CESifo WORKING PAPERS

7674
2019

May 2019

# Estimating macroeconomic uncertainty and discord using info-metrics

*Kajal Lahiri, Wuwei Wang*

CESifo

**Impressum:**

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
www.cesifo-group.org/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: www.CESifo-group.org/wp

CESifo *Working Paper No. 7674*
*Category 12: Empirical and Theoretical Methods*

# Estimating macroeconomic uncertainty and discord using info-metrics

## Abstract

We apply generalized beta and triangular distributions to histograms from the Survey of Professional Forecasters (SPF) to estimate forecast uncertainty, shocks and discord using information framework, and compare these with moment-based estimates. We find these two approaches to produce analogous results, except in cases where the underlying densities deviate significantly from normality. Even though the Shannon entropy is more inclusive of different facets of a forecast density, we find that with SPF forecasts it is largely driven by the variance of the densities. We use Jenson-Shannon Information to measure ex ante "news" or "uncertainty shocks" in real time, and find that this 'news' is closely related to revisions in forecast means, countercyclical, and raises uncertainty. Using standard vector auto-regression analysis, we confirm that uncertainty affects the economy negatively.

JEL-Codes: E370.

Keywords: density forecasts, uncertainty, disagreement, entropy measures, Jensen-Shannon information, Survey of Professional Forecasters.

*Kajal Lahiri*
*Department of Economics*
*University at Albany: SUNY*
*Albany / NY / USA*
*klahiri@albany.edu*

*Wuwei Wang*
*Southwestern University of Finance and Economics*
*Chengdu / Sichuan / P. R. China*
*wangwuwei@swufe.edu.cn*

We are thankful to two anonymous readers and the co-editors for many constructive comments and suggestions on an earlier version of the chapter. We, however, are responsible for any remaining errors and omissions.

## 1. Introduction

For informed decision making in businesses and governments, forecasts as well as their uncertainties are equally important. Not surprisingly, policy makers in diverse fields are increasingly being interested in the uncertainty surrounding point forecasts. There is a rich literature in economics and finance developing models of *ex post* uncertainty by analyzing different functions of forecast errors in vector autoregressive and volatility models. However, the decision makers need *ex ante* uncertainty in real time before observing the outcome variable. Density forecasts obtained from surveys of experts are particularly suited to produce such information. There have been attempts to compare these subjective uncertainties with those generated from estimated time series models.[2]

Before the availability of survey density forecasts, variance of point forecasts or disagreement was used as a convenient proxy for uncertainty. However, using density forecasts from the Surveys of Professional Forecasters (SPF), Zarnowitz and Lambros (1987) and Lahiri et al. (1988) distinguished between aggregate forecast uncertainty and disagreement. This literature has attracted increasing attention in recent years.[3] Most recently, a number of interesting studies on the methodology of using info-metrics in economic forecasting have evolved.[4] Shoja and Soofi (2017) decompose entropy of the consensus density forecast into average uncertainty and disagreement, and use information

[2] See Lahiri and Liu (2005), Clements (2014), and Knuppel (2014). For various problems in generating *ex ante* forecast uncertainty from time series models, see Fresoli et al. (2015) and Mazzeu (2018).
[3] See, for example, Lahiri and Sheng (2010), Boero, Smith and Wallis (2008, 2013), and Abel et al. (2016).
[4] See Mitchell and Hall (2005), Lahiri and Liu (2006, 2009), Mitchell and Wallis (2011), Rich and Tracy (2010), Kenny et al. (2015) and references therein.

divergence as a proxy for disagreement. In this chapter, we utilize the info-metrics approach to estimate uncertainty, and decompose it into components that include a disagreement term. We compare the conventional moment-based measures with info-metrics based measures, including entropy and information divergence.

Our approach involves fitting continuous distributions to the histogram data. Many previous studies have utilized raw histograms, assuming that the probability mass is located on the midpoint of the bin. This approach essentially treats each forecast as a discrete distribution. For various reasons, the alternative of fitting continuous distributions to the histograms has become common in recent years. For output and inflation forecasts, it is reasonable to assume that the true underlying distributions are indeed continuous, even though surveys are recorded in the form of histograms (i.e. probabilities attached to the bins) for convenience. However, changing from a discrete to a continuous system may involve trade-offs, especially when the variable of interest is uncertainty, cf. Golan (1991). Furthermore, there are a certain number of observations in the survey where forecasters attached probabilities to only a few bins. In these cases, assumption on distribution is more tenuous, and may have indeterminate impact at the individual level. These all remain challenges, though some of the issues do not affect the stylized facts after averaging over individuals. However, given that the target variables are continuous, we choose to adopt the method of fitting continuous distributions to the histograms. Following Giordani and Soderlind (2003), some researchers have assumed normal distributions or that the probability mass in a bin is uniformly distributed. The later assumption is inconsistent with distributions that are globally unimodal. Since many of the histograms are not symmetric and exhibit widely different shapes, we follow Engelberg et al. (2006) and use generalized beta as our preferred choice.

We are the first to apply continuous distributions to histograms while using info-metrics to estimate uncertainty, after painstakingly approximating all the individual histograms that have gone through occasional changes in survey bin sizes and number of bins. We further adjust for different forecast horizons to estimate a quarterly time series of aggregate uncertainty. The estimated information measures are then utilized successfully in several macroeconomic applications, including vector auto-regression (VAR) models to estimate the effect of uncertainty on the macro economy. We use Jenson-Shannon Information to measure ex ante "news" or "uncertainty shocks" and study its effect on uncertainty in real time.

The chapter is organized as follows: in Section 2 we briefly introduce the dataset and the key features of the recorded density forecasts. In Section 3 we compare different measures of uncertainty by analyzing variances and confirm previous findings that disagreement regarding point forecast should not be used as a sole proxy for uncertainty. In Section 4 we employ info-metrics to estimate uncertainty and study the difference between two approaches – the moment-based and the entropy approach. In Section 5 we correct for horizons to get a time series of uncertainty and compare our measures to other popular uncertainty indices. In Section 6 we apply Jensen-Shannon information divergence to individual densities to estimate "uncertainty shocks" or "news" from successive fixed target forecasts, and study the impact of these on uncertainty. In Section 7 we evaluate the effects of uncertainty on macroeconomic variables using VAR. Finally, Section 8 summarizes the main conclusions of the study.

## 2. The data

The U.S. Survey of Professional Forecasters (SPF), spearheaded by Victor Zarnowitz[5] in the late 1960's, is a unique dataset that provides probability forecasts from a changing panel of professional forecasters. The forecasters are mostly from financial institutions, universities, research institutions, consulting firms, forecasting firms, and manufacturing. SPF was formerly conducted jointly by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER). It started in the fourth quarter of 1968 and was taken over by the Federal Reserve Bank of Philadelphia from the second quarter of 1990. Similar siblings of the survey, though considerably much younger, include the Bank of England Survey of External Forecasters, and the European Central Bank's Survey of Professional Forecasters. The timing of the survey has been planned carefully such that the forecasters will have the initial estimate of selected quarterly macroeconomic variables at the time of forecasting and at the same time issue forecasts in a timely manner. For instance, while providing forecasts in the middle of quarter 3, a forecaster has the information of the first estimate of the real GDP for quarter 2.

The survey asks professional forecasters about their predictions of a number of macro variables, including real GDP, nominal GDP, GDP deflator, and unemployment rate. Forecasters provide their point forecasts for these variables for the current year and the next year, and for the current quarter and next four quarters. Besides the point forecasts, they also provide density forecasts for output and price level growth for the current and next year. The definition of the 'output' variable has changed in the survey several times.

[5] See Zarnowitz (2008) for a remarkable autobiography.

It was defined as nominal GNP before 1981 Q3, and as real GNP from 1981 Q3 to 1991 Q4. After 1991 Q4, it is real GDP. The definition of the inflation variable 'Price of output' or GDP price deflator (PGDP) has changed several times as well. From 1968 Q4 to 1991 Q4, it stood for the implicit GNP deflator. From 1992 Q1 to 1995 Q4, it was implicit GDP deflator. From 1996 Q1 onward, it stands for GDP price index. The density forecasts for these two variables (output and price of GDP) are fixed-target probability forecasts of their annual growth rates in percentages for predefined intervals (bins) set by the survey. In this chapter, the numerical values of closed "bins" or "intervals" like "+1.0 to +1.9 percent" will be defined as [1, 2), and an open interval like "decline more than 2%" will be defined as an open bin (-∞, -2). We have utilized all density forecasts ever recorded in SPF during 1968Q4 to 2017Q3.[6]

## 3. Aggregate uncertainty, aggregate variance and their components

In the absence of density forecasts, the early literature used variance of the point forecasts or disagreement as proxy for uncertainty. In this section, we calculate aggregate variance (i.e., the variance of the consensus distribution), average of the individual variances and forecaster discord (i.e., disagreement) as the variance of the mean forecasts to highlight the decomposition and their trends over last few decades. Unlike many previous studies where the recorded histograms are treated as discrete data or approximated

[6] Initially we had a total 24,011 forecast histograms, but our final sample is 23, 867 after dropping a few forecasts due to bi-modality, confusion regarding target years and horizons, etc.
The Fed is unsure about the correct horizon of the density forecasts for 1985Q1 and 1986Q1. In order to salvage these two rounds, we compared forecast uncertainty measures of each respondent from each survey to that of the same person in adjacent quarters, and concluded that the 1985Q1 density forecasts were for horizon 0 (i.e., current quarter) and the 1986Q1 density forecasts were for horizon 4. That is, both were current year forecasts. This enabled us to derive a continuous series for the variables of interest from the beginning of the survey until 2017Q3.

by normal distributions, we fit generalized beta distributions or triangular distributions to the density forecast histograms following Engelberg, Manski and Williams (2009). For density forecasts with more than two intervals, we fit generalized beta distributions. When the forecaster attaches probabilities to only one or two bins, we assume that the subjective distribution has the shape of an isosceles triangle. There are many different ways to generalize beta distribution to generate other distributions.[7] The generalized beta distribution we choose has four parameters and its probability density function is defined as follows:

$$f(x;\alpha,\beta,l,r)=\frac{1}{\mathrm{B}(\alpha,\beta)(r-l)^{\alpha+\beta-1}}(x-l)^{\alpha-1}(r-x)^{\beta-1},\ l\le x\le r,\ \alpha>0,\ \beta>0 \qquad (1)$$

where $\mathrm{B}$ is the beta function. We further restrict $\alpha$ and $\beta$ to be greater than 1 to maintain uni-modality of the fitted individual density distribution.

The two parameters $\alpha$ and $\beta$ define the shape of the distribution, and the other two parameters $l$ and $r$ define the support. The generalized beta are the preferred forms of distributions for the recorded histograms for several reasons. First, when the histograms are treated as discrete distributions with the usual assumption that the probability mass within an interval is concentrated at the mid-point of each interval, it does not reflect the expected continuity and uni-modality of the true underlying distributions. Second, compared to the normal distribution, the generalized beta distribution is more flexible to accommodate different shapes in the histograms. The histograms often display excess skewness as well as different degrees of kurtosis. Finally, generalized beta distributions are truncated at both sides, while the normal distribution is defined over an open interval (-∞,

[7] See, for instance, Gordy (1998) and Alexander et al. (2012).

+∞), which is not true with most of the histograms and counterintuitive to the fact that the target variables have historical bounds.

We have adopted the triangular distributions when only 1 bin or 2 bins have positive probability masses (nearly 8% of our histograms). Normal distributions will shrink to a degenerate distribution in these cases. While the use of triangular distribution yields a unique solution for each observation, we should be cognizant of possible limitations of the assumption. The triangular distribution may exaggerate the spread and uncertainty imbedded in the distribution. In the fitting process we restrict the triangular distribution to be isosceles and allow the support to cover the whole bin which has a probability not less than 50%. There are triangular distributions we could fit with shorter supports and smaller variances if we change the restrictions and assumptions. In fact, Liu and Sheng (2018) find that the triangular distribution tends to overestimate the associated uncertainty. However, to avoid multiple solutions for densities with only one or two bins and in the absence of additional information, we choose isosceles triangular distributions.

**Fitting continuous distributions to histogram forecasts**

Suppose forecaster $i$ ($i$ is forecaster ID number) makes a point forecast $y_{i,t,h}$ for the variable $y_t$ with a horizon of $h$ quarters for the target year $t$ and a probability forecast $p_i$ (for convenience subscripts $t$ and $h$ are omitted for now), where $p_i$ is a $k \times 1$ vector of probabilities (in percentage points) corresponding to $k$ pre-defined intervals such that $\sum_{j=1}^{k} p_i(j) = 100$. The target variable $y$ is annual growth rates of output or price level in percentage points. We fit a continuous distribution $f_i$ to the histogram of $p_i$. The fitting process is to minimize the sum of squared differences between the cumulative probabilities at the nodes. As mentioned above, the choice between generalized beta and triangular distribution depends on whether there are more than two bins with positive probabilities. Histograms fitted to generalized beta are also divided into four different cases depending on whether the open bin on either end of the support has positive probabilities. If all probabilities are attached to closed bins then for the generalized beta distribution whose density is $f(x, \alpha, \beta, l, r)$, $l$ and $r$ are set to be the lower bound and upper bound of the bins which have positive probabilities. Then in the fitting process only the shape parameter $\alpha$ and $\beta$ need to be solved in the minimization problem. If the left (right) open bin has positive probabilities, then $l$ ($r$) needs to be solved in the minimization problem.

**Uncertainty decomposition – decomposing aggregate variance**

The mean and variance of $f_i$ are $\mu_i$ and $\sigma_i^2$ respectively (subscripts $t$ and $h$ are omitted for convenience). For individual $i$, $\sigma_i^2$ is the forecast uncertainty. If we take the average of $\sigma_i^2$ over all forecasters, we get the average uncertainty $\overline{\sigma_i^2}$. If we take the average of all the means we get the mean consensus forecast $\overline{\mu_i}$. The variance of the means $\mu_i$ ($N$ is the number of forecasters in that quarter), $\sigma_\mu^2 = \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \overline{\mu_i})^2$, is the disagreement about means. We also pool all individual distributions into a consensus distribution, and obtain the aggregate variance as the variance of that consensus (aggregate) distribution. Lahiri, Teigland and Zaporowsky (1988) show the following identity:

$$\textit{Aggregate Variance} = \textit{disagreement in means} + \overline{\sigma_i^2}. \qquad (2)$$

Boero, Smith and Wallis (2013) have corroborated the above identity by characterizing the aggregate density as a finite mixture.[8] Multiplying both sides by *N*, we get another representation of (2) in terms of total variation:

$$\sum_{i=1}^{N}\int f_i(x)(x-\overline{\mu}_i)^2\,dx=\sum_{i=1}^{N}(\mu_i-\overline{\mu}_i)^2+\sum_{i=1}^{N}\sigma_i^2 \qquad (3)$$

The left hand side of (3) is the total variation, which is the sum of the variation over bins across all forecasters from the consensus mean. On the right hand side we have the sum of squares of means deviations and sum of individual variances, which is *N* times of disagreement and average uncertainty respectively. Obviously, equations (2) and (3) are equivalent, and show that the variance of the aggregate distribution is the sum of the forecaster discord regarding their means and average of the individual variances. Therefore, disagreement is only one component of the aggregate uncertainty and may not be a good proxy for uncertainty if the average variance turns out to be the dominant component during a period and varies over time, cf. Lahiri and Sheng (2010). We now proceed to calculate these components from SPF density forecasts and look at their variations and relative importance.

**Estimation of variance:**

To compute the aggregate variance, we could follow two approaches. In approach 1, we can get the consensus histogram $p_c=\frac{1}{N}\sum_{i=1}^{N}p_i$ first, then fit a generalized beta distribution $f_{p_c}$ to the consensus histogram, and obtain the variance of $f_{p_c}$ denoted by $\sigma^2_{f_{p_c}}$. In approach 2, we utilize the individual generalized beta and triangular distributions that we

[8] Interestingly, the mixture model formulation and this decomposition is well known in the Bayesian forecast combination literature for some time, see Draper (1995).

have fitted to the individual histograms. We take the average of them to get the consensus distribution $f_c = \frac{1}{N}\sum_{i=1}^{N} f_i$, and compute the variance of $f_c$, denoted by $\sigma^2_{f_c}$. The order of completing these two procedures will yield different results. Since the individual distributions are not linear, it is obvious that $f_{p_c} \neq f_c$. Whereas $f_{p_c}$ is a uni-modal distribution, $f_c$ is a mixture distribution as the average of many generalized beta and triangular distributions, and may have multiple modes. Moreover, in approach 2, identity (2) strictly holds while in approach 1 it does not. Indeed, in approach 1 for several quarters in our sample period the aggregate variance was smaller than its disagreement component. For this reason, we adopt approach 2 in computing the aggregate variance in the remaining sections.

**Correcting for bin size for variance decomposition**

Before reporting our variance decomposition results, we have to deal with a complication in the survey design that between 1981Q3 and 1991Q4 the predefined intervals had length of 2.0 percentage points rather than 1.0 percentage point as in other quarters (or 0.5 percentage points for inflation forecasts in more recent quarters).

First, we show that bin-length affects the computed levels of uncertainty, especially when uncertainty is low. Smaller number of bins and longer bin sizes reduce accuracy and information in the forecasts, and can in principle inflate calculated variances, cf. Shoja and Soofi (2017). For periods before 1981 and after 1991, bin-length was 1.0 for GDP growth and inflation. For inflation, the bin-length has changed further to 0.5 after 2014Q1. For periods when bin size is 1.0, we combine probabilities in adjacent bins from the original survey histograms to create new histograms with 2-point bins, re-fit continuous distributions to them, and then re-calculate all relevant measures, (cf. Rich and Tracy 2010).

Figure 1 uses a scatter plot of observations for all quarters to show that, for the same density forecast, 2-point bin produces higher aggregate (upper panel) and average (lower panel) variances compared to 1-point bins. This over-estimation may partly be due to the fact that in the process of conversion from 1-point bin to 2-point bin histograms, we will have more number of cases with only 1 or 2 bin of positive reported probabilities that would necessitate fitting triangular distributions, which may overstate the variances (see Liu and Sheng, 2018). However, by comparing variances based on triangular distributions after converting all histograms to 2-bin histograms, we found that this concern is not an issue in our calculations.

In order to adjust the uncertainty measures during 1981-91 with a wider bin, we run OLS regressions to approximately convert the uncertainty measures recorded estimated with 2% bins to the same we would have obtained had the bins been 1%, like the rest of the sample. We also correct for bin-size for 2014Q1-2017Q3 inflation forecasts by combining adjacent 0.5% bins into 1% bins and re-calculate uncertainty measures from the new histograms. Thus, our estimated uncertainty series will have an underlying bin size of 1% point throughout the sample.

We use two regressions to correct average variances for bin-length. In these regressions, dependent variable is the uncertainty measures computed using 1-point bin settings ($y$), and independent variable is the uncertainty measures computed using 2-point bin settings ($x$). We regress $y$ on $x$ for the output forecast sample {post-1991Q4} and inflation forecast sample {pre-1981Q3 and 1992Q1-2013Q4} respectively, and use these coefficients to calculate $\hat{y}$ from $x$ for samples in the period of {1981Q3 to 1991Q4}. In this way, estimates from surveys when bin-length is 2 points could be adjusted to their 1-point-bin-length equivalents. The regression results are presented in Table 1.

With the above regressions, average variances in most quarters during 1981Q3 and 1991Q4 are adjusted a little lower by around 0.1 (range - 0.09 to 0.13). Since the mapping coefficients were very similar with average variance, we adjusted the aggregate variances by the same magnitude that left the disagreement measures unchanged. During 1981Q3-1991Q4, uncertainty measures in most quarters were adjusted a bit lower, but compared to the era after, the values are still high, and are not attributed to the larger bin size.

**Variance decomposition results**

The time series of aggregate variance, disagreement and average individual variance for forecast horizons from 1 to 8 quarters are presented in Figures 2A and 2B for output and inflation forecasts, respectively. The sixteen graphs in Figures 2A and 2B show how the aggregate variance and its two components evolve over time by forecast horizon.

We summarize the main results as follows:

1) For the period between 1981Q3 and 1991Q4 output growth uncertainty as well as inflation forecast uncertainty were exceptionally high. This elevated level of historical uncertainty cannot be explained by the size of the bin being 2.0 rather than 1.0 percentage point.

3) The recent two decades have witnessed persistently low levels of uncertainty and disagreement for both output and inflation forecasts. Forecaster discord tends to soar quickly and can exceed uncertainty component during recessions and structural breaks. It can be argued that during these periods, forecasters facing conflicting news, interpret them

differentially and generate higher disagreement. We also note that output uncertainty and disagreement are significantly higher than those of inflation because the former is more difficult to predict, cf. Lahiri and Sheng (2010).

4) When we look at the two components of aggregate variance, we find that before 1981 disagreement accounted for most of the aggregate uncertainty, while after 1981, it plays a less prominent role. Before 1981, most spikes in aggregate variance were due to sharp increases in disagreement, while between 1981 and 1991, most spikes in aggregate variance are a result of higher average variances.

5) Average variance is much less volatile than disagreement over the full sample, and picks up only a little during the 1980s. Again, as we showed before, this is not an artifact due to the length of interval being 2.0% rather than 1.0% during 1981-1991.

6) Uncertainty is higher when forecast horizon is longer. Generally, the average values of all three series tend to decrease as forecast horizon shortens, but over horizons 8 to 6 uncertainly does not decline much. The decline, however, is dramatic as the horizon falls from 5 to 1. We expected this since as more information ('news') comes in related directly to the current year, forecasters become increasingly more certain of current year's outturn.[9]

Noting the different trends of the three series, we can conclude that disagreement is not a good proxy for uncertainty, especially since the early 1990s. Furthermore, the variance of the point forecasts as a measure of uncertainty has another limitation. Lahiri and Liu (2009), Engelberg et al. (2009), and Clements (2012) find that point forecasts can deviate from the means/medians of density forecasts, and that forecasters tend to provide

[9] Note that for some quarters before 1981Q3 we do not have data for 1-quarter and longer horizon forecasts (6 ~ 8 quarters) -- as a result we cannot see the potentially high spikes in these volatile quarters, particularly for the longer horizon graphs.

more favorable point forecasts than their corresponding density central tendencies. Elliot et al. (2005) attribute this to asymmetry in forecaster loss functions. Thus, disagreement in point forecasts may even be a worse proxy than disagreement in density means as a measure of uncertainty.

## 4. Uncertainty and information measures

Recently info-metrics has been incorporated in forecast analysis.[10] Soofi and Retzer (2002) present a number of different information measures and estimation methodologies. Rich and Tracy (2010) calculate entropy of forecast densities and show uncertainty should not be proxied by disagreement. Shoja and Soofi (2017) show that entropy of the consensus distribution can be decomposed into average entropy and disagreement in terms of information divergence, and the decomposition of the aggregate variance could be incorporated in a Maximum Entropy model based on the first two moments. In this section, we build on their contribution, and compute entropies and information measures based on fitted generalized beta and triangular distributions that we estimated in the previous section. We decompose the uncertainty in the info-metrics framework, but unlike Shoja and Soofi (2017), use fitted continuous distributions. In info-metrics, entropy reflects all information contained in a distribution, independent of its shape. The Shannon Entropy is defined as

$$H(f) = -\int f(y)\log(f(y))\,dy\,. \tag{4}$$

It measures how close a distribution is to a uniform distribution on the same support. The higher the entropy, the less information the distribution contains, and hence the higher the level of uncertainty. The entropy is akin to the variance of the distribution, but embodies

[10] Golan (2018), especially chapter 4, contains the necessary background literature.

more characteristics of the distribution. Different distributions may have the same variance, but their shapes may be different and thus have different entropies. López-Pérez (2015) shows that entropy satisfies several properties as a "coherent risk measure" whereas moment based measures such as variance does not. Entropy is superior to variance, especially in cases when the underlying distribution is not unimodal and potentially discontinuous.

The divergence between two distributions is related to the entropy. A popular divergence measure is the Kullback-Leibler information measure (KL) or Kullback-Leibler divergence (Kullback and Leibler 1951). For two distributions whose pdf are *f(x)* and *g(x),* the KL divergence is defined as

$$KL(f(x), g(x)) = \int f(x) \log \frac{f(x)}{g(x)} dx. \tag{5}$$

The KL information measure is valid only when *f(x)* is absolutely continuous with respect to *g(x)*. This has some limitations while working with beta shaped distributions. Besides, the KL information measure as it is used widely is not symmetric to the order of the two component density distributions, i.e. $KL(f(x),g(x)) \neq KL(g(x),f(x))$, if certain conditions are not met.[11]

We adopt another information measure – the Jensen-Shannon divergence (JS). Shoja and Soofi (2017) adopts the JS divergence for mixture models as a measure of disagreement as well as the expected uncertainty reduction in info-metrics. For two distributions *f(x)* and *g(x)*, the JS divergence is defined as

$$JS(f(x), g(x)) = \frac{1}{2}\left(\int f(x) \log \frac{f(x)}{m(x)} dx + \int g(x) \log \frac{g(x)}{m(x)} dx\right), \tag{6}$$

[11] The original KL paper introduced a symmetric function also.

where $m(x)=½×(f(x)+g(x))$. The JS information measure is symmetric to the order of the two component density distributions and is suitable for cases when the densities $f(x)$ and $g(x)$ are defined over different intervals. The JS divergence has been extensively studied by statisticians since the 1990s (see Lin 1991 and Minka 1998).

The information measure can be viewed as a measure of disagreement between forecasters, but contains more information than 'disagreement in means'. It captures the differences in all aspects of the distributions rather than the difference in the means only. Following Shoja and Soofi (2017), we develop a parallel analysis between variance decomposition and entropy decomposition.

*Entropy of aggregate distribution=Average individual entropy+Information measure* (7)

**Entropy decomposition:**

We first obtain an estimate of the entropy of the consensus or aggregate distribution in two steps similar to approach 2 in Section 3. We utilize the individual generalized beta and triangular distributions that we have fitted to the individual histograms. We take the average of them to get the consensus distribution $f_c = \frac{1}{N}\sum_{i=1}^{N} f_i$, and compute the entropy of $f_c$ denoted by $H(f_c)$. The first term of right hand side of equation (7), 'average individual entropy' is obtained by taking the average of entropies of $f_i$ across forecasters.

The third term in equation (7), the 'information measure', captures disagreement among $f_i$ in a more holistic manner. With N forecasters, we can compute the JS information measure for $\frac{N(N-1)}{2}$ pairs of distributions, but this approach is computationally inconvenient and unwieldy. Rather, we use the generalized Jensen-Shannon information measure, which is defined for more than two distributions as:

$$JS(f_1, f_2, ...f_N) = H(f_c) - \frac{1}{N}\sum_{i=1}^{N} H(f_i) \quad (8)$$

where *H(.)* is the Shannon entropy. This form of JS information measure is closely related to Kullback-Leibler information measure:

$$JS(f_1, f_2, ...f_N) = \frac{1}{N}\sum_{i=1}^{N} KL(f_i, f_c) \quad (9)$$

We follow equation (8) and take the difference between $H(f_c)$ and average entropy $\frac{1}{N}\sum_{i=1}^{N} H(f_i)$ to get the information measure.

**Correcting for bin-size across surveys**

Similar to section 3 where the variance approach is used, we examine if the heightened entropy during 1981- 1991 can be attributable to the longer bin length of 2% in this period? To see the effect of this longer bin length on entropy, we re-calculated the uncertainty of output and inflation forecasts in periods when bin length is 1% assuming they all had a two-point bin length as well (in a similar way as with variances in section 3). We combine probabilities in adjacent bins for the original survey histograms to create a new histograms with 2-point bins, re-fit a continuous distribution to each of them, and then re-calculate all relevant measures.

In Figure 3, we plot the aggregate entropy with 1-point bin length against the same recalculated with 2-points bin-length for 2-quarter-ahead forecasts for the sake of illustration. Since we suspect the effects of bin size change is different for different levels of uncertainty, and uncertainty was high in the 1970s, we include pre-1981Q3 output forecasts as well in this figure even though the output variable was defined as nominal GNP. Uncertainty, assuming 2-point bin length, is uniformly higher than that using a 1-

point after 1991. However, in the 1970s, the use of 2-point bin length does not cause the uncertainty to be much different. These results are very similar for both GDP growth and inflation. Similar to Figure 1, a plot of uncertainty calculated from the two settings indicates that when uncertainty is low, the structure of bins (bin length) will have a larger effect on the outcome, and will cause the entropy measure to be higher. An explanation is that when entropy is high, the densities span over many intervals and after a change of the bin length from 1 point to 2 points, there are still enough number of bins with positive probabilities to provide adequate information to approximate the distribution well. However if entropy is low, after the change, the number of bins may shrink to 1 or 2, which may alter the choice of the fitted continuous distribution and the estimate of entropy.

Similar to what we did in last section, we regressed average entropy measures calculated from the 1-point bin length ($y$) on entropy measures re-calculated using 2-point bin-length ($x$) for the sample that has 1-point bin size. We then use the coefficients to calculate $\hat{y}$ using $x$ for sample in the period 1981Q3 - 1991Q4, and in this way our estimates from surveys with 2-points bin-length are adjusted to their 1-point bin-length equivalents. We also correct for bin-size for 2014Q1-2017Q3 inflation forecasts by combining adjacent 0.5% bins into 1% bins and re-calculate uncertainty measures from the new histograms. We presented these regression results in Table 2. After the correction for bin length for average entropies, we adjust the aggregate entropy of each quarter by the same values, thus leaving information divergence unchanged.

In Figures 4A (output forecasts) and 4B (inflation forecasts), the aggregate entropy, average entropy, and the information measures are displayed by forecast horizon. In these graphs, aggregate entropy is $H(f_c)$, average entropy is $\frac{1}{N}\sum_{i=1}^{N} H(f_i)$, and information measure is obtained from equation (8). During 1981 - 1991, both aggregate entropy and average entropy are now smaller in magnitudes than those without any adjustment for bin length, and are not as high as the in the 1970s.

**Findings from entropy decomposition:**

Figure 5 suggests a highly significant linear relationship between individual entropies and the logarithm of the variances for majority of the observations. It is not surprising in view of the well-known result that for normally distributed densities with variances $\sigma^2$,

$$H = \frac{1}{2}\log(2\pi e\sigma^2) = \frac{1}{2}\log(2\pi e) + \frac{1}{2}\log\sigma^2 = 1.42 + 0.5\log\sigma^2, \qquad (10)$$

cf. Ebrahimi et al. (1999). However, many of the observations in Figure 5 are located below the diagonal line as well. We checked the properties of these distributions and found that they are significantly more skewed and longer horizon forecasts than those on the diagonal. We also regressed the entropies on the first four moments from the same individual densities and obtained the following regression (standard errors in parentheses, subscripts *i, t, h* are ignored):

$$\underset{(.1293)}{H = 1.36} + \underset{(.0001)}{0.0003\,\mu} + \underset{(.0004)}{0.494\,log\,(\sigma^2)} + \underset{(.0019)}{0.014\,Skewness} - \underset{(.0015)}{0.038\,Excess\ Kurtosis}; \; R^2 = .995 \qquad (11)$$

The significance of the skewness and excess kurtosis in the above regression suggests non-normality of some of the forecast distributions, even though the total marginal contribution of these two factors is only 3%. Ebrahimi et al. (1999) used the Legendre series expansion to reveal that entropy may be related to high-order moments of a distribution, which unlike the variance, could offer a much closer characterization of the density distribution. In the above regression, other parameter estimates are consistent with our expectations, i.e., and the intercept and the coefficient of log ($\sigma^2$) are close to 1.42 and 0.5 respectively.

We can now summarize the main characteristic features of the calculated entropies of the consensus distributions, average entropies and information measures for GDP growth (1981-2017) and inflation (1968-2017) for the eight horizons separately.

1) Output and inflation forecasts have similar cyclical variations in uncertainty across all forecast horizons over the sample. Pre-1981Q3 period for inflation is characterized by the high overall uncertainty (high aggregate entropy), and high disagreement (high information measure). The 1981Q3-1991Q4 period is characterized by a big contribution of average entropy to the aggregate entropy. The post-1992 era sees smaller aggregate entropy.

2) As horizon decreases, uncertainty falls, but the quarterly declines from horizon 8 to horizon 5 is quite small. Only from horizon 4 to 1, that uncertainty declines sharply. It means that as the quarterly values of the target variable in the current year get announced, part of the target variable becomes known, and hence the uncertainty with the remaining part of the target variable becomes less. It does not suggest that forecasters are getting better and more efficient over the quarters. Even though we see new information reducing uncertainty consistently in all eight rounds of forecasting, news coming in the current year does not reduce uncertainty very much for next year forecasts. This is true for both output and inflation.[12]

3) Over all, entropy and information measure in Figures 4A & 4B are similar to aggregate variance and disagreement in Figures 2A & 2B. By comparing these figures

[12] Clements (2014) and Knüppel (2014) find that the SPF *ex ante* survey uncertainties underestimate the *ex post* RMSE-based uncertainty for the current year forecasts, but overestimate for next year forecasts.

more closely, we find that from longer to shorter horizons, and from 1970s to the 2000s, uncertainty as measured by variance falls more precipitously than those based on entropy. The highly volatile disagreement in the moments-based measure are much muted in the information measure during the 1970's, primarily because the later reflects more than just disagreement in means. As a result, the entropy measures are more stable and less volatile than variance measures. The decline in trend uncertainty using entropy is not as significant and sustained as the variance-based measures. We also note that disagreement in means contributed more than 80% of the aggregate uncertainty in certain quarters, but disagreement in distributions (information measure) never contributed more than 50% of the aggregate entropy. Thus, the relative importance of disagreement towards aggregate uncertainty is more limited with entropy measures compared to variance. These findings are very noteworthy regarding the use of info-metrics in measuring uncertainty and forecaster discord.

**Entropy based on continuous distributions vs. discrete distributions**

We conclude Section 4 by pointing out that the calculation of entropy using fitted continuous distributions rather than discrete distributions may produce quite different results with or without the adjustment for the unequal number of bins (normalized entropy index) as used by Shoja and Soofi (2017) for discrete distributions. For each calculated entropy, the upper panel in Figure 6 reports two entropy values for output forecasts using discrete distributions – one without the division by the log of the number of bins (denoted by red *) and the other with the number of bins adjustment (blue •), against the entropy calculated from fitted continuous distributions. The latter are adjusted individually for bin sizes following the logic in the previous section. The lower panel of Figure 6 reports the same scattered diagram for inflation forecasts. For a vast majority of distributions in the upper panel, the red *'s are close to the 45° line, indicating entropies from discrete distributions match well with the estimated entropies based on continuous distributions. However, there is a group of red *'s that run parallel but significantly below the diagonal line. We identified the latter group of observations as those coming from histograms between 1981Q3 and 1991Q4 (when survey has 6 bins and bin size 2%). Clearly, for these histograms, entropy for discrete distributions is lower than that from a continuous fit. With the correction for the number of bins, the blue •'s are not separated into two blocks any more, meaning the correction is successful. The slope of the block of blue •'s is smaller than one – this is mostly due to the fact that the correction multiplier is greater than 1.

In the lower panel of Figure 6 (inflation), there are three clusters of red *'s. The group of *'s above the 45° diagonal are from the 2014Q1-2017Q3 period (when bin size is 0.5%), the group below the 45° diagonal are from the 1981Q3-1991Q4 period (when bin size is 2%), and the group close to the diagonal are from 1968Q4 - 1981Q2 and 1992Q1-2013Q4 (when the bin size is 1.0%). After applying the normalized entropy correction as used by Shoja and Soofi (2017), the group below and on the 45° line merge into the bulk of blue •'s; however, the group of *'s for 2014Q1-2017Q3 still form a separate group. The correction of dividing by *ln (number of bins)* does not match the entropy values from continuous distributions. This is because the calculations based on discrete distributions cannot adjust for the bin size as two discrete distributions that have the same height but two different bin widths will have exactly the same entropy. We present such an example in Figure 7. It is only coincidental that the normalized entropy successfully corrects the observations during 1981Q3-1991Q4 because a smaller number of bins was accompanied by a larger bin size in this period. The number of bins during 2014Q1-2017Q3 is 10 and is similar to that in the majority of the histograms, but the bin size in the last sample period is 0.5%. As a result, the correction does not take into account this smaller bin size and fails to adjust these entropies at par with others. Thus, even after correcting for the number of bins, discrete entropies are not directly comparable when two histograms have different bin sizes. It is obvious that a larger bin size, *ceteris paribus*, makes the entropy based on discrete distributions smaller than it should be.

To be consistent with 1981-1991, Rich and Tracy (2010) converted the bin lengths in all other sub-periods to 2.0%, thereby inflating the entropy. For inflation forecasts, substantial information will be lost if the data since 2014Q1 are adjusted into 2%-bin-size equivalents from 0.5% bin size as it would require combining 4 bins into one. Another disadvantage of the discrete method is that, the estimates are very sensitive to the number of forecasters who attach a probability mass to only one bin. As shown in Figure 5 of Shoja and Soofi (2017), such sensitivity creates abrupt spikes in disagreement.

## 5. Time series of uncertainty measures

After obtaining various uncertainty measures and correcting for bin length for different horizons, we adjust these values for horizons to obtain a single time series of uncertainty. The mean value of uncertainty measures for each horizon is subtracted and then the average of horizon 4 is added to make all observations horizon 4-equivalent. This way we can have a complete time profile of the aggregate macroeconomic uncertainty in the last few decades. We have displayed horizon-corrected uncertainty measures for output and inflation forecasts in Figure 8, where we report quarterly average uncertainty as the overall uncertainty measure.

Based on both entropy and variance, output uncertainty fell steadily from the peak in 1980s to a trough around 1995, but this decline is more dramatic with variance measure. The fall of uncertainty in the post-1990 era seems to be permanent, and is consistent with the macro-economic uncertainty series generated by a factor stochastic volatility model by Jo and Sekkel (2017).

Uncertainty increased during the 2008 financial crisis, and peaked at the end of 2008, which is coincident with the dramatic events of that quarter - Lehman Brother's bankruptcy, and U.S. government's bailout. A concerning trend now is that uncertainty of output forecasts based on entropy remains somewhat elevated during last two years, suggesting risk build up in the economy.

For inflation forecasts, uncertainty remained high from 1970s until early 1990s. Then it went into a downward trend. Similar to the output forecast uncertainty, the fall of inflation uncertainty in the 1990s seems to be permanent, and the new peak in the 2008 financial crisis is quite low compared to 1970/80 levels. Unlike output forecasts, inflation uncertainty has taken a slow but steady downward trend beginning 2010, reflecting a persistently low and stable inflation environment in the U.S. economy.

For the sake of comparison, we have plotted our output entropy measure (aggregate entropy) together with two other popular uncertainty indices from the literature (viz., Jurado, Ludvigson and Ng 2015 (JLN) and Baker, Bloom and Davis 2016 (BBD)) in Figure 9. While doing this we have normalized each series in terms of their means and variances. JLN measures general macroeconomic uncertainty, whereas BBD measures economic policy uncertainty. Since these three uncertainty measures are related to three different underlying target variables, they are not strictly comparable. Still, Figure 9 reveals certain common features in these series. The overall cyclical movement of all three series are very similar except that JNL is smoother and BBD exhibits very high and variable values in the post great recession era. If we smooth the entropy series, it would be very similar to JNL over the whole sample. Like our entropy measure of uncertainty, both JNL and BBD reached their troughs in the mid-1990s and spiked during the great recession of 2008. However, the magnitude and duration of the spikes are quite different between the three measures. Our entropy measure did not reach the unprecedented high levels of JLN and BBD during the 2008 crisis. BBD's high policy uncertainty is also persistent in the last few years even when the economy has recovered. The extraordinary policy environment and subsequent policy changes that existed at the federal level following the financial crisis can explain the unique gyrations in BBD.

## 6. Information measure and "news"

Economists frequently study how "news" affects the economy. The conventional approach in macroeconomics is to generate "news" from estimated VAR models requiring values of the forecast errors. Revisions in fixed target forecasts as measures of news was first suggested by Davies and Lahiri (1995, 1999), and come closest to the theoretical concept that are obtained in real time using forecasts alone without requiring values of the target variables. The Jensen-Shannon information measure is a more comprehensive measure of the same derived from info-metrics. In SPF, forecasters make fixed-target density forecasts, making it possible to check how forecasters update their forecasts quarter to quarter. If a forecaster updates his forecasts for the same target variable by moving from $f_{i,t,h+1}$ to $f_{i,t,h}$, we can obtain the information divergence of these distributions, based on $JS(f_{i,t,h+1}, f_{i,t,h})$. This measure can be viewed as 'news' or 'shocks to uncertainty'. It is new information that makes the forecaster update his forecast. Since each JS is computed using the same forecasters in two adjacent quarters, the 'news' series will be independent of the varying composition of the SPF panel, cf. Engelberg et al. (2011).

In Figure 10, we report news or uncertainty shocks based on Jensen-Shannon information measure for both output and inflation. We find that forecasters were faced with more news in the tumultuous 1970s. Since the mid-1980s the arrival of news has become less volatile, consistent with the so-called 'moderation hypothesis'. Exceptions are the three recessions of early 1980s, 2000/01 and 2008/09. 'News' for real output shows a slightly countercyclical property. It spikes in the recessions when economic data exhibit more negative surprises that lead to large forecasts revisions, and correspondingly large values of "news" as information measure. The time series on inflation news looks quite different from that of real output news, and the correlation between read GDP and inflation news over the whole sample period is only 0.354. The sign and magnitude of this correlation is expected to be time varying, and will be determined by the nature of the shocks (demand or supply, for example) affecting the real and the monetary sectors of the economy.

**Determinants of "news"**

The info-metric measure of "news" is quite similar in concept to "revisions in point forecasts". In order to see the relationship between JS information divergence and revisions in mean forecasts, in Figure 11, we plot "news" against absolute values of forecast revisions at the individual level for current year output forecasts. There is a strong positive correlation between the two, but the association far from perfect. We also see that, because Jensen-Shannon information divergence has an upper bound of *ln2*, as the revisions in forecast means take large values, the relationship gets more variable and the dots eventually get aligned with the upper bound to form a horizontal line. As a local approximation, we regressed "news" on absolute values of first difference of the first four moments at the individual level (first difference between $f_{i,t,h+1}$ and $f_{i,t,h}$), and found that "news" is significantly correlated with the differences (i.e. forecast revisions) in means, variances, skewness and kurtosis of the density forecasts. The forecast revisions in the means explain most of the info-meric "news", i.e., a shift in the forecast density's mean causes the biggest kick in the information divergence, while changes of variance, skewness or kurtosis only explain less than 3% of the variation, even though their coefficients are statistically significant. This evidence indicates that even though information-based 'news' theoretically reflects changes in higher moments of the forecast densities, in the SPF data it is well approximated by changes in mean forecasts alone.

**"News" and its impact on uncertainty**

Engle and Ng (1993) first introduced the "News impact curve". In the last two decades, many authors have studied how "news' impacts on volatility, using mostly high frequency data from the stock market. Davies and Lahiri (1995) showed a positive relationship between news and inflation volatility and a greater effect of bad news on volatility. With our carefully constructed information-based uncertainty measure, we regressed our uncertainty measure on two types of news to estimate such effects. Our regression model is

$$H_{t,h} = \alpha + \beta_1 \cdot news\_good_{t,h} + \beta_2 \cdot news\_bad_{t,h} + \beta_3 \cdot H_{t,h+1} + \varepsilon_{t,h}, \quad (12)$$

where $H_{t,h}$ is entropy of the consensus forecast density for target year t and horizon h. $News\_good_{t,h}$ is average of "news" across forecasters for target year $t$ and horizon $h$, when the mean forecast revision is positive. $News\_bad_{t,h}$ is average of "news" across forecasters for target year $t$ and horizon $h$, when the mean forecast revision is negative. $\varepsilon_{t,h}$ is error term with zero mean. The results of the regression are given in Table 3.

For output forecasts, $\beta_2$ is greater than $\beta_1$, which is consistent with Davies and Lahiri (1995), indicating greater impact of bad news on output forecast uncertainty. However, the signs of the coefficients $\beta_1$ and $\beta_2$ are both positive indicating both good news and bad news increase uncertainty. Chen and Ghysels (2011) found that in stock market both very good news (unusual high positive returns) and bad news (negative returns) increase volatility, with the latter having a more severe impact. For inflation forecasts, $\beta_1$ is not statistically different than $\beta_2$, indicating bad news does not cause more severe impact on uncertainty than good news of similar size, and both are highly significant. The coefficient of lagged entropy is significant in both regressions, but inflation uncertainty ($\beta_3$ = 0.57) seems to be more persistent than output uncertainty ($\beta_3$ = 0.21). It is interesting to note that conventional information theory suggests that news is associated with uncertainty reduction. But in our case with fixed target forecasts, $A_t = y_{i,t,h} + \varepsilon_{i,t,h}$, where $A_t$ is the actual realized value of the target variable and $\varepsilon_{i,t,h}$ is the forecast error, with the variance of $A_t$ the same over horizons. As $h$ decreases $y_{i,t,h}$ absorbs more information, and hence the forecast variance will necessarily increase. However, with variance of $A_t$ the same over $h$, an increase in the entropy of $y_{i,t,h}$ will be compensated by a corresponding reduction in forecast error variance.

## 7. Impact of Real GDP uncertainty on macroeconomic variables

In this section, we explore how our measure of entropy as output forecast uncertainty affects the macro economy. Bloom et al. (2007), Bloom (2009, 2014), and Bloom et al. (2014) explore several channels through which uncertainty affects the real economy. Following the literature and particularly Baker et al. (2016) and Jurado et al. (2015), we use a standard vector autoregression (VAR) model with Cholesky-decomposed shocks to study the effects. We estimate a VAR with the following five quarterly variables: entropy of output forecasts, log of real output (RGDP), log of Non-Farm payroll (NFP), log of private domestic investment (PDI), and Federal Funds rate (FFR). The last variable is an indicator for monetary policy stance – it is increased in order to slow down the economy to avoid future inflation. We include four lags of each variable that minimized the overall AIC. We drew the impulse responses of all five variables to a one standard deviation change in the entropy, keeping the Cholesky ordering the same as above. They are presented in Figure 12. We find that an increase of output uncertainty has negative effects on real GDP, payroll employment, and private investment significantly over next 5-10 quarters, after which they tend to revert back to their original values over next 5 quarters. Thus, uncertainty shocks have a long lasting effect of the real sector of the economy. Uncertainty itself converges back quickly after the initial shock. The dampening effect of uncertainty shock on FFR suggests the reaction of the monetary authority to combat the consequent negative effects on the real variables. A change in the ordering of variables does not alter the results. Specifications with fewer variables produce similar results that output uncertainty has negative effects on real GDP growth. The negative relationship between uncertainty and economic activity has been widely reported in recent macroeconomic research, see Bloom (2009, 2014), Bloom et al. (2014), and Jurado et al. (2015).

## 8. Summary and conclusions

We approximate histogram forecasts reported in the Survey of Professional Forecasters (SPF) by continuous distributions and decompose moment-based and entropy-based aggregate uncertainty into average uncertainty and disagreement. We adjust for bin sizes and horizon effects, and report consistent time series of uncertainty in real output and inflation forecasts over last few decades. We find that adjustments for changes in the SPF survey design are difficult to implement while working directly with discrete distributions. Even though the broad cyclical movements in uncertainty measures provided by the variance and the entropy of the forecast densities are similar, and show significant moderation in the post 1990 era, the former is significantly more variable and erratic than the entropy-based uncertainty. In addition, the variance of mean forecasts as a measure of disagreement is a much larger and a more variable component of the variance of the consensus density compared with the Jensen-Shannon measure based on aggregate entropy. These results are very similar for both output and inflation.

The cyclical dynamics of our entropy-based output uncertainty is consistent with Jurado et al. (2015), despite the fact that the latter is based on forecast errors of a very large number of macro indicators. Following Baker et al. (2016) and Jurado et al. (2015), we also examine the importance of our output entropy in a 5-variable VAR and find that higher uncertainty has significant negative effects on real GDP, employment, investment and federal funds rate, that lasts anywhere from 5 to 10 quarters. These impulse response results are consistent with recent empirical evidence on the pervasive role uncertainty has on the economy, and lend credibility to the entropy-based uncertainty measure we presented in this chapter.

The other significant empirical findings are summarized as follows:

First, since SPF density forecasts provide a sequence of eight quarterly forecasts for a fixed target, the Jensen-Shannon information divergence between two successive forecast densities yields a natural measure of "uncertainty shocks" or "news" at the individual level in a true *ex ante* sense. We find both bad and good news increase uncertainty and are countercyclical, which is consistent with the previous literature. It is noteworthy that new information in our context increases the horizon-adjusted forecast uncertainty, but correspondingly should reduce the *ex post* forecast error uncertainty. Somewhat surprisingly, we find that the entropy-based "news" is conditioned mostly by the changes in the means of the underlying densities and very little by changes in other higher moments of the density forecasts.

Second, we find that the historical changes in the SPF survey design in the number of bins and bin sizes, and how one adjusts for these changes affect uncertainty measures. Bigger bin sizes, *ceteris paribus*, inflate uncertainty estimates and, more importantly, the over-estimation will be relatively higher during low uncertainty regimes. We also find the normalized entropy index for correcting the unequal number of bins by dividing the discrete entropies by the natural log of the number of bins may have certain unexpected distortions. Compared to the entropies based on continuous distributions, the normalized entropies for discrete distributions fail to adjust for bin size, resulting in overestimation during periods where bin size is small.

Finally, even though the Shannon entropy is more inclusive of different facets of a forecast density than just variance, we find that with SPF forecasts it is largely driven by the variance of the densities. An initial indication of this evidence can be found in Shoja and Soofi (2017). However, skewness and kurtosis continue to be statistically significant determinants of entropy, but the total marginal contribution of these two factors is less than 5% in a regression of average entropy on the first four moments of the distribution. There is enough number of non-normal densities in the sample that make these high-order moments non-ignorable components of aggregate entropy. All in all, our analysis of individual SPF densities suggest that uncertainty, news shocks and disagreement measured based on info-metrics are useful for macro-economic policy analysis because these are *ex ante*, never revised and reflect information on the whole forecast distributions.

**References:**

Abel, Joshua, Robert Rich, Joseph Song and Joseph Tracy (2016). "The Measurement and Behavior of Uncertainty: Evidence from the ECB Survey of Professional Forecasters." Journal of Applied Econometrics, 31: 533–550.

Alexander, C., G.M. Cordeiro, E. Ortega, and J.M. Sarabia (2012). Generalized Beta-generated distributions, Computational Statistics and Data Analysis, 56 (6), 1880-1897.

Baker, Scott R., Nicholas Bloom and Steven J. Davis (2016). "Measuring Economic Policy Uncertainty." www.policyuncertainty.com.

Bloom, Nicholas, Stephen Bond and John Van Reenen (2007). "Uncertainty and Investment Dynamics." Review of Economic Studies 74, 391- 415.

Bloom, Nicholas (2009). "The Impact of Uncertainty Shocks." Econometrica, Vol. 77, No. 3, pp. 623-685.

Bloom, Nicholas (2014). "Fluctuations in Uncertainty." Journal of Economic Perspectives, Vol. 28, No. 2, pp. 153-176.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten and Stephen J. Terry (2014). "Really Uncertain Business Cycles." Econometrica, 86 (3),1031-1065.

Boero, Gianna, Jeremy Smith and Kenneth F. Wallis (2008). "Uncertainty and disagreement in economic prediction: the Bank of England Survey of External Forecasters." Economic Journal, 118, 1107-1127.

Boero, Gianna, Jeremy Smith and Kenneth F. Wallis (2013). "The Measurement and Characteristics of Professional Forecasters' Uncertainty." Journal of Applied Econometrics, Vol. 30, issue 7, 1029-1046.

Chen, Xilong and Ghysels, Eric (2011) "News - Good or Bad - and Its Impact on Volatility Predictions over Multiple Horizons." Review of Financial Studies, January 2011, vol. 24, issue 1, pp. 46-81

Cheong, Chongcheul, Gi-Hong Kim and Jan M. Podivinsky (2010) "The impact of inflation uncertainty on interest rates." African Econometric Society 15th Annual Conference, Cairo

Clements, Michael P. (2014). "US inflation expectations and heterogeneous loss functions, 1968–2010." Journal of Forecasting, 35 (1), 1-14.

Clements, Michael P. (2014). "Forecast uncertainty- ex ante and ex Post: U.S. inflation and output growth," Journal of Business and Economic Statistics, 32, 206-216.

D'Amico, Stefania and Athanasios Orphanides (2008). "Uncertainty and Disagreement in Economic Forecasting." FEDS working paper 2008-56.

Davies, Anthony and Kajal Lahiri (1995). "A new framework for analyzing survey forecasts using three-dimensional panel data." Journal of Econometrics, 1995, vol. 68, issue 1, 205-227

Davies, Anthony and Kajal Lahiri (1999). "Re-examining the Rational Expectations Hypothesis Using Panel Data on Multiperiod Forecasts." In Analysis of Panels and Limited Dependent Variable Models, (Eds. C. Hsiao, K. Lahiri, L-F Lee, and H.M. Pesaran), Cambridge Univ. Press, 1999, 226-254.

Draper, David (1995). "Assessment and propagation of model uncertainty", Journal of the Royal Statistical Society, B 57, 45-97.

Ebrahimi Nader, Esfandiar Maasoumi and Ehsan S. Soofi (1999). "Ordering univariate distributions by entropy and variance." Journal of Econometrics 90: 317—336.

Elliott, Graham, Ivana Komunjer, and Allan Timmermann (2005). "Estimation and Testing of Forecast Rationality under Flexible Loss." Review of Economic Studies, Oxford University Press, vol. 72(4), 1107-1125.

Engelberg, Joseph, Charles F. Manski and Jared Williams (2009). "Comparing the Point Predictions and Subjective Probability Distributions of Professional Forecasters." Journal of Business and Economic Statistics, 27, 30-41.

Engelberg, Joseph, Charles F. Manski and Jared Williams (2011). "Assessing the Temporal Variation of Macroeconomic Forecasts by a Panel of Changing Composition." Journal of Applied Econometrics, 26, 1059-1078.

Engle, Robert F. and Ng, Victor K. (1993) "Measuring and Testing the Impact of News on Volatility." Journal of Finance, December, vol. 48, issue 5, pp. 1749-1778.

Fresoli, D., E. Ruiz, and L. Pascual (2015). "Bootstrap Multi-step Forecasts of Non-Gaussian VAR Models." International Journal of Forecasting, 31(3), 834-848.

Giordani, Paolo and Paul Söderlind (2003). "Inflation forecast uncertainty." European Economic Review, 2003, vol. 47, issue 6: 1037-1059

Golan, Amos (1991). "The discrete continuous choice of economic modeling or quantum economic chaos", Mathematical Social Sciences, Vol. 21, Issue 3: 261-286.

Golan, Amos (2018). Foundations of Info-Metrics: Modeling, Inference, and Imperfect Information. Oxford University Press: Oxford, UK.

Gordy, M.B. (1998). "A generalization of generalized beta distributions", Board of Governors of the Federal Reserve System, Washington DC.

Henrique, J. H.G., Ruiz, E., and H. Veiga (2018). "Uncertainty and Density Forecasts of ARIMA Models: Comparison of Asymptotic, Bayesian, and Bootstrap Procedures", Journal of Economic Surveys, Vol. 32, No. 2, pp. 388–419.

Jo, Soojin and Rodrigo Sekkel (2017): "Macroeconomic Uncertainty through the Lens of Professional Forecasters." Journal of Business & Economic Statistics, DOI: 10.1080/07350015.2017.1356729

Jurado, Kyle, Sydney C. Ludvigson and Serena Ng (2015). "Measuring Uncertainty." American Economic Review 2015, 105(3): 1177–1216

Kenny, Geoff, Thomas Kostka and Federico Masera (2015). "Density Characteristics and Density Forecast Performance: A Panel Analysis." Empirical Economics, Springer, vol. 48(3): 1203-1231.

Kinal, Terrence, Kajal Lahiri and Fushang Liu (2005). "Measuring Macroeconomic News and Volatility using Kullback-Leibler Information from Density Forecasts." SUNY Albany Department of Economics Working paper.

Knüppel, Malte, (2014). "Efficient estimation of forecast uncertainty based on recent forecast errors", International Journal of Forecasting, 30 (2), 257-267.

Kullback, S. and R. A. Leibler (1951), "On information and sufficiency," *Ann. Math. Statist.,* vol. 22, pp. 79-86.

Lahiri, Kajal, Christie Teigland and Mark Zaporowski (1988). "Interest Rates and the Subjective Probability Distribution of Inflation Forecasts." Journal of Money, Credit, and Banking, Vol. 20, No. 2.

Lahiri, Kajal and Fushang Liu (2006). "Modeling Multi-Period Inflation Uncertainty Using a Panel of Density Forecasts" Journal of Applied Econometrics 21, 1199-1220.

Lahiri, Kajal and Fushang Liu (2005). "ARCH models for multi-period forecast uncertainty – a reality check using a panel of density forecasts". Advances in Econometrics (Vol. 20), Econometric Analysis of Economic and Financial Time Series, eds: T. Fomby and D. Terrell, 321-363.

Lahiri, Kajal and Fushang Liu (2009). "On the Use of Density Forecasts to Identify Asymmetry in Forecasters' Loss Functions", Proceedings of the Joint Statistical Meetings, Business and Economic Statistics, 2396-2408.

Lahiri, Kajal and Xuguang Sheng (2010), "Measuring Forecast Uncertainty by Disagreement: The Missing Link", Journal of Applied Econometrics, 25: 514–538.

Lahiri, Kajal and Xuguang Sheng (2010), "Learning and Heterogeneity in GDP and Inflation Forecasts". International Journal of forecasting (special issue on Bayesian Forecasting in Economics), 26, 2010, 265–292.

Lin, Jianhua (1991). "Divergence measures based on the Shannon entropy." IEEE Trans. Info. Theory, 37(1):145–151.

Liu, Yang and Xuguang Simon Sheng, (2018). "The Measurement and Transmission of Macroeconomic Uncertainty: Evidence from the U.S. and BRIC Countries". International Journal of Forecasting, Forthcoming.

López-Pérez, Víctor (2015). "Measures of macroeconomic uncertainty for the ECB's survey of professional forecasters." In: Donduran M, Uzunöz M, Bulut E, Çadirci TO, Aksoy T (eds) Proceedings of the 1st annual international conference on social sciences, pp 600–614

Minka, Thomas P. (1998). “Bayesian inference, entropy, and the multinomial distribution.” https://tminka.github.io/papers/multinomial.html

Mitchell, James and Stephen G. Hall (2005). Evaluating, comparing and combining density forecasts using the KLIC with an application to the Bank of England and NIESR ‘fan’ charts of inflation. Oxford Bulletin of Economics and Statistics 67: 995–1033.

Mitchell, James and Kenneth F. Wallis (2011). “Evaluating Density Forecasts: Forecast Combinations, Model Mixtures, Calibration and Sharpness.” Journal of Applied Econometrics, September-October 2011, v. 26, issue 6, pp. 1023-40

Rich, Robert and Joseph Tracy (2010). “The relationships among expected inflation, disagreement and uncertainty: evidence from matched point and density forecasts.” The Review of Economics and Statistics, 92: 200–207.

Shoja, Mehdi and Ehsan S. Soofi (2017). “Uncertainty, Information, and Disagreement of Economic Forecasters.” Econometric Reviews, vol. 36: 796-817

Soofi, E, S and J. J. Retzer (2002). “Information indices: unification and applications.” Journal of Econometrics, vol. 107: 17–40

Zarnowitz, Victor. (2008). Surviving the Gulag, and Arriving in the Free World: My life and times.  Praeger Publishers: Westport, CT.

Zarnowitz, Victor and Louis A. Lambros (1987). “Consensus and uncertainty in economic prediction.” Journal of Political Economy, 95, 591-621.

Table 1: Regression results with variance for mapping 2-point bins to 1-point bins: $y=a+bx+u$

| | *a* | *b* |
|---|---|---|
| Average variance, output forecasts (Sample period: 1992Q1-2017Q3) | -0.1468 (0.0147) | 1.0375 (0.0167) |
| Average variance, inflation forecasts (Sample period: pre-1981Q3 plus 1992Q1-2013Q4) | -0.1312 (0.0133) | 1.0085 (0.0199) |

Note: $y$ is variance when bin-length is 1%; $x$ is the measured variance after converting bin-length to 2%. Standard errors are in parentheses.

Table 2: Regression results with entropy for mapping 2-point bins to 1-point bins: $y=a+bx+u$

| | *a* | *b* |
|---|---|---|
| Average entropy, output forecasts (Sample period: 1992Q1-2017Q3) | -0.6987 (0.0415) | 1.4615 (0.0362) |
| Average entropy, inflation forecasts (Sample period: pre-1981Q3 plus 1992Q1-2013Q4) | -0.5859 (0.0476) | 1.3657 (0.0457) |

Note: $y$ is entropy measured when bin-length is 1%; $x$ is calculated entropy after converting bin-length to 2%. Standard errors are in parentheses.

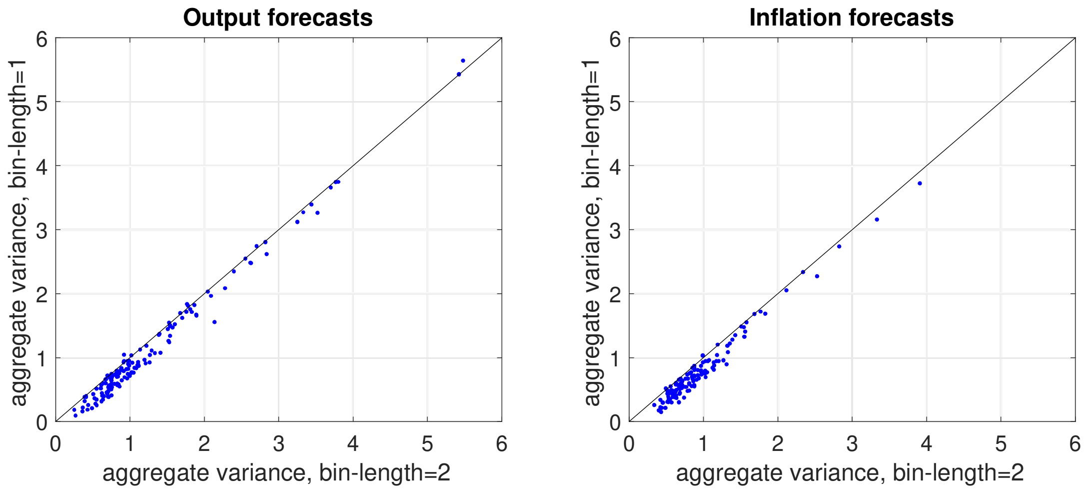
Table 3: Regression of news on uncertainty

$$H_{t,h} = \alpha + \beta_1 \cdot news\_good_{t,h} + \beta_2 \cdot news\_bad_{t,h} + \beta_3 \cdot H_{t,h+1} + \mu_{t,h}$$

| Output forecasts | | | Inflation forecasts | | |
|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Parameter | Estimate | p-value |
| $\alpha$ | 0.8563 | 0.0000 | $\alpha$ | 0.3827 | 0.0000 |
| $\beta_1$ | 0.7791 | 0.0377 | $\beta_1$ | 0.8693 | 0.0342 |
| $\beta_2$ | 0.9997 | 0.0020 | $\beta_2$ | 0.8708 | 0.0233 |
| $\beta_3$ | 0.2059 | 0.0055 | $\beta_3$ | 0.5735 | 0.0000 |

Figure 1: Effect of bin size on uncertainty, all quarters

Aggregate variance:




Average variance:

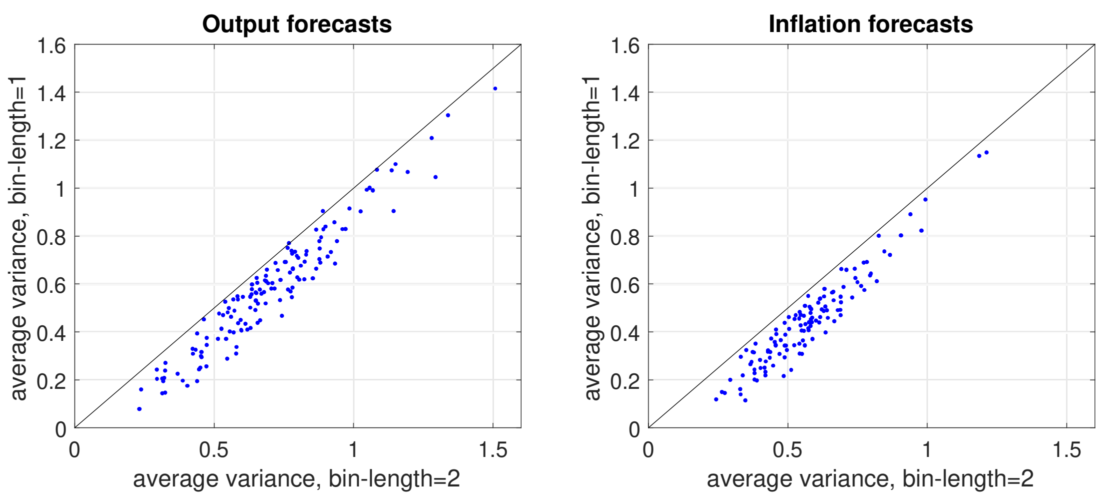


Figure 2A: Output Variance and its components, corrected for bin-length, 1981Q3 to 2017Q3.

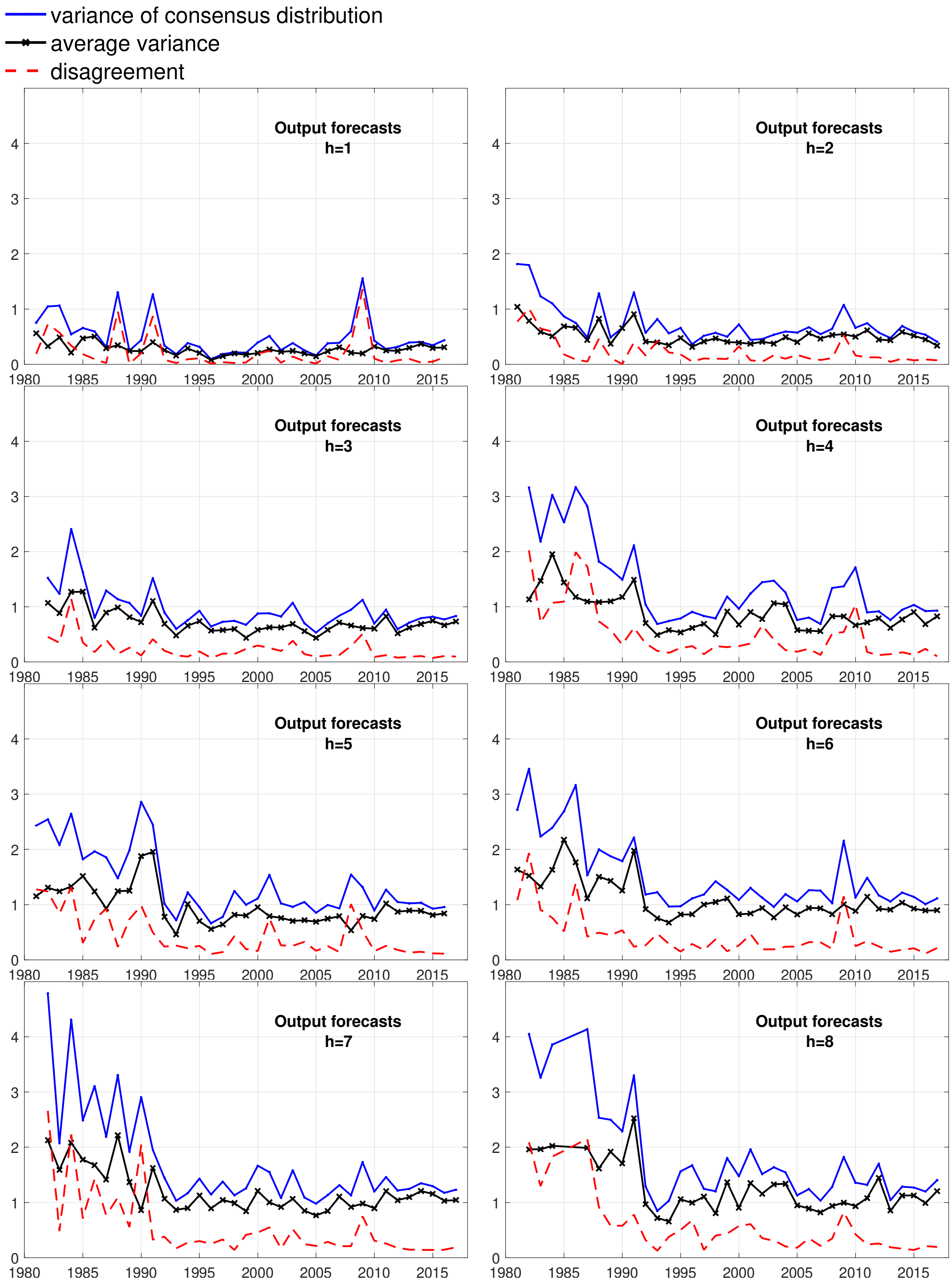


Figure 2B: Inflation variance and its components, corrected for bin-length, 1968 Q4 -2017 Q3.

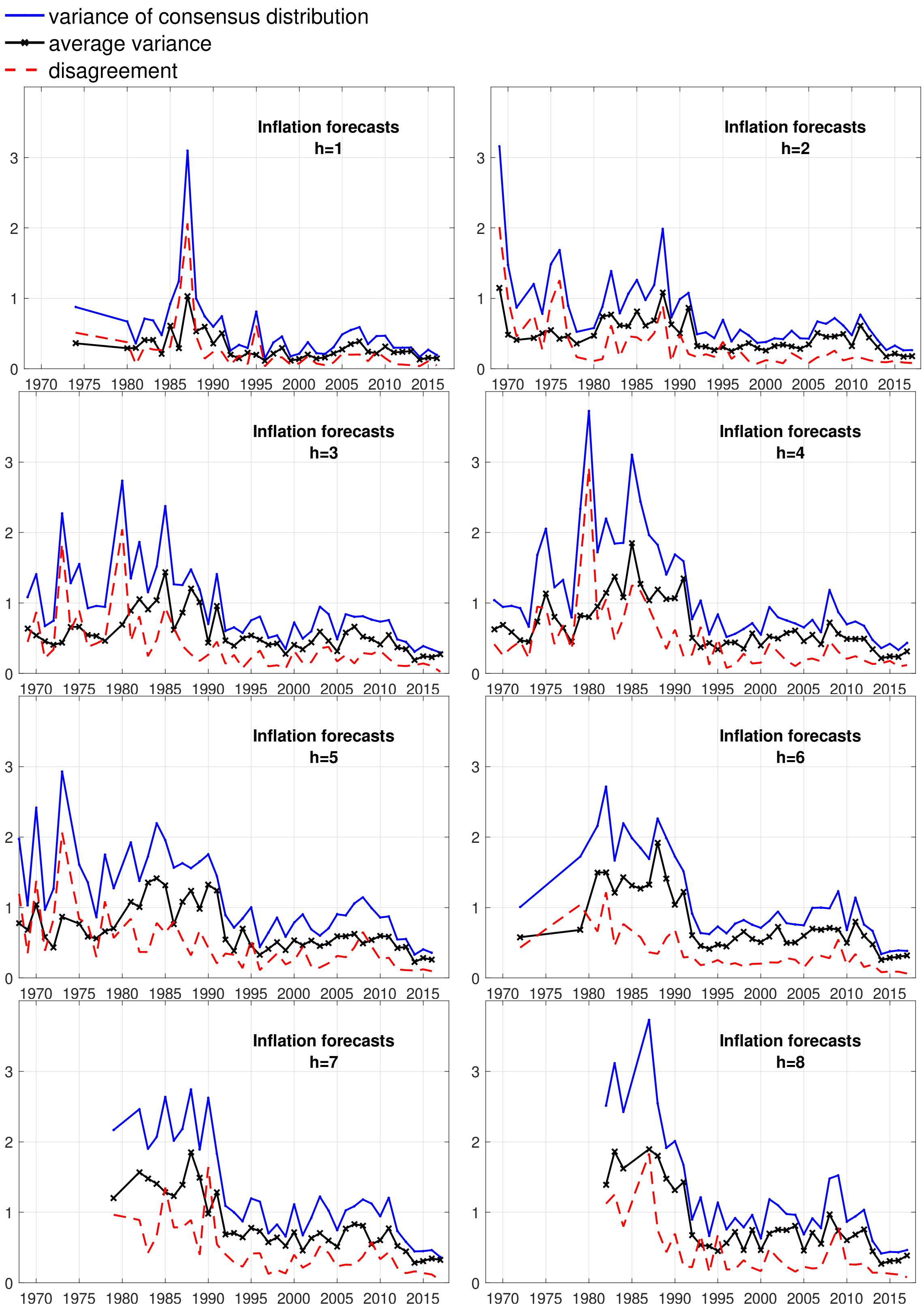
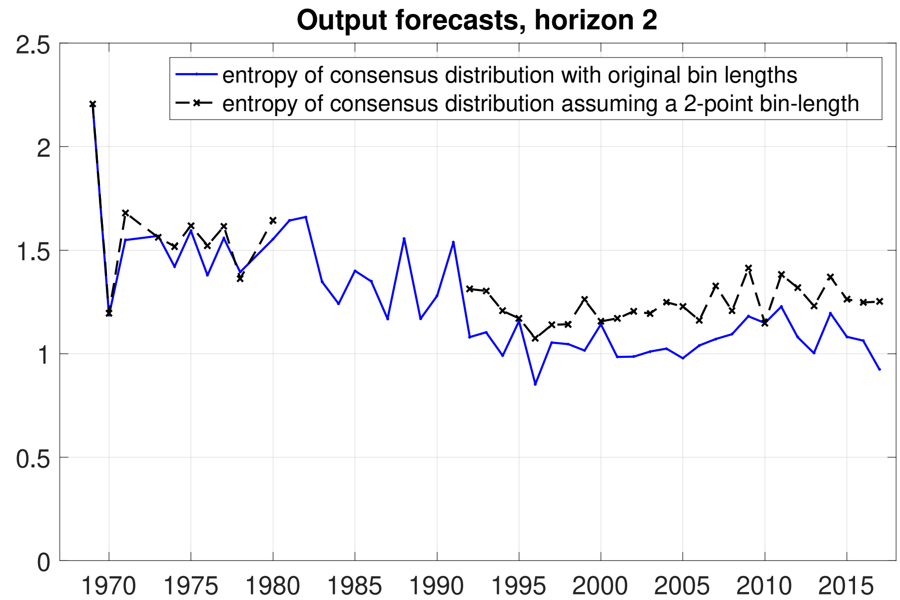


Figure 3: Effects of bin-length on uncertainty measures and the correction



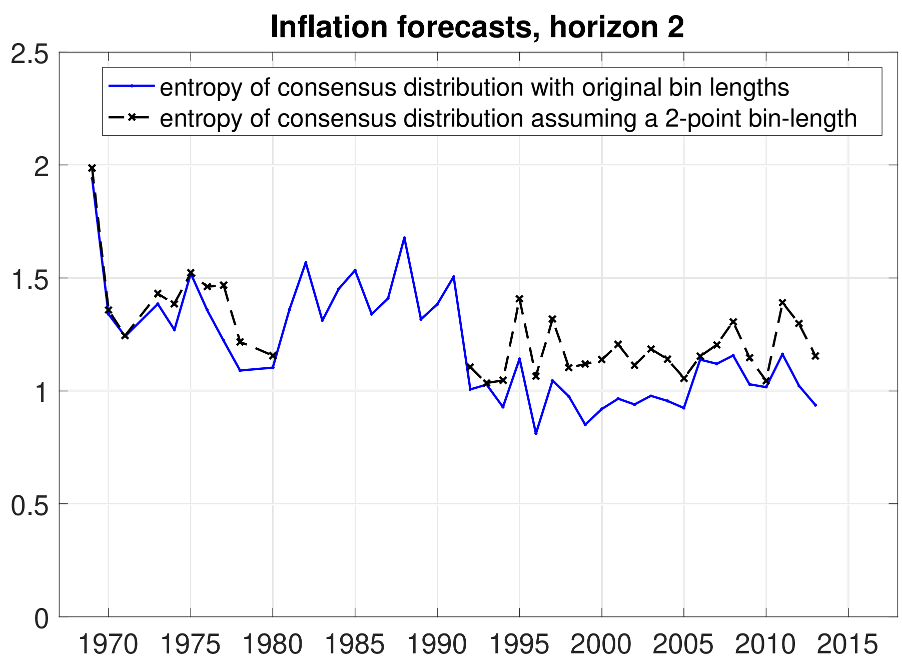



Figure 4A: Output entropy and its components, corrected for bin-length, 1981Q3 to 2017Q3.

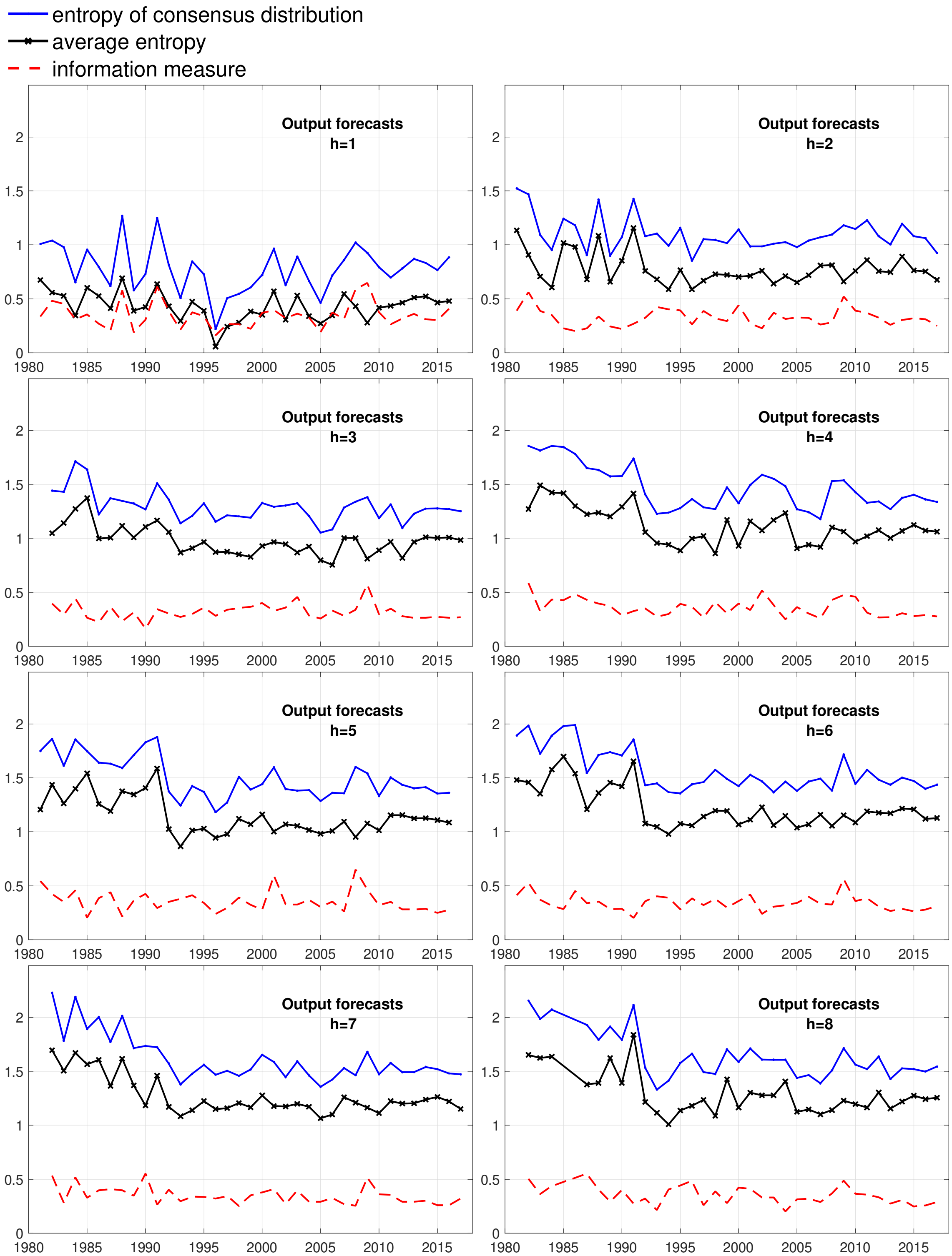


Figure 4B: Inflation entropy and its components corrected for bin-length, 1968Q4 to 2017Q3.

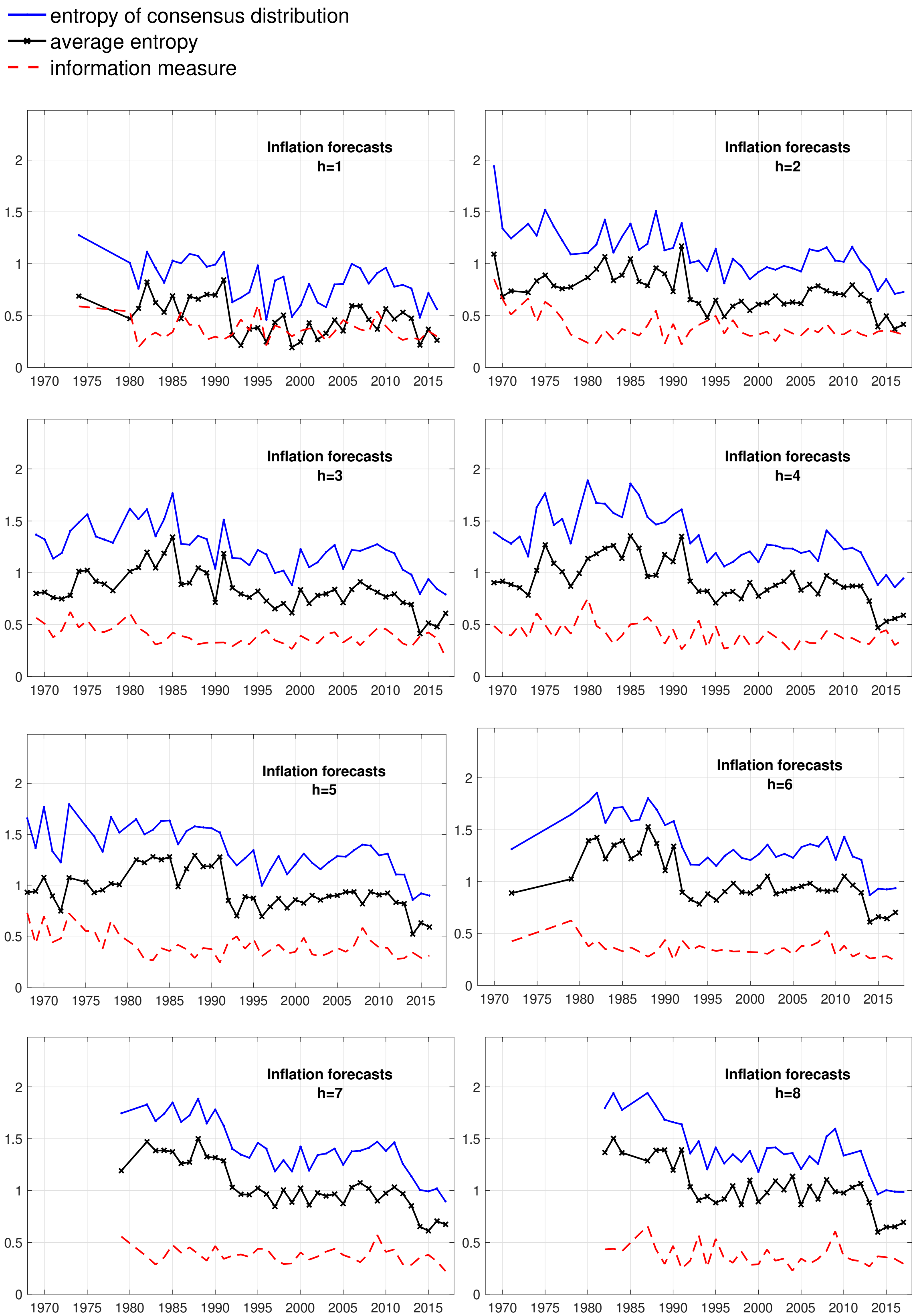


Figure 5: Entropy vs. log (Variance)

X axis: individual level current year output forecasts log (variance)

Y axis: individual level current year output forecasts (Entropy)

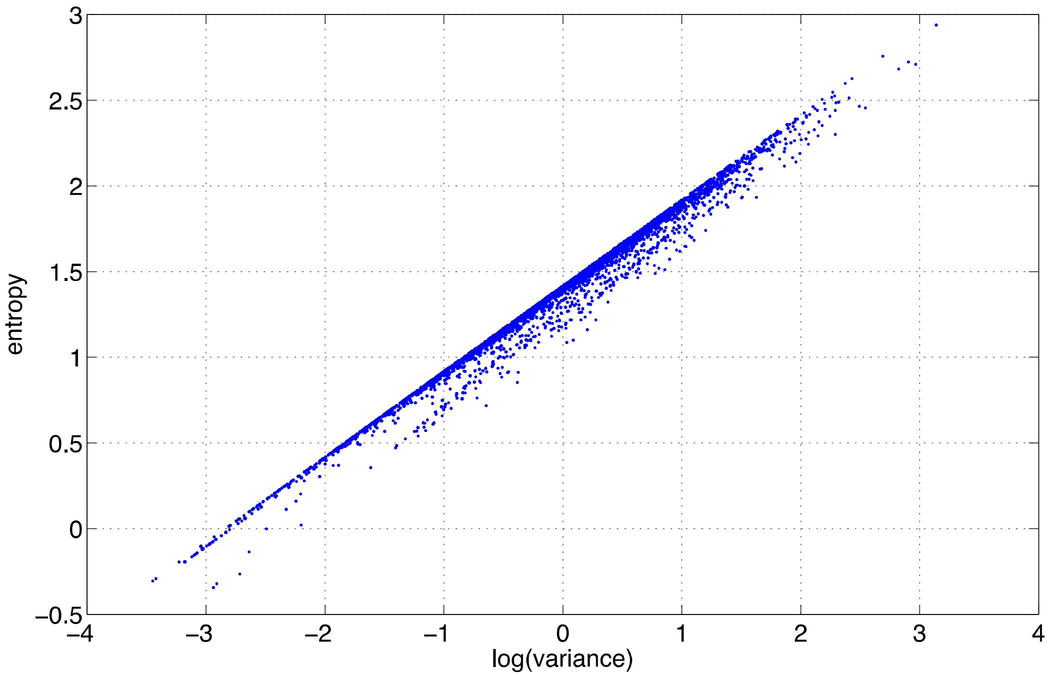
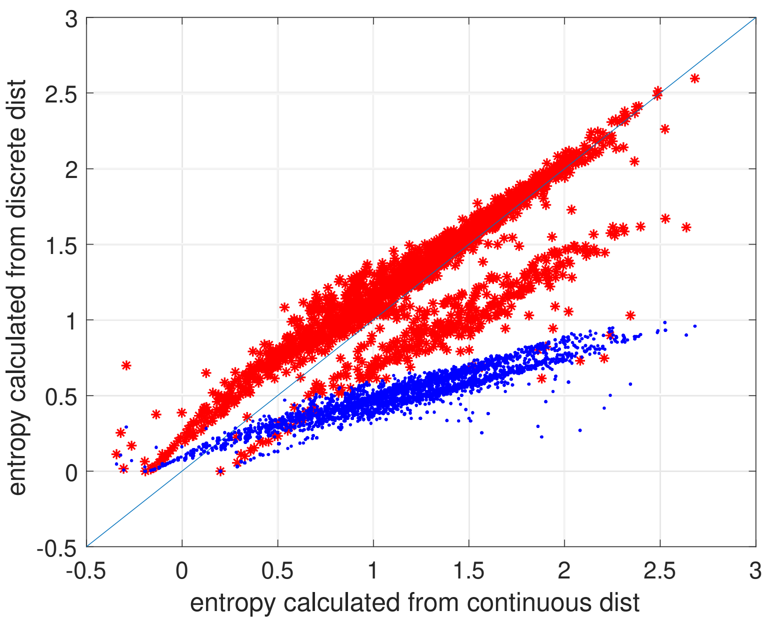


Figure 6: Scattered plot of entropy from alternative methods (continuous vs. discrete)

Red * (stars): X-axis: entropy values calculated from continuous distributions
Y-axis: entropy values calculated from discrete distributions
Blue • (dots): X-axis: entropy values calculated from continuous distributions
Y-axis: entropy values calculated from discrete distributions, with adjustment for number of bins.
Upper panel: output forecasts 1968Q4-2017Q3
Lower panel: inflation forecasts 1968Q4-2017Q3



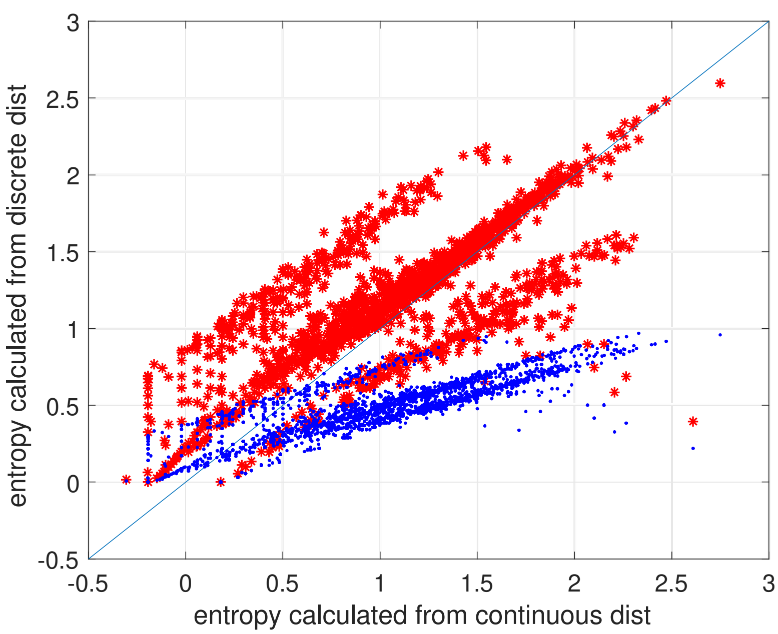
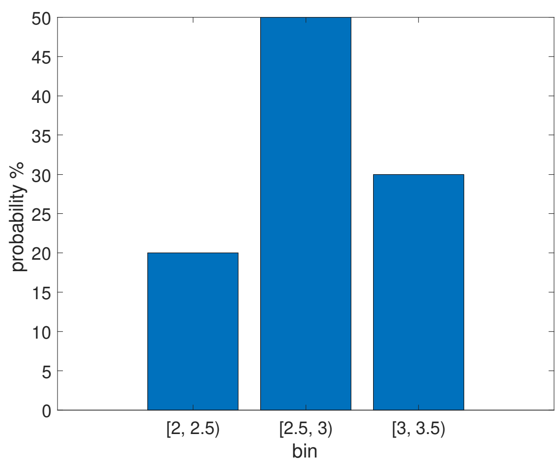



Figure 7: Two discrete distributions with same entropy but different levels of uncertainty

The following two observations have the same entropy under discrete distribution, viz. :

$H=-(0.2\ln 0.2+0.5\ln 0.5+0.3\ln 0.3)=1.0297$

However they imply different levels of uncertainty. The forecaster for the left forecast is certain that there is a 100% probability that the target variable will be between [2%, 3.5%] while the right forecast implies only a 50% chance that the target variable will fall between [2%, 4%].



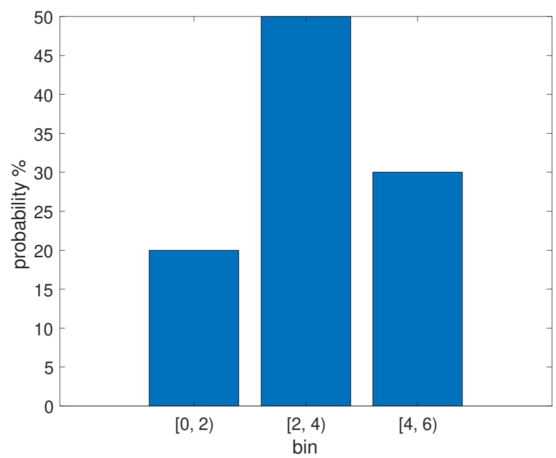



Figure 8: Uncertainty (Entropy and Variance) - corrected for bin length and horizon

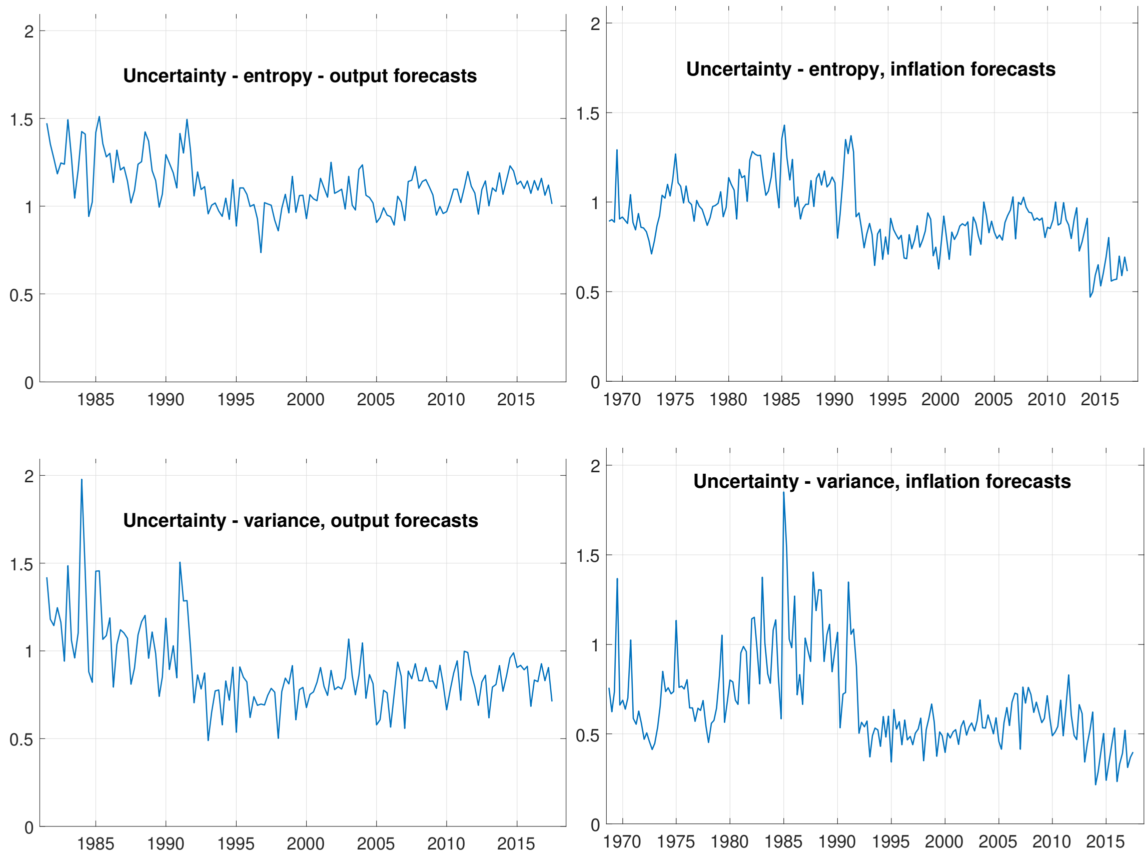


Figure 9: Uncertainty compared with other works

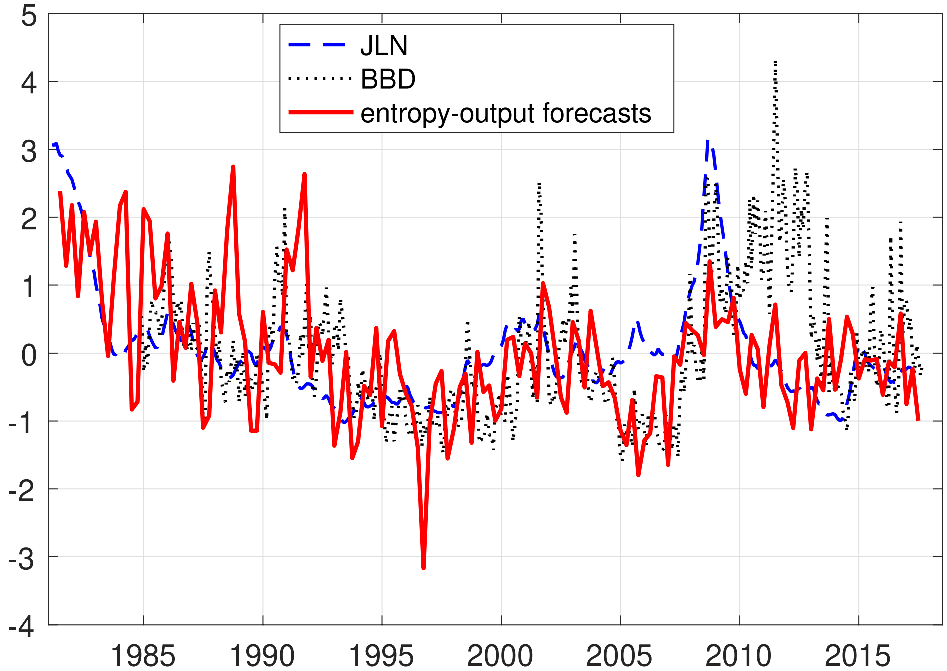
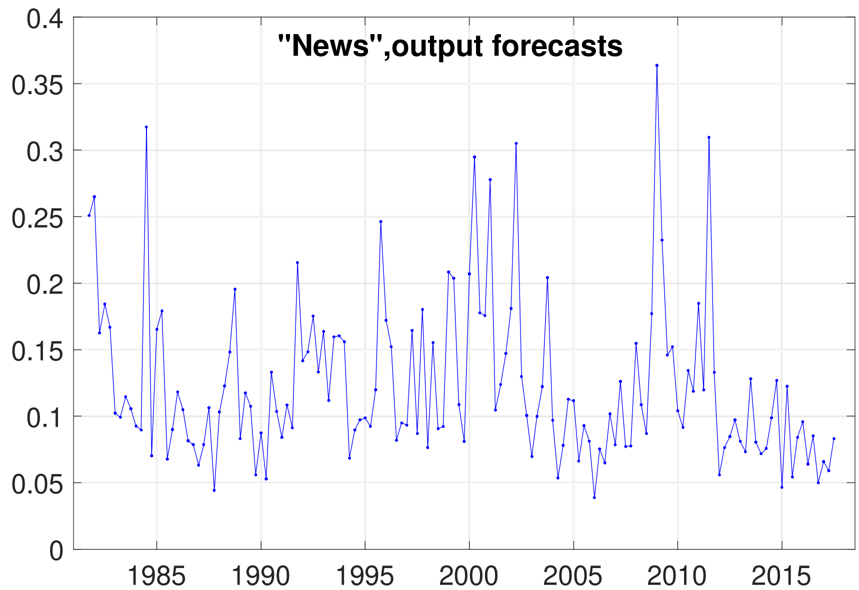


Figure 10: Uncertainty shocks or “news” based on J-S Information



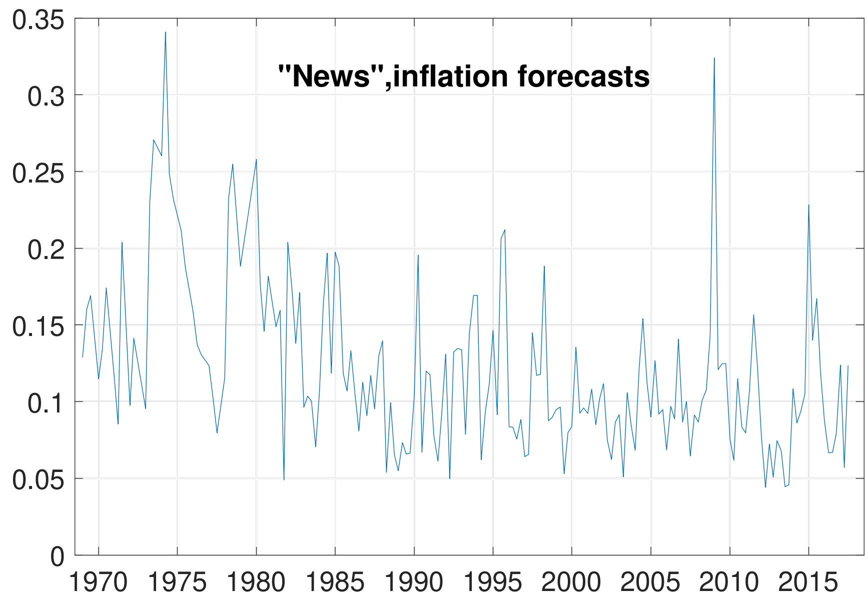



Figure 11: Information "news" vs. forecast revisions, current year output forecasts:

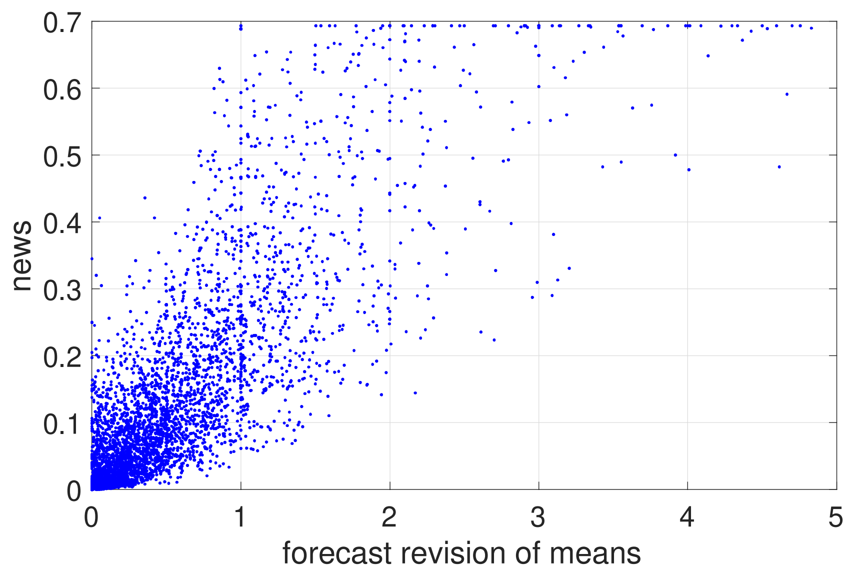
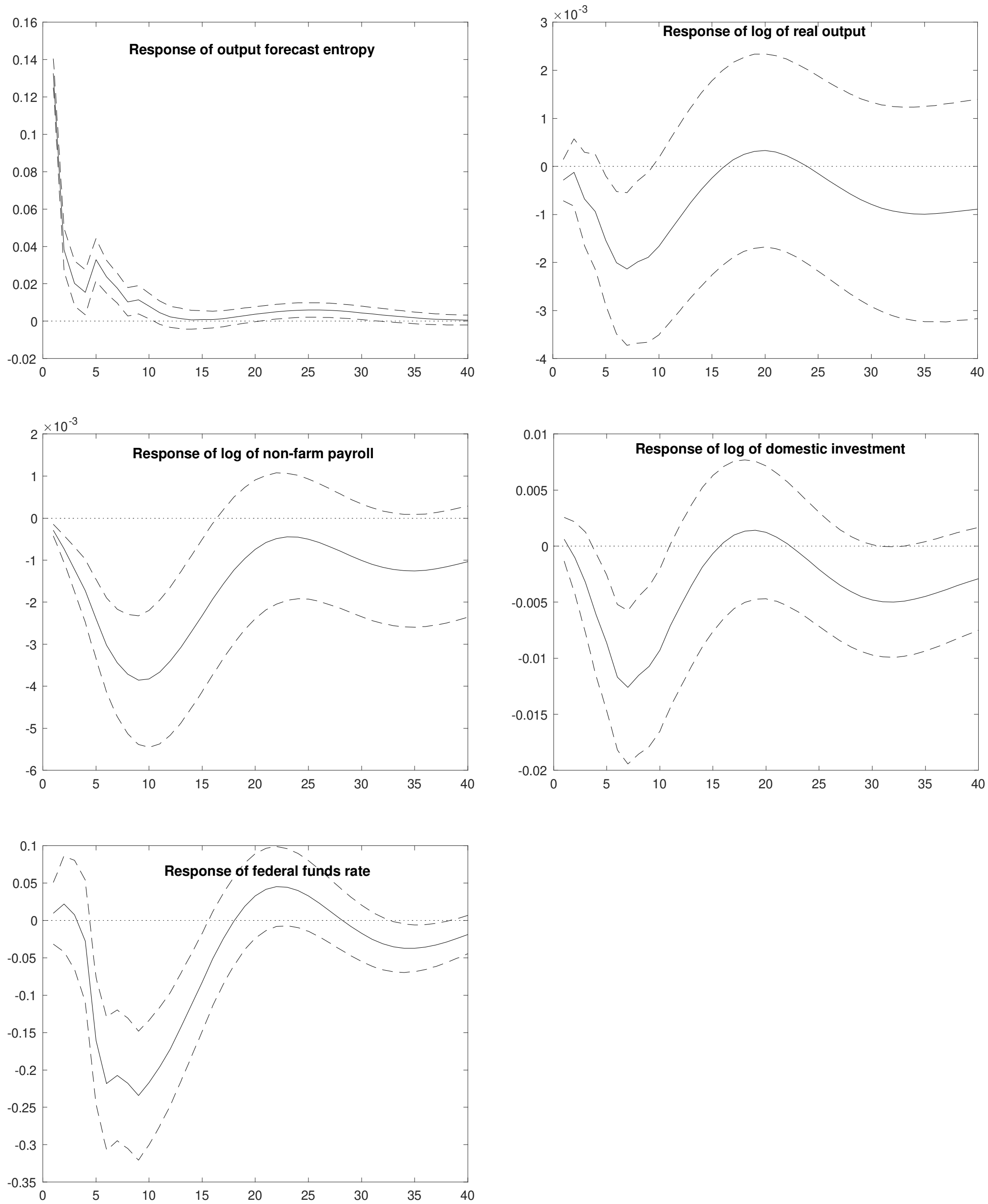


Figure 12: Impulse response functions in a 5-variable VAR to a one S.D. shock of output entropy, (1981Q3 – 2017Q3)




Notes: The 5 variables are entropy of output, log of real output, log of Non-farm payroll, log of private domestic investment, and Federal Funds rate (FFR).
Dashed lines are one standard deviation bands. X-axis is in quarters.