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Abstract

Changes in product characteristics on the extensive margin are an important and hitherto neglected dimension of quality change. Standard techniques for quality-adjusting price indices cannot handle such changes satisfactorily, which leads to an economically and statistically significant bias in the measurement of prices and real output. We combine theoretical insights from index numbers and demand for characteristics to develop a new method for incorporating changes on the extensive characteristic margin. Applied to data on new car sales in the U.K., our method leads to revisions in estimated inflation rates for this commodity group that are both plausible and quantitatively important.

JEL-Codes: C900, C910, C920, D030.

Keywords: extensive and intensive margins of consumption, characteristics model, quality change, Sato-Vartia-Feenstra index numbers.

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1 Introduction

Failing to adjust price indices for new goods and for quality improvements to existing goods has long been known to be an important potential source of mismeasurement in prices and real output. For example, the report of the Boskin Commission, Boskin et al. (1996), estimated that the U.S. consumer price index overestimated the true rate of inflation by 1.1% per year; of this, failing to account for new goods and quality change accounted for over half, 0.60, percentage points.[1] A recent survey by Feldstein (2017) argues that these failures persist, and that the resulting overestimate of the rate of inflation implies systematic underestimation of changes in living standards and aggregate output, and may be a significant source of the "productivity puzzle", the slowdown in measured productivity growth in advanced economies over recent decades.

While the problems of bias arising from new goods and quality change are not yet adequately taken into account in practice, there are well-understood methods of taking them into account in principle, as we discuss below. The same cannot be said of a different problem that we call *new-characteristics bias*: a failure to account for new features in existing products, i.e., for quality change at the extensive margin of product characteristics rather than the intensive margin. A number of authors have commented on this in general terms. Hausman (2003) points out that proper quality adjustment in price indices should allow for changes in available product attributes, and Varian (2018) discusses how the new features included in smartphones are a source of quality change not captured by official measures. However, to date there is no conceptual framework for understanding this problem, and no systematic attempt to quantifying it.

This paper aims to fill this gap. We provide a conceptual framework for understanding new-characteristics bias, and propose and implement an explicit solution for it. In more detail, we show that changes to the attributes of existing products on the extensive margin

[1] The other sources of bias were substitution bias (largely eliminated, according to Gordon (1999), by a subsequent shift from a fixed-weight to a variable-weight index-number formula), and outlet-substitution bias, arising from insufficiently accounting for shifts to lower price outlets.

(changes in the range of features incorporated into a product or service) are potentially as important as changes on the intensive margin (upgrades of existing features). We show that the method of quality-adjustment currently in most widespread use does not properly account for changes on the extensive margin of characteristics and that this is a source of systematic bias in price measurements. We introduce a new method to correct for this which is based on the linear characteristics model developed by Gorman in 1956 (Gorman (1980) and Lancaster (1966)), and the mix-adjusted extension of the Sato-Vartia price index (Sato (1976) and Vartia (1976)) developed by Feenstra (1994). Finally, to illustrate this method, we show the effect of allowing for changes on the extensive margin of characteristics on the quality-adjusted price index for new car sales in the U.K. Our approach is theoretically well-founded and yet straightforward to apply in practice: national statistical agencies could easily adopt it, since it can be applied without necessarily requiring any additional estimation steps over and above those normally required for hedonic methods.

To give a recent concrete example of innovation on the extensive margin, consider camera phones.[2] Nowadays most mobile telephones feature a digital camera but it was not always so. There is some debate about whether the first camera phone was the Samsung SCH-V200 (released in South Korea in June) or the J-SH04 by Sharp (released in Japan in November of the same year).[3] The Samsung was able to take photos at a resolution of 0.35-megapixels (MP) but its storage was limited to just 20 photographs.[4] The Sharp had less good resolution (0.11MP) but it allowed users to transfer and transmit their photos electronically. The first U.S. camera phone was the Sanyo SCP-5300. which was released in November 2002. It cost $400 and had a 0.3MP capability. It also had a basic flash, white balance control, self-timer, digital zoom, and various filter effects like sepia, black and white, and negative colours. By the end of 2003, camera phones were really taking off in the U.S. and over 80 million had

[2]See `https://www.digitaltrends.com/mobile/camera-phone-history/`, from which some of what follows is taken.

[3]The point at issue regarding priority is that the camera and the phone inside the Samsung were in fact separate devices which were simply accommodated in the same housing, whereas the Sharp was fully integrated.

[4]They had to be downloaded, using a hard link, to a PC if the user wanted to take any more.

already been sold worldwide. Over half of the phones sold worldwide in the first 9 months of 2004 had cameras in them, and two-thirds of all the phones shipped in the third quarter were camera phones. Leading the way was Finnish manufacturer, Nokia (the Nokia N90 had a 2MP camera, Carl Zeiss optics, autofocus, an LED flash, and a rotating screen mimicking a camcorder).

The next phase of innovation started to move towards the intensive margin – there was a race for ever greater resolution. 1.3MP arrived in the US with the Audiovox PM8920. It had a dedicated camera button and a number of pre-set settings for different lighting conditions, a multi-shot feature and the ability for users to record their own shutter sound. It was available for $299 RRP – although as part of various bundles it could be had for as little as $190 up-front. The Sony Ericsson K800i was released in 2006 with a 3.2MP camera with auto-focus, image stabilisation, and a Xenon flash. Nokia naturally retaliated with models like the 3.2MP N73, and then in 2007 with the first 5MP camera phone in the Nokia N95. This model also represented an innovation on the extensive margin: it could record video at 30 frames-per-second. Unfortunately (for Nokia) the smartphone era was about to arrive. The original iPhone was launched a few months after the N95, in June 2007. It had a mere 2MP camera, no flash or auto-focus and could not take video. Camera phones continued to improve but the race stalled somewhat as smartphones took off. The iPhone, with its relatively basic camera, proved that there were more important features than the camera.

Not all innovations turned out to be popular with consumers. Some notable misses include attempts by HTC and LG to interest consumers in 3D – they released phones with dual stereoscopic 5MP cameras but there was no real demand. Manufacturers were more successful with shifting their focus to software features beginning with motion control (since resolutions were so high that users without tripods were unable to hold the camera steady enough to take advantage), and then in-built filters and image-processing effects.

This piece of recent economic history serves to make three main points. Firstly, product innovation occurs on both margins: the race for ever higher resolution exemplifies the intensive margin, whilst the huge expansion of features and the capabilities of handsets relates to the extensive margin. Secondly, in the case of handsets, the really significant innovations came via the changes on the extensive margin. Thirdly, it shows that some characteristics fail and are withdrawn so changes on the extensive margin are not simply the accretion of ever more features; other features, like 3D, die away. We will return to these points in our empirical application below.

In the face of product innovation, national statistical offices and institutes quality-adjust the prices used in the construction of their consumer price indices by means of hedonic methods.[5] Of the two main approaches to hedonic pricing, the "time-dummy method" *can* allow for changes on the extensive margin, but the much more widely-used "indirect method" *cannot*.[6]

The time-dummy method essentially works by pooling data across products and periods and regressing (log) prices on a set of product attributes and a sequence of time-dummies. The attributes control for quality-related price changes and the coefficients on the time dummies pick up the pure period-to-period price change. Since the regression is run over data which is pooled across time periods, any product characteristic which features in at least some good in some period can be included even if it is not present in all periods. This makes it straightforward to accommodate product characteristics which come and go. However, the time-dummy method is not used by statistical offices because it suffers from a major drawback: it requires that the historical consumer price index series be completely

[5]Introduced in Griliches (1961), hedonic quality adjustment of prices is now a firmly-established method used by national statistical offices and institutes the world over. In the U.K., hedonic methods were first introduced in the CPI in 2003 (for personal computers) and subsequently their use has expanded to include digital cameras, laptops, tablet PC's and handsets for pay-as-you-go mobiles and smartphones. The US (which started using hedonics to adjust rental values as far back as the first half of the last century) now uses hedonics for Clothing, Footwear, Refrigerators, Washing Machines, Clothes Dryers, Ranges and Cooktops, Microwave Ovens, TVs and DVD Players;. Sweden uses the approach for for 20 Clothing and 12 Footwear items; Canada for PCs, Laptops, Printers, Monitors and Internet Services. Other countries which use these methods include Australia, New Zealand, Denmark, Finland, Germany, the Netherlands and Switzerland. See Wells and Restieaux (2013) for a review of practice in the UK and internationally.

[6]The time-dummy method is based on the repackaging model of Fisher and Shell (1968); the indirect method on the characteristics model of Gorman (1980) and Lancaster (1966).

revised every time the series is up-dated to incorporate new data.[7]

The indirect method works by running a sequence of separate within-period regressions of prices on characteristics. The estimated regression coefficients represent the implicit or shadow price for each of the included characteristics in each period. Applying the shadow prices from period $t + 1$ to the specification of the product in period t generates a price that would apply if the shadow prices had changed but the quality (as measured by its attributes) of the product had remained constant. By contrast with the time-dummy model, this approach simply adds new data to the end of the series and does not require any revisions; it is therefore the practical alternative which is used by statistical offices. However, this method of quality adjustment suffers from a problem analogous to the new-goods problem: it does not account for changes in economic welfare associated with product innovation on the extensive margin of characteristics. To do so would require an estimate of the shadow prices of the characteristic in both the base and comparison periods; but if a characteristic or feature is simply not present in the market in one of those periods then its shadow price cannot be recovered for that period – there is no variation to exploit. If such innovations are important, then hedonic methods, as currently applied in practice, will overstate the true rate of price changes, and consequently under-estimate the rate of growth of real GDP and consumption.

The plan of the paper is as follows. In Section 2 we describe the linear characteristics model, the economic theory underlying the indirect method of hedonic adjustment, and develop a method of accounting for changes in characteristics based on the theory of household behavior under rationing. In Section 3 we show how this method can be operationalized using a hedonic version of the price index developed by Sato (1976) and Vartia (1976), and subsequently extended to allow for changes in the range of goods consumed by Feenstra (1994). Section 4 introduces the data we use in our empirical application, Section 5 presents the results, while Section 6 concludes.

[7]See, for example, the discussion of this drawback by Ball and Allen (2003).

2 Characteristics Models and Hedonic Price Indices

The characteristics model was developed in 1956 by Gorman (1980) and subsequently popularized by Lancaster (1966). There are K varieties of products with quantities denoted by x^k and prices by p^k. These market goods are differentiated by J product characteristics. It is typically assumed that there are fewer characteristics than market goods so that the characteristics model normally entails a degree of dimension-reduction relative to the preferences-for-goods model. We will return to this issue when we come to estimation.

In the linear characteristics model the total amount of a given characteristic present in a bundle of varieties $\left\{x_t^1, ..., x_t^K\right\}$ observed in period t is

$$z_t^j = \sum_k a_t^{kj} x_t^k$$

where a_t^{kj} represents the amount of characteristic j present in one unit of product k according to the product specification in period t. Prices, budgets and the product specification are time-varying (indexed by t) but they are assumed not to be choice variables for the individual consumer. Utility is taken to be continuously differentiable.

The model hypothesises that the consumer has preferences over characteristics, rather than products *per se*, and that she selects her preferred mix of products subject to her budget constraint and the requirement that the selected varieties map to the characteristics according to the current specifications given by a_t^{kj}. Thus the optimizing model is

$$\max_{x^1,...,x^K} v(z^1, ..., z^J) \text{ subject to } \sum_k p_t^k x^k = y_t \text{ and } z^j = \sum_k a_t^{kj} x^k$$

where y_t is the consumer's budget. Maximizing behavior and the linear characteristics structure imply the first-order conditions

$$u_t^k = \sum_j a_t^{kj} v_t^j = \lambda_t p_t^k \quad \forall\ k, t$$

where u_t^k and v_t^j denote the marginal utilities of the k'th good and the j'th characteristic in period t and where we assume for simplicity that the consumer is either consuming the good or just indifferent between consuming and not consuming it.[8] Following Gorman (1980), the shadow prices of characteristics are defined as the ratio of the marginal utility of the characteristics to the marginal utility of income:

$$\pi_t^j \equiv \frac{1}{\lambda_t} v_t^j$$

We will assume that the marginal utilities of characteristics are weakly positive and therefore, given a positive marginal utility of income, the shadow prices are also weakly positive

$$\pi_t^j \geq 0 \quad \forall\, j, t$$

The first-order conditions then become

$$p_t^k = \sum_j a_t^{kj} \pi_t^j \quad \forall k, t$$

This is the hedonic pricing equation which says that the market price of the product in a given period is a linear function of its characteristics, weighted by the consumer's willingness to pay for each characteristic. This implies that the prices are inside the column space of the technology matrix and it therefore follows that, for the linear characteristics model, the budget constraint adds up to total expenditure whether it is expressed in terms of the prices and quantities of market goods or in terms of the shadow prices and characteristics:[9]

$$\sum_k p_t^k x_t^k = \sum_j \pi_t^j z_t^j = y_t$$

Consequently there are two ways to think about the consumer's choice problem: one as a

[8]See Blow, Browning, and Crawford (2008) for a discussion of corner solutions.

[9]See Blow, Browning, and Crawford (2008).

demand-for-products problem in product space

$$\{x^1, ..., x^K\} = \underset{x^1,...,x^K}{\arg\max} \left\{ u(x^1, ..., x^K) \;\middle|\; \sum_k p_t^k x^k = y_t \right\};$$

the other as a demand-for-characteristics problem in characteristics space subject to a budget constraint expressed in terms of shadow prices and the same total budget.

$$\{z^1, ..., z^J\} = \underset{z^1,...,z^J}{\arg\max} \left\{ v(z^1, ..., z^J) \;\middle|\; \sum_j \pi_t^j z^j = y_t \right\}$$

The two being linked by the linear mapping between goods and characteristics: $u(x^1, ..., x^K) = v(\sum_k a_t^{k1} x^k, ..., \sum_k a_t^{kJ} x^k)$.

The demand-for-characteristics version of the model immediately provides the hedonic expenditure function

$$c(\boldsymbol{\pi}_t, v) = \min_{z^1,...,z^J} \sum_j \pi_t^j z^j \text{ subject to } v(z^1, ..., z^J) = v,$$

and the constant-utility Konüs hedonic price index is defined as

$$P_K(\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, v) = \frac{c(\boldsymbol{\pi}_1, v)}{c(\boldsymbol{\pi}_0, v)}$$

where, in contrast to the preferences-for-products model, utility is thought of as being derived from characteristics. Note that this formulation allows for substitution between different combinations of characteristics. Standard index number theory (see Diewert (1981) for an authoritative account) then provides a number of ways of either approximating or, subject to functional form assumptions regarding $v(.)$, exactly computing the hedonic index. For example, denoting by s_t^j the hedonic budget share for the j'th characteristic $\pi_t^j z_t^j / y_t$, the hedonic Laspeyres

$$P_L = \sum_j \left(\frac{\pi_1^j}{\pi_0^j} \right) s_0^j$$

and Paasche indices

$$P_P = \left[\sum_j \left(\frac{\pi_1^j}{\pi_0^j} \right)^{-1} s_1^j \right]^{-1}$$

may be regarded as, respectively, either upper and lower approximations to the hedonic Konüs or as exact hedonic indices if preferences for characteristics are assumed to be either linear or Leontief. The hedonic Sato-Vartia[10] index

$$P_{SV} = \prod_j \left(\frac{\pi_1^j}{\pi_0^j} \right)^{\omega^j}$$

(where the weights ω^j are the normalized logarithmic means of the period 0 and period 1 hedonic budget shares) is likewise exact if preferences are CES and approaches the hedonic Paasche and Laspeyres depending on the degree of substitution between characteristics in the limit. The hedonic Fisher index, which is the geometric mean of the hedonic Paasche and Laspeyres, is exact for quadratic preferences over characteristics. Moreover, unlike the Sato-Vartia, it provides a second-order approximation to any twice continuously differentiable homothetic expenditure function and is therefore described as "superlative" Diewert (1976). Finally the hedonic Tornqvist

$$P_T = \prod_j \left(\frac{\pi_1^j}{\pi_0^j} \right)^{\frac{1}{2}(s_0^j + s_1^j)}$$

is also superlative and can approximate any twice continuously differentiable cost function, whether homothetic or not. It is important to note that that all of these hedonic price indices are functions of the shadow price-relatives π_1^j/π_0^j and hence require knowledge of the shadow prices in both the base and comparison periods.

To connect theory to practice, the "indirect method" used by statistical agencies appends an econometric error to the hedonic pricing equation

$$p_t^k = \sum_j a_t^{kj} \hat{\pi}_t^j + e_t^k$$

[10] Sato (1976), Vartia (1976).

and identifies the shadow prices by regressing prices on product characteristics using within-period cross-variety covariation in product specifications and product prices. The dimension reduction $J < K$ is material here as it is required in order to identify the shadow prices. The resulting shadow prices and the appropriate characteristics are then substituted into the chosen index number formula.

A sequence of hedonic regressions are run on each period of data to estimate $\hat{\pi}_t^j$ using cross-sectional/cross-product variation in prices and product specifications. As the data for new periods become available changes in product characteristics on the intensive margin can be handled by changing the value of a_t^{kj} to reflect the current product specification, and changes on the extensive margin can be allowed for by changing the set of characteristics in the regression and simply adding/deleting characteristics as regressors. For these reasons (flexibility and the non-revisions aspect) this method is the one favoured by all statistical offices which use hedonic methods.

However, there is a problem: when the set of characteristics changes it is no longer possible to form a hedonic price index because the shadow price-relatives π_1^j/π_0^j cannot be formed. Product characteristics which existed in period 0 but disappear from the market in period 1 cannot be priced in the later period, and characteristics which are first introduced in the later period but did not exist in period 0 cannot be priced in the base period. As a result either the denominator (in the case of new characteristics) or the numerator (in the case of disappearing characteristics) of the shadow price-relative is missing, and this causes problems for the hedonic index number formulae.

The standard response is to ignore this by ignoring any changes on the extensive margin and omitting them from the pricing equation and the index. Whilst this stance cannot sensibly be maintained forever without the product specification becoming out-of-date, it can be adapted to form a chained-hedonic index calculated on the basis of the characteristics common to each pair of adjacent periods. Chaining in this manner is routinely used in preferences-for-goods contexts to allow for the entry/exit of products, and it can equally be applied to hedonic price indices to allow for the entry/exit of characteristics.[11] However, two potential problems remain, one econometric, the other economic. The econometric problem is omitted variable bias; if the new characteristic is correlated with other existing characteristics (for example if a new feature is introduced into varieties with existing high-end specifications) this correlation will bias the estimates of the shadow prices of the rest of the characteristics. The economic problem is that changes on the extensive margin affect economic welfare. For example, if a new characteristic becomes available then it becomes cheaper, other things being equal, for the consumer to reach a given level of utility. This is because something akin to a rationing constraint has been removed. This is analogous to the new goods problem in preference-for-products index numbers (see Hausman (1997)). Ignoring changes in the extensive margin of characteristics will therefore bias the hedonic price index, even in a chained index.

To see this, consider the constrained hedonic cost function in which characteristic 1 is fixed, in period t, at the level $\bar{z}_t^1$:

$$c(\boldsymbol{\pi}_t, v, \bar{z}_t^1) = \min_{z^2,\ldots,z^J} \pi_t^1 \bar{z}_t^1 + \sum_{j=2} \pi_t^j z^j \text{ subject to } v(\bar{z}^1, \ldots, z^J) = v$$

Note that whereas the unconstrained hedonic cost function is defined effectively for all v, the constrained hedonic cost function is only defined at utility levels which are attainable under the additional constraint on the characteristic. Given this the constrained cost function satisfies Sheppard's Lemma

$$\nabla_{\pi^j} c(\boldsymbol{\pi}_t, v, \bar{z}_t^1) = \bar{z}_t^1$$

$$\nabla_{\pi^j} c(\boldsymbol{\pi}_t, v, \bar{z}_t^1) = z^j(\boldsymbol{\pi}_t, v, \bar{z}_t^1) \;\; \forall j = 2, ..J$$

[11]Keynes argued strongly in favour of the chaining method: "We are not in a position to weigh the satisfactions for similar persons of Pharaoh's slaves against Fifth Avenue's motor cars, or dear fuel and cheap ice to Laplanders against cheap fuel and dear ice to Hottentots ... We cannot hope to find a ratio of equivalent substitution for gladiators against cinemas, or for the conveniences of being able to buy motor cars against the conveniences of being able to buy slaves" (Keynes (1930)).

The relationship between the constrained and unconstrained hedonic cost functions is given by Neary and Roberts (1980):

$$
\begin{aligned}
c(\boldsymbol{\pi}_t, v, \bar{z}_t^1) &= \pi_t^1 \bar{z}_t^1 + \sum_{j=2} \pi_t^j z^j(\boldsymbol{\pi}_t, v, \bar{z}_t^1) \\
&= \pi_t^1 z^1(\tilde{\boldsymbol{\pi}}_t, v) + \sum_{j=2} \pi_t^j z^j(\tilde{\boldsymbol{\pi}}_t, v) \\
&= c(\tilde{\boldsymbol{\pi}}_t, v) + \left(\pi_t^1 - \tilde{\pi}_t^1\right) \bar{z}_t^1
\end{aligned}
$$

where $\tilde{\pi}_t^1$ is the virtual price of the constrained characteristic and $\tilde{\boldsymbol{\pi}}_t = \left[\tilde{\pi}_t^1, \pi_t^2, ..., \pi_t^J\right]$ is the shadow support price vector. The virtual price is defined implicitly as a function of the constraint level, the shadow prices of the unconstrained characteristics and the utility level. The shadow support prices are those which would precisely induce the consumer to choose the constrained characteristics vector

$$
\bar{z}_t^1 = z^1(\tilde{\boldsymbol{\pi}}_t, v)
$$

The welfare cost of the constraint on the characteristic is therefore

$$
\begin{aligned}
\frac{\partial c(\boldsymbol{\pi}_t, v, \bar{z}_t^1)}{\partial \bar{z}_t^1} &= \frac{\partial c(\tilde{\boldsymbol{\pi}}_t, v)}{\partial \pi_t^1} \frac{\partial \pi_t^1(\boldsymbol{\pi}_t, v, \bar{z}_t^1)}{\partial \bar{z}_t^1} - \frac{\partial \pi_t^1(\boldsymbol{\pi}_t, v, \bar{z}_t^1)}{\partial \bar{z}_t^1} \bar{z}_t^1 + \left(\pi_t^1 - \tilde{\pi}_t^1\right) \\
&= \pi_t^1 - \tilde{\pi}_t^1
\end{aligned}
$$

where the last equality follows from Sheppard's Lemma. Thus the change in welfare associated with a change in the constraint is the difference between the shadow price of the characteristic and its virtual counterpart.

To look at the implications of this in the particular case of changes in the extensive margin of product characteristics consider two adjacent periods $t \in \{0, 1\}$ and suppose that the first characteristic is not available in the base period ($\bar{z}_0^1 = 0$) but is added to the product specification in the following period. Then in the base period the constrained cost function is

$$c(\boldsymbol{\pi}_0, v, \bar{z}_0^1) = c(\tilde{\boldsymbol{\pi}}_0, v) + \left(\pi_0^1 - \tilde{\pi}_t^1\right) \bar{z}_t^1$$

which becomes

$$c(\boldsymbol{\pi}_0, v, 0) = c(\tilde{\boldsymbol{\pi}}_0, v).$$

Consequently, the Konüs hedonic cost-of-living index can be expressed conveniently as the ratio of *unconstrained* hedonic cost functions where the base prices are given by the virtual prices.

$$P_K(\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, v) = \frac{c(\boldsymbol{\pi}_1, v)}{c(\tilde{\boldsymbol{\pi}}_0, v)}$$

The fact that everything can be expressed in terms of the unconstrained cost function means that all of approximate/exact/superlative formulations for the index described above are available. But all of them require that the shadow price relative $\pi_1^1/\tilde{\pi}_0^1$ is known. In principle recovering the virtual shadow price of the characteristic in the base period is a matter of solving

$$0 = z^1(\boldsymbol{\pi}, v)$$

for $\tilde{\pi}^1$.

This is the approach taken in the preference-for-goods context by Hausman (1997) where a demand function is fitted to observations in which all goods are available (and therefore not subject to the constraint) and then evaluated at the point where demand for the good of interest is driven down to zero. In principle, the Hausman (1997) approach could be adapted to the characteristics model but this would be econometrically demanding and the method has proved somewhat controversial. To avoid this, Broda and Weinstein (2006) make a functional form assumption on preferences which allows them to incorporate new goods by means of Feenstra's (1994) mix-adjusted Sato-Vartia index. Recently Redding and Weinstein (2018), also in a preferences-for-goods model, build on the Sato-Vartia index to allow for idiosyncratic taste shocks, and show how this allows a decomposition of the effects of exogenous shocks on consumer welfare.

3 Allowing for New Characteristics

Following Feenstra (1994), Broda and Weinstein (2006) and Redding and Weinstein (2018), assume that preferences for characteristics exhibit a constant elasticity of substitution (CES). Thus

$$v(\mathbf{z}) = \left[\sum_j \left(\beta^j z^j \right)^{\theta} \right]^{\frac{1}{\theta}}$$

where $\theta \in [-\infty, 1]$. The taste parameters β^j reflect perceived quality change at the intensive margin.[12] Their presence implies that preferences need not be symmetric across characteristics. The corresponding cost function is

$$c(\boldsymbol{\pi}, u) = uc(\boldsymbol{\pi}) \quad \text{where:} \quad c(\boldsymbol{\pi}) \equiv \left[\sum_j (\pi^j / \beta^j)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where the elasticity of substitution $\sigma = (1 - \theta)^{-1}$ and $\sigma \in [0, \infty]$. Because preferences are homothetic, the unit cost function $c(\boldsymbol{\pi})$ is independent of the level of utility. It equals the true price level in a given period; and the ratio of these price levels for two periods gives the Konüs true price index between those periods. However, $c(\boldsymbol{\pi})$ depends on the unobservable taste parameters, β^j. To eliminate these, we use the approach of Sato and Vartia, as extended to allow for changes in the number of goods by Feenstra, which expresses the true index in terms of observable budget shares.

By the logarithmic version of Shephard's Lemma, the log budget share for the j'th characteristic is

$$\ln s^j = (1 - \sigma) \left[\ln \pi^j - \ln c(\boldsymbol{\pi}) - \ln \beta^j \right]$$

[12] Baldwin and Harrigan (2011) call this multiplicative specification "box-size quality": the utility derived from two boxes of unit quality equals that from a single box with quality equal to two. The same specification is embodied in the equivalence scale model of household composition of Barten (1964).

Now consider two periods $t \in \{0, 1\}$ and suppose that the set of characteristics changes from $\mathcal{J}_0$ in the base period to $\mathcal{J}_1$ in the comparison period. The intersection $\mathcal{J} = \mathcal{J}_0 \cap \mathcal{J}_1 \neq \varnothing$ contains the set of characteristics which are available in both periods. Taking the difference in log budget shares for the j'th characteristic between periods gives:

$$\ln s_1^j(\mathcal{J}_1) - \ln s_0^j(\mathcal{J}_0) = (1-\sigma)\left[\left(\ln \pi_1^j - \ln \pi_0^j\right) - \left(\ln c(\boldsymbol{\pi}_1) - \ln c(\boldsymbol{\pi}_0)\right)\right]$$

where the taste parameter β^j cancels. Now take a weighted sum (with weights ω^j) over the subset of characteristics available in both periods $\mathcal{J}$ and rearrange for the difference in the log unit cost

$$\ln c(\boldsymbol{\pi}_1) - \ln c(\boldsymbol{\pi}_0) = \sum_{j \in \mathcal{J}} \omega^j \left(\ln \pi_1^j - \ln \pi_0^j\right) - \frac{1}{\sigma - 1} \sum_{j \in \mathcal{J}} \omega^j \left(\ln s_1^j(\mathcal{J}_1) - \ln s_0^j(\mathcal{J}_0)\right)$$

Define the characteristic budget shares with respect to expenditure on the goods in this common set as

$$s_t^j(\mathcal{J}_t) = s_t^j(\mathcal{J})\eta_t$$

where

$$\eta_t = \frac{\sum_{j \in \mathcal{J}} \pi_t^j z_t^j}{\sum_{j \in \mathcal{J}_t} \pi_t^j z_t^j}$$

is the share of shadow expenditure in period t spend on common characteristics. This gives

$$\ln c(\boldsymbol{\pi}_1) - \ln c(\boldsymbol{\pi}_0) = \sum_{j \in \mathcal{J}} \frac{\mu_j}{\mu} \left(\ln \pi_1^j - \ln \pi_0^j\right) - \frac{1}{\sigma - 1} \left(\ln \eta_1 - \ln \eta_0\right)$$

$$\mu_j = \frac{s_1^j(\mathcal{J}_1) - s_0^j(\mathcal{J}_0)}{\ln s_1^j(\mathcal{J}_1) - \ln s_0^j(\mathcal{J}_0)}$$

so that the weights are

$$\omega^j = \frac{\mu^j}{\sum_{j'} \mu^{j'}}$$

Finally expressing the difference in log unit costs in levels gives the mix-adjusted Sato-Vartia index which we term the Sato-Vartia-Feenstra index.

$$P_{SVF} = \frac{c(\boldsymbol{\pi}_1)}{c(\boldsymbol{\pi}_0)} = \left(\frac{\eta_1}{\eta_0}\right)^{\frac{1}{\sigma-1}} \prod_{j \in \mathcal{J}} \left(\frac{\pi_1^j}{\pi_0^j}\right)^{\omega^j}$$

Thus the Sato-Vartia-Feenstra index is equal to the conventional Sato-Vartia hedonic index defined over the characteristics observed in both period, multiplied by the adjustment factor

$$\left(\frac{\eta_1}{\eta_0}\right)^{\frac{1}{\sigma-1}}$$

The adjustment factor takes into account the changing set of characteristics on the extensive margin in two ways. The first depends on the share of new/old characteristics in the bundle and how that changes between periods. If, for example many new characteristics are introduced then η_1 will tend to be small and so the hedonic price index will be lower. Removing existing characteristics will have a countervailing effect on the denominator. The second relates to the substitutability between characteristics. This is governed by the σ parameter. The correction factor is less important the higher is the substitution parameter σ, i.e., ignoring new characteristics matters less if they are close substitutes for existing ones. Equally, the bias will be worse whenever the new characteristics are not easily substitutable with existing characteristics.

The adjustment factor also has a simple interpretation as the proportional bias arising from ignoring new characteristics in the construction of the index and calculating it over the over-lapping set $\mathcal{J}$.

$$\left(\frac{\eta_1}{\eta_0}\right)^{\frac{1}{\sigma-1}} = \frac{P_{SVF}}{P_{SV}(\mathcal{J})}$$

4 The Data

The data were collected by the Office for National Statistics from the magazine *What Car*, and give detailed information on the all major new car models that became available over the period 1988 to mid 1995. The data on market shares and sales are from the Competition Commission.[13] Basic descriptive statistics are given in Table 1. The data covers 36 models of cars. Prices and specifications are measured quarterly. Prices are recorded in current, nominal pounds. Air conditioning to driver's airbag are dummy variables. The acceleration variable records the 0-60mph time and is measured in seconds. Fuel consumption is measured in liters per 100 km. Engine size is recorded in cc's. Brake horsepower and torque are measured at a fixed engine speed and torque is measured in newton metres (N m). Finally length is in meters.

Figure 1 shows how the distribution of prices evolved over the period. The left hand panel shows the density of prices in each quarter (later periods are indicated by lighter line shading). The price distribution moves to the right as prices rise over time. There is also a bi-modal characteristic in the earlier years which is somewhat lost in later years where the market is less obviously split into normal/family versus luxury cars. The right hand panel shows a contour plot of the evolution of the price distribution over time. This illustrates the general up-shift in new car prices of the period. The solid line overlaid on the contours shows the quarterly average price. We can see that the average price of new cars during this period was somewhat over nine thousand pounds. Over the period average prices of new cars rose by about 50% per cent from from £7,350 in the first quarter of 1988 to £10,853 by the end of the second quarter in 1995.

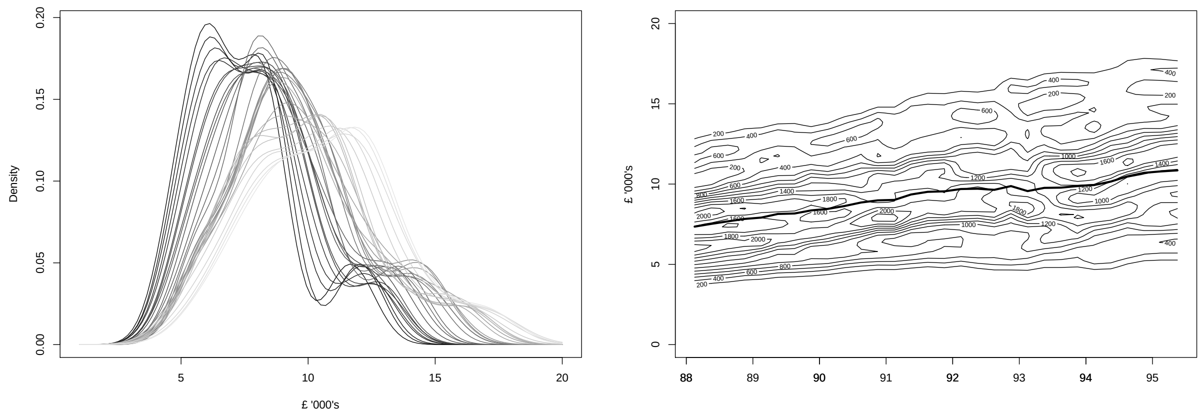
The specifications of new cars generally changed twice per year in the U.K. during the period studied: at the beginning of the fourth quarter, which marks the time when new annual registration (number-plate) letters were issued in the U.K., and the beginning of the

[13] http://webarchive.nationalarchives.gov.uk/20111202195250/http:/competition-commission.org.uk/rep_pub/reports/2000/439cars.htm

Table 1: Desciptive Statistics

Statistic	N	Mean	St. Dev.	Min	Max
Year	3,160	1991	2.12	1988	1995
Price (£)	3,160	9,168	2,629	4,215	17,409
Air conditioning	3,160	0.01	0.09	0	1
Central locking	3,160	0.48	0.50	0	1
Manual sun-roof	3,160	0.29	0.46	0	1
Electric sun-roof	3,160	0.09	0.29	0	1
Electric front windows	3,160	0.28	0.45	0	1
All-electric windows	3,160	0.04	0.19	0	1
Powered (heated) seat	3,160	0.14	0.35	0	1
Head-lamp cleaners	3,160	0.01	0.12	0	1
Electric mirrors	3,160	0.14	0.35	0	1
Trip-computer	3,160	0.01	0.07	0	1
Split-fold seats	3,160	0.65	0.48	0	1
Seat adjustment	3,160	0.29	0.45	0	1
Radio	3,160	0.11	0.31	0	1
Radio cassette	3,160	0.77	0.42	0	1
CD player	3,160	0.003	0.06	0	1
Steering adjustment	3,160	0.20	0.40	0	1
Driver's airbag	3,160	0.11	0.32	0	1
Acceleration	3,160	13.42	2.72	8.70	21.40
Fuel consumption at 90 kph	3,160	5.42	0.61	3.62	7.06
Engine size (cc)	3,160	1,391	256	988	1,998
Brake horsepower	3,160	73	21	41	118
Torque	3,160	81	19	50	134
Length (m)	3,160	3.98	0.35	3.05	4.93
French	3,160	0.17	0.37	0	1
German	3,160	0.14	0.35	0	1
Italian	3,160	0.08	0.28	0	1
Japanese	3,160	0.08	0.28	0	1
Spanish	3,160	0.03	0.16	0	1
Swedish	3,160	0.03	0.16	0	1
UK	3,160	0.14	0.35	0	1
US	3,160	0.33	0.47	0	1

Figure 1: The Density of New Car Prices over Time



second quarter, which normally coincides with changes in vehicle excise duty and taxes on motor fuels in the government's Finance Bill.

Looking at the intensive margin first, there were quality improvements in terms of performance across the board: 0-60mph acceleration times improved over the period by 6.5% whilst fuel economy improved by 2.8%. Both torque and brake horsepower improved by 5.2% on average (torque and BHP are proportional at a fixed engine speed). The availability of features also improved: central locking was present in a quarter of cars at the start of the period; this rose to nearly two-thirds by the end. Manual sun-roofs went from being available on 17.6% of models to 25%. Electric front-windows were uncommon at the beginning with 8% market share, but by the end 40% of new models had them. Electrically operated door mirrors went from a similar level to being available on a quarter of new cars, and split-fold rear seats increased from being specified in around half of new cars to nearly three-quarters.

On the extensive margin there were a number of changes to the set of characteristics over the period. In terms of new characteristics: electronic seat and steering-adjustability both became available in the fourth quarter of 1988; electric sun-roofs were introduced in the first quarter of 1990; air conditioning was introduced in the fourth quarter of 1993 as were powered (heated) seats and drivers' airbags. CD-players were introduced in the second quarter of 1994. These appeared along side radio-cassette players which were available throughout but, by the time CD-players were introduced, simple radio-only models had gone – they disappeared from the new car market by the fourth quarter of 1992. In the case of in-car entertainment there was a clear sequence from radios (gone by the end of 1992), via radio-cassettes towards CD-players (from 1994). Other features were short-lived and simply came and went: head-lamp cleaners were available in some cars (Volvos) at the start of our period of observation, but disappeared at the start of 1990 only to reappear briefly from mid 1993 to mid 1994 (Alfa's and Volvo's again) before disappearing again. Similarly early trip-computers became available (also on Alfa's) but disappeared only to be offered briefly by Rover before again disappearing. Trip computers are now completely standard but they did not take off until after the end of our period of observation.

5 Results

We estimate the following equation by weighted least squares[14]

$$p_t^k = \sum_j a_t^{kj} \hat{\pi}_t^j + e_t^k$$

subject to the restrictions from the underlying theory that shadow prices are weakly positive ($\hat{\pi}_t^j \geq 0$) and that the hedonic demands satisfy adding-up

$$\sum_k p_t^k x_t^k = \sum_j \hat{\pi}_t^j z_t^j$$

The hedonic regressions are estimated separately for each quarter using the characteristics listed in Table 1. Where a characteristic is missing, because it is either yet to be introduced or because it has been withdrawn, it is omitted from the regression model. Other than that, the specification is kept fixed.

For comparison we also estimate the index from the time-dummy method. In this case

[14] The weights are market shares.

the data in Table 1 are pooled across years but the model is re-estimated and the index up-dated quarter by quarter to mimic the way in which statistical offices would need to apply the method. As discussed in the introduction, this results in revisions to the index as each new period of data is added and the model up-dated. The model underlying the time-dummy method and the impact of the revisions on the empirical index are discussed in the appendix.

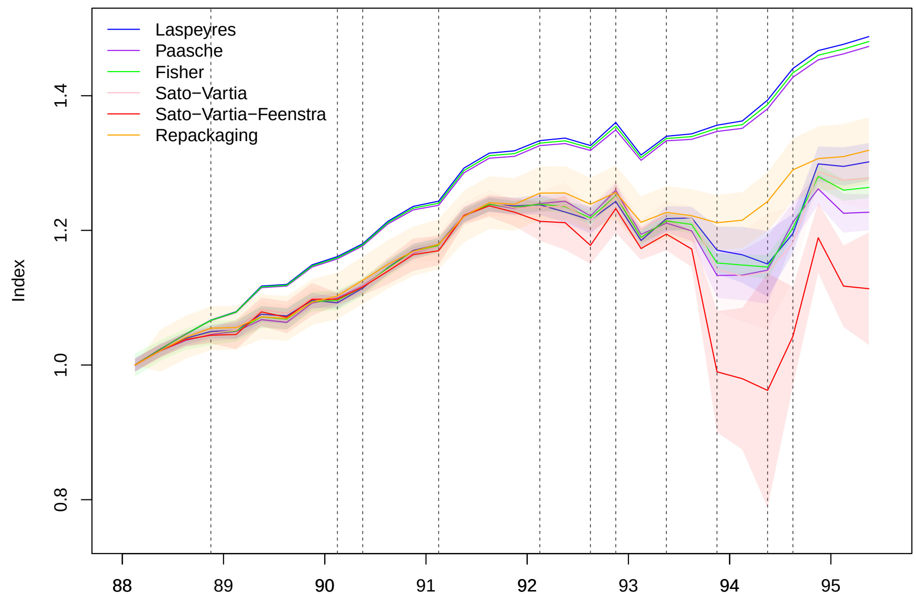
Figure 2 shows a range of fixed-base price indices. The base period is quarter 1 of 1988.[15] The cluster of indices which rise the fastest are the standard, quality-unadjusted Laspeyres, Paasche and Fisher indices. These indicate a quality-unadjusted price rise for new cars over the period of between 47.32% and 48.78%. This matches closely the rise in average prices illustrated in Figure 1.

The rest of the indices are the hedonic/quality-adjusted series for which shaded 95% confidence intervals are provided. The cluster of price indices showing a range of price increases between 22.69% and 31.89% by the end of the period are the hedonic Laspeyres, Paasche, Fisher and Sato-Vartia indices and the (final) Time-Dummy Method index. Generally they are ordered Paasche, Fisher/Sato-Vartia, Laspeyres, Time Dummy (lowest to highest) over the period. Depending on the choice of index number formula they show that controlling for quality change by holding the characteristics fixed at their 1988q1 levels whilst allowing shadow prices to evolve according to innovations on the intensive margin reduces the rate of measured price increase for new cars by a factor of about a half.

The effect of adjusting for quality change on the *extensive* margin as well as the intensive margin is given by the mix-adjusted Sato-Vartia-Feenstra index. We calculate it for the estimated value of $\hat{\sigma} = 1.612$. It shows an overall price increase of around only 8% by the second quarter of 1995. This is significantly lower than both the quality-unadjusted index numbers and also the standard hedonic indices. The vertical dashed lines indicate periods in which changes on the extensive margin occurred. As we can see the major point of

[15] Alternative bases are available from the authors.

Figure 2: Fixed Base Price Indices

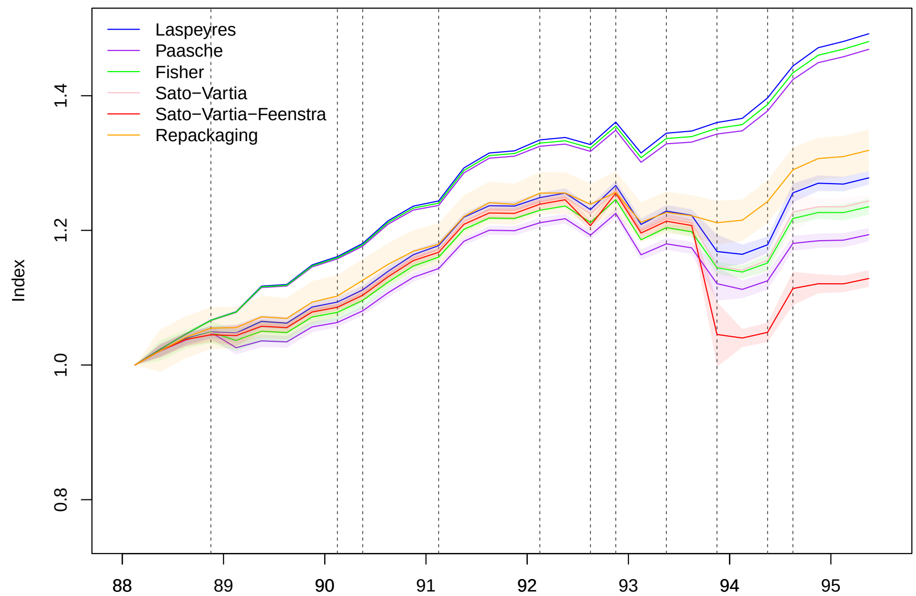


Notes: 1988q4: adjustable seats and adjustable steering introduced; 1990q1: electric sun roofs introduced; 1990q2: headlamp cleaners withdrawn; 1991q1: trip computers introduced; 1992q1: trip computers withdrawn; 1992q3: trip computers re-introduced for one quarter; 1992q4 radio-only withdrawn; 1993q2: headlamp cleaners re-introduced; 1993q4: air conditioning, power seats and driver's airbags introduced; 1994q2 CD players available; 1993q4: headlamp cleaners finally withdrawn.

departure between the Sato-Vartia-Feenstra index and the other hedonic methods occurs in the fourth quarter of 1993. This is when there were three simultaneous significant innovations relating to comfort and safety: air conditioning, powered (heated) seats and divers' airbags were all introduced to coincide with new car registrations in the autumn of that year. It is also noticeable however, that changes in the extensive margin are not a one-way street: the withdrawal of features such as radio-only (1992q4) and headlamp cleaners (1993q4) are associated with a slight catching-up of the Sato-Vartia-Feenstra index in those periods.

The benefit of fixed-base as opposed to chained index numbers is that it is clear what is being compared to what – the set of characteristics in the case of hedonic indices, or the products in the case of quality-unadjusted index numbers, are being held constant across the entire period. Chained indices lack this clarity of interpretation. Nonetheless they have

Figure 3: Chained Price Indices

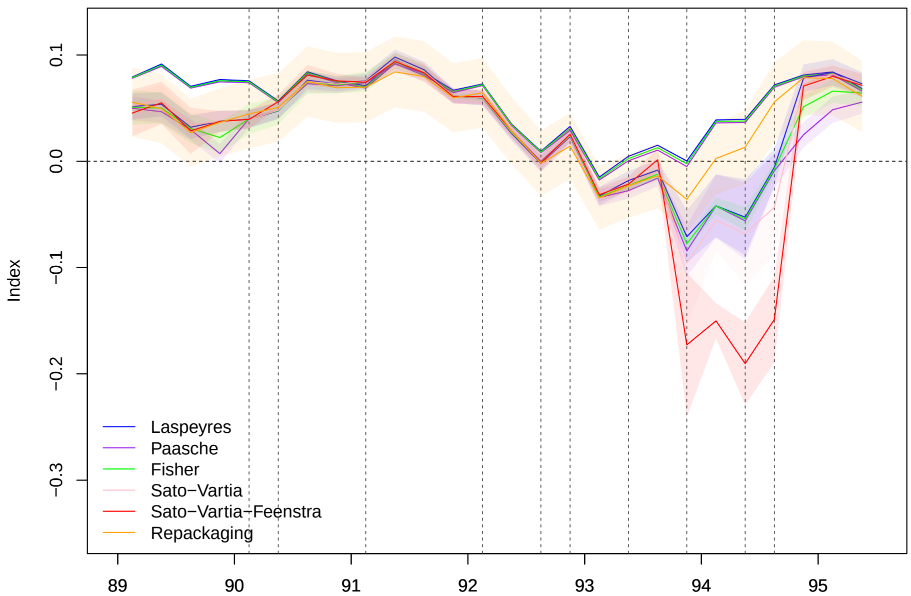


Notes: 1988q4: adjustable seats and adjustable steering introduced; 1990q1: electric sun roofs introduced; 1990q2: headlamp cleaners withdrawn; 1991q1: trip computers introduced; 1992q1: trip computers withdrawn; 1992q3: trip computers re-introduced for one quarter; 1992q4 radio-only withdrawn; 1993q2: headlamp cleaners re-introduced; 1993q4: air conditioning, power seats and driver's airbags introduced; 1994q2 CD players available; 1993q4: headlamp cleaners finally withdrawn.

the significant advantage, discussed above, of allowing the mix of products or characteristics to change over time. Figure 3 shows the chained indices for the period studied. When we calculate chain indices, rather than fixed-base indices, only adjacent periods are compared directly. As a result, only period-to-period changes in the extensive margin matter and, since period-to-period changes can be confidently expected to be less numerous than changes cumulated over a longer period, we should expect the effects of the mix-adjustment in the Sato-Vartia-Feenstra index to be less pronounced.

Figure 3 shows that the chained mix-adjusted index does not depart significantly from the other chained hedonic indices during the early part of the study period. The quarter-to-quarter changes in specifications on the extensive margin were evidently too minor to matter greatly. However, later on it is clear that chaining does not necessarily provide an empirical fix for the new characteristics problem. The burst of innovations which occurred together in

Figure 4: Annual Inflation Rates, Hedonic Price Indices; adjusted and unadjusted



Notes: 1988q4: adjustable seats and adjustable steering introduced; 1990q1: electric sun roofs introduced; 1990q2: headlamp cleaners withdrawn; 1991q1: trip computers introduced; 1992q1: trip computers withdrawn; 1992q1: trip computers re-introduced for one quarter; 1992q4 radio-only withdrawn; 1993q2: headlamp cleaners re-introduced; 1993q4: air conditioning, power seats and driver's airbags introduced; 1994q2 CD players available; 1993q4: headlamp cleaners finally withdrawn.

1993 (air conditioning, power (heated) seats and drivers' airbags) are not ignorable even over the short-term, and hence the Sato-Vartia-Feenstra index adjusts accordingly. By comparing Figure 2 and Figure 3 we can see that, by the end of the our period of study, the overall effects of adjusting for the characteristics mix is similar regardless of whether we choose a fixed-base or a chained index approach.

The annual inflation rates corresponding to the chained index are presented in Figure 4. Note that the inflation rates in Figure 4 are fourth-differenced versions of the quarterly data shown in Figure 3. We see a significant departure of the Sato-Vartia-Feenstra index from the other hedonic models reflecting the numerous specification changes that took place towards the end of the period. These results show that the apparent deflation in new car prices once changes in the extensive margin have been accounted for which we saw in Figure 2 is not an artefact of a comparison being made over a long time period (1988q1 and 1993/1994) in which many innovations accrue; it is also evident over a much shorter period of comparison. Chaining does not, therefore, necessarily fix the new-characteristics problem.

6 Conclusions

Hedonic regression methods of quality adjustment of prices are now an established part of national statistical offices' toolkit. Since the time-dummy method based on the repackaging model cannot be used by statistical offices because of the revisions it entails, the only method in widespread use is the "indirect method" or variations on it. Hedonic indices based on the indirect method suffer from a problem akin to new-goods bias when there are changes in product specifications on the extensive margin (new features are introduced, old features are withdrawn). We have shown that applying the approach developed by Feenstra (1994) for mix-adjusting the Sato-Vartia index in characteristics space, provides a theoretically-consistent, effective method of allowing for such changes. We have also shown in our application to UK new car prices that it makes a quantitatively important difference to estimated inflation rates. Very roughly, if we take the Boskin Commission's estimate of the bias in measured inflation from new goods and quality change of 0.6 percentage points per year, our results suggest that allowing for changes in the extensive margin of characteristics could add half as much again to that, at least for those commodity groups in which proliferation of product features is important.

Two potential drawbacks of our approach need to be borne in mind. The first is that it is based on a theoretical assumption – that preferences for product characteristics are asymmetric CES. The asymmetric CES is "not-quite-superlative";[16] that is to say, it is not a flexible functional form and therefore we cannot necessarily be confident that the approximation to true preferences will be good in the vicinity of the data (although see Hill (2006)). The second is that, just as the Sato-Vartia index is not-quite-superlative, the Sato-Vartia-Feenstra index is "not-quite-exact". A price index number is said to be

[16] W. E. Diewert, personal communication.

exact if its numerical value always corresponds to that of a Konüs cost-of-living index but its calculation only depends on the observed price and quantity data and does not require knowledge of preference-parameter values. The Sato-Vartia-Feenstra index does not require the set of characteristics parameters in the CES to be known, but it does require the value of the elasticity of substitution parameter σ. In this paper we estimated this by fitting CES demand-for-characteristics functions. This represents the one additional piece of estimation which a statistical office would need to do over-and-above its normal hedonic estimation work. An alternative would be to select suitable parameter values for σ from the literature and conduct a sensitivity analysis.

Despite these shortcomings, our method buys the ability to address the new-goods/new-characteristics problem without any of the difficult econometric work involved in solving estimated characteristics-demand equations for reservation prices. This is, we would argue, a practical plus and our method is one which statistical offices could easily adopt.

Appendices

The Time-Dummy Method

As noted in the introduction, changes in the extensive margin can be tackled using the time-dummy method. This is based on the repackaging model of Fisher and Shell (1968) and Muellbauer (1974)) which assumes that the coefficients on characteristics are time-invariant and that pure price effects come only through time dummies. This model allows statistical agencies to price characteristics when they do not exist by imputing a fixed shadow price for them in all periods. However, using the repackaging/time-dummy method means having to revise the index retrospectively every time new data are appended to the series. It is this feature which effectively rules it out of any practical consideration and we are not aware of any national statistical offices which use the time-dummy method in anything other than experimental indices. We briefly outline the repackaging model below and illustrate the effect of revisions.

The repackaging model assumes that the utility function for different varieties is linear:

$$u\left(x^1, ..., x^K\right) = \sum_k \gamma^k x^k$$

where the γ^k coefficients represent the quality of the k'th variety. Note that they are not time-varying. The first-order conditions for the model

$$\max_{x^1,...,x^K} \sum_k \gamma^k x^k \quad \text{subject to} \quad \sum_k p_t^k x^k \leq y_t$$

are

$$p_t^k = \lambda_t \gamma^k$$

for varieties which are purchased and $\lambda_t > 0$ is the marginal utility of income. Taking logs gives

$$\log p_t^k = \log \lambda_t + \log \gamma^k$$

The step required to take this model to the data is to imagine that log quality is well approximated by a linear function of product characteristics. Let one unit of the k'th variety contain a^{kj} units of the j'th characteristic.

It is assumed that

$$\log \gamma^k \approx \sum_j \pi^j a^{kj}$$

Now suppose that we observe the prices and characteristics of the different varieties over time ($t = 1, ..., T$). Pooling the data over time gives

$$\log p_t^k = \log \lambda_1 d_1 + \log \lambda_2 d_2 + ... + \log \lambda_T d_T + \sum_j \pi^j a^{kj}$$

where d_t is a dummy variable which takes the value 1 if the observation was made in year t. Appending an econometric error term, the model is estimated by OLS regression of log prices on time dummies and data on the characteristics of varieties.

$$\log p_t^k = \widehat{\log} \lambda_1 d_1 + \widehat{\log} \lambda_2 d_2 + ... + \widehat{\log} \lambda_T d_T + \sum_j \hat{\pi}_j a^{kj} + e_t^k$$

The model is then used to predict the expected price in period t conditional on a *fixed* set of reference characteristics given by $\{z_1, ..., z_J\}$

$$\mathbb{E}\left(\log p_t | z_1, ..., z_J\right) = \widehat{\log} \lambda_{\mathbf{t}} + \sum_j \hat{\pi}_j z^j$$

or

$$\mathbb{E}\left(p_t | z_1, ..., z_J\right) = \exp\left(\widehat{\log} \lambda_t + \sum_j \hat{\pi}_j z^j + \frac{1}{2}\widehat{\sigma_e}^2\right)$$

where $\widehat{\sigma_e}$ is the standard error of the regression. The hedonic price index between any two periods $\{s, t\}$ is

$$P_R\left(\widehat{\log \lambda_s}, \widehat{\log \lambda_t}, \hat{\pi}_1, ..., \hat{\pi}_J, z_1, ..., z_J\right) = \frac{\mathbb{E}\left(p_t|z_1, ..., z_J\right)}{\mathbb{E}\left(p_s|z_1, ..., z_J\right)}$$

and is formed by holding characteristics/quality constant as

$$P_R\left(\widehat{\log \lambda_s}, \widehat{\log \lambda_t}, \hat{\pi}_1, ..., \hat{\pi}_J, z_1, ..., z_J\right) = \frac{\exp\left(\widehat{\log \lambda_t} + \Sigma_j \hat{\pi}_j z^j + \frac{1}{2}\widehat{\sigma_e}^2\right)}{\exp\left(\widehat{\log \lambda_s} + \Sigma_j \hat{\pi}_j z^j + \frac{1}{2}\widehat{\sigma_e}^2\right)} = \frac{\exp\left(\widehat{\log \lambda_t}\right)}{\exp\left(\widehat{\log \lambda_s}\right)}$$

Note that the characteristics can be fixed at any level without altering the price index; as long as they are fixed they cancel. As a consequence the hedonic price index is just the ratio of (the exponents of) the time dummies – which gives the approach its name. Note too that chained and fixed base hedonic indices are identical – the cancelling of successive terms in the chained index makes it path-independent.

As a new year of data becomes available it is added to the estimation equation (the new data is pooled with the existing data and a new time dummy is added to the model). New characteristics can therefore be introduced to the model as new data become available as long as at least some varieties do not possess the characteristics, in which case the effect of the new characteristic would be co-linear with the period. Assuming that this is not a problem then, as new data are added, the other coefficients of the model will adjust. Importantly this will include the time dummies and as a consequence the hedonic price index will change retrospectively.

Figures 5 and 6 illustrate the hedonic index calculated via the time-dummy method for our data. The model is re-estimated as each successive quarter of data becomes available to mirror the way in which a statistical agency would proceed. The effects of the revisions are illustrated by each separate time series.

Figure 5: Revisions to the Time-Dummy Model (Levels)

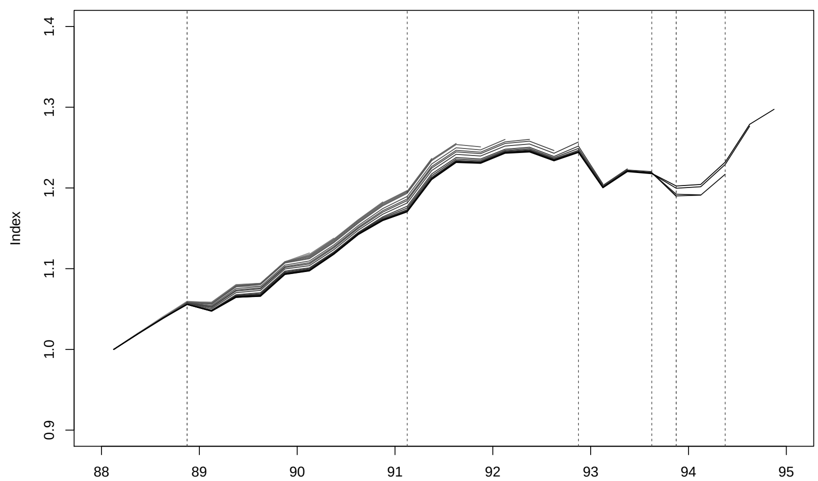
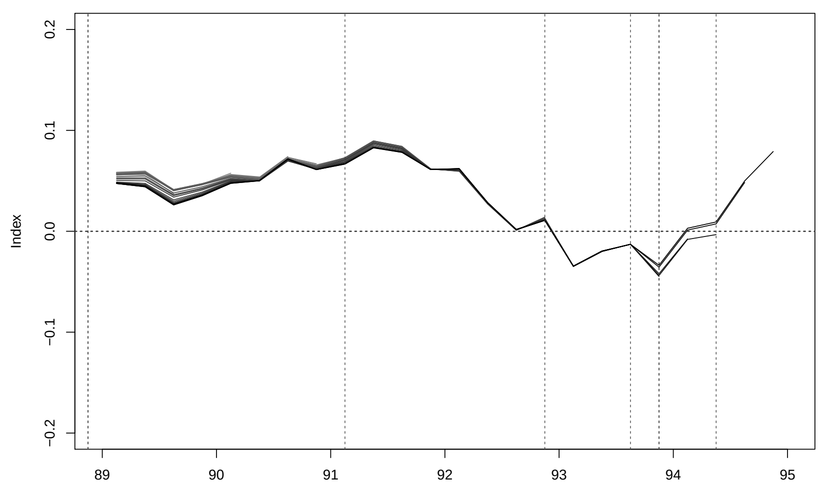


Figure 6: Revisions to the Time-Dummy Model (Changes)



References

Baldwin, R., and J. Harrigan (2011): "Zeros, Quality, and Space: Trade Theory and Trade Evidence," *American Economic Journal: Microeconomics*, 3(2), 60–88.

Ball, A., and A. Allen (2003): "The Introduction of Hedonic Regression Techniques for the Quality Adjustment of Computing Equipment in the Producer Prices Index (PPI) and Harmonised Index of Consumer Prices (HICP)," *Economic Trends*, 592, 30–36.

Barten, A. P. (1964): "Family Composition, Prices, and Expenditure Patterns," in P. Hart, G. Mills, and J. K. Whitaker (eds.): *Econometric Analysis for National Economic Planning*, London: Butterworth.

Blow, L., M. Browning, and I. Crawford (2008): "Revealed Preference Analysis of Characteristics Models," *Review of Economic Studies*, 75(2), 371–389.

Broda, C., and D. E. Weinstein (2006): "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 121(2), 541–585.

Diewert, W. E. (1976): "Exact and Superlative Index Numbers," *Journal of Econometrics*, 4(2), 115–145.

——— (1981): "The Economic Theory of Index Numbers: A Survey," in A. Deaton (ed.): *Essays in the Theory and Measurement of Consumer Behaviour in Honour of Sir Richard Stone*, London: Cambridge University Press, 163-208.

Feenstra, R. C. (1994): "New Product Varieties and the Measurement of International Prices," *American Economic Review*, 84(1), 157–77.

Feldstein, M. (2017): "Underestimating the Real Growth of GDP, Personal Income, and Productivity," *Journal of Economic Perspectives*, 31(2), 145–164.

Fisher, F. M., and K. Shell (1968): "Taste and Quality Change in the Pure Theory of the True Cost of Living Index," in J.N. Wolfe (ed.): *Value, Capital and Growth*, Edinburgh: Edinburgh University Press, 97–139.

Gordon, R. J. (1999): "The Boskin Commission Report and its Aftermath," *Monetary and Economic Studies*, 17(3), 41–68.

Gorman, W. M. (1980): "A Possible Procedure for Analysing Quality Differentials in the Egg Market," *Review of Economic Studies*, 47(5), 843–856.

Griliches, Z. (1961): "Hedonic Price Indexes for Automobiles: An Econometric Analysis of Quality Change," in: *The Price Statistics of the Federal Government: Review, Appraisal, and Recommendations*, New York: National Bureau of Economic Research, General Series 73, pp. 173-96.

Hausman, J. (2003): "Sources of Bias and Solutions to Bias in the Consumer Price Index," *Journal of Economic Perspectives*, 17(1), 23–44.

Hausman, J. A. (1997): "Valuation of New Goods under Perfect and Imperfect Competition," in T.F. Bresnahan and R.J. Gordon (eds.): *The Economics of New Goods*, Chicago: Chicago University Press, 209–247.

Hill, R. J. (2006): "Superlative Index Numbers: Not All of Them are Super," *Journal of Econometrics*, 130(1), 25–43.

Keynes, J. M. (1930): *A Treatise on Money: Volume 1 The Pure Theory of Money.* London: MacMillan.

Lancaster, K. J. (1966): "A New Approach to Consumer Theory," *Journal of Political Economy*, 74(2), 132–157.

Muellbauer, J. (1974): "Household Production Theory, Quality, and the 'Hedonic Technique'," *American Economic Review*, 64(6), 977–994.

Neary, J. P., and K. W. Roberts (1980): "The Theory of Household Behaviour under Rationing," *European Economic Review*, 13(1), 25–42.

Redding, S. J., and D. E. Weinstein (2018): "Measuring Aggregate Price Indexes with Demand Shocks: Theory and Evidence for CES Preferences," NBER Working Paper No. 22479.

Sato, K. (1976): "The Ideal Log-Change Index Number," *Review of Economics and Statistics*, 58(2), 223–228.

Varian, H. (2018): "Measurement Challenges in High Tech: Silicon and Statistics," presented at ESCoE/ONS/BoE Conference on Economic Measurement, Bank of England, London.

Vartia, Y. O. (1976): "Ideal Log-Change Index Numbers," *Scandinavian Journal of Statistics*, 3(3), 121–126.

Wells, J., and A. Restieaux (2013): "Review of Hedonic Quality Adjustment in U.K. Consumer Price Statistics and Internationally," Office for National Statistics, UK, mimeo.