CESifo Working Papers
7619
2019
April 2019

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
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Editor: Clemens Fuest
www.cesifo-group.org/wp

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CESifo *Working Paper No. 7619*
Category 6: Fiscal Policy, Macroeconomics and Growth

Savings externalities and wealth inequality

Abstract

Incomplete markets models imply heterogeneous household savings behaviour which in turn generates pecuniary externalities via the interest rate. Conditional on differences in the processes determining household earnings for distinct groups in the population, these savings externalities may contribute to inequality. Working with an open economy heterogenous agent model, where the interest rate only partially responds to domestic asset supply, we find that differences in the earnings processes of British households with university and non-university educated heads entail savings externalities that increase wealth inequality between the groups and within the group of the non-university educated households. We further find that while the inefficiency effects of these externalities are quantitatively small, the distributional effects are sizeable.

JEL-Codes: E210, E250, H230.

Keywords: incomplete markets, productivity differences, savings externalities.

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April 12, 2019
We would like to thank Fabrice Collard, Thomas Crossley, Harris Dellas, Richard Dennis, Tatiana Kirsanova, Hamish Low, Aditya Goenka, Apostolis Philippopoulos, Peter Sinclair and Ulrich Woitek for helpful comments and suggestions. We also thank seminar participants at Athens, Birmingham, Cyprus, Glasgow and Zurich Universities as well as at the Royal Economic Society Conference, 2018 and the 4th International Conference in Applied Theory, Macro and Empirical Finance, 2018, for helpful comments and suggestions. We are also grateful for financial support from the Economic and Social Research Council, the Scottish Government, as well as the Royal and Scottish Economic Societies. The views expressed here are solely our own.

1 Introduction

Following the contributions by Bewley (1986), Imrohoroglu (1989), Huggett (1993) and Aiyagari (1994), an extensive literature has examined wealth inequality under idiosyncratic earnings shocks when agents cannot fully insure against uncertain income streams. In this framework, market incompleteness implies, via precautionary savings behaviour, inefficient asset accumulation at the aggregate level. Moreover, different histories of earnings shocks received by individuals imply heterogenous choices for asset accumulation, generating wealth inequality in the stationary equilibrium. The benchmark incomplete markets model features *ex ante* identical agents whose earnings are determined by the same stochastic process, leading to differences in savings. This heterogeneity in savings entails pecuniary externalities via the interest rate. For example, Greewald and Stiglitz (1986) in a model of incomplete markets and imperfect information and more recently Davilla *et al.* (2012) in an incomplete markets model with uninsurable idiosyncratic shocks study the efficiency implications of these savings externalities.

When economic agents are not *ex ante* identical, but instead belong to different groups distinguished by a key characteristic such as productivity, saving externalities can link behaviour in one group of agents with outcomes in another. In particular, significant differences in the level of productivity and earnings risk between groups of households could thus imply pecuniary externalities leading to sizeable wealth inequality between and within groups, in addition to potential effects on efficiency. Here, we examine savings externalities arising from skill heterogeneity, and analyse its implications for wealth inequality and efficiency in the UK. To the best of our knowledge, these issues have not been simultaneously addressed in the literature more generally nor have they been confronted for the UK.

We focus on skill heterogeneity motivated by empirical evidence which documents differences between economic agents with respect to their earnings processes. For example, this evidence shows that earnings risk is not homogeneous across different groups in the population (see e.g. Meghir and Pistaferri (2004) and Chang and Kim (2006)) nor are mean earnings the same (see e.g. Heathcote *at al.* (2010) and Blundell and Etheridge (2010)). We approximate differences in ability and skills at the beginning of working life with university education since empirical evidence shows strong wage and earnings premia for university educated workers (see e.g. Blundell and Etheridge (2010) and OECD (2012) for the UK). Moreover, data from the Understanding Society Survey (USoc) 2009-2017 (University of Essex, 2018), suggests clear differences in earnings risk between the group of households whose head is university educated or not. As we explain in our analysis below, the stochastic component of income for university educated households has a higher variance and exhibits more persistence than for the non-university educated.[1]

To investigate how group heterogeneity in mean earnings, risk exposure and persistence (implying saving externalities) contributes to wealth inequality and to the efficiency of the resource allocation in the UK, we specify an incomplete markets model with state-dependent (Markovian) stochastic earnings processes and let households belong to one of two groups. These groups differ in their earnings processes, both in the state-space and in the transition matrix for idiosyncratic earnings shocks. We calibrate the aggregate model using British data and estimate the earnings processes using USoc which is the latest extensive panel dataset for the UK. We then evaluate the model's predictions using wealth data from the Wealth and Assets Survey (WAS) 2006-2016 (ONS, 2018).[2]

Naturally, any effects from savings externalities work via the interest rate in general equilibrium, and are strongest in a closed economy framework. However, the interest rate in the UK is largely determined in international financial markets. Thus, we model the UK as an open economy, where the domestic interest rate differs from a global fixed interest rate by a function of the net foreign asset position of the country (demand minus supply of assets), capturing premia charged by the international financial markets.[3] Using recent advances in theoretical research (see e.g. Acikgoz (2018)), we show that this model has a well-defined partial equilibrium with a unique invariant wealth distribution for each type of household given prices. We further show that an open economy general equilibrium exists, and that for the parameter values chosen in the calibration this is unique.

We find that the model predicts wealth inequality both within and between the university and non-university educated groups that is consistent with the data. More specifically, the university educated group has significantly lower within group wealth inequality than the non-university educated group, despite having more persistent and volatile stochastic earnings processes. The model effectively matches the difference in the wealth Gi-

[1] Analysis of the importance of skills and education for inequality in a historical context for the US can be found in Goldin and Katz (2008). Several studies have also documented differences in earnings risk between groups associated with university education (see e.g. Castro and Coen-Pirani (2008) and Hagedorn *et al.* (2016)).

[2] The WAS dataset covers Great Britain only. For consistency, we use the sub-sample for Great Britain from USoc below. However, the results are very similar if we used the whole sample from USoc.

[3] The mechanism linking the domestic interest rate to the international rate and domestic conditions to close an open economy model is motivated by Kraay and Ventura (2000) and Schmidt-Grohe and Uribe (2003)).

nis between the two groups that are observed in reality and predicts a mean wealth ratio that is close to the data. Therefore, the predictions of the model regarding empirical facts that are of particular interest in this analysis are notably good. As is commonly found using this class of models, the model under-predicts the extent of income inequality at the very top end (top 1 percent). However, it produces very good predictions for the remaining distribution, especially up to the top 5 percent.[4]

The mechanism by which the pecuniary externalities work to affect inequality is as follows. Earnings differences, both in terms of mean earnings and idiosyncratic uncertainty, imply different asset supply functions for the two groups. The equilibrium interest rate is determined by the per capita asset supply function, which is higher (lower) than the asset supply functions for the university (non-university) educated. In other words, the savings of each group move the market interest rate away from the equilibrium level that would be consistent with the asset supply of each group. Consequently, households in the non-university and university educated groups lower and raise their savings respectively. This in turn implies that within group wealth inequality is increased for the non-university and decreased for the university educated, conditional on the earnings shocks that the households in each group receive.

We quantify the effects of the externalities and find that, on average, the two groups increase (university) or decrease (non-university) their equilibrium wealth by about 5 to 6 percent as a result of savings externalities, compared with the counterfactual where the interest for each group was not affected by the actions of the other group. This implies that between group inequality, measured as the ratio of mean wealth, increases by approximately 11 percent. Drilling down below the average effects, externalities induce significant changes in wealth accumulation within each group. For example, the rise and fall in average wealth holdings for the university and non-university groups respectively is roughly 8 to 9 percent for the top quintile and 5 to 7 percent for the bottom quintile. In contrast, for the middle three quintiles, wealth changes range from about 0 to 4 percent. Finally, we find that the inefficiency effects of the externalities are much smaller than the distributional effects since they contribute to an over accumulation of average assets of about 0.8 percent at the aggregate level.

[4] The standard incomplete markets model featuring stochastic labour income, one asset and *ex ante* identical agents captures qualitative properties of the wealth distribution, but quantitatively it under predicts the extent of inequality, especially at the top end of the wealth distribution (see e.g. De Nardi (2015), Quadrini and Rios-Rull (2015) and Krueger *et al.* (2016) who also review extensions that can improve the model's predictions in this respect).

The rest of the paper is organised as follows. We first present the model and data/calibration in Sections 2 and 3 respectively. The model is discussed in some detail to formally introduce the economic environment and clarify the economic quantities used later. We next examine the quantitative implications of the model. We first evaluate the predictions of the model with respect to between and within group wealth inequality in Section 4. We then study the pecuniary externalities mechanism, focusing on its equity and efficiency implications in Section 5. Finally, we present our conclusions in Section 6.

2 Model

We next develop a model emphasising skill differences, pecuniary externalities and wealth inequality. To this end, we consider an economy that is populated by a continuum of infinitely lived agents (households) distributed on the interval $I = [0, 1]$. Time is discrete and denoted by $t = 0, 1, 2, ...$ Households differ in their level of skill. In particular, there are two levels of skill, high and low, and households are randomly and permanently allocated to one of the two. This implies that there are two types of households, high skilled (university educated, u) households, which belong to a set $I^u \subset I$ and low skilled (below university educated, b) households which belong to a set $I^b \subset I$, such that $I^u \cup I^b = I$ and $I^u \cap I^b = \emptyset$. The proportions of high and low skilled households are given respectively by n^u and $n^b = 1 - n^u$. Therefore, there is *ex ante* heterogeneity in the population determined by the skill level of the household, which is assumed to be given.

All households have exogenous labour supply and derive utility from consuming one good that can be acquired by spending either labour income or accumulated savings. Households are identical in their preferences. However, their labour income depends on their skill level, since it determines their productivity. More specifically, households' predictable earnings component differs, reflecting their different skill. This implies that the two groups of households face different effective wage rates. In addition, each household is subject to idiosyncratic shocks, which affect labour income, by determining the residual, unpredictable earnings component. Households draw idiosyncratic shocks independently from a Markov chain which differs for high and low skill households. Both the state-space and corresponding transition matrix differ across the two household types, implying that the level of labour income and the size and persistence of productivity shocks differ for each household type, reflecting different opportunities and earnings risk.

There is a single asset in the economy implying that households cannot fully insure themselves against shocks to labour income. We examine and compute a stationary equilibrium, in which aggregate quantities are constant. In what follows we present the problem for a "typical" high skill educated household and the problem for a "typical" low skill educated household.

2.1 Households

Households have different skill levels ζ^h, $h = u, b$. Denote the idiosyncratic component of labour income of a typical household $h = u, b$ at time t by s_t^h, so that labour income is given by $w\zeta^h s_t^h$, where w is an average wage rate. Therefore, the idiosyncratic earnings shock s_t^h contains shocks that may affect work hours in a time period and/or household productivity.[5] The idiosyncratic earnings shock follows a Markov chain. In particular, we assume that the process s_t^h is an m-state Markov chain with state space S^h and transition matrix Q^h. The state space $S^h = [\overline{s}_1^h, \overline{s}_2^h, ..., \overline{s}_m^h]$ is ordered according to $\overline{s}_1^h > 0$, $\overline{s}_{j+1}^h > \overline{s}_j^h$, $j = 1, ..., m-1$ and has the natural σ-algebra $\mathcal{S}^h$ made up of all subsets of S^h. The elements of the transition matrix Q^h are denoted $\pi^h \left(s_{t+1}^h | s_t^h\right) = Pr(s_{t+1}^h = \overline{s}_{j'}^h | s_t^h = \overline{s}_j^h)$. We follow Acikgoz (2018) and assume that $\pi^h \left(\overline{s}_1^h | \overline{s}_1^h\right) > 0$ and that the Markov chain is *irreducible* and *aperiodic*, i.e. there exists a $k_0 \in \mathbb{N}$ such that $\left[\pi^h \left(s_{t+1}^h | s_t^h\right)\right]^{(k)} > 0$ for all $\left(s_{t+1}^h, s_t^h\right) \in S^h$ and $k > k_0$. This implies that the Markov chain has a unique invariant distribution, with probability measure that we denote by ξ^h.

Households' earnings shock s_t^h is observed at the beginning of period t. They also receive interest income from accumulated assets ra_t^h, and use their income for consumption and to invest in future assets, subject to the budget constraint for each $h = u, b$:

$$c_t^h + a_{t+1}^h = (1 + r) a_t^h + w\zeta^h s_t^h, \tag{1}$$

where $c^h \geq 0$, $a_t^h \geq -\phi^h$ and $-\phi^h < 0$ denotes a borrowing limit on the household. The set comprising a_t^h is defined as $\mathcal{A}^h = [-\phi^h, +\infty)$. The prices (interest rate r and wage rate w) are assumed to be fixed and non-random quantities. This holds if the household's actions take place in a stationary equilibrium, which is defined below. Households assess consumption streams with an intertemporal discount factor $\beta \in (0, 1)$, using a per period utility function $u(c_t^h)$. The utility function $u : [0, +\infty) \to \mathbb{R}$ is bounded, twice continuously differentiable, strictly increasing and strictly concave.[6]

[5] Examples include the quality of the match between employer and employee, health shocks, or changes in personal circumstances.

[6] Boundedness is not needed for equilibrium (see Acikgoz (2018)). In the calibration and computation below we will use a CRRA utility function which is not bounded below.

Furthermore, it satisfies the conditions $\lim_{c\to 0} u_c(c) = +\infty$, $\lim_{c\to\infty} u_c(c) = 0$ and $\lim_{c\to\infty} \inf -\frac{u_{cc}(c)}{u_c(c)} = 0$. These assumptions are typically employed in the literature of partial equilibrium income fluctuation problems (see e.g. Miao (2014, ch. 8)) and in the literature relating to incomplete markets with heterogeneous agents in general equilibrium (see e.g. Aiyagari (1994) and Acikgoz (2018)) to ensure a well-defined stationary equilibrium. The assumption that $\lim_{c\to\infty} \inf -\frac{u_{cc}(c)}{u_c(c)} = 0$ implies that the degree of absolute risk aversion tends to zero as consumption tends to infinity.

The interest rate and wage rate are taken as given and satisfy $r > -1$ and $w > 0$. Moreover, as has been shown (see e.g. Aiyagari (1994), Miao (2014, ch. 8) and Acikgoz (2018)), a necessary condition for an equilibrium with finite assets at the household level in this class of models is that $\beta(1+r) < 1$. Borrowing limits are imposed following e.g. Aiyagari (1994), i.e. assets must satisfy:

$$\begin{aligned} &a_t^h \geq -\phi^h, \text{ where} \\ &\phi^h = \min\left[\gamma, \frac{\overline{s}_1^h \zeta^h w}{r}\right], \text{ if } r > 0 \text{ or} \\ &\phi^h = \gamma, \text{ if } r \leq 0, \end{aligned} \tag{2}$$

and $\gamma > 0$ is arbitrary parameter, capturing an *ad hoc* debt limit. This restriction implies that even if the financial markets have the power to confiscate all of the income of the household, they would never lend so much that the household reaches an asset position where its lifetime labour income (assuming the worst earnings shock is always realised) was not sufficient to repay debt. This requires that $-r\phi^h + w\zeta^h \overline{s}_1^h \geq 0$.

The problem of the typical household $h = u, b$ is summarised as follows. For given values of (w, r) and given initial values $(a_0^h, s_0^h) \in \mathcal{A}^h \times S^h$, the household chooses plans $\left(c_t^h\right)_{t=0}^{\infty}$ and $\left(a_{t+1}^h\right)_{t=0}^{\infty}$ that solve the maximisation problem:

$$V^h(a_0, s_0) = \max_{\left(c_t^h, a_{t+1}^h\right)_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^h), \tag{3}$$

subject to (2), where $\beta \in (0, 1)$, and $c_t^h \geq 0$ is given by (1). To obtain the dynamic programming formulation of the household's problem, let $v^h\left(a_t^h, s_t^h; w, r\right)$ denote the optimal value of the objective function starting from asset-earnings state $\left(a_t^h, s_t^h\right)$ and given the interest and wage rate. The

However, we will work there with a compact set for assets, needed for computation, which, given the continuity of the utility function, implies boundedness.

Bellman equation is:

$$\begin{aligned} v^h\left(a_t^h, s_t^h; w, r\right) = \\ = \max_{\substack{a_{t+1}^h \geq -\phi^h \\ c_t^h \geq 0}} \{u(c_t^i) + \beta \sum_{s_{t+1}^h \in S^h} \pi^h\left(s_{t+1}^h | s_t^h\right) v^h\left(a_{t+1}^h, s_{t+1}^h; w, r\right)\}. \end{aligned} \tag{4}$$

In this case, we aim to find the value function $v^h\left(a_t^h, s_t^h; w, r\right)$ and the policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h; w, r\right)$ and $c_t^h = q^h\left(a_t^h, s_t^h; w, r\right)$, which generate the optimal sequences $\left(a_{t+1}^{*h}\right)_{t=0}^{\infty}$ and $\left(c_t^{*h}\right)_{t=0}^{\infty}$ that solve (3).[7] Standard dynamic programming results imply that the policy functions exist, are unique and continuous.

Following e.g. Stokey *et al.* (1989, ch. 9), we define $\Lambda^h\left[(a,s), A \times B\right] : \left(\mathcal{A}^h \times S^h\right) \times \left(\mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right) \rightarrow [0,1]$, for all $(a,s) \in \mathcal{A}^h \times S^h$, $A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h$, to be the transition functions on $\left(\mathcal{A}^h \times S^h\right)$, induced by the Markov processes $\left(s_t^h\right)_{t=0}^{\infty}$ and the optimal policies $g^h\left(a_t^h, s_t^h\right)$.[8] The transition function is given by:

$$\Lambda^h\left[(a,s), A \times B\right] = \begin{Bmatrix} \Pr\left(s_{t+1}^h \in B | s_t^h = s\right), \text{ if } g^h(a,s) \in A \\ 0, \text{ if } g^h(a,s) \notin A \end{Bmatrix}. \tag{5}$$

In this setup, Proposition 5 in Acikgoz (2018) implies that the Markov process on the joint state-space $\left(\mathcal{A}^h \times S^h\right)$ with transition matrix Λ^h has, for each $h = u, b$, a unique invariant distribution denoted by $\lambda^h(A \times B)$. Furthermore, Proposition 6 in Acikgoz (2018) implies that assets for the typical household tend to infinity when $\beta(1+r) \rightarrow 1$. Moreover, Theorem 1 in Acikgoz (2018) implies that the expected value of assets using the invariant distribution is continuous in the interest rate, r.

2.2 General equilibrium in an open economy

We analyse the general equilibrium in an open economy, following Angelopoulos *et al.* (2019) in modelling the latter within a heterogeneous agent model.

2.2.1 Firm

A representative firm operates the technology to transform borrowed assets from the financial market to capital to be used in production, and an aggregate constant returns to scale production function, using as inputs the

[7] In what follows, we suppress the explicit dependence of the value and policy functions on aggregate prices to simplify notation.

[8] For any set D in some n-dimensional Euclidean space $\mathbb{R}^n$, $\mathcal{B}(D)$ denotes the Borel $\sigma-$algebra of D.

average (per capita) levels of capital K and employment L. The production function is given by $Y = F(K, L)$ and is assumed to satisfy the usual Inada conditions. More specifically, F is continuously differentiable in the interior of its domain, strictly increasing, strictly concave and satisfies: $F(0, L) = 0$, $F_{KL} > 0$, $\lim_{K \to 0} F_K(K, L) \to +\infty$ and $\lim_{K \to \infty} F_K(K, L) \to 0$. The capital stock depreciates at a constant rate $\delta \in (0, 1)$. The firm takes the interest and wage rate as given and chooses capital and employment to maximise profits, which gives the standard first order conditions, defining factor input prices equal to the relevant marginal products:

$$w = \partial F(K, L)/\partial L, \tag{6}$$

$$r = \partial F(K, L)/\partial K - \delta. \tag{7}$$

2.2.2 Open economy setup

The economy trades in global financial markets taking the interest rate as given, which implies that aggregate household savings, A^s, can be above or below the capital demanded by firms, K. The difference between domestic savings and domestic capital will give rise to a non-zero net foreign asset position, $NFA \equiv K - A^s$, for the domestic economy. Given the country's net foreign asset position, the country makes interest payments to foreign households equal to $rNFA$, where r is the interest rate at which the country can borrow from abroad. This determines the economy's aggregate resource constraint as:

$$Y = C + I + rNFA,$$

where C is aggregate consumption and I is aggregate investment.

We assume that each country pays a risk premium on top of a risk-free interest rate r^*. The risk premium is a function of foreign debt (see, e.g. Kraay and Ventura, (2000) or Schmidt-Grohe and Uribe (2003) for debt-elastic interest rate). In particular, we assume that the risk premium is positively correlated with foreign debt relative GDP i.e. with NFA over output:

$$r = r^* + \psi \left[\exp\left(\frac{NFA}{Y} \right) - 1 \right], \tag{8}$$

for $0 < \psi < r^* + \delta$, which is well defined for $r > r^* - \psi$. The parameter ψ measures the elasticity of the country specific interest rate premium relative to the net foreign asset position.[9] The requirement that $r > r^* - \psi$ is automatically satisfied for a country with negative net foreign assets when

[9] Note that $\psi < r^* + \delta$, implying $r^* - \psi > -\delta$, ensures that domestic firm's demand is finite in the international market, and also guarantees that $r > -1$.

$\psi > 0$, as is the case in the calibration for the UK below. Household optimisation and (8) jointly define a constraint set for the interest rate in general equilibrium, R^{ge}, given by $r \in R^{ge} = \left(r^* - \psi, \frac{1}{\beta} - 1\right)$.

2.2.3 General equilibrium

In Appendix A we define formally the stationary general equilibrium in the open economy and show existence. We also present the computational algorithm. Note that while uniqueness of general equilibrium cannot be guaranteed in general, as is commonly the case in this class of models (see e.g. Aiyagari (1994) and Acikgoz (2018)), it is straightforward to confirm uniqueness for a specific calibration. We compute asset demand, as well as the invariant cross-sectional distribution and mean of asset supply for a typical household, for a range of interest rates consistent with the model, and confirm that the demand and supply curves intersect once (see Figure 2 below).

3 Data and calibration

We approximate the skill level of the households with the education level of the head of the household. More specifically, we consider two groups of households, those whose head has university education, and those whose head does not.[10] At the age of 25, which is the minimum age for heads of households in our sample, the education level is predetermined for the households in the sample, hence all households belong to one of the two types.

We estimate the parameters relating to the Markov processes for the idiosyncratic shocks for the university and non-university groups of households using data on net labour income from USoc. We use net labour income as the relevant quantity to calibrate the earnings processes, as this measure coheres well to earnings in the model. We then evaluate the predictions of the model regarding wealth inequality against data form the WAS.

3.1 Earnings dynamics

Household net labour income is our main measure of income that we use to estimate the extent and persistence of idiosyncratic earnings uncertainty since

[10] See also Blundell *et al.* (2008) for a similar classification of households into two groups. Note that we also control below for the educational level of the spouse as part of potential observable variation of earnings within the groups of "university" and "non-university" groups of households.

wealth inequality is measured using household-level data.[11] We estimate the parameters pertaining to idiosyncratic earnings uncertainty separately for the university and non-university educated groups.

3.1.1 USoc data

USoc is a large longitudinal survey which follows more than 25,000 households (on average in the first 8 waves) in the UK. USoc provides extensive information on sources of income for individuals and households, as well as on socio-economic characteristics, demographics and even health condition of the respondents. Data collection for each wave takes place over a 24-month period and the first wave occurred between January 2009 and January 2011. Even though the periods of waves overlap, the individual respondents are interviewed around the same time each year. Thus, there is no respondent who is interviewed twice within a wave or a calendar year (see Knies (2018)). Our main sample consists of the General Population Sample plus the former British Household Panel Survey sample (BHPS), and we exclude the Ethnic Minority Boost Sample and the Immigrant and Ethnic Minority Boost Sample. For consistency with the WAS dataset, we also drop the households located in Northern Ireland. The inclusion of the boost samples and Northern Ireland sample, or the exclusion of the former BHPS sample does not change our results either quantitatively or qualitatively.

We define net labour income as gross household labour income for employment or self employment net of taxes and national insurance contributions, plus social benefits and private transfers. Households are defined as the family or group of individuals who live in the same residence. The head is defined as the member of the household in whose name the accommodation is owned or rented, or is otherwise responsible for the accommodation. We focus on households whose the head is between 25-59 years and report positive net labour income. Furthermore, we trim the top and bottom 0.5 percent of observations of net labour income distribution in each year, to avoid extreme cases or possible outliers in recorded income. Then, we only keep households who are in the sample for at least three consecutive periods. The final sample consists of 38,844 observations from 7,665 unique households. In Appendix B, we report more information on the net labour income series and sample selection process.

[11] Note that in what follows, net labour income and earnings are used synonymously.

3.1.2 Idiosyncratic shocks

To focus on the idiosyncratic component of income, we follow the literature (see e.g. Meghir and Pistaferri (2004), Blundell *et al.* (2008), and Blundell and Etheridge (2010)) and assume that household net labour income is composed of three components, an element capturing aggregate conditions common to all households, a deterministic part depending on observable characteristics and the idiosyncratic component. By denoting the natural logarithm of the measure of income in period t as $y_{i,t}^h$, for $h = u, b$, we assume that it follows the process:

$$y_{i,t}^h = D_t^h + g(x_{i,t}) + \varepsilon_{i,t}^h, \tag{9}$$

where $g(x_{i,t})$ is a linear deterministic function of the observables, $x_{i,t}$, i.e. $g(x_{i,t}) = b^h x_{i,t}$. The vector of parameters for each h is given by b^h and $x_{i,t}$ is a set of dummy variables for experience (approximated by age), region of residence, gender of the head of household, marital status and the educational level of the spouse (if married). Note that the educational level of the spouse is defined in a similar way to the heads i.e. University educated and below University educated. Following Blundell and Etheridge (2010) we also include as a regressor the logarithm of the household size. Function D_t^h captures the aggregate conditions common to all households and is specified as calendar year time effects, i.e. $D_t^h = \sum_{t=2009}^{2017} \mathbf{1}_t d_t^h$, where $\mathbf{1}_t$ is an indicator function which is one when a household i is present at time t and zero otherwise.

For the region dummies we use the UK Government Office Regions classification which corresponds with the highest tier of sub-national division in England, Scotland and Wales. Furthermore, following Meghir and Pistaferri (2004) and to be consistent with our model, we estimate (9) separately for the households whose head has University education and those households whose head does not. Finally, since in our econometric analysis we employ household quantities for the arguments in (9), we define all the variables, apart from the spouse's educational level, in terms of the head of the household.

We next retain the residuals $\varepsilon_{i,t}^h$ for each t as a proxy for the unobserved component of $y_{i,t}^h$ and assume that they are determined by an exogenous $AR(1)$ process (see e.g. Chang and Kim (2006)):

$$\varepsilon_{i,t+1}^h = \rho^h \varepsilon_{i,t}^h + \mu_{i,t+1}^h, \tag{10}$$

where $\left|\rho^h\right| < 1$ and $\mu_{i,t}^h$ is a white noise process with variance $\left(\sigma_\mu^h\right)^2$. We further assume that the $AR(1)$ process is covariance-stationary with a zero mean and variance $\left(\sigma_\varepsilon^h\right)^2 = \frac{\left(\sigma_\mu^h\right)^2}{1-\left(\rho^h\right)^2}$.[12]

[12] We have also modelled the idiosyncratic component as consisting of a persistent and

Following Chang and Kim (2006, 2007), we estimate (10) via OLS and we summarise the results for the Uni, Non-Uni and the whole sample in Table 1. This table shows that the estimated variance of shocks to net labour income for the Uni group is higher than that for the Non-Uni group. We approximate (10) by a discrete state-space process, by applying Rouwenhorst 's (1995) method to build a Markov chain with 15-states (see e.g. Kopecky and Suen (2010) and Krueger *et al.* (2016)).

Table 1: Markov Process Parameters

	Uni	Non-Uni
ρ	0.715	0.692
CI_{90}	[0.703,0.727]	[0.684,0.700]
$\frac{\sigma_\mu}{\sqrt{1-\rho^2}}$	0.445	0.431

The model predictions regarding earnings inequality in the stationary distribution resulting from this approximation are summarised in Table 2, which shows the Gini coefficient, Coefficient of Variation (CV) and variance of logarithms predicted by the Markov Chains with their counterparts calculated using the residuals earnings from equation (9). The AR(1) model and 15-state approximation capture well the quantitative differences in within group earnings inequality, as well as the overall level of earnings inequality in each group.

Table 2: Earnings Inequality

	Data		Model	
	Uni	Non-Uni	Uni	Non-Uni
Gini	0.230	0.226	0.244	0.237
$\sqrt{VarLog}$	0.459	0.439	0.445	0.431
CV	0.448	0.429	0.464	0.448

3.2 Wealth inequality

The WAS is a longitudinal survey for GB reporting information on earnings, income, the ownership of assets (financial assets, physical assets and property), pensions, savings and debt, as well as on socio-economic characteristics of the respondents over five waves between 2006 and 2016.[13] The sample corresponds to the households included in the wave, but the interviews in each wave are carried over a two year period, with the respondents providing information for the year of the interview.

transitory component, but we found that this does not improve the model's approximation of residual earnings inequality, nor its predictions with respect to wealth inequality.

[13]The WAS does not provide information for Northern Ireland.

An important feature of WAS is that it uses a 'probability proportional to size' method of sampling cases. This means that the probability of an address being selected is proportional to the number of addresses within a given geographic area, with a higher number of addresses being selected from densely populated areas. The design of WAS recognizes the fact that wealth is highly skewed, with a small proportion of households owning a large share of the wealth. Thus, WAS over-samples addresses likely to be in the wealthiest 10 percent of households at a rate three times the average. Moreover, the large overall sample size (around 20,000 households) provides robust cross-sectional estimates. These features ensure both good coverage of the very wealthy and more precise estimates of overall household wealth. However, as in similar surveys, the very rich (e.g. Forbes 400) are not typically included and this can affect the estimates of the top 1 percent.

We harmonise the definition of the household and of the head of household as it is defined in the previous section. We select household heads between 25-59 years of age. We discard the households with imputed net income or missing educational information. We use household net worth as our measure for wealth. It is the sum of assets minus debt for all household members.[14] Net worth also admits a substantial proportion of the population which have negative current wealth. Details on the wealth data are in Appendix B, which includes key statistics summarising the wealth distributions for all five waves in Table B1.

3.3 Model parameters

The model parameters that do not relate to the Markov chains are summarised in Table 3. Regarding preferences, following the literature we use a CRRA utility function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{11}$$

and set $\sigma = 1.5$, which is the mid-point of values typically employed in calibration studies for the UK (see also Harrison and Oomen (2010) who econometrically estimate $\sigma = 1.52$).

The annual depreciation rate is set to $\delta = 0.0983$ which implies that the capital over income ratio, given the interest rate (see below), is 2.5 at the equilibrium.[15] We use a Cobb-Douglas production function with constant

[14] We do not add pension wealth to our measure of net-worth. This allows us to maintain comparability with the infinite horizon incomplete markets literature that generally excludes pension wealth. Further note that pension wealth is highly imputed in WAS.

[15] This is also very close to the values in Faccini *et al.* (2013) and Harrison and Oomen (2010).

returns to scale with respect to its inputs:

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{12}$$

We normalise $A = 1$ and set α to 0.3 (see, e.g. Faccini *et al.* (2013) and Harrison and Oomen (2010)). The value of n_u is set to 0.3 based on information on the percentage of university educated households in either WAS dataset or USoc dataset. Finally, we make use of the ratio of the predicted earnings components between the two groups to obtain the ratio ζ^u / ζ^b. We further normalise ζ^b to be equal to one. Note that for the computation we normalise the aggregate labour supply to one, and hence, the units of ζ^u and ζ^b do not matter, but only the ratio. Moreover, we set the international interest rate, r^*, to 0.0215 which is the average value of the real short-term yields in the data for 17 countries for the period 1990-2013 (see Carvalho *et al.* 2016).

Conditional of the above parameters, we calibrate β, ϕ and ψ to match the following data: (i) the value of debt over GDP $\frac{K_t - A_t}{Y_t} = 8.1$ percent which is the average value in the data for UK for the period 1990-2013 in the extended External Wealth of Nations Mark II database (see also Lane and Milesi-Ferretti (2007)); (ii) the percentage of indebted households (i.e. those with zero or negative net-worth) in the WAS data, which is 18.5 percent; and (iii) the interest rate in equilibrium, $r = 0.0217$, which is the average value of the real short-term yields in the data for UK for the period 1990-2013 (see Carvalho *et al.* 2016). However, note that given $\frac{K_t - A_t}{Y_t} = 8.1$ percent and $r = 2.17$ percent, and given $r^* = 2.15$ percent; ψ is determined by $\psi = \frac{r - r^*}{\left[\exp(\frac{NFA}{Y}) - 1\right]}$. Therefore, in effect we calibrate ϕ and β to match $\frac{K_t - A_t}{Y_t}$ and the percentage of indebted households.

Table 3: Model Parameters

β	σ	δ	A	α	n_u	ϕ	ψ	r^*	ζ^u/ζ^b
0.9718	1.50	0.0983	1.00	0.30	0.30	1.33	0.0024	0.0215	1.461

4 Wealth inequality: model vs. data

We first examine the model's predictions regarding wealth inequality within and between the groups of university and non-university educated and compare these to the data for the UK. We summarise the data and model predictions for key statistics of wealth inequality in Table 4, following standard practice in the choice of these statistics, see e.g. Quadrini and Rios-Rull (2015) and Krueger *et al.* (2016).

We complement this Table by Figure 1, which provides a graphical representation of the wealth distributions using the quintile measures of the proportion of total wealth owned by households in the relevant quintile (the first column) and the Lorenz curves (the second column). We also report summary measures of wealth inequality at the aggregate level in the last rows of Table 4 to contextualise the discussion on within and between group wealth inequality.

Table 4: Wealth distributions by group

	WAS Data		Model	
	Uni	Non-Uni	Uni	Non-Uni
Q1 share	-0.006	-0.015	-0.015	-0.051
Q2 share	0.037	0.003	0.066	0.045
Q3 share	0.101	0.075	0.154	0.148
Q4 share	0.205	0.226	0.272	0.283
Q5 share	0.663	0.712	0.523	0.575
T 90-95%	0.136	0.153	0.133	0.144
T 95-99%	0.191	0.205	0.139	0.155
T 1%	0.155	0.148	0.050	0.056
Gini	0.661	0.731	0.545	0.633
a_u/a_b	2.270		1.873	
Gini Total	0.720		0.615	

Note: "WAS Data" refers to the average statistics over waves 1-5.

The first two columns in Table 4 summarise wealth distributions in the data, by presenting the averages of the relevant quantities across the five waves of WAS. The main observation is that households whose head is university educated (denoted as Uni) has lower wealth inequality than households whose head is not university educated (non-Uni). This can be seen in Table 4 by comparing the wealth distributions (approximated by the quintile statistics), wealth ownership at the upper tail and the Gini indices.

[Figure 1]

The quintile shares suggest a relatively smaller concentration of wealth in the lower three quintiles and a relatively higher concentration of wealth in the upper two quintiles for the non-university educated. Given the implied spread between the lower and upper parts of the wealth distributions, all of these observations suggest that wealth inequality is higher for the non-university than for the university educated groups, which is confirmed by the summary Gini measures. Further note that the group of university educated has higher wealth on average, compared with the non-university educated, i.e. the relative wealth ratio, a_u/a_b, is at 2.27 on average across the five waves of data.

The next two columns in Table 4 summarise the predictions of the model in Section 2 and calibrated in Section 3. The calibration implies an average wealth ratio of Uni to Non-Uni households predicted by the model of about 1.9, which is consistent with (but lower than) between group wealth inequality in the data. Importantly, the model coheres with key properties of within group wealth inequality for the two groups, i.e. higher wealth inequality for the Non-Uni group relative to the Uni group. This result can be seen by comparing the Gini indices, but is more comprehensively demonstrated by examining the relative rankings of the measures of wealth ownership for the two groups. The model predictions track those in the data. When the quintile shares are higher in the data for the Uni group (the Q1, Q2 and Q3 shares), they are also higher in the model. Whereas, when the quintile measures are higher in the data for the Non-Uni group they are also higher in the model. Overall, the model predicts a Gini index for the non-university educated that is significantly higher than the respective index for the university educated.

The model's predictions regarding the extent of wealth inequality relative to the data are close for both groups, with the exception of the predictions for the top 5 percent, and especially the top 1 percent, where the model significantly underestimates wealth inequality, consistent with other models of this class in the literature. The first column in Figure 1 shows the wealth distribution approximated by the quintile shares for the USoc calibration in Table 4. Both show that the model magnitudes are similar to the data for both groups. The second column of Figure 1 suggests that the level of predicted inequality within each group is lower compared with the data, reflecting that overall the model quantitatively under-predicts the extent of wealth inequality. This can also been seen by referring to the Gini index implied by the model for the aggregate economy in the last row of Table 4.

In contrast to the WAS data, the model predicts slightly higher wealth concentrations for the top 1 percent of the Non-Uni relative to the Uni groups. However, a closer look at each of the WAS waves shows that the wealth concentration ranking for the top 1 percent is not consistent over all the waves (see Appendix B). For example, in the first three waves, wealth ownership by the top 1 percent is higher for the Non-Uni while it is higher for the Uni in the last two waves.[16] In contrast, the ranking of the remaining statistics between the two groups in Table 1 does not change over the waves. On the other hand, the model's predictions regarding the relative ranking of the group wealth concentrations in the top percentiles below the top 1 percent

[16] For example, the ratios of the Non-Uni top 1 percent to the Uni top 1 percent for Waves 1-3 are 1.029, 1.13 and 1.06 respectively. Whereas the corresponding ratios for Waves 4-5 are 0.812 and 0.873 respectively.

(i.e. the shares owned by the top 90-95 percent and 95-99 percent) are very similar to the data.

Overall, the model's predictions regarding wealth inequality capture the main differences between the two groups and the overall extent of inequality, for the majority of the distribution. As is well known in the literature, this class of standard incomplete markets models does not match quantitatively the extent of wealth inequality that we observe in the data with respect to wealth ownership at the very top end.

5 Equality and efficiency implications

We next quantitatively analyse the equality and efficiency implications of the pecuniary externalities associated with the skill heterogeneity in an open economy context for the UK.

5.1 Equality

We first examine the mechanism by which pecuniary externalities generates wealth inequality and then evaluate the effects of these quantitatively. In particular, we investigate the importance of savings externalities in generating the within and between group inequalities that we observe in Table 4, by comparing inequality and key aggregate quantities for the model analysed above with those obtained in artificial economies. In these economies the two types of households do not interact via the financial market, thus eliminating the pecuniary externalities working via the interest rate.[17]

[Figure 2]

We start with the model analysed above and in Figure 2 we plot the asset supply curves for a typical household in both groups of university and non-university educated, as well as the asset supply and demand functions for the aggregate economy.[18] We summarise key quantitative information relating to this Figure in Table 5 under the column "Base". In addition, we add in Table 5 key statistics that capture model predicted earnings and

[17] Strictly speaking, the economies without market interaction also shut down externalities via the wage rate. To control for this, we have repeated the experiments in this section by adjusting the wage rate for each group to be the same as in the baseline economy, and the results are very similar quantitatively, suggesting the savings externalities in this model economy work predominantly via the interest rate and not via wages.

[18] Note that the group-level and aggregate-level supply and demand functions are in per capita units. Thus, they refer to mean asset supply and demand functions.

wealth inequality. More specifically, we report the earnings inequality that is implied by the calibration in Section 3 and the wealth inequality in general equilibrium. The general equilibrium is obtained at the intersection point of the aggregate-level supply and demand curves for assets, giving an interest rate of $r^* = 0.0217$ and capital stock of $a^* = 3.583$.

Table 5: Pecuniary externalities and inequality per group

	Base	NI_u	NI_b
r^*	0.0217	0.0212	0.0220
a^*	3.583		
a_u^*	5.317	5.015	
a_b^*	2.839		2.980
Wealth Gini Uni	0.545	0.555	
Wealth Gini Non-Uni	0.633		0.620

Notes: (i) the NI_h models are based on the same earning processes as in the Base model; (ii) $\frac{a_u}{a_b}$=2.27 in the data; (iii) $\frac{a_u^*}{a_b^*}$=1.87 for the model; and(iii) $\frac{a_u^*}{a_b^*}$=1.68 for NI.

In Figure 3, we again plot the supply and demand curves for this model, which provide the equilibrium (already shown in Figure 2) when the two groups interact via the market in a single economy. We complement this by plotting the asset supply curves for a typical household in each group, which capture mean asset supply per group, together with the mean asset demand curves that would apply if these two groups did not interact. In other words, we treat the two groups as separate economies, each populated with the *ex ante* identical university or non-university educated agents. We denote these as NI (non-interaction) supply and demand. The intersection points of the respective asset supply and demand curves represent the equilibrium interest rate and assets in the absence of group interaction, which are reported in Table 5 under the NI_h, $h = u, b$ columns.

[Figure 3]

The asset supply curves for a typical household in each group in the Base model encapsulate their optimal policy functions and thus choices for savings given aggregate outcomes under market incompleteness. Therefore, from Figure 3 and Table 5, we can see that in the Base model the equilibrium interest rate $r^* = 0.0217$ implies mean assets for the Uni group that are equal to $a_u^* = 5.317$ and for the Non-Uni group that are equal to $a_b^* = 2.839$. Hence, compared with the case where the groups' savings do not affect each other (i.e. $r^* = 0.0212 \Rightarrow a_u^* = 5.015$ and $r^* = 0.0220 \Rightarrow a_b^* = 2.980$), the asset supply of the other group in the general equilibrium of Base economy, works to lower (increase) the interest rate for the Non-Uni (Uni) groups respectively.

Viewed from the perspective of the Non-Uni (Uni) group, the reduction (increase) in the interest rate resulting from pecuniary externalities, reduces (increases) their respective incentives to save.[19] Hence, mean assets are reduced (increased) for the Non-Uni (Uni) group, leading to an increase in the ratio of mean wealth by about 11 percent. In turn, this under-accumulation (over-accumulation) of assets works to increase (decrease) wealth inequality in each group, by increasing (decreasing) the exposure to earnings variability. To illustrate the effect of the change in the interest rate on asset accumulation and inequality for a given group (in partial equilibrium), we plot in Figure 4 mean assets and the within group Gini index for wealth inequality for a range of interest rates, holding earnings risk and all other parameters fixed, for the non-Uni group. As can be seen, an increase in the interest rate, *ceteris paribus*, increases mean group savings and decreases within group inequality.

[Figure 4]

Therefore, the asset supply of each group creates savings externalities in the financial market which affects inequality in the other group. To quantify the externalities effect, we first summarise in Table 5 wealth inequality for the two groups in these two scenaria. Comparing the NI_h equilibria to the Base model equilibrium, the latter implies higher wealth inequality within the non-university educated, and lower wealth inequality within the university educated. We then further decompose the changes in the Gini index for the two groups in Table 5 into the changes in wealth implied per quintile.

In Table 6, we report mean wealth per quintile for the Base economy and for the NI_h equilibria, and the percentage change due to pecuniary externalities. As can be seen, within the Uni (non-Uni) group, the top and bottom quintiles have significantly higher (lower) wealth accumulation under externalities (i.e. about 8 to 9 percent and 5 to 7 percent respectively), whereas the middle three quintiles have lower changes in wealth (i.e. about 0 to 4 percent). Note that the increase in the interest rate generates income and substitution effects for a typical household in the Uni group and the results indicate that the substitution effects dominate at the tails of the distribution, whereas the income effects are stronger in the middle. The changes in the tails are strong enough to determine the positive change in the mean, shown in the last line in Table 6.

[19] Note that the (decrease) increase in the interest rate also creates income, in addition to substitution, effects. In this case, the substitution effects dominate in terms of mean savings (see also below for a decomposition).

The situation is reversed for the non-Uni group. For example, for the bottom quintile and the top two quintiles the decrease in the interest rate, due to pecuniary externalities, implies lower wealth accumulation (the effects are bigger for the top and bottom quintile). For the second and third quintile, the income effects dominate so that asset accumulation increases. However, the decrease in the other three quintiles is stronger and determines the negative change in the mean for the group. On average, the two groups increase or decrease their equilibrium wealth by about 5 to 6 percent as a result of pecuniary externalities. For example, given average net worth of £273,000 for the Uni group and £121,000 for the non-Uni group across the five waves in the WAS, the results suggest that pecuniary externalities contributes to the average asset accumulation of the Uni by about £16,500 and decreases the average asset accumulation for the non-Uni by about £6,000.

Table 6: Mean assets per quintile by group

	Base		NI		% change $\left(\frac{NI-Base}{\lvert NI \rvert}\right)$	
	Uni	Non-Uni	Uni	Non-Uni	Uni	Non-Uni
$\overline{a}_{Q1}$	-0.495	-0.717	-0.530	-0.684	6.60	-4.82
$\overline{a}_{Q2}$	0.637	0.959	0.665	0.938	-4.21	2.24
$\overline{a}_{Q3}$	2.107	2.716	2.162	2.707	-2.55	0.33
$\overline{a}_{Q4}$	4.968	4.752	4.974	4.864	-0.12	-2.30
$\overline{a}_{Q5}$	19.37	6.486	17.80	7.074	8.82	-8.31
$\overline{a}$	5.317	2.839	5.015	2.980	6.02	-4.76

5.2 Efficiency

We next investigate the efficiency effects of savings externalities and whether they lead to higher or lower aggregate savings compared with an equivalent market allocation where externalities are not present.

The model in Section 2, taking the international markets and skill heterogeneity as part of the institutional setup, incorporates two main sources of inefficiency. The first inefficiency arises irrespective of *ex ante* skill heterogeneity (i.e. even in the case of *ex ante* identical households), as a result of incomplete financial markets, which imply that idiosyncratic earnings shocks lead to income and savings inequality and precautionary savings. This has been analysed extensively in the literature (see e.g. Aiyagari (1994) for theoretical and quantitative analysis in the class of general equilibrium models). These heterogeneous savings imply pecuniary externalities between the households, working from high savers to low savers and vice versa, via the financial markets and, in particular, the interest rate. The efficiency implications of pecuniary externalities incorporated in incomplete market models have been noted since Greewald and Stiglitz (1986) and examined in detail in Davila *et al.* (2012), who have shown that, depending on the stochastic environment, they can work to increase or decrease aggregate savings relative to a *constrained efficiency* benchmark where savings are chosen optimally to maximise aggregate welfare.

The second inefficiency arises because of skill heterogeneity, and also works via the interest rate. In this framework, as we saw in the previous sub-section, the higher savings of the high skill group tends to decrease the market interest rate, thus affecting savings of the low skill group (and vice versa for the savings of the low skill group). Here, we examine whether externalities tend to increase or decrease aggregate savings relative to a situation where in the same market economy savings are chosen optimally without externalities due to skill heterogeneity, and thus whether (and by how much) externalities generates additional inefficiency at the aggregate level.

To this end, we compute the aggregate quantities that characterise the equilibrium of an economy where consumption and savings are chosen to maximise the utility of a typical household in an economy with *ex ante* identical agents, i.e. of households who face the same earnings process, implying that they face the same mean earnings and earnings risk. This model is solved for the same parameter values as the model with the skill heterogeneity, except for those pertaining to the common stochastic process governing earnings for the *ex ante* identical household. To obtain these, we set $\zeta^u = \zeta^b = \zeta$, implying $\zeta = 1$, and assume that the earnings process for the typical household is given by:

$$\varepsilon_{i,t+1} = \rho\varepsilon_{i,t} + \mu_{i,t+1}, \tag{13}$$

where $\sigma_\mu^2 = n^u\left(\sigma_\mu^u\right)^2 + n^b\left(\sigma_\mu^b\right)^2$ and $\rho = \frac{n^u\rho^u(\sigma_\varepsilon^u)^2 + n^b\rho^b\left(\sigma_\varepsilon^b\right)^2}{n^u(\sigma_\varepsilon^u)^2 + n^b\left(\sigma_\varepsilon^b\right)^2}$. This gives $\rho = 0.699$ and $\sigma_\varepsilon^2 = 0.435$.

The results from this economy are summarised in Table 7 under the column "Identical". We also repeat for convenience in Table 7 the respective quantities from the base model. As can be seen, pecuniary externalities implies an increase in mean assets by about 0.8 percent compared to a model economy that eliminates this inefficiency. Given an average mean net worth across the four waves in the data from the WAS of about £166,000, this implies that about £1,300 of the average wealth accumulation is driven by pecuniary externalities. Compared with the inequality implications, the in-

Table 7: Inefficiency

	Identical	Base
r^*	0.0217	0.0217
a^*	3.556	3.583

efficiency arising from savings externalities is much smaller.

6 Conclusions

This paper set out to quantify the inequality and inefficiency implications of externalities due to the heterogenous savings behaviour of different groups in the population. To this end, we developed an open economy incomplete markets model with state dependent (Markovian) stochastic earnings processes and *ex ante* heterogeneity corresponding to being university educated or not. The two groups were allowed to differ in their earnings processes, both in the state-space and in the transition matrix for idiosyncratic earnings shocks.

Using the Understanding Society and the Wealth and Assets Survey for Great Britain, we found that this model predicted wealth inequality both within and between the university and non-university educated groups that was consistent with the data. Although the university educated group faces higher risk in terms of the persistence and volatility of the idiosyncratic component of net labour income, the model predicts that it has significantly lower within group wealth inequality, consistent with the data. In fact, the model predicted a difference in the wealth Ginis between the two groups that is similar to that observed in the data and, more generally, it produced very good predictions for the wealth distribution up to the top 5 percent. Moreover, the model's predictions regarding between group inequality, captured by the mean wealth ratio, were close to the data.

The savings of the two groups generate pecuniary externalities which work via the financial market to increase (decrease) savings for the university (non-university) educated groups. This leads, at the aggregate level, to an inefficient increase in the accumulation of assets, which we find to be relatively small quantitatively, at about 0.8 percent. However, externalities also lead to an increase in inequality between the groups, and within the group of non-university educated, and to a decrease in wealth inequality within the group of university educated. These effects are sizeable with the ratio of mean wealth between the two groups increasing by approximately 11 percent due to the savings externalities. Moreover, there is a heterogeneous response in wealth accumulation within the groups, leading to the significant within group inequality effects. For example, the rise and fall in wealth for the university and non-university groups respectively was 8 to 9 percent for the top quintile and 5 to 7 percent for the bottom quintile. Overall, therefore, the inequality implications of pecuniary externalities are much bigger than their effects on efficiency.

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7 Appendix A

We define a stationary recursive equilibrium following e.g. Miao (2014, ch. 17) and Acikgoz (2018).[20]

`Stationary Recursive General Equilibrium`

For $h = u, b$, a *Stationary Recursive Equilibrium* is stationary distributions $\lambda^h (A \times B)$, policy functions $a_{t+1}^h = g^h \left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathcal{A}^h$, $c_t^h = q^h \left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathbb{R}_+$, value functions $v^h \left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathbb{R}$, and positive real numbers $K, w(K), r(K)$ such that

1. The firm maximises its profits given prices, so that the latter satisfy (6) and (7).

2. The policy functions $a_{t+1}^h = g^h \left(a_t^h, s_t^h\right)$ and $c_t^h = q^h \left(a_t^h, s_t^h\right)$ solve the households' optimum problems in (4) given prices and aggregate quantities, and the value functions $v^h \left(a_t^h, s_t^h\right)$ solve equations (4).

3. $\lambda^h (A \times B)$ is a stationary distribution:

$$\lambda^h (A \times B) = \int_{\mathcal{A}^h \times S^h} \Lambda^h \left[(a, s), A \times B\right] \lambda^h (da, ds),$$

for all $A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h$, where $\Lambda^h \left[(a, s), A \times B\right] : \left(\mathcal{A}^h \times S^h\right) \times \left(\mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right) \to [0, 1]$ are transition functions on $\left(\mathcal{A}^h \times S^h\right)$ induced by the Markov process $\left(s_t^h\right)_{t=0}^{\infty}$ and the optimal policy $g^h \left(a_t^h, s_t^h\right)$.

4. When $\lambda^h (A \times B)$ describe the cross-section of households at each date, i.e. $\overline{\lambda}^h (A \times B) = \lambda^h (A \times B)$, markets clear. In particular, the labour market clears, i.e. $L = L^s = 1$, where

$$L^s = n^u \zeta^u \sum_{j \in S^u} \overline{s}_j^u \xi^u \left(\overline{s}_j^u\right) + n^b \zeta^b \sum_{j \in S^b} \overline{s}_j^b \xi^b \left(\overline{s}_j^b\right),$$

the world asset market clears, i.e.

$$r = r^* + \psi \left[\exp\left(\frac{K - A^s}{F(K, L)}\right) - 1\right],$$

where

$$A^s = n^u \int_{\mathcal{A}^u \times S^u} g^u (a, s) \lambda^u (da, ds) + n^b \int_{\mathcal{A}^b \times S^b} g^b (a, s) \lambda^b (da, ds),$$

[20] Aggregation over the households can be obtained by using the methods discussed e.g. in (see e.g. Uhlig (1996) and Al-Najjar (2004), Acemoglu and Jensen (2015)).

and the goods market clears, which, using factor input market clearing, implies:

$$F(K,1)-\delta K-r(K-A)=$$
$$=n^u\int_{\mathcal{A}^u\times S^u}q^u\left(a,s\right)\lambda^u\left(da,ds\right)+n^b\int_{\mathcal{A}^b\times S^b}q^b\left(a,s\right)\lambda^b\left(da,ds\right).$$

Following standard arguments (commonly used in this class of models since Aiyagari (1994)), it can be shown that continuity of the asset supply and demand functions at the aggregate level with respect to the interest rate as well as the limit properties of supply and demand for assets, imply that a general equilibrium exists.[21] Using results in Acikgoz (2018) and adapting arguments from Angelopoulos *et al.* (2019), we can show the existence of a general equilibrium in the open economy with a unique stationary distribution at the household level that also determines aggregate quantities.

`Proposition 1`

For ψ sufficiently large, $\psi>\psi^{\min}$ satisfying $\frac{K}{Y}\left(r\right)>\ln\left(\frac{r-r^*+\psi^{\min}}{\psi^{\min}}\right)$, a stationary recursive general equilibrium exists.

Proof: The properties of the production function imply that the wage rate is a monotonic function of the interest rate, and, given that $L=1$, K is a decreasing function of r, as are the ratios Y and $\frac{K}{Y}$. Given the interest rate, firm demand implies a demand for assets over labour via (8), given by:

$$A^d=\left[\left(\frac{K}{Y}\right)-\ln\left(\frac{r-r^*+\psi}{\psi}\right)\right]Y,$$

which is a continuous function in r. When $\frac{r-r^*+\psi}{\psi}$ is small enough such that $\frac{K}{Y}>\ln\left(\frac{r-r^*+\psi}{\psi}\right)$, $\frac{dA^d}{dr}<0$. Moreover, when $r\to\frac{1}{\beta}-1$, $A^d\to A^{\min}<+\infty$, whereas when $r\to r^*-\psi$, $A^d\to+\infty$. Given r (and $w\left(r\right)$), there is a unique partial equilibrium, implying a unique aggregate supply of assets, A^s. As shown in Acikgoz (2018), this is continuous with respect to r and when $r\to\frac{1}{\beta}-1$, $A^s\to+\infty$.[22] Moreover, when $r\to-1$, $A^s\to0$. Therefore, an intersection point of the supply and demand curves A^s and A^d exists. ■

Note that the sufficient condition $\phi>\phi^{\min}$ is easy to satisfy for realistic calibrations for developed economies, where the interest rate r does not differ

[21] A general proof of existence of equilibrium for this class of models can be found in Acemoglu and Jensen (2015).

[22] For details see Acikgoz (2018), Theorem 1. Further note that continuity of mean assets with respect to the interest rate, for each type of household, also implies continuity for the weighted average between households.

much from the international interest rate and the capital to output ratio is higher than two, implying values for $\phi^{\min}$ in the third decimal point above zero.

`Computation`

To compute the stationary general equilibrium, we implement the following algorithm:

1. Guess a value for r^n, which, given the first-order conditions (6) and (7) implies a value for K^n, Y^n and w^n.

2. Calculate the demand for domestic assets to labour implied by the international asset markets via (8), given by

$$A^n = \left[K^n - \ln\left(r^n - r^* + \phi\right) + \ln\phi\right] Y^n.$$

3. Given r^n and w^n, solve the "typical" households' problem to obtain $g^h\left(a_t^h, s_t^h\right)$, for $h = u, b$.

4. Use $g^h\left(a_t^h, s_t^h\right)$ and the properties of the Markov processes $\left(s_t^h\right)$ to construct the transition functions $\Lambda_{K_j}^h$. Using $\Lambda_{K_j}^h$, calculate the stationary distributions λ^h.

5. Using λ^h, compute the aggregate values of $A^s\left(r^n\right)$ that is supplied by the domestic economy and the updated value of

$$r^{n^*} = r^* + \phi\left[\exp\left(\frac{K^n - A^s\left(r^n\right)}{Y^n}\right) - 1\right].$$

6. If $|A^s - A^n| < \varepsilon$, where ε is a pre-specified tolerance level, a stationary open economy general equilibrium has been found. If not, go back to step 1, and update $r^{n+1} = \left(1 - \varsigma\right) r^n + \varsigma r^{n^*}$ with $0 < \varsigma \leq 1$.

To solve the household problem we use the Endogenous Grid Method (Carroll (2006)). To implement this algorithm we first choose $a^{\min} = -\phi$. We then let $a^{\max} = 50$, which implies that, in the solution, the probability of asset holdings greater than 40 is less than $3.1 * 10^{-5}$. We discretise the space of household assets $\left[a^{\min}, a^{\max}\right]$ with a log scale by allowing for 1000 points. We have found that the obtained wealth distribution is robust to increasing $K^{\max}$ up to 100 and to decreasing it down to 40.

8 Appendix B

The WAS started in July 2006 with a first wave of interviews carried out over two years to June 2008. The WAS interviewed approximately 30,500 households including 53,300 adult household members in Wave 1. The same households were approached again for a Wave 2 interview between July 2008 and June 2010. In this wave 20,170 households responded (around 70 percent success) including 35,000 adult household members. Waves 3-5 covered the periods between July and June for the years 2010-12, 2012-14 and 2014-16 respectively. After Wave 2, due to sample attrition, the WAS started implementing boost samples in each wave to keep the number of interviewed households around 20,000 and 35,000-40,000 adult household members.[23]

USoc is a large longitudinal survey which follows approximately 40,000 households (at Wave 1) in the UK. USoc covers a wide range of social, economic and behavioural factors making it relevant to a wide range of researchers and policy makers. Data collection for each wave takes place over a 24-month period and the first wave occurred between January 2009 and January 2011. Note that the periods of waves overlap, but the individual respondents are interviewed around the same time each year. Thus, there is no respondent who is interviewed twice within a wave or a calendar year (see e.g. Knies (2018)).

8.1 Demographics (WAS)

1. **Head of the Household**: We define the head of household as the principal owner or renter of the property, and, when there is more than one head, the eldest takes precedence. This follows the reference person definition in USoc. We use of the following variables: (HhldrW), (HiHNumW), (DVAGEw) and/or (DVAge17w).

2. **Education level**: There are two educational attainment variables in the WAS. The first is the TEAw, which is the age that the individual completed education. The second is the EdLevelw which is a derived variable of the education level and represents the highest educational level that respondent has achieved. EdLevelw provides three categories: (i) degree level or above; (ii) below degree qualifications (iii) no qualifications. The TEAw has the disadvantage that it cannot distinguish the type of qualification that the respondent had achieved. Moreover,

[23] The WAS and USoc data sets employed in this paper refer to the free "End User Licence" versions of the datasets (i.e. WAS: SN-7215 and Understanding Society: Waves 1-8, 2009-2017, SN: 6614).

33 percent of the TEAw observations of working-age adults have either missing values or partial answers. Thus, we choose to work with the EdLevelw which is a derived variable and has only 2,942 missing values, i.e. around 2.7 percent of working-age adult observations. However, using EdLevelw, we note that there are respondents for whom educational attainment changes in a way that indicates misreporting. For example, for some respondents, there is an increase of educational attainment just for one wave and then a return back to the previous level of education in subsequent waves. Thus, we have chosen to make some corrections to the educational level when a respondent's educational attainment changes. In particular, if we observe a respondent for all the 5 waves, we replace her educational attainment with the level that was reported the most times across the 5 waves. We follow a similar procedure if a respondent changes her educational attainment just once. More specifically, we require the respondents being present in the sample for at least 3 waves and we use the most commonly recorded education level across waves. These corrections were applied to 4,873 observations out of 107,320 total amount observations of adult respondents (around 4.5 percent) and only half of these 4,873 observations correspond to a head of a household. Despite these corrections, the results are very similar when they are not made.

8.2 Definition of wealth (WAS)

1. **Net property wealth**:[24] is the sum of all property values minus the value of all mortgages and amounts owed as a result of equity release. (HPROPWW).

2. **Net financial wealth**: is the sum of the values of formal and informal financial assets, plus the value of certain assets held in the names of children, plus the value of endowments purchased to repay mortgages, less the value of non-mortgage debt. The informal financial assets exclude very small amounts (less than £250) and the financial liabilities are the sum of current account overdrafts plus amounts owed on credit cards, store cards, mail order, hire purchase and loans plus amounts owed in arrears. Finally, money held in Trusts, other than Child Trust Funds, is not included. (HFINWNTW_sum)

3. **Net Worth:** is the sum of the net property wealth and net financial wealth.

[24] All monetary values are expressed in 2012 prices as measured by CPIH.

Table B1: Wealth Inequality in Great Britain

	Gini	$\frac{sd}{mean}$	$\frac{mean}{median}$	top 10%	$\frac{a_u}{a_b}$
			WAS (wave 1)		
Uni	0.644	1.948	1.846	0.460	
Non-Uni	0.702	1.972	2.073	0.480	2.085
Total	0.696	2.121	2.000	0.492	
			WAS (wave 2)		
Uni	0.632	1.697	1.798	0.442	
Non-Uni	0.714	1.983	2.404	0.481	2.148
Total	0.699	1.977	2.140	0.487	
			WAS (wave 3)		
Uni	0.655	1.995	1.997	0.476	
Non-Uni	0.733	2.488	2.619	0.507	2.247
Total	0.718	2.385	2.301	0.516	
			WAS (wave 4)		
Uni	0.691	2.854	2.267	0.522	
Non-Uni	0.748	2.315	3.410	0.530	2.499
Total	0.742	3.048	2.733	0.555	
			WAS (wave 5)		
Uni	0.685	2.359	2.281	0.514	
Non-Uni	0.761	2.400	3.849	0.538	2.372
Total	0.742	2.628	2.817	0.547	

8.3 Sample selection (WAS)

Table B2: WAS Sample selection, household observations per selection step

selection step	Uni	Non-Uni	Total
1. Whole sample of households			110,963
2. Drop households with misreported age variable			110,937
3. Drop households with duplicate hh grid numbers			110,910
4. Keep if heads' age $\geq$25, $\leq$59			59,457
5. Drop if no or misreported head's educational info	17,490	41,056	58,546
6. Drop if earnings of household members are imputed	17,037	40,235	57,272
Average net worth obs per wave	3,407	8,047	11,454

Table B2 shows the various sample selection steps. The household heads must be between 25-59 years of age, have full information for the relevant demographic information and their household earnings should be reported and not imputed.

8.4 Demographics (USoc)

1. **Head of the Household**: We use the USoc definition of the head of household. The head of household is defined as the principal owner or renter of the property, and, where there is more than one head, the eldest takes precedence. (whrpid, where the prefix w denotes wave)

2. **Education level**: We have used the variable whiqual_dv. To examine the potential heterogeneity of earnings risk in the main text, the sample is split into degree holders and non-degree holders. The former are the individuals who hold either a Higher Degree or 1st Degree, while the latter are the individuals who hold other highers or A-levels/AS level/Highers or GCSE/O level/other qualification or they have no qualifications.

3. **Marital Status**: Marital status of the head of the household. (wmastat_dv)

8.5 Definition of net income (USoc)

Household net labour income: is defined as household net labour earnings plus benefits, plus private transfers. It is equal to household total annual earnings, plus social benefits, plus annual transfers income minus taxes, NI contributions. Private transfers income totals all receipts from other transfers (including education grants, sickness insurance, maintenance, foster allowance and payments from TU/Friendly societies, from absent family members). Social benefits income totals all receipts from state benefits including national insurance retirement pensions. Household Net Labour Income=Net Labour Income (fihhmnlabnet_dv) + Private Transfers (fihhmnprben_dv and fihhmnmisc_dv) + Public Benefits (fihhmnsben_dv).

8.6 Sample selection (USoc)

Our sample selection for USoc is reported in Table B3. The household heads must be between 25-59 years of age, report non-zero net income and their household earnings should be reported and not imputed. Moreover, the head must not have missing values for region and educational attainment. We trim the top and bottom 0.5 percent of observations of net labour income distribution in each year, to avoid extreme cases or possible outliers in recorded income. We also require the households to be observed with positive incomes for at least 3 consecutive waves. As in the WAS, we exclude Northern Ireland.

Table B3: Households and household members USoc

selection step	Uni	Non-Uni	Total
1. Whole sample			208,200
2. Drop proxy & non-full interviews			157,187
3. Original sample & BHPS sample			122,193
4. Drop if no head's educational info			122,023
5. Drop if head's region missing			121,977
6. Drop if head's region is N. Ireland			121,958
7. Keep if heads' age $\geq$25, $\leq$60			68,003
8. Drop if head's marital status missing			67,913
9. Drop if gross labour income is missing or imputed			59,043
10. Drop if net labour income is zero	17,273	40,860	58,133
11. Drop top and bottom 0.5% of observations	17,107	40,461	57,568
12. Drop if they change educational groups	16,770	40,192	56,962
13. keep if present at least at 3 consecutive waves	11,783	27,061	38,844
Average obs per wave	1,472	3,383	4,855
Number of unique households	2,250	5,415	7,665

Figure 1: Quintle Shares and Lorenz Curves of the Wealth Distribution by Group

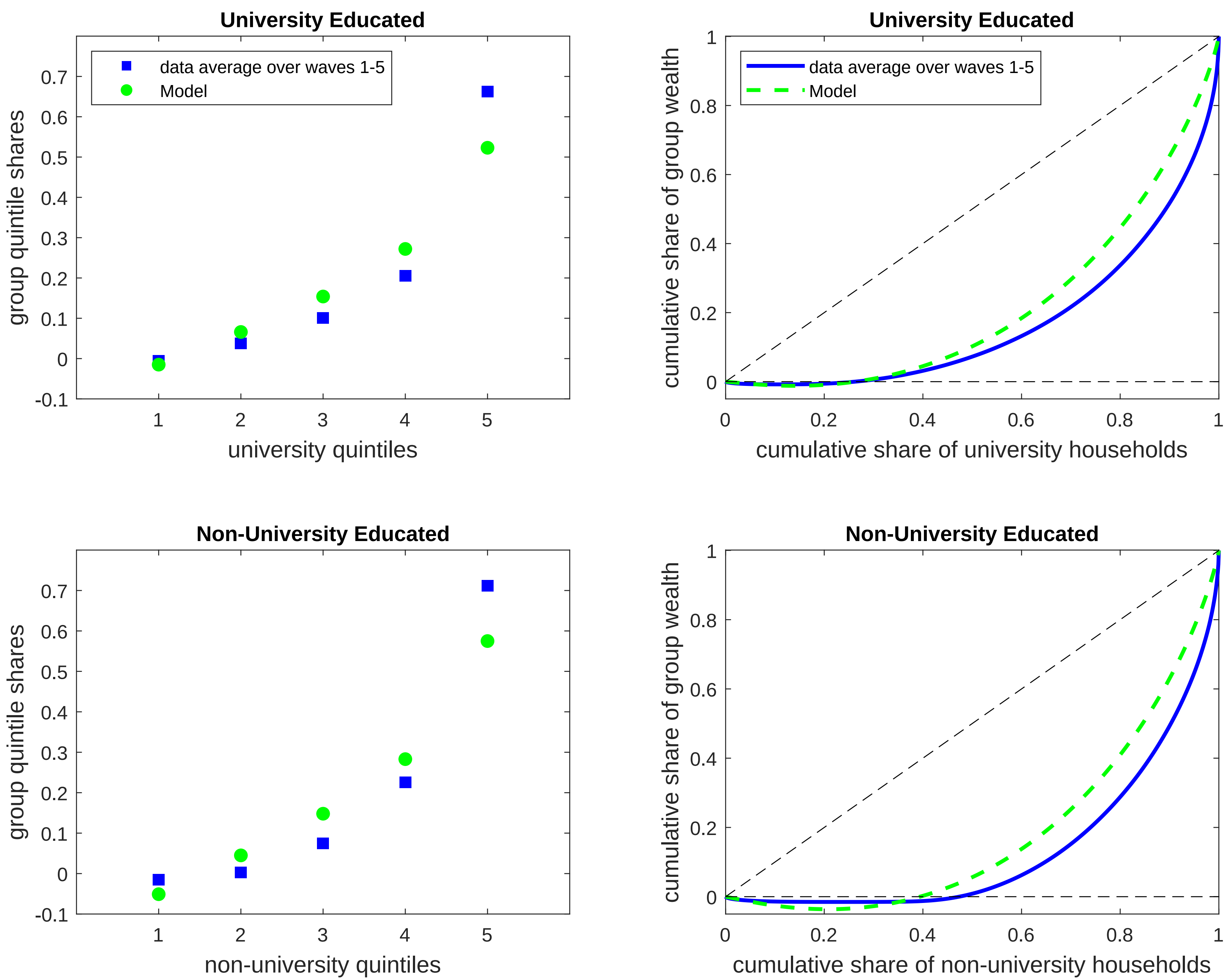

Figure 2: General Equilibrium

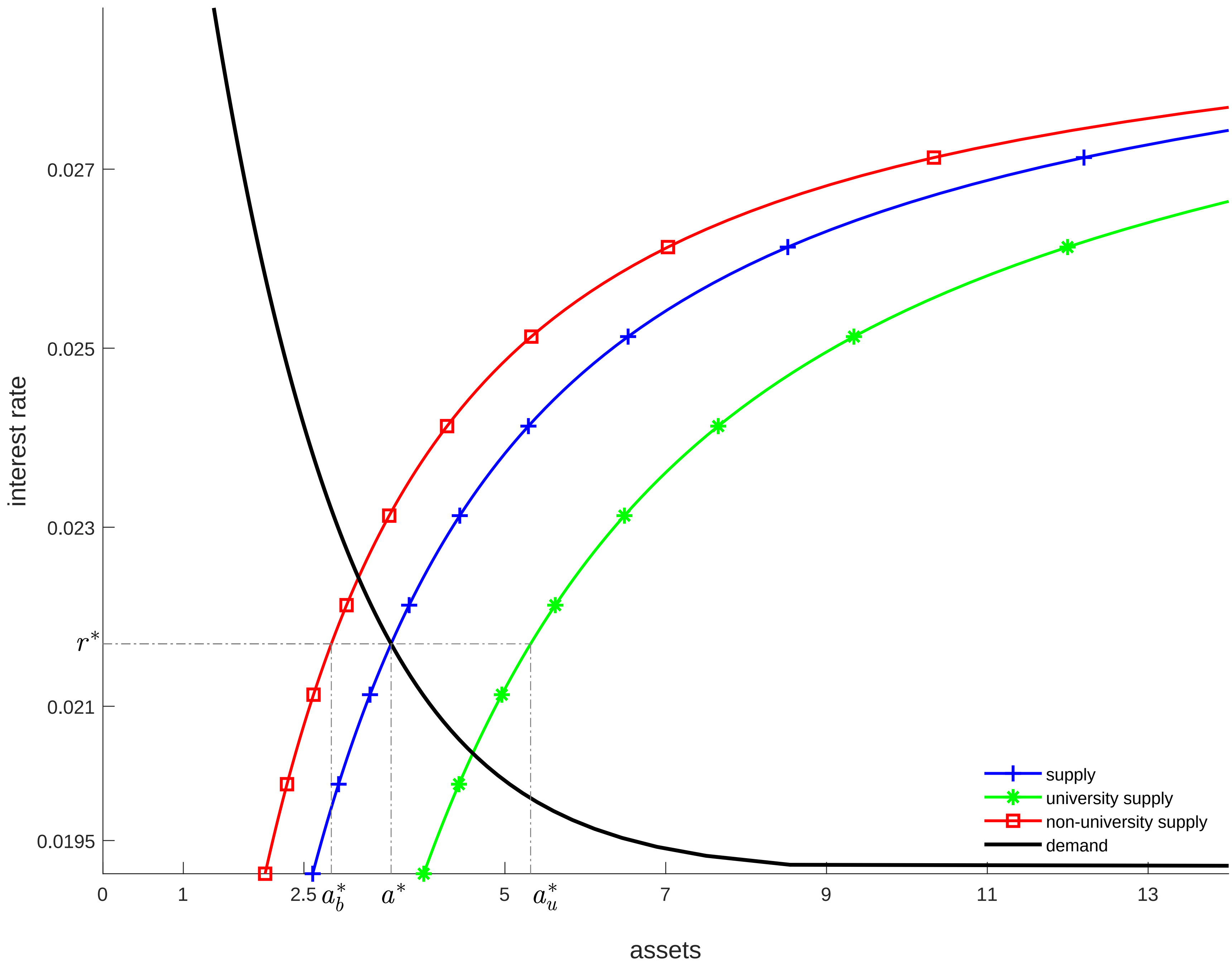

Figure 3: Externalities From Skill Heterogeneity

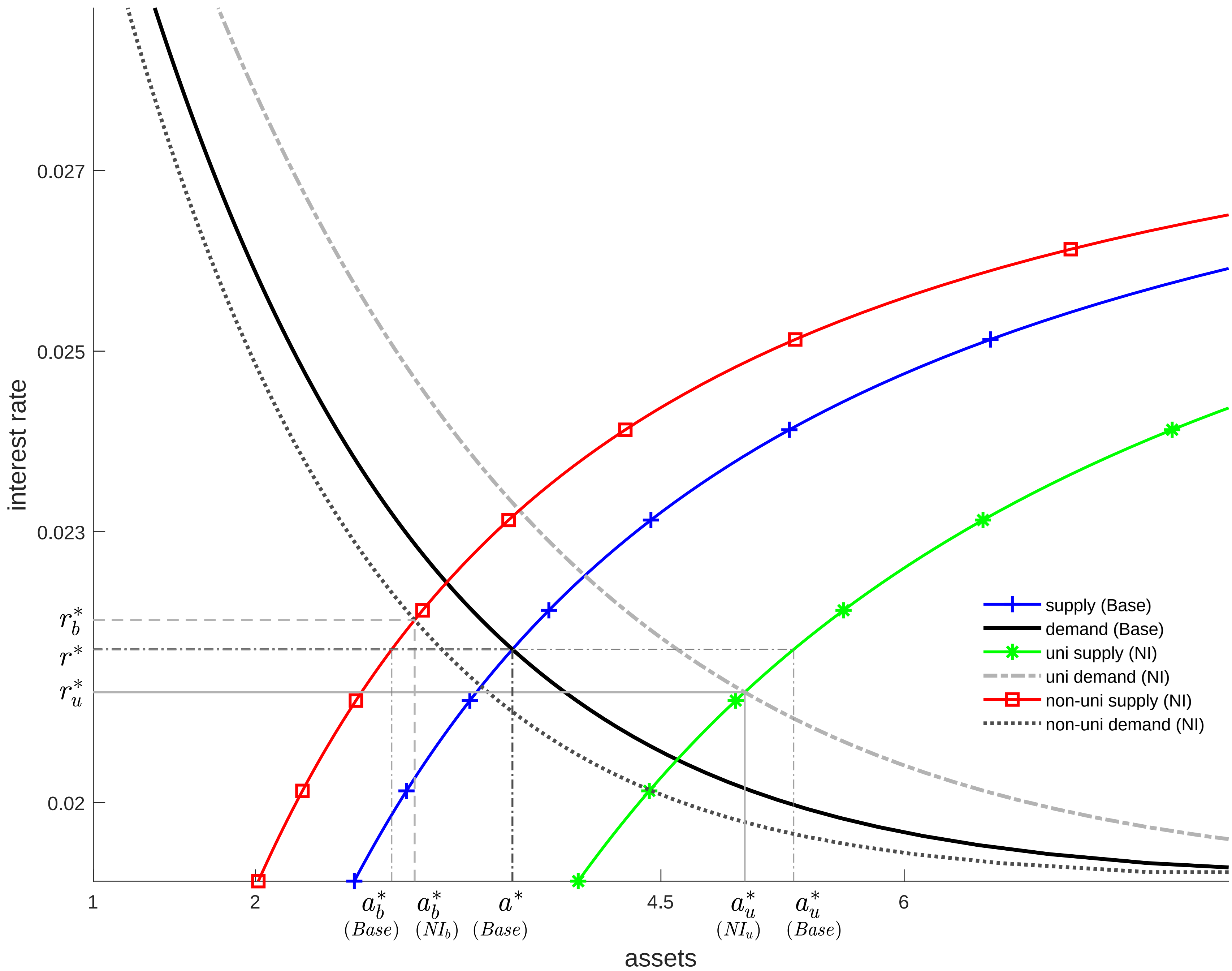

Figure 4: Interest Rate Comparative Statics (non-Uni Group)