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Abstract

Technological innovations facilitating e-commerce have well-documented effects on consumer behavior and firm organization in the retail sector, but the effects of these new transaction technologies on fiscal systems remain unknown. By extending models of commodity tax competition to include urban spatial structure (agglomeration) and online commerce, one can analyze strategic tax-policy interactions among neighboring localities. Consumers buy different types of commodities, sold either by traditional or by online vendors. When the cost of online shopping falls, we show that equilibrium tax rates and revenues increase in small jurisdictions and decrease in large jurisdictions with retail shopping centers. Policy commentators warn that e-commerce erodes tax revenue - true enough for some localities - but, more accurately, changing transaction costs can generate entirely new commercial and fiscal equilibria that ultimately "redistribute" tax revenues from localities with concentrations of traditional vendors toward other, typically smaller, localities.

JEL-Codes: H250, H710, H730, L810, R500.

Keywords: sales tax, retail shopping, agglomeration, e-commerce, fiscal competition.

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How does online shopping affect local sales tax rates and revenues? The advent of new mechanisms to engage in retail transactions creates challenges and opportunities for fiscal authorities. Throughout history, major changes in the structure of commerce and industry have affected fiscal systems in complex and sometimes unforeseen ways (Keen and Slemrod 2015). The analysis presented below, which examines the simultaneous adjustment of tax policies in a system of governments resulting from increased opportunities for online commerce, predicts complex effects leading to *higher* tax rates and tax revenues for some governments and to *lower* tax rates and tax revenues for others.

The difficulty of collecting sales taxes on online transactions is widely recognized by policymakers and the press. Although tax enforcement is a challenge for all types of taxation, it has especially garnered popular attention in the context of online commerce, even raising anxieties about the long-run viability of sales taxation. Concerns about the demise of retail sales taxation may very well be overblown, however. Evolving business practices, e-commerce technology, administrative simplifications, and legal requirements all promise to enhance sales tax collection and indeed have already done so.[1] Many large online retailers are collecting sales taxes on more and more retail transactions; for instance, Amazon now remits sales taxes in all 50 states. By some estimates, 60% (and growing) of transactions from large online vendors are now taxed in the average state.

The rise of Internet commerce is a conspicuous recent instance in which technological change has disrupted retail commerce, but other examples spring readily to mind. In comparatively recent times, innovations such as the refrigerator, the automobile, and changes in packaging – precipitated to some degree by changing technologies in the energy, transportation, and communications sectors – have had major impacts on where and how frequently people go shopping, how goods are transported from suppliers to retailers, and the organization of upstream transactions ("supply chains"). These technological innovations allowed grocery stores, for example, to increase in size in the 1920s because people could make longer trips, could buy larger quantities in one trip, and could buy fresher products (Basker, Vickers and Ziebarth 2017). Analogous to online shopping today, the emergence of large chain stores led many to worry about threats to small businesses in the 1920s and 1930s. Many states passed "chain taxes" because large retailers were driving out "mom and pop" stores – policy responses that, if ultimately triggered by underlying technological change, were nonetheless implemented through political institutions

[1] Technological change – such as online technologies that track consumer purchases, including the precise locations of both buyers and sellers – may ultimately facilitate rather than hinder the taxation of online transactions. On the legal front, issues of nexus (an online vendor must remit state and local retail sales taxes if they have a physical presence in the consumer's state) are rapidly evolving. Additionally, many states have also entered into an interstate compact designed to ease compliance burdens on vendors; related federal legislation is also pending. The jury is still out (figuratively and literally) on these issues, which we revisit in the next section and concluding remarks.

responding to political pressures, subject to judicial review.[2]

Similarly, the "Walmart economy" – characterized by a large variety of relatively cheap goods, large retailers, and, of course, off-shoring of suppliers – has again changed traditional commerce (Holmes and Singer 2017). Walmart and other firms can now more effectively streamline and consolidate shipments, organizing distribution networks using warehouses that were once located in urban areas but are now found increasingly in rural areas (Holmes 2001; Holmes 2011). And still more recently, online shopping, a principal focus of the present paper, is again reshaping the retail sector (Goolsbee 2000). Although the academic literature has studied the effect of technological innovations on consumer behavior and firm organization,[3] there is a dearth of research on the effects of these new transaction technologies on fiscal systems. Our paper tackles this issue by presenting a theoretical analysis of the effects of changes in transaction technologies on indirect taxation of the retail sector, particularly by subnational governments such as cities, counties, and states.

This analysis builds upon earlier research, especially the now-standard Hotelling-type linear model (Kanbur and Keen 1993; Nielsen 2001), in several ways. These models were originally designed to study cross-border shopping across international borders. In order to reflect better the *spatial structure* of *local* retail commerce as it has evolved over the past half-century and longer, and to reflect the fact consumers routinely purchase some types of goods and services in their immediate neighborhoods while other goods and services lend themselves to greater spatial separation (including remote commerce) between buyers and sellers, our model distinguishes two main types of commodities. As in Kanbur and Keen (1993) and Nielsen (2001), the first commodity category represents those goods and services that are commonly purchased either in close proximity to where people reside or, if desired, across the boundary of a neighboring jurisdiction where sales are more lightly taxed. The second type is a category of goods and services of a more specialized nature that have traditionally been purchased at centralized points of agglomeration such as downtown shopping districts or major shopping malls that serve consumers throughout a larger region containing many localities. Sales of these "specialized" commodities constitute part of the sales tax base only for the jurisdiction that contains this agglomeration. This feature of the model captures the spatial asymmetry of retail commerce in metropolitan areas in the post-World War II era, as clearly revealed in Figure 1. This figure shows that large cities – the largest 1 percent by population, in

[2]Huey Long of Louisiana advocated a state tax on chains proportional to the number of outlets *nationwide*, although only Louisiana branches had to pay the tax. The Supreme Court wrote: "If the competitive advantages of a chain increase with the number of its component links, it is hard to see how these advantages cease at the State boundary."

[3]See, for example, Brown and Goolsbee (2002), Jin and Kato (2007), Ellison and Ellison (2009), Lewis (2011), and Goldmanis et al. (2010).

the Figure – accounted for 76% of all municipal sales tax revenues in the 1970s.[4] This share has declined gradually to about 63% of all such revenues today, highlighting that interjurisdictional tax revenue differentials are declining. In addition, our model allows for Internet (or, in a different interpretation, mail-order) purchases of these specialized commodities by consumers, which constitute an important and rapidly-rising share of retail commerce. As in Kanbur and Keen (1993) and Nielsen (2001), we postulate that local governments within the local region (a metropolitan area) must compete for customers with other nearby localities, and that they do so in a strategic fashion.

The model contains several parameters that represent transportation, online access, and other transactions costs associated with traditional and online commerce, and it supports several different types of (Nash non-cooperative) equilibrium tax regimes and transactions patterns. These include a regime (a pre-Internet world) in which all consumers, both resident and non-resident, take advantage of opportunities to obtain specialized commodities in the central agglomeration, and add to the tax base of the locality where the agglomeration is situated. It also includes a regime (a hypothetical future world) in which the costs of online commerce fall so far, relative to traditional commerce, that all transactions involving specialized commodities are executed online. Our main interest, however, is an intermediate case – corresponding to the structure of commerce that we presently observe – bracketed by these other cases, in which traditional and online commerce co-exist. Let us stress that although the equilibrium is determined, in a fundamental sense, by exogenous underlying costs of transactions, it is also the result of the endogenously- and simultaneously-determined tax policies of the local governments, as consumers adjust their shopping patters in response to local taxes. Strategic interactions are likely to be relevant in the context of sales taxation because each locality competes with a comparatively small number of neighboring jurisdictions, and thus an explicitly game-theoretic approach seems especially important, with or without online shopping.

The goal of the analysis, then, is to show how different types of equilibria can arise in situations with different constellations of transaction costs, and to derive economically-important characteristics of these equilibria, notably, equilibrium shopping patterns, tax rates, and tax revenues. The results of the analysis provide insight into the evolution of equilibrium policies in response to disruptive changes in the costs of commerce, offering a guideline for empirical analysis and contributing to our understanding of the non-trivial policy implications of these changes. We highlight a few of the findings:

1. By comparison with the standard model of Kanbur and Keen (1993) and Nielsen (2001), in which localities differ only in absolute size (geographic or population),

[4] Such spatial asymmetries of agglomeration do not arise in the international context, for which these spatial models of commodity taxation were initially developed.

the spatial asymmetry resulting from the presence of large retail shopping centers in an agglomerated center *widens* interjurisdictional tax differentials. Intuitively, agglomeration yields taxable rents which a locality can exploit by raising its sales tax rate relative to a neighboring locality with no such agglomeration.

2. Increased opportunity for online shopping *reduces* interjurisdictional tax differentials. Intuitively, online shopping limits the advantages of agglomeration and provides new opportunities for smaller localities to exploit sales taxes. Consequently, online shopping can also shift tax revenues from large to small jurisdictions. This shift arises because of simultaneous changes in tax rates and the distribution of commercial activity within the region.

3. Requiring online vendors to remit retail sales taxes (for example, by passing the Marketplace Fairness Act) cannot entirely prevent the decline of sales tax revenue. As the cost of online shopping falls, even with online transactions taxed according the destination principle, a large jurisdiction *loses* revenue and a small jurisdiction *gains* revenue, but aggregate revenue *declines*. This decline occurs because the tax base shifts from high-tax jurisdictions containing retail shopping centers to smaller low-tax jurisdictions, which, under the destination principle, are able to tax their residents' online purchases.

The next section of the paper discusses some critical institutional and empirical features of local sales taxation that our model should incorporate. We then proceed by discussing the structure of the model and its equilibrium solutions, and by showing how key properties of these equilibria depend upon the underlying fundamental parameters.

1 Sales Tax Institutions

Taxation of commodities in the United States is decentralized and (aside from tariffs and duties on international commerce) falls under the jurisdiction of state and local governments. Because thirty-four states permit one or more classes of localities (counties, towns, and special districts) to collect sales taxes, there are today more than 15,000 taxing jurisdictions in total. With some exceptions, these localities cohabit the same tax bases (i.e. categories of taxable transactions) as their state governments. Figure 2 shows the change (from 2003 to 2011) in the average tax rate for each county in the United States, inclusive of the (population-weighted) local tax rates. As the figure shows, local sales taxation in the United States is widespread and growing, but not universal.

Commodity taxes in the U.S. take two distinct but closely-connected forms: sales taxes and use taxes. Sales taxes are imposed when purchasers and vendors are in the same location when the transaction occurs. Vendors are obliged to remit these taxes to the fiscal authorities in these situations. Use taxes, by contrast, may come into play when a purchasers and vendor are in different locations. Specifically, use taxes on cross-border transactions (including mail-order and online sales) must be remitted by *purchasers* to the tax authorities in their home jurisdictions, provided that the purchases have not already been subject to sales tax. Because the use taxes shifts the tax remitting responsibility to the purchasers, state and local use tax collection is especially low for business to consumer transactions. In any event, use taxes are collected on a “destination basis”, that is, they are paid to the jurisdictions in which purchasers are located.

Compared to sales taxes, states and their subordinate local governments have not achieved high levels of compliance with use taxes, no doubt partly because the monitoring and recording of consumer purchases from remote vendors is difficult and also partly because of enforcement policies. The direct imposition of state and local sales taxes on remote vendors has been limited by judicial rulings. In particular, the U.S. Supreme Court, in *Quill Corp v. North Dakota* (504 US 298 (1992)) (a case involving mail-order transactions), found that states and localities exceed their constitutional taxing powers if they require “out-of-state” vendors to remit taxes on sales to their residents, unless the vendors have sufficient “nexus” with the taxing state. The Court ruled that nexus, in the context of sales taxation, requires a vendor to be “physically present” in a state – for instance, because it has production, distribution, retail, or other facilities there. Otherwise, fiscal authorities within the state are precluded from imposing sales taxes on that vendor’s sales to consumers within the state. In such instances, a state and its localities may nonetheless require consumers (but not vendors) to remit use taxes, which thus provide states with an administratively distinct (if likely more cumbersome) alternative to the imposition of sales taxes on cross-border transactions.

A key point for our theoretical model is that nexus is a state concept: once nexus is established in a particular state, the firm has nexus in *every* locality within that state and therefore must remit appropriate municipal sales taxes. Because nexus is a state-level concept, taxes (both state and local) on online transactions that feature a buyer and an online seller in the same state are remitted by the seller and the destination tax rate prevails.[5] In practice, vendors remit the local sales taxes to the state government, which distributes them to the municipality based on the final destination of the sale. This dramatically reduces the compliance costs of vendors and eases local tax administration, thus obviating many of the concerns that might otherwise arise regarding the capacity of local governments to enforce taxes on remote transactions from vendors with nexus.

[5]Note that in the vast majority of states, the seller remits the tax based on the municipal address of the buyer, even if the seller does not have a store in that locality. However, in some states, within state remote transactions have taxes assessed at origin. In these states, shipments within that state are taxed at the state and local rate of the vendor’s location, regardless of the tax rate at the destination.

The upshot is that only taxes on transactions between a buyer and a seller located in two *different* states need to be remitted by the buyers. Given that use tax evasion is common, transactions between a buyer and online vendor in different states are effectively tax-free; these types of transactions are frequently studied (Goolsbee 2000; Ellison and Ellison 2009; Hortaçsu, Martínez-Jerez and Douglas 2009; Goolsbee, Lovenheim and Slemrod 2010; Einav et al. 2014).[6]

The distinction between taxable and non-taxable online transactions has become more important as e-commerce has grown and as more and more transactions occur on websites with nexus in a state. Figure 3 shows quarterly data on the percent of total sales that occur online. Due to the growth in e-commerce, 8.5% of transactions now occur online. This growth shows no evidence of slowing: The right panel shows that the growth rate relative to the previous quarter has remained relatively constant at around 5% per quarter. This growth rate well exceeds that of traditional sales.

How much of this online shopping occurs on websites with nexus and without nexus? We have little empirical evidence on this topic, however, best estimates indicate 13-19% occurred on Amazon (Einav et al. 2014). Amazon is now collecting taxes in every state with a sales tax; only two years ago (2015), it had nexus in only half of the states. Data on nexus are not readily available. Bruce, Fox and Luna (2015) identify the approximately 300 largest online retailers and attempt to purchase an item from each of the retailer's websites using an address from each state in the country. They find that even as early as 2006, 50 of the largest 100 firms had nexus in the average state. Furthermore, by 2012, approximately 60% of transactions from the 300 companies in their data were subject to retail sales tax in the average state. These data suggest that the volume of taxable online shopping may now exceed that of tax-free sales. As states have expanded the definitions of nexus and as more online firms are entering into local markets, these numbers will continue to rise rapidly over time.

This upward trend of increased taxation of e-commerce is a result of several factors. First, business practices are evolving and many large online retailers believe that consumers value rapid order fulfillment, which requires firms to have distribution centers in close proximity to markets, a consequence of which is that these firms establish nexus. Second, states are actively working to simplify their tax systems – as exemplified by the Streamlined Sales and Use Tax Agreement – so as to ease compliance costs for all vendors, including remote vendors. Third, several states have broadened the definition of what constitutes nexus, such as by passing affiliate nexus or click-through-nexus laws, and are acting more aggressively to enforce taxes on online transactions. Fourth, as technology continues to evolve, states are finding new ways to monitor commerce for tax purposes.[7]

[6] Zodrow (2006) shows that tax exemption of electronic commerce is unlikely to be optimal.

[7] Although tangential to our application, there are similarities to the rest of the world. European Union (E.U.) countries recently reformed taxation of digital products. One simplification designed to

Despite the importance of these changes for taxation, however, no theoretical analyses have yet examined the implications of the increase in *taxable* remote transactions. The analysis below helps to initiate investigation of this important topic.

2 Model Setup

2.1 Geography

We construct a model of commodity tax competition with two local jurisdictions and two taxed commodities. Our model substantially expands Nielsen (2001), which is a simpler variant of the classic Kanbur and Keen (1993) model.[8] We postulate a monocentric region centered on point 0 with a radius of length H, where all transportation that occurs is radial. There are two spatially proximate local jurisdictions located on a Hotelling line segment on the interval $[0, H]$, with one jurisdiction extending from 0 to $h > \frac{H}{2}$ and the other jurisdiction running from h to H. For simplicity, we sometimes call the larger jurisdiction "the city" and the smaller jurisdiction "the hinterland." In general, however, the large jurisdiction need not be a city, but could be any centrally located jurisdiction in a metropolitan area, containing ample shopping opportunities. The small jurisdiction is a peripheral jurisdiction that could lie outside of the metropolitan area or could be a smaller suburb. Within the region, the distribution of residents is uniform such that the population of the city is h and the population of the hinterland is $H - h$.

2.2 Goods, Firms and Consumer Decisions

2.2.1 The Standard Model: Only Non-specialized Goods

The model features two types of goods: a "non-specialized" good and a "specialized" good. The model has three types of firms: "Type A" (selling non-specialized goods) and "Type B" and "C" (selling specialized goods using traditional or online methods, as discussed below). As in Kanbur and Keen (1993), the Type A firms selling non-specialized goods are potentially located everywhere along the line segment, and never sell these goods online. We think of non-specialized goods as being taxable commodities available in local general stores, convenience stores, or pharmacies. These commodities might include household items like paper towels, taxable food purchases, liquor and cigarettes, gasoline or beauty products.

achieve destination taxation is that the E.U established a Mini One-Stop-Shop where firms file one value added tax return in their home country, reporting transactions to all E.U. countries. The home country disperses tax revenue appropriately. The net effect is to reduce enforcement and compliance costs.

[8]Others include Mintz and Tulkens (1986), Braid (1987), de Crombrugghe and Tulkens (1990), Braid (1993), Lockwood (1993), Haufler (1996), Haufler (1998), Wang (1999), Lockwood (2001), Devereux, Lockwood and Redoano (2007), Keen and Konrad (2013), Aiura and Ogawa (2013), Braid (2013), Kessing and Koldert (2013), and Agrawal (2015).

Supply and demand of the non-specialized good is modeled simply. Type A firms are characterized by perfect competition such that there is free entry and exit; as a result firms can potentially locate at any point along the line segment. In equilibrium, however, the distribution of firms need not be uniform, as the supply of commodities will respond to consumer demand. Given perfect competition, pre-tax prices will be set equal to marginal cost, which we normalize to 1. We suppose that each consumer's demand for the non-specialized good is perfectly inelastic and normalized to one unit. Consumer choice arises over where to purchase the good. Each household may buy the good from firms where it resides, in which case, it incurs no transportation cost and simply pays the tax-inclusive price in its home locality. The household may also cross-border shop, in which case it buys the good from the first non-specialized store over the border and incurs a transportation cost of d per mile resulting in a total transport cost of dx_N where x_N is the distance traveled by the consumer to buy the non-specialized good. Such cross-border transactions are taxed at the point of sale. Denote the local sales tax rate of the large and small localities as T and t, respectively.[9] When $T \geq t$, a resident of the city will then cross-border shop for the non-specialized good if the total cost, inclusive of transportation, is cheaper abroad than at home, if

$$x_N \leq x_N^* := \frac{T - t}{d}, \tag{1}$$

where x_N^* is defined as the critical distance from the border for which a resident will engage in cross-border shopping for the non-specialized good when $T \geq t$. A similar cutoff rule can be defined for residents of the hinterland if $T < t$; they will cross-border shop if they are at a distance $x_N \leq= \frac{t-T}{d} = -x_N^*$ units from the border. The geography of the model with some online shopping and $T \geq t$ is shown in figure 4.

2.2.2 More General Model: Specialized Goods and Agglomeration

Although appealing for their elegance, Kanbur and Keen (1993) style models – which were developed for studying commodity tax competition across countries – may not be entirely appropriate for studying local commodity tax competition.[10] While goods like food, gasoline, and other convenience store purchases may be available in many towns in the United States from the local "general store," many goods cannot ultimately be purchased locally (in one's place of residence). Instead, consumers must often drive to

[9] Recall that the enforcement of use taxes on local cross-border transactions is extremely difficult and notoriously under-enforced. Even with technological innovations that help the tax authority, it is likely that taxation of these local cross-border purchases will remain effectively origin based.

[10] Janeba and Osterloh (2013) and Koethenbuerger (2011) argue it is important to appropriately modify models of tax competition when considering municipal and county level governments.

larger jurisdictions to avail themselves of larger shopping malls containing what we call "specialized goods." To model local commodity tax competition, we introduce an urban spatial structure and a composite "specialized" good to the model, which is taxable at the same rate as the non-specialized good. Examples of the specialized good might include large electronic purchases, automobile purchases, department store clothing, or other larger "high-end" non-perishable products. In countries with a value added tax, these specialized goods might also include symphony or opera tickets. Specialized goods can be purchased from Type B firms (brick-and mortar) or Type C firms (online vendors).

The specialized brick-and-mortar firms (Type B firms) only locate in the city, consistent with the stylized fact that larger jurisdictions often have major shopping centers or districts. We assume that many specialized firms locate at the center of the metro area – or point 0. Cities have historically flourished as focal points for high-end department stores (e.g., Selfridges revolutionized the department store industry when it established a store in downtown London on Oxford Street in 1909). Although we have called the large jurisdiction in our model "the city", our model would be equally applicable to studying suburban-rural tax interactions. Thus, our model now adds a component of realism beyond the role of jurisdiction size: many goods cannot be purchased where one lives; consumers must travel to large shopping agglomerations to obtain some goods.

The exogenous location of the agglomeration at point 0 merits further discussion. Up until now, we have presented the model as a two-jurisdiction model, but it applies to a monocentric urban area. First, all of the results in our paper generalize, by symmetry, to the case where the city runs from $-h$ to h and is surrounded by identical suburbs on both its western border ($-H$ to $-h$) and eastern border (h to H). The western and eastern suburb each set identical tax rates, the city is now twice as large, and none of the analysis below requires modification. Thus, we consider our model running from 0 to H to be an algebraic simplification of a more general model that runs from $-H$ to H with tax competition occurring on both sides of the city. Generalizing further, the model can be interpreted as a monocentric city model with a uniform population distribution where all transportation is radial, such that various hinterland communities do not compete with each other for non-specialized consumers. Appendix A.1 shows a formal proof. Against this backdrop, the central point 0 represents a central business district for retail shopping. Figure 1 provides support that retail firms disproportionately locate in major cities.

Like the non-specialized good, we model supply and demand of the specialized good as simply as possible. Type B firms are characterized by perfect competition such that there is free entry and exit; although these firms can only locate at point zero, they do not have market power. This assumption can be rationalized by a large retail shopping center or district (such as a mall, or a row of car dealers) composed of many individually small vendors selling many similar products competitively. Given perfect competition pre-tax prices will be set equal to marginal cost, which we again normalize to 1.

With respect to demand, consumers generally purchase some fixed amount $S > 0$ units of the specialized (composite) good.[11] As a technical matter, we will need to introduce an elasticity condition on the specialized good necessary to guarantee existence of an equilibrium. Although introduced for technical reasons, this assumption is, in fact, economically more natural than the customary assumption that demand is perfectly price inelastic. Defining P_S as the tax-inclusive price and letting $\overline{P_S} > 0$ denote a threshold price, each household's demand for the specialized goods is given by $S(P_S)$, where $S(P_S) = S$, a constant, for all $P_S \leq \overline{P_S}$ and where, for all $P_S \geq \overline{P_S}$, we assume that $\epsilon_S := d\log S(P_S)/d\log P_S \leq 0$ is "sufficiently small" (i.e., sufficiently elastic). As discussed in Appendix B.5 below, $\epsilon_S \leq -1$ is sufficient (but not necessary) for our results.[12]

In the absence of online shopping, all consumers must drive to the city center to purchase the specialized good. Letting x_S denote the distance to the city center, and thus the household's location on the line, they incur a transportation cost of D dollars per mile such that the total cost is Dx_S. These transportation costs include other transaction-related costs including search costs or may include delivery costs for a larger purchase transported by the vendor and for this reason it need not equal d.

2.2.3 The Most General Model: Adding Online Shopping

To make the model appropriate for studying online shopping, we add a third category of firms (Type C) that sell the specialized good online.[13] We focus on one type of online shopping – purchases from online firms with nexus in the state of residence. Thus, type C firms have a physical presence within the state containing the city and hinterland we are studying; however, consumers have the option to buy from the firm's website.[14] Because the firm has nexus in the state, it must collect taxes based on where it ships the

[11] Consumers cannot bundle trips for the specialized good; in particular, they do not pick up the non-specialized good on their way to buy the specialized good. This is justifiable when the specialized and non-specialized products are significantly different, for instance due to perishability, durability, and other product attributes that result in trips that cannot easily be combined.

[12] Recall that S is a composite of many heterogeneous commodities. The demand for each may be a decreasing function of the tax imposed on it. These commodities may have different reservation prices, some lower than others. Then, ϵ_S captures the reduction in demand for the composite good as taxes rise. In order to lighten the notational burden, when writing out the revenue functions and sketching figures subsequently, we treat S as a constant and omit explicit reference to regions above the threshold $\overline{P_S}$; all proofs, however, account explicitly for the general form of the demand function.

[13] Despite rapid technological change, theoretical models of tax competition have not considered the effect of online shopping on tax rates (with the exception of an empirical study: Agrawal (2016)).

[14] The type C firms may simply represent the online operations of the same type B firms that operate at the city-center; this helps to justify our assumptions on tax compliance.

good (according to the destination principle) as discussed in section 1.[15] Furthermore, we assume that only the specialized good can be purchased online. This is consistent with the stylized fact that most online shopping does not include routine smaller-ticket purchases like groceries, gasoline and the like. Type C firms sell specialized goods from online platforms and the consumer incurs a cost of E to obtain the product.[16]

With this structure in hand, we can solve the consumer problem in the presence of online shopping. Up until now, consumers had to purchase the specialized good in the city. Now the consumer can buy the specialized good online. Residents of the city choose where to purchase the specialized good – whether from online or brick-and-mortar vendors – but these consumers pay the city tax rate in either case. Thus, for purposes of the city's tax revenue function, where consumers in the city buy the specialized good is irrelevant. For completeness, note that a city household will buy online if $(1+T)S+E < (1+T)S+Dx_S$. A hinterland resident faces a more complex problem. If the hinterland household drives to the city it will pay a tax rate of T but if the purchase is made from an online vendor, the vendor collects the tax based on the household's location, which means that it pays a tax rate of t. This implies a simple cutoff rule where the consumer will buy online if

$$x_S \geq x_S^* := \frac{E + (t - T)S}{D} \tag{2}$$

where x_S^* is defined as the critical distance from the city center for which a resident will engage in online shopping for the specialized good. Then, the geography of the model, assuming that $T \geq t$ and some online shopping occurs is given by figure 5.

2.3 Governments and Tax Competition

As is common in this literature, governments are assumed to maximize tax revenue and to set local tax rates in a Nash non-cooperative game. The early literature was characterized by general models (see for example, Mintz and Tulkens (1986), Haufler (1998)), with few restrictions on consumer preferences and and government objective functions. However, in the absence of simplifying assumptions, it is difficult to obtain clear-cut results or to ensure existence of the equilibrium. Given that we have added a second good, an urban spatial structure, and online shopping, we follow the approach taken in the models of Nielsen (2001) and Kanbur and Keen (1993) and maintain the now standard assumption

[15] Although online transactions from firms with nexus in the state are sourced at destination, cross-border shopping trips still result in taxes being sourced at the location of purchase. This is consistent with the regime in the United States: online stores with nexus collect at destination but enforcing destination taxation on local cross-border shopping trips is prohibitively costly.

[16] This cost could include shipping costs, but E could also incorporate any search costs of online shopping, the costs of Internet access, etc.

of revenue maximization. This allows us to derive closed-form solutions, verify existence, and conduct comparative statics.

Although revenue maximization is certainly not the only possible specification of government objectives, we note that various justifications have been suggested for it. One is a welfarist interpretation, where consumers place high marginal valuation on public expenditures (Kanbur and Keen 1993). A second perspective is that governments act as Leviathans attempting to extract maximal rents from their taxpayers. In a variant on this approach, bureaucrats, politicians, or the constituencies that they represent, seek to maximize their budgets. Although these political economy considerations are important, they are not our main focus; our primary goal is to analyze how *changing technologies* may affect local government policies. For our purposes, revenue maximization is one comparatively simple hypothesis that allows us to tackle this problem. We have no reason to believe that other possible approaches, such as a median voter model or welfare maximization, would qualitatively change the nature of our results, although this remains an interesting open question for future research.[17]

We also assume that governments compete in a Nash non-cooperative game. Local commodity taxation is a scenario where local pairwise strategic interactions are naturally expected to arise because of the inherently spatial nature of the problem.[18] Indeed, the empirical evidence suggests that nearby local jurisdictions are rational competitors with respect to their local sales tax rates (Agrawal 2015).

Having solved the consumers' problem of where to purchase the goods, the revenue functions can be constructed. One must recognize, however, that either locality may choose any non-negative tax rate, and that for some choices of these tax rates, either (1) all hinterland residents may shop online for the specialized good, (2) no hinterland residents may shop online for the specialized good, or (3) some hinterland residents may buy the specialized good online. Furthermore, for some (very high) tax rates, (4) all city residents may purchase the nonspecialized good in the hinterland, (5) all hinterland residents may purchase the nonspecialized good in the city, but for reasonable rates, (6) some but not all residents of one of the localities will cross-border shop for the non-specialized goods. To encompass all of these possibilities simultaneously, it is helpful to express each of the two components of the tax base for each locality, that is, the volume of taxable sales of the nonspecialized good and the volume of taxable sales of the specialized good, in a general form. Specifically, let B_N and B_S denote the volume of taxable sales for each of the two types of commodities for the large jurisdiction and let b_N and b_S denote the corresponding bases for the small jurisdiction. For sufficiently low or sufficiently high

[17] Trandel (1994) shows that welfare maximization preserves the results in Kanbur and Keen (1993).

[18] Pairwise strategic interactions are less likely in cases where governments compete for globally-mobile resources like capital (Wilson 1986; Zodrow and Mieszkowkski 1986).

tax rates, a locality can attract or repel all transactions involving the non-specialized good, and, similarly, there are lower and upper rates at which a locality attracts or repels maximal amounts of transactions involving specialized commodities, which are implicitly defined by the cutoff values of x_N^* and x_S^*. Using the definition of x_N^* given in (1), for the non-specialized good we obtain tax bases:

$$B_N(T,t) = \begin{cases} H & \text{if } x_N^* \leq -(H-h) & \text{case "all city"} \\ h + \frac{t-T}{d} & \text{if } x_N^* \in [-(H-h),\, h] & \text{case "interior"} \\ 0 & \text{if } x_N^* \geq h & \text{case "all hinterland"} \end{cases} \tag{3}$$

and, because $B_N + b_N = H$, we may equivalently define

$$b_N(t,T) = \begin{cases} 0 & \text{if } x_N^* \leq -(H-h) & \text{case "all city"} \\ H - h + \frac{T-t}{d} & \text{if } x_N^* \in [-(H-h),\, h] & \text{case "interior"} \\ H & \text{if } x_N^* \geq h & \text{case "all hinterland"} \end{cases} \tag{4}$$

for the small locality. The middle branch of (3) says that the non-specialized tax base of the city is equal to its population plus (if $t \geq T$) or minus (if $T \geq t$) the number of people who cross-border shop. The first observation is that cross-border shopping can occur in either direction depending on the pattern of tax rates; the model places no restriction on this. In particular, some people in a jurisdiction will cross-border shop and others will buy at home for all values of the tax rates where x_N^* is between the (negative of) the hinterland population and the city population (case: interior"). If the tax rate in the city falls very far below the hinterland tax rate, then the first branch says that the city can obtain all of the non-specialized tax base (case: "all city"). The final branch says that if the city tax rate rises very far above the hinterland tax rate, then the entire non-specialized base is lost by the city (case: "all hinterland"). When $S = 0$, these corner solutions can be assumed away, but we cannot do that given the presence of the second good.

For the specialized good, assuming for exposition that the specialized good is always purchased in the fixed amount S, the tax base of the city is given by

$$B_S(T,t) = \begin{cases} SH & \text{if } x_S^* \geq H & \text{Regime I: "Past"} \\ S\left(\frac{E+(t-T)S}{D}\right) & \text{if } x_S^* \in [h,\, H] & \text{Regime II: "Present"} \\ Sh & \text{if } x_S^* \leq h & \text{Regime III: "Future"} \end{cases} \tag{5}$$

and for the hinterland we have

$$b_S(t,T) = \begin{cases} 0 & \text{if } x_S^* \geq H \qquad \text{Regime I: "Past"} \\ SH - S\left(\frac{E+(t-T)S}{D}\right) & \text{if } x_S^* \in [h,\, H] \qquad \text{Regime II: "Present"} \\ S(H-h) & \text{if } x_S^* \leq h. \qquad \text{Regime III: "Future"} \end{cases} \tag{6}$$

Unlike the non-specialized good, each of the branches for the specialized good has important economic meaning which we divide up to discuss the model's applicability to the "past", "present", and "future." Consider the upper branches of (5) and (6). Keeping in mind that x_S^* is a function of the cost of buying online, this branch says that if the cost of buying online is especially high, then no one will do so. Rather, everyone will drive to the city center to buy S units of the good. This upper branch corresponds to a "past" regime when online shopping was not possible or was prohibitively costly. The middle branch characterizes a mixed tax base where some of the hinterland residents – in particular, those with the largest costs of driving to the city center – buy online but where some of the residents still buy at the city center. Using the definition of the cutoff rule in (2), this case will arise if $Dh + (T-t)S \leq E \leq DH + (T-t)S$. Intuitively, the this regime will only occur if the cost of buying online is at some intermediate value. This branch corresponds to the "present" regime where some people buy online while others don't. The final branch says that if online shopping costs are sufficiently low, then everyone in the hinterland will buy online. In turn, the city obtains revenue only from its residents while the hinterland obtains tax revenue from all of its residents. This corresponds to a "future" regime where people commonly buy online.[19] The past regime could arise simply because online shopping did not exist, and the future regime could arise if online shopping drives all specialized brick-and-mortar retailers out of business. To fix notation define the branches of the specialized tax base, as B_S^r and b_S^r, where $r = I,\, II,\, III$ indexes the regimes given in (5) and (6). We denote each of these regimes based on their chronological ordering: regime I denotes the past, regime II denotes the present and regime III denotes the future.

We observe that the tax base functions are each continuous and piecewise linear functions of the tax rates that are decreasing in the own-jurisdiction tax rate and increasing in the neighboring jurisdiction's tax rate. We define revenue functions for all tax

[19]The past regime will arise if $E > DH + (T-t)S$ and the future regime will arise if $E < Dh + (T-t)S$. Of course, as explained below, these regimes are also determined – endogenously – by the tax rates chosen by the competing localities, *conditional* on the technological parameters.

rates $(T,\, t) \in \mathbb{R}^2_+$ and for all parameter values $\pi = (h,\, H,\, d,\, D,\, E,\, S) \in \mathbb{R}^6_+$ as

$$R(T, t, \pi) = T(B_N[T, t, \pi] + B_S[T, t, \pi]) \tag{7}$$

$$r(t, T, \pi) = t(b_N[t, T, \pi] + b_S[t, T, \pi]). \tag{8}$$

To characterize the Nash non-cooperative equilibria of this model, we proceed sequentially by first solving the model without a specialized good. Then, we introduce the specialized good and solve the model for each of the past, future, and present regimes.

3 Solution with Tax Competition

3.1 Basic Model: The Past with No Agglomeration

To introduce our model and to relate it to the prior literature, assume for the moment that $S = 0$. Our model then reduces to a variant of Kanbur and Keen (1993) and Nielsen (2001). Noting that an equilibrium cannot exist if one jurisdiction has no non-specialized base, the revenue functions for the large and small jurisdictions are, respectively.

$$\begin{array}{rcl} R(T, t, \pi) & = & T\left(h + \frac{t-T}{d}\right) \\ r(t, T, \pi) & = & t\left(H - h + \frac{T-t}{d}\right), \end{array} \tag{9}$$

for the large and small jurisdictions. This is illustrated in figure 4 assuming that $T > t$. With only non-specialized firms, the Nash equilibrium is superscripted by N:

$$\begin{array}{rcl} T^N & = & \frac{1}{3}d(H + h) \\ t^N & = & \frac{1}{3}d(2H - h), \end{array} \tag{10}$$

which implies that the tax differential is

$$\Delta^N \equiv T^N - t^N = \frac{1}{3}d(2h - H) > 0. \tag{11}$$

Equilibrium revenues in this case are

$$\begin{array}{rcl} R^N(T^N, t^N, \pi) & = & \frac{1}{9}d(H + h)^2 \\ r^N(t^N, T^N, \pi) & = & \frac{1}{9}d(2H - h)^2. \end{array} \tag{12}$$

Proposition 1. *When $S = 0$ and consumers only purchase non-specialized goods, the city sets a higher tax rate than the hinterland and the city collects more tax revenue.*

Proof. The tax differential is $\Delta^N = \frac{1}{3}d(2h - H) > 0$ because $h > \frac{H}{2}$. With respect to tax revenues, $\frac{1}{9}d(H + h)^2 - \frac{1}{9}d(2H - h)^2 = \frac{1}{3}d(2hH - H^2) > 0$ because $2h > H$. □

As already shown in previous literature, the larger jurisdiction sets a higher tax rate than the smaller jurisdiction. The intuition follows a standard Ramsey-type rule: starting from equal tax rates, the large jurisdiction perceives a smaller elasticity of its tax base and sets a higher tax rate. While there have been many extensions to the standard model – such as making the distribution of residents non-uniform, considering multiple jurisdictions, and assuming welfare maximization – the results of the standard Kanbur and Keen (1993) model have largely been preserved. With this simple case as a reference point, we now investigate the implications of major departures – agglomerations and online shopping – from the standard formulation.

3.2 Nash Equilibria: Analysis of Regimes I and III

Although the revenue functions 7 and 8 cover all possible configurations of shopping patters, we are especially interested in isolating specific cases. For instance, from a historical perspective, it is interesting to study regime I (a world with no online shopping). In particular, one would expect that there exist parameter configurations (e.g., extremely high values of E in relation to D, corresponding to a world with no Internet or highly costly remote transaction technologies), in which no remote transactions would be observed in equilibrium. Identifying values of the parameters that make remote transactions "prohibitively costly" is difficult because the equilibrium regimes depend partly on the tax policies set by the jurisdictions, which are endogenous. However, there are clearly values of E/D sufficiently high, conditional on other parameters, that ensure that no locality would or could choose tax policies that would drive the system away from regime I. The same is true for regime III when E/D is sufficiently small; this regime is also interesting because it corresponds to a hypothetical future where online shopping predominates for important categories of commodities. These two extreme cases, with sufficiently high or low values of E/D, are much simpler to analyze than regime II (the present) where both transaction technologies are simultaneously in use, even though government policies could hypothetically drive the system to the extreme outcomes of regime I or III.

In the next two sections, we are going to talk about equilibria where only regime I or III is possible. When considering these specific equilibria of the past and future, we characterize a "regime-conditional" Nash equilibrium in which all shopping for the specialized good occurs in the large jurisdiction ("past") or all shopping of the specialized good, for residents of the small jurisdiction, occurs online ("future"). Formally we define:

Definition 1. Given a parameter vector π that makes it impossible for any regime other than either regime I or III to arise, a *regime-conditional Nash equilibrium* is a pair of tax rates $(T^*[\pi], t^*[\pi])$ such that

$$T^*[\pi] = \text{argmax}_{<T>} T(B_N[T, t^*[\pi], \pi] + B_S^r[T, t^*[\pi], \pi]) \tag{13}$$
$$t^*[\pi] = \text{argmax}_{<t>} t(b_N[t, T^*[\pi], \pi] + b_S^r[t, T^*[\pi], \pi]) \tag{14}$$

where the non-specialized bases are defined in (3) and (4) and the specialized base is the branch of (5) and (6) corresponding to regime $r = I$ or III.

This definition says that the parameter values of the model are such that a given regime is the only one possible. Of course, not all parameter values are such that they make other regimes infeasible for all conceivable tax policies; this is discussed later.

3.2.1 The Past with Agglomeration

In this section, we assume that E/D is so high that no online shopping arises, irrespective of tax rates, focusing on a regime-conditional equilibrium. To characterize the equilibrium when all specialized shopping occurs in the city, we consider only the upper branches in (5) and (6), along with the non-specialized bases. The revenue functions simplify to

$$\begin{array}{rcccl} R(T,t,\pi) & = & T\left(B_N + B_S^I\right) & = & T\left(B_N + SH\right) \\ r(t,T,\pi) & = & t\left(b_N + b_S^I\right) & = & t\left(b_N\right). \end{array} \tag{15}$$

In our opinion, the most interesting solution is when the city or hinterland have some (but not all or none) of its residents cross-border shopping the nonspecialized good. This occurs when $x_N^* \in [-(H-h),\, h]$. In the appendix C.1, we show that such equilibria exist for sufficiently high values of E and values of S, H, and h satisfying $SH < H + h$. Given that such equilibria exist, we can obtain closed-form solutions for the Nash equilibrium tax rates. First, we obtain the relevant portions of the best response functions, which are $T = \frac{1}{2}(dSH + dh + t)$ and $t = \frac{1}{2}((H-h)d + T)$.[20] These are solved simultaneously for the Nash tax rates denoted $(T^I,\ t^I)$:

$$\begin{array}{rcl} T^I & = & \frac{1}{3}d(H+h) + \frac{2}{3}dSH = T^N + \frac{2}{3}dSH \\ t^I & = & \frac{1}{3}d(2H-h) + \frac{1}{3}dSH = t^N + \frac{1}{3}dSH. \end{array} \tag{16}$$

We now discuss some of the properties of these equilibria and their revenue implications. First, it is interesting to note that the tax rates and the tax differential can be expressed in terms of the equilibria from the standard model (see section 3.1) without any specialized

[20] It is clear from the best response functions that $S > 0$ shifts the large jurisdiction best response function upward as the large jurisdiction can now exploit its even "larger" size agglomeration. The other portions of the piecewise best response function are ruled out in the Appendix.

goods or agglomeration. The tax differential now becomes

$$\Delta^I \equiv T^I - t^I = \frac{1}{3}d(2h - H) + \frac{1}{3}dSH = T^N - t^N + \frac{1}{3}dSH > 0 \tag{17}$$

which is larger than the tax differential in the standard model. Equilibrium revenues are

$$\begin{aligned} R^I(T^I, t^I, \pi) &= \tfrac{1}{9}d(2HS + H + h)^2 \\ r^I(t^I, T^I, \pi) &= \tfrac{1}{9}d(HS + 2H - h)^2. \end{aligned} \tag{18}$$

Proposition 2. *In the past regime, when online shopping is costly (E/D sufficiently high), the equilibrium tax rates are characterized by the city setting a higher tax rate than the hinterland. As the specialized good becomes more important (as S increases), the city and hinterland tax rates both increase, but the rate differential widens. Revenues are larger in the city than in the hinterland and both are increasing in the magnitude of the specialized good.*

Proof. These results follow immediately from inspection of the equilibrium tax rates, tax differential, and revenues, noting that the revenue differential depends on $(2HS + H + h) - (HS + 2H - h) = 2h - H + SH > 0$ and recalling that $2h > H$. $\square$

Note that (16) indicates that although the hinterland tax rate increases relative to (10), it increases by less than the city tax rate, which in turn magnifies inter-jurisdictional tax differentials. Intuitively, the agglomeration in the city gives it an added size advantage which puts upward pressure on its tax rates. The competitive pressures in the game allow the hinterland to also raise its tax rate, but not by as much. Starting from the equilibrium tax rates in the standard model given in (10), the revenue elasticity in the city is substantially less than one, which implies the city should increase its tax rate. This higher tax rate results in more city residents cross-border shopping to buy the non-specialized good in the hinterland. Given this inflow of cross-border shopping, the hinterland can also increase its tax rate to export some of its tax burden. However, because the effects of tax competition are second order, the small jurisdiction increases its tax rate by less. For this reason, and because the taxes on the specialized good are captured entirely by the city, the pattern of tax revenues results in the city collecting more revenues than the hinterland.

To summarize, big towns can "tax export" some of the tax burden to non-residents and capture "agglomeration rents," but agglomeration affects the tax rate (revenues) of the small town; changes in shopping patterns for non-specialized goods also allow the small jurisdiction to engage in tax exporting. Although a small literature on capital tax competition[21] indicates that agglomeration makes it harder for firms to relocate in response to tax increases, these agglomeration channels have been entirely omitted from commodity tax competition models, in part due to their focus on international rather than local tax competition. Thus, although the empirical literature has studied whether mobility is lower in the presence of agglomeration,[22] our model suggests that the tax differential does not rise as much as it would because of the presence of pairwise strategic interactions.

3.2.2 The Future: Extensive Online Shopping

In this section, we assume that E/D is so low that everyone in the hinterland will always want to buy online regardless of the tax rates. In a regime-conditional equilibrium, the cost of online shopping is sufficiently low that the upper two branches of (5) and (6) are no longer relevant at very low values of E. This could also arise if D were to rise substantially due to congestion or other travel costs. We search for a regime-conditional equilibrium assuming that E is sufficiently low that the city or hinterland could not deviate to the past or present, so that the revenue functions simplify to

$$\begin{array}{rcccl} R(T,t,\pi) & = & T\left(B_N + B_S^{III}\right) & = & T\left(B_N + Sh\right) \\ r(t,T,\pi) & = & t\left(b_N + b_S^{III}\right) & = & t\left(b_N + S(H-h)\right). \end{array} \tag{19}$$

Again, in our opinion, the most interesting solution is when the city or hinterland have some but not all of its residents cross-border shopping, which occurs when $x_N^* \in [-(H-h),\, h]$. In the appendix C.2, we show that such equilibria exist for sufficiently low values of E and values of S, H, and h satisfying $S(2h-H) < H+h$. Given that such equilibria exist, we can again obtain closed-form solutions for the Nash equilibrium tax rates using the relevant portions of the best response functions. These are again solved simultaneously for the Nash tax rates in the future regime, denoted by $(T^{III},\, t^{III})$,

$$\begin{array}{rcl} T^{III} & = & \frac{1}{3}d(H+h) + \frac{1}{3}dS(H+h) = T^N + \frac{1}{3}dS(H+h) = T^I - \frac{1}{3}dS(H-h) \\ t^{III} & = & \frac{1}{3}d(2H-h) + \frac{1}{3}dS(2H-h) = t^N + \frac{1}{3}dS(2H-h) = t^I + \frac{1}{3}dS(H-h). \end{array} \tag{20}$$

In the future, the Nash tax rates nest the formulae for the tax rates in the prior regime.

[21]See Kind, Knarvik and Schjelderup (2000), Borck and Pflüger (2006) and Baldwin and Krugman (2004).

[22]See Brülhart, Jametti and Schmidheiny (2012) for a study relating to capital tax competition. We know of no empirical studies concerning the role of agglomeration on commodity tax rates although Burge and Rogers (2011) show that large regional centers are important.

Using this observation, the tax differential is

$$\Delta^{III} \equiv T^{III} - t^{III} = \frac{1}{3}d(1+S)(2h-H) = \Delta^N + \frac{1}{3}dS(2h-H) = \Delta^I - \frac{2}{3}dS(H-h). \quad (21)$$

Revenues in the future equilibria are

$$\begin{aligned} R^{III}(T^{III}, t^{III}, \pi) &= \tfrac{1}{9}d((H+h)S + H + h)^2 \\ r^{III}(t^{III}, T^{III}, \pi) &= \tfrac{1}{9}d((2H-h)S + 2H - h)^2. \end{aligned} \quad (22)$$

Proposition 3. *In the future, when online shopping is cheap (E/D sufficiently low), the equilibrium tax rates are characterized by the city setting a higher tax rate than the hinterland. As the specialized good becomes more important (S increases), both the city and hinterland tax rate increase, but the rate differential widens. Revenues are larger in the city than in the hinterland and both are increasing in the magnitude of the specialized good.*

Proof. The tax differential is given by $\Delta^{III} = \frac{1}{3}d(S+1)(2h-H) > 0$. Comparative statics show $\frac{\partial T^{III}}{\partial S} = \frac{1}{3}d(H+h) > \frac{\partial t^{III}}{\partial S} = \frac{1}{3}d(2H-h) > 0$. These inequalities follow from $2h > H$. With respect to revenues, both terms in the parenthesis in (22) are positive and $((H+h)S + H + h) - ((2H-h)S + 2H - h) = (S+1)(2h-H) > 0$. □

Relative to a world when online shopping is impossible, it is clear from (20) that tax rates in the city are lower and tax rates in the hinterland are higher. Thus, Internet penetration in the presence of taxable online sales *reduces* the inter-jurisdictional tax differential. Part of this reduction occurs by the city lowering its tax rate. The presence of online sales lowers the city's "comparative advantage" in the tax competition game – making it the relative loser (in terms of the level of its tax rate) in this game. However, despite this, the city still collects more tax revenue than the hinterland. This is because the city tax rate remains higher than the hinterland and because its specialized tax base is larger because of its geographic size advantage.

3.3 Nash Equilibria in Regime II: The Present

In this section we focus on values of E/D such that, in equilibrium, both transaction technologies are *simultaneously* in use. However, for certain parameter configurations, government policy could hypothetically extinguish brick-and-mortar or online transactions, and we must therefore consider the possibility that either government could choose a tax policy that would cause the system to shift away from the shopping patterns of the "present."

Definition 2. Given a parameter vector π, a *Nash equilibrium* is a pair of tax rates $(T^*[\pi], t^*[\pi])$ such that

$$T^*[\pi] = \text{argmax}_{<T>} R(T; t^*[\pi], \pi) \tag{23}$$

$$t^*[\pi] = \text{argmax}_{<t>} r(t; T^*[\pi], \pi), \tag{24}$$

where revenues are defined by (7) and (8).

When E/D is at an intermediate value, the middle branches of (5) and (6) apply and characterize an equilibrium with some online shopping. In particular, in regime II, some residents of the hinterland buy the good online while some residents buy the specialized good at the city center. (Of course, some residents of the city may also shop online, but from the perspective of the city government, the same tax is collected for its residents regardless of whether online or from the brick-and-mortar stores at the city center.) We continue to focus on cases where the non-specialized base is "interior."

We relegate a formal proof of existence of Nash equilibria – subject to some restrictions on parameter values – to Appendix B. Figure 6 shows the intuition for a particular vector of parameters. From this figure, it is easy to see that the global revenue maximum – holding fixed the other jurisdiction's tax rate at its equilibrium value – is in the present regime. In the case of the large jurisdiction, we utilize the elasticity condition on the specialized base, which rules out the explosive part of the revenue function where the large jurisdiction's residents constitute a captive base giving it an (unrealistic) incentive to raise its tax rate without meaningful limits.

To state our main existence proposition, define $\hat{\pi} = (h, H, d, D, E) \in \mathbb{R}^5_+$ and let $\hat{\pi}(\lambda) = (h, H, \lambda d, \lambda D, \lambda E)$ denote a vector in which λ scales the transaction cost parameters, preserving their relative values.

Proposition 4. *There exists a vector of parameter values $\hat{\pi}^0 = (h^0, H^0, d^0, D^0, E^0) \in \mathbb{R}^5_{++}$ such that, for every $S \in [0, 1]$ and for every $\lambda > 0$, there exists a unique regime-II Nash equilibrium $\forall\ \hat{\pi} = \hat{\pi}^0(\lambda)$ in which some but not all residents of the small jurisdiction buy the specialized good online and some but not all residents cross-border shop for the non-specialized good. Furthermore, for some number $\epsilon > 0$, and for every $S \in [0, 1]$ and every $\lambda > 0$, there exists a unique regime-II Nash equilibrium for all $\hat{\pi}$ such that $\|\hat{\pi} - \hat{\pi}^0(\lambda)\| < \epsilon$, i.e., for all points in the parameter space within an ϵ-ball around $\hat{\pi}^0(\lambda)$.*

Proof. See appendix B.5. The second statement follows from the first as an application of the implicit function theorem. This follows because the Jacobian of the two-equation system of first-order conditions characterizing the equilibrium is non-vanishing. □

Although we do not establish existence of a regime-II equilibrium for arbitrarily large values of S, we do allow for S to take any value in the interval $[0,1]$. In economic terms, this means that we permit household consumption of specialized goods to be as large as consumption of the non-specialized goods ($S = 1$), and by including $S = 0$ we encompass the standard model discussed in section 3.1. Furthermore, although we do fix a parameter vector of the three transaction costs, we need not restrict their levels, only their relative values. Intuitively, absolute sizes and absolute transactions costs are not the fundamental economic parameters; relative sizes and relative costs of cross-border shopping and online shopping determine the key features of equilibria. In particular, changes in λ determine the absolute levels of equilibrium tax rates and tax revenues, but not the equilibrium shopping patterns. This feature of our model implies that by selecting an appropriate value of λ, we obtain equilibrium tax rates of any desired scale.

Strictly speaking, the implicit function theorem only ensures that the parameters (h, H, d, D, E) must remain within a "neighborhood" of a particular value $\hat{\pi}^0$. It is worth noting, however, that the region of the parameter space within which a regime-II equilibrium holds is defined by functions which are continuous in the parameter vector $\hat{\pi}$. This a closed and bounded region of positive volume. To attempt to delimit this region in its entirety is a technical and not economically enlightening exercise, which we do not conduct here.[23] We do know, however, from simulations, that these equilibria do exist for wide ranges of parameters. Figure 6 shows one simulation yielding existence of a regime-II equilibrium.[24]

Having shown that Nash equilibria exist and that they are unique (for a given π), we characterize their properties. We obtain closed-form solutions for the equilibrium tax rates using the relevant portions of the best response functions. These are solved simultaneously for the Nash tax rates (T^{II}, t^{II}):

$$\begin{aligned} T^{II} &= \tfrac{1}{3}d\frac{(H+h)+2SH+S\left(\frac{E}{D}-H\right)}{(dS^2+D)/D} = \frac{1}{(dS^2+D)/D}\left[T^I + \tfrac{1}{3}dS\left(\tfrac{E}{D}-H\right)\right] \\ t^{II} &= \tfrac{1}{3}d\frac{(2H-h)+SH+S\left(H-\frac{E}{D}\right)}{(dS^2+D)/D} = \frac{1}{(dS^2+D)/D}\left[t^I + \tfrac{1}{3}dS\left(H-\tfrac{E}{D}\right)\right]. \end{aligned} \tag{25}$$

Noting that the tax rates nest the tax rates of the past regime (and therefore, also the rates in the future regime), the tax differentials between the two jurisdictions becomes

$$\Delta^{II} \equiv T^{II} - t^{II} = \frac{\frac{1}{3}d(2h-H) - \frac{1}{3}dSH + \frac{2}{3}dS\left(\frac{E}{D}\right)}{(dS^2+D)/D} = \frac{1}{(dS^2+D)/D}\left[\Delta^I + \frac{2}{3}dS\left(\frac{E}{D}-H\right)\right]. \tag{26}$$

[23] Appendix C.3 shows that a more general existence proof is not possible by proving that for various parameter configurations, no Nash equilibrium exists in regime I, II or III.

[24] Animations, available from the authors, confirm existence for a wide range of values.

Then, using these tax rates, the equilibrium amount of revenue is given by:

$$\begin{aligned} R^{II}(T^{II}, t^{II}, \pi) &= \tfrac{1}{9} d \frac{(HS + \frac{E}{D} S + H + h)^2}{(dS^2 + D)/D} \\ r^{II}(t^{II}, T^{II}, \pi) &= \tfrac{1}{9} d \frac{(2HS - \frac{E}{D} S + 2H - h)^2}{(dS^2 + D)/D}. \end{aligned} \tag{27}$$

We note several interesting properties of the tax rates.[25] First, as $\frac{E}{D} \to H$, T^{II} is simply proportional to T^{I}, where the factor of proportionality is less than one. As $\frac{E}{D} \to h$, the term in parenthesis T^{II} is simply proportional to T^{III}. Of course, it is also possible that $\frac{E}{D}$ is above H, in which case the relationship to T^{I} is ambiguous because the tax rate contains two offsetting effects. Similarly, for the small jurisdiction, as $\frac{E}{D} \to h$ then the term in parenthesis converges to t^{III}, but the relationship to t^{I} cannot be determined. Again, this ambiguity is driven by two effects: (1) the change of the small jurisdiction tax base and (2) added tax competition. Regarding the first effect, changes in the hinterland specialized base may move in opposite directions of the nonspecialized base. Nonetheless, effect (1) is the familiar result from the literature: tax rates increase as the size of the jurisdiction increases. Regarding effect (2), as soon as the specialized tax base comes into play, it causes a discrete change in the intensity of fiscal competition.

The second effect is driven by the term $\frac{D}{(dS^2+D)}$. This effect is a result of heightened tax competition, as can be seen by considering the fiscal externality that one government imposes on the other. The fiscal externality is given by $\frac{\partial R}{\partial t}$ and $\frac{\partial r}{\partial T}$. In regime I and III, these fiscal externalities are $\frac{\partial R}{\partial t} = \frac{T}{d} > 0$ and $\frac{\partial r}{\partial T} = \frac{t}{d} > 0$ evaluated at the local tax rates. However, in the present regime, the fiscal externalities become

$$\begin{aligned} \frac{\partial R}{\partial t} &= T\left(\frac{1}{d} + \frac{S^2}{D}\right) = \frac{T}{d}\left(\frac{dS^2+D}{D}\right) > 0 \\ \frac{\partial r}{\partial T} &= t\left(\frac{1}{d} + \frac{S^2}{D}\right) = \frac{t}{d}\left(\frac{dS^2+D}{D}\right) > 0, \end{aligned} \tag{28}$$

where each of these expressions contains an additional term relative to regime I and III that is due to competition for online shoppers. The signs of these terms suggest that tax rates are "too low" relative to what a global (revenue maximizing) planner would set. The first term in the fiscal externality ($\frac{1}{d}$) says taxes are too low because each of the governments ignores the fact that lowering its tax rate reduces revenue in the other jurisdiction because of cross-border shopping. Similarly, the second new term ($\frac{S^2}{D}$), says that lowering the tax rate reduces tax revenue in the other jurisdiction due to the

[25] One might consider the Nash tax rates when governments can impose non-uniform taxes on the non-specialized and specialized bases. The nonspecialized rate would always be given by (10). In regime I or III, the specialized base is fixed and the governments would (unrealistically) set tax rates at their highest possible value. More interestingly, in regime II, the Nash tax rates on the specialized base are $(2DH + E)/3S$ and $(2DH - E)/3S$ for the large and small jurisdiction, respectively. Importantly, the signs of the comparative statics with respect to E, presented below, are unchanged under these circumstances.

presence of online shopping. In particular, if the large jurisdiction lowers its tax rate, the hinterland will receive less revenue from online shoppers, while if the hinterland lowers its tax rate, it obtains tax revenue at the expense of the city. This second term results from competition over the specialized tax base, which is not present in the other regimes, and heightens tax competition and the fiscal externality. It shows how responsive – and implicitly how elastic – the own-tax base is to the neighboring tax rate. The larger is this term, the more responsive is the tax base and, following an inverse elasticity rule, the lower in the tax rate. We summarize in:

Proposition 5. *In the present regime, when online shopping is of intermediate cost (intermediate E/D), the Nash equilibrium in tax rates is characterized by the city setting a higher tax rate than the hinterland. Revenues are larger in the city than in the hinterland.*

Proof. The proof hinges on $h \leq \frac{E}{D} \leq 2H$ as shown in appendix B.4.1. Substituting for Δ^I yields $\Delta^{II} = \Delta^I + \frac{2}{3}dS\left(\frac{E}{D} - H\right) = \frac{1}{3}d(2h - H) + \frac{1}{3}dSH + \frac{2}{3}dS\left(\frac{E}{D} - H\right) = \frac{1}{3}d(2h - H) + \frac{1}{3}dS(2\frac{E}{D} - H) > (2h - H) + \frac{1}{3}dS(2h - H) > 0$. Regarding revenue, $(HS + \frac{E}{D}S + H + h) - (2HS - \frac{E}{D}S + 2H - h) = (2h - H) + S(2\frac{E}{D} - H) > 0$. □

Intuitively, although some online shopping occurs by residents of the hinterland (and possibly by residents of the city), the city maintains its size advantage in the tax competition game because, a priori, its non-specialized base is larger and because any residents of the city that engage in online shopping contribute tax revenue to the city.[26] This size advantage then implies, following an inverse elasticity rule, that the city tax rate should be higher than the hinterland. These higher taxes coupled with the preserved size advantage result in more tax revenue in the city than the small jurisdiction. Of course, more interesting are the subsequent comparative statics regarding how the city and hinterland tax rates change as the parameters change.

4 Global Comparisons Across Regimes

The presence of three sequential regimes allows us to compare the equilibrium tax and revenues across regimes.[27] From an economic history perspective, this is a useful exercise

[26]Recall that our model allows for both residents of the city and the small jurisdiction to buy online, but that we do not need to model the shopping decision of residents of the city. This is because regardless of where they shop (the point of agglomeration or online) their taxes are always paid to the city.

[27]The dependence of the equilibrium values of tax rates, tax revenues, the spatial structure of transactions, and other variables *within* regimes is shown by (differential) comparative statics analysis in the next section. We can also characterize differences *between* regimes. Of course, real economies do not exhibit sharp breaks in the structure of commercial activities in response to changes in the underlying costs of executing transactions. We do not believe that our model, taken literally, is well-suited to the dynamics of transitions between regimes, as it predicts clear-cut differences in transaction patterns at specific critical values of technological and other parameters. Subject to this qualification, however, the comparisons of different regimes is still informative.

because it is informative about how local public finance has changed over time. It also allows us to anticipate what interjurisdictional policy differentials would look like in the future if all specialized shopping were online.[28] In particular, we can order the potential equilibria on the basis of the size of the tax differentials and revenue. In order to do this, we compare the equilibrium values in the two extreme cases of the past and present. To compare these to the standard model in section 3.1, we multiply tax revenues in that model by $1 + S$; we do this because in that model, consumers only buy one unit of nonspecialized commodities, but in our model they buy an additional S units of the specialized good. This allows us to compare the models assuming that in section 3.1, consumers buy the same number of commodities as in our model.

Proposition 6. *For $S > 0$, the size of the interjurisdictional tax differentials in regime I, III, and in the standard model (N) can be ordered as $\Delta^I > \Delta^{III} > \Delta^N > 0$. The equilibrium tax rates satisfy $T^I > T^{III} > T^N$ and $t^{III} > t^I > t^N$. With respect to equilibrium tax revenue, we know that $R^I > R^{III} > R^N(1+S)$ and $r^{III} > r^I > r^N(1+S)$.*

Proof. The relationship to Δ^N is obvious. $\Delta^I > \Delta^{III}$ because $H < 2h < 2H$, which implies that $2h - H < H$. Regarding tax rates, $\frac{2}{3}dSH > \frac{1}{3}dS(H + h)$ and $\frac{1}{3}dSH < \frac{1}{3}dS(2H - h)$. Tax revenue comparisons can be easily verified. □

Based on this analysis we conclude that for the extreme cases of past and future, the equilibrium tax rates in the city will fall in the future relative to their past tax rates, but in the hinterland taxes will rise in the future relative to their past tax rates. Although tax rates will rise in one jurisdiction and fall in the other over time, the tax differential is falling. The implication is that the extreme case of complete online shopping in the hinterland erodes the city's tax base, but not enough to flip the sign of the tax differential. Furthermore, tax revenues fall in the city over this "time" horizon as both the city tax rate fall and its specialized base contracts, although these reductions are somewhat offset with fewer non-specialized cross-border shoppers (as the tax differential shrinks). For the hinterland, both the tax rate and the size of its specialized base increase, but, as the tax differential shrinks, the revenue gains are tempered by fewer non-specialized shoppers. The first two effects dominate and revenue rises. Thus, the transition from the extreme world with no online shopping to the world where everyone shops online benefits the hinterland while harming the city. The intuition is clear: destination taxation provides small jurisdictions lacking an agglomeration with windfall gains while eroding some of the previously inelastic tax base of larger jurisdictions.

[28] Of course, we do not claim that in the future all shopping would be online, but only that it is possible that costs and consumers change in a way where large retail shopping centers become a thing of the past.

This proposition gives us some insights about the historical (and future) progression of tax rates as online shopping develops. However, the proposition does not include comparisons with regime II because a simple comparison of the level of the tax rates is sensitive to the parameters of the model. As discussed above, this ambiguity can best be seen by thinking about the smaller jurisdiction tax rate. Going from regime I to regime II, the size of the equilibrium hinterland tax base changes ($\frac{S}{3}\left(H-\frac{E}{D}\right)$ in (25)). For $\frac{E}{D} < H$, this larger tax base will put upward pressure on the tax rate; downward pressure is possible if the opposite inequality holds.[29] However, tax competition is also more intense in regime II because the city and the hinterland now compete both for the non-specialized base and for the the specialized tax base ($\frac{1}{(dS^2+D)/dD}$ in (25), which is the inverse of a change in the tax base with respect to the neighboring jurisdiction's tax rate). Thus, a simple comparison of levels in the present regime is not apparent because of these offsetting effects. To make progress, we know that a Nash equilibrium in the present regime must satisfy $x_S^*(T^{II}, t^{II}) \in [h, H]$. Evaluating x_S^* at the Nash tax rates, and setting it equal to these endpoints allows us to solve for the upper and lower values of E that satisfy this equilibrium, which we define as E_h and E_H (see appendix B.4.1). A value of E in this range is necessary, but not sufficient, for existence.

Proposition 7. *Suppose $S > 0$. For all parameter values π for which regime-II equilibria exist, we can order $\Delta^I > \Delta^{II}$ and $T^I > T^{II}$ for all values of $E \in [E_h, E_H]$ while $t^I > t^{II}$ for values of E sufficiently close to E_H. Further, $\Delta^{II} < \Delta^{III}$ and $T^{II} < T^{III}$ for values of E sufficiently close to E_h while $t^{II} < t^{III}$ for all values $E \in [E_h, E_H]$.*

Proof. See appendix A.2. □

Intuitively, the value of E influences the relative elasticities of the specialized tax base and the non-specialized tax base. This, in turn, influences how large the tax change will be across the regimes. The value of S plays a critical role; for some values of S, for example, "sufficiently close" to E_h or E_H in the proposition may include the entire range of E. However, we can conclude that tax differentials are unambiguously smaller with online shopping than with no online shopping. A comparison of levels is only useful when comparing *large* changes from the past and the future to the present. More interesting is how *marginal* changes in transactions technologies have influenced tax rates in periods where we observe the coexistence of online and brick-and-mortar commercial activities,

[29]This is because this term captures two effects: the change in the specialized base and the change in the non-specialized base. If the hinterland tax rate goes down, the specialized base increases but the non-specialized base may decrease depending which jurisdiction rate changes more. Thus, the relative elasticities of the two bases matter. More technically, in this model, the tax rates always equal one over the off-diagonal element of the Jacobian (the tax base externality) all times the equilibrium tax rate. Then, $\frac{t^I}{d}$ is the equilibrium tax base in regime I and $\frac{t^I}{d} + \frac{S}{3}\left(H-\frac{E}{D}\right)$ is the equilibrium tax base in regime II; thus the term $\frac{S}{3}\left(H-\frac{E}{D}\right)$ is the change equilibrium tax base going from regime I to III.

such as the last two decades and for the foreseeable future. How do perturbations of parameter values *within* regime II affect the endogenously determined variables? Because we have established existence for a significant range of parameter values, we know that this is a meaningful (not a knife-edge) exercise and we can thus conduct comparative statics in the neighborhood of regime-II equilibria.

5 Comparative Statics: Present Regime

5.1 Decline in Online Shopping Costs

The past twenty years have witnessed ever-declining costs and ever-increasing opportunities for online commerce. We can investigate the implications of this evolution by studying how regime-II equilibria change as the parameter E falls. Keeping in mind that we are interested in a *decline* in E, comparative statics show that:

$$\begin{array}{rcl} -\frac{\partial T^{II}}{\partial E} & = & -\frac{dS}{3(dS^2+D)} < 0 \\ -\frac{\partial t^{II}}{\partial E} & = & \frac{dS}{3(dS^2+D)} > 0 \end{array}, \tag{29}$$

which implies $\frac{\partial \Delta^{II}}{\partial E} < 0$. This yields what is perhaps an initially unexpected result that a decline in the cost of buying goods online lowers tax rates in the city, raises tax rates in the periphery and reduces interjurisdictional tax differentials – despite the fact that tax rates are strategic complements in the model. Even taxable online transactions create "tax haven" (downward) pressures in agglomerated jurisdictions and create "anti-haven" (upward) pressures in small jurisdictions. Intuitively, the Internet expands the specialized tax base of the hinterland and facilitates the collection of taxes based on the destination principle. The "conventional wisdom" is that online shopping threatens destination taxation, but that argument is usually made assuming that online transactions escape taxation. We find that taxable online transactions raise tax rates for small jurisdictions by enforcing destination taxation. The Internet effectively lowers the cost of raising revenue through the sales tax in small (non-agglomeration) jurisdictions. For thousands of towns the U.S., those with populations near or below the median (1 thousand people), a decline in the cost of using the Internet delivers windfall gains by expanding their tax bases. However, for the biggest jurisdictions, even though tax rates are strategic complements, taxes rates fall because the first-order effect is the decline of the city tax base and the erosion of its ability to extract taxable rents from the shopping center. With online transactions, the old advantages of the city are in decline, consistent with the broad trends in figure 1.

How does the decline in online shopping costs depend on the specialized good? In particular, will the downward and upward pressure on tax rates be more or less intense depending on the value of S? Differentiating with respect to S, the cross-derivative is

$$-\frac{\partial^2 t^{II}}{\partial E \partial S} = \frac{d(D - dS^2)}{3(dS^2 + D)^2} = \frac{\partial^2 T^{II}}{\partial E \partial S}, \tag{30}$$

which clearly depends on the initial size of the specialized good. If S is small, this expression is positive, but if dS^2 is large relative to D, then it will be negative. When it is positive, as E falls, the tax rates will converge faster as S increases. Intuitively, this makes sense: as the specialized good becomes more important, the small jurisdiction realizes even larger gains from a decline in the cost of buying online. However, at some point, this pattern flips because equilibrium tax revenues are not linear in S.

Falling online transaction costs also affect equilibrium revenues:

$$\begin{aligned} -\tfrac{\partial R^{II}}{\partial E} &= -\tfrac{2}{9} dS \left[\tfrac{S(H+\frac{E}{D})+H+h}{(dS^2+D)/D}\right] < 0 \\ -\tfrac{\partial r^{II}}{\partial E} &= \tfrac{2}{9} dS \left[\tfrac{S(2H-\frac{E}{D})+2H-h}{(dS^2+D)/D}\right] > 0. \end{aligned} \tag{31}$$

The sign of the first term is obvious. The sign of the second term follows because, as shown in appendix B.4.1, $\frac{E}{D}$ must lie between between h and $2H$. The implication is that tax revenues follow the same pattern as tax rates. Intuitively, this is because in the large jurisdiction, taxes are falling and the specialized good tax base is shrinking as online costs fall. In the small jurisdiction, taxes are rising and the specialized good tax base is rising. The changes in the non-specialized good tax base result in more cross-border shopping from the large jurisdiction which also expands the tax base.

We may ask, further, about the effect of changes in E on the combined tax revenues of the two localities. Would a "social planner" want to lower online shopping costs? To consider this, we add up the equilibrium revenues, assuming that tax rates are set at their Nash equilibrium values, and differentiate with respect to E. This yields

$$-\tfrac{\partial(R^{II}+r^{II})}{\partial E} = \tfrac{2}{9} dS \left[\tfrac{S(H-2\frac{E}{D})+H-2h}{(dS^2+D)/D}\right] < 0 \ . \tag{32}$$

Because $\frac{E}{D}$ lies in $[h, 2H]$, this expression is clearly negative for all possible values of $\frac{E}{D}$. The implication is that the tax revenue declines from online shopping are larger in the city than the tax revenue gains in the hinterland. This arises because, although the Internet partially equalizes tax revenues, it shifts some of the specialized tax base to a lower tax jurisdiction and also results in the city taxing revenues at a lower tax rate.

5.2 Increases in Specialized Good Consumption

Although our model features a fixed consumption of S units of the specialized good, one may still ask how a shift towards the consumption of specialized goods would affect the equilibrium tax rates in the presence of online shopping. (We might think of this as the Internet affecting relative consumer demand patterns.) An increase in S will imply:

$$\begin{array}{rclcl} \frac{\partial T^{II}}{\partial S} & = & \frac{d(DH+E)}{3(dS^2+D)} - \frac{2d^2S(DH+ES+2DHS+Dh)}{3(dS^2+D)^2} & = & \frac{1}{3}d\left[\frac{(D-S^2d)(H+\frac{E}{D})-2Sd(H+h)}{(dS^2+D)^2/D}\right] \\ \frac{\partial t^{II}}{\partial S} & = & \frac{d(2DH-E)}{3(dS^2+D)} - \frac{2d^2S(2DH-ES+2DHS-Dh)}{3(dS^2+D)^2} & = & \frac{1}{3}d\left[\frac{(D-S^2d)(2H-\frac{E}{D})-2Sd(2H-h)}{(dS^2+D)^2/D}\right]. \end{array} \tag{33}$$

Taking the limit as $S \to 0$, we can see that both of these expressions are positive for all other parameter values. Regardless of other parameters, when S is small, an increase in the importance of the specialized good will put upward pressure on tax rates. This is clear, because introducing the specialized good in the model raises both of the tax rates relative to the Nielsen (2001) model. This result continues to hold in the presence of online shopping when S is small. But, as S increases, non-linearities in revenue prevent us from unambiguously signing the expression for larger values. But, recalling that $2H > \frac{E}{D}$, if $D < S^2d$, then both comparative statics are negative. However, if $D = d$, this would require $S > 1$, but at these high values a regime II equilibria may not exist.

We study the effect of changes in the specialized good on tax revenues:

$$\begin{array}{rcl} \frac{\partial R^{II}}{\partial S} & = & \frac{2}{9}d\frac{\left[H+h+S(H+\frac{E}{D})\right]\left[H+\frac{E}{D}-\frac{Sd}{D}(H+h)\right]}{(dS^2+D)^2/D^2} \\ \frac{\partial r^{II}}{\partial S} & = & \frac{2}{9}d\frac{\left[2H-h+S(2H-\frac{E}{D})\right]\left[2H-\frac{E}{D}-\frac{Sd}{D}(2H-h)\right]}{(dS^2+D)/D^2}. \end{array} \tag{34}$$

We are unable to sign the effect on tax revenues for all possible parameter values. However, if $\frac{Sd}{D} < 1$, the upper branch will definitely be positive because $h < \frac{E}{D} < 2H$, although the lower branch is ambiguous in sign. In the limit as $S \to 0$, we can see that both of these expressions are positive for all other parameter values. Again, even in the presence of online shopping, for small S, an increase in S has a similar effect to introducing S to the Nielsen (2001) model. The general ambiguity of S is driven by the fact that increases in S increase both the per person demand and the number of cross-border shoppers.

5.3 Summary

Proposition 8. *In the neighborhood of a Nash equilibrium in regime II:*

(a) A decline in the cost of buying online lowers the Nash equilibrium tax rate (T^{II}) and revenue (R^{II}) in the large jurisdiction, but raises the Nash equilibrium tax rate (t^{II}) and revenue (r^{II}) in the small jurisdiction.

(b) For sufficiently small quantities of the specialized good, an increase in the

relative importance of the specialized good (S increases) places upward pressure on tax rates and revenue in both the city and hinterland.

Proof. These results follow from the discussion in the prior two sections. $\square$

6 An Inverse-Elasticity Rule for Equilibria

In closing our analysis, it may be valuable to note that the equilibrium tax structure can be given an intuitive inverse-elasticity characterization that does not depend on our specific functional forms previously assumed. In this model, the tax authority in each jurisdiction has access to two separate tax bases. Denote the city's tax base for the non-specialized good as $B_N(T,t)$. Similarly, the city's tax base for the specialized good can be defined as $B_S(T,t)$. Define the total base as $B = B_N + B_S$. Similar terms can be defined for the small jurisdiction, where we use lowercase $b = b_N + b_S$ to indicate its tax base. Each base is declining in its own tax rate and increasing in the neighboring jurisdiction tax rate. Both jurisdictions maximize tax revenue

$$\begin{aligned} R(T,t) &= T \cdot B = T(B_N + B_S) \\ r(t,T) &= t \cdot b = t\,(b_N + b_S)\,. \end{aligned} \tag{35}$$

Differentiating with respect to each jurisdiction's tax rate yields inverse elasticity rules:

$$\begin{aligned} \frac{T}{1+T} &= \frac{1}{-\frac{1+T}{B}\frac{\partial B}{\partial T}} = \frac{1}{\epsilon_B} \\ \frac{t}{1+t} &= \frac{1}{-\frac{1+t}{b}\frac{\partial b}{\partial t}} = \frac{1}{\epsilon_b} \end{aligned} \implies \frac{\frac{T}{1+T}}{\frac{t}{1+t}} = \frac{\epsilon_b}{\epsilon_B}, \tag{36}$$

where ϵ_B and ϵ_b are the elasticities of each of the town's (total) tax bases.

We can apply this formula in the context of the model we developed above. Consider the present regime. Starting from equal taxes, $T = t$, $B_S > b_S$, and furthermore, there is no cross-border shopping of the non-specialized good, implying $B_N > b_N$. As the large jurisdiction's tax base is larger than the small jurisdiction's, $\epsilon_B < \epsilon_b$, which implies that the large jurisdiction should raise its tax rate relative to the small jurisdiction. This establishes that $T > t$ in a regime-II equilibrium.

We decompose the elasticity of the composite base into the the two base elasticities:

$$\begin{aligned} \frac{T}{1+T} &= \frac{1}{-\frac{1+T}{B_N}\frac{\partial B_N}{\partial T}\frac{B_N}{B} - \frac{1+T}{B_S}\frac{\partial B_S}{\partial T}\frac{B_S}{B}} = \frac{1}{\frac{B_N}{B}\epsilon_{B_N} + \frac{B_S}{B}\epsilon_{B_S}} = \frac{1}{\Omega\epsilon_{B_N} + (1-\Omega)\epsilon_{B_S}} \\ \frac{t}{1+t} &= \frac{1}{-\frac{1+t}{b_N}\frac{\partial b_N}{\partial t}\frac{b_N}{b} - \frac{1+t}{b_S}\frac{\partial b_S}{\partial t}\frac{b_S}{b}} = \frac{1}{\frac{b_N}{b}\epsilon_{b_N} + \frac{b_S}{b}\epsilon_{b_S}} = \frac{1}{\omega\epsilon_{b_N} + (1-\omega)\epsilon_{b_S}}. \end{aligned} \tag{37}$$

Notice that taxes become a weighted function of the elasticities of the specialized and nonspecialized bases where Ω is the share of the non-specialized base in the aggregate base of the large jurisdiction and the corresponding weight for the small jurisdiction is ω. As $S \to 0$, these weights both tend to one.

This characterization of the equilibrium tax structure, as is typical of inverse-elasticity formulations of tax rules, is highly implicit. It is useful into gaining economic insight regarding the properties of an equilibrium, but, remembering that the elasticities are variable and not constant in general, the inverse-elasticity formulae cannot yield clear-cut comparative statics results. For that purpose it is necessary to impose reasonable added structure on the problem, which we have done in the preceding sections.

7 Conclusion

We modify the classic commodity tax competition model to allow for multiple goods, spatial agglomeration in a central city, and online shopping. Here we summarize some of the key results and discuss some of their broader implications. First, recognizing the existence of urban agglomeration:

- Agglomeration yields taxable rents that allow a jurisdiction with a retail center to raise its local sales tax rate substantially and widen interjurisdictional tax rate and revenue differences. Revenues are higher in the presence of agglomeration for both jurisdictions relative to a model with only firms located everywhere.

Urban agglomeration results from many different forces, both political and economic. Our paper highlights some of the potential fiscal implications of agglomeration, in particular, the ability of large urbanized areas to use local sales taxes to capture some agglomeration rents. What are the implications? If the increase in tax revenues in the city is simply a result of political rent capture (see Ades and Glaeser 1995), this may be an unwelcome development from a welfare viewpoint. We also note that the increased capacity of central cities to raise revenues occurs partially at the expense of non-resident households; this is because some of the city's tax-burden is "exported" to non-residents (in our model, peripheral residents that must purchase goods in the city, but more generally, this may include commuters). Although tax exporting can sometimes be viewed as problematic, such as when cities "take advantage" of surrounding regions, it could alternatively be viewed as a form of compensation for the services provided at some cost by the city, which accrue to the benefit of residents and non-residents alike. A proper assessment of the normative implications are beyond the scope of this paper, but increases in agglomeration result in the outlying regions benefiting from higher tax rates and revenues: in this case, tax competition is not necessarily a zero-sum game.

Second, technological changes affect local tax policy. Increases in the propensity to conduct taxable transactions online affects rates and revenues:

- Tax rates decrease in agglomerated jurisdictions but tax rates rise in small jurisdictions, contributing to tax rate convergence. Revenues follow a similar pattern, shifting tax revenue towards more outlying areas; the combined effect of the two is to reduce aggregate sales tax revenues.

Thus, consistent with and possibly magnifying the long-run trend of a higher fraction of revenue being raised in smaller jurisdictions (see Figure 1), increases in Internet usage will create fiscal winners and losers *even* when online transactions are all taxed. Even though revenues rise in smaller jurisdictions, aggregate revenue for both jurisdictions declines because the city tax rate falls, but not enough to prevent shifting of the specialized good to a lower-tax neighboring jurisdiction. This shifting of the tax base from higher tax to lower tax jurisdictions contributes to a reduction of aggregate revenues. In this sense, falling costs of online commerce turn tax competition into a negative-sum game. This results in a reduction of the ability of central locations to exploit their agglomeration advantages and to engage in tax exporting. As noted above, from a welfare viewpoint, these changes may be advantageous or disadvantageous. Large jurisdictions are not the only "losers." As the cost of buying online falls, firms in large jurisdictions will continue to suffer from declining sales as they lose some customers to the web. This could continue to transform the retail trade industry as well as fiscal systems.

The results also have important empirical implications. For example, regarding the predictions of agglomeration, researchers could test whether municipalities with retail shopping centers set higher tax rates than otherwise identical municipalities without retail shopping centers. This could be done by using the entry of a large shopping malls/chains or the national bankruptcy (exit) of retail chains. Of course, entry of retail chains is not random, so the researcher would need to compare the towns that "win" the chain store with the runner-up towns that were being considered but were not selected. Our model also has empirical predictions concerning the effect of online shopping on local tax rates. Some empirical research has begun; for example Agrawal (2016) uses Internet penetration (entry of Internet-service providers) as an exogenous shock to the cost of using the Internet. This research shows that online shopping places downward pressure on tax rates in large jurisdictions and upward pressure tax rates in small jurisdictions in states where more firms are collecting retail sales taxes, a finding which is consistent with the theoretical predictions discussed above.

Structural economic changes can also precipitate important institutional and policy changes. Although many states allow for local sales taxes, others do not; once granted the ability to set these taxes, some localities choose to impose them, and other others do not. The landscape of commodity taxation has changed dramatically over time, but far from uniformly across states and localities. What accounts for these changes and their timing? Undoubtedly, institutions and policies reflect underlying fundamentals such as urban spatial structure and sprawl, the costs of alternative transactions, transportation, and the cost of administering and enforcing different tax instruments. Understanding how policies change over time is obviously important but difficult, as it involves the complex interplay of economic, legal, and social forces. Our model identifies some arguably exogenous determinants of policy evolution, which we hope may help shed light on the temporal and spatial changes displayed in figure 1 and 2.

We end by recalling that local sales taxes are but one component of the fiscal systems of state and local governments. These operation of these taxes simultaneously depends upon and impinges upon many other aspects of public policy, at all levels of government. For instance, the Marketplace Fairness Act is federal legislation that would require remote vendors to remit retail sales taxes even without a physical presence in a state. Our analysis suggests that such legislation would give rise to asymmetric effects across local jurisdictions as e-commerce continues to grow. Similar effects may well arise as states attempt to broaden their nexus rules and expand reporting of remote transactions (perhaps by credit card companies), and as businesses continue to increase their sales tax exposure, even under existing nexus standards, by entering more states. Federal and state-level policies are likely to interact with lower-level government policymaking, representing an important area of future research. These regulatory changes may concomitantly affect both state and local tax rates (Keen 1998), giving rise to further interactions as localities adjust their policies in response to state tax rate changes. These changes may also trigger adjustments in fiscal systems more broadly. Changes in local property taxes, zoning rules, and local economic development policies, may accentuate or offset the effects we have identified. Changes in the revenue flows of local governments may also give rise to offsetting intergovernmental transfers. In short, the structural transformation of retail trade presents many challenges and opportunities for federal, state, and local policymakers, as well as many open questions for future research.

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Figure 1: Percent of Sales Tax Revenue Raised by the Large/Small Jurisdictions

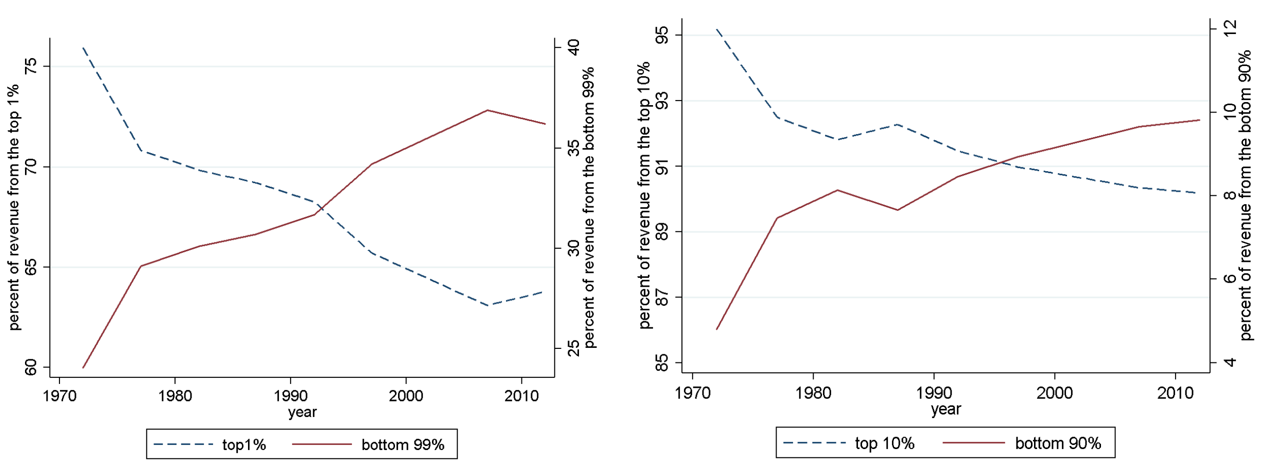
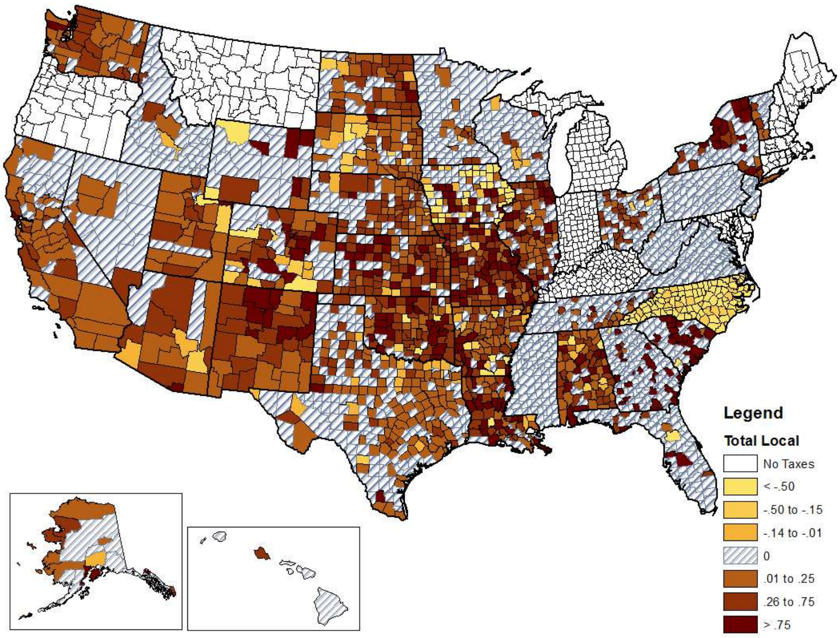


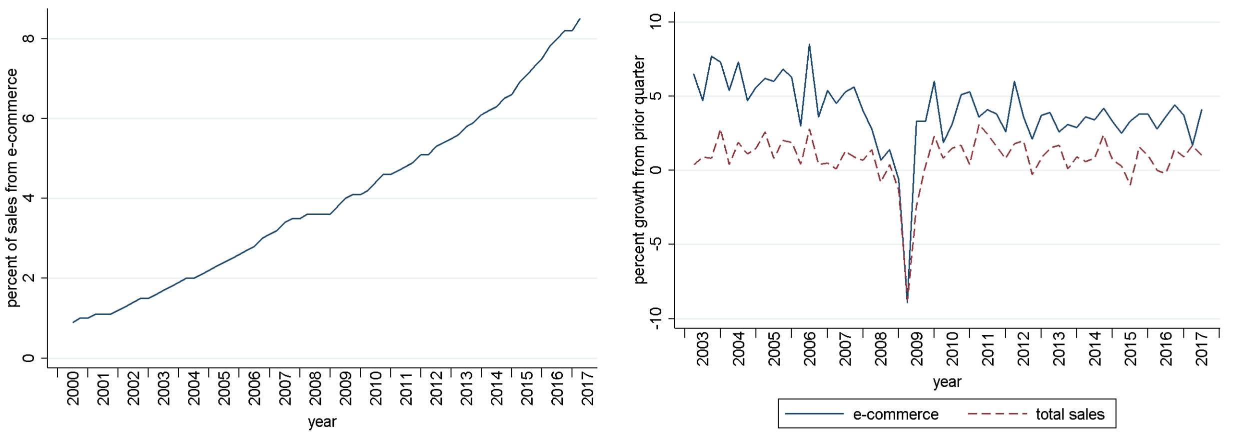
Figure constructed by authors using the Census of Governments. Each figure shows the fraction of municipal sales tax revenue raised by the largest and smallest jurisdictions in the country. The left figure separately shows the percent of revenue raised by the largest (by population) 1% of jurisdictions in the United States. The top 1% largest jurisdictions are those that are approximately above 90,000 people The left figure separately shows the percent of revenue raised by the largest (by population) 10% of jurisdictions in the United States. The top 10% largest jurisdictions are those that are approximately above 15,000 people. We use 2012 population numbers to determine the percentiles.

Figure 2: Sales Tax Rate Changes in the United States (2003-2011)



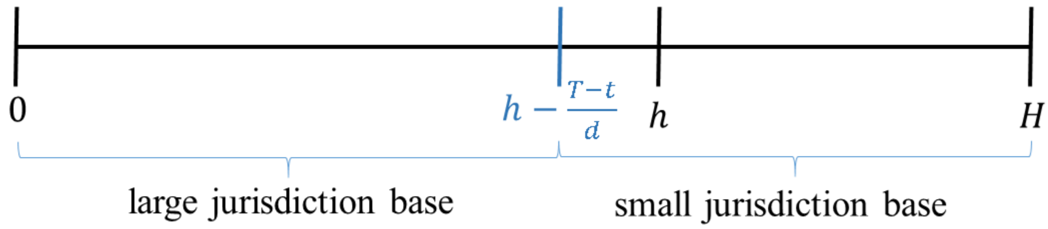
Data Source: Agrawal (2015). This figure shows the tax change from 2003 to 2011 of the county tax rate inclusive of the population weighted town tax rates. White areas are those with no local sales taxes. Hashed areas had the same tax rate in 2011 as in 2003. Light colors are tax decreases and dark colors are tax increases.

Figure 3: Rise of e-Commerce: 2000 to 2017



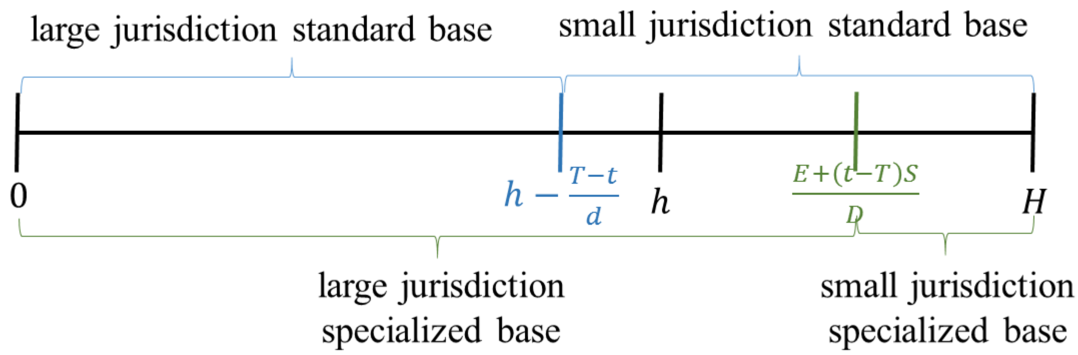
Data Source: Census Quarterly e-Commerce Report (Adjusted Sales). Figure constructed by authors. The left figure shows the percent of total sales that occur on the Internet or an online system. Sales are adjusted for seasonal variation but not for price changes. The right figure shows the growth rate relative to the previous quarter for e-commerce transactions and regular sales.

Figure 4: Geography of the Standard Model



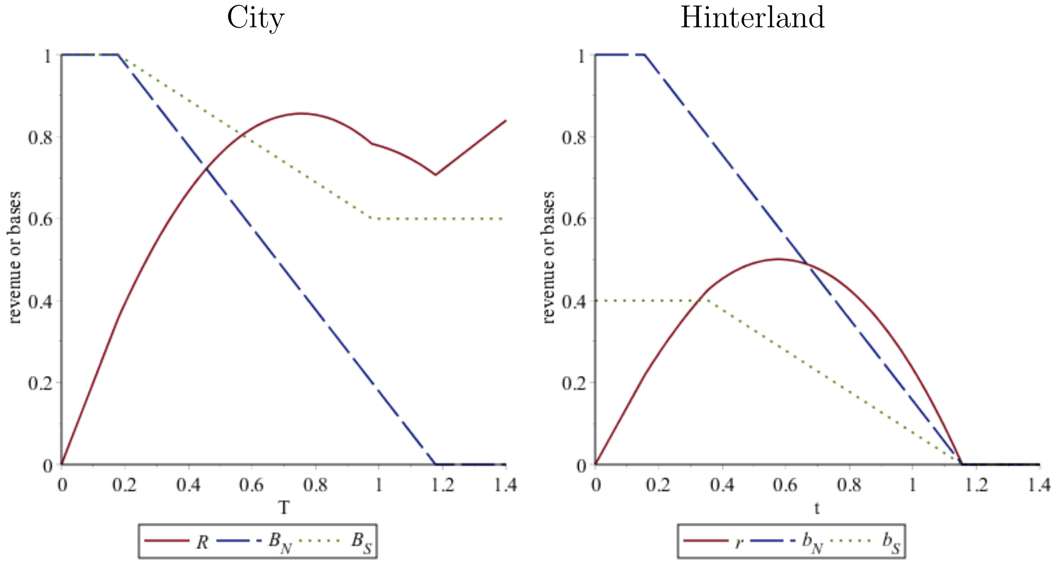
This figure shows the geography of the model along a ray extending from the origin of a circle. Point 0 is the center of the city. Then H is the radius of the circle, h is the distance from the center at which the small jurisdictions start. The figure shows the tax base of the large and small jurisdiction where the cutoff rule for cross border shopping is given by a distance from the center of $h - \frac{T-t}{d}$ where $T > t$ are the local tax rates.

Figure 5: Geography of the Model with online Shopping and Agglomeration



This figure shows the geography of the model along a ray extending from the origin of a circle. Point 0 is where the specialized firms locate. Then H is the radius of the circle, h is the distance from the center at which the small jurisdiction starts. The figure shows the nonspecialized tax base where the cutoff rule for cross border shopping is given by a distance from the center of $h - \frac{T-t}{d}$ where $T > t$ are the local tax rates. The specialized tax base is depicted where the cutoff rule for the specialized base, $\frac{E+(t-T)S}{D}$, implies some residents of the hinterland shop online; some residents of the city may also shop online, but this is not depicted.

Figure 6: Example of Nash Equilibrium for the Present



This figure shows the tax revenues for parameter values of $h = 0.6, H = 1, E = 1.6, D = 2, d = 1, S = 1$. Each revenue function is drawn assuming the other jurisdiction sets the equilibrium tax rate T^{II} or t^{II} so that the reader can verify if a global deviation exists. To ease the partitioning of the cases, we present in dashed lines, the size of the specialized tax base and the non-specialized base. Notice a clear maximum in $T^{II} = 0.75, t^{II} = 0.58$ in Regime II. The revenue functions are drawn without an elasticity condition (fixed S) to show that it is necessary that demand is sufficiently elastic above $\overline{P_S}$; when demand is sufficiently elastic, the explosive portion of the revenue function does not apply.

8 Appendix (For Online Publication)

The appendix to the paper proceeds in three sub-parts. Part "A" focuses on proofs of the propositions and derivations of the model that are not related to existence or uniqueness of an equilibrium. This includes a proof of the model's generalization to the monocentric (circular) city and proofs of the signs of interjurisdictional tax and revenue differentials. Part "B" of the appendix explicitly focuses on existence and uniqueness of the equilibrium, including issues of non-existence. However, this appendix only discusses issues related to the most important regime (Regime II: Present). Appendix "C" focuses on issues of existence and uniqueness of a Regime I or Regime II equilibrium. In addition, this appendix presents a formal proof of non-existence of an equilibrium under certain parameter values.

Notation defined in the text remains applicable in all sub-parts. Some new notation is also introduced in each sub-part for specific purposes, which, although consistent within each part, may be applied for different purposes in different parts.

A Appendix "A"

A.1 Symmetry of the Model

In our model, we assume that the point of agglomeration is at point 0. We think of this as a valid assumption because underlying our model is the idea that the jurisdiction is symmetric on the other side of zero. In this sense, the large shopping mall is centrally located as if it were in the central business district of a monocentric city model. In order to convince the reader that symmetry applies around 0, we sketch a model that explicitly takes this symmetry into account. Consider regime I; the results can similarly be derived for all other regimes. In the text, we wrote revenues as

$$\begin{aligned} R(T,t,\pi) &= T\left(h+\tfrac{t-T}{d}+SH\right) \\ r(t,T,\pi) &= t\left(H-h+\tfrac{T-t}{d}\right) \end{aligned} \tag{A.1}$$

when the city stretched from point 0 to h. To proceed we first consider the case where the two small jurisdictions coordinate and act as one jurisdiction. Imagine the city runs from $-h$ to h and the hinterland occupies mass from $-h$ to $-H$ and from h to H. The point of agglomeration remains at point 0. We can use this as a benchmark before proceeding to discuss the case with two independent cities. This type of configuration could be justified if the city were the doughnut hole and the hinterland a single beltway around, which would likely arise if the remote areas were restricted to set a common tax policy.

As a result of symmetry on both sides of the zero and because we have only two tax rates, we can construct the revenue functions in this case – noted by tildes – as:

$$\widetilde{R}(T,t,\pi) = 2T\left(h+\frac{t-T}{d}+SH\right) = 2R \tag{A.2}$$

$$\widetilde{r}(t,T,\pi) = 2t\left(H-h+\frac{T-t}{d}\right) = 2r. \tag{A.3}$$

By symmetry, the city and the hinterland are now twice as large in size. The hinterland looses twice as many people in cross-border shopping and twice as many residents by the specialized good at the city center. Thus, both revenue functions are simply scaled by two, which has no effect on the Nash equilibrium tax rates.

Now imagine that the city were surrounded by two independent small jurisdictions, or more generally, many independent hinterlands which only have radial transportation to the central city. The city still runs from $-h$ to h and has a point of agglomeration at point 0. The western hinterland runs from $-H$ to $-h$ and sets a tax rate t_w (for west). The eastern hinterland runs from h to H and sets a tax rate t_e (east). This could be thought of as a monocentric city model where the city is surrounded by many small jurisdictions but where all shopping only occurs along a ray running through the origin. As such, this model could be generalized to two dimensions if the hinterlands never interact with each other; it is also assumed that the city is large enough that the eastern and western jurisdiction will not be able to capture cross-border shoppers from the eastern jurisdiction. The revenue functions, denoted by hats, are:

$$\widehat{R}(T, t_w, t_e, \pi) = T(2h + \frac{t_e - T}{d} + \frac{t_w - T}{d} + 2SH) \tag{A.4}$$

$$\widehat{r_e}(t_e, T, t_w, \pi) = t_e((H - h) + \frac{T - t_e}{d}) \tag{A.5}$$

$$\widehat{r_w}(t_w, T, t_e, \pi) = t_w((H - h) + \frac{T - t_w}{d}). \tag{A.6}$$

Note that because there are now two independent hinterlands we have a total of three Nash equilibrium tax rates to solve for in equilibrium. However, if $t_e = t_w$, we have $\widehat{R} = \widetilde{R} = 2R$ and $\widehat{r_e} = \widehat{r_w} = \widetilde{r}/2 = r$. In particular, each revenue function would be (relative to the initial model) scaled by a different factor (2 vs. 1). But, this does not matter because these scalars cancel when deriving the best response functions. So, we can conclude that if $t_e = t_w$, the Nash equilibrium tax rates are unchanged. In particular, note that the number of jurisdictions does not heighten tax competition in this setting because the hinterlands do not compete with other hinterlands; because the city is also larger, issues of existence remain unchanged. Tedious algebra shows that indeed $\hat{t_e} = \hat{t_w}$ and the tax rates equal to those derived using the revenue functions in (A.1). This can easily be verified, as well, in the presence of online shopping in regime II and III.

A.2 Proof of Proposition 7

Proof. In regime II, it must be the case that $h \leq \frac{E}{D} + \frac{(t^{II} - T^{II})S}{D} \leq H$. Using the equilibrium values of the tax rates, we can solve for the values of E that place x_S^* exactly at these boundaries. Appendix B.4.1 does this. Denote the lowest possible value of E that could sustain a type II equilibrium as E_h and the highest possible value as E_H, so that for all $E \in [E_h,\ E_H]$, $h \leq x_S^* \leq H$. A regime II equilibrium can only exist for such values of E, a property that is utilized in the following.

- *Demonstrating* $\Delta^I - \Delta^{II} > 0$ *and* $T^I - T^{II} > 0$.

Evaluating these expressions at E_H implies $\Delta^I(E_H) - \Delta^{II}(E_H) = \frac{d^2 S^2}{3(dS^2 + 3D)}(2h - H + HS) > 0$ because $2h - H > 0$. Evaluating these expressions at E_h implies $\Delta^I(E_h) -$

$\Delta^{II}(E_h) = \frac{dS}{3(dS^2+3D)}(dS(2h-H)+6D(H-h)+dS^2H) > 0$ because $2h-H > 0$. Thus, $\Delta^I - \Delta^{II} > 0$ for all possible values of E that could sustain a regime II equilibrium. Similarly, $T^I(E_H) - T^{II}(E_H)$ is always positive and $T^I(E_h) - T^{II}(E_h)$ is always positive using the fact that $8H > 3h$ and $H > h$.

- *Demonstrating $t^I - t^{II} > 0$ only for values of E sufficiently close to E_H.*

Evaluating these expressions at E_H implies $t^I(E_H) - t^{II}(E_H)$ is positively proportional to $(2H-h)S^2d + (5H-h)D + (dS^2+4S)SH > 0$. Evaluating at E_h implies $t^I(E_h) - t^{II}(E_h)$ is proportional to $-(HS^4d^2 + (h-2H)S^3d^2 - (H+3h)S^2dD + (4H+h)SdD + 3D^2(H-h))$, which is of ambiguous sign. Given $t^I - t^{II} > 0$ at the highest possible value of E, but of ambiguous sign at the lowest possible value, and that t^{II} is monotonic in E, for certain (but not all) other parameter values, there may exist an $\widehat{E} \in [E_h, E_H]$ where $t^I(E_H) - t^{II}(E_H)$ flips signs.

- *Demonstrating $\Delta^{II} - \Delta^{III} < 0$ and $T^{III} - T^{II} > 0$ for values of E close to E_h .*

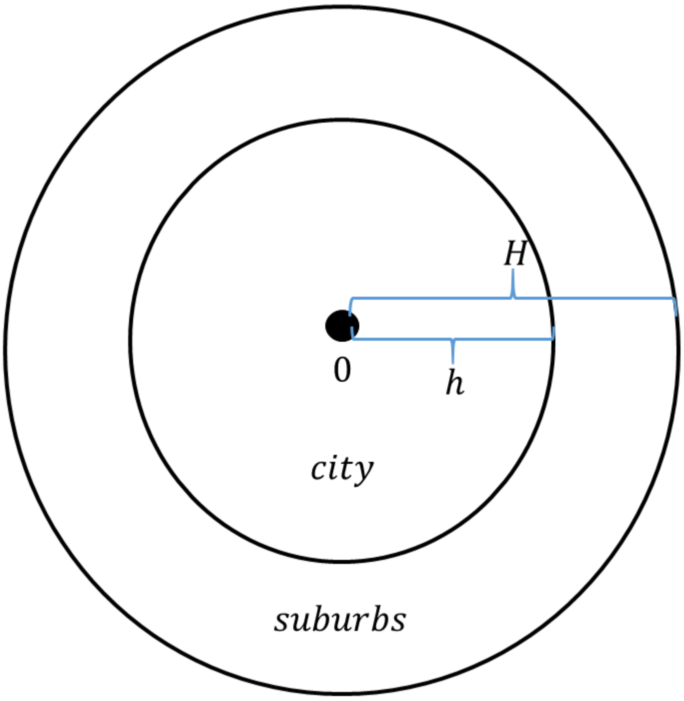
Evaluating these expressions at E_h implies $\Delta^{II}(E_h) - \Delta^{III}(E_h) = \frac{d^2S^2}{3(dS^2+3D)}((S^2+1)(H-2h)) < 0$. Evaluating these expressions at E_H implies $\Delta^{II}(E_H) - \Delta^{III}(E_H) = \frac{dS}{3(dS^2+3D)}((S^2d+Sd)(H-2h)+6D(H-h))$ but this is ambiguous in sign; for small values of S it will be positive for large S it will be negative. Given $\Delta^{II} - \Delta^{III} < 0$ at the lowest possible value of E, but of ambiguous sign at the higher possible value, and that Δ^{II} is monotonic in E, for certain (but not all) other parameter values, there may exists an $\widehat{E} \in [E_h, E_H]$ where $\Delta^{II} - \Delta^{III}$ flips signs. Similarly, $T^{III}(E_h) - T^{II}(E_h)$ is always positive but $T^{III}(E_H) - T^{II}(E_H)$ is only positive for small S.

- *Demonstrating $t^{III} - t^{II} > 0$.*

Evaluating these expressions at E_H implies $t^{III}(E_H) - t^{II}(E_H)$ is positively proportional to $(2H-h)(S^4d^2+S^3d^2+4DS^2d)+(5H-h)DSd+3D^2(H-h) > 0$. Evaluating at E_h implies $t^{III}(E_h) - t^{II}(E_h)$ is proportional to $(2H-h)(S^3d+S^2d)+(5H-h)(DS+D) > 0$.

□

Figure A.1: Model with Symmetry



Our model considers a city competing with the suburbs or hinterlands (or possibly many different suburbs) where we assume that all travel (and thus cross-border shopping) occurs along rays from the origin to the suburb. For this reason, we can solve our model linearly.

B Appendix "B"

The text presents comparative-statics results showing how key endogenous variables – tax rates, revenues, and others – vary with respect to critical parameters in different equilibria. Such results of course require existence and possibly uniqueness of equilibria. The problems of existence and uniqueness of equilibria raise many technical complexities that are often sidestepped in the literature (for example, assuming existence of an equilibrium), or that are finessed by symmetry and other strong assumptions. Because this paper develops a completely novel model, it is perhaps especially important to address these technical questions. This appendix analyzes the existence and uniqueness of equilibrium for a wide range of the model's parameter values, encompassing many economically interesting cases. We hasten to note, however, that we do not provide a general existence proof for all possible parameter values. Indeed, it can be verified – by example, as discussed further below – that no such proof is possible. Our analysis of existence and uniqueness is not exhaustive, but the results presented here provide a firm foundation for the comparative statics analysis results presented above, and demonstrate that the basic modeling approach raises no insuperable technical obstacles.

B.1 Preliminaries

As might be expected, and as is shown below, the existence of one type of regime or another is highly parameter-dependent. For example, for fixed values of other parameters, the cost of online transactions can be made prohibitively high (the "past") by making the cost parameter E sufficiently large, whereas online transactions can be made costless by setting $E = 0$ (the "future"). Thus, to obtain conditions for the existence of type II equilibria, it is plausible – and we show below – that E must take on values in some intermediate range, the limits of which depend on the other parameters of the model.

Like E, the parameter S plays a critical economic role in our analysis. In particular, when $S = 0$, the model effectively reduces to, or nests, the standard Kanbur and Keen (1993)/Nielsen (2001) model, with no agglomeration and with no online transactions. For this reason, it is important for the model to allow for values of S that range from zero to a nontrivial magnitude. Although we do not establish existence of a regime II equilibrium for arbitrarily large values of S, we do henceforth allow for S to take any value in the interval $[0, 1]$. Thinking of the magnitude of $S = S/1$ as the ratio of consumption of specialized to standard commodities, this range of S values allows for a very wide range of consumption patterns.

As a matter of notation, let $\pi = (h, H, d, D, E, S)$ denote the full set of model parameters. One may view $\hat{\pi} = (h, H, d, D, E)$ as underlying non-negative parameters whose values define the spatial structure of the economy and its transaction technology. Of course E is of special interest and S will subsequently be allowed to vary over a range of values. Two other elements, mainly of technical importance, must also be specified.

First, households must have sufficient resources to survive or, equivalently stated, they must have non-zero surplus. This condition, which need not discussed further, can always be satisfied, for any possible configuration of parameters.[30]

Second, in order to avoid economically-uninteresting pathological cases, the demand for the specialized goods must become price elastic above some threshold price. In the absence of this condition, the large locality could raise its tax rate indefinitely, collecting more and more taxes from its own residents whose specialized goods purchases are always subject to tax. In order to avoid this unrealistic and uninteresting outcome, while maintaining the simplicity of the assumption of inelastic demand where possible,

[30]This condition is certainly met if household incomes are sufficient to cover all consumption expenditures, transactions costs, and taxes under any possible spatial and policy configuration.

we assume that there is some threshold price $\bar{P}_S$ above which the demand for specialized goods becomes "sufficiently elastic", and below which (i.e., "in the relevant range") it is perfectly inelastic, where the tax-inclusive price is denoted by P_S, given by $1 + T$ for city residents and by $1 + t$ for residents in the small locality. More formally stated, each household's demand for the specialized goods is given by $S(P_S)$, where $S(P_S) = S$ for all $P_S \leq \bar{P}_S$ and where, for all $P_S \geq \bar{P}_S$, we assume that $\epsilon_S := d\log S(P_S)/d\log P_S \leq 0$ is "sufficiently small" (i.e., sufficiently elastic). As discussed further below, $\epsilon_S \leq -1$ is sufficient (but not necessary) for our results.[31] This elasticity condition is "technical" in the sense that it pertains to portions of the demand function that are never observed in equilibrium, but some version of it is needed to rule out the possibility that a locality could hypothetically raise its revenues indefinitely through an ever-increasing tax rate applied to a perfectly price-inelastic and captive tax base.

B.2 An Example of Non-Existence

Before tackling the problem of existence, however, it is useful to begin by recognizing the limits of the analysis. Although the discussion to follow shows that equilibria do exist in economically-interesting cases, no general proof of existence of equilibrium is possible for all values of π. In particular, detailed calculations show that for the parameter values $h = .6$, $H = 1$, $S = 1$, $d = 40$, $D = 7.7$, $E = 11$, there can be no Nash equilibrium tax rates, i.e., for any possible pair of tax rates, at least one locality can increase its revenues by changing its own tax rate, given the tax rate of the other locality. A formal proof of non-existence is in Appendix C.3. Figure C.3 shows the intuition of this result.

B.3 Nash Equilibrium: A Formal Definition

Under the assumption that some but not all hinterland households purchase the specialized good online, and some households purchase the non-specialized good in both localities, each locality's revenue function is inverse quadratic in its own tax rate and its best-response function is a linear increasing function of the other locality's tax rate with a slope of 1/2; the unique intersection of the best-response functions determines our *candidate* Regime-II Nash equilibrium tax rates, $(T^{II}[\pi], t^{II}[\pi])$, given by (25). These tax rates are derived assuming that the both jurisdictions tax both bases so that, in the neighborhood of the equilibrium, the revenue functions become:

[31] Recall that "specialized goods" are an aggregate of many goods, each possibly with its own demand function, dependent on its own price. By normalization of units, we may set each of these per-unit prices equal to 1. The household decision to purchase any one of these goods, and the amount to be purchased, depends on its tax-inclusive price P_S. It is independent of transactions costs, however, which are an indivisible overhead cost of market access, except when (and if) demand vanishes completely.

$$R(T,t,\pi) = T\left(h + \frac{t-T}{d} + S\frac{E+(t-T)S}{D}\right) \tag{B.1}$$

and

$$r(t,T,\pi) = t\left(H - h + \frac{T-t}{d} + S\left(H - \frac{E+(t-T)S}{D}\right)\right). \tag{B.2}$$

However, this revenue function does not capture all of the possible shopping patterns of the non-specialized base. To encompass all possibilities simultaneously, it is helpful to express each of the two components of the tax base for each locality, that is, the volume of taxable sales of the non-specialized good and the volume of taxable sales of the specialized good, in a general form.

For sufficiently low or sufficiently high tax rates, a locality can attract or repel all transactions involving the non-specialized good; similarly, there are lower and upper rates at which a locality attracts or repels maximal amounts of transactions involving specialized commodities. These tax rates, denoted by $\underline{\Gamma}_N$, $\overline{\Gamma}_N$, $\underline{\Gamma}_S$, $\overline{\Gamma}_S$ for the large locality and by $\underline{\Gamma}_n$, $\overline{\Gamma}_n$, $\underline{\Gamma}_s$, $\overline{\Gamma}_s$ for the small locality, maximize or minimize the respective tax bases for each jurisdiction. These equations, already presented in the text, are reproduced here using this notation. For the non-specialized good,

$$B_N(T,\, t,\, \pi) = \begin{cases} H & \text{if} \quad T \le \underline{\Gamma}_N := t - d(H-h) \\ 0 & \text{if} \quad T \ge \overline{\Gamma}_N := t + dh \\ h + \frac{t-T}{d} & \text{if} \quad T \in [\underline{\Gamma}_N, \overline{\Gamma}_N]. \end{cases} \tag{B.3}$$

Because $B_N + b_N = H$, we may equivalently define

$$b_N(t,\, T,\, \pi) = \begin{cases} H & \text{if} \quad t \le \underline{\Gamma}_n := T - dh \\ 0 & \text{if} \quad t \ge \overline{\Gamma}_n := T + d(H-h) \\ H - h + \frac{T-t}{d} & \text{if} \quad t \in [\underline{\Gamma}_n, \overline{\Gamma}_n]. \end{cases} \tag{B.4}$$

For the specialized good, the tax base of the large jurisdiction is

$$B_S(T,\, t,\, \pi) = \begin{cases} HS & \text{if} \quad T \le \underline{\Gamma}_S := t + \frac{E}{S} - \frac{DH}{S} \\ hS & \text{if} \quad T \ge \overline{\Gamma}_S := t + \frac{E}{S} - \frac{Dh}{S} \\ \left(\frac{E}{D} + [t-T]\frac{S}{D}\right) S & \text{if} \quad T \in [\underline{\Gamma}_S, \overline{\Gamma}_S]; \end{cases} \tag{B.5}$$

for the small jurisdiction, the base is

$$b_S(t,\,T,\,\pi) = \begin{cases} (H-h)S & \text{if} \quad t \le \underline{\Gamma}_s := T - \left[\frac{E}{S} - \frac{Dh}{S}\right] \\ 0 & \text{if} \quad t \ge \overline{\Gamma}_s := T - \left(\frac{E}{S} - \frac{DH}{S}\right) \\ \left(H - \left[\frac{E}{D} + (t-T)\frac{S}{D}\right]\right) S & \text{if} \quad t \in [\underline{\Gamma}_s, \overline{\Gamma}_s]. \end{cases} \tag{B.6}$$

In both cases, demands for the specialized goods are given by the demand function S which, strictly speaking, depends on the prices faced by households in each locality and thus on the tax rates. However, in order to simplify writing, and in accordance with the elasticity assumption mentioned above and discussed formally below, we may suppose that the threshold price $\bar{P}_S$ is sufficiently large that S may be treated as a constant in these expressions and in related expressions below.

Each of these expressions has been written so as to emphasize that the tax bases of each locality depend, first, on its own tax rate, and secondly, on the other jurisdiction's tax rate and on other parameters of the model. Observe that the functions $B_N(T;t,\pi)$ and $B_S(T;t,\pi)$ are each continuous and piecewise linear functions of the tax rates, decreasing in T and increasing in t, and likewise (*mutatis mutandis*) for $b_N(t;T,\pi)$ and $b_S(t;T,\pi)$.

We may now define *general* revenue functions, for all tax rates $(T,t) \in \mathbb{R}^2_+$ and for all parameter values $\pi \in \mathbb{R}^6_+$ as

$$R^G(T;t,\pi) = T\left(B_N[T;t,\pi] + B_S[T;t,\pi]\right) \tag{B.7}$$

$$r^G(t;T,\pi) = t\left(b_n[t;T,\pi] + b_s[t;T,\pi]\right). \tag{B.8}$$

Unlike the revenue functions (B.1) and (B.2), these revenue functions account for shopping patterns other than those assumed for Regime II. For tax rates and parameters that yield the shopping patterns assumed in Regime II, the general revenue functions (R^G, r^G) are identical to those defined in (B.1) and (B.2), but they may diverge otherwise. As illustrated in Figure B.1 for the case of the large jurisdiction, there may be values of T at which $R^G(T;\cdot) > R(T;\cdot)$ and values where the reverse is true. At tax rates where these regime II revenue functions lie below the general revenue functions (e.g., as shown in the figure, for the large locality, when it chooses low values of T), the regime-conditional revenue functions overstate the revenue potentially obtainable by choosing a tax rate that shifts spatial purchase patterns away from Regime II. Where the regime-conditional revenue functions lie above the general revenue functions (e.g., as shown in the figure, for the large locality, when it chooses high values of T), the revenues shown by the regime-conditional revenue functions understate the revenue by choosing tax rates that preclude Regime-II shopping patterns. The two functions coincide for intermediate values.

Equipped with these revenue functions, we restate the definition from the text:

Definition: Given a parameter vector π, a *Nash equilibrium* is a pair of tax rates $(T^*[\pi], t^*[\pi])$ such that[32]

$$\begin{aligned} T^*(\pi) \quad &= \text{argmax}_{<T>}\ R^G(T; t^*[\pi], \pi) \\ \text{and} \quad & \\ t^*(\pi) \quad &= \text{argmax}_{<t>}\ r^G(t; T^*[\pi], \pi). \end{aligned}$$

To tackle the question of existence, observe, first, that each general revenue function is *continuous* in both tax rates and in all of the parameters. These revenue functions are also piecewise diffentiable, but *not* continuously differentiable, in the tax rates and parameters. The reason for this is obvious from (B.3), (B.4), (B.5), and (B.6): each component of the tax bases varies continuously but not differentiably with the tax rates and parameters. For this reason, and in contrast to simpler models with only one tax base, no agglomerated spatial structure, and only one type of transactions technology, there is no *a priori* guarantee that each locality has a continuous best-response function. There is therefore no guarantee that a Nash equilibrium exists at all, and, if there is a Nash equilibrium, that it satisfies the conditions for regime II. It is therefore a non-trivial task to show that there are some regions of the parameter space for which a regime-II equilibrium does exist.

B.4 Finding a Regime II Equilibrium

To begin with, it is obvious that there are values of the key technology parameter E for which such an equilibrium is not possible; it is plausible to conjecture, and we now show, that a regime-II equilibrium – the regime between the "past" and the "future" – can only occur for "intermediate" values of the parameter.

B.4.1 The Range $[E_h, E_H]$ of Admissible Values of E

Whether a regime II equilibrium can occur clearly depends on the cost of accessing the internet, E. In order to insure that some but not all small jurisdiction residents choose to purchase online, it must be the case that $h \leq x_S^* \leq H$ when $(T, t) = (T^{II}, t^{II})$. Because

$$x_S^* = \frac{E}{D} - (T^{II} - t^{II})\frac{S}{D}, \tag{B.9}$$

[32] We may safely ignore non-negativity constraints on (T, t): a locality's revenue, which is to be maximized, is at least 0 when its tax rate is zero.

and recalling the expression $\Delta^N = T^N - t^N$ from (10), we have

$$\begin{aligned} x_S^* = & \left(\frac{dS^2+D-\frac{2}{3}dS^2}{S^2+D}\right)\frac{E}{D} - \frac{\Delta^N S}{S^2d+D} + \frac{1}{3}\left(\frac{dS^2}{S^2d+D}\right)H \\ = & \ \frac{1}{3}\left(\frac{dS^2}{S^2d+D}\right)\left(\frac{E}{D}+H\right) + \left(\frac{D}{dS^2+D}\right)\left(\frac{E}{D} - \Delta^N\frac{S}{D}\right). \end{aligned} \quad \text{(B.10)}$$

Observe that x_S^* is linearly increasing in E and we may therefore solve (B.10) for the values of E at which x_S^* reaches its lower and upper bounds of h and H for regime II:

$$E_h = \frac{1}{\frac{1}{3}dS^2 + D}\left(\left[dS^2 + D\right]h + \Delta^N S - \frac{1}{3}dS^2 H\right)D \quad \text{(B.11)}$$

and

$$E_H = \frac{1}{\frac{1}{3}dS^2 + D}\left(\left[dS^2 + D\right]H + \Delta^N S - \frac{1}{3}dS^2 H\right)D. \quad \text{(B.12)}$$

These expressions, define lower and upper bounds on the parameter E, showing that for any configuration of other parameters, no type-II equilibrium can exist unless $E_h \leq E \leq E_H$. This is of course a *necessary* condition for equilibrium, not a *sufficient* one.

We note from (B.11 and (B.12) that

$$E_H - E_h > 0 \quad \text{(B.13)}$$

i.e., $[E_h, E_H]$ is a non-degenerate interval – and which reduces to $[h, H]$ when $S = 0$. We also note, from (B.11), that $\frac{E_h}{D} - h \geq 0$ because $h \geq 1/2$, and, from (B.12), that $E_H/D \to 2H$ as $S \to \infty$. In establishing restrictions on parameters sufficient for existence of an equilibrium for regime II, we may henceforth limit attention to values of $E \in [E_h, E_H]$.

B.5 Existence of Regime II Equilibrium

To show existence, we first demonstrate that a Regime II equilibrium exists for one *particular* set of parameters, namely, $\pi^0 := (h^0, H^0, d^0, D^0, E^0, S) = (6/10, 1, 1, 2, 8/5, S)$, for any $S \in [0, 1]$. By allowing for S to take any value between 0 and 1, one may note that this set of parameter values is uncountably infinite, although it is of course of measure zero in the set of possible parameters $\pi \in \mathbb{R}_+^6$. This specific set of parameter values provides a starting point from which it will follow easily that equilibria also exist in a neighborhood of π^0 in $\mathbb{R}_+^6$ with positive measure.

To begin, consider the tax rates $(T^{II}(\pi^0), t^{II}(\pi^0))) := (T^0, t^0)$ obtained from the Regime-II best-response functions, shown in (25), for $\pi = \pi^0$. These are the unique solutions to the "restricted" revenue-maximization problem defined in (B.1) and (B.2). It can easily be shown that $T^0 \in [\underline{\Gamma}_N, \overline{\Gamma}_N]] \cap [[\underline{\Gamma}_S, \overline{\Gamma}_S]$ and $t^0 \in [\underline{\Gamma}_n, \overline{\Gamma}_n]] \cap [[\underline{\Gamma}_s, \overline{\Gamma}_s]$ for any $S \in [0,1]$. Then, To show that these are Nash equilibrium tax rates we need to show that T^0 maximizes $R^G(T, t^0)$ for all $T \geq 0$ and that t^0 maximizes $r(t, T^0)$ for all $t \geq 0$.

B.5.1 Large Locality

To check that these conditions are satisfied for our parameter values, consider first the large jurisdiction. Observe that R^G and (B.1) coincide for all $T \in [\underline{\Gamma}_N, \overline{\Gamma}_N] \cap [\underline{\Gamma}_S, \overline{\Gamma}_S]$. It remains to show that $R^G(T^0, t^0) \geq R^G(T, t^0)$ for all $T \in [0, \max\{\underline{\Gamma}_N, \underline{\Gamma}_S\}] \cup [\min\left\{\overline{\Gamma}_N, \overline{\Gamma}_S\right\}, \infty]$. The first of these intervals represents possible "downward deviations" by the large locality, in which it selects a tax rate sufficiently small that it captures the maximum feasible amount(s) of one or both of the tax bases; the second represents possible "upward deviations" in which it chooses a tax rate sufficiently high that it retains only the minimum feasible amount(s) of one or both of the two bases.

- *Downward Deviations:* $T \in [0, \max\{\underline{\Gamma}_N, \underline{\Gamma}_S\}]$

For $\pi = \pi^0$, $\max\{\underline{\Gamma}_N, \underline{\Gamma}_S\} = \max\{t^0 - 2/5, t^0 - (2/5)/S\} = t^0 - 2/5$. For any $T \leq t^0 - 2/5$, $B_N = 1$ and $B_S \leq S$, and therefore $R^G(T, t^0) \leq T(1+S) \leq (t^0 - 2/5)(1+S)$ for all downward deviations. Explicit calculations show that

$$R^G(T^0, t^0) - (t^0 - 2/5)(1+S) = \frac{2}{225}\frac{45S^3 + 36S^2 + 39S + 49}{S^2 + 2} > 0.$$

Thus, the large locality cannot increase its revenues by a downward deviation from T^0.

- *Upward Deviations:* $T \in [min\left\{\overline{\Gamma}_N, \overline{\Gamma}_S\right\}, \infty]$.

We must show that the large locality cannot raise its revenues by an upward deviation.

Beginning at $T = T^0$, increases in T cause both $B_N(T, t^0)$ and $B_S(T, t^0)$ to decline. At sufficiently high values of T, one or the other of these bases reaches its minimum value, i.e., either $B_N = 0$ or $B_S = 6S/10$ (or possibly both), as the case may be.

Because $\overline{\Gamma}_N - \overline{\Gamma}_S = (3/5) - (2/5)/S$, $\min\left\{\overline{\Gamma}_N, \overline{\Gamma}_S\right\} = \overline{\Gamma}_N$ if $S \leq 2/3$, and $\min\left\{\overline{\Gamma}_N, \overline{\Gamma}_S\right\} = \overline{\Gamma}_S$ if $S \geq 2/3$. Each case must be considered separately.

Upward Deviations: Case A: $0 \leq S \leq 2/3$.

In this case, $TB_N(T, t^0) = 0$, and thus $R^G(T, t^0) = TB_S(T, t^0)$, for all $T \geq \overline{\Gamma}_N$.

The function $B_S(T, t^0)$ is decreasing in T up to the tax rate $\hat{T}_S$ at which it falls to its minimum value of $3S/5$; it remains constant for all larger values of T. $R^G(T, t^0)$ is a (negative) quadratic function of S that is increasing in T for all $T \in [\overline{\Gamma}_N, \hat{T}_S]$. Indeed,

$$\frac{\partial R^G(T, t^0)}{\partial T} = \frac{1}{15}\frac{S(12 - 7S)}{(S^2 + 2)} > 0 \; \forall T \in [\overline{\Gamma}_N, \hat{T}_S],$$

i.e., the maximum value of $R^G(T, t^0)$ over the interval $[\overline{\Gamma}_N, \hat{T}_S]$ is achieved at $T = \hat{T}_S$, given that $S \leq 2/3$ in the case under consideration. We may now calculate

$$R^G(T^0, t^0) - R^G(\hat{T}_S, t^0) = \frac{2}{225}\frac{81S + 10}{S^2 + 2} > 0,$$

and therefore the large locality cannot increase its revenue by raising its tax rate above T^0 before its base $B_S(T, t^0)$ falls to its minimum value. At this point, revenue becomes a positive linear function of the tax rate, namely $R^G(T, t^0) = T(3S/5)$, and, of course, this exceeds the amount of revenue in the regime-II Nash equilibrium for sufficiently high values of T. Let us momentarily postpone further discussion of this possibility, which requires discussion of the elasticity condition.

Upward Deviations: Case B: $2/3 \leq S \leq 1$.

In this case, $B_S(T, t^0) = hS = 3S/5$ for all $T \geq \overline{\Gamma}_S$ and thus $R^G(T, t^0) = T(3S/5) + TB_N(T, t^0)$, which is the sum of a positive linear function of T and a negative quadratic function of T, up to the value $T = \overline{\Gamma}_N$. For $T \geq \overline{\Gamma}_N$, $R^G(T, t^0) = T(3S/5)$.

For $T \leq \overline{\Gamma}_N$, the maximum of $R^G(T, t^0)$ occurs at $T = \hat{\hat{T}}$ at which

$$\frac{\partial R^G(T, t^0)}{\partial T} = 0,$$

provided that $T = \hat{\hat{T}} \leq \overline{\Gamma}_N$. Solving this condition explicitly, we find that $\hat{\hat{T}} < \overline{\Gamma}_S < \Gamma_N$, and therefore $R^G(T, t^0)$ is a decreasing function of T for all $T \in [\Gamma_S, \Gamma_N]$. It attains its maximum, over this interval, at $T = \Gamma_S$. We therefore calculate

$$R^G(T^0, t^0) - R^G(\Gamma_S, t^0) = \frac{2}{225}\frac{S^2 - 12S + 36}{S^2(S^2 + 2)},$$

which is clearly positive for all $S \in [2/3, 1]$. Once again, therefore, we see that the large locality cannot increase its revenue by raising its tax rate to any value above T^0 before its tax base falls to its minimum value of $B_S(T, t^0) = 3S/5$. At this point, revenue again becomes a positive linear function of the tax rate, namely $R^G(T, t^0) = T(3S/5)$, and, of course, this exceeds the amount of revenue in the regime-II Nash equilibrium for sufficiently high values of T. We next discuss this possibility.

- *The Determination of $\overline{P}_S$*

The argument so far has shown that the large locality cannot increase its revenue by an upward deviation from its regime-II tax rate, except by raising the tax rate to such a high level that $B_N(T, t^0) = 0$ and $B_S(T, t^0) = 3S/5$, at which point $R^G(T, t^0) = T(3S/5)$.

It is at this point that we invoke the condition that the demand for the specialized goods becomes sufficiently elastic at the threshold price of $\overline{P}_S$. It remains to be shown that there exists such a threshold. Given that $\hat{\pi} = \hat{\pi}^0$ this can easily be determined, for any $S \in [0,1]$, by setting $\overline{P}_S(\hat{\pi}^0, S) = 1 + \max[\overline{\Gamma}_N(\hat{\pi}^0, S), \overline{\Gamma}_S(\hat{\pi}^0, S)]$. Above this threshold, we impose the condition discussed in section B.1, namely that the elasticity of demand for the specialized good is "sufficiently elastic." We can now make clear that "sufficiently elastic" means that revenue is a declining function of the tax rate for $1+T > \overline{P}_S(\hat{\pi}^0, S)$. This condition is clearly satisfied if the absolute value of the elasticity of demand is greater than unity. It is clear that this threshold price is sufficiently high such that every household is on the perfectly inelastic portion of its demand curve for the specialized commodity for all regime II equilibrium tax rates, which now justifies why we have ignored demand variations in the preceding analysis.

Although a condition like $\epsilon_S < -1$ for $P_S > \overline{P}_S$ is sufficient for existence, it is certainly not necessary. Letting $\overline{T}$ be the tax rate that achieves $\overline{P}_S$, because $R^G(\overline{T}, t^0) < R^G(T^0, t^0)$, it is possible for R^G to increase, at least somewhat, for tax rates in excess of $\overline{T}$. Necessity merely requires that $R^G(T, t^0) \leq R^G(T^0, t^0)$ for all $T \geq \overline{T}$. The condition that $\epsilon_S < -1$ suffices to insure that this is true because it means that revenue declines monotonically for $T > \overline{T}$, but it is not necessary to insist on monotonicity: the revenue function could be mildly increasing – i.e., the demand functions for the specialized goods could exhibit ranges with $\epsilon_S > -1$ – through one or some ranges of T values above $\overline{T}$. These are quite weak restrictions on the demand functions.

B.5.2 Small Locality

The situation for the small locality is easier to analyze because it has no "captive" tax base. Once again, it is necessary to consider both downward and upward deviations from the regime-II Nash tax rate of $t = t^0$, keeping T fixed at T^0.

- *Downward Deviations:* $t \in [0, \max\{\underline{\Gamma}_n, \underline{\Gamma}_s\}]$

Beginning at $t = t^0$, reductions in t cause both $b_N(t, T^0)$ and $b_S(t, T^0)$ to rise. At sufficiently low values of t, one or the other of these bases reaches its maximum value, i.e., either $b_N = 1$ or $b_S = 4S/10$ (or possibly both), as the case may be.

Because $\underline{\Gamma}_n - \underline{\Gamma}_s = -(3/5) + (2/5)/S$, $\min\{\underline{\Gamma}_n, \underline{\Gamma}_s\} = \underline{\Gamma}_s$ if $S \leq 2/3$, which defines Case A, and $\min\{\underline{\Gamma}_n, \underline{\Gamma}_s\} = \underline{\Gamma}_n$ if $S \geq 2/3$, which defines Case B.

In either case, the tax base can be no greater than $1 + 2S/5$ and the tax rate can be no greater than $\max\{\underline{\Gamma}_n, \underline{\Gamma}_s\}$. The revenue from a downward deviation is thus no greater than the upper limit $r_A = \underline{\Gamma}_s(1 + 2S/5)$ in Case A and no greater than the upper limit $r_B = \underline{\Gamma}_n(1 + 2S/5)$ in Case B. Each of these limits can be compared to equilibrium revenue in regime II, i.e., to $r(t^0, T^0)$. Explicit calculations show that

$$r(t^0, T^0) - r_A = -\frac{2}{225}\frac{54S^2 + 35S - 90}{S(S^2+2)}$$

and

$$r(t^0, T^0) - r_B = \frac{1}{225}\frac{54S^3 + 99S^2 - 90S + 128}{S^2+2},$$

both of which are strictly positive for $S \in [0,1]$. Thus, the small locality cannot increase its revenues by a downward deviation from $t = t^0$.

- *Upward Deviations:* $t \in [\min\left\{\overline{\Gamma}_n, \overline{\Gamma}_s\right\}, \infty]$.

Note first that $\min\left\{\overline{\Gamma}_n, \overline{\Gamma}_s\right\} = \overline{\Gamma}_n$ because $S \in [0,1]$. Hence, for any upward deviation, $b_n(t, T^0) = 0$, and therefore $r^G(t, T^0) = tb_S(t, T^0)$. Let $\hat{t}_S = \text{argmax}_{<t>} tb_s(t, T^0)$. Solving explicitly for this tax rate,

$$\hat{t}_S = \frac{2}{15}\frac{6S^2 + 4S + 3}{S(S^2+2)},$$

Even though $b_n(, \hat{t}_S, T^0)$ may be positive, we can use $\hat{t}_S$ because any other tax rate solving this problem with a constraint will produce even lower revenue. We that the maximum possible revenue that can be obtained by taxing only the specialized goods is

$$\hat{t}_S b(\hat{t}_S, T^0) = \frac{2}{15}\frac{6S^2 + 4S + 3)^2}{S(S^2+2)}.$$

Again calculating explicitly, for any $t \geq \overline{\Gamma}_n$,

$$r^G(t^0, T^0) - r^G(t, T^0) \geq r^G(t^0, T^0) - \hat{t}_S b_s(\hat{t}_S, T^0) = \frac{2}{225}\frac{36S^3 + 69S^2 + 144S + 89}{(S^2+2)^2} > 0,$$

that is, the small locality cannot increase its revenue by an upward deviation from $t = t^0$.

This finally concludes the demonstration that there is a regime-II Nash equilibrium for the parameter vector $\pi^0 := (h^0, H, d^0, D^0, E^0, S^0) = (6/10, 1, 1, 2, 8/5, 1/2)$.

B.6 Rescale by λ

We now show how to construct a Nash equilibrium for any value of $\lambda > 0$ given the existence of an equilibrium with $\lambda = 1$, as established in the proceeding section. To do this, we need only note that when the equilibrium tax rates vary in proportion to λ, all of the other equilibrium conditions of the model continue to be satisfied. This can easily be seen because the base functions are homogeneous of degree zero in λ and both tax rates. The revenue functions are homogeneous of degree one in the tax rates and degree zero in λ. Then, all of the analysis for the case of $\lambda = 1$ can be reconstructed because the equilibrium conditions are still satisfied at tax rates that vary in proportion to λ.

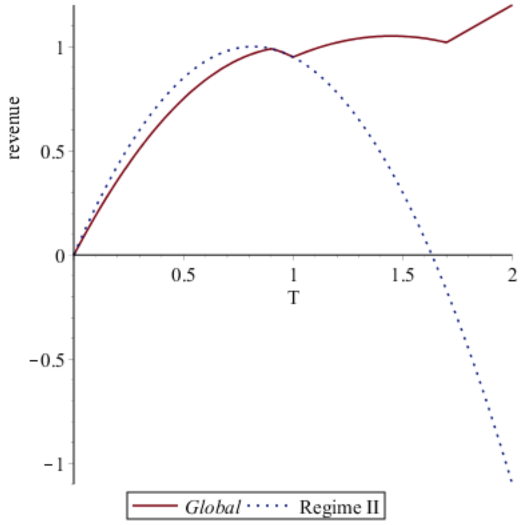
B.7 Existence of Neighborhood Around $\hat{\pi}^0$

The second part of Proposition 4 follows because the Jacobian of the two-equation system of first-order conditions characterizing the equilibrium is non-vanishing. The Jacobian in the region of a regime II equilibrium is given by,

$$\begin{aligned} J(T,t) &= \begin{bmatrix} \frac{\partial^2 R(T,t)}{\partial T^2} & \frac{\partial^2 R(T,t)}{\partial T \partial t} \\ \frac{\partial^2 r(t,T)}{\partial t \partial T} & \frac{\partial^2 r(t,T)}{\partial t^2} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{2}{d} - \frac{2S^2}{D} & \frac{1}{d} + \frac{S^2}{D} \\ \frac{1}{d} + \frac{S^2}{D} & -\frac{2}{d} - \frac{2S^2}{D} \end{bmatrix}, \end{aligned} \tag{B.14}$$

which is of course, negative definite guaranteeing that it is valid to imply the Implicit Function Theorem around an equilibrium point.

Figure B.1: Difference Between Global Revenue Function and Constrained Revenue Function



In solid colors we plot the global revenue function given by (B.7). In the dotted line we plot the constrained revenue function (for regime II) given by (B.1). The figure assumes that $\pi = (h, H, d, D, E, S) = (.6, 1, 1, 2, 1.6, 2.5)$, with $t = 0.4$. It is clear that depending on the parameter values, the global revenue function may be different than the regime II function at low or high tax rates.

C Appendix "C"

C.1 Proof of Existence in Regime I (the "Past")

Proposition 9. *For E/D sufficiently large and for $S < \frac{H+h}{H}$, a regime-conditional Nash equilibrium of type I exists where $x_N^* \in [0,\, h]$.*

Proof. We wish to show that a regime-conditional Nash equilibrium exists when the non-specialized base is "interior" and the specialized base is in regime I. The candidate Nash equilibrium is (T^I, t^I) and is given by (16). A regime-conditional Nash equilibrium means we only need to verify that jurisdictions do not have an incentive to unilaterally deviate from the "interior" case ($j = b$) to the "all city" case ($j = a$) or "all hinterland" case ($j = c$) in (3) and (4). For ease of notation index each of these cases by j, as noted.

- *Satisfying the relevant constraints.*

Before we show that there is no profitable deviation, we need to verify that all constraints on the problem are satisfied at (T^I, t^I). The two constraints are that $x_S^* > H$ and $x_N^* \in [-(H-h),\, h]$. When E/D is very large, it is clear that $x_S^* > H$; in particular this is true if $E > HD + (T^I - t^I)S$. Given that $T^I > t^I$ in equilibrium, the solution must have cross-border shopping from the city to hinterland, i.e. $x_N^* < h$, which means that in equilibrium, substituting (T^I, t^I) into x_N^* yields $\frac{SH-H+2h}{3} < h$ which is satisfied if $S < \frac{H+h}{H}$. Having verified that the tax rates (T^I, t^I) support "interior" shopping patterns, it remains to show that these tax rates maximize each of the revenue functions. Because each locality's regime-conditional revenue function is an inverse quadratic function of its own tax rate, the only question is whether either could profitable deviate from the "interior" case $j = b$.

- *Deviations by the small jurisdiction: Upward deviations.*

If the small jurisdiction deviates upward to case "all city", it obtains no tax revenue and thus will not deviate.

- *Deviations by the small jurisdiction: Downward deviations.*

To deviate to case "all hinterland" the jurisdiction must lower its tax rate. Revenue in case $j = c$ will be maximized at the highest possible tax rate that satisfies that case, which is given by $t = T^I - dh$ and it will obtain tax revenue of $r^c = (T^I - dh)H$. This can be compared to revenue in case $j = b$ which is given by $r^b = t^I(H - h + \frac{T^I - t^I}{d})$. Then $r^b - r^c = \frac{d}{9}(HS - H - h)^2 > 0$, which guarantees the small jurisdiction does not deviate.

- *Deviations by the large jurisdiction: Downward deviations.*

If the large jurisdiction deviates to case "all city", the deviation must be downward. Revenue in the case $j = a$ is maximized at the highest possible tax rate that support that case, which is given by $T = t^I - d(H - h)$. It then obtains revenue $R^a = (t^I - d(H - h))(H + SH)$, as opposed to $R^b = T^I(h + \frac{t^I - T^I}{d} + SH)$. It can be verified that $R^b - R^a = \frac{d}{9}(HS + 2H - h)^2 > 0$ and no profitable deviation exists to this regime.

- *Deviations by the large jurisdiction: Upward deviations.*

Finally, consider a deviation to regime "all hinterland". Here, the city has given up the entire non-specialized base but has a captive specialized base. If the specialized tax base were inelastic then the city could just keep running up its tax rate to infinity (or to a household non-negative surplus constraint). However, as noted in the text, the specialized base becomes elastic at a sufficiently high price. The minimum deviation to case $j = c$ requires a tax rate of $T = t^I + dh > T^I$ yielding tax revenue of $R^c = (t^I + dh)SH$. Then, at that minimum deviation the revenue differential is $R^b - R^c = \frac{d}{9}(HS - H - h)^2 > 0$. Setting $\overline{P_S} = 1 + T = 1 + (t^I + dh)$, and postulating the demand for the specialized good is sufficiently elastic above this threshold, any deviation to a tax rate above $\overline{P_S} - 1$ results in a reduction in revenue. For this value of $\overline{P_S}$, when $T = T^I$, an individual will still buy S units of the specialized good because $1 + T^I \leq 1 + t^I + dh$. □

- *Example.*

We give an illustrative example of this for the parameter values of $h = 0.6, H = 1, E = 100, D = 0.2, d = 0.2, S = 1$, which results in equilibrium tax rates of $T^I = 0.24, t^I = 0.16$. See figure C.1. The elasticity condition on the specialized tax base eliminates a deviation to the explosive portion of the revenue function.

C.2 Proof of Existence in Regime III (the "Future")

Proposition 10. *For E/D sufficiently small and for $S < \frac{H+h}{2h-H}$, a regime-conditional Nash equilibrium of type III exists where $x_N^* \in [0,\ h]$.*

Proof. We wish to show that a regime-conditional Nash equilibrium to the revenue function exists when the non-specialized base is "interior" and the specialized base is in regime III. The candidate Nash equilibrium is (T^{III}, t^{III}) and is given by (20). A regime-conditional Nash equilibrium means we only need to verify that jurisdictions do not have an incentive to unilaterally deviate from the "interior" case ($j = b$) to the "all city" case ($j = a$) or "all hinterland" case ($j = c$) in (3) and (4). For ease of notation index each of these cases by j as noted.

- *Satisfying the relevant constraints.*

Before we show that there is no profitable deviation, we need to verify that all constraints on the problem are satisfied at (T^{III}, t^{III}). The two constraints are that $x_S^* < h$ and $x_N^* \in [-(H-h), h]$. When E/D is very small, it is clear that $x_S^* < h$; in particular, this is true if $E < hD + (T^{III} - t^{III})S$. Given that $T^{III} > t^{III}$ in equilibrium, the solution must have cross-border shopping from the city to hinterland, i.e., $x_N^* < h$. This means that in equilibrium $\frac{\Delta^{III}}{d} < h$ which is satisfied if $S < \frac{H+h}{2h-H}$. As in Appendix C.1, it remains to show that neither locality can profitably deviate away from the "interior" case $j = b$.

- *Deviations by the small jurisdiction: Downward deviations.*

For the small jurisdiction to deviate to the case "all hinterland" ($j = c$), it must be a downward deviation to at least $t = T^{III} - dh$ in order to capture all of the non-specialized base. Then it obtains revenue of $r^c = (T^{III} - dh)(H + S(H-h))$ which is maximized at the tax rate that just puts the jurisdiction into the case. This can be compared to revenue in case $j = b$ which is given by $r^b = t^{III}(H - h + \frac{T^{III} - t^{III}}{d} + S(H-h))$. Then $r^b - r^c = \frac{d}{9}(H + h + HS - 2Sh)^2 > 0$.

- *Deviations by the small jurisdiction: Upward deviations.*

To deviate to the case "all city" the small jurisdiction must raise its tax rate. Note that the minimum deviation to case $j = a$ requires a tax rate of $t = T^{III} + d(H-h) > t^{III}$. At that minimum deviation, $r^b - r^a = \frac{d}{9}(HS - 2H - 2Sh + h)^2 > 0$. However, given that the cost of buying online is so low (the hinterland will give away the entire non-specialized base first), the hinterland could potentially begin raising its tax rate until it begins to lose the specialized base. Set $\overline{P_S} = 1 + t = 1 + T^{III} + d(H-h)$, where one is the pre-tax price and $T^{III} + d(H-h)$ is the deviation tax rate. Any deviation to a tax rate above $T^{III} + d(H-h)$ implies a fall in revenue because the demand for the specialized good is sufficiently elastic in this range. When $T = T^{III} > t^{III}$, an individual still buys S units of the specialized good because $T^{III} < T^{III} + d(H-h)$.

- *Deviations by the large jurisdiction: Downward deviations.*

If the large jurisdiction deviates to the case "all city" ($j = a$), the deviation must be downward. Revenue in case $j = a$ will be maximized at the highest possible tax rate that satisfies that case, which will be given by $T = t^I - d(H-h)$. Then it obtains revenue $R^a = (t^I - d(H-h))(H + Sh)$ as opposed to $R^b = T^{III}(h + \frac{t^{III} - T^{III}}{d} + S(H-h))$. It can be verified that $R^b - R^a = \frac{d}{9}(HS - 2H + h - 2Sh)^2 > 0$.

- *Deviations by the large jurisdiction: Upward deviations.*

Finally, consider a deviation to the case "all hinterland" ($j = c$). Here, the city has given up the entire non-specialized base but has a captive specialized base given that even with online shopping the hinterland cannot obtain revenue from the city's residents. The minimum deviation to case $j = c$ requires a tax rate of $T = t^{III} + dh > T^{III}$. Then, at that minimum deviation, $R^b - R^c = \frac{d}{9}(HS + H - 2Sh + h)^2 > 0$. Set $\overline{P_S} = 1 + T = 1 + (t^{III} + dh)$, where one is the pre-tax price and $t^{III} + dh$ is the deviation tax rate. Any deviation to a tax rate above $t^{III} + dh$ then result in less revenue by our elasticity condition. When $T = T^{III}$, an individual still buys S units of the specialized good because because $t^{III} < T^{III} \leq t^{III} + dh$ using the conditions noted above to guarantee $x_N^* < h$. □

- *Example.*

We give an illustrative example of this for the parameter values of $h = 0.6, H = 1, E = 0.01, D = 0.3, d = 0.3, S = 1$, which results in equilibrium tax rates of $T^{III} = 0.32, t^{III} = 0.28$. See figure C.2. The elasticity condition on the specialized tax base eliminates a deviation to the explosive portions of the revenue function.

C.3 Proof of Non-existence

Proposition 11. *There exists a vector of parameter values $\tilde{\pi}^1 = (h^1, H^1, d^1, D^1, E^1) \in \mathbb{R}^5_{++}$ and a value of $S = S^1$, such that no Nash equilibrium exists.*

Proof. In this section, we show that for certain parameter values, no Nash equilibrium will exist. We proceed by example. Consider the case where $h = 0.6,\ H = 1,\ S = 1,\ D = 7.7,\ E = 11,\ d = 40$. With these parameter values and our elasticity condition, no Nash equilibrium exists. To show this, we evaluate the revenue functions at these parameter values, for case $j = b$ i.e., those where some but not all households cross-border shop for the non-specialized good ($x_N^* \in [-(H - h),\ h]$),

$$R(T,t) = \begin{cases} T\left(\frac{8}{5} + \frac{1}{40}(t - T)\right) & \text{if } x_S^* > H & \text{Past: } I.b \\ T\left(\frac{71}{35} + \frac{477}{3080}(t - T)\right) & \text{if } h < x_S^* < H & \text{Present: } II.b \\ T\left(\frac{6}{5} + \frac{1}{40}(t - T)\right) & \text{if } h > x_S^* & \text{Future: } III.b \end{cases} \tag{C.1}$$

and

$$r(t,T) = \begin{cases} t\left(\frac{2}{5} + \frac{1}{40}(T - t)\right) & \text{if } x_S^* > H & \text{Past: } I.b \\ t\left(\frac{477}{3080}(T - t) - \frac{1}{35}\right) & \text{if } h < x_S^* < H & \text{Present: } II.b \\ t\left(\frac{4}{5} + \frac{1}{40}(T - t)\right) & \text{if } h > x_S^* & \text{Future: } III.b. \end{cases} \tag{C.2}$$

There are still more possibilities to consider, however. When $t > T$, we may also have a situation (case $j = a$) where all all residents of the hinterland cross-border shop:

$$R(T,t) = \begin{cases} 2T & \text{if } x_S^* > H \text{ and } x_N^* > H-h \text{ regime } I.a \\ T\left(\frac{17}{7} + \frac{10}{77}(t-T)\right) & \text{if } h < x_S^* < H \text{ and } x_N^* > H-h \text{ regime } II.a \\ \frac{8}{5}T & \text{if } h > x_S^* \text{ and } x_N^* > H-h \text{ regime } III.a \end{cases} \tag{C.3}$$

and

$$r(t,T) = \begin{cases} 0 & \text{if } x_S^* > H \text{ and } x_N^* > H-h \text{ regime } I.a \\ t\left(\frac{10}{77}(T-t) - \frac{3}{7}\right) & \text{if } h < x_S^* < H \text{ and } x_N^* > H-h \text{ regime } II.a \\ \frac{2}{5}t & \text{if } h > x_S^* \text{ and } x_N^* > H-h \text{ regime } III.a. \end{cases} \tag{C.4}$$

If, on the other hand, when $T \geq t$ we may also have a situation (case $j = c$) where all all residents of the city cross-border shop:

$$R(T,t) = \begin{cases} T & \text{if } x_S^* > H \text{ and } x_N^* > h \text{ regime } I.c \\ \frac{10}{77}T\,(11+t-T) & \text{if } h < x_S^* < H \text{ and } x_N^* > h \text{ regime } II.c \\ \frac{3}{5}T & \text{if } h > x_S^* \text{ and } x_N^* > h \text{ regime } III.c \end{cases} \tag{C.5}$$

and

$$r(t,T) = \begin{cases} t & \text{if } x_S^* > H \text{ and } x_N^* > h \text{ regime } I.c \\ t\left(\frac{4}{7} + \frac{10}{77}(T-t)\right) & \text{if } h < x_S^* < H \text{ and } x_N^* > h \text{ regime } II.c \\ \frac{7}{5}t & \text{if } h > x_S^* \text{ and } x_N^* > h \text{ regime } III.c. \end{cases} \tag{C.6}$$

To differentiate all of these cases, denote a regime-conditional *candidate* equilibrium by $(\widehat{T^r}, \widehat{t^r})$ and the corresponding revenue payoffs as $(\widehat{R^r}, \widehat{r^r})$, where r indexes the regime being considered. We must consider deviations by the large jurisdiction to any alternative regime $k \neq r$ which corresponds to playing a tax rate $\widetilde{T^k}$ with payoff $\widetilde{R^k}(\widetilde{T^k}, \widehat{t^r})$. Similarly, deviations by the small jurisdiction to a tax rate $\widetilde{t^k}$ yields a payoff $\widetilde{r^k}(\widetilde{t^k}, \widehat{T^r})$. We now consider each of the cases in turn, starting with the easiest cases to dismiss.

- *"Degenerate" cases:*

We can always find a $\overline{P_S}$ and a sufficiently elastic demand function for S such that no equilibrium can exist for cases $I.a$, $I.c$, $III.a$, and $III.c$. It remains to show that no Nash equilibrium exists in regime $I.b$, $II.b$, $III.b$, or $II.a/c$. We consider each case in turn.

- *Regime II.a/c deviation.*

Start from the regime $II.a/c$. In these cases, the large government maximizes tax revenue but must satisfy the constraints that $0.6 < x_S^*$, $x_S^* < 1$, and $x_N^* > 0.6$ if $T > t$ or $-x_N^* > 0.4$ if $T < t$, where $x_N^* = \frac{T-t}{d}$. Notice that $x_S^* < 1$, which implies that $T > \frac{33}{10} + t$; likewise, $x_S^* > 0.6$ implies that $T < \frac{319}{50} + t$. Then the constraint on cross-border shopping for the non-specialized good implies that $T > 24 + t$ if $T > t$, or $T < t - 16$ if $T < t$. All of these conditions, when binding, imply that T and t differ by a constant, but by *different* constants. In particular, when $T > t$ it is impossible that both $T < \frac{319}{50} + t$ and $T > 24 + t$. This rules out the possibility of a regime $II.c$ equilibrium. Likewise, when $t < T$, it is impossible that both $T > \frac{33}{10} + t$ and $T < t - 16$. This rules out a possibility of a regime $II.a$ equilibrium.

In the next sections, we consider deviations from the past, present, and future, when some but not all residents cross-border shop for the non-specialized goods (regimes $I.b$, $II.b$, $III.b$). To simplify notation, we suppress further reference to case "b."

- *Regime II.b deviation.*

Start from the regime II (some hinterland residents shop online). Differentiating (C.1) and (C.2) with respect to the tax rates and solving the system of best responses yields a candidate equilibrium of $(\widehat{T^{II}} = \frac{4136}{466}, \widehat{t^{II}} = \frac{2024}{477})$. Tax revenues are $(\widehat{R^{II}} = 11.64, \widehat{r^{II}} = 2.79)$. For this to be a Nash equilibrium, no profitable unilateral deviation can exist. Consider a potential deviation to regime III by the large jurisdiction. Taking the small jurisdiction's tax rate as fixed, if the large jurisdiction deviates to regime III, it obtains revenue $\widetilde{R^{III}}(T, \widehat{t^{II}}) = \widetilde{T^{III}} \left(\frac{6}{5} + \frac{1}{40}(\frac{2024}{477} - \widetilde{T^{III}})\right)$, where the small jurisdiction tax rate remains fixed at $\widehat{t^k} = \frac{2024}{477}$ and $\widetilde{T^{III}}$ is the tax rate that maximizes the regime III revenue function given the small jurisdiction's tax rate. Revenue in regime III following a unilateral deviation is maximized at $\widetilde{T^{III}} = 26.12$ and this tax rate satisfies all of the constraints necessary to be in regime III. This results in tax revenue of $\widetilde{R^{III}} = 17.06 > \widehat{R^{II}} = 11.64$, which shows that the large jurisdiction can profit by deviating.

- *Regime I.b deviation.*

Next, consider a regime I (no online shopping) potential equilibrium. Following the same procedure, differentiating (C.1) and (C.2) with respect to the tax rates and solving the system of best responses yields a candidate equilibrium of $(\widehat{T^I} = 48, \widehat{t^I} = 32)$. However, it can be verified that this candidate equilibrium does not satisfy $x_S^* > H$ and violates some of the necessary constraints on the problem. To proceed we need to solve the governments' "constrained" problems. In particular, we need to maximize $R^I(T, t)$ and

$r^I(t,T)$ subject to the inequality constraint $x_S^* > H$, which yields piecewise best response functions for the large and small jurisdiction:

$$T = \begin{cases} 32 + \frac{t}{2} & \text{if } t > \frac{287}{5} \\ \frac{33}{10} + t & \text{otherwise} \end{cases} \tag{C.7}$$

and

$$t = \begin{cases} 8 + \frac{T}{2} & \text{if } T < \frac{113}{5} \\ T - \frac{33}{10} & \text{otherwise.} \end{cases} \tag{C.8}$$

These two piecewise best responses are colinear over a particular range and there are multiple candidate Nash equilibrium that range anywhere on the line between $(\widehat{T^I} = 22.6, \widehat{t^I} = 19.3)$ and $(\widehat{T^I} = 60.7, \widehat{t^I} = 57.4)$. For no regime I equilibrium to exist, it must be the case that there is a profitable deviation for all of these tax rates. To check for profitable deviations, notice that $\widehat{r^I} = \widehat{t^I}\left(\frac{2}{5} + \frac{1}{40}(\widehat{T^I} - \widehat{t^I})\right) \in [9.31, 27.70]$. If this jurisdiction were to deviate to regime III, it would deviate to $\widetilde{t^{III}}$ to obtain $\widetilde{r^{III}}(t, \widehat{T^I}) = \widetilde{t^{III}}\left(\frac{4}{5} + \frac{1}{40}(\widehat{T^I} - \widetilde{t^{III}})\right) \in [\widetilde{t^{III}}(1.365 - \frac{\widetilde{t^{III}}}{40}), \widetilde{t^{III}}(2.318 - \frac{\widetilde{t^{III}}}{40})]$ where $\widetilde{t^{III}}$ is the tax rate that maximizes the revenue function subject to the constraints. Notice that any deviation to regime III must involve a lower tax rate. Along the colinear portions of the best response functions, tax revenue is monotonically increasing in $\widehat{t^I}$ because $\widehat{r^I} = \widehat{t^I}\left(\frac{2}{5} + \frac{1}{40}((\frac{33}{10} + \widehat{t^I}) - \widehat{t^I})\right) = \frac{193}{400}\widehat{t^I}$. Now, using the facts that $\widehat{T^I} = \frac{33}{10} + \widehat{t^I}$ and that the optimal deviation tax rate is $\widetilde{t^{III}} = \widehat{T^I} - \frac{319}{50}$, we have $\widetilde{r^{III}}(\widetilde{t^{III}}, \widehat{T^I}) - \widehat{r^I} = -2.955 + .47\widehat{t^I}$, which is positive for all tax rates in $\widehat{t^I} \in [9.31, 27.70]$. Thus, no Nash equilibrium exists in regime I.

- *Regime III.b deviation.*

Next, consider a regime III (all online shopping) potential equilibrium. Following the same procedure, differentiating (C.1) and (C.2) with respect to the tax rates and solving the system of best responses yields a candidate equilibrium of $(\widehat{T^{III}} = \frac{128}{3}, \widehat{t^{III}} = \frac{112}{3})$. This candidate equilibrium does not satisfy $x_S^* < h$. To proceed, we need to solve the governments' "constrained" problems. In particular, we need to maximize $R^{III}(T,t)$ and $r^{III}(t,T)$ subject to the inequality constraint $x_S^* < h$, which yields piecewise best response functions for the large and small jurisdictions:

$$T = \begin{cases} 24 + \frac{t}{2} & \text{if } t < \frac{881}{25} \\ \frac{33}{10} + t & \text{otherwise} \end{cases} \tag{C.9}$$

and

$$t = \begin{cases} 16 + \frac{T}{2} & \text{if } T > \frac{1119}{25} \\ T - \frac{33}{10} & \text{otherwise.} \end{cases} \tag{C.10}$$

The simultaneous solution of these two piecewise best responses indicates that there are multiple candidate Nash equilibrium on the line between the points ($\widehat{T^{III}} = 41.62, \widehat{t^{III}} = 35.24$) and ($\widehat{T^{III}} = 44.76, \widehat{t^{III}} = 38.38$). For no regime III equilibrium to exist, it must be the case that there is a profitable deviation for all of these tax rates. To check for a profitable deviations notice that $\widehat{R^{III}} = \widehat{T^{III}}\left(\frac{6}{5} + \frac{1}{40}(\widehat{t^{III}} - \widehat{T^{III}})\right) \in [43.31, 46.57]$, but if the large jurisdiction were to deviate to regime I, it would deviate to $\widetilde{T^{I}}$ to obtain $\widetilde{R^{I}}(T, \widehat{t^{III}}) = \widetilde{T^{I}}\left(\frac{8}{5} + \frac{1}{40}(\widehat{t^{III}} - \widetilde{T^{I}})\right) \in [\widetilde{T^{I}}(2.481 - \frac{\widetilde{T^{I}}}{40}), \widetilde{T^{I}}(2.560 - \frac{\widetilde{T^{I}}}{40})]$ where $\widetilde{T^{I}}$ maximizes the revenue function subject to the given constraints. Along the colinear portions of the best response functions, tax revenue is monotonically increasing in $\widehat{T^{III}}$ because $\widehat{R^{III}} = \widehat{T^{III}}\left(\frac{6}{5} + \frac{1}{40}((\widehat{T^{III}} - \frac{319}{50}) - \widehat{T^{III}})\right) = \frac{2081}{2000}\widehat{T^{III}}$. Then using the facts that $\widehat{T^{III}} - \frac{319}{50} = \widehat{t^{III}}$ and that the tax rate that maximizes revenue is $\widetilde{t^{III}} = \widehat{T^{I}} + \frac{33}{10}$, we have $\widetilde{R^{I}}(\widetilde{T^{I}}, \widehat{t^{III}}) - \widehat{R^{III}} = -674 + .477\widehat{T^{III}}$, which is positive for all tax rates in $\widehat{T^{III}} \in [38.6, 44.7]$. Thus, no equilibrium exists in regime III.

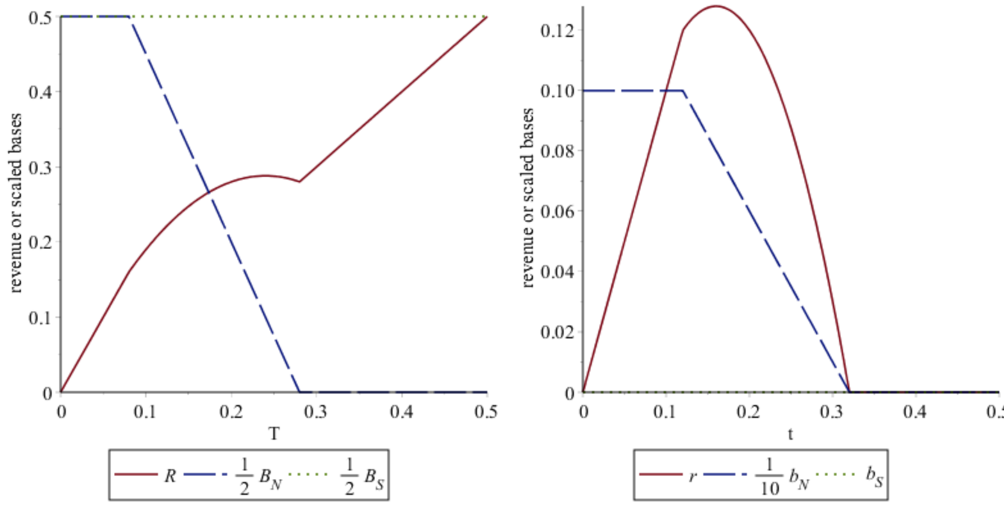
This completes the proof that no Nash equilibrium exists for some parameter values. □

- *Visualization.*

A simple illustration shows convincingly how existence can fail for some parameter values. We continue to use the parameter values of $h = 0.6,\ H = 1,\ S = 1,\ D = 7.7,\ E = 11,\ d = 40$, showing that a profitable deviation exists for at least one jurisdiction in each regime. See figure C.3.

Figure C.1: Example of Regime-Conditional Nash Equilibrium for the Past

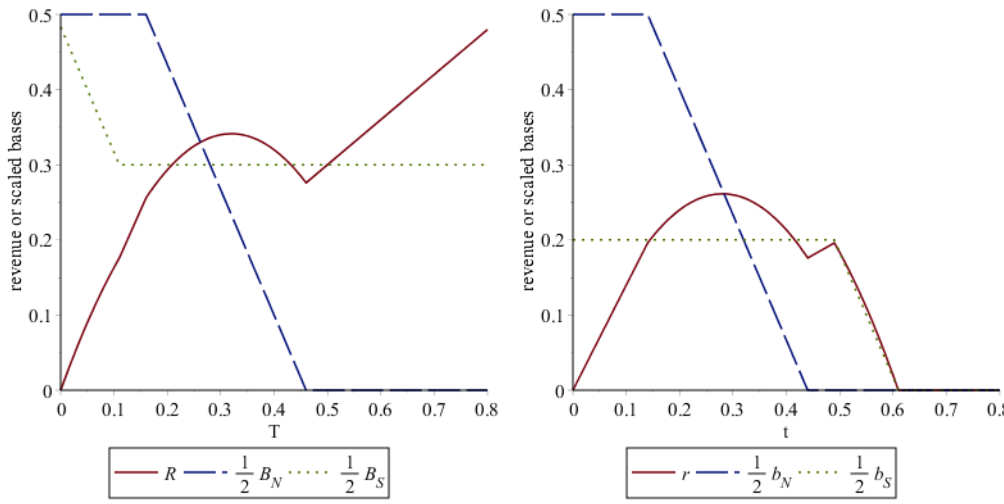
City Hinterland



We use parameter values of $h = 0.6, H = 1, E = 100, D = 0.2, d = 0.2, S = 1$. Each revenue function is drawn assuming the other jurisdiction sets the equilibrium tax rate T^I or t^I. We present in dashed lines, the size (appropriately scaled to appear on the same vertical axis as revenue) of the specialized tax base and the non-specialized base. We are interested in an equilibrium where the non-specialized base has some people cross-border shopping (the downward-sloping part of the dashed line). There are clearly revenue maxima at $T^I = 0.24, t^I = 0.16$ so long as there us a suitable $\overline{P_S}$ when $B_N = 0$ above which the demand for the good is sufficiently elastic such that the linearly increasing part of the function does not apply.

Figure C.2: Example of Regime-Conditional Nash Equilibrium for the Future

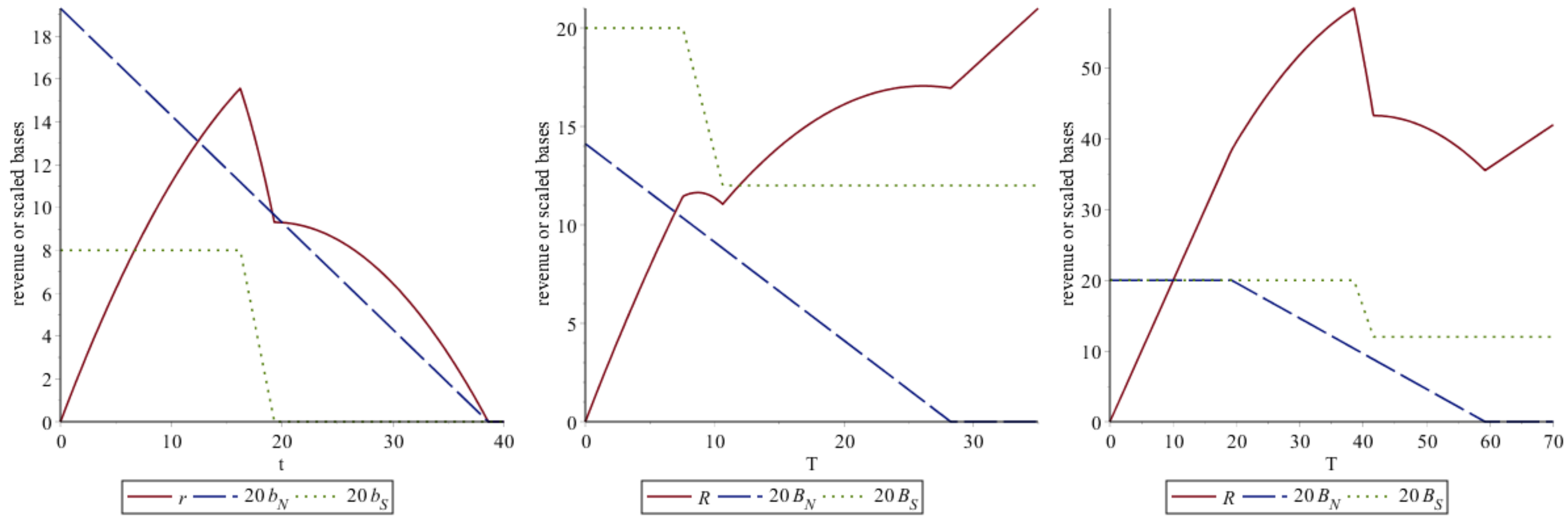
City Hinterland



We use parameter values of $h = 0.6, H = 1, E = 0.01, D = 0.3, d = 0.3, S = 1$. Each revenue function is drawn assuming the other jurisdiction sets the equilibrium tax rate T^{III} or t^{III} so that the reader can verify if a global deviation exists. We present in dashed lines, the size (appropriately scaled to appear on the same vertical axis as revenue) of the specialized tax base and the non-specialized base. We are interested in an equilibrium where the non-specialized base has some people cross-border shopping (the downward-sloping part of the dashed line). Notice a clear maximum in $T^{III} = 0.32, t^{III} = 0.28$ so long as there us a suitable $\overline{P_S}$ when $B_N = 0$ above which the demand for the good is sufficiently elastic such that the linearly increasing part of the function does not apply.

Figure C.3: Example of Nonexistence of an Equilibrium

Regime I (Hinterland Deviation) Regime II (City Deviation) Regime III (City Deviation)



This figure shows the tax revenues for parameter values of $h = 0.6$, $H = 1$, $S = 1$, $D = 7.7$, $E = 11$, $d = 40$. Each revenue function is drawn assuming that the other jurisdiction sets a candidate equilibrium tax rate, so that one can verify whether a global deviation exists. To ease the partitioning of the cases, we present in dashed lines, the size (appropriately scaled to appear on the same vertical axis as revenue) of the specialized tax base and the non-specialized base. In the leftmost figure, we show that fixing the city tax rate at its regime I optimum, the hinterland deviates from its locally optimal tax rate in regime I (hinterland specialized base is zero) to regime III (all online shopping). In the middle figure, we show that fixing the hinterland's tax rate at its regime II optimum, the city deviates from its locally optimal tax rate in regime II (some online shopping) to regime III (all online shopping). In the rightmost figure, we show that fixing the hinterland's tax rate at its regime III optimum, the city deviates from its locally optimal tax rate in regime III (all online shopping) to regime I (no online shopping).