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Joint Income-Wealth Inequality: An Application Using Administrative Tax Data

Abstract

Using tax data from the Swiss canton of Lucerne, we study how measures of economic inequality change if they account for income and wealth rather than income alone. The joint distribution of income and wealth displays strong tail dependence at the top and a negative association for negative net wealth. Joint income-wealth, the sum of labor income and annuitized wealth, serves as a measure of combined inequality of income and wealth. Inequality measured using joint income-wealth is higher than measured using income alone. We refine existing annuitization techniques by introducing heterogeneous returns. A decomposition shows that the underlying marginal distributions of labor income and annuitized wealth account for most of joint income-wealth inequality, whereas their association matters only in the tails.

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1 Introduction

Income is not the only resource available to households. People's material well-being also depends on their stock of wealth. They can use their financial assets to finance their consumption and they can borrow against real assets such as housing wealth. A combined measure of income and wealth would, thus, allow to assess inequality of consumption possibilities more comprehensively than one based only on income. Although there is an emerging literature on wealth inequality, we still know far less about the wealth distribution than about the income distribution. Our knowledge of the joint distribution of income and wealth is even more limited, as most existing studies only look at either income or wealth inequality.[1] How does our assessment of inequality change if we account for income and wealth at the same time?

In this study, we exploit a data set that covers the universe of taxpaying households in this Swiss canton of Lucerne over the years 2005-15. Given that Switzerland is one of few countries besides Norway (Halvorsen and Thoresen, 2019; Fagereng et al., 2019) and Spain (Durán-Cabré et al., 2019) that still have a wealth tax, we observe administrative information on both income and wealth.[2] This feature of the Swiss setting has so far been touched upon by few studies in different contexts (Foellmi and Martínez, 2017; Krapf, 2018). We are, however, the first to use Swiss data to provide detailed descriptions of joint copula distributions. Most of the existing literature on wealth inequality, in contrast, relies either on survey data (Kennickell, 2009) or estimates wealth from estate tax data (Kopczuk and Saez, 2004), from capital income data (Saez and Zucman, 2016), or from rich lists (Vermeulen, 2018). These previous studies, thus, could observe wealth only at a more aggregate level and, often, focused on the top of the distribution. Most importantly, our data allow us to document how wealth covaries with income, which is largely still a puzzle.

We use the concept of annuitized wealth developed by Weisbrod and Hansen (1968) to

[1]One exception is Roine and Waldenström (2008), who are able to calculate wealth share for the top 1% of the income distribution and the top 1% of the wealth distribution in Sweden. The wealth share of high income earners is lower than that of the very wealthy, but far greater than under independence of income and wealth. Another exception is Jäntti et al. (2015), who find a positive association between income and wealth across countries using survey data.

[2]For trends in the distributions of income and wealth in Switzerland, see, for example, Dell et al. (2007) or Foellmi and Martínez (2017). While inequality has been going up in Switzerland since around 1980, this trend is less pronounced than in Anglo-Saxon countries.

convert wealth into an item that is comparable to income flows. Annuitized wealth measures how much somebody could consume if they were to reduce their wealth stock to zero by the end of their expected remaining lifetime. The sum of labor income and annuitized wealth provides a more complete picture of consumption possibilities than regular taxable income. We further expand this measure of consumption possibilities to allow for variation of returns across different asset classes and across the distribution of financial assets.

By introducing heterogeneous returns, we address a shortcoming of the previous literature, notably the so-called 'capitalization method' (Saez and Zucman, 2016), which largely ignored that not all wealth is productive capital (McGrattan, 2015). We are able to create a more realistic measure of consumption possibilities due to wealth by introducing variation in returns across different asset classes into Weisbrod and Hansen's formula. We also allow for returns to increase along the distribution of financial asset holdings following recent evidence that wealthier individuals tend to be able to generate higher returns to their assets (Bach et al., 2016; Fagereng et al., 2019).

Our documentation of combined inequality of income and wealth proceeds in two steps. Following, for example, Brandolini et al. (2010) and Kuypers and Marx (2018), we first, investigate the joint distribution of income and wealth and of labor income and annuitized wealth. In the second step, we examine distributional properties of joint income-wealth, the sum of labor income and annuitized wealth.

In the first step, we estimate non-parametric copulas by quantifying and visualizing 100×100 association matrices between rank pairs. This approach differs from Brandolini et al. and from Kuypers and Marx, who measure the share of jointly income-poor and asset-poor households.[3] Our methodology is more closely related to Aaberge et al. (2018) and to Chetty et al. (2017). An important previous study in this literature is Jäntti et al. (2015), who parametrically model the joint distribution of income and wealth using a mixture model, in which they distinguish between negative wealth, zero wealth, and positive wealth. While the data that are available to us allow us to relax their parametric assumptions, distinguishing between negative, zero, and positive wealth remains important.

In line with the previous literature, we find strong tail dependence. The top 1% of the

[3]This is similar to the counting method by Alkire and Foster (2011a,b), used in Peichl and Pestel (2013).

wealth distribution are highly likely to be among the top 1% of the income distribution. This strong dependence is, unsurprisingly, driven by strong association of wealth and capital income. The wealthiest individuals in our data, however, also tend to be among the group of taxpayers with the highest labor income. The dependence patterns between income and wealth and, in particular, between labor income alone and wealth, are less pronounced in parts of the distribution further away from the tails. The joint copula density of labor income and annuitized wealth displays patterns that are similar to the patterns between income and wealth. It is, however, less characterized by tail dependence.

Most strikingly, we show that the correlation between income and wealth switches signs and becomes negative for negative wealth. This relationship is driven by a positive association between debt and high labor income. We thus provide a more detailed documentation of a pattern that has been found in other settings. Rios-Rull and Kuhn (2016) document that households with negative net wealth tend to receive relatively high incomes in the Survey of Consumer Finances.[4] Krapf (2018) finds the same phenomenon in tax data from the Swiss canton of Bern. Krapf shows not only that the correlation between wealth and income turns negative for negative net wealth, but also that taxpayers with negative net wealth experience large gains on average in both income and wealth in subsequent years.

In the second step, we examine joint income-wealth, i.e. the sum of labor income and annuitized wealth, and compare it to standard taxable income, i.e. the sum of labor income and disbursed taxable capital income. We find that joint income-wealth is distributed more unequally than taxable income and apply a decomposition method by Rothe (2015) to distinguish between the contributions of the underlying marginal distributions of labor income and annuitized wealth and their association to joint income-wealth inequality. The greater inequality of joint-income wealth is mostly due to annuitized wealth being higher, on average, than taxable disbursed capital income, but similarly concentrated in the upper tail. Another important factor behind joint income-wealth inequality is the positive interdependence between labor income and positive net wealth, which dominates the negative interdependence for negative net wealth. The difference between joint income-wealth inequality and taxable income inequality is most pronounced for individuals aged 65 and older who have the highest

[4]See Table 8 in Rios-Rull and Kuhn (2016).

levels of annuitized wealth, but not limited to this age group, though.

The remainder of this paper is organized as follows. Section 2 explains the concept of annuitized wealth, outlines the methodology we use to assess joint distributions. Section 3 provides an overview of the institutional setting and describes the construction of the variables used in this study. In Section 4, we visualize the joint distributions, first between income and wealth, then between labor income and annuitized wealth, and describe the patterns we find. In Section 5, we examine the distribution of joint income-wealth. Section 6 concludes.

2 Analytical framework

2.1 Annuitization and joint income-wealth

While we will also examine the joint distribution of total income and the stock of wealth, our focus will be on labor income, annuitized wealth, and the sum of the two, referred to as *joint income-wealth* (Kuypers and Marx, 2018). Annuitization transforms wealth into a hypothetical income stream assuming that households receive not only returns on their assets but consume their wealth stock over their expected remaining life time. The wealth stock, thus, turns into a flow that is directly comparable to labor income. This allows us to add up the two items to arrive at joint income-wealth, which we use to assess combined inequality of income and wealth.

That we focus on labor income and annuitize wealth comes with two further advantages. First, the dependence between labor income and annuitized wealth is not driven by capital income, which is a function of wealth. Second, annuitized wealth accounts for accrued capital gains, which we cannot observe because they are not taxed in Switzerland.

We build on the annuitization method developed by Weisbrod and Hansen (1968), who define annuitized wealth as

$$Y_a^N = \left[\frac{r}{1-(1+r)^{-n}}\right] \cdot W_N, \tag{1}$$

where W_N is the stock of net wealth, r is the interest rate and n is the life expectancy and, thus, the expected duration of the annuity. A shorter life expectancy or a higher interest rate would increase the value of the annuity. Given the same stock of wealth and interest rate, younger people with more years left in expectation have less annuitized wealth than elderly people.

We apply Weisbrod and Hansen's annuitization formula not to the overall stock of net wealth, but distinguish between different asset classes, which may generate different rates of return. Following Fagereng et al. (2016) and Bach et al. (2016), we allow for variation in return rates not only across asset classes but also within the class of financial assets. The resulting measure of annuitized wealth Y_a takes the form

$$Y_a = \left[\frac{r_f^p}{1-(1+r_f^p)^{-n}}\right] \cdot W_f + \left[\frac{r_h}{1-(1+r_h)^{-n}}\right] \cdot W_h - \left[\frac{r_d}{1-(1+r_d)^{-n}}\right] \cdot D, \qquad (2)$$

where r_f^p is a percentile-specific return on financial assets W_f, r_h is the return on housing wealth and real estate W_h, r_d is the interest rate on debt D and n is remaining life expectancy.[5]

We define joint income-wealth Y_j as the sum of labor income Y_l and annuitized wealth Y_a. The one-dimensional nature of joint income-wealth will allow us to assess inequality of income and wealth combined. It could, however, blur complex dependencies between labor income and annuitized wealth. This is why we will additionally assess the joint distribution of the latter two variables.

The advantages of annuitized wealth compared to other measures of capital income and wealth come at a cost (see Brandolini et al., 2010; Kuypers and Marx, 2018). First, annuitization imposes structure to the measurement of inequality by imposing assumptions about the annuities' length and interest rates, changing particularly the relative position of the elderly. Second, while annuitization of wealth stocks is, in principle, available, we do not argue that households generally make use of it by purchasing annuities. In the framework of this paper we use annuities as a hypothetical measure of yearly consumption possibilities rather than actual income streams over the expected remaining years of life.[6]

[5] As we will show in Section 3, our assumption that $W_N = W_f + W_h - D$ works reasonably well. The sum of financial assets and housing wealth minus debt is equal to around 96% of net wealth in our data. The gap is explained by items such as cars that are not likely to produce returns.

[6] There is a recent literature, which argues that households should in fact purchase annuities to smooth consumption and to insure against longevity risk. Brown et al. (2017), for example, examine behavioral biases that may reduce people's ability to correctly value annuities. While reverse mortgages are, in principle, available, relative illiquidity of housing wealth is another impediment to annuitization (Pashchenko, 2013).

2.2 Non-parametric analysis of joint distributions

Following Jäntti et al. (2015), we analyze the joint distributions by describing the respective marginal distributions and their dependence structure separately. This approach is motivated by Sklar's Theorem (Sklar, 1959) according to which we can represent the joint distribution of two continuous variables X_1 and X_2 as unique copula function of the variable's marginal distributions. The copula function is itself a joint distribution function of the two uniformly distributed rank variables

$$F_{X_1,X_2}(x_1, x_2) = C_{X_1X_2}(F_{X_1}(x_1), F_{X_2}(x_2)).$$

Yet, our variables of interest, income and wealth or labor income and annuitized wealth, are not strictly continuous. They exhibit important mass points at zero as non-negligible parts of the population do either not receive any income or have zero net wealth. As a consequence, there is a range of quantile ranks linked to zero that renders it impossible to attribute these ranks uniquely to another variable's rank. In other words, we cannot infer a uniquely defined copula across mass points.

We discretely approximate the copula densities by association matrices like Aaberge et al. (2018) and Chetty et al. (2017) and assume the ranks at mass points to be uniformly distributed in order to circumvent the problem of indeterminacy. An alternative would have been to impose parametric restrictions over the entire support of the variables like Jäntti et al. (2015). However, a parametric model for the entire support would be overly restrictive. We have access to large administrative data set that allows us to estimate the dependency structure for the continuous parts of the distribution non-parametrically. We provide additional technical details on our non-parametric methodology in Appendix A.

3 Data

Our data cover the universe of tax returns in the Swiss canton of Lucerne between 2005 and 2015. In total, we have access to 2.63 million entries. The key feature of our data is that

On the other hand, while insurance companies generally tend to be better at managing longevity risk than their customers, an increasing number of pension plans are underfunded (Tang et al., 2010).

they contain not only income but also wealth. We also observe most items that wealth is composed of, such as debt, real estate and financial assets, and that income is composed of, such as income from self-employment or from dependent employment, interest, dividends and pensions. We also observe a number of items that are relevant for taxation, such as age and marital status, but not education and other socio-economic background variables. Brülhart et al. (2019) use the same data set to assess behavioral responses to wealth taxation.

Swiss tax authorities treat married as well as registered same-sex couples as one entity.[7] We divide all variables for married couples by two and assign one half to each spouse. Thereby, we imply that economic resources within couples are shared equally. This is a strong but nonetheless conservative assumption as it tends to underestimate inequality (see e.g. Piketty et al., 2018, pp. 590-94). After splitting couples we end up with 3.22 million observations. Note that to assure anonymity, all variables in the raw data are truncated from above. Stock variables are truncated at 40 million Swiss francs and flow variables are truncated at 2 million Swiss francs. We have information on the means of the truncated variables over all individuals to whom truncation applies in a given year and we use these yearly means instead of the unobserved true values. Since we only analyze distributions across percentiles and the truncation is always far above twice the lower bound of the top percentile, the truncation from above is not problematic for our analysis even though we divide wealth and income by two for married couples.

Our data have the advantage of providing information on wealth across the entire distribution.[8] Throughout this paper, wealth will be defined as household net wealth, which we measure using the data item *Reinvermögen*, rather than gross wealth or assets. The tax

[7]Same-sex couple cannot formally marry in Switzerland but they can register which entails treatment equivalent to married couples in terms of taxation. We only observe the gender of the "household head" and, therefore, treat all couples as one man and one woman.

[8]Kopczuk (2015) provides an overview of other methods to measure wealth at the top of the distribution that are available in settings without a wealth tax. One is to use survey data such as the Survey of Consumer Finances (SCF) for the U.S. (e.g. Bricker et al., 2016; Rios-Rull and Kuhn, 2016) or the SOEP for Germany (Frick et al., 2007). An alternative method, employed by Piketty and Zucman (2014) and Saez and Zucman (2016), is the capitalization method, which uses income tax data and imputes wealth from capital income flows. Notably, studies that rely upon the capitalization method tend to find greater wealth inequality than survey-based studies and conclude that inequality has been increasing more dramatically. Possible reasons for this discrepancy are the unclear distinction between labor and capital income and assumptions underlying the estimation of the capitalization factor, which links income flows to wealth stocks (Kopczuk, 2015, 2016; Fagereng et al., 2016). A third method is to use estate tax multipliers (Kopczuk and Saez, 2004) and a fourth is to use rich lists (Vermeulen, 2018).

administration computes *Reinvermögen* by subtracting household debt from total household wealth if the difference between the two is positive and sets it to zero otherwise. We also observe gross wealth and debt separately and replace raw net wealth with gross wealth minus debt if raw net wealth equals zero and debt is greater than gross wealth.[9] Gross wealth includes everything a taxpayer owns, such as cash, financial assets, real estate, shares of non-incorporated firms or cars.[10] The wealth components, real estate, financial assets and business wealth, that we use to impute returns on average add up to 96% of gross wealth. We assume the remainder is wealth that does not yield returns such as cars.

The only items that households do not have to report to the tax administration are household inventory and pension savings in employer covered or private pension plans. After retirement, taxpayers have the choice whether to cash-out their pension savings, in which case they become taxable wealth, or whether to have them paid out in the form of monthly installments over the rest of their life, in which case they become taxable income. Wealth would be distributed less unequally if we could include these pension savings. Simulations by Foellmi and Martínez (2017), however, show that including these would not fundamentally change the distribution.[11]

Households must collect and provide information on income from self-employment themselves, and submit statements on income from dependent employment or from pensions with their tax declaration. This, in combination with the banking secrecy, may induce underreporting (Alstadsæter et al., 2019). Households also report their financial assets themselves by submitting their own bank statements when filing their tax returns. There is, however, a 35% withholding tax that is applied to all income from interest and dividends. These tax payments are returned upon declaration of financial assets, which provides an incentive to

[9] *Reinvermögen* is the only variable in our data that is truncated from below. The tax administrations of different Swiss cantons follow different procedures. The closely related paper by Krapf (2018), for example, uses a similar data set for the canton of Bern, in which raw net wealth is not truncated either from below or from above.

[10] Note that real estate is the only item that is taxed based on location rather than based on residence. We, therefore, do not observe real estate owned by residents of the canton of Lucerne that is located outside the canton.

[11] Pension savings can also be used before retirement to purchase owner-occupied housing or to start self-employment. Apart from these exceptions, pension plans are not directly marketable and cannot be used to finance current consumption. One can, therefore, argue even from a theoretical perspective that they are not wealth in a narrow sense. Saez and Zucman (2019), similarly, made the case for not including the present-value of future government transfers such as the public pension AHV.

report correctly. The only third-party reported component of wealth is real estate, which is assessed by the cantonal administration.

Note that cantons do not generally assess housing wealth at market values but apply a discount and rarely update real estate valuations.[12] In the following, we will evaluate real estate wealth at face value. We will, however, also provide robustness checks that correct for the undervaluation by inflating real estate assets in the tax data by 150%. This inflation factor follows Brülhart et al. (2018), according to whom tax value of real estate in Switzerland correspond, on average, to two thirds of their market value.

We apply different methods to quantify capital income that fall into two broad categories. The measures from the first category are based on disbursed taxable capital income, whereas the measures from category two use the methodology in Section 2.1 to annuitize stocks. In each of the two categories, we will focus on one specific measure, but we will also provide assessments using the alternatives in appendices to this paper. The choice of method to asses capital income affects our measure of total income and may even affect labor income.

Our first measure of capital income consists of disbursed taxable capital income. We further follow Atkinson and Lakner (2017) who assign two thirds of income from self-employment to labor income and one third to capital income. Labor income, hence, is the sum of income from dependent employment, two thirds of income from self-employment and pension income. Capital income is the sum of income from interest, from dividends, from real estate and housing property as well as the remaining third of income from self-employment. All components of capital income can be negative, the item most likely to be negative is interest. We define total income as the sum of labor income and capital income. See Appendix B.A for additional detail on these measures and for an alternative split of income from self-employment.

Annuitized wealth Y_a, our second type of measure of capital income, is stock-based as described in Equation (2). It assumes different returns across asset classes and across percentiles of the distribution of financial asset holdings that remain constant over time. Our computation of annuitized wealth exploits that we directly observe stocks of different assets.

[12] Lucerne's tax law applies a 25% discount on the land register values, which already tend to be below market values, of owner-occupied housing. Moreover, agricultural land is assessed based on expected returns. However, our data do not allow us to distinguish between owner-occupied housing, agricultural land or other types of real estate assets.

We construct W_f from the sum of financial assets ("Wertschriftenvermögen") and shares of non-incorporated firms ("Betriebsvermögen") that we directly observe in our data for each taxpayer. To measure r_f^p, we rely on Fagereng et al. (2019)'s findings, which are for Norway but over the same period as in our data, 2005-15. We use publicly available information by consultancy *Wüest Partner* for the area "Central Switzerland," which Lucerne is part of, to assess real estate income and publicly available information from the Swiss National Bank (SNB) to assess interest rates on debt, resulting in $r_h = 3.90\%$ and $r_d = 2.78\%$. See to Appendix B.B for further details on the sources and construction of these interest rates and returns.

The Swiss Federal Office of Statistics provides official figures on gender- and age-specific life expectancy for years 2008-13. To get a measure of n, we assign these official numbers to the individuals in our data based on their age and marital status. We use life expectancy of the female spouse for both partners of married couples.[13] Since we only observe age of the 'household head,' i.e. generally the male spouse, we subtract two years from this number to get the relevant age for both spouses. Like other variables, we split annuitized wealth in half between partners of married or registered couples. For single households, we use the age recorded in our tax data and the gender-specific life expectancy from the mortality tables to measure n.

Appendix B.B outlines two alternative measures of annuitized wealth that we will examine in robustness checks. We will use the sum of labor income, including two-thirds of income from self-employment, and annuitized wealth to construct joint income-wealth, our measure of combined consumption possibilities outlined in Section 2.

As the relationship between wealth and income changes over the life cycle, we will show how the joint distributions differ across age groups. Again, we subtract 2 years from the age of the male household head to impute age of married women. The youngest age group in our data consists of 763,000 individuals that are between 20 and 34 years old. We obtain 901,000 individuals that are 35-49 years old, 770,000 that are 50-64 years old and, finally,

[13] Brandolini et al. (2010) and Kuypers and Marx (2018) set $n = T_1 + (T_2 - T_1)/b$ for couples, where b is a reduction of the equivalence scale and T_1, T_2 are the life expediencies of the partners. Since our focus is on individuals and, we do not apply an equivalence scale and set $b = 1$.

682,000 that are 65 or older.[14] Differences in distributional patterns between the 1.56 million men and 1.65 million women in our individual data are not a focus of this paper as gender differences may, to a large extent, be driven by our assumption of equal division of resources within couples.

Our data do not suffer from changes in the tax base that often affect the assessment of factor incomes (Bartels and Jenderny, 2015). While our data were processed by the sub-federal tax administration of the canton of Lucerne, the tax base is largely comparable across cantons. Another broadly discussed topic is the extent to which personal income tax returns capture 'business income,' which may also change over time (Alstadsæter et al., 2016). While it is not entirely clear to what extent income from self-employment includes business income, wealth stocks in our data include self-reported business assets.

4 Joint density copula functions

4.1 Income and wealth

Figure 1 shows that wealth in Lucerne is distributed more unequally than income. We estimate the densities of the income and wealth distributions separately for the positive and negative domains using local polynomial regression. To improve visualization, we scale the x-axis by an inverse hyperbolic sine transformation.[15] The income distribution in Lucerne is right skewed but not very unequal and moderately unequal with a Gini coefficient of 0.372. Top income shares do not exhibit very extreme outcomes, either.[16]

The distribution of net wealth is much more stretched out. Wealth inequality by far exceeds income inequality as indicated by a Gini coefficient of 0.883. The wealth distribution is bimodal. The first mode is located in the negative domain among those 13% of the popu-

[14]There are also 102,000 taxpayers in our data that are 19 years old or younger and 400 whose age is over 100. We keep those in our overall sample, but do not create separate age groups for them.

[15]Inverse hyperbolic sine transformation: $y = \ln\left(\sqrt{x^2+1}+x\right)$.

[16]Table C.1 in Appendix C displays corresponding summary statistics including Gini coefficients and top shares. Appendix D provides alternative visualizations to Figure 1 in the form of Pen's parades. Note that the distributional properties of the three outcomes in Figure 1 vary somewhat across age groups. Average income top income shares increase with age in the working age population (see Table E.1 in Appendix E). Income of individuals aged 65 and older is, on average, lower than in the working age population, whereas inequality attains similar levels.

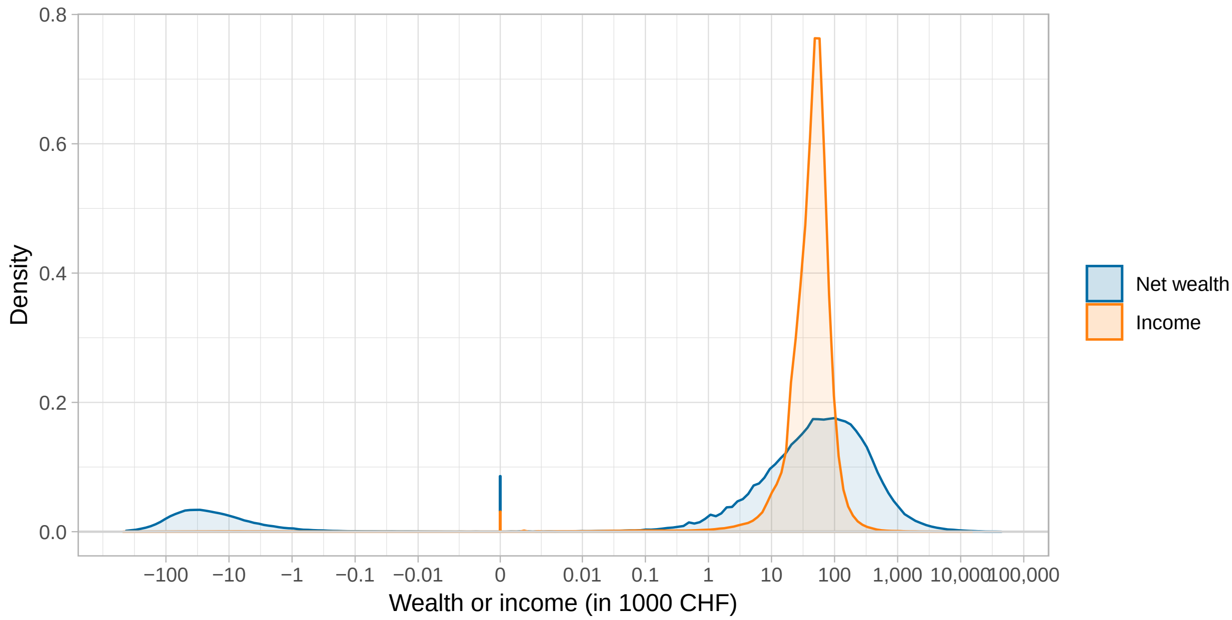

Figure 1: Local polynomial density estimates of the marginal distributions of income, and net wealth in Lucerne, 2005-2015. X-Axis is scaled by an inverse hyperbolic sine transformation.

lation whose debt is larger than their gross wealth stock.[17] While net wealth of the majority of taxpayers is positive, in most cases it is relatively low with a median of 34,016 CHF, which is less than median income. Finally, there is an important mass point at zero. 7% have net wealth that is neither negative nor positive.[18]

The high degree of wealth inequality suggests that overall economic inequality in the canton of Lucerne will likely be greater if we use a measure that accounts for both income and wealth, such as joint income-wealth, compared to a measure based only on income. The extent of joint income-wealth inequality, however, depends not only the marginal distributions but also on the dependence of income and wealth. So, do the income rich also have high

[17]One might suspect that negative net wealth is related to undervaluation of real estate. The picture does, however, not change completely if we adjust for the undervaluation of real estate wealth in the tax data. Inflating real estate wealth by 150% leads to a lower share of individuals with negative wealth and an increase in wealth stocks above the median (see Figure D.2 in Appendix D). However, the Gini coefficient and top income shares of the adjusted wealth measure are only slightly smaller than those of the unadjusted wealth measure (see Table C.4 in Appendix C). Thus, the undervaluation of real estate does not fundamentally undermine our assessment of wealth inequality.

[18]Wealth varies substantially across age groups. The average wealth of individuals 65 and older is 448,021 CHF. This is much more than the average wealth of the rest of the population and particularly than the average wealth of the 20-to-30-year-old which amounts to 38,511 CHF (see Table E.1). High wealth inequality is, however, not entirely due to large differences between generations. Inequality as measured both by the Gini coefficient as well as by top income shares attain high levels within all age groups.

wealth?

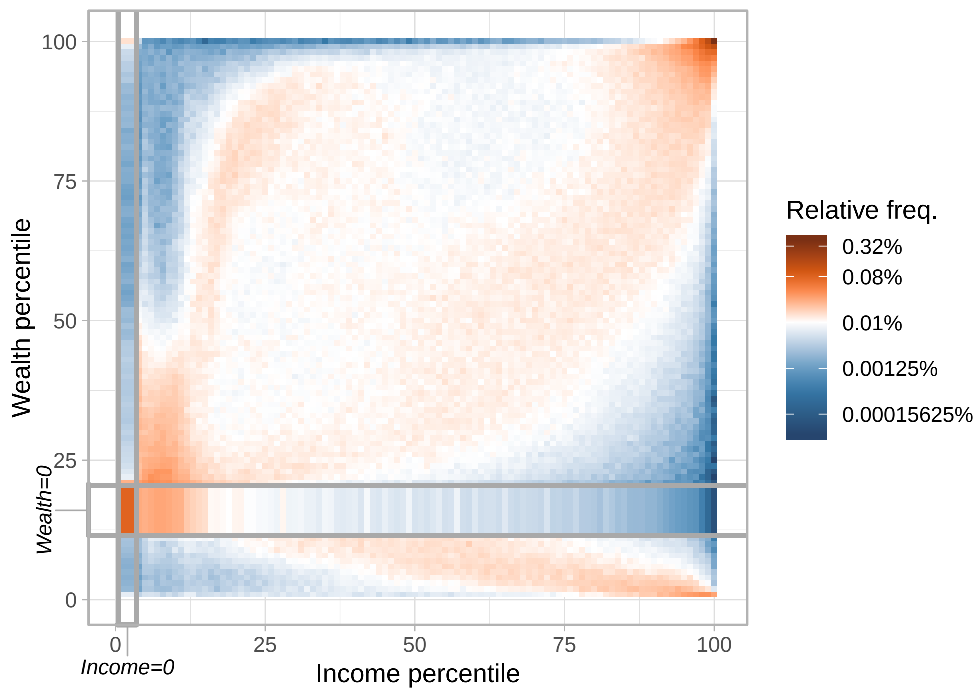

Figure 2: Association matrix between income and net wealth percentile ranks. Every cell represents the relative frequency of a percentile rank combination. Bold lines separate percentile range with zero labor income and annuitized wealth, respectively. Uniform distribution of ranks assumed over mass points at zero.

Figure 2 shows the association matrix between percentile ranks of income and wealth. As a discrete approximation to the empirical copula density, every cell of the matrix represents the relative sample frequency of a percentile combination. For illustration, take the uppermost and rightmost cell, which indicates the relative frequency of observations among the top 1% of both the income and wealth distributions. We observe that $\hat{p}_{1,1} = 0.0041$, i.e. 0.41% of all taxpayers are simultaneously in the top percentiles of both the income and wealth distributions.

The association matrix displays a positive dependence between income and wealth for positive wealth ranks. High income ranks tend to be associated with high wealth ranks. The dependence pattern is not very strong, though. Spearman's rank correlation coefficient attains a relatively low level of 0.321 (see Table 1 for this and other association measures). For most positive wealth percentiles, conditional income rank densities look pretty similar.

Table 1: Association measures

	Income (A) wealth (B)	Labor income (A) annuitized wealth (B)
Spearman's rank correlation		
All observations	0.265	0.134
Obs. with $B > 0$	0.321	0.118
Obs. with $B < 0$	-0.276	-0.238
Tail dependence		
% of top 1% of A in top 1% of B	40.9	13.1
% of top 10% of B in top 1% of A	80.3	48.8
% of top 10% of A in top 1% of B	79.2	28.8
% of top 10% of A in top 10% of B	35.3	19.0

Yet, there is pronounced tail dependence. As noted above, 41% among the top 1% of the income distribution are simultaneously among the top 1% of the wealth distribution. This is a striking discrepancy compared to the 1% one would expect if income and wealth were independent from each other.

The relationship between income and wealth, however, switches signs as we move from positive to negative net wealth. Individuals with negative wealth tend to have high incomes and the more negative their wealth, the higher the incomes we see in our data. We observe hardly any combinations of negative net wealth with low incomes. There are two possible reasons for this association between debt and high incomes. Either high income earners are more creditworthy, or people use debt to finance investments that generate high capital income. Negative net wealth is, however, only negatively correlated with labor income (Spearman coefficient -0.277) and not with disbursed capital income (Spearman coefficient 0.038). Thus, heavily indebted individuals tend to have negative capital incomes.[19]

The negative relationship between income and wealth for negative wealth ranks is weaker when we correct for the undervaluation of real estate in the tax data. There are fewer

[19]For association matrices between income and wealth by age group, see Figure E.4. Negative net wealth is most prevalent in age group 35-49. Hardly any retirees have negative net wealth.

individuals with negative wealth than without the correction and the dependence pattern for negative wealth values takes on a convex rather than a linear form (see Figure F.4 in Appendix F). This suggests that part of the negative association is due to real estate being undervalued compared to (mortgage) debt. For positive wealth, the correction for real estate undervaluation does not lead to a significantly different dependence pattern between income and wealth.[20]

The dependence structure we observe has two competing effects on the inequality of joint income-wealth. On the one hand, the positive association of positive net wealth increases inequality, particularly at the top. On the other hand, the negative dependence for negative wealth dampens inequality. Some high income receivers are actually heavily indebted. Even though both income and wealth of these indebted individuals tend to increase substantially over subsequent years (Krapf, 2018), one would overestimate their resources by looking at their income alone.

4.2 Labor income and annuitized wealth

Estimation of the joint distribution of labor income and annuitized wealth offers several advantages over estimation of the joint distribution of income and wealth. First, there is no mechanical link between labor income and wealth as labor income does not include capital income that accrues to wealth owners. Second, our annuitized wealth measure is not limited to disbursed capital income. Third, annuitized wealth can be thought of as a flow variable that is more directly comparable to income.

We first compare the densities of labor income, including two-thirds of income from self-employment, capital income including the remaining third of income from self-employment, and annuitized wealth. Figure 3 shows local polynomial density estimates. Labor income is distributed more evenly than capital income and annuitized wealth. The Gini coefficients and the top income shares in Appendix Table C.2 confirm this. Annuitized wealth is a relevant

[20] As mentioned above, there may be underreporting of wealth in our data. While underreporting may, to a certain extent, be responsible for the large number of individuals with negative net wealth in our data, it does likely not explain why the association between income and wealth switches signs at zero net wealth. As Krapf (2018) argued, tax evasion would, if anything, provide a rationale for a discontinuity at the taxable wealth threshold and not at zero net wealth.

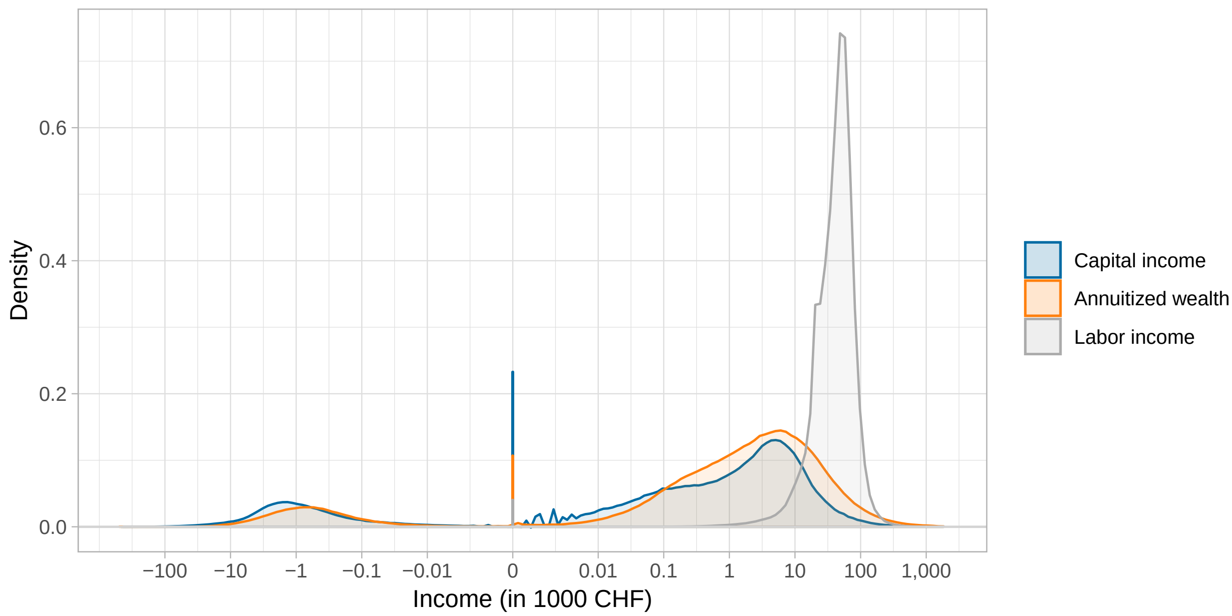

Figure 3: Local polynomial density estimates of the marginal distributions of capital income, annuitized wealth and labor income in Lucerne, 2005-2015. X-Axis is scaled by an inverse hyperbolic sine transformation.

source of potential income for only very few people. The top 1% of the annuitized wealth distribution could increase their consumption compared to a situation in which they only spent their labor income on average by 468,293 CHF. Median annuitized wealth, however, is a mere 1,148 CHF. 21% of all observations have negative or no annuitized wealth.

Annuitized wealth is on average higher than capital income and there are fewer individuals with non-positive annuitized wealth than with non-positive capital income. Partly due to the larger number of non-positive values, capital income is more unequal when measured by the Gini coefficient. Annuitized wealth is, however, more heavily concentrated in the upper tail. The annuitized wealth shares of top annuitized wealth holders are markedly larger than the corresponding top capital income shares of top capital income recipients.[21]

We now turn to the joint density of labor income and annuitized wealth in Figure 4.[22]

[21] As before, we verify how sensitive these statistics are to potential undervaluation of real estate and how they vary across age groups. Our conclusions do not change fundamentally if we inflate real estate by a factor 1.5 (see Table C.4 in Appendix C). Annuitized wealth is on average much higher among individuals 65 and older than among the working age population. Differences across age groups are, however, not the only explanation for high inequality of annuitized wealth. Inequality attains high levels in all age groups and is even highest among the 20-to-35-years-old (see Table E.2).

[22] Appendix F displays corresponding figures with association matrices for other combinations of all alternative definitions of labor and capital income in Section 3 and Appendix B.

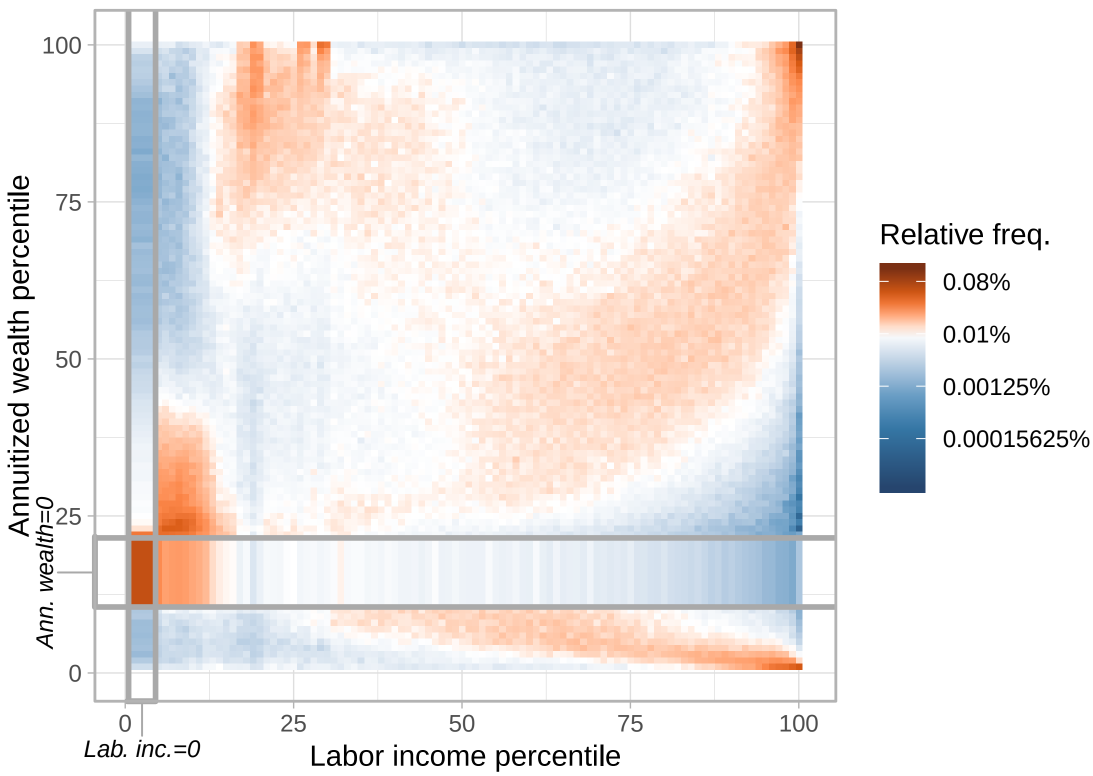

Figure 4: Association matrix between labor income and annuitized wealth percentile ranks. Every cell represents the relative frequency of a percentile rank combination. Bold lines separate percentile range with zero labor income and annuitized wealth, respectively. Uniform distribution of ranks assumed over mass points at zero.

There is a positive association between labor income and annuitized wealth for positive values of annuitized wealth. The association is less pronounced than the one we observed between income and wealth, though. The dependence between labor income and annuitized wealth is strongest in the upper tail. The top 1% of labor income is very likely to be part of the top 1% of annuitized wealth. For lower ranks, there is a weak and asymmetric association. This becomes evident from the convex shaped heat mass in the right half of the association matrix. Individuals in higher labor income percentiles tend to rank in similar but slightly lower annuitized wealth percentiles. We observe a similar asymmetry at the top, too. It is more likely to find an individual of the top 10% of annuitized wealth among the top 1% of labor income than an individual from the top 10% of labor income among the top 1% of annuitized wealth (see Table 1). These asymmetries reflect the fact that savings out of high labor income are not the only pathway to high annuitized wealth. Persistently high capital incomes, bequests or disbursed capital from pension plans are, potentially, an even more important source of high annuitized wealth.

"Classical capitalists," i.e. rentiers with very high annuitized wealth but no labor income are rare. Only 0.006% of all observations are part of the top 1% of the annuitized wealth distribution and do not receive any labor income. There is, however, a significant number of low labor income earners within high wealth ranks as indicated by the heat mass in the upper-left part of Figure 4. One half of the top 1% of annuitized wealth owners, for example, receive labor income that is below the median. This negative relationship between low labor income and high annuitized wealth is driven by differences across age groups. Many retirees have high wealth but receive labor income only in form of the relatively small public pension AHV (see Moser, 2019), which leads to the high mass in the upper-left corner of Figure 4.

Consequently, we do not observe high frequencies in rank combinations between low labor income and high annuitized wealth when we plot the association matrices separately for different age groups in Figure 5. The dependence between labor income and annuitized wealth for positive wealth exhibits largely symmetric positive patterns in all age group. For the oldest age group the pattern is still positive but somewhat noisier. A possible explanation is the system of professional pension savings in Switzerland mentioned above. Individuals who choose to cash-out their pension savings upon retirement have relatively high taxable wealth, whereas individuals who choose to convert their pension savings into a life-long monthly pension have relatively high taxable (labor) income.

There is also substantial dependence in the lower tail of the distribution for positive annuitized wealth. Figure 4 shows a large share of people with low labor income and low annuitized wealth. In numbers, 86% of the bottom 10% of the labor income distribution have an annuitized wealth below the median. We also observe the strong association between low labor income and low annuitized wealth ranks in the high number of individuals such as students or social welfare recipients who have neither labor income nor annuitized wealth.[23] They make up 3% of all observations.

Again, we find a negative association for negative values of the annuitized wealth variable

[23]Note that most transfer payments like pensions or unemployment benefits are treated as labor income. Social welfare benefits are, however, tax exempt and, thus, not included in the data.

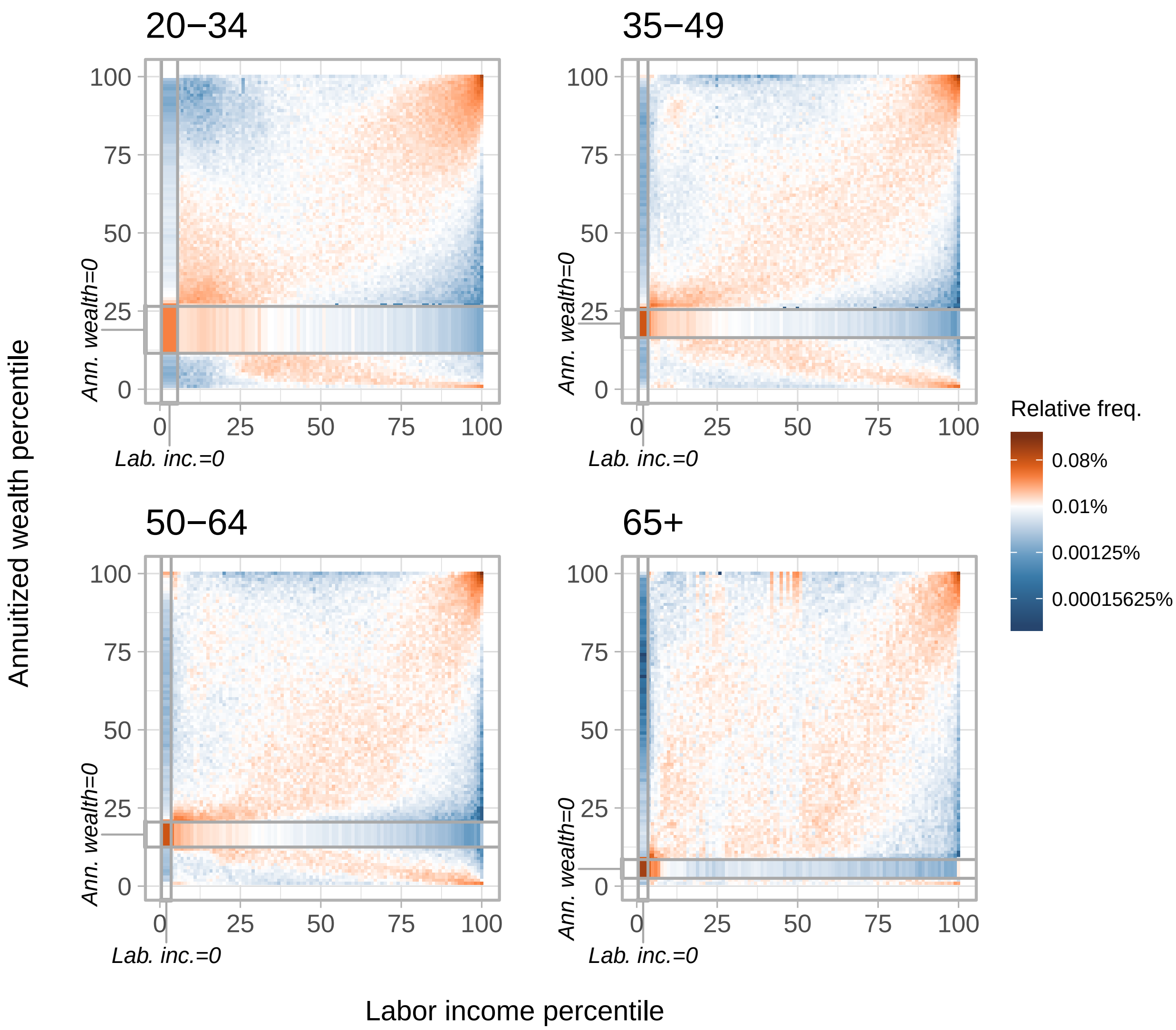

Figure 5: Association matrices between labor income and annuitized wealth percentile ranks by age group. Every cell represents the relative frequency of a percentile rank combination. Note that percentiles refer to conditional percentiles of respective age groups. Bold lines separate percentile range with zero labor income and annuitized wealth, respectively. Uniform distribution of ranks assumed over mass points at zero.

and labor income in Figure 4 for the entire population, as well as for each single age group in Figure 5. Labor income ranks tend to rise as we move down the annuitized wealth distribution for negative annuitized wealth. This relationship is again attenuated but does not disappear when we correct for real estate undervaluation (see Figure F.5 in Appendix F).[24]

The negative association for negative annuitized wealth, which somewhat reduces inequal-

[24]The negative association between income and wealth ranks for negative net wealth persists across all alternative definitions of labor and capital income in Appendix F.

ity, stands in contrast to the generally positive association between labor income and annuitized wealth. Wealth tends to increase inequality created in the labor market not only through its marginal distribution but also through its mostly positive interdependence with labor income. To quantify the extent to which wealth compounds inequality, we will next compare joint income-wealth (i.e. the sum of labor income and annuitized wealth) to income (i.e. the sum of labor income and capital income).

5 Joint income-wealth inequality

5.1 Comparison joint income-wealth vs. taxable income

Figure 6 shows density estimates for joint income-wealth as well as for income. We see that the joint income-wealth distribution is located generally to the right of the income distribution and has a heavier tail at the top. The greater variation in joint income-wealth compared to income implies that the the difference between the two measures increases as we move up the distribution. While the difference between joint income-wealth and income is 11,124 CHF on average and a mere 3,873 CHF between the medians of the two distributions, it increases to 111,698 CHF between the averages of the top percentiles of the two distributions. Mean joint income-wealth in the top 1% of joint income-wealth is, on average, almost twice as large as mean income in the top 1% of the income distribution.[25]

In Figure 7, we perform a comparison based on "anonymous and non-anonymous growth incidence curves" suggested by Bourguignon (2011) and "income mobility profiles" of van Kerm (2009). This figure plots two versions of the average percentile differences between joint income-wealth and income. The blue circled line shows the difference between the averages for each percentile of the respective distributions. This version ignores that individuals in, say, the top percentile of joint income-wealth are not necessarily the same as individuals in the top percentile of the income distribution. It further visualizes the increasing difference between joint income-wealth and income mentioned above. The second, orange pattern in

[25]See Appendix Table C.3 for additional descriptive statistics including Gini coefficients, which confirm this picture. Appendix Figure D.4 plots average income by percentile of the income distribution and the average joint-income wealth by average of the joint income-wealth distribution. This graph further visualizes how the difference between joint income-wealth and income increases with higher percentiles.

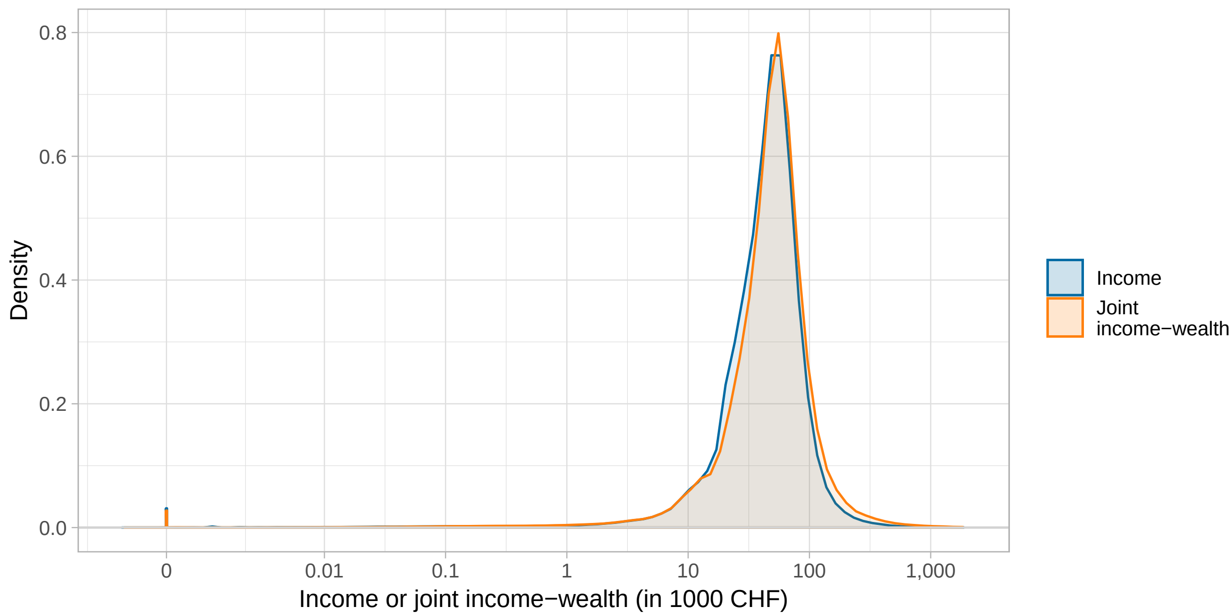

Figure 6: Local polynomial density estimates of the marginal distributions of income and joint income-wealth in Lucerne, 2005-2015. X-Axis is scaled by an inverse hyperbolic sine transformation.

Figure 7, which connects triangles, compares joint income-wealth and income by percentiles of the income distribution. Here, we thus hold individuals constant, which allows us to infer which income percentiles benefit on average if we use annuitized wealth to measure consumption possibilities rather than capital income, and which lose.

Again, we see that the difference is highest at top. Mean joint income-wealth of the top 1% of the income distribution is 321,140 CHF or 85% higher than their mean income. The difference between the two income concepts decreases between the 15th and the 65th percentile. We, hence, tend to underestimate the consumption possibilities of individuals around the first quartile of the income distribution more than those of individuals around the median if we only rely on taxable income and ignore annuitized wealth.

The primary reason why individuals at the bottom of the distribution tend to benefit more if we account for annuitized wealth is, again, the presence of retirees in our data. Retirees have lower incomes but more wealth on average than the working-age population. Their lower remaining life expectancy further drives up their annuitized wealth compared to younger generations. Figure 8 plots percentile-specific averages of joint income-wealth and of income for four different age groups. Even though, average joint income-wealth is higher

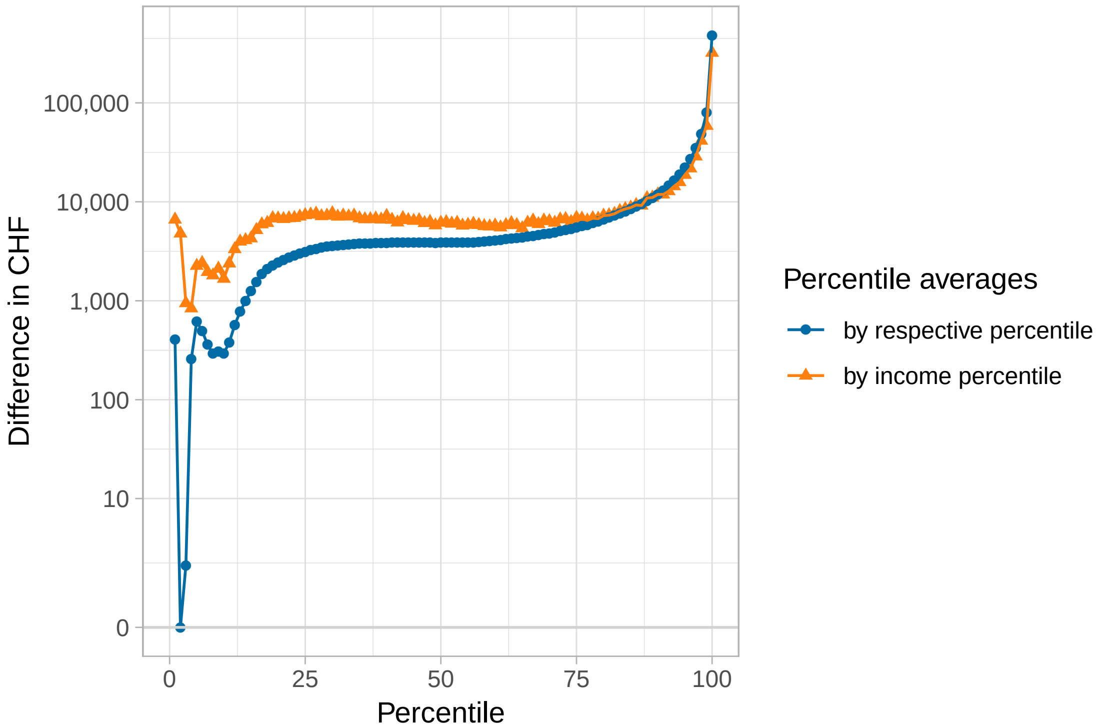

Figure 7: Differences between average joint income-wealth and income by percentile of respective distribution and differences between averages between average joint income-wealth and income by percentile of the income distribution, respectively. Y-Axis is scaled by an inverse hyperbolic sine transformation.

for almost every percentile in every age group, the gap increases with age and percentile. The difference between the two income concepts is by far largest in the top percentile of individuals 65 and older. Joint income-wealth of the most high-income retirees exceeds their income by more than 1 million CHF. Note, however, that not all retirees become rich once we account for annuitized wealth. For example, the difference between joint income-wealth and income at the median of the distribution for retirees is merely 11,270 CHF, which corresponds to 34% of the median income of retirees.[26]

[26]In a similar vein, the difference between income inequality and joint income-wealth inequality is also highest for individuals 65 and older (see also Table E.1 in Appendix E). For example, the top 1% income share more than doubles if we include annuitized wealth into our inequality assessment. For the working age population, the differnce is smaller and most significant for individuals between 50-64.

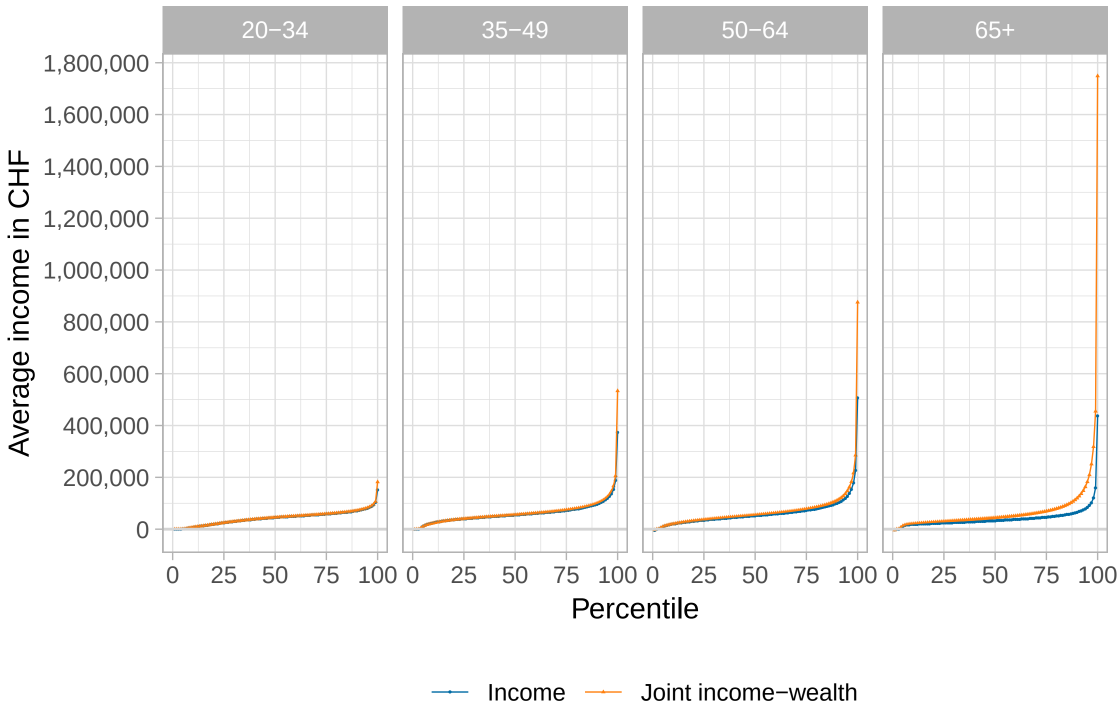

Figure 8: Average income and joint income-wealth distribution by respective quantile and age group. See Figure E.3 in Appendix E for differences between average joint income-wealth and income by percentile of respective distribution.

5.2 Decomposition of joint income-wealth

Having shown above that joint income-wealth is distributed more unequally than income, in this section we ask what factors are responsible for joint income-wealth inequality.[27] To address this question, we rely on the decomposition method developed by Rothe (2015). Appendix G outlines formally how Rothe's method allows us to distinguish between the contributions of the marginal distributions of labor income and annuitized wealth, the contribution of the interaction between the marginal distributions of the factors and the contribution of the dependence structure.

Table 2 shows aggregate results. Labor income and annuitized wealth are of similar importance for explaining joint income-wealth inequality, although mean annuitized wealth is merely a third of mean labor income. The copula plays only a minor role. The Gini coefficient of joint income-wealth would be 2.6 base points or around 6% smaller if labor income and

[27]In Appendix H, we perform this analysis for the Atkinson and Lakner (2017) measures of capital income and income. The results are qualitatively similar.

Table 2: Decomposition of Distributional Statistics of Joint Income-Wealth

	Total	Lab. income	Ann. wealth	Interaction	Dependence
Mean (in CHF)	62,594	46,516	16,078	0	0
Gini	0.434	0.175	0.143	0.090	0.026
Top 1% share	0.138	0.007	0.081	0.030	0.010
Top 10% share	0.345	0.095	0.073	0.063	0.014

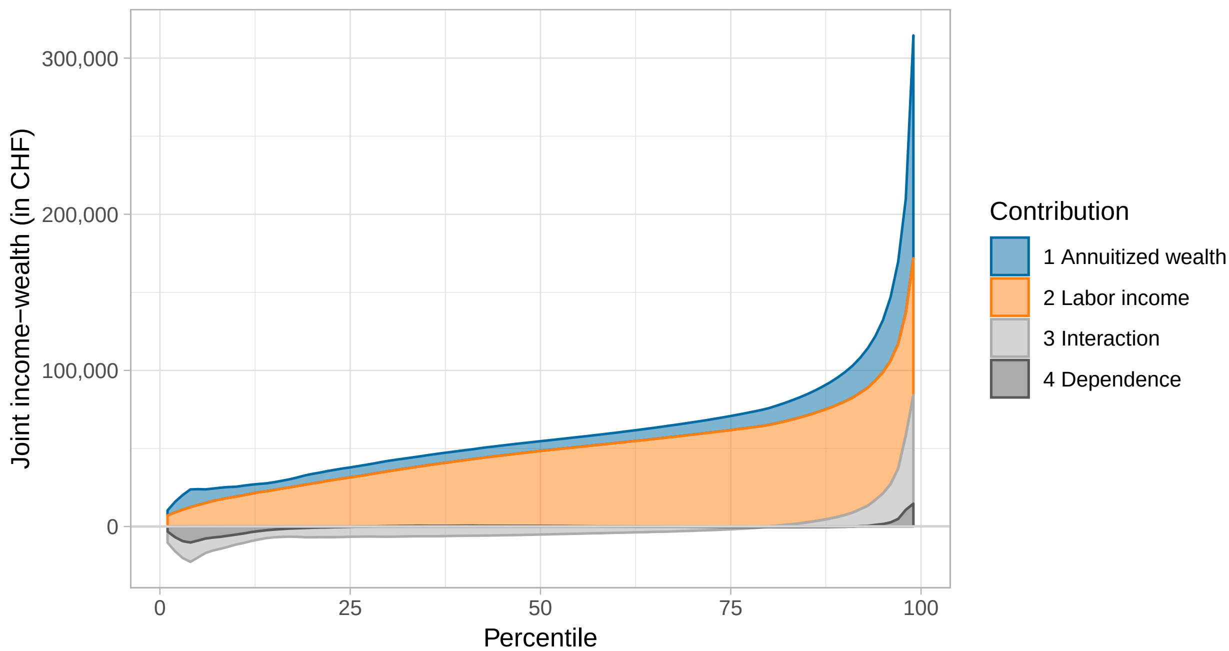

Figure 9: Decomposition of joint income-wealth by percentile into the contribution of the marginal distribution of labor income, the marginal distribution of annuitized wealth, the interaction between the marginals and the dependence between marginals, respectively.

annuitized wealth were independent. The dependence effect on top income shares is similarly modest. The interaction component is more important. Around 20% of joint income-wealth inequality measured either by the Gini coefficient or by top income shares can be linked with the mutually reinforcing inequality of the two income factors. Still, the interaction and dependence taken together explain less than the direct marginal effect of annuitized wealth. High labor income earners simultaneously having high annuitized wealth is less important for joint income-wealth inequality than inequality of annuitized wealth per se.

Figure 9 plots the contributions of the four components to average joint income-wealth by percentile. It shows that the interaction contributes particularly to inequality at the top of the distribution. Below the 80th percentile, the interaction effect is negative. The pattern for the interaction effect follows mechanically from the fact that joint income-wealth is the sum of two income factors both with right-skewed and heavy-tailed distributions. It also implies that there would be high labor income earners who have high annuitized wealth even if the two income sources were independent.

While the dependence effect substantially contributes to higher joint income-wealth at the top, it has a small negative effect on incomes in the first two deciles of the joint income-wealth distribution. Both phenomena contribute to increased joint income-wealth inequality and reflect the strong tail dependence discussed above. For most percentiles in between, the dependence component is negligible.

Figure 9 further confirms that labor income is highly important throughout the distribution, whereas annuitized wealth is a significant source of income only at the top. Note that, in the top percentile, the direct marginal effect of annuitized wealth is larger than the interaction and dependence effects combined. This implies that also high annuitized wealth of individuals with low or medium labor income is an important reason for the concentration of joint income-wealth at the top.

Table 3: Decomposition of Gini Coefficient of Joint Income-Wealth by Age Group

	all	20-34	35-49	50-64	65+
Gini	0.434	0.333	0.339	0.415	0.542
Contribution in %:					
Lab. income	40.3	79.5	60.3	43.0	9.3
Ann. wealth	33.0	-6.0	11.7	28.3	71.8
Interaction	20.6	15.1	19.0	21.2	14.1
Dependence	6.0	11.4	9.0	7.5	4.8

Table 3 distinguishes between the contributions of the four different factors by age group. It shows that the contribution of annuitized wealth to joint income-wealth inequality is strongest in the oldest age group, in which there is a high share of individuals with low labor income in the form of retirement benefits, but with high annuitized wealth. Labor income, on the other hand, is the most relevant factor for inequality in the age groups below 65. Annuitized wealth not only increases levels and inequality of consumption possibilities among the elderly but also among individuals aged 50-64. The contribution of the dependence between labor income and annuitized wealth is highest for inequality among individuals aged 20-34. But even in that group, the dependence pattern explains only 11% of inequality as measured by the Gini coefficient.

6 Conclusion

In this paper, we apply methods from several recent literatures to assess joint income-wealth inequality using tax data from the Swiss canton of Lucerne. To improve measures of consumption possibilities related to wealth, we introduce heterogeneous returns (Fagereng et al., 2019) into the estimation of annuitized wealth (Weisbrod and Hansen, 1968). The size and quality of our data allow us to use non-parametric techniques in the spirit of Aaberge et al. (2018) or Chetty et al. (2017) in our estimation of joint distributions. Finally, we use Rothe's (2015) method to decompose joint income-wealth into four contributing factors.

Inequality in the Swiss canton of Lucerne increases if we base our assessment on both income and wealth rather than just on income alone. The marginal distribution of wealth being more unequal than the income distribution is the most important factor explaining this increase in inequality. Positive dependence between income and wealth contributes substantially to inequality in the upper tail of the distribution. Income and wealth are, however, negatively associated for negative net wealth. We reach similar conclusions if we look at labor income and annuitized wealth instead of income and wealth. Annuitized wealth accounts for a large share of consumption possibilities among the elderly, who tend to have high wealth-to-income ratios. But even among the elderly, wealth is highly concentrated at the top of the distribution. The most affluent working-age individuals, however, have significantly higher consumption possibilities than an assessment of their income alone would suggest, too.

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A Technical detail on non-parametric analyses of joint distributions

An association matrix consists of empirical probabilities of the variables' joint percentiles. We estimate the empirical joint probability for every combination of percentiles $u, v \in Q = \{0.01, 0.02, \ldots, 1\}$

$$\hat{p}_{u,v} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}\{\hat{F}_{X_1}^{-1}(u - 0.01) \leq x_{1,i} < \hat{F}_{X_1}^{-1}(u)\} \mathbb{1}\{\hat{F}_{X_2}^{-1}(v - 0.01) \leq x_{2,i} < \hat{F}_{X_2}^{-1}(v)\}.$$

Given a mass point in X_2, we will uniformly distribute the conditional percentile probabilities of variable X_1 over all ranks corresponding to that mass point. We do this by changing the association matrix entries of percentile bins covering mass points to

$$\hat{p}^*_{u,w} = \hat{p}_{u,w} \frac{0.01}{\hat{p}_w},$$

where we treat a bin of rank w as mass point covering if $\hat{p}_w = \sum_{u \in Q} \hat{p}_{u,w} > 0.01$. The cells of the next percentile rank $w + 0.01$ cover the same mass point entirely or partly because these neighboring percentile bins have the same lower bound, i.e. $\hat{F}^{-1}(w) = \hat{F}^{-1}(w + 0.01)$. Thus, we aslo observe $\hat{p}_{w+0.01} > 0.01$ and adjust probabilities associated with $w + 0.01$ by

$$\hat{p}^*_{u,w+0.01} = \hat{p}_{u,w+0.01} - \hat{p}^*_{u,w}.$$

We proceed iteratively by correcting for the lowest mass point covering bins first before adjusting the subsequent ones using the corrected probabilities from the previous step. Mass points on variable X_1 are adjusted analogously. The adjustment guarantees that every column and row of the association matrix adds up to 0.01 and the marginal probabilities of the rank variables for which the mass points are not adjusted remain unchanged.

We illustrate the association matrices with heatmaps and will focus our analysis on the continuous parts of the copula. The partial copula between positive values of income and wealth as well as on the partial copula between positive income and negative wealth are of particular interest as they cover the largest part of the population and, hence, are most relevant for inequality.[28]

[28] We understand our copula model as a finite mixture of non-overlapping partial copulas. These are partial copulas spanning the non-zero variable combinations (i.e. positive income/positive wealth, positive income/negative wealth, negative income/positive wealth and negative income/negative wealth, respectively) and the partial copulas covering zero support of the two variables, respectively.

Our analysis of the marginal distributions will be non-parametric as well. We will present a set of descriptive statistics, which was suggested by Jenkins and Jäntti (2005) to study wealth distributions and will separately estimate densities for the positive and negative domain with local polynomial regressions.[29]

[29]We use the `locfit` package in R and fit second-order polynomials (see Ledolter, 2013, p.55-66). The bandwidth is set to the one minimizing the generalized cross-validation (GCV) statistic (i.e. the average squared prediction error). The mass point at zero is estimated by the sample probabilities of observations with 0.

B Additional information on income measures

Appendix B.A Measures based on disbursed taxable capital income

In the paper by Atkinson and Lakner (2017) that we rely upon in Section 3, they suggest alternative measures of labor and capital income to account for the dependence between capital and labor income induced by the split of income from self-employment. These follow the same methodology, but assign all income from self-employment to capital income and none to labor income. Atkinson and Lakner's methods lead to higher measures of housing income and lower measures of financial income if applied to our data than to data from other countries. Income from housing wealth includes imputed rents for people who live in homes they own themselves, a feature that is, to our knowledge, unique to Switzerland. Our data do not allow us to distinguish between imputed rents and actual income flows if the property is rented to others. These measures of capital income, hence, consist of more property income than would be the case with similar data from other countries because of imputed rents. Additionally, capital income includes neither accrued nor realized capital gains, which are not taxed in Switzerland.

Appendix B.B Measures based on annuitized wealth

Annuitized wealth Y_a, which we use as an alternative measure of capital income, overcomes the shortcomings of disbursed taxable capital income mentioned above and is thus better comparable to measures from other countries. Alternatively to different asset types, which we use based on Equation (2), we could also use the stock of net wealth. Fagereng et al. (2019) additionally provide returns that vary along the distribution of net wealth rather than financial assets and real estate as an alternative. Returns on net wealth are, however, potentially less comparable to returns in Switzerland because of different rates of home ownership, real estate valuation and indebtedness.

As we mention in Section 3, we take returns on financial assets r_f^p drirectly from Fagereng et al. (2019). These can be found in Panel A of Figure 2 of their paper. We approximate these patterns by a piecewise linear function, where we assume that returns are 0.77% in the first percentile of financial asset holding, decrease linearly from 0.4% to 0 between the second and 18th percentile, remain 0 for the 19th and 20th percentiles, then increase linearly to 0.15% for the 30th percentile, to 0.35% for the 40th percentile, to 0.6% for the 50th percentile, to 0.9% for the 60th percentile, to

1.06% for the 64th percentile, to 1.3% for the 70th percentile, to 1.8% for the 80th percentile, to 2.8% for the 90th percentile, to 4% for the 95th percentile, to 4.9% for the 97th percentile to 6.2% for the 99th percentile and, finally, to 6.4% for financial asset holdings in the 100th percentile.

We retrieved *Wüest Partner* data from `https://www.wuestpartner.com` to assess real estate income and SNB data from `https://data.snb.ch` to assess interest rates on debt. Returns on real estate vary between -1.12% in 2007 and 10.31% in 2012, with a geometric mean of the years of 3.90%. Fagereng et al. (2019) use a return that is constant over time at 2.88%. Since we do not observe debt composition, we use interest on amounts due from domestic customers, which is available only from 2007 onwards and declined from 4.01% to 2.17% over that period. For the remaining years 2005 and 2006, we use the mean ratio of interest on debt and debt stock observed in our data, which are 3.27% and 3.22%. The interest rates over all eleven years result in a geometric mean of 2.78%.

We retrieved the mortality tables from `https://www.bfs.admin.ch/` to measure remaining life expectancy. The mean age gap for married couples in Switzerland is 2.4 years and has not changed much over time. A possible extension in future work would be to use income- and wealth-dependent life expectancy rates. As argued by Saez and Zucman (2019), accounting for income-dependent life expectancy rates leads to large variation in estimates of tax revenue from a wealth tax for the United States.

We will provide results based on two variants of annuitized wealth. The first will use Equation (2) but ignore the denominators of the terms in brackets and simply multiply asset stocks with returns. This measure is of interest because it corresponds to an 'inverse-capitalization' method. By imputing capital income from stocks of household assets and subtracting interest payments, it takes the opposite path of Piketty and Zucman (2014) and Saez and Zucman (2016), albeit relaxing the assumption of a constant capitalization factor. This approach is similar to Larrimore et al. (2016), who impute accrued capital gains in order to correct for the incomplete nature of realized capital gains. The second variant will use Equation (2) but multiply W_h with 1.5 based on Brülhart et al. (2018) to account for undervaluation of housing wealth.

C Descriptives

Table C.1: Summary Statistics Net Wealth and Income

	Net Wealth	Income	Difference
% obs. with y < 0	11.6	0.2	-11.4
% obs. with y = 0	8.6	3.1	-5.5
% obs. with y > 0	79.8	96.7	16.9
Mean (in CHF)	207,805	51,470	-156,335
10th perc. (in CHF)	-3,542	13,668	17,211
50th perc. (in CHF)	32,956	45,677	12,721
90th perc. (in CHF)	383,475	86,220	-297,256
99th perc. (in CHF)	2,357,216	202,873	-2,154,342
Gini	0.883	0.372	-0.510
Adj. Gini*	0.863	0.372	-0.491
Top 1% share	0.408	0.074	-0.334
Top 10% share	0.749	0.273	-0.476

*Gini coefficient adjusted for negative values and normalized to $[0, 1]$-range (see Chen et al., 1982).

Table C.2: Summary Statistics Labor Income, Capital Income and Annuitized Wealth

	Labor Income	Capital Income	Annuitized Wealth
% obs. with y < 0	0.1	13.2	10.7
% obs. with y = 0	4.0	23.3	10.7
% obs. with y > 0	95.9	63.5	78.6
Mean (in CHF)	46,516	4,954	16,078
10th perc. (in CHF)	12,784	-288	-38
50th perc. (in CHF)	42,852	119	1,148
90th perc. (in CHF)	80,099	10,684	26,178
99th perc. (in CHF)	157,201	74,876	212,340
Gini	0.356	1.032	0.907
Adj. Gini*	0.356	0.924	0.896
Top 1% share	0.053	0.412	0.460
Top 10% share	0.247	0.843	0.818

*Gini coefficient adjusted for negative values and normalized to $[0, 1]$-range (see Chen et al., 1982).

Table C.3: Summary Statistics of Joint Income-Wealth and Income

	Joint Income-Wealth	Income	Difference
% obs. with y < 0	0.2	0.2	0.0
% obs. with y = 0	2.7	3.1	0.4
% obs. with y > 0	97.1	96.7	-0.4
Mean (in CHF)	62,594	51,470	-11,124
10th perc. (in CHF)	13,951	13,668	-283
50th perc. (in CHF)	49,550	45,677	-3,873
90th perc. (in CHF)	98,672	86,220	-12,452
99th perc. (in CHF)	314,571	202,873	-111,698
Gini	0.434	0.372	-0.061
Adj. Gini*	0.434	0.372	-0.061
Top 1% share	0.138	0.074	-0.064
Top 10% share	0.345	0.273	-0.072

*Gini coefficient adjusted for negative values and normalized to $[0,1]$-range (see Chen et al., 1982).

Table C.4: Summary Statistics of Wealth and Joint Income-Wealth with and without 150% Adjustment of Real Estate Wealth for Undervaluation

	Net Wealth	Adj. Net Wealth	Joint Income-Wealth	Adj. Joint Income-Wealth
% obs. with y < 0	11.6	5.7	0.2	0.2
% obs. with y = 0	8.6	8.6	2.7	2.7
% obs. with y > 0	79.8	85.7	97.1	97.1
Mean (in CHF)	207,805	277,740	62,594	67,302
10th perc. (in CHF)	-3,542	0	13,951	14,272
50th perc. (in CHF)	32,956	57,038	49,550	51,749
90th perc. (in CHF)	383,475	524,405	98,672	106,973
99th perc. (in CHF)	2,357,216	3,000,000	314,571	359,209
Gini	0.883	0.816	0.434	0.448
Adj. Gini*	0.863	0.813	0.434	0.448
Top 1% share	0.408	0.354	0.138	0.145
Top 10% share	0.749	0.690	0.345	0.359

*Gini coefficient adjusted for negative values and normalized to $[0, 1]$-range (see Chen et al., 1982).

D Pen's parades

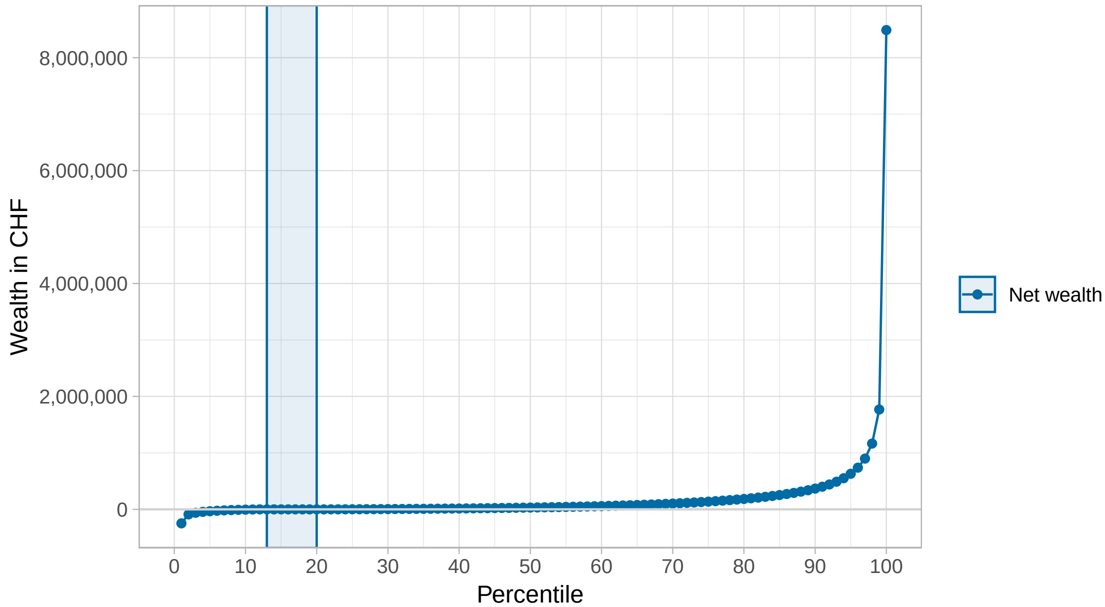

Figure D.1: Average net wealth by percentile. Percentiles with zero net wealth indicated by shaded area.

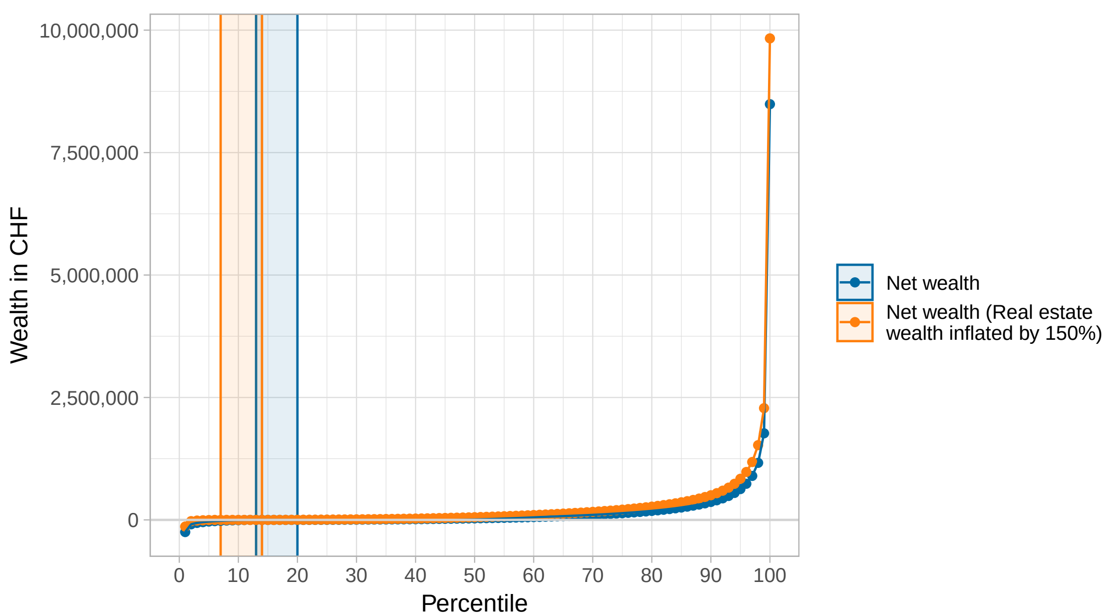

Figure D.2: Average net wealth and average net wealth with real estate wealth inflated by 150% to adjust for undervaluation, respectively, by percentile. Percentiles with zero net wealth indicated by shaded area.

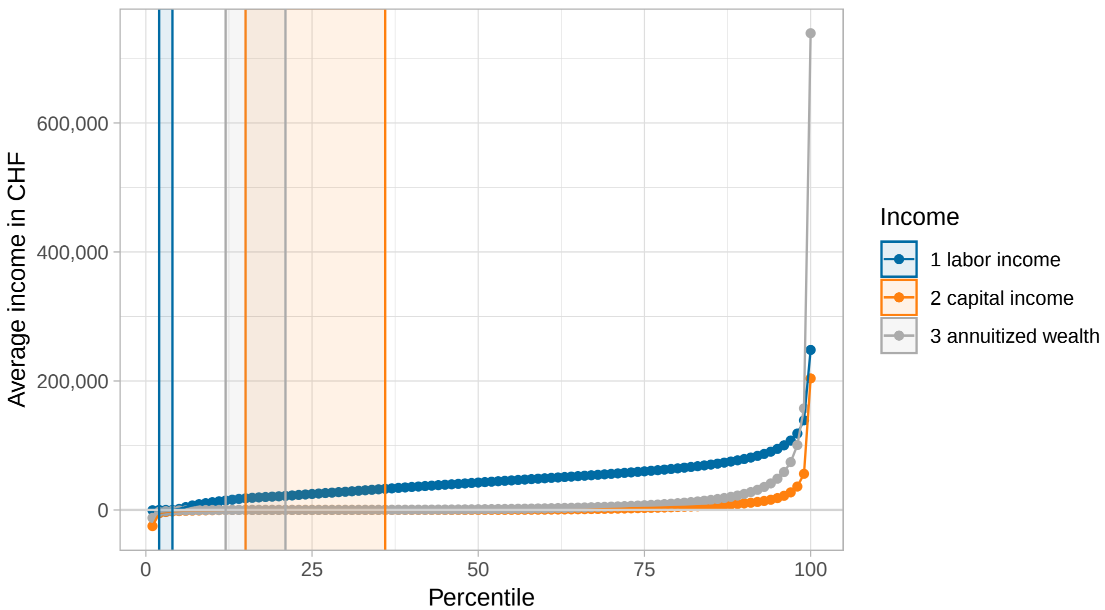

Figure D.3: Average labor income, average capital income (tax definition) and average annuitized wealth by percentile of respective distribution. Percentiles with zero income indicated by shaded areas.

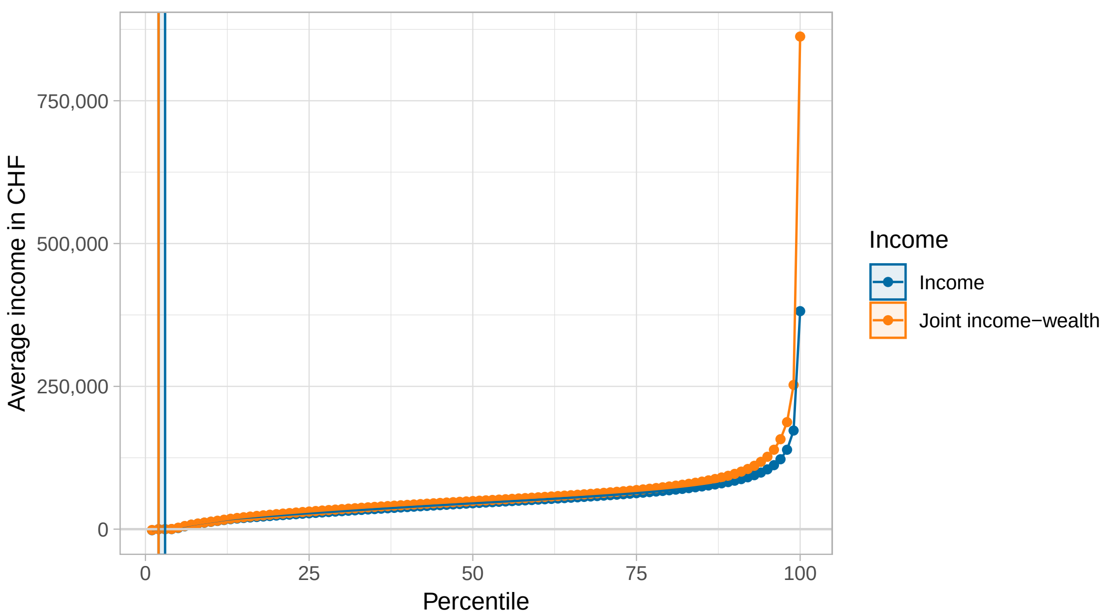

Figure D.4: Average income (tax definition) and average joint income-wealth by percentile of respective distribution. Percentiles with zero income indicated by shaded areas.

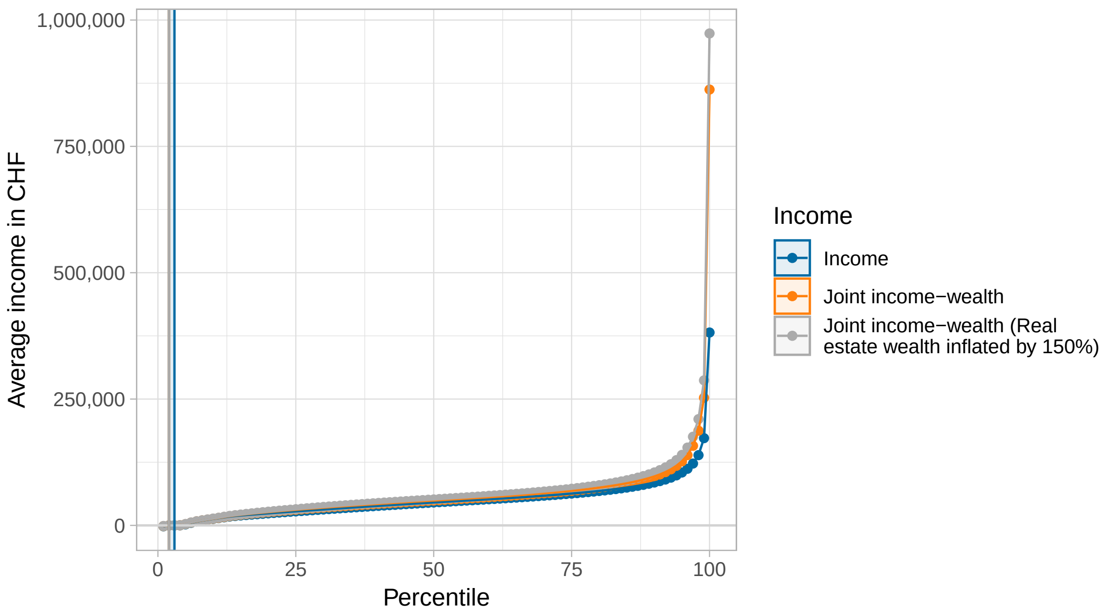

Figure D.5: Average income (tax definition), average joint income-wealth and average joint income-wealth with real estate wealth adjusted by 150% for undervaluation, respectively, by percentile. Percentiles with zero income indicated by shaded areas.

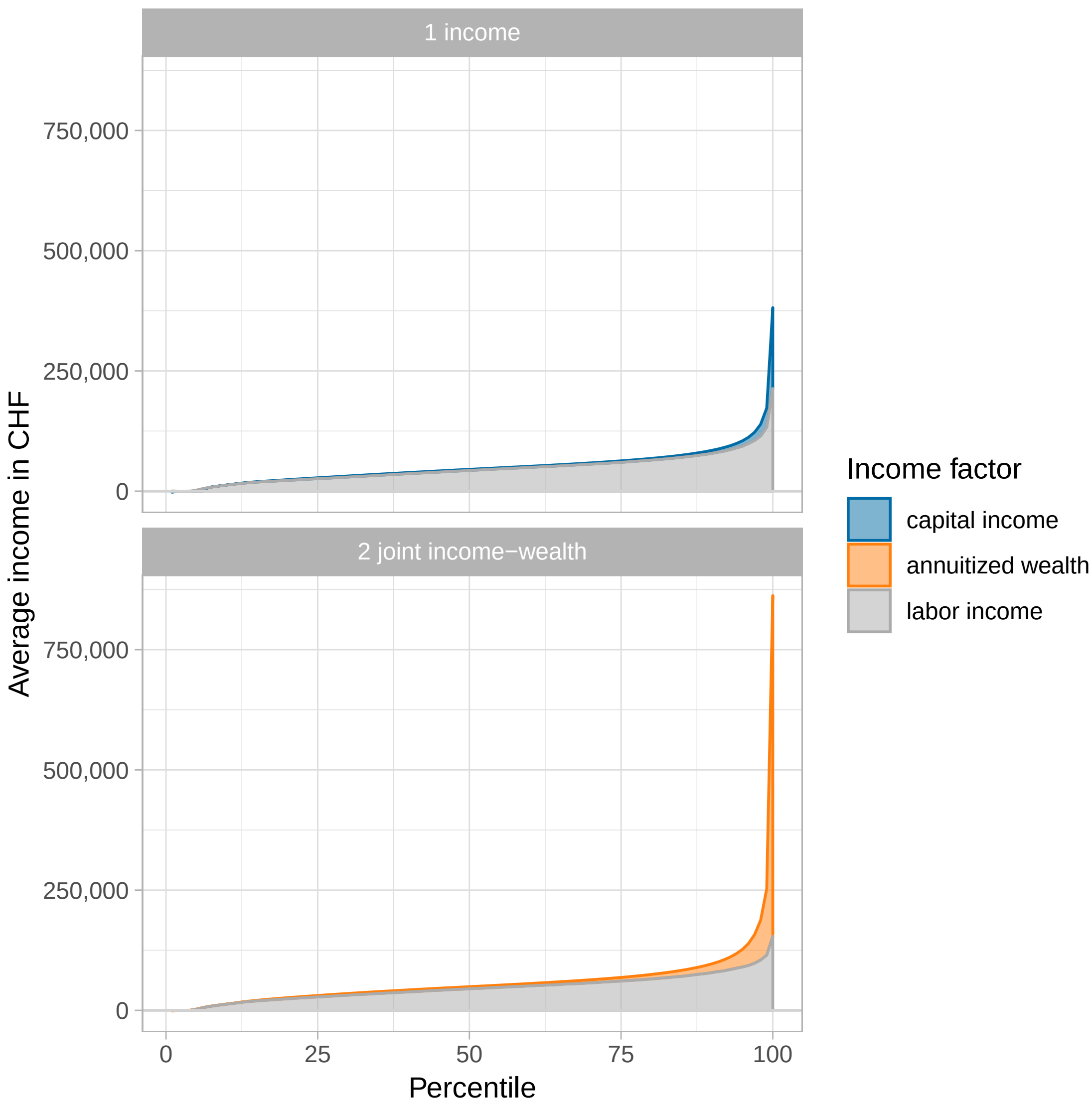

Figure D.6: Average factor income by percentile of income distribution and joint income-wealth distribution.

E Patterns by age group

Table E.1: Summary Statistics of Wealth, Income and Joint Income-Wealth by Age Group

	All	20-34	35-49	50-64	65+
Net Wealth					
% obs. with y < 0	11.6	10.1	20.0	13.3	2.0
% obs. with y = 0	8.6	12.6	6.7	6.2	5.7
Mean (in CHF)	207,805	38,511	135,446	273,543	448,021
Adj. Gini*	0.863	0.841	0.886	0.839	0.769
Top 1% share	0.408	0.331	0.421	0.376	0.351
Income					
% obs. with y < 0	0.2	0.1	0.3	0.3	0.1
% obs. with y = 0	3.1	3.6	2.6	2.4	3.0
Mean (in CHF)	51,470	43,271	61,426	61,835	41,930
Adj. Gini*	0.372	0.327	0.314	0.370	0.370
Top 1% share	0.074	0.035	0.061	0.082	0.104
Joint Income-Wealth					
% obs. with y < 0	0.2	0.2	0.2	0.3	0.1
% obs. with y = 0	2.7	3.0	2.4	2.1	2.7
Mean (in CHF)	62,594	44,038	64,626	70,907	78,842
Adj. Gini*	0.434	0.333	0.339	0.414	0.542
Top 1% share	0.138	0.041	0.083	0.124	0.222

*Gini coefficient adjusted for negative values and normalized to $[0, 1]$-range (see Chen et al., 1982).

Table E.2: Summary Statistics of Labor Income, Capital Income and Annuitized Wealth by Age Group

	All	20-34	35-49	50-64	65+
Labor Income					
% obs. with y < 0	0.1	0.0	0.1	0.2	0.0
% obs. with y = 0	4.0	5.0	3.1	3.2	3.2
Mean (in CHF)	46,516	42,680	57,339	54,253	33,192
Adj. Gini*	0.356	0.325	0.303	0.348	0.301
Top 1% share	0.053	0.032	0.049	0.057	0.058
Capital Income					
% obs. with y < 0	10.0	12.7	14.7	9.8	2.2
% obs. with y = 0	12.6	19.4	10.5	9.3	7.2
Mean (in CHF)	9,297	1,044	6,503	12,947	19,465
Adj. Gini*	0.902	0.962	0.890	0.868	0.847
Top 1% share	0.477	0.620	0.473	0.428	0.405
Annuitized Wealth					
% obs. with y < 0	10.7	11.3	16.8	12.0	2.1
% obs. with y = 0	10.7	15.7	9.2	8.3	6.4
Mean (in CHF)	16,078	1,358	7,287	16,654	45,650
Adj. Gini*	0.896	0.916	0.891	0.859	0.802
Top 1% share	0.460	0.503	0.456	0.405	0.364

*Gini coefficient adjusted for negative values and normalized to $[0, 1]$-range (see Chen et al., 1982).

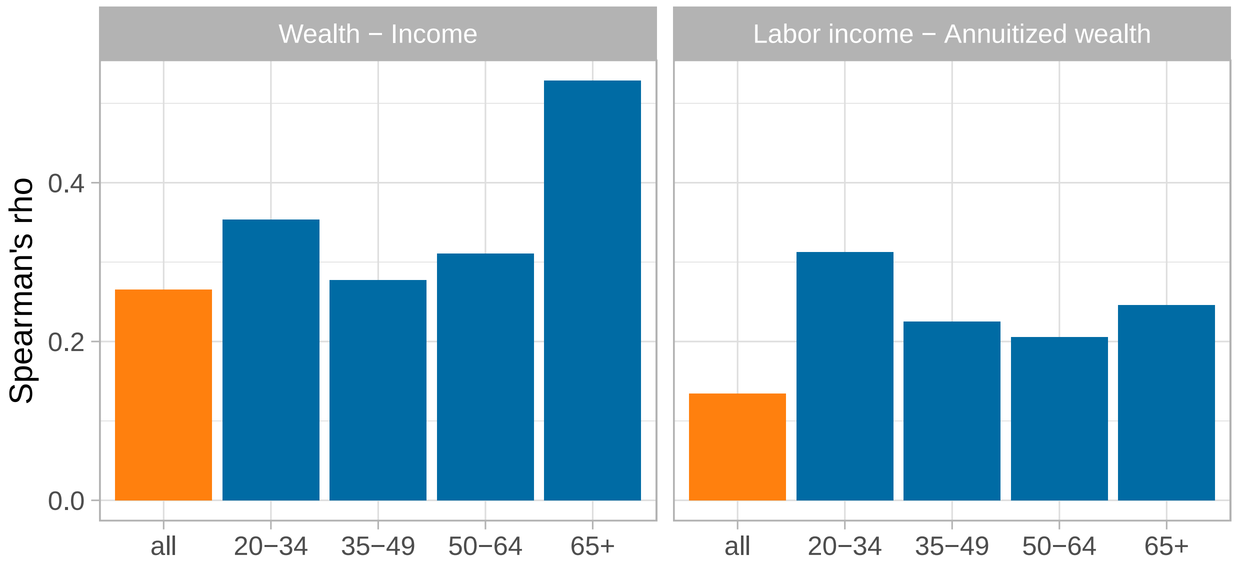

Figure E.1: Rank correlations by age group, entire distribution.

Figure E.2: Rank correlations by age group, for positive (annuitized) wealth ranks and negative (annuitized) wealth ranks separately.

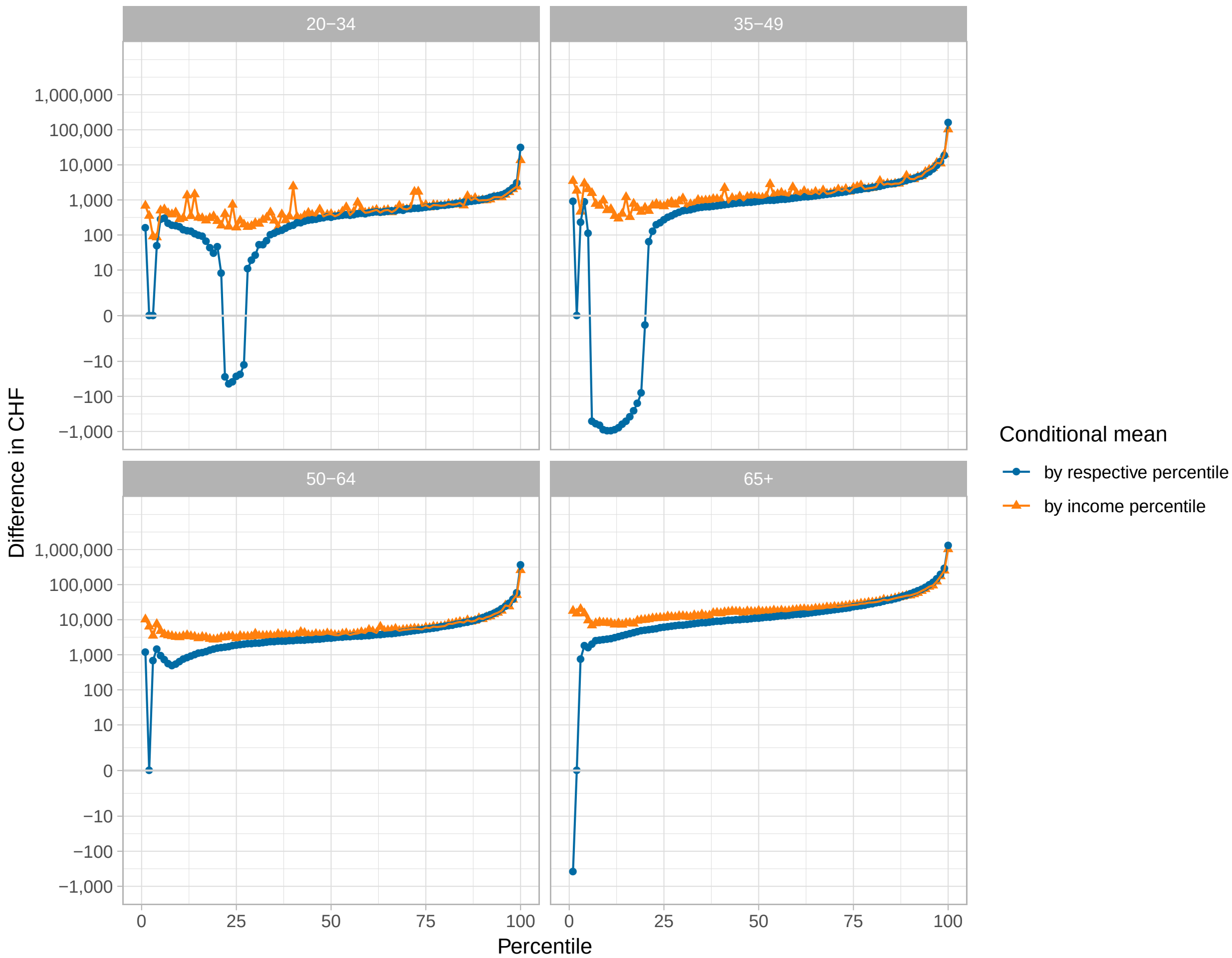

Figure E.3: Differences between average joint income-wealth and income by percentile of respective distribution and differences between averages between average joint income-wealth and income by percentile of the income distribution, respectively. Y-Axis is scaled by an inverse hyperbolic sine transformation.

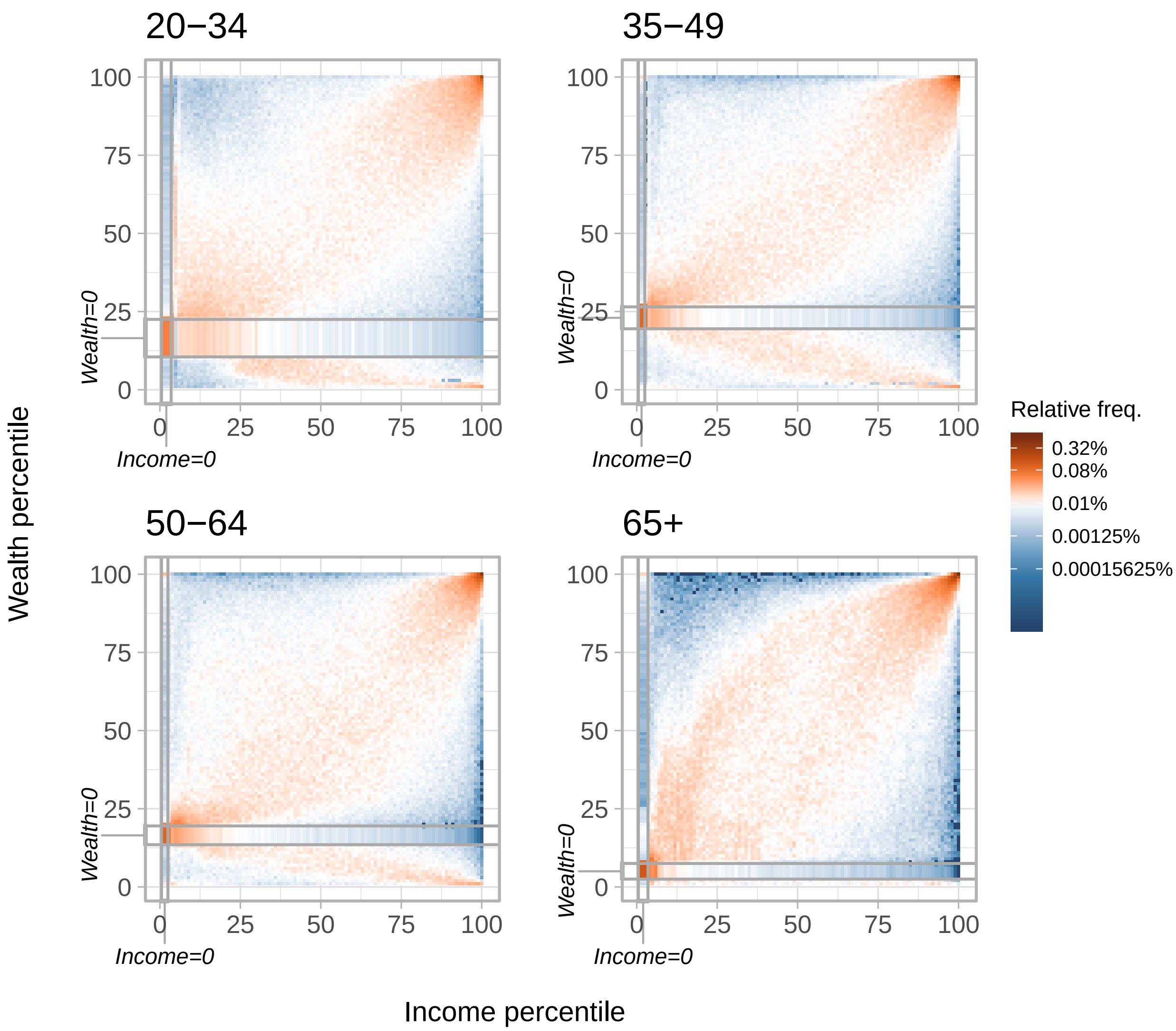

Figure E.4: Association matrices between income and wealth by age group. Every cell represents the relative frequency of a percentile rank combination. Note that percentiles refer to conditional percentiles of respective age groups. Bold lines separate percentile range with zero income and wealth, respectively. Uniform distribution of ranks assumed over mass points at zero.

F Copulas with alternative labor and capital income definitions

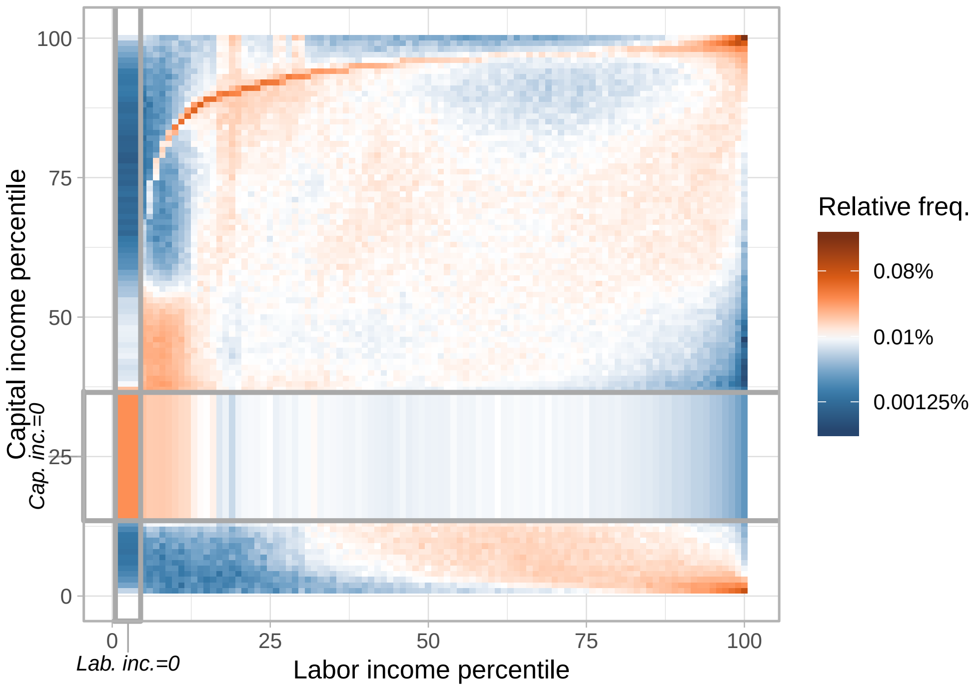

Figure F.1: Alternative capital income definition I: Association matrix between labor and capital income percentile ranks. Two thirds of self-employment income assigned to labor income and the remaining third to capital income. Every cell represents the relative frequency of a percentile rank combination. Bold lines separate percentile range with zero labor income and capital income, respectively. Uniform distribution of ranks assumed over mass points at zero.

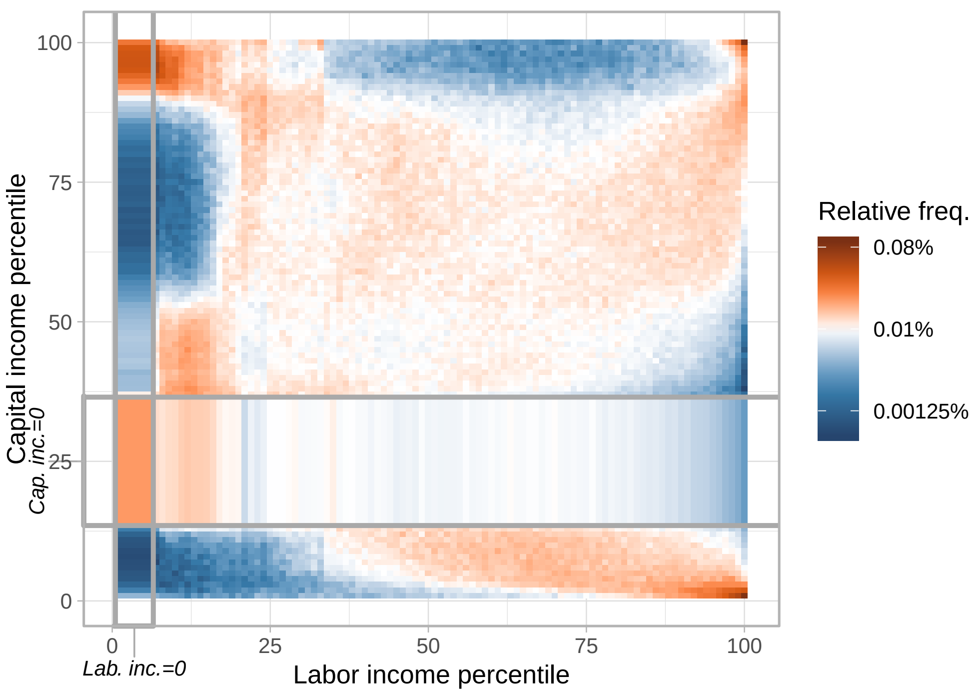

Figure F.2: Alternative capital income definition II: Association matrix between labor and capital income percentile ranks. Self-employment income entirely assigned to capital income. Every cell represents the relative frequency of a percentile rank combination. Bold lines separate percentile range with zero labor income and capital income, respectively. Uniform distribution of ranks assumed over mass points at zero.

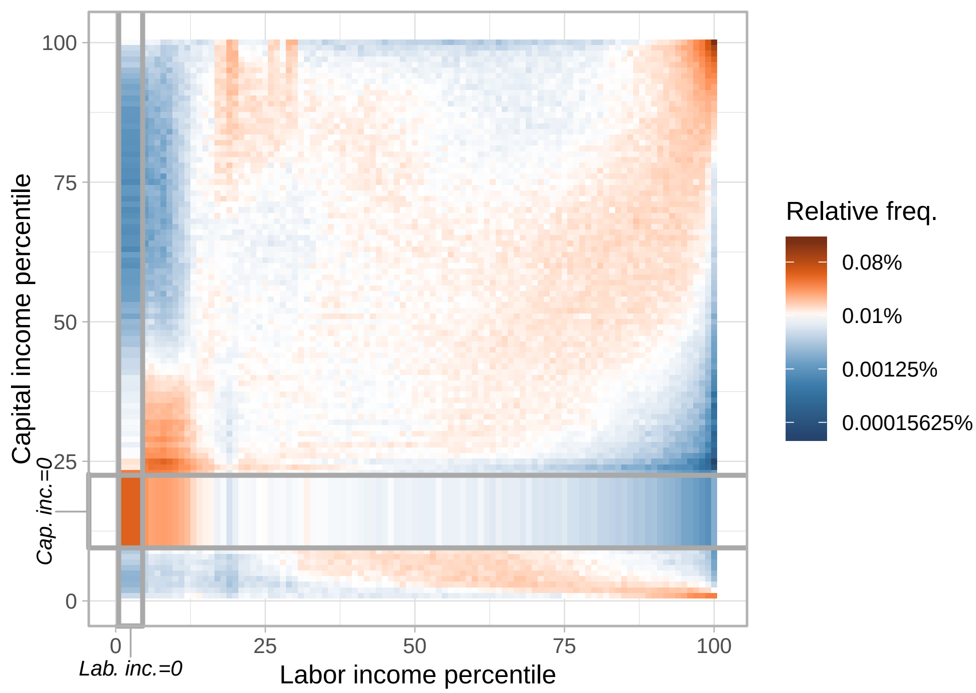

Figure F.3: Alternative capital income definition III: Association matrix between labor and capital income percentile ranks. Capital income derived from wealth stocks. Every cell represents the relative frequency of a percentile rank combination. Bold lines separate percentile range with zero labor income and capital income, respectively. Uniform distribution of ranks assumed over mass points at zero.

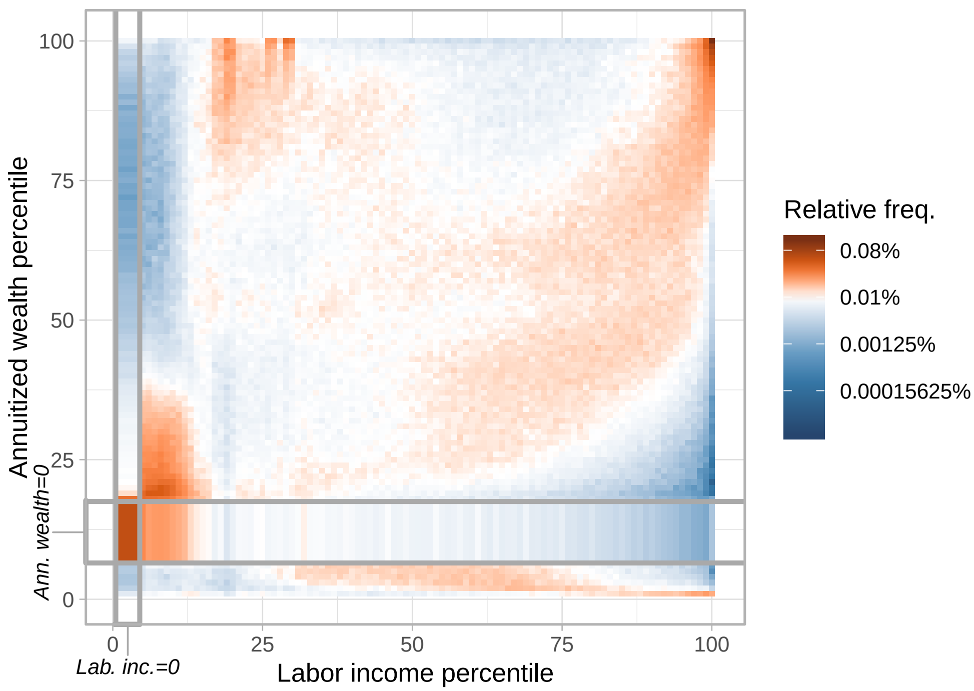

Figure F.4: Alternative annuitized wealth definition: Association matrix between labor and annuitized wealth percentile ranks. Annuitized wealth based on wealth with 150% adjustment of real estate wealth for undervaluation. Every cell represents the relative frequency of a percentile rank combination. Bold lines separate percentile range with zero labor income and annuitized wealth, respectively. Uniform distribution of ranks assumed over mass points at zero.

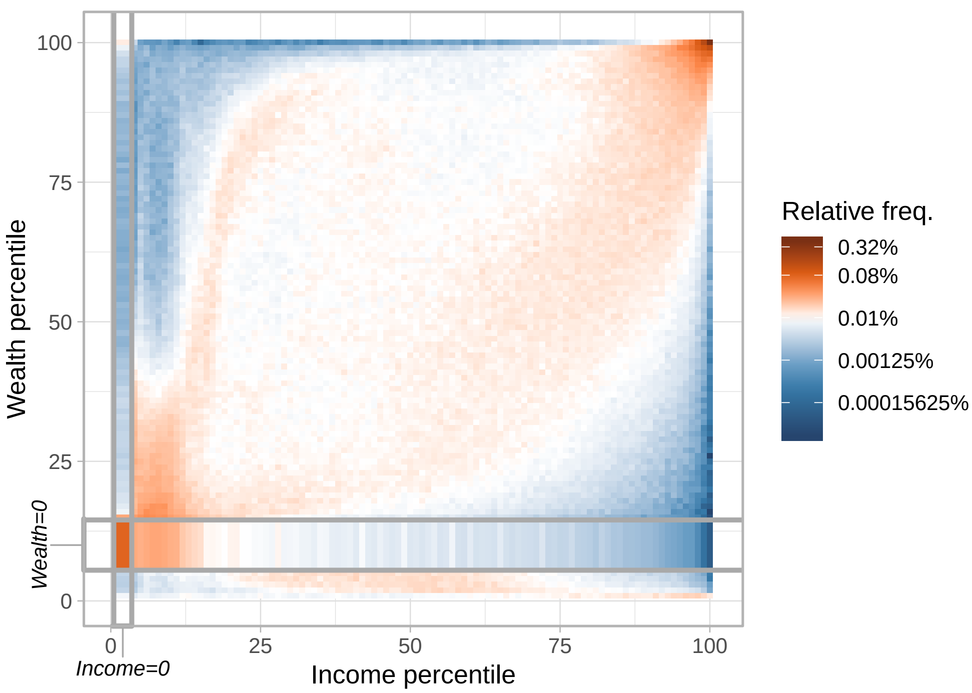

Figure F.5: Alternative net wealth definition: Association matrix between total income and net wealth percentile ranks. Net wealth computed with 150% adjustment of real estate wealth for undervaluation. Every cell represents the relative frequency of a percentile rank combination. Bold lines separate percentile range with zero labor income and capital income, respectively. Uniform distribution of ranks assumed over mass points at zero.

G Decomposition of joint income-wealth

To what extent is the distribution of joint income-wealth driven by its underlying components and by the dependence structure between these components? We apply the method proposed by Rothe (2015) to quantify the contributions of the marginal distributions of underlying characteristics and of the copula between those characteristics to the distribution of an outcome variable. In our case, we are interested in outcome joint income wealth Y_j and its underlying factors labor income Y_l and annuitized wealth Y_a, which is why, in terms of notation, we will now replace X_1 and X_2 from Section 2.2 with Y_l and Y_a. Rothe builds on Sklar's theorem to express any distributional statistic $\nu(F_{Y_j}(y_j))$, such as the mean or quantiles, of outcome Y_j, as a function $\phi(\cdot)$ of the marginal distributions and the copula.[30] In our case, we write

$$\nu(F_{Y_j}(y_j)) = \phi(F_{Y_l,Y_a}(y_l, y_a)) = \phi(C(F_{Y_l}(y_l), F_{Y_a}(y_a))),$$

where the outcome variable joint income-wealth Y_j is the sum of the covariates labor income Y_l and annuitized wealth Y_a.

As our comparison scenario, we choose the hypothetical distribution of joint income-wealth $F^E_{Y_j}(y_j)$ where all individuals receive the mean joint income-wealth or, to put it differently, where all labor income and annuitized wealth are equally distributed. Rothe (2015) suggests to decompose the difference between the distributional statistics of observed and the comparison group by constructing counterfactual distributions that differ either in the marginal distributions, in the copula or the structural function $\phi(\cdot)$ that links the covariates to the outcome variable's statistic.

With two income factors and the same structure function, we can decompose the overall difference Δ^ν_O into four terms

$$\begin{aligned}\Delta^\nu_O &= \nu(F_{Y_j}(y_j)) - \nu(F^E_{Y_j}(y_j)) \\ &= \Delta^\nu_D + \Delta^\nu_{M_l} + \Delta^\nu_{M_a} + \Delta^\nu_I,\end{aligned}$$

which capture the four components mentioned above.[31] Note that the structural function is the same for the actual distribution and the equality-and-independence-scenario as both link two income

[30] Actually, Rothe expresses the outcome distribution as conditional distribution integrated over the joint distribution of covariates: $F_{Y_j}(y_j) = \int F_{Y_j|Y_l,Y_l}(y_j|y_l, y_a)dC(F_{Y_l}(y_l), F_{Y_a}(y_a))$.

[31] With different structure functions linking covariates' distribution to the outcome variable, we would identify a "structure effect" as a fifth component.

factors to the distributional statistics of their sum.

The first term is the "dependence effect," which can be derived by combining the independence copula with the observed marginals

$$\Delta_D^\nu = \nu(F_{Y_j}(y_j)) - \phi(C^E(F_{Y_l}(y_l), F_{Y_a}(y_a))).$$

This first term captures the distributional effect of the actual dependence structure in contrast to the distribution with independent marginals.

The "direct marginal distribution effects" $\Delta_{M_l}^\nu$ and $\Delta_{M_a}^\nu$ are defined as the difference that would be observed if the actual marginal distributions $F_{Y_l}(Y_l)$ or $F_{Y_a}(Y_a)$, were replaced, respectively, with the marginals of the equality-and-independence-scenario

$$\begin{aligned} \Delta_{M_l}^\nu &= \phi(C^E(F_{Y_l}(y_l), F_{Y_a}(y_a))) - \phi(C^E(F_{Y_l}^E(y_l), F_{Y_a}(y_a))) \\ \text{and} \quad \Delta_{M_a}^\nu &= \phi(C^E(F_{Y_l}(y_l), F_{Y_a}(y_a))) - \phi(C^E(F_{Y_l}(y_l), F_{Y_a}^E(y_a))), \end{aligned}$$

where, as defined above, $F_{Y_l}^E(y_l)$ and $F_{Y_a}^E(y_a)$ correspond to the marginals if every individual were to receive the same factor income.

The "pure interaction effect" Δ_I^ν, finally, measures the remaining differences caused by the interaction between the marginal distributions that are not related to the dependence structure.

$$\Delta_I^\nu = \phi(C^E(F_{Y_l}^E(y_l), F_{Y_a}^E(y_a))) - \Delta_{M_l}^\nu - \Delta_{M_a}^\nu - \nu(F_{Y_j}^E(y_j)),$$

where $\nu(F_{Y_j}^E(y_j))$ captures the distributional statistics of the joint income-wealth in the equality-and-independence-scenario.

To illustrate the intuition behind the four decomposition terms, assume two countries A and B with country A having distributions of labor income and capital income, which both Lorenz-dominate the corresponding factor income distributions in country B. Additionally, let there be a positive dependence such that top labor income earners in country A are more likely among top capital income ranks than top labor income earners in country B. When comparing total income inequality between A and B, the more unequal factor distributions in A would be attributed to the "direct marginal distribution effects". The "interaction effect" measures the additional contribution from both labor income and capital income being more unequally distributed, such that the total income gap between any two arbitrary factor income rank combinations is wider in country A.[32] Finally, the "dependence effect" accounts for the concentration of top labor income earners among top capital income receivers.

As suggested by Rothe (2015, p. 329), we simulate the counterfactual distributions by first estimating the empirical marginal distribution functions of labor and capital income. The copula counterfactual is estimated by combining the empirical marginal distributions with the independence copula. As mean income we just use the sample averages. The distributional statistics are then estimated with the simulated distributions.

[32]E.g. the total income of an individual at the 10th labor income percentile and the 20th capital income percentile in contrast to the total income of an individual at the 90th labor income percentile and the 85th capital income percentile.

H Decomposition with traditional capital income

Table H.1: Copula decomposition of income

	Total	Lab. income	Cap. income	Interaction	Dependence
Mean	51,470	46,516	4,954	0	0
Gini	0.372	0.244	0.022	0.077	0.029
Top 1% share	0.074	0.023	0.006	0.024	0.011
Top 10% share	0.273	0.166	-0.057	0.048	0.016

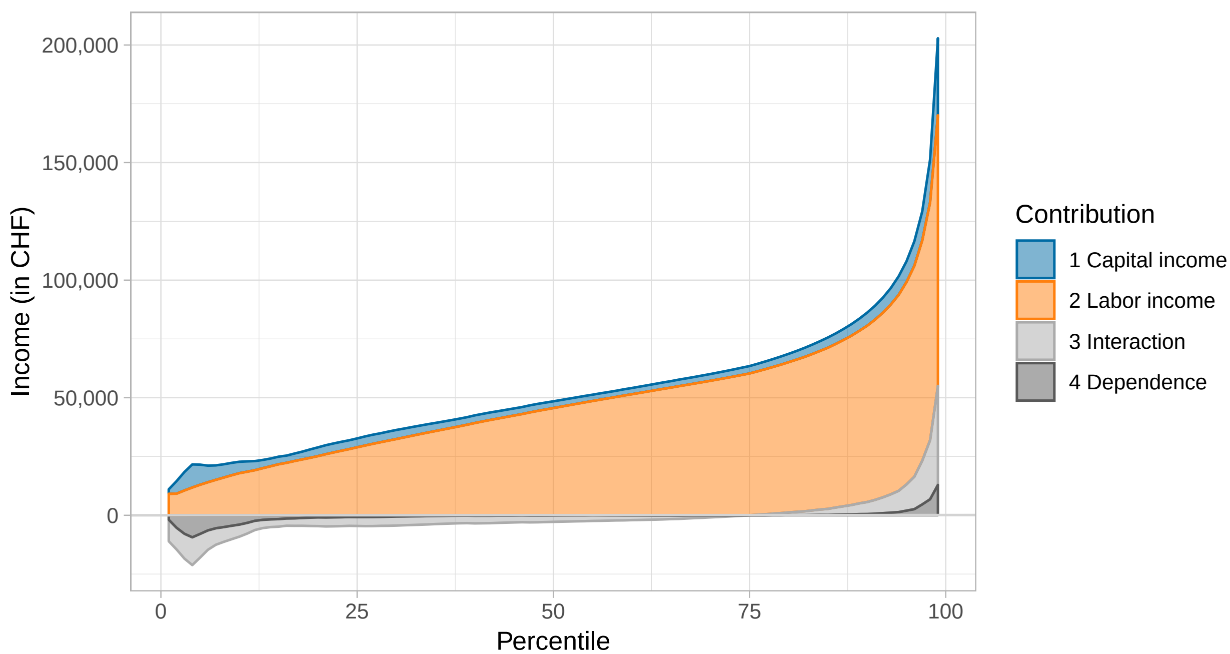

Figure H.1: Decomposition income into contribution of the marginals of labor income, capital income and their dependence by quantile.