cdot)$, $f(\cdot)$ and $\sigma(\cdot,\cdot)$ where $v(x) = c(x) - c(0)$ for all $x \in \mathbb{R}$, $f(\sigma(x,x)) = 0$ for all $x \in \mathbb{R}$, and $\sigma(x,y) = k\ln(|v(x) - v(y)|)$ with $k > 0$ and $f(\sigma) = \delta^{-\sigma}$ with $\delta = e^{\frac{1-r}{k}} \in (0,1)$ for all $x, y \in \mathbb{R}$ with $x \neq y$.*[11]

Proof. Given (14),

$$\Psi(x,y) = \begin{cases} (c(x) - c(y))^r & \text{if } x \geq y, \\ -(c(y) - c(x))^r & \text{if } x < y. \end{cases} \tag{15}$$

Given the salience specification in Proposition 1,

$$\Psi^{ST}(x,y) = \begin{cases} \delta^{-k\ln(|v(x)-v(y)|)}[v(x) - v(y)] & \text{if } x \neq y, \\ 0 & \text{if } x = y. \end{cases} \tag{16}$$

To show that $\Psi^{ST}(x,y) = \Psi(x,y)$ for all $x, y \in \mathbb{R}$, let $\Delta := v(x) - v(y) = c(x) - c(y)$. If $x = y$, then $\Psi(x,y) = \Psi^{ST}(x,y)$ holds irrespective of the value of δ. Next, if $x > y$, then

$$\Psi(x,y) = \Psi^{ST}(x,y) \iff \Delta^r = \delta^{-k\ln(\Delta)}\Delta \iff \delta = e^{\frac{1-r}{k}}.$$

Analogously, if $x < y$, then $\Psi(x,y) = \Psi^{ST}(x,y)$ if and only if $\delta = e^{\frac{1-r}{k}}$. Finally, $0 < e^{\frac{1-r}{k}} < 1$ for all $r > 1$ by the properties of the exponential function. □

According to Proposition 1, an original regret theory representation with power regret function is behaviorally equivalent to a smooth salience specification with a logarithmic salience function. While different from the salience functions that are typically used in the literature that followed in the wake of Bordalo, Gennaioli, and Shleifer (2012), the salience function specified in Proposition 1 allows for a nice interpretation: The salience of a given state—i.e., the extent of the triggered stimuli—does not depend on the difference of numerical outcomes, but on the differences in the utilities associated with these outcomes. The perceived difference in experienced utilities, however, is a logarithmic transformation of actual change in utilities rather than the actual change itself, which is in line with the Weber-Fechner law of psychophysics (Weber, 1835; Fechner, 1858).

Proposition 1 reveals that there exists a specification of the value function and the salience function such that "standard" smooth salience theory is behaviorally equivalent to the prominent specification of original regret theory with a power regret function. Specifically, there is a one-to-one mapping from the degree of local thinking δ (i.e., the salience bias) to the degree of convexity of the regret function r (i.e., the degree of regret

[11] The function $\sigma(x,y) = k\ln(|v(x) - v(y)|)$ is not defined for $x = y$. However, for this specification of $\sigma(\cdot,\cdot)$ we have $\lim_{x\to y} \delta^{-\sigma(x,y)} = 0$ for any $\delta \in (0,1)$. Thus, setting $f(\sigma(x,x)) = 0$ for all $x \in \mathbb{R}$ is the natural continuous extension of the above salience specification to the limit case with $x = y$.

aversion). While the actual values of these two parameters depend on the specifications of other parameters (e.g. on k) and the normalizations of "utilities" (e.g. $c(x)$), the variations of δ and r and whether they are significantly different from 1 (such that predicted behavior significantly diverges from EUT) can be directly compared.[12]

6. Conclusion

In this paper, we investigate the overlap between two prominent context-dependent theories for choice under risk, regret theory and salience theory. Specifically, we show that original regret theory (Loomes and Sugden, 1982) is a special case of salience theory (Bordalo, Gennaioli, and Shleifer, 2012), which itself is a special case of generalized regret theory (Loomes and Sugden, 1987). Furthermore, we establish that salience theory's property of ordering implies the convexity property of generalized regret theory and we identify the restrictions that salience theory's properties of diminishing sensitivity and reflection impose on the choiceless utility function of original regret theory.

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[12] For instance, r is measured by Bleichrodt, Cillo, and Diecidue (2010) and δ is measured by Königsheim, Lukas, and Nöth (2019). Königsheim, Lukas, and Nöth (2019), however, use rank-based salience theory and a different salience function, which is why the comparison of estimates has to be interpreted with caution.

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A. Extensions and Robustness

In this appendix, we first relate the more general salience model introduced in Section 3 to smooth and rank-based salience theory as introduced by Bordalo, Gennaioli, and Shleifer (2012). Thereafter, we briefly compare the existing approaches that extend regret theory and salience theory beyond pairwise choice.

A.1. Smooth Salience Theory and Rank-Based Salience Theory

Bordalo, Gennaioli, and Shleifer (2012, p.1251-1252, 1255) propose two formalizations of how a salience-biased perception translates into decision weights: *smooth salience theory* and *rank-based salience theory.*

Under *smooth salience theory*, the decision maker's preference relation is characterized by

$$L^x \succeq L^y \iff \sum_{s=1}^{S} p_s \delta^{-\sigma(x_s,y_s)}[v(x)-v(y)] \geq 0, \tag{A.1}$$

where $\delta \in (0,1)$ is an (inverse) measure of the strength of the "salience bias". For $\delta \to 1$ the model coincides with EUT. With regard to the notation introduced in Section 3, smooth salience theory corresponds to assuming that $f(\sigma) = \delta^{-\sigma}$.

Under *rank-based salience theory*, the decision maker's preference relation is characterized by

$$L^x \succeq L^y \iff \sum_{s=1}^{S} p_s \delta^{k_s}[v(x)-v(y)] \geq 0, \tag{A.2}$$

where $k_s \in \mathbb{N}_{>0}$ denotes the salience rank of state $s \in \mathcal{S}$. The salience ranking of the states $1,\ldots,S$ starts at 1, has no jumps, and $k_s \lesseqgtr k_{s'}$ if and only if $\sigma(x_s,y_s) \gtreqless \sigma(x_{s'},y_{s'})$. Again, $\delta \in (0,1)$ denotes the (inverse) degree of the salience induced distortion. The advantage of rank-based salience theory is its analytical tractability in choice problems with only few outcome combinations.[13] As, however, already noted by Bordalo, Gennaioli, and Shleifer (2012, p.1255), the discrete nature of rank-based salience theory leads to "states with similar [though not identical] salience obtain[ing] very different weights," which may create discontinuities in valuation. As later formally analyzed by Kontek (2016), these discontinuities in valuation under rank-based salience theory entail that an act's certainty equivalent may not be well defined.

Notably, the salience ranking under rank-based salience theory depends on the entire set of outcome combinations for all S states of the world; i.e., the salience rank assigned to state $s \in \mathcal{S}$ is specific to the probability-independent aspect of the choice problem.

[13]For example, in the analysis of the common consequence effect and the common ratio effect in Bordalo, Gennaioli, and Shleifer (2012), rank-based salience theory (unlike smooth salience theory) allows for a crisp characterization of behavior in terms of the salience parameter δ.

In other words, the salience rank k_s of state s depends on all outcome combinations $(x_1, y_1), \dots, (x_S, y_S)$ and not only on (x_s, y_s) (even though the salience function $\sigma(\cdot, \cdot)$ is a function of (x_s, y_s) alone).[14] In fact, it is this choice-problem specificity that gives rise to the aforementioned conceptual difficulties of rank-based salience theory.

What are the implications for the bi-variate function $\Psi^{ST}(\cdot, \cdot)$ that was used to establish Theorem 1? The salience-induced decision weights $q_1(L^x, L^y), \dots, q_n(L^x, L^y)$ in (5) depend on the specification of the choice problem under both smooth and rank-based salience theory. Under smooth salience theory, however, the associated bi-variate function $\Psi^{ST}(\cdot, \cdot)$ is independent of the specifics of the choice problem. Under rank-based salience theory, in contrast, the function $\Psi^{ST}(\cdot, \cdot)$ depends on the whole probability-independent aspect of the choice problem because the salience ranks of the S possible states depend on it. Thus, while the function $\Psi^{ST}(\cdot, \cdot)$ is universally invariant to variations of the choice situation under smooth salience theory, under rank-based salience theory such invariance prevails only if the variation is restricted to the vector P of occurrence probabilities. With the key insights of salience theory holding under both formalizations, the answer to the question whether to apply smooth or rank-based salience theory is based on trading off analytical convenience and conceptual coherence.

A.2. More than Two Choice Options

Both salience theory and original regret theory belong to the general class of non-transitive theories, i.e., both theories are special cases of generalized regret theory. As pointedly stated by Bleichrodt and Wakker (2015, p.507) "a limitation of regret theory, as of any intransitive theory of binary choice, is that it is unclear how to extend the theory to choices among three or more actions." This conceptual dificulty is the reason why these theories were defined with a rather narrow focus on pairwise choice and have been tested primarily for pairwise choices. Nevertheless, various ideas how to extend these theories beyond pairwise choice are discussed in the extant literature. To convey these ideas concisely, in what follows we consider a decision maker who faces the choice set $\mathcal{C} = \{L^1, \dots, L^n\}$, were $L^i = (x^i_1, \dots, x^i_S)$ with x^i_s denoting the outcome the decision maker receives if she opted for act L^i and state $s \in \mathcal{S}$ is realized.

Regarding regret theory, the applied literature typically follows the approach proposed by Quiggin (1994), who argued that preferences should satisfy *irrelevance of statewise dominated alternatives (ISDA).* The ISDA axiom entails that the outcome received in a given state $s \in \mathcal{S}$ is compared only to the best possible outcome $x^{max}_s := \max\{x^1_s, \dots, x^n_s\}$ in that state, such that the decision maker experiences only (maximum) regret and never

[14] Formally, letting $\Gamma = \langle \mathcal{S}, L^x, L^y \rangle$ denote the probability-independent aspect of the pairwise choice problem under consideration, the salience rank k_s of state s, which results in payoff combination (x_s, y_s), takes the form $k_s = k(\sigma(x_s, y_s)|\Gamma)$.

experiences rejoicing.[15] The decision maker's preference relation under this approach can be described by

$$L^i \succeq L^j \iff \sum_{s=1}^{S} p_s[M(x_s^i, x_s^{max}) - M(x_s^j, x_s^{max})] \geq 0. \tag{A.3}$$

As becomes apparent from (A.3), under the approach proposed by Quiggin (1994) the left-hand side of the inequality that governs the decision maker's choice corresponds to a probability-weighted sum of the values of a function that depends only on consequences.

This latter feature is not necessarily shared by the approach which was already proposed in the seminal contribution by Loomes and Sugden (1982). Under this alternative approach it is hypothesized that the decision maker forms action weights $a_1, \ldots, a_n$ with $a_i > 0$ for all $i = 1, \ldots, n$ and $\sum_{i=1}^n a_i = 1$, which enter the evaluation of act L^i in the context of the choice set $\mathcal{C}$ as follows: $V(L^i|\mathcal{C}) = \sum_{k \in \{1,\ldots,n\} \setminus \{i\}} \frac{a_k}{1-a_i} V^{RT}(L^i|\langle L^i, L^k \rangle)$, where $V^{RT}(\cdot|\langle \cdot, \cdot \rangle)$ is defined in (1). Thus, the evaluation of act L^i in the choice context $\mathcal{C}$ corresponds to the action-weight-weighted sum of the evaluation of act L^i in the $n-1$ pairwise choices between act L^i and any other feasible act $L^j \in \mathcal{C} \setminus \{L^i\}$. The preference relation of the decision maker under this approach can be described as follows:

$$\begin{aligned} L^i \succeq L^j \iff \sum_{s=1}^{S} p_s \Bigg[& \sum_{k \in \{1,\ldots,n\} \setminus \{i\}} \frac{a_k}{1-a_i} M(x_s^i, x_s^k) \\ & - \sum_{k \in \{1,\ldots,n\} \setminus \{j\}} \frac{a_k}{1-a_j} M(x_s^j, x_s^k) \Bigg] \geq 0. \end{aligned} \tag{A.4}$$

Loomes and Sugden (1982, p.816) "hope[d] in the future to formulate a theory of action weights" within the framework of regret theory, but, to the best of our knowledge, such a theory never came to be. In the simplest possible case with identical action weights for all acts (i.e., $a_i = a_j = 1/n$ for all $i, j = 1, \ldots, n$), the left-hand side of the inequality that governs the decision maker's choice again corresponds to a probability-weighted sum of the values of a function that depends only on consequences—as under the approach proposed by Quiggin (1994). The same holds true if action weights depend only on the consequences of the acts in choice set $\mathcal{C}$. If, in contrast, action weights also depend on the occurrence probabilities of the states in state space $\mathcal{S}$, then such a multiplicative separability does not prevail.

In fact, such multiplicative separability typically does not prevail in the approach to extend salience theory beyond pairwise choice proposed in the Web-Appendix in Bordalo, Gennaioli, and Shleifer (2012). When evaluating act L^i, the salience of state s

[15] While appealing from a normative perspective, the ISDA axiom is at odds with evidence (Bordalo, 2011) that documents an *asymmetric dominance decoy effect*, i.e., a shift in demand induced by adding a third "decoy" option to a binary choice set where the decoy option in terms of outcomes is state-wise dominated by one but not the other of the initial choice options. A similar axiom is also defined and applied by Hayashi (2008).

depends on how the outcome x_s^i under act L^i in that state compares to the average $\bar{x}_s^{-i} := \frac{\sum_{k\in\{1,\dots,n\}\setminus\{i\}} x_s^k}{n-1}$ of the outcomes of the other feasible acts in that state. The evaluation of act L^i is given by $V^{ST}(L^i|\mathcal{C}) = \sum_{s=1}^{S} \frac{f(\sigma(x_s^i,\bar{x}_s^{-i}))}{\sum_{r=1}^{S} f(\sigma(x_r^i,\bar{x}_r^{-i}))p_r}\, p_s v(x_s^i)$ such that

$$L^i \succeq L^j \iff \sum_{s=1}^{S} p_s \left[\frac{f(\sigma(x_s^i,\bar{x}_s^{-i}))}{\sum_{r=1}^{S} f(\sigma(x_r^i,\bar{x}_r^{-i}))p_r}\, v(x_s^i) - \frac{f(\sigma(x_s^j,\bar{x}_s^{-j}))}{\sum_{r=1}^{S} f(\sigma(x_r^j,\bar{x}_r^{-j}))p_r}\, v(x_s^j) \right] \geq 0. \quad \text{(A.5)}$$

As becomes apparent from (A.5), under the approach proposed by Bordalo, Gennaioli, and Shleifer (2012), multiplicative separability prevails only if $\sum_{\tau=1}^{S} f(\sigma(x_\tau^i,\bar{x}_\tau^{-i}))p_\tau$ is identical for all acts. While indeed being satisfied for choice sets with only two elements, this condition typically does not hold for larger choice sets. In other words, a peculiarity of this formalization is that the weight the decision maker attaches to a state is act dependent.

With applications of both regret theory and salience theory being mainly confined to binary choices, little is known regarding how well the different approaches outlined above predict choice behavior from richer choice sets. Furthermore, except for the arguments made by Quiggin (1994), the normative implications of salience theory and regret theory for choices among more than two options are under-explored. Therefore, due to a lack of theoretical and empirical guidance, at this point regret theory and salience theory cannot be easily compared regarding choice between three or more acts.