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Vincent Martinet, Pedro Gajardo, Michel De Lara

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Abstract

We frame sustainability problems as bargaining problems among stakeholders who have to agree on a common development path. For infinite alternatives, the set of feasible payoffs is unknown, limiting the possibility to apply classical bargaining theory and mechanisms. We define a framework accounting for the *economic environment*, which specifies how the set of alternatives and payoff structure underlie the set of feasible payoffs and disagreement point. A mechanism satisfying the axioms of *Independence of Non-Efficient Alternatives* and *Independence of Redundant Alternatives* can be applied to a reduced set of alternatives producing all Pareto-efficient outcomes of the initial problem, and produces the same outcome. We exhibit monotonicity conditions under which such a subset of alternatives is computable. We apply our framework to dynamic sustainability problems. We characterize a “satisficing” decision rule achieving any Pareto-efficient outcome. This rule is renegotiation-proof and generates a time-consistent path under the axiom of *Individual Rationality*.

Keywords: social choice, axiomatic bargaining theory, economic environment, monotonicity, dynamics, sustainability, intergenerational equity, maximin.

Vincent Martinet
Université Paris Saclay
Economic Publique, AgroParisTech, INRA
France - 78850 Thiverval-Grignon
vincent.martinet@inra.fr

Pedro Gajardo
Departamento de Matemática
Universidad Técnica Federico Santa María
Avenida España 1680 Casilla 110-V
Valparaíso / Chile
pedro.gajardo@usm.cl

Michel De Lara
Université Paris Est - CERMICS
6-8 avenue Blaise Pascal, Champs-sur-Marne
France - 77455 Marne la Vallée Cedex 2
delara@cermics.enpc.fr

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1 Introduction

Ongoing debates on global changes, such as biodiversity erosion and climate change, illustrate tensions among environmental, social, and economic issues, on the one hand, and between short and long terms, on the other hand. The ambition of sustainability is to tackle such social choice problems and to propose development paths that trade off these issues.

As two sides of the same coin, the axiomatic bargaining theory and welfare economics are two different ways to describe social choice problems and characterize their solutions (Kaneko, 1980). The formalization of the problem differs, however, and brings different views. In welfare economics, an emphasis is put on the welfare function used to rank alternative options. In the axiomatic bargaining approach, an emphasis is put on the set of feasible payoffs for a group of stakeholders and on the characterization through axioms of mechanisms selecting specific solutions.

A striking point is that actual sustainability problems have been largely addressed in welfare economics (e.g., for climate change, refer to Nordhaus, 2007; Stern, 2008), but not in the axiomatic bargaining theory.[1] A reason may be that the classical bargaining theory is mainly based on the set of feasible payoffs, with the usual assumption that this set is given. Thus, it is necessary to characterize this set before applying an axiomatic bargaining approach. This abstract framework may be too restrictive to address complex social choice problems, which do not aim at selecting a payoff vector directly, but an alternative among a set of feasible ones (e.g., decisions to drive the economic development path). The set of feasible payoffs may even be unknown when formulating the social choice problem, as it may be the case for sustainability problems involving complex dynamics.

Bargaining problems on dynamical systems have received little attention.[2] Fershtman (1983) introduced "dynamic bargaining problems" in which stakeholders have to agree on a time path of decisions for the system, with intertemporal payoffs depending on the resulting economic path. In such problems, computing the set of feasible payoffs is a formidable challenge as it would require exploring an infinity of possible sequences of decisions and associated economic dynamics. This constitutes an obstacle to the analysis of these problems in the bargaining approach. The axiomatic bargaining theory can, however, be mobilized to provide interesting insights to the social choice problem of trading off environmental, economic, and other issues over time, in particular when defining a sustainability criterion (i.e., a representation of social preferences for sustainability) is difficult and controversial (Fleurbaey, 2015). Stakeholders can be seen as the carriers of separate environmental, social, and economic issues. The shape of

[1]In the sustainability literature, axioms have been used to define welfare functions (see Asheim, 2010, for a survey). In this framework, economic and environmental issues are first encompassed within the utility of each generation, which are subsequently aggregated by the (axiomatized) intertemporal welfare function to give a present social value to alternative development paths. We adopt a different approach, considering that each issue is represented by a stakeholder, and the bargaining takes place at the current time among stakeholders each having their own intertemporal objective.

[2]In the dynamic framework, attention has been devoted to i) repeated or iterative static bargaining (Abreu, 1988), possibly with an endogenous status quo (e.g., Anesi and Duggan, 2018), ii) the timing of the bargaining protocol and its influence on the timing of payoffs and the cost of delay (e.g., Schweighofer-Kodritsch, 2018), or iii) dynamic games in which each player has a decision parameter (which is not the case when stakeholders bargain over a common set of decisions for a dynamic system). Fleurbaey and Roemer (2011) proposed a dynamic justification of the axiomatic bargaining theory, without accounting for the economic dynamics. These frameworks differ from ours.

the Pareto frontier of the set of feasible payoffs provides important information on the tension among these issues. Axioms can be used to characterize solutions that satisfy meaningful properties.

We lay out a formalism to study bargaining problems over alternatives (and not directly over payoffs), which can be used to study sustainability problems and characterize their Pareto frontier. For this purpose, we proceed in two steps. In Section 2 and 3, we propose a general framework, and related results, to discuss bargaining problems in *economic environments*. In Section 4, we formalize sustainability problems in this framework and discuss bargaining solutions and their time-consistency.

Roemer (1986, 1988) criticized the classical axiomatic bargaining approach as it does not account for the economic environment, that is, the nature of the goods to be shared and the preferences of the stakeholders. A bargaining problem can be enriched by specifying how the set of alternatives being bargained over and the payoff structure underlie the set of feasible payoffs and the disagreement outcome (Roemer, 1988).[3] The central element of the classical bargaining theory, the set of feasible payoffs, must be characterized from the set of feasible alternatives. Thus, we focus on this set of alternatives and the selection of alternatives satisfying some desirable properties.

In the axiomatic bargaining literature, a problem can have different possible solutions, depending on the axioms assumed (Border and Segal, 1997). Our purpose here is not to characterize a particular mechanism but to provide conditions, in the form of axioms, under which a bargaining mechanism can be applied to sets of alternatives. We introduce two axioms specific to our framework, *Independence of Non-Efficient Alternatives* (INEA) and *Independence of Redundant Alternatives* (IRA). We show that a bargaining mechanism satisfying these axioms can be applied to any reduced set of alternatives yielding the Pareto-efficient outcomes of the initial bargaining problem, and produces the same outcome. We formulate monotonicity conditions for economic environments under which such a subset of efficient alternatives can be computed. This provides a practical tool to apply the axiomatic bargaining theory to bargaining problems in economic environments.

Subsequently, we study *dynamic bargaining problems* that can represent sustainability problems.[4] Each sustainability issue (e.g., environmental, social, and economic) is embodied by a stakeholder who aims at sustaining an *ad hoc* indicator over time (e.g., consumption, GDP, employment rate, the atmospheric concentration of green-

[3]In this framework, the problem is not to allocate utility among stakeholders, but commodities (distribution problems). Characterizing a particular mechanism is then relatively more demanding than on the unrestricted domain of outcomes (Roemer, 1988). Nieto (1992) characterized a resource egalitarian solution corresponding to the lexicographic extension of the maximin criterion defined on economic environments. Chen and Maskin (1999) enriched the economic environment of the bargaining problem by considering the possibility of production, focusing on the egalitarian mechanism. We aim at describing a problem in which the bargaining is not over the division of a fixed aggregate endowment of goods, but over the common decisions driving the economy. For this purpose, the whole economic dynamics should be accounted for in the description of the economic environment. As such, our definition of the economic environment slightly differs from that of the literature.

[4]While sustainable development is a natural field of application for the described dynamic bargaining problems, the scope of application is not limited to sustainability or environmental economics. Other bargaining problems may involve stakeholders who want to "sustain" something over time, that is, to keep it above the highest possible level. For example, stakeholders involved in a firm's management may have conflicting preferences over sustaining profits, market shares, employment, and shareholders' dividend. In a political-economy setting, administrations may be interested in sustaining their budget, citizens may be concerned about the sustained level of public services, and tax payers would focus on the tax level.

house gases (GHG), or the abundance of an endangered species). The various indicators, which depend on the economic state and decisions at each time, differ and are expressed in different units. The stakeholders bargain over a common sequence of decisions (for instance, the intertemporal path of consumption and investment), thereby influencing the evolution of the economy through the dynamics representing production possibilities. Intertemporal payoffs depend on the resulting economic trajectory. If no agreement is reached, then stakeholders would get a reference payoff given by the status-quo or business-as-usual economic trajectory (disagreement point). Fershtman (1983) studied the case of intertemporal payoffs defined as discounted utility. Discounted utility has been criticized and qualified as a "dictatorship of the present" in the sustainability literature (Chichilnisky, 1996). Maximin is an alternative criterion that treats all generations with anonymity and expresses the idea to "sustain" something over time (usually consumption or utility but possibly environmental indicators; see Cairns and Long, 2006).[5] Thus, we consider intertemporal maximin payoffs to capture the tension between short- and long-term, the payoff of each stakeholder being defined as the minimal level over time of the corresponding indicator.[6] This is consistent with the way environmental issues are addressed in practice, with the social choice of thresholds environmental indicators should not overshoot.[7] As "any social decision is the ultimate outcome of some kind of collective bargaining process" (Kalai et al., 1976, p.233), the definition of such thresholds can be seen as the result of a dynamic bargaining problem with maximin intertemporal preferences, in a society in which stakeholders are concerned by the extremal levels of indicators.

We formalize these bargaining problems in our general framework and introduce monotonicity properties, called MonDAI (Monotonicity of the dynamics and indicators). In essence, these properties correspond to a requirement that i) capital stocks are "productive" (higher capital stocks produce more and do not reduce the payoff of stakeholders) and ii) some (not all) indicators depend on the decisions in a monotonic way (i.e., these indicators increase with "lower" decisions). The stakeholders whose payoffs depend on these monotonic indicators belong to an *interest group*; the others are called *outsiders*. When there is such an interest group, we show that the Pareto frontier of the set of feasible payoffs can be parametrized by as many variables as there are outsiders. Hence, in a sense, this Pareto frontier is of a lower dimension than the number of stakeholders. Finally, as these dynamic problems are subject to

[5]Maximin may represent preferences with extreme aversion to inequalities or complementarity. It has been used to address justice and distributive concerns both in static (Engelmann and Strobel, 2004; Mármol and Ponsatí, 2008) and dynamic frameworks (Solow, 1974; Burmeister and Hammond, 1977).

[6]Without loss of generality, it is always possible to take the negative level of an indicator representing a "bad" (e.g., pollution) to be able to consider that the payoff is the minimal level over time of the indicator.

[7]The climate change issue is addressed by defining an upper limit on the atmospheric concentration of GHG (UN, 1998). A similar approach is applied for biodiversity, with the creation of reserves to protect natural habitats (UN, 2010). These reserves put constraints on the development of alternative land-uses such as agriculture or urban development. Other examples include minimal stock sizes for fisheries (FAO, 2005) or thresholds for air or water pollution. Although these thresholds often have a scientific basis (as the Intergovernmental Panel on Climate Change provides a view on the climate change issue), they also account for economic and social issues. A clear argument showing that sustainability thresholds are socially chosen is that they differ among countries, particularly with the level of development. Environmental standards are higher in developed countries with high income than in developing countries (Dasgupta et al., 2001).

time-consistency concerns (Fershtman, 1983), we discuss the evolution of the bargaining solution over time. We characterize a parametrized decision rule that achieves any Pareto-efficient outcome, and is renegotiation-proof under the axiom of *Strong Individual Rationality*, resulting in a time consistent sustainable development path.

To the best of our knowledge, such dynamic bargaining problems with intertemporal maximin payoffs have never been studied. We believe that this study offers a useful framework for formalizing sustainability problems, and is an original contribution to the bargaining literature, with clear potential applications. Some studies are loosely related to our study. Fershtman (1983) introduced and studied problems of dynamic bargaining on a sequence of decisions, but with payoffs defined by discounted utility. Lu (2016) examined negotiations over the sharing of a given, infinite stream of surplus, focusing on the timing of acceptance within an alternating proposals protocol, but without considering production dynamics. Long (2006) considered a dynamic game with intertemporal maximin preferences in which each player can select own action and the solution would depend on the strategic interactions between players. Another interesting connection with the bargaining literature appears in the analysis of Mármol and Ponsatí (2008). They examined bargaining problems involving several issues when preferences are maximin – for example, a bargain over ingredients of some recipes wherein all stakeholders use different proportions of these ingredients in their own recipe and the individual output is given by the limiting ingredient of each recipe. Our work can be considered the intertemporal, dynamic counterpart of this problem; however, our study involves more complexity because it requires to account for the production of the various assets and economic dynamics.

2 Bargaining in economic environments

In standard bargaining theory, a bargaining problem involving H stakeholders is characterized by a *set of feasible payoffs* $\mathcal{O} \subset \mathbb{R}^H$ and a *disagreement outcome* $\theta^d \in \mathbb{R}^H$. A bargaining mechanism (sometimes called solution) is a correspondence that defines a subset of desirable outcomes to a bargaining problem.[8]

We introduce bargaining in economic environments to study problems in which the economic information underlying $\mathcal{O}$ and θ^d is explicitly considered. In this framework, a bargaining mechanism selects *alternatives* within a set of feasible alternatives characterizing the economic environment, instead of allocating utility among stakeholders directly.

The framework that we propose can be described as follows. A *bargaining domain* is a set of bargaining problems sharing common properties; a *bargaining mechanism* can be applied to any problem in this domain. This domain is defined by a given, finite number of stakeholders who have well-defined preferences over a set of potential *alternatives*, so that a vector of payoffs is associated with any alternative. A specific bargaining problem within this domain is characterized by a subset of feasible alternatives and a disagreement alternative, which define the *economic environment* of this problem. A bargaining mechanism assigns a subset of alternatives to any economic environment within a domain. To illustrate this overall architecture, let us refer to a

[8]Formally, in the standard framework, a mechanism is a correspondence μ defining a subset $\mu(\mathcal{O}, \theta^d) \subset \mathcal{O}$ to a problem characterized only by a couple $(\mathcal{O}, \theta^d)$. A mechanism can reduce to a mapping defining a solution vector $\mu(\mathcal{O}, \theta^d) = \theta^* \in \mathcal{O}$.

simple example. Consider the choice of an apartment (an alternative) by the members of a household (stakeholders). This is a common choice that will impact the payoff of all the members of the household. An apartment can be described by different characteristics, including its location, surface, natural environment, and price. These characteristics would be valued differently by different members of the household, according to their preferences. Even if each member of the household has well-defined preferences for these characteristics and can rank any possible pair of apartments, in practice, the household will not be faced with the choice of an apartment within the (virtual) set of all possible apartments, but only within a subset of feasible alternatives corresponding to the current offers on the real estate market. This choice set is associated with a set of feasible payoffs and a disagreement outcome (e.g., staying in the current apartment). This constitutes the economic environment of the bargaining problem. The household can, however, be soon confronted with a slightly different bargaining problem when the goods on the real estate market change. As the two bargaining problems are characterized by the same stakeholders with given preferences and the some object, they can be considered to correspond to two economic environments belonging to the same bargaining domain. The same bargaining mechanism (that is, the decision-making pattern within the household) can apply to both problems and be characterized by axioms.

Notations: Let $\mathbb{N}$ be the set of natural numbers, $\mathbb{N}^*$ be the set of positive natural numbers, and $\mathbb{R}$ the set of real numbers. We denote the power set (i.e., the set of subsets) of a set S by 2^S. When needed, the set $\mathbb{R}^q$, where $q \in \mathbb{N}^*$ is equipped with the following componentwise order: $y' = (y'_1, \ldots, y'_q) \geq y = (y_1, \ldots, y_q) \iff y'_i \geq y_i\ ,\ \forall i = 1, \ldots, q$.

All proofs of Theorems and Propositions are in the Appendix.

2.1 The bargaining problem

Let $H \in \mathbb{N}^*$ and $n \in \mathbb{N}^* \cup \{\infty\}$ be fixed. $\mathbb{A}$ is a subset of $\mathbb{R}^n$ when $n \in \mathbb{N}^*$, or a subset of $\mathbb{R}^\infty = \mathbb{R}^\mathbb{N}$ (the set of all mappings from $\mathbb{N}$ to $\mathbb{R}$).

Consider H stakeholders whose preferences over a set of *potential alternatives* $\mathbb{A}$ are characterized by the *payoff function* $\pi : \mathbb{A} \to \mathbb{R}^H$. By convention, for $\mathbb{A}' \subset \mathbb{A}$, $\pi(\mathbb{A}') = \{\pi(a) \mid a \in \mathbb{A}'\} \subset \mathbb{R}^H$.

With this payoff function π and its underlying elements (the set $\mathbb{A}$ and the number of stakeholders H), we define the *bargaining domain* $\Xi = \{H\} \times \{\pi\} \times 2^\mathbb{A} \times \mathbb{A}$. Within this bargaining domain, the stakeholders may be faced with different bargaining situations, called economic environments.

An *economic environment* $\xi = [H; \pi; \mathbb{A}_\xi; a^d_\xi] \in \Xi$ is characterized by a set of *feasible alternatives* $\mathbb{A}_\xi \subset \mathbb{A}$ and a *disagreement alternative* $a^d_\xi \in \mathbb{A}$. As H and π are fixed within a bargaining domain, we simplify the notation of economic environments and write $\xi = \langle \mathbb{A}_\xi; a^d_\xi \rangle = [H; \pi; \mathbb{A}_\xi; a^d_\xi] \in \Xi$.

In our analysis, we will consider *reduced economic environments* ξ', which differ from a given environment ξ only by their set of considered alternatives, with $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$.

Definition 1 (Reduced economic environment)
An economic environment $\xi' \in \Xi$ is said to be a reduced economic environment *with respect to environment $\xi = \langle \mathbb{A}_\xi; a^d_\xi \rangle$ if $\xi' = \langle \mathbb{A}_{\xi'}; a^d_\xi \rangle$, with $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$.*

Within this framework, the *set of feasible payoffs* of a bargaining problem in the economic environment $\xi = \langle \mathbb{A}_\xi; a_\xi^d \rangle$ is $\pi(\mathbb{A}_\xi)$ and the *disagreement outcome* is $\pi(a_\xi^d)$. The explicit characterization of the set of feasible payoffs $\pi(\mathbb{A}_\xi)$ is a difficult task if $\mathbb{A}_\xi$ is a large (possibly infinite) set. There may be no straightforward way to determine the set of feasible payoffs from the description of the economic environment, except by exploring the whole alternatives, which may not be feasible.

In the traditional axiomatic bargaining theory, a bargaining mechanism allocates utility among stakeholders. In our framework, outcomes are associated with alternatives through the payoff function. Thus, we define a *bargaining mechanism* as a process to select alternatives.

Definition 2 (Bargaining mechanism)
A bargaining mechanism *is a correspondence $\mu : \Xi \to 2^{\mathbb{A}}$, which assigns to any economic environment $\xi = \langle \mathbb{A}_\xi; a_\xi^d \rangle \in \Xi$ a subset of acceptable solutions $\mu(\xi) \subset \mathbb{A}_\xi$ within the set of feasible alternatives $\mathbb{A}_\xi$ of the environment.*

We provide conditions on mechanisms making the bargaining problem tractable. To do so, we consider the following properties and axioms.

2.2 Efficient outcomes and efficient alternatives

We introduce the properties of (weak and strong) *Efficiency* for outcomes (i.e., vectors of payoffs in $\mathbb{R}^H$, for bargaining problems in economic environments in Ξ). These properties are formulated as correspondences, which can be used either to reduce the set of payoff vectors worth considering (i.e., presolutions ; see Thomson, 2001, p. 354) or to characterize mechanisms with corresponding axioms. Such axioms will be specified later.[9]

Definition 3 (Weakly Pareto Efficient outcomes)
Let $H \in \mathbb{N}^*$ and $\mathcal{O} \subset \mathbb{R}^H$. An outcome $\theta = (\theta_1, \dots, \theta_H) \in \mathcal{O}$ is weakly Pareto efficient *– weakly efficient for short – on $\mathcal{O}$ if, for any $\tilde{\theta} = (\tilde{\theta}_1, \dots \tilde{\theta}_H)$ such that $\tilde{\theta}_h > \theta_h$ for all $h = 1, \dots, H$, one has $\tilde{\theta} \notin \mathcal{O}$.*

We denote by $\mathcal{E}^w(\mathcal{O}) \subset \mathcal{O}$ the set of all weakly efficient outcomes of $\mathcal{O}$, and call it the *weak Pareto frontier* of $\mathcal{O}$.

Definition 4 (Strongly Pareto Efficient outcomes)
Let $H \in \mathbb{N}^*$ and $\mathcal{O} \subset \mathbb{R}^H$. An outcome $\theta = (\theta_1, \dots, \theta_H) \in \mathcal{O} \subset \mathbb{R}^H$ is strongly Pareto efficient *– efficient for short – on $\mathcal{O}$ if, for any $\tilde{\theta} = (\tilde{\theta}_1, \dots \tilde{\theta}_H)$ such that $\tilde{\theta}_h \geq \theta_h$ for all $h = 1, \dots, H$ and $\tilde{\theta}_h > \theta_h$ for some h, one has $\tilde{\theta} \notin \mathcal{O}$.*

We denote by $\mathcal{E}(\mathcal{O}) \subset \mathcal{O}$ the set of all efficient outcomes of $\mathcal{O}$, and call it the *Pareto frontier* of $\mathcal{O}$. We have $\mathcal{E}(\mathcal{O}) \subset \mathcal{E}^w(\mathcal{O})$. An outcome $(\theta_1, \dots, \theta_H) \in \mathcal{E}^w(\mathcal{O}) \backslash \mathcal{E}(\mathcal{O})$ is dominated in the sense that one can increase the payoff of at least one stakeholder without decreasing that of the others.

Efficient outcomes are generated by *efficient alternatives*. We define the set of efficient alternatives for an economic environment ξ as follows.

[9] The properties of (weak and strong) *Individual Rationality* and corresponding axioms will be introduced in Section 4.3. These properties are not introduced here because they are used only to study time-consistency in dynamic problems.

Definition 5 (Efficient Alternatives)
For the economic environment $\xi = \langle \mathbb{A}_\xi; a^d_\xi \rangle \in \Xi$, *the set of* efficient alternatives *is the subset* $\mathbb{A}^\star_\xi$ *of alternatives in* $\mathbb{A}_\xi$ *yielding efficient outcomes, that is,* $\mathbb{A}^\star_\xi = \{a \in \mathbb{A}_\xi \mid \pi(a) \in \mathcal{E}(\pi(\mathbb{A}_\xi))\}$. *We define the corresponding reduced economic environment* $\xi^\star = \langle \mathbb{A}^\star_\xi; a^d_\xi \rangle$, *which contains only efficient alternatives* $\mathbb{A}^\star_\xi \subset \mathbb{A}_\xi$.

The Appendix gathers propositions characterizing such efficient alternatives (Proposition 4), their existence (Proposition 5), and the properties of efficient alternatives for reduced economic environments (Proposition 6). These properties are used to prove our main Theorems.

2.3 Bargaining mechanisms: Axiomatic properties

We are now ready to discuss bargaining mechanisms and the possibility to apply them to bargaining problems in economic environments. We introduce the axiom of *Pareto efficiency* as a well-known benchmark, and consider the following two axioms specific to our framework: *Independence of Non-Efficient Alternatives* and *Independence of Redundant Alternatives.*[10]

Axiom 1 (Pareto Efficiency)
A bargaining mechanism $\mu : \Xi \to 2^{\mathbb{A}}$ *satisfies the axiom of* Pareto Efficiency *if, for any* $\xi = \langle \mathbb{A}_\xi; a^d_\xi \rangle \in \Xi$, *one has* $\pi(\mu(\xi)) \subset \mathcal{E}(\pi(\mathbb{A}_\xi))$, *which is equivalent to* $\mu(\xi) \subset \mathbb{A}^\star_\xi$.

The axiom of Pareto Efficiency has the usual interpretation – a bargaining mechanism is Pareto-efficient if all the alternatives it selects are associated with Pareto-efficient outcomes.

Characterizing the efficient outcomes of a bargaining problem in economic environments as well as some alternatives to achieve these outcomes are challenging and non-trivial tasks. We will show that these tasks can be simplified if the bargaining mechanism satisfies the axioms of *Independence of Non-Efficient Alternatives* (INEA) and *Independence of Redundant Alternatives* (IRA), defined as follows.

Axiom 2 (Independence of Non-Efficient Alternatives – INEA)
A bargaining mechanism $\mu : \Xi \to 2^{\mathbb{A}}$ *satisfies the axiom of* Independence of Non-Efficient Alternatives *if, for any couple of economic environments* $\xi = \langle \mathbb{A}_\xi; a^d_\xi \rangle \in \Xi$ *and* $\xi' = \langle \mathbb{A}_{\xi'}; a^d_\xi \rangle \in \Xi$, *we have* $\mathbb{A}^\star_\xi \subset \mathbb{A}_{\xi'} \subset \mathbb{A}_\xi \Rightarrow \mu(\xi') = \mu(\xi)$.

The INEA axiom requires that the mechanism acts independent of alternatives that do not yield efficient outcomes in the sense that dropping any subset of alternatives that are not efficient does not change the bargaining solution.

The axiom is more demanding than Pareto Efficiency in the sense that INEA implies Pareto Efficiency[11] while imposing independence to some alternatives. This is, however, a much less demanding axiom than the usual axiom of *Independence of Irrelevant Alternatives* (IIA).[12] Any bargaining mechanism satisfying IIA in the classical bargaining theory framework also satisfies INEA when considering economic

[10] In Section 4.3, we will also consider the axioms of weak and strong *Individual Rationality.*

[11] A mechanism satisfying INEA would satisfy $\mu(\xi) \subset \mathbb{A}^\star_\xi$ as by applying the definition of the axiom to the subset $\xi' = \langle \mathbb{A}_{\xi'}; a^d_\xi \rangle$, where $\mathbb{A}_{\xi'} = \mathbb{A}^\star_\xi$ implies $\mu(\xi) = \mu(\xi') \subset \mathbb{A}_{\xi'} = \mathbb{A}^\star_\xi$.

[12] A mechanism is IIA if, whenever $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$ and $\mu(\xi) \subset \mathbb{A}_{\xi'}$, it follows that $\mu(\xi') = \mu(\xi)$.

environments (e.g., the Nash mechanism or the egalitarian mechanism). Some classical mechanisms, that do not satisfy IIA, can satisfy INEA as long as they depend only on efficient outcomes and the disagreement outcome (e.g., the Kalai-Smorodinsky mechanism). Under INEA, the mechanism may be sensitive to some non-optimal alternatives (Karni and Schmeidler, 1976) if these alternatives are associated with efficient outcomes.

When a bargaining mechanism satisfies INEA, bargaining over the efficient alternatives $\mathbb{A}^\star_\xi$ (Definition 5) leads to the same result as bargaining over the full set of alternatives.

Remark 1 (Bargaining over efficient alternatives)
If a mechanism $\mu : \Xi \to 2^{\mathbb{A}}$ satisfies the axiom of Independence of Non-Efficient Alternatives, *then for any $\xi = \langle \mathbb{A}_\xi; a^d_\xi \rangle \in \Xi$, one has $\mu(\xi^\star) = \mu(\xi)$, where $\xi^\star = \langle \mathbb{A}^\star_\xi; a^d_\xi \rangle$ is the reduced environment containing only efficient alternatives $\mathbb{A}^\star_\xi \subset \mathbb{A}_\xi$.*

We now introduce the axiom of *Independence of Redundant Alternatives*.

Axiom 3 (Independence of Redundant Alternatives – IRA)
A bargaining mechanism $\mu : \Xi \to 2^{\mathbb{A}}$ satisfies the axiom of Independence of Redundant Alternatives *if, for any couple of economic environments $\xi = \langle \mathbb{A}_\xi; a^d_\xi \rangle \in \Xi$ and $\xi' = \langle \mathbb{A}_{\xi'}; a^d_\xi \rangle \in \Xi$, we have $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$ and $\pi(\mathbb{A}_{\xi'}) = \pi(\mathbb{A}_\xi) \Rightarrow \pi\big(\mu(\xi')\big) = \pi\big(\mu(\xi)\big)$.*

This axiom requires that the mechanism acts independent of redundant alternatives in the sense that dropping any subset of alternatives without reducing the set of feasible payoffs does not change the bargained outcomes. This may, however, modify the set of alternatives selected by the mechanism.[13] It is satisfied by all the mechanisms defined in the classical bargaining theory framework as they depend only on the set of feasible payoffs.

The following theorem states that, if the mechanism satisfies the INEA and IRA axioms, the bargaining can take place over a subset of efficient alternatives only.

Theorem 1 (Bargaining over a subset of efficient alternatives)
Let $\mu : \Xi \to 2^{\mathbb{A}}$ be a bargaining mechanism satisfying the axioms of Independence of Non-Efficient Alternatives *and* Independence of Redundant Alternatives. *Suppose that the set of alternatives $\mathbb{A}$ is a metric space and the payoff mapping $\pi = (\pi_1, \ldots, \pi_H) : \mathbb{A} \to \mathbb{R}^H$ is composed of upper semicontinuous functions $\pi_h : \mathbb{A} \to \mathbb{R}$, $h = 1, \ldots, H$. Consider any economic environment $\xi = \langle \mathbb{A}_\xi; a^d_\xi \rangle \in \Xi$ such that $\mathbb{A}_\xi \subset \mathbb{A}$ is compact. Then, for any reduced economic environments $\xi' \in \Xi$ satisfying $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$, we have $\mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \subset \pi(\mathbb{A}_{\xi'}) \Rightarrow \pi\big(\mu(\xi')\big) = \pi\big(\mu(\xi)\big)$.*

If the bargaining mechanism satisfies the INEA and IRA axioms, a bargaining problem can be reduced to a simpler bargaining problem over any subset of alternatives generating the Pareto frontier of the set of feasible payoffs. *Within a given bargaining domain*, the outcome of the bargaining mechanism depends only on the set of efficient payoffs.[14] This result can be used to simplify bargaining problems in economic environments. This is the case in *monotonic* economic environments.

[13]Under the axiom of IRA, the mechanism μ does not need to select all the alternatives that achieve the solution outcomes.

[14]Our framework implies a form of welfarism within a bargaining domain (i.e., within a set of problems characterized by economic environments sharing several features). The domain, however, depends on the type of alternatives (through the definition of $\mathbb{A}$) and on the preferences of the

3 Bargaining problems in monotonic economic environments

In this section, we focus on specific bargaining problems in economic environments satisfying some monotonicity properties. For these problems, we provide a computational method to determine the set of efficient outcomes and a set of alternatives that produce these outcomes.

3.1 Monotonic economic environments

The payoff of some stakeholders may satisfy some monotonicity properties, in the sense that when alternatives can be ranked, the payoff of these stakeholders is higher for "lower" alternatives. We qualify such stakeholders as an *interest group*, the other stakeholders being *outsiders* (of the interest group).

Definition 6 (MonEE)
Suppose that the set $\{1, \dots, H\}$ of stakeholders is partitioned[15] *in an* interest group i *and* outsiders o, *yielding sub-groups payoff functions* $\pi = (\pi^i, \pi^o)$, $\pi^i : \mathbb{A} \to \mathbb{R}^{|i|}$, $\pi^o : \mathbb{A} \to \mathbb{R}^{|o|}$, *where $|i|$ and $|o|$ indicate the cardinality of subsets i and o. An economic environment $\xi = \langle \mathbb{A}_\xi ; a^d_\xi \rangle \in \Xi$ is said to be (i, o)*-monotonic *if*

I. the set $\mathbb{A}_\xi$ is equipped with an order $\leq$,

II. the sub-groups payoff functions are such that

a. the mapping $\pi^i : \mathbb{A}_\xi \to \mathbb{R}^{|i|}$ is non-increasing, that is, $a^\flat \leq a^\sharp \Rightarrow \pi^i(a^\flat) \geq \pi^i(a^\sharp)$,

b. for all $\theta^o \in \mathbb{R}^{|o|}$, the set $\{a \in \mathbb{A}_\xi \mid \pi^o(a) \geq \theta^o\}$ is either empty or admits a minimum, that is, an $\bar{a} \in \mathbb{A}_\xi$ such that

$$\pi^o(\bar{a}) \geq \theta^o \text{ and } (a \in \mathbb{A}_\xi \ , \ \pi^o(a) \geq \theta^o) \Rightarrow a \geq \bar{a} \ . \tag{1}$$

Item I means that, to define a monotonic economic environment, we must be able to partially order its alternatives, in the sense that some of them can be ranked. Item IIa means that if an alternative is lower than others, it is preferred by all the members of the interest group. Item IIb is a strong assumption (refer to the discussion in subsection 4.2); it means that, for any payoff θ^o that can be guaranteed to the outsiders (in the sense that their actual payoff can be at least equal to that level), there is a lowest alternative among the subset of alternatives yielding at least this payoff.

3.2 Bargaining over an efficient reduced economic environment

For monotonic economic environments satisfying the MonEE properties, it is possible to identify a subset of alternatives $\mathbb{A}^o_\xi \subset \mathbb{A}_\xi$ yielding efficient outcomes, consistent with Theorem 1. This set of *satisficing alternatives* is defined as follows.

stakeholders over these alternatives (through the definition of π), and not only on the set of feasible payoffs, as in classical bargaining theory. The definition of the bargaining domain can be restrictive, and some mechanisms can be designed to produce different solutions to two bargaining problems with the same set of feasible payoffs but belonging to different domains. This makes it possible to avoid welfarism across domains.

[15] (i, o) is a partition of $\{1, \dots, H\}$ if i and o are subsets of $\{1, \dots, H\}$, $i \neq \emptyset \neq o$, $i \cup o = \{1, \dots, H\}$, and $i \cap o = \emptyset$.

Definition 7 (Satisficing alternatives)
For a (i,o)-monotone economic environment $\xi = \langle \mathbb{A}_\xi ; a^d_\xi \rangle$, we define

I. the set of satisficing payoffs for outsiders *by*

$$\mathcal{O}^o_\xi = \{\theta^o \in \mathbb{R}^{|o|} \mid \{a \in \mathbb{A}_\xi \mid \pi^o(a) \geq \theta^o\} \neq \emptyset\} \subset \mathbb{R}^{|o|} \ , \tag{2}$$

II. the satisficing alternative *associated to a* satisficing payoff *as*

$$\forall \ \theta^o \in \mathcal{O}^o_\xi \ , \ \ a_\xi(\theta^o) = \min\{a \in \mathbb{A}_\xi \mid \pi^o(a) \geq \theta^o\} \in \mathbb{A}_\xi \ , \tag{3}$$

III. the reduced economic environment $\xi^o = \langle \mathbb{A}^o_\xi ; a^d_\xi \rangle$, where the set of alternatives $\mathbb{A}^o_\xi$ is the set of all satisficing alternatives

$$\mathbb{A}^o_\xi = \{a \in \mathbb{A}_\xi \mid \exists \theta^o \in \mathcal{O}^o_\xi \ , \ \ a = a_\xi(\theta^o)\} \subset \mathbb{A}_\xi \ . \tag{4}$$

A *satisficing payoff* for outsiders is a payoff level that can be guaranteed to outsiders in the sense that there is one (or more) alternative(s) yielding at least that payoff for them. The set of such payoffs is defined by eq. (2). The *satisficing alternative* associated with a satisficing payoff (eq. 3) is the "lowest" alternative among those yielding at least the given payoff for outsiders. Item IIa of the Definition 6 of monotonic economic environments specifies that the payoff of the stakeholders in the interest group is non-increasing in the alternatives. This means that the satisficing alternatives maximize the payoff of the members of the interest group given a satisficing payoff for the outsiders. This satisficing alternative is defined for any feasible satisficing payoff for outsiders. We can use it to define a subset of alternatives parametrized by the vector of satisficing payoffs for outsiders, and the associated reduced economic environment (eq. 4).

We show in Theorem 2 that, for a monotonic economic environment ξ, the set of payoffs associated with satisficing alternatives includes the Pareto-efficient outcomes of $\pi(\mathbb{A}_\xi)$.

Theorem 2 (Efficient alternatives for (i,o)-MonEE)
For a (i,o)-monotonic economic environment $\xi = \langle \mathbb{A}_\xi ; a^d_\xi \rangle$, we have $\mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \subset \pi(\mathbb{A}^o_\xi) \subset \mathcal{E}^w\big(\pi(\mathbb{A}_\xi)\big)$, where the set $\mathbb{A}^o_\xi$ of satisficing alternatives is defined in eq. (4).

According to Theorem 2, the reduced environment $\xi^o = \langle \mathbb{A}^o_\xi ; a^d_\xi \rangle$ satisfies the assumptions of Theorem 1. We conclude that a bargaining mechanism satisfying the INEA and IRA axioms can be applied to this reduced economic environment, and produces the same outcome as the initial problem.

Interest group and low dimensional Pareto frontier. These results have practical implications. In large dimension bargaining problems in monotonic economic environments, if the bargaining mechanism satisfies the INEA and IRA axioms, the satisficing alternatives of Definition 7 can be used to construct a computable, parametrized set of alternatives that yield all the efficient outcomes of the bargaining problem, and thus defines a reduced problem to which the bargaining mechanism can be applied. When θ^o ranges over $\mathcal{O}^o_\xi$, the outcomes $\pi(\mathbb{A}^o_\xi) = \pi \circ a_\xi(\mathcal{O}^o_\xi)$ include the Pareto frontier $\mathcal{E}(\pi(\mathbb{A}_\xi))$ of the outcomes of the bargaining in the economic environment ξ (Theorem 2). As $\mathcal{O}^o_\xi \subset \mathbb{R}^{|o|}$, we obtain a parametrization of the Pareto frontier with a dimension of at most $|o| = H - |i|$, the number of outsiders. This kind of group of interest is somehow similar to alliances, as described by Manzini and Mariotti (2005), in which stakeholders have the same preferences over the control parameters even if their payoffs are different. As in the case of alliances, the payoffs of the $|i|$ stakeholders in the interest group are co-monotonic, making it possible to represent them in the set of Pareto outcomes by a single dimension. The larger the interest group, the smaller will be the dimensions to explore to characterize the reduced environment ξ^o.

Figure 1 illustrates how to generate this set, for $H = 3$ stakeholders, $i = \{1, 2\}$, and $o = \{3\}$. The Pareto-frontier is parametrized by the payoff of stakeholder 3.

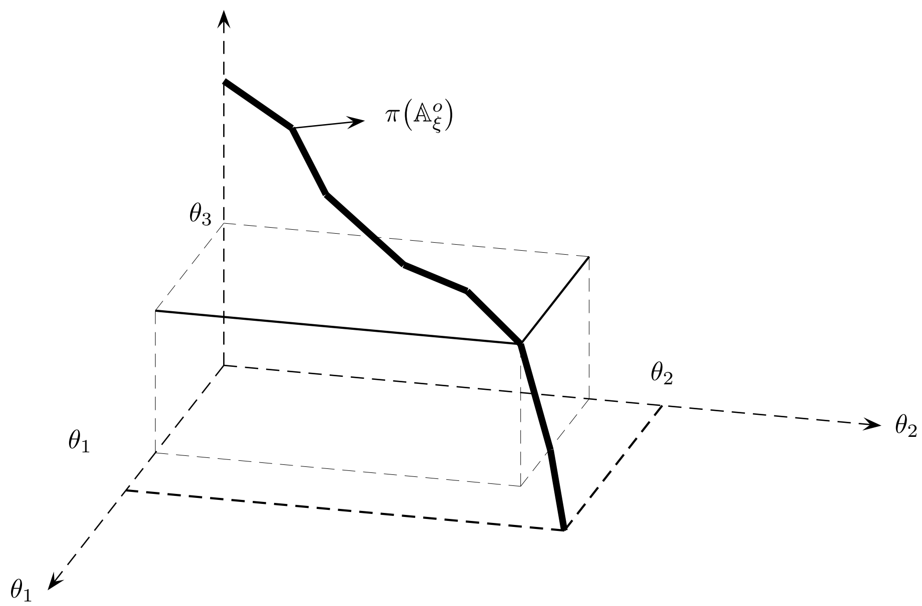

Figure 1: "One-dimensional" Pareto frontier, parametrized by θ^o, with $(\theta_1, \theta_2, \theta_3) = \pi(a_\xi(\theta^o))$ for any $\theta^o \in \mathcal{O}_\xi^o \subset \mathbb{R}$

Until now, we presented a framework to account for the economic environment (and, particularly, for alternatives) in bargaining problems. We provided two axioms under which a bargaining mechanism can be applied to a reduced bargaining problem over a subset of alternatives; we also identified a way to compute such subsets for monotonic economic environments. We now investigate how Theorems 1 and 2 provide some operational way to study dynamic bargaining problems with intertemporal maximin payoffs.

4 Dynamic bargaining problems with intertemporal maximin payoffs (IMP)

To formalize dynamic bargaining problems with IMP in our framework, we first characterize the domain of economic environments on which Theorem 1 will be applied. Subsequently, we specify monotonicity properties that some dynamic problems satisfy, in association with Theorem 2. This allows us to propose a computable characterization of dynamic bargaining problems with IMP under monotonicity properties. Finally, we address a specific concern related to the temporal nature of dynamic problems: time consistency.

4.1 Economic environments with IMP

To characterize the bargaining domain, we must specify the nature of alternatives as well as the payoff function.

Consider a finite number $H \in \mathbb{N}^*$ of stakeholders ($H \geq 2$), identified as $h = 1, \dots, H$, who are bargaining over the trajectory of the economy. They must agree on a path of common decisions, which will drive the evolution of the economic state from current state, according to the economic dynamics. The resulting economic trajectory will determine the individual payoffs. This may represent the kind of bargaining problems at stake in the conference of the parties (COP) for climate change negotiations.

The evolution of the economy is described by a non-linear discrete-time dynamical control system, given by the following ingredients:

- the *time* $t \in \mathbb{N}$;
- the *state variable* $x(t)$ belongs to the finite dimensional *state space* $\mathbb{X} \subset \mathbb{R}^{n_{\mathbb{X}}}$;
- the *control variable* $c(t)$ is an element of the *control set* $\mathbb{C} \subset \mathbb{R}^{n_{\mathbb{C}}}$;
- the *dynamics* is a mapping $G : \mathbb{X} \times \mathbb{C} \to \mathbb{X}$.[16]

The economic model is general. The state variable can encompass usual economic variables (capital and labor) as well as natural resources stocks and environmental quality. The decision variable can encompass consumption decisions, resource extraction, and pollutant emission. The dynamics can correspond to production technologies or to the dynamics of natural resources and the environment.

The dynamics of the economy is given by the system

$$\begin{cases} x(t+1) = G\big(x(t), c(t)\big), \quad t = t_0, t_0+1, \dots \\ x(t_0) = x_0 \ \text{given}, \end{cases} \tag{5}$$

for any *initial time* $t_0 \in \mathbb{N}$ and any *initial state* $x_0 \in \mathbb{X}$. In this context, given an initial state $x(t_0) = x_0$, the trajectory of the economy if fully defined by a sequence of decisions $c(\cdot) = \big(c(t_0), c(t_0+1), \dots\big)$. The bargaining problem aims at defining such a common sequence of decisions.[17]

We assume that the stakeholders have IMP: An *indicator* $I_h : \mathbb{X} \times \mathbb{C} \to \mathbb{R}$ is a function of the economic state and decisions that represents the measurement of the h^{th} stakeholder's interest. Indicators may have different units (e.g., utility, the concentration of GHG, or a biodiversity index), which makes direct transfers between stakeholders impossible. Stakeholders aim to maximize their intertemporal payoff,

[16] If the image $G(x, c)$ is not defined for some state-control couples (x, c), then cemetery points can be added to both state and control sets, and the dynamics can be extended adequately – any cemetery state or cemetery control is mapped to a cemetery state by the dynamics.

[17] A bargained sequence $c(\cdot)$ of decisions can be defined either directly as *open-loop*, that is, by a function of time $t \to c(t)$, or indirectly as *closed-loop*, that is, by a state-dependent decision rule (policy), namely a mapping $\mathfrak{C} : \mathbb{X} \to \mathbb{C}$ giving controls as a function of the state in eq. (5) by $c(t) = \mathfrak{C}\big(x(t)\big)$.

which is the minimal value over time of their indicator.[18] Given an initial time $t_0 \in \mathbb{N}$, an initial state $x_0 \in \mathbb{X}$, and a sequence $c(\cdot)$ of decisions, the *intertemporal payoff* of the h^{th} stakeholder is defined by

$$J_h\big(x_0, c(\cdot)\big) = \inf_{t \geq t_0} I_h\big(x(t), c(t)\big) \ , \quad h = 1, \ldots, H \ , \tag{6}$$

where $x(\cdot)$ is defined by the dynamic system (5), with the sequence of decisions $c(\cdot)$. Payoffs are not directly transferable between agents and/or over time, and are entirely given by the bargained economic trajectory. If there is no agreement, then the economy would remain in the business-as-usual trajectory, generated by a given sequence $c^d(\cdot)$ of decisions.

Bargaining domain and economic environments. To express this problem in our general framework, we define the bargaining domain $\Xi = \{H\} \times \{\pi\} \times 2^{\mathbb{A}} \times \mathbb{A}$ where

- H is the number of stakeholders,
- the set of all potential economic trajectories given the dynamics (5) is defined by $\mathbb{A} = \mathbb{X} \times \mathbb{C}^{\mathbb{N}}$.
- the function $\pi : \mathbb{X} \times \mathbb{C}^{\mathbb{N}} \to \mathbb{R}^H$ defines the payoff of all stakeholders for any trajectory, with $\pi\big(x, c(\cdot)\big) = \Big(J_1\big(x, c(\cdot)\big), \ldots, J_H\big(x, c(\cdot)\big)\Big)$, where the $J_h\big(x, c(\cdot)\big)$ are defined by eq. (6).

Within this bargaining domain, stakeholders can be faced with different bargaining situations, constrained by the current state of the economy $x \in \mathbb{X}$ and the corresponding feasible development paths. As our problem is dynamic and time autonomous, we can define the corresponding economic environments, which we index by the current state $x \in \mathbb{X}$, as $\xi_x = \langle \mathbb{A}_x; a_x^d \rangle$, where the set of feasible alternatives $\mathbb{A}_x$ is the set of development paths starting from state x, i.e., $\mathbb{A}_x = \{x\} \times \mathbb{C}^{\mathbb{N}} \subset \mathbb{A}$. The disagreement alternative corresponds to the trajectory starting from the current state x and generated by the business-as-usual sequence of decisions $a_x^d = \big(x, c^d(\cdot)\big) \in \mathbb{A}_x$. The set of feasible payoffs $\pi(\mathbb{A}_x) \subset \mathbb{R}^H$ corresponds to the set of achievable intertemporal payoffs for an economy starting from x.

According to Theorem 1, in such economic environments, whenever the mechanism satisfies the INEA and IRA axioms, a bargaining mechanism could be applied to a reduced subset of economic development paths producing all the efficient outcomes.

4.2 Monotonic economic environments with IMP

Now, we consider specific dynamic bargaining problems with IMP satisfying some monotonicity properties, called MonDAI (MONotonicity of Dynamics And Indicators). Under these monotonicity assumptions, the economic environment is monotonic in the sense of Definition 6.

[18] This corresponds to a maximin problem (Solow, 1974; Burmeister and Hammond, 1977). In any case, the best payoff that the stakeholder h can get is the maximin level $\bar{J}_h(x_0) = \sup_{c(\cdot)} \big(\inf_{t=t_0,t_0+1,\ldots} I_h\big(x(t), c(t)\big)\big)$.

4.2.1 Monotonicity assumptions

Some discrete-time dynamical models have the following qualitative properties (*ceteris paribus*): (i) the higher the state vector at a period, the higher it will be at the following period; (ii) the higher the decisions at a period, the lower the state vector will be at the following period. This is the case for economic problems in which capital stocks are productive[19] and the decisions reduce the capital stocks. As we put a particular focus on environmental issues, let us emphasize that these properties are satisfied for problems of air quality dynamics and pollutant emissions[20] and for problems of natural resources extraction/harvesting.[21] More generally, economic models in which decisions (e.g., consumption) reduce investment satisfy these properties.

Indicators may also exhibit monotonicity properties. If all capital stocks are defined as "goods," then indicators will usually increase with the state: the larger the state vector, the higher will be the indicators.[22] Some indicators may monotonically respond to the decisions too. This is the case for environmental indicators that are monotonically non-increasing with the decisions, such as pollutant emissions or resource extraction.

Formally, we say that a mapping[23] $f : \mathbb{X} \times \mathbb{C} \to \mathbb{R}^q$ — defined for state and decision variables with values in $\mathbb{R}^q$ — is

- *non-decreasing with respect to the state* if $x' \geq x \Rightarrow f(x', c) \geq f(x, c)$, for all $(x, x', c) \in \mathbb{X} \times \mathbb{X} \times \mathbb{C}$,
- *non-increasing with respect to the decision* if $c' \geq c \Rightarrow f(x, c') \leq f(x, c)$, for all $(x, c, c') \in \mathbb{X} \times \mathbb{C} \times \mathbb{C}$.

A function that does not depend on the state or the decision is both non-decreasing and non-increasing with respect to that variable.

At this point, we can formalize the monotonicity properties that we consider for dynamic bargaining problems with IMP.

MonDAI property: Monotonicity of the dynamics and of $|i|$ indicators. In what follows, we will consider vectors $c = (c_1, \ldots, c_{n_{\mathbb{C}}})$ of bounded decisions, with $c_j \in \mathbb{C}_j = [c_j^\flat, c_j^\sharp]$ for every $j = 1, \ldots, n_{\mathbb{C}}$. We also assume that the decision set $\mathbb{C}$ is a bounded product set of the form $\mathbb{C} = \mathbb{C}_1 \times \cdots \times \mathbb{C}_{n_{\mathbb{C}}} = [c_1^\flat, c_1^\sharp] \times \cdots \times [c_{n_{\mathbb{C}}}^\flat, c_{n_{\mathbb{C}}}^\sharp]$.

[19] In the sense that more "capital" induces more "production." It requires that the various components of the capital vector have no negative effect one on the others.

[20] The better the air quality at one period, the better it will be at the following period (*ceteris paribus*). The higher the pollutant emission at one period, the worse the air quality will be at the following period. This works for the climate change issue and greenhouse gases emissions, taking the negative level of CO_2 atmospheric concentration as a state.

[21] The larger the resource stock at one period, the larger it will be at the following period (*ceteris paribus*). The larger the extraction or harvesting, the lower will be the resource stock at the following period. Note that these assumptions are not satisfied for multispecies ecological models when there is a prey-predator relationship; this is because a larger predator stock may reduce the prey stock in the next period.

[22] This is true for economic indicators, which may depend for instance on capital stocks, knowledge / human capital, or infrastructures. This is also true for ecological indicators as long as the environmental capital stocks are properly defined, by accounting for "bads" (pollution stock for instance) by their negative level.

[23] Such a mapping can represent either the dynamics or the indicator functions, with $q = n_{\mathbb{X}}$ in the case of the dynamics, and $q = 1$ in the case of an indicator.

Definition 8 (MonDAI)
Suppose that the set $\{1, \ldots, H\}$ of stakeholders is partitioned in an interest group i *and* outsiders o, *of cardinality $|i|$ and $|o|$. In a dynamic bargaining problem with IMP, an economic environment is MonDAI if:*

- *the dynamics $G : \mathbb{X} \times \mathbb{C} \longrightarrow \mathbb{X}$ is non-decreasing in the state variable and non-increasing in the decision;*
- *for $h = 1, \ldots, H$, all the indicators $I_h : \mathbb{X} \times \mathbb{C} \longrightarrow \mathbb{R}$ are continuous and non-decreasing in the state variable;*
- *the indicators of the stakeholders of the* interest group $I_{h\in i}$ *are non-increasing in the decisions;*
- *the indicators of the* outsiders $I_{h\in o}$ *depends on at most one decision component among $\{c_1, \ldots, c_{n_{\mathbb{C}}}\}$.*

The first three conditions correspond to the monotonicity features broadly described above – capital stocks are "good" and productive, decisions correspond to foregone investments, and the interest group favors "lower" decisions. Let us be more explicit about the last property required for the outsiders. For every decision component $j = 1, \ldots, n_{\mathbb{C}}$, we can define the subgroup of outsiders whose indicator depends on decision c_j as $\Lambda(j) = \{h \in o \mid I_h \text{ depends on } c_j\}$. The last assumption in the MonDAI definition requires $r \neq j \Rightarrow \Lambda(j) \cap \Lambda(r) = \emptyset$. A model of pollutant emission sharing, in which the interest group is composed of environmental stakeholders (who favor low emissions) and the outsiders are individual polluters who are solely concerned about their individual emission rights, satisfies these properties.

The stakeholders of the interest group have an interest in keeping the decisions at low levels for two reasons. First, their indicators are non-increasing when the decision variables increase. Second, as the dynamics is non-increasing in the decision, lower decisions favor higher capital stocks, and all the indicators are non-decreasing with the state variable. The stakeholders of the interest group have an interest in reducing the level of the decision variables at all periods. Conversely, the indicators of outsiders do not depend on the decisions in a particular way (even if some of the indicators may be increasing with the decisions, unlike the indicators of the interest group).

A dynamic bargaining problem with IMP satisfying the MonDAI properties also satisfies the assumptions of Theorem 1 and the monotonicity assumptions MonEE of Definition 6 (see Appendix A.4).

Proposition 1 (MonDAI implies MonEE)
In a bargaining problem with IMP, if the dynamics and the indicators satisfy the MonDAI monotonicity properties (Definition 8), economic environments ξ_x are (i, o)–monotonic (Definition 6).

Thus, the result of Theorem 2 applies to these bargaining problems. As such, it is of interest to define a set of alternatives (development paths) generating the efficient payoffs of the bargaining problem.

4.2.2 Satisficing decision rule and efficient outcomes

The following family of decision rules, which are parametrized by outsiders payoffs, can be used to characterize the efficient outcomes of a MonDAI economic environment.

Definition 9 (Satisficing decision rule)
Consider a MonDAI economic environment and a vector of satisficing payoff for outsiders $\theta^o \in \mathbb{R}^{|o|}$. For each decision component index $j = 1, \ldots, n_{\mathbb{C}}$ and every state x for which the following expression is well-defined,[24] *we define the feedback decision rule $\mathfrak{C}_j^{\theta^o}(x) = \inf\{c_j \in \mathbb{C}_j \mid I_{\Lambda(j)}(x, c_j) \geq \theta^{\Lambda(j)}\}$, where $\Lambda(j) \subset o$ is the group of outsiders whose indicator depends on decision j, $I_{\Lambda(j)}(x, c_j)$ is the vector of their indicator's value, and $\theta^{\Lambda(j)}$ is the corresponding vector of satisficing payoffs in θ^o. Then, we define the* satisficing decision rule *$\mathfrak{C}^{\theta^o}$ by*

$$\mathfrak{C}^{\theta^o} = \left(\mathfrak{C}_1^{\theta^o}, \ldots, \mathfrak{C}_{n_{\mathbb{C}}}^{\theta^o}\right) . \tag{7}$$

Given this decision rule, one can define a corresponding set of alternatives $\mathbb{A}_x^o$, parametrized by outsiders satisficing payoffs, in the spirit of eq. (4) for the general case. Indeed, if for $x \in \mathbb{X}$ and $\theta^o \in \mathbb{R}^{|o|}$ we define $c_x^{\theta^o}(\cdot) \in \mathbb{C}^{\mathbb{N}}$ by $c_x^{\theta^o}(t) = \mathfrak{C}^{\theta^o}(x^\star(t))$ for $t \geq t_0$, where $x^\star(t)$ is given by

$$x^\star(t_0) = x \ , \ \ x^\star(t+1) = G\left(x^\star(t), \mathfrak{C}^{\theta^o}(x^\star(t))\right) \ , \ \ t = t_0, t_0+1, \ldots \tag{8}$$

then the set

$$\mathbb{A}_x^o = \left\{(x, c_x^{\theta^o}(\cdot)) \in \mathbb{A}_x \,\middle|\, \theta^o \in \mathbb{R}^{|o|}\right\} \tag{9}$$

will correspond to the set defined in (4) for our dynamic problem. The corresponding reduced environment is $\xi_x^o = \langle \mathbb{A}_x^o; a_x^d \rangle$, with $\mathbb{A}_x^o \subset \mathbb{A}_x$.

According to Theorem 2, the set of payoffs $\pi\big(\mathbb{A}_x^o\big)$ associated with the reduced economic environment $\mathbb{A}_x^o$ satisfies the following condition

$$\mathcal{E}(\pi(\mathbb{A}_x)) \subset \pi\big(\mathbb{A}_x^o\big) \subset \mathcal{E}^w(\pi(\mathbb{A}_x)) \ . \tag{10}$$

The set $\pi(\mathbb{A}_x^o) \subset \mathbb{R}^H$ of payoffs of the satisficing alternatives is parametrized by the satisficing payoff $\theta^o \in \mathbb{R}^{|o|}$ of outsiders, and is at most of dimension $|o|$. This corresponds to a low dimensional Pareto frontier when there is an interest group. As the set of satisficing alternatives $\mathbb{A}_x^o$ and the corresponding payoffs $\pi(\mathbb{A}_x^o)$ are easier to characterize than the set of feasible payoffs $\pi(\mathbb{A}_x)$, bargaining in the reduced environment ξ_x^o is easier than bargaining in the environment ξ_x for problems satisfying MonDAI monotonicity properties. We get the following Corollary of Theorem 1.

Corollary 1
If a bargaining mechanism μ satisfies the axioms of INEA and IRA, for dynamic bargaining problems with IMP satisfying MonDAI monotonicity properties, one has $\pi(\mu(\xi_x)) = \pi(\mu(\xi_x^o))$, where $\xi_x^o = \langle \mathbb{A}_x^o; a_x^d \rangle$, with $\mathbb{A}_x^o \subset \mathbb{A}_x$ defined by eq. (9) and $\mathcal{E}(\pi(\mathbb{A}_x)) \subset \pi\big(\mathbb{A}_x^o\big)$.

4.3 Time consistency

Finally, we address a concern that is specific to the dynamic nature of the studied problem: time-consistency.

Contrary to static bargaining problems, breaking the agreement does not bring the stakeholders back to the initial situation as the state of the economy and thus

[24] Indeed, $\mathfrak{C}_j^{\theta^o}(x)$ is not defined for states x such that $\{c_j \in \mathbb{C}_j \mid I_h(x, c_j) \geq \theta_h, \ h \in \Lambda(j)\} = \emptyset$.

the set of feasible payoffs evolve over time. Fershtman (1983) is concerned with the stability of the bargaining solution for dynamic bargaining problems. In his model, where stakeholders have intertemporal payoffs corresponding to discounted utility, one stakeholder may reconsider the agreement at some time $t > t_0$ if he has received most of his planned payoff at that time. In our model, the stability concern also arises – stakeholders may want to bargain again after some time. We examine how the payoff of stakeholders evolve over time and show that, in MonDAI economic environments, a bargained solution based on a satisficing decision rule is renegotiation-proof if the bargaining mechanism satisfies (a strong version of) the axiom of *Individual Rationality.*

4.3.1 Axiomatic: Individual rationality and its consequences

We complete the axiomatic discussion of Section 2.3 by introducing the properties of *(weak and strong) Individual Rationality* and related axioms, along with some results.

Definition 10 (Individually Rational outcomes)
Let $\mathcal{O} \subset \mathbb{R}^H$ and $\theta^d \in \mathbb{R}^H$ be the disagreement outcome vector. A vector of outcomes $\theta = (\theta_1, \ldots, \theta_H) \in \mathcal{O}$ is individually rational *with respect to θ^d if $\theta_h \geq \theta_h^d$ for all $h = 1, \ldots, H$.*

We denote by $\mathcal{R}(\mathcal{O}, \theta^d) \subset \mathcal{O}$ the set of all individually rational outcomes. It contains all the outcomes with a payoff that is at least as large as the disagreement outcome for all stakeholders.

We also introduce a stronger version of the individual rationality property.

Definition 11 (Strongly Individually Rational outcomes)
Let $\mathcal{O} \subset \mathbb{R}^H$ be the set of feasible outcomes, and $\theta^d \in \mathbb{R}^H$ be the disagreement outcome vector. A vector of outcomes $\theta = (\theta_1, \ldots, \theta_H) \in \mathcal{O}$ is strongly individually rational *with respect to θ^d if $\theta_h > \theta_h^d$ for all $h = 1, \ldots, H$.*

We denote by $\mathcal{R}^s(\mathcal{O}, \theta^d) \subset \mathcal{O}$ the set of all strongly individually rational outcomes. It contains all the outcomes with a payoff that is strictly larger than the disagreement outcome for all stakeholders.

The following results are obtained directly by combining efficiency and individual rationality properties (proofs are omitted).

Lemma 1 (Efficient status quo)
$\theta^d \in \mathcal{E}(\mathcal{O})$ if and only if $\mathcal{R}(\mathcal{O}, \theta^d) = \{\theta^d\}$.

If the disagreement outcome is efficient, there is no other Individually Rational outcome than the disagreement outcome.

Lemma 2 (Lack of bargaining incentives)
$\theta^d \in \mathcal{E}^w(\mathcal{O})$ if and only if $\mathcal{R}^s(\mathcal{O}, \theta^d) = \emptyset$.

If the disagreement outcome is weakly efficient, then there will be no Strongly Individually Rational outcome.

Individual rationality properties can characterize bargaining mechanisms through the following axioms.

Axiom 4 (Individual Rationality)
A bargaining mechanism $\mu : \Xi \to 2^{\mathbb{A}}$ *satisfies the axiom of* Individual Rationality *if, for any* $\xi = \langle \mathbb{A}_\xi ; a^d_\xi \rangle \in \Xi$, $\pi\big(\mu(\xi)\big) \subset \mathcal{R}(\pi(\mathbb{A}_\xi), \pi(a^d_\xi))$.

Axiom 5 (Strong Individual Rationality)
A bargaining mechanism $\mu : \Xi \to 2^{\mathbb{A}}$ *satisfies the axiom of* Strong Individual Rationality *if, for any* $\xi = \langle \mathbb{A}_\xi ; a^d_\xi \rangle \in \Xi$, $\pi\big(\mu(\xi)\big) \subset \mathcal{R}^s(\pi(\mathbb{A}_\xi), \pi(a^d_\xi))$.

Under *Individual Rationality*, no stakeholder should accept an alternative that would result in a payoff lower than the disagreement outcome. A mechanism satisfying *Strong Individual Rationality* implies that only solutions that strictly improve the payoffs of all stakeholders with respect to the *status quo* are considered. Otherwise, some stakeholders have no incentive to bargain over alternatives to the status quo. We shall see that, regarding time-consistency, Individual Rationality may not be sufficient to ensure renegotiation-proofness in all cases, while Strong Individual Rationality is sufficient.

4.3.2 Dynamic efficiency of bargained solutions and renegotiation-proofness

In our dynamic problem, the economic environment $\xi_{x(t)}$ evolves over time as the state $x(t)$ of the economy changes. The set of feasible alternatives $\mathbb{A}_{x(t)} = \{x(t)\} \times \mathbb{C}^{\mathbb{N}} \subset \mathbb{A}$ changes over time in a well-defined way. The disagreement alternative also changes in a way we need to specify. As we are interested in studying the time-consistency of a bargained dynamic path, we consider the case in which the outcome of the current bargaining situation would become the status quo for future bargaining situations. This feature is used by Kalai (1977) in the axiomatic bargaining framework, and is common in the literature on legislative bargaining in which "today's status quo policy is the policy enacted in the last period" (see Bowen and Zahran, 2012).[25] This leads to defining the following dynamic disagreement outcome.

Assumption 1 (Dynamic disagreement outcome)
For an economic environment $\xi_x = \langle \mathbb{A}_x ; a^d_x \rangle$, *a bargained sequence of decisions* $c^\star(\cdot)$ *with* $(x, c^\star(\cdot)) \in \mu(\xi_x) \subset \mathbb{A}_x$ *would define the new business-as-usual trajectory and the corresponding disagreement point.*

If bargaining takes place again, it may result in time inconsistent trajectories. The stability of a bargained development path will depend on how the payoff of the stakeholders evolves over time with respect to the payoff of alternative options. If the dynamic disagreement outcome is not on the Pareto frontier of the dynamic set of feasible outcomes $\pi(\mathbb{A}_{x(t)})$, it will be possible to find a new solution that improves the payoff of all stakeholders with respect to the status quo. In this case, the initially bargained path is not followed, and there is a time-inconsistency. Conversely, if the initial solution and the associated decision rule result in Pareto efficient dynamic outcomes, then this solution would be renegotiation-proof under the axiom of Strong Individual Rationality, in accordance with Lemma 2.

[25] Alternative features can be considered, but it would require specifying how the disagreement alternative would evolve with the state of the economy.

The dynamic disagreement outcome corresponds to the actual payoffs along the initial bargained trajectory, which is non-decreasing over time,[26] but may become inefficient.[27]

We prove that, for MonDAI economic environments, the outcome of the trajectory determined by a satisficing decision rule (Definition 9) is always on the Pareto frontier of the set of feasible outcomes.

PROPOSITION 2 (EFFICIENT DYNAMIC DISAGREEMENT OUTCOME)
Consider a bargaining problem in a MonDAI economic environment ξ_x, with outsiders o, and a bargained sequence of satisficing decisions $c^\star(\cdot) = \mathfrak{C}^{\theta^o}$ parametrized by some $\theta^o \in \mathbb{R}^{|o|}$. For $t \geq t_0$, the dynamic disagreement outcome

$$\theta^d(t) = \big(J_1\big(x(t), c^\star(\cdot)\big), \ldots, J_H\big(x(t), c^\star(\cdot)\big)\big) \ , \tag{11}$$

where, for all $s \geq t_0$, $c^\star(s) = \mathfrak{C}^{\theta^o}(x(s))$ and $x(s+1) = G\left(x(s), c^\star(s)\right)$, with $x(t_0) = x$, is such that $\theta^d(t) \in \mathcal{E}^w(\pi(\mathbb{A}_{x(t)}))$, $\forall t \geq t_0$.

We can establish renegotiation-proofness under Assumption 1 and the axiom of *Strong Individual Rationality.*

PROPOSITION 3 (RENEGOTIATION-PROOFNESS)
For bargaining problems in MonDAI economic environments, the satisficing decision rule $\mathfrak{C}$ of Definition 9 is renegotiation-proof under Assumption 1 on the dynamic disagreement outcome and the axiom of Strong Individual Rationality*.*

This proposition is a direct application of Lemma 2. According to Proposition 2, the dynamic disagreement outcome remains on the (weak) Pareto frontier of the dynamic set of feasible outcomes at all times. It is not possible to increase the payoff of all stakeholders, and there is a lack of bargaining incentives if the bargaining mechanism satisfies the axiom of *Strong Individual Rationality.* The decision rule is never changed and results in a time consistent economic trajectory. Whenever the set of feasible outcomes is such that $\mathcal{E}^w\big(\pi(\mathbb{A}_{x(t)})\big) = \mathcal{E}\big(\pi(\mathbb{A}_{x(t)})\big)$ at all times, renegotiation-proofness is achieved under the weaker axiom of *Individual Rationality* (Lemma 1).

5 Final remarks

We developed a framework to discuss bargaining problems when the objects of the social choice are alternatives rather than outcomes. We showed that, when a bargaining mechanism satisfies the axioms of *Independence of Non-Efficient Alternatives* and *Independence of Redundant Alternatives*, it can be applied to any reduced set of alternatives yielding the Pareto-efficient outcomes of the initial bargaining problem, and produces the same outcome. Additionally, we showed that, for monotonic problems, such a reduced set of alternatives is computable. When there is an "interest

[26]Unlike in the discounted-utility framework, in which the payoff of some stakeholders may decline over time (Fershtman, 1983), in the maximin case, the payoff of all stakeholders is always non-decreasing over time due to the properties of the inf function.

[27]As the disagreement outcome increases over time along a given trajectory, delaying the bargaining process may modify the solution. Considering such temporal strategies is out of the scope of this study, as it would require assuming different time preferences for the stakeholders than the intertemporal maximin.

group," that is, a subset of stakeholders who rank the alternatives in the same order, the payoffs of the interest group members is co-monotonic and the Pareto frontier of the set of feasible payoffs is of a lower dimension than the number of stakeholders. From a practical point of view, this simplifies the computation of the Pareto-efficient solutions.

Subsequently, we discussed dynamic bargaining problems, in which i) stakeholders have to agree on a time path of decisions that influence the dynamic state of the economy, and ii) their payoff is defined as the minimal level over time of some indicators that depend on the evolution of the economic state and decisions (intertemporal maximin payoff). A set of efficient alternatives (i.e., development paths) can be determined under some monotonicity properties, and we exhibit a common decision rule to achieve any Pareto-efficient outcome. Under the axiom of *Strong Individual Rationality*, the studied decision rule is renegotiation-proof and generates a time-consistent dynamic economic path.

Our results help characterizing the Pareto frontier of bargaining problems in economic environments. This frontier provides information on the necessary trade-offs among stakeholders or among sustainability issues in a dynamic framework. How society makes its final choice among Pareto efficient solutions is beyond the scope of this study. Imposing other axioms to restrict the set of choices would require strong assumptions on the comparability of the payoffs of the various stakeholders. How to interpret, for example, *Symmetry* between sustained consumption and a minimal biodiversity level to be preserved? Sustainability raises equity concerns, both among generations and among different issues. Roemer (1986, 1988) argued that the bargaining theory is not sufficient to address distributive justice mainly because it is, in its original formulation, "context free" and neglects preferences and needs. We tried to avoid this drawback.

As emphasized in the Introduction, the axiomatic bargaining theory and welfare economics are two complementary ways to formalize social choice problems. Any efficient solution of our bargaining problem can be the optimum of a "social welfare ordering" represented by a strictly increasing real-valued function that obeys the Pareto principle (Kaneko, 1980; Denicolò and Mariotti, 2000; Mariotti, 2000). The property of time-consistency is specific to the solution of the bargaining problem. It may not emerge from a social choice problem based on a stationary welfare function $W(\theta_1, \ldots, \theta_H)$. In fact, for a decision-maker with stationary preferences, "Individual Rationality" is not a relevant property. The decision-maker may find it desirable to reduce the payoff of some stakeholders to increase the payoff of some others if this decision increases social welfare. In the sustainability context, it would mean that some environmental thresholds may be modified when the set of feasible payoffs evolves (Martinet, 2011). In a sense, it may not be a concern as new trade-offs may be made as the economic context and associated opportunities change. For instance, the ceiling constraint on the atmospheric concentration of GHG may change over the next decades. Time-inconsistency is increasingly considered as unavoidable for collective dynamic choices (Jackson and Yariv, 2015) or when aiming at intergenerational equity (Asheim and Mitra, 2018).

We examined the case of dynamic bargaining problems with intertemporal maximin payoffs. Another interesting case would combine stakeholders having different forms of intertemporal payoffs (e.g., when some stakeholders have discounted utility payoffs and others have maximin payoffs). Another possibility for extensions is related to the dynamic nature of the solution. We focused on bargaining solutions and their time-consistency, assuming that a solution is selected at an initial time. However, there may be a strategic interest for some stakeholders to delay the bargaining process and remain on the status quo trajectory for some time if this strategy places them in a better position to bargain in the future. This could be the case if their disagreement payoff or their maximal potential payoff increase in a favorable way relative to that of other stakeholders. Rubinstein (1982) initiated a stream of literature examining how the cost of time influences the bargaining solution. More recently, Lu (2016) studied the timing of bargaining resolution when considering infinite streams of surplus sharing. In our bargaining problem, which considers the economic environment and its evolution, the "cost" of delay can be endogenous and measured by the loss (or gain if the "cost" is negative) of maximal potential outcome for example. Future research can examine such strategic delay of the bargained solution.

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A Appendix

A.1 Technical results on efficient alternatives

The following results will be useful in the proofs of the Theorems and Propositions of the paper.

A.1.1 Characterization of efficient alternatives

PROPOSITION 4
The set $\mathbb{A}^\star_\xi$ of efficient alternatives of environment ξ has the following properties.

1. *The set $\mathbb{A}^\star_\xi$ is the largest subset A of the set $\mathbb{A}_\xi$ such that $\pi(A) \subset \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big)$, that is, for all subset A of $\mathbb{A}_\xi$,*

$$A \subset \mathbb{A}_\xi \text{ and } \pi(A) \subset \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \iff A \subset \mathbb{A}^\star_\xi . \tag{12}$$

2. *The set $\mathbb{A}^\star_\xi$ is the smallest subset A of the set $\mathbb{A}_\xi$ such that $\pi(\mathbb{A}_\xi \backslash A) \cap \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) = \emptyset$, that is, for all subset A of $\mathbb{A}_\xi$,*

$$A \subset \mathbb{A}_\xi \text{ and } \pi(\mathbb{A}_\xi \backslash A) \cap \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) = \emptyset \iff \mathbb{A}^\star_\xi \subset A . \tag{13}$$

3. *The set* $\mathbb{A}^{\star}_{\xi}$ *is characterized as follows*

$$A \subset \mathbb{A}_{\xi} \text{ and } \pi(A) \subset \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) \text{ and } \pi(\mathbb{A}_{\xi} \backslash A) \cap \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) = \emptyset \\ \iff A = \mathbb{A}^{\star}_{\xi} \; . \tag{14}$$

4. *We have that, for all subset* A *of* $\mathbb{A}_{\xi}$*,*

$$A \subset \mathbb{A}_{\xi} \text{ and } \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) \subset \pi(A) \Rightarrow \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) \subset \pi(A \cap \mathbb{A}^{\star}_{\xi}) \; . \tag{15}$$

5. *We have that*

$$\pi(\mathbb{A}^{\star}_{\xi}) = \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) \text{ and } \pi(\mathbb{A}_{\xi} \backslash \mathbb{A}^{\star}_{\xi}) = \pi(\mathbb{A}_{\xi}) \backslash \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) \; . \tag{16}$$

Proof

1. The set of subsets $A \subset \mathbb{A}$ such that $\pi(A) \subset \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big)$ is closed under union $\cup$. By Definition 5, $\mathbb{A}^{\star}_{\xi}$ contains any such subset A, and hence is the union of all such subsets A, hence is the largest subset A of the set $\mathbb{A}_{\xi}$ of alternatives such that $\pi(A) \subset \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big)$.

2. The set of subsets $A \subset \mathbb{A}_{\xi}$ such that $\pi(\mathbb{A}_{\xi} \backslash A) \cap \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) = \emptyset$ is closed under intersection $\cap$. From the definition of $\mathbb{A}^{\star}_{\xi}$ (Definition 5), one can check that $\mathbb{A}^{\star}_{\xi}$ is contained in any such subset A. Therefore, $\mathbb{A}^{\star}$ is the intersection of all such subsets A, hence it is the smallest subset A of the set $\mathbb{A}_{\xi}$ of alternatives such that $\pi(\mathbb{A}_{\xi} \backslash A) \cap \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) = \emptyset$.

3. This is a direct consequence of the two previous items.

4. Let $A \subset \mathbb{A}_{\xi}$ be such that $\mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) \subset \pi(A)$. Therefore, any element of $\mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big)$ can be written as $\pi(a)$, where $a \in A$. Now, by the definition of $\mathbb{A}^{\star}_{\xi}$ (Definition 5), we deduce that $a \in \mathbb{A}^{\star}_{\xi}$. We conclude that $a \in A \cap \mathbb{A}^{\star}_{\xi}$ and that (15) holds true.

5. Equation (16) is made of two equalities.

   (a) We prove that $\mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) = \pi(\mathbb{A}^{\star}_{\xi})$ by two inclusions. On the one hand, by the definition of $\mathbb{A}^{\star}_{\xi}$ (Definition 5), we have that $\pi(\mathbb{A}^{\star}_{\xi}) \subset \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big)$. On the other hand, the reverse inclusion $\mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) \subset \pi(\mathbb{A}^{\star}_{\xi})$ is a consequence of (15) with $A = \mathbb{A}_{\xi}$.

   (b) We prove that $\pi(\mathbb{A}_{\xi} \backslash \mathbb{A}^{\star}_{\xi}) = \pi(\mathbb{A}_{\xi}) \backslash \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big)$ by two inclusions. On the one hand, we use (13) with $A = \mathbb{A}_{\xi}$ and obtain that $\pi(\mathbb{A}_{\xi} \backslash \mathbb{A}^{\star}_{\xi}) \subset \pi(\mathbb{A}_{\xi}) \backslash \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big)$. On the other hand, consider $a \in \mathbb{A}_{\xi}$ such that $\pi(a) \notin \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big)$. By the definition of $\mathbb{A}^{\star}_{\xi}$ (Definition 5), we deduce that $a \notin \mathbb{A}^{\star}_{\xi}$. Therefore, $a \in \mathbb{A}_{\xi} \backslash \mathbb{A}^{\star}_{\xi}$ and $\pi(a) \in \pi(\mathbb{A}_{\xi}) \backslash \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big)$. We conclude that $\pi(\mathbb{A}_{\xi}) \backslash \mathcal{E}\big(\pi(\mathbb{A}_{\xi})\big) \subset \pi(\mathbb{A}_{\xi} \backslash \mathbb{A}^{\star}_{\xi})$.

   Hence, we have obtained (16).

A.1.2 Existence of efficient alternatives

Proposition 5

Let $\mathbb{A}$ be a nonempty metric space. Suppose that $\pi = (\pi_1, \ldots, \pi_H) : \mathbb{A} \to \mathbb{R}^H$ is an upper semicontinuous function (usc) in the sense that $\pi_h : \mathbb{A} \to \mathbb{R}$ is upper semicontinuous (usc) for all $h = 1, \ldots, H$, that is, for all $a \in \mathbb{A}$ and for all $a_k \to a$, one has $\limsup_{k\to\infty} \pi_h(a_k) \leq \pi_h(a)$. Then, for all nonempty compact set $\mathbb{A}_\xi \subset \mathbb{A}$, and $\theta \in \pi(\mathbb{A}_\xi)$, there exists $a^* \in \mathbb{A}_\xi$ such that $\pi(a^*) = \theta^* \in \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \cap (\theta + \mathbb{R}^H_+)$, is an efficient point above θ. As a consequence, the set $\mathbb{A}^\star_\xi = \big\{a \in \mathbb{A}_\xi \mid \pi(a) \in \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big)\big\}$ (Definition in 5) is not empty.

Proof The proof is in two steps. In the following, for two vectors $\theta = (\theta_1, \ldots, \theta_H)$ and $\theta' = (\theta'_1, \ldots, \theta'_H)$ in $\mathbb{R}^H$, we will use the distance $\delta(\theta, \theta') = \sum_{h=1}^H |\theta_h - \theta'_h|$.

1. First, we prove that, for every $\theta \in \pi(\mathbb{A}_\xi)$, there exists $a^* \in \mathbb{A}_\xi$ such that $\theta^* = \pi(a^*) \in \pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}^H_+)$ and

$$\delta(\theta, \theta^*) = \alpha_\theta = \sup_{\theta' \in \pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}^H_+)} \delta(\theta, \theta') \ . \tag{17}$$

For this purpose, consider a sequence $(\theta^k)_{k\in\mathbb{N}^*}$ in $\pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}^H_+)$ such that

$$\delta(\theta, \theta^k) \to \alpha_\theta \ \text{ as } \ k \to \infty \ . \tag{18}$$

Since $\theta^k \in \pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}^H_+)$, then, for each $k \in \mathbb{N}^*$, there exist $a^k \in \mathbb{A}_\xi$ and $v^k = (v^k_1, \ldots, v^k_H) \in \mathbb{R}^H_+$ such that

$$\theta^k = \theta + v^k = \pi(a^k) = \big(\pi_1(a^k), \ldots, \pi_H(a^k)\big) \ . \tag{19}$$

As the set $\mathbb{A}_\xi$ is metric compact, there exist $a^* \in \mathbb{A}_\xi$ and a subsequence $(a^{k_j})_{j\in\mathbb{N}^*}$ such that

$$a^{k_j} \to a^* \ \text{ as } \ j \to \infty \ . \tag{20}$$

We put

$$\theta^* = (\theta^*_1, \ldots, \theta^*_H) \ \text{ where } \ \theta^*_h = \pi_h(a^*) \ , \ \ \forall h = 1, \ldots, H \ , \tag{21}$$

and we now prove that $\theta^* \in \pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}^H_+)$ and solves (17).

   (a) By (21), we have that $\theta^* \in \pi(\mathbb{A}_\xi)$.

   (b) We also have that $\theta^* \in (\theta + \mathbb{R}^H_+)$. Indeed, from (20) and the upper semicontinuity of the functions $\pi_h : \mathbb{A} \to \mathbb{R}$, for $h = 1, \ldots, H$, we obtain that

$$\limsup_{j\to\infty} \pi_h(a^{k_j}) \leq \pi_h(a^*) = \theta^*_h \ , \ \ \forall h = 1, \ldots, H \ . \tag{22}$$

   Therefore, by (19) where $v^k = (v^k_1, \ldots, v^k_H) \in \mathbb{R}^H_+$, we deduce that

$$0 \leq \limsup_{j\to\infty} v^{k_j}_h = \limsup_{j\to\infty} \pi_h(a^{k_j}) - \theta_h \leq \theta^*_h - \theta_h \ , \tag{23}$$

   for all $h = 1, \ldots, H$, that is, $\theta^* \in (\theta + \mathbb{R}^H_+)$.

(c) Finally, we show that θ^* solves (17). We have that

$$\begin{aligned}
\alpha_\theta &= \lim_{j\to\infty} \delta(\theta, \theta^{k_j}) \text{ by (18)} \\
&= \limsup_{j\to\infty} \delta(\theta, \theta^{k_j}) \\
&= \limsup_{j\to\infty} \sum_{h=1}^{H} |\theta_h - \theta_h^{k_j}| \\
&= \limsup_{j\to\infty} \sum_{h=1}^{H} |v_h^{k_j}| \text{ by (19)} \\
&= \limsup_{j\to\infty} \sum_{h=1}^{H} v_h^{k_j} \text{ because } v^k = (v_1^k, \dots, v_H^k) \in \mathbb{R}_+^H \\
&\le \sum_{h=1}^{H} \limsup_{j\to\infty} v_h^{k_j} \\
&\le \sum_{h=1}^{H} (\theta_h^* - \theta_h) \text{ by (23)} \\
&= \delta(\theta, \theta^*) \ .
\end{aligned}$$

Thus, we obtain that $\delta(\theta, \theta^*) \ge \alpha_\theta = \sup_{\theta' \in \pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}_+^H)} \delta(\theta, \theta')$. Now, as $\theta^* = \pi(a^*) \in \pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}_+^H)$ by (22), we conclude that (17) holds true.

2. Second, we prove that, for any $\theta \in \pi(\mathbb{A}_\xi)$ and $\theta^* \in \pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}_+^H)$ such that $\alpha_\theta = \delta(\theta, \theta^*)$ (α_θ defined in (17)), then $\theta^* \in \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big)$.

   The proof is obtained by contradiction. Suppose $\theta^* \notin \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big)$. Then, there exists $\tilde{\theta} \in \pi(\mathbb{A}_\xi) \cap (\theta^* + \mathbb{R}_+^H \setminus \{0\})$. Expressing that $\theta^* \in (\theta + \mathbb{R}_+^H)$ and that $\tilde{\theta} \in \pi(\mathbb{A}_\xi) \cap (\theta^* + \mathbb{R}_+^H \setminus \{0\})$, we get that $\theta^* = \theta + v$ and $\tilde{\theta} = \theta^* + w$ for some $v = (v_1, \dots, v_H) \in \mathbb{R}_+^H$ and $w = (w_1, \dots, w_H) \in \mathbb{R}_+^H \setminus \{0\}$ (i.e., $w_h > 0$ for some $h \in \{1, \dots, H\}$). We easily deduce that $\tilde{\theta} \in \pi(\mathbb{A}_\xi) \cap (\theta + \mathbb{R}_+^H)$. Now, we have that $\alpha_\theta = \delta(\theta, \theta^*) = \sum_{h=1}^{H} v_h < \sum_{h=1}^{H} (v_h + w_h) = \delta(\theta, \tilde{\theta})$, which contradicts the definition of α_θ (eq. 17).

We have shown that, for every $\theta \in \pi(\mathbb{A}_\xi)$, there exists $a^* \in \mathbb{A}_\xi$ such that $\pi(a^*) = \theta^* \in \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \cap (\theta + \mathbb{R}_+^H)$. As a consequence, the set $\mathbb{A}_\xi^\star = \big\{a \in \mathbb{A}_\xi \mid \pi(a) \in \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big)\big\}$ is not empty.

A.1.3 Efficient alternatives of reduced economic environments

Proposition 6

Let $(\xi', \xi) \in \Xi^2$ be a couple of economic environments such that

$$\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi \tag{25}$$

$$\mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \subset \pi(\mathbb{A}_{\xi'}) \ . \tag{26}$$

Suppose that the set $\mathbb{A}$ of alternatives is a metric space, $\mathbb{A}_\xi \subset \mathbb{A}$ is compact, and the payoff mapping $\pi : \mathbb{A} \to \mathbb{R}^H$ is upper semicontinuous. Then, we have that $\mathbb{A}_{\xi'}^\star \subset \mathbb{A}_\xi^\star$ and $\pi(\mathbb{A}_{\xi'}^\star) = \pi(\mathbb{A}_\xi^\star)$.

Proof Consider a couple $(\xi', \xi) \in \Xi^2$ of economic environments such that $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$ and $\mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \subset \pi(\mathbb{A}_{\xi'})$.

1. First, we prove that
$$\pi(\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi) = \pi(\mathbb{A}^\star_\xi) \ . \tag{27}$$
From (16), we have that $\mathcal{E}(\pi(\mathbb{A}_\xi)) = \pi(\mathbb{A}^\star_\xi)$. As, by assumption (26), $\mathcal{E}(\pi(\mathbb{A}_\xi)) \subset \pi(\mathbb{A}_{\xi'})$, we deduce from (15) that $\mathcal{E}(\pi(\mathbb{A}_\xi)) \subset \pi(\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi)$. Linking these two results, we obtain $\pi(\mathbb{A}^\star_\xi) = \mathcal{E}(\pi(\mathbb{A}_\xi)) \subset \pi(\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi) \subset \pi(\mathbb{A}^\star_\xi)$, and we conclude that all terms are equal, so that $\pi(\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi) = \pi(\mathbb{A}^\star_\xi)$.

2. Second, we prove that
$$\pi(\mathbb{A}^\star_\xi) \subset \pi(\mathbb{A}^\star_{\xi'}) \ . \tag{28}$$
For this purpose, we will use the following property, which follows from the definition of Pareto sets:
$$\bar{\mathcal{O}} \subset \mathcal{O} \subset \mathbb{R}^H \Rightarrow \mathcal{E}(\mathcal{O}) \cap \bar{\mathcal{O}} \subset \mathcal{E}(\bar{\mathcal{O}}) \ . \tag{29}$$
We have
$$\begin{aligned} \pi(\mathbb{A}^\star_\xi) &= \pi(\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi) \ \text{ by (27)} \\ &\subset \pi(\mathbb{A}_{\xi'}) \cap \pi(\mathbb{A}^\star_\xi) \\ &= \pi(\mathbb{A}_{\xi'}) \cap \mathcal{E}(\pi(\mathbb{A}_\xi)) \ \text{ by (16)} \\ &\subset \mathcal{E}\big(\pi(\mathbb{A}_{\xi'})\big) \ \text{ by (29)} \end{aligned}$$
since $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$, by assumption (25), hence $\pi(\mathbb{A}_{\xi'}) \subset \pi(\mathbb{A}_\xi)$. Therefore, we conclude that $\pi(\mathbb{A}^\star_\xi) \subset \mathcal{E}\big(\pi(\mathbb{A}_{\xi'})\big) = \pi(\mathbb{A}^\star_{\xi'})$ by (16).

3. Third, we prove that $\mathbb{A}^\star_{\xi'} = \mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi$. For this purpose, we will use the equivalence (14) that requires three conditions. We establish that these three conditions hold true.

   (a) We establish that $\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi \subset \mathbb{A}_{\xi'}$. This is obvious.

   (b) We establish that $\pi(\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi) \subset \mathcal{E}\big(\pi(\mathbb{A}_{\xi'})\big)$. Indeed,
$$\begin{aligned} \pi(\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi) &= \pi(\mathbb{A}^\star_\xi) \ \text{ by (27)} \\ &\subset \pi(\mathbb{A}^\star_{\xi'}) \ \text{ by (28)} \\ &= \mathcal{E}\big(\pi(\mathbb{A}_{\xi'})\big) \ \text{ by (16).} \end{aligned}$$

   (c) We establish that $\pi(\mathbb{A}_{\xi'} \backslash (\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi)) \cap \mathcal{E}\big(\pi(\mathbb{A}_{\xi'})\big) = \emptyset$. This is where the assumptions that the set $\mathbb{A}_\xi$ of alternatives is compact and that the payoff mapping $\pi : \mathbb{A} \to \mathbb{R}^H$ is upper semicontinuous play a role, as they make it possible to use Proposition 5. Consider an alternative $a \in \mathbb{A}_{\xi'} \backslash (\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi)$. By Proposition 5, there exists an alternative $\tilde{a} \in \mathbb{A}_\xi$ such that $\pi(\tilde{a}) > \pi(a)$, with $\pi(\tilde{a}) \in \mathcal{E}(\pi(\mathbb{A}_\xi))$. As $\mathcal{E}(\pi(\mathbb{A}_\xi)) \subseteq \pi(\mathbb{A}_{\xi'})$, we deduce that there exists an alternative $\bar{a} \in \mathbb{A}_{\xi'}$ such that $\pi(\bar{a}) = \pi(\tilde{a}) > \pi(a)$. As a consequence, $a \notin \mathcal{E}\big(\pi(\mathbb{A}_{\xi'})\big)$.
   We have thus proven that $\pi(\mathbb{A}_{\xi'} \backslash (\mathbb{A}_{\xi'} \cap \mathbb{A}^\star_\xi)) \cap \mathcal{E}\big(\pi(\mathbb{A}_{\xi'})\big) = \emptyset$.

A.2 Proof of Theorem 1

The proof is in three steps. Consider a couple $(\xi', \xi) \in \Xi^2$ of economic environments such that $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$ and $\mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \subset \pi(\mathbb{A}_{\xi'})$. We use the environments $\xi^\star$ and $\xi'^\star$, and their sets $\mathbb{A}_{\xi^\star} = \mathbb{A}^\star_\xi$ and $\mathbb{A}_{\xi'^\star} = \mathbb{A}^\star_{\xi'}$ of efficient alternatives, as introduced in Definition 5.

1. The assumptions of Proposition 6 are satisfied, as $\mathbb{A}_{\xi'} \subset \mathbb{A}_\xi$ and $\mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \subset \pi(\mathbb{A}_{\xi'})$ correspond to equations (25) and (26). From Proposition 6, we obtain that $\mathbb{A}^\star_{\xi'} = \mathbb{A}_{\xi'^\star} \subset \mathbb{A}_{\xi^\star} = \mathbb{A}^\star_\xi$ and $\pi(\mathbb{A}_{\xi'^\star}) = \pi(\mathbb{A}_{\xi^\star})$. As the mechanism μ is IRA, we deduce from Definition 2 that
$$\pi\big(\mu(\xi^\star)\big) = \pi\big(\mu(\xi'^\star)\big) \ . \tag{32}$$

2. The mechanism μ is INEA. As $\mathbb{A}_{\xi^\star} = \mathbb{A}^\star_\xi \subset \mathbb{A}_\xi$, we deduce from Definition 3 that
$$\mu(\xi^\star) = \mu(\xi) \ . \tag{33}$$
Repeating the argument with the environment $\xi'^\star$, we obtain that
$$\mu(\xi'^\star) = \mu(\xi') \ . \tag{34}$$

3. We conclude that
$$\begin{aligned} \pi\big(\mu(\xi)\big) &= \pi\big(\mu(\xi^\star)\big) \text{ as deduced from (33)} \\ &= \pi\big(\mu(\xi'^\star)\big) \text{ by (32)} \\ &= \pi\big(\mu(\xi')\big) \text{ as deduced from (34).} \end{aligned}$$

A.3 Proof of Theorem 2

We need a preliminary Lemma.

Lemma 3
Consider a given (i, o)-monotone economic environment $\xi = \langle \mathbb{A}_\xi ; a^d_\xi \rangle$, as in Definition 6. Then, for all $(\theta^o, \bar\theta^o) \in \mathcal{O}^o_\xi \times \mathcal{O}^o_\xi$ and all $\bar a \in \mathbb{A}_\xi$, we have
$$\theta^o \leq \pi^o(\bar a) \Rightarrow \pi^i(\bar a) \leq \pi^i\big(a_\xi(\theta^o)\big) \ , \tag{36a}$$
$$\theta^o \leq \pi^o\big(a_\xi(\bar\theta^o)\big) \Rightarrow \pi^i\big(a_\xi(\bar\theta^o)\big) \leq \pi^i\big(a_\xi(\theta^o)\big) \ , \tag{36b}$$
$$\theta^o \leq \bar\theta^o \Rightarrow \pi^i\big(a_\xi(\bar\theta^o)\big) \leq \pi^i\big(a_\xi(\theta^o)\big) \ . \tag{36c}$$

Proof of Lemma 3 Let $(\theta^o, \bar\theta^o) \in \mathcal{O}^o_\xi \times \mathcal{O}^o_\xi$ and $\bar a \in \mathbb{A}_\xi$.

1. Suppose that $\theta^o \leq \pi^o(\bar a)$. By eq. (3) and by definition of a minimum of the set $\{a \in \mathbb{A}_\xi \mid \pi^o(a) \geq \theta^o\}$ (eq. 1), we deduce that $\bar a \geq a_\xi(\theta^o)$. Now, the mapping $\pi^i : \mathbb{A}_\xi \to \mathbb{R}^{|i|}$ is non-increasing by item IIa in the Definition 6 of MonEE. We deduce that $\pi^i(\bar a) \leq \pi^i\big(a_\xi(\theta^o)\big)$. Hence, (36a) is proven.

2. Use (36a) with $\bar a = a_\xi(\bar\theta^o)$ to derive (36b)

3. Let $\theta^o \leq \bar\theta^o$. As $\bar\theta^o \in \mathcal{O}^o_\xi$, by definition of the set $\mathcal{O}^o_\xi$ of satisficing payoffs for outsiders (eq. 2), and by eq. (3), we have that $\bar\theta^o \leq \pi^o\big(a_\xi(\bar\theta^o)\big)$. Therefore, we get that $\theta^o \leq \pi^o\big(a_\xi(\bar\theta^o)\big)$ and we use (36b) to obtain (36c).

Proof of Theorem 2 First, we prove that

$$\big(\theta^i,\theta^o\big) \in \pi(\mathbb{A}_\xi) \Rightarrow \theta^o \in \mathcal{O}^o_\xi \ , \ \ \theta^i \le \pi^i\big(a_\xi(\theta^o)\big) \ , \ \ \theta^o \le \pi^o\big(a_\xi(\theta^o)\big) \ . \tag{37}$$

By definition of $\pi(\mathbb{A}_\xi)$, there exists an alternative $\bar{a} \in \mathbb{A}_\xi$, such that $\big(\theta^i,\theta^o\big) = \theta = \pi(\bar{a}) = \big(\pi^i(\bar{a}),\pi^o(\bar{a})\big)$. On the one hand, as $\theta^o = \pi^o(\bar{a})$, we deduce from item IIb in the Definition 6 of MonEE (eq. 2) that $\theta^o \in \mathcal{O}^o_\xi$, and that $\theta^i = \pi^i(\bar{a}) \le \pi^i\big(a_\xi(\theta^o)\big)$ by (36a). On the other hand, by definition of the set $\mathcal{O}^o_\xi$ of satisficing payoffs for outsiders (eq. 2), and by eq. (3), we obtain that $\theta^o \in \mathcal{O}^o_\xi \Rightarrow \theta^o \le \pi^o\big(a_\xi(\theta^o)\big)$.

Second, we prove that $\mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \subset \pi(\mathbb{A}^o_\xi)$. Let $\theta \in \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big)$. By Definition 4 of a Pareto set, we have that $\theta \in \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big) \subset \pi(\mathbb{A}_\xi)$. By (37), we obtain that $\theta^i \le \pi^i\big(a_\xi(\theta^o)\big)$ and that $\theta^o \le \pi^o\big(a_\xi(\theta^o)\big)$. Putting $\bar{\theta} = \big(\pi^i\big(a_\xi(\theta^o)\big),\pi^o\big(a_\xi(\theta^o)\big)\big)$, we deduce that $\theta \le \bar{\theta}$. As $\bar{\theta} \in \pi(\mathbb{A}_\xi)$ and $\theta \in \mathcal{E}\big(\pi(\mathbb{A}_\xi)\big)$, we obtain that $\bar{\theta} = \theta$, by definition of a (strong) Pareto set. By definition of $\mathbb{A}^o_\xi$ (eq. 4), we conclude that $\theta = \bar{\theta} = \big(\pi^i\big(a_\xi(\theta^o)\big),\pi^o\big(a_\xi(\theta^o)\big)\big) \in \pi(\mathbb{A}^o_\xi)$.

Third, we prove that $\pi(\mathbb{A}^o_\xi) \subset \mathcal{E}^w\big(\pi(\mathbb{A}_\xi)\big)$. The proof is obtained by contradiction. Let $\theta \in \pi(\mathbb{A}^o_\xi)$ and suppose that there exists $\bar{\theta} \in \pi(\mathbb{A}_\xi)$ such that $\theta < \bar{\theta}$ (*i.e*, a strict inequality component by component). On the one hand, by definition of $\mathbb{A}^o_\xi$ (eq. 4), there exists $\hat{\theta}^o \in \mathcal{O}^o_\xi$ such that $\theta = \pi\big(a_\xi(\hat{\theta}^o)\big)$. On the other hand, we have that $\bar{\theta} \le \pi\big(a_\xi(\bar{\theta}^o)\big)$, by (37). We deduce that

$$\pi\big(a_\xi(\hat{\theta}^o)\big) = \theta < \bar{\theta} \le \pi\big(a_\xi(\bar{\theta}^o)\big) \ . \tag{38}$$

As already seen, we have that $\hat{\theta}^o \le \pi^o\big(a_\xi(\hat{\theta}^o)\big)$. Together with inequality (38), this yields $\hat{\theta}^o < \pi^o\big(a_\xi(\bar{\theta}^o)\big)$. By (36b), we deduce that $\pi^i\big(a_\xi(\bar{\theta}^o)\big) \le \pi^i\big(a_\xi(\hat{\theta}^o)\big)$. Now, using inequality (38), we get that $\bar{\theta}^i \le \pi^i\big(a_\xi(\bar{\theta}^o)\big) \le \pi^i\big(a_\xi(\hat{\theta}^o)\big) = \theta^i$. However, this contradicts $\theta < \bar{\theta}$. Therefore, no such $\bar{\theta}$ exists and we conclude that $\theta \in \mathcal{E}^w\big(\pi(\mathbb{A}_\xi)\big)$.

A.4 Proofs for bargaining problems with intertemporal maximin payoffs (IMP)

We prove here that the bargaining problem with IMP defined in Section 4 satisfies the assumptions of Theorem 1. In addition, we prove that if the bargaining problem is MonDAI (Definition 8), then the monotonicity hypotheses MonEE in Definition 6 hold.

Let $\mathbb{C} \subset \mathbb{R}^{n_\mathbb{C}}$ be a nonempty compact set. Consider a finite number $H \in \mathbb{N}^*$ of stakeholders ($H \ge 2$), identified by $h = 1,\dots,H$, bargaining over a sequence of decisions $c(\cdot) = \big(c(t_0), c(t_0+1),\dots\big) \in \mathbb{C}^\mathbb{N}$ that will define the economic trajectory, given the current economic state $x \in \mathbb{X}$.

For a fixed initial state $x \in \mathbb{X}$, we define the set of alternatives $\mathbb{A}_x = \{x\}\times\mathbb{C}^\mathbb{N}$. Thus, with dynamics (5) and instantaneous indicators $I_h : \mathbb{X}\times\mathbb{C} \to \mathbb{R}$, $h = 1,\dots,H$, we obtain the intertemporal payoff $J_h\big(x,c(\cdot)\big)$ in (6) for each stakeholder, hence the payoff mapping $\pi : \mathbb{A}_x \to \mathbb{R}^H$, $(x,c(\cdot)) \mapsto (J_1\big(x,c(\cdot)\big),\dots,\ J_H\big(x,c(\cdot)\big))$ where $J_h\big(x,c(\cdot)\big) = \inf_{t\ge t_0} I_h\big(x(t),c(t)\big)$, $h = 1,\dots,H$.

Recall that we are considering continuous dynamics $G : \mathbb{X}\times\mathbb{C} \to \mathbb{X}$ in (5) and continuous instantaneous indicators $I_h : \mathbb{X}\times\mathbb{C} \to \mathbb{R}$, $h = 1,\dots,H$.

A.4.1 Compacity of the set of alternatives and upper semicontinuity of payoff mapping

Since $\mathbb{C} \subset \mathbb{R}^{n_\mathbb{C}}$ is a nonempty compact metric space, considering $\mathbb{R}^{n_\mathbb{C}}$ is endowed with a metric defined by any norm, the set of alternatives $\mathbb{A}_x = \{x\} \times \mathbb{C}^\mathbb{N}$, for a given initial state of the economy x, will also be a compact metric space.[28]

On the other hand, for a given $h \in \{1, \ldots, H\}$ and $t \geq t_0$, the mapping $(x, c(\cdot)) \in \mathbb{A}_x \mapsto I_h\big(x(t), c(t)\big) \in \mathbb{R}$ is continuous considering the product topology in $\mathbb{A}_x = \{x\} \times \mathbb{C}^\mathbb{N}$, due to the continuity of the following functions: $c(\cdot) \mapsto x(t)$ (from the continuity of dynamics G); $c(\cdot) \mapsto c(t)$ (from the definition of the product topology in $\mathbb{A}_x$); and $(x(t), c(t)) \mapsto I_h\big(x(t), c(t)\big)$ (from the continuity of I_h). Therefore, the function $J_h\big(x, c(\cdot)\big) = \inf_{t \geq t_0} I_h\big(x(t), c(t)\big)$ is upper semicontinuous because it is an infimum of a collection of continuous functions. Thus, it can be concluded that the bargaining problem with IMP satisfies assumptions of Theorem 1, for the payoff function $\pi(x, c(\cdot)) = \big(J_1\big(x, c(\cdot)\big), \ldots, J_H\big(x, c(\cdot)\big)\big)$.

A.4.2 Proof of Proposition 1 (MonDAI implies MonEE properties)

Consider a MonDAI economic environment (Definition 8), with the interest group i and outsiders o being a partition of the set $\{1, \ldots, H\}$ (Definition 8). Under the MonDAI assumption of Definition 8, we obtain that the mapping π^i is non-increasing and that, for any $\theta^o \in \mathbb{R}^{|o|}$, the set $\{a \in \mathbb{A}_x \mid \pi^o(a) \geq \theta^o\} = \{x\} \times \{c(\cdot) \in \mathbb{C}^\mathbb{N} \mid I_h\big(x(t), c(t)\big) \geq \theta_h \ , \ \forall h \in o \ , \ \forall t = t_0, t_0 + 1, \ldots\}$ is empty or it has a minimal element, which is $a^\star_{\theta^o} = (x, c_x^{\theta^o}(\cdot))$ composed of the initial state of the economy x and the control trajectory $c_x^{\theta^o}(\cdot)$, which is generated by the satisficing decision rule $\mathfrak{C}^{\theta^o}$ (Definition 9).

Since the set $\mathbb{A}_x = \{x\} \times \mathbb{C}^\mathbb{N}$ is equipped with the component-wise order $\leq$, we conclude that the economic environment satisfies hypotheses of MonEE in Definition 6.

A.4.3 Proof of Proposition 2

This proof is a direct application of Theorem 2. Indeed, if we consider a bargaining problem in a MonDAI economic environment ξ_x, with outsiders o, and a bargained sequence of satisficing decisions $c^\star(\cdot) = \mathfrak{C}^{\theta^o}$ (see Definition 9) parametrized by some $\theta^o \in \mathbb{R}^{|o|}$, for $t \geq t_0$, then the corresponding dynamic disagreement outcome (eq. 11) is $\theta^d(t) = \big(J_1\big(x(t), c^\star(\cdot)\big), \ldots, J_H\big(x(t), c^\star(\cdot)\big)\big)$, where, for all $s \geq t_0$, $c^\star(s) = \mathfrak{C}^{\theta^o}(x(s))$ and $x(s+1) = G\left(x(s), c^\star(s)\right)$ with $x(t_0) = x$.

Since $(x(t), c^\star(\cdot)) \in \mathbb{A}^o_{x(t)}$ (eq. 9), then $\theta^d(t) \in \pi\big(\mathbb{A}^o_{x(t)}\big)$, and therefore, from Theorem 2 (and eq. 10), one obtains $\theta^d(t) \in \mathcal{E}^w(\pi(\mathbb{A}_{x(t)}))$, $\forall t \geq t_0$.

[28]Recall that the product of any collection of compact topological spaces is compact with respect to the product topology (Tychonoff's theorem). If the collection is countable and composed by compact metric spaces, one can define a distance in the product space inducing the product topology.