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Taxes, Incorporation, and Productivity

Abstract

U.S. businesses can choose to be C-corporations or pass-through entities in the forms of S-corporations, partnerships (notably LLCs), and sole proprietorships. C-corporate status conveys benefits from perpetual legal identity, limited liability, potential for public trading of shares, and ability to retain earnings. However, legal changes have enhanced pass-through alternatives, for example, through the invention of the S-corporation in 1958 and the improved legal status of LLCs at the end of the 1980s. C-corporate form is subject to a time varying tax wedge, which offsets the productivity benefits. In a theoretical framework, firms' productivities associated with C-corporate and pass-through status are distributed as bivariate log-normal. The tax wedge then determines the fraction of firms that opt for C-corporate status, the level of economy-wide output (productivity), the share of total output generated by C-corporations, and the sensitivity of this share to the tax wedge. This framework underlies the empirical analysis of C-corporate shares of business economic activity. Long-difference regressions for 1968-2013 show that a higher tax wedge reduces the C-corporate share of net capital stocks, equity (book value), gross assets, and positive net income, as well as the corporate share of gross investment. The C-corporate shares also exhibit downward trends, likely reflecting underlying legal changes. We infer from the quantitative findings that the downward movement in the tax wedge since 1968 has expanded economy-wide productivity by about 4%.

JEL-Codes: H200, H300, L100, E600.

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As discussed in Barro and Furman (2018)—henceforth, BF—there are sharp differences in views among economists and policymakers about the macroeconomic effects of the 2017 U.S. tax package. Some observers think that the likely effects on economic growth were important; others think not.

BF considered effects from changes in business and individual taxation. The bottom-line forecast was a boost to the growth rate of real GDP by 1.1 percentage points per year over 2018-2019, the two years following the enactment of the tax reform. Over the longer term, say 2020-2028, the estimated growth effect was an increase by about 0.2 percentage point per year. The bulk of the predicted short-run growth response reflected the cuts in individual marginal income-tax rates. The comparatively small predicted rise in the long-run growth rate came from the cuts in business taxes, particularly those applying to C-corporations. For present purposes, we focus on business taxes.

Most of the analysis in BF assessed the incentives to invest in capital within a given legal form of organization: C-corporation or the various forms of pass-through business. However, part of the analysis considered the impact of differential taxation of C-corporations versus pass-through entities on the incentives to choose one legal form versus another. Reductions in the relevant tax wedge, as in the 2017 law, tend to raise the frequency of C-corporate ownership. Moreover, if there are typically productivity advantages associated with C-corporate form—as must be the case if many businesses have chosen this form despite the often large tax penalty—then shifts in the form of legal ownership affect the economy's overall productivity. This paper assesses the effects of business taxation on choices of legal form and, thereby, on productivity.

Previous Research and Framework

Previous research on choices of C-corporate versus pass-through status includes Mackie-Mason and Gordon (1997), Goolsbee (1998), Goolsbee (2004), and Prisinzano and Pearce (2017). Following Mackie-Mason and Gordon, suppose that firm i has output (or productivity) $Y_c(i) > 0$ in corporate (meaning C-corporate) form and $Y_p(i) > 0$ in non-corporate or pass-through form. The respective tax rates, taken as proportionate to output, are $\tau_c < 1$ and $\tau_p < 1$. Negative tax rates, constituting subsidies, can be admitted. The tax rates for each legal form are assumed to be the same for all firms, although they could instead be allowed to vary across firms.[1] Firm i opts for corporate form if

(1) $$(1-\tau_c)Y_c(i) > (1-\tau_p)Y_p(i).$$

This condition is analogous to the one derived by Mackie-Mason and Gordon (1997, equations [1] and [2]).

We can rewrite equation (1) as

(2) $$y(i) \equiv \log\left[\frac{Y_c(i)}{Y_p(i)}\right] > \log\left[\frac{(1-\tau_p)}{(1-\tau_c)}\right] \equiv \tau$$

The term on the far right is the tax wedge, τ; that is, the tax penalty for being corporate rather than pass-through. Equation (2) says that if this wedge is positive, a business has to enjoy at least the offsetting proportionate productivity advantage, *y(i)*, in order to opt for corporate form. If the magnitudes of τ_c and τ_p are much less than one, then $\tau \approx \tau_c - \tau_p$. Generally, τ is increasing in τ_c and decreasing in τ_p.

[1]However, owners would tend to be selected for having tax rates that are comparatively low for the given legal form of business. For example, owners of dividend-paying stocks would tend to have relatively low tax rates on dividends.

If tax rates are the same for all firms, the key matter for choices of legal form is the frequency distribution of the proportionate productivity advantage, *y(i)*. In the overall population of firms, the fraction opting to be corporate is one minus the cumulative density of *y(i)* evaluated at the cutoff τ.

Comments on the Setup

The framework treats a firm's potential outputs, $Y_c(i)$ and $Y_p(i)$, as dependent only on the choice of legal form of organization. We could extend the model to include variable factor inputs, such as labor and capital chosen by corporate and pass-through businesses. The tax rates τ_c and τ_p would then have the usual effects on quantities of inputs demanded. If taxes are levied on net business income with labor payments fully expensed, then the tax rates would not directly distort the margins associated with labor input. Similarly, if capital outlays are fully expensed (with loss realizations fully allowed), there would be no direct distortion on the margins associated with capital input (as in King and Fullerton [1984]). This last result applies also to labor when labor has capital-like dimensions; for example, when output and sales depend on lagged labor input.

In the present setup, the only distortion comes from the difference in the tax rates levied on corporations and pass-through businesses; that is, the tax wedge τ given in equation (2). In this setting, the first-best outcome will correspond to $\tau=0$, because distortions arise only when the two forms of business organization are taxed at different rates. The levels of taxation—τ_c and τ_p—will not matter here as long as the two tax rates are equal. We could extend the framework to have distortions from levels of tax rates as well as from differences between the rates. However, that extension would leave the distortion associated with differential taxation, and that effect is likely to be largely separable from those involving the levels of tax rates.

Hence, it seems desirable to focus on a framework that abstracts from choices of factor inputs and in which only differential taxation matters.

The Tax Wedge

To implement equation (2) for U.S. business data, we need a time series for the tax wedge, τ, which depends on the tax rates τ_c on C-corporations and τ_p on pass-through businesses. Figure 1 shows an approximate empirical measure of τ based on concepts of τ_c and τ_p that are available for the United States back to 1914. The tax rate τ_c is represented by the top federal rate on C-corporate profits.[2] The rate τ_p is represented by the labor-income weighted average marginal tax rate from the federal individual income tax, as constructed by Barro and Sahasakul (1983), Barro and Redlick (2011), and the Tax Policy Center.[3] The tax wedge, τ, is then computed from equation (2) as $\log[\frac{(1-\tau_p)}{(1-\tau_c)}]$. Although the empirical measures of τ_c and τ_p are imperfect (for example, neglecting taxation of dividends and capital gains in the construction of τ_c), the computed τ is likely to capture the main elements of differential taxation. Moreover, this concept turns out to work reasonably well empirically.

Figure 1 shows the U.S. time series for τ_c, τ_p, and τ. Note that the wedge, τ, peaked at 0.48 in 1954 and 1968 and has since had a strong downward trend. To understand the post-1968

[2]Data are in IRS, *Statistics of Income Bulletin*, Fall 2003, and in recent issues of IRS *Statistics of Income*. The use of the top rate neglects graduation in the corporate-profits tax schedule. This assumption seems reasonable because most of taxable profits appears to be taxed at the top federal rate. We are not including the taxation of corporate profits by state governments—Barro and Furman (2018) estimated the average state tax rate on corporate profits in 2017 to be 0.04, compared to the top federal rate of 0.35. Goolsbee (2004) has examined corporate-profits taxes across states. More importantly, the construction neglects taxation of C-corporate owners on dividends and capital gains. The effective tax rates on these forms of income tend to be low because of deferrals of income realizations (at the corporate level through retained earnings and at the individual level by delaying sales of assets) and because owners tend to be selected in accordance with their tax rates—for example, dividend-paying stocks tending to be owned by entities with low tax rates on dividends.

[3]Since potential owners of pass-through businesses tend to have relatively high incomes, this measure may understate the pass-through tax rate. However, the labor-income-weighted average marginal tax rate is well above a simple average of marginal rates (see Barro and Sahasakul [1983]). The measure also does not count levies other than federal income taxes (such as self-employment taxes) that apply to pass-through income.

pattern, note first that, in 1968, the C-corporate tax rate, τ_c=0.53, was well above the pass-through rate, τ_p=0.24, so that τ was very high, 0.48. Then, up to 1986, τ_c fell to 0.46, while τ_p rose to 0.26—both changes contributed to a fall in τ, which reached 0.32. The Reagan 1986 tax reform, applying between 1986 and 1988, is well-known to have lowered individual marginal income-tax rates. However, the fall in the C-corporate rate, to τ_c=0.34, more than offset the fall in τ_p to 0.20, so that τ fell further, to 0.20. From 1988 to 2017, τ_c was virtually unchanged, but τ_p rose back to 0.24, as the Reagan tax changes were substantially undone. Therefore, τ fell again, to 0.15. Finally, in 2018, the large cut in τ_c, to 0.21, more than offset the fall in τ_p, which reached 0.22. Consequently, τ became negative, -0.01, for the first time.

In the earlier period, the main elements leading to the upward trend in τ were the increases in τ_c in World War I, the late 1930s, World War II, and the Korean War. The wedge, τ, did not change appreciably between 1954 and 1969.

The Corporate Productivity Advantage

We start with a list of productivity benefits that associate qualitatively with corporate legal ownership. We include historical context on how U.S. legal changes have shifted these benefits when compared to those arising from pass-through alternatives.

1. A corporation is a distinct and perpetual legal entity, the structure of which—unlike partnerships—is not compromised substantially by the departure of its owner(s). Kuran (2004) argues that the lack of independent corporate identity was a major constraint on economic development of Muslim countries after the Industrial Revolution. The Ottoman Empire instituted its first law of corporations only in 1908, and Egypt followed within a year.

2. C-corporations offer the potential for convenient public trading of shares, often on organized markets. This public trading is important for the raising of capital and for gaining information from market prices. Notably, this information is useful for executive-compensation decisions, bond financings, and valuing interests of departing owners. Public trading eventually became available in the United States for limited partners in a few sectors as master limited partnerships (MLPs) or publicly traded partnerships (PTPs). These legal forms started in 1981 with Apache Petroleum and eventually even included the Boston Celtics basketball team. Subsequently, these forms of ownership were available mainly for companies operating in the energy sector, due to provisions of a 1987 federal law on the sources of a company's income. However, some financial firms, such as Blackstone, qualified for MLP status. (Blackstone is currently considering a shift to C-corporate status, following the 2017 tax reform and the apparently successful transformation of KKR into a C-corporation in 2018.[4])

3. Limited liability for C-corporations. This status applies also to pass-through S-corporations, which were created in 1958. However, S-corporations have major limitations on numbers and types of shareholders—although the allowable number increased substantially over time from the original 10 to the current 100.[5] S-corporations are also restricted to have only one class of stock with respect to rights to distributions and liquidation proceeds. The major pass-through alternative to the S-corporation is the partnership, which does not feature the restrictions on ownership that apply to S-corporations. The partnership form

[4]See *Institutional Investor*, July 19, 2018.
[5]The number rose to 15 in 1976, 25 in 1981, 35 in 1983, 75 in 1997, and 100 in 2004. See Sicular (2014).

has a long history, but a key innovation was the invention of the limited liability company (LLC) in Wyoming in 1977. LLCs, which offer limited liability for partner-owners, are regulated at the state level and became increasingly popular after the IRS determined in 1988 that LLCs could be taxed as pass-through partnerships.

4. The retention of earnings is permissible in C-corporations but not in pass-through businesses. This retention is useful for financing of investment and for deferring taxes on dividends.

5. C-corporations and various forms of pass-through businesses have numerous differences in filing requirements, regulations, and government supervision.

We now describe our formal treatment of the frequency distribution of the corporate productivity advantage, $y \equiv \log(\frac{Y_c}{Y_p})$, which appears in equation (2) (where we now drop the index i). We consider this distribution at a point in time, which features a given legal/regulatory regime applying to C-corporations and pass-through alternatives. Over time, changes in laws and regulations can shift the entire distribution of y. Implicitly, we are also holding constant the structure of production across sectors—changes in this composition can also affect the distribution of y; that is, corporate benefits may be more important in some types of business than in others.

We assume that $log(Y_c)$ and $log(Y_p)$ are distributed bivariate normal with respective means and standard deviations of μ_c, σ_c, μ_p, and σ_p. The correlation coefficient between the two random variables is ρ. This specification implies that $y \equiv \log(\frac{Y_c}{Y_p})$ is distributed normally with mean $\mu = \mu_c - \mu_p$ and variance $\sigma^2 = \sigma_c^2 + \sigma_p^2 - 2\rho\sigma_c\sigma_p$. The fraction of firms that opt to be pass-through is the cumulative normal value for y corresponding to the cutoff τ, and the fraction corporate is one minus this cumulative normal value.[6]

We want to assess the impact of τ on overall output (productivity) and the fraction of output generated by the corporate sector. To make these calculations, we have to work out the expectations of corporate output, Y_c, conditional on $y \equiv \log(\frac{Y_c}{Y_p}) \geq \tau$, and of pass-through output, Y_p, conditional on $y < \tau$. The appendix shows that the expectation of Y_c, conditional on y, is given by:

$$(3) \qquad E(Y_c|\text{y}) = \exp\{\mu_c + \left(\frac{1}{\sigma^2}\right) \cdot \left[\sigma_c \cdot \left(\sigma_c - \rho\sigma_p\right) \cdot (y - \mu) + 0.5 \cdot (1 - \rho^2)\sigma_c^2\sigma_p^2\right]\}.$$

Using equation (3), the appendix shows that output generated by the corporate sector is given by:

$$(4) \qquad Prob.(y \geq \tau) \cdot E(Y_c|y \geq \tau) = [\exp(\mu_c + 0.5\sigma_c^2)] \cdot [1 - \Phi(\tau')],$$

where $\tau' = \left(\frac{1}{\sigma}\right)\left[\tau - \mu - \sigma_c\left(\sigma_c - \rho\sigma_p\right)\right]$, Prob. denotes probability, and $\Phi(\cdot)$ is the cumulative standard normal density. The formula for output generated by the pass-through sector is analogous except that the parameters related to *c* and *p* are switched, and τ is replaced by $-\tau$ in the expression for τ'.

The quantitative results involve five parameters: μ_c, σ_c, μ_p, σ_p, and ρ. However, one parameter can be eliminated as a normalization related to the level of overall output. We choose a normalization so that the maximum output, corresponding to a zero tax wedge, τ=0, equals 1.0. We know in the data (described in detail below) that the C-corporate share of economic activity is high—in a range of something like 0.6 to 0.9—even when the tax wedge, τ, is as high as 0.5. To replicate this pattern, the model requires μ_c to be well above μ_p—that is, corporate

[6]The fraction corporate is $1-\Phi[(\tau-\mu)/\sigma]$, where $\Phi(\cdot)$ is the cumulative standard normal density.

productivity must typically be well above pass-through productivity. We assume a gap of μ_c-μ_p=0.75 in the baseline analysis. (The levels of the two μ parameters are chosen to get the peak level of output equal to 1.0, given the values of the other parameters.) We assume $\rho>0$ and set it to 0.25 in the baseline. Finally, we assume $\sigma_c=\sigma_p=0.5$ in the baseline. Some of these parameters are not well pinned down, and we consider later how the results vary with changes in the parameters.

Figures 2-6 give the model's results for the baseline parameter values: μ_c=-0.146, μ_p=-0.896, σ_c= σ_p=0.5, and ρ=0.25. Figure 2 shows that the fraction of firms opting to be corporate declines monotonically with the tax wedge, τ. For τ between 0 and 0.5 (the empirical range indicated in Figure 1), the corporate share of numbers is between 0.66 and 0.89. However, it is unclear that the number of firms is an empirically meaningful concept, and we focus on results related to corporate and total output.

Figure 3 shows the relation of economy-wide output (productivity) to the tax wedge, τ. Because the only distortion in the model is this tax wedge, the maximum of output occurs at $\tau=0$. (The parameters were chosen, as a normalization, so that this maximal output equals 1.0.) Figure 4 shows explicitly the marginal effect of τ on output (and productivity). This marginal effect is positive when $\tau<0$ and negative when $\tau>0$.

Note that we are using the revealed preference of business owners to infer the effects of the tax wedge, τ, on economy-wide output and productivity. Specifically, when $\tau>0$, a firm opts to be corporate only if the productivity advantage associated with corporate form is sufficient to justify the tax penalty. Moreover, a firm at the margin must have a productivity advantage that exactly compensates for the tax penalty.

Figure 5 shows the relation of the corporate share of output to the tax wedge, τ. Note that, at a given τ, the computed corporate output share exceeds the corporate share of numbers, shown in Figure 2. This result arises because—in the specification where $\mu_c > \mu_p$ and $\sigma_c = \sigma_p$—the typical corporate firm is more productive that the typical pass-through firm. The results for the corporate output share in Figure 5 can be matched with data, described below, on the C-corporate share of economic activity. In particular, the model (with the baseline parameters) implies that the corporate output share is between 0.79 and 0.89 when τ in Figure 1 is in the range 0.13 to 0.48 that applies in the main regression sample from 1968 to 2013. This predicted range of corporate output shares accords well with the data—but, of course, the relative values of the parameters μ_c and μ_p were chosen to deliver this match.

Figure 6 shows the marginal effect of the tax wedge, τ, on the corporate share of output. Consistent with Figure 5, this marginal effect is negative throughout. Quantitatively, the marginal effect in Figure 6 is between -0.24 and -0.35 when τ in Figure 1 is in the range from 0.13 to 0.48 that applies in the regression sample from 1968 to 2013. These results suggest that a linear relation between the corporate output share and the tax wedge may be a reasonable approximation; that is, the slope does not vary greatly over the relevant range of τ. The marginal effects in the model (Figure 6) should correspond to regression coefficients in a relation between the C-corporate share of economic activity and the tax wedge. The magnitudes of coefficients found in the empirical regressions turn out to accord reasonably well with those generated by the model, using the baseline parameters.

The results in Figures 2-6 depend on the baseline parameter settings, some of which are not well pinned down. Table 1 shows how the predictions about the corporate output share, Y_c/Y, and the slope of this share with respect to τ change when different parameter values are assumed. The table considers alternative values of σ_c, σ_p, and ρ. With the baseline parameters, the marginal effect of τ varied between -0.24 and -0.35 when τ was between 0.13 and 0.48. Now a wider range of values applies. For example, the magnitude of the slope can be as low as 0.11 and as high as 0.53.

C-Corporate Share of Economic Activity

We have several empirical measures of the C-corporate share of economic activity, based on IRS data and mostly covering the period 1958 to 2013. (See the various items under Internal Revenue Service in the References.) Figures 7-9 apply to stock measures of economic activity—net capital stocks, equity (book value), and gross assets. These variables are available for C-corporations, S-corporations, and partnerships, but not for sole proprietorships. The partnership numbers on net capital stocks and equity were interpolated based on data available every two years from 1959 to 1975 and annually for 1977-1982 and 1988-2013.

Figure 7 has the shares of business net capital stocks (in the total that includes C-corporations, S-corporations, and partnerships). The C-corporate share was 0.95 in 1958 and trended downward to 0.53 in 2013. The main offsetting increase, shown in the figure, was in the partnership share, which went from 0.04 in 1958 to 0.40 in 2013. Legal changes noted before, especially for LLCs, are likely responsible for part of this trend, but it is hard to be precise on the timing. The share for S-corporations was 0.004 in 1958 (the first year of existence), rose to 0.025 in 1986, then jumped upward to 0.074 by 1999. The share then fell to 0.067 in 2013, probably because of increased competition from partnership form, especially LLCs.

Figure 8 has the shares of business equity (book value). The trends are similar to those for net capital stocks, although the share of C-corporations in equity at 0.69 in 2013 is notably higher than that for net capital stocks. Correspondingly, the partnership share in 2013 was 0.29.

There is also more of an indication that S-corporations are being driven out of the market, with the share down to 0.024 in 2013, compared to a peak of 0.038 in 1990. Eventually, the attractiveness of the LLC may make the S-corporation obsolete.

Figure 9 has the shares for business gross assets. This concept of C-corporate share was used by Mackie-Mason and Gordon (1997). The trends in business gross assets are similar to those for net capital stocks and equity, but the C-corporate share of gross assets has not declined as much—the share in 2013 was 0.75, whereas that for partnerships was 0.22. The S-corporation share of gross assets in 2013 was 0.033, compared to 0.037 in 1990.

Figures 10 and 11 apply to business net income, which is a flow measure of economic activity. Variants of these data were used by Mackie-Mason and Gordon (1997) and Prisinzano and Pearce (2017). Figure 10 shows shares in positive net income (excluding businesses with losses). In this case, data are available from 1917 to 2013 and for sole proprietorships as well as C-corporations, S-corporations, and partnerships. The shares of positive net income are highly volatile because of the strong sensitivity of the various forms of this net income to the business cycle. However, recent trends are similar to those shown in Figures 7-9. From 1979 to 2013, the C-corporate share of positive net income fell from 0.71 to 0.43, the partnership share rose from 0.09 to 0.31, the S-corporate share increased from 0.02 to 0.15 (but has been flat since 2001), and the sole proprietor share fell from 0.18 to 0.11.

Figure 11 shows shares in overall business net income—thereby including businesses with losses. These share measures are even more volatile than those in Figure 10 because of the extreme sensitivity of the various forms of negative net income to the business cycle.

Figure 12 shows data from the Bureau of Economic Analysis (BEA) on the corporate share of business gross investment.[7] These data, available from 1901 to 2017, have the drawback of combining C- corporations with S-corporations (originating in 1958). On the plus side, the BEA data may be a useful supplement to the IRS information because the two agencies draw mostly from different sources.[8] Surprisingly, the BEA data indicate that the corporate share of gross investment shows no clear trend over the period that coincides with the various IRS series. For example, the BEA's corporate investment share varies only between 84% and 89% from 1974 to 2017.

Regression Framework

The main regression analysis relates the C-corporate shares of net capital stock (Figure 7), equity (Figure 8), gross assets (Figure 9), and positive net income (Figure 10) to the measured tax wedge, τ (Figure 1). This wedge depends on the federal C-corporate top tax rate, our representation of the C-corporate tax rate, τ_c, and the federal average marginal tax rate on individual income, which is our measure of the pass-through tax rate, τ_p. Specifically, $\tau = \log[\frac{(1-\tau_p)}{(1-\tau_c)}]$ from equation (2). The regressions enter the two parts of τ separately—as $\log(1-\tau_c)$ and $\log(1-\tau_p)$—and then check whether the sum of the two estimated coefficients differs

[7]The BEA also reports the share of the business capital stock held by corporations (a combination of C- and S-corporations), based on a perpetual-inventory method. However, these data are problematic because they do not pick up, in a timely way, the effects on business ownership of stocks of capital that arise from changes in ownership; for example, when a business shifts from C-corporation to LLC or vice versa. In the BEA data, these ownership changes do not show up contemporaneously as shifts in ownership of capital stocks—that get reflected only over time when investment outlays are associated with the new form of ownership. See Bureau of Economic Analysis (2003, p. M-26). Goolsbee (1998) says that he used the BEA data on corporate share of capital stock from 1900 to 1939. However, he actually used, apparently because of confusion over the BEA table headings, the data on corporate share of gross investment.
[8]The BEA data are gathered primarily from the Economic Census conducted every five years by the Census Bureau. The BEA interpolates these data over non-Census years in a sophisticated manner involving data from a variety of sources, including the BEA's survey of plant & equipment expenditures, IRS data, and multiple other government surveys that focus on specific sectors. See Bureau of Economic Analysis (2003, 2019).

significantly from the theoretical value of zero. Additional regressions use the corporate shares of gross investment (Figure 12), although these data combine C-corporations with S-corporations.

In principle, we would like to isolate variables, such as changes in the legal/regulatory system, that influence the relative attractiveness of C-corporate and pass-through forms. We think that most legal changes over recent decades have gone in the direction of favoring pass-through alternatives to C-corporations—notable here are the invention of the S-corporation in 1958 and the IRS tax ruling in 1988 that favored LLCs. However, we have been unable to quantify the timing of these influences. In the present regressions, we allow for linear and quadratic trends as crude ways to proxy for these kinds of omitted determinants.

Level regressions for C-corporate shares, as implemented in Mackie-Mason and Gordon (1997, Table III), are probably not meaningful. Specifically, as is evident from Figures 7-10, the C-corporate share measures have strong persistence and may be non-stationary. This problem was noted by Prisinzano and Pearce (2017), who emphasized regressions (for variables based on C-corporate shares of net income) with annual first-differences (their Tables IV and V). However, this specification is likely to be heavily influenced by measurement error, particularly because the timing between changes in the tax system and changes in C-corporate shares are not precisely determined. Given these concerns, our empirical analysis relies on long-difference estimation; specifically, on 10-year changes in C-corporate or corporate shares and the tax-rate variables. Because this procedure creates serial dependence in the error terms for the overlapping data, we used the Newey-West procedure with a bandwidth of ten or more years to construct standard errors of the estimated coefficients.

Table 2, parts 1 and 2, has regressions where the dependent variable is the ten-year difference of the C-corporate share of net business capital stocks (columns 1 and 2), of equity or book value (columns 3 and 4), of gross assets (columns 5 and 6), and of positive net income (columns 7 and 8).[9] The sample period is 1968 to 2013. In these regressions, the only regressors are a constant (which picks up a trend in the levels) and the ten-year changes in the two tax-rate variables, $\log(1-\tau_c)$ and $\log(1-\tau_p)$. Columns 2, 4, 6, and 8 add a year variable, which allows for a quadratic trend in the levels.

Consider first the regressions without a quadratic trend (columns 1, 3, 5, and 7). The estimated coefficients on $\log(1-\tau_c)$ are all positive, as predicted; that is, the estimated effects of τ_c on the C-corporate shares are negative. These estimated coefficients are statistically significantly from zero at the 5% level for net capital stock, equity, and gross assets (columns 1, 3, and 5), but not for positive net income (column 7). The magnitudes of these estimated coefficients are in a range between 0.2 and 0.4. The estimated coefficients on $\log(1-\tau_p)$ are all negative, as predicted; that is, the estimated effects of τ_p on the C-corporate shares are positive. These estimated coefficients are all statistically significantly different from zero at the 1% level. The magnitudes of these estimated coefficients are in a range from 0.3 to 1.0.

The hypothesis of equal magnitudes of the two tax coefficients (as implied by the form of the tax wedge, τ, in equation [2]) is rejected at a p-value less than 0.01 in Table 2, columns 1, 5, and 7, and with a p-value of 0.07 in column 3. Thus, the econometric results deviate from the precise theoretical restriction—possibly because the empirical measures of τ_c and τ_p are

[9]Mackie-Mason and Gordon (1997, Table III) and Prisinzano and Pearce (2017, Tables II-V) note that tax effects would have the opposite sign for business negative net income if these losses were expected ex ante (and to the extent that losses can be taken for tax purposes). Hence, they look separately at regressions involving positive or negative net income. We find tax effects of roughly zero if we look at C-corporate shares of business negative net income.

imperfect. Nevertheless, in a broader sense, the estimated magnitudes of the coefficients accord reasonably well with those predicted by the model. As noted before, the predicted range of marginal tax-wedge effects in Figure 6 was between 0.24 and 0.35 when the tax wedge, τ, was in the range from 0.13 to 0.48 that applies in the regression sample, 1968 to 2013.

Regressions that allow for a quadratic trend are in Table 2, columns 2, 4, 6, and 8. The magnitudes of the estimated tax coefficients are smaller than before, but the broad nature of the results does not change. For example, the estimated coefficients on $\log(1-\tau_c)$ are still all positive, but statistically significantly different from zero at the 1% level only for C-corporate shares of equity and gross assets (columns 2 and 6). The estimated coefficients on $\log(1-\tau_p)$ are still all negative, and all are statistically significantly different from zero at least at the 5% level. As before, the hypothesis of equal magnitudes of the two tax coefficients is rejected, now in all cases at the 5% level. But the magnitudes of the estimated coefficients still accord roughly with those predicted by the model (Figure 6).

The estimated trends in C-corporate shares (corresponding to the constant terms in the regressions) are significantly negative at the 1% level for net capital stocks, equity, and gross assets (columns 1, 3, and 5 of Table 2). These results accord with the patterns shown in Figures 7-9, although the negative trend for equity seems to set in only around 1980. The estimated trend is significantly negative at the 5% level for positive net income (column 7). This result accords with Figure 10, which shows a downward trend only since around 1980. The quadratic trend terms in C-corporate shares (corresponding to the year variables in the regressions) are significantly negative at the 1% level for equity and gross assets (columns 2 and 4), at the 5% level for positive net income (column 8), and at the 10% level for net capital stocks (column 2).

The results show clearly that there are unexplained trend-like changes in the various concepts of C-corporate shares of economic activity over the regression sample, 1968-2013. At this point, we have not related these trends to fundamental forces, such as the legal/regulatory changes that have been discussed qualitatively or to changes in the composition of output.[10] On a positive note, the estimated tax effects on C-corporate shares emerge even when the trend terms are held constant. And these estimated tax effects accord in a rough way with theoretical expectations.

We should stress that the estimated effects of the tax changes from 1968 to 2013 involve a sharp overall drop in the tax wedge, τ (see Figure 1). Thus, this tax effect goes against the estimated trends, which associate with declining C-corporate shares of economic activity (Figures 7-10). That is, on their own, the tax changes from 1968 to 2013 should have led to substantial increases in the C-corporate share of economic activity.

Table 2, part 3, has results using the BEA data on corporate share of business gross investment (Figure 12). As already noted, the BEA information combines C- and S-corporations; hence, the tax variables do not match up precisely with the corporate data (which partly include pass-through businesses). However, this consideration may not be too important because the data on S-corporate shares of net business capital stocks in Figure 7 suggest that S-corporations would not comprise a major share of business gross investment.

The estimated tax coefficients with the BEA data in columns 9 and 10 align well with those found with the IRS data in columns 1-8. These results obtain even though the estimated trends are very different for the BEA and IRS data. For example, in column 9, the estimated

[10]We have a sectoral breakdown from the IRS for business gross assets and positive net income. The eight sectors are agriculture, construction, finance-insurance-real estate, manufacturing, mining, services, trade, and transportation. We have not found any clear effects from changing sectoral composition on C-corporate shares of gross assets or positive net income.

trend for the BEA corporate gross investment share is nil, consistent with the pattern in Figure 12, whereas the corresponding trends for the IRS C-corporate share data (columns 1, 3, 5, and 7 and Figures 7-10) were strongly negative.

The results on business gross investment in Columns 11 and 12 apply to the period 1924-2017, which can be used because of the long-term availability of the BEA data. The estimated coefficients accord roughly with those from the shorter sample, 1968-2013, although the fit of the regression turns out to be poor for 1924-1967.

As mentioned, the 10-year difference estimates shown in Table 2 calculate the standard errors of the estimated coefficients by the Newey-West procedure with a bandwidth of 10 years. The results change little if the bandwidth is raised to 15 years. For example, the estimated standard errors on the two tax coefficients were 0.096 and 0.107 in column 1 (with a 10-year bandwidth). These values change with the use of a 15-year bandwidth to 0.101 and 0.109, respectively. Similar changes apply to the other regression results in Table 2.

Historical Productivity Effects

Table 3 shows the implications of the model for the productivity effects from the sizable cut in the tax wedge, τ, that occurred over recent decades. We gauge the effects of τ on overall productivity (output) from the results shown in Figure 3—with the regression results providing some evidence that these modeling predictions would be informative. Note that these effects on productivity reflect only the associated changes in legal form of organization—the present analysis does not deal with effects on capital accumulation (the focus of Barro and Furman [2018]).

Table 3 shows the main historical tax changes—discussed before—that affected the tax wedge, τ, since the start of the regression sample in 1968. In 1968, when τ was at 0.476, the model's associated level of productivity, corresponding to Figure 3, is 0.964 (relative to the peak, which was normalized to equal 1.0). By 1986, the cut in τ to 0.318 is estimated to have raised productivity to 0.986 or by 2.3% compared to 1968. The further cut in τ to 0.199 in 1988 is then estimated to raise productivity to 0.996 or by another 1.0%. In 2017, τ reached 0.151, which implied a 0.2% rise in productivity compared to 1988. Finally, the 2017 tax reform implied that τ fell to -0.006 in 2018, which raised estimated productivity by another 0.2%. Note that, although the cut in τ in 2018 is large (from 0.151 to -0.006), the estimated response of productivity (due to the shift in legal form of organization) is only moderate. A key point here is that the comparatively low level of the initial τ (0.15 in 2017) implies that the economy is gauged to be operating in a range where the sensitivity of productivity to the tax wedge is small (Figures 3 and 4).

In terms of cumulative effects, the full cut in the tax wedge, τ, from 1968 to 2018 is estimated to have raised economy-wide productivity by 3.7%. Thus, this cumulative effect is substantial. Moreover, in the model, this change corresponds to reduced distortion and, hence, to a gain in efficiency. However, in terms of annual productivity growth, the contribution from this 50-year cumulation of cuts in tax wedges—and the resulting shifts in legal form of organization—would be only around 0.1 percentage points per year.

In the background, there are also substantial trend effects, which led to declines in the various measures of C-corporate shares of economic activity. However, since we have not related these trend changes to fundamentals, we cannot assess these changes from a welfare standpoint.

References

Barro, Robert J. and Jason Furman. 2018. "Macroeconomic Effects of the 2017 Tax Reform." *Brookings Papers on Economic Activity* Spring: 257-345.

Barro, Robert J. and Charles J. Redlick. 2011. "Macroeconomic Effects from Government Purchases and Taxes." *Quarterly Journal of Economics* 126(1): 51-102.

Barro, Robert J. and Chaipat Sahasakul. 1983. "Measuring the Average Marginal Tax Rate from the Individual Income Tax." *Journal of Business* 56(4): 419-452.

Bureau of Economic Analysis. 2003. *Fixed Assets and Consumer Durable Goods in the United States, 1925–99*. Washington, DC: U.S. Government Printing Office, September.

Bureau of Economic Analysis. 2019. "Fixed Assets Accounts Table." Web resource: https://apps.bea.gov/itable/index_FA.cfm. Accessed January 22.

Goolsbee, Austan. 1998. "Taxes, Organizational Form, and the Deadweight Loss of the Corporate Income Tax." *Journal of Public Economics* 69(1):143-152.

Goolsbee, Austan. 2004. "The Impact of the Corporate Income Tax: Evidence from State Organizational Form Data." *Journal of Public Economics* 88: 2283-2299.

Internal Revenue Service. 1958-2013. *Statistics of Income: Corporation Income Tax Returns*. Washington, D.C.: Government Printing Office.

Internal Revenue Service. 1958-1976. *Statistics of Income: Business Income Tax Returns*. Washington, D.C.: Government Printing Office.

Internal Revenue Service. 1977-1981. *Statistics of Income: Sole Proprietorship Returns*. Washington, D.C.: Government Printing Office.

Internal Revenue Service. 1982-2013. *Statistics of Income Bulletin: Sole Proprietorship Returns*. Washington, D.C.: Government Printing Office.

Internal Revenue Service. 1977-1982. *Statistics of Income: Partnership Returns*. Washington, D.C.: Government Printing Office.

Internal Revenue Service. 1983-2013. *Statistics of Income Bulletin: Partnership Returns*. Washington, D.C.: Government Printing Office.

King, Mervyn A., and Don Fullerton. 1984. "Chapter 2: The Theoretical Framework." In *The Taxation of Income from Capital: A Comparative Study of the United States, the United Kingdom, Sweden, and West Germany*. Chicago: University of Chicago Press.

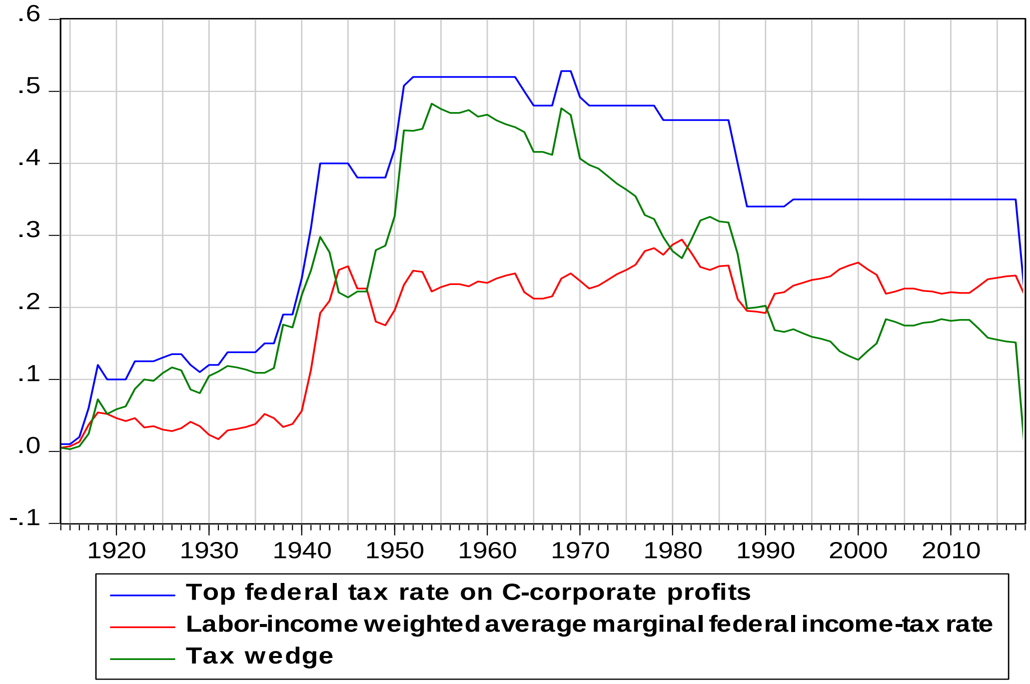
Kuran, Timur. 2004. “Why the Middle East Is Economically Underdeveloped: Historical Mechanisms of Institutional Stagnation.” *Journal of Economic Perspectives* 18(3): 71-90.

Mackie-Mason, Jeffrey K. and Roger H. Gordon. 1997. “How Much Do Taxes Discourage Incorporation?” *Journal of Finance*. 52(2): 477-505.

Prisinzano, Richard and James Pearce. 2017. “Tax-Based Switching of Business Income.” unpublished, Penn-Wharton Budget Model and Office of Tax Analysis.

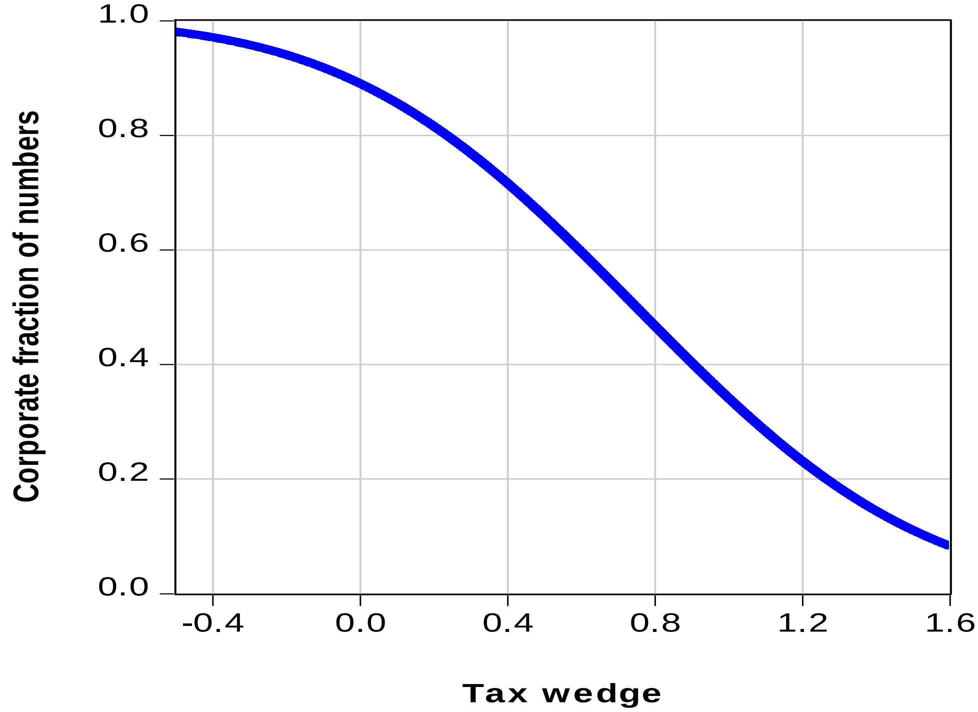
Sicular, David R. 2014. “Supchapter S at 55—Has Time Passed this Passthrough By? Maybe Not.” *Tax Lawyer*, 68(1), Fall, 185-238.

Figure 1 Estimated U.S. Tax Wedge, 1914-2018



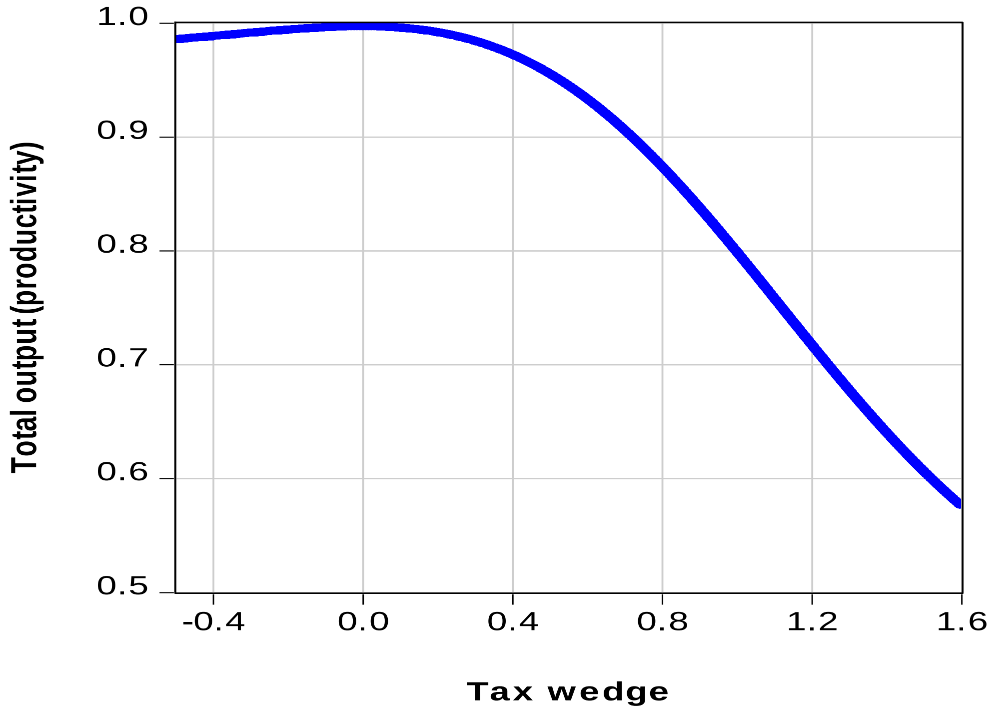
Note: τ_c (measured by the top federal tax rate on C-corporate profits) corresponds to the blue graph and τ_p (measured by the labor-income weighted average marginal tax rate from the federal income tax) to the red graph. The tax wedge, τ, which equals $\log[(1-\tau_p)/(1-\tau_c)]$ from equation (2), corresponds to the green graph.

Figure 2 Corporate Fraction of Numbers of Firms as Function of Tax Wedge, τ



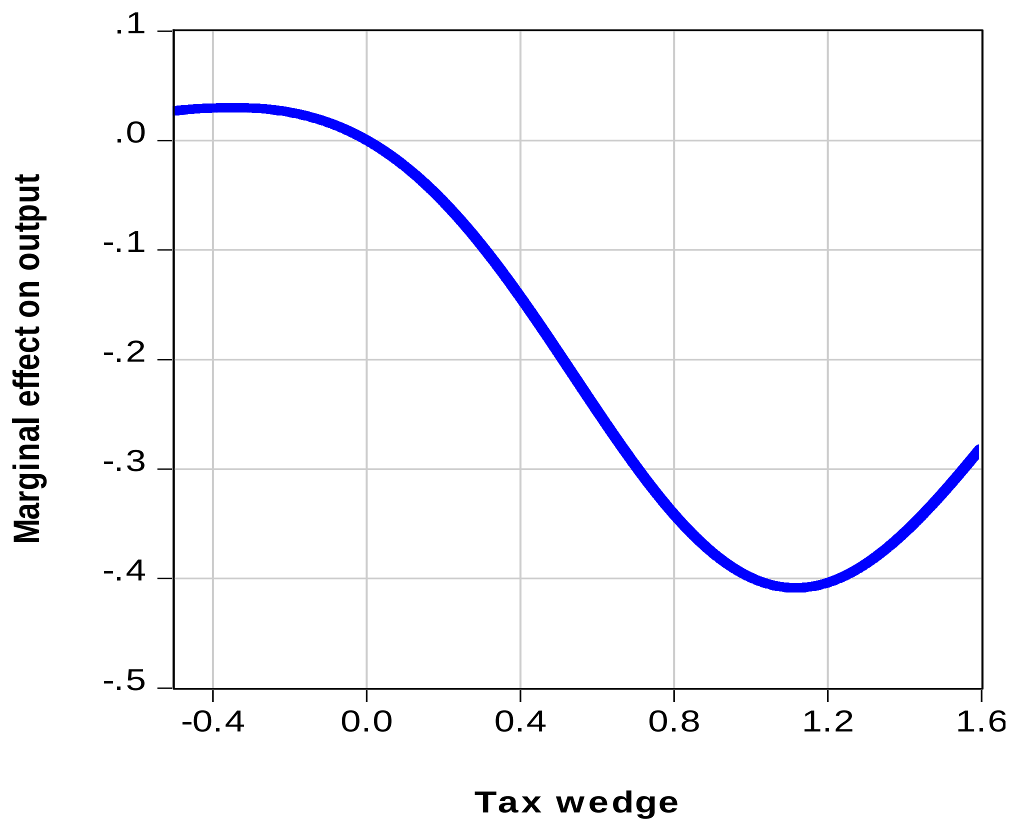
Note: This graph uses the baseline parameter values: μ_c=-0.146, μ_p=-0.896, σ_c= σ_p=0.5, and ρ=0.25. The corporate share of numbers of firms declines monotonically with the tax wedge, $\tau = \log[\frac{(1-\tau_p)}{(1-\tau_c)}]$. This share approaches 1 as τ approaches -∞ (as τ_p approaches 1) and approaches 0 as τ approaches ∞ (as τ_c approaches 1).

Figure 3 Total Output (Productivity) as Function of Tax Wedge, τ



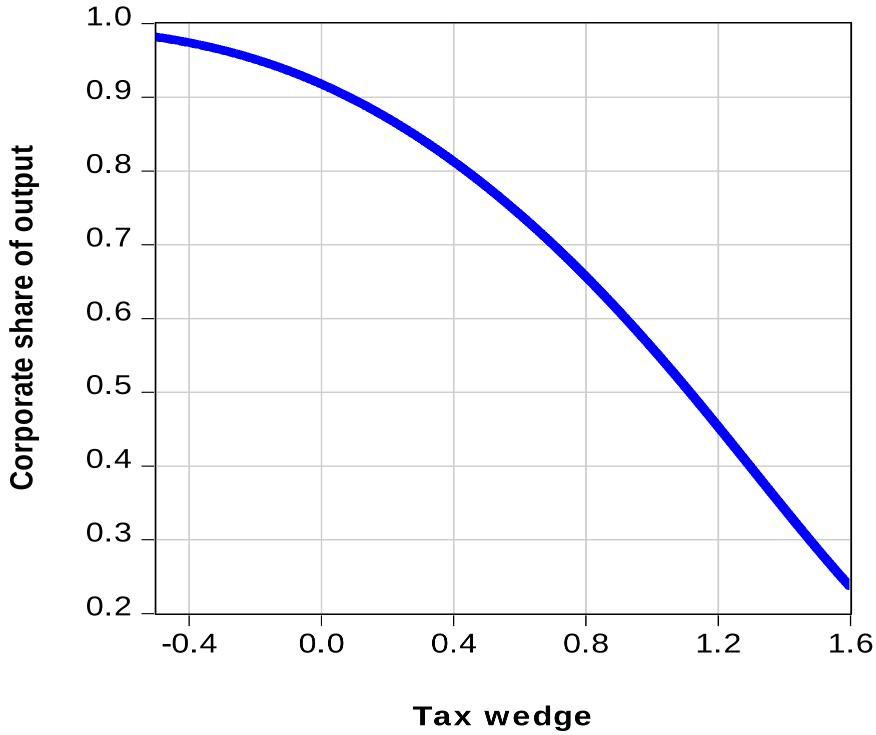
Note: This graph uses the baseline parameter values: μ_c=-0.146, μ_p=-0.896, σ_c= σ_p=0.5, and ρ=0.25. Total output peaks at a value normalized to 1.0 at a tax wedge, τ, of 0. Total output falls with τ when τ>0 and rises with τ when τ<0.

Figure 4 Marginal Effect of τ on Total Output (Productivity)



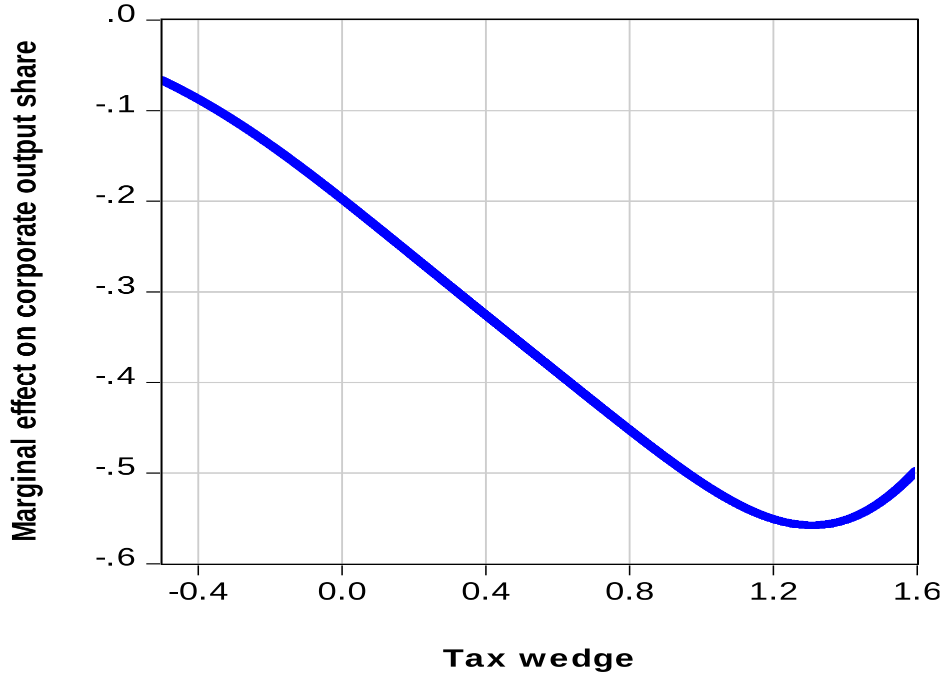
Note: This graph uses the baseline parameter values: μ_c=-0.146, μ_p=-0.896, σ_c= σ_p=0.5, and ρ=0.25. The marginal effect of the tax wedge, τ, on total output is positive for τ<0 and negative for τ>0.

Figure 5 Corporate Share of Output as Function of Tax Wedge, τ



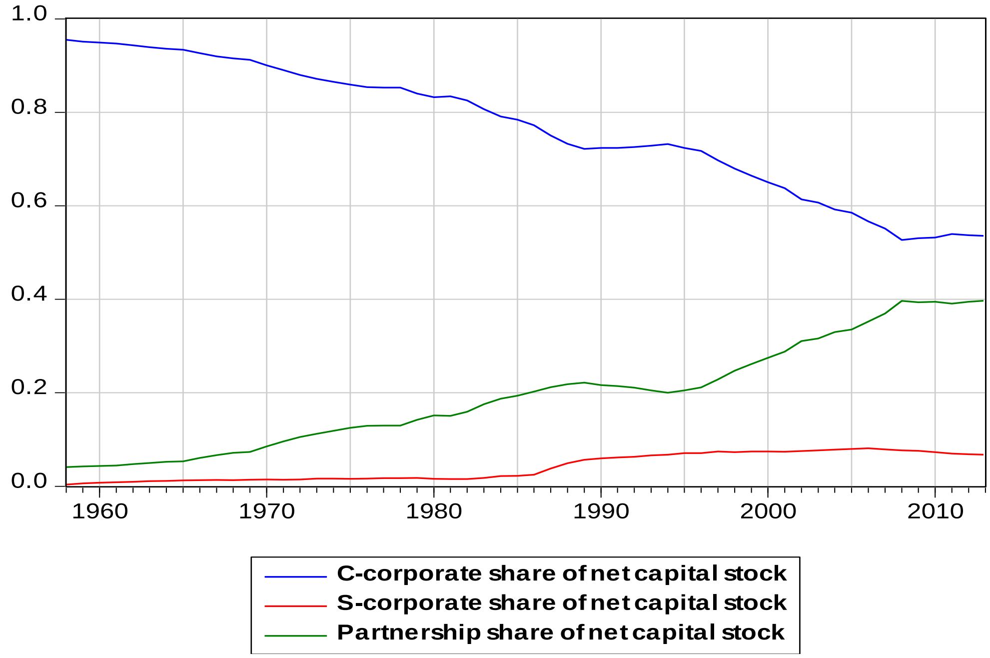
Note: This graph uses the baseline parameter values: μ_c=-0.146, μ_p=-0.896, σ_c= σ_p=0.5, and ρ=0.25. The corporate share of output declines monotonically with the tax wedge, $\tau = \log[\frac{(1-\tau_p)}{(1-\tau_c)}]$. This share approaches 1 as τ approaches -∞ (as τ_p approaches 1) and approaches 0 as τ approaches ∞ (as τ_c approaches 1). For τ between 0.13 and 0.48 (as in Figure 1 for the regression period 1968-2013 used later), the corporate output share is between 0.79 and 0.89.

Figure 6 Marginal Effect of τ on Corporate Output Share



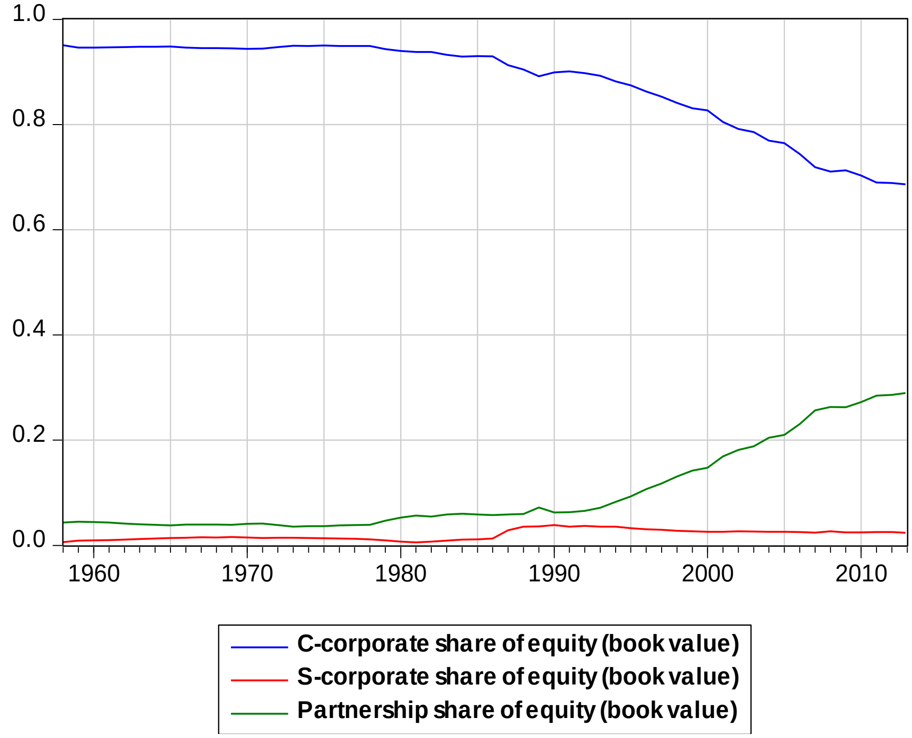
Note: This graph uses the baseline parameter values: μ_c=-0.146, μ_p=-0.896, σ_c= σ_p=0.5, and ρ=0.25. The marginal effect of the tax wedge, τ, on the corporate output share is negative throughout. For τ between 0.13 and 0.48 (as in Figure 1 for the regression period 1968-2013 used later), the slope is between -0.24 and -0.35.

Figure 7 Shares of Business Net Capital Stock



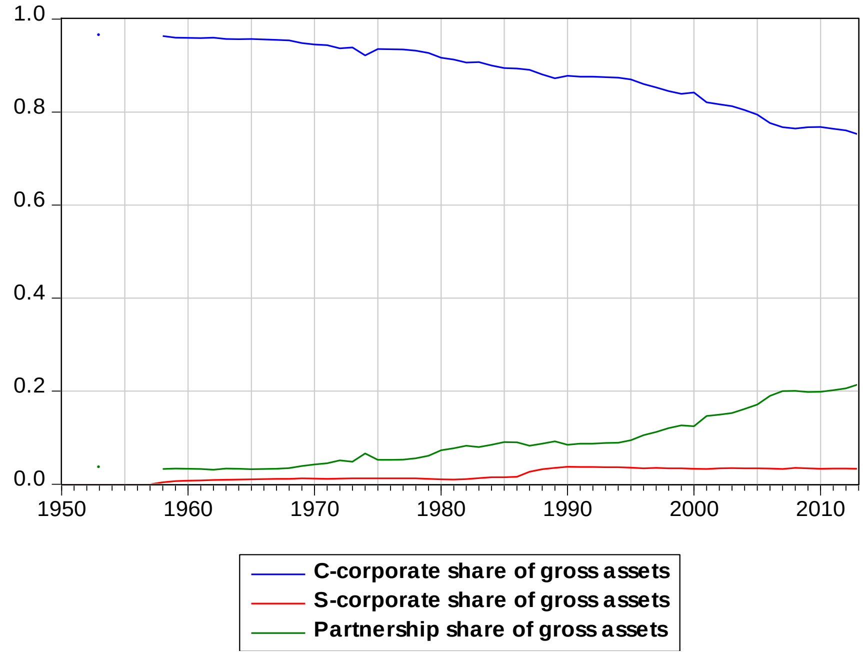
Note: The underlying data on business capital stocks net of depreciation are from various IRS sources, noted in the references. Data for sole proprietorships are unavailable. The partnership numbers are interpolated based on data available every two years from 1959 to 1975 and annually for 1977-1982 and 1988-2013.

Figure 8 Shares of Business Equity (Book Value)



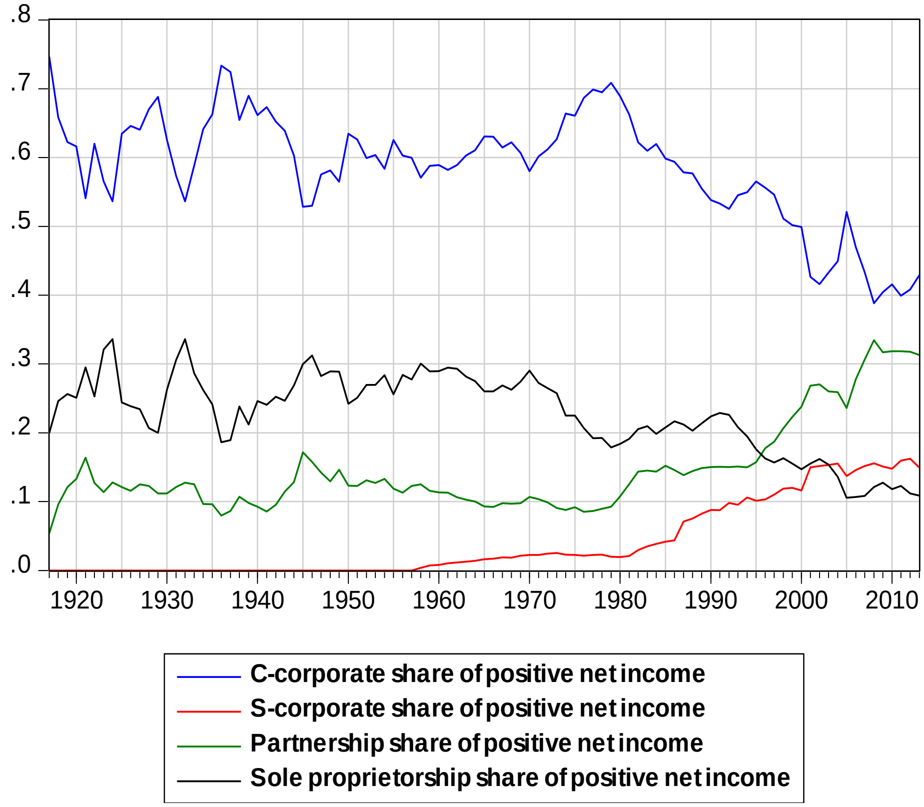
Note: The underlying data on business equity (book value) are from various IRS sources, noted in the references. Data for sole proprietorships are unavailable. The partnership numbers are interpolated based on data available every two years from 1959 to 1975 and annually for 1977-1982 and 1988-2013.

Figure 9 Shares of Business Gross Assets



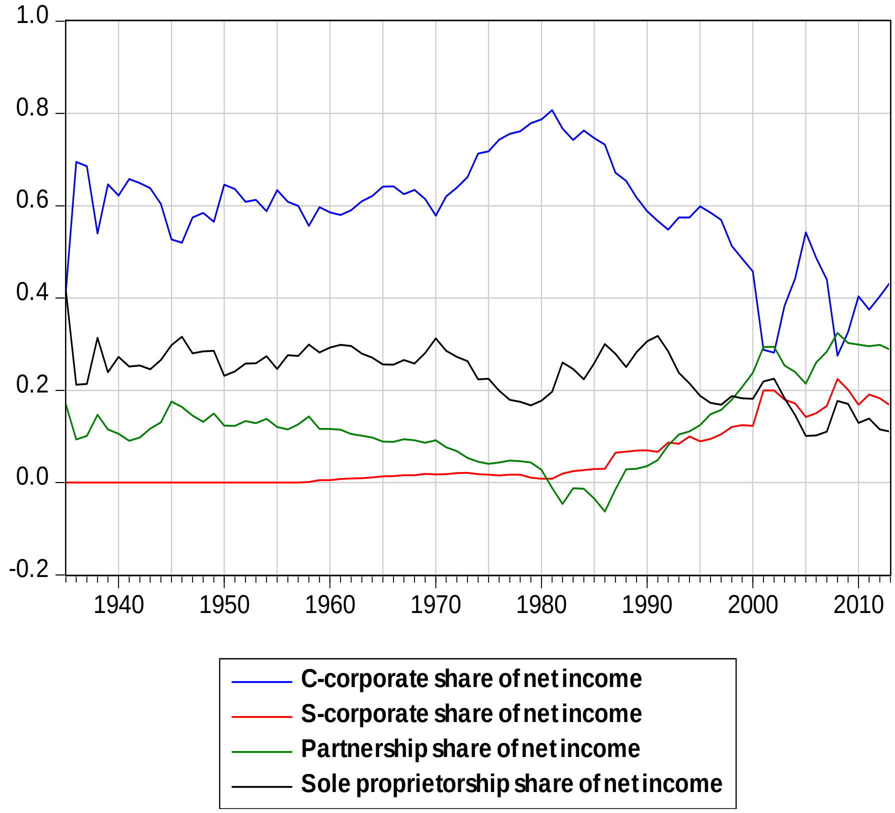
Note: Data are available for 1953 and for 1958-2013. The underlying data on business gross assets are from various IRS sources, noted in the references. Data for sole proprietorships are unavailable.

Figure 10 Shares of Business Positive Net Income



Note: The underlying data on business positive net income are from various IRS sources, noted in the references.

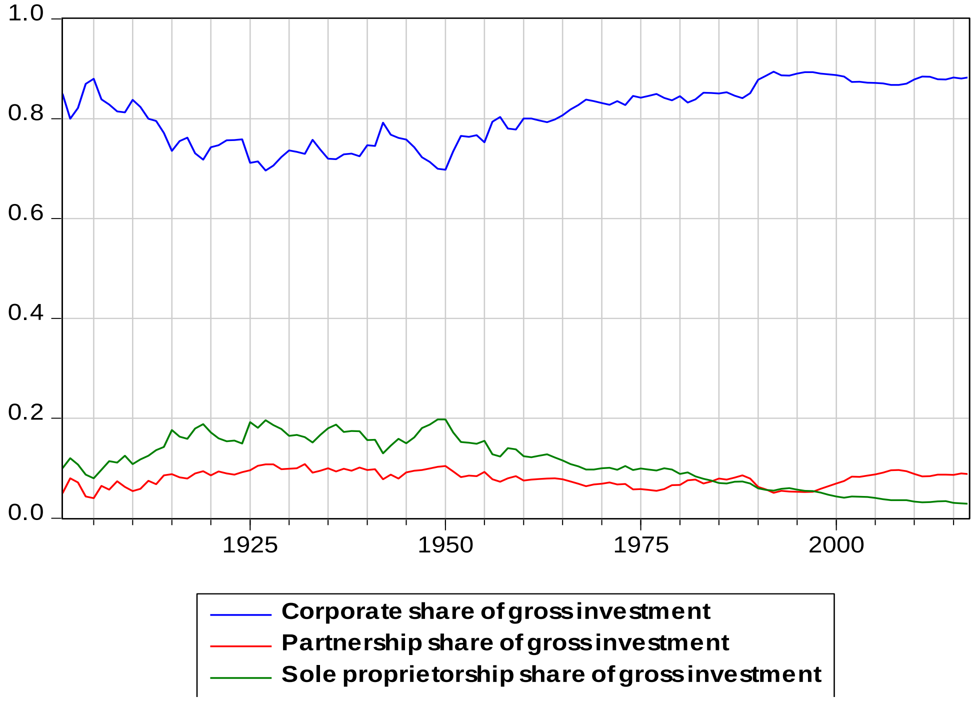
Figure 11 Shares of Business Net Income



Note: The underlying data on business net income are from various IRS sources, noted in the references.

Figure 12

Shares of Business Gross Investment (BEA Data)



Note: The underlying data are from Bureau of Economic Analysis (2019). These corporate numbers combine C-corporations with S-corporations (which originated in 1958).

Table 1

Model Predictions on Corporate Output Share with Alternative Parameters

σ_c	σ_p	ρ	τ	Y_c/Y	Slope
0.5	0.5	0.25	0	0.917	-0.20
"	"	"	0.13	0.889	-0.24
"	"	"	0.48	0.786	-0.35
0.25	0.25	0.25	0	0.995	-0.04
"	"	"	0.13	0.986	-0.11
"	"	"	0.48	0.885	-0.53
0.5	0.5	0.5	0	0.951	-0.17
"	"	"	0.13	0.925	-0.23
"	"	"	0.48	0.812	-0.42
0.5	0.25	0.25	0	0.968	-0.12
"	"	"	0.13	0.949	-0.17
"	"	"	0.48	0.860	-0.34
0.25	0.5	0.25	0	0.926	-0.23
"	"	"	0.13	0.892	-0.30
"	"	"	0.48	0.753	-0.49

Note: These results for corporate output share, Y_c/Y, and the marginal effect of the tax wedge, τ, on this share correspond to the model described in the text and to the results shown in Figures 5 and 6. The results are for alternative values of the underlying parameters, as shown. The first specification is the baseline used in Figures 5 and 6. The mean parameters are set throughout at μ_c=-0.146 and μ_p=-0.896. These values generate a peak level of output, Y, equal to 1.0 (given the other baseline parameters set at the first specification above).

Table 2, part 1

Regressions for C-Corporate Shares of Net Business Capital Stock and Equity (Book Value)

	(1)	(2)	(3)	(4)
Dependent variable:	**C-corporate share of net business capital stock**		**C-corporate share of equity (book value)**	
Independent variables:				
Constant (trend)	-0.0970*** (0.0113)	-0.0719*** (0.0143)	-0.0731*** (0.0187)	0.0059 (0.0065)
C-corporate top federal tax rate, log(1-τ_c)	0.217** (0.096)	0.137 (0.097)	0.410*** (0.149)	0.158*** (0.032)
AMTR federal individual income tax, log(1-τ_p)	-0.493*** (0.107)	-0.370*** (0.135)	-0.593*** (0.185)	-0.206*** (0.039)
Years since 1968 (quadratic trend)	--	-0.00091* (0.00047)	--	-0.00287*** (0.00022)
p-value for equal magnitude of tax coefficients	0.0000	0.0003	0.073	0.026
R-squared	0.49	0.61	0.41	0.95
s.e. of regression	0.0231	0.0206	0.0362	0.0115

***Significant at 1%, **significant at 5%, *significant at 10%.

Table 2, part 2

Regressions for C-Corporate Shares of Gross Assets and Positive Net Income

	(5)	(6)	(7)	(8)
Dependent variable:	**C-corporate share of gross assets**		**C-corporate share of positive net income**	
Independent variables:				
Constant (trend)	-0.0513*** (0.0081)	-0.0227*** (0.0055)	-0.0483** (0.0212)	0.0172 (0.0164)
C-corporate top federal tax rate, $\log(1-\tau_c)$	0.188*** (0.061)	0.098** (0.042)	0.224 (0.162)	0.015 (0.128)
AMTR federal individual income tax, $\log(1-\tau_p)$	-0.289*** (0.063)	-0.150** (0.058)	-1.024*** (0.285)	-0.703** (0.273)
Years since 1968 (quadratic trend)	--	-0.00104*** (0.00018)	--	-0.00238** (0.00089)
p-value for equal magnitude of tax coefficients	0.0001	0.032	0.002	0.003
R-squared	0.46	0.80	0.48	0.63
s.e. of regression	0.0157	0.0096	0.0524	0.0444

Table 2, part 3

Regressions for Corporate Shares of Gross Investment (BEA data)

	(9)	(10)	(11)	(12)
Dependent variable:	**Corporate share of gross investment, 1968-2013**		**Corporate share of gross investment, 1924-2017**	
Independent variables:				
Constant (trend)	-0.0008 (0.0118)	0.0320** (0.0119)	0.0062 (0.0076)	0.0040 (0.0193)
C-corporate top federal tax rate, log(1-τ_c)	0.254** (0.099)	0.150 (0.094)	0.123* (0.070)	0.117 (0.078)
AMTR federal individual income tax, log(1-τ_p)	-0.390*** (0.104)	-0.229* (0.127)	-0.261** (0.120)	-0.258** (0.121)
Years since 1968 or 1924 (quadratic trend)	--	-0.00119*** (0.00038)	--	0.00004 (0.00035)
p-value for equal magnitude of tax coefficients	0.001	0.083	0.050	0.053
R-squared	0.41	0.63	0.14	0.15
s.e. of regression	0.0234	0.0187	0.0327	0.0328

***Significant at 1%, **significant at 5%, *significant at 10%.

Notes to Table 2: Variables in the regressions are 10-year differences. The sample periods are 1968-2013 in columns 1-10, 1924-2017 in columns 11-12. Standard errors, shown in parentheses, are calculated from the Newey-West method with a 10-year bandwidth. The dependent variable in columns 1 and 2 is the C-corporate share of business net capital stocks (Figure 7), in columns 3 and 4 is the C-corporate share of equity or book value (Figure 8), in columns 5 and 6 is the C-corporate share of gross assets (Figure 9), in columns 7 and 8 is the C-corporate share of positive net income (Figure 10), and in columns 9-12 is the corporate share of business gross investment from BEA data (Figure 12). The business totals in columns 1-6 comprise C-corporations, S-corporations, and partnerships. In columns 7 and 8, sole proprietorships are also included in the business totals. In columns 9-12, the BEA corporate data combine C-corporations with S-corporations; the BEA business totals include sole proprietorships. The C-corporate top federal tax rate, τ_c, and the AMTR for the federal individual income tax, τ_p, are in Figure 1. The tax variables enter (as in equation [2]) as log(1-τ_c) and log(1-τ_p). The p-values are for tests that the sum of the two coefficients add to zero, as implied by the model.

Table 3

Estimated Productivity Effects from Major U.S. Historical Tax Changes

- 1968, τ_c=0.53, $\tau_p = 0.240$, τ=0.476, estimated productivity=0.964;
- 1986, τ_c=0.46, $\tau_p = 0.258$, τ=0.318, estimated productivity=0.986, up 2.3%;
- 1988, τ_c=0.34, $\tau_p = 0.195$, τ=0.199, estimated productivity=0.996, up 1.0%;
- 2017, τ_c=0.35, $\tau_p = 0.244$, τ=0.151, estimated productivity=0.998, up 0.2%;
- 2018, τ_c=0.21, $\tau_p = 0.215$, τ=-0.006, estimated productivity=1.000, up 0.2%.

Note: In Figure 1, τ_c equals the top federal corporate-profits tax rate and τ_p equals the labor-income weighted average tax rate from the federal individual income tax. The tax wedge, τ, is calculated from equation (2) as $\tau = \log[(1 - \tau_c)/(1 - \tau_p)$. The estimated productivity, normalized to 1.0 at the peak, corresponds to Figure 3.

Appendix

Derivation of Expectations of Corporate and Pass-Through Output

We start with the derivation of equation (3), which gives the expectation of corporate output, Y_c. The setup is that $\log(Y_c)$ and $\log(Y_p)$ are bivariate normal with respective means and standard deviations of μ_c, σ_c, μ_p, and σ_p. The correlation coefficient between the two random variables is ρ. This specification implies that $y \equiv \log(\frac{Y_c}{Y_p})$ is distributed normally with mean μ=μ_c- μ_p and variance $\sigma^2 = \sigma_c^2 + \sigma_p^2 - 2\rho\sigma_c\sigma_p$.

The distribution of $\log(Y_c)$, conditional on y, is normal with respective mean and standard deviation of $\tilde{\mu}_c=\mu_c + \left(\frac{\sigma_c}{\sigma}\right) \cdot \left(\frac{\sigma_c-\rho\sigma_p}{\sigma}\right) \cdot (y-\mu)$ and $\tilde{\sigma}_c^2 = \sigma_c^2\sigma_p^2(1-\rho^2)/\sigma^2$ (see Hogg and Craig [1965, pp. 102-104]). The expectation of Y_c conditional on y is $\exp(\tilde{\mu}_c+0.5\cdot\tilde{\sigma}_c^2)$. The expectation of Y_c is then

$$Prob.(y \geq \tau) \cdot E(Y_c|y \geq \tau) = \frac{1}{\sigma\sqrt{2\pi}}\int_{\tau}^{\infty} \exp(\tilde{\mu}_c + 0.5 \cdot \tilde{\sigma}_c^2) \cdot \exp[\frac{(y-\mu)^2}{2\sigma^2}]dy$$

Using the expressions for $\tilde{\mu}_c$ and $\tilde{\sigma}_c^2$, the result can be written as:

$$Prob.(y \geq \tau) \cdot E(Y_c|y \geq \tau) = \frac{\exp(\mu_c+0.5\sigma_c^2)}{\sigma\sqrt{2\pi}}\int_{\tau}^{\infty} \exp[(\frac{-1}{2\sigma^2}) \cdot [y-\mu-\sigma_c(\sigma_c-\rho\sigma_p)]^2 dy.$$

Finally, using the change of variable $z=\left[y-\mu-\sigma_c\left(\sigma_c-\rho\sigma_p\right)\right]/\sigma$, the lower limit of integration becomes $\tau' = \left(\frac{1}{\sigma}\right) \cdot [\tau-\mu-\sigma_c\left(\sigma_c-\rho\sigma_p\right)]$. We then get equation (4):

$$Prob.(y \geq \tau) \cdot E(Y_c|y \geq \tau) = [\exp(\mu_c + 0.5\sigma_c^2)] \cdot [1-\Phi(\tau')],$$

where $\Phi(\cdot)$ is the cumulative standard normal density.

The result for pass-through output, $Prob.(y < \tau) \cdot E\left(Y_p\middle|y < \tau\right)$, is analogous, with the parameters for c and p switched (including that μ is now μ_p-μ_c) and –τ replaced by τ in the expression for τ'.