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Abstract

This paper analyzes taxation of an Internet platform attracting users from different jurisdictions. When corporate income tax rates are different in the two jurisdictions, the platform distorts prices and outputs in order to shift profit to the low-tax country. We analyze the comparative statics effects of an increase in the tax rate of one country. When cross-effects are present in both countries, the platform has an incentive to increase the number of users in the high-tax country and decrease the number of users in the low-tax country. When externalities only flow from one market to another, an increase in the corporate tax rate results either in a decrease or an increase in the number of users in both countries depending on the direction of externalities. We compare the baseline regime of separate accounting (SA) with a regime of formula apportionment (FA), where the tax bill is apportioned in proportion to the number of users in the two countries. Under FA, an increase in the corporate tax rate increases the number of users in the low-tax country and decreases the number of users in the high-tax country. We use a numerical simulation to show that the high-tax country prefers SA to FA whereas the low-tax country prefers FA to SA.

JEL-Codes: H320, H250, L120, L140.

Keywords: digital platforms, multinational firms, corporate income taxation, formula apportionment, separate accounting.

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1 Introduction

Internet platforms often connect agents living under different fiscal jurisdictions. Facebook and Google users receive targeted advertising from companies headquartered outside their country of residence. Sellers and buyers on E-bay transact with agents living in different countries. Booking or Expedia users book flights and hotels all over the world. As users benefit from positive exernalities due to the presence of other users on the platform, the surplus created by the platform cannot easily be ascribed to a specific fiscal jurisdiction, raising difficult issues when the jurisdictions involved fix different corporate income tax rates. In this paper, our objective is precisely to analyze the effect of differences in corporate tax rates on the behavior of Internet platforms operating in different jurisdictions.

The issue of taxation of the profits of Internet platforms has recently received considerable attention from policy-makers. Digital companies are known to pay very low corporate income taxes in the countries in which they operate. Recognizing this fact, the OECD has launched a specific action to address the tax challenges in the digital economy in the Base Erosion and Profit Shifting (BEPS) project. Following an initial report in 2015 (OECD 2015), the OECD has defined a new set of proposals contained in an action plan, unveiled in early 2019, (OECD, 2019). These proposals include a reform of the definition of "permanent establishment" to allow countries where Internet platforms operate to tax corporate income and new sharing rules based on revenues and users to allocate profit across jurisdictions. In the meantime, while these proposals are still under discussion, some European countries, namely France, the United Kingdom, Austria, Spain and Italy are contemplating imposing taxes on the revenues of Internet platform unilaterally. In July 2019, one of these countries – France – passed a law imposing a tax of 3% on the revenues of the 27 largest digital companies.

In this paper, we contribute to the study of profit-shifting and profit-sharing of digital platforms, by considering a simple model where a monopolistic platform operates in two countries. Users in one country benefit from positive externalities generated by the users in the other country. This general formulation covers two-sided markets, where users on the two sides of the market are located in different countries (e.g. search engines and digital social media, where advertisers are located in a small, low-tax country whereas users are located in a large, high-tax country) as well as peer-to-peer markets, where users located in the two countries can both be buyers and sellers (e.g. on-line market places and auction sites, matching and file-sharing platforms). In the baseline model (Separate Accounting – SA), we assume that each country taxes the revenues made in that country. We suppose that users are immobile, or more generally, that the users' location decisions are independent of the corporate tax rates imposed on the platform. We also assume zero marginal cost and abstract from fixed investment costs. Hence transfer pricing through royalties on intangible assets is not incorporated into the analysis. Our interest bears on the impact of different tax rates on the platform's policy in the two countries.[1]

Our first result shows that, even in the absence of transfer pricing, the platform can use externalities to shift profits from a high-tax jurisdiction to a low-tax jurisdiction. More precisely, we analyze the effect of an increase in the corporate tax rate of one of the jurisdictions on the behavior of the platform, and distinguish between direct and indirect effects. The platform has an incentive to reduce the profit made in the high-tax country relative to the profit in the low-tax country. Considering the change in each country separately, this is achieved by an increase in the number of users in the high-tax country and a decrease the number of users in the low-tax country. These direct effects must however be balanced with indirect effects: by increasing the number of users in the high-tax country, the platform generates a change in demand in the low-tax country and, by decreasing the number of users in the low-tax country, the platform generates a change in demand in the high-tax country. When markets are "substitutes", an increase in the demand in one country induces a decrease in the demand of the other. In the more plausible case where markets are "complements", an increase in the demand in one country results in an increase in the demand of the other. In the latter case, direct and indirect effects have different directions, and the indirect effect may dominate the direct effect. To determine how an increase in tax affects in the end the number of users in both countries, we first examine two particular situations.

We show that when markets are symmetric and balanced – users from both countries benefit from externalities from users in the two countries – and tax rates are similar, the direct effect always dominates the indirect effect so that an increase in the tax rate increases the number of users and decrease prices in the high-tax country and reduces the number of users and increases prices in the low-tax jurisdiction. This situation corresponds for example to peer-to-peer platforms operating in two similar, large countries, such as France and Germany.

We also consider the situation where externalities only flow from one country to another. This corresponds to situations where one side of the market (e.g. advertisers) benefits from positive externalities whereas the other side of the market (e.g. users) does not benefit from the number of users in the other country. Suppose first that externalities flow from the low-tax country to the high-tax country. The optimal number of users in the high-tax country is then independent of the tax rate, so that the direct effect of an increase in the tax rate on the demand in the high-tax country is null while the direct effect on the demand of the low-tax country is negative. This implies that the number of users in the low-tax country always decreases. The number of users in the high-tax country increases when markets are substitutes and decreases

[1] The model could easily be adapted to deal with direct rather than indirect taxes, as we assume that there is no transfer pricing and no marginal cost of production.

when markets are complements. When externalities flow from the high-tax country to the low-tax country, the opposite result is obtained. An increase in the corporate tax rate of the high-tax country always results in an increase in the number of users in the high-tax country, and induces an increase or a decrease in the number of users in the low-tax country depending on whether markets are complements or substitutes.

To summarize, when externalities only flow from one country to another, the effect of an increase in the corporate tax rate of the high-tax country may result in a variety of configurations, except for one. One cannot observe that the number of users in the high-tax country decreases while the number of users in the low-tax country increases. This impossibility holds more generally when externalities flow in both directions. Apart from that property, the interplay between externalities, market size and corporate tax rates is too complex to allow for a full analysis. We thus use a linear model to compute precisely the optimal choices of the platform, and discuss the comparative statics effects of changes in the parameters on the optimal prices, quantities and the tax revenues of the two countries.

We then analyze a profit-sharing rule for peer-to-peer platform where profit is apportioned according to the number of users in jurisdiction rather than the revenues (Formula Apportionment – FA). We show that the direction of the direct effect of an increase in the corporate tax rate on the number of users in the two countries are reversed with respect to Separate Accounting. In order to lower its tax bill, the platform has an incentive to increase the number of users in the low-tax jurisdiction and decrease the number of users in the high-tax jurisdiction. Externalities across jurisdictions and the apportionment key give rise to an indirect effect: if the number of users in the low-tax country increases, demand in the high-tax country increases, pushing up the number of users in the high-tax country. We characterize two situations where the direct effect dominates the indirect effect around identical tax rates: when countries are symmetric and when externalities across jurisdictions disappear. In both these situations, a higher tax rate in the corporate tax rate results in a decrease in the number of users and an increase in price in the high-tax country and in an increase in the number of users and a decrease in price in the low-tax country.

We finally use a numerical simulation to compare profits and tax revenues of a peer-to-peer platform under Separate Accounting and Formula Apportionment. We first show that the comparative statics results obtained locally around identical tax rates are robust, and that the direct effect dominates the indirect effect for the entire range of tax rate values. Distortions in the number of users are stronger under FA than under SA, resulting in lower pre-tax profit. However, the tax bill of the platform is higher under SA than under FA, so that post-tax profits are comparable in the two régimes. Tax revenues in the high-tax country are higher under SA than under FA, but tax revenues in the low-tax country are higher under FA than under SA, suggesting that the two countries will disagree on the profit-sharing régime.

The rest of the paper is organized as follows. We discuss the related literature in the next subsection. We present the model in Section 2. In Section 3, we analyze the benchmark model under Separate Accounting. The model with Formula Apportionment is discussed in Section 4. Section 5 presents the results of a numerical simulation comparing the two régimes. Section 6 concludes.

1.1 Relation to the literature

This paper is related to two different strands of the literature: the literature on taxation of two-sided platforms and the literature on formula apportionment. Optimal taxation of two-sided monopolistic platforms have been studied by Kind et al. (2008, 2009, 2010, 2013) and Bourreau, Caillaud and de Nijs (2018). In Kind et al. (2008, 2009, 2010, 2013), the two sides of the market are located in the same jurisdiction and tax rates are equal on revenues generated by the two sides of the market. The studies of Kind et al. (2008, 2009, 2010, 2013) have generated two main results. First, they show that ad valorem taxes (like VAT) do not necessarily dominate unit taxes. The classical result in public finance on the domination of ad valorem taxes no longer holds for two-sided markets. Second, the price of a good may decrease with the ad valorem tax. The introduction of a tax on the value added for one side of the market can lead to a change in the entire business model of the platform. For example, the increase in VAT on the price of access for users could induce the platform to set a zero price for Internet access and switch all its revenues to the advertisers side. Bourreau, Caillaud and de Nijs (2018) supplement the model of the two-sided platform by considering data collection and letting consumers select the flow of data uploaded to the platform. They allow for different taxes levied on the two sides of the market (a tax based on data uploaded by users and an ad valorem tax paid by advertisers). Their main result shows that a small increase in the tax rate on data collection above zero results in an increase in fiscal revenues and an increase in the prices and quantities of the platform. By contrast to our paper, they do not provide a general analysis of the comparative statics effect of changes on the tax rate on one side of the market.

Kotsogiannis and Serfes (2010) study competition between two platforms located in different countries. Countries choose both a tax rate and a level of public good. Consumers and businesses choose which platform to go to, taking into account the tax rate and public good provision in both jurisdictions. If the difference in public good provision in the two countries is large, each platform specializes in one segment of the population. If the difference in public good provision is small, competition between platforms is fierce, and all consumers and all businesses may choose to join a single platform. Comparative statics results show that an increase in externalities between the two sides of the market may lead to a decrease in the tax rate in both jurisdictions, an increase in the number of firms on platform A and a decrease in the number of firms on platform B. The model of Kotsogiannis and Serfes (2010) differs from ours in several respects. First they consider perfectly mobile users on the two sides of the market, second they assume competition between two platforms located in the two countries. Finally, they consider taxation on firms and businesses whereas we analyze corporate income taxes paid by a monopolistic platform.

The literature on formula apportionment originates with a paper by Gordon and Wilson (1986) who show that the formula used in the United States, which puts positive weight on sales, wages and assets induces distortions in the optimal choice of inputs by the firms. In addition, it results in discriminatory treatment of companies in the same jurisdiction as they will face different effective tax rates. Anand and Sansing (2000) analyze a model where two states bargain over the weights to place on different indicators and show that the weights placed on sales and inputs are typically inefficient in a decentralized equilibrium. Nielsen, Raimondos Moller and Schjelderup (2003) compare SA and FA in a model where transfer prices are used to manipulate the behavior of a subsidiary in an oligopolistic market. Kind, Midelfart and Schjelederup (2005) extend the model by considering a first stage of tax competition where two countries simultaneously select their corporate income tax rate to maximize fiscal revenues. Nielsen, Raimondos-Moller and Schjelderup (2010) analyze capital investment decisions of a multinational under the two régimes of SA and FA around symmetric tax rates. Finally, Gresik (2010) compares SA and FA when the production cost of the intermediate output is privately known by the multinational. None of the literature on formula apportionment has considered externalities in demand across jurisdictions as we do in this paper.

2 The model

We analyze the strategies of a monopolistic Internet platform with activities in two jurisdictions with possibly different tax rates. The services provided by Internet platforms are very diverse, ranging from on-line retailing (like Amazon, which connects customers and sellers, Booking, which connects customers and hotels) to social media (like Facebook, which allows users to be connected and connect advertisers to users), search engines (like Google, which connects advertisers to users), collaborative and peer-to-peer platforms (like E-bay, Meetic, Spotify, Airbnb, which connect users.) The simple model we introduce in the next section captures some of these situations (but not all).

We distinguish between platforms according to their type of users. Some platforms connect users of the same type residing in different jurisdictions, like peer-to-peer or collaborative platforms. Other platforms connect two types of users and the market is intrinsically two-sided, as in the case of advertisers and consumers or hotels and consumers. For two-sided platforms, we assume that users on the same side of the market are all located in the same jurisdiction. This situation arises for example if customers only search for hotels located in a foreign country for if advertisers all locate in a small low-tax country, whereas consumers reside in a large high-tax country. We analyze a situation where users are immobile (either because their moving costs are too high or because they have already moved and we thus start the analysis after the mobility decision has been made, assuming that users will not choose to relocate).

2.1 Utilities of users and pre-tax profit of the platform

We consider a monopolistic digital platform with users living in two separate jurisdictions, denoted A and B. Users is a generic term, which represents different types of agents according to the specific platform. We do not distinguish between types of users and only track down the total number of users in the two jurisdictions, x_A and x_B. We suppose that the platform follows a business model whereby all users pay a fixed fee to access the platform. The platform can discriminate according to the residence of users, and charges a fee p_A for users in country A and a fee p_B for users in country B. The volume of use of the platform is supposed to be fixed and identical across users. In each jurisdiction, the utility of users is the sum of two components: an idiosyncratic utility for the platform, which is heterogeneous across users, and a positive externality term which depends on the number of other users in the platform, distinguishing between users in jurisdictions A and B. Formally,

$$\begin{aligned} U_A &= \theta_A + u_A(x_A, x_B) - p_A, \\ U_B &= \theta_B + u_B(x_A, x_B) - p_B \end{aligned}$$

where θ_A is distributed according to a continuous distribution with full support F_A on $[\underline{\theta}, \overline{\theta}]$, and θ_B is distributed according to a continuous distribution with full support F_B on $[\underline{\eta}, \overline{\eta}]$.

Externalities across jurisdictions are always nonnegative: u_A is weakly increasing in x_B and u_B is weakly increasing in x_A. Externalities arising from the participation of users in the same jurisdiction can either be positive or negative. To illustrate, we map out the model with some examples.

1- Peer-to-peer platforms: The users only care about a weighted total number of users: The externality for users in A and in B are described by weakly increasing functions $u_A(x_A + bx_B)$ and $u_B(ax_A + x_B)$, where a and b represents the weight placed on the users from abroad. There are positive externalities both across and within jurisdictions.

2- Social media and search engines. Users are located in A and advertisers in B. Users of a social media are positively affected by the number of users of the media: there are positive externalities within A. If they do not care about advertising, there are no externalities from B to A: The externality for users in A is described by an increasing function $u_A(x_A)$. As for the advertisers, they benefit positively from a large number of users, but negatively from other advertisers, due to a competition effect which is not modeled explicitly: The externality $u_B(x_A, x_B)$ is weakly increasing in x_A and weakly decreasing in x_B. Externalities only flow from jurisdiction A to jurisdiction B. For a search engine, the model is similar, but with no externalities among users within A.

3- Online retailers and online intermediaries. Customers reside in A and suppliers in B. There are positive externalities across jurisdictions but negligible externalities within each country: they are described by weakly increasing functions $u_A(x_B)$ and $u_B(x_A)$.

We now derive the demand associated to (p_A, p_B). Assume that users in country A have an expectation x_B over the number of users in country B. There is an indifferent consumer given by

$$\widehat{\theta_A} = p_A - u_A(x_A, x_B).$$

provided $p_A - u_A(x_A, x_B)$ is in the support of F_A; otherwise we take $\widehat{\theta_A}$ at one of the extreme values (if the market is covered or not covered at all). We compute similarly the indifferent user in country B. We normalize the measure of users in jurisdiction B to 1, and let s denote the measure of users in jurisdiction A. Assuming rational expectations on the participation decisions of users, the demand thus satisfies

$$x_A = s(1 - F_A(p_A - u_A(x_A, x_B)),$$

Similarly,

$$x_B = 1 - F_B(p_B - u_B(x_A, x_B)).$$

There is a one-to-one relationship between the prices chosen by the monopolistic platform p_A and p_B and the number of users x_A and x_B. As argued by Weyl (2010), it will prove easier to write the profit in terms of numbers of users instead of prices.[2] The interpretation is that

[2]Such a construction does not work for competitive platforms, as in the case of a single market where competition in prices (Bertrand) leads to different results than competition in quantities (Cournot). See Belleflamme

the platform chooses x_A and x_B, knowing the prices for which the numbers of users will be x_A and x_B. From the computations above, the prices are[3]

$$\begin{aligned} P_A(x_A, x_B) &= u_A(x_A, x_B) + F_A^{-1}(1 - \frac{x_A}{s}), && (1) \\ P_B(x_A, x_B) &= u_B(x_A, x_B) + F_B^{-1}(1 - x_B). && (2) \end{aligned}$$

Because externalities are positive, the price $P_A(x_A, x_B)$ is always increasing in x_B. We suppose that externalities are not too strong, so that the price $P_A(x_A, x_B)$ is decreasing in x_A. This assumption is always satisfied if externalities are non-positive within A (u_A is decreasing in x_A). It is also satisfied when externalities are positive but the marginal effect of an increase in x_A on u_A is sufficiently small relative to the marginal effect on the distribution F ($\partial u_A/\partial x_A F'(1 - x_A) \leq 1$) Similarly, in jurisdiction B the price function $P_B(x_A, x_B)$ is ncreasing in x_A and assumed to be decreasing in x_B.

The user surplus in jurisdiction A is computed as

$$\begin{aligned} CS_A &= \int_{p_A - u(x_A, x_B)}^{\underline{\theta}} [\theta + u_A(x_A, x_B) - p_A] f(\theta) d\theta, \\ &= \int_{F_A^{-1}(1-x_A)}^{\underline{\theta}} [\theta - F_A^{-1}(1 - x_A)] f(\theta) d\theta \end{aligned}$$

Hence, taking into account the participation decision of the users, the user surplus in jurisdiction A can be written as a function only of the number of participants in jurisdiction A, x_A. Furthermore, it is easy to check that

$$\begin{aligned} \frac{\partial CS_A}{\partial x_A} &= \int_{F_A^{-1}(1-x_A)}^{\underline{\theta}} [\frac{1}{f[F_A^{-1}(1 - x_A)]} d\theta, \\ &= \frac{1 - F_A^{-1}(1 - x_A)}{f[F_A^{-1}(1 - x_A)]}, \\ &> 0. \end{aligned}$$

Hence, as intuition suggests, an increase in the number of participants x_A results in an increase in user surplus. Similarly, we obtain

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and Toulemonde (2016) for a study of two platforms competing in prices.

[3]The price functions are akin to inverse demand functions, with some subtleties: due to coordination issues, a given couple of prices (p_A, p_B) could lead to different demands Here we always select the largest demands.

$$CS_B = \int_{F_B^{-1}(1-x_B)}^{\bar{\theta}} [\theta - F_B^{-1}(1-x_B)] f(\theta) d\theta,$$

and

$$\frac{\partial CS_B}{\partial x_B} = \frac{1 - F_B^{-1}(1-x_B)}{f[F_B^{-1}(1-x_B)]} > 0.$$

We suppose, following empirical evidence, that the operating costs of the platform are negligible so that the pre-tax profit in each jurisdiction is given by

$$\begin{aligned} V_A &= x_A P_A(x_A, x_B), \\ V_B &= x_B P_B(x_A, x_B) \end{aligned}$$

and the total pre-tax profit as

$$V = V_A + V_B.$$

The choices of the platform depend on how marginal revenues in a jurisdiction depend on the number of users in the other jurisdiction, i.e. on the signs of the cross-partial derivatives $\frac{\partial^2 V_A}{\partial x_A \partial x_B}$ and $\frac{\partial^2 V_B}{\partial x_A \partial x_B}$. We compute

$$\frac{\partial^2 V_A}{\partial x_A \partial x_B} = \frac{\partial P_A}{\partial x_B} + x_A \frac{\partial^2 P_A}{\partial x_A \partial x_B}.$$

The first term $\frac{\partial P_A}{\partial x_B}$ represents the positive marginal effect of the number of users in B on the price in A. The cross-derivative is positive when this marginal effect does not decrease too fast with x_A, more precisely when the elasticity of $\frac{\partial P_A}{\partial x_B}$ with respect to x_A is larger than -1.

We relate this condition to the fundamentals in country A. Using (1) when the market is not covered, we obtain

$$\frac{\partial^2 V_A}{\partial x_A \partial x_B} = \frac{\partial u_A}{\partial x_B} + x_A \frac{\partial^2 u_A}{\partial x_A \partial x_B}.$$

The first term $\frac{\partial u_A}{\partial x_B}$ is positive due to positive cross-externalities across jurisdictions . The second term may be positive or negative. The sum of the two terms is positive when the marginal cross-externality in market A, $\frac{\partial u_A}{\partial x_B}$ increases, or decreases with the number of users in A with an elasticity less than 1. It is negative when the marginal cross-externality $\frac{\partial u_A}{\partial x_B}$ decreases with the number of users in A at a very fast rate, with an elasticity greater than 1.

We say that the two markets are *complements* when both $\frac{\partial^2 V_A}{\partial x_A \partial x_B}$ and $\frac{\partial^2 V_B}{\partial x_A \partial x_B}$ are non-negative. In the opposite case, when both $\frac{\partial^2 V_A}{\partial x_A \partial x_B}$ and $\frac{\partial^2 V_B}{\partial x_A \partial x_B}$ are non-positive, we say that the two markets are *substitutes*.

The case of market complements might be the most natural case. Markets are complements whenever the cross-derivatives $\frac{\partial^2 u_A}{\partial x_A \partial x_B}$ and $\frac{\partial^2 u_B}{\partial x_A \partial x_B}$ are non-negative. This of course includes the case where u_A and u_B are linear and the cross-derivatives vanish. When there are no externalities from users in the same jurisdiction (u_A does not depend on x_A and u_B does not depend on x_B), the cross-derivatives are also equal to zero, so markets are complements. In the peer-to-peer model, where $u_A(x_A, x_B) = u(x_A + bx_B)$, markets are complements whenever the elasticity of the function u is smaller than 1.[4]

2.2 Separate Accounting and Formula Apportionment

We suppose that the two jurisdictions charge corporate income tax rates t_A and t_B. We consider two regimes of profit-sharing. Under Separate Accounting (SA), the platform pays taxes according to the profit declared in each jurisdiction. The post-tax profit of the platform is then given by

$$\Pi = (1 - t_A)V_A + (1 - t_B)V_B.$$

and the fiscal revenues of the two countries are computed as

$$\begin{aligned} R_A &= t_A V_A, \\ R_B &= t_B V_B. \end{aligned}$$

Under Formula Apportionment (FA), the total profit of the platform, V is attributed to each jurisdiction using the ratio of users, so that the post-tax profit is given by

$$\Pi = V[1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}],$$

[4] To see this, notice that

$$\frac{\partial^2 V_A}{\partial x_A \partial x_B} = u'(x_A + bx_B) + x_A u''(x_A + bx_B)$$

Since u is concave,

$$\frac{\partial^2 V_A}{\partial x_A \partial x_B} \geq u'(x_A + bx_B) + (x_A + bx_B)u''(x_A + bx_B)$$

Thus the cross-derivative is positive whenever the elasticity of u', $-x\frac{u''}{u'}$, is smaller than 1.

and the fiscal revenues of the two countries are computed as

$$
\begin{aligned}
R_A &= t_A V \frac{x_A}{x_A + x_B}, \\
R_B &= t_B V \frac{x_B}{x_A + x_B}.
\end{aligned}
$$

Formula apportionment only makes sense when the apportionment key uses the same type of users in the two countries. Hence FA is an appropriate profit-sharing regime for peer-to-peer platforms, but not for two-sided platforms, where different types of users reside in different jurisdictions.

3 Separate accounting

Our objective in this Section is to compute the optimal choices of the platform under SA for a fixed choice of tax rates (t_A, t_B) and analyze the comparative statics effects of an increase in one of the corporate tax rates, t_A. Because of externalities across jurisdictions, the optimal numbers of users in countries A and B are interdependent. We introduce two "reaction functions" ϕ_A and ϕ_B to define the optimal number of users in jurisdictions A and B. Formally, let $\phi_A(x_B; t_A, t_B)$ denote the value x_A that maximizes the platform's profit given x_B and the tax rates (t_A, t_B) and $\phi_B(x_A; t_A, t_B)$ the value x_B that maximizes platform's profit given x_A and the tax rates (t_A, t_B). As in a game between two players, the optimal numbers of users for the platform are obtained at the intersection of these two reaction functions. Denoting them by X_A and X_B, they satisfy:

$$X_A(t_A, t_B) = \phi_A(X_B(t_A, t_B); t_A, t_B) \text{ and } X_B(t_A, t_B) = \phi_B(X_A(t_A, t_B); t_A, t_B). \tag{3}$$

We derive the comparative statics effects of a change in the corporate tax rate t_A on the optimal choices of the platform:

$$\frac{dX_A}{dt_A} = \frac{\frac{\partial \phi_A}{\partial t_A} + \frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial t_A}}{1 - \frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial x_A}}, \tag{4}$$

$$\frac{dX_B}{dt_A} = \frac{\frac{\partial \phi_B}{\partial t_A} + \frac{\partial \phi_B}{\partial x_A}\frac{\partial \phi_A}{\partial t_A}}{1 - \frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial x_A}}. \tag{5}$$

To sign these terms, we assume that the profit's platform is concave in x_A and x_B so that the optimal interior values are characterized by the first-order conditions:

$$\begin{aligned} x_A = \phi_A(x_B; t_A, t_B) \text{ if } \frac{\partial \Pi}{\partial x_A} = (1-t_A)\frac{\partial V_A}{\partial x_A}(x_A, x_B) + (1-t_B)\frac{\partial V_B}{\partial x_A}(x_A, x_B) &= 0, \\ x_B = \phi_B(x_A; t_A, t_B) \text{ if } \frac{\partial \Pi}{\partial x_B} = (1-t_A)\frac{\partial V_A}{\partial x_B}(x_A, x_B) + (1-t_B)\frac{\partial V_B}{\partial x_B}(x_A, x_B) &= 0. \end{aligned}$$

Using these implicit equations, we compute

$$\frac{\partial \phi_A}{\partial t_A} = \frac{\frac{\partial V_A}{\partial x_A}}{\frac{\partial^2 \Pi}{\partial x_A^2}} \quad \text{and} \quad \frac{\partial \phi_B}{\partial t_A} = \frac{\frac{\partial V_A}{\partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B^2}} \tag{6}$$

$$\frac{\partial \phi_A}{\partial x_B} = -\frac{\frac{\partial^2 \Pi}{\partial x_A x_B}}{\frac{\partial^2 \Pi}{\partial x_A^2}} \quad \text{and} \quad \frac{\partial \phi_B}{\partial x_A} = -\frac{\frac{\partial^2 \Pi}{\partial x_A x_B}}{\frac{\partial^2 \Pi}{\partial x_B^2}} \tag{7}$$

$$\text{where } \frac{\partial^2 \Pi}{\partial x_A x_B} = (1-t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1-t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B} \tag{8}$$

Due to the concavity of profit, the product $\frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial x_A}$ is less than 1. Hence, from (4) and (5), the signs of the effect of a change in t_A on the number of users in country A and B are respectively the same as the signs of

$$\frac{\partial \phi_A}{\partial t_A} + \frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial t_A} \text{ and } \frac{\partial \phi_B}{\partial t_A} + \frac{\partial \phi_B}{\partial x_A}\frac{\partial \phi_A}{\partial t_A}.$$

We can thus decompose the effect of an increase in the tax rate t_A on the number of users in country $i = A, B$ into (i) a direct effect $\frac{\partial \phi_i}{\partial t_A}$ and (ii) an indirect effect linked to the change in the number of users in the other country, $\frac{\partial \phi_i}{\partial x_j}\frac{\partial \phi_j}{\partial t_A}$.

Consider first the direct effects, $\frac{\partial \phi_A}{\partial t_A}$ and $\frac{\partial \phi_B}{\partial t_A}$, given in (6). We claim that

> *The direct effect of an increase in the corporate tax rate t_A is positive on the number of users in jurisdiction A and negative on the number of users in jurisdiction B.*

To show this, observe first that, by concavity, $\frac{\partial^2 \Pi}{\partial x_A^2}$ and $\frac{\partial^2 \Pi}{\partial x_B^2}$ are both negative. Because externalities across jurisdictions are positive, $\frac{\partial V_B}{\partial x_A} = x_B \frac{\partial P_B}{\partial x_A} > 0$ and $\frac{\partial V_A}{\partial x_B} = x_A \frac{\partial P_A}{\partial x_B} > 0$. The latter inequality implies $\frac{\partial \phi_B}{\partial t_A} < 0$. As for $\frac{\partial \phi_A}{\partial t_A}$, at the optimum

$$\frac{\partial V_A}{\partial x_A} = -\frac{1-t_B}{1-t_A}\frac{\partial V_B}{\partial x_A} < 0.$$

To understand the signs of the direct effects, recall that, when t_A increases, the platform has an incentive to shift profit from jurisdiction A to jurisdiction B. To do so, the platform should increase the number of users in jurisdiction A, thereby increasing profit V_B in the low-tax jurisdiction through the positive externality of users of jurisdiction A on the price P_B. Similarly, because of positive externalities, the platform should reduce the number of users in jurisdiction B in order to reduce the price P_A and hence the profit V_A in the high-tax jurisdiction.

Consider now the indirect effects, $\frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial t_A}$ and $\frac{\partial \phi_B}{\partial x_A}\frac{\partial \phi_A}{\partial t_A}$ generated by the marginal reactions of the demand in one country to the demand in the other. From (7) the signs generated by these marginal reactions, $\frac{\partial \phi_A}{\partial x_B}$ and $\frac{\partial \phi_B}{\partial x_A}$, are both identical to the sign of the cross-derivative $\frac{\partial^2 \Pi}{\partial x_A \partial x_B}$, which from (8) depends on the cross-derivatives $\frac{\partial^2 V_A}{\partial x_A \partial x_B}$ and $\frac{\partial^2 V_B}{\partial x_A \partial x_B}$. If the markets are complements, both cross-derivatives are positive: the reaction functions are increasing. When the number of users in one country increases, the optimal number of users in the other country increases. If, on the other hand, the markets are substitutes, both cross-derivatives are negative and the two reaction functions are decreasing. An increase in the number of users in one country leads the platform to decrease the number of users in the other country. Accounting for both direct and indirect effects, we assert the following.

Proposition 1 *Suppose that markets are substitutes. Then a higher corporate tax rate t_A always results in an increase in the number of users in country A and a decrease in the number of users in country B. Suppose that markets are complements, then a higher corporate tax rate t_A never results in a decrease in the number of users in country A together with an increase in the number of users in country B.*

The direct effects result in an increase in x_A and a decrease in x_B. When markets are substitutes, the direct and indirect effects reinforce each other: the decrease in x_B induces an additional increase in x_A and a decrease in x_B, which induces further reactions in the same directions. When markets are complements, the direct and indirect effects work in opposite directions. However, due to the concavity of the profit, the indirect effect cannot outweigh the indirect effect on both markets, as proved in the appendix. Apart from this impossibility, an increase in t_A may result in any possible effects on the number of users in both jurisdictions, as we will see in the next section.

Figure 1 illustrates the effect of a change in the corporate tax rate t_A on the two reaction functions ϕ_A and ϕ_B when markets are complements (left panel) and substitutes (right panel). The curves in black depict the functions ϕ_A and ϕ_B when the tax rates are identical. The intersection of the two curves gives the optimal numbers of users. The curves in red and green describe the same functions following an increase in t_A. The new choices of the platform are given by the intersection of the red and green curves. In both cases, an increase in t_A leads to an increase in ϕ_A and a decrease in ϕ_B. When markets are complements, the direct and indirect

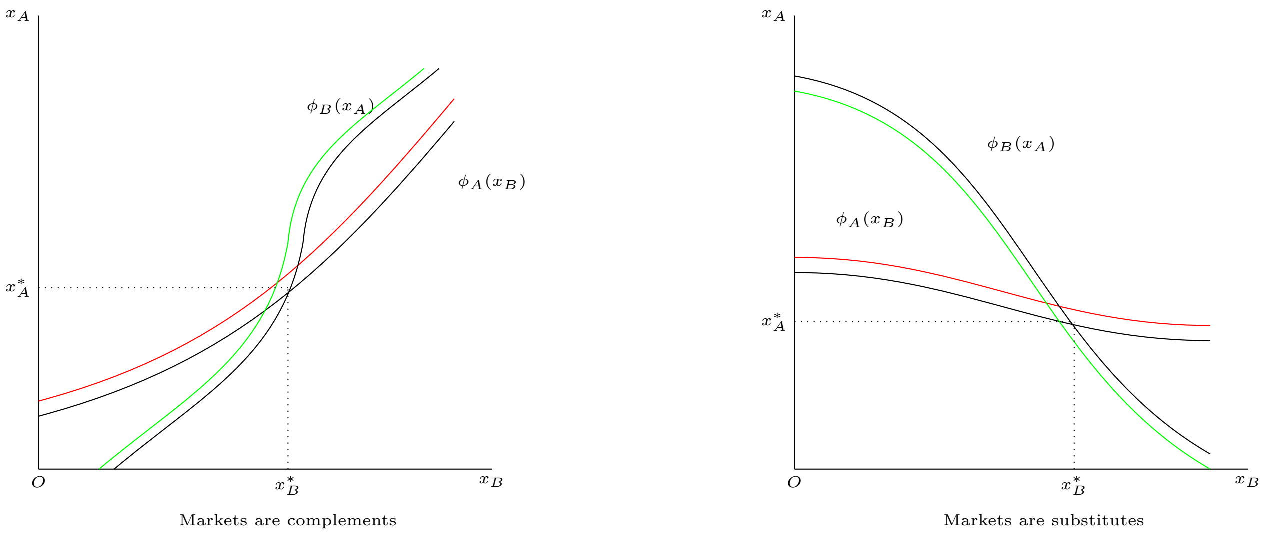

Figure 1: Effect of an increase in t_A under SA

effects work in opposite directions: the direct effect of an increase in t_A on x_A (respectively x_B) is positive (respectively negative), whereas the indirect effect is negative (respectively positive). Which of the two dominates depends on the exact specifications of the model. When, on the other hand, markets are substitutes, direct and indirect effects work in the same direction. An increase in t_A unambiguously results in an increase in the number of users in jurisdiction A and a decrease in the number of users in jurisdiction B.

When the reaction functions are decreasing or when the direct effect dominates the indirect effect, an increase in t_A results in an increase in x_A and a decrease in x_B. Hence the price in market A, P_A goes down while the price in market B, P_B, goes up. When the tax rate t_A is sufficiently high, market A will end up being covered, and the fee p_A will converge to zero. The platform eventually chooses to extract revenues from users in the low-tax jurisdiction, while not charging any fee to participants in the high-tax jurisdiction. This corresponds to a business model where the platform only charges participants on one side of the market, as in the case of search engines and digital social networks.

We also observe that an increase in the corporate tax rate t_A always results in a reduction in the profit of the platform. By the envelope theorem, only the direct effect of t_A on the post-tax profit matters, and this direct effect is given by $\frac{\partial \Pi}{\partial t_A} = -V_A$ and is always negative.

Consider next the effect of an increase in t_A on the tax revenues of country A,

$$\begin{aligned}\frac{\partial R_A}{\partial t_A} &= V_A + \frac{\partial V_A}{\partial t_A}, \\ &= V_A + \frac{\partial V_A}{\partial x_A}\frac{\partial x_A}{\partial t_A} + \frac{\partial V_A}{\partial x_B}\frac{\partial x_B}{\partial t_A}\end{aligned}$$

An increase in the tax rate t_A has two effects on tax revenues: a positive direct effect (measured by the first term V_A) and an effect on the tax base (measured by the second term $\frac{\partial V_A}{\partial x_A}\frac{\partial x_A}{\partial t_A} + \frac{\partial V_A}{\partial x_B}\frac{\partial x_B}{\partial t_A}$). When markets are substitutes, or when markets are complements and the direct effect dominates the indirect effect, x_A is increasing and x_B decreasing in t_A, so that the effect on the tax base is negative. An increase in the corporate tax rate then has an ambiguous effect on the tax revenues of country A.

Similarly, we compute the effect of an increase in t_A on the tax revenues of country B as

$$\frac{\partial R_B}{\partial t_A} = \frac{\partial V_B}{\partial t_A} = \frac{\partial V_B}{\partial x_A}\frac{\partial x_A}{\partial t_A} + \frac{\partial V_B}{\partial x_B}\frac{\partial x_B}{\partial t_A}.$$

The positive effect disappears and the only effect is the effect on the tax base. When the direct effect dominates, x_A is increasing and x_B decreasing in t_A, so that the effect on the tax base is positive. An increase in the corporate tax rate in country A increases the tax base in country B, resulting in an increase in the tax revenues of country B.

We now investigate in detail the magnitude of the direct and indirect effects in two simple situations: when externalities are one-sided and when the two countries are symmetric and corporate tax rates are initially set at the same rate t.

3.1 One-sided externalities

We suppose that externalities are one-sided so that only one type of users experiences positive externalities from the presence of the other users. We first consider the case where users in jurisdiction A do not benefit from the presence of users in jurisdiction B on the platform, $\frac{\partial P_A}{\partial x_B} = 0$. The platform's profit in country A is independent of the number of users in country B, $\frac{\partial V_A}{\partial x_B} = 0$. This implies that the optimal choice of the platform in country B is given by $\frac{\partial V_B}{\partial x_B} = 0$, so that a change in the corporate tax rate t_A has no direct effect on the optimal number of users in country B, $\frac{\partial \phi_B}{\partial t_A} = 0$. Hence a change in the tax rate t_A only has a direct effect on the number of users in country A, and has an indirect effect only on the number of users in country B. We conclude that a higher t_A corresponds to an increase in the number of users in country A, an increase in the number of users in country B when markets are complements and a decrease in the number of users in country B when markets are substitutes. Furthermore as

$\frac{\partial V_B}{\partial x_B} = 0$, $\frac{\partial V_B}{\partial x_A} > 0$ and $\frac{\partial x_A}{\partial t_A} > 0$, the tax revenues of country B are increasing in t_A.

Conversely, suppose that users in jurisdiction B fo not benefit from the presence of users in jurisdiction A on the platform, $\frac{\partial P_B}{\partial x_A} = 0$. By a similar reasoning, the optimal choice of the platform in country A is given by $\frac{\partial V_A}{\partial x_A} = 0$ and is independent of the tax rate t_A. An increase in the corporate tax rate t_A only has a direct effect on the number of users in country B, and an indirect effect on the number of users in country A. An increase in the corporate tax rate t_A thus always results in a decrease in the number of users in country B, a decrease in the number of users in country A when markets are complements, and an increase in the number of users in country A when markets are substitutes. Furthermore, as $\frac{\partial V_B}{\partial x_A} = 0$, $\frac{\partial V_B}{\partial x_B} < 0$ and $\frac{\partial x_B}{\partial t_A} < 0$, the tax revenues of country B are increasing in t_A. We summarize in the following Proposition

Proposition 2 *(one-sided) Suppose that externalities only flow from market A to market B. A higher corporate tax rate t_A always results in an increase in the number of users in country A. It results in an increase in the number of users in country B if markets are complements and a decrease in the number of users in country B if markets are substitutes. Conversely, if externalities only flow from market B to market A, a higher corporate tax rate t_A always results in a decrease in the number of users in country B. It results in a decrease in the number of users in country A if markets are complements and an increase in the number of users in country A if markets are substitutes. In both cases, the tax revenues of country B are increasing in t_A while the effect of an increase in t_A on the tax revenues of country A is ambiguous.*

Proposition 2 can be applied to analyze the optimal number of users of a two-sided platform where one side (advertisers) care about the number of users on the other side but not vice versa. If advertisers are located in a low-tax country, any increase in the difference between tax rates will result in lower prices and higher number of users in both countries under the assumption that markets are complements. On the other hand, if the side of the market which experiences positive externalities is located in the high-tax country, an increase in the corporate tax difference results in higher prices and lower number of users in both countries when markets are complements.

3.2 Symmetric countries

We next consider a situation where demands are symmetric, $P_A(x_A, x_B) = P_B(x_B, x_A)$ for all x_A, x_B, and tax rates are identical, $t_A = t_B = t$. This corresponds to peer-to-peer markets where externalities are balanced and the sizes of the markets are identical. At that point, by symmetry, the direct effects of an increase in the corporate tax rate t_A on the number of users in the two countries have opposite signs, but the same magnitude. Furthermore, concavity implies that the slope of the reaction functions $\phi_A(x_B; t_A, t_B)$ and $\phi_B(x_A; t_A, t_B)$ are smaller than 1. Hence when markets are complements, the direct effect of a change in the corporate tax rate t_A always dominates the indirect effect. When markets are substitutes, the two effects always work in the same direction. We thus obtain the following Proposition

Proposition 3 *(symmetric) Suppose that countries are symmetric and $t_A = t_B = t$. A small increase in the corporate tax rate t_A always results in an increase in the number of users in country A and a decrease in the number of users in country B. Hence prices in country A decrease, prices in country B increase, tax revenues in country B increase while the effect on tax revenues in country A is ambiguous.*

Proposition 3 shows that when similar countries apply similar corporate tax rates on digital peer-to-peer platforms, a marginal increase in the corporate tax rate of one country results in an increase in the number of users in that country and a reduction in the number of users in the other country. This in turn has an immediate effect on prices, yielding lower prices in the country with a higher tax rate and higher prices in the country with lower tax rates. The tax base will be shifted to the low-tax country, so that tax revenues in that country increase.

3.3 A linear model

When externalities are two-sided and the markets are not symmetric, the comparison of the direct and indirect effects becomes intractable. We thus consider a special linear model, by assuming that u_A and u_B are linear in the number of users and the distributions of the parameters θ_A and θ_B are uniform over $[0, 1]$. More specifically, utilities are given by

$$u_A(x_A, x_B) = ax_A + \beta x_B \text{ and } u_B(x_A, x_B) = bx_B + \alpha x_A.$$

where the parameters α and β are non-negative, reflecting the cross-externalities from A to B and B to A. The parameters a and b reflect the externalities within jurisdictions A and B and can either be positive or negative depending on the applications. Recalling that the size of market B is normalized to 1 and the size of market A is given by s, the numbers of users x_A and x_B must satisfy the constraints: $0 \leq x_A \leq s$ and $0 \leq x_B \leq 1$. A market is said to be 'covered' when the upper-bound is reached, meaning that all users in the jurisdiction participate in the platform: hence market A is covered if $x_A = s$ and market B is covered if $x_B = 1$.

Following the computations presented in Section 2.1, the inverse demand functions for $x_A \leq s$

and $x_B \leq 1$ are given by:

$$P_A(x_A, x_B) = 1 - \sigma_A x_A + \beta x_B \text{ with } \sigma_A = \frac{1}{s} - a \tag{9}$$

$$P_B(x_A, x_B) = 1 - \sigma_B x_B + \alpha x_A \text{ with } \sigma_B = 1 - b. \tag{10}$$

The parameters σ_A and σ_B measure the sensitivity of the price in jurisdiction A (respectively B) to the number of users in jurisdiction A (respectively B). These parameters are positive, given that we assume that the price in a jurisdiction is decreasing in the number of users in that jurisdiction. According to the expression above, this sensitivity is decreasing both in the size of the market and in the externalities within the jurisdiction.

Effect of tax distortions on output The optimal choices of the platform depend on the tax levels only through the ratio $\rho = \frac{1-t_B}{1-t_A}$, which is increasing in t_A and decreasing in t_B. We let A denote the high-tax country so that $\rho \geq 1$. In line with the well-known evidence on corporate tax rates, country A is likely to be larger than country B, so we assume $s \geq 1$, although the next proposition does not require this assumption.

Following the approach introduced in Section 3, we compute the optimal number of users in each jurisdiction for a fixed number of users in the other jurisdiction, the reaction functions ϕ_A and ϕ_B. For a fixed x_B, the profit is concave in x_A. Thus, given ρ and x_B, the optimal number of users in A, $\phi_A(x_B)$, is given by

$$\begin{aligned} \phi_A(x_B) &= \frac{1}{2\sigma_A}[1 + (\rho\alpha + \beta)x_B] \text{ if it is less than } s, \\ &= s \text{ otherwise.} \end{aligned} \tag{11}$$

Similarly, given ρ and x_A, the optimal number of users in B, $\phi_B(x_A)$, is given by

$$\begin{aligned} \phi_B(x_A) &= \frac{1}{2\sigma_B}[1 + \frac{1}{\rho}(\rho\alpha + \beta)x_A] \text{ if it is less than } 1, \\ &= 1 \text{ otherwise.} \end{aligned} \tag{12}$$

The functions ϕ_A and ϕ_B are non-decreasing, as markets are complements when utility functions are linear. The numbers of users at the intersection of the two curves, abstracting from the bounds, are given by $(X_A(\rho), X_B(\rho))$ where

$$X_A(\rho) = \frac{1}{2\sigma_A}[\frac{1+\frac{\rho\alpha+\beta}{2\sigma_B}}{1-\frac{(\rho\alpha+\beta)^2}{4\rho\sigma_A\sigma_B}}], \tag{13}$$

$$X_B(\rho) = \frac{1}{2\sigma_B}[\frac{1+\frac{1}{\rho}\frac{\rho\alpha+\beta}{2\sigma_A}}{1-\frac{(\rho\alpha+\beta)^2}{4\rho\sigma_A\sigma_B}}]. \tag{14}$$

These numbers are admissible when they are respectively within $[0, s]$ and $[0, 1]$. We assume that the parameters satisfy two conditions. (i) The platform's profit is concave and (ii) the platform's optimal choice is interior when there are no tax distortions: $\rho = 1$. We next establish conditions on the parameters for which the optimal number of users are interior. We first consider the lower bounds. The numbers $X_A(\rho)$ and $X_B(\rho)$ are negative when $1 - \frac{(\rho\alpha+\beta)^2}{4\rho\sigma_A\sigma_B}$ is negative. This situation arises when the profit is not concave, as concavity of profit holds if and only if $(\rho\alpha + \beta)^2 < 4\rho\sigma_A\sigma_B$.

We next consider the upper bounds. The numbers $X_A(\rho)$ and $X_B(\rho)$ are positive. When both are admissible, they are the optimal choices (due to the concavity of Π). When at least one number is greater than the maximum market share, one of the markets must be covered.

When market A is the only covered market, the optimal number of users in B is given by $\phi_B(s)$ which is less than 1. Hence

$$X^*_A(\rho) = s, \text{ and } X^*_B(\rho) = \frac{1}{2\sigma_B}[1 + \frac{1}{\rho}(\rho\alpha + \beta)s] \text{ if it is less than 1.} \tag{15}$$

Similarly, when market B is the only covered market, the optimal number of users in A satisfy

$$X^*_A(\rho) = \frac{1}{2\sigma_A}[1 + (\rho\alpha + \beta)] \text{ if it is less than } s, \text{ and } X^*_B(\rho) = 1. \tag{16}$$

When both markets are fully covered

$$X^*_A(\rho) = s \text{ and } X^*_B(\rho) = 1. \tag{17}$$

When ρ is sufficiently large, one of the two markets must be covered. We observe that market A is necessarily covered for ρ large enough. If B were the only fully covered, the optimal number x_A in A given by (16), wouldbe increasing in ρ and eventually reach s.

In general, identifying which market is fully covered when $X_A(\rho)$ and $X_B(\rho)$ are not admissible, requires a careful analysis and depends on the parameters. The following proposition summarizes the optimal platform's choice as a function of the externalities parameters, α and

β, the sensitivity parameters σ_A and σ_B, and the tax ratio ρ.

Proposition 4 *Assume that the profit is concave and the optimal solutions are interior at $\rho = 1$: $(\alpha + \beta)^2 < 4\sigma_A\sigma_B$ and $0 < X_A(1) < s$ and $0 < X_B(1) < 1$. Let ρ_A be the minimum value of ρ, $\rho > 1$ for which $X_A(\rho) \geq s$ and ρ_B be the minimum value of ρ for which $X_B(\rho) \leq 1$. The optimal number of users in the two jurisdictions $(X_A^*(\rho), X_B^*(\rho))$ is characterized as follows.*

For $\rho < \min\{\rho_A, \rho_B\}$, none of the markets is fully covered and the optimal quantities are given by: $X_A^*(\rho) = X_A(\rho), X_B^*(\rho) = X_B(\rho)$.

For $\rho \geq \min\{\rho_A, \rho_B\}$, two configurations arise:

1. *$\rho_A \leq \rho_B$. Then for any $\rho \geq \rho_A$, market A is fully covered, but not market B, with numbers of users given by (15): market B is never covered.*

2. *$\rho_A > \rho_B$. Then, for values ρ larger than ρ_B, market B is first fully covered, but not market A, with numbers of users given by (16). As ρ increases, the number of users in market A increases until both markets are covered. As ρ increases further, when $\frac{1}{2\sigma_B}[1+\frac{1}{\rho}(\rho\alpha+\beta)s] < 1$, only market A is covered and the number of users on market B goes down. (Notice that this last situation only which happens when $1 + \alpha s < 2\sigma_B$.)*

Proposition 4 characterizes the optimal choices of the platform under different parameter configurations. When the difference in tax rates is sufficiently small, none of the markets are covered and the optimal number of users is computed as $X_A(\rho)$ and $X_B(\rho)$. When the difference in tax rates becomes large, one of the markets ends up being fully covered. Which market becomes fully covered first depends on the parameters.

If ρ_A is smaller than ρ_B, market A is the first market to be covered. In that case, an increase in ρ unambiguously decreases X_B. The reasoning is the following: in order to shift profit from market A to market B, the platform can no longer increase the number of users in market A and will only reduce the number of users in market B, which is too large with respect to the efficient level in B, i.e. the level maximizing the sole profit in B, given that $x_A = s$.

If ρ_A is larger than ρ_B, market B is the first market to be covered. When ρ increases, the platform increases the number of users in platform A to shift profit towards B. This eventually leads to both markets being fully covered for ρ sufficiently large. Then two situations may arise, as explained below. For ρ large enough, the platform seeks to maximize the profit in jurisdiction B, given full coverage of market A. The platform has an incentive to reduce coverage in market B, if covering market B given $x_A = s$ is not efficient, i.e. it does not maximize the profit in B. In that case, when ρ becomes large enough, the platform reduces coverage in B, whereas market A is fully covered. If on the other hand, given that market A is fully covered, the platform would choose to cover market B in order to maximize the profit in B, then both markets end up being fully covered in equilibrium.

An inspection of the optimal choices shows that they may be increasing or decreasing in ρ depending on the parameters, and the type of equilibrium. Their behavior is easy to analyze when at least one market is covered, as discussed above because we know that the number of users in A is increasing in ρ when B is covered whereas the number of users in B is decreasing in ρ when A is covered. Hence, the differences in taxes are large enough, the marginal effect of an increase in ρ is easy to sign. When instead the difference in tax levels is small, no market is covered, and the effect of an increase in ρ is more complex due to the presence of both direct and indirect effects. To illustrate this point, consider the impact of an increase in ρ when ρ is close to one. Easy computations show that:

$$\begin{aligned}\frac{X'_A(1)}{X_A(1)} &= \frac{\alpha}{2+\alpha+\beta} + \frac{\alpha^2-\beta^2}{4\sigma_A\sigma_B-(\alpha+\beta)^2},\\ \frac{X'_B(1)}{X_B(1)} &= -\frac{\beta}{2+\alpha+\beta} + \frac{\alpha^2-\beta^2}{4\sigma_A\sigma_B-(\alpha+\beta)^2}\end{aligned}$$

As expected, when externalities are symmetric, $(\alpha = \beta)$, an increase in ρ results in an increase in X_A and a decrease in X_B. When externalities only flow from A to B, $\beta = 0$ and $\alpha > 0$, an increase in ρ results in an increase in both X_A and X_B. The result extends to the case where externalities from A to B are sufficiently strong relative to externalities from B to A, in particular when $\alpha > \beta$. Similarly, when externalities only flow from market B to A, $\alpha = 0$ and $\beta > 0$, an increase in ρ results in a decrease in both X_A and X_B. The result extends to the case where externalities from B to A are sufficiently strong relative to A to B, $\alpha < \beta$.

If the price sensitivities σ_A and σ_B are sufficiently small, an increase in t_A leads to a reduction in the number of users on both markets. We relate σ_A and σ_B to the externalities inside each country, $\sigma_A = \frac{1}{s} - a$ and $\sigma_B = 1 - b$. Without externalities within jurisdictions, $a = b = 0$, the larger the size of the high-tax rate country A, the more likely an increase in its tax level leads to a reduction of the number of users in both markets. This situation corresponds to example 3 of section 2.1. An online intermediary connects tourists in the large and high-tax rate country A with hotels in the small and low-tax rate country B. This arises when externalities are positive across markets, and negligible within markets.

4 Formula apportionment

We now turn to the second profit-sharing rule, formula apportionment, and characterize the optimal choice of the platform. Assuming that the post-tax profit function is concave in x_A and x_B, the optimal number of users is given by the solution to the two equations:

$$
\begin{aligned}
(1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}) \frac{\partial V}{\partial x_A} - \frac{x_B(t_A - t_B)}{(x_A + x_B)^2} V &= 0, \qquad (18) \\
(1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}) \frac{\partial V}{\partial x_B} + \frac{x_A(t_A - t_B)}{(x_A + x_B)^2} V &= 0. \qquad (19)
\end{aligned}
$$

Under formula apportionment, a change in the number of users in any of the two countries affects the platform's post-tax profits through two channels. First, it changes the apportionment key, modifying the tax bases and the tax burdens in the two jurisdictions. Second, it changes the pre-tax profit V. Even in the absence of externalities between users in the two countries, the first effect creates an interdependence between the optimal choices of the platform in the two jurisdictions. The coexistence of these two channels also greatly complicates the analysis of the optimal choice of the platform when the jurisdictions set different corporate tax rates.

As in the case of Separate Accounting, let $\psi_A(x_B; t_A, t_B)$ and $\psi_B(x_A; t_A, t_B)$ be the implicit functions defined by equations (18) and (19). The intersection of these two reaction functions define the platform's choices. The signs of the slopes of the reaction functions are given by

$$
\begin{aligned}
\frac{\partial \psi_A}{\partial x_B} &= -\frac{\frac{(1-t_A)\frac{\partial V}{\partial x_A} + (1-t_B)\frac{\partial V}{\partial x_B}}{(x_A + x_B)} + (1 - \frac{t_A x_A}{x_A + x_B} - \frac{t_B x_B}{x_A + x_B}) \frac{\partial^2 V}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}}, \\
\frac{\partial \psi_B}{\partial x_A} &= -\frac{\frac{(1-t_A)\frac{\partial V}{\partial x_A} + (1-t_B)\frac{\partial V}{\partial x_B}}{(x_A + x_B)} + (1 - \frac{t_A x_A}{x_A + x_B} - \frac{t_B x_B}{x_A + x_B}) \frac{\partial^2 V}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B \partial x_B}}.
\end{aligned}
$$

The functions $\psi_A(x_B)$ and $\psi_B(x_A)$ are not necessarily monotonic. The sign of the derivative $\frac{\partial \psi_A}{\partial x_B}$ depends on the sign of two terms. The first term, $(1 - t_A)\frac{\partial V}{\partial x_A} + (1 - t_B)\frac{\partial V}{\partial x_B}$ can either be positive or negative, depending on the tax rates t_A and t_B and the optimal choices of the numbers of users x_A and x_B. This term vanishes when the tax rates are equal, when there is no distortion in the optimal number of users and $\frac{\partial V}{\partial x_A} = \frac{\partial V}{\partial x_B} = 0$. It is positive whenever $t_A > t_B$ and the number of users in country B is greater than the number of users in country A. The sign of the second term $(1 - \frac{t_A x_A}{x_A + x_B} - \frac{t_B x_B}{x_A + x_B}) \frac{\partial^2 V}{\partial x_A \partial x_B}$ depends on the sign of the cross-derivative $\frac{\partial^2 V}{\partial x_A \partial x_B}$. It is positive when markets are complements but negative when markets are substitutes.

. We thus observe that the reaction functions are increasing when (i) markets are complements and (ii) the number of users in country B is at least as large as the number of users in country A. In all other cases, it is not possible to ascertain whether the reaction functions are increasing or decreasing.

As in the case of Separate Accounting, we can decompose the effect of a change in the corporate tax rate t_A on the number of users in jurisdiction i, X_i into a direct and indirect effect, as $\frac{dX_i}{dt_A}$ has the same sign as

$$\frac{\partial \psi_i}{\partial t_A} + \frac{\partial \psi_i}{\partial x_j}\frac{\partial \psi_j}{\partial t_A}.$$

We compute the direct effect using equations (18) and (19):

$$\begin{aligned}
\frac{\partial \psi_A}{\partial t_A} &= \frac{\frac{x_A}{x_A+x_B}\frac{\partial V}{\partial x_A} + \frac{V x_B}{(x_A+x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}}, \\
\frac{\partial \psi_B}{\partial t_A} &= \frac{\frac{x_A}{x_A+x_B}\frac{\partial V}{\partial x_B} - \frac{V x_A}{(x_A+x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_B \partial x_B}},
\end{aligned}$$

The direct effect of an increase in t_A is negative on the number of users in jurisdiction A, and positive on the number of users in jurisdiction B.

To see this, recall that, by equation (18),

$$(1 - t_A\frac{x_A}{x_A+x_B} - t_B\frac{x_B}{x_A+x_B})\frac{\partial V}{\partial x_A} = \frac{x_B(t_A - t_B)}{(x_A+x_B)^2},$$

so that when $t_A \geq t_B$, $\frac{\partial V}{\partial x_A} > 0$. An increase in the tax rate t_A results in a downward shift of the optimal choice on market A. By a similar computation, $\frac{\partial V}{\partial x_B} < 0$, so that an increase in the tax rate t_A results in an upward shift of the optimal choice on market B.

Under FA, an increase in the corporate tax rate in country A induces the platform to reduce its coverage in the high-tax country and increase its coverage in the low-tax country. This is easily explained: when the number of users in the low-tax country is fixed, the platform has an incentive to lower the number of users in the high-tax country in order to reduce the share of profit allocated to the high-tax jurisdiction. This first order effect dominates the second-order effect of a reduction in total profit due to the distortion in output. By a similar reasoning, when the number of users in the high-tax country is fixed, the platform has an incentive to increase the number of users in the low-tax country.

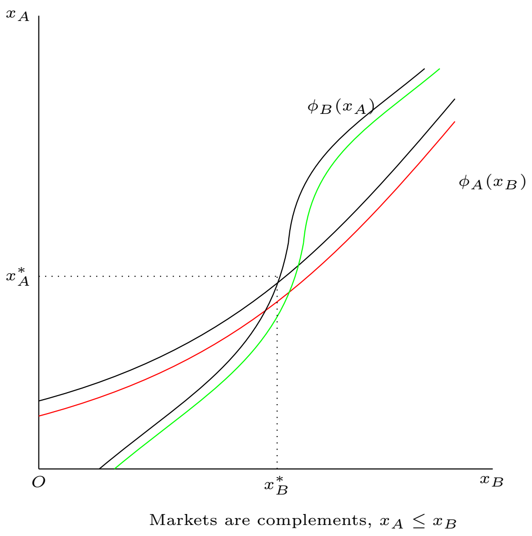

Figure 2: Effect of an increase in t_A under FA

We thus observe that a change in the corporate tax rate t_A has *opposite direct effects under SA and under FA*. Under SA, it leads to an increase in x_A and a reduction in x_B whereas under FA, it results in a decrease in x_A and an increase in x_B.

Figure 2 illustrates the effect of an increase in the corporate tax rate on the choice of the platform when the reaction functions are increasing. As opposed to the case of Separate Accounting, an increase in the corporate tax rate in country A shifts the reaction function in country A downwards (the curve in red is below the curve in black) and the reaction function in country B upwards (the curve in green is to the right of the curve in black). The total effect on equilibrium depends on the balance between the direct and indirect effects which have opposite signs. In Figure 2, the direct effect dominates the indirect effect so that the platform reduces the number of users in jurisdiction A and increases the number of users in jurisdiction B in response to the increase in the corporate tax rate.

As in the case of Separate Accounting, an increase in the corporate tax rate t_A always reduces the profit of the platform. By the envelope theorem, an increase in t_A affects the profit only through the direct effect $\frac{\partial \Pi}{\partial t_A} = -\frac{x_A}{x_A+x_B}V < 0$.

When the corporate tax rate t_A increases, the variation of tax revenues in country A is given by

$$\frac{\partial R_A}{\partial t_A} = \frac{x_A}{x_A + x_B}V + t_A \frac{x_A}{x_A + x_B}\frac{\partial V}{\partial t_A} + \frac{t_A}{(x_A + x_B)^2}(x_B \frac{\partial x_A}{\partial t_A} - x_A \frac{\partial x_B}{\partial t_A})V$$

The first term captures the direct effect which is always positive. The second effect captures the effect on the tax base. When $t_A \geq t_B$, an increase in t_A increases the distortions on the platform's choice, and hence reduces the pre-tax profit V. Hence the second effect is negative. The third term captures the effect on the apportionment key. When the reaction functions are increasing and the direct effect dominates the indirect effect, this effect is also negative. Hence the effect of an increase in t_A on the tax revenues of country A is ambiguous.

Consider next the effect of an increase in t_A on the tax revenues of country B:

$$\frac{\partial R_B}{\partial t_A} = t_B \frac{x_B}{x_A + x_B} \frac{\partial V}{\partial t_A} - \frac{t_B}{(x_A + x_B)^2} (x_B \frac{\partial x_A}{\partial t_A} - x_A \frac{\partial x_B}{\partial t_A}) V$$

There is no direct effect, and the only two effects are (i) the negative effect on the tax base and (ii) the positive effect (when the reaction functions are increasing and the direct effect dominates the indirect effect) on the apportionment key.

In order to make progress, we consider two specific models: one where externalities across jurisdictions are absent and interdependence of choice only results from the effect of the apportionment key, and one where countries are symmetric and one country contemplates an increase above an identical tax rate and a model with linear inverse demands.[5]

4.1 No externalities

Suppose that there are no externalities across jurisdictions, so that the cross-derivative $\frac{\partial V}{\partial x_A} \partial x_B$ is equal to zero. When tax rates are equal, the optimal choice of the number of users in the two jurisdictions are independent. Hence, an increase in the corporate tax rate t_A only affects the optimal number of users through the direct effects. Furthermore, when tax rates are equal, the effect of a change in the tax rate t_A on the tax base is negligible, so that the tax revenues in country B are increasing when t_A increases. We summarize this discussion in the following Proposition.

Proposition 5 *(no externalities) In a situation with no externalities across jurisdictions, a small increase in the tax rate t_A above an identical tax rate t results in a decrease in the number of users in country A and an increase in the number of users in country B. In addition, an increase in t_A always results in an increase in the tax revenues of country B.*

[5]Under FA, the situation with one-sided externalities is not fundamentally different from the situation with two-sided externalities. Even if externalities do not flow from B to A, the number of users in market B affects the choice of users in market A because the platform maximizes total profit, taking into account the fact that the allocation key is the ratio of the number of users.

4.2 A symmetric model

We next consider a model where the two jurisdictions are symmetric, and the inverse demand functions satisfy $P_A(x_A, x_B) = P_B(x_B, x_A)$ for all x_A, x_B. Country A contemplates an increase in the corporate tax rate t_A, starting from an identical tax rate $t_A = t_B = t$. When the tax rates are equal, the sign of the slope of the reaction functions only depends on the cross-derivative $\frac{\partial^2 \Pi}{\partial x_A \partial x_B}$. The reaction functions are increasing when markets are complements and decreasing when markets are substitutes. In addition, concavity implies that, when markets are complements, the slope of the reaction function is smaller than one, so that the direct effect always dominates the indirect effect. We conclude that an increase in t_A always results in a decrease in x_A and an increase in x_B. In addition, when $t_A = t_B$, the platform's choice of pre-tax profit is optimal, so that a small increase in t_A has a negligible effect on the pre-tax profit. Hence, tax revenues in country B are always increasing when the corporate tax rate in country A increases. The following Proposition summarizes our findings.

Proposition 6 *(symmetric) In the symmetric model, a small increase in the tax rate t_A above an identical tax rate t results in a decrease in the number of users in country A and an increase in the number of users in country B. In addition, an increase in t_A always results in an increase in the tax revenues of country B.*

5 A comparison between Separate Accounting and Formula Apportionment

In this Section, we use a numerical simulation to compare equilibrium outcomes under Separate Accounting and Formula Apportionment for a peer-to-peer platform. Each user's utility depends on the sum of the number of users in the two jurisdictions, with

$$u_A(x_A, x_B) = u_B(x_A, x_B) = \gamma(x_A + x_B).$$

Valuations are drawn from a uniform distribution over $[0, 1]$ in the two markets, and the markets have the same size, $s = 1$. The inverse demand function is thus given by

$$P_i(x_i, x_j) = 1 - (1 - \gamma)x_i + \gamma x_j,$$

We compute the optimal choices of the platform under Separate Accounting and Formula Apportionment, assuming that $\gamma = 0.2$ and $t_B = 0.2$, and let the tax rate of country A increase from the identical tax rate $t_A = 0.2$ to 0.4. Figures 3 and 4 show how different economic variables vary with changes in t_A under Separate Accounting (in red) and Formula Apportionment (in blue).[6]

Figure 3 illustrates how number of users and prices in the two counties vary when the tax rate t_A increases. The direct effect dominates the indirect effect in the two régimes, not only at identical tax rates (as indicated by Propositions 3 and 6), but for the entire range of tax rates. Hence, an increase in t_A results in an increase in the number of users in country A and a decrease in the number of users in country B under SA, and a decrease in the number of users in country A and an increase in the number of users in country B under FA. As a consequence, prices in jurisdiction A are decreasing in t_A under SA but increasing under FA. Prices in jurisdiction B follow the opposite trend: they are increasing in t_A under SA but decreasing in t_A under FA.

Figure 4 shows how profits and tax revenues are affected by an increase in the corporate tax rate t_A. The left upper panel considers pre-tax profit. Clearly, output distortions in response to the differences in corporate tax rate move the platform away from its optimal pre-tax profit. The effect becomes stronger when the difference in tax rates increases. The computations show that the effect is stronger under FA than under SA. As the number of users affects the apportionment key in addition to profit, output distortions are larger under FA than under SA.

When one considers post-tax profits however, the difference between the two régimes becomes less stark. As shown in the upper right panel, the post-tax profit of the platform falls at a similar rate under SA than under FA. As pre-tax profits fall faster under FA than under SA, this suggests that the total tax bill of the platform is lower under FA than under SA.

To assess the effect of an increase in t_A on the tax bill, we decompose the tax revenues into tax revenues received by country A (lower left panel) and country B (lower right panel). In country A, we find that the direct effect of an increase in the tax level dominates the tax base effect, so that tax revenues are increasing in t_A under both régines. In addition, it appears that the tax revenues increase faster under SA than under FA, as the platform reacts more strongly to the increase of the corporate tax rate under FA than under SA. Hence, *the high-tax country prefers the régime of Separate Accounting to Formula Apportionment.*

As indicated by Propositions 3 and 6 (for an identical tax rate), an increase in t_A always results in an increase in the tax revenues of country B. This increase is of much smaller magnitude than the increase in tax revenues of country A. Interestingly, the increase is larger under FA than under SA, so that *the low-tax country prefers the régime of Formula Apportionment to Separate Accounting.*

Hence, in the taxation of peer-to-peer platforms, the high-tax and low-tax countries have opposite preferences over the two régimes of profit-sharing. However, as most of the tax revenues

[6]Robustness checks show that similar pictures are obtained for different values of s, γ and t_B.

increase accrues to the high-tax country, the sum of tax revenues is higher under SA than under FA, indicating that the two countries could agree on the Separate Accounting régime with appropriate compensations to the low-tax country.

6 Conclusion

This paper analyzes taxation of an Internet platform attracting users from different jurisdictions. When corporate income tax rates are different in the two jurisdictions, the platform distorts prices and outputs in order to shift profit to the low-tax country. We analyze the comparative statics effects of an increase in the tax rate of one country. When cross-effects are present in both countries, the platform has an incentive to increase the number of users in the high-tax country and decrease the number of users in the low-tax country. When externalities only flow from one market to another, an increase in the corporate tax rate results either in a decrease or an increase in the number of users in both countries depending on the direction of externalities. We compare the baseline regime of separate accounting (SA) with a regime of formula apportionment (FA), where the tax bill is apportioned in proportion to the number of users in the two countries. Under FA, an increase in the corporate tax rate increases the number of users in the low-tax country and decreases the number of users in the high-tax country. We use a numerical simulation to show that the high-tax country prefers SA to FA whereas the low-tax country prefers FA to SA.

The analysis relies on the presence of externalities in demand applies to any multinational firm operating in different jurisdictions with positive network effects. In the particular context of Internet platforms, several restrictive assumptions have been made and need to be relaxed in future work. First, we need to allow for several types of users in each jurisdiction, and extend the model to an arbitrary number of countries. Second, we should allow users to be mobile, and analyze the effect of tax policies on the location of users. Third, we need to allow governments to strategically choose corporate tax rates in a model of tax competition. Finally, we should pay close attention to transfer pricing through royalties on intangible assets and rules of profit repatriation to the home country of the platform. In order to get a better understanding of the reaction of Internet platforms to tax policies, we need to include all these elements in future research.

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8 Appendix

Proof of Proposition 1: The signs of the effect of a change in t_A on the number of users in country A and B are respectively the same as the signs of

$$\frac{\partial \phi_A}{\partial t_A} + \frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial t_A} \text{ and } \frac{\partial \phi_B}{\partial t_A} + \frac{\partial \phi_B}{\partial x_A}\frac{\partial \phi_A}{\partial t_A}.$$

The direct effects are $\frac{\partial \phi_A}{\partial t_A} > 0$ and $\frac{\partial \phi_B}{\partial t_A} < 0$.

When markets are substitutes, $\frac{\partial \phi_A}{\partial x_B}$ and $\frac{\partial \phi_B}{\partial x_A}$ are negative so that the indirect effects $\frac{\partial \phi_B}{\partial t_A}\frac{\partial \phi_A}{\partial x_B}$ and $\frac{\partial \phi_A}{\partial t_A}\frac{\partial \phi_B}{\partial x_A}$ are of the same sign as the direct effects. This proves the first part of the proposition.

When markets are complements, we prove by contradiction that X_A cannot decrease while X_B increases in t_A.

X_A decreases iff $\frac{\partial \phi_A}{\partial t_A} + \frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial t_A} < 0$, which, accounting for the negativity of $\frac{\partial \phi_B}{\partial t_A}$, writes:

$$\frac{\partial \phi_A}{\partial x_B} > -\frac{\frac{\partial \phi_A}{\partial t_A}}{\frac{\partial \phi_B}{\partial t_A}}$$

X_B increases iff $\frac{\partial \phi_B}{\partial t_A} + \frac{\partial \phi_B}{\partial x_A}\frac{\partial \phi_A}{\partial t_A} > 0$, which writes, accounting for the positivity of $\frac{\partial \phi_B}{\partial x_A}$ and negativity of $\frac{\partial \phi_B}{\partial t_A}$,

$$-\frac{\frac{\partial \phi_A}{\partial t_A}}{\frac{\partial \phi_B}{\partial t_A}} > \frac{1}{\frac{\partial \phi_B}{\partial x_A}}$$

Thus X_A is decreasing and X_B increasing only if

$$\frac{\partial \phi_A}{\partial x_B}\frac{\partial \phi_B}{\partial x_A} \geq 1,$$

but we have proved in the text that the product is lower than 1 due to the concavity of profit. This contradiction completes the proof of the Proposition. ∎

Proof of Proposition 3: Remember that the sign of $\frac{\partial X_A}{\partial t_A}$ is the same as the sign of

$$S = \frac{\partial V_A}{\partial x_A}\frac{\partial \Pi^2}{\partial x_B^2} - \frac{\partial V_A}{\partial x_B}[(1-t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1-t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B}].$$

Now, suppose that $t_A = t_B = t$. By the first order condition,

$$\frac{\partial V_A}{\partial x_A} = -\frac{\partial V_B}{\partial x_A}.$$

and by symmetry,

$$\frac{\partial V_B}{\partial x_A} = \frac{\partial V_A}{\partial x_B},$$

so that

$$\frac{\partial V_B}{\partial x_A} = -\frac{\partial V_A}{\partial x_A}.$$

Note also that, as $t_A = t_B = t$,

$$\begin{aligned}
[(1-t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1-t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B}] &= (1-t)(\frac{\partial^2 V_A}{\partial x_A \partial x_B} + \frac{\partial^2 V_B}{\partial x_A \partial x_B}), \\
&= \frac{\partial^2 \Pi}{\partial x_A \partial x_B}.
\end{aligned}$$

Finally, by concavity

$$\frac{\partial^2 \Pi}{\partial x_A^2}\frac{\partial^2 \Pi}{\partial x_B^2} > (\frac{\partial^2 \Pi}{\partial x_A \partial x_B})^2,$$

and by symmetry $\frac{\partial^2 \Pi}{\partial x_A^2} = \frac{\partial^2 \Pi}{\partial x_B^2}$ so that

$$-\frac{\partial^2 \Pi}{\partial x_B^2} > |\frac{\partial^2 \Pi}{\partial x_A \partial x_B}|.$$

We find that

$$\begin{aligned}
S &= \frac{\partial V_A}{\partial x_A}\frac{\partial \Pi^2}{\partial x_B^2} - \frac{\partial V_B}{\partial x_A}\frac{\partial \Pi^2}{\partial x_A \partial x_B} \\
&= \frac{\partial V_A}{\partial x_A}(\frac{\partial \Pi^2}{\partial x_B^2} + \frac{\partial \Pi^2}{\partial x_A \partial x_B}) \\
&> 0,
\end{aligned}$$

showing that $\frac{\partial X_A}{\partial t_A} > 0$.

Now it is easy to check that $\frac{\partial X_B}{\partial t_A} = -\frac{\partial X_A}{\partial t_A} < 0$. The effect of an increase in t_A on the prices P_A, P_B and the tax revenues R_A and R_B are immediately obtained. ■

Proof of Proposition 4: We have computed the reaction functions in the text. It is convenient to introduce the following functions, which give the reaction functions provided the bounds on the markets are satisfied, and make explicit the dependence with respect to ρ:

$$\chi_A(\rho, x_B) = \frac{1}{2\sigma_A}[1 + (\rho\alpha + \beta)x_B] \quad (20)$$

$$\chi_B(\rho, x_A) = \frac{1}{2\sigma_B}[1 + \frac{1}{\rho}(\rho\alpha + \beta)x_A] \quad (21)$$

From (21), given ρ, $\phi_B(x_A) = \min(\chi_A(\rho, x), s)$ and $\chi_B(x_A) = \min(\chi_B(\rho, x_A), 1)$.

We first look for interior solutions of the optimization problem, when the market coverage constraints are not binding. The quantities are given by (13) and (14) when they are positive and smaller than s and 1 respectively. The quantities are positive if $1 - \frac{(\rho\alpha+\beta)^2}{4\rho\sigma_A\sigma_B} > 0$, which is equivalent to the concavity of profit. The inequality can be written as requiring that a quadratic function in ρ is negative. This quadratic function has a positive coefficient in ρ^2 and is negative at $\rho = 1$ by assumption. Thus there is value $\rho_{min} > 1$ such that the profit is concave if and only if $\rho < \rho_{min}$.

Assuming $\rho < \rho_{min}$, consider $X_A(\rho)$ and $X_B(\rho)$. The inequality $X_A(\rho) < s$ can be written as requiring that a quadratic function in ρ with a positive coefficient in ρ^2 is negative. By a similar argument as above, there is unique value ρ_A, $1 < \rho_A < \rho_{min}$, such that $X_A(\rho) \leq s$ for $\rho < \rho_{min}$ if and only if $\rho \leq \rho_A$. The same argument can be used for the function $X_B(\rho)$ and provides the existence of a unique value ρ_B, $1 < \rho_B < 1$, such that $X_B(\rho_B) \leq 1$ for $\rho < \rho_{min}$ if and only if $\rho \leq \rho_B$.

This proves the first part of Proposition 4: the users' numbers $(X_A(\rho), X_B(\rho))$ are the optimal platform choices for $\rho < \min(\rho_A, \rho_B)$.

We now consider the situation where the optimal solution is not interior.

Case 1: $\rho_A < \rho_B$. When ρ increases, the market coverage constraint binds first for A is but not for B. At $\rho = \rho_A$, $\chi_A(\rho_A, x_B) = s$ and $x_B = \chi_B(\rho_A, s) < 1$. Consider $\rho > \rho_A$. Since $\chi_B(\rho, s)$ is decreasing in ρ, $x_B = \chi_B(\rho, s) < 1$ holds. As for $x_A = \chi_A(\rho, x_B)$, x_A is larger than s: if not, x_A, x_B would be an interior solution, in contradiction with $\rho > \rho_A$: Market A is covered but not B, with quantities given by (15).

Case 2:$\rho_A > \rho_B$. When ρ increases, the market coverage constraint binds first for B is but not for A. At $\rho = \rho_B$, $\chi_A(\rho_B, 1) < s$ and $x_B = \chi_B(\rho_B, x_A) = 1$.

Consider $\rho > \rho_B$. Since $\chi_A(\rho, 1)$ is increasing in ρ, there is a value $\widehat{\rho}$ such that $x_A = \chi_A(\rho, 1) < s$ holds for $\rho \in [\rho_B, \widehat{\rho}[$ and $\chi_A(\widehat{\rho}, 1) = s$. The inequality $\chi_A(\rho, 1) < s$ for $\rho \in [\rho_B, \widehat{\rho}[$ implies $\widehat{\rho} < \rho_A$.

Let $\rho \in [\rho_B, \widehat{\rho}[$. We surely have $\chi_B(\rho, x_A) \geq 1$ at $x_A = \chi_A(\rho, 1)$ because otherwise (x_A, x_B) would be an interior solution, in contradiction with $\rho > \rho_B$. This also implies that $\widehat{\rho} < \rho_A$.

Thus for $\rho \in [\rho_B, \widehat{\rho}[$ market B is covered but not A, with quantities given by (16).

Consider now $\rho \geq \widehat{\rho} = \frac{1}{\alpha}[2\sigma_A s - \beta - 1]$. We have $1 < \widehat{\rho} < \rho_A$. At $\rho = \widehat{\rho}$, $\chi_A(\rho, 1) = s$ and $\chi_B(\rho, x_A) \geq 1$ so that both markets are covered. Increasing ρ, both markets are covered as long as $\chi_A(\rho, 1) > s$ and $x_B = \chi_B(\rho, s) > 1$. The first inequality surely holds since χ_A is increasing in ρ. As for the second inequality, $\chi_B(\rho, s)$ is decreasing with ρ with limit $\frac{1}{2\sigma_B}[1 + \frac{1}{\rho}(\rho\alpha + \beta)s]$. Thus $x_B = \chi_B(\rho, s) > 1$ always holds if the limit is at least one; otherwise, $2\sigma_B < 1 + \alpha s$. it becomes optimal to decrease the number of users at the value ρ for which $\chi_B(\rho, s) = 1$ with an optimal number of users is given by $\chi_B(\rho, s)$.

We have worked by increasing ρ, analyzing the solutions to the first order conditions. Since the profit is not always concave, it remains to check that there are no multiple solutions. We know that an interior solution is the unique optimum. We thus need to check that we cannot have two solutions, each with at least one covered market. This is proved as follows. Let a solution with B covered. Assume A is covered as well: covering a market is the best response to the other being covered, hence it is the unique solution with one covered market at least. Assume now that A is not covered. An alternative solution could be that A is covered but not B, which implies $\chi_A(\rho, x_B) \geq s$ with $x_B = \chi_A(\rho, s) < 1$. As $\chi_A(\rho, x_B)$ is increasing in x_B, we must have $\chi_A(\rho, 1) > s$: it is optimal to cover A if B is covered, in contradiction with the initial assumption. ■

Proof of Proposition 6: When the countries are symmetric and $t_A = t_B$, then $x_A = x_B$ and $\frac{\partial V}{\partial x_A} = \frac{\partial V}{\partial x_B} = 0$. This implies that

$$\begin{aligned} \frac{\partial \psi_A}{\partial x_B} &= -\frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A^2}}, \\ \frac{\partial \psi_B}{\partial x_A} &= -\frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B^2}}. \end{aligned}$$

Hence

$$\frac{\partial X_A}{\partial t_A} = \frac{\frac{V x_B}{(x_A + x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_A^2}} + \frac{\frac{V x_A}{(x_A + x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_B^2}} \frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A^2}}.$$

Now by symmetry $\frac{\partial^2 \Pi}{\partial x_A^2} = \frac{\partial^2 \Pi}{\partial x_B^2}$ and, by concavity,

$$(\frac{\partial^2 \Pi}{\partial x_A^2})^2 - (\frac{\partial^2 \Pi}{\partial x_A \partial x_B})^2 > 0.$$

Hence

$$|\frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A^2}}| < 1,$$

which guarantees that

$$\begin{aligned}\frac{\partial X_A}{\partial t_A} &< \frac{\frac{V x_B}{(x_A+x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_A^2}} - \frac{\frac{V x_B}{(x_A+x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_A^2}}, \\ &< 0,\end{aligned}$$

so that X_A is decreasing in t_A. A similar computation shows that x_B is decreasing in t_A. Next observe that $\frac{\partial V}{\partial t_A} = 0$ at $t_A = t_B = t$, so that an increase in the corporate tax rate always increases the tax revenues of country B, concluding the proof. ∎

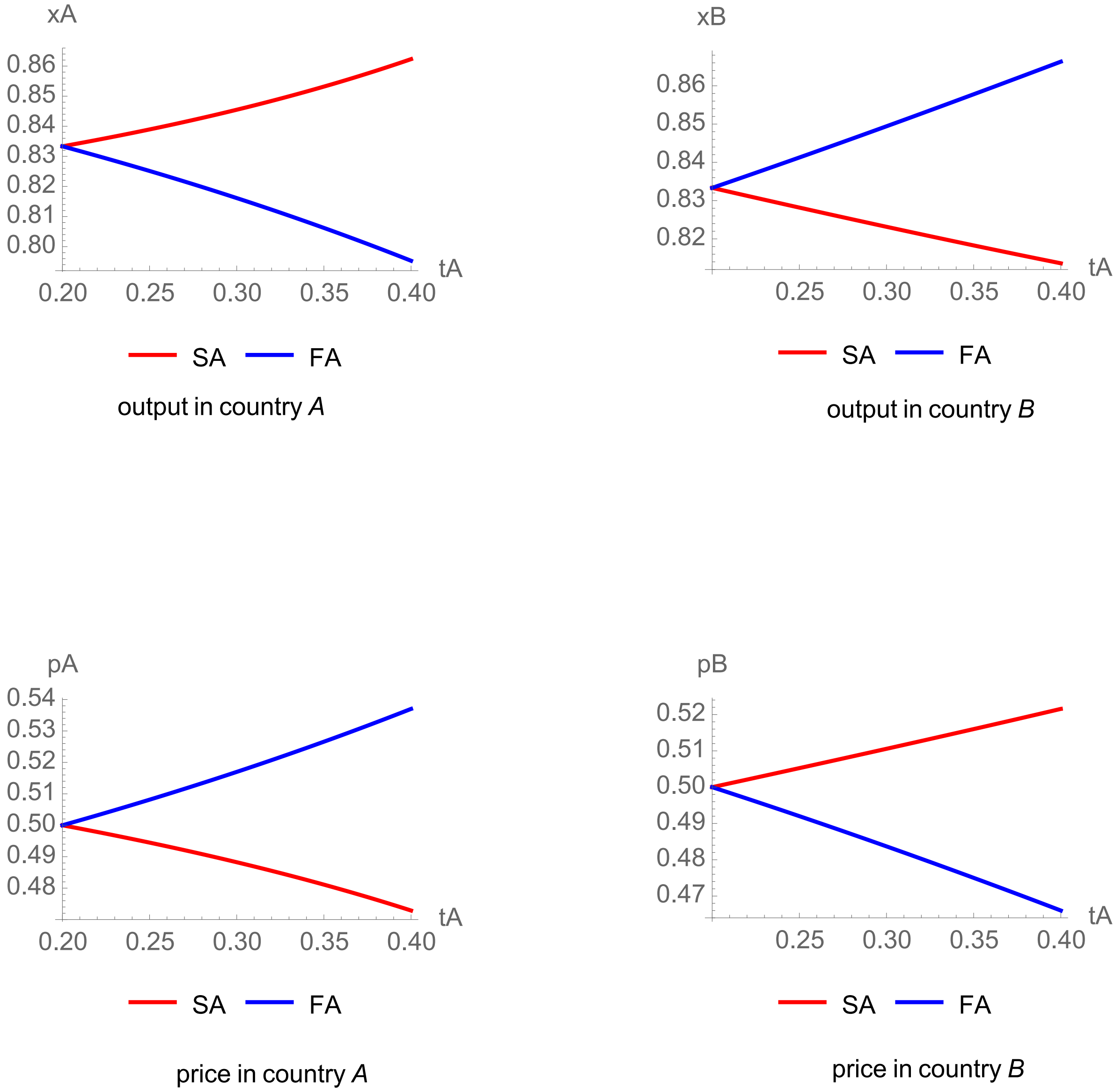

Figure 3: Outputs and prices

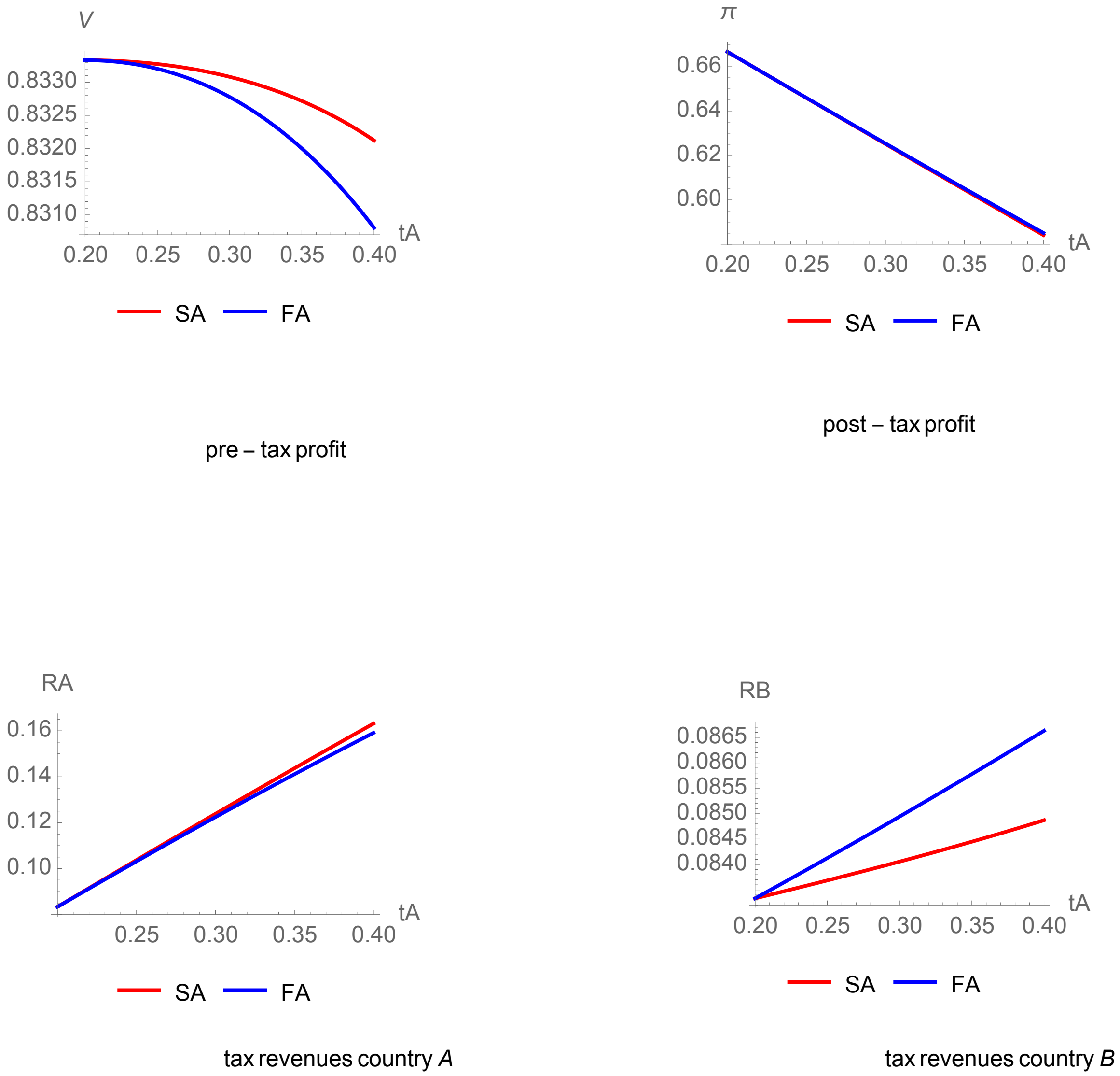

Figure 4: Profits and revenues