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Revelation Principle with Persistent Correlated Types: Impossibility Result

Abstract

This paper studies the revelation principle for mechanisms with limited commitment when agents have correlated persistent types over the infinite horizon. After characterizing necessary and sufficient conditions to construct a mechanism with same ex-ante payoffs and interim beliefs to all players as in the given mechanism that offers the type space as the message space to all players after every history, the paper shows that they are not satisfied by every mechanism with persistent correlated types. Unless one can construct a mechanism with only the same ex-ante payoffs and not the same interim beliefs for all remaining cases, offering the type space as the message space to all agents every period is with loss of generality and leads to a strictly smaller set of equilibrium payoffs.

JEL-Codes: D820.

Keywords: mechanism design, limited commitment, revelation principle, informed-principal problem, persistence, correlated types.

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1 Introduction

This paper focuses on mechanism design with limited commitment when agents' types are correlated across agents and persistent across time periods. Examples of correlated persistent types are easier to find with firms. For example, unless the liquidity holding requirement and the overnight interbank loans are slack constraints, all banks regulated by the same authority share common parameters in their operation. Also, any firm using semi-conductors is subject to the same cost shock. If there are commonly shared parameters in each firm's revenue-maximization problem, then as long as these parameters enter binding constraints, all such firms' payoffs are correlated within the period. As for persistence over time, revenue maximization over multiple periods requires taking into account exogenous parameters across different time periods. History dependence in optimal mechanisms or equilibrium strategies means that the past realizations of exogenous parameters matter for the continuation game. This is in general true in the mechanism design literature and the repeated games literature. It is far more unusual if an optimal mechanism or an equilibrium strategy is not history dependent without assuming it in the model, e.g., Markovian strategies. The precise channels of history dependence can be through payoff-relevant states, but it can equally be just the history-contingent strategy that induces correlated persistent types.

Mechanism design with limited commitment in the sense that the mechanism designer offers a new mechanism each period goes back to the 80s. See for example the literature on ratchet effect. However, once the mechanism designer has limited commitment, there are several other assumptions that matter for optimal mechanisms. First of all, given that the designer has limited commitment, one must choose an equilibrium notion for the game. However, when there are multiple agents, there are two assumptions, which together matter more than the equilibrium notion; these are whether agents' types are i.i.d. or not and whether the mechanism designer has any capacity constraint.[1]

[1] See Kwon (2019a) for the characterization of equilibrium payoffs when types are i.i.d. and the mechanism designer doesn't have any resource constraint.

This paper doesn't consider any resource constraint of the mechanism designer but relaxes the i.i.d. assumption of multiple agents' types.

In a dynamic setting, identically and independently distributed types require more qualifiers. The distribution of the type profile matters within period for all agents in the same period; and the realized type profile this period matters for the distribution in the following period. I allow for both correlation within each period and persistence over multiple periods.

I focus on the revelation principle in this paper. If we already know the model satisfies a sufficient condition to obtain full-commitment solutions with limited commitment, one can invoke the revelation principle with full commitment, i.e., it is sufficient to consider mechanisms that offer the type space as the message space for each agent. However, if we don't already know that full-commitment solutions can be obtained with limited commitment, then one needs a version of revelation principle to characterize any mechanism with a qualifier such as "optimal." The revelation principle in the usual sense typically brings down the set of all mechanisms one needs to consider; instead of having any "set" as the message space each agent can choose from, each agent (mis)reports his type to the mechanism designer, and it is sufficient to consider this type of mechanisms to find a welfare-maximizing mechanism, revenue-maximizing mechanism and so forth. Therefore, if we don't know whether full-commitment solutions can be attained with limited commitment in a given class of mechanisms, then one cannot assert any optimality without a version of revelation principle.

I characterize two necessary and sufficient conditions under which one can construct an alternative mechanism that offers the type space as the message space and obtains the same ex-ante payoff and interim beliefs for each player as in the original mechanism. If neither applies, then one can only hope for payoff equivalence and cannot construct an alternative mechanism with same interim beliefs and ex-ante payoffs for all players that offer type spaces as message spaces.

In case anyone is already familiar with the proof of Bester-Strausz (2001), the first condition I characterize has the same spirit as theirs; all players' beliefs, including the mechanism designer and all agents, remain the same as in the original mechanism, and the weighted distribution over allocation for each type profile remains the same when the type space is offered as the message space for each agent. However, one of the immediate restrictions on this class of mechanisms is that when the priors of agents after observing only their types are the same, then two different type profiles, that are unobservable to any player, must lead to the same distribution over allocations. In a static setting, this statement doesn't have a bite with independent types but matters with correlated types which is a key assumption of my model. However, in a dynamic setting, this is again different from just having two different distributions with the same marginals, if agents don't learn the type profile perfectly at the end of each period.

What is more surprising, or hasn't been shown in the literature, is the second condition I show. The second condition doesn't hold the distribution of allocation for each type profile the same as in the original mechanism. Loosely speaking, agents' beliefs on the type profile are perpendicular to the weighted difference in allocation, between the original mechanism and the one that offers the type space as the message space. This has to hold for each type profile and allocation. In particular, it immediately follows that if only a fraction of agents have private types and the total number of agents is bigger than the total number of type profiles, this condition never holds unless agents' beliefs are linearly dependent.

Combining two conditions for each pair of type profile and allocation, one can characterize a necessary set of conditions to be able to offer the type space as the message space. If none of them holds after any history on the equilibrium path, there is no mechanism that attains the same ex-ante payoffs and interim beliefs as in the given mechanism while offering the type space as the message space. What this suggests is that if one only considers mechanisms that offer the type space as the message space, one cannot verify that all "possible" mechanisms are considered in any comparison, without constructing alternative mechanisms for all remaining cases that only attain the same ex-ante payoffs and not interim beliefs. However, this requires finding the set of all equilibrium payoffs without restricting message spaces to type spaces.

The rest of the paper is organized as follows. After introducing related literature, section 2 presents the model. Results are in section 3, and section 4 concludes.

1.1 Related Literature

1.1.1 Revelation Principle with Limited Commitment

The most closely related paper is Bester-Strausz (2001) which shows that the principal with limited commitment can offer the set of types as the message space in any optimal mechanism if there is a single agent in finite horizon. However, the agent need not report truthfully with probability 1 every period. This paper shows what happens with infinite horizon, informed principal and (partially) persistent correlated types with multiple agents.

One should note first that Bester-Strausz (2001) is a statement on optimal mechanisms. Once there exists an optimal mechanism, one can construct an equilibrium in which the message space for the agent is the set of types. The proof of Bester-Strausz (2001) constructs the reporting strategy of the agent. As is the case with all versions of revelation principle, there exists "an" equilibrium with the desired property. With full commitment of the principal, it is straightforward to see why the revelation principle cannot hold in every equilibrium in the literal sense; just let type 1 always report type 2 and vice versa for type 2. In Bester-Strausz (2001), there exists an equilibrium that achieves the same payoffs and the agent uses a mixed strategy over the set of types.

I should also emphasize what changes in the settings I consider in this paper. With full commitment of the principal, the revelation principle makes it without loss of generality only to study mechanisms whose message space is the set of types. In this case, regardless of the property of the mechanism one wants to characterize, one can ask the agent to report his type in the beginning of the period and choose allocations and transfers jointly. When the principal has limited commitment, Bester-Strausz (2001) works for a single agent in finite horizon for "optimal" mechanism. To put it differently, in order to characterize optimal mechanisms, one can ask the agent to report his type each period and choose allocations and transfers simultaneously; however, the agent need not report truthfully every period. With full commitment, one could focus on equilibria in which agents report truthfully every period. In the setting of Bester-Strausz (2001), one cannot assume truthful reporting every period.

Without any type of revelation principle, if the mechanism designer can offer any set as a message space, then the number of mechanisms one can offer is at least as big as the number of "sets," and if the objective is to characterize one mechanism with an equilibrium that has the desired property, one can still try to construct an example. However, any statement alongs the lines of "there exists no mechanism with the following property" or "in the set of all mechanisms one can offer" requires the comparison to any mechanism that can possibly be offered. Therefore, unless one were to literally construct a mechanism with an equilibrium with some property, one must be able to compare to all mechanisms that can be offered, and any result on "optimal" mechanism cannot be validated without this comparison.

1.1.2 (Partially) Persistent Correlated Types

Correlated types by itself are allowed in any model with the common prior assumption that doesn't assume independence across different players or agents. In the dynamic mechanism design literature, there are more papers in the past decade that allow for partially persistent types, see for example Escobar-Toikka (2013) and Pavan, Segal, and Toikka (2014), but it certainly dates back to Harris-Holmström (1982). Fully persistent types with limited commitment go back to the ratchet-effect literature starting in the 80s including Laffont-Tirole (1988).

For the fact that the mechanism designer faces the informed-principal problem in the beginning of every period after the first doesn't depend on whether it is correlated types or interdependent values. What matters is that either the information or the payoff-relevant type is correlated across all agents and partially persistent so that the mechanism designer has private information in the beginning of the following period. Then due to lack of commitment, the mechanism designer faces the informed-principal problem. The informed-principal problem in the class of mechanisms considered in this paper is studied further in Kwon (2019b).

1.1.3 Mechanisms with Limited Commitment

There is a difference between durable goods and nondurable goods. In general, if agents receive utility from an allocation only in a given period, one can consider it as nondurable good, and this is a more common assumption than the capacity constraint, which is always the case in auctions. This literature is overall more recent than the ones with partially persistent states. Escobar-Toikka (2013) is techanically speaking a game and doesn't have a mechanism designer in the main model. Otherwise, there aren't many publications on mechanisms with limited commitment. Bernheim-Madsen (2017) is one of the few. In terms of working papers, there is Gerardi-Maestri (2018), Kwon (2019) among others. In a more applied context, there is Halac-Yared (2018) for example. Papers mentioned so far all involve nondurable good or allocation every period, and Liu, Mierendorff, Shi and Zhong (2018) is on auctions.

2 Model

A mechanism designer offers a mechanism to $N > 1$ agents every period over the infinite horizon $t = 1, 2, \cdots$. The set of agents are denoted by $\mathcal{N} = \{1, 2, \cdots, N\}$. The mechanism designer has limited commitment and can only commit to the allocation within the period. The common discount factor is $\delta \in (0, 1)$. Each agent i has a private type $\theta_t^i \in \Theta^i$ in period t; Θ^i is finite. In period 1, types are drawn by P_0, and from the following period onwards, the types follow a first-order Markov chain $P(\theta_{t+1}|\theta_t)$ where $\theta_t = (\theta_t^1, \cdots, \theta_t^N)$. This allows for correlated types and (partially) persistent types. I assume full support that after any type profile realization, each type profile has a strictly positive probability which implies that the mechanism designer never detects a deviation in reporting strategy. Each agent assigns a strictly positive probability on all types in his type set.

In the beginning of the first period, the mechanism designer and all agents share the common prior P_0. At the beginning of any subsequent period, the mechanism designer has his prior based on the messages sent in previous periods. Each agent's prior is given by his type in the previous period and the past allocations. I assume messages are private so that only the mechanism designer sees all messages. The set of messages for agent i in period t is denoted by $\mathcal{M}_t^i$ and will be specified in theorem 1. Allocations are public in each period.

I assume the mechanism designer faces no capacity constraint in any period. At the end of each period, the mechanism designer assigns an allocation $x_t = (x_t^1, \cdots, x_t^N)$ from a finite set $\mathcal{X} = \times_i \mathcal{X}^i$[2]. The allocation is nondurable in a sense that agents receive the utility only within the period. Each agent's utility function is $u(x_t^i|\theta_t^i) \in [0, \bar{u}]$. This assumes that other agents' types are only relevant through allocation, and agents have private values. I assume that for given $\Theta_i, \mathcal{X}_i$, the range of $u(x_t^i|\theta_t^i)$ is bounded. I also assume that the mechanism designer's objective function is additively separable across agents, but this is not necessary. The objective function itself can be either welfare or revenue. The mechanism designer's payoff in any given period is also weakly positive and bounded. If an agent doesn't participate in a given period, both the agent's utility and the mechanism designer's payoff from that agent are 0.

Essentially, I assume utility functions and the designer's payoff are bounded because then one can invoke continuity at infinity. This doesn't have to be bounded for each individual utility function and the designer's payoff; for each player, if the continuation value is bounded in expectation, this is sufficient. I also assume that any utility level is weakly higher than the no-participation payoff. This again in the light of repeated games or stochastic games can be in expectation. However, if the utility function is quasilinear, this can be automatically satisfied by shifting the transfers across periods on the equilibrium

[2] This can be generalized to a metric space. The proof of theorem 2 will then have to be done with measures instead of matrices.

path and therefore is not a restriction.

The timing of events within each period is as follows. At the beginning of the period, each agent privately observes his type. Then the mechanism designer offers a mechanism to agents, and each agent decides whether to participate. If an agent accepts, he sends a message to the mechanism designer and receives the allocation. The types transit at the end of the period. The mechanism offered by the mechanism designer and each agent's participating decision are publicly observable. When the mechanism designer asks each agent to send a message, messages are private, and only the mechanism designer knows the message profile. Otherwise if messages are public, the informed principal problem is irrelevant. Without the mechanism designer receiving private signals, the informed-principal problem arises solely through the fact that the message profile is private and informative about the type distribution in the following period. Allocations are public.

With limited commitment, the equilibrium definition is perfect Bayesian equilibrium, except that the designer commits to allocation within the same period. I assume full support for agents, and therefore, the off-the-equilibrium path only applies to the mechanism designer offering a mechanism that has zero probability for any of the agents given his private information up until the beginning of the period or any agent accepting (rejecting) the mechanism he is supposed to reject (accept).

A private history of agent i is $h^{t,i} = (\theta^i_1, \Omega_1, m^i_1, x_1, \cdots, \theta^i_t, \Omega_t, m^i_t, x_t) \in \mathcal{H}^i_t$, and $\mathcal{H}^i = \cup_t \mathcal{H}^i_t$ where Ω_t is the mechanism offered in period t and $m^i_t \in \mathcal{M}^i_t$ is the message sent. If agent i rejects the mechanism in period t, denote $m^i_t = x^i_t = \emptyset$. The same applies to the private history of the mechanism designer. A private history of the mechanism designer is $h^{t,m} = (\Omega_1, m^1_1, \cdots, m^N_1, x_1, \cdots, \Omega_t, m^1_t, \cdots, m^N_t, x_t) \in \mathcal{H}^m_t$, $\mathcal{H}^m = \cup_t \mathcal{H}^m_t$. As mentioned already, since messages are private, the mechanism designer has private information. Primitives of the model, i.e., the initial prior P_0, type spaces Θ^i, type transition $P(\cdot|\cdot)$, the set of allocations $\mathcal{X}$, utility functions $u(\cdot|\cdot)$ are common knowledge across the mechanism designer and all agents.

3 Results

Before stating the revelation principle, let me point out that after Bester-Strausz (2001), there are a few intermediate versions of revelation principle that should have been done. Theorem 1 considers the case where there are multiple agents with correlated persistent types over infinite horizon and the mechanism designer has limited commitment. Compared to the version in Bester-Strausz (2001) where there is a single agent over finite horizon, the differences are (i) infinite horizon and (ii) multiple agents with correlated persistent types. I will not prove all intermediate versions.

I should also point out that the revelation principle means (i) given a mechanism satisfying conditions in the statement, there's at least one equilibrium with the stated property, and not every equilibrium has to satisfy the property, (ii) there exists at least one mechanism that has the stated property, e.g., optimality, and offers the set of messages as stated in the theorem.

Bester-Strausz (2001) is for "optimal" mechanism with limited commitment. (ii) means that there exists at least one optimal mechanism that offers the set of types as the set of messages when the principal has limited commitment, there is only one agent, and the time horizon is finite. (i) means that once such a mechanism is offered, there exists at least one equilibrium that the principal obtains the optimal payoff. Their theorem states that the agent need not report truthfully in any equilibrium.

Theorem 1 (Revelation Principle)**.** *Given any mechanism, there exists a mechanism such that (i) the message space for each agent in every period is their type space and (ii) there exists an equilibrium with the same expected payoffs for the mechanism designer and all agents as in the original mechanism, if after every history on the equilibrium path, for each pair of θ, x, either*

1. *the allocation rule within each period $\Sigma(x|\theta)$ satisfies*

$$\Sigma(x|\theta)\Sigma'(x|\theta^j) = \Sigma'(x|\theta)\Sigma(x|\theta^j),\ \forall\theta^j,$$

2. *or $(\pi_i(\theta^j|\theta_i))_{\theta^j}$ for each i is perpendicular to $(\Sigma(x|\theta)\Sigma'(x|\theta^j)-\Sigma'(x|\theta)\Sigma(x|\theta^j))_{\theta^j}$.*

They are necessary and sufficient for existence of alternative mechanism such that (i) the message space for each agent in every period is their type space and (ii) there exists an equilibrium with the same ex-ante payoffs and ***interim beliefs*** *for the mechanism designer and all agents as in the original mechanism.*

Theorem 2 (Impossibility)**.** *There exists no mechanism such that (i) attains same ex-ante payoffs and interim beliefs as in the given mechanism and (ii) offers the type space as the message space to all players every period if none of the following conditions holds after any history on the equilibrium path: (i) all type profiles have the same distribution over allocation, (ii) there exists a PBE of the extensive-form game this period when agents are asked to report a type such that* $\Sigma(x|\theta) = \Sigma'(x|\theta)$*, or (iii) there exists at least one vector in* $\mathbb{R}^{|\Theta|}$ *that is perpendicular to all agents' interim beliefs after observing their own types in the original mechanism.*

Generically, conditions (i)-(iii) are not satisfied by every mechanism with persistent correlated types.

Theorem 1 shows two necessary and sufficient conditions to construct a mechanism that offers the type space as the message space and attains same ex-ante payoffs and interim beliefs to all players. The revelation principle as in Bester-Strausz (2001) allows one to restrict attention to mechanisms that ask the agent to report his type. For the purpose of characterizing any optimal mechanism, we don't necessarily need the same interim beliefs for all players; same ex-ante payoffs should suffice. In that sense, theorem 1 characterizes sufficient conditions to be able to ask all agents to report their types and obtain same payoffs. They are necessary and sufficient once interim beliefs are required to remain the same.

I will first discuss the conditions, together with theorem 2 that shows when one can never construct such a mechanism, before discussing why interim beliefs matter with persistent correlated types.

The first condition is when the distribution of allocation for each allocation is scaled the same for all type profiles when agents are asked to report their types. In my model, each agent only observes their own type, and messages are private. Once the allocation is observed, all agents can update their beliefs about the type profile, but no allocation needs to reveal the type profile. The mechanism designer on the other hand only observes messages, and given that with a single agent in finite horizons the agent need not report truthfully, the mechanism designer doesn't necessarily learn the type profile either. Therefore, any type profiles that are indistinguishable to players' given their private histories must have the same distribution over allocations within the same mechanism.

The second condition is when the agents' beliefs after observing their own types are perpendicular to the difference vector of weighted allocation for all type profiles. This doesn't pin down the allocation in the new mechanism to one specific distribution, but the agents' beliefs are already given in the original mechanism, and the scope of adjusting allocations is restricted.

Theorem 1 shows when one can construct a mechanism satisfying the conditions. However, unlike in mechanisms with full commitment or mechanisms with limited commitment, single agent over finite horizon, these conditions don't cover every mechanism with limited commitment and persistent correlated types. Theorem 2 shows when there exists no mechanism satisfying the conditions; this actually matters for the purpose of finding a mechanism among all possible mechanisms, e.g., any optimal mechanism irrespective of the objective function. One must be able to compare the given mechanism to all mechanisms that do not have an alternative mechanism witfth same ex-ante payoffs, interim beliefs and type spaces as message spaces.

Theorem 2 shows that at least one of three conditions must be satisfied after every history on the equilibrium path to be able to construct a mechanism satisfying the conditions. (i) just means that if all type profiles get the same distribution, there is no need to ask for any message. (ii) follows from the fact that if the first condition in theorem 1 holds for all type profiles, then either (i) holds, or the distribution of allocation must be scaled by 1 for every allocation; the mechanism we want should offer the same distribution over allocation as the original mechanism for each type profile. For (iii), if there exists at least one pair of type profile and allocation for which the first condition doesn't hold, then the second condition should hold and the difference in allocation should be perpendicular to interim beliefs of all agents. At the very minimum, we need existence of a vector that is perpendicular to all interim beliefs.

I will next explain the role of interim beliefs then show both theorems in turn. Theorem 1 characterizes necessary and sufficient conditions for keeping not only ex-ante payoffs but also interim beliefs of all players the same. One might ask why the theorem doesn't characterize necessary and sufficient condition for keeping only the ex-ante payoffs the same. With one agent and i.i.d. types, the designer's belief over the agent's type is the same at the beginning of every period, regardless of the history. When multiple agents have persistent correlated types, each agent has his belief over the type profile at the beginning of a period, given his private history. He updates his belief after observing his own type. Correlated type, or prior, means that each agent's belief over the type profile depends on his own type. Persistent type means that the past realizations of the type profile matter for this period; in particular, with Markovian types as I assume in this paper, the type profile itself only depends on the last period's type profile. However, since each agent only observes his own type, unless some allocation fully reveals the type profile, each agent has a belief over the type distribution at the end of the period, which gets updated with the Markov chain in the beginning of the following period. Therefore, in any period, each agent's belief over the type profile depends on his entire private history, unless there is an allocation that fully reveals the type profile after some history; with such allocations, the persistence of types matters. I focus on first-order Markov chains in this paper.

What this implies is that with persistent correlated types, if we don't already know properties of the mechanism we want to characterize, interim beliefs of agents after observing their own types only can be any distribution in $\mathbb{R}^{|\Theta|}$. We also cannot choose the number of messages each type of given agent is sending, except that the only relevant cases are when at least one type of some agent sends more messages than the number of his types. When we ask agents to report a type, we are changing the number of actions for each player in a normal-form game; we also cannot restrict the correlated prior. Generically, one cannot arbitrarily change the number of actions and keep posteriors from correlated prior unaffected. The only way of keeping the posterior the same is to take mixtures of the following form, $\sum_j \alpha_j \sigma_i(j|k)$, which means that all types of one agent scale the probability he sends a particular message by the same constant. The rest of theorem 2 follows from the designer's incentives to keep the same distribution over allocation.

I should also note that condition (iii) in theorem 2 doesn't hold in particular classes of mechanisms with persistent correlated types. Condition (i) is a more difficult one to satisfy.

Proof of Theorem 1. **Step 1: If we already know full-commitment solutions can be implemented with limited commitment**

For the purpose of finding a mechanism with the desired payoffs, if we already know full-commitment solutions can be obtained with limited commitment, then one can characterize the full-commitment solution directly. Therefore, the revelation principle with full commitment applies, and it is sufficient to consider all mechanisms that offer the type space as the message space to each agent.

Step 2: If there exists at least one type of some agent who randomizes over more messages than the number of his types

If every type of every agent never sends more messages than the number of his types with a strictly positive probability in a given period, then one can relabel all messages with types in his type space. There exists at least one equilibrium in which the agents' reporting strategies treat the messages as being relabelled, and the equilibrium provides the same expected payoffs to all players including the mechanism designer. Relabelling does change the mechanism itself, but as long as there exists one equilibrium that preserves the expected payoffs, it suffices for our goal.

Therefore, we just need to characterize the case in which at least one type of some agent sends more messages than the number of his types in some period in the given mechanism.

Step 3: Necessary and sufficient conditions to keep all players' beliefs the same as in the original mechanism

Before characterizing the necessary and sufficient conditions to keep beliefs the same, let me point out that this is sufficient but not necessary for constructing an equilibrium with same expected payoffs.

Once a mechanism is given, if one can construct reporting strategies and the allocation strategy that keep all beliefs and expected payoffs the same, then priors in the beginning of each period are the same as in the original mechanism. Therefore, it becomes a static problem.

I will abstract away from inference from the mechanism itself when all agents are asked to report a type; if on the equilibrium path, there was another private history of the mechanism designer after which he offers the type space as the message space to all agents, the agents now need to make inferences about the designer's private history.

Denote $|\Theta^i| = n_i$. The total number of type profiles is $\Pi_{i=1}^N n_i$. If there exists an equilibrium of the mechanism whose message spaces are the type space for each agent, then the total number of posterior beliefs of the mechanism designer that can happen after agents send messages is $\Pi_{i=1}^N n_i$. When allowing for mixed strategies of agents by itself is not enough to construct an equilibrium in which the mechanism designer's posterior beliefs after messages are sent are the same as in the optimal mechanism with limited commitment we want to replicate, then one needs to change the allocations themselves.

Correlated persistent types also make it difficult to just modify allocations and reporting strategies within the period so that the continuation values remain invariant. Given t and $\Theta_i = \{\theta_i^1, \cdots, \theta_i^{n_i}\}$ for each i, denote the set of messages that are sent with a strictly positive probability in period t by $\{\bar{m}_i^1, \cdots, \bar{m}_i^{N_i}\}$. Also denote the prior of agent i in the beginning of period t as π_i. If types were only correlated within the same period, all $\pi_i = \pi$ which is common knowledge. With correlated persistent types, each agent has his own prior because he only knows his own type realization each period, none of other agents' reports which still might not be completely informative about the type profile anyway, and allocations at the end of each period need not be completely informative about the type profile either. Given $\pi_i, \{\bar{m}_i^1, \cdots, \bar{m}_i^{N_i}\}$, denote the probability that type θ_i^n of agent i will send message $\bar{m}_i^k$ this period by $\sigma_i(k|n)$. Bayesian updating requires that the designer updates his prior π^P to

$$\bar{\pi}^P(\theta|\bar{m}) = \frac{\pi^P(\theta)\Pi_{i=1}^N \sigma_i(k_i|j_i)}{\sum_{\theta' \in \Theta} \pi^P(\theta')\Pi_{i=1}^N \sigma_i(k_i|j_i')}$$

where $(\theta_1^{j_1}, \cdots, \theta_N^{j_N}) = \theta \in \Theta = \Pi_{i=1}^N \Theta_i$ is the type profile in period t and $\bar{m} = (\bar{m}_1^{k_1}, \cdots, \bar{m}_N^{k_N})$ is the message profile sent by agents. The total number of message profiles is $\Pi_{i=1}^N N_i$. Now, denote the probability of allocation x given $\bar{m}$ as $\mu(x|\bar{m})$. Denote each agent's belief on the type distribution after observing their own type as $\pi_i(\theta|\theta_i)$. Agents update their posterior beliefs after observing the allocation as

$$\bar{\pi}_i(\theta|x) = \frac{\pi_i(\theta|\theta_i)\sum_{\bar{m}\in\bar{M}} \mu(x|\bar{m})\Pi_{i=1}^N \sigma_i(k_i|j_i)}{\sum_{\theta'\in\Theta} \pi_i(\theta'|\theta_i)\sum_{\bar{m}\in\bar{M}} \mu(x|\bar{m})\Pi_{i=1}^N \sigma_i(k_i|j_i')}$$

where $\bar{M} = \Pi_{i=1}^N \{\bar{m}_i^1, \cdots, \bar{m}_i^{N_i}\}$.

The difference from Bester-Strausz (2001) is $\pi_i(\theta|\theta_i)$ which reflects the correlated persistent types. It is history dependent, as it depends on past realizations of types and allocations, and the posterior on the joint distribution needs to be updated after observing own type. When all sets are finite, one can rewrite the equation above as

$$\begin{aligned}\bar{\pi}_i(\theta|x) &= \frac{\pi_i(\theta|\theta_i)\Sigma(x|\theta)}{\sum_{\theta'\in\Theta} \pi_i(\theta'|\theta_i)\Sigma(x|\theta')} \\ &= \frac{\pi_i(\theta|\theta_i)}{\sum_{\theta'\in\Theta} \pi_i(\theta'|\theta_i)\frac{\Sigma(x|\theta')}{\Sigma(x|\theta)}},\end{aligned}$$

and any non-zero $\bar{\pi}_i(\theta|x)$ given σ_i, μ is the same as those given σ_i', μ' if and only if

$$\sum_{\theta'\in\Theta} \pi_i(\theta'|\theta_i)\frac{\Sigma(x|\theta')}{\Sigma(x|\theta)} = \sum_{\theta'\in\Theta} \pi_i(\theta'|\theta_i)\frac{\Sigma'(x|\theta')}{\Sigma'(x|\theta)}, \ \forall i, \theta, x$$

$$\Leftrightarrow \qquad \pi\Sigma = \pi\Sigma',\ \forall\theta, x$$

where π is a matrix whose (i, j)-th element is $\pi_i(\theta^j|\theta_i)$ and the j-th element of Σ is $\frac{\Sigma(x|\theta^j)}{\Sigma(x|\theta)}$. We can rewrite it as $\pi(\Sigma - \Sigma') = \vec{0}$, and either $\Sigma = \Sigma'$ or each agent i's belief on the distribution of type profile is perpendicular to $\Sigma - \Sigma'$. The first case means $\frac{\Sigma(x|\theta^j)}{\Sigma(x|\theta)} = \frac{\Sigma'(x|\theta^j)}{\Sigma'(x|\theta)}$ for all θ^j given θ, x. The latter implies that in $\mathbb{R}^{|\Theta|}$, agents' beliefs given θ lie in $\mathbb{R}^{|\Theta|-1}$. □

Proof of Theorem 2. Conditions in theorem 1 hold for each pair of θ, x.

From

$$\frac{\Sigma(x|\theta^j)}{\Sigma(x|\theta)} = \frac{\Sigma'(x|\theta^j)}{\Sigma'(x|\theta)}$$
$$\Sigma'(x|\theta^j) = \frac{\Sigma'(x|\theta)}{\Sigma(x|\theta)}\Sigma(x|\theta^j)$$

for all θ^j given θ, x, Σ' allocates x in proportion with all type profiles. If this were to hold for every θ, x, then there exists $\lambda(x)$ such that $\Sigma'(x|\theta) = \lambda(x)\Sigma(x|\theta)$ for all θ. And for each θ, $\Sigma(x|\theta)$ given the designer's belief integrates over x to at most 1; the designer's belief is the same for both Σ, Σ'.

With finite sets, one can consider a matrix whose rows are distributions over x for each θ. The statement above says one can multiply positive real numbers to each column so that each entry is still in $[0, 1]$, and each row adds up to at most 1. If we define no allocation as one of x, then each row sums up to 1 after multiplying λ's. Consider a 2x2 example. We have

$$\begin{pmatrix} a & 1-a \\ b & 1-b \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda a & \lambda'(1-a) \\ \lambda b & \lambda'(1-b) \end{pmatrix}$$
$$\Rightarrow \begin{cases} \lambda a + \lambda'(1-a) = 1 \\ \lambda b + \lambda'(1-b) = 1 \end{cases}$$
$$\Rightarrow (\lambda - \lambda')(a - b) = 0.$$

Therefore, if there are only two type profiles and two allocations, then the first case can hold for both profiles and both allocations if and only if (i)

$\Sigma(x|\theta) = \Sigma'(x|\theta),\ \forall\theta, x$ or (ii) two type profiles have the same distribution over allocation.

The same result holds for any finite number of type profiles and allocations. Therefore, the first condition holds for all θ, x if and only if (i) $\Sigma(x|\theta) = \Sigma'(x|\theta),\ \forall\theta, x$ or (ii) all types have the same distribution over allocation.

If neither condition in the previous paragraph holds, there exists at least one pair of θ, x that satisfies the second condition and not the first, i.e., all agents' beliefs are perpendicular to $\Sigma - \Sigma'$. For the purpose of constructing Σ', $\pi_i(\theta^j|\theta_i)$'s are already given, and this requires that there exists at least one vector in $\mathbb{R}^{|\Theta|}$ that is perpendicular to all agents' beliefs after observing their own types in the original mechanism.

This shows that if in the original mechanism after some history, none of the following holds, then there exists no mechanism with same beliefs and expected payoffs for all players as in the original mechanism and offers the type space as the message space to all agents after every history: (i) $\Sigma(x|\theta) = \Sigma'(x|\theta),\ \forall\theta, x$, (ii) all types have the same distribution over allocation, or (iii) for any (θ, x) that $(\Sigma(x|\theta)\Sigma'(x|\theta^j) - \Sigma'(x|\theta)\Sigma(x|\theta^j))_{\theta^j} \neq \vec{0}$, there exists at least one vector in $\mathbb{R}^{|\Theta|}$ that is perpendicular to all agents' beliefs after observing their own types in the original mechanism.

□

4 Conclusion

I studied the revelation principle for mechanisms with limited commitment when agents have persistent correlated types in this paper. I characterize necessary and sufficient conditions to construct a mechanism that attains same ex-ante payoffs, interim beliefs and offers type spaces as message spaces to all agents. These are sufficient but not necessary if one were to only require same ex-ante payoffs and not interim beliefs. These conditions readily lead to necessary conditions in theorem 2 such that if none of them holds after one history on the equilibrium path, then one cannot construct a desired mechanism as in theorem 1.

When the mechanism designer has private information, it is not obvious construcing an equilibrium in which only one of the mechanisms on the equilibrium path are replaced is the best thing to do. If all message spaces are replaced with type spaces, then allocations are the only way of signalling the designer's private information. I abstracted away from agents' inference and equilibrium strategies in theorem 1. However, the impossibility result in theorem 2 holds regardless.

Theorem 1 shows necessary and sufficient conditions to be able to elicit agents' types. However, theorem 2 points at a problem that unlike in mechanisms with full commitment or with limited commitment, single agent over finite horizon, there is loss of generality to ask agents to report their types, if types are persistent and correlated. I mentioned that if we already know full-commitment solutions can be attained with limited commitment, one can characterize payoffs directly. Otherwise, theorem 2 shows that one needs to find the set of equilibrium payoffs without restricting the message space in order to confirm whether offering the type space as the message space can generate the same set of payoffs. If one already knows that an optimal mechanism does satisfy conditions in theorem 1, then one can characterize the mechanism by offering the type space as the message space.

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