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Abstract

We find that segments of society who have shorter life expectancy can expect a lower retirement income and lifetime utility due to the longevity of other groups participating in the same pension scheme. Linking retirement age to average life expectancy magnifies the negative effect on the lifetime utility of those who suffer low longevity. Furthermore, when the income of those with greater longevity increases, those with shorter life expectancy become even worse off. Conversely, when the income of those with shorter life expectancy increases, they end up paying more into the pension scheme, which benefits those who live longer. The relative sizes of the low and high longevity groups in the population determine the magnitude of these effects. We calibrate the model based on data on differences in life expectancy of men and women and find that males suffer from a 10 percent drop in the amount of pension benefits from being forced to pay into the same scheme as females.

JEL-Codes: E210, E240.

Keywords: longevity, pension age, retirement, inequality.

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1. Introduction

Everything points towards an increase in life expectancy for the elderly, i.e., individuals aged 65 and above. A typical estimate today is that the remaining life expectancy for a 65-year old will increase by a year every ten years (OECD, 2017). This raises a number of economic issues: One concerns public finances, which will come under pressure as pension and elderly care expenses increase. Therefore, the practice of linking retirement age to life expectancy has become quite common (Jensen et al., 2019).

While simple and logical, such longevity indexation rules pose a number of challenges. One of them, and the theme addressed in this paper, is that they are designed in terms of average measures. In Denmark, for example, the official pension age increases in line with changes in average longevity (Andersen, 2015). However, differences in life expectancy between high- and low-education workers, or high-income and low-income workers, as well as between men and women, are well known. Therefore, changes in the pension age based on an increase in average life expectancy may affect different socioeconomic groups differently. Such intra-generational differences may have important implications for lifetime utility for different groups in society.

So, if average figures cover over a high degree of intra-generational disparity, longevity indexation may widen inequality among the elderly. Such unintended effects may seriously jeopardize the egalitarian objectives pursued by, say, Scandinavian welfare states. For example, if longevity indexation would reduce lifetime utility of blue-collar workers with health issues, due to wearing-out following a long working-life with physically demanding routine work, unlike that of white-collar workers with a long education and a shorter working-life, the broad political support behind longevity adjustment might well gradually disappear.

In this paper, we study these issues in further detail. Our key theme is the implications of a longevity-indexed retirement age in an economy where some segments of the population live longer than others, in order to explore whether a longevity-indexed retirement age implicitly leads to intra-generational disparities. We believe this fills an important gap in the academic literature. While a few papers have addressed aspects related to intergenerational redistribution following the introduction of longevity adjustment (see, e.g., Andersen, 2010; Jensen and Jørgensen, 2008), the intra-generational dimension has, to our best knowledge, not been seriously studied so far.

From here, the paper proceeds as follows: We start by providing some additional evidence of differences in life expectancy of observably different groups of individuals. We then derive an overlapping-generations model for groups that differ in life expectancy but face a common retirement age. Next, we use the model to explore the externalities between the groups associated with their differing life expectancies. Thereafter, we calibrate our model based on the different life expectancy of men and women in order to get an estimate of the effect of the common retirement age on the utility of men and women. Finally, we summarize our findings, point out some implications for social and pension policy, and make suggestions for future research.

2. Heterogeneous life expectancy

In order to set the scene for this paper, we provide some additional evidence of differences in life expectancy of observably different groups of individuals who share the same system of a (flow) state pension or belong to the same pension fund. The example that may first come to mind is that of men and women. Differences in the life expectancy of men and women are well known and widely documented. The United Nations published data on developed and developing countries in 2015 where significant differences can be seen, with women having longer life expectancy than men. Thus, the life expectancy for men is 80.9 in Japan, 79.4 in Spain, 81.9 in Sweden and 80.0 in Denmark while the corresponding numbers for women are 86.6 in Japan, 85.1 in Spain, 83.7 in Sweden and 81.9 in Denmark (United Nations, 2015). But these numbers also reflect infant mortality, which is not a part of the issues addressed in this paper. According to the OECD, the life expectancy at age 65 for men in these countries is 19.6 in Japan, 19.4 in Spain, 19.1 in Sweden and 18.2 in Denmark while the corresponding numbers for women are 24.4 in Japan, 23.6 in Spain, 21.5 in Sweden and 20.8 in Denmark (OECD, 2019). Thus, a woman at age 65 can expect to live 4.8 more years than a man in Japan, 4.2 in Spain, 2.4 in Sweden and 2.6 in Denmark.

There are also differences in life expectancy across income groups. Table 1 shows life expectancy in Denmark at age 60 by income quantiles. The difference between the life expectancy of men in the top and bottom income group at age 60 was 5.9 years in 1996 and grew to 6.0 years in 2016. Similar numbers for women are 5.2 years in 1996 and 3.8 years in 2016. Thus, the gap between low-income and high-income women was becoming smaller while the gap for men increased slightly.

Table 1. Life expectancy at 60 by income quantiles, Denmark

	Q1	Q2	Q3	Q4
	Men			
1996	14.9	17.6	18.9	20.8
2016	18.9	21.3	23.1	24.9
	Women			
1996	18.8	21.7	22.4	24.0
2016	23.5	23.9	24.9	27.3

Source: Danish Ministry of Finance.

In addition, there are differences between skill groups. Table 2 illustrates life expectancy at 60 by skill groups in Denmark.

Table 2. Life expectancy at 60 by income quantiles, Denmark

	Unskilled	Skilled	Shorter higher education	Longer higher education
Men				
2002	18.5	19.1	20.5	21.4
2016	20.6	22.0	23.3	24.0
Women				
2002	21.8	22.8	23.6	23.8
2016	23.8	25.2	26.0	26.3

Source: Danish Ministry of Finance.

Comparing with income groups, the differences are smaller. For men, the difference in life expectancy between those having longer higher education and those who are unskilled was 3.4 years in 2016 and 2.9 years in 2002. For women, the difference was 2.5 years in 2016 and 2.0 years in 2002. In this case, the gap between the two groups – those unskilled and those with long higher education – is becoming larger.

Not surprisingly, the differences between the life expectancy of high-income and low-income workers are larger in the US. Waldron (2007) found differences in life expectancy of the rich and the poor in the US and that this is a gradient across the socioeconomic scale. More recently, Chetty et al. (2016) showed that the life expectancy of the richest 1 percent in the U.S. is 14 years longer than that of the poorest 1 percent and the top income quartile can expect to live about a decade longer than the bottom quartile. These are much bigger differences than those found for Denmark (see Table 1).

Furthermore, both studies have found that the spread in life expectancy between income groups is increasing. Case and Deaton (2017) find an increase in mortality and morbidity among white non-Hispanic Americans in midlife (35-59) since the beginning of this century, continuing until at least 2015 due to increases in drug overdoses, suicides and alcohol-related liver mortality. Case and Deaton attribute this development to progressively worsening labor market opportunities of whites with low levels of education at the time of entry into the labor market, which is magnified by the over prescription of opioids and other drugs. Educational differences in mortality among whites are increasing to such an extent that mortality has risen for those without a college degree while decreasing for those with a college degree.

The gap between income groups is evidently not as wide in Europe. The OECD (2017) reported that the average gap in life expectancy in Europe between those with tertiary education and those below upper secondary education is 2.7 years. For example, is gap is only 1.5 years for Denmark. Moreover, Case and Deaton show how mortality rates have continued to drop in Europe, especially for those with lower levels of education. Thus, European countries had an average rate of decline of age-adjusted mortality of 2.0 percent per year between 1990 and 2015, while non-Hispanic whites without a college degree in the US saw that same decline only until the late 1990s, when mortality started to increase for those without a college degree.

Differences in longevity also exist between a wide array of other social groups, such as those defined on the basis of race, country of origin in the case of immigrants, professions, and so forth. However, in this paper we will calibrate our model for genders since they are an obvious example of groups that pay the same amount into pension schemes but have different life expectancies.

3. A model with overlapping generations and a heterogeneous population

In this section, we explore the implications of differences in longevity across groups and lifetime utility when there is a common retirement age for the whole population. We set up a overlapping generations (OLG), stated in continuous time, and with a heterogeneous population based on Andersen and Gestsson (2016) and Gestsson and Zoega (2018). We, as well as these two papers, depart from Blanchard (1985) by assuming that the probability of dying increases with age. Thus, the old differ from the young in facing a higher probability of death and there is a maximum possible age for every cohort.

We depart from the basic model in Andersen and Gestsson (2016) by introducing a more realistic mortality profile and splitting the population into two heterogeneous groups. In that paper, no one dies before reaching retirement age and thereafter, the size of a cohort gradually reduces until no one is left. In contrast, we start with data on actual mortality profiles and calibrate our theoretical model to fit this profile. In addition, and this is the key contribution of our paper, one half of the population, denoted by H, enjoys high life expectancy, while the other half, denoted by L, suffers low life expectancy. Both work until age R, making consumption and saving decisions while paying into a pay-as-you-go (PAYG) pension scheme. After age R they consume their savings and pension benefits. Agents can die at any time, but their instantaneous death probability, or hazard rate, is dependent on their age and which societal group they belong to. The maximum age possible is A. We can then use the model to explore the effect of increased longevity of one group on the lifetime utility of the other, the effect of changes in the retirement effect on the utility of both groups, and, finally, the effect of changes in the income of both groups on their lifetime utility.

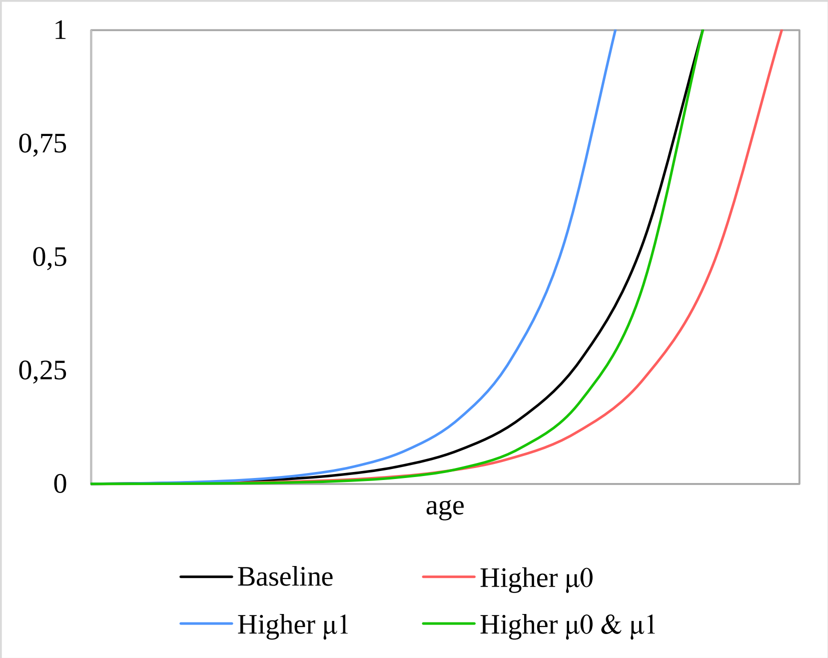
3.1. Age-dependent death probability

The cumulative distribution function (CDF) captures the chance of being dead at age a, where $i \in \{H, L\}$. The superscript denotes which group each agent belongs to. We adopt the realistic mortality structure introduced by Boucekkine et al (2002). The CDF of time of death (D) takes the following form:

$$\Phi^i(a) = \Pr(D \leq a) = \int_D^a \varphi^i(D)dD = \frac{e^{\mu_1^i a} - 1}{\mu_0^i - 1}, i = H, L \tag{1}$$

The parameters $\mu_0^i > 1$ and $\mu_1^i > 0$ determine the shape of the CDF, see Figure 1. An increase in μ_0^i makes the denominator of the CDF greater, proportionally increasing the probability of being dead at each age. Importantly, any change in μ_1^i has an age-dependent positive effect on the numerator. Therefore, a manipulation of both parameters allows us to change the slope and reach of the CDF, effectively creating a mortality profile that closely resembles reality, see Figure 1. Equation (1) can be used to approximate the empirical survival curves shown in Figure A1 (appendix). In general, we have to assume that the group that suffers from a shorter expected lifespan has either a lower value of μ_0^i or a higher value of μ_1^i (or both) than the high longevity group.

Figure 1. Cumulative distribution function of time of death



From the CDF we can find that the chance of being alive at a given age, a, denoted by $m^i(a)$, is:

$$m^i(a) = 1 - \Phi^i(a) = \frac{\mu_0^i - e^{\mu_1^i a}}{\mu_0^i - 1} \tag{2}$$

The maximum age for each group, $A^i = \ln(\mu_0^i)/\mu_1^i$, can be found through the CDF. We see that the chance of being alive is strictly decreasing and strictly concave at an increased age.

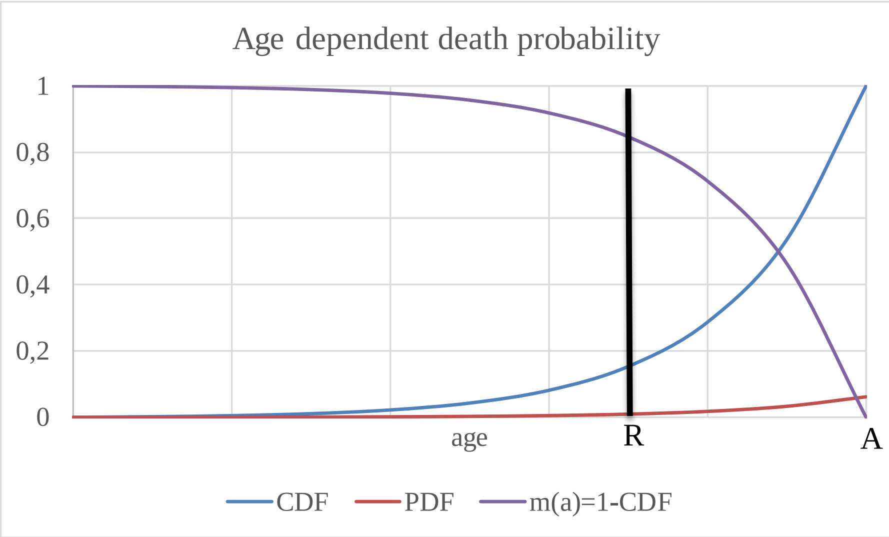
$$\frac{\partial m^i(a)}{\partial a} < 0 \quad , \quad \frac{\partial^2 m^i(a)}{\partial a^2} < 0 \quad and \quad \frac{\partial m^i(a)}{\partial A^i} > 0$$

Finally, the probability density function (PDF) of death, is found by differentiation:

$$\varphi^i(a) = \frac{\mu_1^i e^{\mu_1^i a}}{\mu_0^i - 1} \tag{3}$$

These functions are depicted in Figure 2 along with a hypothetical retirement age, R, and the maximum possible age A.

Figure 2. Mortality over the lifespan



3.2. Private insurance/pension system

In line with (Yaari, 1965), we introduce an actuarially fair insurance company that provides agents with actuarial notes. The purchaser of an actuarial note gets a constant stream of payment until his death. The notes are in a sense an annuity, which, at the time of the purchaser's death, leave the insurance company free of any obligations. This mitigates the loss of utility caused by the uncertainty of death, as all of the agent's private assets are held in these notes. These notes pay the rate $R^i(a) = \int_0^a r + \beta^i(z)dz$, where $\beta^i(z)$ is the instantaneous death probability of agent aged a:

$$\beta^i(a) = \frac{\varphi^i(a)}{1 - \Phi^i(a)} = \frac{\mu_1^i e^{\mu_1^i a}}{\mu_0^i - e^{\mu_1^i a}} \qquad (4)$$

Here, the small open economy setting, where r is exogenous, has been adopted. Equation (4) implies the insurance company can observe which group each agent belongs to, paying group L a higher rate of return, because their instantaneous death probability is higher than those of group H. Therefore, assets held in these notes can be interpreted as a private pension fund or a life insurance company. From this we get the rate of return:

$$R^i(a) = \int_0^a r + \beta^i(z)dz = ar - \ln\left(\mu_0^i - e^{\mu_1^i a}\right) + \ln\left(\mu_0^i - 1\right) \qquad (5)$$

which implies: $e^{-R^i(a)} = e^{-ar}\frac{\mu_0^i - e^{\mu_1^i a}}{\mu_0^i - 1} = e^{-ar}m^i(a)$.

It can directly be observed that the mortality profile influences the rate of return and therefore plays a key role in the consumption-saving decisions of agents.

3.3. Beveridgean-type public PAYG pension system

Agents are forced to pay into a government-run PAYG pension scheme. Contrary to the private insurance company, the government, and therefore this pension scheme, cannot "see" which group each agent belongs to. So, all agents pay the same amount into the PAYG system while working and receive identical pension benefits after retirement, providing that the agent is alive. This system represents the social security system of the economy.

The pension system is run on a balanced budget basis:

$$\Pi = \frac{\mathrm{T}N_w}{N_o} \tag{6}$$

Here we operate in the defined contributions (DC) case, where we treat pension benefits, Π, as endogenous, implying a defined contribution scheme, and the pension contributions, T, as exogenous. Conversely, in the defined benefits (DB) case, we treat pension contributions as endogenous and the benefits as exogenous. Here both groups pay the same amount into the pension scheme. In section 5 we will wage-index the pension transfers and allow for an asymmetric income distribution. For simplicity, we start with a uniform income across the groups causing any wage-indexation to yield identical results as in the case where the groups pay the same dollar-amount.

The subscript denotes which group an agent belongs to; N_o and N_w represent the number of retired (old) agents and working agents, respectively. R represents the retirement age. We can find the number of working agents and retired agents:

$$\begin{aligned} N_w = N_w^L + N_w^H &= \sigma \int_0^R m^L(a)da + (1-\sigma)\int_0^R m^H(a)da \\ &= \sigma \frac{e^{\mu_1^L R} - R\mu_1^L \mu_0^L - 1}{\mu_1^L - \mu_1^L \mu_0^L} + (1-\sigma)\frac{e^{\mu_1^H R} - R\mu_1^H \mu_0^H - 1}{\mu_1^H - \mu_1^H \mu_0^H} \end{aligned} \tag{7}$$

$$N_o = N_o^L + N_o^H = \sigma \int_R^{A^L} m^L(a)da + (1-\sigma)\int_R^{A^H} m^H(a)da$$

$$= \sigma \frac{\mu_0^L - \ln(\mu_0^L)\,\mu_0^L - e^{\mu_1^L R} + R\mu_1^L\mu_0^L}{\mu_1^L - \mu_1^L\mu_0^L} + (1 - \sigma)\frac{\mu_0^H - \ln(\mu_0^H)\,\mu_0^H - e^{\mu_1^H R} + R\mu_1^H\mu_0^H}{\mu_1^H - \mu_1^H\mu_0^H} \quad (8)$$

We impose $0 < \sigma < 1$, where σ and $1 - \sigma$ are the proportional size of the new-born cohorts of group L and H, respectively. Agents in this economy are continuously dying and new agents are continuously being born. The whole population is of size:

$$N = N^L + N^H = \sigma \frac{\mu_0^L - \ln(\mu_0^L)\,\mu_0^L - 1}{\mu_1^L - \mu_1^L\mu_0^L} + (1-\sigma)\frac{\mu_0^H - \ln(\mu_0^H)\,\mu_0^H - 1}{\mu_1^H - \mu_1^H\mu_0^H} \quad (9)$$

Population size is therefore dependent on the life expectancy of each group. Naturally, the population size is not a function of the retirement age and is not affected by the pension system. However, the structure of the pension scheme depends on the demographic structure.

3.4. Utility maximization

Agents maximize expected lifetime utility:

$$E(U) = \int_0^{A^i} e^{-\delta a} m^i(a) u(c(a))\, da \quad (10)$$

where $u(c(a))$ is utility derived from consumption at age a and δ is the discount rate. The lifetime budget constraint becomes:

$$\int_0^R (y^i - \mathrm{T})\, e^{-R^i(a)} da + \int_R^{A^i} \Pi e^{-R^i(a)} da = \int_0^{A^i} c(a)\, e^{-R^i(a)} da \quad (11)$$

Here y^i is the income of agents. The first order condition of the utility maximization problem w.r.t. $c(a)$ yields:

$$e^{-\delta a} m^i(a)\, u'\big(c(a)\big) = \gamma e^{-ar} m^i(a) \quad (12)$$

where γ is the Lagrange multiplier. By assuming the real interest rate, r, equals the subjective rate of time preference, δ, we get:[1]

[1] We can do this as we are not interested in the lifetime consumption *path* of each group, but rather the *total* lifetime consumption of each group.

$$u'(c(a)) = \gamma \quad \forall a \in [0, A^i] \tag{13}$$

which implies: $c(a) = c^i \quad \forall a \in [0, A^i]$.

Finally, from this realization, the budget constraint, and applying the identity of the PAYG-pension system ($\Pi = \frac{\mathrm{T}N_w}{N_o}$), we get that each agent consumes according to:

$$c^i \int_0^{A^i} e^{-ar}\, m^i(a)da = (y^i - \mathrm{T}) \int_0^{R} e^{-ar}\, m^i(a)da + \frac{\mathrm{T}N_w}{N_o} \int_R^{A^i} e^{-ar} m^i(a)da \tag{14}$$

By rearranging we arrive at:

$$c^i = (y^i - \mathrm{T}) \frac{\int_0^R e^{-ar}\, m^i(a)da}{\int_0^{A^i} e^{-ar}\, m^i(a)da} + \frac{\mathrm{T}N_w}{N_o} \frac{\int_R^{A^i} e^{-ar}\, m^i(a)da}{\int_0^{A^i} e^{-ar}\, m^i(a)da} \tag{15}$$

The consumption of the agent is dependent on the relative portions of his expected lifetime spent working, captured by the first fraction in equation (15), and retired, captured by the second fraction. When planning his consumption, the agent accounts for after-tax income while young and expected pension transfers while retired, both of which are influenced by the demographic structure of society. Therefore, the agent has to account for his own life expectancy and the life expectancy of the whole population when making consumption decisions.

The old-age dependency ratio dictates the contributions/benefits structure of the pension scheme. This is an obvious way in which the demographic structure of society influences the consumption plan of the agent. These effects are analyzed in sections 4.1 and 4.2. More subtly, as the retirement age is uniform across the population, any changes in the life expectancy of one group will raise the average life expectancy of the whole population, which might lead to a rise in the retirement age. In sections 4.3 and 4.4 we elaborate further on the implications of retirement age hikes.

4. Experiments

Having presented our analytical framework, we next study some of the results that can be derived from it. We concentrate on longevity shocks and changes in the retirement age.

4.1. Asymmetric longevity shock

We now turn our attention towards the implications of increased longevity by deriving the effect of a widening of the gap between the life expectancies of the two groups. The widening of the gap between the life expectancies of the two groups can either manifest itself in a lowering of μ_1^H or increases in μ_0^H causing the high longevity group (H) to live even longer.

We begin by lowering μ_1^H and keep μ_0^H, μ_1^L, μ_1^L and R constant. This rise will therefore not affect the mortality profile of the L group but will change the consumption pattern of the L group through the pension system.

$$\frac{\partial N_w}{\partial \mu_1^H} = (1-\sigma)\frac{e^{\mu_1^H R}(1-R\mu_1^H)-1}{(\mu_1^H)^2(\mu_0^H-1)} < 0 \tag{16}$$

$$\frac{\partial N_o}{\partial \mu_1^H} = (1-\sigma)\frac{\mu_0^H - \ln(\mu_0^H)\,\mu_0^H - e^{\mu_1^H R}(1-R\mu_1^H)}{(\mu_1^H)^2(\mu_0^H-1)} < 0 \tag{17}$$

We are not only interested in the effect on each population, but also the effect on the ratio of old to young. We find that the old population grows more than the young population when we raise longevity via μ_1^H. This implies that pension benefits will decrease for both groups. Therefore, group L will receive a lower return for their contributions, because the drop in pension benefits is not caused by an increase in their own lifespan. The expected lifetime consumption of members of the H group will rise, since their expected pension benefits will increase because of their increased life expectancy. Therefore, a positive longevity shock on one group has a negative financial effect on the other.

We can also simulate the asymmetric shock by increasing μ_0^H while keeping all other parameters in the mortality profile, μ_1^H, μ_0^L and μ_1^L, and the retirement age, R, constant.

$$\frac{\partial N_w}{\partial \mu_0^H} = (1-\sigma)\frac{e^{\mu_1^H R} - R\mu_1^H - 1}{\mu_1^H(\mu_0^H-1)^2} > 0 \tag{18}$$

$$\frac{\partial N_o}{\partial \mu_0^H} = (1-\sigma)\frac{\mu_0^H + R\mu_1^H - \ln(\mu_0^H) - e^{\mu_1^H R}}{\mu_1^H(\mu_0^H-1)^2} > 0 \tag{19}$$

We observe the same effects in this case; the expected lifetime consumption of the H group increases at the cost of the L group.

4.2. Population shock

We have seen that the longevity of one group has an effect on the welfare of the other through the PAYG scheme. This welfare effect depends on the relative sizes of the H and L groups. Let's define the PAYG equality, from equation (6), as [2]

$$\Pi = \mathrm{T}\frac{N_w}{N_o} = \mathrm{T}\frac{\sigma\widetilde{N}_w^L + (1-\sigma)\widetilde{N}_w^H}{\sigma\widetilde{N}_o^L + (1-\sigma)\widetilde{N}_o^H} \tag{20}$$

where: $\widetilde{N}_w^L \equiv \int_0^R m^L(a)da$, $\widetilde{N}_o^L \equiv \int_R^{A^L} m^L(a)da$, $\widetilde{N}_w^H \equiv \int_0^R m^H(a)da$ and $\widetilde{N}_o^H \equiv \int_R^{A^H} m^H(a)da$. Now we can see what happens to the PAYG benefits (or contributions in the DB case) when the relative sizes of the group changes:

$$\frac{\partial\Pi}{\partial\sigma} = \mathrm{T}\frac{(\widetilde{N}_w^L - \widetilde{N}_w^H)\big(\sigma\widetilde{N}_o^L + (1-\sigma)\widetilde{N}_o^H\big) - (\widetilde{N}_o^L - \widetilde{N}_o^H)(\sigma\widetilde{N}_w^L + (1-\sigma)\widetilde{N}_w^H)}{\big(\sigma\widetilde{N}_o^L + (1-\sigma)\widetilde{N}_o^H\big)^2} > 0 \tag{21}$$

The sign of equation (21) can be determined by solving the numerator:

$$(1-\sigma)\widetilde{N}_w^L\widetilde{N}_o^H + \sigma\widetilde{N}_w^L\widetilde{N}_o^H - (1-\sigma)\widetilde{N}_w^H\widetilde{N}_o^L - \sigma\widetilde{N}_w^H\widetilde{N}_o^L = \frac{\widetilde{N}_o^H}{\widetilde{N}_w^H} - \frac{\widetilde{N}_o^L}{\widetilde{N}_w^L} > 0 \tag{22}$$

This implies that when the size of the L group increases, the pension benefits for both groups increase. This is due to the fact that the L group pays proportionally more into the PAYG scheme than it receives as benefits. So, as the number of L agents increases, there are proportionally fewer long-lived agents who collect benefits in old age. Because the PAYG scheme is balanced at each time, this translates into either a drop in the contributions, T, in the DB case or a rise in the benefits, Π, in the DC case as described in equation (21).

This implies that when the H group gets smaller the remaining members get even richer through the PAYG scheme. Conversely, as the L group gets smaller its members are even worse off.

4.3. Rise in the retirement age

Having derived the effects of a longevity shock we now turn to changes in the retirement age. At first glance, we see that the probability of surviving until retirement is always lower for members of the L group than the H group. Furthermore, by differentiation of the survival chance, we get:

[2] To analyze the effect on the size of one group on the pension transfers we have separated σ from the definition of the relative size of each population group, see equations (7) and (8).

$$\frac{\partial m^i(R)}{\partial R} = -\frac{\mu_1^i e^{\mu_1^i R}}{\mu_0^i - 1} < 0 \tag{23}$$

From this expression, we see that any change in the retirement age will have a greater impact on the probability of reaching retirement on members of the L group. The L group already suffers from a lower chance of survival to retirement, but any raising of the retirement age will widen this gap in survival probabilities.

The effects of the pension system are not only captured in the probability of reaching retirement, but also with the relative population size of retirees. The rise in the retirement age entails:

$$\frac{\partial N_w}{\partial R} = \sigma \frac{e^{\mu_1^L R} - \mu_0^L}{1 - \mu_0^L} + (1 - \sigma) \frac{e^{\mu_1^H R} - \mu_0^H}{1 - \mu_0^H} > 0 \tag{24}$$

$$\frac{\partial N_o}{\partial R} = -\frac{\partial N_w}{\partial R} = \sigma \frac{\mu_0^L - e^{\mu_1^L R}}{1 - \mu_0^L} + (1 - \sigma) \frac{\mu_0^H - e^{\mu_1^H R}}{1 - \mu_0^H} < 0 \tag{25}$$

Since we treat the pension contributions, T, as exogenous in the DC case, the pension benefits increase with the retirement age. Regardless of whether the difference in longevity is caused by $\mu_1^L > \mu_1^H$ or $\mu_0^L < \mu_0^H$ we can see that the following holds (assuming that $\sigma = 0.5$):

$$\frac{\partial N_w^L}{\partial R} = \sigma \frac{e^{\mu_1^L R} - \mu_0^L}{1 - \mu_0^L} < (1 - \sigma) \frac{e^{\mu_1^H R} - \mu_0^H}{1 - \mu_0^H} = \frac{\partial N_w^H}{\partial R} \tag{26}$$

$$\frac{\partial N_o^L}{\partial R} = \sigma \frac{\mu_0^L - e^{\mu_1^L R}}{1 - \mu_0^L} > (1 - \sigma) \frac{\mu_0^H - e^{\mu_1^H R}}{1 - \mu_0^H} = \frac{\partial N_w^H}{\partial R} \tag{27}$$

This implies that the change in the size of the working (or retired) population, when the retirement age increases, is greater for the H group. But this is solely due to the fact that the population of the H group is higher at retirement, as because members of the L group are more likely to die before then.

The ratio between working population and retired population of each group will shed further light on this. Let's define the dependency ratio, Γ^i for $i \in \{H, L\}$ as:

$$\Gamma^i \equiv \frac{N_o^i}{N_W^i} = \frac{\mu_0^i - \ln(\mu_0^i)\,\mu_0^i - e^{\mu_1^i R} + R\mu_1^i \mu_0^i}{e^{\mu_1^i R} - R\mu_1^i \mu_0^i - 1} = \frac{\mu_0^i - \ln(\mu_0^i)\,\mu_0^i - 1}{e^{\mu_1^i R} - R\mu_1^i \mu_0^i - 1} - 1 \tag{28}$$

We see from the denominator of the fraction on the right-hand side that the size of the impact of a retirement age hike is greater when μ_1 is bigger. This implies that the impact on the dependency ratio of the population of the *L* group is higher than the *H* group. Next, we find the effect of an increase in the retirement age:

$$\frac{\partial \Gamma^i}{\partial R} = \frac{\mu_1^i(1 + \ln(\mu_0^i)\,\mu_0^i - \mu_0^i)(e^{\mu_1^i R} - \mu_0^i)}{\left(R\mu_1^i\mu_0^i - e^{\mu_1^i R} + 1\right)^2} < 0 \tag{29}$$

We conclude that when the retirement age, R, increases, the dependency ratio decreases. From the expression above we also see that this impact is greater for the *L* group, which suffers from high values of μ_1. An increase in the retirement age will therefore affect the dependency ratio of both groups, but it will affect the *L* group disproportionately. Agents in the *L* group will be forced to work longer, and the gap between the *H* and *L* group in the probability of reaching retirement will widen. This leads to the negative financial effect associated with the longevity shock becoming even greater. A rise in the retirement age will exacerbate the negative financial effect caused by the disparities in longevity between the groups. The *L* group will end up contributing more than before to the PAYG scheme while getting lower benefits, since they are less likely to survive into retirement.[3] We arrive at the same results when simulating the asymmetric longevity through μ_0.[4]

4.4. Longevity indexed retirement age

In the previous section, we explored an exogenous rise in the retirement age increase. Now we treat the retirement age as endogenous by linking it to the average life expectancy of the whole population. The life expectancy of each agent when entering the labor market, Λ, is:

$$\Lambda^{\mathrm{i}} = \frac{\ln(\mu_0^i)\,\mu_0^i + 1 - \mu_0^i}{\mu_1^i(\mu_0^i - 1)} \tag{30}$$

The retirement age follows;

$$R(\Lambda) = \lambda\left(\sigma\frac{\ln(\mu_0^L)\,\mu_0^L + 1 - \mu_0^L}{\mu_1^L(\mu_0^L - 1)} + (1-\sigma)\frac{\ln(\mu_0^H)\,\mu_0^H + 1 - \mu_0^H}{\mu_1^H(\mu_0^H - 1)}\right) \tag{31}$$

where the average life expectancy of the whole population when entering the labor-market is $\Lambda = \sigma\Lambda^{\mathrm{L}} + (1-\sigma)\Lambda^H$ and λ is the proportional indexation parameter. We generally expect λ to be greater than zero and less than one. For example, a population that has the life

[3] The members of the *L* group that reach retirement also have shorter life expectancies at retirement.
[4] A numerical exercise was used to explore the effects when the asymmetric longevity structure is driven by μ_0.

expectancy of 80 years, that enters the labor market at age 21 and retires at age 65 would have an indexation parameter of: $\lambda = \frac{65-21}{80-21} = 0.75$. As we treat λ as exogenous, any rise in life expectancy would affect the retirement age through equation (31).

Whether longevity increases of the H group are driven by decreases in μ_1^i or increase in μ_0^i they will lead to higher life expectancy. This will in turn affect the retirement age:

$$\frac{\partial \mathrm{R}}{\partial \mu_1^H} = \lambda(1-\sigma)\frac{\partial \Lambda^{\mathrm{H}}}{\partial \mu_1^H} < 0 \qquad \text{and} \qquad \frac{\partial \mathrm{R}}{\partial \mu_0^H} = \lambda(1-\sigma)\frac{\partial \Lambda^{\mathrm{H}}}{\partial \mu_0^H} > 0$$

In the case of a longevity-indexed retirement age we therefore observe that as the H group enjoys greater longevity, they will both affect the L group through the pension scheme, as seen in section 4.1, and through a rise in the retirement age, as seen in section 4.3.

We can summarize the findings so far as follows: A positive longevity shock to one group has a negative financial effect on the other. Moreover, any longevity adjustment of the retirement age based on the average life expectancy of the whole population will exacerbate the effect on the group that didn't enjoy the increased longevity. Both of these effects are then magnified by the relative sizes of the H and L groups. As the L group gets smaller, the members are even worse off. Conversely, as the H group gets smaller, its remaining members benefit.

5. Income inequality and pension transfers

So far, the re-distributional effects of pension schemes that operate based on average life expectancy and old-age dependency ratios of a heterogeneous population have been analyzed. For simplicity, we assumed that all agents, regardless of longevity, paid the same contributions and received the same benefits from the pension scheme, provided they were alive. By introducing contribution and benefits proportional to wages, we can further deepen the analysis of intra-generational transfers due to pension systems. This allows us to demonstrate intra-generational transfers imposed by the pension system that are driven by, and exacerbate, income inequality, hence adding to the results of section 4.

5.1. Wage indexed pension contributions

We replace the contribution in the Beveridge case with proportional taxation as introduced by Bismarck in late 19th century Germany. Each agent pays into the PAYG scheme while young, according to τy^i and receives πy^i after retirement. Here $0 < \pi < 1$ is the replacement rate of the pension benefits and $0 < \tau < 1$ is the proportional wage tax used to finance the benefits to the retired. We assume that the L group has the same wage replacement rate as the

H group (π and τ are uniform between groups). However, we allow for distinct income levels across groups. The PAYG scheme is balanced each time (aggregate inflows equal aggregate outflows).

$$\pi(y^H N_o^H + y^L N_o^L) = \tau(y^H N_w^H + y^L N_w^L) \tag{32}$$

On the left hand, we have the outflows from the PAYG scheme. This is composed of members the H group, N_o^H, receiving the pension of πy^H and members of the L group, N_o^L, receiving the pension of πy^L. On the right-hand side, we have flows into the PAYG scheme; both groups pay the same proportion of their wages to the scheme. Total inflows from the H and L groups are $\tau y^H N_w^H$ and $\tau y^L N_w^L$, respectively.

We arrive at a new equation for the relationship between pension benefits and pension contributions:

$$\pi = \tau \frac{y^H N_w^H + y^L N_w^L}{y^H N_o^H + y^L N_o^L} \tag{33}$$

In the case of perfect income equality, $y^H = y^L$, equation (30) reduces to the same PAYG equality as in equation (6). In the case of income inequality, $y^H \neq y^L$, the utility maximization follows near-identical steps as in section 3.4, yielding:

$$c^i = y^i \left((1-\tau) \frac{\int_0^R e^{-ar}\, m^i(a)da}{\int_0^{A^i} e^{-ar}\, m^i(a)da} + \tau \frac{y^H N_w^H + y^L N_w^L}{y^H N_o^H + y^L N_o^L} \frac{\int_R^{A^i} e^{-ar}\, m^i(a)da}{\int_0^{A^i} e^{-ar}\, m^i(a)da} \right) \tag{34}$$

In addition to the effects observed in sections 4.1-4.4, we now see from equation (33) and (34) that the income of one group has an effect on the pension contributions paid out to the other, which ultimately affects consumption.

5.2. Wealth shock to high longevity group

To study this in further detail, let's look first at the PAYG scheme under defined contributions (DC). In this case τ is exogenous and π endogenous. The effect of changes in the demographic or income distribution on the PAYG scheme is therefore captured through changes in π. When the income of the group that enjoys greater longevity increases, we see that:

$$\frac{\partial \pi}{\partial y^H} = \tau \frac{y^L(N_w^H N_o^L - N_w^L N_o^H)}{(y^H N_o^H + y^L N_o^L)^2} < 0 \tag{35}$$

Notice that if there was no difference in the mortality profiles of the two groups ($N_w^H N_o^L - N_w^L N_o^H = 0$) an increase in the income of one group would not affect the pension benefits/contributions of the other. However, since we impose different mortality profiles, any change in the income of one group imposes externalities on the other through the PAYG scheme.

The old age dependency ratio is higher for the H group because they enjoy higher longevity, implying that the sign of equation (35) is negative. We see that when the income of the H group increases, the pension replacement rate π decreases, causing the pension benefits paid to the L group to decrease – even though they did not enjoy a rise in their income. The H group pays more into the PAYG scheme while young but also receives more benefits during retirement. This would not have any effect on the replacement rate if the dependency ratio was identical for both groups – the increased benefits would be exactly financed by the increase in contributions. But because the H group enjoys higher longevity than the L group, they collect benefits disproportional to their contributions. When their income goes up they are entitled to more benefits, πy^H, during retirement than can be financed by their current levels of the contributions, τy^H. In order for the PAYG scheme to remain balanced, the income replacement rate, π, drops for both the H and L group, thereby causing members of the L group to get lower pension benefits than before the H group's income rose. This effect adds to the effect tied to the demographic structure as described in section 4.

5.3. Wealth shock to low longevity group

Now let's look at the effects of an increase in the income of the L group. The derivative is similar to the one we saw above:

$$\frac{\partial \pi}{\partial y^L} = \tau \frac{y^H (N_w^L N_o^H - N_w^H N_o^L)}{(y^H N_o^H + y^L N_o^L)^2} > 0 \tag{36}$$

Just as above, we see that the sign of the derivative is driven by the relative sizes of the retired and working cohorts of each group. However, in this case we see that an increase in the wages of the L group actually increases π.

When the L group's income rises, they pay more into the PAYG scheme, just as in the case where the H group's income rose. But the L group pays proportionally more than they receive out of the PAYG scheme. These new funds entering the PAYG scheme allow for an increase in the benefits paid out to the retired, causing π to increase. Therefore, the H group pays the same into the PAYG scheme, but receives more as a result of an increase in the income of the L group. The H group is better off when the L group's income rises.

In the defined benefits (DB) case the replacement rate, π, is treated as exogenous and τ as endogenous and is defined as:

$$\tau = \pi \frac{y^H N_o^H + y^L N_o^L}{y^H N_w^H + y^L N_w^L} \tag{37}$$

And the derivatives become:

$$\frac{\partial \tau}{\partial y^H} = \pi \frac{y^L (N_o^H N_w^L - N_o^L N_w^H)}{(y^H N_w^H + y^L N_w^L)^2} > 0 \tag{38}$$

$$\frac{\partial \tau}{\partial y^L} = \pi \frac{y^H (N_o^L N_w^H - N_o^H N_w^L)}{(y^H N_w^H + y^L N_w^L)^2} < 0 \tag{39}$$

We see similar effects here as in the DC case, as the income of the H group (L group) goes up, the L group (H group) is worse off (better off) through negative (positive) externalities of the PAYG system.

To summarize, both in the DC or the DB case, when the income of those that have greater longevity goes up, those that have shorter life expectancy are made even worse off. Conversely, when the income of those who have shorter life expectancy goes up they end up paying more into the PAYG scheme, which benefits those that live longer.

6. Calibration of the mortality profile

To determine the size of the effect on lifetime consumption we turn to a calibration of the mortality profile. The chief parameters are essentially two, μ_0^i and μ_1^i, which determine the maximum age and the chance of death at each time. We apply parameter values to μ_1^i and μ_0^i, which can be done in a convincing way by matching empirical survival functions.

Two calibrations of the mortality profile are needed when comparing the effects on each group. From the mortality profile of each group we can calculate the age structure of the total population, this provides valuable insights into the externalities the groups inflict on each other.

6.1. The survival functions of men and women

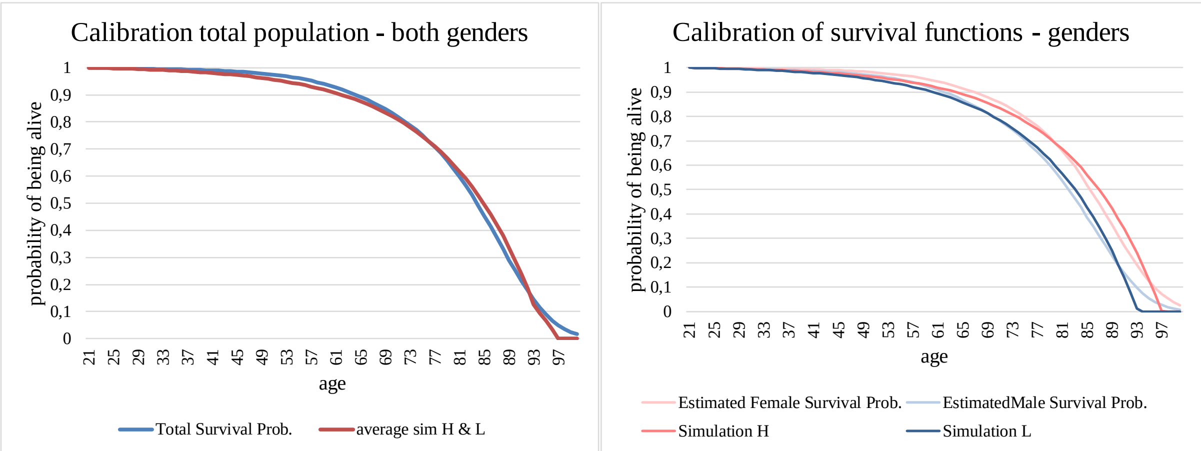
The calibration seeks to minimize the error in average survival probability for all ages across the population while still maintaining each group's observed life expectancy. We assume agents of both groups enter the labor market at the physical age of 21 years. The baseline calibration is based on the life expectancy of newborns in Denmark in 2016, the H and L groups are calibrated to match females and males respectively.

Table 4. Calibrated parameters

	Women (H)	Men (L)
μ_1	0,068	0,068
μ_0	176	135
Simulated life expectancy	83 years	79 years
Observed life expectancy	83 years	79 years

The graph on right in Figure 4 depicts the survival functions of each gender while the graph on the left depicts the survival function of the total population for both genders.

Figure 4. Survival functions



Note: Total Survival Probability and Estimated Female/Male Survival Probability
Source: Human Mortality Database, 2018

Now we can see to which extent paying into a joint pension scheme will have a negative (positive) effect on the males (females) compared to the case where there are distinct pension schemes for each gender. We assume that retirement age is at 66 years, both genders earn the same wages and they pay 10% of their wages to the PAYG pension scheme. We find that males suffer from a 10% drop in pension benefits from paying into the same scheme as females, while females enjoy an increase in the pension benefits by approximately the same amount. Note that here we are in the fixed contribution setting and assume a uniform wage structure between the genders. In line with our findings, these effects increase as retirement age rises.

7. Concluding remarks

We have found that groups who have lower life expectancy suffer a drop in lifetime income when forced to share a pension scheme with others who have longer life expectancies. In effect, the lower life-expectancy group pays as much into the PAYG scheme, provided they reach retirement, but receives less during retirement due to lower life expectancy.

When the retirement age is increased to reflect the increased longevity of the higher life-expectancy group the lower life-expectancy group suffers because even fewer of them will reach retirement age. Also, when contributions during working life are made proportional to wage income, as is typically the case for PAYG schemes, it follows that an increase in the income of the group that enjoys greater longevity will reduce the pension benefits and lifetime utility of the group with less longevity.

These findings may have important implications for pension policy in economies where PAYG is the dominant scheme. Indeed, the spread between life expectancies between, say, academics and, say, blue-collar workers, could reach a certain critical level, that it would hardly be controversial to allow for differentiated pension ages. In practice, schemes designed to link the retirement age to changes in longevity, and which operate on average figures, should be extended to allow for variability in physical and mental disabilities across different groups in society.

One model for implementing this could be to grant a right to receive old-age pension benefits before the official pension age, subject to means-testing. Alternatively, one could allow the elderly to start receiving public pension benefits some years before the official pension age without means-testing. However, at the individual level, the public pension should then be based on an actuarial principle, so that the total pension benefits received during the entire life as a retiree is not affected by the time chosen to retire.

There is also the possibility that the disutility of work may differ between groups, so that the group with lower life expectancy also has a greater disutility of work, and that this disutility may increase with age.[5] The shorter life expectancy and the rising disutility of work may both stem from the depreciation in health and human capital that workers experience due to more difficult tasks and working conditions. In this case, the short-lived group is adversely affected by a common retirement age, due both to the redistribution of retirement income shown in this

[5] See Böckerman, Petri and Pekka Ilmakunnas (2019).

paper to those who, on average, live longer and because their lifetime utility is adversely affected by raising the retirement age due to the higher disutility of work. This latter effect remains a topic of future research.

To sum up, introducing a longevity-indexed retirement age is critical for keeping fiscal policy on a sustainable track in economies that are subject to population ageing. That is why it is important to maintain a broad support of such schemes, both in the population and across a broad political majority. If the legitimacy and credibility of introducing such key welfare reforms critically depends on easier access to earlier retirement age for citizens with lower life expectancy and diminished (physical and mental) ability to work, then such adjustments to the reforms might well be worth advocating.

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Appendix

Figure A1