CESifo WORKING PAPERS

7837
2019

September 2019

Taxes on Unhealthy Food and Externalities in the Parental Choice of Children's Diet

Zarko Y. Kalamov, Marco Runkel

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
www.cesifo-group.org/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: www.CESifo-group.org/wp

CESifo *Working Paper No. 7837*
Category 1: Public Finance

Taxes on Unhealthy Food and Externalities in the Parental Choice of Children's Diet

Abstract

This paper addresses the question whether taxes on unhealthy food are suitable for internalizing intergenerational externalities inflicted by parents when they decide on their children's diet. Within an OLG model with an imperfectly altruistic parent, the optimal steady state tax rate on unhealthy food is strictly positive. However, it is only second best since it not only reduces food consumption of the child but also distorts the parent's food consumption. Surprisingly, the optimal tax may under- or overinternalize the marginal damage.

JEL-Codes: D110, D620, H210, I120.

Keywords: obesity, fat tax, altruism.

Zarko Y. Kalamov
University of Technology Berlin
Faculty of Economics and Management
Straße des 17. Juni 135
Germany – 10623 Berlin
zarko.y.kalamov@tu-berlin.de

Marco Runkel
University of Technology Berlin
Faculty of Economics and Management
Straße des 17. Juni 135
Germany – 10623 Berlin
marko.runkel@tu-berlin.de

1 Introduction

This short paper addresses the question whether taxes on unhealthy food (fat, sugar, soda, ...) are suitable for internalizing intergenerational externalities inflicted by parents when they decide on their children's diet. According to OECD (2017), one out of six children is overweight or obese, implying a higher risk of obesity in adulthood and related noncommunicable – often chronic – diseases like, e.g., diabetes. Parents have a large impact on their children's diet, and they are often not perfectly altruistic vis-à-vis their children. The most direct evidence is provided by Bruhin and Winkelmann (2009) who study how the happiness of children impacts the utility of their parents and estimate that only 21% to 27% of the parents are altruistic. Hence, in choosing their children's diet, a large part of parents do not fully take into account the future health costs of their children and, thus, inflict an intergenerational externality.

We investigate this externality in an overlapping generations (OLG) model of a family in which the parent chooses both its own and its child's diet. Food consumption in childhood increases weight and health costs in adulthood. It also creates habits that raise the marginal utility of food consumption in adulthood. The parent is imperfectly altruistic and takes into account only a part of the child's future utility and health costs. We find that the optimal steady state tax rate on unhealthy food is indeed strictly positive. However, it is only second best since it not only reduces food consumption of the child but also distorts the parent's food consumption, which is not associated with an externality. Surprisingly, the optimal tax rate may under- or overinternalize the marginal damage. A tax rate increase in a given period reduces the parent's food consumption in this period and thereby gives rise to underinternalization. However, the corresponding fall in the child's consumption *ceteris paribus* reduces weight in adulthood such that parent consumption in the next period may go up. If this effect is large enough, the optimal tax overinternalizes the marginal damage.

In the economic literature, taxes on unhealthy food (or sin taxes in general) are justified with self control problems of individuals (O'Donoghue and Rabin, 2003, 2006), misperceived health costs (Cremer et al., 2016) or negative cost externalities through health insurance (Allcott et al., 2019). In contrast to our paper, however, the previous literature largely ignores intergenerational externalities between parents and children. A remarkable exception is the study of Goulão and Pérez-Barahona (2014). They model a family where parents choose an unhealthy activity (e.g., food consumption) that influences health capital, which is later inherited by their children. Parents are not altruistic and do not take the children's future utility into account when deciding on the unhealthy activity. The optimal tax on the unhealthy activity is strictly positive, as in our paper. However, in their analysis the optimal tax is always first best since they do not include a second margin which the tax erroneously distorts. Hence, the important contribution of our paper is to highlight that the parent food consumption is such a second margin which renders the tax on unhealthy food only second best.

2 Model

Consider an OLG model of a representative family.[1] In each period, the family consists of a parent and a child and each individual lives two periods, childhood and adulthood. In period $t \in \{0, 1, 2, \ldots\}$, the child's utility from consuming x_t^c units of food reads

$$u_t^c = V^c(x_t^c), \tag{1}$$

with $V_x^c(x_t^c) > 0 > V_{xx}^c(x_t^c)$. The child's consumption is not chosen by the child itself, but by its parent. The parent in period t, in turn, receives consumption utility $z_t^p + V^p(x_t^p, s_t)$ from the consumption of a numerairé good z_t^p and own food consumption x_t^p. Utility of food consumption $V^p(x_t^p, s_t)$ is influenced by habits defined as

$$s_t = x_{t-1}^c. \tag{2}$$

Hence, habits equal food consumption during childhood. In addition, the parent in period t has to bear health costs $C(q_t)$ that are positively correlated with weight

$$q_t = x_t^p + \gamma x_{t-1}^c. \tag{3}$$

The parent's weight equals food consumption during adulthood plus a share $\gamma \in [0, 1]$ of food consumption during childhood. The net utility of the parent in t reads

$$u_t^p = z_t^p + V^p(x_t^p, s_t) - C(q_t). \tag{4}$$

The utility function V^p exhibits positive and declining marginal utility of food consumption, i.e. $V_x^p(x_t^p, s_t) > 0 > V_{xx}^p(x_t^p, s_t)$. The marginal utility of habits is assumed to be negative and declining, so $V_s^p(x_t^p, s_t) < 0$ and $V_{ss}^p(x_t^p, s_t) < 0$. Moreover, the utility function satisfies $V_{xs}^p(x_t^p, s_t) > 0$. Hence, the parent's marginal utility of food

[1] All our results also hold in a model with family heterogeneity, provided there is a positive mass of families with imperfectly altruistic parents. The optimal tax then turns out to be third best only.

consumption is increasing in past food consumption and *ceteris paribus* gives the adult incentives to consume more when it has eaten more during childhood.[2] Finally, the marginal health costs are positive and increasing, i.e., $C_q(q_t) > 0$ and $C_{qq}(q_t) \geq 0$.

Long-term utility of the parent in period t equals

$$W_t = u_t^c + u_t^p + \alpha W_{t+1}, \tag{5}$$

where W_{t+1} is long-term utility of a child born in period t when it becomes a parent in period $t+1$. The weight $\alpha \in [0,1]$ determines the degree of (intergenerational) altruism. If $\alpha = 1$, the parent takes fully into account the long-term utility of its child and is perfectly altruistic. For $\alpha = 0$, the parent is non-altruistic. If $\alpha \in]0,1[$, the parent is imperfectly altruistic. Lastly, the family's budget constraint in period t reads

$$z_t^p + (1+\tau_t)(x_t^p + x_t^c) = e + \ell_t, \tag{6}$$

where e is a given income, ℓ_t represents a lump sum transfer received from the government and τ_t is the tax rate on food consumption. For simplicity, we subsequently refer to this tax as fat tax. All producer prices are normalized to unity.

3 Consumption choice of the parent

In period t, the parent chooses its own consumption z_t^p and x_t^p as well as the child's consumption x_t^c in order to maximize utility (5) subject to (1)-(4) and the budget constraint (6) for period t and all periods thereafter. In so doing, the parent in period t takes as given habits $s_t = x_{t-1}^c$. The first-order conditions are (see Appendix A)

$$V_x^p(x_t^{p*}, x_{t-1}^{c*}) - 1 - \tau_t - C_q(x_t^{p*} + \gamma x_{t-1}^{c*}) = 0, \tag{7}$$

$$V_x^c(x_t^{c*}) - 1 - \tau_t + \alpha\Big[V_s^p(x_{t+1}^{p*}, x_t^{c*}) - \gamma C_q(x_{t+1}^{p*} + \gamma x_t^{c*})\Big] = 0, \tag{8}$$

where the asterisk indicates optimal values. According to (7), the parent chooses own consumption such that the net marginal utility, $V_x^p - 1$, equals the sum of marginal health costs, C_q, and the fat tax, τ_t. Equation (8) states that the parent sets the child's consumption where the net marginal utility, $V_x^c - 1$, equals the perceived long-term marginal costs, $\alpha(\gamma C_q - V_s^p) > 0$, plus the fat tax, τ_t. Hence, in case of a zero tax

[2]These properties of the utility function with respect to habits are satisfied for the most commonly used specifications of habits, namely the subtractive habit specification $V^p(x,s) = v(x - \theta s)$ with $v' > 0 > v''$ and $\theta \in]0,1[$, see Lahiri and Puhakka (1998) and Carroll (2000), and the multiplicative habit specification $V^p(x,s) = v(x/s^\theta)$ with $v' > 0 > v''$ and $\theta \in]0,1[$ pioneered by Abel (1990).

$\tau_t = 0$, the parent takes into account only a part of the child's future costs and creates an externality reflected by the share of marginal costs that it ignores, $(1-\alpha)(\gamma C_q - V_s^p)$.

Lagging equations (7) by one period yields

$$V_x^p(x_{t+1}^{p*}, x_t^{c*}) - 1 - \tau_{t+1} - C_q(x_{t+1}^{p*} + \gamma x_t^{c*}) = 0. \tag{9}$$

For each period $t \in \{0, 1, 2, \ldots\}$, (8) and (9) form a system of equations that determines child consumption in period t and parent consumption in period $t+1$ as functions of the tax rates in period t and period $t+1$. Formally, we obtain $x_t^{c*} = X^c(\tau_t, \tau_{t+1})$ and $x_{t+1}^{p*} = X^p(\tau_t, \tau_{t+1})$.[3] Differentiating (8) and (9) gives the comparative static results

$$\frac{\partial x_{t+1}^{p*}}{\partial \tau_{t+1}} = \frac{V_{xx}^c + \alpha(V_{ss}^p - \gamma^2 C_{qq})}{\Delta} < 0, \quad \frac{\partial x_t^{c*}}{\partial \tau_t} = \frac{V_{xx}^p - C_{qq}}{\Delta} < 0, \tag{10}$$

$$\frac{\partial x_{t+1}^{p*}}{\partial \tau_t} = -\frac{V_{xs}^p - \gamma C_{qq}}{\Delta} \lesseqgtr 0, \quad \frac{\partial x_t^{c*}}{\partial \tau_{t+1}} = -\frac{\alpha(V_{xs}^p - \gamma C_{qq})}{\Delta} \lesseqgtr 0. \tag{11}$$

with $\Delta := (V_{xx}^p - C_{qq})[V_{xx}^c + \alpha(V_{ss}^p - \gamma^2 C_{qq})] - \alpha(V_{xs}^p - \gamma C_{qq})^2 > 0$ due to stability reasons. An increase in the tax rate in a given period raises the marginal costs of food consumption and thereby reduces child and parent consumption in that period, as shown in (10). The fall in period t child consumption, following from an increase in the period t tax rate, in turn, has two opposing effects on parent consumption in $t+1$: On the one hand, it reduces weight of the parent in $t+1$, so the parent in $t+1$ may increase its consumption during adulthood (due to $\gamma C_{qq} > 0$). On the other hand, the reduction in child consumption in t weakens habits in $t+1$ and, thus, gives the parent in $t+1$ an incentive to lower its own consumption (due to $-V_{xs}^p < 0$). Taking both effects together, the first expression in (11) shows that the impact of the period t tax rate on parent consumption in $t+1$ is ambiguous. Similarly, the reduction in parent consumption in $t+1$, following from an increase in the period $t+1$ tax rate, reduces the long-term marginal costs perceived by the parent in t by lowering the marginal health costs, and increases the long-term marginal costs of stronger habits. Due to $\alpha\gamma C_{qq} > 0$ and $-\alpha V_{xs}^p < 0$ these changes in the perceived marginal costs translate into opposing effects on child consumption in t, so the parent in t may increase or decrease child consumption in t if the period $t+1$ tax rate goes up, as shown by the second expression in (11).

[3] In period 0, we obtain from (7) the additional condition $V_x^p(x_0^{p*}, x_{-1}^c) - 1 - \tau_0 - C_q(x_0^{p*} + \gamma x_{-1}^c) = 0$ where x_{-1}^c is predetermined. This condition yields x_0^{p*} as a function of τ_0. As we subsequently focus on the steady state only, we can safely ignore this condition from the initial period.

4 Optimal policy

The present value of welfare can be written as $W = \sum_{t=0}^{\infty}(u_t^p + u_t^c)$. Inserting (1)–(4) and (6) as well as the public budget constraint $\ell_t = \tau_t(x_t^{p*} + x_t^{c*})$ yields

$$W = \sum_{t=0}^{\infty} \Big[e - x_t^{p*} - x_t^{c*} + V^p(x_t^{p*}, x_{t-1}^{c*}) - C(x_t^{p*} + \gamma x_{t-1}^{c*}) + V^c(x_t^{c*}) \Big]. \tag{12}$$

The optimal policy maximizes this welfare function, taking into account the comparative static effects (10) and (11). In determining the optimal fat tax rate in period t, we have to take into account the effects on period t child consumption $x_t^{c*} = X^c(\tau_t, \tau_{t+1})$ and period $t+1$ parent consumption $x_{t+1}^{p*} = X^p(\tau_t, \tau_{t+1})$. Moreover, the period t tax rate also influences period $t-1$ child consumption $x_{t-1}^{c*} = X^c(\tau_{t-1}, \tau_t)$ and period t parent consumption $x_t^{p*} = X^p(\tau_{t-1}, \tau_t)$. Differentiating (12) with respect to τ_t and taking into account all these effects as well as (8) and (9) we obtain for $t \in \{1, 2, ...\}$[4]

$$\begin{aligned} \frac{\partial W}{\partial \tau_t} &= \Big\{ \tau_{t-1} - (1-\alpha) \Big[\gamma C_q(x_t^{p*} + \gamma x_{t-1}^{c*}) - V_s^p(x_t^{p*}, x_{t-1}^{c*}) \Big] \Big\} \frac{\partial x_{t-1}^{c*}}{\partial \tau_t} + \tau_t \frac{\partial x_t^{p*}}{\partial \tau_t} \\ &+ \Big\{ \tau_t - (1-\alpha) \Big[\gamma C_q(x_{t+1}^{p*} + \gamma x_t^{c*}) - V_s^p(x_{t+1}^{p*}, x_t^{c*}) \Big] \Big\} \frac{\partial x_t^{c*}}{\partial \tau_t} + \tau_{t+1} \frac{\partial x_{t+1}^{p*}}{\partial \tau_t} = 0. \end{aligned} \tag{13}$$

As Goulão and Pérez-Barahona (2014) we focus on the properties of the optimal tax in the steady state with $\tau_{t-1} = \tau_t = \tau_{t+1} =: \tau^*$. Inserting into (13) and solving gives

$$\tau^* = (1-\alpha)(\gamma C_q - V_s^p)\,\Omega \quad \text{with} \quad \Omega := \frac{\dfrac{\partial x_{t-1}^{c*}}{\partial \tau_t} + \dfrac{\partial x_t^{c*}}{\partial \tau_t}}{\dfrac{\partial x_{t-1}^{c*}}{\partial \tau_t} + \dfrac{\partial x_t^{c*}}{\partial \tau_t} + \dfrac{\partial x_t^{p*}}{\partial \tau_t} + \dfrac{\partial x_{t+1}^{p*}}{\partial \tau_t}}. \tag{14}$$

Using this expression, we prove in Appendix B the following result.

Proposition. *For any $\alpha \in [0, 1[$ the optimal steady state fat tax rate τ^* is strictly positive. In general, however, it deviates from the first best policy and is only second best. We obtain underinternalization (overinternalization) iff*

$$V_{xx}^c + \alpha(V_{ss}^p - \gamma^2 C_{qq}) - (V_{xs}^p - \gamma C_{qq}) < (>) 0. \tag{15}$$

Increasing the fat tax reduces child consumption. This effect is intended since child consumption creates an externality. At the same time, the increase in the fat tax also

[4] In $t = 0$, the first term in (13) vanishes, since x_{-1}^c is predetermined. We can ignore this difference between $t = 0$ and all other periods, since we subsequently focus on the steady state only.

changes parent consumption, which is not intended since parent consumption does not cause an externality. But this latter effect is of second order only, implying that the optimal fat tax rate is strictly positive, as stated in the first part of the proposition.

The unintended distortion of parent consumption explains why the optimal tax is not first best, as stated in the second part of the proposition. A tax rate increase in a given period reduces parent consumption in this period and, at first glance, one may conjecture that the optimal tax rate has to underinternalize the external marginal costs $(1-\alpha)(\gamma C_q - V_s^p)$ in order to mitigate the unintended reduction in parent consumption. But beside the *intra*temporal effect on parent consumption there is also an *inter*temporal effect on parent consumption in the next period which may lead to overinternalization: The intratemporal effect is reflected by $\partial x_{t+1}^{p^*}/\partial \tau_{t+1}$ in (10), and $\partial x_{t+1}^{p^*}/\partial \tau_t$ in (11) gives the intertemporal effect, all expressions evaluated at the steady state. The intratemporal effect is negative, while the intertemporal effect is ambiguous; remember that it may be positive because the decline of consumption during childhood and the corresponding fall in weight in adulthood *ceteris paribus* induces the parent to eat more during adulthood. If the intertemporal effect is positive and larger in absolute terms than the intratemporal effect, then the fat tax has an unintended positive effect on steady state consumption of the parent and the optimal fat tax overinternalizes the external costs. In terms of the model primitives V^p, V^c and C, the conditions for under- and overinternalization are given in (15). An example with overinternalization is obtained if parents are non-altruistic ($\alpha = 0$) and habits are absent ($V_{xs}^p = V_{ss}^p = 0$). For $V^c(x) = ax - bx^2/2$ and $C(q) = cq^2$, overinternalization occurs if $\gamma c > b$.

5 Conclusion

In this paper, we develop an OLG model to analyze non-altruism within the family as a rationale for fat taxes. We show that non-altruism is an argument for taxation of unhealthy food, indeed, but the optimal tax rate is only second best and may under- or overinternalize the intergenerational externality. Of course, this latter result relies on our implicit assumption that there is a uniform tax on parent and child consumption. If taxation may discriminate between parent and child consumption, a zero tax on the former and a tax equal to the marginal costs on the latter would do the job. However, in practice it is often difficult or even impossible to tax parents and children food consumption differently. Such discrimination may be possible if we take into account further margins. A thorough analysis of such margins is left for future research, though.

References

Abel, A. (1990). Asset prices under habit formation and catching up with the joneses. *American Economic Review*, 80(2):38–42.

Allcott, H., Lockwood, B., and Taubinsky, D. (2019). Should we Tax Sugar-Sweetened Beverages? An Overview of Theory and Evidence. *Journal of Economic Pespectives*, forthcoming.

Bruhin, A. and Winkelmann, R. (2009). Happiness functions with preference interdependence and heterogeneity: the case of altruism within the family. *Journal of Population Economics*, 22(4):1063–1080.

Carroll, C. (2000). Solving consumption models with multiplicative habits. *Economics Letters*, 68(1):67–77.

Cremer, H., Goulão, C., and Roeder, K. (2016). Earmarking and the political support of fat taxes. *Journal of Health Economics*, 50(C):258–267.

Goulão, C. and Pérez-Barahona, A. (2014). Intergenerational transmission of noncommunicable chronic diseases. *Journal of Public Economic Theory*, 16(3):467–490.

Lahiri, A. and Puhakka, M. (1998). Habit persistence in overlapping generations economies under pure exchange. *Journal of Economic Theory*, 78(1):176–186.

O'Donoghue, T. and Rabin, M. (2003). Studying Optimal Paternalism, Illustrated by a Model of Sin Taxes. *American Economic Review*, 93(2):186–191.

O'Donoghue, T. and Rabin, M. (2006). Optimal sin taxes. *Journal of Public Economics*, 90(10-11):1825–1849.

OECD (2017). Obesity Update. OECD Publishing, https://www.oecd.org/els/health-systems/Obesity-Update-2017.pdf.

A Derivation of Equations (7) and (8)

Iteratively inserting W_{t+1} in the objective function (5) yields

$$W_t \;=\; \sum_{i=0}^{\infty} \alpha^i (u^p_{t+i} + u^c_{t+i}). \tag{A.1}$$

The parent in t maximizes (A.1) over x^p_t and x^c_t subject to (1)-(4) and (6), taking into account that it may affect its descendants' choices x^{j*}_{t+i} for $j = p, c$ and $i \geq 1$. It takes as given habits $s_t = x^c_{t-1}$ already determined in $t-1$. The first-order conditions are

$$\begin{aligned}
\frac{\partial W_t}{\partial x^p_t} &= V^p_x(x^p_t, x^c_{t-1}) - 1 - \tau_t - C_q(x^p_t + \gamma x^c_{t-1}) \\
&+ \sum_{i=1}^{\infty} \alpha^i \left\{ \left[V^p_x(x^p_{t+i}, x^c_{t-1+i}) - 1 - \tau_{t+i} - C_q(x^p_{t+i} + \gamma x^c_{t-1+i}) \right] \frac{dx^{p*}_{t+i}}{dx^p_t} \right. \\
&+ \left. \left[\alpha \left[V^p_s(x^p_{t+1+i}, x^c_{t+i}) - \gamma C_q(x^p_{t+1+i} + \gamma x^c_{t+i}) \right] + V^c_x(x^c_{t+i}) - 1 - \tau_{t+i} \right] \frac{dx^{c*}_{t+i}}{dx^p_t} \right\} = 0,
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
\frac{\partial W_t}{\partial x^c_t} &= V^c_x(x^c_t) - 1 - \tau_t + \alpha \left[V^p_s(x^p_{t+1}, x^c_t) - \gamma C_q(x^p_{t+1} + \gamma x^c_t) \right] \\
&+ \sum_{i=1}^{\infty} \alpha^i \left\{ \left[V^p_x(x^p_{t+i}, x^c_{t-1+i}) - 1 - \tau_{t+i} - C_q(x^p_{t+i} + \gamma x^c_{t-1+i}) \right] \frac{dx^{p*}_{t+i}}{dx^c_t} \right. \\
&+ \left. \left[\alpha \left[V^p_s(x^p_{t+1+i}, x^c_{t+i}) - \gamma C_q(x^p_{t+1+i} + \gamma x^c_{t+i}) \right] + V^c_x(x^c_{t+i}) - 1 - \tau_{t+i} \right] \frac{dx^{c*}_{t+i}}{dx^c_t} \right\} = 0,
\end{aligned} \tag{A.3}$$

where we have replaced s_t and q_t by (2) and (3), respectively. If we iterate (A.2) and (A.3) to periods after t, we obtain the first-order conditions of the descendants' optimal choices and see that these first-order conditions do not depend on x^p_t. It follows

$$\frac{dx^{p*}_{t+i}}{dx^p_t} = \frac{dx^{c*}_{t+i}}{dx^p_t} = 0 \quad \text{for all } i \geq 1. \tag{A.4}$$

Inserting this back into (A.2) proves (7). Moreover, using (7) simplifies (A.3) to

$$\begin{aligned}
\frac{\partial W_t}{\partial x^c_t} \;&=\; V^c_x(x^c_t) - 1 - \tau_t + \alpha \left[V^p_s(x^p_{t+1}, x^c_t) - \gamma C_q(x^p_{t+1} + \gamma x^c_t) \right] \\
&+ \sum_{i=1}^{\infty} \alpha^i \left\{ \alpha \left[V^p_s(x^p_{t+1+i}, x^c_{t+i}) - \gamma C_q(x^p_{t+1+i} + \gamma x^c_{t+i}) \right] \right. \\
&\qquad \left. + V^c_x(x^c_{t+i}) - 1 - \tau_{t+i} \right\} \frac{dx^{c*}_{t+i}}{dx^c_t} = 0.
\end{aligned} \tag{A.5}$$

Iterating (A.5) to periods after period t, we see that the resulting expression does not directly depend on x^c_t. They contain x^p_{t+1+i} for $i \geq 1$. By the iterated version of (7), however, x^p_{t+1+i} only depends on x^c_{t+i}. Hence, the iterated version of (A.5) also does not depend indirectly on x^c_t (via x^p_{t+1+i} for $i \geq 1$) and it follows

$$\frac{dx^{c*}_{t+i}}{dx^c_t} = 0 \quad \text{for all } i \geq 1. \tag{A.6}$$

Inserting (A.6) into (A.5) proves (8).

B Proof of the Proposition

Inserting the comparative static results (10) and (11) into Ω from (14) yields

$$\Omega = \frac{V^p_{xx} - \alpha V^p_{xs} - (1-\alpha\gamma)C_{qq}}{V^p_{xx} - \alpha V^p_{xs} - (1-\gamma)(1-\alpha\gamma)C_{qq} + V^c_{xx} + \alpha V^p_{ss} - V^p_{xs}}. \tag{B.1}$$

The properties of V^p, V^c and C as well as the conditions $\alpha \in [0,1[$ and $\gamma \in [0,1]$ imply $\Omega > 0$ and, thus, a positive tax rate $\tau^* > 0$, as stated in the first part of the proposition. The first best policy is obtained by differentiating the welfare function

$$W = \sum_{t=0}^{\infty} \Big[e - x^p_t - x^c_t + V^p(x^p_t, x^c_{t-1}) - C(x^p_t + \gamma x^c_{t-1}) + V^c(x^c_t) \Big] \tag{B.2}$$

directly with respect to x^p_t and x^c_t. Denoting the first best steady state values by x^{po} and x^{co}, respectively, the steady state first-order conditions read

$$V^p_x(x^{po}, x^{co}) - 1 - C_q(x^{po} + \gamma x^{co}) = 0, \tag{B.3}$$

$$V^c_x(x^{co}) - 1 + V^p_s(x^{po}, x^{co}) - \gamma C_q(x^{po} + \gamma x^{co}) = 0. \tag{B.4}$$

The steady state consumption levels x^{p*} and x^{c*} chosen by the parent satisfy (7) and (8). In the steady state, these conditions can be rewritten as

$$V^p_x(x^{p*}, x^{c*}) - 1 - \tau^* - C_q(x^{p*} + \gamma x^{c*}) = 0, \tag{B.5}$$

$$V^c_x(x^{c*}) - 1 - \tau^* + \alpha \Big[V^p_s(x^{p*}, x^{c*}) - \gamma C_q(x^{p*} + \gamma x^{c*}) \Big] = 0. \tag{B.6}$$

If we insert $\tau^* = (1-\alpha)(\gamma C_q - V^p_s)\Omega$ from (14), we see that (B.5) does not coincide with (B.3). Hence, the optimal steady state tax rate is only second best. The first best requires a tax $\tau^{po} = 0$ on parent consumption and $\tau^{co} = (1-\alpha)(\gamma C_q - V^p_s)$ on child consumption. Under-(over-)internalization with respect to child consumption occurs if $\tau^* < (>)(1-\alpha)(\gamma C_q - V^p_s)$ or, equivalently, $\Omega < (>)1$. Using (B.1) and rearranging gives (15) and completes the proof of the proposition.