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# Distributive justice and social conflict in an AK model

## Abstract

We introduce distributive justice into a simple model of growth and distribution. Two groups ('classes') of otherwise identical, capital-rich and capital-poor individuals ('capitalists') and ('workers') are in conflict over factor (labour-capital) shares. Capitalists' (workers') ideal labour share is low (high) – but always tempered by the recognition that everyone supplies one unit of labour inelastically and desires a wage; and that the labour share impacts growth negatively in our 'AK' production economy. Social conflict is defined as the difference between the ideal labour shares of the two classes. This conflict is resolved by the two positive and three normative criteria we consider. Thus, the macroeconomy (growth, factor shares, distribution), social conflict and the methods of its resolution are jointly determined in a complete socio-economic equilibrium. We believe both this approach and our rich set of results are novel.

We consider two positive (probabilistic voting and Nash bargaining, encapsulating electoral politics and socio-political bargaining) and two normative (justice) criteria (utilitarian and Rawlsian) of conflict resolution. Greater impatience, intensified status comparisons and negative consumption externalities, greater wealth inequality and a decline in productivity exacerbate social conflict. Status comparisons and wealth inequality tend to raise the labour share under all positive and normative criteria. Finally, we propose and analyse a criterion of 'justice as minimal social friction'. Under the plausible assumption that the capitalists' overall socio-political influence (numerical strength aside) is at least as high as that of workers, all positive methods imply a smaller labour share and more inequality than all our three criteria of distributive justice. We offer a numerical illustration of the key points.

JEL-Codes: O410, O430, E250, P160, Z130.

Keywords: growth, factor shares, status, distributive justice, social conflict, social contract.

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# 1. Introduction

This paper introduces distributive justice into a simple model of growth and distribution. The paper is motivated by the general observation that inequality is generally rising among advanced economies (e.g. Piketty, 2014; Atkinson, 2015). Associated with inequality is distributive social conflict. This conflict is multi-faceted but ever-present (Bernard, 1983; Dahrendorf, 2007) whereby various actors or groups attempt to capture a larger share of the output either directly (through wage negotiations or price rises given the nominal wages) or indirectly by manipulating the political system to achieve taxes, favourable transfers, regulations, and other redistributive policies. In particular, we are concerned with the ‘functional’ distribution of income, i.e. its division into labour or capital income. The capital-labour split is often thought to be one of the ‘constants’ of macroeconomics and growth theory at about one-third; recently, however, there have been indications that the capital share is rising at the expense of the labour share (Piketty, 2014; Karabarbounis and Neiman, 2014), a development for which a consensus explanation has yet to emerge. The paper is based on the premise that factor shares are determined by social conflict, a concept to which we give a precise meaning. These shares are not only among the key determinants of income distribution; additionally, they determine the marginal revenue productivity of capital, growth and welfare. But, as Bowles (2008) emphasises, conflict breeds compromise. Our focus is on the arrangements that resolve this conflict. The positive arrangements capture key features of institutions such as the nature of electoral politics but also broader socio-political bargaining between classes. The normative criteria capture distributive justice. Our departure point is that the macroeconomy and its key aspects (labour share, functional distribution and growth) interacts with social conflict and the methods of its resolution, encompassing politics and justice.

As has been often commented, social conflict may be at the heart of many recent political developments such as the Brexit vote, the rise to power of President Trump and the rise of anti-

systemic parties in various parts of Europe. The social conflict perspective has been explored in sociology and politics, particularly in analyses of inflation (Goldthorpe, 1978, 1984; Hirsch, 1978). In macroeconomics, issues involving social conflict have received less attention; notable exceptions include Rowthorn (1977), Benhabib and Rustichini (1996) and Eggert, Itaya and Mino (2011); while income distribution is thoroughly examined in Bertola, Foellmi and Zweimüller (2006). Acemoglu, Johnson and Robinson (2002) summarises an important programme of research which analyses how the distributional conflict has historically been resolved by institutions; these institutions may be regarded as the historical embodiment of the positive and normative resolutions of the social conflict that we consider here. Most closely to our analysis here, a couple of models of the functional distribution of income (Alesina and Rodrik, 1994; Bertola, 1993) have pointed out a basic conflict of interest between workers and owners of capital and analyse its implications for tax policy. In a similar manner, Tornell and Lane (1999) analyse the conflict over fiscal policy and access to a common resource that gives rise to a commons-type problem ('voracity'). To address these issues, we build on Tsoukis and Tournemaine (2011), which provides a formal model of social conflict. By exploring the determinants of conflict, we help to cast light on the nature of such developments.

More concretely, we study a socio-macroeconomic equilibrium in which growth, income distribution and welfare are jointly determined with the functional distribution of income (factor shares). Developing the Pareto-efficient 'utility possibility frontier' (UPF), we show the basic trade-off between growth and equality as well as conflict of interest between groups.<sup>1</sup> Following Benabou (2000), we refer to the institutional arrangements that society adopts over the factor shares (our key distributional variable) and the resulting growth, inequality and welfares as the 'social contract'. Our main objective is to analyse the social contract that emerges from the application of various positive and normative welfare criteria such as voting, the Nash bargaining solution, Rawlsian justice and the utilitarian outcome. We are not aware of any previous study that applies these criteria of distributive

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<sup>1</sup> Evidence on a growth-equality trade-off is mixed, but if there is no trade-off one might ask, why do societies not support unlimited redistribution? See Aghion, Caroli and Garcia-Penalosa (1999) and Ostry, Berg and Tsangarides (2014) for reviews of the debate.

justice to a growth model. Since the economy is a production one (albeit simple), there are two-way interactions between the social contract and economic outcomes: The social contract affects growth and welfare via the marginal revenue product of capital while the economy affects the socio-political arrangements in place under the various criteria. Thus, the study of social conflict, distributive justice and the determination of factor shares in a complete socio-economic equilibrium is our focus.<sup>2</sup> We derive a number of novel results which contribute towards the understanding of factor shares and the determinants of social conflict. A second broad contribution is to propose and study the properties of a novel normative criterion of distributive justice in addition to the above, namely that of ‘minimal social friction’.

The setup is a simple AK model with agent heterogeneity over asset endowments. The lack of dynamics lends this model tractability as the initial heterogeneity of assets, which include only physical capital, is perpetuated for ever. In every other respect, all agents are identical: they all provide inelastically one unit of labour and have a common rate of time preference. In particular, there are two classes of otherwise identical capital-rich and capital-poor agents (‘capitalists’ and ‘workers’). Agents consume their identical wage, equal to the labour share (as agents are of unit mass), plus a constant fraction of their assets (physical capital); this is standard in an AK model. The higher labour share however decreases the marginal profitability of capital and thereby growth, consistently with the Euler equation of consumption growth. So, a basic trade-off arises from the fact that a higher labour income increases the wage but decreases growth; both matter for intertemporal welfare. Because capital-poor (-rich) agents rely relatively more (less) on labour income, they prefer a higher (lower) labour share. This is the main source of conflict in this model. The ‘utility possibility frontier’ (UPF) shows the trade-off in intertemporal welfare between the two groups as the labour share varies; the various justice criteria pick up different points on the UPF.

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<sup>2</sup> Focusing on factor shares rather than a particular distributive instrument, e.g. a tax, is more general as there are various mechanisms that can deliver the same factor shares (taxes, laws, market structures), as explained above. Thus, we can focus on notions of justice and not on specifics. Additionally, our analysis is connected to Piketty (2014) and the important findings there.

The positive criteria that determine the labour share and resolve the conflict are based on the probabilistic voting model and the Nash bargaining solution. In the former, the electoral outcomes reflect on average the partisan preferences of the two classes according to their numerical strengths. In the latter, two parties that represent ‘capitalists’ and ‘workers’ (capital-rich and -poor agents, respectively) bargain directly but the class sizes do not map neatly into electoral outcomes; rather, other intermediate mechanisms like culture, media, the organisational strength of ‘corporatist’ associations such as labour or employer unions, play a role. We abstract from details and only postulate a socio-political bargaining strength (given numerical strength) that affects outcomes. In addition to these positive arrangements, we also analyse normative criteria of justice such as the utilitarian criterion applied by a benevolent dictator; and Rawls’s (1971) maximin criterion of distributive justice that entails maximising the welfare of the weakest.<sup>3</sup> Furthermore, we propose a novel normative criterion of justice as ‘minimum social friction’, ‘friction’ defined as difference in welfare between the two groups normalised by aggregate welfare gains over a certain benchmark. The benevolent dictator that might adopt this criterion will need to find the optimal labour share that reduces welfare differences without compromising aggregate welfare too much. We map these criteria in terms of labour shares and growth. To pre-ambly, almost by definition, the Rawlsian criterion is the most egalitarian (giving the highest labour share), followed by our ‘minimal social friction’ criterion, followed by the utilitarian outcome and finally by the probabilistic voting outcome. Thus, a complete pecking order arises and this is one of our key results. The Nash solution will be lower than the utilitarian at least if the socio-political power of capitalists is higher than that of workers. Under this condition, our framework shows that all positive criteria imply a lower labour share than all the normative criteria which we consider.

Individual utility features social interactions (Manski, 2000) through status comparisons in consumption (‘keeping up with the Joneses’); these negative externalities have been amply

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<sup>3</sup> We apply the maximin criterion to intertemporal welfare rather than current income (*contra* Rawls, 1971).

highlighted in macroeconomics (Abel, 1990; Gali, 1994; Tsoukis, 2007; Tournemaine and Tsoukis, 2008, 2009). We show how such sociological considerations interact with the various socio-economic outcomes. *Ceteris paribus*, the strengthening of such status-related consumption externalities increase the labour share under any distributive criterion, as a way of compensating for higher conflict. Additionally, as the social contract is part of institutional design and constitutions, we contribute to these strands of literature, too (Acemoglu, 2006; Aghion, Alesina and Trebbi, 2004; Persson and Tabellini, 2004; Przeworski and Wallerstein, 1982; Tricchi and Vindigni, 2010).

We provide a clear measure of social conflict as the difference between the best outcome that the two groups would unilaterally determine for themselves; we analyse its main determinants. We show that a productivity slow-down, a rise in impatience, a rise in the intensity of status comparisons and negative consumption externalities and greater asset inequality all lead to heightened social conflict; all are candidates for understanding the conflict that has potentially led to the recent political developments mentioned above. The remainder of the paper is organised as follows. In Section 2, we develop the model. In Section 3, we highlight the nature of the conflict via the emerging UPF. In Sections 4 and 5, we analyse the positive and normative outcomes. Section 6 illustrates numerically some of the results. We conclude in Section 7. An Appendix expands on the nature of our suggested criterion of distributive justice, that of ‘minimal social friction’. An Online Appendix gives proofs of the Propositions.

## **2. A model of growth and the functional distribution of income**

We postulate an economy in continuous time with an AK production technology and a unit mass of infinitely-lived agents. The agents are identical in all respects except their endowments of physical capital (which is the only form of asset). All workers supply inelastically one unit of labour for which they receive the remuneration for labour. Let us indicate the constant share that labour



receives in production by  $0 < \gamma < 1$  (equivalently, the capital share is  $1 - \gamma$ ); thus, the remuneration of labour is  $\gamma y$ , where  $y$  (without any superscript) is aggregate output. All agents also receive remuneration for their capital holdings, equal to a fraction  $1 - \gamma$  of output times their relative capital holdings.  $k$  is physical capital, so that  $k^m/k$  is relative capital.<sup>4</sup>  $l \equiv 1$ , to be introduced later, is labour, identically equal to unity, at both individual and aggregate levels.

Total income is either consumed or invested; there is no depreciation. So, the budget constraint of individual  $m$  is:

$$\dot{k}^m = \gamma y + (1 - \gamma) \left( \frac{k^m}{k} \right) y - c^m . \quad (1)$$

In proportion to their capital endowments, the agents own the unit mass of identical firms in the economy. Because of the unit mass, the distinction between firm-specific and aggregate variables is immaterial, so we postulate an augmented AK production function which applies identically to both the aggregate and firm levels:

$$y = A k l^\lambda , \quad (2)$$

where  $0 < \lambda < 1$  is the elasticity of labour in production. Firms hire passively all factor supplies at the implicit factor prices that are conditional on the labour share. There is no government or external sector.

In this model, the capital and labour shares are indeterminate. Our point of departure is that these factor shares are not (necessarily) determined in any competitive or Walrasian equilibrium; instead,

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<sup>4</sup> The notational convention is that individual variables are with superscripts, e.g.  $c^m$ , whereas means of variables (which equal the aggregate due to unit mass) are without superscripts, e.g.  $c$ . Also, time ( $t$ ) is omitted except when strictly necessary.

they are determined through the political process or via corporatist (worker-employer union) interactions. The time-invariant labour share ( $0 < \gamma < 1$ ) is key: Our crucial assumption is that this choice variable is the object of conflict and is endogenously determined by the application of the various criteria of distributive justice we consider below.<sup>5</sup>

Therefore, the budget constraint of agent  $m$  becomes:

$$\dot{k}^m = \gamma Ak + (1 - \gamma)Ak^m - c^m . \quad (1')$$

All agents maximise intertemporal utility:

$$U^m = \int_0^{\infty} \exp\{-\rho t\} u^m(t) dt, \quad \rho > 0 . \quad (3)$$

Instantaneous utility,  $u^m(t)$ , is:

$$u^m(t) = \log(c^m(t) - \alpha c(t)), \quad 0 < \alpha < 1 . \quad (4)$$

By  $0 < \alpha < 1$  we indicate the strength of the status or 'keeping up with the Joneses' externality relative to mean consumption. Maximisation takes place subject to (1), the initial levels of  $k^m$  for all  $m$  and the standard transversality condition:

$$\lim_{t \rightarrow \infty} \exp\{-\rho t\} k^m(t) = 0 .$$

The resulting Euler equation is:

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<sup>5</sup> Walrasian equilibrium is the special case of our framework in which  $\gamma = \lambda$ .

$$\frac{\dot{c}^m - \alpha \dot{c}}{c^m - \alpha c} = (1 - \gamma)A - \rho . \quad (5)$$

The only admissible possibility is a balanced-growth path with a constant growth rate,  $g$ , common across all individuals ( $m$ ), and across both consumption and capital.<sup>6</sup> So, the status-induced consumption externality has no implication in the absence of flexible labour (see e.g. Tsoukis, 2007). These results are all standard in the AK setup. The constant growth rate, denoted by  $g$ , in particular, greatly simplifies the analysis in a number of respects: It implies both the absence of transitional dynamics and constant relative capital holdings. Thus, the initial heterogeneity in asset endowments across agents is preserved unaltered for ever: The individual capital ownership at any time equals the relative endowment ( $\chi^m \equiv k^m/k$ ) multiplied by aggregate capital ( $k$ ), which in turn grows at a constant rate ( $g$ ). As mentioned, firms readily hire all capital.

Moreover, the factor shares, however determined, are time-invariant: the chosen  $\gamma$  stays constant through time. Thus, considering  $g \equiv \frac{\dot{c}^m}{c^m} = \frac{\dot{c}}{c} = \frac{\dot{k}^m}{k^m} = \frac{\dot{k}}{k}$ , growth becomes:

$$g = (1 - \gamma)A - \rho . \quad (5')$$

Note that the labour share,  $\gamma$ , is taken as given by any individual. Inserting (5') back into the individual budget constraint (1'), we have:

$$c^m = \gamma Ak + (1 - \gamma)Ak^m - gk^m = \gamma Ak + \rho k^m . \quad (6)$$

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<sup>6</sup> In other words, the economy jumps immediately to a path characterised by  $\frac{\dot{c}^m}{c^m} = \frac{\dot{c}}{c}$  without transitional dynamics. A unique balanced growth rate is required as different steady-state growth rates would imply that the individual with the highest growth rate would asymptotically own the capital of the entire economy; while the absence of transitional dynamics is delivered by the form of the Euler equation (5) where only growth rates appear.

It is well known that in this economy, the optimal consumption of each individual is given by their fraction of the labour share ( $\gamma Ak$ ) plus a constant fraction of their capital holdings determined by their rate of impatience (Bertola, Foellmi and Zweimüller, 2006). To appreciate the results that follow, it is worth noting from (6) that, if each individual were to choose  $\gamma$  unilaterally, their optimal choice would depend inversely on their capital ownership.

We now describe capital endowments, the only source of agent heterogeneity. To ensure tractability, we assume that individuals belong to two classes of fixed size,  $m=i,j$ . Without loss of generality, class  $i$  (the ‘capitalists’) are relatively well endowed with capital, whereas class  $j$  (the ‘workers’) are poorly so; thus, the time-invariant relative capital holdings ( $\chi^m \equiv k^m / k$ ) are such that  $\chi^i > 1 > \chi^j > 0$ . Noting the unit size of the population, we have the identity  $\theta\chi^i + (1-\theta)\chi^j = 1$ . We assume that the worker class, of relative size  $1-\theta$ , is more numerous than the capitalist class of size  $\theta$ , so  $1-\theta > 0.5$ . All this and a couple of regularity restrictions are crystallised below:

**Assumption 1:**  $\rho < \theta < 0.5$

This assumption is very weak as empirically  $\rho$  is of the order of 0.02, smaller than the size of any class worth considering.

**Assumption 2:**  $\chi^j - \alpha > 0$

As shown by the data in Section 5, this assumption is also quite weak:  $\alpha$  may be of the order of 0.1 while  $\chi^j$  may be of the order of 0.4 if the capitalist class is taken to be the top 20% of the population and the worker class the rest 80%.

Using  $\chi^m \equiv k^m / k$ , (6) may be re-expressed as:

$$c^m = [\gamma A + \rho \chi^m] k . \tag{6'}$$

Aggregating over time as instructed by (3) and (4), in view of the constant growth rate (5') and the level of consumption (6), we have:

$$U_0^m = \int_0^\infty \exp\{-\rho t\} \log[\gamma A(1-\alpha)k(t) + \rho(\chi^m - \alpha)k(t)] dt . \quad (7)$$

As individual utility (4) depends negatively on aggregate consumption due to status, we have used  $c = \theta[\gamma A + \rho\chi^i]k + (1-\theta)[\gamma A + \rho\chi^j]k = [\gamma A + \rho]k$ ; aggregate consumption is exogenous to the individual. Considering that  $k(t) = \exp\{gt\}k_0$  and normalising initial aggregate capital to  $k_0=1$ , (7) becomes:

$$U_0^m = \int_0^\infty \exp\{-\rho t\} \{ \log[\gamma A(1-\alpha) + \rho(\chi^m - \alpha)] + gt \} dt .$$

Integration with (5') readily gives:

$$U_0^m = \frac{\log[\gamma A(1-\alpha) + \rho(\chi^m - \alpha)]}{\rho} + \frac{(1-\gamma)A - \rho}{\rho^2} . \quad (7')$$

As in Tournemaine and Tsoukis (2011), we find it instructive to normalise welfare to what will be achieved in Walrasian equilibrium, when the labour share is  $\gamma=\lambda$ . Thus, quasi-subtracting (7) from its Walrasian counterpart with  $\gamma=\lambda$  and using (5), we replace welfare in (7') by:<sup>7</sup>

$$W_0^m \equiv A(\lambda - \gamma) / \rho^2 + \log [A(\gamma - \lambda)(1 - \alpha) + \rho(\chi^m - \alpha)] / \rho . \quad (8)$$

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<sup>7</sup> By taking a Taylor expansion of  $U_0^m$  around  $\gamma=\lambda$  and of  $W_0^m$  around  $\gamma-\lambda=0$ , it is easy to see that, except the unimportant constant  $\frac{A(1-\lambda)-\rho}{\rho^2}$ , these are equal to a first-order approximation.

The advantage of (8) is that it allows the labour share determined below to be anchored on the Walrasian rate. Note that  $(\chi^j - \alpha)/(1 - \alpha) < 1$ . To ensure  $A(\gamma - \lambda)(1 - \alpha) + \rho(\chi^j - \alpha) > 0$ , we restrict attention to:

**Assumption 3:**  $\gamma \in (\gamma^0, 1)$ , where  $0 < \gamma^0 \equiv \lambda - \rho(\chi^j - \alpha)/(1 - \alpha)A < 1$ .

As discussed in relation to Assumption 2, this is a mild assumption too.

In the next Section, we shall show how the labour share,  $\gamma$ , affects the welfare of the two classes when growth (5') is taken into account. The central point of (7') is that a greater labour share elicits two effects: It increases consumption (6') and instantaneous utility (4) but it decreases growth (5'); therefore the effect on lifetime utility (7') or welfare (8) is uncertain. The balance between the two effects determines the optimal labour share for each class. As we show next, because labour income is equally shared whereas capital income depends on unequal asset ownership, the worker (capitalist) class prefers a higher (lower) labour share. However, even the capitalist class would like a labour share above the minimum because they also earn a labour income (in the real world, they want some of the revenues of the business to be distributed as remuneration for labour); symmetrically, the worker class does not desire a maximum labour share ( $\gamma=1$ ) as they also care about growth.

### 3. Distributive conflict and the Utility Possibility Frontier

#### 3.1: Partisan solutions

For future reference, it is useful to establish the solutions that each class would select unilaterally if it cared only about its own welfare; this arises from maximising (8) for  $m=i,j$ , with respect to  $\gamma$ , respectively. The worker class  $j$ , for instance would set:

$$\text{Max}_\gamma \quad W_0^j = A(\lambda - \gamma) / \rho^2 + \log \left[ A(\gamma - \lambda)(1 - \alpha) + \rho(\chi^j - \alpha) \right] / \rho ,$$

leading to

$$\rho \frac{\partial W_0^j}{\partial \gamma} = -\frac{A}{\rho} + \frac{A(1 - \alpha)}{A(\gamma - \lambda)(1 - \alpha) + \rho(\chi^j - \alpha)} = 0. \quad (8')$$

The two effects of the labour share ( $\gamma$ ) are, firstly, the negative effect on growth and its cumulative impact over the lifetime welfare and secondly the effect on instantaneous utility. The equality between the marginal utility of saving/growth and the marginal utility of instantaneous consumption yields the following solutions from unilateral optimisation – we call them the ‘partisan’ solutions, to be denoted by an asterisk:

$$\gamma_j^* = \lambda + \frac{\rho(1 - \chi^j)}{A(1 - \alpha)} = \lambda + \frac{\theta\rho(\chi^i - \chi^j)}{A(1 - \alpha)} > \lambda, \quad (9a)$$

Where the third expression follows from using  $\theta\chi^i + (1 - \theta)\chi^j = 1$ . Symmetrically, for the capitalist class i:

$$\gamma_i^* = \lambda + \frac{\rho(1 - \chi^i)}{A(1 - \alpha)} = \lambda - \frac{(1 - \theta)\rho(\chi^i - \chi^j)}{A(1 - \alpha)} < \lambda. \quad (9b)$$

A couple of remarks are in order:

**Remark 1:** Comparison of (9a, b) reveals that  $\gamma_i^* < \gamma_j^*$ : As mentioned, the partisan labour shares rank in reverse order to capital holdings of the two classes. Intuitively, this is because the capitalist

class enjoys higher consumption due to higher capital income, hence the second effect (essentially marginal utility) is smaller.

**Remark 2:** Assumptions 1 and 3, (9b), and (9a) imply that:

$$\gamma_i^* < \gamma^0 < \lambda < \gamma_j^*. \quad (10)$$

Recall the definition  $0 < \gamma^0 \equiv \lambda - \frac{\rho(\chi^j - \alpha)}{(1-\alpha)A} < 1$ .

It is worth emphasising that (9a, b) are unilateral, partisan solutions; in other words, everyone's preferences are for the actually chosen labour share to be  $\gamma \in [\gamma^0, \gamma_j^*]$ .<sup>8</sup> Thus, these represent the bounds of effective social conflict (no-one wishes to be outside those) and we therefore restrict the distributive justice criteria to select a labour share in that range. Though the partisan solutions will in general not be attained (at least one of them will not and probably neither if there is a compromise), their difference is indicative of social conflict, to be formalised below. Both partisan solutions (9a, b) are firstly anchored on the Walrasian outcome ( $\lambda$ ), which is useful in capturing the effects of deep fundamentals such as the nature of technology or globalisation (on which we do not dwell further). The partisan solutions are either side of the Walrasian benchmark,  $\gamma_i^* < \lambda < \gamma_j^*$ ; this implies that the latter is a contentious outcome, therefore below we compare it with the various normative and positive criteria. From (9a, b) we also see that a number of factors affect the partisan shares in a symmetrically opposite way: (a) the rate of time preference,  $\rho$ ; (b) productivity,  $A$ ; and (c) the capital holdings of each individual. Since  $\chi^i > \chi^j$ , we get  $\gamma_j^* > \gamma_i^*$ , the worker class prefers a higher labour share as it is more dependent on labour income (cf. Remark 1). Viewed slightly differently, asset inequality,  $\chi^i - \chi^j$ , widens the gap between the partisan labour shares of the two classes and therefore intensifies social conflict. Finally, the effect of status ( $\alpha$ ) is to widen the gap

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<sup>8</sup> Note the restriction placed by Assumption 3 and Remark 2.



between the partisan solutions, as the consumption differential due to heterogeneous asset holdings matters more. Formally, these effects are as follows:

**Proposition 1: Properties of the partisan solutions:**

- a)  $0 < \gamma_i^* < \lambda < \gamma_j^*$ ,
- b)  $\partial \gamma_j^* / \partial \lambda = \partial \gamma_i^* / \partial \lambda = 1$ ,
- c)  $\partial \gamma_j^* / \partial \rho > 0$ ,            and     $\partial \gamma_i^* / \partial \rho < 0$ ,
- d)  $\partial \gamma_j^* / \partial A < 0$ ,            and     $\partial \gamma_i^* / \partial A > 0$ ,
- e)  $\partial \gamma_j^* / \partial \alpha > 0$ ,            and     $\partial \gamma_i^* / \partial \alpha < 0$ ,
- f)  $\partial \gamma_j^* / \partial (\chi^i - \chi^j) > 0$ , and  $\partial \gamma_i^* / \partial (\chi^i - \chi^j) < 0$ .

Part a) replicates (10); part b) suggests that the Walrasian labour share ( $\lambda$ ), as a natural benchmark, increases commensurately both partisan shares. Clauses (c-f) follow from straightforward differentiation of (9a, b), using Assumption 3. They suggest that the rate of time preference ( $\rho$ ) increases the gap between the partisan solutions as the relevance of the common growth declines (note the  $g/\rho^2$  term in (7')), while the component of consumption financed by the remuneration of capital increases (which varies across classes); in other words, the rise in  $\rho$  heightens the importance of the source of inequality, so it pushes the partisan optimal shares in opposite directions. The next two effects are best appreciated by considering (8') and its counterpart for  $i$ , showing the marginal instantaneous utilities of  $\gamma$ . An intensified consumption externality (status,  $\alpha$ ) changes the marginal utility of instantaneous consumption for both classes - corresponding to the second component of the middle term of (8') - but in opposite ways; for the worker (capitalist) class, the marginal utility increases (decreases). This increase alters the equality of that with the marginal cost of foregone growth, leading to altered partisan labour shares. In a similar vein, a

greater asset inequality ( $\chi^i - \chi^j$ ) changes marginal utilities and therefore the partisan shares in opposite ways; here, the key is that the asset differential alters the balance between the capital-financed own consumption and the average consumption which causes the status-related externality. Finally, productivity (A) changes the marginal utility of instantaneous consumption asymmetrically; it decreases (increases) the marginal utility of the worker (capitalist) class. Combined with the positive effect on growth, this produces the effect in part d) of Proposition 1. In essence, a rise in productivity increases both the immediate consumption effect of the labour share but also enhances growth through (5'); for workers (capitalists), the former (latter) effect is of greater significance, which leads them to demand a higher (lower) labour share.

A further set of points follows as a corollary. The distance between the partisan solutions,  $\gamma_*^j - \gamma_*^i > 0$ , is a natural measure of potential social conflict. We define:

**Definition D1: The degree of social conflict is defined as  $0 < SC \equiv \gamma_j^* - \gamma_i^* < 1$ .**

The following Corollary of Proposition 1 and analyses the determinants of social conflict:

**Corollary 1: On the determinants of social conflict:**

- a)  $\partial SC / \partial \lambda = 0$ ,
- b)  $\partial SC / \partial \rho > 0$ ,
- c)  $\partial SC / \partial \alpha > 0$ ,
- d)  $\partial SC / \partial (\chi^i - \chi^j) > 0$ , and
- e)  $\partial SC / \partial A < 0$ .

Thus, potential conflict is invariant to the Walrasian benchmark. A rise in productivity (A) alleviates it: Good times moderate social conflict, thereby promoting social cohesion. More impatient societies (higher  $\rho$ ) are more prone to conflict. There is a parallel with well-known results from game theory: A greater horizon introduces repeated games and facilitates cooperate outcomes; here, a shortening of the horizon due to greater impatience gives rise to more conflict, which is akin to less coordination. Though it is outside our framework, this suggests that aging societies, in the sense that there is a greater proportion of old with less life expectancy at their point in life and less horizon, leads effectively to a greater discount rate and thereby to greater social conflict:

**Corollary 2:**

Aging societies with a greater proportion of old with less remaining life expectancy are prone to more social conflict, all else equal.

The intensity of the status motive ( $\alpha$ ) and a rise in asset inequality ( $\chi^i - \chi^j$ ) also exacerbate social conflict. These results are interesting in their own right, intuitive and novel. Currently, news headlines are dominated by Brexit, President Trump and the rise of anti-systemic parties (among others), developments that many commentators agree are signs of deeper societal anxieties and conflicts; our analysis can shed useful lights on the nature of such developments. For instance, a widening in wealth inequality (exogenous here) could be one of the causes, as it increases social conflict (part d) of Corollary 1). We next turn to the utility possibility frontier that connects these partisan solutions, along which a compromise may be sought.

*3.2: Welfare and the Utility Possibility Frontier (UPF)*

As mentioned, social conflict exists only in the range  $\gamma \in [\gamma^0, \gamma_j^*]$ . From the concavity of welfare

(8) and Assumptions 1-3, we note that  $\frac{\partial W_0^i}{\partial \gamma} < 0$  as  $\gamma > \gamma_i^*$  and  $\frac{\partial W_0^j}{\partial \gamma} < 0$  as  $\gamma < \gamma_j^*$ ; the signs of

derivatives are mixed only for  $\gamma_i^* < \gamma < \gamma_j^*$ . Outside this range, their choices coincide. Therefore, invoking Assumption 3 and Remark 2, we restrict attention to the range of labour shares:

$$\gamma^0 < \gamma < \gamma_j^*.$$

A diagrammatic exposition is given in Figure 1 below in  $(W_0^i, W_0^j)$  space. The maximal permissible welfare levels for the two classes are  $W_*^j \equiv W_0^j|_{\gamma^j=\gamma_j^*}$  and  $W_*^i \equiv W_0^i|_{\gamma^i=\gamma^0}$ . Indicative indifference curves are also shown. Point V is defined as  $V \equiv (W_{\gamma_*^j}^i, W_{\gamma^0}^j)$ , where  $W_{\gamma_*^j}^i$  and  $W_{\gamma^0}^j$  are the welfares of the two respective classes at the labour share that is the partisan optimal for the other class. The restriction  $\gamma^0 < \gamma < \gamma_j^*$  implies that we only consider the region northeast of the dotted lines.

In the region of interest, as the labour share varies, there is a trade-off between the welfares of the two classes – the Utility Possibility Frontier (UPF). Figure 1 depicts the UPF as a downward-sloping concave curve. The welfare of the capitalist class is on the vertical axis, hence the higher up we are on the UPF, the lower the labour share. The entire area of interest is above the  $45^0$ , reflecting the fact that by definition the welfare of the capitalist class (i) is higher than that of the worker class (j). Hence, even with their partisan labour share, workers will be worse off than capitalists; at point R (Rawlsian – see below), corresponding to  $\gamma = \gamma_j^*$ , they will have narrowed the welfare gap as much as possible. It follows from (8') and the corresponding equation for i that the slope of the UPF is:

$$\frac{dW_0^i}{dW_0^j} = \frac{\partial W_0^i / \partial \gamma}{\partial W_0^j / \partial \gamma} = \frac{-\frac{1}{\rho} + \frac{1-\alpha}{A(\gamma-\lambda)(1-\alpha) + \rho(\chi^i - \alpha)}}{-\frac{1}{\rho} + \frac{1-\alpha}{A(\gamma-\lambda)(1-\alpha) + \rho(\chi^j - \alpha)}} < 0, \quad \text{for } \gamma \in [\gamma^0, \gamma_j^*]. \quad (11)$$

Proposition 2 dwells on the slope of the UPF as that will be important in gaining intuition on more substantive results below:

**Proposition 2: On the slope of the UPF (11):** For  $\gamma \in [\gamma^0, \gamma_j^*]$ :

- a)  $\frac{dW_0^i}{dW_0^j} < 0$ ,
- b)  $d \frac{dW_0^i}{dW_0^j} / d\gamma < 0$ ,
- c)  $\text{sgn} \left\{ \partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial A \right\} = -\text{sgn} \{ \gamma - \lambda \}$ ,
- d)  $\partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial \alpha > 0$ ,
- e)  $\partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial (\chi^i - \chi^j) > 0$ .

The concavity of UPF is important as several of the solutions below intuitively pick up the point of tangency between the UPF and an indifference curve of the criterion in question. All clauses follow from straightforward comparative statics analysis of (11) using (9a, b) and Assumptions 1-3 as necessary. Part a) suggests that there is conflict between the classes over the labour (equivalently: capital) share in the region of interest for the labour share. Part b) shows that the UPF is concave – it becomes steeper as  $\gamma$  increases, therefore as we move downwards. Part c) suggests that a rise in productivity has an ambiguous effect on the slope of the UPF, depending on whether  $\gamma > \lambda$  or not. This is because a rise in A raises growth but lowers (raises) marginal utility from instantaneous consumption if  $\gamma > (<) \lambda$ . For a labour share above the Walrasian level ( $\gamma > \lambda$ ), marginal utility falls for both classes but relatively more for the workers whose consumption is smaller. Thus,  $\left| \frac{dW_0^i}{dW_0^j} \right|$  rises.

The opposite happens if the labour share is below the Walrasian level. Thus, a rise in productivity

makes the UPF steeper (flatter) if  $\gamma > (<) \lambda$ . A rise in the intensity of the status motive makes the UPF flatter as implied by part d); finally, a rise in the inequality of asset endowments also makes the UPF flatter. The last two effects are particularly interesting; as will be shown later, they entail a higher labour share under any given distributive criterion.

**INSERT FIGURE 1 HERE**

The main question of the paper is what point on the UPF is picked up; this choice depends on what criterion of distributive justice will be followed, or as we have termed it, the nature of the social contract. We present and compare two positive and two normative criteria below.

#### **4. Positive and normative criteria**

In this Section, we analyse the points on the UPF, therefore labour share, that the various positive and normative criteria pick up. Recall that the labour share increases as we go down the UPF.

##### *4.1: Two positive outcomes: Electoral competition and the Nash bargaining solution*

The basic model of electoral competition is based on the median voter; however, this analysis is trivial here. As there are two classes of identical individuals, the more numerous class (here, workers) would always win the political contest; the median-voter model trivially predicts  $\gamma = \gamma_j^*$ . Instead, the probabilistic voting model of Lindbeck and Weibull (1987) and Persson and Tabellini (2000, Chapter 3) assumes that voter preferences map stochastically onto their vote for the two parties in a two-party electoral competition. The model predicts that the equilibrium implements the maximum of weighted-average welfare, where the weights measure voter responsiveness multiplied by group size. In this vein, and with equal voter responsiveness, we may argue that electoral competition on average returns the two partisan choices,  $\gamma_i^*$  and  $\gamma_j^*$ , weighted by the sizes of the two classes:

$$\gamma^{PV} = \theta\gamma_i^* + (1-\theta)\gamma_j^* . \quad (12)$$

The average voter outcome is denoted ‘PV’ in Figure 2. Using (9a, b), we first establish:

$$\gamma^{PV} = \theta\gamma_i^* + (1-\theta)\gamma_j^* = \lambda \quad (12')$$

Strikingly, the labour share that the probabilistic-voting electoral competition model delivers is the Walrasian outcome.

We next turn attention to the Nash bargaining solution (see e.g. Roemer, 1996, Chapter 2). We take this solution to capture the underlying fundamentals of bargain between the two groups: Apart from numerical strength, this is also based on broad ideological influence (e.g. through the media, think-tanks, education), political cohesion (affine parties and trade-unions or employer organisations) and organisational strength such as ability to do lobbying or fund campaigns and institutions.<sup>9</sup> Our argument is that, in addition to numerical strength, these fundamentals determine the policies (e.g. redistributive taxation), laws (such as minimum wages and anti-monopoly regulation) and institutions (such as those in the labour market) that in turn determine outcomes. In other words, this is a positive criterion, predicting what happens on the basis of the broad strength of the two classes. We capture the per capita relative political, organisational and ideological strength of the capitalist class by  $0 < \phi < 1$ :  $\phi \rightarrow 1$  (0) signifies that capitalists (workers) are omnipotent, while the case of  $\phi = 0.5$  signifies equal per capita strength.

Analytically, the Nash bargaining solution involves the maximisation of the product of welfare improvements over a disagreement point. Here it is important to recall that when we moved from

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<sup>9</sup> Much of this influence may in fact be endogenous, depending on the capital endowment, but we ignore this point here.

(7') to (8), we effectively removed the constant  $\frac{A(1-\lambda)-\rho}{\rho^2}$  from welfare (cf. Footnote 7). It is important to re-instate that into welfare, to get the following welfare improvements for  $m=i,j$ :<sup>10</sup>

$$W_0^m - \bar{W}_0^m + \Lambda / \rho^2, \quad \Lambda \equiv A(1-\lambda) - \rho > 0$$

The overbar indicates the welfare in the ‘outside option’, i.e. when disagreement occurs and bargaining breaks down. In that case, the assumption we make is that there is no production and individuals subsist by consuming their capital endowment. Schematically,  $A=0$  so that welfare in the case of disagreement becomes:

$$\bar{W}_0^m = \log [\rho(\chi^m - \alpha)] / \rho, \quad m=i,j.$$

Taking a Taylor approximation around  $A=0$ , welfare is written as:

$$W_0^m \approx \frac{\log [\rho(\chi^m - \alpha)]}{\rho} + \frac{A(\lambda - \gamma)}{\rho^2} + \frac{A(\gamma - \lambda)(1 - \alpha)}{\rho^2(\chi^m - \alpha)}$$

Therefore the improvement over the ‘outside option’ is:

$$W_0^m - \bar{W}_0^m + \frac{\Lambda}{\rho^2} = \frac{A(\gamma - \lambda)(1 - \chi^m)}{\rho^2(\chi^m - \alpha)} + \frac{\Lambda}{\rho^2} > 0 \quad (13)$$

Accordingly, the Nash bargaining criterion is given by:

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<sup>10</sup> The reason why we should add  $\frac{A(1-\lambda)-\rho}{\rho^2}$  is that without, it welfare improvements may be of either sign, thus invalidating the Nash bargaining approach which requires that there be positive improvements over the ‘outside option’.



$$\text{Max}_\gamma (W_0^i - \bar{W}_0^i + \Lambda / \rho^2)^{\phi\theta} (W_0^j - \bar{W}_0^j + \Lambda / \rho^2)^{(1-\phi)(1-\theta)}, \quad 0 < \phi < 1,$$

s.t. the UPF.

As per above, the capitalists' overall power is  $\theta\phi$ , the product of numerical and socio-political relative per capita strength, while that of the workers is  $(1-\phi)(1-\theta)$ . We call the iso- $(W_0^i - \bar{W}_0^i + \Lambda)^{\phi\theta} (W_0^j - \bar{W}_0^j + \Lambda)^{(1-\phi)(1-\theta)}$  curve in  $(W_0^i, W_0^j)$  space the indifference curve of the Nash product ('Nash indifference curve' for short); it is given by:

$$-\phi\theta \frac{dW_0^i}{(W_0^i - \bar{W}_0^i + \Lambda / \rho^2)} + (1-\phi)(1-\theta) \frac{dW_0^j}{(W_0^j - \bar{W}_0^j + \Lambda / \rho^2)} = 0, \quad (14)$$

With (13) and (14), the slope of the Nash indifference curve is:

$$\frac{dW_0^i}{dW_0^j} = - \frac{(1-\phi)(1-\theta)}{\phi\theta} \frac{\frac{\chi^j - \alpha}{A(\gamma - \lambda)(1 - \chi^j) + \Lambda(\chi^j - \alpha)}}{\frac{\chi^i - \alpha}{A(\gamma - \lambda)(1 - \chi^i) + \Lambda(\chi^i - \alpha)}} < 0. \quad (14')$$

This is a convex curve against the origin as by (13) both numerator and denominator are positive.

Its properties are summarised below:

**Proposition 3: On the slope of the Nash indifference curve, (14'):**

- a)  $\frac{dW_0^i}{dW_0^j} < 0,$
- b)  $\partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial \gamma > 0,$

- c)  $\text{sgn} \left\{ \partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial A \right\} = \text{sgn} \{ \gamma - \lambda \},$
- d)  $\text{sgn} \left\{ \partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial \alpha \right\} = \text{sgn} \{ \gamma - \lambda \},$
- e)  $\text{sgn} \left\{ \partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial \chi^i \right\} = \text{sgn} \{ \gamma - \lambda \},$
- f)  $\text{sgn} \left\{ \partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial \chi^j \right\} = -\text{sgn} \{ \gamma - \lambda \},$
- g)  $\partial \left( \frac{dW_0^i}{dW_0^j} \right) / \partial \phi > 0.$

The first-order condition for the Nash bargaining solution requires that the Nash indifference curve be tangent to the UPF; the respective slopes (14') and (11) must be equalised. The Nash bargaining solution is denoted 'N' in Figure 2 and in superscripts. Then, Propositions 2 and 3 yield:

**Proposition 4: Determinants of the Nash bargaining solution,  $\gamma^N$ :**

- a)  $\text{sgn} \{ \partial \gamma^N / \partial A \} = -\text{sgn} \{ \gamma^N - \lambda \},$
- b) To a first approximation, i.e. using  $g \approx \rho$ ,  $\partial \gamma^N / \partial \alpha > 0$ , if:
  - i) either  $\gamma^N < \lambda$ ,
  - ii) or  $\gamma^N > \lambda$ , at least if (sufficiency condition)  $\alpha < \chi^j$ ,
- c) To a first approximation, i.e. using  $g \approx \rho$ ,  $\partial \gamma^N / \partial (\chi^i - \chi^j) > 0$  if:
  - i) either  $\gamma^N < \lambda$ ,
  - ii) or  $\gamma^N > \lambda$ , at least if (sufficiency condition)  $\alpha < \chi^j < 1 - \theta(1 - \alpha)$ ,
- d)  $\partial \gamma^N / \partial \phi < 0$ ,
- e)  $\lim_{\phi \rightarrow 0} \gamma^N = \gamma_j^*$  and  $\lim_{\phi \rightarrow 1} \gamma^N = \gamma_0$ ,

f) There exists a  $0.5 < \bar{\phi} < 1$  such that  $\gamma^N|_{\phi=\bar{\phi}} = \lambda$ ,

g)  $\gamma^N < (>) \lambda$  iff  $\phi > (<) \bar{\phi}$ .

The key point behind Proposition 4 is that when the UPF becomes flatter (steeper) and the indifference curve of the Nash product steeper (flatter), the labour share of the Nash bargaining solution ( $\gamma^N$ ) becomes higher (less). The results show that the effect of productivity is ambiguous, as the UPF becomes steeper (flatter) with A if  $\gamma^N > (<) \lambda$ ; since the Nash indifference curve becomes flatter with A, the labour share decreases at least if  $\gamma^N > \lambda$ . The effects of status and asset inequality generally lead to a flatter UPF and steeper Nash indifference curve, leading to a higher labour share, thus offsetting to some extent the effects of either by a greater reliance on the egalitarian labour share. While these results are straightforward in the case of  $\gamma^N < \lambda$ , they require a mild auxiliary assumption and sufficiency conditions in the more important case of  $\gamma^N > \lambda$ ; notably, they are approximate (derived under  $g \approx \rho$ ) and complemented by sufficiency conditions:  $\alpha < \chi^j$ , i.e. capital inequality is not too high, or  $\alpha < \chi^j < 1 - \theta(1 - \alpha)$ , that inequality is not extreme (neither too little or too low).<sup>11</sup> Finally, the capitalist influence ( $\phi$ ) makes the Nash indifference curve flatter; this decreases the labour share.

Part (a) of Proposition (4) has an interesting implication in terms of the growth-equality trade-off. As mentioned in the Introduction, this has been of some interest in the literature, with ambiguous results. Our analysis provides an explanation for a possible such trade-off. If we take productivity as the main exogenous determinant of growth, a rise in A generates a higher growth rate but may also imply a lower labour share (if  $\gamma^N > \lambda$ ), and therefore a more unequal society. This is essentially because a higher productivity boosts current production and encourages the social partners to reach

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<sup>11</sup> The underlying reason for the extra complications in these results (clauses b and c) is that the Nash solution fixes the proportions of welfare improvements over the outside option, and the effect of parameters on welfare at that point as well as welfare at the solution must be traced.

a compromise that favours growth more, at the expense of equality. To our knowledge, this political-economy interpretation of the growth-equality trade-off is novel.

**Corollary 3: On the growth-equality trade-off:**

According to the Nash bargaining solution, if  $\gamma^N > \lambda$ , a rise in productivity increases both growth and inequality.

*4.2: Two normative criteria: ‘Rawlsian’ justice and the utilitarian outcome*

Rawls’s (1971) celebrated maximin criterion suggests that, behind the ‘veil of ignorance’, all individuals agree to adopt policies that maximise the welfare of the poorer, worker class (j), as in that state, no individual knows what class they belong to. Therefore, the criterion entails:

$$\text{Max}_\gamma W_0^j, \quad \text{s.t. the UPF,}$$

and hence, the Rawlsian outcome (indicated by R in Figures 1 and 2) coincides with the partisan choice of group j (9a):

$$\gamma^R = \gamma_j^* \tag{15}$$

Under the utilitarian outcome, on the other hand, a weighted average of welfares is maximised, with the weights being the group sizes. This may be rationalised as the choice made by a benevolent dictator caring about the ‘public interest’. The criterion entails:

$$\text{Max}_\gamma \theta W_0^i + (1-\theta)W_0^j, \quad \text{s.t. the UPF.}$$

Again, the iso-welfare line (called the ‘Utilitarian line’) is given by:

$$\theta \frac{\partial W_0^i}{\partial \gamma} + (1-\theta) \frac{\partial W_0^j}{\partial \gamma} = 0. \quad (16)$$

Therefore the indifference curve is a straight line with slope:

$$\frac{dW_0^i}{dW_0^j} = -\frac{1-\theta}{\theta} \quad (16')$$

The first-order condition requires that the Utilitarian line be tangent to the UPF; and that the slopes (11) and (16') be equalised. The Utilitarian outcome is indicated as 'U' in superscripts and in Figure 2 below. Application of Proposition 2, noting the straight indifference curve (16'), leads to the following results:

**Proposition 5: On the Utilitarian labour share,  $\gamma^U$ :**

- a)  $\gamma^U > \lambda$ ,
- b)  $\partial \gamma^U / \partial A < 0$ ,
- c)  $\partial \gamma^U / \partial \alpha > 0$ ,
- d)  $\partial \gamma^U / \partial (\chi^i - \chi^j) > 0$ .

All results follow from the linear indifference curve (16') and the UPF (11). Part (a) follows from a few tedious manipulations, relegated to the Online Appendix; note that the result is unambiguous. Thus, the benevolent planner would dictate a higher labour share than the competitive outcome. The other parts follow directly from the relevant parts of Proposition 2, keeping in mind that  $\gamma^U > \lambda$ . A rise in productivity will lead the planner to reduce the labour share as current consumption has increased, thereby now boosting growth. Not surprisingly, to offset partly the negative externality

arising from social comparisons, one must have a higher labour share which is relatively more egalitarian, therefore offers less scope for social comparisons. Likewise, s/he would want to offset a more unequal distribution of the exogenous capital endowments in the same way, i.e. by shifting more income towards labour which is equally distributed. Thus, these results are qualitatively similar to those under the Nash bargaining solution.

#### 4.3: Comparison

The first comparison will be between the electoral and utilitarian outcomes. (12') and part (a) of Proposition (5) immediately yield:

$$\gamma^U > \gamma^{PV} = \lambda \tag{17}$$

Second is the comparison between the electoral (PV) and Nash solution outcomes. Parts (f) and (g) of Proposition 4 may be summarised as:  $\gamma^N(\phi) < (>) \lambda = \gamma^{PV}$  iff  $\phi > (<) \bar{\phi}$ , with  $0.5 < \bar{\phi} < 1$ . In other words, a higher per capita socio-political strength of capitalists relative to that of the workers is required in order for the Nash bargain solution to emulate the electoral outcome based on the probabilistic voting model (where  $\gamma = \lambda$ , cf. 12'). This seemingly counterintuitive result is due to the fact that capitalists have a higher welfare at the outside option, where there is no production and individuals simply consume their capital endowment. As a result of their worse position in the outside option, keeping numbers and socio-political influence constant, workers bargain harder.<sup>12</sup> Thus, the capitalists' required degree of socio-political strength needs to be relatively higher in order for direct bargain to emulate the electoral result. Putting it more broadly, the democratic-electoral outcome in our context is less egalitarian than the direct socio-political bargain, in the sense that it delivers the same labour share that the latter would determine under a relatively higher socio-political influence of the capitalists. Although the Nash solution is a positive and not a

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<sup>12</sup> In the words of Binmore (2005, Ch. 11, p. 180): 'People in need get more because their desperation leads them to bargain harder.'

normative criterion, this comparison is rather striking and novel to our knowledge. In any case, it is worth investigating further in more general contexts.

**Corollary 4: On the comparison between the electoral outcome and the Nash solution:**

The electoral outcome (based on the probabilistic-voting model) is less egalitarian than the Nash bargaining solution in the sense that the former determines the same labour share as the latter under a relatively higher socio-political influence of the capitalists.

Furthermore, the comparison between the Nash bargaining solution and the utilitarian outcome can be done with reference to the respective indifference curves. Re-writing (14') and equating to (16'), we have:

$$\frac{(1-\phi)(1-\theta)}{\phi\theta} \frac{-A(\gamma-\lambda)(\chi^i-1)/(\chi^i-\alpha)+\Lambda}{A(\gamma-\lambda)(1-\chi^j)/(\chi^j-\alpha)+\Lambda} = \frac{1-\theta}{\theta}$$

It is easily seen that at least if  $0.5 < \phi < \bar{\phi}$ , then the absolute slope of the Nash indifference curve (the left side of the equality) is smaller than that of the Utilitarian counterpart (the right side);<sup>13</sup> therefore the Nash indifference curve is flatter than the Utilitarian one, implying that  $\gamma^N < \gamma^U$ . On the other hand, for  $\bar{\phi} < \phi < 1$ , then  $\gamma^N < \lambda$  by Proposition (4g), so that by (17), again  $\gamma^N < \gamma^U$ .

It follows that  $\gamma^N < \gamma^U$  at least for (sufficient condition)  $\phi > 0.5$ . In other words, if capitalists' per capita influence is greater than that of the workers', then this is sufficient for the socio-political Nash bargain to deliver a labour share that is lower than that of the utilitarian outcome. We bring all these results together below:

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<sup>13</sup> Note that both numerator and denominator are positive as improvements over the welfare of the disagreement, or 'outside', option; by (13), they are both positive.

**Proposition 6: Relation between labour shares under different criteria:**

- a)  $\gamma_i^* < \gamma^{PV} = \lambda < \gamma^U < \gamma^R = \gamma_j^*$ ,
- b) At least if  $\phi > 0.5$ , then  $\gamma^N < \gamma^U$ ,
- c) At least if  $\phi > 0.5$ ,  $\max\{\gamma^{PV}, \gamma^N\} < \min\{\gamma^U, \gamma^R\}$ .

The higher relative influence of the capitalists is a realistic case; one may think of media influence, party-political donations, etc. Thus, the plausible possibility emerges that both positive criteria deliver lower labour shares than under both normative criteria; this is the case depicted in Figure 2.

**INSERT FIGURE 2 HERE**

## **5. Justice as minimal social friction**

We finally introduce a third normative criterion and concept of justice, based on minimising ‘social friction’, defined below. To our knowledge, this concept has not been proposed before and is therefore another key contribution of this paper. ‘Social friction’ may be defined as the degree of conflict divided by the degree of synergy in society. Conflict is captured by the (group size-weighted) difference in welfare between the rich and the poor; while synergy is the sum of welfares over and above a benchmark (point V, below). In other words, synergy is the aggregate welfare above a threshold, while conflict is the difference between classes. Thus, social friction may be defined as:

**Definition 2: Social friction F:**



$$F \equiv \frac{(\theta W_0^i - (1-\theta)W_0^j)^2}{(\theta W_0^i + (1-\theta)W_0^j - \bar{V})^2}. \quad (18)$$

Social friction is defined as the difference in group welfares divided by the improvement in aggregate welfare over the reference point:  $\bar{V} > 0$  is a benchmark aggregate welfare, defined as:

$$\bar{V} \equiv \theta W_{\gamma_j^*}^i + (1 - \theta)W_{\gamma^0}^j. \quad (19)$$

The benchmark  $\bar{V}$  aggregates  $W_{\gamma_j^*}^i$  and  $W_{\gamma^0}^j$  in proportion to group size. The square of the numerator in (18) is taken in order to rule out negative quantities; and the denominator is squared for symmetry.

Based on this, distributive justice as minimal social friction may be defined as,

$$\text{Min}_{\gamma} F, \quad \text{s.t. the UPF},$$

i.e., as the point on the UPF minimises this social friction. This criterion combines both the maximisation of joint welfare implicit in the utilitarian criterion and the minimisation of difference implicit in the Rawlsian criterion. It is motivated by the aim to minimise welfare differences (egalitarianism) whilst taking into account the effects on growth and welfare improvement (efficiency implications). So, there are two ways to reduce social friction: One is to reduce inequality, while the other to increase the average. Policies that increase inequality may be tolerated if they deliver a sufficiently strong increase in the total ‘pie’. These issues can be meaningfully analysed in the present production economy.

Setting a labour share that minimises F yields a point of the UPF where the following holds:

$$\frac{dW_0^i}{dW_0^j} = \frac{(1+\sqrt{F})(1-\theta)}{(1-\sqrt{F})\theta} . \quad (20)$$

The Appendix shows the indifference map described by (20); it is shown that the indifference curves are downward-sloping and convex in the quadrant northeast of point V' of Figure B1. We have  $dW_0^i / dW_0^j < 0$  in the area of interest. Importantly also, F falls as we move outwards in the northeast region of the diagram, so the usual tangency-point geometry applies. Thus, in that region,

F is high enough that  $1-\sqrt{F} < 0$ . Since the Utilitarian solution involves  $\frac{dW_0^i}{dW_0^j} = -\frac{1-\theta}{\theta}$  and since

$(1+\sqrt{F})/(1-\sqrt{F}) < -1$ , we have that the ‘minimal social friction’ criterion gives a point (denoted by F) with a steeper slope along the UPF than the utilitarian outcome, therefore the labour share thus determined will be between the utilitarian and Rawlsian criteria. In other words, the ‘minimal friction’ criterion is more egalitarian than the utilitarian criterion, even though it also pays attention to the size of the pie as well as its distribution and incorporates the fact that a higher labour share hurts growth and welfare; the resulting higher labour share is due to the fact that the minimal friction criterion pays more attention than the utilitarian one to the differences between classes. Thus, this criterion may be seen as a half-way house between the equality-neutral utilitarian criterion and the extremely inequality-averse Rawlsian criterion. We summarise:

**Proposition 7: The labour share according to the ‘minimal social friction’ criterion:**

$$\gamma^U < \gamma^F < \gamma^R .$$

As a corollary, we can extend Proposition 6 part (c) to argue that all our normative criteria require a higher labour share, and more equality, than all the positive criteria at least if  $\phi > 0.5$ :

### **Corollary 5: Relation between positive and normative outcomes**

At least if  $\phi > 0.5$ , then we have:  $\max\{\gamma^{PV}, \gamma^N\} < \min\{\gamma^U, \gamma^F, \gamma^R\}$ .

## **5. Numerical illustration**

This Section provides a simple numerical illustration of various points made above. Table 1 describes our basic wealth data for a few economies. Countries are in column 1; in all cases, the unit where this applies is the household except in the US where it is the family. Column 2 gives the wealth share of the 5<sup>th</sup> quantile from Davies, Sandström, Shorrocks and Wolff (2011, T.7); the time is around 2000. We focus on the 5<sup>th</sup> quantile as we assume schematically a ‘80-20’ size ratio between the two classes:  $\theta=0.2$ . To see that the US and India have a great degree of wealth concentration is no surprise; the surprise here is the great such concentration shown by Sweden. Column 3 has wealth Gini coefficients from the same source (T.9). Columns 2, 3 and 7 (on which we comment below) are the only data we use; the rest of Table 1 as well as Table 2 show our calculations. Columns 4 and 5 gives the implied average capital holding of the 1<sup>st</sup> and 5<sup>th</sup> quintile, respectively, calculated in the way shown. Columns 6 and 7 are explained below.

### **INSERT TABLE 1 HERE**

Table 2 then proceeds to calculate a number of parameters; the underlying assumptions are:

- $\theta=0.2$  as mentioned;
- $\rho=0.02$ , as is standard;
- $\varphi=2$  (so that capitalist influence is twice the size of the class);
- $A=1$ ;
- $\alpha=0.1$ ; this parameter is perhaps the one where the least empirical guidance is available in the literature;

- $\lambda=0.60$ ; with US data (1953-98), Klump, McAdam and Willman (2007, Fig. 1) estimate the competitive capital share (i.e. net of pure economic profit) close to 0.2; but this seems to be somewhat on the low side of what is commonly assumed; e.g., with international data, Caselli (2007, Fig. VII) estimates capital shares between 0.2 to 0.5, so 1/3 is closer to an average, leading to the present assumption which is on the conservative side. ILO and OECD (2015, Figure 4) gives  $\lambda=0.65$ . Jones (2016, Fig. 2016), using data from Karabarbounis and Neiman (2014), shows that the capital share has been increasing since 2000 in the US, approaching 40% in about 2010, and commensurately the labour share has been falling.

Columns 2 and 3 calculate the partisan shares,  $\gamma^*_j$  and  $\gamma^*_i$ ; they are mostly of the order of 0.6 to 0.67. But note that  $\gamma_0=0.60$  is imposed as the lower bound of  $\gamma$ , so that  $\gamma^*_i$  is not used, as discussed in the text. The estimate of social conflict SC (Definition D1) is shown in column 3 (which is the percentage difference between the partisan shares). An alternative definition (SC1 - not shown) would record the percentage difference between welfares at the partisan points (using  $\gamma^*_j$  and  $\gamma_0$ ) – column 4. While the social conflict shown by SC is rather small (of the order of 0.1-0.15, an indication of the fact that even partisan shares are not greatly different), the conflict in maximum potential welfares (SC1) is an order of magnitude larger, of the order of 0.4, showing the importance of the wealth holdings for welfare but also the importance of even small differences in the labour shares. Note that a greater wealth share of the 5<sup>th</sup> quantile implies a lower optimal labour share and a greater welfare for that group. It also implies a greater social conflict, however measured; thus, economies with a greater wealth inequality such as US and India show a greater degree of conflict.

**INSERT TABLE 2 HERE**

Columns 6 and 7 of Table 2 show the labour share under the utilitarian and Nash bargaining solutions, respectively. The two-decimal truncation imposed here allows hardly any variation; but there is a slight variation: The utilitarian criterion (that is the same as the Nash solution under  $\phi=1$ , i.e. a smaller capitalist influence) yields a marginally higher labour share as expected; and more unequal societies have larger labour shares under both criteria, consistently with Propositions 4 and 5, parts (d). Furthermore, not shown are the estimates of social friction, which are ordered as follows:

$$\sqrt{F(\gamma^U)} < \sqrt{F(\gamma^{*j})} < \sqrt{F(\gamma^N)} < \sqrt{F(\gamma_0)} .$$

This demonstrates the fact that this criterion does not pick up the Rawlsian outcome, which has a higher degree of friction, but an outcome closer perhaps to the utilitarian criterion.

Finally, based on the utilitarian labour shares of column 6 of Table 2, we can construct the relative incomes (gross of investment) of the 1<sup>st</sup> quintile ('lower' group) according to the formula:

$$\frac{y^j}{y} = \frac{(1-\gamma)Ak + \gamma Ak \chi^j}{(1-\gamma)Ak + \gamma Ak} = (1 - \gamma) + \gamma \chi^j .$$

We can then proceed to construct the Gini coefficient of income. Using a formula due to Dorfman (1979, eq. 8) for the Gini coefficient in step densities, as is the case here, with  $E(\chi)=1$ , the Gini (G) of the above ratio is given by the convenient formula:

$$G = \gamma(1-\theta)(1 - \chi^j) .$$

The Gini thus calculated is given in column 6 of Table 1; it is juxtaposed to the real income Gini (column 7) from OECD data (2014). This reveals that the calculated Gini underestimates the actual

income inequality; in other words, our assumption of a two-class split of society is simplistic in this respect. Work already started aims to redress this deficiency by allowing for more heterogeneity than the two-class society hypothesised here allows; more details are given in Conclusions.

## **6. Conclusions**

At the interface between economics, philosophy and other social disciplines, distributive justice is gaining attention as inequality is arguably rising in most advanced economies. Our contribution is to examine it in a simple aggregate production economy which captures the growth implications of any distributive arrangements. In particular, we focus on the ‘functional’ division of income into capital and labour shares as a key aspect of income distribution; considered as fixed at about one to three, the ratio of factor shares is now reportedly changing. Our analysis casts light on factor shares and their development but the paper is primarily about wider issues related to distributive justice. We take a social conflict approach to all these issues: There is a multi-faceted social conflict among the beneficiaries of the capital and labour shares. The main thrust of the paper is to explore the arrangements, positive and normative, that resolve the conflict. The normative arrangements are informed by notions of distributive justice, while the positive ones capture key features of political institutions. Additionally, we give a precise meaning to this conflict, analyse its nature and consider various positive and normative resolutions of it. We emphasise the interaction between macroeconomic outcomes (growth, welfare) and such possible resolutions. We call the totality of the institutional/constitutional arrangements and their macroeconomic implications the ‘social contract’. Despite the importance of these topics, there is as yet only limited integration of such issues into standard macroeconomic theory, with notable exceptions; this paper’s overall contribution is in this direction.

More specifically, we introduce conflict about factor shares into a standard AK model. We highlight a dual trade-off, namely between the welfare of different groups and between growth and equality. The main part of our analysis involves considering various normative and positive criteria of justice by which the labour (equivalently: capital) share is determined. We analyse the implications of these criteria on growth and welfare. We define social conflict as the difference between the ‘partisan’ labour shares preferred by ‘capitalists’ and ‘workers’. An important set of results concerns the determinants of social conflict. A further contribution is to propose a novel (normative) criterion, that of minimising ‘social friction’.

There is a unit mass of identical firms and two classes of otherwise identical capital-rich (‘capitalists’) and capital-poor (‘workers’) individuals; all supply one unit of labour inelastically. For concreteness, we assume that ‘workers’ are more numerous. As is standard in the AK model with fixed labour, all individuals consume the labour share plus a fraction of their capital wealth; thus, capitalists consume more and have a higher period utility. All agents are subject to a status (or ‘keeping up with the Joneses’) consumption externality. The key point, and the main result of the capital heterogeneity, is the conflict in preferences of the two classes concerning the fraction of income that is awarded as a labour share (and therefore taken away from the capital share). A higher labour share fuels current consumption but also harms growth as it reduces the marginal return to capital (from the Euler equation). As the capitalists’ consumption is based relatively more on wealth, they do like a labour share but are relatively more keen to safeguard growth, therefore prefer a lower (but non-zero) labour share; while the workers are keen to have a higher labour share (though not unity, as they are aware of the growth consequences of that). Thus, both classes have non-trivial partisan preferences over the labour share, with the workers preferring more of it. We derive a Utility Possibility Frontier that shows the trade-off and conflict between the welfares of the two groups evaluated intertemporally (so that growth is taken into account).

Thus, our setup incorporates a distributive social conflict in a simple production and growth setup. In the AK model with fixed labour adopted here, the functional distribution of income is parametric. We consider alternative positive and normative criteria as to how the labour share could or should be determined and compare their properties. The implicit Walrasian outcome serves as a useful benchmark. We consider two positive outcomes, the probabilistic voting model and the Nash bargaining solution that is determined by a multi-faceted socio-political bargain between the two groups. In normative terms, we consider the Rawlsian minimax criterion of maximising the welfare of the poorest, and the utilitarian outcome. We also propose a novel normative criterion which is another contribution of this paper; namely the criterion of minimising of ‘social friction’, defined as the welfare difference among the groups divided by aggregate welfare gains from a benchmark (as a measure of social synergy or cooperation). Thus, both distributive and growth considerations are incorporated in this criterion, which may be further argued to be a hybrid between the extremely equality-minded Rawlsian criterion and the ‘common good’ utilitarian approach. We explore its nature in the Appendix.

In a series of Propositions, we derive a range of novel results on the determinants of social conflict and of the various normative and positive criteria. Social conflict is exacerbated by greater impatience (the rate of time preference), intensified status comparisons and negative consumption externalities, greater wealth inequality and a decline in productivity. These are interesting results in their own right and, arguably, highly topical. We also find that status comparisons and wealth inequality tend to raise the labour share under all positive and normative criteria (with a couple of qualifications in the case of the Nash solution), as labour income is more egalitarian and this helps alleviate the social pressures arising from more inequality and more intense comparisons. We also provide a ranking of most of those criteria: We show that the highest labour share is proposed by the Rawlsian criterion, which is equal to the partisan workers’ preference – almost by definition. The second highest labour share is given by our proposed ‘minimum social friction’ criterion,



followed by the utilitarian one. The lowest labour share is given by the probabilistic voting solution which equals the implicit Walrasian outcome. Comparing the Nash bargaining solution with the utilitarian criterion, the former involves a lower labour share at least under the assumption that the capitalists' overall per capita influence (social, ideological and political) is greater than that of workers. If so, both positive criteria deliver a lower share, and are therefore less egalitarian, than all our three normative criteria.

A simple numerical illustration for a handful of diverse economies shows that, with a set of data on wealth distribution and reasonable assumptions, the model can emulate the labour share found in the data quite well, proving the usefulness of this framework. The dissonance is related to the income Gini which seems below what is found in the data. This suggests that our sharp exogenous division of individuals between two otherwise identical classes of exogenously fixed sizes is too restrictive. Inelastic labour is another assumption made for tractability. Building on parallel work on identity in macroeconomics (Tsoukis and Tournemaine, 2018), one can incorporate a continuous distribution with two classes emerging endogenously. Incorporation of this plus flexible labour will give a fuller model of the macroeconomy with social conflict. Additionally, we aim to clarify the relations between conflict, cooperation and friction, building on Bowles (2004, Chapters 1 and 5) and Axelrod (1984). Work on these points is under way.

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## Appendix: The 'iso-F' indifference map

We start from the expression for the indifference curves given by (20):

$$dW_0^i / dW_0^j = \frac{(1 + \sqrt{F})(1 - \theta)}{(1 - \sqrt{F})\theta} \quad (20)$$

It is straightforward to check that:

$$\frac{1 + \sqrt{F}}{1 - \sqrt{F}} = \frac{2\theta W_0^i - \bar{V}}{-2(1 - \theta)W_0^j + \bar{V}}.$$

The indifference map is shown in Figure A1 below; this is the same as Figure 1 of the main text with the exception that the reference point is  $V' \equiv \left(\frac{\bar{V}}{2\theta}, \frac{\bar{V}}{2(1-\theta)}\right)$ . Note that the region northeast of  $V'$  does not correspond exactly with the boundaries of the UPF (the boundaries of the northeast quadrant can be either higher or lower (left or right) than the boundaries of the area under the UPF). It is readily checked that these indifference curves are convex in the quadrant northeast of  $V'$ .

Importantly,  $F$  falls as we move outwards in the northeast region. Consider any ray that extends northeastwards from the  $V'$  point, i.e. such that the ratio of welfares remains constant; alongside any such ray,  $\sqrt{F}$  equals

$$\sqrt{F} = \frac{W_0^i(\theta + (1-\theta)\omega)}{W_0^i(\theta + (1-\theta)\omega) - \bar{V}}$$

Where the ratio of utilities  $W_0^j/W_0^i$  is constant along the ray. Then, it is easy to check that  $\sqrt{F}$  falls with  $W_0^i$  so that, as the iso-curves do not cross, an outer curve involves lower friction than an inner one. It follows that a tangency point (F in Figure B) minimises  $F$ .

**INSERT FIGURE B1 HERE**

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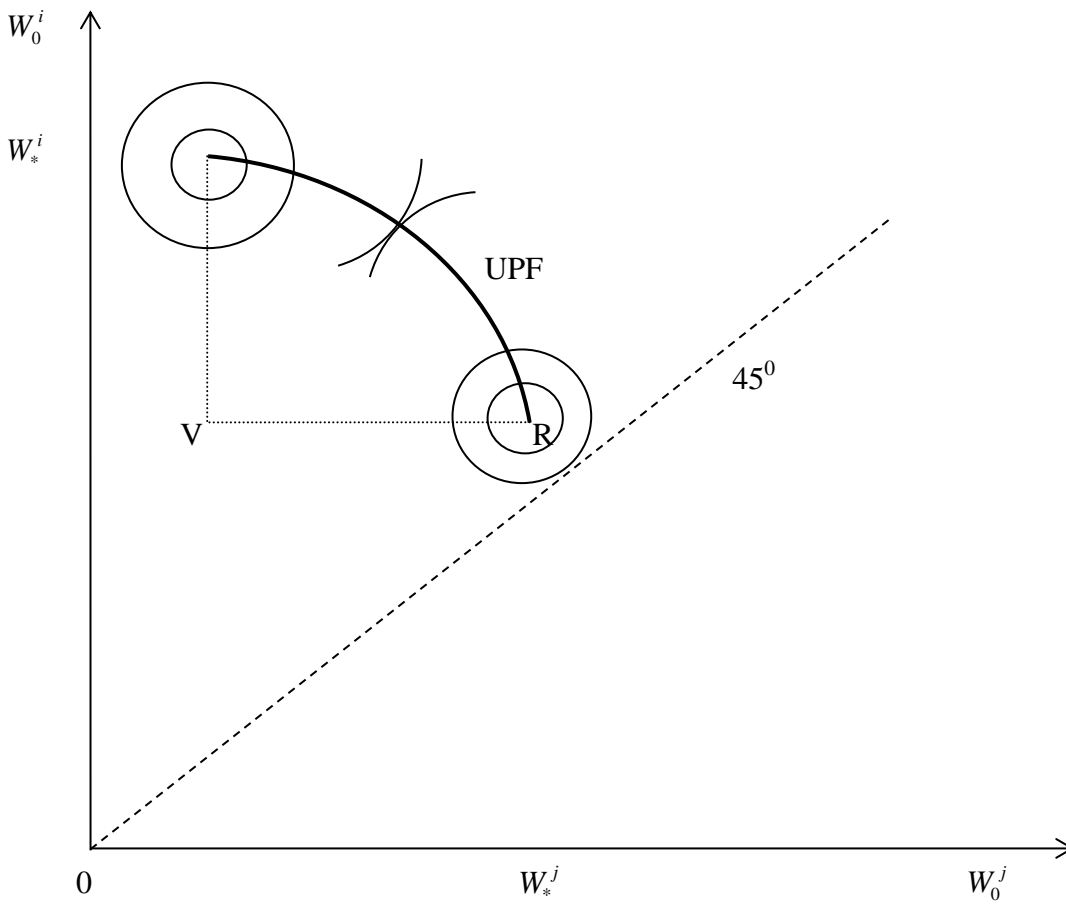
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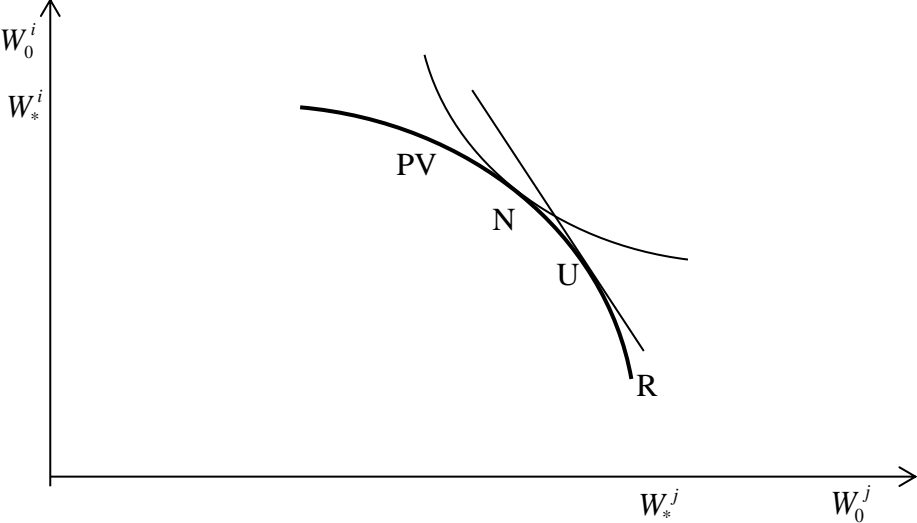
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## FIGURES AND TABLES

**Figure 1: The partisan solutions, the indifference map and UPF**



**Figure 2: Comparison of different outcomes and criteria under  $0.5 < \phi < \bar{\phi}$**



**Table 1: Data and implications**

1	2	3	4	5	6	7
Country	Share of 5 <sup>th</sup> quintile ( $s^5$ )	Wealth Gini	Implied $\chi^i = s^5/0.2$	Implied $\chi^j = (1-s^5)/0.8$	Implied income Gini	Actual income Gini
Australia	0.63	0.62	3.15	$\approx 0.46$	0.16	0.39
Finland	0.61		3.05	$\approx 0.49$	0.15	0.304
Germany	0.66	0.67	3.3	$\approx 0.43$	0.17	0.344
India	0.7	0.67	3.5	$\approx 0.38$	0.18	0.495 (2011)
Japan	0.58	0.55	2.94	$\approx 0.53$	0.14	0.352 (2012)
Sweden	0.8		4	$\approx 0.25$	0.22	0.303
US (family)	0.83	0.8	4.15	$\approx 0.21$	0.23	0.433

Source: Davies, Sandström, Shorrocks and Wolff (2011, T.7) for columns 2 and 3; OECD data (2014) for column 7 (<http://stats.oecd.org/Index.aspx?DataSetCode=IDD#>); and authors' calculations for columns 4-6.



**Table 2: Labour shares and social conflict**

1	2	3	4	5	6	7
Country	$\gamma^*_j$	$\gamma^*_i$	SC	SC1	$\gamma^U$	$\gamma^N (\phi=2)$
Australia	0.61	0.55	0.11	0.39	0.61	0.61
Finland	0.61	0.55	0.10	0.38	0.61	0.61
Germany	0.61	0.55	0.12	0.41	0.61	0.61
India	0.61	0.54	0.13	0.43	0.61	0.61
Japan	0.61	0.56	0.10	0.37	0.61	0.60
Sweden	0.62	0.53	0.15	0.47	0.61	0.61
US	0.62	0.53	0.16	0.49	0.61	0.61

Source: Authors' calculations; please see text for details.

**Figure B1: The iso-F indifference map**

