Costs of energy efficiency mandates can reverse the sign of rebound

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Abstract

Improvements in energy efficiency can reduce costs of consuming services from cars and appliances and result in positive rebound that offsets part of the direct energy reduction. Our analytical general equilibrium model decomposes rebound into direct and indirect effects. A costless technology shock has positive rebound as in prior literature, but a pre-existing energy efficiency standard can negate direct energy savings from that shock. For increased stringency of energy efficiency standards, however, we show how income effects reduce energy use for both services and other goods. We show exactly when those increased costs imply negative total rebound.

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New energy efficiency technology can reduce electricity or fuel use needed to get the same services such as cooling from an air conditioner or refrigerator, heat from a furnace, or miles driven in a car. It reduces the marginal cost of those services, so it encourages consumers to make more use of those appliances – and thus causes a rebound effect that offsets at least part of the energy savings. Many papers demonstrate this positive rebound effect, both in empirical partial equilibrium models and in theoretical general equilibrium models. Here, we demonstrate potential problems interpreting those results and their implications for energy policy. In particular, these papers analyze effects of a costless technology shock (CTS) and correctly shows the important economic effects of technology. But policymakers cannot require a costless technology shock. They can require that firms use more resources to achieve more energy efficiency in the appliances they produce, but the economic effects of such requirements cannot be inferred from studies of a CTS. A few papers consider costs of energy efficiency in partial equilibrium or computable models. None provide analytical general equilibrium results for a costless technology shock and for increased stringency of an energy efficiency standard (EES) in a world with a pre-existing standard that is already binding and therefore costly.

In other words, we focus on the costs of policies that require energy efficiency in a model that incorporates economy-wide resource constraints and where these policies face increasing marginal cost of achieving greater energy efficiency. Our general equilibrium model is solved analytically to decompose rebound into direct effects and indirect effects, each of which is explained by income and substitution effects. The closed-form solutions show exactly how each effect depends on parameters. Then we also calibrate the model to provide numerical illustrations. We show that this view of the policy experiment can indeed improve energy efficiency and has the same kind of direct efficiency effect of reducing energy use, but the sign of rebound likely switches from positive to negative. What do we mean by "likely"? The sign is still formally ambiguous, but our theoretical results show exactly the conditions under which rebound is negative. And the most plausible cases we consider in our numerical illustrations have negative rebound.

The intuition is clear: the CTS is an exogenous improvement in technology that makes more possible with less, expanding the production possibility frontier (PPF). It raises real income, which induces consumers to purchase more heating or cooling services. In contrast, the EES requires a move along the PPF. With no pre-existing mandate, introducing a small EES has small costs. But, most developed economies have substantial energy efficiency

requirements. If technology is given, and consumers are constrained to buy more-expensive appliances with more energy efficiency than they would if unconstrained, then an increase in stringency can have rising marginal costs that reduce real income. Those extra costs could well be justified by benefits of reduced negative externalities from energy use, but costs of complying with the mandate still reduce household incomes and thus have negative income effects on the purchases of all goods including appliance services. We show how the negative income effect offsets part of the positive direct effect on energy used for those services, and it creates a large negative indirect rebound effect on energy used in production of other goods.

Scores of economics papers have provided theoretical and empirical analyses of rebound effects from energy efficiency improvements, but the size of rebound is still under debate. These papers include both partial equilibrium (PE) and general equilibrium (GE) models, but most focus on a costless technology improvement in energy efficiency and find positive rebound.[1] Some research considers the costs of energy efficiency improvements in PE models, and they find smaller rebound effects than in the case of a costless technology change.[2] But, PE studies do not usually incorporate economy-wide resource constraints. A few papers include costs of energy efficiency in numerical results using quantitative, multi-sectoral, dynamic, GE growth models.[3] While these PE or GE papers may consider the costs of greater energy efficiency, however, they are not clear about whether these costs are incurred voluntarily by firms and consumers or are mandated by government. Yet, voluntary costs can be presumed to *raise* consumer welfare, with positive overall effects on real income. We calculate rebound effects from a mandated increase in costly energy efficiency, and we compare them to effects of a costless technological improvement. We also show how the effect of either such shock is altered by pre-existing costly efficiency standards.[4]

Our simple general equilibrium model has many identical consumers who get utility

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[1] For reviews, see Greening *et al*. (2000), Sorrell and Dimitropoulos (2008), Sorrell *et al.* (2009), and Gillingham *et al*. (2016). For PE examples, see Thomas and Azevedo (2013) and Chan and Gillingham (2015). GE models include Grepperud and Rasmussen (2004), Wei (2007, 2010), Abdessalem and Labidi (2016), and Lemoine (2017).

[2] See Mizobuchi (2008) and Nässén and Holmberg (2009). In Borenstein (2015), the price of electricity exceeds social marginal cost, a pre-existing distortion that encourages energy efficiency. He considers the consumer's choice to spend more for additional energy efficiency, a cost that reduces income and can reduce use of energy.

[3] See Allan *et al.* (2007), Barker *et al.* (2007, 2009), Turner (2009), and Chang *et al.* (2018).

[4] The review by Gillingham *et al.* (2016) points out the distinction between rebound from a costless technology shock and a costly policy shock, and it describes how empirical estimates mix the two. Interestingly, however, none of the papers in this large rebound literature has analyzed an energy efficiency improvement – either costless or costly – as we do here in a world with a pre-existing policy that is both costly and binding.

from vehicle or appliance services and from another composite good. They own a single factor of production that they can sell to firms for income to buy appliances, energy to run them, and the other composite good. We model production, all resource constraints, and zero profit conditions of competitive firms. We differentiate all equations to linearize the model, and then we analyze three kinds of exogenous shocks. First, we consider a costless technology shock with no change in spending on appliances. That shock raises welfare and reduces direct energy use – with positive rebound as in prior literature. Second, we consider a CTS in a world with a pre-existing efficiency mandate that is binding and costly. When consumers can change all their other choices, we show how the CTS allows them to reduce their expenditures that were formerly necessary to satisfy the mandate. The result is no gain in energy efficiency, which still barely satisfies the unchanged mandate, but the CTS does have a positive income effect and so still yields positive rebound. Third, and finally, we consider the essential policy question: what are the effects of increasing the energy efficiency requirement of a pre-existing mandate? The answer depends on the curvature of the cost function for acquiring additional energy efficiency (given existing technology). If that extra efficiency is cheap, then overall welfare costs are low, but we consider the likely case that the marginal cost of achieving additional energy efficiency is not only significantly positive but rising – as policymakers attempt to achieve greater energy efficiency.

Following sections describe the model (section 1), our linearization (2), theoretical results for the three types of shocks just described (sections 3, 4, and 5), our calibration (6), numerical results (7), sensitivity analysis (8), extensions (9), and conclusions (10). All appendices are included below, but later will be online only. Our section on extensions discusses rebound with: (a) negative effects of energy use on productivity, (b) upward-sloping energy supply, (c) energy efficiency standards in the other sector, and (d) disaggregation.

1. The Analytical General Equilibrium Model

For simplicity, we assume a static, one-period, closed economy with competitive markets and a large number n of identical consumers (or households). These consumers each own and supply a single primary factor K, which can be labor, capital, or a composite of both. As specified below, K is used in production of energy E, appliances A, and a composite of all other goods X (such as clothing, food, and shelter). With this aggregation for tractability, we define A to include air conditioners, furnaces, and all other consumer durables that use energy – including vehicles. Then a costless technological discovery might increase a refrigerator's

cooling per kilowatt hour of electricity, or it might increase a car's miles per gallon. In fact, one of our major examples of a mandate is the federal corporate average fuel economy (CAFE) requirement that manufactures meet a minimum fleet-average miles per gallon. Several nations have such requirements and might increase their stringency.

In a form of home production, each household produces services S from purchases of appliances A and energy E_S (that is, energy, E, used for services, S). These services include refrigeration, cooling in summer, heat in winter, and miles driven. Consumers get utility from these services, and from the composite good, and they get disutility from the economy-wide aggregate use of energy. Thus, each consumer's utility function takes the following form:

$$U = U(S, X; nE) \tag{1.1}$$

where U is twice continuously differentiable, quasi-concave, and homothetic. It is increasing in the first two arguments and decreasing in nE, aggregate energy use.[5] This public good or bad is separable in utility, so changes in nE do not affect consumer choices of S or X.

Our simple model cannot consider the discrete choice of whether to buy an appliance or vehicle. Instead, the household can choose any amount of the aggregate "A" commodity. Moreover, conceptually, features of a car not related to energy efficiency such as leather seats are best considered to be part of the other good, X. Similarly, a home's double-pane insulated glass window is part of A, but the stained-wood window frame is part of X. Thus, we use A not to represent total cars and appliances, but only the portion devoted to energy efficiency.[6]

With those clarifications, the home-production function is:

$$S = \epsilon A E_S \tag{1.2}$$

where ϵ is a technology scalar, and ϵA is energy efficiency. If ϵA is in miles per gallon, for example, then multiplication by E_S in gallons of gasoline yields S in miles. Or, if ϵA is in cooling per kwh, and E_S is kwh of electricity, then S is measured in degrees of cooling. With only one type of A, consumers who want more energy efficiency must buy more A. Then, with this model, we can study a small exogenous increase in the technology scalar, ϵ, to solve

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[5] If n is large enough, an individual household disregards its own contribution to aggregate energy use and takes as fixed the third argument in utility (nE). Except for an extension in Section 9, we do not make use of this third argument in our analysis, but include it primarily as a reminder of the reason to have an energy efficiency policy.

[6] With no data to identify the portion spent on fuel efficiency, our calibration below uses alternative assumptions. Also, our static model abstracts from the timing of durable purchases, vintages that affect energy use, and the distinction between short run and long run effects. We compare long run equilibria. But, we capture the key fact that either a CTS or EES would reduce the price of services and thus give rise to rebound.

for rebound effects from a costless technology shock (CTS), and we can study a small increase in required spending on A to analyze an energy efficiency standard (EES).

Extra energy efficiency in A can be purchased, and it is produced by firms using extra resources, so the total cost of making appliances or cars more energy efficient is expressed in units of the composite input, K_A (that is, composite input, K, used in production of A):

$$K_A = B(A - A_0)^{\beta}, \text{ with } 0 \leq A_0 \leq A, \ B > 0, \text{ and } \beta \geq 1 \qquad (1.3)$$

The scale parameter B converts units (from A to K), and the exponent β represents cost curvature. If $\beta = 1$, then cost is linear, so additional fuel efficiency can be achieved at a flat marginal cost. But we generally assume $\beta > 1$, to capture the likely case that additional efficiency requires using successively more expensive technologies or materials. The parameter A_0 provides flexibility to shift the intercept of the cost function; this shifted function is used in most engineering studies reviewed below and in our calibration appendix.[7] The general idea is that any car must generate some positive miles per gallon (mpg), even when *no* costs are incurred trying to raise mpg. Some energy efficiency (A_0) comes with any car, while costs of additional efficiency could be quadratic (e.g., β=2).

The model abstracts from various taxes on inputs or outputs, just as it abstracts from government expenditures.[8] The essential function of government modeled here includes only a required target total energy efficiency of household appliances, a target that can be stated in miles/gallon or degrees of cooling/kwh. That standard can be represented as $\epsilon A \geq \eta$, for a policy scalar η. We assume the existing policy is binding, so fuel efficiency matches the scalar: $\epsilon A = \eta$. For a given value of the technology parameter, government has essentially set A, which costs K_A. Thus, a new regulation that raises required fuel efficiency must raise costs in this model, where the cost of A rises at an increasing rate (increasing marginal cost).[9]

Because the initial equilibrium has a pre-existing efficiency standard that is both

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[7] Those studies include NRC (2002, 2015), DOE (2011, 2016a, 2016b), and Lutsey *et al.* (2017). Our cost function allows A_0 to be zero, and we later determine whether a positive A_0 is necessary to fit the data.

[8] Our model could be extended to include other taxes and second-best effects, but those topics are well covered already (see papers collected in Goulder, 2002). Such extensions would complicate our analysis and modify our solutions by adding more rebound terms of either sign, but it would not remove the key terms we discuss below. It might confuse readers to discuss old second-best results that are not the topic of this paper. We prefer the simplest model necessary to demonstrate our new results (as summarized above and in propositions below).

[9] These costs follow from our assumption that consumers are well-informed optimizers facing a costly mandate. In a behavioral model where consumers miss opportunities to reduce their own overall costs by choosing more energy efficiency, a mandate could raise welfare – with effects very similar to the CTS analyzed below. Here we use the simplest model to show how increased stringency can raise costs and thus have negative rebound.

costly and binding, we have no need to describe how a consumer facing no standard would maximize $U = U(\epsilon A E_S, X; nE)$ by their choices of A, E_S, and X. Instead, we assume that the government "distorts" those choices by requiring more A than unconstrained consumers would choose – presumably to reduce the negative effect on U from total energy use nE. Our linearization below can analyze small changes, so we also assume that a small increase in technology ϵ would not be enough to make the policy non-binding (discussed more below).

In Proposition 2 below, we solve for a condition on parameters that must hold for our assumption that rational and fully-informed consumers are being forced to purchase more A than they would desire if unconstrained. When we assume this condition holds, then any increase in the stringency of the energy efficiency requirement must be costly and therefore must reduce real income (ignoring the benefits of reduced external damages).[10]

Since ϵ and A are given, and K_A is a fixed cost, the household chooses only energy use E_S (at price P_E) and other goods X (at price P_X). Their only income is from a fixed total factor endowment, $\overline{K}$, which earns the rate of return P_K. Thus, the budget constraint is:

$$I = P_K \overline{K} = P_X X + P_K K_A + P_E E_S$$

Then, because $S = \epsilon A E_S$, the maximization of $U(S, X)$ subject to this budget yields first order conditions that can be solved for the marginal rate of substitution: $\frac{\partial U}{\partial X} / \frac{\partial U}{\partial S} = P_X / (P_E / \epsilon A)$. In other words, given the fixed cost $P_K K_A$, the *effective marginal price of appliance services* is $P_E / \epsilon A$. For example, if P_E is the price of gasoline in \$/gallon, and ϵA is fuel efficiency in miles/gallon, then the cost of an additional mile $(P_E / \epsilon A)$ is in dollars per mile.

This model abstracts from the fact that the choice of car or air conditioner takes place before the choice of miles or cooling. It collapses that dynamic problem to a year in which the economy is in long-run equilibrium, so the household pays the annualized cost of the car plus annual cost of miles. But this model captures exactly the aspect of the problem that gives rise to rebound: once the car or appliance is purchased with a particular energy efficiency, the only marginal cost is energy use. Energy efficiency may rise through a costless shock to ϵ, or a requirement that raises the fixed cost of A, but either such shock *reduces* marginal cost $(P_E / \epsilon A)$ per mile driven or per degree of cooling from the air conditioner.

On the producer side, competitive firms are price takers in all markets, with constant returns to scale (CRTS) production. Firms in sector X use K_X and energy E_X as inputs. Energy

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[10] Or, if that condition is not satisfied, then a policy to increase energy efficiency can raise real income. That case would require a model of how consumers choose energy efficiency (A) that does not maximize utility.

E is generated from input of factor K_E. We choose units of measurement such that one unit of the primary factor K_E can produce one unit of E. Thus, the production functions are:

$$X = X(K_X, E_X) \tag{1.4}$$

$$E = K_E \tag{1.5}$$

Perfect competition and CRTS imply zero-profit conditions stating that the value of each sector's output produced and sold must equal the sum of amounts spent on inputs:

$$P_X X = P_K K_X + P_E E_X \tag{1.6}$$

$$P_E E = P_K K_E \tag{1.7}$$

All markets must clear in equilibrium. That is, energy supply must equal the sum of all demands, and the factor endowment must equal the sum of all factor uses:

$$E = E_X + E_S \ , \tag{1.8}$$

$$\overline{K} = K_X + K_A + K_E \ \ . \tag{1.9}$$

2. Linearization

We totally differentiate and linearize all equations at the initial equilibrium, and we use a "hat" to denote a proportional change (e.g., $\hat{X} \equiv dX/X$). The resulting N linear equations are solved in later sections for N unknowns, the changes in quantities and prices that result from a small exogenous change in technology ($\hat{\epsilon}$) or policy ($\hat{\eta}$). The analysis of small changes does not mean results are small, however. Pre-existing standards act like taxes that raise the cost of appliances, so they create deadweight losses that rise disproportionately with the implicit tax rate. While the initial small standard or tax has only second-order effects on welfare, successive increases have first-order effects. Moreover, our linearization captures exactly the sort of the policy debates about small legislative changes that are most common.

Large new regulations are rare, as actual policy proceeds incrementally. When first enacted in 1975, for example, CAFE rules required new passenger cars by 1978 to average 18 miles per gallon (mpg). The standards increased to 27.5 mpg for model year 1985, and they were raised again in 2011 (NHTSA, 2011a). By model year 2025, vehicles with footprint over 55 square feet need to meet a standard of 46 mpg, while those 41 square feet or smaller must achieve 60 mpg (NHTSA, 2011b). Rules for household appliances similarly face periodic debates about incremental changes.[11]

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[11] A standard enacted in 1988 required top-loading clothes washers manufactured between 1988 and 1994 to have an unheated rinse option, and it required those manufactured after 1994 to have a "modified energy factor"

To proceed, total differentiation of production functions for the three goods shows how the change in each output is determined from changes in each set of inputs:

$$\hat{S} = \hat{\epsilon} + \hat{A} + \widehat{E_S} \tag{2.1}$$

$$\hat{X} = \theta_{KX}\widehat{K_X} + \theta_{EX}\widehat{E_X} \tag{2.2}$$

$$\hat{E} = \widehat{K_E} . \tag{2.3}$$

For sector X, the factor share for input K is θ_{KX}, the factor share for energy is θ_{EX}, and $\theta_{KX} + \theta_{EX} = 1$. The elasticity of substitution in sector X between inputs to production is defined as σ_X, the percentage change in the input use ratio in response to a one percent change in the input price ratio. For small changes, the definition of σ_X implies:

$$\widehat{K_X} - \widehat{E_X} = \sigma_X\left(\widehat{P_E} - \widehat{P_K}\right). \tag{2.4}$$

Since the marginal price of services S is $P_E/\epsilon A$, differentiation yields the proportional change in that price as $(\widehat{P_E} - \hat{A} - \hat{\epsilon})$. The elasticity of substitution in utility between X and S is σ_U, defined as the percentage change in the ratio of those quantities for a one percent change in the marginal price ratio. For small changes, we get:

$$\hat{X} - \hat{S} = \sigma_U\left((\widehat{P_E} - \hat{A} - \hat{\epsilon}) - \widehat{P_X}\right) . \tag{2.5}$$

Then, we differentiate $K_A = B(A - A_0)^\beta$ and manipulate:

$$\widehat{K_A} = \frac{\beta}{1-\alpha}\hat{A} , \tag{2.6}$$

where $\alpha \equiv A_0/A$ is minimum energy efficiency as a fraction of total initial energy efficiency ($0 \leq \alpha < 1$). We interpret $\beta/(1 - \alpha)$ in this equation as the "cost elasticity of energy efficiency", because it is the percent change in cost for one percent more efficiency. Next, totally differentiate zero-profit equations (1.6) - (1.7), and use the firm's FOC's:

$$\widehat{P_X} = \theta_{KX}\widehat{P_K} + \theta_{EX}\widehat{P_E} \tag{2.7}$$

$$\widehat{P_E} = \widehat{P_K} \tag{2.8}$$

Finally, total differentiation and linearization of the market-clearing condition in (1.8) and the resource constraint in (1.9) yield:

$$\hat{E} = \lambda_X\widehat{E_X} + \lambda_S\widehat{E_S} \tag{2.9}$$

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of at least 1.18 (cu.ft./kWh/cycle). This DOE standard was raised in 2007 to 1.26, in 2015 to 1.29, and in 2018 to 1.57. See https://www.gpo.gov/fdsys/browse/collectionCfr.action?selectedYearFrom=2018&go=Go . For another example, the 1992 standard for split system central air conditioners required a "seasonal energy efficiency ratio" (SEER) of at least 10. It was raised in 2006 to 13, and in many states again in 2015 to 14.

$$0 = \gamma_X \widehat{K_X} + \gamma_A \widehat{K_A} + \gamma_E \widehat{K_E} \qquad (2.10)$$

where λ_i denotes the ratio of energy use in sector i to the total use of energy ($\lambda_X + \lambda_S = 1$), and γ_i is the ratio of capital used in sector i to total capital in all sectors ($\gamma_X + \gamma_A + \gamma_E = 1$). We define the primary factor K as numeraire, so $\widehat{P_K} = 0$.

3. Rebound from a Costless Technological Shock with No Binding Mandate

A costless technology shock (CTS) is represented here by a small exogenous change in efficiency technology, $\hat{\epsilon} > 0$. Many prior papers study rebound from this kind of shock in a world with no mandate and assuming no change in spending on appliances. This section is designed to represent prior literature by similarly assuming no mandate nor change in appliance spending. This case is *not* "general equilibrium" in nature, because we assume no change in A. Yet, consumers do react to greater energy efficiency by changing their fuel use, services, and other goods.[12] We use this representation of prior literature below for direct comparisons with results for GE rebound from either a CTS or a policy shock in a world *with* a pre-existing energy efficiency mandate that is both costly and binding. This section also provides a detailed walkthrough of how we decompose rebound into its components.

Thirteen changes ($\hat{X}, \hat{S}, \hat{A}, \hat{E}, \widehat{E_X}, \widehat{E_S}, \widehat{K_X}, \widehat{K_A}, \widehat{K_E}, \widehat{P_X}, \widehat{P_E}, \widehat{P_K}, \hat{\epsilon}$) appear in the ten linearized equations (2.1) – (2.10). This section takes $\hat{\epsilon} > 0$ as exogenous and assumes that A is unchanged ($\hat{A} = 0$). Thus, with K as numeraire ($\widehat{P_K} = 0$), we can solve for the other ten outcomes (in proportional changes). In addition, we unpack the solution for $\hat{E}$ into different terms. Each such term can show a particular rebound effect as a function of parameters and of the exogenous increase in the efficiency scalar, $\hat{\epsilon} > 0$.

Some of the solutions are easy. Production of E uses only the numeraire primary factor, so our first result is $\widehat{P_E} = \widehat{P_K} = 0$. Production of X uses both K and E, but neither input price changes, so CRTS implies $\widehat{P_X} = 0$. Also, the two inputs change in the same proportion as output, $\widehat{E_X} = \widehat{K_X} = \hat{X}$. The effective marginal price of S is $P_E/\epsilon A$, but ϵ rises while A and P_E are unchanged. Therefore, the cost per additional unit of S falls ($\widehat{P_E} - \hat{\epsilon} - \hat{A} = -\hat{\epsilon}$). Appendix A shows derivations for all outcomes, but here are solutions for four of them:

$$\hat{X} = \left(-(\gamma_A + \gamma_E \lambda_S)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} \right) \hat{\epsilon} \qquad (3.1)$$

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[12] Our simple model is not designed to determine consumer choice of A optimally in a world with no mandate. Instead, we want to compare rebound from the CTS with and without a mandate, "all else equal" (including A).

$$\hat{S} = \left((\gamma_X + \gamma_E \lambda_X) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} \right) \hat{\epsilon} \tag{3.2}$$

$$\widehat{E_S} = \left((\gamma_X + \gamma_E \lambda_X) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - 1 \right) \hat{\epsilon} \tag{3.3}$$

$$\hat{E} = \left((\lambda_S \gamma_X - \lambda_X \gamma_A) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \lambda_S \right) \hat{\epsilon} \tag{3.4}$$

The last equation for the overall change in energy use ($\hat{E}$) will be re-arranged to decompose it into key components, but we first gain some intuition by looking at the other outcomes.

The positive technology shock increases real incomes, so consumers buy more goods and services.[13] They also respond to the fall in the marginal cost of services by substituting toward S from X (in a way that depends on σ_U). The simplicity of our model allows us to separate income from substitution effects for both services and the other good. Specifically, in Appendix B, we derive the substitution effect as the change in consumption while consumers face the new prices but are as happy as in the old equilibrium. The income effect is the remaining change in consumption. In fact, Appendix B shows that the first term in equation (3.1) is the substitution effect on X, which depends on σ_U and is negative. The income effect on X is the remaining term in (3.1) and is positive. Similarly, the first term in (3.2) is the positive substitution effect on S. The income effect on S is the remaining term in (3.2), and it matches the positive income effect on X (because of homothetic preferences).

Next, our model enables us to solve for the welfare gain from this shock, given by the overall change in utility (dU) divided by the marginal utility of income (μ). This dollar value of the change in utility is then divided by total income (I) to express it in relative terms. As shown in Appendix C, this measure of the change in welfare is:

$$\frac{dU}{\mu I} = \frac{P_X X}{I} \hat{X} + \frac{P_K K_A + P_E E_S}{I} \hat{S}$$

$$= (\gamma_X + \gamma_E \lambda_X) \hat{X} + (\gamma_A + \gamma_E \lambda_S) \hat{S} = \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} \hat{\epsilon} > 0$$

In the first line, the relative change in welfare is a weighted average of the changes in consumption of goods and services, where the weight for each is its share of income. Using solutions for $\hat{X}$ and $\hat{S}$ above, the closed-form solution for $dU/(\mu I)$ is shown on the far right. As confirmation of these two derivations, we note that this relative change in real income is

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[13] Utility is homothetic, so both the commodity X and services S are normal goods.

identical to the earlier-derived income effect on goods and on services.

The substitution effect increases E_S by the first term in (3.3), and the income effect increases E_S by the second term in (3.3). But (3.3) for $\widehat{E_S}$ has a third term that reduces energy use for appliance services (by $-\hat{\epsilon}$). This last term is the direct effect of the CTS that allows consumers to produce the same services using less energy.

The solution for $\hat{E}$ in (3.4) has an ambiguous sign, and it is hard to interpret, so we unpack it into three major components: a direct efficiency effect, a direct rebound, and an indirect rebound. We then further decompose both the direct and indirect rebound effects into income and substitution effects, in general equilibrium.[14] These components appear in the first column of Table 1, Panel A, which decomposes effects of a CTS on total energy use in this case (with no policy, and fixed A). The sum of all terms in the first column is $\hat{E}$ in equation (3.4). The other columns and panel – discussed later – show GE effects of a CTS *with* a pre-existing standard, and effects of increased stringency of an energy efficiency standard (EES).

In column (1), with no policy, consumers could exploit the efficiency improvement to get the same services using less energy. That is, even if households were to choose unchanged X and S, the energy used to produce this bundle would fall. We define this energy saving as the "direct efficiency effect" (DEE), identified in the first term of the column. This energy saving would be the only effect if relative prices and real incomes held constant – as it might be calculated by engineers. Thus, the DEE reduces energy used *for services* by the full amount of the technology shock ($-\hat{\epsilon}$). It reduces total energy use in the economy by the shock times the fraction of total energy used for those services ($-\lambda_S \hat{\epsilon} < 0$).

Next, in column (1) of Panel A, we define the "direct rebound effect" (DRE) as the change in E from the combined substitution and income effects on the demand for services S. The income effect component of that DRE is the next term in the column (λ_S times the income effect on E_S in eq. 3.3). The real income gain allows consumers to get more S using more E_S. The DRE through the substitution effect is the next term down. It is λ_S times the substitution effect on E_S in (3.3), and it is also positive. Its size depends on σ_U. The DRE is the sum of the income and substitution effects and thus must be positive.

The "indirect rebound effect" (IRE) then refers to the effect of the shock on the

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[14] Most PE empirical studies estimate only the uncompensated direct rebound effect, without separate income and substitution effects. Thomas and Azevedo (2013) and Borenstein (2015) use PE theory models to decompose direct and indirect rebound effects into substitution and income effects. We follow their lead, but in a GE model.

equilibrium change in demand for the *other* good, X, and thus on *its* energy use, E_X. The second-to-last term in this first column of Panel A is the change in energy use from the income effect on demand for X. It is $\lambda_X \equiv E_X/E$ times the income effect on E_X in (3.1), and it is positive. The last term is the change in energy use through the substitution effect away from X. The total IRE is the sum of a positive income effect and negative substitution effect:

$$\frac{\lambda_X\big((\gamma_X + \gamma_E\lambda_X)\sigma_U\gamma_A + \lambda_S\gamma_E\big)}{1-\gamma_A}\hat{\epsilon} - \lambda_X(\gamma_A + \gamma_E\lambda_S)\sigma_U\hat{\epsilon} = \frac{\lambda_X\lambda_S\gamma_E(1-\sigma_U)}{1-\gamma_A}\hat{\epsilon}.$$

The sign of this expression depends on the size of σ_U. If goods X and S are not substitutable enough ($\sigma_U<1$), then the income effect dominates, and indirect rebound is positive. If the substitution effect dominates ($\sigma_U>1$), however, then the IRE is negative.

Alternatively, we can decompose rebound into an "overall income effect" and an "overall substitution effect". The real income gain increases demands for both E_X and E_S and thus always adds positively to rebound. The sum of the two income effects in the first column of Table 1 is the total income effect, the second term in the solution for $\hat{E}$ in (3.4). Similarly, we can calculate an overall substitution effect on rebound through both E_S and E_X by adding the two substitution effects. This sum is the first term in (3.4). It has an ambiguous sign because the substitution effect increases E_S but decreases E_X.

We can show that $\lambda_S\gamma_X - \lambda_X\gamma_A > 0$ implies that appliance services are more energy-intensive than the composite good.[15] If this inequality holds, then the overall income effect and overall substitution effect both increase energy use. If S is less energy-intensive, however, then the substitution effect is a negative component of rebound, and the sign of total rebound depends on whether the income effect dominates the substitution effect.

These analytical results and their signs in the second column of Table 1 are consistent with results in prior literature. When technological progress improves energy efficiency, its DEE reduces total energy use. The DRE is expected to be positive, through both substitution and income effects, while the IRE can be positive or negative. We next distinguish these effects of a CTS with no mandate to effects of a CTS with a pre-existing and binding mandate (and later, to effects of increasing stringency of a binding EES).

4. Rebound from a Costless Technological Shock with a Binding Mandate

Unlike existing papers, we now study GE rebound effects from a costless technology

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[15] Rearrange the inequality $\lambda_S\gamma_X - \lambda_X\gamma_A > 0$ to get $\frac{\lambda_S}{\gamma_A} > \frac{\lambda_X}{\gamma_X}$, or $\frac{E_S/E}{K_A/\bar{K}} > \frac{E_X/E}{K_X/\bar{K}}$. Thus, $\frac{E_S}{K_A} > \frac{E_X}{K_X}$, which implies that the S sector is more energy intensive than sector X.

shock in the case with a pre-existing efficiency mandate that is both costly and binding. The CTS is still modeled as an increase in the technology parameter, $\hat{\epsilon} > 0$, but we assume that a small change in ϵ does not make the policy non-binding.[16]

In general equilibrium, if consumers face a minimum energy efficiency ϵA and are given a free increase in ϵ, then they can choose to spend less on A. With a costly mandate that remains binding, then consumers still have to purchase more A than desired, so they respond to the technology shock by cutting back as much as possible on purchase of A. If consumers spend only enough to satisfy the unchanged mandate, then the CTS results in no additional energy efficiency at all.[17] The unchanged policy still requires $\epsilon A \geq \eta$, so consumers can reduce purchase of costly A such that $\hat{A} = -\hat{\epsilon}$, and energy efficiency ϵA is unchanged.[18]

We use the ten linearized equations in Section 2 to solve for the same ten unknowns as above. The exogenous shock is still $\hat{\epsilon} > 0$, and the numeraire is still $\widehat{P_K} = 0$, but all outcomes are now different because $\hat{A} = -\hat{\epsilon}$. Appendix D derives a closed-form expression for each of these ten equilibrium changes resulting from a small CTS with a binding EES, including the change in total energy use, $\hat{E}$, but then we unpack that solution for $\hat{E}$ into the same rebound effects as before, as shown in column (3) of Table 1.

Many of the results are both easy to solve and intuitive. As before, $\widehat{P_E} = \widehat{P_K} = 0$. Neither input price changes, so CRTS implies $\widehat{P_X} = 0$. Therefore, the two inputs change in the same proportion as output, $\widehat{E_X} = \widehat{K_X} = \hat{X}$. Next, the CTS does not change overall energy efficiency, ϵA, so $\hat{S} = \widehat{E_S}$. The effective marginal price of S is $P_E/\epsilon A$, where $\widehat{P_E} = 0$, and A falls by the same percentage that ϵ rises. Therefore, the marginal cost of S does not change ($\widehat{P_E} - \hat{\epsilon} - \hat{A} = 0$). Unchanged relative prices of services S and good X implies no substitution between these goods. Yet the CTS helps consumers to cut spending on A while they still meet the unchanged standard. Thus, it has a positive income effect that increases X and S by the same percentage as the income gain (as shown in Appendix D):

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[16] If an actual shock *were* large enough to make the policy non-binding, then the outcome would include some effects of this section with a binding mandate and some effects of the previous section with no binding mandate.

[17] In a similar result, though not about rebound, Goulder *et al* (2012) look at a subset of states that adopt a vehicle fuel efficiency standard that is more stringent than the federal standard requiring automakers to meet a nationwide corporate average fuel economy (CAFE). Those automakers can sell more fuel efficient cars in the more-stringent states and sell less efficient cars in other states, with no overall change in average fuel efficiency.

[18] Empirical evidence in Knittel (2012) suggests that improvements in fuel efficiency technology over time were offset by spending on other vehicle characteristics that reduce fuel efficiency, such as vehicle weight and power.

$$\hat{X} = \hat{S} = \frac{\gamma_A \beta}{(1-\gamma_A)(1-\alpha)} \hat{\epsilon} > 0.$$

As a consequence, the relative welfare gain, measured by the dollar value of the change in utility divided by total income as in Appendix C, is:

$$\frac{dU}{\mu I} = \frac{P_X X}{I} \hat{X} + \frac{P_K K_A + P_E E_S}{I} \hat{S} = \hat{X} = \hat{S} = \frac{\gamma_A \beta}{(1-\gamma_A)(1-\alpha)} \hat{\epsilon}$$

Increases in consumption of X and S lead to an increase in energy use by the same percentage:

$$\hat{E} = \widehat{E_X} = \widehat{E_S} = \frac{\gamma_A \beta}{(1-\gamma_A)(1-\alpha)} \hat{\epsilon} > 0 .$$

These positive rebound effects depend positively on the fraction of K used in the appliance sector ($\gamma_A \equiv K_A/\overline{K}$), and on the cost elasticity of additional energy efficiency [$\beta/(1-\alpha)$]. This elasticity depends on the curvature of the cost function β and the fraction $\alpha \equiv A_0/A$.

In fact, all effects on energy use in column (3) of Table 1 are either zero or positive. The economy stays at the same mandated efficiency level after the technology shock, so the DEE is zero. And rebound through substitution effects is zero, so the DRE and IRE have only their positive income effect. The *total* effect of the CTS is to increase total energy use (as shown in Table 1). This result could be defined as a "backfire", since the positive rebound effect swamps the zero DEE. We state these results formally in Proposition 1.

PROPOSITION 1*: For a positive costless technology shock (CTS) in this model with a pre-existing energy efficiency mandate that is costly and still binding after the shock, then:*

    (i) The direct efficiency effect is zero.
    (ii) Rebound substitution effects are zero.
    (iii) The only nonzero rebounds are through income effects and are positive.

The proof for Proposition 1 is the derivation above and in Appendix D. It follows from the assumption that the unchanged EES is still binding after the CTS.

These results are strikingly different from those in prior rebound literature. For the CTS with no pre-existing mandate, the prior literature shows that the DEE is a reduction in energy use, only partly offset by a positive DRE (as in the first two columns of Table 1). For the CTS *with* a pre-existing mandate, the next two columns show the CTS has no direct energy savings at all, and no rebound through substitution effects. In essence, the energy efficiency innovation reduces the cost of achieving the unchanged pre-existing standard. Then "direct rebound" is the positive income effect on energy for S, and "indirect rebound" is the positive income effect on energy for production of other goods, X.

A glance down column (3) shows that if the fraction of K in services (λ_S) is less than the fraction of K used in other goods (λ_X), as could be expected, then the indirect rebound effect (on E_X) exceeds the direct rebound effect (on E_S). Most empirical PE models measure only direct rebound, using the elasticity of demand for services, but they ignore mandates. In contrast, our GE model can show indirect rebound. For the CTS with a pre-existing efficiency mandate, the positive indirect effect likely exceeds the positive direct rebound effect.

An additional possibility ignored here is that policy makers set standards based on costs and benefits, such that this reduction of costs could induce them eventually to tighten the standard. We do not analyze endogenous policy, but we do analyze a tighter standard (in the next section). We also ignore endogenous technology, but another possibility is that the costly policy can *induce* improvements in technology that then cause positive rebound effects.

5. Rebound Effects from a Change in the Energy Efficiency Standard

We now solve for effects of a costly increase in EES stringency, decompose it into types of rebound, and compare these results to those above. Since the policy is represented by $\epsilon A \geq \eta$, we model the policy shock as a small exogenous change, $\hat{\eta} > 0$ (with no change in technology, $\hat{\epsilon} = 0$). Because purchase of A is costly, and consumers are already required to purchase more than they would if unconstrained, they will not buy more than necessary to satisfy the new requirement. Therefore the chosen $\hat{A}$ will exactly equal the required $\hat{\eta}$.

Here again, we use the ten linearized equations (2.1) - (2.10) to solve for equilibrium changes in energy consumption, $\hat{E}$, and nine other unknowns. Then we decompose that effect on total energy use into separate terms. As before, $\widehat{P_E} = \widehat{P_K} = 0$ and $\widehat{P_X} = 0$. Inputs change in the same proportion as output: $\widehat{E_X} = \widehat{K_X} = \hat{X}$. The marginal cost of S falls in the same proportion as energy efficiency rises ($\widehat{P_E} - \hat{A} - \hat{\epsilon} = -\hat{\eta} < 0$). Appendix E explains in detail the derivations for all unknowns. Here, we show the general solutions for only four of them:

$$\hat{X} = \left(-(\gamma_A + \gamma_E \lambda_S)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \right) \hat{\eta} \tag{5.1}$$

$$\hat{S} = \left((\gamma_X + \gamma_E \lambda_X)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \right) \hat{\eta} \tag{5.2}$$

$$\widehat{E_S} = \left((\gamma_X + \gamma_E \lambda_X)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} - 1 \right) \hat{\eta} \tag{5.3}$$

$$\hat{E} = \left((\lambda_S \gamma_X - \lambda_X \gamma_A)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} - \lambda_S \right) \hat{\eta} \tag{5.4}$$

Again we discuss outcomes in (5.1)-(5.3) to help understand effects on total energy in (5.4).

Similar to the analysis of the CTS, the reduction in marginal cost of S has both income and substitution effects on X and S. Substitution leads to more consumption of services and less of other goods. The first term in (5.1) is the substitution effect on X, and it is negative. The substitution effect on S is the first term in (5.2), and it is positive. The size of substitution effects depends positively on σ_U. In addition, the total income effect has two terms. On the one hand, consumers get a positive income effect from the service cost reduction, captured by the second terms in (5.1) and (5.2). On the other hand, they get a negative income effect because the more stringent policy ($\hat{\eta} > 0$) also requires more resources K_A to produce A. This portion of the income effect ($-\frac{\gamma_A \beta}{(1-\gamma_A)(1-\alpha)}\hat{\eta}<0$) has a magnitude that depends on existing resource use for fuel efficiency ($\gamma_A \equiv K_A/\overline{K}$), curvature of its cost β, and the fraction α.

The policy shock alters relative prices, the consumption bundle, and thus energy use. The change in production of X in (5.1) results in the same percentage change in E_X. The change in E_S in (5.3) is ascribable to both the change in S and an additional negative term, $-\hat{\eta}$. This additional term is the direct energy savings from the mandated increase in efficiency.

To interpret the effect of the EES on total energy use, we now re-arrange (5.4) into the components shown in column (5) of Table 1, in Panel B. The first entry shows the direct efficiency effect, $-\lambda_S\hat{\eta}$, which is the reduction in use of energy with more-efficient appliances that would occur if households were to consume an unchanged bundle of X and S. This DEE reduces energy use in production of S, and thus reduces the economy-wide aggregate energy consumption by $\lambda_S \equiv E_S/E$ times the policy shock $\hat{\eta}$.

But households do change their bundle of X and S. We define rebound effects of the EES in the same way as for the CTS, which enables us to compare the same concept for the two different shocks. Both the CTS and the EES shocks allow consumers to get the same services using less energy, so results in the fifth column are consistent with existing findings that the substitution effect increases consumption of S but reduces X. In particular, the DRE through the substitution effect is positive (see the third entry in the fifth column). The last entry in that column is the negative IRE through the substitution effect on X. Substitution effects decrease E_X but increase E_S. Adding the two substitution terms in the fifth column yields the overall substitution effect on energy use, $(\lambda_S\gamma_X - \lambda_X\gamma_A)\sigma_U\hat{\eta}$, which has ambiguous sign. Thus, if services S are more energy-intensive than X, $(\lambda_S\gamma_X - \lambda_X\gamma_A)>0$, then the overall substitution effect increases E, with a net positive rebound effect. But if the X-sector is *more*

energy intensive than the S-sector, the substitution effect increases E_S less than it decreases E_X, so the overall substitution effect could reduce total energy use.

The income effect is the main difference between the EES and CTS (with or without pre-existing mandate). The income effect for the CTS is always positive, as consumers get "free money" from reduced need to spend on A or E_S. Assuming the EES is binding and costly, however, the real income effect of this policy shock must be negative.

As shown in Appendix C, using solutions for $\hat{X}$ and $\hat{S}$ above, we can measure the change in real income as:

$$\frac{dU}{\mu I} = \frac{P_X X}{I}\hat{X} + \frac{P_K K_A + P_E E_S}{I}\hat{S}$$

$$= (\gamma_X + \gamma_E \lambda_X)\hat{X} + (\gamma_A + \gamma_E \lambda_S)\hat{S} = \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)}$$

Our assumption that the policy is binding therefore means that the change in real income on the far right must be negative. A slight re-arrangement of this condition implies:

PROPOSITION 2: *In this model, the pre-existing energy efficiency standard is costly and binding if and only if* $\gamma_A \beta > (1 - \alpha)\big((\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E\big).$

Our assumption that the EES is binding means that the inequality holds. It means we assume a large enough fraction of resources is used for energy efficiency (γ_A), or a large enough curvature in its cost function (β), or both. If so, then the total income effect is negative.

The direct rebound effect is the sum of substitution and income effects on E_S. The substitution effect on S and E_S is always positive, but the income effect is negative. Therefore, the DRE can be negative or positive. The indirect rebound effect on E_X includes a negative substitution effect and a negative income effect, so its sign is clearly negative.

Also, some prior studies find evidence for "backfire" from energy efficiency – where the direct energy savings from increased energy efficiency are more than offset by positive rebound effects (Khazzoom, 1980; Brannlund *et al*., 2007; Fouquet and Pearson, 2012; Saunders, 2013). This backfire is less likely for a *mandated* increase in energy efficiency, because the mandate is costly and thus has negative income effects on rebound. Here, we show exact conditions under which backfire is impossible for the efficiency mandate.

PROPOSITION 3: *For increased stringency of an energy efficiency standard (EES) in this model with a pre-existing efficiency mandate that is costly and always binding:*

(i) *The direct rebound is negative if and only if* $\gamma_A\beta > (1-\alpha)\big((\gamma_E\lambda_X + \gamma_X)\sigma_U + \gamma_E\lambda_S\big)$ *(in which case,* $\gamma_A\beta$ *is even larger than required for the EES to bind).*

(ii) *The total rebound is negative if and only if* $\gamma_A\beta > (1-\alpha)(\lambda_S\gamma_X\sigma_U + \gamma_E\lambda_S)$ *(less strict than in (i)). A sufficient condition is when X is energy-intensive* $(\lambda_S\gamma_X - \lambda_X\gamma_A<0)$.

(iii) *Backfire is impossible if and only if* $\gamma_A\beta > (1-\alpha)\lambda_S\gamma_X(\sigma_U - 1)$ *(less strict than in (ii)). Sufficient conditions are X is energy-intensive* $(\lambda_S\gamma_X - \lambda_X\gamma_A<0)$, *or* $\sigma_U < 1$.

Appendix F provides a detailed proof, but here we provide some intuition. The direct rebound effect on S is the sum of a positive substitution effect and a negative income effect, so it will be negative if the income effect dominates (a large enough γ_A or β, a small enough σ_U, or a combination of these). Total rebound is the sum of that DRE and the unambiguously negative IRE, so it can be negative under a looser condition. It is trivial to show that the condition in (*i*) is stricter than the one in (*ii*), and either is more strict than in (*iii*). If (*ii*) holds, so the *total* rebound effect (TRE) is negative, then direct energy savings are augmented by further energy savings, and backfire is impossible. The condition $\sigma_U < 1$ is sufficient to rule out backfire, because the right side of the inequality in (*iii*) is negative, while the left side is positive. With $\sigma_U < 1$, the positive substitution effect is small and is swamped by the negative income effect, so the total rebound effect cannot offset direct energy savings.[19]

Empirical studies of rebound often ignore or underestimate the indirect rebound effect.[20] Yet we find above for the CTS that it can exceed the direct rebound effect. The same can hold for increased stringency of the EES. Proposition 3 shows that the condition under which the DRE is positive is compatible with the condition where the TRE is negative. If so, then the magnitude of the negative IRE must be greater that of the positive DRE. Thus, again, the IRE can swamp the DRE. Moreover, as shown in Appendix F, we can use the proposition to find conditions under which indirect rebound is more negative than direct rebound.

6. Calibration

As shown above, direct and rebound effects from an energy efficiency improvement depend on elasticities, shares, and other parameters. To illustrate numerical magnitudes, we choose plausible values for these parameters and use them in our formulas. Our benchmark

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[19] A different special case also can guarantee that total rebound is negative. The costs of a more stringent EES certainly make the income effect negative, but if the X-sector is more energy-intensive than the S-sector, as well, then E_X falls by more than E_S rises, and the overall substitution effect must also be negative.

[20] One difficulty for empirical studies is to define a complete set of substitutes and complements to a particular product. Second, most ignore indirect income effects and use only cross-price elasticity estimates.

dataset approximates the U.S. economy in 2015, the most recent year with relevant data.

Our model has one primary factor, K, and two final outputs: appliance services S and a composite good X. Two other outputs are used as intermediate inputs, including energy E and appliances A. Yet, an actual car or appliance is a combination of energy efficiency features plus many aesthetic and functional features not related to energy use. We observe spending on appliances, but not on each feature. Added costs to improve energy efficiency may also vary by appliance. Yet, the energy efficiency mandate requires that $\epsilon A \geq \eta$, for policy scalar η, so spending on A is best calibrated and interpreted as spending on energy efficiency rather than on other features not related to energy use. Here, we arbitrarily assume initially that half of actual expenditures on appliances is for energy efficiency (e.g. extra insulation or higher quality parts).[21] That is, if an Energy Star freezer-refrigerator costs $600, then $300 is spent on energy efficiency (A). The remaining $300 is part of the other composite good, X.

Using PE models, Borenstein (2015) or Chan and Gillingham (2015) can calculate rebound for particular vehicles, electric lighting, or refrigerators. For a broader perspective, our GE model is most useful for a large sector. Therefore, we analyze energy efficiency in a general way for all household appliances, an aggregation of everything of households that use energy – including washer, dryer, furnace, refrigerators, lights, air conditioners, and electric space heaters. Since household vehicles are also subject to fuel efficiency standards, they are also included.[22] Then energy for appliance services (E_S) is an aggregation of all household purchase of electricity, natural gas, heating oil, propane, gasoline, and other motor fuels.

We get residential energy expenditure data from State Energy Data System (SEDS), through which the Energy Information Administration (EIA) provides time series of energy production, consumption, prices, and expenditures across sectors.[23] We get expenditure data on motor fuels from the Bureau of Economic Analysis (BEA).[24] Energy expenditure by households for 2015 is 248 ($B), and expenditure on motor fuels is 270 ($B), for a total of 518 ($B). We define a unit of E as the amount that costs $1, so $P_E = 1$ (dollars per unit of E).

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[21] Accounting figures for explicit costs of energy efficiency technology are undoubtedly less than half of total spending on cars or appliances, but manufacturers may change other design features to help reduce fuel costs (e.g., expensive lightweight structural materials). In any case, we test sensitivity of results to this assumption.

[22] Actual appliances also have standards with different stringency. With more disaggregation and complexity, a computational GE model could represent each of them separately. Those with more stringency will have more negative income effects and negative rebound. In an extension below, we analyze cars and appliances separately.

[23] https://www.eia.gov/state/seds/sep_sum/html/pdf/sum_ex_res.pdf

[24] https://www.bea.gov/national/consumer_spending.htm

Energy E_S is the sum of all fuels used by households, so the initial $E_S = 518$ billion units.

To calculate the annual capital cost of appliances for energy efficiency, $P_K K_A$, we start with household expenditures on new cars and appliances for 2015 from the BEA (675 $B) and then perform a user cost calculation to arrive at their annualized cost.[25] We assume annual depreciation (δ) equal to 10% of total appliances, and annual maintenance (ω) equal to another 5%. We also assume no growth, so that all new appliance purchases are replacement investment (10% of the existing appliance stock each year). Then the existing stock is 675×10=6,750 ($B). If the annual discount rate (ρ) is 5%, then the annual user cost of cars and appliances in $B is 6,750×($\delta + \omega + \rho$)=1,350 ($B).[26] For the portion representing energy efficiency features, the annual user cost is half of that: $P_K K_A = 675$ ($B).

Next, we use the EIA's SEDS to obtain economy-wide energy use for 2015 as $1,127 billion.[27] The ratio of energy use for cars and appliances to total energy use ($\lambda_S \equiv E_S/E$) is 518/1,127 = 0.460. Then, $\lambda_X \equiv E_X/E = 1 - \lambda_S = 0.540$ is the fraction of energy used in production of the composite good X. Next, we define a unit of K as the amount that earns $1 per year, so the initial $P_K = 1$, and national income is $P_K \bar{K}$. Using U.S. gross domestic product (GDP) for 2015 from BEA, $I = 18,037$ ($B).[28] The fractions of K used in production of energy and for energy efficiency are:

$$\gamma_E \equiv \frac{K_E}{\bar{K}} = \frac{P_K K_E}{P_K \bar{K}} = \frac{P_E E}{P_K \bar{K}} = \frac{1,127}{18,037} = 0.063$$

$$\gamma_A \equiv \frac{K_A}{\bar{K}} = \frac{P_K K_A}{P_K \bar{K}} = \frac{675}{18,037} = 0.037$$

Since the fractions of K used in all sectors add to one, $\gamma_X = 1 - \gamma_A - \gamma_E = 0.900$.

No one has estimated the elasticity of substitution in utility between appliance services and all other goods. To calibrate this σ_U, we use estimates of the own-price demand elasticity for appliance services, ξ_{SS}. As shown in Baylis *et al.* (2014), these parameters are related to each other by $\xi_{SS} = -(\zeta_S + \sigma_U(1 - \zeta_S))$, where ζ_S is the calibrated share of income spent on

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[25] These assets include televisions, major household appliances (e.g., refrigerator, freezer, washer, dryer, stove, range, and air conditioners), small electric appliances (e.g., vacuum cleaner and electric kettle), equipment for house and garden (e.g., lawn mowers and propane grills), plus new motor vehicles and parts.

[26] Here, we assume perfect markets and ignore taxes, so the market rental price per year would be the same as the annualized cost to the owner of the car or appliance.

[27] https://www.eia.gov/state/seds/sep_sum/html/pdf/sum_ex_tot.pdf

[28] Available at:
https://www.bea.gov/iTable/iTable.cfm?ReqID=9&step=1#reqid=9&step=3&isuri=1&904=2015&903=5&906=a&905=2015&910=x&911=0

services. Estimates range between -0.1 to -0.3, both for the price elasticity of vehicle-miles-traveled and for the price elasticity of electricity demand.[29] Using that range, the calculated elasticity of substitution in utility ranges from 0.05 to 0.25. For our central parameter, we use the center of that range, $\sigma_U = 0.15$, and we vary it to see the sensitivity of results.

For our cost function $K_A = B(A - A_0)^\beta$, linearized equations depend on the parameter $\alpha \equiv A_0/A$. We do not need to calibrate B, because it drops out of $\widehat{K_A} = \frac{\beta}{1-\alpha}\hat{A}$. Appendix G describes in detail the calibration of α and β using engineering studies of the added costs associated with adding to energy efficiency. These costs are calculated for selected vehicles by the National Research Council (NRC, 2002, 2015) and the International Council on Clean Transportation (Lutsey *et al.*, 2017), and for selected appliances by the Department of Energy (DOE, 2011, 2016a, 2016b). Looking at specific measures of energy efficiency (such as mpg), our overall strategy is as follows. (1) Choose A_0 in the low-efficiency range for the vehicle or appliance, and use observed energy efficiency A to calculate the ratio α. (2) Vary that choice to check sensitivity, and include $\alpha = 0$ to see if we could dispense with this extra parameter. (3) Use their calculations of changes in cost for changes in energy efficiency to recover β. (4) Show in appendix tables the various resulting combinations of A_0, α, and β. (5) Choose the most reasonable combination of α and β for our primary parameters. (6) Use those values for our aggregation of cars and appliances, and show sensitivity of results to alternatives.

The value of β can vary along a single cost curve, and the whole curve can differ between datasets, cars, and different appliances. This heterogeneity is confirmed across five selected vehicles and appliances for which engineering studies are used in Appendix G to calibrate α and β shown in Tables G1-G5.[30] The two tables for cars and light trucks each show two methods, so the five tables show seven different calculations. These tables are not available for every appliance and vehicle, so these examples only show a plausible range of values. We choose a pair α and β to best represent all appliances and cars in our aggregation. Our numerical results below are only illustrative; the same calculation could be undertaken for any particular vehicle or household appliance, as in a partial equilibrium model.

In the five tables, we see that β can range widely, though all exceed one. Empirical

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[29] See Allcott (2011), Gillingham (2014), Ito (2014), and Deryugina *et al.* (2017).

[30] The examples in Appendix G are based on two different datasets for vehicles, which yield somewhat different results, plus three different appliances (central air conditioners, furnaces, and refrigerator-freezers).

studies of energy efficiency costs often find $\beta=2$ (quadratic costs).[31] But appendix tables also show how α depends on β, so they need to be chosen in concert.[32] Starting with $\alpha=0$ in these tables, the implied values of β are unstable, varying from 1.25 to 5.82. Indeed, this instability is what induced us to add the intercept A_0 to the cost function. When α is about one-half, the values of β vary less, from about 1.75 to 2.75. The implied elasticity $\frac{\beta}{1-\alpha}$ ranges from 3.5 to 5.5, but that range in our model would yield costly standards and large negative rebound. To be conservative, we choose $\alpha=0.5$ and $\beta=1.5$ (so $\frac{\beta}{1-\alpha}$ is only 3.0).[33] This choice is equivalent to the combination where α is one-third and $\beta=2$, since the implied $\frac{\beta}{1-\alpha}$ is still 3.0.

Finally, we check the condition in Proposition 2 to make sure that the EES is binding as our model assumes. The chosen parameter values imply $\gamma_A\beta$ is 0.06, while $(1-\alpha)\times[(\gamma_X+\gamma_E\lambda_X)\sigma_U\gamma_A+\lambda_S\gamma_E)]$ is 0.02, so these values are indeed consistent with our assumption that the EES is binding and costly. Also, with all the calibrated parameters, β must be at least 0.45 to ensure that the EES is binding. For sensitivity analysis, we vary β from 1 to 5.

7. Numerical Illustrations

For any costless technology shock, we assume a 10% increase in energy efficiency technology, $\hat{\epsilon}=0.10$. In the CTS case with no policy and no change in purchased energy efficiency ($\hat{A}=0$), 10% better technology implies 10% better energy efficiency. For the EES, we assume the government raises the efficiency standard for all appliances by 10%, so it generates the same impact on energy efficiency. Inserting all calibrated parameters into each expression in Table 1 yields the size of each effect in percentage points. The direct efficiency effect (DEE) is calculated as if relative prices and incomes were constant. For example, a +10% change in ϵ leads to a –4.6% DEE (because $\lambda_S \equiv E_S/E = 0.460$). If the total rebound

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[31] Greene and DeCicco (2000) find that a quadratic cost function for fuel economy improvement can be a good fit to the data. Also see NRC (2002) for various estimates of fuel economy cost curves. Our calibration of β uses more recent studies from the NRC and the International Council on Clean Transportation.

[32] In the tables, a higher α is always associated with a smaller β. We take as given the engineering calculation of the incremental cost of improving energy efficiency, but matching that incremental cost with a higher α requires a smaller β. But, the overall cost elasticity $\frac{\beta}{1-\alpha}$ does not vary as much as α or β individually. Appendix Tables G1-G5 show that virtually any combination of α and β yield an overall elasticity of 3.0 or higher.

[33] If the overall cost elasticity of 3.0 seems large, remember that it is the percent change in cost for a one percent increase in *total* energy efficiency. In the vehicle example, suppose the required mpg rises from $\epsilon A=30.0$ to 30.3 (a 1% increase in mpg). Manufacturers had no extra cost of achieving the initial "minimum" 15 mpg, so our cost function applies to the mpg in excess of $A_0=15$. The same increase of 0.3 mpg is a 2% increase in the *extra* 15 mpg (from 15 to 30 mpg). With our $\beta=1.5$, that 2% addition means a 3% increase in cost.

effect (TRE) is about +1%, then total use of energy E falls by the difference (–3.6%).

Using our calibrated parameters, we calculate and plot in Figure 1 not only the DEE, but also the direct rebound effect (DRE), the indirect rebound effect (IRE), the TRE, and the total effect (TE). The first five bars on the left are the effects from the CTS with no mandate. The five bars in the middle show the CTS case with a binding EES, and the last five bars are the effects on energy use from increased stringency of the EES policy.

For the CTS with no policy, the fourth bar shows that the free 10% gain in energy efficiency leads to a –4.6% "direct effect" on total energy use. More than one-fifth of that energy saving is offset by the positive TRE in the third bar. Comparing the DRE in the first bar to the TRE in the third bar suggests that nearly 90% of total rebound is from rebound on appliance services. Though not shown in Figure 1, both the income and substitution effects increase consumption of S. For the other goods in X, however, the substitution effect reduces consumption, while the income effect increases it. The IRE in the second bar is net positive, which means the positive income effect dominates the negative substitution effect on X. As intended, these numerical results are consistent with findings in the prior rebound literature that study the exogenous costless technology change.

New in this paper is a pre-existing mandate that remains binding after the costless technology shock (in the middle bars of Figure 1). Then the CTS allows consumers to pay less for energy efficiency but still meet the unchanged energy efficiency requirement, ϵA. The reduced spending on A perfectly offsets the increase in technology (so $\hat{A} = -\hat{\epsilon}$). The savings can be spent on more energy for services (E_S) and on other goods (X). Visually, in the middle bars for the CTS with existing policy, the bar for DEE seems missing, but energy efficiency stays at the original level, so the DEE is literally zero. The total effect on energy is exactly the total rebound effect, +1.17%. Both the rebound effects are positive, but the IRE is slightly greater than the DRE. The income effect through the cost savings on energy efficiency explains all rebound (i.e., no substitution effect).

The "stricter EES" case shows the effects on energy use when the pre-existing EES becomes 10% more stringent. As normalized, the CTS with no policy and the stricter EES have identical direct energy efficiency effects – as might be calculated by engineers. But the DRE from the EES (the dotted bar on the right) is less than half of the DRE from the CTS with no policy (the dotted bar on the left). The IRE from the EES case (the diagonal stripe bar on the right) is negative and larger in magnitude than the positive DRE. Therefore, total

rebound is negative: total energy saving in the last bar is greater than direct energy saving. Substitution effects are the same for both the EES and CTS with no policy. Therefore, the real income effect is a main difference in numerical results between the CTS and EES shocks. The CTS provides an income gain, while policy presents a real income loss.

Since the income effect is a key factor that distinguishes the CTS and the EES, we next consider sensitivity of results first for β and γ_A and then for other parameters.

8. Sensitivity Analysis

Figure 2A investigates sensitivity of rebound effects as we vary the assumed curvature of the cost function for energy efficiency (β). Then Figure 2B shows these same rebound effects through income and substitution effects. We use the central-case values for all other parameters. In Figure 2 and others, we generally use dash-dot lines for the CTS with no policy, dashed lines for the CTS with policy, and solid lines for the EES.

For the costless energy efficiency gain with *no* pre-existing policy, Figure 2A shows that the DRE and the IRE are completely flat: they do not depend on the cost of policy (β). Similarly, the income rebound effect (IncRE) and the substitution rebound effect (SubRE) in Figure 2B also do not depend on β. These results for the CTS with no existing policy match the prior literature that ignores the cost of requiring more energy efficiency.

By contrast, the rebound effects from the CTS *with* pre-existing policy and from the EES shock are more responsive to β, but they respond in opposite directions. First, for the CTS with policy (dashed lines), the DRE and IRE are positive and rise with β. Why? A larger β means that more is already being spent on energy efficiency, and the CTS allows consumers to meet the unchanged mandate by use of new free technology instead, so it provides a larger positive income shock (allowing more use of energy for both S and X). In contrast, for the stricter EES (solid lines), the DRE and IRE both fall with β. A larger β in this case means that the additional required energy efficiency is more expensive, which causes a larger negative income shock and negative rebound effects. In fact, Figure 2B shows that all sensitivity of the DRE and IRE comes from the income effect. Substitution effects for the CTS with no policy and for the EES are the same and are completely flat lines (solid flat line in Figure 2B).

Figure 2 does not show the direct efficiency effect, because it does not depend on β. The DEE from the CTS with policy is zero. For both the CTS with no policy and EES shock,

the DEE are normalized for all value of β to be the same 4.6% reduction in energy use.[34]

Next, Figures 3A and 3B show results as the intercept in the cost function, $\alpha \equiv A_0/A$, varies between 0.2 and 0.8. Results are almost identical to those in Figure 2, because α and β affect only the overall added cost in $\widehat{K_A} = \frac{\beta}{1-\alpha}\hat{A}$. The difference is that β affects that added cost linearly (as also shown in Figure 2), whereas α affects it nonlinearly (as also shown in Figure 3). Just as for β in Figure 2, the choice of α in Figure 3 does not affect the CTS with no existing policy. In both figures, when $\frac{\beta}{1-\alpha}$ gets larger, rebound effects are more positive for a CTS with existing policy and more negative for an EES. Again, all of this sensitivity occurs through the income effect. The substitution effect is completely flat (orange solid line in 3B). Sensitivity results in the figures show that our central values of β=1.5 and α=0.5 appear to be both reasonable and conservative, as well as compatible with each other.

Next, we undertake sensitivity analysis for the arbitrary assumption that one-half of expenditures on appliances is for features that enhance energy efficiency. This assumed fraction primarily affects $\gamma_A \equiv K_A/\overline{K}$, the fraction of K used for energy efficiency.[35] Because the relationship between this assumed fraction and γ_A is monotonic, however, our sensitivity analysis for γ_A is equivalent to testing the sensitivity of results to this assumed fraction.

In our central calibration above, γ_A is 0.037, but Figure 4 varies it from 0.02 to 0.06 (or equivalently, the assumed fraction of appliance spending that is for energy efficiency varies from 0.25 to 0.8). As shown, rebound effects from the CTS with no policy do not depend on the value of γ_A, because again this parameter pertains to the cost of policy. In contrast, rebound effects from both the EES and the CTS with policy are responsive to γ_A. Figure 4 looks a lot like Figure 2, because γ_A affects the cost of policy as does β. Total rebound from the EES shock can be negative if γ_A is larger than 0.036, and our central estimate is 0.037. As depicted in Figure 4B, the substitution effect (orange solid line) has almost no slope, so the income effect explains most of how rebounds depend on γ_A.

Prior studies focus overwhelmingly on the responsiveness of rebound effects to the elasticity of substitution between energy services and other goods, and to the elasticity of substitution between appliances and energy in the production of energy services. They ignore the cost curvature parameter β and the fraction γ_A, which we show here can explain much of

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[34] Backfire is impossible in this case; positive rebound from substitution is offset by the negative income effect.

[35] This assumption also affects $\gamma_X \equiv 1 - \gamma_A - \gamma_E$. We directly observe $\gamma_E \equiv K_E/\overline{K}$, but $\gamma_X \equiv K_X/\overline{K}$ is assumed to include the fraction of total appliance spending that is not for energy efficiency.

the difference between rebound effects from the CTS and EES.

Nonetheless, to compare our results with prior literature, Figure 5 plots rebound effects from the CTS and EES shock against the elasticity of substitution in utility (σ_U). First, recall that rebound effects from the CTS with a binding policy depend only on income effects, not on substitution effects, so its rebound effects are flat in this figure.

In contrast, rebound effects from both the CTS without policy and the EES are indeed sensitive to σ_U. In Figure 5A, both direct rebound effects rise steeply with σ_U, which is both intuitive and consistent with prior literature. Regardless of whether it is cost-free or costly, an increase in energy efficiency reduces the marginal cost of energy services and thus causes a positive direct rebound effect that rises with σ_U. Also in Figure 5A, the indirect rebound effects in these two cases change only slightly with σ_U, because of offsetting income and substitution effects on energy use in X. In addition, Figure 5B shows that the income effect from the EES shock is always negative, while the income effects from both CTS cases are always positive. Thus, for every value of σ_U in 5A, the DRE from the EES shock is always smaller than the DRE from the CTS without policy.[36]

9. Extensions

Our model must be fairly simple to be able to solve for GE effects analytically, but the advantage is that mathematical expressions show exactly how each effect depends on parameters and policies. We explained several caveats along the way. In this section, we extend our model to relax four of our key assumptions and to see how they affect our main results: (a) negative effects of total energy use on productivity; (b) upward-sloping supply of energy; (c) mandates in the business sector, and (d) disaggregation.

First, we study the case where emissions from energy use affect productivity (e.g., Williams, 2002). If so, reducing energy use could have positive income effects on rebound. Then equation (1.9) of our basic model becomes $G(E)\overline{K} = K_X + K_A + K_E$, with $\partial G / \partial E < 0$, so that more emissions reduce productivity of the primary factor K. In the initial equilibrium, $G(E^0) = 1$, where E^0 is initial energy use. We linearize that equation to get:

$$\delta\widehat{E} = \gamma_X\widehat{K_X} + \gamma_A\widehat{K_A} + \gamma_E\widehat{K_E} \tag{9.1}$$

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[36] In Figure 5B, income effects (in all three black lines) slightly rise with σ_U, because consumers are better able to substitute towards cheaper energy services. But most of the response to σ_U comes through the substitution effect, as the orange solid line in Figure 5B is strongly upward sloping.

where $\delta \equiv \frac{\partial G}{\partial E} \frac{E}{G(E)} < 0$ is the elasticity of productivity with respect to energy use. Appendix H

derives all outcomes for a stricter EES, but here we show only the effect on total energy, $\hat{E}$:

$$\hat{E} = -\lambda_S \hat{\eta} + \frac{1}{(1 - \gamma_A - \delta)} \left(\gamma_X \lambda_S \sigma_U + \lambda_S \gamma_E - \lambda_S \delta - \frac{\gamma_A \beta}{1 - \alpha} \right) \hat{\eta} \qquad (9.2)$$

The first term, $-\lambda_S \hat{\eta}$, is the direct efficiency effect, and the rest is total rebound. The new

parameter $\delta < 0$ appears in two locations and therefore affects rebound, but it does not affect

the final term reflecting the cost of the policy – which generates a negative income effect with

a magnitude that depends on the cost curvature parameter β.

The new condition for total rebound to be negative can be written as:

$$\gamma_A \beta > (1 - \alpha)(\gamma_X \lambda_S \sigma_U + \lambda_S \gamma_E - \lambda_S \delta) , \qquad (9.3)$$

which almost matches the condition in Proposition 3.*ii* from before, but adds $-\lambda_S \delta > 0$ to the

right hand side. This term makes it harder for total rebound to be negative. Clearly, the

productivity gain from reducing energy consumption through the stricter EES generates a

positive income effect and thus a positive effect on rebound. The sign of the TRE is still

ambiguous, however. With a high cost of the mandate (β), rebound can be negative.

Second, to consider upward-sloping supply of energy, the production function $E = K_E$

is replaced by $E = E(K_E, \bar{Q})$, where $\bar{Q}$ is a fixed natural resource.[37] Then σ_E is the elasticity of

substitution in the production of energy, and θ_{QE} is the cost share of the fixed factor $\bar{Q}$. This

extension introduces one new unknown variable P_Q, the return to the fixed factor $\bar{Q}$, and thus

it raises the number of equations and unknowns from 10 to 11. All derivations are in

Appendix I. The result for $\hat{E}$ in the case with increased stringency of the EES is:

$$\hat{E} = -\lambda_S \hat{\eta} + \frac{1}{D} \left((\sigma_E \theta_{KE} + \theta_{QE} \lambda_S \theta_{KX}) \gamma_X \lambda_S \sigma_U + \theta_{QE} \lambda_S \gamma_X \sigma_X (1 - \lambda_S \theta_{KX}) + \lambda_S \gamma_E \sigma_E - \sigma_E \theta_{KE} \frac{\gamma_A \beta}{1 - \alpha} \right) \hat{\eta}$$

where $D \equiv \theta_{QE} \left(\sigma_E (\theta_{KE} \gamma_X + \gamma_E)/\theta_{QE} + \gamma_X \lambda_S \theta_{KX} \sigma_U + \gamma_X \sigma_X (1 - \lambda_S \theta_{KX}) \right) > 0.$ (9.4)

The first term is the DEE, $-\lambda_S \hat{\eta}$, and the rest is the TRE (with both substitution and income

effects from the reduced price of energy).[38] After accounting for these effects, can the cost of

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[37] Gillingham *et al.* (2016) note that an upward-sloping supply of oil would imply that an EES could cut demand for oil, reduce its equilibrium price, and thus encourage other uses of oil.

[38] The change in energy price is $\widehat{P_E} = -\frac{\theta_{QE}}{D} \left(\gamma_X \lambda_S (1 - \sigma_U) + \frac{\gamma_A \beta}{1 - \alpha} \right) \hat{\eta}$, so $\widehat{P_E}$ is negative unless σ_U is very large. When the price of S falls, a large $\sigma_U > 1$ means a strong shift toward energy-intensive services, S, and thus more demand for energy. With an upward-sloping energy supply and $\sigma_U > 1$, more demand for E implies an increase in price P_E. Estimated demand elasticities are low, however, and our calibrated σ_U is only 0.15.

the stricter EES still make the TRE negative? The last term in the big parenthesis shows the negative rebound effect from the costly policy $(-\sigma_E\theta_{KE}(\gamma_A\beta)/(1-\alpha))$, but other terms are positive. Thus, the sign of the TRE is still ambiguous. The condition for a negative TRE is:

$$\gamma_A\beta > (1-\alpha)\left(\lambda_S\gamma_X\sigma_U + \gamma_E\lambda_S + \frac{\lambda_S\theta_{KX}\lambda_S\gamma_X\sigma_U\theta_{QE}}{\sigma_E\theta_{KE}} + \frac{\gamma_E\lambda_S\theta_{QE}}{\theta_{KE}} + \frac{\lambda_S(1-\lambda_S\theta_{KX})\gamma_X\sigma_X\theta_{QE}}{\sigma_E\theta_{KE}}\right) \qquad (9.5)$$

which is stricter than the condition in Proposition 3.*ii*. Thus, adding slope to the energy supply curve makes it somewhat more difficult for negative rebound effects to outweigh positive rebound effects. But, it does not negate our point that the cost of the mandate introduces negative effects on rebound. Thus, results of prior analyses of the costless technology shock should not be used directly to infer effects of energy efficiency policy.

Third, firms in sector X also face energy standards on their business equipment and vehicles. A complete model of that fact would include several more equations, lengthy expressions, and more effects in our solutions. Using an *ad hoc* approach, however, we can approximate those complicated effects within our simple model. Figure 6 plots rebound effects for larger values of the parameter λ_S, the fraction of total energy used with regulated appliances to produce energy services. Our central parameter values include λ_S of about one-half, so Figure 6 varies λ_S between one-third and two-thirds. It shows a steeply falling grey line for the DEE, indicating that more energy subjected to efficiency standards means more reduction in energy use. But the other lines all indicate that percentage rebound effects from either the CTS or the EES shock are not very sensitive to λ_S.

Fourth, disaggregation and heterogeneity could matter for several reasons. First, the PE model of Chan and Gillingham (2015) includes a list of goods with different cross-price effects and energy intensities. Thus, a change in the price of services from one appliance leads to more of some goods and less of others, each with a positive or negative effect on energy use.[39] Indeed, our results highlight income effects, so different income elasticities would also matter. Second, disaggregation could capture details of policy: vehicle CAFE standards in the U.S. implicitly subsidize fuel-efficient cars and tax fuel-inefficient cars. Here, we capture only the overall costs of such a policy on the average car with a single fuel efficiency. Third, of course, heterogeneity would be important if high-income household expenditures differ from those of low-income households – where policy places more burden on one or the other.

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[39] West *et al.* (2017) show that a change in fuel efficiency also changes important vehicle attributes such as size, safety, and performance. While the reduced cost per mile can increase driving, changes in those other attributes could reduce it. Conceptually, these other attributes are "other goods" in utility, with cross-price elasticities.

We cannot disaggregate goods in GE without a computational model, but a GE model is not necessary to analyze a policy that applies only to freezers that constitute 0.1% of the economy; it would have tiny indirect effects on the other 99.9% of the economy. Again, however, we can use an *ad hoc* approach to see key differences between cars and appliances. Using calibration procedures exactly analogous to those in section 6 above, Appendix J derives share parameters (γ_A and λ_S) for household vehicles and energy use (motor fuel). It also aggregates all household appliances other than vehicles, such as refrigerators, furnaces, and air conditioners. The calculated share parameters are shown in the first two columns of Table 2. We continue to use the previous cost function parameters for both cases.[40]

Using the parameters in Table 2 for vehicles separately from appliances, we calculate bar graphs just like Figure 1 (showing bars for the DEE, DRE, IRE, TRE, and total energy use). Those bar graphs are in Appendix J, rather than here, because they look exactly like Figure 1 but for the vertical scale. Table 2 shows that share parameters for vehicles are very nearly half the size of the shares for appliances and vehicles together (and so appliances alone are the other half). Thus, bars for the DEE extend down to 2.3% less energy, instead of 4.6% less energy. Each type of positive rebound extends up to 0.5% more energy instead of 1.0% more energy; and each type of negative rebound is also half the size of those in Figure 1.

The lesson here is that the size of the sector does not affect the relative size of effects. Both share parameters (γ_A and λ_S) are halved, so all results are halved. The relative sizes of the DEE would differ if *only* λ_S changed (as in Figure 6); the relative sizes of other effects would differ if only γ_A changed (as in Figure 4). The relative sizes of the effects for appliances vs. vehicles would also differ if those two categories faced different mandate stringency (i.e., β, as in Figure 2), or have different demand elasticities (based on σ_U, varied in Figure 5). The point here is that the relative size of each rebound effect depends in a primary way on whether the improvement in energy efficiency derives from a technology shock or a mandate, and whether the technology shock takes place with or without a pre-existing mandate.

10. Conclusions

We use a simple analytical general equilibrium model to analyze and to compare rebound effects from an EES to those from a CTS. In both cases, we consider the economy

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[40] The calibration of α and β in Appendix G for cars and appliances are not much different from each other. In four calculations for cars and light trucks, when α is about one-half, β varies from 1.75 to 2.71. Three cases for appliances with α about one-half yield β from 2.18 to 2.78. Sensitivity above shows how α or β affects results.

with a pre-existing EES that is both costly and binding. Also, we decompose each total effect on the use of energy into various components, including a direct efficiency effect, a direct rebound effect, and an indirect rebound effect. Each rebound term is composed of substitution and income effects. Results show that the magnitude and sign of rebound for *both* the CTS and EES depend strongly on the costliness of efficiency mandates.

As found in prior literature, a CTS raises real incomes and thus has positive rebound effects on energy use that offset part of the reduction from the improvement in technology. Some studies have interpreted such results to indicate effects of energy efficiency policies. Here, however, we show that energy efficiency standards have costs and therefore reduce income, which can reduce energy use and cause negative rebound. Also, the prior literature gives much attention to rebound through the substitution effect. Here, with a costly energy efficiency policy, the curvature of the cost curve for energy efficiency improvements gives rise to income effects that help determine the magnitude and sign of rebound.

Numerical illustrations use reasonable values for all parameters, including elasticities, factor shares, and cost parameters. With these plausible parameters, total rebound from a CTS is positive, but total rebound from an EES are negative. The analytical findings and numerical example both suggest that using estimated rebound from a zero-cost efficiency gain as the effect of an energy efficiency mandate could greatly overstate rebound from the mandate.

Our results do not contradict prior empirical estimates of rebound in PE models, but they do demand a reinterpretation of those results. If a PE model of a costless efficiency improvement finds 10% positive rebound (as we do in Section 3), then the author or reader needs to add a sentence. "If these estimated behaviors were embedded into a GE model and used to calculate the effects of a costly mandate to improve energy efficiency, then negative income effects could reduce energy use not only for appliance services but also in the other sector of the economy; total rebound could well be negative."

Table 1: Decomposition of the Effect of Energy Efficiency on Total Energy Use

Panel A: Costless Efficiency Improvement ($\hat{\epsilon} > 0$)

Effects on Energy Use	No Existing Policy (with $\hat{A} = 0$)		Pre-existing policy (so $\hat{A} = -\hat{\epsilon}$)	
	(1)	(2)	(3)	(4)
Direct Efficiency Effect	$-\lambda_S \hat{\epsilon}$	$(-)$	0	0
Direct Rebound Effect		$(+)$	$(+)$	$(+)$
Income Effect	$\dfrac{\lambda_S\big((\gamma_X + \gamma_E\lambda_X)\sigma_U\gamma_A + \lambda_S\gamma_E\big)}{1 - \gamma_A}\hat{\epsilon}$	$(+)$	$\dfrac{\lambda_S\gamma_A\beta}{(1 - \gamma_A)(1 - \alpha)}\hat{\epsilon}$	$(+)$
Substitution Effect	$\lambda_S(\gamma_X + \gamma_E\lambda_X)\sigma_U\hat{\epsilon}$	$(+)$	0	0
Indirect Rebound Effect		$(+/-)$	$(+)$	$(+)$
Income Effect	$\dfrac{\lambda_X\big((\gamma_X + \gamma_E\lambda_X)\sigma_U\gamma_A + \lambda_S\gamma_E\big)}{1 - \gamma_A}\hat{\epsilon}$	$(+)$	$\dfrac{\lambda_X\gamma_A\beta}{(1 - \gamma_A)(1 - \alpha)}\hat{\epsilon}$	$(+)$
Substitution Effect	$-\lambda_X(\gamma_A + \gamma_E\lambda_S)\sigma_U\hat{\epsilon}$	$(-)$	0	0

Panel B: Efficiency Mandate ($\hat{\eta} > 0$)

Effects on Energy Use	Pre-existing policy (so $\hat{A} = \hat{\eta} > 0$)	
	(5)	(6)
Direct Efficiency Effect	$-\lambda_S\hat{\eta}$	$(-)$
Direct Rebound Effect		$(+/-)$
Income Effect	$\left(\dfrac{\lambda_S\big((\gamma_X + \gamma_E\lambda_X)\sigma_U\gamma_A + \lambda_S\gamma_E\big)}{1 - \gamma_A} - \dfrac{\lambda_S\gamma_A\beta}{(1 - \gamma_A)(1 - \alpha)}\right)\hat{\eta}$	$(+/-)$
Substitution Effect	$\lambda_S(\gamma_X + \gamma_E\lambda_X)\sigma_U\hat{\eta}$	$(+)$
Indirect Rebound Effect		$(+/-)$
Income Effect	$\left(\dfrac{\lambda_X\big((\gamma_X + \gamma_E\lambda_X)\sigma_U\gamma_A + \lambda_S\gamma_E\big)}{1 - \gamma_A} - \dfrac{\lambda_X\gamma_A\beta}{(1 - \gamma_A)(1 - \alpha)}\right)\hat{\eta}$	$(+/-)$
Substitution Effect	$-\lambda_X(\gamma_A + \gamma_E\lambda_S)\sigma_U\hat{\eta}$	$(-)$

Table 2: Parameters for Vehicles only and Household Appliances Only

	Share of Capital $\gamma_A \equiv K_A/\overline{K}$	Share of Energy $\lambda_S \equiv E_S/E$	Elasticity Parameters		
			α	β	σ_U
Both together	0.037	0.460	0.5	1.5	0.15
Vehicles only	0.019	0.240	0.5	1.5	0.15
Appliances only	0.018	0.220	0.5	1.5	0.15

Figure 1: Effects on Energy Use from the CTS and EES

(central parameter values)
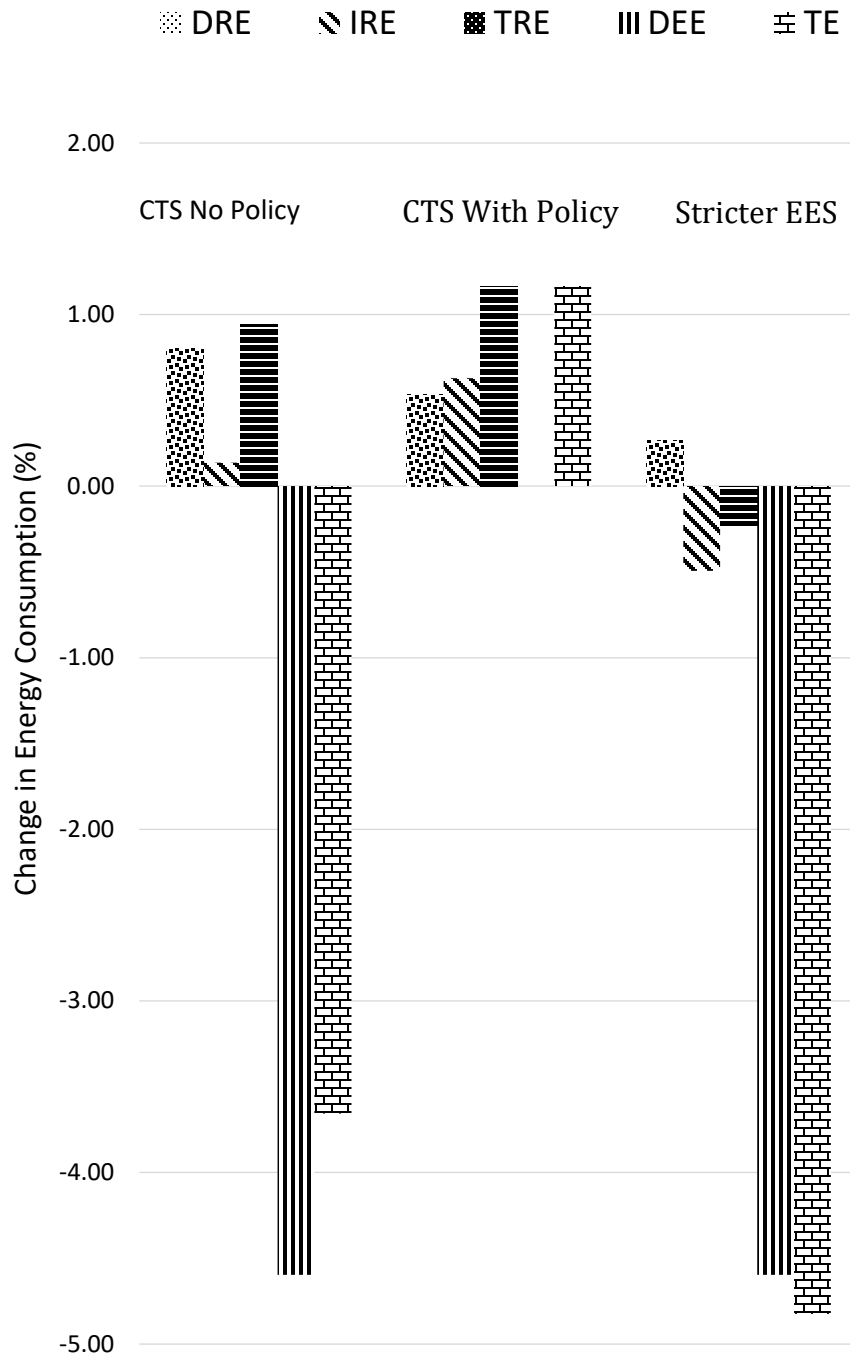
DRE: Direct Rebound Effect   DEE: Direct Efficiency Effect
IRE: Indirect Rebound Effect   TE: Total Energy
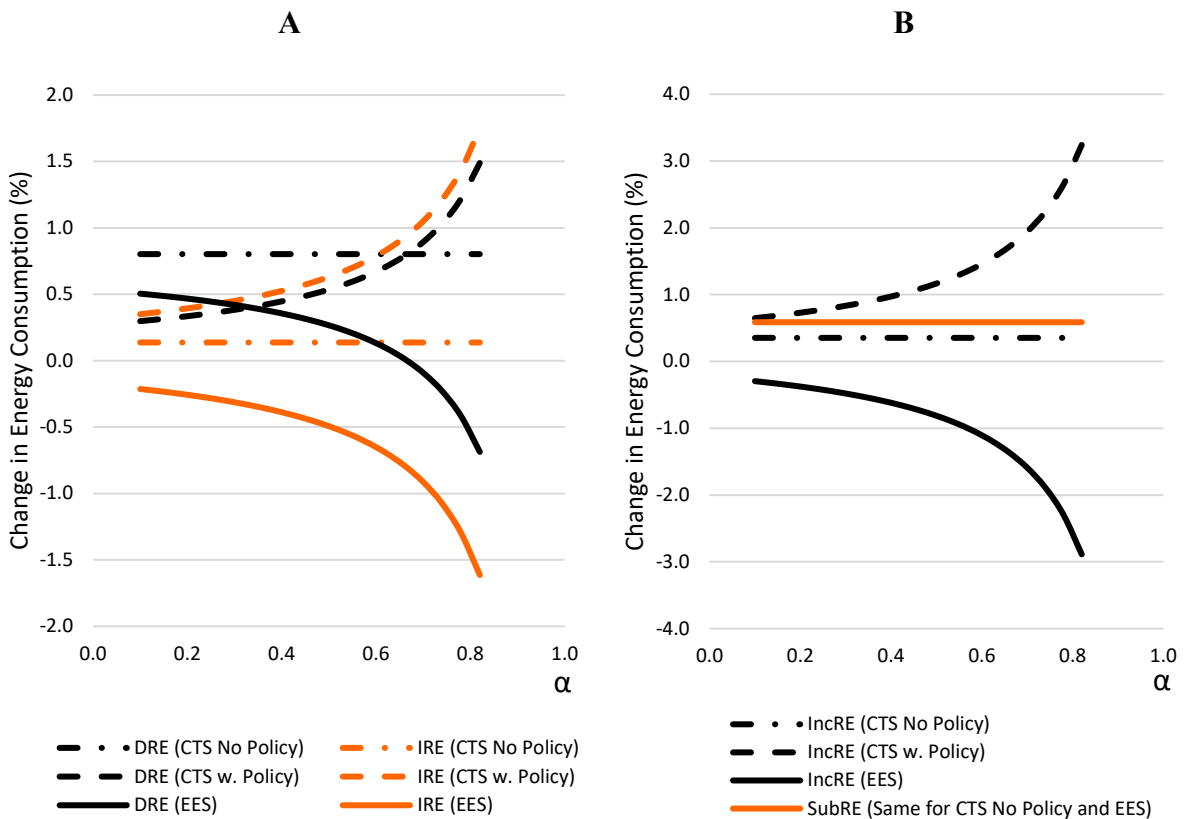TRE: Total Rebound Effect

Figure 2: Rebound Effects from the CTS and EES, Varying β
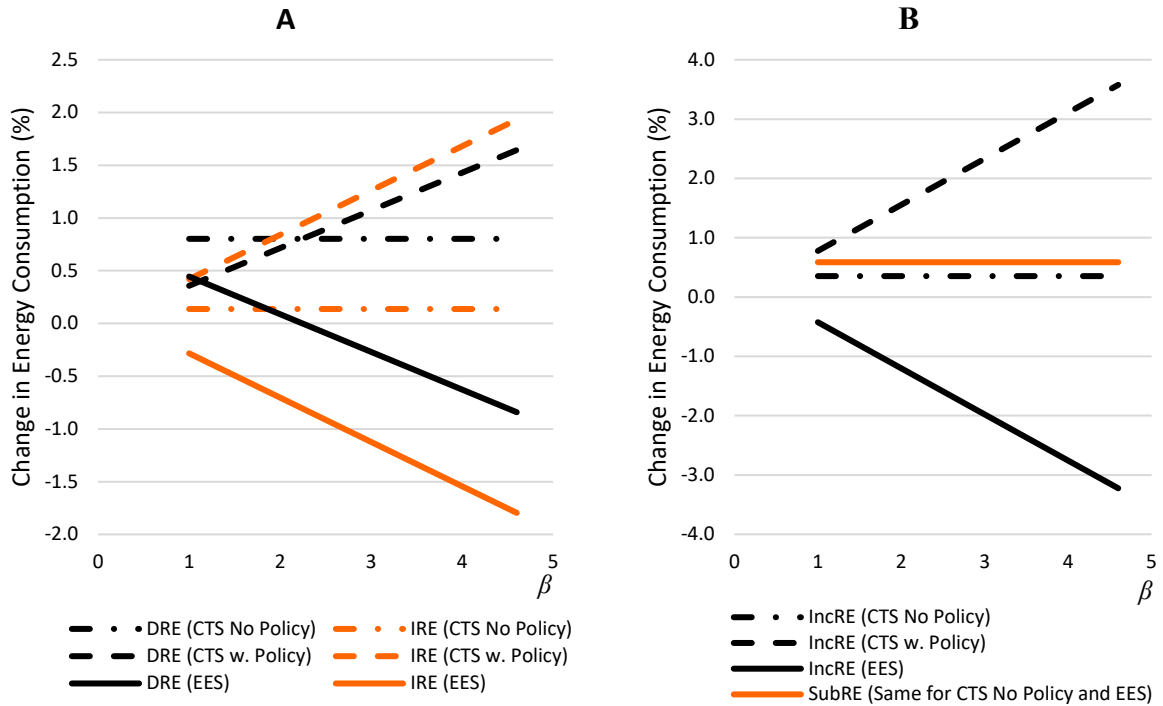
Figure 3: Rebound Effects from the CTS and EES, Varying α

Figure 4: Rebound Effects from the CTS and EES, Varying γ_A
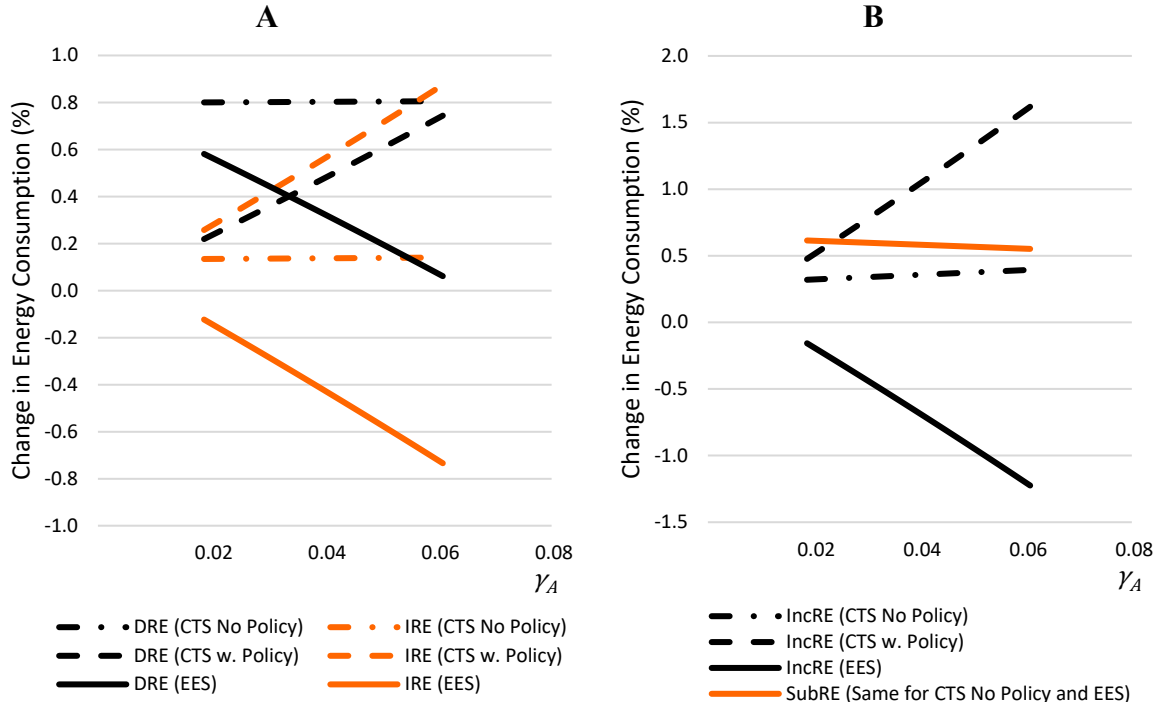
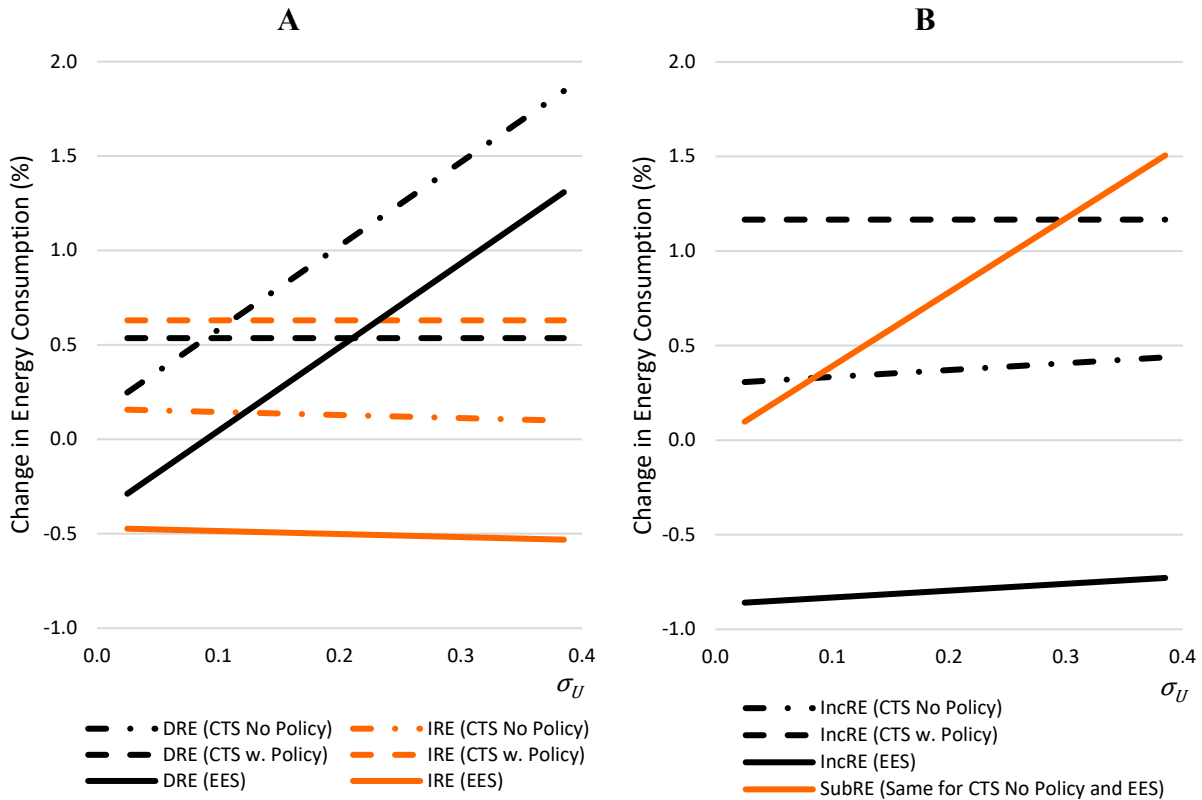**Figure 5: Rebound Effects from the CTS and EES, Varying σ_U**
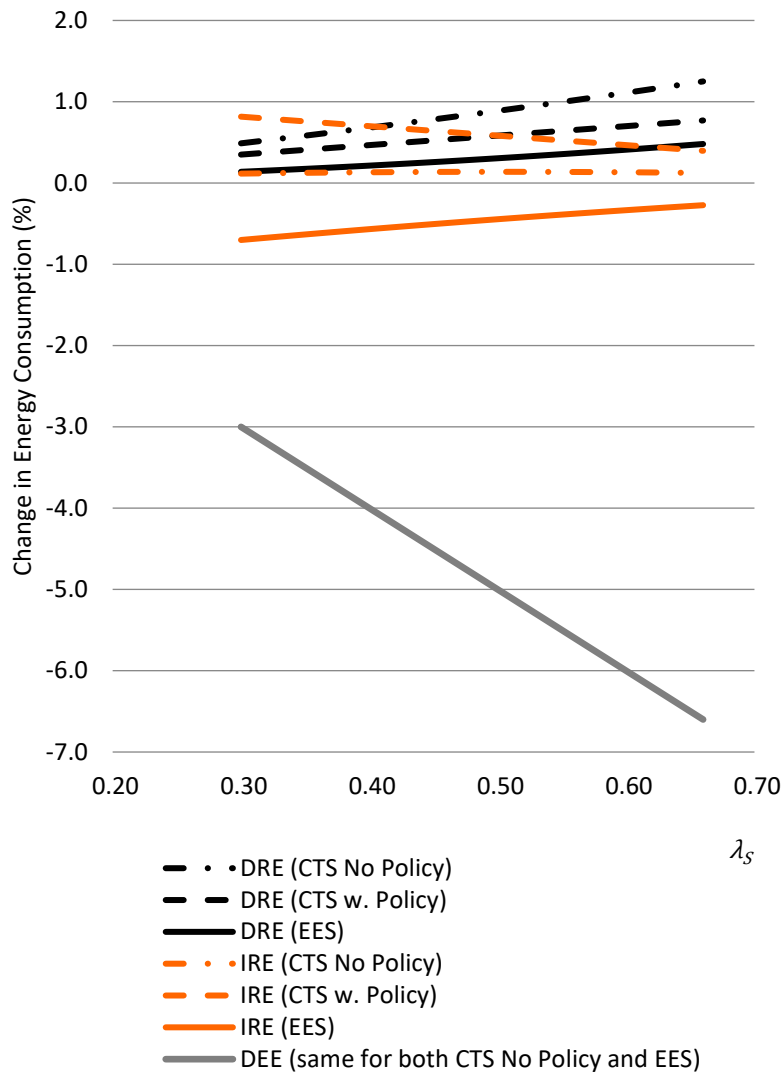
Figure 6: Rebound Effects from the CTS and the EES, Varying λ_S

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Appendices to Be Placed Online

Appendix A. Solve for Changes in Energy from a CTS with No Policy

This appendix uses equations (2.1) - (2.10) in the text to solve for changes in prices and quantities that result from a small CTS, $\hat{\epsilon} > 0$, but to compare these results with the previous literature, we calculate the effects of the alternative assumption that: $\hat{A} = \widehat{K_A} = 0$. With the numeraire choice, $\widehat{P_K} = 0$, relative prices of E and X does not change because the production of E uses only K and the production of X uses K and E. Substitute $\widehat{P_E} = \widehat{P_X} = 0$ into (2.2), (2.4), (2.7), and (2.8) and manipulate to derive:

$$\widehat{K_X} = \widehat{E_X} = \hat{X} = -\sigma_U\hat{\epsilon} + \hat{\epsilon} + \hat{A} + \widehat{E_S} = (1 - \sigma_U)\hat{\epsilon} + \widehat{E_S} \tag{A.1}$$

Then substitute (A.1) into (2.9):

$$\hat{E} = \lambda_X\hat{\epsilon}(1 - \sigma_U) + \widehat{E_S} \tag{A.2}$$

Substitute (A.1), (A.2), and $\widehat{K_A} = 0$ into (2.10) to obtain the closed form solution for $\widehat{E_S}$ (equation (3.3) in the text):

$$\widehat{E_S} = \frac{(\gamma_X + \gamma_E\lambda_X)(\sigma_U - 1)}{1 - \gamma_A}\hat{\epsilon} = \left((\gamma_X + \gamma_E\lambda_X)\sigma_U + \frac{(\gamma_X + \gamma_E\lambda_X)\sigma_U\gamma_A + \lambda_S\gamma_E}{1 - \gamma_A} - 1\right)\hat{\epsilon}$$

$$= \left((\gamma_X + \gamma_E\lambda_X)\sigma_U + \frac{(\gamma_X + \gamma_E\lambda_X)\sigma_U\gamma_A + \lambda_S\gamma_E}{1 - \gamma_A} - 1\right)\hat{\epsilon} \tag{A.3}$$

Substitute (A.3) into (2.1), (A.1) and (A.2) to obtain equations (3.1), (3.2), and (3.4) in the text for the closed form solutions of $\hat{X}$, $\hat{S}$ and $\hat{E}$.

Appendix B: Separating Income and Substitution Effects for the CTS

We take the overall effect on energy use from a small exogenous CTS and separate it into income and substitution effects. This shock changes consumers' costs, real incomes, and consumption choices. We measure the substitution effect for each commodity as a change in that quantity within the overall consumption bundle that makes the consumers as happy as before the price change but while facing new prices. The income effect is the remaining change in the consumption bundle due to the change in real income.

We use superscript "0" to denote the initial equilibrium and superscript "1" to denote the new equilibrium. Also, the superscript "C" refers to the "compensated" quantity needed to measure the substitution effect, and the superscript "I" refers to the remaining change in quantity from the income effect. We start with equations from Appendix A that show

solutions for $\hat{X}$ and $\hat{S}$ as functions of the shock and parameters:

$$\hat{X} = \left(-(\gamma_A + \gamma_E \lambda_S)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} \right) \hat{\epsilon}$$

$$\hat{S} = \left((\gamma_X + \gamma_E \lambda_X)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} \right) \hat{\epsilon}$$

Decompose the effect of the shock on S into two components:

$$\hat{S} \equiv \frac{S^1 - S^0}{S^0} = \tilde{S}^C + \tilde{S}^I \tag{B.1}$$

where $\tilde{S}^C \equiv (S^C - S^0)/S^0$ defines the substitution effect, and $\tilde{S}^I \equiv (S^1 - S^C)/S^0$ is the income effect. We define $\tilde{X}^C$ and $\tilde{X}^I$ similarly as the substitution and income effects on $\hat{X}$:

$$\hat{X} = \frac{X^1 - X^0}{X^0} = \tilde{X}^C + \tilde{X}^I \tag{B.2}$$

By definition, the substitution effect implies that consumers stay at their original utility level facing the new price, so we observe zero change in utility:

$$dU^C = \frac{\partial U}{\partial X}(X^C - X^0) + \frac{\partial U}{\partial S}(S^C - S^0) = 0$$

$$dU^C = \frac{\partial U}{\partial X} X^0 \tilde{X}^C + \frac{\partial U}{\partial S} S^0 \tilde{S}^C = 0$$

$$\frac{dU^C}{\mu I} = \frac{P_X^0 X^0}{I} \tilde{X}^C + \frac{P_K^0 K_A^0 + P_E^0 E_S^0}{I} \tilde{S}^C = 0$$

$$\frac{dU^C}{\mu I} = (\gamma_X + \gamma_E \lambda_X)\tilde{X}^C + (\gamma_A + \gamma_E \lambda_S)\tilde{S}^C = 0 \tag{B.3}$$

where μ is the marginal utility of income, and we use first order conditions from consumer optimization ($\partial U / \partial X = \mu P_X$).

With homothetic preferences, the income effect changes the consumption bundle proportionately, so we get:[1]

$$\tilde{X}^I = \tilde{S}^I \tag{B.4}$$

Equations (B.1) - (B.4) yield a system of four linear equations and four unknowns ($\tilde{S}^C, \tilde{S}^I, \tilde{X}^C$ and $\tilde{X}^I$). We solve this system of linear equations by successive substitution:

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[1] Homotheticity implies: $\frac{X^C - X^1}{X^1} = \frac{S^C - S^1}{S^1}$, which can written as $\tilde{X}^I \frac{X^0}{X^1} = \tilde{S}^I \frac{S^0}{S^1}$, so $\frac{\tilde{X}^I}{\tilde{S}^I} = \frac{X^1}{X^0} / \frac{S^1}{S^0} = \frac{\hat{X} + 1}{\hat{S} + 1} \approx 1$.

$$\tilde{S}^C = (\gamma_X + \gamma_E \lambda_X)\sigma_U \hat{\epsilon}$$

$$\tilde{S}^I = \tilde{X}^I = \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A}\hat{\epsilon}$$

$$\tilde{X}^C = -(\gamma_A + \gamma_E \lambda_S)\sigma_U \hat{\epsilon}$$

Appendix C: Derivation of the Welfare Change from a CTS

Here, we derive the welfare change from the CTS. Consumers maximize utility by their choices of the composite good, X, and appliance services, S, subject to their budget:

$$\max_{X,S} U(S, X; nE) \text{ subject to } P_X X + P_E E_S + P_K K_A \leq I$$

Assuming interior solutions (the budget constraint holds as an equality), first order conditions for this maximizing problem are:

$$U_X - \mu P_X = 0 \text{ or } U_X/\mu = P_X \tag{C.1}$$

$$U_S - \mu P_E/(\epsilon A) = 0 \text{ or } U_S/\mu = P_E/(\epsilon A) \tag{C.2}$$

where a subscript on U indicates a partial derivative (marginal utility), and μ is the shadow price on the budget constraint (marginal utility of income).

The CTS affects prices and outputs, so it affects utility. Totally differentiate the utility function and divide by μ to obtain the dollar value of the change in utility:

$$\frac{dU}{\mu} = \frac{U_X}{\mu}dX + \frac{U_S}{\mu}dS$$

Substitute (C.1) and (C.2) into the above equation and divide by the total income (I). Then multiply and divide by appropriate terms, and rearrange:

$$\frac{dU}{\mu I} = \frac{P_K K_X + P_E E_X}{I}\hat{X} + \frac{P_K K_A + P_E E_S}{I}\hat{S} = \frac{P_K K_X + P_E E_X}{P_K \overline{K}}\hat{X} + \frac{P_K K_A + P_E E_S}{P_K \overline{K}}\hat{S}$$

$$= (\gamma_X + \gamma_E \lambda_X)\hat{X} + (\gamma_A + \gamma_E \lambda_S)\hat{S}$$

The left-hand side is a measure of the welfare change from the shock, the dollar value of the change in utility divided by total income.[2] Substitute the closed form solutions for $\hat{X}$ and $\hat{S}$ into the above equation to get the closed form change in welfare:

$$\frac{dU}{\mu I} = (\gamma_X + \gamma_E \lambda_X)\hat{X} + (\gamma_A + \gamma_E \lambda_S)\hat{S} = \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A}\hat{\epsilon}$$

---

[2] We can show that $(\gamma_X + \gamma_E \lambda_X)$ is the income share of the composite good X: $\gamma_X + \gamma_E \lambda_X = \frac{K_X}{\overline{K}} + \frac{K_E}{\overline{K}}\frac{E_X}{E} = \frac{K_X + E_X}{\overline{K}} = \frac{P_K K_X + P_E E_X}{P_K \overline{K}} = \frac{P_X X}{I}$. Similarly, $(\gamma_A + \gamma_E \lambda_S) = \frac{K_A}{\overline{K}} + \frac{K_E}{\overline{K}}\frac{E_S}{E} = \frac{P_K K_A + P_E E_S}{I}$ is the income share for S.

Appendix D: Solve for Changes in Energy from a CTS with a Pre-existing EES

To solve for changes in prices and quantities from a small CTS, $\hat{\epsilon} > 0$, we assume no change in the EES. The pre-existing EES is still binding, so we get:

$$\widehat{\epsilon A} = \hat{\eta} = 0 \text{ or } \hat{A} = -\hat{\epsilon} \qquad (D.1)$$

Given (D.1), linearized equations (2.1) - (2.10) yield a system of 10 equations and 10 unknowns $(\hat{E}, \hat{X}, \hat{S}, \widehat{E_X}, \widehat{E_S}, \widehat{K_X}, \widehat{K_A}, \widehat{K_E}, \widehat{P_X}, \widehat{P_E})$. We are interested in closed-form solutions for changes in energy use in sectors S and X as functions of parameters and the shock $\hat{\epsilon} > 0$. The easiest way to solve this system is to find expressions for changes in all quantities in term of $\widehat{E_S}$ and $\hat{\epsilon}$, and then substitute those expressions into equation (2.10) to get the closed-form solution of $\widehat{E_S}$.

Our numeraire choice ($\widehat{P_K} = 0$) simplifies the mathematical derivations significantly, and combined with (2.7) - (2.8), it yields: $\widehat{P_K} = \widehat{P_E} = \widehat{P_X} = 0$. We substitute these zero changes and (D.1) into into (2.1), (2.2), (2.4), and (2.5)) to obtain:

$$\widehat{K_X} = \widehat{E_X} = \hat{X} = \hat{S} = \widehat{E_S} \qquad (D.2)$$

Thus, the percentage change in E equals the percentage change in E_X and E_S, then use (2.3) and (2.9) to get:

$$\widehat{K_E} = \hat{E} = \lambda_X \widehat{E_X} + \lambda_S \widehat{E_S} = \widehat{E_X} = \widehat{E_S} \qquad (D.3)$$

Combine (D.1) and (2.6) to obtain:

$$\widehat{K_A} = -\beta \hat{\epsilon} \qquad (D.4)$$

Substitute (D.2) - (D.4) into (2.10) and solve for $\widehat{E_S}$:

$$\widehat{K_X} = \widehat{E_X} = \hat{X} = \hat{S} = \hat{E} = \widehat{E_S} = \frac{\gamma_A \beta}{(1-\gamma_A)(1-\alpha)} \hat{\epsilon}$$

We can solve for the welfare gain from this shock by following the same step in Appendix C, given by the overall change in utility (dU) divided by the marginal utility of income (μ). This dollar value of the change in utility is then divided by total income (I) to express it in relative terms. This measure of welfare is:

$$\frac{dU}{\mu I} = \frac{P_X X}{I} \hat{X} + \frac{P_K K_A + P_E E_S}{I} \hat{S}$$

$$= (\gamma_X + \gamma_E \lambda_X)\hat{X} + (\gamma_A + \gamma_E \lambda_S)\hat{S} = \hat{X} = \hat{S} = \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \hat{\epsilon} > 0$$

Appendix E: Solve for Changes in Energy from a Small EES Policy Shock

This appendix uses equations (2.1) - (2.10) in the text to solve for changes in prices and quantities that result from a tighter efficiency standard ($\hat{\eta} > 0$). We assume no change in energy efficient technology, so $\hat{\epsilon} = 0$. Because purchase of A is costly, households will not buy more than necessary to satisfy the new requirement. Therefore the chosen $\hat{A}$ will equal the required $\hat{\eta}$. Substitute the choice of numeraire ($\widehat{P_K} = 0$) into (2.7) - (2.8) to get $\widehat{P_E} = \widehat{P_X} = 0$. We want the closed-form expression for $\hat{E}$ as a function of parameters and the policy shock $\hat{\eta}$. To do so, we first find all quantity changes in terms of $\widehat{E_S}$ and $\hat{\eta}$, and then substitute those expressions into equation (2.10) to get the closed-form solution for $\widehat{E_S}$.

From (2.2) and (2.4), no change in relative prices of K, E, and X implies that:

$$\widehat{K_X} = \widehat{E_X} = \hat{X}$$

Then substitute (2.1) into (2.5) to get:

$$\widehat{K_X} = \widehat{E_X} = \hat{X} = -\sigma_U \hat{\eta} + \hat{\eta} + \widehat{E_S} \tag{E.1}$$

Substitute (E.1) into (2.9) and (2.3):

$$\widehat{K_E} = \hat{E} = \lambda_X(-\sigma_U \hat{\eta} + \hat{\eta}) + \widehat{E_S} \tag{E.2}$$

Substitute (E.1), (E.2), and (2.6) into (2.10) to solve for $\widehat{E_S}$ (equation (5.3) in the text):

$$0 = \gamma_X(\sigma_U \hat{\eta} + \hat{\eta} + \widehat{E_S}) + \frac{\gamma_A \beta}{(1-\alpha)}\hat{\eta} + \gamma_E \lambda_X(-\sigma_U \hat{\eta} + \hat{\eta}) + \gamma_E \widehat{E_S}$$

$$\widehat{E_S} = \left(\frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U - (\gamma_X + \gamma_E \lambda_X)}{(\gamma_E + \gamma_X)} - \frac{\gamma_A \beta}{(\gamma_E + \gamma_X)(1-\alpha)}\right)\hat{\eta}$$

$$= \left((\gamma_X + \gamma_E \lambda_X)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1-\gamma_A} - \frac{\gamma_A \beta}{(1-\gamma_A)(1-\alpha)} - 1\right)\hat{\eta} \tag{E.3}$$

Substitute (E.3) into (2.1), (E.1), and (E.2) to get equations (5.1), (5.2), and (5.4) in the text.

Appendix F: Proof of Proposition 3

Proof of (3.i):

The DRE from the EES shock is the sum of the substitution and income effects on E, so it can be written and shorten as:

$$\text{DRE} = \left(\frac{\lambda_S\left((\gamma_X+\gamma_E\lambda_X)\sigma_U\gamma_A+\lambda_S\gamma_E\right)}{1-\gamma_A} - \frac{\lambda_S\gamma_A\beta}{(1-\gamma_A)(1-\alpha)}\right)\hat{\eta} + \lambda_S(\gamma_X + \gamma_E\lambda_X)\sigma_U\hat{\eta}$$

$$= \left(\lambda_S\left((\gamma_X + \gamma_E\lambda_X)\sigma_U\gamma_A + \lambda_S\gamma_E\right) - \frac{\lambda_S\gamma_A\beta}{(1-\alpha)} + \lambda_S(\gamma_X + \gamma_E\lambda_X)\sigma_U(1-\gamma_A)\right)\frac{\hat{\eta}}{1-\gamma_A}$$

$$= \left(\lambda_S\gamma_E - \lambda_S\gamma_E\lambda_X - \frac{\lambda_S\gamma_A\beta}{(1-\alpha)} + \lambda_S\gamma_X\sigma_U + \lambda_S\gamma_E\lambda_X\sigma_U\right)\frac{\hat{\eta}}{1-\gamma_A}$$

$$= \left((\gamma_E\lambda_X + \gamma_X)\sigma_U + \gamma_E\lambda_S - \frac{\gamma_A\beta}{(1-\alpha)}\right)\frac{\lambda_S\hat{\eta}}{1-\gamma_A}$$

Because $1 - \gamma_A > 0$, $\lambda_S > 0$, and $\hat{\eta} > 0$, the sign of DRE has the sign of the whole expression in big parentheses. That is, if $\gamma_A\beta > (1-\alpha)\left((\gamma_E\lambda_X + \gamma_X)\sigma_U + \gamma_E\lambda_S\right)$, then DRE$< 0$. In such a case, $\gamma_A\beta$ is even larger than required for the EES to bind as

$$\gamma_A\beta > (1-\alpha)\left((\gamma_E\lambda_X + \gamma_X)\sigma_U + \gamma_E\lambda_S\right) > (1-\alpha)\left((\gamma_E\lambda_X + \gamma_X)\gamma_A\sigma_U + \gamma_E\lambda_S\right).$$

We can also show that under the condition $\gamma_A\beta > (1-\alpha)\left((\gamma_E\lambda_X + \gamma_X)\sigma_U + \gamma_E\lambda_S\right)$, the IRE is also negative and more negative that the DRE. We can write the IRE as:

$$\text{IRE} = \left(\frac{\lambda_X[(\gamma_X+\gamma_E\lambda_X)\sigma_U\gamma_A+\lambda_S\gamma_E]}{1-\gamma_A} - \frac{\lambda_X\gamma_A\beta}{(1-\gamma_A)(1-\alpha)}\right)\hat{\eta} - \lambda_X(\gamma_A + \gamma_E\lambda_S)\sigma_U\hat{\eta}$$

$$= \frac{\hat{\eta}\lambda_X}{1-\gamma_A}\left((\gamma_X + \gamma_E\lambda_X + \gamma_A + \gamma_E\lambda_S)\sigma_U\gamma_A + \lambda_S\gamma_E - \frac{\gamma_A\beta}{(1-\alpha)} - (\gamma_A + \gamma_E\lambda_S)\sigma_U\right)$$

$$= \frac{\hat{\eta}\lambda_X}{1-\gamma_A}\left(\lambda_S\gamma_E - \gamma_E\lambda_S\sigma_U - \frac{\gamma_A\beta}{(1-\alpha)}\right) < 0$$

Subtract DRE by IRE and consider the result:

$$\text{DRE} - \text{IRE} = \left((\gamma_E\lambda_X + \gamma_X)\sigma_U + \gamma_E\lambda_S - \frac{\gamma_A\beta}{(1-\alpha)}\right)\frac{\lambda_S\hat{\eta}}{1-\gamma_A} - \frac{\hat{\eta}\lambda_X}{1-\gamma_A}\left(\lambda_S\gamma_E - \gamma_E\lambda_S\sigma_U - \frac{\gamma_A\beta}{(1-\alpha)}\right)$$

$$= \left((\gamma_E\lambda_X + \gamma_X + \gamma_E)\lambda_S\sigma_U + (\lambda_S - \lambda_X)\gamma_E\lambda_S + \frac{\lambda_X\gamma_A\beta}{(1-\alpha)}\right)\frac{\hat{\eta}}{1-\gamma_A}$$

If $(1-\alpha)\left((\gamma_E\lambda_X + \gamma_X)\sigma_U + \gamma_E\lambda_S\right) < \gamma_A\beta$, then

$$\text{DRE} - \text{IRE} > \left((\gamma_E\lambda_X + \gamma_X + \gamma_E)\lambda_S\sigma_U + (\lambda_S - \lambda_X)\gamma_E\lambda_S + \lambda_X\left((\gamma_E\lambda_X + \gamma_X)\sigma_U + \gamma_E\lambda_S\right)\right)\frac{\hat{\eta}}{1-\gamma_A}$$

and $\text{DRE} - \text{IRE} > \left((\gamma_E + \gamma_X)\sigma_U + \lambda_S\gamma_E\lambda_S\right)\frac{\hat{\eta}}{1-\gamma_A} > 0$

Because DRE<0 and IRE<0, then the absolute size of the indirect rebound effect is greater than the absolute size of the direct rebound effect.

Proof of (3.ii):

From Column (5) of Table 1, the TRE is the sum of direct and indirect substitution and

income effects:

$$\text{TRE} = \left((\lambda_S \gamma_X - \lambda_X \gamma_A)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \right) \hat{\eta}$$

$$= \frac{\hat{\eta}}{1 - \gamma_A} \left((\lambda_S \gamma_X - \lambda_X \gamma_A)\sigma_U + \lambda_X \sigma_U \gamma_A + \lambda_S \gamma_E - \frac{\gamma_A \beta}{(1 - \alpha)} \right)$$

$$= \frac{\hat{\eta}}{1 - \gamma_A} \left(\lambda_S \gamma_X \sigma_U + \lambda_S \gamma_E - \frac{\gamma_A \beta}{(1 - \alpha)} \right)$$

The fraction of K used in production of A to $\bar{K}$, γ_A, is less than one, so $1 - \gamma_A > 0$. The sign of TRE depends on the term in big parentheses. That is, if $\gamma_A \beta > (1 - \alpha)(\lambda_S \gamma_X \sigma_U + \gamma_E \lambda_S)$, then TRE < 0.

First, we must meet the minimum requirement such that the pre-existing energy efficiency standard is costly and binding: that is, $\gamma_A \beta > (1 - \alpha)\big((\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E\big)$. If energy service sector is less energy intensive (i.e., $\lambda_S \gamma_X - \lambda_X \gamma_A < 0$), then we can show:

$$\lambda_S \gamma_X \sigma_U + \gamma_E \lambda_S < (\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A - \lambda_S \gamma_E$$

by subtracting the left hand side terms from the right hand side terms in the above inequality and rearranging the result as :

$$\lambda_S \gamma_X \sigma_U + \gamma_E \lambda_S - (\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A - \lambda_S \gamma_E = \sigma_U (1 - \gamma_A)(\lambda_S \gamma_X - \lambda_X \gamma_A) < 0$$

Then in such a case, the minimum requirement to ensure the pre-existing energy efficiency standard is costly and binding implies the condition such that $TRE < 0$.

If energy service sector is more energy intensive (i.e., $\lambda_S \gamma_X - \lambda_X \gamma_A > 0$), then the minimum costly and binding EES condition is less strict than the condition that is required for the TRE to be negative:

$$\gamma_A \beta > (1 - \alpha)(\lambda_S \gamma_X \sigma_U + \gamma_E \lambda_S) > (1 - \alpha)\big((\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A - \lambda_S \gamma_E\big)$$

Proof of (3.iii):

Backfire occurs when the direct energy savings from increased energy efficiency are more than offset by the positive rebound effects, or the total effect on energy use is positive. It's trivial to show that if the energy service sector is less energy intensive than the other good sector, backfire is impossible as the total rebound effect is actually a negative rebound.

Consider if the energy service sector is more energy intensive, then from equation (5.4), the total effect on energy use can be shortened as:

$$\hat{E} = \left((\lambda_S \gamma_X - \lambda_X \gamma_A)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} - \lambda_S \right) \hat{\eta}$$

$$= \frac{\hat{\eta}}{1-\gamma_A}\left((\lambda_S\gamma_X - \lambda_X\gamma_A)\sigma_U + \lambda_X\sigma_U\gamma_A - \frac{\gamma_A\beta}{(1-\alpha)} - \lambda_S\gamma_X\right) = \frac{\hat{\eta}}{1-\gamma_A}\left(\lambda_S\gamma_X(\sigma_U - 1) - \frac{\gamma_A\beta}{(1-\alpha)}\right)$$

If $\gamma_A\beta > (1-\alpha)\lambda_S\gamma_X(\sigma_U - 1)$, $\hat{E} < 0$ and there is no backfire!

One special case is that if $\sigma_U < 1$, then $\lambda_S\gamma_X(\sigma_U - 1) - \frac{\gamma_A\beta}{(1-\alpha)} < -\frac{\gamma_A\beta}{(1-\alpha)} < 0$. Since

$1 - \gamma_A > 0$ and $\hat{\eta} > 0$, $\hat{E} < 0$. So, there exists no backfire if $\sigma_U < 1$.

Appendix G: Calibration of Cost Function Parameters (α and β)

Recall that the annual cost, K_A, of achieving more energy efficiency, A, is a function

of the extra energy efficiency beyond some A_0 which we call "minimum" energy efficiency:

$K_A = B(A - A_0)^\beta$, with $B > 0$ and $\beta \geq 1$

This minimum A_0 is essentially an engineering concept, but it is better interpreted as a

parameter of the cost function – a fixed number that needs to be calibrated for the model to fit

the facts we want to capture.[3] Various sources below are used to set α, defined as $\alpha \equiv A_0/A$.

For convenience of notation, we define $A_1 \equiv A - A_0$. Then the above cost equation

can be written as: $K_A = BA_1^\beta$, and it can be differentiated with respect to the chosen extra

energy efficiency A_1 to obtain the marginal cost of energy efficiency:

$$MC \equiv \frac{dK_A}{dA} = \frac{dK_A}{dA_1} = B\beta A_1^{\beta-1} \qquad (G.1)$$

Note that $dA_1 = dA$, since A_0 is a fixed parameter. Totally differentiate equation (G.1) with

respect to A_1 to get:

$$dMC = B\beta(\beta - 1)A_1^{\beta-2}dA_1 \qquad (G.2)$$

Divide equation (G.2) by equation (G.1) to obtain:

$$\frac{dMC}{MC} = \frac{(\beta - 1)dA_1}{A_1}$$

$$\widehat{MC} = (\beta - 1)\widehat{A_1}$$

$$\beta = \frac{\widehat{MC}}{\widehat{A_1}} + 1 \qquad (G.3)$$

where we can rewrite $\widehat{A_1} \equiv dA_1/A_1 = dA/(A - A_0) = \hat{A}/(1 - \alpha)$. If we have enough data

on both the *levels* and *changes* in both marginal cost and energy efficiency, then we could use

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[3] This A_0 is not the cost-minimizing energy efficiency that un-regulated firms and consumers would choose. Existing mandates mean that firms start with A above A_0, and our model is used only for small changes. This A_0 just shifts the cost curve, so that A_0 and β can be chosen jointly to obtain the best fit for available information.

equation (G.3) to calibrate our cost curve parameter β.

For light-duty vehicles, given some initial fuel efficiency A in miles per gallon (mpg), calculations of incremental costs (dK_A) associated with improved fuel efficiency (dA) are provided by the International Council on Clean Transportation (Lutsey *et al.*, 2017) and the National Research Council (NRC, 2015). For several major appliances, the Department of Energy (DOE, 2011, 2016a, and 2016b) provides similar cost calculations for changes in energy efficiency (as measured for appliances). We use those additional costs to calculate the marginal cost of energy efficiency (MC, per unit dA) and its percentage change ($\widehat{MC}$).

Then, to calibrate β using equation (G.3), we also need to know $\widehat{A_1} = dA/(A - A_0)$, where A_0 is not observed. In the model, A and A_0 are in services S per unit of energy, where S is an aggregation. In the data, energy efficiency is reported in mpg for cars, or Annual Fuel Utilization Efficiency (AFUE) for furnaces, or Seasonal Energy Efficiency Ratio (SEER) for air conditioners. Our text lists the six steps of our approach.

Table G1 shows calibration results for midsize cars with spark-ignition (SI) engines, as studied in NRC (2015). Panel A shows their "low-technology cost scenario", and panel B shows their "high-technology cost scenario". Fuel efficiency improvements and associated incremental costs in the first two columns of Table G1 are taken from NRC (2015). All incremental costs are relative to the "null vehicle" which has the lowest technology in the 2008 model year.[4] The third column shows our calculated marginal cost of energy efficiency, MC, the change in incremental cost divided by the change in fuel economy. Using the first two rows, we have (1,388–321)/(52.1–36.5) = 68 (in dollars per additional mpg). Then we calculate the proportional *change* in marginal cost ($\widehat{MC} = dMC/MC$). For the same example, the percentage change in the marginal cost from 36.5 to 52.1 mpg is (153–68)/68 = 1.24 (or, 124%). Our calibrated β in either the low- or high-cost scenarios is only a point calculation (at 36.5 mpg or 36.4 mpg). These mpg's are between the 2015 CAFE standard and the 2016 CAFE standard for passenger cars (i.e., between 36.2 mpg and 37.8 mpg).

The last four columns of Table G1 show the calculation of β for each A_0 and α. The table shows that a smaller minimum fuel economy α tends to yield a larger β. The calculated β is generally higher in the low-cost scenario than in the high-cost scenario. In either panel, and in our other calculations below, α=0 can yield high and unstable values of β. When α is

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[4] Technologies are added to improve the fuel economy of the null vehicle in order to meet the current and future CAFE standards. See NRC (2015) for more technical descriptions of the null vehicle.

near one-half, tables show a range of β that are all around 2 (quadratic costs).

Table G1: Calibration of β for Midsize Cars with SI Engine

A. Low Cost

Fuel Economy (mpg)	Incremental Cost ($)	MC ($/mpg)	$\widehat{MC}$	$A_0 = 0$ $(\alpha = 0)$	$A_0 = 10$ $(\alpha = .27)$	$A_0 = 15$ $(\alpha = .41)$	$A_0 = 20$ $(\alpha = .55)$
						β	
36.5	321	68	124%	3.91	3.11	2.71	2.32
52.1	1388	153					
57.9	2278						

B. High Cost

Fuel Economy (mpg)	Incremental Cost ($)	MC ($/mpg)	$\widehat{MC}$	$A_0 = 0$ $(\alpha = 0)$	$A_0 = 10$ $(\alpha = .27)$	$A_0 = 15$ $(\alpha = .41)$	$A_0 = 20$ $(\alpha = .55)$
						β	
36.4	325	104	79%	2.84	2.33	2.08	1.83
52.1	1956	186					
56.1	2701						

Note: Fuel economy improvements and their associated incremental costs are extracted from Tables 8.4a and 8.4b in NRC (2015). Panel A shows their low-cost scenario, and panel B is their high-cost scenario. According to U.S. EPA statistics, the combined mpg of the least-efficient midsize cars sold in U.S. in 2018 is 14. Choices of A_0 for sensitivity analysis are 0, 10, 15, and 20.

Table G2: Calibration of β for Passenger Cars and Light Trucks

A. Passenger Car

Fuel Economy (mpg)	Incremental Cost ($)	MC ($/mpg)	$\widehat{MC}$	$A_0 = 0$ $(\alpha = 0)$	$A_0 = 10$ $(\alpha = .28)$	$A_0 = 15$ $(\alpha = .42)$	$A_0 = 20$ $(\alpha = .56)$
						β	
35.7	378	92	24%	2.80	2.30	2.05	1.79
40.4	811	113					
44.4	1255						

B. Light Truck

Fuel Economy (mpg)	Incremental Cost ($)	MC ($/mpg)	$\widehat{MC}$	$A_0 = 0$ $(\alpha = 0)$	$A_0 = 10$ $(\alpha = .26)$	$A_0 = 15$ $(\alpha = .42)$	$A_0 = 20$ $(\alpha = .58)$
						β	
28.8	416	131	47%	2.98	2.43	2.09	1.75
35.7	900	193					
44.1	1621						

Note: Fuel economy improvements and their associated incremental costs are extracted from Tables A.1 – A.4 in Lutsey *et al.* (2017). Panel A shows calculations for passenger cars, and panel B is for light trucks. According to U.S. EPA statistics, the combined mpg of the least-efficient passenger cars and light-duty trucks is about 11 to 14. Choices of A_0 for passenger cars are 0, 10, 15, and 20. For light trucks, choices of A_0 are 0, 8, 13, and 18.

Table G2 shows our calibration for passenger cars and light trucks using results from Lutsey *et al.* (2017), where targets are stated as percentages of carbon dioxide emission

reductions of 0% up to 100% from the 2008 baseline year. We convert those emission reduction targets into equivalent mpg goals using the common conversion factor of 8,887 grams of CO_2 emissions per gallon of gasoline consumed.[5] We report equivalent mpg goals in the first column of Panels A and B. Incremental costs are taken from Lutsey *et al.* (2017), from which we calculate marginal costs and percentage changes in MC. For these passenger cars and light trucks, we vary A_0 just as in the case of midsize cars above. Our calculated β shown in the last four columns of Panel A and B are very close to those in Table G1.

Table G3: Calibration of β for Residential Packaged Central Air Conditioner

Energy Efficiency (SEER)	Installed Cost ($)	MC ($/SEER)	$\widehat{MC}$	β			
				$A_0 = 0$ ($\alpha = 0$)	$A_0 = 8$ ($\alpha = .57$)	$A_0 = 9$ ($\alpha = .64$)	$A_0 = 10$ ($\alpha = .71$)
14.0	4779	156	26%	4.66	2.57	2.31	2.05
15.0	4935	197					
17.5	5427						

Note: Data are taken from the DOE (2016a). SEER stands for Seasonal Energy Efficiency Ratio, which is a measure of an air conditioner's efficiency. A SEER rating is the ratio of cooling in British thermal unit (BTU) to the energy consumed in watt-hours. The federally regulated minimum SEER for air conditioners are 13 or 14 depending on geographical locations. Choices of A_0 for sensitivity analysis are 0, 8, 9, and 10.

Next, we show calibration results for residential central air conditioning units in Table G3, furnaces in Table G4, and refrigerator-freezers in Table G5. These selections are major appliances that consume much household energy, and ones for which incremental costs are available from DOE (2011, 2016a, 2016b). The first column in Tables G3 and G4 shows the measured energy efficiency (the SEER rating for air conditioners and AFUE rating for furnaces).[6] Their associated installed costs are provided by DOE (2016a, 2016b) and shown in the second column of both tables. To calculate marginal costs of energy efficiency in the third column, we divide the difference between installed costs in the second column by the increase in energy efficiency in the first column. For example, in the third column of Table G3, the marginal cost 156 = (4,935 – 4,779)/(15.0-14.0). Then, the table shows the percentage change in marginal costs, followed by the cost curve parameter β (in the last four columns). The minimum energy efficiency A_0 is a technological parameter, not related to policy, so we vary

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[5] The conversion rate can be found on the U.S. Environmental Protection Agency (EPA)'s website: https://www.epa.gov/energy/greenhouse-gases-equivalencies-calculator-calculations-and-references.

[6] A SEER rating is the ratio of cooling in British thermal unit (BTU) to the energy consumed in watt-hours. A furnace with an AFUE rating of 90 would mean that it can convert 90% of its fuel into usable heat.

it below the federally required minimum energy efficiency. For $A_0>0$, our calculated β for central air conditioners and furnaces varies around 2.

Table G4: Calibration of β for Residential Non-weatherized Furnace

Energy Efficiency (AFUE)	Installed Cost ($)	MC ($/AFUE)	$\widehat{MC}$	β $A_0 = 0$ ($\alpha = 0$)	$A_0 = 50$ ($\alpha = .54$)	$A_0= 60$ ($\alpha = .65$)	$A_0= 70$ ($\alpha = .76$)
92	2635	36	8%	3.58	2.18	1.90	1.62
95	2742	39					
98	2858						

Note: Data are taken from the DOE (2016b). AFUE stands for Annual Fuel Utilization Efficiency, which is a measure of furnace efficiency. A furnace with an AFUE rating of 90 would mean that it can convert 90% of its fuel into usable heat. A low-efficiency heating system has 56% to 70% AFUE. The minimum federal energy efficiency standard for residential furnaces is 80. Choices of A_0 for sensitivity analysis are 50, 60, and 70.

Table G5: Calibration of β for 16 Cubic Foot Top-Mount Refrigerator-Freezers

Maximum Energy Use [kWh/yr]	Energy Efficiency [ft3/(kWh/day)]	Incremental Cost [$]	MC [$/(ft3/kWh/day)]	$\widehat{MC}$	β $A_0= 0$ ($\alpha = 0$)	$A_0= 600$ ($\alpha = .63$)	$A_0= 550$ ($\alpha = .68$)	$A_0= 500$ ($\alpha = .74$)
410	14.27	6.52	15.03	96%	5.82	2.78	2.53	2.23
341	17.13	49.41	29.50					
296	19.76	127.14						

Note: For each maximum energy use (first column), DOE (2011) calculates incremental cost (third column). The text describes how we convert their maximum energy use to a measure of required energy efficiency for our model (second column), in cubic feet cooled per kWh per day. The last four columns show our calculated β for alternative assumptions about A_0 in kWh/year (namely, 0, 500, 550, and 600).

Table G5 presents calculations for refrigerator-freezers, where efficiency is measured as the annual energy use. The federal standard before September 15, 2014 for top-mount refrigerator-freezers is stated as a maximum energy use, for example 432.8 kWh per year.[7] DOE (2011) provides calculations for each annual energy consumption (kWh/year) shown in the first column. In our analytical model, however, energy efficiency is measured as services per unit of energy ($\epsilon A = S/E$). Therefore, we convert the efficiency of refrigerator-freezers to volume cooled per unit of electricity per day. To do so, we multiply the size of the appliance (16 cubic feet) by the number of days per year (365.25) and divide by annual energy use. Results are shown in the second column of Table G5. The third column is the incremental cost of DOE (2011), from which we calculate marginal cost in the fourth column (and change in marginal cost in the fifth column). Using four alternatives for A_0 (in kWh per year), the last

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[7] This figure is calculated as 9.80 times the adjusted volume of the appliance in cubic feet, plus 276. If the volume is 16 cubic feet, then the maximum energy use is 9.8 ×16+276= 432.8 kWh per year.

four columns show the calibrated β. We generally vary the minimum energy efficiency A_0 below the federally required minimum efficiency for each appliance.

In general, Tables G1-G5 confirm that cost parameters can vary across vehicles and appliances and also depend on methods and assumptions. Our calibration section describes how we choose a pair of parameters that best fit the calculations appearing in all these tables.

Appendix H: Negative Effects of Energy Consumption on Productivity

This appendix uses equations (2.1) - (2.9) and (9.1) from the text to solve for changes in prices and quantities that result from a tighter efficiency standard ($\hat{\eta} > 0$). We follow the exact same steps as in Appendix E, and we get:

$$\widehat{P_E} = \widehat{P_X} = 0.$$

$$\widehat{K_X} = \widehat{E_X} = \hat{X} = -\sigma_U\hat{\eta} + \hat{\eta} + \widehat{E_S} \tag{H.1}$$

$$\widehat{K_E} = \hat{E} = \lambda_X(-\sigma_U\hat{\eta} + \hat{\eta}) + \widehat{E_S} \tag{H.2}$$

To solve for $\widehat{E_S}$, first substitute (H.1), (H.2), and (2.6) into (9.1) to get:

$$\delta\lambda_X(-\sigma_U\hat{\eta} + \hat{\eta}) + \delta\widehat{E_S} = \gamma_X(\sigma_U\hat{\eta} + \hat{\eta} + \widehat{E_S}) + \frac{\gamma_A\beta}{(1-\alpha)}\hat{\eta} + \gamma_E\lambda_X(-\sigma_U\hat{\eta} + \hat{\eta}) + \gamma_E\widehat{E_S}$$

$$\widehat{E_S} = \left((\gamma_E\lambda_X + \gamma_X)\sigma_U + \left(\frac{(\mu+\gamma_E)\lambda_S + (\gamma_X+\gamma_E\lambda_X)\gamma_A\sigma_U + (\gamma_A\lambda_X - \gamma_X\lambda_S)\mu\sigma_U}{(\mu+1-\gamma_A)} - \frac{1}{(\mu+1-\gamma_A)}\frac{\beta\gamma_A}{1-\alpha}\right) - 1\right)\hat{\eta} \tag{H.3}$$

Here, we decompose the effect of the EES on energy use in S into a substitution rebound effect, an income rebound effect, and a DEE. Then, substitute (H.3) into (H.1) to get the closed-form solution for $\widehat{E_X}$:

$$\widehat{E_X} = \left(-(\gamma_E\lambda_S + \gamma_A)\sigma_U + \left(\frac{(\mu+\gamma_E)\lambda_S + (\gamma_X+\gamma_E\lambda_X)\gamma_A\sigma_U + (\gamma_A\lambda_X - \gamma_X\lambda_S)\mu\sigma_U}{(\mu+1-\gamma_A)} - \frac{1}{(\mu+1-\gamma_A)}\frac{\beta\gamma_A}{1-\alpha}\right)\right)\hat{\eta}$$

Substitute (H.3) into (H.2) to get the closed-form solution for $\hat{E}$ in equation (9.2) in the text.

Appendix I: An Upward-Sloping Supply of Energy

This appendix considers an upward-sloping supply of energy, and it solves for changes in prices and quantities that result from a tighter efficiency standard ($\hat{\eta} > 0$). Like Appendix E, we assume $\hat{\epsilon} = 0$ while $\hat{A} = \hat{\eta}$. We want the closed-form expression for $\hat{E}$ as a function of parameters and the policy shock $\hat{\eta}$. To do so, we first find all quantity changes in terms of $\widehat{P_E}$ and $\hat{\eta}$, and then substitute those expressions into equation (2.10) to get the closed-form

solution for $\widehat{P_E}$.

First, linearize the production function of energy to get:

$$\hat{E} = \theta_{KE}\widehat{K_E} \tag{I.1}$$

Then, because $\widehat{P_K}=0$, the linearized definition of elasticity substitution between K_E and $\bar{Q}$ is:

$$\widehat{K_E} = \sigma_E\widehat{P_Q} \tag{I.2}$$

Rearrange equations (2.8) and substitute it successively into (2.11) and then (2.3) to get:

$$\widehat{K_E} = \sigma_E\widehat{P_Q} = \frac{\sigma_E}{\theta_{QE}}\widehat{P_E} \tag{I.3}$$

$$\hat{E} = \theta_{KE}\widehat{K_E} = \frac{\sigma_E\theta_{KE}}{\theta_{QE}}\widehat{P_E} \tag{I.4}$$

Rearrange and manipulate equations (2.2), (2.4), (2.1), (2.5), (2.9), (I.4) to obtain:

$$\hat{X} = \theta_{KX}\widehat{K_X} + \theta_{EX}\widehat{E_X} = \theta_{KX}\sigma_X\widehat{P_E} + \widehat{E_X}$$

$$\theta_{KX}(\sigma_U - \sigma_X)\widehat{P_E} + (1 - \sigma_U)\hat{\eta} = \widehat{E_X} - \widehat{E_S}$$

$$\widehat{E_X} = \hat{E} + \lambda_S(\widehat{E_X} - \widehat{E_S}) = \sigma_E\frac{\theta_{KE}}{\theta_{QE}}\widehat{P_E} + \lambda_S\theta_{KX}(\sigma_U - \sigma_X)\widehat{P_E} + \lambda_S(1 - \sigma_U)\hat{\eta}$$

$$\widehat{K_X} = \widehat{E_X} + \sigma_X\widehat{P_E} = \left(\sigma_E\frac{\theta_{KE}}{\theta_{QE}} + \lambda_S\theta_{KX}(\sigma_U - \sigma_X) + \sigma_X\right)\widehat{P_E} + \lambda_S(1 - \sigma_U)\hat{\eta} \tag{I.5}$$

Then, to solve for $\widehat{P_E}$, substitute (2.6), (I.3), and (I.5) into (2.10):

$$0 = \gamma_X\left(\left(\sigma_E\frac{\theta_{KE}}{\theta_{QE}} + \lambda_S\theta_{KX}(\sigma_U - \sigma_X) + \sigma_X\right)\widehat{P_E} + \lambda_S(1 - \sigma_U)\hat{\eta}\right) + \frac{\gamma_A\beta}{(1-\alpha)}\hat{\eta} + \gamma_E\frac{\sigma_E}{\theta_{QE}}\widehat{P_E}$$

$$\widehat{P_E} = -\frac{\theta_{QE}}{D}\left(\gamma_X\lambda_S(1 - \sigma_U) + \gamma_A\frac{\beta}{1-\alpha}\right)\hat{\eta} \tag{I.6}$$

where $D \equiv \sigma_E(\theta_{KE}\gamma_X + \gamma_E) + \gamma_X\lambda_S\theta_{KX}\theta_{QE}\sigma_U + \gamma_X\sigma_X\theta_{QE}(1 - \lambda_S\theta_{KX})$

Finally, Substitute (I.6) into (I.4) and rearrange to get equation (9.4) in the text.

Appendix J: Calibration for Vehicles Separately from Household Appliances

As in the main calibration of section 6, we obtain 2015 residential energy expenditure data from the U.S. Energy Information Administration (EIA). Data on gasoline and other motor fuels are from the Bureau of Economic Analysis (BEA).

For household vehicles and parts, we use EIA data to calculate that the annualized cost for energy efficiency is $P_K K_A = 344.342$ ($B). Thus, the fraction of K used for energy

efficiency ($\gamma_A \equiv K_A/\overline{K}$) is 344.342/18,037 = 0.019. Total expenditure on gasoline and other motor fuels is 270 ($B). The ratio of this energy use to total energy use is $\lambda_S \equiv E_S/E =$ 270/1,127 = 0.240, and the fraction of energy used in production of the composite good X is 0.760. We keep other parameters the same as in the case with all cars and appliances together, and we check the condition in Proposition 2 to make sure that the fuel efficiency standard is binding. All direct and rebound effects are shown in the figure below.

For household appliances other than vehicles, our calibrated annual cost for energy efficiency is $P_K K_A$ = 330.658 ($B), so the fraction of K used for energy efficiency is $\gamma_A \equiv$ $K_A/\overline{K}$ = 330.658/18,037 = 0.018. The total energy used for these appliance services (without motor fuel costs) is 248 ($B). Thus the ratio of this energy use to total energy use is $\lambda_S \equiv$ E_S/E = 248/1,127 = 0.220, and the fraction of energy used in production of the composite good is 0.780. We keep other parameters the same as in the case with household vehicles, and we check the condition in Proposition 2 to make sure that the appliance standard is binding. All direct and rebound effects are shown in another figure below.
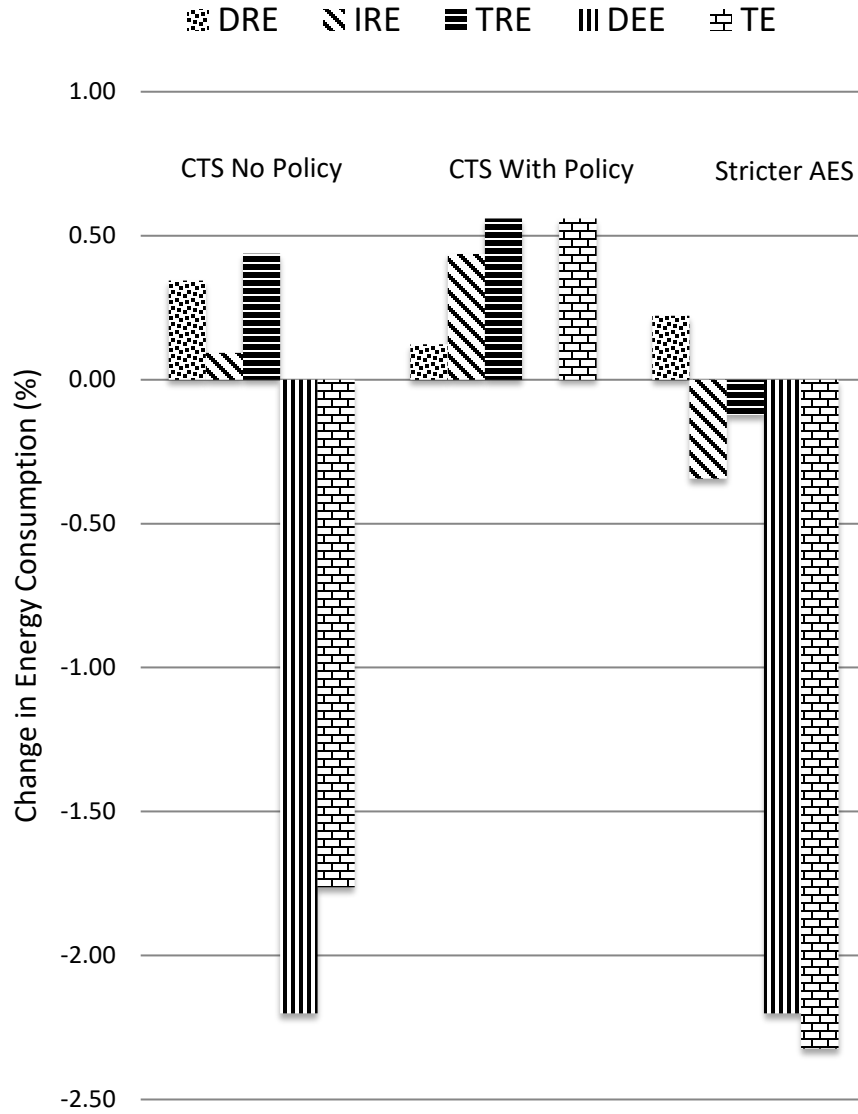
Effects on Energy Use from the CTS and CAFE Standards – Vehicles Only
(central parameter values)
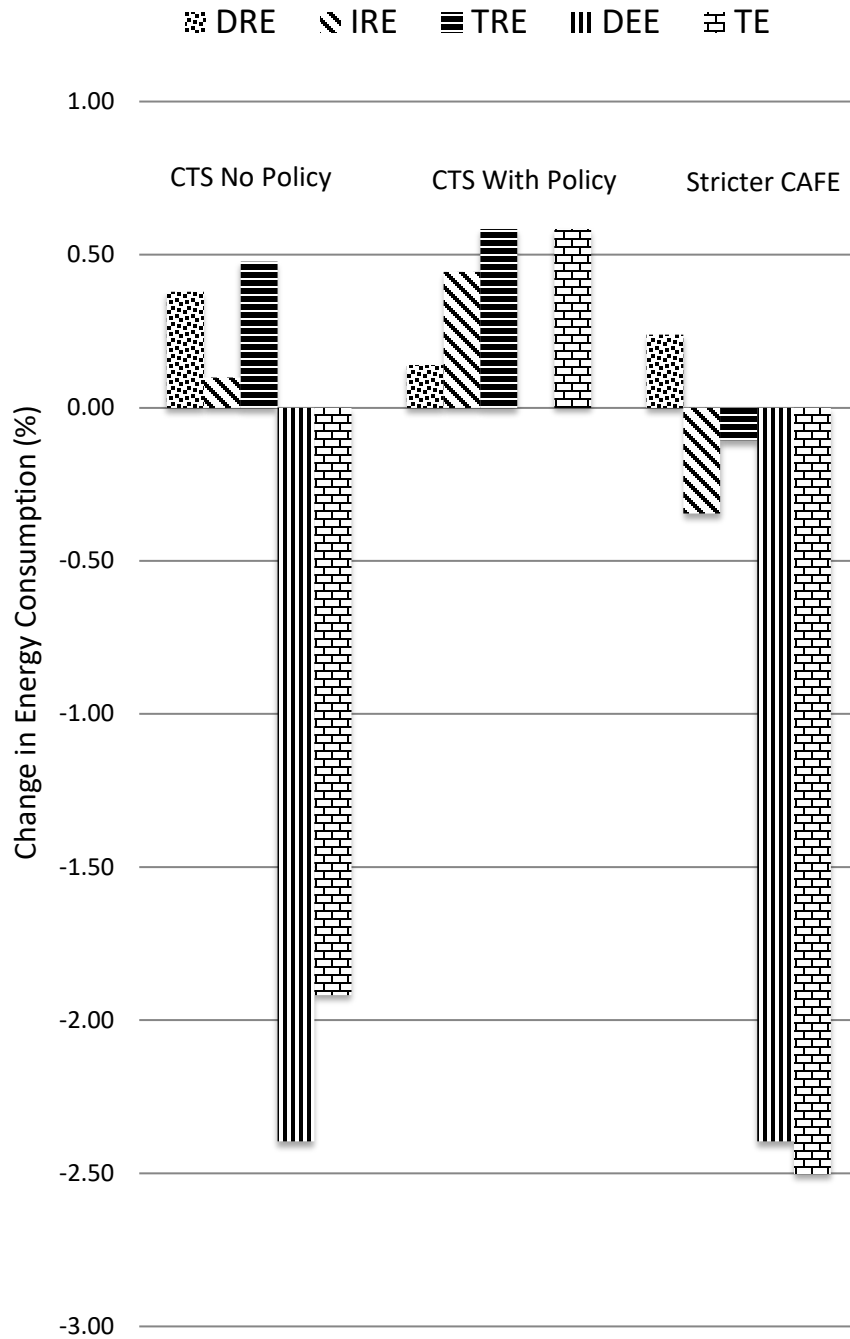
DRE: Direct Rebound Effect   DEE: Direct Efficiency Effect
IRE: Indirect Rebound Effect   TE: Total Energy
TRE: Total Rebound Effect

Effects on Energy Use from the CTS and EES – for Appliances other than Vehicles (central parameter values)

DRE: Direct Rebound Effect   DEE: Direct Efficiency Effect
IRE: Indirect Rebound Effect   TE: Total Energy
TRE: Total Rebound Effect