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Robust Information Aggregation Through Voting

Abstract

Numerous theoretical studies have shown that information aggregation through voting is fragile. We consider a model of information aggregation with vote-contingent payoffs and generically characterize voting behavior in large committees. We use this characterization to identify the set of vote-contingent payoffs that lead to a unique outcome that robustly aggregates information. Generally, it is not sufficient to simply reward agents for matching their vote to the true state of the world. Instead, robust and unique information aggregation can be achieved with vote-contingent payoffs whose size varies depending on which option the committee chooses, and whether the committee decision is correct.

JEL-Codes: D710, D720.

Keywords: information aggregation, voting, vote-contingent payoffs.

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1 Introduction

Many important social decisions are made collectively by a majority vote. Ideally, collective decisions improve on individual decision-making by aggregating the private information of all committee members. However, it is not clear that decision-making through voting provides the proper incentives for individuals to vote in a manner that results in collective decisions that are more informed than individual decisions. Accordingly, a classic question in political economy considers whether a decision taken by majority rule selects the option that is optimal, given the aggregate information contained in the private signals of the voting body. This question was first addressed by Condorcet (1785), who showed that if all individuals hold private information that is more likely to be "right" than "wrong", and if all individuals vote according to their private information, then a sufficiently large committee that votes via a majority rule will choose the "right" option with arbitrary precision.

As subsequent research has shown, however, information aggregation through voting is fragile in the classic Condorcet setting—since the probability that any agent's vote influences the committee's decision becomes arbitrarily small in a large committee, voting behavior is very sensitive to information and payoff structures.[1] Therefore, even very small perturbations of agents' payoffs can distort information aggregation; for example, Dal Bo (2007) shows that if agents receive arbitrarily small payoffs that condition on their *individual* vote, then any committee decision can be supported as an equilibrium independent of the private information of the committee members.

This implies that information aggregation can be distorted by vote-contingent payoffs, examples of which include vote buying (Dal Bo (2007)), expressive motives (Feddersen et al. (2009), Morgan and Vardy (2012)), a desire to vote for the winner (Callander (2007, 2008)), and reputation payoffs (Visser and Swank (2007), Levy (2007), and Midjord et al. (2017)). Another salient example of vote-contingent payoffs occurs in committees composed of elected representatives: Many pundits identified Hillary Clinton's vote supporting military intervention in Iraq, which was predicated on the incorrect assumption that Iraq possessed sizable stores of weapons of mass destruction, as a key factor in her 2008 primary loss to Barak Obama, who voted against the war. This anecdote illustrates that the ex-post correctness of a representative's vote can have an instrumental effect by impacting the probability of re-election, or election to higher office.

Given the prevalence of vote-contingent payoffs in real-world settings and the sensitivity of voting behavior to these payoffs, an important question is whether large committees can aggregate information *robustly*. That is, whether information aggregation through voting can survive small perturbations of the incentives of committee members. In this paper, we study

[1] Information aggregation in large committees has been shown to fail due to, among others, the decision rule (Feddersen and Pesendorfer, 1997), vote-contingent payoffs (Dal Bo, 2007, and Morgan and Vardy, 2011), uncertainty regarding the signal structure (Mandler, 2012), a failure of preference monotonicity (Bhattacharya, 2013), and uncertainty regarding the size of the population (Ekmekci and Lauermann, 2018).

a general model of voting behavior with vote-contingent payoffs and characterize the incentive structures that lead to a unique and robust equilibrium prediction that information will be aggregated in large committees.

Generally, we find that the only payoff structures that admit equilibria that aggregate information provide agents with (relative) rewards for matching their individual vote to the state of the world. However, it is not generally sufficient to simply reward agents for matching their vote to the true state of the world. As we will show in detail below, vote-contingent payoffs must be balanced relative to agents' prior regarding the state of the world. If one state is more likely ex ante, then robust information aggregation can be achieved by rewarding voters more for voting correctly in cases when the committee chooses the less likely option—that is, vote-contingent payoffs that depend on both the correctness of the individual vote, *and* which option is chosen by the committee.

For a simple example, assume agents get a payoff of 1 for matching their vote to the state, which is either equal to α or β. Moreover, assume agents receive a private signal of a or b, and that the probability that they receive a signal of a (b) given a state of α (β) is equal to $2/3$. In this simple example, information will be aggregated if agents have an incentive to vote their signal (sincere voting). Whether the vote-contingent payoff of 1 for matching their vote to the state incentivizes sincere voting depends on the underlying prior, $\Pr(\alpha)$: If the prior is uninformative ($\Pr(\alpha) = 1/2$), then agents maximize their expected vote-contingent payoffs by voting sincerely. If the prior is sufficiently biased towards α (say, $\Pr(\alpha) = 3/4$), then agents maximize their expected vote-contingent payoffs by disregarding their individual signal and voting according to the prior regardless of whether their signal is a or b. That is, since the informativeness of the private signal is not sufficient to out-weight the prior (applying Bayes rule shows that $\Pr(\alpha|s_i = b) = 3/5$), all agents maximize their expected vote-contingent payoffs by voting according to the prior rather than according to their signal.

In the latter case with $\Pr(\alpha) = 3/4$, given a large enough committee it will be a unique equilibrium for all agents to vote according to the prior, since as the size of the committee goes to infinity, expected payoffs that accrue based on the probability of influencing the committee decision approach zero, and voting behavior is driven by the vote-contingent payoffs. This implies that despite receiving a reward for matching their vote to the state, these vote-contingent payoffs are not sufficient to generally result in equilibria that aggregate information.

Instead, the existence of a sequence of equilibria that robustly aggregate information as the size of the committee approaches infinity can be achieved when the relative size of vote-contingent payoffs condition on which option is chosen by the committee: As we establish formally below, information aggregation is achieved when the reward for individually voting correctly when the committee selects the ex ante less likely option is large, but not too large, relative to the reward for voting correctly when the committee selects the ex ante more likely option.

While conditioning vote-contingent payoffs on which option the committee chooses is sufficient to ensure robust information aggregation, it is not sufficient to ensure that equilibria that aggregate information are *unique*. Interestingly, the existence of an equilibrium that aggregates information depends *only* on the relative payoffs agents receive for voting for the correct option, given that the committee also selects the correct option. The intuition for this result stems from the fact that if the committee decision aggregates information, then as the number of committee members approaches infinity, the probability that the committee selects the incorrect option approaches zero. Therefore, the impact of vote-contingent payoffs that realize when the committee selects the incorrect option fade out as the size of the committee approaches infinity.

However, the uniqueness of equilibria that aggregates information depends on the vote-contingent payoffs that realize when the committee decision does not match the state. If there are no vote-contingent payoffs when the committee selects the incorrect option, then a non-responsive equilibrium exists where agents all vote for the same option, no agent is pivotal, and all agents have a strict incentive to vote for the committee decision since they receive (positive) vote-contingent payoffs only when the committee decision is ex-post correct. Therefore, for an equilibrium that aggregates information to be unique, agents must incur a relatively large punishment for voting for the incorrect option in instances when the committee selects the incorrect option. This shows the general existence of a unique limit outcome that robustly aggregates information can be achieved if vote-contingent payoffs condition on which option the committee chooses, which ensures robust information aggregation—i.e. the rewards for voting correctly when the committee chooses correctly are balanced relative to the prior—and whether the committee decision is correct—i.e. the punishment for voting incorrectly when the committee decision is incorrect is large enough.

Our primary contribution is to the literature on information aggregation in committees. In contrast to other papers, which have largely explored specific instances in which vote-contingent payoffs cause information aggregation to fail in large committees, Dal Bo (2007), Feddersen et al. (2009), Morgan and Vardy (2012), Callander (2007, 2008), we take a general approach and provide a simple method for characterizing the set of all equilibria for large committees with vote-contingent payoffs that applies generically.[2] We then use this characterization result to identify the set of vote-contingent payoffs that lead to unique equilibria with robust information aggregation.[3] Accordingly, our model nests the settings that have been explored by previous papers, and fully specifies in which instances vote-contingent payoffs distort outcomes.

In order to carry out our characterization we define a *limit outcome* as a pair of conditional (committee) decision probabilities—conditional on each of the two possible states of the world—

[2]Our characterization result is generic, rather than general, in the sense that there are certain border cases to which it does not apply. However, we show that the set of vote-contingent payoffs to which the characterization result does not apply has a measure of zero, and that these border cases do not impact our main results.

[3]In a related paper, Breitmoser and Valasek (2017) show that information aggregation is robust to vote-contingent payoffs if committee members have access to cheap talk prior to voting *and* decisions are taken via a unanimity rule. This paper takes a different approach, and characterizes the set of vote-contingent payoffs that result in information aggregation in large committees, where pre-vote cheap talk may not be feasible.

with the property that if agents in a hypothetical committee with infinitely many members anticipate that these are in fact the conditional decision probabilities of the committee, then they best respond (either in pure or mixed strategies) in a way that can in fact, in the aggregate, give rise to these decision probabilities. Because in such a hypothetical committee individual members have no incidence over the decision probabilities, and their vote can never be pivotal, solving for all *limit outcomes* is a linear programming problem, analytically more tractable than directly characterizing equilibria of games involving finite committees (finite games). To argue that limit outcomes indeed map out the aggregate behavior of large enough committees, we show that: (1) The decision probabilities associated to any sequence of equilibria of finite games must converge to the set of limit outcomes, and (2) generically, given a limit outcome, one can construct a sequence of equilibria of the finite games such that its associated sequence of conditional decision probabilities converges to that limit outcome.

This approach is closely related to the method, used in other areas of game theory (see Bodoh-Creed (2013), Bodoh-Creed et.al. (2016)), of defining and solving an often more tractable game with infinitely many players in order to understand equilibrium behavior in games with a large number of players, first carefully arguing that the equilibria of the artificial game are arbitrarily good approximations of the equilibria of the finite games for a sufficiently large number of players. As we are not interested in a game involving infinitely many players per se, we don't fully specify the game with infinitely many players, but rather, directly specify what the limit outcomes (in terms of decision probabilities) would look like, and then show that these indeed approximate arbitrarily well the outcomes of games with a sufficiently large number of players. To the best of our knowledge our formal argument justifying this kind of approximation to large, finite games, is a contribution to the literature on the Condorcet jury theorem. It is worth noting that this justification can serve as the basis to characterize the outcomes of voting in large committees, under (generically) any incentive structure entailing vote-contingent payoffs.

The paper proceeds as follows. Section 2 introduces the model and discusses corresponding real-world examples. In Section 3.1 we consider our benchmark and the case of a single vote-contingent payoff. Section 3.2 presents our general method of characterizing equilibrium outcomes. Section 3.3 presents our main results and in Section 3.4 we study partial information aggregation.

2 Model

Our model is based on the standard model of information aggregation introduced by Austen-Smith and Banks (1996). There are two states of the world $\omega \in \{\alpha, \beta\}$ where $Pr(\alpha) \in (0, 1)$. A committee of $n > 2$ agents indexed by $i \in \{1, ..., n\}$ makes a decision between two choices, $x \in \{a, b\}$ by majority rule: If strictly more than half of the agents vote for a then $x = a$, and otherwise $x = b$. Each agent receives a private signal, $s_i \in \{a, b\}$, and then votes for either a

($v_i = a$) or b ($v_i = b$). The signals are i.i.d. conditional on ω and $Pr(a|\alpha) = Pr(b|\beta) = 1 - \varepsilon$, where $\varepsilon \in (0, \frac{1}{2})$.

Agents receive both "common-value" and "vote-contingent" payoffs. Common-value payoffs condition only on the committee decision and the state of the world: all agents receive a payoff of one if the committee decision matches the state of the world, and a payoff of zero otherwise. Additionally, we consider the case of payoffs that are linked to each agent's individual vote; i.e. vote-contingent payoffs. We allow these vote-contingent payoffs to condition not only on the agent's vote, but to also interact with the committee decision and the state of the world. That is, vote-contingent payoffs are represented by a function $k(v_i, x, \omega) : \{a, b\} \times \{a, b\} \times \{\alpha, \beta\} \to \mathbb{R}$. As we detail below, this form of vote-contingent payoffs is general enough to capture all our motivating examples, and to allow for payoffs that condition on both which option the committee chooses, and whether the committee decision is correct.

Agents' payoffs are represented in the following expression (abusing notation we denote $x = \omega$ when $(x, \omega) = (a, \alpha)$, or $(x, \omega) = (b, \beta)$):

$$U(v_i, x, \omega) = \mathbb{1}(x = \omega) + k(v_i, x, \omega),$$

where the first term represents common-value payoffs, and $k(v_i, x, \omega)$ represents vote-contingent payoffs.

Note that $k(v_i, x, \omega)$ can be represented by a vector of eight different values (we use the notation $(k(v_i, x, \omega))$ to refer to this vector). Moreover, given the structure of the model and as we later show in detail, it turns out that asymptotically (as $n \to \infty$) we can normalize four of these values to zero, and represent the vote-contingent payoffs as the *relative* payoff for voting for, say, option a given a certain committee decision and state of the world. Therefore, in what follows we simplify our notation for vote-contingent payoffs to $k_{\omega,x}$, where:

$$k_{\omega,x} \equiv k(v_i = \omega, x, \omega) - k(v_i \neq \omega, x, \omega). \tag{1}$$

For convenience, we define $k_{\omega,x}$ as the relative vote-contingent payoff for voting "correctly" (with the true state of the world); e.g. $k_{\alpha,b}$ is the relative vote-contingent payoff of voting for a given a majority decision for b, and a realized state of the world α.

A strategy for agent i is denoted by $\sigma_i = (\sigma_i(a), \sigma_i(b))$ such that $\sigma_i(a)$ is the probability that $v_i = a$ given $s_i = a$ and $\sigma_i(b)$ is the probability that $v_i = a$ given $s_i = b$. Given some n and strategy profile σ we let $Z^n_\alpha = Pr(x = a|\sigma, \alpha)$ indicate the probability that the committee chooses a when the state is α and $Z^n_\beta = Pr(x = a|\sigma, \beta)$ be the probability that the committee chooses a when the state is β. The pair (Z^n_α, Z^n_β) is denoted by Z^n.

Throughout the analysis we rely on the concept of symmetric bayesian Nash equilibrium:

Definition **1** (Symmetric Equilibrium). *A pair σ^* is a symmetric equilibrium if, and only if, for all $i \in \{1, 2, ..., n\}$, $s_i \in \{a, b\}$, and σ_i: $E_\sigma[U(\sigma^*, x, \omega)|s_i] \geq E_\sigma[U(\sigma^*_{-i}, \sigma_i, x, \omega)|s_i]$.*

Next, we define our concept of robust information aggregation. Intuitively we say that information aggregation is a robust prediction at a given incentive structure if arbitrarily accurate decision-making can be achieved in equilibrium by a large enough committees under that incentive structure, and at all small enough perturbations of that incentive structure. That is, information aggregation is robust if a sequence of equilibria exists where the committee decision approaches the state of the world as $n \to \infty$ for vote-contingent payoffs $k(v_i, x, \omega)$ *and* for small perturbations in $k(v_i, x, \omega)$:

Definition **2** (Robust information aggregation). *Information aggregation is robust for a given vector of parameters* $(k(v_i, x, \omega))$ *if there exists a neighborhood of* $(k(v_i, x, \omega))$ *such that for all vote-contingent payoffs within the neighborhood, there exists a sequence of equilibria such that* $Z^n \to (1, 0)$ *as* $n \to \infty$.

In addition to robust information aggregation, we are also interested in uniqueness. Models of information aggregation in committees often admit multiple equilibria, resulting in imprecise theoretical predictions regarding committee behavior. Therefore, we are also interested in characterizing the set of parameters for which the prediction of information aggregation is unique:

Definition **3** (Unique information aggregation). *Information aggregation is unique for a given vector of parameters* $(k(v_i, x, \omega))$ *if for all sequences of equilibria with* $n \to \infty$, $Z^n \to (1, 0)$.

Next we introduce a main object of interest in our analysis, namely the relative expected utility that agent i receives from voting for a given i's signal and the strategy of all other agents. Formally, we denote this value by $\Phi^n_{s_i}$:

$$\Phi^n_{s_i}(\sigma_{-i}) \equiv E_{\sigma_{-i}}[U(v_i = a, x, \omega)|s_i] - E_{\sigma_{-i}}[U(v_i = b, x, \omega)|s_i].$$

In the following expression, we present a simplified equation for $\Phi^n_{s_i}$—letting piv_i indicate the event that agent i is pivotal for the final decision we get:

$$\begin{aligned}\Phi^n_{s_i}(\sigma_{-i}) =& Pr(\alpha|s_i)\big[(1 + k(a, a, \alpha) - k(a, b, \alpha))Pr(piv_i|\alpha) + (k_{\alpha,a} - k_{\alpha,b})Pr(a, \neg piv_i|\alpha) + k_{\alpha,b}\big]\\ &-Pr(\beta|s_i)\big[(1 + k(a, b, \beta) - k(a, a, \beta))Pr(piv_i|\beta) + (k_{\beta,a} - k_{\beta,b})Pr(a, \neg piv_i|\beta) + k_{\beta,b}\big]\end{aligned}$$

The term $\Phi^n_{s_i}(\sigma_{-i})$ will feature heavily in our analysis below. However, before we begin with the analysis, we discuss several real-world examples that are captured by the model we introduce above.

2.1 Economic interpretation of vote-contingent payoffs

In the following, we highlight several examples of settings where voters may be subject to vote-contingent payoffs, and illustrate how they are captured by our model.

Getting the individual vote right: Agents may receive a positive relative vote-contingent payoff for matching their vote to the true state of the world. This may be due to external reputation concerns or reelection payoffs (see Levy (2007), Visser and Swank (2007)), an intrinsic utility from getting the vote right, or avoiding voting for the incorrect option. Increasing utility from getting the individual vote right (or disutility from getting it wrong) corresponds to increases in $k_{\omega,x}$.

The payoff for a correct individual vote is not necessarily symmetric. The committee's decision may be more salient in one state of the world—for example, it has been suggested that regret for voting to convict an innocent defendant is higher than voting to let a guilty defendant free, Kaplan (1968), and in certain cases, the state of the world, say the quality of a pharmaceutical drug, is unlikely to be revealed if the committee votes not to approve the drug, Midjord et al. (2017).

Expressive voting/vote buying: In certain cases, agents may receive payoffs from voting for a particular option, either due a direct monetary incentive (i.e. vote-buying; see Dal Bo, 2007), or due to expressive motives (see Feddersen et al. (2009), Morgan and Vardy (2012), and Spenkuch et al. (2017)). Given our notation, an expressive motive linked to voting for a increases $k_{\alpha,a}$ and $k_{\alpha,b}$ and lowers $k_{\beta,a}$ and $k_{\beta,b}$.

Conformity/voting for the winner: Similar to an expressive payoff, in certain situations agents may receive a payoff for voting along with the majority, either due to an incentive to conform, or a desire to vote for the "winner" (see Callander (2007, 2008)). Incentives to conform translates into increases in $k_{\alpha,a}$ and $k_{\beta,b}$ and decreases in $k_{\alpha,b}$ and $k_{\beta,a}$.

Anti-conformity/blame-guilt aversion: Conversely, agents may receive a payoff for voting against the majority. This may be due to anti-conformity motives (see references to social psychology in Callander (2008)) or because agents seek to avoid personal responsibility for an incorrect committee decision (Midjord et al. 2017). This corresponds to increases in $k_{\alpha,b}$ and $k_{\beta,a}$ and decreases in $k_{\alpha,a}$ and $k_{\beta,b}$.

3 Analysis

We begin this section with our benchmark and an example illustrating the non-robustness of information aggregation in the setup of the standard Condorcet model. In Section 3.1 we present a novel methodological development and formally establish a general method for characterizing equilibria of games of information aggregation in large committees with vote-contingent payoffs. In section 3.2, we present our main results on robust information aggregation and explain the necessary and sufficient conditions for robust information aggregation and unique robust information aggregation. Section 3.3 applies the method we develop in section 3.1 to generically characterize information aggregation in the case when voters are rewarded for voting according to the true state of the world or, equivalently, punished for voting incorrectly (i.e. $k_{\omega,x} > 0$ for all (x, ω)). The objective of this section is to offer an idea of the kinds of outcomes that can be obtained when perfect information aggregation fails.

Benchmark with no vote-contingent payoffs, $k(v_i, x, \omega) = 0$ for all (v_i, x, ω): First, we consider a benchmark of the classic Condorcet model with no vote-contingent payoffs. In this case, agents only consider the impact of their vote on the committee decision, and hence base their voting decision on the event that their vote is pivotal. Since $k(v_i, x, \omega) = 0$ the game is of common interest with diverse information and optimal equilibria yield asymptotically perfect decisions (McLennan, 1998). This leads us to the following result stemming from McLennan (1998) and Theorem 3 in Feddersen and Pesendorfer (1997).[4]

PROPOSITION **1** (No vote-contingent payoffs). *Given $k(v_i, x, \omega) = 0$ for all (v_i, x, ω) there exists a sequence of equilibria (σ^{n*}) such that $Z^n \to (1, 0)$ as $n \to \infty$.*

The easiest way to explain the intuition behind Proposition 1 is when $Pr(\alpha) = \frac{1}{2}$ and n is uneven. Suppose all agents vote sincerely and thus when agent i is pivotal there are exactly $\frac{n-1}{2}$ signals for a and $\frac{n-1}{2}$ signals for b among all agents other than i. In this case, it is strictly optimal for agent i to vote sincerely as s_i determines which option is supported by the most signals. In all other cases (where i is not pivotal), the vote from agent i is inconsequential and the sincere strategy is then optimal. Given the sincere strategy profile and the law of large numbers the committee's mistake probability converges to zero as $n \to \infty$.

Single vote-contingent payoff, $k_{\omega,x} = \delta > 0$: Next, we consider the robustness of Proposition 1 by considering a "small" perturbation to the payoffs of the benchmark case. Take $k_{\alpha,a} = \delta > 0$, and all other vote-contingent payoffs equal to zero. This gives us the following expression for $\Phi^n_{s_i}(\sigma_{-i})$:

$$\begin{aligned}\Phi^n_{s_i}(\sigma_{-i}) =& (1+\delta)Pr(piv_i|\alpha)Pr(\alpha|s_i) - Pr(piv_i|\beta)Pr(\beta|s_i) \\ &+ \delta Pr(a, \neg piv_i|\alpha)Pr(\alpha|s_i)\end{aligned}$$

Consider some sequence of equilibria $\sigma^{n*} \neq (1, 1)$ whereby $Pr(a, \neg piv_i|\alpha)$ is bounded away from zero for all $n > n'$. For n large enough this gives us a contradiction as the pivotal probability converges uniformly to zero as $n \to \infty$ and $\delta Pr(a, \neg piv_i|\alpha)Pr(\alpha|s_i)$ is bounded away from zero and thus optimal behavior prescribes $\sigma^n = (1, 1)$. It follows that under this incentive structure, large committees either choose a in both states of the world with probability close to 1 or choose b in both states of the world with probability close to 1.

The intuition is that for a large enough committee, with a shrinking pivotal probability, the common value component is dominated by the vote-contingent payoff: Agents will do anything

[4] Proof can be requested from the authors as supplementary material to Midjord et al. (2017).

to try and get for the extra payoff δ, by voting a, in the event that the committee decides for a and the state is α. This gives the following result, which shows that information aggregation in the Condorcet model is not robust to small vote-contingent payoffs.

PROPOSITION **2** (Non-robustness Condorcet). *For any $k_{\alpha,a} = \delta > 0$ (and all other vote-contingent payoffs being zero) any sequence of equilibria (σ^{n*}) have $Z^n \to (0,0)$ or $Z^n \to (1,1)$ as $n \to \infty$.*

Our Proposition 2 builds on the same type of logic as the introductory examples in Morgan and Vardy (2012) showing how large voting bodies perform no better than a coin flip in selecting the correct outcome when adding (small) expressive payoffs. In Section 3.2 we show how Condorcet's positive result (i.e. the probability that increasingly large committees chooses the better alternative approaches 1) can be reestablished as a robust and unique outcome by considering a full-fledged analysis of vote-contingent payoffs.

3.1 Characterizing equilibria in large committees with vote-contingent payoffs

Before characterizing the set of vote-contingent payoffs that result in robust information aggregation, we first present a novel approach that allows us to generically characterize equilibria of large committees with vote-contingent payoffs. This approach allows us to characterize the set of equilibrium outcomes in a straightforward manner by identifying the probabilities $\Pr(X = a|\alpha)$ and $\Pr(X = a|\beta)$ that satisfy a simple set of conditions on the expression $\Phi_{s_i}(Z)$, which we introduce next. Also, to illustrate the simplicity of applying our approach, we replicate the limit results of Callander (2008) at the end of this section.

Loosely, $\Phi_{s_i}(Z)$ can be thought of as the limiting expression of the relative payoff of voting for option a as $n \to \infty$:

$$\Phi_{s_i}(Z) = Pr(\alpha|s_i)\big[k_{\alpha,a}Z_\alpha + k_{\alpha,b}(1 - Z_\alpha)\big] - Pr(\beta|s_i)\big[k_{\beta,a}Z_\beta + k_{\beta,b}(1 - Z_\beta)\big], \qquad (2)$$

where (Z_α, Z_β) are used to distinguish the "limiting" values of $Z^n_\alpha = \Pr(X = a|\alpha)$ and $Z^n_\beta = \Pr(X = a|\beta)$. More precisely, all probability terms involving piv_i converge uniformly to 0 as $n \to \infty$. Therefore, given a sequence of strategies (σ^n) such that $(Z^n_\alpha, Z^n_\beta) \to (Z_\alpha, Z_\beta)$, the relative willingness to vote for a is asymptotically equivalent to Expression 2.

This structure allows us to define a *limit outcome* as a pair of conditional decision probabilities (Z_α, Z_β) that are consistent with the limiting values of a sequence of strategies σ^n that are best responses by the agents, given the expression for the limiting relative payoff of voting for option a, $\Phi_{s_i}(Z)$:

DEFINITION **4** (Limit Outcome). *Given vote-contingent payoffs $k_{\omega,x}$, a pair $(Z_\alpha, Z_\beta) \in [0,1]^2$ is a limit outcome if, and only if, the following conditions are met:*

$$Z_\alpha = 1 \quad \text{if } \Phi_a(Z_\alpha, Z_\beta) > 0, \qquad Z_\beta = 1 \quad \text{if } \Phi_b(Z_\alpha, Z_\beta) > 0,$$
$$Z_\alpha \in [0,1] \quad \text{if } \Phi_a(Z_\alpha, Z_\beta) = 0, \qquad Z_\beta \in [0,1] \quad \text{if } \Phi_b(Z_\alpha, Z_\beta) = 0,$$
$$Z_\alpha = 0 \quad \text{if } \Phi_a(Z_\alpha, Z_\beta) < 0. \qquad Z_\beta = 0 \quad \text{if } \Phi_b(Z_\alpha, Z_\beta) < 0.$$

To fully understand the idea behind this definition consider the example of a limit outcome with $Z_\alpha = 1$ and $Z_\beta \in [0, 1]$. Presumably, for there to exist a sequence of equilibrium strategies (σ^{n*}) such that $(Z^n_\alpha, Z^n_\beta) \to (Z_\alpha = 1, Z_\beta \in [0, 1])$, it must be the case that, in the limit, agents with a signal of a must have a best response of voting for a, so that $Z_\alpha = 1$, and agents with a signal of b must be indifferent between voting for a and b, so that $Z_\beta \in [0, 1]$. That is, intuitively, for $Z_\alpha = 1$ and $Z_\beta \in [0, 1]$ to be the limiting outcome of a sequence of equilibrium voting strategies, it must be the case that $\Phi_a(Z_\alpha, Z_\beta) > 0$ and $\Phi_b(Z_\alpha, Z_\beta) = 0$ for these values of (Z_α, Z_β).

This intuition suggests that the set of limiting values of (Z^n_α, Z^n_β) that result from limit equilibria is equivalent to the set of limit outcomes. However, this result must be proved formally, and is derived in the following result that establishes that (1) any limit outcome is arbitrarily close to an equilibrium outcome of a large committee and that (2) any equilibrium outcome of a large committee is arbitrarily close to a limit outcome—that is, (Z_α, Z_β) is a limit outcome if, and only if, it corresponds to the limit of (Z^n_α, Z^n_β) for a convergent sequence of finite equilibria.

Theorem **1** (Approximation of outcomes of large committees)**.** *Generically, in the space of all payoff vectors,*[5]

(1) Given any limit outcome (Z_α, Z_β), there exists a sequence of equilibria of the finite games, (σ^{n*}) such that the associated sequences of decision probabilities Z^n_α and Z^n_β converge to Z_α and Z_β.

(2) The sequence of decision probabilities, (Z^n_α, Z^n_β), associated to any sequence of equilibria of the finite games, (σ^{n*}), must converge to the set of limit outcomes.[6]

The proof of Theorem 1, and the proofs of all following formal results can be found in the Appendix. Intuitively, Theorem 1 results from the fact that, as $n \to \infty$, the terms in agent i's best response function that condition on i being pivotal converge uniformly to 0 for any

[5]That is, any payoff vector $(k_{\omega,x})$ to which the theorem does not apply is arbitrarily close to vectors to which it does apply. In particular, given any $\epsilon > 0$, the ϵ ball around $(k_{\omega,x})$ contains payoff vectors to which the theorem does apply.

[6]Convergence of a sequence to the set of limit outcomes means that for any ϵ, there exists large enough n^*, so that for any given $n > n^*$ there is some limit outcome (Z_α, Z_β) within ϵ of (Z^n_α, Z^n_β). Notice that this does not imply convergence of the sequence to a given limit outcome, as crucially, the specific limit outcome to which the n^{th} term in the sequence is close to, might be different from the limit outcome to which the $(n+1)^{th}$ term is close to. Indeed, one can construct sequences of equilibria with associated sequences of decision probabilities that do not converge, for instance by having the even and odd terms in the sequence converge to different limit outcomes.

set of strategies, resulting in an equivalence of the limiting "fixed points" of the best response functions represented by $\Phi^n_{s_i}$ and the fixed points of Φ_{s_i}. However, showing a full equivalence of the set of limit outcomes and the set of limiting outcomes of (Z^n_α, Z^n_β) corresponding to any convergent sequence of equilibria must be shown mechanically on a case-by-case basis, and we therefore constrain this exercise to the Appendix.

While Theorem 1 generically applies to the set of vote-contingent payoffs, there are certain (non-generic) border cases to which it does not apply. Proposition 4 in the appendix fully characterizes the set of these points and shows that it has measure zero.[7] Importantly, however, Lemma 1 below implies that these non-generic cases do not impact our main result of characterizing the set of vote-contingent payoffs that lead to robust information aggregation. That is, the proof of this Lemma relies on the fact that all such non-generic points have the property of being arbitrarily close to points at which Theorem 1 does apply, and which do not support equilibria which aggregate information perfectly. It follows that no such point is a candidate for supporting robust information aggregation.

Example: Voters who like to win (Callander (2008)): Theorem 1 provides a simple method for characterizing all limit outcomes of games of information aggregation with vote-contingent payoffs. We illustrate this method by characterizing limit outcomes when voters like to win, replicating the limit results of Callander (2008) for an uninformative prior.

For this example, we assume that $\Pr(\alpha) = 1/2$ (a tight election in the terminology of Callander (2008)). A payoff for voting for the winning candidate translates into the following vote-contingent payoffs:

$$k_{\alpha,a} = k_{\beta,b} = k,$$
$$k_{\alpha,b} = k_{\beta,a} = -k.$$

Using Expression 2, it is easy to show that $Z = (1,1)$ and $Z = (0,0)$ are limit outcomes for any $k > 0$ since Expression 2 is positive (negative) for $\Phi_{s_i}(1,1)$ ($\Phi_{s_i}(0,0)$). Similarly, $Z = (1,0)$ and $Z = (0,1)$ are limit outcomes as Expression 2 is positive (negative) for $\Phi_a(1,0)$ ($\Phi_b(1,0)$) and Expression 2 is positive (negative) for $\Phi_b(0,1)$ ($\Phi_a(0,1)$).

Moreover, it is also easy to identify mixed limit outcomes, where either Z_α or Z_β are between zero and one. First, taking $Z_\alpha = 1$ and $Z_\beta \in (0,1)$, Theorem 1 shows that this is a limit outcome if and only if $\Phi_a(Z) > 0$ and $\Phi_b(Z) = 0$. This allows us to identify whether a

[7] Specifically, Theorem 1 does not apply to payoff vectors $(k_{\omega,x})$ such that (i) $\Phi_{s_i}(Z_\alpha \in \{0,1\}, Z_\beta \in \{0,1\}) = 0$, (ii) $\Phi_{s_i}(Z_\alpha \in \{0,1\}, Z_\beta \in (0,1)) = 0\ \forall s_i$, and (iii) $\Phi_a(Z_\alpha \in (0,1), Z_\beta \in \{0,1\}) = 0\ \forall s_i$. Note that the set of cases where Theorem 1 does not apply has measure zero.

mixed limit outcome exists by solving for:

$$\Phi_b(1, Z') = \Pr(\alpha|s_i = b)[k] - \Pr(\beta|s_i = b)[-kZ' + k(1 - Z')] = 0$$
$$\rightarrow Z' = \frac{1 - \frac{\varepsilon}{1-\varepsilon}}{2}$$

Which shows that a mixed limit outcome, $Z = (1, Z')$ exists as $Z' \in (0, 1)$ and $\Phi_a(1, Z') > 0$. Also, since the model is symmetric, a mixed limit outcome also exists with $Z_\alpha = 1 - Z'$.[8]

3.2 Robust and unique information aggregation

In the following section, we utilize the characterization result of Theorem 1 to lay out the necessary and sufficient conditions for robust and unique information aggregation. To begin, we present a result that allows us to restrict attention to sincere strategies when identifying the conditions for robust information aggregation.

Lemma **1** (Sincere Voting)**.** *Information aggregation is robust for a given vector of parameters* $(k_{\omega,x})$ *if and only if* $Z = (1, 0)$ *is a limit outcome under* $(k_{\omega,x})$ *with* $\Phi_a > 0$ *and* $\Phi_b < 0$*.*

Lemma 1 shows that the payoff vectors that result in robust information aggregation are equivalent to the payoff vectors that give agents a strict incentive to vote sincerely, defined as $v_i = s_i$, as $n \rightarrow \infty$ given $Z = (1, 0)$. Note that voting sincerely will always aggregate information in the limit, since the proportion of agents that receive signals equal to the true state of the world, and hence vote for the true state of the world, approaches $\Pr(s_i = \omega|\omega) > 1/2$ as $n \rightarrow \infty$.

Also note that, combined with the result of Theorem 1, Lemma 1 implies that if information aggregation is robust for a given vector of parameters $(k_{\omega,x})$, then there exists a sincere strategy equilibrium for all large enough n. This highlights an important distinction between our model and the pure common value setting: As showed by Feddersen and Pessendorfer (1997), in the Condorcet model equilibria that aggregate information often entail a sequence of signal-responsive mixed strategy profiles. In contrast, in our setting, sequences of equilibria that robustly aggregate information achieve sincere voting in finite n.

Given Lemma 1, we are able to characterize the set of vote-contingent payoffs that result in robust information aggregation by identifying the set of payoffs that satisfy $\Phi_a > 0$ and $\Phi_b < 0$:

Theorem **2** (Robust information aggregation)**.** *Information aggregation is robust if, and only if,* $k_{\alpha,a} > 0$*,* $k_{\beta,b} > 0$*, and:*

$$\frac{k_{\alpha,a}}{k_{\beta,b}} \in \left(\frac{Pr(\beta|s_i = a)}{Pr(\alpha|s_i = a)}, \frac{Pr(\beta|s_i = b)}{Pr(\alpha|s_i = b)} \right).$$

[8] In this example there are also mixing outcomes with $Z = (0, Z')$ and $Z = (1 - Z', 1)$ resulting in negative information aggregation. Moreover, we have a double mixing outcome with $Z = (\frac{1}{2}, \frac{1}{2})$.

Theorem 2 states the conditions on vote contingent payoffs that are necessary and sufficient for robust information aggregation. We should emphasize that it is only the relative size of the vote-contingent payoffs that matter—robust information aggregation can be achieved with vote-contingent payoffs that are very small (or very large), as long as they are balanced relative to the prior. In what follows we refer to any payoff vector which satisfies the conditions of Theorem 2 as *RIA* (Robust Information Aggregation) payoffs.

Notice that none of the conditions in Theorem 2 depend on vote-contingent payoffs that realize when the committee is wrong. Intuitively, given $Z^n \to (1,0)$, the probability of the committee making an error and not matching the committee decision to the state becomes vanishingly small as $n \to \infty$. Therefore, no matter the sign or size of vote-contingent payoffs that realize when the committee decision is incorrect, they do not impact voting behavior for committees that are large enough, *given* $Z^n \to (1,0)$. Therefore, as long $k_{\alpha,a}$ and $k_{\beta,b}$ are greater than zero and balanced, then an equilibrium with information aggregation that robustly aggregates information exists for any $k_{\alpha,b}$ and $k_{\beta,a}$.

In addition to being strictly positive, $k_{\alpha,a}$ and $k_{\beta,b}$ must be balanced *relative to the prior* for an equilibrium with robust information aggregation to exist. This also implies that a simple vote-contingent payoff that rewards agents for matching their vote to the state, $k_{\omega,x} = k$ for all ω, x, is not sufficient to generally admit an equilibrium that aggregates information. We illustrate this result in the following example.

Example: Suppose $Pr(\alpha) = 3/4$, and $\varepsilon = 1/3$. First, assume that vote-contingent payoffs only depend on voting in line with the state of the world; e.g. $k_{\omega,x} = k$ for all ω, x. To check to see whether there is a limit outcome with robust information aggregation, we check whether $Z = (1,0)$ satisfies the conditions of Definition 4. First, note that:

$$\Phi_{s_i}(1,0) = Pr(\alpha|s_i)[k] - Pr(\beta|s_i)[k].$$

This shows that $Z_\alpha = 1$ is an admissible limit outcome, since $Pr(\alpha|a) > Pr(\beta|a)$ and $\Phi_a(1,0) > 0$. However, given $Pr(\alpha) = 3/4$, and $\varepsilon = 1/3$, $Pr(\alpha|b) = 3/5 > 2/5 = Pr(\beta|b)$. This implies that $\Phi_b(1,0) > 0$, which means that $Z^\beta = 0$ is not an admissible limit outcome. That is, $k_{\omega,x} = k$ is not a sufficient condition to generally ensure a limit outcome that robustly aggregates information.

Next, assume that vote-contingent payoffs also condition on *which* option the committee chooses, and are higher when $x = b$; e.g. $k_{\alpha,a} = k_{\beta,a} = k$ and $k_{\beta,b} = k_{\alpha,b} = 2k$. In this case, given $Z = (1,0)$:

$$\Phi_{s_i}(1,0) = Pr(\alpha|s_i)[k] - Pr(\beta|s_i)[2k].$$

Therefore, $\Phi_a(1,0) = 4/7{*}k > 0$ and $\Phi_b(1,0) = -1/5{*}k < 0$, which implies that a limit outcome that robustly aggregates information exists. This shows that in cases where the private signal is not strong enough to overturn the prior for one of the two private signals, it is still possible for vote-contingent payoffs to result in RIA, as long as voters receive a higher reward for voting correctly when the committee chooses option b.

This example illustrates that only vote-contingent payoffs that are balanced relative to the prior robustly aggregate information. Therefore, vote-contingent payoffs that reward agents for matching their vote to the state are not sufficient to generally achieve an equilibrium with robust information aggregation. Instead, robust information aggregation can be achieved if agents receive a higher reward for voting correctly when the committee chooses the option that is ex ante less likely.

While the second set of payoffs introduced in the above example are sufficient for the existence of a limit outcome that robustly aggregates information, they do not guarantee a *unique* limit outcome. That is, given $k_{\alpha,a} = k_{\beta,a} = k$ and $k_{\beta,b} = k_{\alpha,b} = 2k$, it is also an equilibrium for agents to vote for a for any signal—if all agents vote a, then the committee never selects $x = b$ and $k_{\beta,b}$ and $k_{\alpha,b}$ never realize. Therefore, it is a best reply for each agent to vote a and receive $k_{\alpha,a}$ with a conditional probability greater than 1/2 for each signal. This example highlights that RIA preferences are not sufficient to ensure that information is aggregated in large committees. Our next result addresses uniqueness and details the conditions under which there is a unique limit outcome with robust information aggregation. First, however, we make a short remark regarding the robustness of Theorem 2 to heterogeneous vote-contingent payoffs.

Remark on heterogeneous vote-contingent payoffs: Theorem 2 shows that given homogeneous vote-contingent payoffs that satisfy RIA, there exists a limit outcome that aggregates information and is robust to small deviations in the homogeneous payoffs. However, the result is actually stronger—for any set of heterogeneous vote-contingent payoffs distributed within a small neighborhood of a set of homogeneous vote-contingent payoffs that satisfy RIA, there exists an equilibrium with sincere voting for a large enough committee. This implies that a sequence of equilibria exist with $(Z^n_\alpha, Z^n_\beta) \to (1, 0)$ as $n \to \infty$.

The intuition for this robustness to small, individual deviations in payoffs is straightforward: given that agent i has individual vote-contingent payoffs close enough to RIA payoffs, i has a strict incentive to vote sincerely for large enough n as long as $(Z^n_\alpha, Z^n_\beta) \to (1, 0)$. Therefore, Theorem 2 shows that RIA payoffs results in a limit outcome that aggregates information and is robust to small payoff deviations, regardless of whether the payoff deviations are homogeneous or heterogeneous.

The following result details the conditions under which there is a unique limit outcome with robust information aggregation.

Theorem 3 (Uniqueness). *Suppose the conditions in Theorem 2 are met. Any sequence of equilibria* (σ^{n*}) *have* $Z^n \to (1, 0)$ *if, and only if:*

$$\frac{k_{\beta,a}}{k_{\alpha,a}} > \frac{Pr(\alpha|s_i = b)}{Pr(\beta|s_i = b)}, \qquad \frac{k_{\alpha,b}}{k_{\beta,b}} > \frac{Pr(\beta|s_i = a)}{Pr(\alpha|s_i = a)}.$$

Theorem 3 shows that to achieve a unique limit outcome that robustly aggregates information, agents must receive vote-contingent payoffs both when the committee decision matches the state (for robust information aggregation) and when the committee does not match the state (for uniqueness). Moreover, in contrast with Theorem 2, which shows that the vote-contingent payoffs that realize when the committee is right must be balanced relative to the prior, Theorem 3 shows that the vote-contingent payoffs that realize when the committee is wrong do not need to be balanced for uniqueness—they only need to be relatively large.

The intuition behind uniqueness can be explained in terms of how the conditions in Theorem 3 destabilize so-called "non-responsive" equilibria, where all agents vote for either option a or option b. For example, assume that payoffs satisfy the conditions of Theorems 2 and 3 and that agents vote for a under both signals (non-responsive voting). In this case, consider the choice of agent i with a signal of b. Since all agents vote for a, the committee decision is equal to a with certainty. Therefore, if agent i votes a, then they receive $k_{\alpha,a}$ with probability $\Pr(\alpha|s_i = b)$, and if agent i votes b, then they receive $k_{\beta,a}$ with probability $\Pr(\beta|s_i = b)$. By the first condition of Theorem 3, voting b given a signal of b yields a strictly higher expected payoff, which implies that voting b is a best reply for agent i if all other agents vote for option a.

This example shows that, given the conditions of Theorem 3, all agents voting for a cannot constitute an equilibrium of the model for large committees. The same logic can be applied to all agents voting for b given the second condition of Theorem 3. Moreover, it can easily be shown that if payoffs satisfy the conditions of Theorems 2 and 3, then by Theorem 1, the only admissible limit outcome of the game is that $Z = (1, 0)$.

Lastly, note that Theorem 3 implies that vote-contingent payoffs that only condition which option the committee chooses are not generally sufficient for there to exist a unique limit outcome that robustly aggregates information. That is, it is not always the case that there exists a set of vote-contingent payoffs such that $k_{\alpha,a} = k_{\beta,a}$ and $k_{\beta,b} = k_{\alpha,b}$ and the conditions of Theorems 2 and 3 are satisfied. Instead, robust and unique information aggregation can be achieved with vote-contingent payoffs that condition on which option the committee chooses and whether that decision is correct. We illustrate this result in the following example:

Example: Continuing with the same example as above, suppose $Pr(\alpha) = 3/4$, and $\varepsilon = 1/3$. Moreover, assume $k_{\alpha,a} = k_{\beta,a} = k$ and $k_{\beta,b} = k_{\beta,a} = 2k$; as we showed above, $k_{\alpha,a}$ and $k_{\beta,b}$ satisfy the conditions of Theorem 2, and a limit outcome exists that robustly aggregates information. However, as we mentioned above, $Z = (1, 1)$ is also a limit outcome given these vote-contingent payoffs since:

$$\Phi_{s_i}(1, 1) = Pr(\alpha|s_i)[k] - Pr(\beta|s_i)[k],$$

which is positive for $s_i = a, b$. Instead, uniqueness can be assured if vote-contingent payoffs condition on which option the committee chooses and whether the committee decision is correct.

For example, take $k_{\alpha,a} = k$, $k_{\beta,a} = 2k$ and $k_{\beta,b} = k$, $k_{\alpha,b} = 2k$. In this case, neither $Z = (1,1)$ or $Z = (0,0)$ are limit outcomes since:

$$\Phi_{s_i}(1,1) = \Phi_{s_i}(0,0) = Pr(\alpha|s_i)[k] - Pr(\beta|s_i)[2k],$$

which as we showed in the first example, is positive for $s_i = a$ and negative for $s_i = b$.

The above example illustrates that the domain of vote-contingent payoffs that admit equilibria that aggregate information consists of the set of payoffs that reward agents for matching their individual vote to the state of the world when the committee also selects the correct option—however, for vote-contingent payoffs to result in a unique equilibrium that robustly aggregates information, these payoffs need to be appropriately calibrated to the underlying information structure, which can be achieved with vote-contingent payoffs that, in addition to the individual vote, also condition on the committee outcome.

3.3 Rewards for Being Right: Characterization of the case $k_{\omega,x} > 0$

Above, we show that to uniquely and robustly aggregate information, vote-contingent payoffs must be in the domain where agents are rewarded for matching their vote to the state of the world, but that these rewards must be balanced relative to the prior, which can be achieved with vote-contingent payoffs that depend on the committee outcome. Here, we characterize responsive limit equilibria that may occur when the conditions for Theorem 2 do not hold. We show that in certain cases, equilibria exist that partially aggregate information in the sense that, e.g., the probability that the committee selects a when the state is α approaches a value strictly between zero and one. In other cases only non-responsive equilibria exist. Lastly, we highlight that perturbations of *RIA* payoffs in some directions lead to smooth departures from perfect information outcomes $(Z_\alpha, Z_\beta) = (1,0)$, whereas perturbations in some other directions lead to abrupt breakdowns of information aggregation altogether.[9]

To simplify the exposition, we focus on the case of $\Pr(\alpha) = 1/2$ and assume that $k_{\alpha,a} \geq k_{\beta,b}$. These assumptions are not completely without loss of generality; however, we provide and prove the analogous general results in the Appendix. First, note that given $\Pr(\alpha) = 1/2$ and $k_{\alpha,a} \geq k_{\beta,b}$, all limit outcomes that partially aggregate information will have $Z_\alpha = 1$ (there is no limit outcome with partial information aggregation and $Z_\alpha \in (0,1)$).

Next, define $P_{\alpha/\beta}$ as follows:

$$P_{\alpha/\beta} = \frac{Pr(\alpha|s_i = b)}{Pr(\beta|s_i = b)}.$$

By applying Definition 4 and the result of Theorem 1, we can characterize the set of vote-

[9] It is important to note that by definition of robust information aggregation, payoffs obtained by sufficiently small perturbations of *RIA* payoffs always support perfect information aggregation. So in this subsection we are discussing small but not arbitrarily small perturbations.

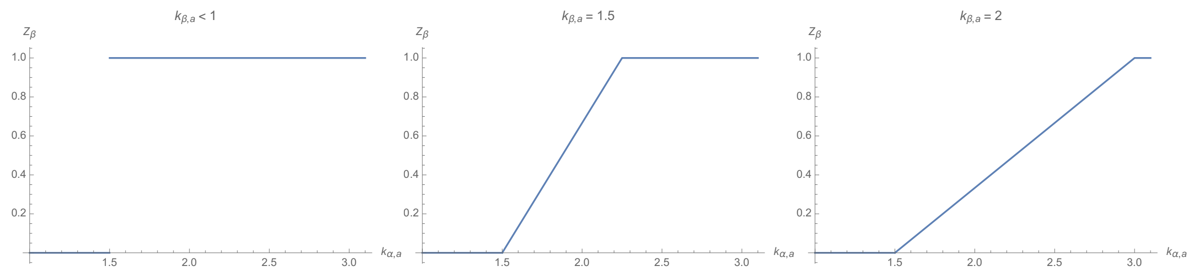

Figure 1: Z_β as a function of $k_{\alpha,a}$ for different levels of $k_{\beta,a}$, given $\Pr(\alpha) = 1/2$, $\varepsilon = 2/5$, $k_{\beta,b} = 1$. Since $Z_\alpha = 1$ for the domain covered in all figures, information is aggregated if $Z_\beta = 0$, and all limit outcome are non-responsive when $Z_\beta = 1$.

contingent payoffs that admit a limit outcome that partially aggregates information:

Proposition **3.** *Partial information aggregation with $Z_\beta \in (0,1)$: There exists a sequence of equilibria (σ^{n*}) such that $Z^n(\sigma^{n*}) \to (1, Z_\beta \in (0,1))$ as $n \to \infty$ if, and only if, $k_{\beta,a} \neq k_{\beta,b}$ and:*

$$\frac{P_{\alpha/\beta} k_{\alpha,a} - k_{\beta,b}}{k_{\beta,a} - k_{\beta,b}} = Z_\beta \in (0,1) \tag{3}$$

Since $\Pr(\alpha) = 1/2$ and $k_{\alpha,a} \geq k_{\beta,b}$, a limit outcome with RIA exists whenever the numerator of Expression 3 is negative. Since we are interested in characterizing the set of parameters for which there is no limit outcome with robust information aggregation, the relevant range of parameters satisfy $P_{\alpha/\beta} k_{\alpha,a} > k_{\beta,b}$

Proposition 3 shows that a necessary condition for a limit outcome with partial information aggregation (outside of the range of RIA preferences) is that the denominator of Expression 3 is positive; i.e. $k_{\beta,a} > k_{\beta,b}$. Proposition 3 also shows that conditional on $k_{\beta,a} > k_{\beta,b}$, limit outcomes with partial information aggregation exist for a range of vote-contingent payoffs with $P_{\alpha/\beta} k_{\alpha,a}$ greater than, but close to, $k_{\beta,b}$.

We illustrate these results in Figure 1, which shows Z_β as a function of $k_{\alpha,a}$, fixing all other vote-contingent payoffs. Note that the size of the range of payoffs for which we get a limit outcome with partial information aggregation depends on $k_{\beta,a} - k_{\beta,b}$: If $k_{\beta,a} - k_{\beta,b}$ is negative, then there is no range of payoffs with partial information aggregation and there exists a discontinuity in Z_β at $P_{\alpha/\beta} k_{\alpha,a} = k_{\beta,b}$. If $k_{\beta,a} - k_{\beta,b}$ is positive, however, then Z_β transitions continuously from 0 to 1 as $k_{\alpha,a}$ increases.

Moreover, we see that the range of $k_{\alpha,a}$ that support partial information aggregation is increasing in $k_{\beta,a} - k_{\beta,b}$; for larger values of $k_{\beta,a} - k_{\beta,b}$, the distortion of information aggregation is weakly smaller for a given value of $P_{\alpha/\beta} k_{\alpha,a} > k_{\beta,b}$. This shows that vote-contingent payoffs that realize when the committee makes the incorrect decision (e.g. $k_{\beta,a}$) are not only important for determining the uniqueness of limit outcomes that robustly aggregate information, they also decrease the distortion of information aggregation for the range of parameters outside of the range that support RIA.

4 Conclusion

In this paper, we highlight the fragility of information aggregation by voting in large populations under vote-contingent payoffs. In a general setting, we develop a method for characterizing all limit outcomes and use this method to identify the set of vote-contingent payoffs that lead to robust information aggregation (*RIA* payoffs, above). The existence of robust information aggregation is established by balanced rewards for individually voting for the correct option when the committee selects the correct option. The balance in vote-contingent rewards is required to offset a favorite-bias whereby voters would opt for the ex ante favorite regardless of their private information. The uniqueness of robust information aggregation is guaranteed by sufficiently large penalties for voting incorrectly when the committee is wrong. Such penalties work effectively to destabilize non-responsive equilibria with no information aggregation.

Our method relies on defining what would be the equilibrium outcomes of a hypothetical committee with infinitely many players, and showing that these outcomes are arbitrarily good approximations of the equilibrium outcomes of sufficiently large finite committees. We also apply our method in order to illustrate the kinds of outcomes that can arise when robust information aggregation is not supported (when payoffs are not *RIA*), by characterizing the equilibrium outcomes of large committees when the net payoffs to agents from voting for the correct alternative are strictly positive.

A Appendix: Proofs of Formal Results

Before presenting the proof for Theorem 1, we introduce some notation that will be helpful for the proof: we let $\mu_{\omega,n} = \sigma^n(a)(1-\varepsilon) + \sigma^n(b)\varepsilon$ be the probability that a randomly chosen agent votes for a given σ^n and ω. In the limit as $n \to \infty$ we have $\mu_\alpha = \sigma(a)(1-\varepsilon) + \sigma(b)\varepsilon$ and $\mu_\beta = \sigma(a)\varepsilon + \sigma(b)(1-\varepsilon)$.

Proof of Theorem 1: We prove part (1) of Theorem 1 by addressing three different kinds of *limit outcomes* separately. Lemma 2 addresses limit outcomes (Z_α, Z_β) at which both inequalities hold strictly, $\Phi_a \lesseqgtr 0$ and $\Phi_b \lesseqgtr 0$. Lemma 3 addresses limit outcomes at which exactly one of the inequalities holds with equality, at least one of Z_α and Z_β is interior (belongs to $(0,1)$) and its slope in the inequality that holds with equality is non-zero. Lemma 4 addresses the case in which both inequalities hold with equality and both Z_α and Z_β are interior. Proposition 4, presented immediately after this proof, shows that the set of points to which none of Lemmas 2, 3 and 4 apply has measure 0, thus establishing that the theorem holds generically, as pointed out by its statement.

Part (2) of Theorem (1) is proved in Lemma 5.

Lemma 2 If (Z_α, Z_β) is a limit outcome at which the two inequalities (associated to Φ_a

and Φ_b) hold strictly, then there exists a sequence of equilibria of the finite games (σ^{n*}) such that the associated sequences of decision probabilities (Z^n_α) and (Z^n_β) converge to Z_α and Z_β.

Proof of Lemma 2: If both inequalities hold strictly, then let $\sigma^{n*}(a) = 1$ if $\Phi_a(1,0) > 0$, $\sigma^{n*}(a) = 0$ if $\Phi_a(1,0) < 0$, $\sigma^{n*}(b) = 1$ if $\Phi_b(1,0) > 0$, $\sigma^{n*}(b) = 0$ if $\Phi_b(1,0) < 0$.

Due to convergence of $\Phi^n_{s_i}$ to Φ_{s_i} for all sufficiently large n, the inequalities associated to each of Φ^n_a and Φ^n_b will hold strictly when evaluated at $(\sigma^{n*}(a), \sigma^{n*}(b))$. This is the case, because given that $\varepsilon < 1/2$, (Z^n_α) and (Z^n_β) will converge to Z_α and Z_β as given in Definition 4 (of limit outcomes). Thus, for all sufficiently large n, σ^{n*} will be an equilibrium and (Z^n_α) and (Z^n_β) will converge to Z_α and Z_β as required.■

Lemma 3. If (Z_α, Z_β) is a limit outcome at which only one of two inequalities holds with equality (call it Φ_s), at least one of $Z_\alpha \in (0,1)$ or $Z_\beta \in (0,1)$ holds (call it Z_ω), and the slope of Z_ω in Φ_s is non-zero, then there exists a sequence of equilibria of the finite games (σ^n) such that the associated sequences of decision probabilities (Z^n_α) and (Z^n_β) converge to Z_α and Z_β.

Proof of Lemma 3: Assume that the inequality associated to Φ_s holds with equality and the one associated to $\Phi_{s'}$ holds strictly and let $Z_\omega \in (0,1)$. We denote the state of the world different from ω by ω'. Then for all sufficiently small δ the inequality associated to $\Phi_{s'}$ evaluated at $(Z_\omega + \delta, Z_{\omega'})$, continues to hold strictly (and has the same direction as when evaluated at $(Z_\omega, Z_{\omega'})$), and $\Phi_s > 0$; and evaluated at $(Z_\omega - \delta, Z_{\omega'})$, the inequality associated to $\Phi_{s'}$ continues to hold strictly (and has the same direction as when evaluated at $(Z_\omega, Z_{\omega'})$), and $\Phi_s < 0$. This is because of the fact that Φ_a and Φ_b are linear functions of Z_α and Z_β and the slope of Z_ω in Φ_s is non-zero.[10]

Now, for all sufficiently large n it must be the case if we fix $Z^n = (Z_\omega - \delta, Z_{\omega'})$, or $Z^n = (Z_\omega + \delta, Z_{\omega'})$ then the inequalities above hold for Φ^n_a and Φ^n_b independently of the strategy σ_n that we use to evaluate the additional terms in Φ^n_a and Φ^n_b associated to the probabilities of the event piv_i. This is the case because of the uniform convergence to 0 of these probabilities. We split the rest of the proof into cases: (case I) $\Phi_a > 0$, $\Phi_b = 0$, (case II) $\Phi_a = 0$, $\Phi_b < 0$, (case III) $\Phi_a < 0$, $\Phi_b = 0$ and (case IV) $\Phi_a = 0$, $\Phi_b > 0$.

(case I) ($\Phi_a > 0$, $\Phi_b = 0$)
Then $\sigma(a) = 1$ and $Z_\alpha = 1$ and therefore $Z_\beta \in (0,1)$. Notice that regardless of what $\sigma_n(b)$ is, we can have Z^n_α approximate 1 as well as we want by choosing n sufficiently large. Furthermore the quality of the approximation is increasing in $\sigma^n(b)$.[11] Evaluated at $(\sigma_n(a), \sigma_n(b)) = (1,0)$, $\mu_\beta < 1/2$ and at $(\sigma_n(a), \sigma_n(b)) = (1,1)$, $\mu_\beta > 1/2$ so for all sufficiently large n, Z^n_β is smaller than $Z_\beta + \delta$ when evaluated at $(1,0)$ and larger than $Z_\beta + \delta$ when evaluated at $(1,1)$. By continuity we can find $\bar{\sigma}_n(b)$ such that $Z^n_\beta = Z_\beta + \delta$. It follows that for all large enough n,

[10] Note that the δ may need to be negative; when we say "for all small enough δ" we mean in absolute value.
[11] So we can establish the required threshold for n by considering $\sigma^n(b) = 0$.

$\Phi_a^n > 0$ and $\Phi_b^n > 0$ when evaluated at $(1, \bar{\sigma}_n(b))$. The key is that for any ϵ we can find an N such that for all $n > N$ we get $Z_\alpha^n > 1 - \epsilon$ and $Z_\beta^n = Z_\beta + \delta$ when evaluated at $(1, \bar{\sigma}_n(b))$.

Similarly, we can find $\underline{\sigma}_n(b)$ such that $Z_\beta^n = Z_\omega - \delta$ so that for all large enough n, $\Phi_a^n > 0$ and $\Phi_b^n < 0$ when evaluated in $(1, \underline{\sigma}_n(b))$. It follows that there exists $\sigma_n(b) \in (\underline{\sigma}_n(b), \bar{\sigma}_n(b))$ such that $\Phi_a^n > 0$ and $\Phi_b^n = 0$ when evaluated at $(1, \sigma_n(b))$. Pick n_1 large enough such that this is the case and notice that $(1, \sigma_n(b))$ is an equilibrium of the game with n_1 players. Furthermore, note that $Z_{n_1}^\alpha > 1 - \epsilon$ and $Z_{n_1}^\beta \in (Z_\beta - \delta, Z_\beta + \delta)$.[12] We now repeat all the process above but starting with $\delta/2$ instead of δ and $\epsilon/2$ instead of ϵ, and construct $n_2 > n_1$ and $\sigma_{n_2}(b))$. In step k we repeat the process above but starting with δ/k instead of δ and ϵ/k instead of ϵ, and construct $n_k > n_{k-1}$ and $\sigma_{n_k}(b))$. We thus obtain a subsequence of committee sizes and equilibria $(1, \sigma_{n_k}(b))$. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{k-1} for all games with committees of sizes between n_{k-1} and n_k. By construction $Z^n \to Z$.

(case II) $(\Phi_b < 0,\, \Phi_a = 0)$

Then $\sigma(b) = 0$ and $Z_\beta = 0$ and therefore $Z_\alpha \in (0, 1)$. Notice that regardless of what $\sigma_n(a)$ is, we can have Z_β^n approximate 0 as well as we want by choosing n sufficiently large. Furthermore the quality of the approximation is decreasing in $\sigma^n(a)$.[13] Evaluated at $(\sigma_n(a), \sigma_n(b)) = (0, 0)$, $\mu_\alpha < 1/2$ and at $(\sigma_n(a), \sigma_n(b)) = (1, 0)$, $\mu_\alpha > 1/2$ so for all sufficiently large n, Z_α^n is smaller than $Z_\alpha + \delta$ when evaluated at $(0, 0)$ and larger than $Z_\alpha + \delta$ when evaluated at $(1, 0)$. By continuity we can find $\bar{\sigma}_n(a)$ such that $Z_\alpha^n = Z_\alpha + \delta$. It follows that for all large enough n, $\Phi_a^n > 0$ and $\Phi_b^n < 0$ when evaluated at $(\bar{\sigma}_n(a), 0)$. The key is that for any ϵ we can find an N such that for all $n > N$ we get $Z_\beta^n < \epsilon$ and $Z_\alpha^n = Z_\alpha + \delta$ when evaluated in $(\bar{\sigma}_n(a), 0)$.

Similarly, we can find $\underline{\sigma}_n(a)$ such that $Z_\alpha^n = Z_\alpha - \delta$ so that for all large enough n, $\Phi_a^n < 0$ and $\Phi_b^n < 0$ when evaluated in $(\underline{\sigma}_n(a), 0)$. It follows that there exists $\sigma_n(a) \in (\underline{\sigma}_n(a), \bar{\sigma}_n(a))$ such that $\Phi_a^n = 0$ and $\Phi_b^n < 0$ when evaluated at $(\sigma_n(a), 0)$. Pick n_1 large enough such that this is the case and notice $(\sigma_n(a), 0)$ is an equilibrium of the game with n_1 players. Furthermore, note that $Z_{n_1}^\beta < \epsilon$ and $Z_{n_1}^\alpha \in (Z_\alpha - \delta, Z_\alpha + \delta)$[14] We now repeat all the process above but starting with $\delta/2$ instead of δ and $\epsilon/2$ instead of ϵ, and construct $n_2 > n_1$ and $\sigma_{n_2}(a))$. In step k we repeat the process above but starting with δ/k instead of δ and ϵ/k instead of ϵ, and construct $n_k > n_{k-1}$ and $\sigma_{n_k}(a))$. We thus obtain a subsequence of committee sizes and equilibria $(\sigma_{n_k}(a), 0)$. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{k-1} for all games with committees of sizes between n_{k-1} and n_k. By construction $Z^n \to Z$.

(case III) $(\Phi_a < 0,\, \Phi_b = 0)$

Then $\sigma(a) = 0$ and $Z_\alpha = 0$ and therefore $Z_\beta \in (0, 1)$. Notice that regardless of what $\sigma_n(b)$ is, we can have Z_α^n approximate 0 as well as we want by choosing n sufficiently large. Furthermore

[12]Notice that δ was not necessarily positive. If $\delta < 0$ then the correct interval is just $(Z_\beta + \delta, Z_\beta - \delta)$.

[13]So we can establish the required threshold for n by considering $\sigma^n(a) = 1$.

[14]Notice that δ was not necessarily positive. If $\delta < 0$ then the correct interval is just $(Z_\beta + \delta, Z_\beta - \delta)$.

the quality of the approximation is decreasing in $\sigma^n(b)$.[15] Evaluated at $(\sigma_n(a), \sigma_n(b)) = (0,0)$, $\mu_\beta < 1/2$ and at $(\sigma_n(a), \sigma_n(b)) = (0,1)$, $\mu_\beta > 1/2$ so for all sufficiently large n, Z^n_β is smaller than $Z_\beta + \delta$ when evaluated at $(0,0)$ and larger than $Z_\beta + \delta$ when evaluated at $(0,1)$. By continuity we can find $\bar{\sigma}_n(b)$ such that $Z^n_\beta = Z_\beta + \delta$. It follows that for all large enough n, $\Phi^n_a < 0$ and $\Phi^n_b > 0$ when evaluated at $(0, \bar{\sigma}_n(b))$. The key is that for any ϵ we can find an N such that for all $n > N$ we get $Z^n_\alpha < \epsilon$ and $Z^n_\beta = Z_\beta + \delta$ when evaluated in $(0, \bar{\sigma}_n(b))$.

Similarly, we can find $\underline{\sigma}_n(b)$ such that $Z^n_\beta = Z_\omega - \delta$ so that for all large enough n, $\Phi^n_a < 0$ and $\Phi^n_b < 0$ when evaluated in $(0, \underline{\sigma}_n(b))$. It follows that there exists $\sigma_n(b) \in (\underline{\sigma}_n(b), \bar{\sigma}_n(b))$ such that $\Phi^n_a < 0$ and $\Phi^n_b = 0$ when evaluated at $(0, \sigma_n(b))$. Pick n_1 large enough such that this is the case and notice that $(0, \sigma_n(b))$ is an equilibrium of the game with n_1 players. Furthermore, note that $Z^\alpha_{n_1} <$ and $Z^\beta_{n_1} \in (Z_\beta - \delta, Z_\beta + \delta)$.[16] We now repeat all the process above but starting with $\delta/2$ instead of δ and $\epsilon/2$ instead of ϵ, and construct $n_2 > n_1$ and $\sigma_{n_2}(b))$. In step k we repeat the process above but starting with δ/k instead of δ and ϵ/k instead of ϵ, and construct $n_k > n_{k-1}$ and $\sigma_{n_k}(b))$. We thus obtain a subsequence of committee sizes and equilibria $(0, \sigma_{n_k}(b))$. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{k-1} for all games with committees of sizes between n_{k-1} and n_k. By construction $Z^n \to Z$.

(case IV) $(\Phi_b > 0,\ \Phi_a = 0)$
Then $\sigma(b) = 1$ and $Z_\beta = 1$ and therefore $Z_\alpha \in (0,1)$. Notice that regardless of what $\sigma_n(a)$ is, we can have Z^n_β approximate 1 as well as we want by choosing n sufficiently large. Furthermore the quality of the approximation is increasing in $\sigma^n(a)$.[17] Evaluated at $(\sigma_n(a), \sigma_n(b)) = (0,1)$, $\mu_\alpha < 1/2$ and at $(\sigma_n(a), \sigma_n(b)) = (1,1)$, $\mu_\alpha > 1/2$ so for all sufficiently large n, Z^n_α is smaller than $Z_\alpha + \delta$ when evaluated at $(0,1)$ and larger than $Z_\alpha + \delta$ when evaluated at $(1,1)$. By continuity we can find $\bar{\sigma}_n(a)$ such that $Z^n_\alpha = Z_\alpha + \delta$. It follows that for all large enough n, $\Phi^n_a > 0$ and $\Phi^n_b > 0$ when evaluated at $(\bar{\sigma}_n(a), 1)$. The key is that for any ϵ we can find an N such that for all $n > N$ we get $Z^n_\beta > 1 - \epsilon$ and $Z^n_\alpha = Z_\alpha + \delta$ when evaluated in $(\bar{\sigma}_n(a), 1)$.

Similarly, we can find $\underline{\sigma}_n(a)$ such that $Z^n_\alpha = Z_\alpha - \delta$ so that for all large enough n, $\Phi^n_a < 0$ and $\Phi^n_b > 0$ when evaluated in $(\underline{\sigma}_n(a), 1)$. It follows that there exists $\sigma_n(a) \in (\underline{\sigma}_n(a), \bar{\sigma}_n(a))$ such that $\Phi^n_a = 0$ and $\Phi^n_b > 0$ when evaluated at $(\sigma_n(a), 1)$. Pick n_1 large enough such that this is the case and notice $(\sigma_n(a), 1)$ is an equilibrium of the game with n_1 players. Furthermore, note that $Z^\beta_{n_1} < \epsilon$ and $Z^\alpha_{n_1} \in (Z_\alpha - \delta, Z_\alpha + \delta)$.[18] We now repeat all the process above but starting with $\delta/2$ instead of δ and $\epsilon/2$ instead of ϵ, and construct $n_2 > n_1$ and $\sigma_{n_2}(a))$. In step k we repeat the process above but starting with δ/k instead of δ and ϵ/k instead of ϵ, and construct $n_k > n_{k-1}$ and $\sigma_{n_k}(a))$. We thus obtain a subsequence of committee sizes and equilibria $(\sigma_{n_k}(a), 1)$. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{k-1} for all games with

[15] So we can establish the required threshold for n by considering $\sigma^n(b) = 1$.
[16] Notice that δ was not necessarily positive. If $\delta < 0$ then the correct interval is just $(Z_\beta + \delta, Z_\beta - \delta)$.
[17] So we can establish the required threshold for n by considering $\sigma^n(a) = 0$.
[18] Notice that δ was not necessarily positive. If $\delta < 0$ then the correct interval is just $(Z_\beta + \delta, Z_\beta - \delta)$.

committees of sizes between n_{k-1} and n_k. By construction $Z^n \to Z$.■

Lemma 4. Given any limit outcome (Z^*_α, Z^*_β), such that

(1) The two inequalities (associated to Φ_a and Φ_b) hold with equality.

(2) $Z_\alpha \in (0,1)$ and $Z_\beta \in (0,1)$,

then there exists a sequence of equilibria of the finite games (σ^{n*}) such that the associated sequences of decision probabilities (Z^n_α) and (Z^n_β) converge to Z^*_α and Z^*_β.

Proof of Lemma 4: Note that Φ_a and Φ_b are both linearly increasing, or decreasing, in Z_α and Z_β where the slope (which is non-zero as $k_{\alpha,a} - k_{\alpha,b} = 0$ or $k_{\beta,b} - k_{\beta,a} = 0$ rules out (Z^*, σ^*)) of Φ_a (Φ_b) is steeper with respect to Z_α (Z_β). By this, there exists a constant δ such that $(Z^*_\alpha + \delta), (Z^*_\alpha - \delta) \in (0,1)$ and $Z'_\beta, Z''_\beta \in (0,1)$ such that $\Phi_a = (Z^*_\alpha + \delta)(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|s_i = a) + Z'_\beta(k_{\beta,b} - k_{\beta,a})Pr(\beta|s_i = a) + k_{\alpha,b}Pr(\alpha|s_i = a) - k_{\beta,b}Pr(\beta|s_i = a) = 0$ and $\Phi_a = (Z^*_\alpha - \delta)(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|s_i = a) + Z''_\beta(k_{\beta,b} - k_{\beta,a})Pr(\beta|s_i = a) + k_{\alpha,b}Pr(\alpha|s_i = a) - k_{\beta,b}Pr(\beta|s_i = a) = 0$ whereby $\Phi_b(Z^*_\alpha + \delta, Z'_\beta) > x$ and $\Phi_b(Z^*_\alpha - \delta, Z''_\beta) < -x$, where x is some positive constant. By the same token, there exists $\delta_m = \frac{\delta}{m}$, where $m = 1, 2, ...$, such that $(Z^*_\alpha + \delta_m), (Z^*_\alpha - \delta_m) \in (0,1)$ and $Z'_{\beta m}, Z''_{\beta m} \in (0,1)$ such that, for any m, we have $\Phi_a = (Z^*_\alpha + \delta_m)(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|s_i = a) + Z'_{\beta m}(k_{\beta,b} - k_{\beta,a})Pr(\beta|s_i = a) + k_{\alpha,b}Pr(\alpha|s_i = a) - k_{\beta,b}Pr(\beta|s_i = a) = 0$ and $\Phi_a = (Z^*_\alpha - \delta_m)(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|s_i = a) + Z''_{\beta m}(k_{\beta,b} - k_{\beta,a})Pr(\beta|s_i = a) + k_{\alpha,b}Pr(\alpha|s_i = a) - k_{\beta,b}Pr(\beta|s_i = a) = 0$ whereby $\Phi_b(Z^*_\alpha + \delta_m, Z'_{\beta m}) > x_m$ and $\Phi_b(Z^*_\alpha - \delta_m, Z''_{\beta m}) < -x_m$, where x_m is some positive constant. Moreover, $(Z^*_\alpha + \delta_m)$, $(Z^*_\alpha - \delta_m)$ converge to Z^*_α and $Z'_{\beta m}$, $Z''_{\beta m}$ converge to $Z_{\beta *}$ as $m \to \infty$.

Recall that

$$\begin{aligned}\Phi^n_{s_i}(\sigma) =&(k(a,a,\alpha) - k(a,b,\alpha))Pr(piv_i|\alpha)Pr(\alpha|s_i) - (k(a,b,\beta) - k(a,a,\beta))Pr(piv_i|\beta)Pr(\beta|s_i)\\ &+ (k_{\alpha,a} - k_{\alpha,b})Pr(a, \neg piv_i|\alpha)Pr(\alpha|s_i) + (k_{\beta,b} - k_{\beta,a})Pr(a, \neg piv_i|\beta)Pr(\beta|s_i)\\ &+ k_{\alpha,b}Pr(\alpha|s_i) - k_{\beta,b}Pr(\beta|s_i)\end{aligned}$$

where Φ^n_a and Φ^n_b are continuous in $\sigma(a)$ and $\sigma(b)$ and $Pr(a, \neg piv_i|\alpha)$ is a continuous, and strictly increasing, function of $\mu_\alpha = \sigma(a)(1-\varepsilon) + \sigma(b)\varepsilon$ and $Pr(a, \neg piv_i|\beta)$ is a continuous, and strictly increasing, function of $\mu_\beta = \sigma(a)\varepsilon + \sigma(b)(1-\varepsilon)$. For $\sigma = (1,1)$ we have $Pr(a, \neg piv_i|\alpha) = 1$ and $Pr(a, \neg piv_i|\beta) = 1$ and for $\sigma = (0,0)$ we have $Pr(a, \neg piv_i|\alpha) = 0$ and $Pr(a, \neg piv_i|\beta) = 0$. Consider some n and let $\mu^*_{\alpha,n}$ indicate the unique $\mu_{\alpha,n}$ such that $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha + \delta)$. Given $\mu^*_{\alpha,n}$, the highest possible $\sigma^n(a)$ is attained with $\sigma^n = (1, \frac{\mu^*_{\alpha,n} - (1-\varepsilon)}{\varepsilon})$ if $\mu^*_{\alpha,n} \geq (1-\varepsilon)$ and $\sigma^n = (\frac{\mu^*_{\alpha,n}}{(1-\varepsilon)}, 0)$ if $\mu^*_{\alpha,n} < (1-\varepsilon)$ and the lowest possible $\sigma^n(a)$ is attained with $\sigma^n = (0, \frac{\mu^*_{\alpha,n}}{\varepsilon})$ if $\mu^*_{\alpha,n} \leq \varepsilon$ and $\sigma^n = (\frac{\mu^*_{\alpha,n} - \varepsilon}{(1-\varepsilon)}, 1)$ if $\mu^*_{\alpha,n} > \varepsilon$. Choosing the highest possible $\sigma^n(a)$ gives the lowest feasible $Pr(a, \neg piv_i|\beta)$ and choosing the lowest possible $\sigma^n(a)$ gives the highest feasible $Pr(a, \neg piv_i|\beta)$. As $(Z^*_\alpha + \delta) \in (0,1)$ we must have that $\mu^*_{\alpha,n}$ converges to $\frac{1}{2}$ as $n \to \infty$

and thereby, for all sufficiently large n, the two relevant extremes are $\sigma^n = (\frac{\mu^*_{\alpha,n}}{(1-\varepsilon)}, 0)$ and $\sigma^n = (\frac{\mu^*_{\alpha,n}-\varepsilon}{(1-\varepsilon)}, 1)$. For these two extremes $Pr(a, \neg piv_i|\beta)$ converge to 0 and 1, respectively, as $n \to \infty$, and since $Z'_\beta \in (0,1)$ (because by assumption $k_{\beta,b} - k_{\beta,a} \neq 0$) then, for all sufficiently large n, $\Phi^n_a > 0$ for one of the extremes and $\Phi^n_a < 0$ for the other (note that $\Phi^n_a(\frac{\mu^*_{\alpha,n}}{(1-\varepsilon)}, 0) \to \Phi_a(Z^*_\alpha + \delta, 0)$ and $\Phi^n_a(\frac{\mu^*_{\alpha,n}-\varepsilon}{(1-\varepsilon)}, 1) \to \Phi_a(Z^*_\alpha + \delta, 1)$. By continuity of Φ^n_a in the σ-plane then, for all sufficiently large n, there exists an intermediate σ^n such that $\Phi^n_a(\sigma) = 0$. Moreover, given such σ^n whereby $\Phi^n_a(\sigma) = 0$ we have that

$$\begin{aligned}\Phi^n_a(\sigma) =&(k(a,a,\alpha) - k(a,b,\alpha))Pr(piv_i|\alpha)Pr(\alpha|s_i) - (k(a,b,\beta) - k(a,a,\beta))Pr(piv_i|\beta)Pr(\beta|s_i)\\ &+ (k_{\alpha,a} - k_{\alpha,b})(Z^*_\alpha + \delta)Pr(\alpha|s_i) + (k_{\beta,b} - k_{\beta,a})Pr(a, \neg piv_i|\beta)Pr(\beta|s_i)\\ &+ k_{\alpha,b}Pr(\alpha|s_i) - k_{\beta,b}Pr(\beta|s_i) = 0,\end{aligned}$$

and as $n \to \infty$ the pivotal terms uniformly converge to zero and we can conclude that, given our σ^n such that $\Phi^n_a(\sigma) = 0$, the term $Pr(a, \neg piv_i|\beta)$ must converge to Z'_β (where Z'_β is as defined above ensuring that $\Phi_a = 0$ given $Z_\alpha = (Z^*_\alpha + \delta)$).

By the parallel arguments we can fix $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha - \delta)$ and, for all sufficiently large n, there exists a σ^n such that $\Phi^n_a(\sigma) = 0$ and $Pr(a, \neg piv_i|\beta)$ converges to Z''^β for $n \to \infty$. Similarly if we consider $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha + \delta')$ and $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha - \delta')$ for $\delta' \in [0, \delta]$. For sufficiently large n this constitutes a span of strategies.

Now fix $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha + \delta)$ and $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha - \delta)$ then, for sufficiently large n, call it n_1, there exists σ^{n_1} such that $\Phi^{n_1}_a(\sigma) = 0$ and $\Phi^{n_1}_b(\sigma) < 0$ and another σ^{n_1} such that $\Phi^{n_1}_a(\sigma) = 0$ and $\Phi^{n_1}_b(\sigma) > 0$ and by continuity there exists σ^{n_1} such that $\Phi^{n_1}_a(\sigma) = 0$ and $\Phi^{n_1}_b(\sigma) = 0$. We now repeat the process starting with δ_2 and $-\delta_2$ and we have $n_2 > n_1$ and σ^{n_2} such that $\Phi^{n_2}_a(\sigma) = 0$ and $\Phi^{n_2}_b(\sigma) = 0$. We do this for any δ_m and $-\delta_m$ and construct $n_m > n_{m-1}$ and we have a subsequence of committee sizes and equilibria with associated Z^{n*}_α and Z^{n*}_β converging to Z^*_α and Z^*_β. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{m-1} for all games with committees of sizes between n_{m-1} and n_m. ■

Lemma 5 *The sequence of decision probabilities, $(Z^n_\alpha, Z^n_\beta))$, associated to any sequence of equilibria of the finite games, (σ^n), must converge to the set of limit outcomes.*

Proof of Lemma 5: Let (σ^n) be a sequence of equilibria of the finite games and suppose $(Z^n_\alpha, Z^n_\beta) = (\Pr^n(a|\alpha), (\Pr^n(a|\beta))$ is the associated sequence of decision probabilities. Suppose that there exists $\epsilon > 0$ such that there is an infinite subsequence of terms $(\Pr^n(a|\alpha), (\Pr^n(a|\beta))$ which are at least ε away from any pair (Z_α, Z_β) that is a limit outcome. Because all the terms in this subsequence are bounded above and below (by $(0,0)$ and $(1,1)$), it must have a convergent subsequence. Call it's limit point (Y_α, Y_β). by construction we thus have that for any limit outcome (Z_α, Z_β), $||(Z_\alpha, Z_\beta) - (Y_\alpha, Y_\beta)|| \geq \varepsilon$. So (Y_α, Y_β) must violate at least one of the four conditions that define a limit outcome. Suppose that it violates (1a). That is, suppose that $\Phi_a(Y_\alpha, Y_\beta) > 0$ yet $Y_\alpha < 1$. Let $h = 1 - Y_\alpha$ This means that for all sufficiently large n we must have $\Phi_a(Z^n_\alpha, Z^n_\beta) > 0$ and $Z^n_\alpha < 1 - h/2$, but this is a contradiction, since $\Phi_a(Z^n_\alpha, Z^n_\beta) > 0$ implies that $\sigma_n(a) = 1$ and thus $Z^n_\alpha \to 1$. The other 3 cases are anlogous. ■.

PROPOSITION **4** (Theorem 1 applies generically). *The set of parameter vectors with the property that there exists some limit outcome* (Z_α, Z_β) *at which Theorem 1 does not apply has measure* 0. *That is, Theorem* 1 *holds generically in* $\mathbb{R}^8$.

Proof of Proposition 4: The only cases to which the argument presented in Lemmas 2,3 and 4 do not apply involve one of the following two conditions:

1. A vector of parameters $(k(v_i, x, \omega))$ and a limit outcome such that only one of $\Phi_{s_i}(Z_\alpha, Z_\beta)$ ($s_i = a$ or $s_i = b$) is 0, $Z_\alpha \notin (0,1)$ and $Z_\beta \notin (0,1)$.
2. A vector of parameters $(k(v_i, x, \omega))$ and a limit outcome such that only one of $\Phi_{s_i}(Z_\alpha, Z_\beta)$ ($s_i = a$ or $s_i = b$) is 0, (call it Φ_s), only one of Z_α or Z_β is interior (call it Z_ω) and the multiplier of Z_ω in Φ_s is 0.
3. Both $\Phi_{s_i}(Z_\alpha, Z_\beta) = 0$ (for $s_i = a$ and $s_i = b$) and either (or both) $Z_\alpha \notin (0,1)$ or $Z_\beta \notin (0,1)$.

We proceed by showing that the set of vectors $(k(v_i, x, \omega))$ for which each of the three conditions above can hold is a subspace of $\mathbb{R}^8$ of dimension strictly less than 8, and therefore of measure 0 in $\mathbb{R}^8$. In fact, we will show the stronger property that the sets of vectors $(k_{\omega,x})$ for which each of the conditions above can hold is a subspace of $\mathbb{R}^4$ of dimension strictly less than 4, and therefore of measure 0 in $\mathbb{R}^4$. Since the union of finitely many sets of measure 0 is of measure 0 the result follows.

(Condition 1) Let s be such that $\Phi_s(Z_\alpha, Z_\beta) = 0$. Assume that $Z_\alpha = 0$ and $Z_\beta = 0$. Then it follows that $Pr(\alpha|s)k_{\alpha,b} - Pr(\beta|s)k_{\beta,b} = 0$ which given that $Pr(\alpha|s) > 0$ and $Pr(\beta|s) > 0$ defines a subspace of dimension 3 in $\mathbb{R}^4$. Similarly in case $Z_\alpha = 1$ and $Z_\beta = 0$ then the analogous condition is $Pr(\alpha|s)k_{\alpha,a} - Pr(\beta|s)k_{\beta,b} = 0$. In case $Z_\alpha = 0$ and $Z_\beta = 1$ then it is $Pr(\alpha|s)k_{\alpha,b} - Pr(\beta|s)k_{\beta,a} = 0$. In case $Z_\alpha = 1$ and $Z_\beta = 1$ then it is $Pr(\alpha|s)k_{\alpha,a} - Pr(\beta|s)k_{\beta,a} = 0$.

(Condition 2) Let s be such that $\Phi_s(Z_\alpha, Z_\beta) = 0$. Assume that Z_α is interior and the multiplier of Z_α in Φ_s is 0. Then $k_{\alpha,a} - k_{\alpha,b} = 0$ which defines a subspace of dimension 3 in $\mathbb{R}^4$. Similarly, if Z_β is interior and the multiplier of Z_β in Φ_s is 0 then $k_{\beta,a} - k_{\beta,b} = 0$.

(Condition 3) Suppose that $\Phi_a(Z_\alpha, Z_\beta) = 0$ and $\Phi_b(Z_\alpha, Z_\beta) = 0$ If $k_{\alpha,a} - k_{\alpha,b} = 0$ or $k_{\beta,a} - k_{\beta,b} = 0$ then as above we have subspaces of dimension 3 in $\mathbb{R}^4$. Otherwise, suppose $Z_\alpha \notin (0,1) = 0$. It follows by solving for Z_β in each of the equations $\Phi_a(Z_\alpha, Z_\beta) = 0$ and $\Phi_b(Z_\alpha, Z_\beta) = 0$, that,

$$Z_\beta = \frac{Z_\alpha(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|a) - k_{\beta,b}Pr(\beta|a) + k_{\alpha,b}Pr(\alpha|a)}{(k_{\beta,a} - k_{\beta,b})}$$
$$= \frac{Z_\alpha(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|b) - k_{\beta,b}Pr(\beta|b) + k_{\alpha,b}Pr(\alpha|b)}{(k_{\beta,a} - k_{\beta,b})}$$

If $Z_\alpha = 0$, then $k_{\alpha,b}(Pr(\alpha|a) - Pr(\alpha|b)) + k_{\beta,b}(Pr(\beta|b) - Pr(\beta|a)) = 0$. Given that $Pr(\alpha|a) > Pr(\alpha|b)$ and $Pr(\beta|b) > Pr(\beta|a)$, this equation defines a 3 dimensional subspace in $\mathbb{R}^4$. Similarly, if $Z_\alpha = 1$, then $k_{\alpha,a}(Pr(\alpha|a) - Pr(\alpha|b)) + k_{\beta,b}(Pr(\beta|b) - Pr(\beta|a)) = 0$.

If on the other hand $Z_\beta \notin (0,1) = 0$ we solve for Z_α in each of the equations $\Phi_a(Z_\alpha, Z_\beta) = 0$ and $\Phi_b(Z_\alpha, Z_\beta) = 0$, and proceed just as above. ∎

The following Proposition, which relies on Proposition 4, is the basis for Lemma 1 in the main text.

PROPOSITION **5.** *Given any vector of parameters* $(k(v_i, x, \omega))$ *at which Theorem 1 does not apply for limit outcome* $(Z_\alpha, Z_\beta) = (1, 0)$*, and for any* ε*, there exists a vector of parameters* $(k'(v_i, x, \omega))$ *at which Theorem 1 does apply for* $(Z_\alpha, Z_\beta) = (1, 0)$ *and such that (1)* $\Phi_a(Z = (1, 0)) < 0$ *or* $\Phi_b(Z = (1, 0)) > 0$ *and (2)* $||(k(v_i, x, \omega)) - (k'(v_i, x, \omega))|| < \varepsilon$ *.*

Proof of Proposition 5: Note that,

$$\Phi_{s_i}(Z_\alpha = 1, Z_\beta = 0) = k_{\alpha,a}Pr(\alpha|s_i) - k_{\beta,b}Pr(\beta|s_i)]$$

Suppose that $\Phi_a(Z_\alpha = 1, Z_\beta = 0) \neq 0$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) = 0$. Then by perturbing the parameters just slightly so that $k'_{\beta,b} = k_{\beta,b} - \delta$ (with $\delta > 0$) we can guarantee that at the new parameters, $\Phi_a(Z_\alpha = 1, Z_\beta = 0) \neq 0$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) > 0$. It follows that since both inequalities hold strictly at these new parameters, Theorem 1 holds for $(Z_\alpha, Z_\beta) = (1, 0)$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) > 0$.

Analogously, suppose that $\Phi_a(Z_\alpha = 1, Z_\beta = 0) = 0$ and $\Phi_b(Z_\alpha 1, Z_\beta = 0) \neq 0$.Then by perturbing the parameters just slightly so that $k'_{\alpha,a} = k_{\alpha,a} - \delta$ (with $\delta > 0$) we can guarantee that at the new parameters, $\Phi_a(Z_\alpha = 1, Z_\beta = 0) < 0$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) \neq 0$. It follows that since both inequalities hold strictly at these new parameters, Theorem 1 holds for $(Z_\alpha, Z_\beta) = (1, 0)$ and $\Phi_a(Z_\alpha = 1, Z_\beta = 0) < 0$.

Suppose that $\Phi_a(Z_\alpha = 1, Z_\beta = 0) = 0$ and $\Phi_b(Z_\alpha 1, Z_\beta = 0) = 0$. Then by perturbing the parameters just slightly so that $k'_{\beta,b} = k_{\beta,b} - \delta$ (with $\delta > 0$) we can guarantee that at the new parameters, $\Phi_a(Z_\alpha = 1, Z_\beta = 0) > 0$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) > 0$. It follows that since both inequalities hold strictly at these new parameters, Theorem 1 holds for $(Z_\alpha, Z_\beta) = (1, 0)$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) < 0$

By Proposition 4, with the above we have covered all possible parameter vectors at which Theorem 1 does not apply for limit outcome $(Z_\alpha, Z_\beta) = (1, 0)$. ∎

Remark: As stated in Lemma 1 of the main text it follows from Proposition 5 that information aggregation cannot be robust at payoff vectors for which Theorem 1 does not apply.

Proof of Lemma 1. It is clear that if Theorem 1 applies to a given payoff vector at which $(Z_\alpha, Z_\beta) = (1, 0)$ as a limit outcome at which $\Phi_a(Z = (1, 0)) > 0$ and $\Phi_b(Z = (1, 0)) < 0$, then this vector supports robust information aggregation.

By Theorem 1 any vector of parameters $(k_{\omega,x})$ to which it applies and which does not have $(Z_\alpha, Z_\beta) = (1, 0)$ as a limit outcome does not support information aggregation to begin with, and therefore can't support robust information aggregation either. Furthermore for robust information aggregation to be supported by payoff vectors which do have $(Z_\alpha, Z_\beta) = (1, 0)$ as a limit outcome it must be the case that $\Phi_a(Z = (1, 0)) > 0$ and $\Phi_b(Z = (1, 0)) < 0$. If this is not the case then in any open ball around $(k_{\omega,x})$ there will be a payoff vector $(k'_{\omega,x})$ under which either $\Phi_a(Z = (1, 0)) < 0$ or $\Phi_b(Z = (1, 0)) > 0$ and to which Theorem 1 applies (because it applies generically in the payoffs' space as shown by Proposition 4 above). It follows that $(Z_\alpha, Z_\beta) = (1, 0)$ is not a limit outcome for $(k'_{\omega,x})$. By the argument above $(k'_{\omega,x})$ does not support information aggregation, and therefore $(k_{\omega,x})$ does not support robust information aggregation.

The only possible concern is thus with payoff vectors $(k_{\omega,x})$ at which Theorem 1 does not apply. As shown by Proposition 5 above, in any open ball around $(k_{\omega,x})$ there must exist payoff vectors to which Theorem 1 does apply and at which at least one of $\Phi_a(Z = (1, 0)) < 0$ or $\Phi_b(Z = (1, 0)) > 0$ holds. It follows that robust information aggregation is not supported by $(k_{\omega,x})$.■

Proof of Corollary 1. by Lemma 1, if information aggregation is robust for a given vector of parameters $(k_{\omega,x})$ then $(1, 0)$ must be a limit outcome at this vector, with $\Phi_a(1, 0) > 0$ and $\Phi_b(1, 0) < 0$. Furthermore, it follows that under any sequence of strategy profiles such that $Z^n \to (1, 0)$ it must be the case that $\Phi_a^n \to \Phi_a(1, 0)$ and $\Phi_b^n \to \Phi_b(1, 0)$. It follows that for sufficiently large n it must be the case that $\Phi_b^n < 0$ and $\Phi_a^n > 0$ and thus $\sigma^n(a) = 1$ and $\sigma^n(b) = 0$.■

Proof of Theorem 2. by Lemma 1, we just need to solve for the payoff vectors $(k_{\omega,x})$ such that $(Z_\alpha, Z_\beta) = (1, 0)$ is a limit outcome with $\Phi_a > 0$ and $\Phi_b < 0$. That is, solve for all the payoff vectors such that $\Phi_a(1, 0) > 0$ and $\Phi_b(1, 0) < 0$. A payoff vector solves this problem if and only if the three conditions in the statement of the theorem are met.■

Proof of Theorem 3. When the conditions in Theorem 3 are met (in addition to the conditions in Theorem 2), then there are no limit outcomes other than $(Z_\alpha, Z_\beta) = (1, 0)$. Furthermore, Theorem 1 applies to all payoff vectors within the set described by Theorem 2. It follows that in the light of part (2) of Theorem 1, the sequence of decision probabilities associated to any sequence of equilibria of the finite games must converge to $(1, 0)$.■

Before proceeding with the proof of Proposition 3, we state the general version of the proposition (the proof of Proposition 3 follows as a Corollary of the proposition listed below):

PROPOSITION **6.** *(i) Partial information aggregation with $Z_\beta \in (0,1)$: There exists a sequence of equilibria σ^{n*} such that $Z^n(\sigma^{n*}) \to (1, Z_\beta \in (0,1))$ as $n \to \infty$ if, and only if, $k_{\beta,a} \neq k_{\beta,b}$ and:*

$$Z_\beta = \frac{\frac{Pr(\alpha|s_i=b)}{Pr(\beta|s_i=b)}k_{\alpha,a} - k_{\beta,b}}{k_{\beta,a} - k_{\beta,b}} \in (0,1)$$

(ii) Partial information aggregation with $Z_\alpha \in (0,1)$: There exists a sequence of equilibria σ^{n*} such that $Z^n(\sigma^{n*}) \to (Z_\alpha \in (0,1), 0)$ as $n \to \infty$ if, and only if, $k_{\alpha,a} \neq k_{\alpha,b}$ and:

$$Z_\alpha = \frac{\frac{Pr(\beta|s_i=a)}{Pr(\alpha|s_i=a)}k_{\beta,b} - k_{\alpha,b}}{k_{\alpha,a} - k_{\alpha,b}} \in (0,1).$$

Proof of Proposition 6: By relying on Theorem 1, we solve for the payoff vectors $(k_{\omega,x})$ such that $(1, Z_\beta)$ is a limit outcome where $Z_\beta \in (0,1)$. Note that the definition of limit outcome implies that $Z_\beta \in (0,1)$ requires $\Phi_b(Z_\alpha, Z_\beta) = 0$. We thus obtain the expression for Z_β by solving $\Phi_b(1, Z_\beta) = 0$. Analogously for partial information aggregation with $Z_\alpha \in (0,1)$. ■

Proof of Proposition 5. by relying on Theorem 1, we solve for the payoff vectors $(k_{\omega,x})$ such that $(Z_\alpha, Z_\beta) = (1,1)$ and $(Z_\alpha . Z_\beta) = (0,0)$. Are limit outcomes. ■.

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