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Abstract

In a two-country economy we analyze how tax competition differs from the standard all-Nashian tax competition, if one or both countries are Kantians in Roemer's sense. Kantians are shown to choose a higher tax rate than Nashians for any given tax rate of the other country, which indicates that they seek to mitigate the (Nashian) race to the bottom. We avoid dealing with multiple equilibria by assuming that capital is sufficiently scarce, and we find for symmetric countries that the all-Kantian tax competition is efficient and that the inefficient race to the bottom is weakened in economies with a Nashian and a Kantian. That confirms the intuitive idea that countries following the Kantian categorical imperative avoid or at least soften the socially undesirable impact of (Nashian) self-interest. We also investigate the incentives of opportunistic countries to choose Nashian or Kantian behavior out of self-interest and find that either both governments choose to behave as Kantians or that - under different conditions - the robust Nashian selfinterest supersedes Kantian moral principles such that the inefficient all-Nashian tax competition results.

JEL-Codes: H730, H870, C720.

Keywords: tax competition, best reply, Kantian, Nashian, endogenous behavior selection.

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1 Introduction

In games of international tax competition, all countries usually exhibit 'Nashian behavior' in the sense that their best reply to the other countries' given tax rates is that tax rate which maximizes their welfare. The equilibrium is a state of mutual best replies (Nash equilibrium). In the standard model of capital tax competition that goes back to Zodrow and Mieszkowski (1986), Wilson (1986) and Wildasin (1988) tax competition leads to inefficiently low equilibrium tax rates (also known as race to the bottom) and to an underprovision of public goods. In this paper, we depart from the hypothesis of Nash behavior and assume that some countries follow a Kantian rule of behavior (to be explained below) when they compete for capital. For convenience, we refer to countries as Nashians, if they exhibit Nash behavior, and as Kantians, if they follow a Kantian rule. Our goal is to investigate how the tax competition differs from the standard all-Nashian competition, if some or all countries are Kantians. In addition, we wish to answer the question whether countries that are solely interested in welfare maximization would prefer to act as (if they were) a Nashian or a Kantian.

In contrast to purely self-interested Nashians, Kantians are guided by the moral principle underlying Kant's categorical imperative. In essence, that imperative calls on agents to take those actions and only those actions that they would advocate all others take as well. We follow Roemer (2010, 2015) who suggested a rule of behavior in the spirit of Kant's imperative that has already been applied to different contexts such as environmental problems, public and private good provision and income taxation by Gosh and van Long (2015), Grafton et al. (2017) and Roemer (2017). Roemer's Kantian rule says that an agent's current level of activity "... is morally appropriate if any scaling up or scaling down of that activity level by a factor $\lambda \neq 1$ would make her worse off, were everyone else to scale up or down their activity levels by the same proportion" (Gosh and van Long 2015, p.3). The analytically interesting and important consequence is that a Kantian in Roemer's sense gives a 'Kantian reply' to the other agents' given actions that is optimal (or 'best') from the Kantian's perspective, but differs from the welfare maximizing Nashian best reply. The main result of the above mentioned literature on Kantian economics is that Kantian behavior - unlike Nashian behavior - leads to efficiency in economies with negative or positive externalities.

To some extent, our motivation to investigate the implications of Kantian behavior for tax competition is the increasing evidence of empirical and experimental economic studies in recent years which suggest that the explanatory power of the paradigm of the self-interested homo oeconomicus is limited. As a response to that insight a strand of studies replaced 'selfish' by 'other-regarding' preferences and another line of literature assumes that ethical choices supersede utility maximizing choices.[1] Grafton et al. (2017) point out "... that analysis of Nash behavior may not be entirely appropriate in contexts where some subset of agents are aware of their responsibility toward provision of a public good." Roemer (2015) argues that Kantian behavior may be more appropriate for modelling (solidary) cooperative behavior than Nash behavior where agents optimize in an autarkic manner. In the area of international economic relations, the conclusions of the homo oeconomicus paradigm with regard to coordination or cooperation (free riding, race to the bottom, failure of international agreements) are arguably too pessimistic. There are many barriers to cooperation, in fact, but it is also true that there is a long list of successful cooperation in various fields of international economic relations that is difficult to explain as the work of agents driven by pure self-interest (Barrett 2003).

At the country level non-self-interested Nash behavior is applied in the context of foreign aid (Heinrich 2013), at international environmental agreements (van der Pol et al. 2012), and at redistributive tax competition (Wildasin 1991 and Weichenrieder and Busch 2007). In addition, in the tax competition literature there are different assumptions with regard to the governments' objective function. The standard assumption is that governments maximize the welfare of its representative consumer. Following the public choice literature Edwards and Keen (1996) consider Leviathan governments that maximize tax revenues. At income tax competition Hamilton and Pestieau (2005) suppose Rawlsian governments that maximize the welfare of the worst-off member of society. Finally, when a government mimicks the tax policy of other governments and cares about its performance relative to that of other governments, it maximizes its relative welfare, i.e. the difference between its own and other countries' welfare (Wagener 2013).

In the studies of Kantian economics we are aware of, the subjects of Kantian behavior have been assumed to be individuals. Here we take the view that Kantian behavior may also be relevant for governments. We assume identical consumers, as is common in the tax competition literature, and take the view that a Kantian government is one where the representative consumer is a Kantian.[2]

As already mentioned we wish to investigate how the outcome of tax competition

[1] The Kantian approach does not rely on the practice of behavioral economics to include "ad hoc" arguments into preferences (e.g. altruism or fairness) to model behavior that cannot be explained by self-interest.

[2] We follow the common procedure in the capital tax competition literature to consider countries with identical consumers and represent them by a single consumer.

changes, when we replace some or all Nashians by Kantians. The question to be answered is whether this modification of the tax competition model reduces or even eliminates the under-provision of public goods and the inefficiencies due to fiscal externalities which are the typical results in conventional studies on all-Nashian tax competition. In a two-country economy with simple parametric functions forms, we consider tax competition games that differ according to whether both countries are Nashians (which is then the conventional tax competition game), or both countries are Kantians, or one country is a Nashian and the other is a Kantian. We distinguish these games by referring to them as Nash-Nash game, Kant-Kant game, Kant-Nash game and Nash-Kant game. Correspondingly, an equilibrium of these games is called Nash-Nash equilibrium, Kant-Kant equilibrium, Kant-Nash equilibrium and Nash-Kant equilibrium, respectively.

As is well-known in the kind of model we apply, for a Nashian tax rates are strategic complements which means that its best reply curve is upward-sloping. A Kantian's best reply curve, that we derive from its (unconventional) optimization behavior, turns out to be also upward-sloping, but it is steeper than the Nashian best reply curve. Consequently, a Kantian chooses a higher tax rate than a Nashian for any given tax rate of the other country and thus works against the inefficient race to the bottom that characterizes the all-Nashian tax competition.

Next, we show that the Kant-Kant game has always multiple efficient equilibria characterized by a zero rate of return to capital, whereas the kind of equilibria of all other games crucially depends on parameters that determine the scarcity of capital. If capital is sufficiently scarce, there exists a unique equilibrium with a positive rate of return to capital in each game with at least one Nashian. If capital is sufficiently abundant, all these games[3] have multiple equilibria which are efficient and characterized by a zero rate of return to capital. Our prime focus is on capital scarcity. In this case two Kantians avoid the race to the bottom, and in the games with a Nashian and a Kantian the latter prevents the tax rates from falling as much as in the Nash-Nash game. Since Kant-Kant equilibria are efficient, our analysis reveals that Kantian behavior not only internalizes technological externalities as in Roemer (2010, 2015) but also fiscal externalities.

In order to provide an informative comparison of equilibrium allocations including tax rates and welfares, we restrict our attention to countries with identical characteristics and assume that capital is scarce enough to allow for unique equilibria in all games with at least one Nashian. Furthermore, we select that equilibrium out of the set of efficient Kant-Kant

[3]In these games the equilibrium is a state of mutually best replies.

equilibria which gives both Kantians the same welfare (and thus maximizes the aggregate welfare of both countries). As expected, the Nash-Nash equilibrium leads to the lowest tax rates. The highest tax rate is either that of the Kantian in the Kant-Nash equilibrium or that of the Kantians in the Kant-Kant equilibrium. The Kantian in the Kant-Nash equilibrium has the lowest welfare whereas either the Kantian in the Kant-Kant equilibrium or the Nashian in the Kant-Nash equilibrium has the highest welfare.

Finally, we endogenize Nashian and Kantian behavior by assuming that governments may choose to behave as Kantians or Nasians. This "behavior selection game" is similar in spirit to the timing game proposed by Hamilton and Slutsky (1990) and applied to tax competition by Kempf and Rota Graziosi (2010), Ogawa (2013) and Hindriks and Nishimura (2015). In our behavior selection game the countries' strategies are Nashian behavior and Kantian behavior and their payoffs are the unique equilibrium welfares of the four tax competition games discussed above.[4] We get a subgame in the form of a 2×2 matrix and solving it yields two subgame perfect (SP) equilibria on some subset of economies and a single SP equilibrium in the complementary subset. The two SP equilibria are the Kant-Kant equilibrium and the Nash-Nash equilibrium. Clearly, since the Kant-Kant equilibrium Pareto dominates the Nash-Nash equilibrium we select the former which means that both countries choose to be Kantians out of self-interest. In the complementary subset of economies, the unique SP equilibrium of the behavior selection game is the Nash-Nash equilibrium. In this case, we end up in a prisoners' dilemma. Countries choose to behave as Nashians although each would benefit when both would switch to Kantian behavior.

The paper is organized as follows. Section 2 introduces the model and characterizes the allocation that is efficient under the constraint of financing the public good via capital taxes. Sections 3.1-3.3 analyze uniqueness, multiplicity and (in)efficiency of equilibria in the games with two Nashians, two Kantians, and a Nashian and a Kantian. Section 3.4 compares these games with regard to tax rates, public good provision and welfare levels in the respective equilibria. Section 4 turns to the behavior selection game and studies its SP equilibria. Section 5 concludes.

[4]Here we refer to the unique equilibria of the Kant-Kant game, the Kant-Nash game, the Nash-Kant game and the Nash-Nash game under the assumptions of symmetry and the symmetric equilibrium of the Kant-Kant game.

2 The analytical framework

Our analysis builds on the model of capital tax competition (Zodrow and Mieszkowski 1986, Wilson 1986 and Wildasin 1988) that we will briefly outline here using simple parametric functional forms.[5]

2.1 The model

Consider an economy consisting of two countries, denoted countries i and j. Country i hosts a representative firm that employs k_i units of capital to produce good X according to the production function[6]

$$X^i(k_i) = a_i k_i - \frac{b}{2} k_i^2. \tag{1}$$

Good X can be used either as a private good or as government-provided local public good. The utility of the representative consumer of country i is[7]

$$u_i = x_i + (1+\varepsilon) g_i, \tag{2}$$

where x_i and g_i denote consumption of private and public goods,[8] respectively, and where $\varepsilon \geq 0$ is a parameter reflecting the preference intensity for the public good. There are competitive world markets for capital and for good X. The price of good X is normalized to one and capital is priced at the rate of return on capital, r. Equilibrium on the capital market requires

$$k_i + k_j = 2\bar{k} \tag{3}$$

[5] For an excellent survey of this literature see Keen and Konrad (2013).

[6] To ease the notation, we apply the following convention. If there is a formula, in which only the index 'i' appears, as e.g. in (1), the model contains the same formula with all indexes i replaced by j. To avoid clutter, we do not write down that second formula with indexes 'j', however. Correspondingly, in addition to each formula, in which the two indexes 'i' and 'j' appear, there exists the same formula with all indexes i and j interchanged.

[7] We choose the restrictive functional forms (1) and (2) for reasons of tractability. Production functions of type (1) and/or utility functions of type (2) are employed among others by Keen and Lahiri (1998), Bucovetsky (2009), Kempf and Rota-Graziosi (2010), Ogawa (2013) and Hindriks and Nishimura (2015).

[8] Due to our representative-consumer assumption, there is no analytical difference between a local public good and a publicly provided private good.

with $\bar{k}$ being each country's consumer-owned capital endowment. Throughout the paper we restrict the parameters to[9]

$$a_i \geq a_j > 2b\bar{k} \quad \text{and} \quad \varepsilon > 0. \tag{4}$$

The government of country i levies a tax at rate t_i on capital input of its firm and uses the tax revenue to provide the local public good

$$g_i = t_i k_i. \tag{5}$$

Taking the rate of return on capital r and the tax rate t_i as given, the producer of country i maximizes profits $X^i(k_i) - (r + t_i)k_i$. Combined with (3) the resulting first-order conditions yield

$$r = R(t_i, t_j) := \frac{\alpha}{2} - \frac{t_i + t_j}{2} \geq 0 \quad \text{and} \quad k_i = K^i(t_i, t_j) := \frac{2a_i - \alpha}{2b} + \frac{t_j - t_i}{2b} \geq 0, \tag{6}$$

where $\alpha := a_i + a_j - 2b\bar{k} > 0$ and $2a_i - \alpha > 0$ due to (4). In the absence of taxation, the equilibrium price of capital is $r = \frac{\alpha}{2}$. Taking that price as an indicator of capital scarcity (in the two-country economy), it is obvious from the definition of α, that the scarcity of capital is increasing in the productivity parameters a_i and a_j and decreasing in capital abundance. Capital scarcity as measured by α will turn out to play a crucial role in our subsequent analysis.

The representative consumer's total income consists of profit income $X^i(k_i) - (r + t_i)k_i$ and capital income $r\bar{k}$. All income is spent on private consumption x_i,

$$x_i = X^i(k_i) - t_i k_i + r(\bar{k} - k_i) \geq 0. \tag{7}$$

In view of the equations (3), (5) and (7), the economy's aggregate demand for final goods equals aggregate supply (Walras Law):[10]

$$\sum_h x_h + \sum_h g_h = \sum_h X^h(k_h) \tag{8}$$

We conclude that for every tuple (t_i, t_j) there exists a unique competitive equilibrium in which country i's welfare is given by

$$W^i(t_i, t_j) = X^i(k_i) + r\left(\bar{k} - k_i\right) + \varepsilon t_i k_i, \tag{9}$$

where $k_i = K^i(t_i, t_j)$ and $r = R(t_i, t_j)$ are defined in (6).

[9] The condition $a_i > 2b\bar{k}$ ensures that the marginal productivity $X^i_{k_i}(k_i)$ is always positive even if all capital is employed in one country. The inequality $a_i \geq a_j$ serves to fix ideas without restricting generality. $a_i \geq a_j$ is the only exception to the rule introduced in footnote 6. Throughout the paper we assume that country i is equally productive as or more productive than country j.

[10] $\sum_h z_h$ is short for $z_i + z_j$.

2.2 The efficiency benchmark

The governments pay for the local public goods they provide with the revenue of the distortionary capital tax. It is therefore clear that first-best is unattainable. We will apply the Pareto criterion to identify second-best allocations under the condition of capital tax finance as follows. Given that the local public goods are capital-tax financed, an allocation of the two-country economy with tax rates (t_i, t_j) is said to be Pareto efficient, if there are no tax rates different from (t_i, t_j) which increase the utility of one country's representative consumer without reducing the utility of the other country's representative consumer. To avoid clumsy wording, we will denote Pareto efficient second-best allocations simply as efficient allocations.

In the analysis below we will have to cope with corner solutions that relate to the non-negativity constraints[11] $k_i \geq 0$, $x_i \geq 0$ and/or $r \geq 0$. The extreme case $k_i = 0$ and $k_j = \bar{k}$ does not occur, unless productivities would strongly differ across countries. Since we will focus on relatively small differences in productivities, we need not formally deal with the $k_i = 0$ constraint (although we check in our numerical analysis that capital is used in both countries). Interestingly, $x_i = 0$ is first best with lumpsum taxation, if the preference parameter ε is positive. However, with capital tax finance $x_i = 0$ will not be reached, because increasing the provision of public goods requires increasing tax rates. This turns out to decrease the equilibrium rate of return on capital so strongly, that the boundary $r = 0$ is reached before governments have bought up all output for public good provision. It suffices, therefore, to account for the non-negativity constraint $r \geq 0$. In our model, that constraint is not made for analytical convenience. It is necessary, because capital owners would rather leave their capital idle than pay for offering it to producers at a negative price.

In the two-country market economy with capital taxation, a tuple of tax rates (t_i, t_j) generates an efficient allocation, if it is impossible to increase one country's welfare through variations of tax rates without reducing the other country's welfare. In order to identify such efficient tax rates, we invoke (6) and (9) and consider the derivatives of the welfare functions W^i and W^j,[12]

$$W^i_{t_i} = \frac{(1+2\varepsilon)(a_i - a_j) + 4b\varepsilon\bar{k}}{4b} - \frac{3+4\varepsilon}{4b}t_i + \frac{1+2\varepsilon}{4b}t_j = 0, \tag{10}$$

$$W^i_{t_j} = \frac{(a_i - a_j}{4b} - \frac{1+2\varepsilon}{4b}t_i + \frac{1}{4b}t_j = 0. \tag{11}$$

[11]Non-negativity constraints are also taken into account by e.g. Bucovetsky (2009) and Kempf and Rota-Graziosi (2010).

[12]The derivation of (10) and (11) can be found in the Appendix.

Since $W^i_{t_it_i} < 0$ and $W^i_{t_jt_j} > 0$, function W^i has no maximum. Hence, the $r \geq 0$ constraint is binding, i.e. the maximum feasible welfare is reached if the tax rates satisfy the equation

$$t_i + t_j = \alpha. \tag{12}$$

We account for (12) by considering the welfare functions $\tilde{W}^i$ and $\tilde{W}^j$ defined by

$$\tilde{W}^i(t_i) := W^i(t_i, \alpha - t_i) \quad \text{and} \quad \tilde{W}^j(t_i) := W^j(t_i, \alpha - t_i). \tag{13}$$

The Appendix shows that the first derivatives of the functions $\tilde{W}^i$ and $\tilde{W}^j$ are

$$\tilde{W}^i_{t_i} = \frac{a_i\varepsilon}{b} - \frac{1+2\varepsilon}{b}t_i \quad \text{and} \quad \tilde{W}^j_{t_i} = \frac{(1+2\varepsilon)\alpha - a_j\varepsilon}{b} - \frac{1+2\varepsilon}{b}t_i. \tag{14}$$

We infer from (14) that $\tilde{W}^i$ and $\tilde{W}^j$ are strictly concave. Their respective maximizers are

$$\operatorname{argmax} \tilde{W}^i(t_i) = \frac{a_i\varepsilon}{1+2\varepsilon} =: \underline{t}_i^* \quad \text{and} \quad \operatorname{argmax} \tilde{W}^j(t_i) = \alpha - \frac{a_j\varepsilon}{1+2\varepsilon} =: \overline{t}_i^* \tag{15}$$

Since $a_i = \alpha - (a_j - 2b\bar{k}) < \alpha$ due to (4), we conclude from the definition of α that $\overline{t}_i^* \in \left[0, \alpha\right]$, $\underline{t}_i^* \in \left[0, \alpha\right]$, and $\overline{t}_i^* > \underline{t}_i^*$. The tax rates $\underline{t}_i^*$ and $\overline{t}_i^*$ partition the interval of feasible capital tax rates, $\left[0, \alpha\right]$, into three sub-intervals: $\left[0, \underline{t}_i^*\right[$, $\left[\underline{t}_i^*, \overline{t}_i^*\right]$, and $\left]\overline{t}_i^*, \alpha\right]$. On the interval $\left[0, \underline{t}_i^*\right[$ both $\tilde{W}^i(t_i)$ and $\tilde{W}^j(t_i)$ are increasing in t_i and on the interval $\left]\overline{t}_i^*, \alpha\right]$ both $\tilde{W}^i(t_i)$ and $\tilde{W}^j(t_i)$ are decreasing in t_i. On the intermediate interval $\left[\underline{t}_i^*, \overline{t}_i^*\right]$ $\tilde{W}^i(t_i)$ is increasing and $\tilde{W}^j(t_i)$ is decreasing in t_i such that it is impossible to increase the welfare of one country without reducing the other country's welfare by variations of t_i within that interval. Thus, we established

Proposition 1 .

(i) The competitive market equilibrium with capital-tax financed public goods is efficient, if and only if the tax rates $\left(t_i^*, t_j^*\right)$ *satisfy* $\alpha - t_i^* - t_j^* = 0$ *and* $t_i^* \in \left[\underline{t}_i^*, \overline{t}_i^*\right]$.

(ii) If $(t_{i0}^*, \alpha - t_{i0}^*)$ *and* $(t_{i1}^*, \alpha - t_{i1}^*)$ *are two pairs of efficient tax rates such that* $t_{i1}^* > t_{i0}^*$, *then* $W^i\left(t_{i1}^*, \alpha - t_{i1}^*\right) > W^i\left(t_{i0}^*, \alpha - t_{i0}^*\right)$ *and* $W^j\left(t_{i1}^*, \alpha - t_{i1}^*\right) < W^j\left(t_{i0}^*, \alpha - t_{i0}^*\right)$.

Proposition 1 augments the finding of Bucovetsky (2009) that the efficient tax rate is restricted by the $r = 0$ constraint. While Bucovetsky assumes that the efficient tax rate is identical for all countries, we show that there are multiple efficient equilibria that differ with respect to the distribution of welfares. According to Proposition 1(ii) the country that sets a higher efficient tax rate achieves a higher welfare level. There are two partial effects that underly Proposition 1(ii). Increasing country i's tax rate reduces investments and increases tax revenues for public good provision in country i. The welfare-enhancing latter effect overcompensates the welfare-reducing former effect.

3 Tax competition with Nashian and/or Kantian governments

The standard procedure of modeling capital tax competition is to assume that for every given (feasible) tax rate t_j country i chooses that tax rate t_i which maximizes its welfare (= payoff) $W^i(t_i, t_j)$. Following Grafton et al. (2017), we refer to those countries as Nashians which exhibit conventional best-reply behavior in order to distinguish them from Kantians (Roemer 2010) whose behavior we will specify below in Section 3.2. We will combine Kantians and Nashians to obtain four two-country games with either two Nashians, or two Kantians, or a Nashian and a Kantian. We begin with the conventional tax competition game of two Nashians, which we denote as Nash-Nash game to distinguish it from the other three games.

3.1 Nash-Nash game

The game in which the countries i and j are Nashians is the conventional game of tax competition. The solution of that game, here denoted as Nash-Nash equilibrium, is the tuple of tax rates that constitute the mutually best replies. In case of an interior solution the Nash-Nash equilibrium is implicitly determined by the first-order conditions $W^i_{t_i} = 0$ from (10). Solving these first-order conditions yields the best-reply functions

$$t_i = T^{Ni}(t_j) := \frac{4b\varepsilon\bar{k} + (1+2\varepsilon)(a_i - a_j)}{3+4\varepsilon} + \frac{1+2\varepsilon}{3+4\varepsilon} t_j, \tag{16}$$

that demonstrate the well-known Nash response to an increase in the foreign capital tax. If t_j increases by one unit, country i responds by increasing its tax rate by the fraction $\frac{1+2\varepsilon}{3+4\varepsilon} < 1$ of a unit and thus increases the public good provision $g_i = t_i k_i$ via $\Delta t_i > 0$ and via $\Delta k_i = \frac{1}{2b}(\Delta t_j - \Delta t_i) = \frac{1}{2b}\left(1 - \frac{1+2\varepsilon}{3+4\varepsilon}\right) = \frac{1+\varepsilon}{b(3+4\varepsilon)}$.

Since $T^{Ni}(0) > 0$, $T^{Nj}(0) > 0$ and $T^{Ni}_{t_i} = T^{Nj}_{t_j} = \frac{1+2\varepsilon}{3+4\varepsilon}$, the Nashians' best supply curves have a single point of intersection, which we denote by $\left(t_i^{nn}, t_j^{nn}\right)$. Solving the two equations in (16) yields

$$t_i^{nn} := \frac{2b\varepsilon\bar{k}}{1+\varepsilon} + \frac{1+2\varepsilon(a_i - a_j)}{2(2+3\varepsilon)} \quad \text{and} \quad t_j^{nn} := \frac{2b\varepsilon\bar{k}}{1+\varepsilon} - \frac{1+2\varepsilon(a_i - a_j)}{2(2+3\varepsilon)}. \tag{17}$$

The conclusion is near at hand that $\left(t_i^{nn}, t_j^{nn}\right)$ is the unique Nash-Nash equilibrium. But that is unclear because so far we have ignored the rate of return constraint $r \geq 0$. If we take that constraint into account, the complete Nashian best reply functions consist of two different parts

$$t_i = T^{Ni}(t_j) \text{ for all } t_j \in \left[0, \underline{t}_j^n\right] \quad \text{and} \quad t_i = \alpha - t_j \text{ for all } t_j \in \left[\underline{t}_j^n, \alpha\right],$$
$$t_j = T^{Nj}(t_i) \text{ for all } t_i \in \left[0, \underline{t}_i^n\right] \quad \text{and} \quad t_j = \alpha - t_i \text{ for all } t_i \in \left[\underline{t}_i^n, \alpha\right], \tag{18}$$

where the tuple $\left(\bar{t}_i^n, \underline{t}_j^n\right)$ is defined by $\bar{t}_i^n = \alpha - \underline{t}_j^n = T^{Ni}\left(\underline{t}_j^n\right)$ and the tuple $\left(\bar{t}_j^n, \underline{t}_i^n\right)$ is defined by $\bar{t}_j^n = \alpha - \underline{t}_i^n = T^{Nj}\left(\underline{t}_i^n\right)$. Graphically, $\left(\bar{t}_i^n, \underline{t}_j^n\right)$ and $\left(\bar{t}_j^n, \underline{t}_i^n\right)$ are the intersection points of the $r = 0$ line with the T^{Ni} curve and the T^{Nj} curve, respectively. We establish in the Appendix

Proposition 2 .

(i) If $\alpha \geq \frac{4b\varepsilon\bar{k}}{1+\varepsilon}$, the tuple $\left(t_i^{nn}, t_j^{nn}\right)$ (as defined in (17)) is the unique Nash-Nash equilibrium. That equilibrium is inefficient.

(ii) If $\alpha < \frac{4b\varepsilon\bar{k}}{1+\varepsilon}$, there exist multiple Nash-Nash equilibria characterized by $(t_i, t_j = \alpha - t_i)$ and $t_i \in \left[\underline{t}_i^n, \bar{t}_i^n\right] \neq \emptyset$. All Nash-Nash equilibria are efficient.

The existence of a unique Nash-Nash equilibrium with a positive rate of return and of multiple Nash-Nash equilibria with a zero rate of return have been proved for symmetric countries under more general assumptions on production functions by Laussel and Le Breton (1998). Proposition 2 provides the additional information that unique Nash-Nash equilibria are inefficient and perhaps more surprisingly that multiple Nash-Nash equilibria are efficient.

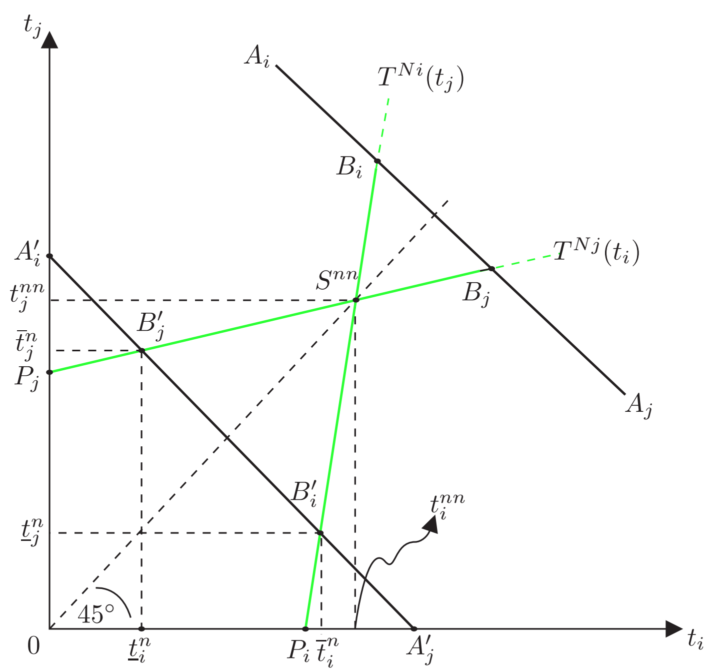

Figure 1: Nash-Nash equilibria

Figure 1 illustrates Proposition 2 for symmetric countries ($a_i = a_j$). The graphs of the functions T^{Ni} and T^{Nj} from (16) are the positively sloped green lines in Figure 1 that intersect in point S^{nn}, whose coordinates are $\left(t_i^{nn}, t_j^{nn}\right)$. For the time being we disregard the negatively sloped straight line $A_i'A_j'$ in Figure 1, and suppose the line A_iA_j represents the $r = 0$ line (12). Then i's and j's best replies (18) are illustrated by the kinked lines $P_iB_iA_i$ and $P_jB_jA_j$, respectively. Since point S^{nn} lies below the A_iA_j line we know that $t_i^{nn} + t_j^{nn} < \alpha$ and hence S^{nn} represents the unique Nash-Nash equilibrium according to Proposition 2(i). Next we disregard the A_iA_j line and assume the $r = 0$ line is given by $A_i'A_j'$. Obviously, the A_iA_j line and $A_i'A_j'$ line differ in that α is smaller in the latter than in the former. The best reply functions (18) are now illustrated by $P_iB_i'A_i'$ for i and by $P_jB_j'A_j'$ for j such that the point S^{nn} has become unattainable. If they exist, Nash-Nash equilibria must be points on the $A_i'A_j'$ line in Figure 1. To illustrate them we consider the interval $\left[0, \alpha\right]$ of feasible tax rates t_i on the t_i axis. That interval is partitioned into the subintervals $\left[0, \underline{t}_i^n\right[$, $\left[\underline{t}_i^n, \overline{t}_i^n\right]$, and $\left]\overline{t}_i^n, \alpha\right]$. On the interval $\left[0, \underline{t}_i^n\right[$, i's best reply is at the $r = 0$ boundary, but j's best reply is not. On the interval $\left]\overline{t}_i^n, \alpha\right]$, j's best reply is at the boundary, but i's best reply is not. There cannot be a Nash-Nash equilibrium on the intervals $\left[0, \underline{t}_i^n\right[$ and $\left]\overline{t}_i^n, \alpha\right]$, because the $r \geq 0$ constraint cannot be binding and non-binding at the same time. On the interval $\left[\underline{t}_i^n, \overline{t}_i^n\right]$, the best replies of both i and j are at the boundary, because the $r = 0$ branches of both Nashians' best reply curves coincide on the segment $B_i'B_j'$ of the $A_i'A_j'$ line. Therefore, each point on the segment $B_i'B_j'$ represents a Nash-Nash equilibrium or - as equivalently stated in Proposition 2(ii) - the tuples $(t_i, t_j = \alpha - t_i)$ define a Nash-Nash equilibrium for every $t_i \in \left[\underline{t}_i^n, \overline{t}_i^n\right] \neq \emptyset$.

Figure 1 illustrates the case of symmetry ($a_i = a_j$). If $a_i > a_j$, the slopes of the functions T^{Ni} and T^{Nj} remain unchanged, but the point $P_j[P_i]$ in Figure 1 moves closer to [farther away from] the origin such that the intersection point S^{nn} moves below the 45° line. The coordinates of the new point S^{nn} satisfy $t_i^{nn} > t_j^{nn}$, which is in line with (17).

3.2 Kant-Kant game

As noted in the introduction, we will apply Roemer's concept of Kantian optimization to governments that are engaged in tax competition.[13] Roemer defines Kantian behavior with

[13]Rather than attempting to clarify the relation between Roemer's concept of Kantian optimization and informal notions of the categorical imperative (see e.g. Elster 2017), we provide a non-technical explanation of that concept. Roemer (2017) considers his approach as a proposal of "how cooperation of economic agents

respect to comparing the present with a certain class of counterfactual alternatives. A player's conventional Nash counterfactual describes the payoffs she would receive if she would deviate while all other players do not. In contrast, its Kantian counterfactual describes the payoff she would get if she would deviate, while all other agents would deviate likewise (Grafton et al. 2017). More specifically, given the tax rates of all other governments, the best reply of a Kantian government is a tax rate yielding a payoff (welfare) no smaller than the payoff it would receive upon a deviation assuming all other players would deviate likewise.

The key to understanding the race to the bottom in the Nash-Nash game is the Nash counterfactual. If a government considers reducing the tax rate and assumes that all other governments' tax rates remain unchanged, then it benefits by attracting foreign capital. However, if the country realizes that it benefits at the expense of the other countries and disapproves of that on moral grounds, it may want to choose its best-reply tax rate based on the Kantian counterfactual. Following Roemer's concept, it may then choose that particular tax rate which satisfies the condition that any scaling up or scaling down of the tax rate by a factor unequal to one would make it worse off, were everyone else to scale up or down their tax rates levels by the same proportion. This Kantian rule is formalized as follows. A Kantian (government) i with welfare $W^i(t_i, t_j)$ chooses that tax rate t_i for any given (feasible) tax rate t_j, which satisfies

$$W^i(\lambda t_i, \lambda t_j) \leq W^i(t_i, t_j) \quad \text{for all } \lambda \geq 0. \tag{19}$$

Although our discussion of efficiency showed that we have to be aware of the $r \geq 0$ constraint we begin with assuming that this constraint is not binding. Accordingly, we maximize with respect to λ the welfare

$$W^i(\lambda t_i, \lambda t_j) = X^i(k_i) + r(\bar{k} - k_i) + \varepsilon \lambda t_i k_i \tag{20}$$

subject to $k_i = K^i(\lambda t_i, \lambda t_j) = \bar{k} + \frac{a_i - a_j + \lambda(t_i - t_j)}{2b}$ and $R(\lambda t_i, \lambda t_j) = \frac{\alpha - \lambda(t_i + t_j)}{2}$. We show in the Appendix that

$$W^i_\lambda(\lambda t_i, \lambda t_j) = \frac{[(3+4\varepsilon)t_i + t_j](t_j - t_i)\lambda + (a_i - a_j)t_j + [4b\varepsilon\bar{k} + (1+2\varepsilon)(a_i - a_j)]t_i}{4b}. \tag{21}$$

Following the Roemer rule, we set $\lambda = 1$ in (21) and prove in the Appendix that solving $W^i_\lambda(\lambda t_i, \lambda t_j)$ for t_i yields

$$t_i = T^{Ki}(t_j) := \frac{T^{Ni}(t_j)}{2} + \frac{(1+\varepsilon)t_j}{2(3+4\varepsilon)} + \sqrt{\left[\frac{T^{Ni}(t_j)}{2} + \frac{(1+\varepsilon)t_j}{2(3+4\varepsilon)}\right]^2 + \frac{(t_j + a_i - a_j)t_j}{3+4\varepsilon}}. \tag{22}$$

can be formalized as a mathematical first cousin of Nash optimization."

Equation (22) is Kantian i's best reply function $t_i = T^{Ki}(t_j)$ under the condition that the $r \geq 0$ constraint is not strictly binding. As long as $T^{Ki}(t_j) + t_j \leq \alpha$, Kantian i responds to the given tax rate t_j with a much higher tax rate t_i than if country i were a Nashian. This is exactly what one would expect. Being a Kantian, country i takes into account that competing down the tax rates - as Nashians do - deteriorates the public good provision in both countries and hence Kantian i resists giving in to self-interest which would lead it to choose the lower tax rate $t_i = T^{Ni}(t_j)$. In the Appendix we characterize the functions T^{Ki} and T^{Kj} as follows. Under the condition $2b\varepsilon\bar{k} > (a_i - a_j)(1+\varepsilon)$ the functions T^{Ki} and T^{Kj} are convex and they satisfy $T^{Ki}(t_j) > t_j$ for all $t_j \geq 0$ and $T^{Kj}(t_i) > t_i$ for all $t_i \geq 0$.[14] Consequently, there is no point of intersection with non-negative coordinates, i.e. there does not exist an internal Kant-Kant equilibrium - no matter how small or large the capital-scarcity parameter α is. In order to specify Kant-Kant equilibria at the $r = 0$ boundary, we proceed as described above in the Nash-Nash game. Analogous to (18) Kantian i's best reply consists of two parts on the domain $\left[0, \alpha\right]$. It is equal to

$$t_i = T^{Ki}(t_j) \text{ for all } t_j \in \left[0, \underline{t}_j^k\right] \quad \text{and} \quad t_i = \alpha - t_j \text{ for all } t_j \in \left[\underline{t}_j^k, \alpha\right], \tag{23}$$

where $\left(\bar{t}_i^k, \underline{t}_j^k\right)$ is the tuple satisfying $\bar{t}_i^k = \alpha - \underline{t}_j^k = T^{Ki}\left(\underline{t}_j^k\right)$.[15] Next we verify that the domain of Kantian j's best reply function is partitioned into the three subintervals $\left[0, \underline{t}_i^k\right[$, $\left[\underline{t}_i^k, \bar{t}_i^k\right]$ and $\left]\bar{t}_i^k, \alpha\right]$. There is no common point of the best reply functions in the subintervals $\left[0, \underline{t}_i^k\right[$ and $\left]\bar{t}_i^k, \alpha\right]$, because in these intervals one Kantian's best reply is constrained, but the other's is not. In the intermediate sub-interval $\left[\underline{t}_i^k, \bar{t}_i^k\right]$ both Kantians' best replies are subject to the $r = 0$ constraint such that the tax rates $(t_i, t_j = \alpha - t_i)$ define a Kant-Kant equilibrium for every $t_i \in \left[\underline{t}_i^k, \bar{t}_i^k\right]$. We summarize our findings in

Proposition 3. *If $2b\varepsilon\bar{k} > (a_i - a_j)(1+\varepsilon)$, there exist multiple Kant-Kant equilibria characterized by $(t_i, t_j = \alpha - t_i)$ and $t_i \in \left[\underline{t}_i^k, \bar{t}_i^k\right] \neq \emptyset$. All Kant-Kant equilibria are efficient.*

According to Proposition 3 all Kant-Kant equilibria are efficient in the sense that they eliminate the fiscal externalities which render inefficient the equilibria in conventional capital-tax competition games. Hence in our model the Kant-Kant equilibria implement the second-best, whereas the applications of Kantian economics we are aware of, e.g. to the fishery or the provision of public goods (Roemer 2010), usually focus on internalizing technological externalities.

[14] $2b\varepsilon\bar{k} > (a_i - a_j)(1+\varepsilon)$ is satisfied for identical countries and also for asymmetric countries as long as the productivity advantage of country i over j is not too large.

[15] Keep in mind that (23) also holds for Kantian j after interchanging all subscripts i and j.

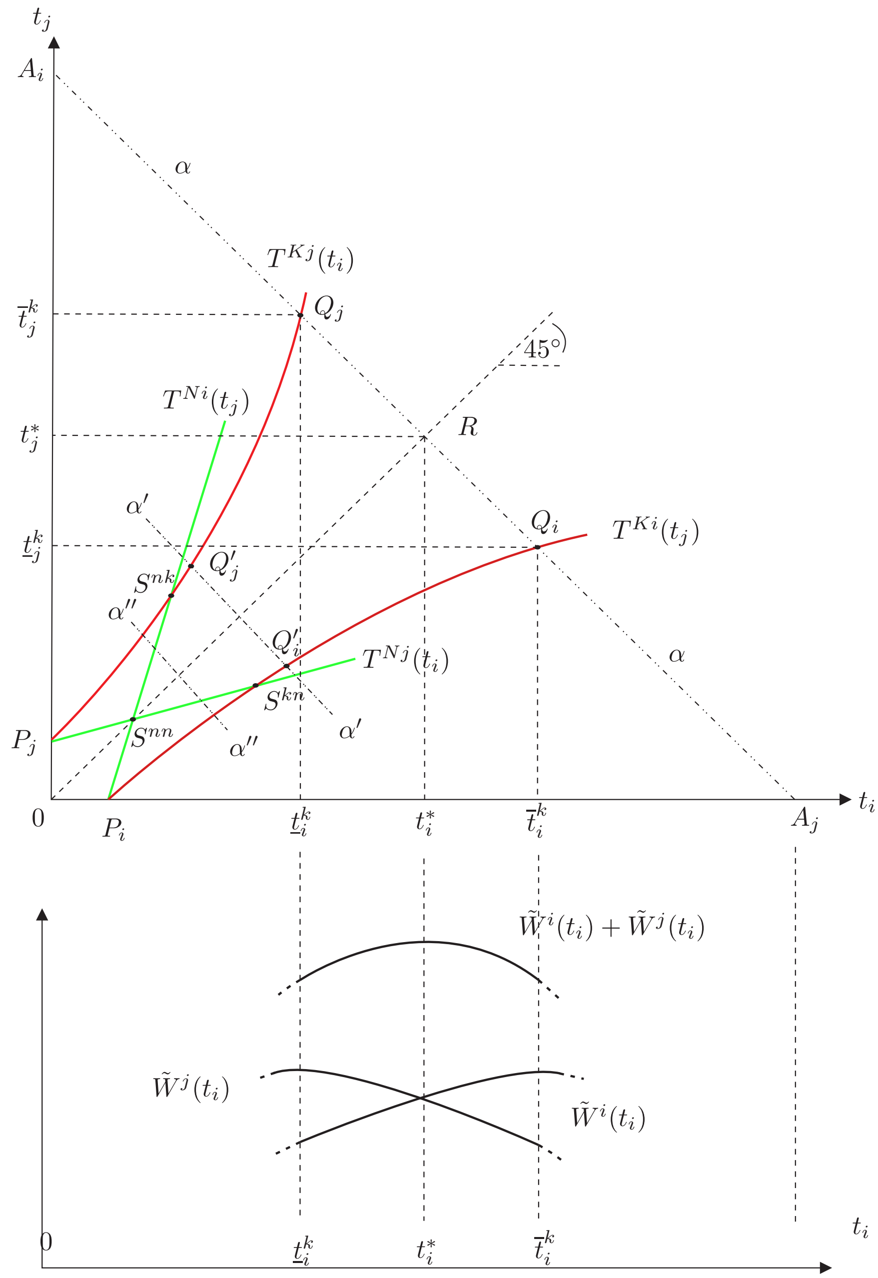

Figure 2: Kant-Kant equilibria

Figure 2 illustrates Proposition 3 for symmetric countries ($a_i = a_j$). We consider first the upper panel and discuss its contents step by step. For the time being, we disregard the negatively sloped straight lines denoted $\alpha\alpha$, $\alpha'\alpha'$ and $\alpha''\alpha''$, and thus implicitly assume that capital is so scarce that the $r = 0$ line is in a great distance from the origin of Figure 2. The positively sloped green lines reproduce the Nashian best reply curves from Figure 1 with their point of intersection S^{nn}, which now represents the unique interior Nash-Nash equilibrium - as described in Figure 1.

Next we focus on the countries i and j as Kantians. In Figure 2 the graphs of the best reply functions (22) are given by the curves $P_iQ_i'Q_i$ and $P_jQ_j'Q_j$, respectively. Unlike the Nashians' best reply curves T^{Ni} and T^{Nj}, the Kantians' curves T^{Ki} and T^{Kj} have no point of intersection at non-negative tax rates, which is obvious in Figure 2 by the observation (proved in the Appendix) that one curve lies completely above and the other completely below the 45° line. In order to identify in Figure 2 the Kant-Kant equilibria specified in Proposition 3, we now reintroduce in Figure 2 the straight $\alpha\alpha$ line (while we still disregard the $\alpha'\alpha'$ and $\alpha''\alpha''$ lines) defined as the locus of all tuples (t_i, t_j) satisfying $t_i + t_j = \alpha$. With this $r = 0$ line, the complete Kantian best reply curves (23) are given by the red lines $P_iQ_i'Q_iA_i$ and $P_jQ_j'Q_jA_j$, respectively. It follows immediately, that these best reply curves have all points in common on the line segment Q_jQ_i in Figure 2. Analogous to the procedure we applied in the Nash-Nash game, we identify the Kant-Kant equilibria in Figure 2 by considering the interval $\left[0, \alpha\right]$ of feasible tax rates on the t_i axis that is partitioned into the subintervals $\left[0, \underline{t}_i^k\right[$, $\left[\underline{t}_i^k, \overline{t}_i^k\right]$, and $\left]\overline{t}_i^k, \alpha\right]$. We conclude that there is no Kant-Kant equilibrium on the intervals $\left[0, \underline{t}_i^k\right[$ and $\left]\overline{t}_i^k, \alpha\right]$, because the rate of return constraint $r \geq 0$ cannot be binding and non-binding at the same time. Hence $(t_i, t_j = \alpha - t_i)$ is a Kant-Kant equilibrium for every $t_i \in \left[\underline{t}_i^k, \overline{t}_i^k\right]$ - as stated in Proposition 3.

The lower panel of Figure 2 illustrates the efficiency property of Kant-Kant equilibria. To see that consider the welfare functions $\tilde{W}^i(t_i)$ and $\tilde{W}^j(t_i)$ defined in (13). Clearly, function $\tilde{W}^i(t_i)$ $[\tilde{W}^j(t_i)]$ is Kantian i's [Kantian j's] welfare if $R(t_i, t_j) = 0$, i.e. if we move along the $\alpha\alpha$ line from $t_i = 0$ to $t_i = \alpha$. The lower panel of Figure 2 shows the corresponding welfare curves. The graph of $\tilde{W}^j$ $[\tilde{W}^i]$ attains its unique maximum at $\underline{t}_i^k$ $[\overline{t}_i^k]$. Consequently, all Kant-Kant equilibria differ with respect to the distribution of welfare across countries. On the interval $[\underline{t}_i^k, \overline{t}_i^k]$, $\tilde{W}^i(t_i)$ is strictly increasing and $\tilde{W}^j(t_i)$ is strictly decreasing in t_i. In addition to the welfares $\tilde{W}^i$ and $\tilde{W}^j$ the lower panel of Figure 2 shows the graph of the aggregate welfare function $\tilde{W}^i + \tilde{W}^j$. Under the assumption of symmetry (made in Figure 2), that graph has a unique maximum at the efficient tax rate $t_i^* = \frac{\underline{t}_i^k + \overline{t}_i^k}{2} \in \left[\underline{t}_i^k, \overline{t}_i^k\right]$. Hence one of the Kant-Kant equilibria is not only efficient, as all Kant-Kant equilibria are, but also maximizes the aggregate welfare of Kantians i and j.

3.3 Games with a Kantian and a Nashian

For convenience of notation, we refer to the game with Kantian i and Nashian j as Kant-Nash game and to the game with Nashian i and Kantian j as Nash-Kant game. We show in the Appendix that there exists a single tuple of tax rates, denoted $\left(t_i^{kn}, t_j^{kn}\right)$ in the Kant-Nash game and $\left(t_i^{nk}, t_j^{nk}\right)$ in the Nash-Kant game, such that $\left(t_i^{kn}, t_j^{kn}\right)$ satisfies $(t_i, t_j) = \left[T^{Ki}(t_j), T^{Nj}(t_i)\right]$ and $\left(t_i^{nk}, t_j^{nk}\right)$ satisfies $(t_i, t_j) = \left[T^{Ni}(t_j), T^{Kj}(t_i)\right]$. In Figure 2 the tuples $\left(t_i^{kn}, t_j^{kn}\right)$ and $\left(t_i^{nk}, t_j^{nk}\right)$ are represented by the points S^{kn} and S^{nk}, respectively. The tuple $\left(t_i^{kn}, t_j^{kn}\right)$ is the unique equilibrium of the Kant-Nash game and $\left(t_i^{nk}, t_j^{nk}\right)$ is the unique equilibrium of the Nash-Kant game, if the rate-of-return constraint is not strictly binding. This case is illustrated in Figure 2, if the $r = 0$ line is represented by the $\alpha\alpha$ line. The solution of the game remains unchanged, if we choose a smaller value of α, e.g. that value which generates the $\alpha'\alpha'$ line in Figure 2. However, if α is even smaller than that, e.g. if the $r = 0$ line is given by the $\alpha''\alpha''$ line in Figure 2, the points S^{kn} and S^{nk} are no equilibria anymore, because now they are unattainable. In that case the games with a Kantian and a Nashian have the following feature in common with the Nash-Nash game analyzed above. If capital is sufficiently abundant (i.e. if α is sufficiently small), then there is no subinterval in the interval $[0, \alpha]$ of feasible tax rates t_i in which both countries choose their tax rate in accordance with the unconstrained branch of their best reply function. The preceding observations are stated rigorously in

Proposition 4.

(i) If $\alpha > \overline{H}(b, \bar{k}, \varepsilon, a_i - a_j)$, the tuple $\left(t_i^{kn}, t_j^{kn}\right)$ is the unique Kant-Nash equilibrium. That equilibrium is inefficient.[16]

(ii) If $\alpha < \underline{H}(b, \bar{k}, \varepsilon, a_i - a_j)$, there exist multiple Kant-Nash equilibria characterized by $(t_i, t_j = \alpha - t_i)$ and $t_i \in \left[\underline{t}_i^k, \bar{t}_i^k\right] \neq \emptyset$. All Kant-Nash equilibria are efficient.

(iii) Nash-Kant equilibria are characterized as in Proposition 4(i) and 4(ii) after interchanging kn with nk and i with j.

To better understand Proposition 4, we consider Figure 3 that displays that enlarged detail of Figure 2, which is relevant for Proposition 4. Since the Kant-Nash game and the Nash-Kant game are mirror-symmetric it suffices to focus on the Kant-Nash game. If

[16] The functions $\overline{H}(b, \bar{k}, \varepsilon, a_i - a_j)$ and $\underline{H}(b, \bar{k}, \varepsilon, a_i - a_j)$ are defined in the proof of Proposition 4 of the Appendix. It holds $\overline{H} \geq \underline{H} > 0$.

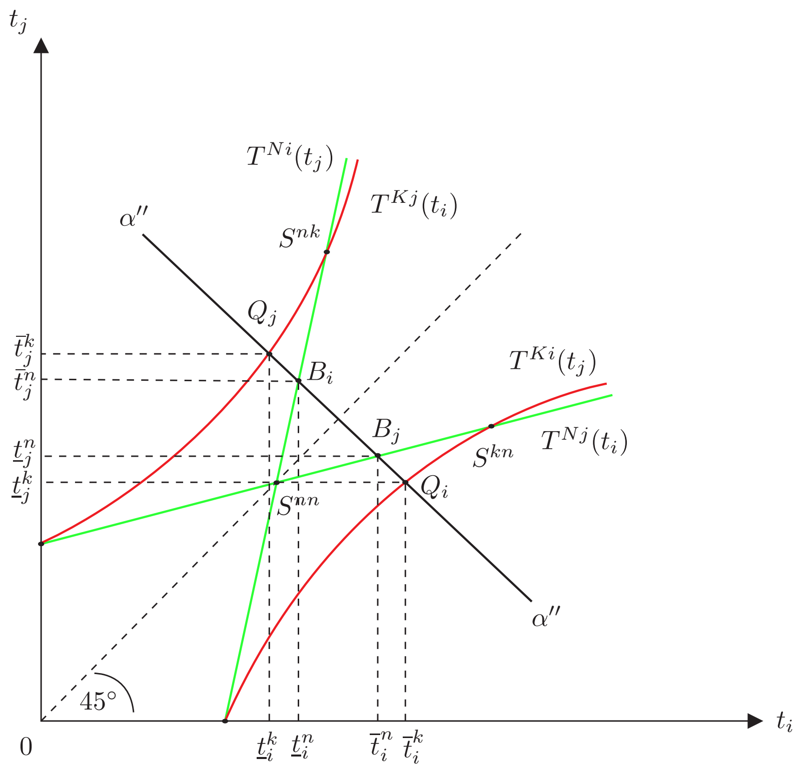

Figure 3: Equilibria in games with a Kantian and a Nashian

the $\alpha''\alpha''$ line applies as drawn in Figures 2 and 3, the interval $\left[0, \alpha\right]$ of feasible tax rates t_i on the t_i axis is partitioned into the subintervals $\left[0, \underline{t}_i^k\right[$, $\left[\underline{t}_i^k, \bar{t}_i^k\right]$ and $\left]\bar{t}_i^k, \alpha\right]$. Applying arguments analoguous to those used in the previous Sections 3.1 and 3.2 we conclude that $\left(t_i^{kn}, t_j^{kn}\right) = (t_i, t_j = \alpha - t_i)$ is a Kant-Nash equilibrium for every $t_i \in \left[\underline{t}_i^k, \bar{t}_i^k\right]$ - as stated in Proposition 4(ii). In Figure 3, each point on the segment B_jQ_i of the $\alpha''\alpha''$ line represents a Kant-Nash equilibrium. Correspondingly, each point on the segment Q_jB_i of the $\alpha''\alpha''$ line represents a Nash-Kant equilibrium.

3.4 Comparing the outcome of the games

In the preceding Sections 3.1 - 3.3 we have analyzed four tax competition games with Nashian and/or Kantian governments. Now we wish to summarize and compare the results. The first surprising observation is that the scarcity of capital - as measured by the size of the parameter α - plays an important role for the equilibria attained in the tax competition games studied above. While the outcome of the Kant-Kant game is efficient no matter how scarce capital is, in the other games with at least one Nashian government the equilibria depend on capital scarcity. We have extensively described and illustrated that dependence of equilibria on capital scarcity in the previous sections by means of the Figures 1 - 3. In a nutshell, the result is that with increasing capital abundance the unique equilibria in the games with at least one Nashian are replaced by (efficient) multiple equilibria. It is obvious that multiple equilibria seriously hamper the comparison of games. We therefore seek relief by restricting the comparison to games with values of the capital-scarcity parameter α large enough to yield unique equilibria in all games with at least one Nashian.

Consequently, it is only the Kant-Kant game, in which multiple equilibria still prevail. Since we assume symmetry in what follows, the straightforward and accepted way to resolve that multiplicity of equilibria is to invoke the principle of equal treatment of equals. The unique Kant-Kant equilibrium satisfying this principle is the tuple

$$\left(t_i^*, t_j^*\right) \quad \text{where } t_j^* = \alpha - t_i^* \quad \text{and} \quad t_i^* := \frac{\underline{t}_i^k + \bar{t}_i^k}{2} \in \left[\underline{t}_i^k, \bar{t}_i^k\right]. \tag{24}$$

The tax rates $\left(t_i^*, t_j^*\right)$ maximize aggregate welfare $\tilde{W}^i(t_i) + \tilde{W}^j(t_i)$ of the two countries and due to symmetry, the welfare is the same across countries. In the upper panel of Figure 2, the Kant-Kant equilibrium (24) is represented by the point R, and the lower panel of Figure 2 shows that the tax rate t_i^* maximizes the aggregate welfare and that $\tilde{W}^i\left(t_i^*\right) = \tilde{W}^j\left(t_i^*\right)$.

Summing up, we will now compare the outcome of the four games presented in Sections 3.1, 3.2 and 3.3 assuming that the equilibria of these games are

$$\left(t_i^{KK}, t_j^{KK}\right) = \left(t_i^*, t_j^*\right), \qquad \left(t_i^{KN}, t_j^{KN}\right) = \left(t_i^{kn}, t_j^{kn}\right), \quad \left(t_i^{NK}, t_j^{NK}\right) = \left(t_i^{nk}, t_j^{nk}\right),$$
$$\left(t_i^{NN}, t_j^{NN}\right) = \left(t_i^{nn}, t_j^{nn}\right).$$

These equilibria correspond to the points R, S^{kn}, S^{nk} and S^{nn} in Figure 2 under the condition that the $r = 0$ line is given by a line such as $A_j A_i$. From Figure 2 we readily infer that[17]

$$\left(t_i^{KK}, t_j^{KK}\right) \gg \left(t_i^{NN}, t_j^{NN}\right), \quad \left(t_i^{KN}, t_j^{KN}\right) \gg \left(t_i^{NN}, t_j^{NN}\right), \quad \left(t_i^{NK}, t_j^{NK}\right) \gg \left(t_i^{NN}, t_j^{NN}\right). \tag{25}$$

In order to compare $\left(t_i^{KK}, t_j^{KK}\right)$ with the equilibria $\left(t_i^{KN}, t_j^{KN}\right)$ and $\left(t_i^{NK}, t_j^{NK}\right)$, note first that the equilibria in the two games with a Nashian and a Kantian are mirror-symmetric such that $t_j^{KN} = t_i^{NK}$ and $t_j^{NK} = t_i^{KN}$. Since we also have $t_j^{KK} = t_i^{KK}$ and $t_j^{NN} = t_i^{NN}$ it suffices to compare the four tax rates t_i^{KK}, t_i^{KN}, t_i^{NK} and t_i^{NN} of country i. Figure 4a displays these tax rates for a numerical example ($a = 10, b = 3.75, \bar{k} = 1$) over the interval $[0.1, 1.02]$ of the preference parameter ε. As expected we find that irrespective of whether j is a Nashian or a Kantian, country i always chooses a lower tax rate when it is a Nashian than when it is a Kantian i.

[17] $\left(t_i^{KK}, t_j^{KK}\right) \gg \left(t_i^{NN}, t_j^{NN}\right)$ means that $t_i^{KK} > t_i^{NN}$ and $t_j^{KK} > t_j^{NN}$.

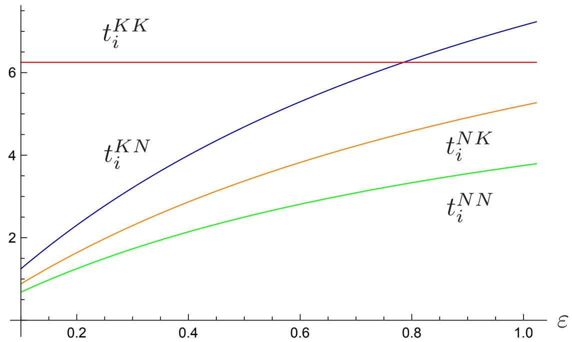

Figure 4a

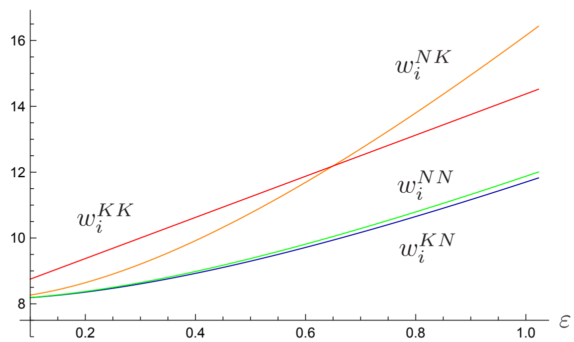

Figure 4b

Figure 4: Tax rates and welfare of country i in a numerical example ($a = 10, b = 3.75, \bar{k} = 1$)

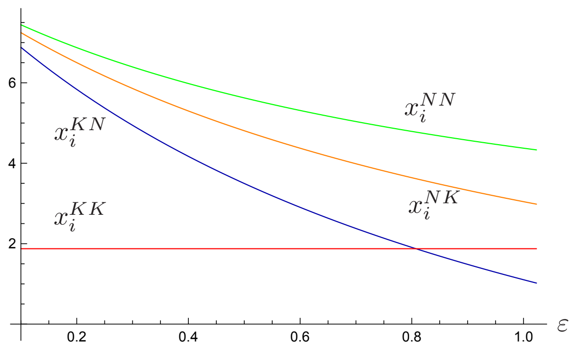

Figure 5a

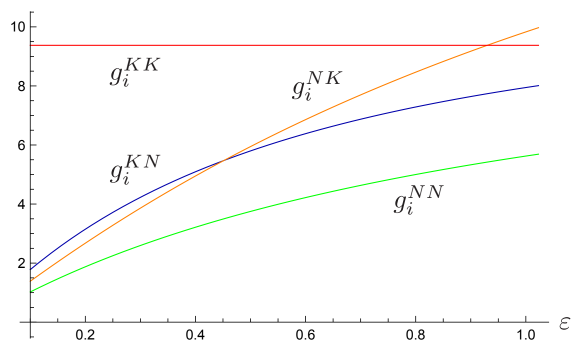

Figure 5b

Figure 5: Public and private good consumption country i in a numerical example

According to Figure 4a, the Kant-Nash equilibrium is characterized by $t_i^{KN} > t_j^{KN}$. The rationale for that ranking is that Nashian j free rides on Kantian i's best-reply tax rates that are higher than those of Nashian j. i's high tax rate allows j to raise its own tax rate (according to $T_{t_i}^{Nj} > 0$) without triggering an outflow of capital. So Kantian i exports and Nashian j imports capital, $k_i^{KN} < \bar{k} < k_j^{KN}$, which makes Nashian j better off than Kantian i, $w_j^{KN} > w_i^{KN}$, as shown in Figure 4b. In games with a Nashian and a Kantian, the Nashian is always better off than the Kantian.

Next we investigate how the games with a Nashian and a Kantian compare with the games in which the players are either two Nashians or two Kantians. As displayed in Figure 5, private consumption of the countries i and j is lower and their public consumption is higher in the Kant-Nash than in the Nash-Nash equilibrium. According to Figure 4b, Nashian j's low welfare from private consumption is overcompensated by high welfare from public consumption such that Nashian j's welfare is higher in the Kant-Nash than in the Nash-Nash equilibrium. Kantian i's private consumption is so low that its welfare drops below the welfare it enjoys in the Nash-Nash equilibrium. Summarizing,

$$w_j^{KN} = w_i^{NK} > w_j^{NN} = w_i^{NN} > w_i^{KN}. \tag{26}$$

The comparison of the Kant-Nash with the Kant-Kant equilibrium is not so clear. For low values of ε, Figure 4a shows the ranking $t_i^{KK} > t_i^{KN} > t_j^{KN}$. Compared to the Kant-Kant equilibrium, private consumption is larger and public consumption is smaller in the Kant-Nash equilibrium (Figure 5), but the larger welfare from private consumption is over-compensated by the smaller welfare from public consumption such that welfares are smaller in the Kant-Kant than in the Kant-Nash equilibrium, $w_i^{KK} > w_i^{NK} = w_j^{KN} > w_i^{KN}$ when ε is small.

For large values of ε, Figure 4a shows the ranking $t_i^{KN} > t_i^{KK} > t_j^{KN}$. Nashian j's incentives to set low tax rates (in order to attract capital) induce Kantian i to choose such high tax rates that its tax rate overshoots the level in the efficient Kant-Kant equilibrium. The high tax rate of Kantian i causes a massive capital flight into country j with the consequence that Nashian j's public good provision increases and Kantian i's public good provision decreases compared to the Kant-Kant equilibrium. Since changes of private consumption are dominated by changes of public consumption in terms of welfare, the ranking of welfares is $w_j^{KN} = w_i^{NK} > w_i^{KK} > w_i^{KN}$ when ε is 'large'. It remains to make precise the conditions under which the inequalities $w_i^{KK} > w_i^{NK}$ and $w_i^{KK} < w_i^{NK}$ are satisfied. To that end we define in the Appendix the set S of all feasible parameters $(a_i = a_j \equiv a, b, \bar{k}, \varepsilon)$ and show that there exists a non-empty subset, denoted S^E, such that

$$w_i^{KK} \begin{Bmatrix} > \\ < \end{Bmatrix} w_i^{NK} \qquad \Longleftrightarrow \qquad (a, b, \bar{k}, \varepsilon) \begin{cases} S^E \\ S \setminus S^E. \end{cases} \tag{27}$$

Figure 6 illustrates the set S^E and its complement $S \setminus S^E$.

So far we referred to the Figures 4 and 5 without having commented on the shape of the curves. Our focus will be on Figure 4a, but the explanations readily extend to all other curves in Figures 4 and 5. In Figure 4a the tax rate t_i^{KK} is a horizontal straight line because $t_i^{KK} = \frac{1}{2}\alpha = \frac{1}{2}(a_i + a_j - 2b\bar{k})$ is independent of ε. The tax rates t_i^{KN}, t_i^{NK} and t_i^{NN} in Figure 4a are increasing in ε, because they are coordinates of the intersection points S^{kn}, S^{nk} and S^{nn}, respectively, and these points move away from the origin with increasing ε. Thus, in all games other than the Kant-Kant game the sum of equilibrium tax rates is increasing in the preference parameter ε which translates into a declining equilibrium rate of return to capital. Specifically, the increasing curves of t_i^{KN} and t_i^{NK} imply not only that $t_i^{KN} + t_i^{NK}$

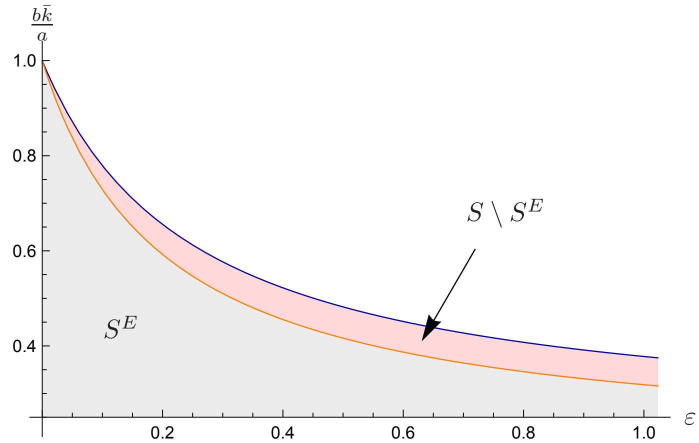

Figure 6: The sets of parameters S^E and $S \setminus S^E$

but also that $t_i^{KN} + t_j^{KN}$ and $t_i^{NK} + t_j^{NK}$ are increasing in ε. The t_i^{KN} curve lies below the t_i^{KK} curve in Figure 4a when ε is small, but t_i^{KN} increases so strongly in ε that it becomes greater than t_i^{KK} at about $\varepsilon = 0.8$. If $\varepsilon = 1.02$, the difference $2t_i^{KK} - (t_i^{KN} + t_i^{NK})$ has become zero, which means that the $r \geq 0$ constraint has become weakly binding. The points S^{nk} and S^{kn} are still the unique equilibrium points of the (mirror-symmetric) Kant-Nash and Nash-Kant games, and they are points on the welfare possibility frontier and hence are efficient equilibria. However, the welfare is not the same across countries. Rather, in the games with a Nashian and a Kantian the Nashian's welfare is higher and the Kantian's welfare is lower than that welfare, which each country enjoys in the efficient Kant-Kant equilibrium.

4 Behavior selection game

Throughout Section 3 our implicit assumption was that a country's government is either a Nashian or a Kantian. This assumption can be interpreted to mean that the government got a mandate from the country's consumer-voters to act either as a Nashian or a Kantian. In this section we still deal with Nashian and Kantian governments, but the research question is different. Now we consider governments - or the mandate-giving consumer-voters - as opportunists. The governments' only guideline is their countries' welfare, as in the standard tax competition literature with Nashian behavior, but now they have the additional option to act as (if they were) a Nashian or as a Kantian. A government chooses to act as a Kantian [Nashian] if the outcome of tax competition under this behavior is a higher level of its country's welfare than that which the country attains if the government acts as a Nashian [Kantian]. Hence we endogenize the countries' Kant-Nash behavior.

In game-theoretic language, we apply the extended game with observable delay introduced by Hamilton and Slutsky (1990). In this game countries decide at a first or preplay stage non-cooperatively and simultaneously whether to behave as a Kantian or a Nashian. Their decision is announced at the end of the preplay stage and countries are committed to their behavioral choice. At the second stage, the countries enter into tax competition, the outcome of which depends on the behavioral decision at the preplay stage. If both countries choose to act as Nashians [Kantians], we have a Nash-Nash [Kant-Kant] game. If one country behaves as Kantian and the other as Nashian we have a Kant-Nash or Nash-Kant game.[18]

		country j	
		K	N
country i	K	w_i^{KK}, w_j^{KK}	w_i^{KN}, w_j^{KN}
	N	w_i^{NK}, w_j^{NK}	w_i^{NN}, w_j^{NN}

Table 1: Normal form of the second-stage subgame of the behavior selection game

To solve the two-stage behavior selection game we consider the reduced normal form of the second-stage subgame in Table 1. In that subgame, the countries' strategies are Kantian or Nashian behavior and their payoffs are the equilibrium welfares of the respective tax competition games analyzed in Section 3. Given that country j has chosen to be a Nashian, country i chooses to be a Nashian [Kantian] if and only if $w_i^{NN} > w_i^{KN}$ $[w_i^{KN} > w_i^{NN}]$. Likewise, given that country j has chosen to be a Kantian, country i chooses to be a Kantian [Nashian] if and only if $w_i^{KK} > w_i^{NK}$ $[w_i^{NK} > w_i^{KK}]$. Invoking (26) and (27), we distinguish two different cases of the subgame in Table 1, characterized by the following rankings of welfares/payoffs.

$$\begin{aligned} &\text{Subgame 1}: \quad w_i^{NN} > w_i^{KN} \quad \wedge \quad w_i^{KK} > w_i^{NK}, \quad \text{if } (a, b, \bar{k}, \varepsilon) \in S^E, \\ &\text{Subgame 2}: \quad w_i^{NN} > w_i^{KN} \quad \wedge \quad w_i^{KK} < w_i^{NK}, \quad \text{if } (a, b, \bar{k}, \varepsilon) \in S \setminus S^E. \end{aligned} \tag{28}$$

Recalling that $w_i^{KN} = w_j^{NK}$, $w_i^{NK} = w_j^{KN}$, $w_i^{NN} = w_j^{NN}$ and $w_i^{KK} = w_j^{KK}$ it is easy to show that Subgame 1 possesses two subgame-perfect (SP) equilibria, namely $\left(w_i^{KK}, w_j^{KK}\right)$ and $\left(w_i^{NN}, w_j^{NN}\right)$. The existence of two SP equilibria makes the outcome of Subgame 1 ambiguous and therefore calls for equilibrium selection. Selecting one of the two SP equilibria is straightforward and widely accepted in the present case. Since the payoffs in the Kant-Kant equilibrium are strictly greater than in the Nash-Nash equilibrium (equation (25)), the outcome of Subgame 1 are the payoffs of the Kant-Kant game. It follows immediately

[18]In Table 1 (K and N stand for the strategies "Kantian behavior" and "Nash behavior", respectively.

that at stage 1 of Subgame 1 both countries choose to behave as Kantians. Consider next Subgame 2. The consequence of the change in sign of the difference $w_i^{KK} - w_i^{NK}$ is that $\left(w_i^{NN}, w_j^{NN}\right)$ is the unique SP equilibrium of Subgame 2. Hence, at stage 1 of Subgame 2 both countries choose to behave as Nashians. These results are summarized in

Proposition 5. *Suppose the governments of the identical countries i and j engage in capital tax competition and make a decision about whether they find it in their countries' interest to act as a Nashian or a Kantian. There exists a non-empty proper subset S^E of the set S of all feasible model parameters such that*

(i) both governments choose to be Kantians, if the parameters belong to the set S^E;

(ii) both governments choose to be Nashians, if the parameters belong to the set $S \setminus S^E$.

According to Proposition 5 it depends on the characteristics of the economy whether opportunistic governments prefer to behave as Nashians or as Kantians. We find it surprising that under certain conditions both governments choose Kantian behavior out of self-interest (Proposition 5(i), because our intuition was that robust Nashian self-interest would always supersede Kantian moral principles. We obtain an interesting side result of the behavior selection game, if we assume that one country, say country j, is a Nashian or a Kantian by conviction, whereas country i chooses Nashian or Kantian behavior opportunistically. Inspection of (28) shows that in all economies in the set S, the opportunistic country i chooses Nashian behavior, if j is a 'convinced' Nashian.[19] If j is a 'convinced' Kantian, i chooses Kantian behavior in economies in the set S^E but Nashian behavior in economies in the set $S \setminus S^E$.

5 Concluding remarks

This paper analyzes how tax competition differs from the standard all-Nashian tax competition, if some or all countries follow Roemer's (2010) moral rule that is meant to reflect the spirit of Kant's categorical imperative. It also investigates the preference to behave as a Nashian or a Kantian of those countries, which are solely interested in welfare maximization. Multiple equilibria render difficult the comparison between the conventional all-Nashian tax competition and tax competition with at least one country behaving as a Kantian. Therefore, in Sections 3.4 and 4 we focus on symmetric economies and unique equilibria and find that the all-Kantian tax competition is efficient and that the inefficient race to the bottom

[19] Figure 4b shows, however, that w_i^{NN} is only slightly greater then w_i^{KN}.

becomes less severe when not all countries are Nashians. These results appear to confirm the intuitive idea of the Kantian categorical imperative that moral behavior tends to soften the impact of 'detrimental' self-interest. We also find that the choice of opportunistic countries to behave as Nashians or Kantians depends on model parameters. It is possible that both governments behave as Kantians out of self-interest and thus implement the efficient allocation. Under different conditions the robust Nashian self-interest supersedes Kantian moral principles, which constitutes a prisoners' dilemma and thus implements the inefficient all-Nashian allocation.

Our very simple analytical framework has been applied in various studies before. The only reason why we make use of it is that we were unable to find a less restrictive model that would allow to derive of informative results. The price is, of course, that the robustness of results remains unclear. Nevertheless, the thrust of the comparison of the four tax competition games of Section 3 is in line with the studies on Kantian economics, cited in the introduction. The puzzling opposite outcomes of the behavior selection game hardly allow to predict what opportunistic countries in the real world would do. Yet they give some support to the *possibility* that Kantian behavior is in the countries' self-interest.

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Appendix

Throughout the Appendix we use the short-cut $\Phi = a_i - a_j$.

Derivation of (10) and (11):

Differentiating W^i from (9) with respect to t_i and t_j, respectively, yields

$$W^i_{t_i} = (1+\varepsilon)t_i K^i_{t_i} + \varepsilon k_i + (\bar{k} - k_i)R_{t_i}, \tag{A1}$$

$$W^i_{t_j} = (1+\varepsilon)t_i K^i_{t_j} + (\bar{k} - k_i)R_{t_j}. \tag{A2}$$

Inserting the functions $K^i(t_i, t_j)$ and $R(t_i, t_j)$ from (6) and its derivatives into (A1) and (A2) we obtain

$$\begin{aligned} W^i_{t_i} &= -\frac{1+\varepsilon}{2b}t_i - \frac{\bar{k}}{2} + \frac{1+2\varepsilon}{2}\left(\frac{2a_i - \alpha}{2b} + \frac{t_j - t_i}{2b}\right) \\ &= \frac{(1+2\varepsilon)(a_i - a_j) + 4b\varepsilon\bar{k}}{4b} - \frac{3+4\varepsilon}{4b}t_i + \frac{1+2\varepsilon}{4b}t_j, \end{aligned} \tag{A3}$$

and

$$W^i_{t_j} = \frac{1+\varepsilon}{2b}t_i - \frac{\bar{k}}{2} + \frac{1}{2}\left(\frac{2a_i - \alpha}{2b} + \frac{t_j - t_i}{2b}\right) = \frac{a_i - a_j}{4b} + \frac{1+2\varepsilon}{4b}t_i + \frac{1}{4b}t_j. \tag{A4}$$

Derivation of (14):

The first derivative of the function $\tilde{W}^i$ is

$$\begin{aligned} \tilde{W}^i_{t_i} &= W^i_{t_i} + W^i_{t_j}\frac{\mathrm{d}(\alpha - t_i)}{\mathrm{d}t_i} = W^i_{t_i} - W^i_{t_j} = \frac{\varepsilon(2a_i - \alpha)}{2b} - \frac{2+3\varepsilon}{2b}t_i + \frac{\varepsilon}{2b}(\alpha - t_i) \\ &= \frac{a_i\varepsilon}{b}t_i - \frac{1+2\varepsilon}{b}t_i. \end{aligned} \tag{A5}$$

Analogously, differentiation of the function $\tilde{W}^j$ yields

$$\tilde{W}^j_{t_i} = W^j_{t_i} + W^j_{t_j}\frac{\mathrm{d}(\alpha - t_i)}{\mathrm{d}t_i} = W^j_{t_i} - W^j_{t_j} = \frac{(1+2\varepsilon)\alpha - a_j\varepsilon}{b} - \frac{1+2\varepsilon}{b}t_i. \tag{A6}$$

Derivation of (17):

We solve $t_i = \frac{4b\varepsilon\bar{k}+(1+2\varepsilon)\Phi}{3+4\varepsilon} + \frac{1+2\varepsilon}{3+4\varepsilon}t_j$ and $t_j = \frac{4b\varepsilon\bar{k}-(1+2\varepsilon)\Phi}{3+4\varepsilon} + \frac{1+2\varepsilon}{3+4\varepsilon}t_i$ in seversal steps:

$$t_i = \frac{4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi}{3+4\varepsilon} + \frac{1+2\varepsilon}{3+4\varepsilon}\left[\frac{4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi}{3+4\varepsilon} + \frac{1+2\varepsilon}{3+4\varepsilon}t_i\right]$$

is equivalent to

$$t_i\left(1 - \frac{(1+2\varepsilon)^2}{(3+4\varepsilon)^2}\right) = \frac{(3+4\varepsilon)\left[4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi\right] + (1+2\varepsilon)\left[4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi\right]}{(3+4\varepsilon)^2}$$

and further rearrangements lead to

$$\begin{aligned} t_i^{nn} &= \frac{4b\varepsilon\bar{k}\left[(3+4\varepsilon)+(1+2\varepsilon)\right] + (1+2\varepsilon)\Phi\left[(3+4\varepsilon)-(1+2\varepsilon)\right]}{(3+4\varepsilon)^2 - (1+2\varepsilon)^2} \\ &= \frac{4b\varepsilon\bar{k}\left[(3+4\varepsilon)+(1+2\varepsilon)\right] + (1+2\varepsilon)\Phi\left[(3+4\varepsilon)-(1+2\varepsilon)\right]}{\left[(3+4\varepsilon)+(1+2\varepsilon)\right]\left[(3+4\varepsilon)-(1+2\varepsilon)\right]} \\ &= \frac{2b\varepsilon\bar{k}}{1+\varepsilon} + \frac{(1+2\varepsilon)\Phi}{2(2+3\varepsilon)}. \end{aligned} \tag{A7}$$

Proof of Proposition 2:

Consider the partition of Nashian i's domain into the subintervals $\left[0, \underline{t}_j^n\right]$, $\left[\underline{t}_j^n, \alpha\right]$. For analytical convenience we 'map' these subintervals from the domain into the range of i's best reply function such that $\left[0, \underline{t}_j^n\right]$ corresponds to $\left[\overline{t}_i^n, \alpha\right]$ and $\left[\underline{t}_j^n, \alpha\right]$ corresponds to $\left[0, \overline{t}_i^n\right]$. Clearly, if i's reply is $t_i \in \left[\overline{t}_i^n, \alpha\right]$, then i is not constrained by $r = 0$, and if the reply is $t_i \in \left[0, \overline{t}_i^n\right]$ then the constraint $r \geq 0$ is binding. When we combine the partition $\left[0, \overline{t}_i^n\right]$, $\left[\overline{t}_i^n, \alpha\right]$ of i's best replies with the partition $\left[0, \overline{t}_i^n\right]$, $\left[\overline{t}_i^n, \alpha\right]$ of the domain of Nashian j's best reply function, the interval $\left[0, \alpha\right]$ is divided into three subintervals. Depending on the sign of the difference $\underline{t}_i^n - \overline{t}_i^n$, the non-empty subintervals are either $\left[0, \underline{t}_i^n\right]$, $\left[\underline{t}_i^n, \overline{t}_i^n\right]$, $\left[\overline{t}_i^n, \alpha\right]$ or $\left[0, \overline{t}_i^n\right]$, $\left[\overline{t}_i^n, \underline{t}_i^n\right]$, $\left[\underline{t}_i^n, \alpha\right]$. Closer inspection reveals that if $\underline{t}_i^n > \overline{t}_i^n$, the best reply functions of i and j have no point in common in the intervals $\left[\overline{t}_i^n, \underline{t}_i^n\right]$ and $\left[\underline{t}_i^n, \alpha\right]$. However, we find that $t_i^{nn} \in \left[0, \overline{t}_i^n\right]$ such that $\left(t_i^{nn}, t_j^{nn}\right)$ from (17) is the unique interior Nash-Nash equilibrium. If $\underline{t}_i^n > \overline{t}_i^n$ there is no common point in the subintervals $\left[0, \underline{t}_i^n\right]$ and $\left[\overline{t}_i^n, \alpha\right]$, because in these intervals one Nashian's best reply is constrained, but the other's is not. In the intermediate sub-interval $\left[\underline{t}_i^n, \overline{t}_i^n\right]$ both Nashians' best replies are subject to the $r = 0$ constraint such that the tax rates $(t_i, t_j = \alpha - t_i)$ define a Nash-Nash equilibrium for every $t_i \in \left[\underline{t}_i^n, \overline{t}_i^n\right]$.

In view of (A7) the Nash-Nash equilibrium is an interior solution, if and only if

$$t_i^{nn} + t_j^{nn} < \alpha \quad \Longleftrightarrow \quad \frac{4b\varepsilon\bar{k}}{1+\varepsilon} < \alpha \quad \Longleftrightarrow \quad 4b\varepsilon\bar{k} < (1+\varepsilon)\alpha.$$

■

Derivation of (21):

Differentiating $W^i(\lambda t_i, \lambda t_j)$ from (20) we obtain

$$\begin{aligned} W_\lambda^i &= X_{k_i}^i K_\lambda^i + \left(\bar{k} - k_i\right) R_\lambda - r K_\lambda^i + \varepsilon t_i k_i + \varepsilon \lambda t_i K_\lambda^i \\ &= \left(X_{k_i}^i - r + \varepsilon \lambda t_i\right) K_\lambda^i + \left(\bar{k} - k_i\right) R_\lambda + \varepsilon t_i k_i = (1+\varepsilon)\lambda t_i K_\lambda^i + \left(\bar{k} - k_i\right) R_\lambda + \varepsilon t_i k_i \\ &= (1+\varepsilon)\lambda t_i \frac{t_j - t_i}{2b} + \left(\frac{\Phi}{2b} + \frac{\lambda(t_j - t_i)}{2b}\right) \frac{t_j + t_i}{2} + \varepsilon t_i \left(\frac{2b\bar{k} + \Phi}{2b} + \frac{\lambda(t_j - t_i)}{2b}\right) \\ &= \frac{\left[(3+4\varepsilon)t_i + t_j\right](t_j - t_i)\lambda + \Phi t_j + \left[4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi\right] t_i}{4b}. \end{aligned} \tag{A8}$$

Derivation of (22):

Setting $\lambda = 1$ in $W_\lambda^i = 0$ yields

$$\begin{aligned} & \left[(3+4\varepsilon)t_i + t_j\right](t_j - t_i) + \Phi t_j + \left[4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi\right] t_i = 0 \\ \Longleftrightarrow \quad & (3+4\varepsilon)t_i^2 - \left[4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi + (2+4\varepsilon)t_j\right] t_i - (t_j + \Phi)t_j = 0 \\ \Longleftrightarrow \quad & t_i^2 - \frac{4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi + (2+4\varepsilon)t_j}{3+4\varepsilon} t_i - \frac{(t_j + \Phi)t_j}{3+4\varepsilon} = 0 \\ \Longleftrightarrow \quad & t_i^2 - \left[T^{Ni}(t_j) + \frac{(1+2\varepsilon)t_j}{3+4\varepsilon}\right] t_i - \frac{(t_j + \Phi)t_j}{3+4\varepsilon} = 0. \end{aligned} \tag{A9}$$

The solution of the last equation is (22).

Proof of Proposition 3:

The best-reply function $T^{Ki}(t_j)$ has the properties

$$\begin{aligned}
T^{Ki}(0) &= \frac{4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi}{3+4\varepsilon}, && \text{(A10)}\\
T^{Ki}(t_j) - t_j &= \frac{-4t_j(1+\varepsilon) + 4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi}{6+8\varepsilon} \\
&\quad + \frac{\sqrt{16t_j^2(1+\varepsilon)^2 + \left[4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi\ \right]^2 + 16t_j\left[b\varepsilon\bar{k}(1+2\varepsilon) + (1+\varepsilon)^2\Phi\right]}}{6+8\varepsilon}, && \text{(A11)}\\
T^{Ki}_{t_j} &= \frac{1 + 2\varepsilon + \frac{4\left[2t_j(1+\varepsilon)^2 + b\varepsilon(1+2\varepsilon)\bar{k} + (1+\varepsilon)\Phi\right]}{\sqrt{16t_j^2(1+\varepsilon)^2 + \left[4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi\ \right]^2 + 16t_j\left[b\varepsilon\bar{k}(1+2\varepsilon) + (1+\varepsilon)^2\Phi\right]}}}{3+4\varepsilon}, && \text{(A12)}\\
T^{Ki}_{t_jt_j} &= \frac{8\left[4b^2\varepsilon^2\bar{k}^2 - (1+\varepsilon)^2\Phi^2\ \right]}{\left[16t_j^2(1+\varepsilon)^2 + \left[4b\varepsilon\bar{k} + (1+2\varepsilon)\Phi\ \right]^2 + 16t_j\left[b\varepsilon\bar{k}(1+2\varepsilon) + (1+\varepsilon)^2\Phi\right]\right]^{\frac{3}{2}}}. && \text{(A13)}
\end{aligned}$$

From (A10) we get $T^{Ki}(0) > 0$. From (A11) we infer $T^{Ki}(t_j) - t_j > 0$. Next, observe that the numerator of (A13) can be written as

$$8\left[4b^2\varepsilon^2\bar{k}^2 - (1+\varepsilon)^2\Phi^2\ \right] = 8\left[2b\varepsilon\bar{k} - (1+\varepsilon)\Phi\right]\left[2b\varepsilon\bar{k} + (1+\varepsilon)\Phi\right]. \quad \text{(A14)}$$

The assumption $2b\varepsilon\bar{k} - (1+\varepsilon)\Phi > 0$ implies $T^{Ki}_{t_jt_j} > 0$.

In case of $T^{Kj}(t_i)$ we get:

$$\begin{aligned}
T^{Kj}(0) &= \frac{4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi}{3+4\varepsilon}, && \text{(A15)}\\
T^{Kj}(t_i) - t_i &= \frac{-4t_j(1+\varepsilon) + 4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi}{6+8\varepsilon} \\
&\quad + \frac{\sqrt{16t_j^2(1+\varepsilon)^2 + \left[4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi\ \right]^2 + 16t_j\left[b\varepsilon\bar{k}(1+2\varepsilon) - (1+\varepsilon)^2\Phi\right]}}{6+8\varepsilon}, && \text{(A16)}\\
T^{Kj}_{t_it_i} &= \frac{8\left[4b^2\varepsilon^2\bar{k}^2 - (1+\varepsilon)^2\Phi^2\ \right]}{\left[16t_j^2(1+\varepsilon)^2 + \left[4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi\ \right]^2 + 16t_j\left[b\varepsilon\bar{k}(1+2\varepsilon) - (1+\varepsilon)^2\Phi\right]\right]^{\frac{3}{2}}}. && \text{(A17)}
\end{aligned}$$

Verify that assumption $2b\varepsilon\bar{k} > (1+\varepsilon)\Phi$ implies $4b\varepsilon\bar{k} > (1+2\varepsilon)\Phi$ and hence we get $T^{Kj}(0) > 0$. Further rearranging (A16) leads to

$$\begin{aligned}
T^{Kj}(t_i) - t_i &= \frac{-4t_j(1+\varepsilon) + 4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi}{6+8\varepsilon} \\
&\quad + \frac{\sqrt{\left[-4t_j(1+\varepsilon) + 4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi\right]^2 + 8t_j(3+4\varepsilon)[2b\varepsilon\bar{k} - (1+\varepsilon)\Phi)]}}{6+8\varepsilon}. && \text{(A18)}
\end{aligned}$$

Due to $2b\varepsilon\bar{k} > (1+\varepsilon)\Phi$ it holds

$$\begin{aligned}
&\sqrt{\left[-4t_j(1+\varepsilon) + 4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi\right]^2 + 8t_j(3+4\varepsilon)[2b\varepsilon\bar{k} - (1+\varepsilon)\Phi)]} \\
&\qquad > -4t_j(1+\varepsilon) + 4b\varepsilon\bar{k} - (1+2\varepsilon)\Phi && \text{(A19)}
\end{aligned}$$

and hence we get $T^{Kj}(t_i) - t_i > 0$. Using the same arguments as in case of T^{Ki} one can show that $T^{Kj}_{t_i t_i} > 0$. The properties that T^{Ki} lies above and T^{Kj} lies below the 45° line in a (t_i, t_j) diagram, formally proven by $T^{Ki}(t_j) - t_j > 0$ and $T^{Kj}(t_i) - t_i > 0$, establishes that the best reply curves do not intersect. ■

Proof of Proposition 4:

Consider exemplarily the Kant-Nash equilibria. Determining with the help of Mathematica the limits of the slope and the curvature of Kantian i's best-reply curve yields

$$\lim_{t_j \to \infty} T^{Ki}_{t_j} = \frac{1 + 2\varepsilon + 2\sqrt{(1+\varepsilon)^2}}{3 + 4\varepsilon} = 1, \tag{A20}$$

$$\lim_{t_j \to \infty} T^{Ki}_{t_j t_j} = 0. \tag{A21}$$

The slope of Nashian j's best-reply curve T^{Nj} is

$$\frac{\mathrm{d}t_i}{\mathrm{d}t_j} = \frac{3 + 4\varepsilon}{1 + 2\varepsilon} > 1. \tag{A22}$$

In view of Figure 2 there may be no point of intersection, one point of intersection or two points of intersection between the best-reply curves depending of the locus of the $r = 0$ constraint. The case of two points of intersection can be ruled out. To see that consider the point of intersection S^{nk} in Figure 2. In S^{nk} the slope of T^{Ni} is larger (and greater than one, see (A22)) than the slope of T^{Kj}. Although T^{Kj} is convex it reaches its maximal slope when t_i converges to infinity. Since this slope is equal to one (see (A21)) and hence smaller than the slope of T^{Ni} the two best reply curves do not intersect twice.

Next, we determine the domain that ensures interior Kant-Nash and interior Nash-Kant equilibria. Solving $t_i = T^{Ki}(t_j)$ and $t_j = T^{Nj}(t_i)$ we obtain the (interior) Kant-Nash equilibrium

$$t_i^{kn} = \frac{(17 + 48\varepsilon + 32\varepsilon^2)b\varepsilon\bar{k} + (1 + 3\varepsilon + 2\varepsilon^2)\Phi}{2(1+\varepsilon)^2(5+8\varepsilon)} + \frac{\sqrt{(3+4\varepsilon)^2[b^2\varepsilon^2\bar{k}^2\Upsilon + 2(3 + 7\varepsilon + 4\varepsilon^2)b\varepsilon\bar{k}\Phi - (1+\varepsilon)^2(1+2\varepsilon)\Phi^2]}}{2(1+\varepsilon)^2(5+8\varepsilon)}, \tag{A23}$$

$$t_j^{kn} = \frac{(57 + 226\varepsilon + 296\varepsilon^2 + 128\varepsilon^3)b\varepsilon\bar{k} - (1+2\varepsilon)(1+\varepsilon)(3+4\varepsilon)^2\Phi}{2(1+\varepsilon)^2(3+4\varepsilon)(5+8\varepsilon)} + \frac{(1+2\varepsilon)\sqrt{(3+4\varepsilon)^2[b^2\varepsilon^2\bar{k}^2\Upsilon + 2(3 + 7\varepsilon + 4\varepsilon^2)b\varepsilon\bar{k}\Phi - (1+\varepsilon)^2(1+2\varepsilon)\Phi^2]}}{2(1+\varepsilon)^2(3+4\varepsilon)(5+8\varepsilon)}, \tag{A24}$$

where $\Upsilon := \sqrt{41 + 104\varepsilon + 64\varepsilon^2}$. Summing up both tax rates we get

$$\begin{aligned} t_i^{kn} + t_j^{kn} &= \frac{(54 + 219\varepsilon + 292\varepsilon^2 + 128\varepsilon^3)b\varepsilon\bar{k} - (1+2\varepsilon)^2(3+7\varepsilon+4\varepsilon^2)\Phi}{(1+\varepsilon)^2(3+4\varepsilon)(5+8\varepsilon)} \\ &+ \frac{(2+3\varepsilon)\sqrt{(3+4\varepsilon)^2[b^2\varepsilon^2\bar{k}^2\Upsilon + 2(3+7\varepsilon+4\varepsilon^2)b\varepsilon\bar{k}\Phi - (1+\varepsilon)(1+2\varepsilon)\Phi^2]}}{(1+\varepsilon)^2(3+4\varepsilon)(5+8\varepsilon)} \\ &=: F(b, \bar{k}, \varepsilon, \Phi). \end{aligned} \tag{A25}$$

Solving $t_i = T^{Ki}(t_j)$ and $t_j = T^{Nj}(t_i)$ we obtain the (interior) Kant-Nash equilibrium

$$\begin{aligned} t_i^{nk} &= \frac{(57 + 226\varepsilon + 296\varepsilon^2 + 128\varepsilon^3)b\varepsilon\bar{k} + (1+2\varepsilon)(1+\varepsilon)(3+4\varepsilon)^2\Phi}{2(1+\varepsilon)^2(3+4\varepsilon)(5+8\varepsilon)} \\ &+ \frac{(1+2\varepsilon)\sqrt{(3+4\varepsilon)^2[b^2\varepsilon^2\bar{k}^2\Upsilon - 2(3+7\varepsilon+4\varepsilon^2)b\varepsilon\bar{k}\Phi - (1+\varepsilon)^2(1+2\varepsilon)\Phi^2]}}{2(1+\varepsilon)^2(3+4\varepsilon)(5+8\varepsilon)}, \end{aligned} \tag{A26}$$

$$\begin{aligned} t_j^{nk} &= \frac{(17 + 48\varepsilon + 32\varepsilon^2)b\varepsilon\bar{k} - (1+3\varepsilon+2\varepsilon^2)\Phi}{2(1+\varepsilon)^2(5+8\varepsilon)} \\ &+ \frac{\sqrt{(3+4\varepsilon)^2[b^2\varepsilon^2\bar{k}^2\Upsilon + 2(3+7\varepsilon+4\varepsilon^2)b\varepsilon\bar{k}\Phi - (1+\varepsilon)^2(1+2\varepsilon)\Phi^2]}}{2(1+\varepsilon)^2(5+8\varepsilon)}. \end{aligned} \tag{A27}$$

Summing up both tax rates we get

$$\begin{aligned} t_i^{nk} + t_j^{nk} &= \frac{(54 + 219\varepsilon + 292\varepsilon^2 + 128\varepsilon^3)b\varepsilon\bar{k} + (1+2\varepsilon)^2(3+7\varepsilon+4\varepsilon^2)\Phi}{(1+\varepsilon)^2(3+4\varepsilon)(5+8\varepsilon)} \\ &+ \frac{(2+3\varepsilon)\sqrt{(3+4\varepsilon)^2[b^2\varepsilon^2\bar{k}^2\Upsilon - 2(3+7\varepsilon+4\varepsilon^2)b\varepsilon\bar{k}\Phi - (1+\varepsilon)(1+2\varepsilon)\Phi^2]}}{(1+\varepsilon)^2(3+4\varepsilon)(5+8\varepsilon)} \\ &=: G(b, \bar{k}, \varepsilon, \Phi). \end{aligned} \tag{A28}$$

Interior Kant-Nash and Nash-Kant equilibria exist, if $\alpha > \overline{H}(b, \bar{k}, \varepsilon, \Phi) := \max\left[F(b, \bar{k}, \varepsilon, \Phi), G(b, \bar{k}, \varepsilon, \Phi)\right]$. In contrast, multiple Kant-Nash and Nash-Kant equilibria exists, if $\alpha < \underline{H}(b, \bar{k}, \varepsilon, \Phi) := \min\left[F(b, \bar{k}, \varepsilon, \Phi), G(b, \bar{k}, \varepsilon, \Phi)\right]$. ■

Symmetric countries:

Nash-Nash-equilibrium. Inserting $a_i = a_j = a$ in (17) we get

$$t_i^{NN} = t_j^{NN} = t_i^{nn} = t_j^{nn} = \frac{2b\varepsilon\bar{k}}{1+\varepsilon}. \tag{A29}$$

The associated welfare levels are

$$w_i^{NN} = w_j^{NN} = a\bar{k} + \frac{(4\varepsilon^2 - 1 - \varepsilon)b\bar{k}^2}{2(1+\varepsilon)}. \tag{A30}$$

Kant-Nash equilibrium. If country i behaves as Kantian and country j as Nashian the interior equilibrium is characterized by the intersection point of $t_i = T^{Ki}(t_j)$ and $t_j = T^{Nj}(t_i)$. Making use of (16), (20) and $a_i = a_j = a$ we obtain

$$t_i^{KN} = t_i^{kn} = \frac{\left[32\varepsilon^2 + 4\left(\Upsilon + 12\right)\varepsilon + 3\Upsilon + 17\right] b\bar{k}\varepsilon}{2(\varepsilon+1)^2(8\varepsilon+5)}, \tag{A31}$$

$$t_j^{KN} = t_j^{kn} = \frac{\left[32\varepsilon^2 + 2\left(\Upsilon + 25\right)\varepsilon + \Upsilon + 19\right] b\bar{k}\varepsilon}{2(\varepsilon+1)^2(8\varepsilon+5)}. \tag{A32}$$

The associated welfare levels are

$$w_i^{KN} = a\bar{k} + \frac{\left[32\varepsilon^4 - 36\varepsilon - 10 + 4\varepsilon^3(8+\Upsilon) + \varepsilon^2(-25+3\Upsilon)\right] b\bar{k}^2}{4(1+\varepsilon)^2(5+8\varepsilon)}, \tag{A33}$$

$$\begin{aligned} w_j^{KN} &= \frac{\left[4a(5+13\varepsilon+8\varepsilon^2)^2 + b\left[-50 - 260\varepsilon + 640\varepsilon^5 + 16\varepsilon^4(79+4\Upsilon)\right.\right.}{4(1+\varepsilon)^2(5+8\varepsilon)}, \\ &\quad + \frac{\left.\left.12\varepsilon^3(48+7\Upsilon) + \varepsilon^2(-265+27\Upsilon)\right]\right] \bar{k}}{4(1+\varepsilon)^2(5+8\varepsilon)}. \end{aligned} \tag{A34}$$

Nash-Kant equilibrium. If $a_i = a_j = a$ and if country i behaves as Nashian and country j as Kantian, the solution of $t_i = T^{Ni}(t_j)$ and $t_j = T^{Kj}(t_i)$ is given by

$$t_i^{NK} = t_i^{nk} = \frac{\left[32\varepsilon^2 + 2\left(\Upsilon + 25\right)\varepsilon + \Upsilon + 19\right] b\bar{k}\epsilon}{2(\varepsilon+1)^2(8\varepsilon+5)}, \tag{A35}$$

$$t_j^{NK} = t_j^{nk} = \frac{\left[32\varepsilon^2 - 4\left(\Upsilon - 12\right)\varepsilon - 3\Upsilon + 17\right] b\bar{k}\varepsilon}{2(\varepsilon+1)^2(8\varepsilon+5)}. \tag{A36}$$

The associated welfare levels are

$$\begin{aligned} w_i^{NK} &= \frac{\left[4a(5+13\varepsilon+8\varepsilon^2)^2 + b\left[-50 - 260\varepsilon + 640\varepsilon^5 + 16\varepsilon^4(79+4\Upsilon)\right.\right.}{4(1+\varepsilon)^2(5+8\varepsilon)}, \\ &\quad + \frac{\left.\left.12\varepsilon^3(48+7\Upsilon) + \varepsilon^2(-265+27\Upsilon)\right]\right] \bar{k}}{4(1+\varepsilon)^2(5+8\varepsilon)}, \end{aligned} \tag{A37}$$

$$w_j^{NK} = a\bar{k} + \frac{\left[32\varepsilon^4 - 36\varepsilon - 10 + 4\varepsilon^3(8+\Upsilon) + \varepsilon^2(-25+3\Upsilon)\right] b\bar{k}^2}{4(1+\varepsilon)^2(5+8\varepsilon)}. \tag{A38}$$

Kant-Kant equilibrium. Finally, the Kant-Kant tax rates are given by

$$t_i^{KK} = t_j^{KK} = \frac{\alpha}{2} = a - b\bar{k} \tag{A39}$$

and the associated welfare levels are

$$w_i^{KK} = w_j^{KK} = a\bar{k}(1+\varepsilon) - \frac{1}{2}b\bar{k}^2(1+2\varepsilon). \tag{A40}$$

Since we restrict our attention to interior Kant-Nash and Nash-Kant equilibria the feasible parameter set is given by

$$S := \left\{ (a, b, \bar{k}, \varepsilon) \middle| \alpha > \overline{H}(b, \bar{k}, \varepsilon, 0) \right\}. \tag{A41}$$

Using the definition of α and rearranging terms (A41) can be rewritten to

$$S = \left\{ (a, b, \bar{k}, \varepsilon) \middle| \frac{b\bar{k}}{a} \leq \frac{2(1+\varepsilon)^2(5+8\varepsilon)}{10 + 54\varepsilon + 91\varepsilon^2 + 48\varepsilon^3 + (2\varepsilon + 3\varepsilon^2)\Upsilon} \right\}. \tag{A42}$$

Comparison of tax rates and welfare levels. Comparing the tax rates we obtain

$$t_i^{KK} - t_i^{NN} = a - \frac{(1+3\varepsilon)b\bar{k}}{1+\varepsilon}, \tag{A43}$$

$$t_i^{KK} - t_i^{KN} = a - \frac{[10 + 48\varepsilon^3 + \varepsilon(53 + 3\Upsilon) + \varepsilon^2(90 + 4\Upsilon)]b\bar{k}}{2(\varepsilon+1)^2(8\varepsilon+5)}, \tag{A44}$$

$$t_i^{KN} - t_i^{NK} = \frac{(\Upsilon - 1)\, b\bar{k}\varepsilon}{(\varepsilon+1)(8\varepsilon+5)} > 0, \tag{A45}$$

$$t_i^{NK} - t_i^{NN} = \frac{(\Upsilon - 1)\,(2\varepsilon+1)b\bar{k}\varepsilon}{2(\varepsilon+1)^2(8\varepsilon+5)} > 0. \tag{A46}$$

$t_i^{KK} > t_i^{NN}$ follows from using the assumption $\alpha > \frac{4b\varepsilon\bar{k}}{1+\varepsilon}$ of Proposition 2(i) in (A43). Next, (A45) and (A46) yield the ranking

$$t_i^{KN} > t_i^{NK} > t_i^{NN} \quad \text{and} \quad t_j^{NK} > t_j^{KN} > t_j^{NN}. \tag{A47}$$

Comparing the welfare levels yields

$$w_j^{KN} - w_i^{KN} = \frac{[37 + 3\Upsilon + 2\varepsilon(77 + 5\Upsilon) + 4\varepsilon^2(53 + 2\Upsilon) + 4\varepsilon^2(53 + 2\Upsilon) + 96\varepsilon^3]b\varepsilon^2\bar{k}^2}{(1+\varepsilon)^2(5+8\varepsilon)^2} > 0, \tag{A48}$$

$$\begin{aligned} w_i^{KK} - w_i^{NK} &= \frac{\left[4a(5 + 13\varepsilon + 8\varepsilon^2)^2 - b\bar{k}\left[100 + 896\varepsilon^4 + 28\varepsilon^2(71 + 3\Upsilon) - 16\varepsilon^3\,(139 + 4\Upsilon)\right.\right.}{4(1+\varepsilon^2)(5+8\varepsilon)^2} \\ &\quad + \frac{\left.\left.-3\varepsilon(251 + 9\Upsilon)\right]\right]\varepsilon\bar{k}}{4(1+\varepsilon)^2(5+8\varepsilon)^2}, \end{aligned} \tag{A49}$$

$$w_i^{NN} - w_i^{KN} = \frac{[23 + 32\varepsilon^2 - 3\Upsilon + 4\varepsilon(14 - \Upsilon)]b\bar{k}^2\varepsilon^2}{4(1+\varepsilon)^2(5+8\varepsilon)^2}. \tag{A50}$$

Observe that the sign of $w_i^{NN} - w_i^{KN}$ depends only on ε and is independent of b, $\bar{k}$. The proof that $w_i^{NN} - w_i^{KN} > 0$ for $0 < \varepsilon \leq 2$ is shown in Figure 7.

Finally, verify that

$$w_i^{KK} > w_i^{NK} \quad \Longleftrightarrow \quad (a, b, \bar{k}, \varepsilon) \in S^E, \tag{A51}$$

where

$$S^E := \left\{ (a, b, \bar{k}, \varepsilon) \middle| \frac{b\bar{k}}{a} \leq \frac{4(5 + 13\varepsilon + 8\varepsilon^2)^2}{100 + 753\varepsilon + 1988\varepsilon^2 + 2224\varepsilon^3 + 896\varepsilon^4 + (27\varepsilon + 84\varepsilon^2 + 64\varepsilon^3)\Upsilon} \right\}.$$

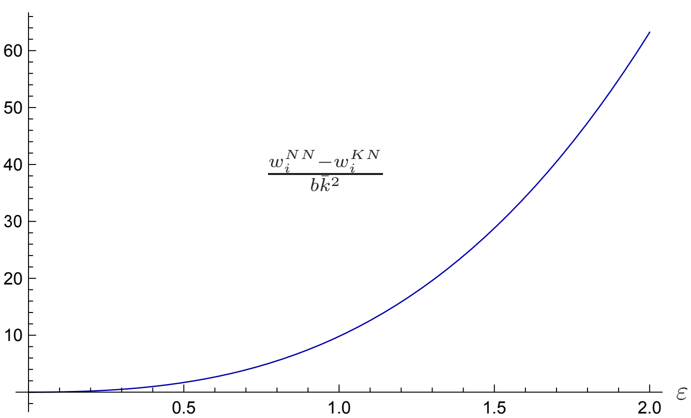

Figure 7: Welfare difference $w_i^{NN} - w_i^{KN}$