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Abstract

The social cost of carbon is the central economic measure for aggregate climate change damages and functions as a metric for optimal carbon prices. Previous literature shows that inequality significantly influences the level of the social cost of carbon, but mostly neglects a major source of inequality - heterogeneity in income below the national level. We characterize the relationship between climate and redistributional policy in an optimal taxation model that explicitly accounts for inequality between and within countries. In particular, we demonstrate that climate and distributional policy cannot be separated when national governments fail to compensate low-income households for climate change damages: Even if only one country does not compensate especially affected households, the social cost of carbon increases globally. Further, we use numerical methods to estimate the scope of these effects. Our results suggest that it is crucial to correct previous estimates of the social cost of carbon for national distributional policies.

JEL-Codes: D300, D610, D630, H210, H230, Q540.

Keywords: optimal taxation, inequality, climate change, social cost of carbon.

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1 Introduction

The social cost of carbon (SCC) is a central measure for climate policy that is used as a guideline for carbon pricing in regulatory impact assessments (Greenstone et al., 2013; US Interagency Group, 2016; Watkiss and Hope, 2011; Environment and Climate Change Canada, 2016; Vinson and Elkins, 2016). The SCC measures the additional damage caused by an extra unit of emissions. Its level is influenced by normative assumptions about distributional justice since preferences for equity can increase the SCC if damages to low-income groups are not compensated. Yet, most estimates of the SCC neglect a major component of inequality: differences between households below the national level. We extend the existing literature in two respects.

First, methodologically: We propose a novel model that accounts for heterogeneity both between and within countries by integrating two levels of governance. At the global level, the SCC is a normative benchmark for efficient climate policy. At the national level, distributional policy determines inequality between households. To capture how distributional and climate policy interact, we use an optimal taxation approach: the SCC is the optimal carbon tax of a global social welfare function. The derived SCC is country-specific and defines how much each country should reduce emissions from the global perspective.[1]

Second, as a consequence of our methodological choices, we find that climate and distributional policies cannot be separated – the SCC depends on the redistribution that occurs within each country. In particular, if low-income households in one country are not compensated for disproportional climate damages and there is aversion to existing inequality at the global level, the SCC increases in all countries. In this case, studies that assume no household heterogeneity at the sub-national level underestimate the SCC. This case is empirically relevant as low-income groups have been shown to be more vulnerable to climate change (Ahmed et al., 2009; Leichenko and Silva, 2014; Letta et al., 2018) and since the existence of institutions that compensate households cannot be taken for granted in all countries.

For policy making, this implies that ignoring heterogeneity within countries when determining climate policy may increase inequality between households. A

[1] Throughout the manuscript we use the terms *social cost of carbon* and *optimal carbon tax* interchangeably. We are aware that technically, if national redistribution is not optimal from the global perspective, carbon taxes are second-best and might not be considered to equal the SCC by some scholars. We therefore extend the definition of the SCC we use in Sec. 2.2.

policy maker with the objective to mitigate global climate change and inequality simultaneously – as in the context of the Sustainable Development Goals (UN, 2018) – needs to account for interactions between these policies.

Our article builds on two strands of literature. The first strand comprises articles on the derivation and the determinants of the level of the SCC (Stern, 2008; Foley et al., 2013; Engstrm and Gars, 2015). The majority of this literature derives estimates of the SCC from Integrated Assessment Models (Greenstone et al., 2013; Metcalf and Stock, 2017), which often aggregate the global economy to one representative agent (Nordhaus, 2014, 2017) or use Negishi-weights in regionalized studies (Nordhaus and Yang, 1996). Chichilnisky and Heal (1994) show that the SCC generally differs between heterogeneous countries when allowing for arbitrary welfare weights. Within this strand, we draw on studies that focus on inequality. Azar and Sterner (1996), Anthoff et al. (2009), Adler et al. (2017) and Anthoff and Emmerling (2019) estimate the SCC when regions differ in their consumption and for different assumptions about social preferences for equity. They show that regional inequality critically influences the level of the SCC.

To avoid a bias in the estimates of the SCC, models need to include heterogeneity within countries (Rausch and Schwarz, 2016; Burke et al., 2016). All studies above, however, assume a representative agent at the global or regional level. As an exception, Dennig et al. (2015) and Budolfson et al. (2017) estimate the optimal global carbon tax under different assumptions about subregional inequality. Our study also accounts for sub-regional heterogeneity, but, contrary to Dennig et al. (2015) and Budolfson et al. (2017), we take into account that household inequality is not a given characteristic. Instead, allocation between households is determined by distributional policies of national governments (Wang et al., 2012) and thus damages to households may be compensated by redistribution. Estimates of the SCC with national redistribution may considerably differ from studies with exogenous distributions.

The second strand determines (optimal) policies under incomplete information (Maggi, 1999; Mirrlees, 1971; Kolev and Prusa, 2002). In contrast to most of this literature, information asymmetries in our model do not arise between governments and individuals or firms, but between the global and national levels of governance. At the global level, the distributions of income, costs and damages of households are known but not the identity of households – this prevents redistribution at the global level. Only the national government can redistribute. To our knowledge this approach is novel to the literature on the social cost of carbon (it is related, however, to considerations of environmental and fiscal federalism, see Williams 2012; Banzhaf and Chupp 2012).

We therefore model two levels of governance: the SCC is determined at the global level; the national level allocates consumption between households and compensates them for climate change damages and abatement costs. As in Chichilnisky and Heal (1994), we explicitly exclude transfers between countries to focus on the more plausible case of heterogeneous SCC across countries, and hence country-specific carbon taxes (see also Bataille et al. 2018).

We completely characterize the SCC in this two-level governance setting. The main analysis focuses on two specific redistribution schemes that are relevant for climate policy. First, national governments choose transfers to maximize a national social welfare function, thus allocating abatement costs between households and compensating for climate damages. Transfers are nationally optimal and mimic the tax and welfare system of each country. We find that the SCC of each country does not deviate from the case of equality within countries[2] under the – in previous literature common – assumption that utility is approximately logarithmic. This is a consequence of the national level compensating the households in its country for excessive damages. The SCC remains almost unchanged irrespective of whether global preferences over inequality align with the national level or whether transfers based on the global normative benchmark demand eliminating inequality between households. Optimal climate policy can approximately be separated from national distributional policy.

In the second scheme, the national level reimburses households exactly what they paid in taxes. National distribution is suboptimal as the distribution of climate damages and abatement costs is not taken into account when transfers are determined. This scheme models settings, in which governments fail to compensate parts of the population, for example due to capacity constraints. We show that when low-income households experience large and uncompensated climate damages, the SCC increases globally given that low-income households receive a global welfare weight that expresses aversion to existing inequality (i.e. weights differ from Negishi weights). Because national governments fail to compensate low-income households, optimal climate policy ambition increases to avoid impacts on these households.

To quantify the influence of household inequality and national redistribu-

[2] I.e. the case of a representative agent at the country level.

tion on the SCC, we use a variant of the NICE model[3] (Dennig et al., 2015). We show that the SCC doubles for some regions when national redistribution is suboptimal and the households in each region experience the same absolute and uncompensated climate damage. The SCC changes moderately when redristribution is nationally optimal with a maximum increase of 20%.

The article is structured as follows: we describe the model in detail Sec. 2. In Sec. 3 we derive the main results analytically. Sec. 4 uses numerical methods to extend the analytical results and Sec. 5 concludes.

2 A social cost of carbon model with inequality between households

This section first describes our optimal taxation model and introduces the choice of households, the choice of the national governance level and the objective at the global governance level. We then extend the concept of the SCC from the literature to account for inequality within countries.

2.1 The model

Our model is based on Chichilnisky and Heal (1994), who study the optimal carbon tax under inequality between countries. We extend the model by making it dynamic and by accounting for inequality between households.[4]

Households: There are N countries and H_k^t households j in each country k at time $t = 0..t_{\text{end}}$.[5] Households derive their utility u from consumption $c_{k,j}^t$ and from the aggregate abatement of a global stock pollutant with zero decay rate. The stock of abatement A^t is hence a global public good and given by the sum of the individual abatement of all households $a_{k,j}^t$ from all previous time steps:

$$A^t = \sum_{T=0}^{t} \sum_{k=1}^{N} \sum_{j=1}^{H_k^T} a_{k,j}^T.$$

[3]NICE is based on the RICE 2010 model, which disaggregates the global economy into twelve regions. NICE further disaggregates each of these regions into income quintiles.

[4]We use the following notation. Lower case letters are variables/parameters at the household level. When they are barred, they are the mean of the variable/parameter over household distributions within a country. Upper case letters are aggregate variables/parameters at the country level. For a complete list of symbols see Appendix F.

[5]Households are represented by their respective income n-tile with the same number of persons per household in every country.

We distinguish, without loss of generality, two kinds of benefits from abatement. First, households experience monetary damages $d_{k,j}^t$ from climate change. Monetary benefits of abatement are equal to avoided damages and are additive to the consumption of households which leads to composite consumption $c_{k,j}^t - d_{k,j}^t(A^t)$. The functional form of damages is given with $\frac{\partial d_{k,j}^t}{\partial A^t} < 0$ and $\frac{\partial^2 d_{k,j}^t}{\partial A^{t2}} \geq 0$. Second, there are non-market benefits, accounted for in the second argument of the utility function $u(c_{k,j}^t - d_{k,j}^t(A^t), A^t)$.

We make the standard assumption of positive and decreasing marginal utility of consumption: $\frac{\partial u(c_{k,j}^t - d_{k,j}^t, A^t)}{\partial c_{k,j}^t} = muc_{k,j}^t > 0, \frac{\partial muc_{k,j}^t}{\partial c_{k,j}^t} = mucc_{k,j}^t < 0$. The same holds for the marginal utility of abatement in non-market benefits, $\frac{\partial u(c_{k,j}^t - d_{k,j}^t, A^t)}{\partial A^t} = mua_{k,j}^t > 0, \frac{\partial mua_{k,j}^t}{\partial A^t} \leq 0$, where the derivative is only with respect to the second argument of the utility function.

Households consume their income $i_{k,j}^t$ net of abatement costs $m_{k,j}^t$ and carbon tax payments, which are a function of the carbon tax rate τ_k^t of each country and their business-as-usual emissions $e_{k,j}^t$ less abatement. In addition, households receive a transfer $\ell_{k,t}^t$, which adds to their disposable income. The budget constraint of households is given by:

$$c_{k,j}^t + \tau_k^t(e_{k,j}^t - a_{k,j}^t) + m_{k,j}^t(a_{k,j}^t) = i_{k,j}^t + \ell_{k,j}^t. \quad (1)$$

Note that households do not anticipate benefits of abatement. The model has the following variables: household abatement $a_{k,j}^t$ (determined by households), the transfer $\ell_{k,j}^t$ each household receives from pollution tax revenue and further redistribution between households (determined at the national level), and the carbon tax rate of each country τ_k^t (determined at the global level).

The carbon tax and transfers are exogenous to households. Optimizing their utility subject to the budget constraint, they perform abatement cost-efficiently:

$$\frac{\partial m_{k,j}^t}{\partial a_{k,j}^t} = \tau_k^t, \quad \forall j. \quad (2)$$

We will represent household choices by the above relationship, so that abatement is not an independent choice variable but implicitly defined by tax rates τ.

National governance level: The national governance level in each country redistributes the revenues from the national carbon tax by adjusting the transfer level each household receives. The sum of transfers in each country has to equal the tax revenue $\sum_j \ell_{k,j}^t = \tau_k^t \cdot \sum_j (e_{k,j}^t - a_{k,j}^t)$, $\forall k, t$.

We describe the choice of redistribution to household j in country k at time t by the national governance level through generic constraints. The transfer is defined by $f^t_{k,j}(\ell^t_{k,1}, \dots, \ell^t_{k,H^t_k}, A^t, \tau^t_k) = 0$, which depends on the decision variables of the model: the level of transfer to all households in that country, the stock of abatement and the tax rate of the country.

Appendix A solves the model for generic constraints. Sec. 3 analyzes nationally optimal and nationally suboptimal transfers.

Global governance level: Optimal climate policy is determined at the global level through the maximization of a global social welfare function (SWF). Optimal policies are constrained Pareto-efficient because the SWF aggregates each household's utility with global welfare weights $w^t_{k,j}$ so that efficiency in the Pareto sense holds (Chichilnisky and Heal, 1994) but the optimization is subject to constraints on redistribution. As in Chichilnisky and Heal (1994) we exclude transfers between countries to focus on the case where optimal carbon taxes differ between countries.[6] Second, national transfers between households in each country are determined through national institutions.

The objective at the global level is given by:

$$\begin{aligned} \max_{\tau^t_k, \ell^t_{p,s}} SWF = \sum_{t=0}^{t_{\text{end}}} \frac{1}{(1+\rho)^t} \sum_{k=1}^{N} \sum_{j=1}^{H^t_k} w^t_{k,j} \cdot u(c^t_{k,t} - d^t_{k,j}(A^t), A^t) \\ \text{s.t.} \qquad f^t_{k,j}(\ell^t_{k,1}, \dots, \ell^t_{k,H^t_k}, A^t, \tau^t_k) = 0, \quad \forall k, j, t \\ \sum_j [\ell^t_{k,j} - \tau^t_k \cdot (e^t_{k,j} - a^t_{k,j})] = 0, \quad \forall k, t, \end{aligned} \tag{3}$$

where household consumption is given by the budget constraint Eq. (1) and abatement by Eq. (2). The parameter ρ is the pure time preference rate.

The maximization is with respect to carbon taxes τ and transfers ℓ. It defines the optimal and country-specific carbon tax rates. The weights w determine the constrained Pareto frontier. A particular set of weights expresses normative preferences for equity between and within countries at the global level. The optimal carbon tax of the model is hence both constrained Pareto-efficient and a normative metric for climate policy. We explicitly implement discounting as a common form of welfare weighting.

[6] Chichilnisky and Heal (1994) show that when allowing for unrestricted international transfers, optimal carbon taxes are equal among countries. We are particularly interested in the consequences of relaxing this assumption.

Inequality and national distributional policy are anticipated at the global level, which is represented by the constraints f (together with the budget constraint of the national level). Transfers ℓ are formally choices at the global level in Eq. (3) but are indirectly determined through the constraints. The constraints can be interpreted as defining second-best settings for deriving the carbon tax.

2.2 The SCC as the optimal carbon tax

We extend the concept of the SCC used in previous literature to inequality within countries. If our model aggregated households at the national level, the optimal tax rate defined by Eq. (3) would be equal to the SCC along an optimal emissions pathway (as in Nordhaus 2017). We refer to the optimal tax rates from Eq. (3) as the SCC, also under sub-national inequality.

In previous literature, the SCC of a country/region is the gain in social welfare from an extra unit of abatement divided by the gain in social welfare from an additional unit of consumption in the respective country/region (Adler et al., 2017):

$$SCC_k^t = \frac{\frac{\partial SWF}{\partial A^t}}{\frac{\partial SWF}{\partial C_k^t}}. \tag{4}$$

In our model, Eq. (4) cannot be readily applied to define the SCC because the consumption level of a country C_k^t is not an independent choice. Rather, the consumption level of a country changes indirectly when household consumption changes for different optimal tax levels. By defining that the SCC equals the optimal tax rate of each country, our model reproduces previous results when households within each country are identical. Results can however change considerably under inequality within countries.

Throughout the manuscript, the formulas for the SCC usually include the consumption elasticities of the marginal utilities:

$$\mu_C = -\frac{\frac{d}{dc}muc}{muc}(c-d) > 0, \mu_{CC} = -\frac{\frac{d}{dc}mucc}{mucc}(c-d),$$
$$\lambda_C = -\frac{\frac{d}{dc}mua}{mua}(c-d), \lambda_{CC} = -\frac{\frac{d^2}{dc^2}mua}{\frac{d}{dc}mua}(c-d).$$

Below, we use the isoelastic utility function as a common special case:

$$u(c-d, A) = \frac{(c-d)^{1-\eta} - 1}{1-\eta}, \tag{5}$$

for which the elasticity of marginal felicity is constant at $\mu_C = \eta$ and for which the other elasticities are $\mu_{CC} = \eta + 1$, $\lambda_C = \lambda_{CC} = 0$.

3 The social cost of carbon with national redistribution

We derive the SCC as the optimal national carbon tax with inequality between countries and between households (see Sec. 2.2). As a reference case, this section first determines the SCC if there is no inequality within countries (Sec. 3.1). The equality case is equivalent to previous SCC estimates that aggregate at the national level. We then continue with the case of inequality at the household level. Appendix A fully characterizes generic distributional constraints, which only permits general conclusions. We therefore analyze two different national redistribution schemes that are relevant for climate policy. First, redistribution is nationally optimal and maximizes a national welfare function (Sec. 3.2.1). Second, redistribution is nationally suboptimal and households are only reimbursed for what they paid in taxes (Sec. 3.2.2).

The rules for the SCC turn out to be quite complex. To analyze the influence of inequality, we approximate the rules around equality within countries (as in Bernstein et al. (2017), see Appendix C). To compare the rules under household inequality with the concept of the SCC given in Eq. (4), we approximate marginal benefits from abatement and marginal utility of consumption separately. The results include the standard deviations of household characteristics and the covariances between these. The standard deviations introduce inequality at the household level. The covariances describe whether the effects of inequality of different characteristics cancel or reinforce each other.

3.1 The SCC for equality within countries

This section derives the SCC for the case of equality within countries, allowing for inequality between countries. This case is equivalent to SCC estimates from most of the previous literature. The SCC will serve as a benchmark when the next two sections introduce inequality at the household level.

Equality means that the households in each country are identical (they may differ across countries). All household characteristics equal the country mean: $c^t_{k,j} = \frac{1}{H^t_k}\sum_j \left(i^t_{k,j} - m^t_{k,j}(a^t_{k,j}) + \ell^t_{k,j} - \tau^t_k(e^t_{k,j} - a^t_{k,j})\right) = \bar{i}^t_k - \bar{m}^t_k = \bar{c}^t_k$ and $d^t_{k,j}(A^t) = \frac{1}{H^t_k}\sum_j d^t_{k,j}(A^t) = \bar{d}^t_k$. The maximization at the global level is:

$$\max_{\tau^t_k} \sum_{T=1}^{t_{\text{end}}} \frac{1}{(1+\rho)^T} \sum_{k,j} w^T_{k,j} \cdot u(\bar{c}^T_k - \bar{d}^T_k, A^T)$$

where abatement $a^t_{k,j}(\tau^t_k)$ is defined by Eq. (2).

Solving the optimization yields the country-specific SCC under equality within countries. Each country p's SCC is:

$$\tau^t_p|_{\text{EQ}} = \frac{\sum_{T=t}^{t_{\text{end}}} \frac{1}{(1+\rho)^{T-t}} \sum_k H^t_k \bar{w}^T_k \left\{ muc^T_k \cdot (-\bar{d}^{T'}_k) + mua^T_k \right\}}{\bar{w}^t_p muc^t_p}. \quad (6)$$

Here, muc^T_k and mua^T_k indicate the value of the functions at mean composite consumption of each county $\bar{c}^t_k - \bar{d}^t_k$. The set of mean welfare weights $\bar{w}^t_k$ determine global preferences for equity between countries. The formula above spells out Eq. (4) in terms of our model.

The SCC is equal to the optimal carbon tax rate. The rule for the SCC reflects the optimality condition, in which marginal benefits of an additional unit of abatement are equal to the marginal costs of providing the extra abatement in each country, both evaluated with the social welfare function. The SCC depends on inequality between countries and on normative preferences at the global level.

To see this, consider marginal benefits. They consist of two parts. First avoided damages add to the consumption of households. At the global level, this change in composite consumption is evaluated with the change in social welfare of that household, which is the weighted marginal utility of consumption of the country this household lives in. Avoided damages tend to have a higher social value in low-income countries because their marginal utility of consumption is higher. However, a lower welfare weight of lower-income countries can offset this effect. Second, marginal benefits include the increase in utility from non-market benefits. The social value of non-market benefits is larger in lower-income countries if $\lambda_C > 0$ and larger in higher-income countries if $\lambda_C < 0$, again if not offset by the welfare weight.

Providing an additional unit of abatement tends to be socially more costly in lower-income countries, again because their marginal utility of consumption is higher. Indeed, Eq. (6) shows that lower-income countries will have a lower SCC if their welfare weight is high enough. If weights tend towards equality between countries, optimal climate policy establishes an implicit redistribution from high- to low-income countries through lower abatement efforts for low-income countries, which benefit from higher abatement by high-income countries (see Chichilnisky and Heal, 1994; Anthoff et al., 2009; Adler et al., 2017).

However, the SCC is the same for all countries if the existing level of inequality among countries is globally preferred and Negishi-weights are chosen (Chichilnisky and Heal, 1994; Nordhaus and Yang, 1996). Lower-income countries receive a lower weight that is inversely proportional to their marginal utility of consumption.

We next analyze how the SCC changes under inequality within countries.

3.2 The SCC for inequality between households

This section introduces inequality between households, which is determined by the national governance level. Modeling this implies adding constraints on redistribution between households (f) to the maximization at the global level. We find two effects that govern why the SCC changes under inequality within countries. We introduce both effects now and refer to them throughout the rest of the analysis.

First, the social value of an extra unit of consumption and of abatement differs from the level at equality. For equal welfare weights, Fig. 1 illustrates this effect by showing the marginal utility of consumption on the left and of abatement on the right hand side. Whether the social value increases or decreases depends on whether the marginal utility functions are convex or concave. The left hand side illustrates the convex case exemplary for the marginal utility of consumption. An extra unit of consumption to two identical households each is socially less valuable than an extra unit of consumption to a high- and a low-income household each, holding aggregate consumption fixed. The right hand side shows the opposite effect when marginal utility is concave, exemplary for the marginal utility of abatement: the social value of an extra unit of abatement in non-market benefits decreases under inequality between households.

Second, the SCC influences inequality within countries (Cremer et al., 2003).

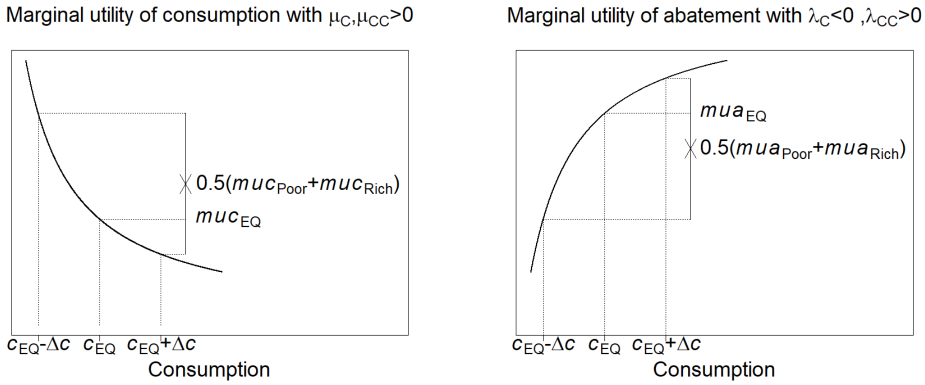

Figure 1: Marginal utility of consumption (left) and of abatement (right) of two households. Under inequality, the sum of marginal utilities of consumption (abatement) is larger (smaller) compared to the equality case. The function muc changes from convex (in the figure) to concave if $\mu_{CC} < 0$. The function mua is also concave (as in the figure) if $\lambda_C > 0$ and $\lambda_{CC} < 0$; it is convex if both elasticities have the same sign.

For a given set of SCC, the national level redistributes according to the constraints f. Fig. 2 illustrates the case when the national level neutralizes disproportional damages and abatement costs on low-income households, the scenario of nationally optimal transfers studied below. The left hand side of Fig. 2 shows a case where the carbon tax incidence is regressive in the sum of abatement costs and tax payments as a share of income. The national level redistributes the tax revenue to render the incidence neutral. On the right hand side of Fig. 2, damages fall disproportionally on low-income households before redistribution. In this example of national transfers, damages are proportional to income after redistribution. Whether the national level neutralizes climate and policy effects fully, partially or not at all is anticipated at the global level and influences the SCC.

The next two sections show how these two effects lead to an SCC that differs from the equality case.

3.2.1 Nationally optimal transfers

In the first scheme the national level chooses transfers to maximize a national welfare function. The main result shows that in this case the SCC hardly deviates from the equality case if two conditions are fulfilled: the utility is isoelastic and the elasticity of marginal felicity η (see Eq. 5) is close to unity. Households

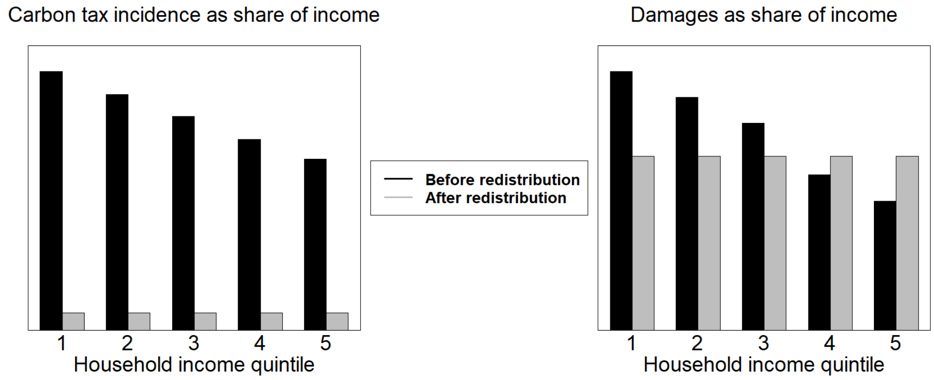

Figure 2: Abatement costs (left) and damages (right) under a set of SCC, as a share of income over income quintiles. The national level adjusts redistribution between households to offset the regressive tax incidence and to offset higher damages to low-income households. Note that the incidence can be regressive, neutral or progressive, depending on the country (Sterner, 2011).

are compensated for excessive climate damages while inequality between them persists. The SCC does not change irrespective of whether national transfers are optimal from the global perspective or if normative preferences at the global level would demand transfers to completely offset inequality within countries.

The national welfare function (NWF) aggregates the utilities of households through a weighted sum, where $z^t_{k,j} > 0$ are the welfare weights the national level assigns to each household. The objective of each country k is:

$$\max_{\ell^t_{k,s}} NWF = \sum_t^{t_{\text{end}}} \sum_j^{H^t_k} z^t_{k,j} \cdot u(c^t_{k,j} - d^t_{k,j}(A^t), A^t)$$

$$\text{s.t.} \quad \sum_j \ell^t_{k,j} = \tau^t_k \sum_j (e^t_{k,j} - a^t_{k,j}(\tau^t_k)) \quad \forall\, t,$$

Without loss of generality, we assume that the average of the national welfare weights is equal to one for each country ($\bar{z}^t_k = 1$). The globally determined carbon taxes τ are exogenous parameters in the optimization of the national governments. Household consumption is defined by the budget constraint (1).

The national level implements a Pareto-optimal distribution among its households based on its preferences for equity. The national weights mimic the tax and welfare system within each country. The resulting redistribution compensates households for excessive climate damages and costs of abatement but leaves a certain level of inequality, for example based on household's income differences.

To achieve its own Pareto-optimum, the national level redistributes until weighted marginal utilities of consumption are equalized.[7] The constraints at the global level are:

$$f^t_{k,j}(\ell^t_{k,j}, A^t, \tau^t_k) = z^t_{k,j} muc^t_{k,j} - z^t_{k,s} muc^t_{k,s} = 0. \tag{8}$$

The weights z determine – together with the shape of the utility function – how much each household's composite consumption differs from the national mean, and hence determine national inequality.

Accounting for Eq. (8) as a constraint, the SCC under nationally optimal transfers for each country p and time t is (see Appendix B for details):

$$\tau^t_p|_{\text{NaOp}} = \frac{\sum_{T=t}^{t_{\text{end}}} \frac{1}{(1+\rho)^{T-t}} \sum_{k,j} \left[\kappa^T_k \cdot (-\bar{d}^{T'}_k) + w^T_{k,j} mua^T_{k,j} - \left(w^T_{k,j} muc^T_{k,j} - \kappa^T_k \right) \frac{muac^T_{k,j}}{mucc^T_{k,j}} \right]}{\kappa^t_p}. \tag{9}$$

with

$$\kappa^t_p = \frac{\sum_j \frac{w^t_{p,j} muc^t_{p,j}}{z^t_{p,j} mucc^t_{p,j}}}{\sum_j \frac{1}{z^t_{p,j} mucc^t_{p,j}}}, \quad \forall\, p, t \tag{10}$$

$muac^T_{k,j} = \frac{\partial}{\partial c^T_{k,j}} mua^T_{k,j}$ are the cross derivatives.

If the national level prefers equality ($z = 1$), the SCC replicates the equality case of the previous section. Changes in the SCC due to inequality between households are driven by national preferences that differ from equality as it is solely the national welfare weights that determine inequality between households. By Eq. (8), irrespective of whether damages or costs fall over- or under-proportionally on low-income households, the national level adjusts transfers so that weighted marginal utilities of consumption are equalized. In effect, the national level distributes total national composite consumption according to

[7] The Lagrangian to the national maximization problem is:

$$\mathcal{L} = \sum_t \sum_{k,j} z^t_{k,j} \cdot u(c^t_{k,j} - d^t_{k,j}, A^t) - \epsilon^t_k \left(\sum_j \ell^t_{k,j} - \tau^t_k \sum_j (e^t_{k,j} - a^t_{k,j}) \right).$$

Setting $\partial\mathcal{L}/\partial\ell^t_{k,j} = 0$ we get the following first-order condition:

$$\epsilon^t_k = z^t_{k,j} muc^t_{k,j} \quad \forall j. \tag{7}$$

By choosing an arbitrary household of each country s, Eqs. (7) leads to Eq. (8).

the national welfare weights, irrespective of the pre-transfer distributions. The SCC is independent of the distribution of income, damages and costs across households and only depends on the welfare weights z.

We now show that the SCC of a country may increase or decrease under inequality within countries, depending on three characteristics: (i) the shape of the utility function with consumption elasticities μ and λ; (ii) inequality within countries, approximated by the standard deviation of national welfare weights $\sigma_k^t(z)$ of each country k and time t; (iii) in how far national and global welfare weights differ, approximated by the covariance between the weights $cov_k^t(w,z)$.

Proposition 1. *For nationally optimal transfers the SCC*

(a) does not change compared to equality within countries if utility is logarithmic ($\eta = 1$).

(b) is for each country p and time t approximated by :

$$\tau_p^t|_{NaOp} \approx \frac{\sum_{T=t}^{t_{end}} \frac{1}{(1+\rho)^{T-t}} \sum_k H_k^T \bar{w}_k^T muc_k^T \cdot (-\bar{d}_k^{T'}) \left[1 + \varphi_k^T\right]}{\bar{w}_p^t muc_p^t \left[1 + \varphi_p^t\right]} \tag{11}$$

if utility is isoelastic. The adjustment factors φ determine the change of the SCC compared to equality within countries and are given by:

$$\varphi_k^t = \left(\sigma_k^t(z)^2 - 2\frac{cov_k^t(w,z)}{\bar{w}_k^t}\right)\frac{1}{2}\frac{\eta - 1}{\eta} \quad \forall k, t.$$

If equality is preferred at the global level ($w_{k,j}^t = \bar{w}_k^t$), the SCC at time t tends to increase for the country with the smallest inequality – i.e. the smallest $\sigma(z_p^t)^2$ – if $\eta > 1$ and decrease if $\eta < 1$. For the country with the largest inequality the reverse holds.

(c) is for each country p and time t generally approximated by:

$$\tau_p^t|_{NaOp} \approx \frac{\sum_{T=t}^{t_{end}} \frac{1}{(1+\rho)^{T-t}} \sum_k H_k^T \bar{w}_k^T \left\{muc_k^T \cdot (-\bar{d}_k^{T'}) \left[1 + \varphi_k^T\right] + mua_k^T \left[1 + \phi_k^T\right]\right\}}{\bar{w}_p^t muc_p^t \left\{1 + \varphi_p^t\right\}}. \tag{12}$$

The adjustment factors φ and ϕ are given by:

$$\varphi_k^t = \left(\sigma_k^t(z)^2 - 2\frac{cov_k^t(w,z)}{\bar{w}_k^t}\right)\left(1 - \frac{1}{2}\frac{(\mu_{CC})_k^t}{(\mu_C)_k^t}\right) \quad \forall\, k,t$$

$$\phi_k^t = \left(\sigma_k^t(z)^2 - 2\frac{cov_k^t(w,z)}{\bar{w}_k^t}\right)\frac{(\lambda_C)_k^t}{(\mu_C)_k^t}\left(1 - \frac{1}{2}\frac{(\lambda_{CC})_k^t}{(\mu_C)_k^t}\right) \quad \forall\, k,t.$$

Here, muc_k^t and mua_k^t as well as $(\mu_C)_k^t$, $(\mu_{CC})_k^t$, $(\lambda_C)_k^t$, $(\lambda_{CC})_k^t$ indicate the value of the functions at the national mean $\bar{c}_k^t - \bar{d}_k^t$.

Proof. *Part (a) follows from showing that κ_p^t is independent of inequality between households when $\eta = 1$. This follows from rearranging the constraints f (see footnote 8) to get $z_{k,j}^t/(c_{k,j}^t - d_{k,j}^t) = H_k^t/(C_k^t - D_k^t)$, $\forall j$. With this we know $muc_{p,j}^t = 1/(c_{p,j}^t - d_{p,j}^t) = H_p^t/(z_{p,j}^t(C_p^t - D_p^t))$, $mucc_{p,j}^t = -(H_p^t)^2/(z_{p,j}^t(C_p^t - D_p^t))^2$. Inserting this in Eq. (10) gives $\kappa_p^t = \bar{w}_p^t/(\bar{c}_p^t - \bar{d}_p^t)$.*

Eq. (12) is derived by applying a second-order Taylor approximation in the variables $(w_{k,1}^t, \ldots, w_{k,H_k^t}^t, z_{k,1}^t, \ldots, z_{k,H_k^t}^t)\ \forall k,t$ to the numerator and denominator in Eq. (9) (see Eq. C.1). The points of approximation are $\frac{1}{H_k^t}\sum_j w_{k,j}^t = \bar{w}_k^t$ and $z_{k,j}^t = 1$, which implies equality within countries. The numerator consists of separate summations over time steps and countries, which can be approximated separately. The complete derivation is in Appendix D.

Eq. (11) is derived by plugging in $\mu_C = \eta, \mu_{CC} = \eta + 1, \lambda_C = \lambda_{CC} = 0$ in Eq. (12). The statements are derived by comparing the φ_k^t for the different cases of η.

□

The proposition shows that a country's SCC depends on the level of inequality in all countries, and that it may increase or decrease under inequality between households compared to the equality case. To understand why, we first discuss the case, in which inequality does not affect the SCC: logarithmic utility. In this case, the national level redistributes until each household consumes a share of total national composite consumption that is equal to its welfare weight.[8] As a result, the two effects described in Figs. 1 and 2 directly offset each other, which is illustrated in Fig. 3. First, the social value of increasing

[8] This can be derived by rearranging Eq. (7) to $c_{k,j}^t - d_{k,j}^t = (z_{k,j}^t)^{1/\eta}(\varepsilon_k^t)^{-1/\eta}$. Summing over j yields $C_k^t - D_k^t = \sum_s (z_{k,s}^t)^{1/\eta}(\varepsilon_k^t)^{-1/\eta}$. Solving the last equation for $(\varepsilon_k^t)^{-1/\eta}$ and inserting yields:

$$c_{k,j}^t - d_{k,j}^t = \frac{(z_{k,j}^t)^{1/\eta}}{\sum_s (z_{k,s}^t)^{1/\eta}}(C_k^t - D_k^t) \tag{13}$$

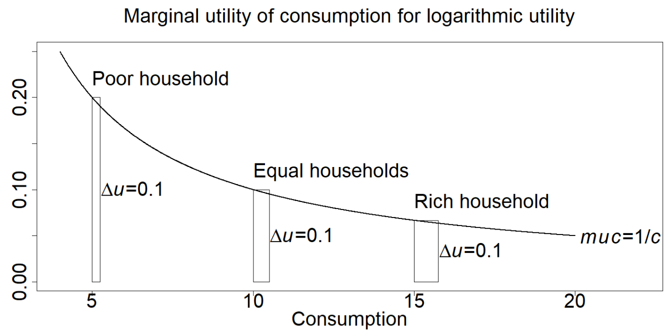

Figure 3: A numerical illustration of the increase in logarithmic utility when the national level distributes one unit of composite consumption. The figure illustrates that the increase in utility of both households does not change under inequality. When both households are equal, i.e. have weights $z = 1$, they each receive a share of 0.5 of the unit and gain 0.1 in utility. Under inequality, the low-consumption household with a weight of 0.5 receives a share of 0.25 of the unit, its utility increases by 0.1. The high-income household with a weight of 1.5 receives a share of 0.75 of the unit, its utility increases by 0.1.

consumption changes (the effect of Fig. 1): a low-consumption household's – one with a lower national welfare weight z – utility increases to a larger degree with one unit of consumption. However, the low-consumption households receive a lower share of an increase in total national consumption through the national distributional policy (the effect visualized in Fig. 2). The same holds v.v. for the high-consumption households. In sum, utility changes of all households are equal. As a result, marginal benefits of abatement and marginal utility of consumption of each country remain at their level of identical households when evaluated at the global level. The SCC does not change with inequality within countries irrespective of global preferences for equality.

For the general isoelastic utility function, the overall effect of inequality on the SCC switches sign with $\eta \lesseqgtr 1$, part (b) of Prop. 1. The effects of Fig. 1 and 2 still offset each other, but not completely. Again, the national level allocates a fixed share of total composite consumption to households, see Footnote 8. Hence, lower-consumption households receive a smaller share of national consumption. If $\eta > 1$, low-consumption households have a over-proportionally larger gain in utility from an additional unit of consumption. Allocating a smaller share of consumption to low-consumption households is beneficial if equality is preferred at the global level. Indeed, Prop. 1 shows that the SCC for the country with the largest inequality tends to decrease in this case. The SCC tends to increase for the country with the lowest inequality. As a result, the low-consumption households in the more unequal countries have larger composite consumption through the abatement efforts of the other countries while saving abatement costs. For $\eta < 1$ the opposite holds. Here, the utility of low-consumption households increases to a lesser extent with consumption so that the small increase in their composite consumption (allocated from the national level) is globally less preferable.

Preferences for equity at the global level that align with the national level offset the effect of inequality on the SCC. In Eq. (11), positive covariances between national and global welfare weights capture how the level of inequality that is implemented by national transfers is actually preferred at the global level. If global welfare weights are equal to Negishi weights, the influence of inequality disappears and the SCC equals aggregate damages.[9]

The intuition is more complex in case of general utility functions. We only briefly discuss it here, leaving a detailed analysis to Appendix E. The SCC in Prop. 1 (c) shows that it is again the shape of the utility function that determines differences to the case of equality in Eq. (6). The adjustment factors φ and ϕ capture the differences. The factors combine the two effects described in Figs. 1 and 2. The consumption elasticities (μ_C in combination with μ_{CC} and λ_C in combination with λ_{CC}) enter the adjustment factors because they determine how the social value of increasing consumption and abatement changes under inequality between households (the effect of Fig. 1). The elasticities also determine how the national level redistributes between households (the effect of Fig. 2). Notably, for general utility functions it is not only the marginal utility of consumption (with elasticities μ_C and μ_{CC}) that determines how the national level redistributes. The stock of abatement also affects the utility of households in non-market benefits. The elasticities of the marginal utility of abatement (λ_C and λ_{CC}) therefore also affect national redistribution.

Prop. 1 demonstrates that the SCC does not differ much from the equality case considered mostly in previous literature if national redistribution is based

[9] Negishi weights are inversely proportional to the marginal utility of consumption: $w^t_{k,j} = B/muc^t_{k,j}$ with a normalization parameter B (Chichilnisky and Heal, 1994; Nordhaus and Yang, 1996). Inserting this relationship in Eqs. (9) and (10) yields $\tau^t_p = \sum_{T=t}^{t_{\text{end}}} \frac{1}{(1+\rho)^{T-t}} \sum_k H^T_k \cdot (-\bar{d}^{T'}_k)$ for the isoelastic utility function. The approximation in Eq. (11) yields the same SCC when applying a second-order Taylor approximation of the global weights $w^t_{k,j} = B/muc^t_{k,j}$ in the national weights $z^t_{k,j}$ and inserting in (11). The approximation of w requires the relationships (D.2) and (D.5) from the appendix.

on a national social welfare function and utility is close to the logarithmic case. Each household is compensated for excessive climate change damages, should they occur. As a result, globally optimal climate policy can approximately be separated from distributional policy at the sub-national level.

The proposition relies on national institutions reaching each household and having the necessary resources available to compensate. The next section shows that results critically change when national institutions do not or cannot compensate households for excessive climate damages.

3.2.2 Nationally suboptimal transfers

Under the second scheme, households are reimbursed exactly the amount they paid in carbon taxes. This is equivalent to the case in which command and control instruments implement the emission reductions that the SCC entails without further redistribution. In general, transfers are nationally suboptimal: households have to bear the costs of abatement and of residual climate change. We study this second-best allocation because it models a situation, in which national institutions fail to account for the distributional consequences of climate policy. Distributional and climate policy interact as a result: we show that if climate damages accrue disproportionally to low-income households in one country, the SCC increases for every country.

The national constraints are: $f^t_{k,j} = \ell^t_{k,j} - \tau^t_k \cdot (e^t_{k,j} - a^t_{k,j}) = 0$. The optimization in Eq. (3) with these constraints leads to the following SCC for each country p:

$$\tau^t_p|_{\text{NaSuOp}} = \frac{\sum_{T=t}^{t_{\text{end}}} \frac{1}{(1+\rho)^{T-t}} \sum_{k,j} w^T_{k,j} \left\{ muc^T_{k,j} \cdot (-d^{T'}_{k,j}) + mua^T_{k,j} \right\}}{\sum_j w^t_{p,j} muc^t_{p,j} \frac{\frac{\partial a^t_{p,j}}{\partial \tau^t_p}}{\sum_s \frac{\partial a^t_{p,s}}{\partial \tau^t_p}}}. \tag{14}$$

The formula for the SCC looks similar as in the equality case in Eq. (6) and is the same if households are identical at the sub-national level.

The SCC can however be quite different in magnitude under inequality. To see this we make inequality at the household level more explicit and assume that households' costs of abatement are directly proportional to income:[10]

$$m_{k,j}^t = \frac{i_{k,j}^t}{\bar{i}_k^t} \bar{m}_k^t(\tau_k^t).$$

Mean national abatement costs are $\bar{m}_k^t = \frac{1}{H_k^t} \sum_j m_{k,j}^t$.

We allow damages to deviate from proportionality to income

$$d_{k,j}^t = \left(\frac{i_{k,j}^t}{\bar{i}_k^t} + \delta_{k,j}^t \right) \cdot \bar{d}_k^t(A^t).$$

$\delta_{k,j}^t$ has a zero mean in each country. Damages accrue over-proportionally to low-income households if δ is negatively correlated with income i, which the literature suggests to be relevant (Ahmed et al., 2009; Leichenko and Silva, 2014; Letta et al., 2018).

Composite consumption of each household is:

$$c_{k,j}^t - d_{k,j}^t = i_{k,j}^t - \frac{i_{k,j}^t}{\bar{i}_k^t} \bar{m}_k^t(\tau_k^t) - \left(\frac{i_{k,j}^t}{\bar{i}_k^t} + \delta_{k,j}^t \right) \bar{d}_k^t(A^t). \tag{15}$$

The next proposition derives the SCC when damages and abatement costs are directly proportional to income (i.e. $\delta = 0$). The approximated SCC depends on the standard deviation in income $\sigma_k^t(i)$ and the covariance $cov_k^t(w, i)$ of global welfare weights and income of households in country k at time t.

Proposition 2. *If abatement costs and damages are directly proportional to income and national governments reimburse households exactly what they paid in taxes, the SCC*

(a) does not change compared to equality within countries if utility is logarithmic ($\eta = 1$).

(b) is for each country p and time t approximated by:

$$\tau_p^t|_{NaSuOpA} \approx \frac{\sum_{T=t}^{t_{end}} \frac{1}{(1+\rho)^{T-t}} \sum_{k=1}^{N} H_k^T \bar{w}_k^T muc_k^T \cdot (-\bar{d}_k^{T'}) \left\{1 + \tilde{\varphi}_k^T\right\}}{\bar{w}_p^t muc_p^t \left\{1 + \tilde{\varphi}_p^t\right\}} \tag{16}$$

[10] Although the literature on the carbon policy incidence often finds regressive effects (i.e. policy costs disproportionally affect low-income groups (Parry et al., 2007), a proportional effect is a good first-order assumption and has been used in previous literature (Dennig et al., 2015).

if utility is isoelastic. The adjustment factors $\tilde{\varphi}$ determine the change of the SCC compared to equality within countries and are given by:

$$\tilde{\varphi}_p^t = (\eta - 1)\left(\frac{\eta}{2}\frac{\sigma_p^t(i)^2}{(\bar{i}_p^t)^2} - \frac{cov_p^t(w,i)}{\bar{w}_p^t i_p^t}\right) \quad \forall\, p, t.$$

If equality is preferred at the global level ($w_{k,j}^t = \bar{w}_k^t$), the SCC tends to increase compared to equality for the country with the smallest income inequality at time t, i.e. smallest $\frac{\sigma_p^t(i)^2}{(\bar{i}_p^t)^2}$, if $\eta > 1$ and to decrease if $\eta < 1$. For the country with the largest inequality the same holds vice versa.

(c) is for each country p and time t generally approximated by:

$$\tau_p^t|_{NaSuOpA} \approx \frac{\sum_{T=t}^{t_{end}} \frac{1}{(1+\rho)^{T-t}} \sum_{k=1}^{N} H_k^T \bar{w}_k^T \left[muc_k^T \cdot (-\bar{d}_k^{T'})\left\{1 + \tilde{\varphi}_k^T\right\} + mua_k^T \left\{1 + \tilde{\phi}_k^T\right\}\right]}{\bar{w}_p^t muc_p^t \left\{1 + \tilde{\varphi}_p^t\right\}}, \tag{17}$$

The adjustment factors $\tilde{\varphi}$ and $\tilde{\phi}$ are given by:

$$\begin{aligned}
\tilde{\varphi}_k^t =& \Big(-2 + (\mu_{CC})_k^t\Big)(\mu_C)_k^t \frac{\frac{1}{2}\sigma_k^t(i)^2}{(\bar{i}_k^t)^2} + \Big(1 - (\mu_C)_k^t\Big)\frac{cov_k^t(w,i)}{\bar{w}_k^t \bar{i}_k^t} \quad \forall\, k, t \\
\tilde{\phi}_k^t =& \frac{1}{2}(\lambda_C)_k^t(\lambda_{CC})_k^t \frac{\sigma_k^t(i)^2}{(\bar{i}_k^t)^2} - (\lambda_C)_k^t \frac{cov_k^t(w,i)}{\bar{w}_k^t \bar{i}_k^t} \quad \forall\, k, t.
\end{aligned}$$

Proof. *The summands of the numerator in Eq.* (14) *can be transformed to:*

$$\sum_j w_{k,j}^t \left\{muc_{k,j}^t(-d_{k,j}^{t'}) + mua_{k,j}^t\right\} \overset{\delta_{k,j}^t = 0}{\overbrace{=}} \sum_j w_{k,j}^t \left(muc_{k,j}^t \cdot (-\bar{d}_k^{t'})\frac{i_{k,j}^t}{\bar{i}_k^t} + mua_{k,j}^t\right).$$

The denominator is

$$\sum_j w_{p,j}^t muc_{p,j}^t \frac{\frac{\partial a_{p,j}^t}{\partial \tau_p^t}}{\sum_s \frac{\partial a_{p,s}^t}{\partial \tau_p^t}} \overset{\frac{\partial a_{p,j}^t}{\partial \tau_p^t} = \frac{1}{\tau_p^t}\frac{i_{p,j}^t}{\bar{i}_p^t}\frac{\partial \bar{m}_p^t}{\partial \tau_p^t}}{\overbrace{=}} \sum_j w_{p,j}^t muc_{p,j}^t \frac{i_{p,j}^t}{H_p^t \bar{i}_p^t},$$

which follows from differentiating abatement costs: $\frac{\partial}{\partial \tau_p^t} m_{p,j}^t(a_{p,j}^t) = \frac{\partial}{\partial \tau_p^t}\left(\frac{i_{k,j}^t}{\bar{i}_k^t}\bar{m}_k^t\right)$ $\rightarrow \tau_p^t \cdot \frac{\partial a_{p,j}^t}{\partial \tau_p^t} = \frac{i_{p,j}^t}{\bar{i}_p^t}\frac{\partial \bar{m}_p^t}{\partial \tau_p^t}$. Both $muc_{k,j}^t$ and $mua_{k,j}^t$ are functions of income through consumption: $c_{k,j}^t - d_{k,j}^t = i_{k,j}^t - \frac{i_{k,j}^t}{\bar{i}_k^t}\bar{m}_k^t(\tau_k^t) - \frac{i_{k,j}^t}{\bar{i}_k^t}\bar{d}_k^t$.

Part (a) is derived by setting $muc_{k,j}^t = 1/(i_{k,j}^t \cdot (1 - \frac{\bar{m}_k^t(\tau_k^t)}{\bar{i}_k^t} - \frac{\bar{d}_k^t}{\bar{i}_k^t}))$ in the numerator *and denominator above.*

Eq. (17) *is derived by applying a second-order Taylor approximation in the variables* $(w^t_{k,1},\dots,w^t_{k,H^t_k}, i^t_{k,1},\dots,i^t_{k,H^t_k})\ \forall k,t$ *to the numerator and denominator above (see Eq. C.1). The points of approximation are* $\frac{1}{H^t_k}\sum_j w^t_{k,j} = \bar{w}^t_k$ *and* $\frac{1}{H^t_k}\sum_j i^t_{k,j} = \bar{i}^t_k$*, which imply household equality at the national level.*

Eq. (16) *is derived by plugging in* $\mu_C = \eta, \mu_{CC} = \eta + 1, \lambda_C = \lambda_{CC} = 0$ *in Eq.* (17)*. The statements follow from comparing* $\tilde{\varphi}^t_k$ *for the different values for* η*.* □

The proposition shows that the SCC of each country depends on the level of inequality in all countries. Again, it may increase or decrease under inequality at the household level. As for nationally optimal transfers, no clear-cut conclusion is generally possible.

In fact, results (a) and (b), that is results for logarithmic and isoelastic utility functions, are the same in Props. 2 and 1 given that damages are proportional to income. In both cases, consumption of households is a fixed share of total composite consumption of their country.[11] Hence, the same reasoning as in Prop. 1 is valid in this case. The SCC hardly deviates from the case of equality if utility is approximately logarithmic.

The effects of inequality for general utility functions as analyzed in Prop. 2 (c) are more complex. First, the SCC influences inequality within countries, the effect in Fig. 2. This is represented by the "-2" summand in $\tilde{\varphi}$. Abatement leads to larger gains for high-income households in absolute terms as avoided damages are proportional to income. The high-income households however also pay larger abatement costs in absolute terms. If equality is preferred at the global level, both effects tend to increase the SCC in the country with the largest inequality because high-income households will pay most of the additional abatement costs. The SCC tends to decrease in countries with less inequality to avoid decreasing damages primarily for high-income households in the unequal countries. Second, the social value of increasing consumption changes under inequality – the effect described in Fig. 1. This is represented by the "μ_{CC}" summand in $\tilde{\varphi}$. This second effect offsets the first one if $\mu_{CC} > 0$. In this case, increasing consumption of a low-income household leads to an over-proportionally larger increase in utility compared to increasing consumption of high-income households. This effect tends to decrease the SCC in the country with the largest inequality to avoid

[11] One can show that the approximations in Eqs. (11) and (16) are the same when setting the welfare weights to $z^t_{k,j} = \frac{H^t_k}{\sum_s (i^t_{k,s})^\eta}(i^t_{k,j})^\eta$ and applying a second-order Taylor approximation of these weights around the income levels $i^t_{k,s}$.

abatement costs for low-income households and increases the SCC in more equal countries to avoid damages to low-income households in unequal countries.

Both effects are offset if preferences differ from equality at the global level. If high-income households receive a higher global weight ($cov(w, i) > 0$), it is socially valuable that they experience larger avoided damages in absolute terms and not socially valuable that they bear larger abatement costs. This third effect is represented by the "1" summand and offsets the first effect. Additionally, the social value of avoided damages or lower abatement costs of low-income households is less (represented by the "$-\mu_C$" summand in $\tilde{\varphi}$), which offsets the second effect. The opposite holds if low-income households receive a higher welfare weight ($cov(w, i) < 0$).

Lastly, consider the aggregation of non-market benefits in $\tilde{\phi}$. Inequality influences the social value of increasing abatement (the effect in Fig. 1). If equality is preferred at the global level and marginal utility of abatement is concave (the term with $\lambda_C \lambda_{CC}$ in $\tilde{\phi}$ is negative), the presence of inequality tends to decrease the aggregate benefit of abatement and hence the SCC for all countries. If marginal utility of abatement is convex, the opposite holds. If inequality is preferred at the global level ($cov(w, i) > 0$), the SCC tends to increase for all countries if high-income households gain more from non-market benefits (the case of $\lambda_C < 0$) because these utility gains receive higher global weight.

Props. 1 and 2 show that the influence of inequality on the SCC is ambiguous. We next show that inequality increases the SCC of all countries when damages disproportionally affect low-income households and are not compensated. This case is important, as empirical research shows that low-income households are more vulnerable to climate change and because national institutions may face capacity constraints when redistributing.

As we have seen so far, the effects of inequality can be countervailed through appropriate choices of the global welfare weights. To single out the influence of a higher burden on low-income households, we set the global weights at the household level to equality for the following proposition, i.e. $w^t_{k,j} = \bar{w}^t_k$. We also discuss the effect of global weights that diverge from equality.

Proposition 3. *Assume that abatement costs are proportional to income, global welfare weights are equal at the household level ($w^t_{k,j} = \bar{w}^t_k$) and damages fall disproportionally on low-income households ($cov^t_k(\delta, i) < 0$). The SCC increases* *for each country additional to the effects of income inequality if damages are small compared to total consumption.*[12] *The SCC is approximated by:*

$$\tau_p^t|_{NaSuOpB} \approx \tau_p^t|_{NaSuOpA} + \underbrace{\frac{\sum_{T=t}^{t_{end}} \frac{1}{(1+\rho)^{T-t}} \sum_k \ H_k^t \bar{w}_k^T muc_k^T \bar{d}_k^{T'} (\mu_C)_k^t \frac{cov_k^t(\delta,i)}{\bar{i}_k^t}}{\bar{w}_p^t muc_p^t \left\{1+\tilde{\varphi}_p^t\right\}}}_{>0} \quad \forall\, p,t.$$

Proof. *The proposition is obtained by applying formula* (C.1) *to the numerator and denominator in* (14)*. The numerator consists of separate summations over the households of each country, which can be approximated separately:*

$$\sum_j w_{k,j}^t \left\{ muc_{k,j}^t(-d_{k,j}^{t'}) + mua_{k,j}^t \right\} \overset{w_{k,j}^t=\bar{w}_{k,j}^t}{\widehat{=}} \bar{w}_k^t \sum_j \left(muc_{k,j}^t \left(\frac{i_{k,j}^t}{\bar{i}_k^t} + \delta_{k,j}^t \right) (-\bar{d}_k^{t'}) + mua_{k,j}^t \right).$$

The numerator is

$$\sum_j w_{p,j}^t muc_{p,j}^t \frac{\frac{\partial a_{p,j}^t}{\partial \tau_p^t}}{\sum_s \frac{\partial a_{p,s}^t}{\partial \tau_p^t}} \overset{w_{p,j}^t=\bar{w}_p^t}{\widehat{=}} \bar{w}_p^t \sum_j muc_{p,j}^t \frac{i_{p,j}^t}{H_p^t \bar{i}_p^t}.$$

Both $muc_{k,j}^t$ *and* $mua_{k,j}^t$ *are functions of income and the parameter* δ *through composite consumption:* $c_{k,j}^t - d_{k,j}^t = i_{k,j}^t - \frac{i_{k,j}^t}{\bar{i}_k^t}\bar{m}_k^t(\tau_k^t) - \left(\frac{i_{k,j}^t}{\bar{i}_k^t} + \delta_{k,j}^t\right)\bar{d}_k^t(A^t)$*. The variables for the approximation are* $\delta_{k,j}^t$ *and* $i_{k,j}^t$*. The respective points of approximation are* 0 *and* $\frac{1}{H_k^t}\sum_j i_{k,j}^t = \bar{i}_k^t$*, which would lead to equality within countries.*

The approximation is

$$\tau_p^t|_{T_{2b}} \approx \frac{1}{\bar{w}_p^t muc_p^t \left\{1+\hat{\varphi}_p^t\right\}} \sum_{T=t}^{t_{end}} \frac{1}{(1+\rho)^{T-t}} \sum_k H_k^t \bar{w}_k^T \Bigg[muc_k^T(-\bar{d}_k^{T'}) \Bigg\{ 1 - (\mu_C)_k^t \left(\frac{cov_k^t(\delta,i)}{\bar{i}_k^t} - \frac{\bar{d}_k^t}{\bar{c}_k^t - \bar{d}_k^t}\sigma_k^t(\delta)^2 \right) + \hat{\varphi}_k^T \Bigg\} + mua_{k,j}^T \left\{1+\hat{\phi}_k^T\right\} \Bigg].$$

[12]The results may be reversed if avoided damages become large compared to overall consumption. In this case, low-income households can consume more than high-income households through abatement, which can change the direction of the influence.

with

$$\hat{\varphi}_k^t = \underbrace{\left((\mu_{CC})_k^t - 2\right)(\mu_C)_k^t \frac{1}{2}\frac{\sigma_k^t(i)^2}{(\bar{i}_k^t)^2}}_{=\tilde{\varphi}|_{cov(w,i)\equiv 0}}$$

$$- (\mu_C)_k^t \frac{\bar{d}_k^t}{\bar{c}_k^t - \bar{d}_k^t}\left[\left((\mu_{CC})_k^t - 1\right)\frac{cov_k^t(\delta,i)}{\bar{i}_k^t} - (\mu_{CC})_k^t \frac{\bar{d}_k^t}{\bar{c}_k^t - \bar{d}_k^t}\frac{1}{2}\sigma_k^t(\delta)^2\right]$$

$$\hat{\phi}_k^t = \underbrace{(\lambda_C)_k^t(\lambda_{CC})_k^t \frac{1}{2}\frac{\sigma_k^t(i)^2}{(\bar{i}_k^t)^2}}_{=\tilde{\phi}|_{cov(w,i)\equiv 0}}$$

$$- (\lambda_C)_k^t \lambda_{CC})_k^t \frac{\bar{d}_k^t}{\bar{c}_k^t - \bar{d}_k^t}\left[\frac{cov_k^t(\delta,i)}{\bar{i}_k^t} - \frac{1}{2}\sigma_k^t(\delta)^2 \frac{\bar{d}_k^t}{\bar{c}_k^t - \bar{d}_k^t}\right]$$

Neglecting all terms with $\frac{\bar{d}_k^t}{\bar{c}_k^t - \bar{d}_k^t}$ *as they are small by assumption of the proposition, we arrive at the approximation.*

□

Prop. 3 shows a clear effect of inequality on the SCC. If damages accrue disproportionally to low-income households in only one country, the SCC of all countries increases compared to Prop. 2. A global increase in the SCC prevents higher inequality by avoiding damages to low-income households. It can additionally be shown that the SCC increases globally, albeit to a lesser extent, as long as global preferences do not exactly align with existing inequality, i.e. global weights are different from Negishi weights.[13]

9

At the global level, the SCC increases because the national level fails to compensate low-income households for excessive climate damages. Therefore, the value of abatement critically increases under inequality.

[13]The proof is straightforward. If global welfare weights differ from $\bar{w}_k^t$, the approximation in Prop. 3 is adjusted by including the welfare weights in the second-order Taylor approximation:

$$\tau_p^t|_{\text{NaSuOpB}} \approx \tau_p^t|_{\text{NaSuOpA}} + \frac{\sum_{T=t}^{t_{\text{end}}} \frac{1}{(1+\rho)^{T-t}} \sum_k H_k^t \bar{w}_k^T muc_k^T \bar{d}_k^{T'} \left[(\mu_C)_k^t \frac{cov_k^t(\delta,i)}{\bar{i}_k^t} - \frac{cov_k^t(\delta,w)}{\bar{w}_k^t}\right]}{\bar{w}_p^t muc_p^t \left\{1 + \tilde{\varphi}_p^t\right\}}.$$

Negishi weights are $w_{k,j}^t = B/muc_{k,j}^t$ with a normalization parameter B (Chichilnisky and Heal, 1994; Nordhaus and Yang, 1996). Applying a second-order Taylor approximation of the weights w in income i and damage parameter δ yields $cov_k^t(\delta,w) = \bar{w}_k^t(\mu_C)_k^t cov_k^t(\delta,i)$, so that the additional summand is zero.

4 Numerical simulations

This section uses the Nested Inequalities Climate Economy model (NICE) (Dennig et al., 2015; Budolfson et al., 2017) to quantitatively assess how the SCC changes under inequality. NICE is based on the Integrated Assessment Model RICE (Nordhaus, 2010), which disaggregates the global economy into twelve regions. In NICE, each of these regions is further disaggregated into its five income quintiles. The income of quintiles is a share of the regional total. The income shares are based on empirical estimates and can be found in Table SI 1 of Dennig et al. (2015). We denote the income share of quintile j in region k as $i\_\text{share}_{k,j}$. This study diverges from Dennig et al. (2015) by including national redistribution and allowing for regionally specific carbon taxes.

The SCC is the maximum of a global social welfare function, which takes the constant elasticity form as in Eq. (5) and equality preferred at the global level:

$$SWF = \sum_t \frac{1}{(1+\rho)^t} \sum_{k,j}^{t} pop_{k,j}^t \left(\frac{c_{k,j}^t - d_{k,j}^t}{pop_{k,j}^t} \right)^{1-\eta} / (1-\eta) \qquad (18)$$

where $pop_{k,j}^t$ is the population size of each quintile j in region k at time t. The pure time preference rate is 1.5%.[14] Income net of mitigation costs determines quintile consumption $c_{k,j}^t$. In the basic NICE model the distribution of climate change damages $d_{k,j}^t$ and abatement costs over quintiles can be varied and enters as an assumption.

We extend the NICE model by implementing the three cases of Sec. 3.

The case of equality (Sec. 3.1): Each quintile gets the same share of regional composite consumption, which is equivalent to studies of the SCC with a representative agent (Nordhaus, 2017; Adler et al., 2017; Ricke et al., 2018).

Nationally optimal transfers (Sec. 3.2.1): We let each income quintile's share of regional composite consumption $c_{k,j}^t - d_{k,j}^t$ follow Eq. (13). The weights $z_{k,j}^t$ for each region k and quintile j are calculated in the following way: Given a regional consumption level, the national maximization leads to each quintile's share of consumption to be proportional to its income share $i\_\text{share}_{k,j}$. This amounts to assuming that the currently observed income distribution is considered optimal by each region and is preserved over the future.[15]

[14]We use a pure time preference rate of 1.5% as a benchmark to study inequality. We are aware that lower discount rates may be needed from an ethical viewpoint (Barrage, 2018).

[15]The Dennig et al. results on a globally uniform carbon tax carry over to our national

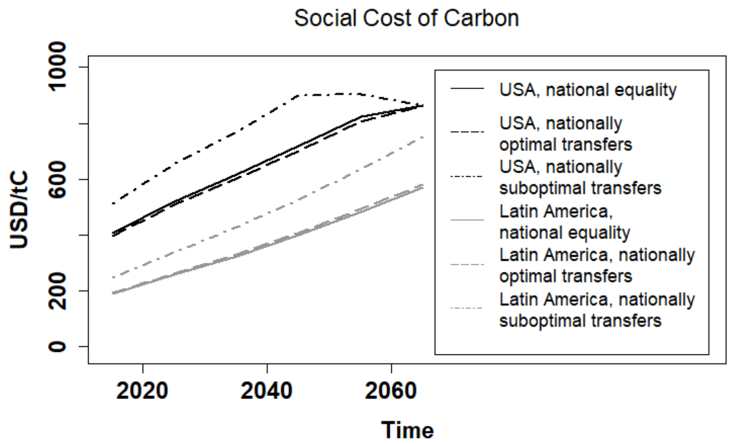

Figure 4: The SCC over time for the USA and Latin America for the following cases: equality between quintiles; inequality and nationally optimal transfers; inequality and nationally suboptimal transfers. The elasticity of marginal felicity is $\eta = 0.5$. The income elasticity of damages is $\xi = 0$, hence we are looking at the case in which damages fall disproportionally on low-income quintiles. We chose these two regions to illustrate our results, since they exhibit differences in income. In theory, any two regions with income differences would be suitable.

Nationally suboptimal transfers (Sec. 3.2.2): We use the same specification of quintile consumption as in (Dennig et al., 2015). Pre-damage consumption of quintiles is proportional to the income share $i\_\text{share}_{k,j}$ of quintiles, which is equivalent to our formulation in Sec. 3.2.2. Each quintile receives a share of the regional climate damages, denoted $d\_\text{share}_{k,j}$. The shares can be varied from being proportional to being more or less than proportional to the income share, which is computed through the income elasticity of damage ξ: $d\_\text{share}_{k,j} \propto (i\_\text{share}_{k,j})^{\xi}$. If $\xi = 1$, damages are proportional to income shares and we have the same setting of NICE as in Prop. 2. If $\xi < 1$, damages fall disproportionally on low-income quintiles and we have the setting of Prop. 3.[16]

For the three cases, Fig. 4 shows the time-path of the SCC for the USA and Latin America until 2065. The SCC grows over time until it reaches the level

redistribution schemes. The proportional damage case in Dennig et al. (2015) is the same as implementing the nationally optimal transfers defined here and nationally suboptimal transfers with proportional damages.

[16]With reference to Footnote 15, the results of (Dennig et al., 2015) on a globally uniform tax with disproportional damages to low-income quintiles carry over to this nationally suboptional transfer scheme.

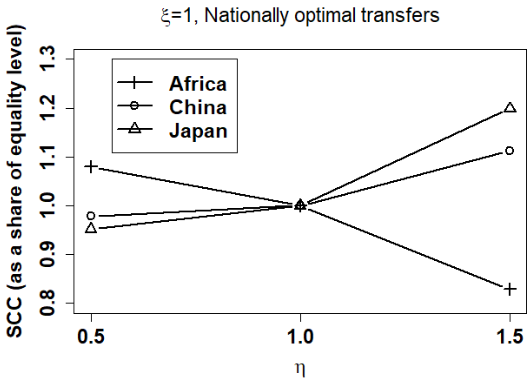

Figure 5: The relative change of the SCC in the year 2035 compared to equality of quintiles for different values of the elasticity of marginal felicity η and nationally optimal transfers. Displayed are NICE region with the highest (Africa), lowest (Japan) and an average (China) level of inequality (measured with the standard deviation in the income shares).

of the backstop technology in the USA. Comparing the SCC paths shows that inequality between quintiles can have a large impact on the SCC when transfers are nationally suboptimal, which we now discuss in detail.

Consider first the case of equality within countries. In Fig. 4, the USA has a larger SCC than Latin America. This shows the quantitative impact of the effect discussed in Sec. 3.1: the SCC of the higher-income USA should be larger than of lower-income Latin America region as equality is preferred at the global level.

Concerning the impact of inequality under nationally optimal transfers, Fig. 4 shows that the SCC decreases moderately for the USA and increases moderately for Latin America. This effect is derived in Prop. 1: the SCC may increase or decrease for each region depending on the relative level of inequality and the consumption elasticity $\eta \lesseqgtr 1$. Nationally suboptimal transfers lead to increases in the SCC in both regions. We detail this case when discussing Figure 6 below.

To further investigate the quantitative effects, Fig. 5 displays the relative change of the SCC under inequality compared to equality at the household level for the regions which exhibit the highest (Africa), lowest (Japan) and an average (China) level of inequality in the NICE model. Inequality is measured with the standard deviation in the income shares. On the horizontal axis, the elasticity of marginal felicity of the isolelastic utility function in Eq. (18) is varied. In line

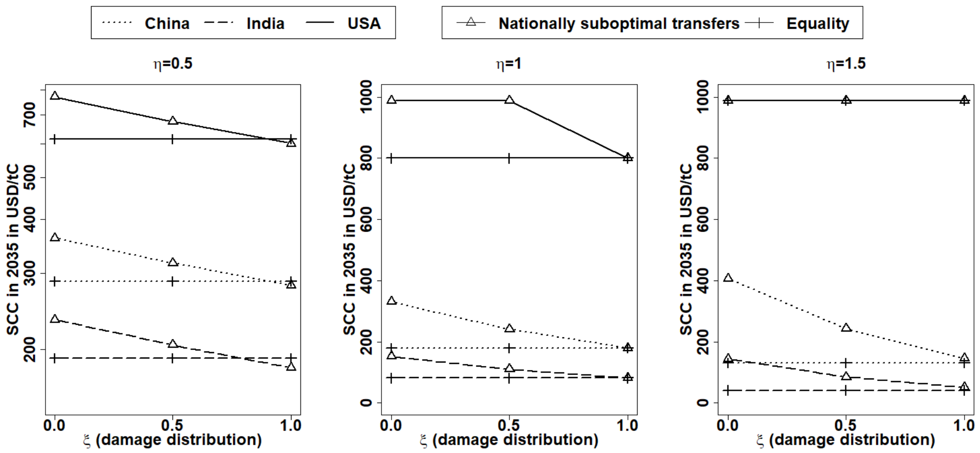

Figure 6: The SCC in the year 2035 for different values of the elasticity of marginal felicity η, different damage distribution parameters ξ, nationally suboptimal transfers and the case of equality between quintiles. Climate damages to the lowest quintiles decrease from left to right.

with Prop. 1, the country with the smallest inequality – Japan – has a larger SCC for $\eta > 1$ and a smaller SCC if $\eta < 1$ compared to equality. The reverse holds for Africa. For China, changes are more moderate but the figure shows that it tends toward the behavior of Japan. The SCC does not change from the equality case if utility is logarithmic. The numerical estimates with NICE show that the magnitude of change can become significant if η diverges from unity. For the regions with the lowest and highest inequality, the SCC changes by roughly 20 % if η increases to 1.5.

Lastly, we estimate the quantitative impact of nationally suboptimal transfers. Fig. 5 also displays the quantitative effects of Prop. 2 if damages are proportional to income (i.e. the income elasticity of damages is $\xi = 1$). In this case, composite consumption of quintiles is proportional to their income share so that the SCC changes compared to equality in the same way under nationally optimal and suboptimal transfers (see discussion below Prop. 2).

Figs. 4 and 6 display a larger increase in the SCC across all regions when damages fall disproportionally on low-income quintiles and are not compensated – the numerical implementation of Prop. 3. The vertical axis of Fig. 6 shows the SCC for three countries (USA, India, China) under nationally suboptimal transfers. The figure compares the SCC to the equality case. If damages are proportional to income ($\xi = 1$), the SCC diverges only moderately from the case of equality. When increasing damages to the low-income quintiles to the extreme case, in which all quintiles experience the same absolute damage ($\xi = 0$), the SCC increases sharply compared to the equality case. For $\eta = 1.5$, the magnitude of change is especially pronounced, in line with Prop. 3, with the SCC more than doubling for India and China. The SCC of the USA reaches the value of the back-stop technology for $\eta = 1.5$ and does not change with introducing inequality within regions.

5 Conclusion

This article is the first to calculate the SCC with heterogeneity between and within countries, when the distribution within countries is endogenous. Traditionally, the SCC has been calculated in frameworks that model countries (or regions) as single representative agents. We identify the cases in which accounting for heterogeneity both between and within countries leads to large differences in the SCC compared to previous estimates.

Modeling heterogeneous households requires distinguishing between a global and a national level of governance. Optimal climate policy is determined at the former, while the latter redistributes between households within its jurisdiction. Redistribution between households is not available at the global level due to an information asymmetry: while the distribution of climate damages, abatement costs and income is known at the global level, the identity of households in connection to these distributions is unknown. Thus, redistribution is left to the national government. We characterize the SCC as a globally optimal set of national carbon taxes in this setting.

Our main findings are as follows. First, we show analytically that the SCC depends on the redistribution taking place within countries. Second, we compare the SCC when households are heterogeneous at the sub-national level to the case of representative national agents (i.e. no heterogeneity within countries). Differences are especially pronounced when climate damages fall disproportionally on lower-income households without compensation. Third, we use numerical methods to quantify the effects for a standard range of parameter values.

These results have immediate relevance for policy makers, since the SCC is a benchmark measure for efficient carbon taxes. Climate and distributional policies can roughly be determined separately only if national institutions compensate households for excessive climate policy costs or climate damages. This holds irrespective of whether national compensation aligns with or differs substantially from distributional preferences at the global level. By contrast, if some national governments fail to compensate low-income households for substantial climate damages, for example due to a lack of institutional capacity, policy interactions are large, and the SCC in other countries can more than double. This can also be interpreted as a delicate balance between national insurance mechanisms against climate damages and globally ambitious mitigation efforts. If one is below optimal levels, the other becomes more important.

One limitation of our framework is that national governments rely on first-best lump-sum transfers for redistributing the carbon tax revenues. We choose this abstraction to highlight the importance of accounting for household heterogeneity when calculating the SCC in the simplest possible way. In the real world, national governments would be information-constrained, and revenue would have to be either redistributed through less optimal transfer and tax systems, public investment or via tax cuts. Future research needs to study more realistic redistributional policies in a two level governance framework.

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Appendices

A Solving the analytical model for a general redistribution scheme

This section derives the SCC for general redistribution. For that purpose we analyze a general functional relationship that defines the level of transfers to each household through the constraint $f^t_{k,j}(\dots,\ell^t_{k,j},\dots)=0$. We let f be a function of the variables of our problem. The transfer to household j in country k is determined by $f^t_{k,j}=f^t_{k,j}(\ell^t_{k,1},\dots,\ell^t_{k,H^t_k},A^t,\tau^t_k)=0$. The determining variables are: (i) the transfer levels $\ell^t_{k,s}$ of all households of country k at time t, (ii) the stock of abatement A^t at time t, (iii) the tax level τ^t_k of country k.

The following optimization procedure determines the SCC at the global level:

$$\max_{\tau_k^t,\ell_{k,j}^t} \sum_{T=1}^{t_{\text{end}}} \frac{1}{(1+\rho)^T} \sum_{k,j} w_{k,j}^T \cdot u(c_{k,j}^T - d_{k,j}^T(A^T), A^T).$$
$$\text{s.t.} \qquad \sum_j [\ell_{k,j}^t - \tau_k^t \cdot (e_{k,j}^t - a_{k,j}^t)] = 0, \quad \forall k, t$$
$$\text{and} \qquad f_{k,j}^t(\ell_{k,1}^t, \ldots, \ell_{k,H_k^t}, A^t, \tau_k^t) = 0, \quad \forall k, j, t.$$

Consumption is given by the budget constraint: $c_{k,j}^T = i_{k,j}^T - \tau_k^T \cdot (e_{k,j}^T - a_{k,j}^T(\tau_k^T)) + \ell_{k,j}^T - m_{k,j}^T(a_{k,j}^T(\tau_k^T))$. The Lagrangian reads:

$$\mathcal{L} = \sum_T \frac{1}{(1+\rho)^T} \sum_{k,j} w_{k,j}^T \cdot u(c_{k,j}^T - d_{k,j}^T(A^T), A^T)$$
$$+ \sum_T \sum_k \zeta_k^T \sum_j [\ell_{k,j}^T - \tau_k^T \cdot (e_{k,j}^T - a_{k,j}^T)] + \sum_T \sum_{k,j} \chi_{k,j}^T f_{k,j}^T.$$

The government's first-order condition, rearranged to give the SCC, are:

$$\tau_p^t = \frac{1}{-\zeta_p^t \sum_s \frac{\partial a_{p,s}^t}{\partial \tau_p^t}} \cdot \left(\sum_{T=t}^{t_{\text{end}}} \frac{1}{(1+\rho)^T} \sum_{k,j} w_{k,j}^T \left(muc_{k,j}^T \cdot (-d_{k,j}^{T'}) + mua_{k,j}^T \right) \sum_s \frac{\partial a_{p,s}^t}{\partial \tau_p^t} \right.$$
$$\left. + \sum_{T=t}^{t_{\text{end}}} \sum_{k,j} \chi_{k,j}^T \sum_s \frac{\partial f_{k,j}}{\partial A^T} \frac{\partial a_{p,s}^t}{\partial \tau_p^t} + \sum_s \chi_{p,s}^t \left[\sum_j (e_{p,j}^t - a_{p,j}^t) \frac{\partial f_{p,s}^t}{\partial \ell_{p,j}^t} + \frac{\partial f_{p,s}^t}{\partial \tau_p^t} \right] \right) \quad \forall p, t$$

$$\zeta_p^t = -\frac{1}{(1+\rho)^t} w_{p,j}^t muc_{p,j}^t - \sum_s \chi_{p,s}^t \frac{\partial f_{p,s}^t}{\partial \ell_{p,j}^t} \quad \forall j \quad \forall p, t$$
$$= \frac{1}{H_p^t} \sum_j \left(-\frac{1}{(1+\rho)^t} w_{p,j}^t muc_{p,j}^t - \sum_s \chi_{p,s}^t \frac{\partial f_{p,s}^t}{\partial \ell_{p,j}^t} \right) \quad \forall\, p, t$$

This expression for the SCC differs notably from the equality case in Eq. (6). There are two drivers of this difference:

1. Increasing consumption or abatement receives a different social value at the global level with inequality between households (see Fig. 1). Since marginal utilities of consumption and abatement are not equalized between households, the denominator and numerator take account of these differences by taking an average across all households. Hence, when the

marginal utilities are convex or concave functions, their sum will generally differ from their value at the mean.

2. The SCC changes inequality between households (see Fig. 2) with national distributional decisions anticipated at the global level. Different levels of the SCC influence the transfer to each household, reflected in the terms that include the Lagrange multipliers χ on the constraints f. The transfer generally changes with (i) the stock of abatement (derivative of f with respect to the stock of abatement) by changing avoided damages and non-market benefits of abatement; (ii) the SCC in a particular country (derivative of f with respect to the carbon tax rate), with different levels of total composite consumption in a country through different abatement costs and redistribution of the national tax revenue.

B Derivation of SCC rule in Sec. 3.2.1

The constraints Eq. (8) can be summarized by setting the weighted marginal utilities of consumption in each country k at time t to a variable ϵ_k^t:

$$z_{k,j}^t muc_{k,j}^t = \epsilon_k^t \quad \forall j$$

which governs how the national level redistributes total consumption $\sum_s c_{k,s}^t = \sum_s i_{k,s}^t - m_{k,s}^t$ among households.

The maximization at the global level is:

$$\begin{aligned}
\max_{c_{p,s}^T, \tau_p^T, \epsilon_p^T} & \sum_{t=0}^{t_{\text{end}}} \frac{1}{(1+\rho)^t} \sum_{k,j} w_{k,j}^t u(c_{k,j}^t - d_{k,j}^t, A^t) \\
\text{s.t.} \quad & z_{k,j}^t muc_{k,j}^t = \epsilon_k^t \quad \forall\, j, k, t \\
& \sum_s c_{k,s}^t = \sum_s i_{k,s}^t - m_{k,s}^t \quad \forall\, k, t.
\end{aligned}$$

The Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{t_{\text{end}}} \left(\frac{1}{(1+\rho)^t} \sum_{k,j} \left(w_{k,j}^t u(c_{k,j}^t - d_{k,j}^t, A^t) + \chi_{k,j}^t (z_{k,j}^t muc_{k,j}^t - \epsilon_k^t) \right.\right.$$
$$\left.\left. + \zeta_k^t (c_{k,j}^t + i_{k,j}^t - m_{k,j}^t) \right) \right)$$

with $\chi^t_{k,j}$ and ζ^t_k the Lagrange-multipliers for the respective constraints. The two conditions for the optimum can be obtained by rearranging the FOCs $\frac{\partial}{\partial c^t_{p,s}}\mathcal{L} = 0$, $\frac{\partial}{\partial \tau^t_p}\mathcal{L} = 0$, $\frac{\partial}{\partial \epsilon^t_p}\mathcal{L} = 0$ and setting $-(1+\rho)^t\zeta^t_k = \kappa^t_k$.

C Approximation of the SCC around equality

In the rules for the SCC, the numerators and denominators generally depend on parameters $\vec{x}$, such as household income or benefits of abatement. Let equality of these parameters within countries be denoted by $\bar{\vec{x}}$. If n_x is the number of parameters, the second-order Taylor approximation of a function y (equal to the denominator or the numerator) is generally given by:

$$\begin{aligned} y(\vec{x}) \approx & y(\vec{x} = \bar{\vec{x}}) + \sum_{n=1}^{n_x} \frac{\partial y}{\partial x_n}|_{\vec{x}=\bar{\vec{x}}}(x_n - \bar{x}_n) \\ & + \frac{1}{2}\sum_{n1=1}^{n_x}\sum_{n2=1}^{n_x} \frac{\partial^2 y}{\partial x_{n1}\partial x_{n2}}|_{\vec{x}=\bar{\vec{x}}}(x_{n1} - \bar{x}_{n1})(x_{n2} - \bar{x}_{n2}) \end{aligned} \tag{C.1}$$

D Derivation of Eq. (12)

The following approximations are usually of functions of the general form $Y^t_k = \sum_j w^t_{k,j} \cdot y(z^t_{k,j}, \epsilon^t_k(\{z^t_{k,s}\}_{s=1..H^t_k})) \cdot n^t_k(\{z^t_{k,s}\}_{s=1..H^t_k})$. Here $\epsilon^t_k(\{z^t_{k,s}\}_{s=1..H^t_k}))$ and $n^t_k(\{z^t_{k,s}\}_{s=1..H^t_k})$ are functions of the set of welfare weights, for which their value and values of the first and second derivatives with respect to the weights are equal at the mean of $z^t_{k,s}$ (see below). For such Y^t_k the general approximation in (C.1) can be simplified:

$$
\begin{aligned}
Y_k^t \approx & Y_k^t|_{w=\bar{w}_k^t, z=1} \\
& + \underbrace{\sum_{j=1}^{H_k^t} \frac{\partial Y_k^t}{\partial w_{k,j}^t}|_{w=\bar{w}_k^t, z=1}(w_{k,j}^t - \bar{w}_{k,j}^t)}_{=0} + \underbrace{\sum_{s=1}^{H_k^t} \frac{\partial Y_k^t}{\partial z_{k,s}^t}|_{w=\bar{w}_k^t, z=1}(z_{k,s}^t - 1)}_{=0} \\
& + \frac{1}{2}\sum_{j=1}^{H_k^t}\sum_{s=1}^{H_k^t} \underbrace{\frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial w_{k,s}^t}|_{w=\bar{w}_k^t, z=1}}_{=0}(w_{k,j}^t - \bar{w}_k^t)(w_{k,s}^t - \bar{w}_k^t) \\
& + \frac{1}{2}\sum_{j=1}^{H_k^t}\sum_{s=1}^{H_k^t} \frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,s}^t}|_{w=\bar{w}_k^t, z=1}(w_{k,j}^t - \bar{w}_k^t)(z_{k,s}^t - 1) \\
& + \frac{1}{2}\sum_{j=1}^{H_k^t}\sum_{s=1}^{H_k^t} \frac{\partial^2 Y_k^t}{\partial z_{k,j}^t \partial w_{k,s}^t}|_{w=\bar{w}_k^t, z=1}(z_{k,j}^t - 1)(w_{k,s}^t - \bar{w}_k^t) \\
& + \frac{1}{2}\sum_{j=1}^{H_k^t}\sum_{s=1}^{H_k^t} \frac{\partial^2 Y_k^t}{\partial z_{k,j}^t \partial z_{k,s}^t}|_{w=\bar{w}_k^t, z=1}(z_{k,j}^t - 1)(z_{k,s}^t - 1)
\end{aligned}
$$

The first and second non-zero sums are the same and can be further manipulated:

$$
\begin{aligned}
& \sum_{j=1}^{H_k^t}\sum_{s=1}^{H_k^t} \frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,s}^t}|_{w=\bar{w}_k^t, z=1}(w_{k,j}^t - \bar{w}_k^t)(z_{k,s}^t - 1) \\
= & \sum_{j=1}^{H_k^t}\sum_{s=1, s\neq j}^{H_k^t} \ldots + \sum_{j=1}^{H_k^t} \frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,j}^t}|_{w=\bar{w}_k^t, z=1}(w_{k,j}^t - \bar{w}_k^t)(z_{k,j}^t - 1) \\
= & \underbrace{\frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,s}^t}|_{w=\bar{w}_k^t, z=1}}_{\text{for any } j\neq s} \left(\underbrace{\sum_{j=1}^{H_k^t}\sum_{s=1}^{H_k^t}(w_{k,j}^t - \bar{w}_k^t)(z_{k,s}^t - 1)}_{=0} - \sum_{j=1}^{H_k^t}(w_{k,j}^t - \bar{w}_k^t)(z_{k,j}^t - 1) \right) \\
& + \sum_{j=1}^{H_k^t} \frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,j}^t}|_{w=\bar{w}_k^t, z=1}(w_{k,j}^t - \bar{w}_k^t)(z_{k,j}^t - 1) \\
= & H_k^t cov_k^t(w, z) \left(\frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,j}^t} - \frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,s}^t}|_{j\neq s} \right) |_{w=\bar{w}_k^t, z=1}.
\end{aligned}
$$

The same type of manipulation for the remaining third non-zero summand above leads to the following formula:

$$
\begin{aligned}
Y_k^t \approx & Y_k^t|_{w=\bar{w}_k^t, z=1} \\
& + H_k^t cov_k^t(w,z) \left(\frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,j}^t} - \frac{\partial^2 Y_k^t}{\partial w_{k,j}^t \partial z_{k,s}^t}|_{j\neq s} \right) |_{w=\bar{w}_k^t, z=1} \\
& + H_k^t \frac{1}{2} (\sigma_k^t(z))^2 \left(\frac{\partial^2 Y_k^t}{(\partial z_{k,j}^t)^2} - \frac{\partial^2 Y_k^t}{\partial z_{k,j}^t \partial z_{k,s}^t}|_{j\neq s} \right) |_{w=\bar{w}_k^t, z=1} \qquad \text{(D.1)}
\end{aligned}
$$

The approximation in Eq. (12) is derived in deviations of $w_{k,j}^t$ and $z_{k,j}^t$ around their means $\bar{w}_k^t$ and 1. Inequality of individual household income, benefits and costs of abatement is not taken into account because only $w_{k,j}^t$ and $z_{k,j}^t$ determine household inequality in composite consumption at the global level, which we show in detail in the following.

The composite consumption level of a household j in country k at time t is determined by the total level of composite consumption $C_k^t - D_k^t = \sum_j (c_{k,j}^t - d_{k,j}^t)$ and its national welfare weight $z_{k,j}^t$ through Eq. (8):

$$
z_{k,j}^t muc_{k,j}^t = z_{k,s}^t muc_{k,s}^t = \epsilon_k^t \quad \forall j, s \qquad \text{(D.2)}
$$

$$
\Rightarrow c_{k,j}^t - d_{k,j}^t = (muc^t)^{-1} \left(\frac{\epsilon_k^t}{z_{k,j}^t} \right) \qquad \text{(D.3)}
$$

$$
\Rightarrow C_k^t - D_k^t = \sum_j (muc^t)^{-1} \left(\frac{\epsilon_k^t}{z_{k,j}^t} \right). \qquad \text{(D.4)}
$$

Here $(muc^t)^{-1}$ is the inverse function of the marginal utility of consumption, which depends on the time index through the second argument of the utility function $u(c_{k,j}^t - d_{k,j}^t, A^t)$. Eq. (D.3) defines household composite consumption based on the set of welfare weights $z_{k,s=1\dots H_k^t}^t$.

Hence, we can express the value of equalized weighted marginal utilities of consumption, ϵ_k^t, as a function of the weights $z_{k,j}^t$ that distribute $C_k^t - D_k^t$ in Eq. (D.4) around the national mean $\bar{c}_k^t - \bar{d}_k^t$ if all weights were equal $z_{k,j}^t = 1$. For the second order approximation below, we will need the following expressions of ϵ_k^t, which can be found by applying the implicit function theorem to Eq. (D.4):

$$\begin{aligned}
\epsilon_k^t|_{z_{k,j}^t=1} &= muc_k^t \\
\frac{\partial \epsilon_k^t}{\partial z_{k,j}^t}|_{z_{k,j}^t=1} &= \frac{1}{H_k^t} muc_k^t \\
\left(\frac{\partial^2 \epsilon_k^t}{(\partial z_{k,j}^t)^2} - \frac{\partial^2 \epsilon_k^t}{\partial z_{k,j}^t \partial z_{k,s}^t}\right)|_{z_{k,j}^t=1} &= -\frac{1}{H_k^t} muc_k^t \left\{\frac{[(muc_k^t)^{-1}]''}{[(muc_k^t)^{-1}]'} muc_k^t + 2\right\}.
\end{aligned} \tag{D.5}$$

Here, $[(muc_k^t)^{-1}]'$ and $[(muc_k^t)^{-1}]''$ are the first and second derivatives of the inverse function to the marginal utility of consumption, taken at mean consumption at household level in country k at time t. They can be calculated through the general law on derivatives of inverse functions as $[(muc)^{-1}]' = \frac{1}{mucc}$ and $[(muc)^{-1}]'' = -\frac{\frac{\partial mucc}{\partial c}}{(mucc)^3}$.

With (D.3) we can hence express the composite consumption level of each household as a function of national weights:

$$c_{k,j}^t - d_{k,j}^t = muc^{-1}\left(\frac{\epsilon_k^t(\{z_{k,s}^t\}_{s=1..H_k^t})}{z_{k,j}^t}\right)$$

With the last Eq. we can now approximate the denominator of (9)

$$\kappa_k^t = \frac{\sum_j \frac{w_{k,j}^t muc_{k,j}^t}{z_{k,j}^t mucc_{k,j}^t}}{\sum_j \frac{1}{z_{k,j}^t mucc_{k,j}^t}}$$

around $w_{k,j}^t = \bar{w}_k^t$ and $z_{k,j}^t = 1$. This is done by applying the approximation in (D.1). The differences in the derivatives are:[17]

[17] To derive the differences in the derivatives it is convenient to first represent the denominator generically because almost all first and second derivatives cancel:

$$\kappa_k^t = \frac{\sum_j \frac{w_{k,j}^t muc_{k,j}^t}{z_{k,j}^t mucc_{k,j}^t}}{\sum_j \frac{1}{z_{k,j}^t mucc_{k,j}^t}} = \sum_j w_{k,j}^t \cdot g(z_{k,j}^t, \epsilon_k^t) \cdot f(z_{k,j}^t, \epsilon_k^t) \cdot \left(\sum_s f(z_{k,s}^t, \epsilon_k^t)\right)^{-1}$$

with $g(z_{k,j}^t, \epsilon_k^t) = muc_{k,j}^t = \frac{\epsilon_k^t}{z_{k,j}^t}$ and $f(z_{k,j}^t, \epsilon_k^t) = \frac{1}{z_{k,j}^t mucc_{k,j}^t}$, where f is a function of consumption: $mucc_{k,j}^t = mucc(c_{k,j}^t - d_{k,j}^t) = mucc(muc^{-1}(\epsilon_k^t / z_{k,j}^t))$. Taking the derivatives of κ_k^t then involves the derivatives of these functions with respect to $z_{k,j}^t$ and $z_{k,s}^t$, keeping in mind that ϵ is a function of these weights.

$$\left(\frac{\partial^2 \kappa_k^t}{\partial w_{k,j}^t \partial z_{k,j}^t} - \frac{\partial^2 \kappa_k^t}{\partial w_{k,j}^t \partial z_{k,s}^t}|_{j\neq s}\right)|_{w_{k,j}^t=\bar{w}_k^t, z_{k,j}^t=1} = \frac{muc_k^t}{H_k^t}\left(\frac{(\mu_{CC})_k^t}{(\mu_C)_k^t} - 2\right)$$

and

$$\left(\frac{\partial^2 \kappa_k^t}{(\partial z_{k,j}^t)^2} - \frac{\partial^2 \kappa_k^t}{\partial z_{k,j}^t \partial z_{k,s}^t}|_{j\neq s}\right)|_{w_{k,j}^t=\bar{w}_k^t, z_{k,j}^t=1} = -\frac{\bar{w}_k^t muc_k^t}{H_k^t}\left(\frac{(\mu_{CC})_k^t}{(\mu_C)_k^t} - 2\right)$$

Hence the approximation of the denominator, written down in Eq. (12), is:

$$\kappa_p^t \approx \bar{w}_p^t muc_p^t \left[1 + \left(\sigma_p^t(z)^2 - 2\frac{cov_p^t(w,z)}{\bar{w}_p^t}\right)\left(1 - \frac{1}{2}\frac{(\mu_{CC})_p^t}{(\mu_C)_p^t}\right)\right] \tag{D.6}$$

The numerator can be split into three parts:

$$\begin{aligned}\sum_{T=t}^{t_{\text{end}}} \frac{1}{(1+\rho)^{T-t}} \Bigg[\sum_k \Bigg(\kappa_k^T H_k^T (-\bar{d}_k^{T'}) + \sum_j w_{k,j}^T mua_{k,j}^T \\ - \sum_j \left(w_{k,j}^T muc_{k,j}^T - \kappa_k^T\right) \frac{muac_{k,j}^T}{mucc_{k,j}^T}\Bigg)\Bigg] \\ = \sum_{T=t}^{t_{\text{end}}} \frac{1}{(1+\rho)^{T-t}} \sum_k \left(V1_k^T + V2_k^T + V3_k^T\right)\end{aligned}$$

The first part can be easily approximated with the approximation of κ_k^t in (D.6):

$$\begin{aligned}V1_k^t &= \kappa_k^t H_k^t(-\bar{d}_k^{T'}) \\ &\approx \bar{w}_p^t muc_p^t \left[1 + \left(\sigma_p^t(z)^2 - 2\frac{cov_p^t(w,z)}{\bar{w}_p^t}\right)\left(1 - \frac{1}{2}\frac{(\mu_{CC})_p^t}{(\mu_C)_p^t}\right)\right] H_k^t(-\bar{d}_k^{T'})\end{aligned}$$

The second part can again be approximated with (D.1):

$$\begin{aligned}V2_k^t &= \sum_j w_{k,j}^t mua_{k,j}^t \\ &\approx H_k^t \bar{w}_k^t mua_k^t \left(1 - \frac{cov_k^t(w,z)}{\bar{w}_k^t}\frac{(\lambda_C)_k^t}{(\mu_C)_k^t} + \frac{1}{2}(\sigma_k^t(z))^2 \frac{(\lambda_C)_k^t (\lambda_{CC})_k^t}{{(\mu_C)_k^t}^2}\right)\end{aligned}$$

The approximation of the third term can be obtained again by applying (D.1):[18]

$$
\begin{aligned}
V3_k^t =&\kappa_k^t \sum_j \frac{muac_{k,j}^t}{mucc_{k,j}^t} - \sum_j w_{k,j}^t muc_{k,j}^t \frac{muac_{k,j}}{mucc_{k,j}} \\
\approx& \, H_k^t \bar{w}_k^t mua_k^t \Bigg[\frac{cov_k^t(w,z)}{\bar{w}_k^t} \bigg(- \frac{(\lambda_C)_k^t}{(\mu_C)_k^t} + \frac{(\lambda_C)_k^t (\lambda_{CC})_k^t}{{(\mu_C)_k^t}^2} \bigg) \\
&- (\sigma_k^t(z))^2 \bigg(- \frac{(\lambda_C)_k^t}{(\mu_C)_k^t} + \frac{(\lambda_C)_k^t (\lambda_{CC})_k^t}{{(\mu_C)_k^t}^2} \bigg) \Bigg]
\end{aligned}
$$

Combining all three parts produces the numerator in Eq. (12).

E Detailed discussion of SCC rule in Eq. (12)

First, consider how the marginal utility of consumption changes with inequality within countries, captured by φ. This factor appears in both the numerator and denominator because inequality changes the social value of increasing composite consumption in a country – either by decreasing damages (in the numerator) or decreasing costs in the country (in the denominator).

The effect of inequality within countries is best understood when we isolate it from the effects of global weights that diverge from equality. Setting global welfare weights to equality at the household level ($cov(w,z) = 0$), we first consider the case of linear marginal utility of consumption when $\mu_{CC} = 0$. Because the marginal utility of consumption is neither convex nor concave, the social value of increasing consumption or abatement does not change under inequality (the effect of Fig. 1 is absent). The SCC will however still impact inequality at

[18] Here it is, like in footnote 17, convenient to first represent the summand generically through:

$$
\begin{aligned}
V3_k^t =&\kappa_k^t \sum_j \frac{muac_{k,j}^t}{mucc_{k,j}^t} - \sum_j w_{k,j}^t muc_{k,j}^t \frac{muac_{k,j}}{mucc_{k,j}} \\
=&\frac{\sum_j \frac{w_{k,j}^t muc_{k,j}^t}{z_{k,j}^t mucc_{k,j}^t}}{\sum_j \frac{1}{z_{k,j}^t mucc_{k,j}^t}} \sum_j \frac{muac_{k,j}^t}{mucc_{k,j}^t} - \sum_j w_{k,j}^t muc_{k,j}^t \frac{muac_{k,j}}{mucc_{k,j}} \\
=&\sum_j w_{k,j}^t \cdot g(z_{k,j}^t, \epsilon_k^t) \cdot f(z_{k,j}^t, \epsilon_k^t) \cdot \left(\sum_s f(z_{k,s}^t, \epsilon_k^t) \right)^{-1} \cdot \sum_s h(z_{k,s}^t, \epsilon_k^t) \cdot f(z_{k,s}^t, \epsilon_k^t) \\
&- \sum_j w_{k,j}^t g(z_{k,j}^t, \epsilon_k^t) \cdot f(z_{k,j}^t, \epsilon_k^t) \cdot h(z_{k,j}^t, \epsilon_k^t)
\end{aligned}
$$

with $g(z_{k,j}^t, \epsilon_k^t) = muc_{k,j}^t = \frac{\epsilon_k^t}{z_{k,j}^t}$, $f(z_{k,j}^t, \epsilon_k^t) = \frac{1}{z_{k,j}^t mucc_{k,j}^t}$ and $h(z_{k,j}^t, \epsilon_k^t) = z_{k,j}^t muac_{k,j}^t$.

the household level (the effect of Fig. 2). If $\mu_{CC} = 0$, the national level will allocate over-proportionally more of a consumption increase to the households with the lowest national welfare weight.[19] With increasing total composite consumption that is available at the national level when the SCC increases from zero, inequality among households hence decreases. Indeed, Eq. (12) shows that the SCC tends to decrease for the country with the largest inequality and increase for the country with the lowest inequality. By assigning the different SCC, inequality in the most unequal country is reduced, which is beneficial if equality is preferred at the global level.

For $\mu_{CC} > 0$ the above identified effect is mitigated. In this case, the marginal utility of consumption of a low-consumption household, the one with a lower national weight, is over-proportionally larger compared to households with higher consumption, the ones with larger weights. To achieve equalization of weighted marginal utilities of consumption, the national governance level has to allocate more composite consumption to households that have a higher weight and less to households with a lower weight compared to $\mu_{CC} = 0$ when total composite consumption increases at the national level. The larger allocation to higher-consumption households is not socially beneficial if equality is preferred at the global level. Therefore, with $\mu_{CC} > 0$ the effect of the previous paragraph is mitigated. For $\mu_{CC} < 0$ the same holds v.v.

Additionally, with $\mu_{CC} > 0$, inequality at the household level changes the social value of increasing consumption (the effect of Fig. 1). Because marginal utility of consumption is convex, low-consumption household's marginal utility is disproportionally larger than of a high-consumption household. Hence, even if the national level only allocates a smaller share of additional national consumption to low-consumption households, its social value is higher at the global level. Again, for $\mu_{CC} < 0$ the same holds v.v. Both effects for $\mu_{CC} \neq 0$ are combined in the rule for the SCC.

[19] For $\mu_{CC} = 0$, marginal utility of consumption is linear: $muc_{k,j}^t = K_1 - K_2 \cdot (c_{k,j}^t - d_{k,j}^t)$. With the constraint that weighted marginal utilities of consumption are equalized in Eq. (8), we can derive the level of consumption of each household as a function of total national composite consumption $C_k^t - D_k^t$ and the national welfare weights:

$$c_{k,j}^t - d_{k,j}^t = \frac{1}{K_2}\left(K_1 - \frac{1}{z_{k,j}^t}\frac{H_k^t K_1 - K_2(C_k^t - D_k^t)}{\sum_s \frac{1}{z_{k,s}^t}}\right)$$

Increasing composite consumption $C_k^t - D_k^t$ of the country, the national level allocates consumption to each household inversely proportional to its weight.

Turning to how inequality within countries changes in the aggregation non-market benefits, the term ϕ, consider again the case where (i) equality is preferred at the global level, $cov(w, z) \equiv 0$; (ii) the elasticity λ_{CC} is zero. Marginal utility of abatement is then linear in composite consumption. Hence, inequality in composite consumption does not change the social value of abatement in non-market benefits at the global level (the effect of Fig. 1 is absent). However, a different level of non-market benefits influences how the national level redistributes between households (the effect of Fig. 2). If the marginal utility of abatement increases with consumption ($\lambda < 0$), how-consumption households gain more from non-market benefits. As the national level puts a higher weight on these households, it has a further incentive to increase their consumption to increase their utility from non-market benefits. This would increase inequality within countries. To counteract this effect, the SCC of all countries decreases at the global level ($\phi < 0$). The same holds v.v. for $\lambda > 0$.

For $\lambda_{CC} > 0$ the effect is again mitigated. With $\lambda_{CC} > 0$, marginal utility of abatement still increases in consumption but with decreasing returns. Hence the national level has a lower incentive to increase consumption of high-consumption households under $\lambda_C < 0$.

In addition the social value of increasing abatement changes with inequality (the effect of Fig. 1). Both effects for $\lambda_{CC} \neq 0$ are combined in the factor ϕ.

Lastly, all discussed effects are mitigated if preferences over inequality align at the global and national level. The covariance between national and global welfare weights is positive in this case, $cov(w, z) > 0$. The changes of the SCC under inequality within countries are mitigated because the distribution between households is actually preferred at the global level.

F List of symbols

t_{end}	number of periods
ρ	rate of pure time preference
t, T	time, runs from $1..t_{\text{end}}$
N	number of countries

k, p	country indices, run from $1..N$
H_k^t	number of households in country k at time t
j, s	household indices, run from $1..H_k^t$
u	utility function of households
$c_{k,j}^t$	consumption of household j in country k at time t
A^t	stock pollutant at time t
$d_{k,j}^t$	monetary damages as a function of abatement of household j in country k at time t
$muc_{k,j}^t$	marginal utility of consumption at composite consumption level $c_{k,j}^t - d_{k,j}^t$ and abatement level A^t
$mucc_{k,j}^t$	second derivative of utility with respect to consumption at composite consumption level $c_{k,j}^t - d_{k,j}^t$ and abatement level A^t
$mua_{k,j}^t$	marginal utility of abatement in non-market benefits at composite consumption level $c_{k,j}^t - d_{k,j}^t$ and abatement level A^t
$muac_{k,j}^t$	derivative of marginal utility of abatement with respect to consumption at composite consumption level $c_{k,j}^t - d_{k,j}^t$ and abatement level A^t
τ_k^t	Social cost of carbon of country k at time t
$a_{k,j}^t$	abatement of household j in country k at time t
$e_{k,j}^t$	emissions of household j in country k at time t
$m_{k,j}^t$	abatement costs of household j in country k at time t
$i_{k,j}^t$	income of household j in country k at time t
$\ell_{k,j}^t$	transfer to household j in country k at time t
$w_{k,j}^t$	global welfare weight of household j in country k at time t
$\bar{(.)}_k^t$	mean of characteristic over households in country k at time t
C_k^t, D_k^t	aggregate consumption and damages in country k at time t
$(\cdot)'$	derivative of damage functions $(d, \bar{d})$ with respect to stock of abatement

$f^t_{k,j}$	constraint defining national transfer to household j in country k at time t
SWF, NWF	welfare function at global and national level, respectively
$\overrightarrow{x}$ x_n	arrays of values and their components, respectively
η	elasticity of marginal felicity in the isoelastic utility function
$(\mu_C)^t_k$, $(\mu_{CC})^t_k$, $(\lambda_C)^t_k$, $(\lambda_{CC})^t_k$	elasticities at average composite consumption level of households in country k at time t
$\mathcal{L}, \zeta^t_p, \chi^t_{k,j}$	Lagrange function and multipliers
κ^t_k	transformation of Lagrange multiplier $\kappa^t_k = -(1+\rho)^t\zeta^t_k$
φ^t_k,ϕ^t_k,$\tilde{\varphi}^t_k$,$\tilde{\phi}^t_k$	adjustment factors to the SCC from the equality case
$\sigma^t_k(\cdot)^2$	standard deviation of specific variable over all households in country k at time t
$cov^t_k(\cdot,\cdot)$	covariance of two specific variables over all households in country k at time t
$z^t_{k,j}$	national welfare weights
$\delta^t_{k,j}$	parameter defining how damages deviate from being proportional to income for household j in country k at time t
ϵ^t_k	weigthed marginal utility of consumption for nationally optimal transfers in country k at time t
$V1..3^t_k$	summands in the aggregated benefits from abatement under nationally optimal transfers
$pop^t_{k,j}$, $i\_\text{share}_{k,j}$, $d\_\text{share}_{k,j}$	population, income and damage shares of the NICE model of quintile j in region k at time t, respectively
ξ	income elasticity of damages in the NICE model