How Do Trade and Communication Costs Shape the Spatial Organization of Firms?

Toshitaka Gokan, Sergey Kichko, Jacques-François Thisse

Impressum:

CESifo Working Papers

ISSN 2364-1428 (electronic version)

Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH

The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute

Poschingerstr. 5, 81679 Munich, Germany

Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de

Editor: Clemens Fuest

www.cesifo-group.org/wp

An electronic version of the paper may be downloaded

- from the SSRN website: www.SSRN.com
- from the RePEc website: www.RePEc.org
- from the CESifo website: www.CESifo-group.org/wp

How Do Trade and Communication Costs Shape the Spatial Organization of Firms

Abstract

We consider an economic geography setting in which firms are free to choose one of the following organizational types: (i) integrated firms, which perform all their activities at the same location, (ii) horizontal firms, which operate several plants producing the same good at different locations, and (iii) vertical firms, which perform distinct activities at separated locations. We show that there exists a unique organizational equilibrium, which typically involves the coexistence of various organizational forms. We also give necessary and sufficient conditions for the three types of firms to coexist within the same region and show that transportation and communication costs have opposite effects on firms' organizational choices. This suggests that, depending on its nature, the supply of a new transportation infrastructure may lead to contrasted locational patterns.

JEL-Codes: F120, F210, R120.

Keywords: region, transportation costs, communication costs, horizontal firm, vertical firm.

Toshitaka Gokan
Institute of Developing Economies – JETRO
Japan - Chiba 261-8545
Toshitaka_Gokan@ide.go.jp

Sergey Kichko
HSE University of Russia
Russia - 190068 Saint Petersburg
skichko@hse.ru

Jacques-François Thisse
UC Louvain / ESPO
Belgium - 1348 Louvain-la-Neuve
jacques.thisse@uclouvain.be

October 1, 2019

We are very grateful to W. Strange and two referees for their insightful comments that have led to significant improvements of our paper. We also thank T. Akamatsu, K. Behrens, H. Egger, M. Fujita, T. Furusawa, M. Hanazono, M. Larch, T. Okubo, H. Pellegrina, F. Robert-Nicoud, A. Tarasov, D.-Z. Zeng, seminar audience at Bayreuth, Nagoya, Saint-Louis, Tohoku, and Tokyo Universities, and participants to the 8th European and 13th American meetings of the UEA for remarks and suggestions. This research was undertaken within the framework of the HSE University Basic Research Program and funded by the Russian Academic Excellence Project .5-100.

1 Introduction

The purpose of this paper is to contribute to a better understanding of the geography of production by showing how the interplay between communication and transportation/trade costs affects the way firms organize their activities across space. Building an economic geography setting in which both costs are taken into account, we investigate why identical firms operating in the same technological and market environment choose different spatial organizational forms and how these choices affect regional prices, outputs, and welfare. While most research in economic geography is based on external drivers, our approach stresses the role of communication costs as an internal driver.

Even since the Industrial Revolution, transportation costs have plummeted. However, firms operate multiple plants to supply spatially separated markets, thus suggesting that distance remains a major impediment to trade in several sectors (Hillberry and Hummels, 2008). What is more, firms are packages of different functions, such as management, R&D, finance, marketing, and production. Due to the development of new information and communication technologies (ICT), firms are able to disperse these functions into geographically separated units in order to benefit from the attributes specific to different locations (Aarland et al., 2007).

Yet, Fink et al. (2005) find that communication costs have a significant impact on trade patterns, especially for differentiated goods. For multi-plant US firms Giroud (2013) shows that the opening of new airline links that reduce the travel time between headquarters and plants generated an increase of 7% in plant productivity. Charnoz et al. (2018) use the development of the high-speed railway network in France to show how the decrease in passenger travel time between headquarters and affiliates allowed a higher concentration of management functions in the headquarters. In the same vein, Kalnins and Lafontaine (2013) observe that a greater distance to the headquarters is associated with shorter establishment longevity. Why is this so? The transmission of knowledge via the new ICT remains incomplete and imperfect (Leamer and Storper, 2001). In addition, face-to-face contacts are still needed between specialized workers operating in spatially separated plants and headquarters because such contacts allow for immediate feedback in non-routine activities (Battiston et al., 2017). The list could go on much further. Thus, despite the ICT revolution, we may safely conclude that the communication curse is still with us.

A firm conducting all its activities under the same roof has what we call a spatially *integrated* structure. When firms are not spatially integrated, we distinguish between the following two types of spatial organization. The firm adopts a *horizontal* structure when several plants produce the same good at different locations. The cost of being horizontal is the loss in scale economies, while the benefit is direct access to each market with zero transportation costs. By contrast, the firm selects a *vertical* structure when it organizes and performs discrete activities at distinct locations, which altogether form a supply chain. The vertical fragmentation of the firm aims to take advantage of differences in market size, but this involves communication costs between the headquarters and plants, as well as transportation costs from the distant region to the domestic one.

The coexistence of the three types of firms is observed in many real world situations. For instance, the Japanese Ministry of Economy, Trade and Industry provides a unique dataset, Kogyo Statistics, that describes the spatial organization of manufacturing firms. Integrated firms account for about 71% of the manufacturing sector and vertical firms with only one plant for about 12%; the remaining 17% are operated by multi-plant firms, which are horizontal (Okubo and Tomiura 2016). The dataset accounts for firms with more than four full-time employees, which probably explains the high share of integrated firms. In Denmark, the average distance between plants and

their headquarters has more than double during the last three decades, thus showing the growing importance of vertical firms, while the share of multi-plant firms has increased (Acosta and Lyngemark, 2019). There is also a vast literature in urban economics that documents the fact that city centers attract management or administrative activities, while operation plants are located in edge cities, hinterland cities of the same country or abroad, where land and/or labor are cheap (Henderson, 1997; Henderson and Ono, 2008; Rossi-Hansberg et al., 2009).

As pointed out by Helpman (2006), the diversity of firms' spatial organizations goes beyond these canonical forms. However, we limit ourselves to the above three forms because considering several types of organizational forms involves a large number of cases to investigate. For example, focusing only on the three canonical types of organization in a two-region setting yields the discouraging number of 49 candidates patterns. This should not come as a surprise since, when studying the geographical separation of front-offices and back-offices in the standard monocentric city model, Ota and Fujita (1993) find that 11 configurations may emerge as an equilibrium according to the values of the main parameters of their model.

Despite a rich literature on multinational enterprises, economic geography has neglected to address the simultaneous occurrence of integrated, vertical, and horizontal firms. This is where we hope to contribute by developing a standard setting in which firms have a headquarters whose location is given and one or two plants whose locations are endogenous. More specifically, we assume that firms are free to choose the number and locations of their plants in the presence of transportation, communication, and fixed production costs. We then use this setting to study how the interregional distribution of activities varies with transportation and communication costs and how various regional magnitudes are affected. Furthermore, since our aim is to study the impact of communication and transportation costs on firms' organizational choices, we control for the main factors that explain differences in firms' organizational forms in the literature on multinational enterprises, i.e., wage inequality and technological differences across space (Antras and Yeaple, 2014). More specifically, we isolate market size as the only *ex ante* difference between the two regions, a variable whose empirical relevance is well known (Head and Mayer, 2004; Redding, 2011).

In a closed economy, when identical monopolistically competitive firms have the option of investing in new technologies to produce at a lower marginal cost, they face a trade-off between the cost of adopting the new technology and market size. Importantly, they all make the same choice and remain *ex post* identical (Elberfeld, 2003). In an open economy, whether firms choose to be integrated or horizontal depends on fixed production costs and the relative size of markets, but the level of transportation costs also matters. Taking communication costs on board allows firms to choose to be vertical or not. These costs interact in a non-trivial way with market size to determine the share of firms that choose a specific organizational form.

Our main findings are as follows. For any given values of transportation, communication, fixed production costs, and the relative market sizes, we first show that *there exists a unique equilibrium in which firms choose their prices, sizes, and types of organization*. However, that the equilibrium is unique does not mean that firms are organized according to the same configuration. Instead, the equilibrium configuration chosen by firms typically changes with the parameter values. Apart from the extreme cases where all firms are horizontal (integrated) because fixed costs are very low (high) relative to market size, *firms choose different organizational forms*. Indeed, (i) not all firms located in the smaller region choose to be integrated because their domestic market is too competitive; (ii) not all firms choose to be vertical because this renders the big market very competitive; and (iii) not all firms choose to be

horizontal because both markets become very competitive.

In this paper, we have chosen to focus primarily on the case where the three organizational forms come together which we call a mixed equilibrium. For this to happen, market sizes must differ, but not by too much. Furthermore, we show that only the smaller region hosts the three types of organizational forms; the larger region's firms remain integrated. In other words, there is one-way offshoring. Some of the smaller region's firms invest in the other region to have a better access to the larger market. At the same time, other firms remain integrated and focus on the smaller market because the establishment of distant plants makes competition in the larger market tougher while the displacement of plants toward the larger region makes competition in the smaller region softer. However, it worth noting that the same holds for most of the other equilibria where the smaller region hosts two or one type of firm. In sum, the larger region's firms are integrated while it pays for the smaller region's firms to be different.

The foregoing results imply that the home market effect holds true when the mass of plants is endogenous: *the bigger (smaller) region hosts a more (less) than proportionate share of plants*. This result is reassuring because it shows that our setting is consistent with the economic geography literature (Baldwin et al., 2003). However, we want to stress the following difference: here the mass of plants is endogenous whereas the home market effect is usually obtained when the mass of (integrated) firms is exogenously given. Therefore, that the home market effect still holds when the mass of plants is endogenous is not a straightforward implication of existing results.

The coexistence of the three organizational forms is socially optimal under conditions similar to those that sustain the market equilibrium. In other words, the coexistence of various organizational forms is not evidence of a market failure. Nevertheless, since a firm's production cost depends on its organizational choice, the cost distribution is now endogenous, which implies that the numbers of firms adopting a specific structure in the equilibrium and optimal outcomes need not be the same. To be precise, we show that too few firms are horizontal while too many firms are vertical. All in all, too few firms invest in the larger region in order to soften competition therein.

Having done this, we study how transportation and communication costs affect the pattern of organizational types at the mixed equilibrium. First, when shipping goods becomes cheaper, the number of plants operating in each region decreases because firms change their organizational form in response to a drop in transportation costs. Our analysis confirms the classical result in the theory of multinational enterprises, that is, *fewer firms go multiregional when shipping goods is cheaper* (Markusen, 2002). More specifically, lowering transportation costs leads to a hike in the number of integrated firms. However, although the number of horizontal firms is reduced, the number of vertical firms rises.

Second, in accordance with Krugman (1995) and others who argued that low communication costs allow the slicing up of the supply chain, we find that *falling communication costs lead to a higher number of multiregional firms*. Even though the total number of plants increases, the smaller region hosts fewer plants. Simply put, lower transportation costs or communication costs have opposite impacts on the spatial patterns of production: in the former more firms are integrated, while in the latter more firms are fragmented. In other words, more efficient transportation fosters the interregional dispersion of production, whereas more efficient communication technologies promote the spatial concentration of production in the larger region. Our results concur with Baldwin (2016) who argued that drops in transportation and communication costs are at the origin of two different phases of globalization.

Third, profits are higher when transportation costs decrease. This runs against the widespread idea that spatial

separation endows firms with market power (Ottaviano and Thisse, 2004). Thus, our analysis shows that, in response to lower transportation costs, *firms are able to restore their profits by rearranging the spatial organization of their activities*. Everything being equal, consumers are, therefore, worse-off because regional markets are less competitive. Even when profits are redistributed to consumers, the impact of a deeper market integration is ambiguous as it varies with the level of transportation, communication, and fixed production costs. On the other hand, lower transportation costs foster a higher GDP in each region, but exacerbate regional disparities, whereas lower communication costs reduce GDP in each region, but promotes regional convergence.

This difference in results has an interesting implication from the policy viewpoint. Indeed, improving accessibility through the provision of new transportation infrastructure is one of the main policy instruments used by governments and international agencies to foster regional development (Redding and Turner, 2015). Policy instruments typically involve the building of highways, high-speed railways, or airports. Our analysis shows that these instruments may have different impacts on the space-economy. Indeed, new highways makes shipping of commodities cheaper. By contrast, new high-speed railways or airports reduce the travel costs of business people and professionals, thus reducing communication costs by making face-to-face contacts easier. As a result, *depending on its nature, a new transportation infrastructure might generate different impacts on the geography of production and the intensity of regional disparities*. When the aim of transport policy is to affect the spatial distribution of activities, there is a need to distinguish between different types of infrastructure, an issue that has been overlooked in the literature.

Last, when firms are exogenously heterogeneous in productivity, it is not clear that firms may want to be differentiated in spatial organizational forms because they are already differentiated in costs. Therefore, we find it natural to investigate what our main findings become when firms are heterogeneous à la Melitz. As in the foregoing, we show that the smaller region hosts the three types of firms under conditions that are similar to those obtained when firms are homogeneous. The most efficient firms always choose to become horizontal because these firms are able to bear the higher fixed costs associated with the operation of two plants. On the other hand, the organizational form selected by the medium efficient firms depends on the relative size of the two markets. When the asymmetry is strong, the medium efficient firms go vertical because their domestic market is too small. Otherwise, they go integrated because their domestic market offers a sufficiently big outlet.

Admittedly, our results are obtained using a bare-bone framework. Accounting for a richer set of effects might turn down some of our conclusions. Nevertheless, by recognizing that firms may choose their organizational form, our paper shows that *the assumption of integrated firms and the emphasis on the transportation costs are not innocuous assumptions* in that they might deliver misleading policy recommendations.

Related literature. Our model is closely linked to one of the workhorses of economic geography, that is, the footloose capital model where firms run a single plant and are spatially integrated (Baldwin et al., 2003). By contrast, we allow firms to choose their organizational forms, that is, the headquarters and plants may or may not collocate, while firms may operate one or several plants. Therefore, our setting can be viewed as the “footloose plant model”. Behrens and Picard (2007) used an economic geography setting to compare integrated and horizontal firms. These authors find that, as transportation costs decrease, multiplant firms close their plants in the small region and serve this market through trade from the large region. Fujita and Thisse (2006) highlighted the role of communication

costs in firms' decisions to go vertical in a setting where all firms are established in the core region. They showed that firms located in the core region are integrated, integrated and vertical, and vertical only when communication costs steadily decrease. Fujita and Gokan (2005) extended this setting to the case where firms may be integrated, horizontal or vertical. Depending on the levels of transportation and communication costs, the equilibrium outcome involves any one firm type or any two firm types. Robert-Nicoud (2008) used a similar model to show that the spatial fragmentation of the supply chain may be beneficial to consumers when communication costs are low.

We differ from these three contributions in at least two major respects. First, in our setting, firms are located in both regions, so that the intensity of competition varies with the organizational choices made by firms located in each region. Second, we investigate a set of issues that these authors do not consider. Yeaple (2003) is closer to us in that he studies the simultaneous emergence of the three organizational forms. To do this, Yeaple considered a 3-country setting and shows that the same firm may choose to go horizontal in one country and vertical in the other.

The paper is organized as follows. The model is described in Section 2. Section 3 deals with the equilibrium and welfare analyses when firms have the same productivity. The impact of market size is discussed in Section 4, while the various effects triggered by lower transportation and communication costs are studied in Section 5. Section 6 discusses what our main findings become when firms differ in productivity. Section 7 concludes.

2 The model and preliminary results

2.1 The economy

The economy features two regions - $i = 1, 2$ -, two sectors - one sector produces a differentiated good and the other a homogeneous good -, and one production factor - labor. The differentiated good is produced under increasing returns by a mass F of Melitz-like heterogeneous firms using labor. Each variety is provided by a single firm and each firm supplies a single variety. The homogeneous good is produced under constant returns and perfect competition using labor. The distribution of firms' total factor productivity in the differentiated good sector is the same in both regions, while the productivity of the homogeneous good sector is the same in the two regions. The homogeneous good is costlessly traded. Therefore, its price, hence the wage w_i paid in region i , is the same in both regions. The unit of the homogeneous good is chosen for the regional wage to be equal to 1 and this good is chosen as the numéraire. There is a mass L of consumers. In the spirit of economic geography, we assume that workers own the firms located in the same region as them.

Each consumer is endowed with one unit of labor, which is supplied inelastically. Consumers are spatially immobile; region i hosts $s_i L$ of consumers with $s_1 > s_2 > 0$ and $s_1 + s_2 = 1$. We denote by $S \equiv s_2/s_1 \in (0, 1)$ the relative size of the two regions. Like in the footloose capital model, we rule out comparative advantage à la Heckscher-Ohlin by assuming that the mass of firms in region i is $s_i F$. Although the mass of firms is exogenous, the mass of plants is endogenous and varies from F to $2F$.

2.2 Consumers

Consumers share the same quasi-linear, logarithmic preferences:

$$U = \alpha \ln \left[\left(\int_0^F x_k^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}} \right] + z, \quad (1)$$

where x_k is the consumption of variety $k \in [0, 1]$, $\sigma > 1$ is the elasticity of substitution between any two varieties, $\alpha \in (0, 1)$ is the salience coefficient of the differentiated good in preferences, and z is the consumption of the homogeneous good, while the budget constraint is given by $\mathbf{X}P + z = 1$, where $\mathbf{X}$ is the consumption bundle in the parentheses of (1) and P is the price index of differentiated good defined below.

Applying the first-order condition yields $\mathbf{X}P = \alpha$, which implies that each consumer spends α units of the numéraire on the differentiated good. In other words, a consumer's budget constraint on the differentiated good boils down to

$$\int_0^F x_k p_k dk = \alpha, \quad (2)$$

where p_k is the consumer price of variety k . By implication, an increase in income generates only an increase in the consumption of the homogeneous good.

The individual demand for variety k is given by

$$x_k = \frac{\alpha p_k^{-\sigma}}{\Delta}, \quad (3)$$

where the market aggregate

$$\Delta \equiv \int_0^F p_k^{-(\sigma-1)} dk = P^{-(\sigma-1)} \quad (4)$$

is a monotone decreasing transformation of the CES-price index

$$P = \left[\int_0^F p_k^{-(\sigma-1)} dk \right]^{-1/(\sigma-1)}.$$

2.3 Producers

A firm involves a *headquarters* (HQ) and one or two production *plants*. By convention, we refer to a firm's location as the location of its HQ. A HQ provides the specialized pre- and post-fabrication services for the good to be processed and delivered to customers. For simplicity, we assume that a HQ needs a fixed number of labor units. Each firm chooses to have a single production facility in one of the two regions or a production site in each region where the same variety is produced. To operate a plant, a θ -firm needs a fixed requirement of $f > 0$ and a marginal requirement of c/θ units of labor, where θ is drawn from a given distribution $G(\theta)$ that is the same in both regions. When firms are homogeneous, the marginal requirement of labor is the same across firms and equal to c .

The "distance" between regions is measured in two different ways. First, in line with the literature, when a firm ships one unit of its variety it incurs an iceberg transportation cost $\tau > 1$; it is costless to ship the variety to its local customers ($\tau = 1$). Second, a firm's HQ provides various specialized inputs to its plant(s), while local managers require regularly pieces of information from their HQs related to specific tasks, unexpected issues, and more. This implies the existence of communication costs that link the two units. Since distance affects productivity in a negative way, it is natural to assume that the plant's marginal cost is higher when the HQ and plant are located in distant

regions. Alternatively, we may interpret communication costs as a “reduced form” for the various management and informational costs generated by spatial separation, such as those studied in the literature on the organization of multi-level enterprises (Helpman, 2006; Antràs and Rossi-Hansberg, 2009).

There is no general agreement about how to model communication costs. According to some authors, the cost of communication is almost entirely a fixed cost because, once a communication device is installed, the marginal cost of sending messages is very low (Antràs and Yeaple, 2014). Others model communication costs as an iceberg cost $\gamma > 1$, with $\gamma = 1$ when plants and HQs are collocated (Duranton and Puga, 2005; Fujita and Thisse, 2006). This can be justified on the following grounds. First, using an iceberg cost implies that communication costs are proportional to the plant output. This is in line with the literature on firms’ organization where managers spend time solving sophisticated tasks arising in distant plants while their working time is proportional to firms’ output (Garicano, 2000; Gumpert, 2018). Second, an iceberg cost may account for both communication costs that are unrelated to distance, as in the case of talks via communication devices, and costs that vary with distance, as in the case of travel costs of engineers and business people. Third, since less efficient firms are likely to experience higher communication costs, linking communication costs to marginal costs leads to an inverse relationship between the former and firm’s productivity when the plant is located away from its HQ. For example, a lower quality of internal resources makes firms more vulnerable when HQs and plants are spatially separated. Last, modeling both frictions in the same way makes it easier to compare their respective impact on firms’ organizational forms, while yielding a wide range of equilibrium outcomes. In what follows, we choose this modeling strategy.

The choice of a specific organizational form determines a firm’s production cost function. In what follows, we describe the cost functions associated with the three types of firms. We denote by q_{ij} the total consumption in region $j = 1, 2$ of a variety produced by a firm headquartered in region $i = 1, 2$.

(i) A θ -firm is said to be *integrated* (**I**) when it operates a single plant which is located together with its HQ; the plant supplies both markets. Hence, the cost function of a **I**-firm with productivity θ located in region $i = 1, 2$ is given by

$$C_i^I(\theta) = f + \frac{c}{\theta} \cdot (q_{ii} + \tau q_{ij}) \quad \text{with } j \neq i. \quad (5)$$

The total output, or size, of this firm is thus equal to $q_i^I \equiv q_{ii} + \tau q_{ij}$.

(ii) A θ -firm is *vertical* (**V**) when it has a single plant, which operates abroad; the plant supplies both regions. A **V**-firm faces an additional cost associated with the operation of a plant set up away from its HQ. As discussed in the introduction, distance implies higher coordination and communication costs between the HQ and its plant. Therefore, the cost function of a **V**-firm located in region i is given by

$$C_i^V(\theta) = f + \frac{c}{\theta} \cdot (\tau \gamma q_{ii} + \gamma q_{ij}) \quad \text{with } j \neq i. \quad (6)$$

This firm’s total output is given by $q_i^V \equiv \tau \gamma q_{ii} + \gamma q_{ij}$.

(iii) Finally, a θ -firm is *horizontal* (**H**) when it has a plant in each region. When a firm splits its production between the two regions, it incurs an additional fixed cost f ; the marginal costs of the domestic and distant plants are, respectively, c/θ and $\gamma c/\theta$.¹ Since both plants supply the same variety, the activity of a **H**-firm entails no

¹To keep things simple, we assume the same level of fixed costs in the two countries. However, our results remain qualitatively the same when these two costs differ but not too much.

interregional trade. The cost function of a **H**-firm located in region i is then given by the following expression:

$$C_i^H(\theta) = 2f + \frac{c}{\theta} \cdot (q_{ii} + \gamma q_{ij}) \quad \text{with } j \neq i, \quad (7)$$

while its total output is equal to $q_i^H \equiv q_{ii} + \gamma q_{ij}$.

The expressions (5)–(7) show that transportation and communication costs affect firms' production costs in different ways according to their organizational form. In particular, the presence of communication costs implies that the location of HQs matters for the definition of firms' cost functions. Though firms are heterogeneous in cost efficiency, firms' heterogeneity is endogenous because firms choose their marginal costs through their organizational choices. Note also that the communication cost γ in (6) cannot be interpreted as a wage wedge between the two regions. Indeed, this interpretation would mean that producing in i is more expensive than in j . However, as C_i^V and C_j^V have the same functional form, this would imply that producing in i would be cheaper than in j , a contradiction.

Note also the following difference between our setting and standard models of economic geography: here the mass of firms (plants) is exogenous (endogenous) and profits are positive; in the latter firms are spatially integrated while the mass of firms is determined by the quantity of labor or capital (Baldwin et al., 2003). Assuming that the mass of firms of *each* type is determined by the zero-profit conditions appears to be a tricky issue. Indeed, we show in Section 3.1 that this condition never holds but for a zero-measure subset of the plane (ϕ, ω) . More generally, we will see that our results are independent of the total mass F of firms.

2.4 Market equilibrium

Since all region- i firms sharing the same productivity θ and the same organizational form $\ell = I, V, H$ choose the same equilibrium consumer price $p_{ii}^\ell(\theta)$ in region i ($p_{ij}^\ell(\theta)$ in region j), (3) implies that the profit function of a θ -firm is given by the following expression:

$$\pi_i^\ell(\theta) = s_i L \cdot \frac{(p_{ii}^\ell(\theta))^{1-\sigma}}{\Delta_i} + s_j L \cdot \frac{(p_{ij}^\ell(\theta))^{1-\sigma}}{\Delta_j} - C_i^\ell(\theta) \quad \text{with } \ell = I, V, H, \quad i, j = 1, 2 \text{ and } j \neq i.$$

The timing of the game is as follows. First, firms choose their organizational forms and, then, their prices and quantities sold in each region.

For notational simplicity, we choose the unit of output for $c = (\sigma - 1)/\sigma < 1$ to hold. Using (3), profit-maximization yields the equilibrium consumer price of a variety produced in region $i = 1, 2$ by a **I**-firm and sold in regions i and j :

$$p_{ii}^I(\theta) = \frac{1}{\theta} \quad p_{ij}^I(\theta) = \frac{\tau}{\theta} > p_{ii}^I \quad \text{with } j \neq i. \quad (8)$$

The consumer prices charged by a **V**-firm located in region i are as follows:

$$p_{ii}^V(\theta) = \frac{\gamma^\tau}{\theta} > p_{ii}^I(\theta) \quad p_{ij}^V(\theta) = \frac{\gamma}{\theta} < p_{ii}^V(\theta) \quad \text{with } j \neq i, \quad (9)$$

while a **H**-firm in i sets prices given by

$$p_{ii}^H(\theta) = \frac{1}{\theta} \quad p_{ij}^H(\theta) = \frac{\gamma}{\theta} > p_{ii}^H(\theta) \quad \text{with } j \neq i. \quad (10)$$

Denote by n_i (v_i or h_i) the mass of integrated (vertical or horizontal) firms in region i , so that

$$n_i + v_i + h_i = s_i F. \quad (11)$$

Using (8)–(10), the market aggregate Δ_i is given by the following expression:

$$\Delta_i = n_i + n_j \phi + v_i \phi \omega + v_j \omega + h_i + h_j \omega,$$

where $0 < \phi \equiv \tau^{-(\sigma-1)} < 1$ and $0 < \omega \equiv \gamma^{-(\sigma-1)} < 1$ whose values measure, respectively, the freeness of trade and the freeness of communication. It follows from (11) that Δ_i can be interpreted as the *effective* mass of plants competing in region i , that is, the mass of plants discounted by the corresponding friction factors ϕ and ω . Indeed, everything works as if the mass of plants located in region i were equal to Δ_i . The price index in *one* region depends on the spatial structure chosen by firms located in *both* regions. As Δ_i rises through lower transportation or communication costs, the price index P_i decreases because the effective mass of plants increases. In other words, *when the organizational structure of firms is given, lowering communication and/or transportation costs renders both regional markets more competitive.*

Using (11), we can rewrite Δ_i as follows:

$$\Delta_i = s_i F + \omega s_j F - (\omega - \phi) n_j - (1 - \phi \omega) v_i, \quad i = 1, 2. \quad (12)$$

Measuring the intensity of competition in a market by the inverse of the corresponding price index, we may conclude as follows. If all region- i firms are integrated ($n_i = s_i$), competition becomes tougher in i and softer in region j because all region- i firms produce home, which protects region- j firms. If all firms are vertical ($v_i = s_i$), competition becomes tougher in region j , and softer in region i because all varieties are imported from j . Last, if all region- i firms are horizontal ($h_i = s_i$), competition gets tougher in both regions because each one hosts a larger mass of plants. In short, the organizational structure of firms affects the intensity of competition in both regions.

Using (3) and (8)–(10), the profits made by a **I**-firm, a **V**-firm and a **H**-firm are, respectively, given by the following expressions:

$$\pi_i^I(\theta) = \alpha L \left[\frac{\theta^{\sigma-1}}{\sigma} \left(\frac{s_i}{\Delta_i} + \phi \frac{s_j}{\Delta_j} \right) - \frac{f}{\alpha L} \right], \quad (13)$$

$$\pi_i^V(\theta) = \alpha L \left[\frac{\theta^{\sigma-1}}{\sigma} \left(\phi \omega \frac{s_i}{\Delta_i} + \omega \frac{s_j}{\Delta_j} \right) - \frac{f}{\alpha L} \right], \quad (14)$$

$$\pi_i^H(\theta) = \alpha L \left[\frac{\theta^{\sigma-1}}{\sigma} \left(\frac{s_i}{\Delta_i} + \omega \frac{s_j}{\Delta_j} \right) - \frac{2f}{\alpha L} \right]. \quad (15)$$

Therefore, without loss of generality, we can normalize αL to 1. In other words, a higher total expenditure on the differentiated good is formerly equivalent to a decrease in the fixed labor requirement.

An *organizational equilibrium* is such that consumers maximize utility, each firm maximizes its profits, markets clear, and profits are non-negative in both regions. Since firms are free to choose the organizational form across space, the equilibrium profits in region $i = 1, 2$ are such that

$$\pi_i^*(\theta) = \max\{\pi_i^I(\theta), \pi_i^V(\theta), \pi_i^H(\theta)\} \geq 0.$$

The following remarks are in order. First, **I**-firms' profits decrease with communication costs because the price indices P_i and P_j fall, while **H**-firms' profits fall for the same reason when transportation costs decrease. Profits

of **V**-firms change with ϕ and ω in more complex ways. Note already the importance of communication costs for the difference between integrated and multiregional firms. If communication costs are prohibitive ($\omega = 0$), all firms are integrated. On the other hand, when communication costs are negligible ($\omega = 1$), the model has a continuum of organizational equilibria (see Appendix A). This is reminiscent of Krugman (1980) where there is a continuum of locational equilibria when $\phi = 1$. In what follows, we eliminate those extreme cases by assuming that $0 < \omega < 1$. Last, if transportation costs are negligible ($\phi = 1$), there are no **H**-firms. Therefore, we assume $\phi < 1$.

The meager literature that aims to compare transportation and communication costs is not conclusive. At best, it suggest that the relationship between τ and γ varies with the type of products (see, e.g. Gallagher, 2013). The two cases where τ is larger or smaller than γ must, therefore, be discussed. When transportation costs are lower than communication costs, plugging $\tau < \gamma$ in (13)-(15) shows that $\pi_i^I(\theta) > \pi_i^V(\theta)$ and $\pi_i^I(\theta) > \pi_i^H(\theta)$ always hold for $i = 1, 2$. As a result, no region- i firm chooses to be vertical or horizontal, which implies that the only organizational equilibrium involves integrated firms in both regions (**I** - **I**). In other words, when $\tau < \gamma$ firms' choices are such that there is interregional trade and no distant investment. Additional motives, such as an interregional wage gap or subsidies provided by local governments to attract investments, must be added to the setting for vertical and/or horizontal firms to emerge. By contrast, when transportation costs are higher than communication costs, we will see that a plethora of equilibria exist according to the values of the different parameters. Since we do observe organizational diversification in the real world, it is reasonable to dismiss the case where $\tau < \gamma$ and to focus on the case $\tau > \gamma$ or, equivalently, $\phi < \omega$.

The expression (12) becomes easy to interpret when $\phi < \omega$. The term $s_i + \omega s_j$ in the right hand-side of (12) is the effective mass of plants in region i when all i -firms are integrated or horizontal. When some region- j firms choose to be horizontal, the price of their varieties is affected by the gap $\omega - \phi > 0$ between communication and transportation costs. Similarly, the term $(1 - \phi\omega)v_i$ accounts for the region- i firms that choose to go vertical, which generates a price gap equal to $1 - \phi\omega$. When communication costs are lower than transportation costs, everything else equal this renders market in region i more competitive because more region- j firms locate their plants in region i .

Last, it follows from (13) and (14) that $s_j/\Delta_j > s_i/\Delta_i$ must hold for some region- i firms to go vertical. Since $s_j/\Delta_j < s_i/\Delta_i$ must also hold for some region- j firms to be vertical, **V**-firms can exist at most in one region.

3 Homogeneous firms

Working with heterogeneous firms could blur the sheer effects that drive firms in their organizational choices in the space-economy. This is why we start with the case of homogeneous firms.

3.1 Equilibrium

The following result is shown in the Supplementary Material.

Proposition 1. *Assume that $0 < \phi < \omega < 1$. Then, there exists a unique organizational equilibrium almost everywhere in $X = \{(S, \sigma f) \mid 0 < S < 1, 0 < \sigma f\}$.*

It is worth stressing that the uniqueness of the equilibrium does not imply that firms choose the same organizational form for all admissible values of the parameters ϕ , ω , S , and f . More specifically, there are 49 possible

configurations, but “only” 10 of them are equilibrium organizational equilibria. They are given by **I - I**, **I - IV**, **I - V**, **I - VH**, **I - IVH**, **I - IH**, **I - H**, **IH - IH**, **IH - H**, and **H - H**. Among these equilibria, we want to stress the following two polar cases. First, if σf is high enough and regions are not too different, then the equilibrium is given by **I - I**, that is, all firms are integrated. In this case, there are no interregional investments and the mass of plants is minimized. At the other extreme of the spectrum, when σf is sufficiently low, all firms are horizontal, that is, the equilibrium is given by **H - H**. There is no trade because the whole range of varieties is produced in each region. In this case, interregional investments are a perfect substitute for trade and the mass of plants is maximized. In Appendix D, we specify the mass of firms for the configurations in which only two types of firms coexist in the smaller region, while region-1 firms are integrated, that is, **I - IH**, **I - HV**, and **I - IV**. We briefly mention the properties of these three equilibria in Sections 4 and 5. Doing this allows us to discuss the robustness of the properties of the mixed equilibrium. All but one of the remaining equilibria involve integrated and horizontal firms. These patterns are studied in the literature on multinationals and we do not have much to add to the existing body of knowledge. The last configuration **I - V** is associated with very low communication costs and dissimilar regions.

As discussed in the introduction, we focus here on the case in which at least one region hosts the three types of firms. We call such a configuration a *mixed* equilibrium. Since both regions cannot host **V**-firms, only one region, say j , can accommodate the three organizational forms. In this case, the equilibrium condition in region j is as follows:

$$\pi_j^I = \pi_j^V = \pi_j^H > 0. \quad (16)$$

In what follows, we determine necessary and sufficient conditions for homogeneous firms located in one region to become heterogeneous in the way they organize their production activities. Hence, *competition alone is sufficient for identical firms to operate under the three organizational forms*. First, as shown in Appendix A, at a mixed equilibrium one region, say i , hosts only integrated firms ($n_i = s_i F$). We then find the mass of region- j firms which choose each organizational form and show that $i = 1$ and $j = 2$. In other words, diversification arises among the smaller region's firms. Last, we determine the necessary and sufficient conditions for the candidate mixed configuration to be an equilibrium.

When $n_i = s_i F$, we may use (16) to determine the corresponding equilibrium values of Δ_i and Δ_j .

1. Using (13) and (15), the condition $\pi_j^H = \pi_j^I$ implies

$$\Delta_i^* = \frac{\omega - \phi}{\sigma f} s_i. \quad (17)$$

Observe that (4) and (17) imply that P_i^* decreases with the size of region i . Similarly, P_i^* decreases when σ and/or f falls because more plants settle in region i when varieties are less differentiated and/or fixed costs are lower.

2. Using (14) and (15), the condition $\pi_j^H = \pi_j^V$ implies

$$\Delta_j^* = \frac{1 - \phi\omega}{\sigma f} s_j. \quad (18)$$

For the three firm-types to coexist in a region, the regional indices Δ_i^* and Δ_j^* must be given by (17) and (18). Note that Δ_i^* and Δ_j^* do not depend on the mass F of firms.

3. The last condition $\pi_i^I = \pi_i^V$ yields

$$\frac{\Delta_i^*}{\Delta_j^*} = \frac{s_i}{s_j} \cdot \frac{\omega - \phi}{1 - \phi\omega}, \quad (19)$$

which follows immediately from (17) and (18). The expression (19) highlights how communication and transportation costs interact in region- j firms' spatial choices through the price indices of the two markets.

If $\omega = 1$, that is, there are no communication costs, (19) becomes

$$\frac{\Delta_i^*}{\Delta_j^*} = \frac{s_i}{s_j},$$

which is identical to the equilibrium condition obtained by Helpman et al. (2004) when firms have the same productivity. In this case, the price index ratio is determined by the relative size of regions $S = s_2/s_1$.

We show in Appendix B that profits are equal across types when the region-2 firms are split into the following three groups:

$$n_2^* = \frac{1}{1+S} \cdot \left(\frac{1+\omega S}{\omega-\phi} F - \frac{1}{\sigma f} \right), \quad (20)$$

$$v_2^* = \frac{1}{1+S} \cdot \left(\frac{\phi+S}{1-\phi\omega} F - \frac{S}{\sigma f} \right), \quad (21)$$

$$h_2^* = \frac{1}{1+S} \cdot \left[\frac{1+S}{\sigma f} - \frac{(1-\phi^2)(1+\omega S)}{(1-\phi\omega)(\omega-\phi)} F \right]. \quad (22)$$

But does a mixed equilibrium exist? Inspecting n_2^* and v_2^* shows immediately that σf must be bounded below for n_2^* and v_2^* to be positive. Otherwise competition is too soft, or fixed costs are too low, to prevent all region-2 firms to be horizontal. Likewise, it follows from h_2^* that σf must be bounded above for h_2^* to be positive. Otherwise competition is too tough, or fixed costs are too high, for some region-2 firms to be able to cover the fixed cost associated with the launching of a second plant. In short, varieties cannot be very poor or very close substitutes, fixed costs cannot be very small or very large, or both.

Using (20)-(22) yields necessary and sufficient conditions for $n_2^* > 0$, $v_2^* > 0$, and $h_2^* > 0$ to hold. Putting these conditions together shows that region 2 hosts the three types of organizational forms if and only if the following condition holds:

$$B_L < \sigma f \cdot F < B_R, \quad (23)$$

where B_L and B_R are bundles of the parameters S , ω , and ϕ defined as follows:

$$B_L \equiv \max \left\{ \frac{\omega-\phi}{1+\omega S}, \frac{(1-\phi\omega)S}{\phi+S} \right\}, \quad B_R \equiv \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)}.$$

Furthermore, for (23) to be feasible, B_R must exceed B_L . We show in Appendix B that there exists a unique value $\bar{S}$ such that $B_L < B_R$ if and only if the size ratio S satisfies the following inequalities:

$$\frac{\phi}{K} < S < \bar{S} < \frac{1}{K} < 1, \quad (24)$$

where

$$K \equiv \frac{1-\omega\phi}{\omega-\phi} > 1. \quad (25)$$

Since S is smaller than 1, it must be that $i = 1$, which means that region 1 hosts only **I**-firms, while the diversified firms are located in region 2.

To sum up, we have:

Proposition 2. *Assume that $0 < \phi < \omega < 1$. Then, there exists a mixed equilibrium if and only if (23) and (24) hold. This equilibrium is given by $n_1^* = s_1$ and (20)-(22).*

Without productivity differences across firms and an interregional wage gap, the region-2 firms are at a disadvantage in accessing the larger market. This is why some of these firms choose to invest in region 1. What is less straightforward is that the three organizational forms coexist even when there is no exogenous heterogeneity across firms and regions but their relative size.

Yet, the intuition behind Proposition 2 is neat. Since the region-1 firms have a direct access to the larger market, they are not incited to differentiate their spatial structures. In other words, the larger region has no **V**-firms and **H**-firms. By contrast, the smaller region accommodates both **V**-firms and **H**-firms in order to have a better access to the larger market. However, for this to happen, the mass of plants established in region 1 cannot be too large relative to the size of this region, for otherwise competition would be very strong. Moreover, the region-1 firms always choose to be integrated while (20)-(22) is the unique equilibrium configuration that prevails in region 2 under (23) and (24). In other words, the equilibrium described in Proposition 2 is the unique mixed equilibrium.

Furthermore, what matters for a mixed equilibrium to arise is the relative size S of the two regions. If they have similar sizes, the region-2 firms have a strong incentive to focus on their domestic market, making **V**-firms unprofitable. By contrast, owing to the fixed cost they have to bear, these firms have little incentive to invest home when region 2 is not big enough, making **H**-firms unprofitable. As a result, the size of region 1 must take on intermediate values for a mixed configuration to arise in equilibrium. In the same vein, the fixed cost associated with the construction of a second plant cannot be very low, for otherwise all the region-2 firms would undertake horizontal investments, neither very large, for otherwise no region-2 firms would undertake such investments. This is precisely what (23) says. In addition, fixed production costs relative to region sizes cannot be too different for **I**-firms to emerge, while they cannot be similar either, for otherwise no firm would be integrated. In short, *full diversification requires transportation between regions which differ in size but not too much*.

To gain more insights, we quantified the conditions of Proposition 2. Bergstrand et al. (2013) found that the value of σ is approximately 7, while Head and Mayer (2004) obtained a value $\phi = 0.2$ in the case of trade between France and Germany. Choosing $\tau = 1.3$, which means that transportation costs account for 30% of manufactured goods' prices, we obtained $\phi = 0.21$. To the best of our knowledge, there is no estimate of γ . Since we expect communication costs to be low, we have chosen $\gamma = 1.05$, so that $\omega = 0.75$. In this case, a mixed equilibrium exists when region 1 is at least 3 times as large as region 2, but not more than 7.5 times. Raising communication costs to 10% of the goods' price ($\gamma = 1.10$), which implies $\omega = 0.56$, the relative size $s_1/s_2 = 1/S$ of the two regions varies from 7.5 to 12. However, regions are likely to be more integrated than nations. Therefore, we also run the same set of simulations for lower transportation costs, i.e., $\tau = 1.2$, which implies $\phi = 0.34$, and $\gamma = 1.05$. In this case, the interval of admissible values for s_1/s_2 becomes (3.0, 5.3), while $\tau = 1.2$ and $\gamma = 1.10$ yields the interval (7.7, 12). These results suggest that regions must be fairly dissimilar in size for a mixed equilibrium to emerge. However, further drops in communication costs lead to the emergence of a mixed equilibrium with less dissimilar regions in size. For instance, when $\gamma = 1.03$, this interval becomes (2, 4.2). In sum, region 1 must be relatively larger than region 2, but by how much may vary substantially with the parameter values.

The equilibrium profits π_1^* and π_2^* at the mixed equilibrium can be obtained by substituting (17)-(18) into (13)-(15)

and equalizing profits among region-2 firms:

$$\begin{aligned}\pi_1^* &= \pi_1^I = \left(\frac{1}{\omega - \phi} + \frac{\phi}{1 - \phi\omega} - 1 \right) f, \\ \pi_2^* &= \pi_2^I = \pi_2^V = \pi_2^H = \left(\frac{1}{1 - \phi\omega} + \frac{\phi}{\omega - \phi} - 1 \right) f.\end{aligned}\tag{26}$$

Clearly, $\pi_1^* > \pi_2^* > 0$, where the second inequality holds since $\phi < \omega < 1$. In other words, firms located in the larger region make higher profits, a result driven by the presence of positive communication costs because $\pi_1^* = \pi_2^*$ when $\omega = 1$. That $\pi_1^* > \pi_2^*$ agrees with the empirical literature that stresses the existence of a robust and positive relationship between profitability and market size (Head and Mayer, 2004; Redding, 2011). Furthermore, all firms cannot make zero profits in the presence of transportation and communication costs. Indeed, the second expression in (26) shows that all region-2 firms earn zero profits if and only if the parameters ϕ and ω are such that the term between parentheses is equal to 0. In other words, (ϕ, ω) must belong a zero-measure set of the positive orthant for $\pi_2^I = \pi_2^V = \pi_2^H = 0$ to hold. As a result, the total mass of firms cannot be determined by the zero-profit condition when F is endogenous. This may come as a surprise since the zero-profit condition is almost ubiquitous in the trade and economic geography literature. This difference in results stems from the presence of communication costs, while HQs are immobile but plants are mobile. Since individual profits are independent of F , we assume from now on that $F = 1$.

We can use the demand (3) and the equilibrium prices (8)–(10) to find the equilibrium size of region-1 firms and the different types of region-2 firms:

$$\begin{aligned}q_1^I &= \left(\frac{\phi}{1 - \phi\omega} + \frac{1}{\omega - \phi} \right) \sigma f, \\ q_2^I &= \left(\frac{1}{1 - \phi\omega} + \frac{\phi}{\omega - \phi} \right) \sigma f = q_2^V = \left(\frac{\phi\omega}{1 - \phi\omega} + \frac{\omega}{\omega - \phi} \right) \sigma f < q_2^H = \left(\frac{1}{1 - \phi\omega} + \frac{\omega}{\omega - \phi} \right) \sigma f.\end{aligned}\tag{27}$$

Hence, the **I**- and **V**-firms have the same size, which is smaller than that of the **H**-firms. However, the **I**- and **V**-firms sell different quantities in each region because they set different consumer prices. Moreover, the integrated region-1 firms are bigger than the integrated region-2 firms. This is because the market size effect ($s_1 > s_2$) dominates the market crowding effect triggered by the higher mass of plants located in region 1.

Let us pause and ask whether other interregional heterogeneities may yield results similar to Propositions 1 and 2. The most natural candidate involves regions that differ in productivity but not in size ($s_1 = s_2$). In this case, region 1 is endowed with a productivity advantage instead of a size advantage. Following the lines of the above argument, it can be shown that region 2 hosts the three types of firms if and only if the productivity gap is neither too wide nor too narrow. Indeed, if the productivity gap is wide enough, no region 2-firm is integrated. On the other hand, if the productivity gap is very narrow, no region 2-firm is vertical.² This condition is similar to the condition we obtained about the relative size S of the two regions.

3.2 Welfare

Does the multiplicity of spatial organizations entail a waste of resources? The benefit of using quasi-linear preferences are reaped in the welfare analysis. Even though we have two groups of individuals who do not face the same prices,

²The proof can be obtained from authors upon request.

that is, the workers in regions 1 and 2, their utilities can be added in a utilitarian welfare function. In other words, the planner chooses the consumption level of each variety and the mass of firm-types in each region so as to maximize the sum of individual utilities under the labor constraints:

$$n_i C_i^I + v_i C_i^V + h_i C_i^H + s_i z_i = s_i \quad i = 1, 2.$$

This is equivalent to maximizing

$$W \equiv \sum_{i=1}^2 s_i U_i - \sum_{i=1}^2 (n_i C_i^I + v_i C_i^V + h_i C_i^H) \quad (28)$$

subject to (11), where we have set:

$$U_i \equiv \frac{\sigma}{\sigma-1} \ln \left[n_i (x_{ii}^I)^{\frac{\sigma-1}{\sigma}} + v_i (x_{ii}^V)^{\frac{\sigma-1}{\sigma}} + h_i (x_{ii}^H)^{\frac{\sigma-1}{\sigma}} + n_j (x_{ji}^I)^{\frac{\sigma-1}{\sigma}} + v_j (x_{ji}^V)^{\frac{\sigma-1}{\sigma}} + h_j (x_{ji}^H)^{\frac{\sigma-1}{\sigma}} \right] + 1,$$

while the cost functions are given by (5)-(7) where $q_{ij} = s_j x_{ij}$.

The next proposition is proven in Appendix C.

Proposition 3. *Assume that $0 < \phi < \omega < 1$. If*

$$B_L < (\sigma - 1)f < B_R, \quad (29)$$

then the social optimum is such that all firms in the larger region are integrated, while the smaller region hosts the three types of organizational forms:

$$n_2^* > n_2^o = \frac{1}{1+S} \cdot \left(\frac{1+\omega S}{\omega-\phi} - \frac{1}{f(\sigma-1)} \right), \quad (30)$$

$$v_2^* > v_2^o = \frac{1}{1+S} \cdot \left(\frac{\phi+S}{1-\phi\omega} - \frac{S}{f(\sigma-1)} \right), \quad (31)$$

$$h_2^* < h_2^o = \frac{1}{1+S} \cdot \left[\frac{1+S}{f(\sigma-1)} - \frac{(1-\phi^2)(1+\omega S)}{(\omega-\phi)(1-\phi\omega)} \right]. \quad (32)$$

Following the same approach as in 3.2, it is readily verified that $n_2^o > 0$, $v_2^o > 0$ and $h_2^o > 0$ if and only if (29) holds.

It follows from (2) that the manufacturing sector operates as in a CES one-sector economy where consumers are endowed with an income equal to α . In this case, the equilibrium and optimum of a one-sector economy coincide even when firms are heterogeneous (Dhingra and Morrow, 2019). Therefore, it is no surprise that the coexistence of different organizational forms is not socially wasteful. Indeed, comparing (23) and (29) shows that both the market equilibrium and the social optimum involve the coexistence of all organizational forms when $B_L/(\sigma-1) < f < B_R/\sigma$. However, the numbers of firm-types in the smaller region need not be the same at the two outcomes. The difference in numbers stems from the fact that the cost distribution is endogenously determined through the noncooperative organizational choices made by firms whereas the cost distribution is exogenously given in Dhingra and Morrow (2019).

Propositions 2 and 3 have the following implication: *the whole economy involves too few plants at the market equilibrium* ($1+h_2^o > 1+h_2^*$). Since $n_2^* > n_2^o$, too few region-2 firms become multiregional under competition because these firms hold back their investments in the larger market to soften competition therein. As a result, competition in the larger region becomes weak enough for this market to host too many **V**-firms. In other words, by delocalizing

their production activities in the larger region, too many region-2 firms do not invest in their home region. Hence, *each region accommodates too few plants at the market outcome*. To put it differently, there is excessive geographical concentration of production. Note also that Proposition 3 shows that the diversity of organizational forms leads to the minimization of total transportation and communication costs associated with the first-best flows of varieties.

4 The effects of size

Our set-up allows us to determine the total mass of plants in the whole economy and their distribution between the two regions. In this section, we show how these masses vary with the absolute and relative sizes of the two regions.

First of all, Proposition 2 implies that the mass of plants located in the larger region is equal to $s_1 + v_2^* + h_2^* > s_1$, while the mass of plants established in the smaller region is $n_2^* + h_2^* = s_2 - v_2^* < s_2$. Consequently, even though firms are distributed proportionally between regions, *the larger region hosts a disproportionately higher mass of plants*. This result echoes the home market effect (HME), which states that the larger region hosts a more than proportionate share of firms. Recall, however, the difference between the two settings: in Baldwin et al. (2003), say, firms are assumed to be spatially integrated so that a firm's HQs moves with its plant; here, firms' plants are mobile but their HQs are immobile.

We now study the impact of the relative size of the two regions on the mass of plants located in region 1 by differentiating $n_1^* + v_2^* + h_2^*$ with respect to $S = s_2/s_1$. First, we have:

$$\frac{dn_1^*}{dS} = -\frac{1}{(1+S)^2} < 0. \quad (33)$$

Second, some tedious calculations show that the following expression holds:

$$\frac{dv_2^*}{dS} + \frac{dh_2^*}{dS} = \frac{1}{(1+S)^2} \left(\frac{1-\phi}{\omega-\phi} - \frac{1}{\sigma f} \right). \quad (34)$$

By implication of (23), we have

$$\sigma f < B_R = \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)} < \frac{\omega-\phi}{1-\phi} \Leftrightarrow \frac{1-\phi}{\omega-\phi} - \frac{1}{\sigma f} < 0,$$

because $(1+S)/(1+\omega S)$ is an increasing function of S while the inequality holds at $S = 1/K$. Therefore, we have:

$$\frac{dv_2^*}{dS} + \frac{dh_2^*}{dS} < 0. \quad (35)$$

Combining (33) and (34) yields

$$\frac{d(n_1^* + v_2^* + h_2^*)}{dS} = \frac{1}{(1+S)^2} \left(\frac{1-\phi}{\omega-\phi} - \frac{1}{\sigma f} - 1 \right) < 0.$$

Since an increase in s_1 amounts to a decrease in S , the share of plants located in the larger region grows with the size of this region. More specifically, a relatively larger region 1 triggers a flow of interregional investments through a higher mass of **V**-firms. This implies a drop in the mass of **I**-firms in the smaller region.

Furthermore, we have:

$$\frac{d(n_2^* + h_2^*)}{dS} = \frac{1}{(1+S)^2} \left[\frac{\phi(1-\omega)}{1-\phi\omega} + \frac{1}{\sigma f} \right] > 0.$$

Combining this expression with (35) implies

$$\frac{d(v_2^* + h_2^*)}{dS} = -\frac{dn_2^*}{dS} < 0 < \frac{d(n_2^* + h_2^*)}{dS}.$$

Hence, when the relative size of the smaller region decreases, it hosts fewer **I**-firms. Moreover, the mass of region 2's **H**-firms decreases, but this drop is more than compensated by the hike in the mass of **V**-firms generated by the larger size of region 1. In other words, region 1 hosts more plants belonging to region-2 firms.

Finally, since

$$\frac{d(n_1^* + v_2^* + h_2^*)}{dS} + \frac{d(n_2^* + h_2^*)}{dS} = \frac{1}{(1+S)^2} \frac{(1-\omega)(1-\phi^2)}{(\omega-\phi)(1-\phi\omega)} > 0,$$

the increase in the mass of region 1's plants is smaller than the decrease in the mass of plants operating in region 2. By implication, the total mass of plants in the economy falls when regions become more dissimilar in size.

The following proposition comprises a summary.

Proposition 4. *Assume that $0 < \phi < \omega < 1$. At a mixed equilibrium, the larger region hosts a more than proportionate share of plants. Furthermore, the mass of plants established in this region increases with its size, but the total mass of plants operating in the economy decreases.*

Consider now the three neighboring configurations discussed in Section 3.1. Under **I - IH**, the mass of plants in region 1 is given by $s_1 + h_2^* > s_1$, whereas it is equal to s_2 in region 2. When **I - HV** prevails, there are $h_2^* = s_2 - v_2^*$ plants in region 2 while region 1 attracts the largest mass of plants, which is equal to $1 > s_1$. Last, when the equilibrium is **I - IV**, region 1 hosts $s_1 + v_2^* > s_1$ while $n_2^* < s_2$ plants are set up in region 2. As a result, in each of those three configurations, *the HME holds*. Put differently, market integration and new ICTs foster the industrialization of the larger region at the expense of the smaller region, which gradually specializes in management activities.

5 The interregional distribution of activities

In this section, we study the effects of transportation and communication costs on the mass of plants and the numbers of each firm-type at the mixed equilibrium. We also show that transportation and communication costs have very different impacts on the market outcome and its welfare properties.

5.1 Spatial frictions and the location of firms

The most popular thought experiment in economic geography deals with the impact of transportation costs on firms' locational choices. Differentiating n_2^* , v_2^* and h_2^* with respect to ϕ , we obtain:

$$0 < \frac{dv_2^*}{d\phi} < \frac{dn_2^*}{d\phi} < -\frac{dh_2^*}{d\phi}.$$

Hence, *fewer firms are multiregional when market integration becomes deeper*, so that the mass of **I**-firms rises. However, the impact on **H**- and **V**-firms are opposite. While a decrease in transportation costs leads to a smaller mass of **H**-firms since the access to region 1 becomes easier from region 2, the mass of **V**-firms rises because importing goods from region 1 to region 2 is cheaper. Since more region-2 firms become vertical, fewer region-2 firms invest

home, which renders market 2 less competitive. Similarly, market 1 becomes less competitive since the drop in the mass of **H**-firms is stronger than the hike in the mass of **V**-firms. Inspecting (17) and (18) where $i = 1$ and $j = 2$ shows that both Δ_1^* and Δ_2^* decrease, hence both P_1^* and P_2^* increase, when ϕ rises. This runs against the standard result in spatial competition that states that lower transportation costs make competition tougher. In our setting, firms are able to overcome this effect by changing their organizational forms. This confirms once more that models with integrated firms only may lead to inaccurate conclusions.

It is well known that a deeper market integration induces the relocation of firms from the smaller to the larger region when firms are spatially integrated (Baldwin et al., 2003). Here, the total mass of plants operating in the larger region decreases faster than in smaller region when transportation costs fall. In other words, a deeper market integration “demagnifies” the HME. Finally, the fact that production is concentrated in a smaller mass of plants when transportation costs decrease concurs with the main message of economic geography, that is, lowering transportation costs fosters the agglomeration of activities.

Furthermore, it follows immediately from (17) and (18) that lowering communication costs have a different impact on the two markets. Indeed, as ω increases, the effective mass of plants competing in the larger region rises, whereas the effective mass of plants competing in the smaller region falls. Consequently, competition is intensified in region 1 and weakened in region 2. More specifically, since making the transfer of information cheaper facilitates the spatial fragmentation of firms, it is readily verified that

$$\frac{dn_2^*}{d\omega} < 0 \quad \frac{dv_2^*}{d\omega} > 0 \quad \frac{dh_2^*}{d\omega} > 0.$$

In other words, *lowering communication costs leads more region-2 firms to go multiregional*, which increases the mass of plants hosted by the larger market, while the mass of plants established in the smaller region decreases. Observe the difference with the impact of lower transportation costs which lead to a drop in the mass of multiregional firms. This difference should not come as a surprise since the two costs affect the proximity-concentration trade-off differently: lowering transportation costs weakens the need for proximity, while lower communication costs weakens the benefits of concentration. Furthermore, whereas lower transportation costs weakens the HME, the total mass of plants located in the larger region increases with ω , hence there is magnification of the HME. That is to say, communication costs play here the same role as transportation costs in the footloose capital model (Baldwin et al., 2003). Since region 2 hosts fewer plants, decreasing communication costs also fosters the deindustrialization of the smaller region through the relocation of manual jobs toward the larger region.

It remains to investigate how the size of each type of firm reacts to a drop in transportation and communication costs. Differentiating (27) with respect to ϕ and ω yields the following inequalities:

$$\begin{aligned} \frac{\partial q_1^I}{\partial \phi} &> \frac{\partial q_2^I}{\partial \phi} = \frac{\partial q_2^V}{\partial \phi} = \frac{\partial q_2^H}{\partial \phi} = \left[\frac{\omega}{(1 - \phi\omega)^2} + \frac{\omega}{(\omega - \phi)^2} \right] \sigma f > 0, \\ \frac{\partial q_1^I}{\partial \omega} &< \frac{\partial q_2^I}{\partial \omega} = \frac{\partial q_2^V}{\partial \omega} = \frac{\partial q_2^H}{\partial \omega} = \left[\frac{\phi}{(1 - \phi\omega)^2} - \frac{\phi}{(\omega - \phi)^2} \right] \sigma f < 0. \end{aligned}$$

Therefore, trade liberalization makes *all* firms bigger, regardless of their type and location, while the ICT revolution generates the reverse. Again, transportation and communication costs have opposite effects.

Our main predictions regarding the geography of production are summarized in the following proposition.

Proposition 5. *Assume that $0 < \phi < \omega < 1$. At a mixed equilibrium, lowering transportation costs makes all firms bigger and leads to a smaller mass of plants, while lower communication costs have the opposite impact.*

Finally, note that Propositions 2 and 3 imply that the optimal and equilibrium masses of firms respond in the same way to shocks on transportation or communication costs. Therefore, the above results are driven by the fundamentals of the model.

As in the above section, we now consider the three configurations **I - IH**, **I - HV**, and **I - IV**. In the first case, differentiating (D.1) in Appendix D with respect to ϕ and ω yields:

$$\frac{dn_2^*}{d\phi} > 0 \quad \frac{dn_2^*}{d\omega} < 0,$$

while the impacts of ϕ and ω on h_2^* have the opposite sign because $n_2 + h_2 = s_2$. Therefore, transportation and communication costs have opposite effects on the masses of integrated and horizontal firms and the signs are the same as in mixed equilibrium. Indeed, when ϕ increases, being located in the larger region becomes less attractive for some **I**-firms, which choose to shut down their plants in this region in order to bring communication costs down to zero. Note here the difference with Behrens and Picard (2007) who come to the opposite prediction in a setting without communication costs. Furthermore, that h_2^* decreases with ϕ also means that lowering transportation costs leads to a smaller mass of plants in region 1, while the mass of plants in region 2 remains the same. As a result, the total mass of plants in the economy decreases when ϕ rises. A drop in communication costs has the opposite effects.

When **I - HV** prevails, differentiating (D.2) with respect to ϕ and ω , we obtain

$$\frac{dh_2^*}{d\phi} < 0 \quad \frac{dh_2^*}{d\omega} < 0.$$

In this case, both costs have the same impact on the mass of horizontal (vertical) firms. This differs from what we have obtained so far. This is because lowering transportation or communication costs makes it less profitable for some region-2 firms to run two plants. Since $dh_2^*/d\phi = -dv_2^*/d\phi$, they prefer to shut down their plant established in their home region and to supply the larger region from a plant located there. This should not come as a surprise because the configuration **I - HV** arises when regions have fairly different sizes. Furthermore, since $h_2 + v_2 = s_2$, the mass of plants in region 1 is unaffected by a drop in transportation or communication costs. Given that $dv_2^*/d\phi > 0$, fewer plants are located in region 2. Consequently, the total mass of plants shrinks when ϕ increases, which agrees with what we saw above. The difference is that here lowering communication costs also fosters a smaller mass of plants.

The case **I - IV** is more difficult to handle because the equilibrium mass of integrated or vertical firms is very cumbersome. Therefore, we use the equilibrium condition $\pi_2^I = \pi_2^V$, together with the condition $n_2 + v_2 = s_2$, and apply the implicit function theorem to determine the sign of $dn_2^*/d\phi$ and $dn_2^*/d\omega$. Tedious calculations show that $dn_2^*/d\omega$ is always negative for the admissible values of the parameters, which means that lessening communication costs leads more region-2 firms to produce in the larger region. By contrast, the sign of $dn_2^*/d\phi$ is ambiguous. More specifically, it can be shown that, as in the mixed equilibrium case, $dn_2^*/d\phi > 0$ holds if

$$\frac{2}{1 + \phi^2} \phi > \omega.$$

When this inequality is reversed, $dn_2^*/d\phi$ remains positive for intermediate values of ϕ . Otherwise, $dn_2^*/d\phi$ may become negative when ϕ takes on low values while S is small enough. In other words, when shipping goods between

dissimilar regions is expensive, a drop in transportation costs may lead to more 2-region firms to produce in the larger region because shipping varieties from region 1 to region 2 tends to become inexpensive.

In the last two configurations, **V**-firms exist because the two regions have fairly different sizes and/or communication costs are low. Therefore, the incentive to offshore production in the larger region becomes stronger than in the mixed equilibrium case.

6 The impact of spatial frictions on welfare and trade

We are equipped to study how the spatial frictions discussed above affect the regional welfare levels and the interregional gap at a mixed equilibrium. It is natural to start by considering the impact of transportation and communication costs on the welfare level achieved in each region. Plugging (3) in (1) yields the indirect utilities:

$$V_1 = \frac{1}{\sigma - 1} \ln \Delta_1^* + \pi_1^* \quad V_2 = \frac{1}{\sigma - 1} \ln \Delta_2^* + \pi_2^*. \quad (36)$$

In (36), the first term is the consumer surplus associated with the consumption of the differentiated good, while the second, which is given by the equilibrium profits, stands for the consumption of the homogeneous good.

Consider first a decrease in transportation costs. It follows immediately from (17) and (18) that both Δ_1^* and Δ_2^* decrease with ϕ . Indeed, since each region hosts a smaller mass of plants, regional competition is softer and the regional price indices are higher. In other words, lowering transportation costs intensifies monopoly distortions. Furthermore, since market integration leads to fewer plants in each region, competition is relaxed in both regions. This allows firms to earn higher profits. Indeed, differentiating the equilibrium profits (26) with respect to ϕ yields:

$$\frac{d\pi_2^*}{d\phi} = \omega \frac{d\pi_1^*}{d\phi} > 0. \quad (37)$$

Thus, *firms can protect themselves against the damaging effects of lower trade barriers by reorganizing their geography of production.*

The impact of transportation costs on V_1 and V_2 is a priori ambiguous. Indeed, the first term in (36) decreases while the second term increases. However, it can be shown that a drop in transportation costs may lead to a welfare loss. For instance, when fixed costs are not too high and communication costs are not too low, welfare in region 1 decreases with transportation costs. Welfare in region 2 falls with ϕ when communication costs are sufficiently low, i.e., when ω is close to 1. To confirm our findings, we appeal to numerical analysis. We choose $\phi = 0.2$ and $\phi = 0.7$, which is the value that Head and Mayer (2004) find about trade in the automotive sector between Canada and the United States. Regarding communication costs, we evaluate the sign of $dV_1/d\phi$ and $dV_2/d\phi$ over the domain defined by (23) and (24) for the range of admissible values of ω .

When $\phi = 0.2$, $dV_1/d\phi > 0$ and $dV_2/d\phi > 0$ always hold. However, when $\phi = 0.7$, $dV_1/d\phi$ becomes negative for an interval of communication cost values. In other words, *a deeper market integration need not be favorable to all consumers*. Whatever the sign of the impact, it is worth stressing that the welfare gains (if any) are driven by the redistribution of profits. Would firms be owned by absentee share-holders, *all* consumers would be worse-off when transportation costs decrease.

Let us now come to the impact of communication costs. It follows from (17) and (18) that Δ_1^* increases with ω ,

while Δ_2^* decreases. Moreover, differentiating (26) with respect to ω shows that

$$\frac{d\pi_1^*}{d\omega} < \frac{d\pi_2^*}{d\omega} < 0. \quad (38)$$

Since the region-1 firms are integrated, they do not benefit from the drop in communication costs. However, they face a higher mass of competitors on their domestic market. Consequently, the region-1 firms and the region-2 vertical and horizontal firms make lower profits in the larger market. Although the smaller market is less competitive because fewer region-2 firms remain integrated, the difference in market sizes is sufficiently big ($s_1 > \bar{s} > s_2$) for the losses incurred in region 1 by the integrated region-2 firms to overcome the gains made in region 2. Since Δ_2^* decreases with ω , we have $dV_2/d\omega < 0$. In other words, at a mixed equilibrium, *lowering communication costs make people in region 2 worse-off*. Unfortunately, the impact of lower communication costs on V_1 is again hard to sign. It can be shown that V_1 decreases with ω when both communication and transportation costs are low enough. On the other hand, our numerical analysis shows that $dV_1/d\omega > 0$ for $\sigma = 7$, $\phi = 0.2$ and 0.7 .

In sum, *reducing spatial frictions need not be welfare-enhancing*. This unexpected result should not be viewed as an exotica. On the contrary, it is our contention that it highlights the following fact: when firms may respond to changes in trade barriers by changing their spatial organization, spatial frictions interact in a complex way to determine the gains of trade and their impact on welfare. Assuming that firms are integrated washes out these effects and yields predictions that might be inaccurate.

We now turn our attention to the impact of the two types of costs on the regional GDPs, which leads to neat conclusions. Regional GDPs are equal to the wage bills plus profits:

$$GDP_1 = s_1 + s_1\pi_1^*, \quad GDP_2 = s_2 + s_2\pi_2^*.$$

Since GDP_i and the GDP per capita $GDP_i/s_i = 1 + \pi_i^*$ vary together, individuals in the larger region are better off than those in the smaller region (recall that $\pi_1^* > \pi_2^*$).

Furthermore, it follows from (37) that a drop in transportation costs generates higher profits, hence a higher GDP, in both regions. Moreover, since $\omega < 1$, (37) implies that profits in region 1 increases faster than in region 2 with ϕ . Consequently, lowering transportation costs fosters regional divergence. By contrast, each region ends up with a lower GDP when communication costs fall. However, (38) implies that the drop is sharper in the larger region than in the smaller one. Therefore, when communication costs decrease, the regional gap shrinks.

To sum up, we have:

Proposition 6. *At a mixed equilibrium, lowering transportation costs promotes regional GDPs, but exacerbates regional disparities. By contrast, decreasing communication costs hampers regional GDPs and weakens regional discrepancies.*

Since the wage bill is given in each region, regional GDPs vary only through changes in profits. Transportation and communication costs have opposite macrospatial impacts because decreasing the transportation costs softens competition, thus raising regional GDPs at the expense of a widening of the regional gap in income per capita. By contrast, decreasing communication costs strengthens competition, thus reducing regional GDPs and narrowing the interregional gap.

Finally, we consider the effect of communication and transportation costs on interregional trade flows. Since the producer price is equal to 1 for all organizational types, the exports X_{12} from region 1 to region 2 is given by the

sum of the quantities produced by the region-1 integrated firms and region-2 vertical firms:

$$X_{12} = (s_1 + \omega v_2^*) \cdot \frac{\phi}{1 - \phi\omega} \cdot \sigma f,$$

while the exports X_{21} from region 2 to region 1 are given by the total output of the region-2 integrated firms:

$$X_{21} = n_2^* \cdot \phi \cdot \frac{s_1}{\Delta_1} = n_2^* \cdot \frac{\phi}{\omega - \phi} \cdot \sigma f.$$

Since $dn_2^*/d\omega > 0$ and $d\left(\frac{\phi}{1-\phi\omega}\right)/d\phi > 0$, we have:

$$\frac{dX_{12}}{d\omega} > 0.$$

Likewise $dn_2^*/d\phi > 0$ and $d\left(\frac{\phi}{\omega-\phi}\right)/d\phi > 0$ yields

$$\frac{dX_{12}}{d\phi} > 0.$$

A similar argument leads to the following inequalities:

$$\frac{dX_{21}}{d\phi} > 0 \quad \frac{dX_{21}}{d\omega} < 0.$$

Therefore, at a mixed equilibrium, *lowering transportation costs intensifies trade in both directions*. This is because lowering transportation costs reduces the advantage of being horizontal, i.e., region-2 firms can exploit increasing returns more intensively within a single plant. In other words, the pro-trade effect associated with a drop in transportation costs holds true even when firms are free to choose their geography of production. Note that trade may expand while welfare in the larger region may decrease with ϕ . In this case, there is wasteful trade.

Decreasing communication costs also has a positive impact on the volume of trade from the larger to the smaller region because more region-2 firms become vertical. However, the effect on trade from the smaller to the larger region has the opposite sign. This is because more 2-region firms have a plant in the larger region. Consequently, unlike transportation costs, at a mixed equilibrium *lowering communication costs need not boost two-way trade*.

7 Heterogeneous firms

In this section, we study what Proposition 2 becomes when firms differ a priori in productivity regardless of the organizational form they choose. In line with the literature, we consider a Pareto distribution. Since we focus on the configuration where region 2 hosts the three types of firms while all region-1 firms are integrated, we assume that G is given by a truncated Pareto distribution $G(\theta) = \delta \cdot [1 - (1/\theta)^\beta]$ defined on $[1, \bar{\theta})$, where $\delta \equiv \bar{\theta}^\beta / (\bar{\theta}^\beta - 1) > 1$, while $\beta > 2$ guarantees that the productivity distribution has a finite variance. Indeed, as shown in Appendix A, all region-1 firms are integrated if $\bar{\theta} \leq K^{\frac{1}{\sigma-1}}$ where K is the constant given by (25). Therefore, for the the cases of homogeneous and heterogeneous firms to be compatible, we must consider a truncated Pareto distribution defined on $[1, \bar{\theta})$.

When firms are heterogeneous, only the most productive firms can afford to invest in two plants. Hence, the **H**-firms (if any) are always the most productive. Consequently, a decrease in transportation costs (communication costs) leads some **H**-firms to become integrated (vertical). Among the firms that choose not to be horizontal, which

ones choose to be integrated or vertical? Two cases may arise. In the first one, the least productive region-2 firms are integrated: $1 < \theta_2^V < \theta_2^H < \bar{\theta}$, where θ_2^V and θ_2^H are the productivity thresholds such that a **I**-firm has a productivity $\theta_2 < \theta_2^V$, a **V**-firm has a productivity $\theta_2^V < \theta_2 < \theta_2^H$, while a **H**-firm has a productivity $\theta_2 > \theta_2^H$. In the second case, the least productive region-2 firms are vertical, i.e., $1 < \theta_2^I < \theta_2^H < \bar{\theta}$. In the former case, the equilibrium conditions are given by $\pi_2^I(\theta_2^V) = \pi_2^V(\theta_2^V)$ and $\pi_2^V(\theta_2^H) = \pi_2^H(\theta_2^H)$ while they are $\pi_2^I(\theta_2^I) = \pi_2^V(\theta_2^I)$ and $\pi_2^I(\theta_2^H) = \pi_2^H(\theta_2^H)$ in the latter.

In either case, the equilibrium conditions are equivalent to

$$\Delta_1^*(\theta_2^H) = \frac{\omega - \phi}{\sigma f} s_1 \cdot (\theta_2^H)^{\sigma-1}, \quad (39)$$

$$\Delta_2^*(\theta_2^H) = \frac{1 - \phi\omega}{\sigma f} s_2 \cdot (\theta_2^H)^{\sigma-1}. \quad (40)$$

Note that (39) and (40) are, respectively, identical to (17) and (18) when firms are homogeneous since $\theta_2^H = 1$.

Using (11), we may rewrite (39)-(40) as follows:

$$\Delta_1^*(\theta_2^H) = A \cdot [s_1 + \omega s_2 - (\omega - \phi)n_2], \quad (41)$$

$$\Delta_2^*(\theta_2^H) = A \cdot [\phi s_1 + s_2 - (1 - \phi\omega)v_2], \quad (42)$$

where A given by

$$A \equiv \frac{\beta}{\beta - \sigma + 1} \cdot \frac{\bar{\theta}^\beta - \bar{\theta}^{\sigma-1}}{\bar{\theta}^\beta - 1} > 0 \quad (43)$$

is a normalization constant that guarantees that $s_i + s_j = F$.

Following the same approach as in the homogeneous firm case, we find that (11) and (39)-(42) yields the following expressions:

$$n_2^*(\theta_2^H) = \frac{1}{1 + S} \cdot \left[\frac{1 + \omega S}{\omega - \phi} - \frac{(\theta_2^H)^{\sigma-1}}{A} \cdot \frac{1}{\sigma f} \right], \quad (44)$$

$$v_2^*(\theta_2^H) = \frac{1}{1 + S} \cdot \left[\frac{\phi + S}{1 - \phi\omega} - \frac{(\theta_2^H)^{\sigma-1}}{A} \cdot \frac{S}{\sigma f} \right], \quad (45)$$

$$h_2^*(\theta_2^H) = \frac{s_2}{A} \cdot \int_{\theta_2^H}^{\bar{\theta}} \theta^{\sigma-1} dG = \frac{1}{1 + S} \cdot \left[\frac{(\theta_2^H)^{\sigma-1}}{A} \cdot \frac{1 + S}{\sigma f} - \frac{(1 + \omega S)(1 - \phi^2)}{(\omega - \phi)(1 - \phi\omega)} \right]. \quad (46)$$

Since the left-hand side of (46) is decreasing and positive at $\theta_2^H = 1$ while the right-hand side is increasing and negative at $\theta_2^H = 1$, (46) has a unique solution. Furthermore, this solution exceeds 1 and is smaller than $\bar{\theta}$. Plugging this solution in (44) and (45) yields the corresponding equilibrium masses of **I**- and **V**-firms. As consequence, there exists at most one equilibrium and the equilibrium value θ_2^H is independent of the respective masses of integrated and vertical firms.

Similar to the homogenous firm case, it can be shown that (44)-(46) imply that region 2 hosts the three types of firms if and only if the following condition holds:

$$B_L < \frac{(\theta_2^H)^{\sigma-1}}{A} \cdot \sigma f < B_R. \quad (47)$$

Similarly, a mixed equilibrium with heterogeneous firms exists when

$$0 < \frac{\phi}{K} < S < \bar{S} < \frac{1}{K} < 1 \quad (48)$$

holds.

Note that the conditions (44)-(46) reduce to (20)-(22), while (47)-(48) reduces to (23)-(24), when firms are homogeneous because $A/(\theta_2^H)^{\sigma-1} = 1$.

It remains to determine whether the least productive region-2 firms are integrated or vertical.

Case 1. Assume that the least productive firms are integrated: $1 < \theta_2^V < \theta_2^H < \bar{\theta}$. Then, the masses of integrated, vertical and horizontal firms are given by:

$$n_2^* = \frac{S}{1+S} \cdot \frac{1 - (\theta_2^V)^{-(\beta-\sigma+1)}}{1 - (\bar{\theta})^{-(\beta-\sigma+1)}}, \quad (49)$$

$$v_2^* = \frac{S}{1+S} \cdot \frac{(\theta_2^V)^{-(\beta-\sigma+1)} - (\theta_2^H)^{-(\beta-\sigma+1)}}{1 - (\bar{\theta})^{-(\beta-\sigma+1)}}, \quad (50)$$

and

$$h_2^* = \frac{S}{1+S} \cdot \frac{(\theta_2^H)^{-(\beta-\sigma+1)} - (\bar{\theta})^{-(\beta-\sigma+1)}}{1 - (\bar{\theta})^{-(\beta-\sigma+1)}}. \quad (51)$$

For the assumed configuration to be an equilibrium, the equations (44)-(46) and (49)-(51) must be consistent. In particular, (50)-(51) and (45)-(46) must be equal. Using (39)-(40), we then obtain the equilibrium conditions corresponding to the configuration $1 < \theta_2^V < \theta_2^H$:

$$(\theta_2^H)^{-(\beta-\sigma+1)} = \frac{\phi K S^2 + (K-1)S - \phi}{(1-\omega\phi)S} \cdot [1 - (\bar{\theta})^{-(\beta-\sigma+1)}] + (1+S) (\theta_2^V)^{-(\beta-\sigma+1)}, \quad (52)$$

and

$$\frac{(\theta_2^H)^{\sigma-1}}{A\sigma f} - \frac{S(\theta_2^H)^{-(\beta-\sigma+1)}}{1+S} \cdot \frac{1}{(\bar{\theta})^{\beta-\sigma+1} - 1} = \frac{(1+\phi K)S + \phi + K}{(1-\phi\omega)(1+S)} - \frac{S}{1+S} \cdot \frac{1}{(\bar{\theta})^{\beta-\sigma+1} - 1}. \quad (53)$$

It remains to determine under which conditions the inequalities $1 < \theta_2^V < \theta_2^H$ hold. We show in Appendix E that this configuration is an equilibrium when $S \in [\phi/K, \underline{S}]$, where the constant $\underline{S}$ is defined in the same appendix.

Case 2. Assume now that the least productive firms are vertical: $1 < \theta_2^I < \theta_2^H < \bar{\theta}$. Hence, n_2^* and v_2^* are given by

$$n_2^* = \frac{S}{1+S} \cdot \frac{(\theta_2^I)^{-(\beta-\sigma+1)} - (\theta_2^H)^{-(\beta-\sigma+1)}}{1 - (\bar{\theta})^{-(\beta-\sigma+1)}},$$

$$v_2^* = \frac{S}{1+S} \cdot \frac{1 - (\theta_2^I)^{-(\beta-\sigma+1)}}{1 - (\bar{\theta})^{-(\beta-\sigma+1)}},$$

while h_2^* is still given by (51).

Following the same approach as in the case above, we obtain the equilibrium conditions corresponding to the configuration $1 < \theta_2^I < \theta_2^H$:

$$S \left(\theta_2^H \right)^{-(\beta-\sigma+1)} = (1+S) \left(\theta_2^I \right)^{-(\beta-\sigma+1)} - \frac{\omega K S^2 + (K-1)S - \phi}{(1-\phi\omega)S} \left[1 - (\bar{\theta})^{-(\beta-\sigma+1)} \right] - 1, \quad (54)$$

$$\frac{\left(\theta_2^H \right)^{\sigma-1}}{A\sigma f} - \frac{S \left(\theta_2^H \right)^{-(\beta-\sigma+1)}}{1+S} \cdot \frac{1}{(\bar{\theta})^{\beta-\sigma+1} - 1} = \frac{(1+\phi K)S + \phi + K}{(1-\phi\omega)(1+S)} - \frac{S}{1+S} \cdot \frac{1}{(\bar{\theta})^{\beta-\sigma+1} - 1}. \quad (55)$$

Observe that (53) and (55) are the same. In other words, the equilibrium mass of **H**-firms is the same in the two configurations. However, the equilibrium masses of **I**- and **V**-firms are not the same because (52) and (54) differ.

It remains to determine under which conditions $1 < \theta_2^I < \theta_2^H < \bar{\theta}$ holds. We show in Appendix F that this configuration is an equilibrium when $S \in [\underline{S}, \bar{S}]$.

Our main findings may be summarized as follows.

Proposition 7. *Assume that firms are cost-heterogeneous. Then, a mixed equilibrium exists if and only if (47) and (48) hold. This equilibrium is such that all region-1 firms are integrated while the most productive region-2 firms are horizontal. Furthermore, when (i) $S \in [\phi/K, \underline{S}]$ the least productive region-2 firms are integrated, and (ii) $S \in [\underline{S}, \bar{S}]$ the least productive region-2 firms are vertical.*

The intuition behind Proposition 7 is easy to grasp. The most productive firms choose to be horizontal because this allows them to avoid paying transportation costs which exceed communication costs. Which organizational form choose the mid-productive firms depends on the relative size of regions. When the asymmetry is relatively high (bullet (i) in Proposition 7) the mid-productive firms go vertical because they are able to provide the large market at lower prices than under the **I**-form. However, if the asymmetry is mild (bullet (ii)), the local market matters more, which leads the mid-productive firms to be integrated because they can supply the local market at lower prices than under the **V**-form.

The effect of lowering transportation and communication costs on the equilibrium configurations is more involved than in the homogeneous firm case. Nevertheless, a few neat results hold true. First of all, we show in Appendix G that θ_2^H always increases with ϕ and decreases with ω . Therefore, as in the homogeneous firm case, the mass of **H**-firms decreases (increases) when transportation costs (communication costs) fall.

Furthermore, for the configuration where the least productive firms are integrated, the first term in the right-hand side of (52) decreases with ω , hence θ_2^V also decreases. Consequently, a drop in communication costs leads to fewer **I**-firms, like in the homogeneous firm case, while the change in the mass of **V**-firms depends on the shape parameter β of productivity distribution. Similarly, decreasing transportation costs leads to hike in θ_2^H , so that the left-hand side of (52) decreases. Since the first term in the right-hand side of (52) increases when $\omega < \bar{\omega}$, with $\bar{\omega} = 2\phi/(1+\phi^2)$, θ_2^V increases, we may conclude that lowering transportation costs makes **I**-firms more profitable (see Appendix G for a proof). Under these circumstances, communication and transportation costs have the same impacts on **I**-firms as in the homogeneous firm case. However, the impact on the mass of **V**-firms is ambiguous.

Finally, regarding the configuration where the least productive firms are vertical, it can be shown that the second term in the right-hand side of (54) increases with ω , so that the impact of ω on θ_2^I is ambiguous. However, a deeper interregional integration leads to an increase in θ_2^I . Similarly to the homogeneous firm case, more firms thus choose

to become vertical when transportation costs decrease if (i) communication costs are low enough, i.e., $\omega > \omega^*$ where $\omega^* > \bar{\omega}$, and (ii) regions are sufficiently asymmetric, i.e., $S \in (S^*, 1/K)$, where $S^* > \underline{S}$.

8 Concluding remarks

Our analysis has shown that neglecting communication costs as a specific determinant of firms' spatial structure is unwarranted. On the contrary, understanding how firms organize their activities across regions requires a clear distinction between communication and transportation costs because these costs may affect firms' choices differently. In particular, both costs often have opposite impacts on the geography of production. Since the social optimum also involves diversification under conditions similar to those obtained at the market equilibrium, the diversification of organizational forms is driven by the same fundamentals of the economy, especially transportation and communication costs.

Furthermore, we have put aside several variables that interact with spatial frictions to determine the geography of production. However, insulating the sole effects of transportation and communication costs has allowed us to uncover that policies which reduce either transportation costs or communication costs need not have the same effects on space-economy. In other words, caution is needed when planning new transportation infrastructure. For example, airports are likely to be more effective than other infrastructure for knowledge-intensive activities in which frequent face-to-face contacts are required. Obviously, more work is called for here.

Last, can our results be useful to the empirical researcher? We believe that the answer is yes. Wage and technological differences are likely to be the main drivers in firms' organizational choices when they compete across unevenly developed countries. However, at the interregional, subnational level, our analysis suggests that competition on the product market is another important force. Although very simple, the setting used in this paper is sufficient to show how competition on the product market affects the way firms choose their organizational forms, thus suggesting to add market imperfections to the list of explanatory variables. Another challenge is to find smart proxies for commuting costs. Nowadays, such costs are primarily associated with moving business people among spatially separated firm units. Therefore, communication costs are related to travel time between locations. One of the possible strategies, proposed by Giroud (2013) and Charnoz et al. (2018), is to link communication costs to the opening new direct airline or high-speed railway routes.

Appendix A

Step 1. Assume that $\phi < \omega < 1$. We show that one region hosts only one type of firms at any mixed equilibrium.

Since only the most productive firms can afford to invest in two plants, the **I**-firms (if any) are the most efficient ones. Consequently, the least productive firms are integrated or vertical. In the first case, the least productive region- j firms are integrated: $1 < \theta_j^V < \theta_j^H < \bar{\theta}$, where θ_j^V and θ_j^H are the productivity thresholds such that a **I**-firm has a productivity $\theta_j < \theta_j^V$, a **V**-firm has a productivity $\theta_j^V < \theta_j < \theta_j^H$, while a **H**-firm has a productivity $\theta_j > \theta_j^H$. The equilibrium conditions are given by $\pi_j^I(\theta_j^V) = \pi_j^V(\theta_j^V)$. In the second, the least productive region- j firms are vertical, i.e., $1 < \theta_j^I < \theta_j^H < \bar{\theta}$. In the former case, and $\pi_j^V(\theta_j^H) = \pi_j^H(\theta_j^H)$; the equilibrium conditions are

$$\pi_j^I(\theta_j^I) = \pi_j^V(\theta_j^I) \text{ and } \pi_j^I(\theta_j^H) = \pi_j^H(\theta_j^H).$$

Using (13)-(15), the equilibrium conditions are in both cases equivalent to

$$\Delta_i^* = \left(\theta_j^H\right)^{\sigma-1} \cdot \frac{\omega - \phi}{\sigma f} s_i, \quad (\text{A.1})$$

$$\Delta_j^* = \left(\theta_j^H\right)^{\sigma-1} \cdot \frac{1 - \phi\omega}{\sigma f} s_j, \quad (\text{A.2})$$

while the mirror-image equations for region i are as follows:

$$\Delta_j^{**} = \left(\theta_i^H\right)^{\sigma-1} \cdot \frac{\omega - \phi}{\sigma f} s_j, \quad (\text{A.3})$$

$$\Delta_i^{**} = \left(\theta_i^H\right)^{\sigma-1} \cdot \frac{1 - \phi\omega}{\sigma f} s_i. \quad (\text{A.4})$$

At any equilibrium in which one region hosts the three types of firms and the other two or three types, at least two of the following conditions must hold: (i) $\Delta_i^* = \Delta_i^{**}$ and (ii) $\Delta_j^* = \Delta_j^{**}$. However, $\omega - \phi \neq 1 - \phi\omega$ because $\omega < 1$. This implies $\Delta_i^* \neq \Delta_i^{**}$ and $\Delta_j^* \neq \Delta_j^{**}$. Hence, we have: (i) $\pi_j^V(\theta_j^H) \neq \pi_j^H(\theta_j^H)$ must hold when $\pi_i^I(\theta_i^H) = \pi_i^H(\theta_i^H)$; (ii) $\pi_j^I(\theta_j^H) \neq \pi_j^H(\theta_j^H)$ when $\pi_i^V(\theta_i^H) = \pi_i^H(\theta_i^H)$; or (iii) $\pi_j^I(\theta_j^H) \neq \pi_j^V(\theta_j^H)$ when $\pi_i^I(\theta_i^H) = \pi_i^V(\theta_i^H)$. As a result, region i can host only one type of firms when region- j firms are fully diversified.

Step 2. Assume now that $\phi < \omega = 1$. It follows from (A.1)-(A.4) that

$$\frac{\Delta_i^*}{\Delta_j^*} = \frac{\Delta_i^{**}}{\Delta_j^{**}} = \frac{s_i}{s_j}, \quad (\text{A.5})$$

which implies $\theta^H \equiv \theta_i^H = \theta_j^H$. Therefore, using (12) and the equilibrium conditions $\pi_i^H(\theta_i^H) = \pi_i^V(\theta_i^V)$ and $\pi_i^V(\theta_i^V) = \pi_i^I(\theta_i^V)$ for $i = 1, 2$, as well as $n_i + v_i + h_i = s_i$ for $i = 1, 2$, we obtain:

$$\Delta_i^* = A \cdot [s_i F + s_j F - (1 - \phi)(n_i^* + v_i^*)] = \left(\theta^H\right)^{\sigma-1} \cdot \frac{1 - \phi}{\sigma f} s_i, \quad (\text{A.6})$$

$$\Delta_j^* = A \cdot [\phi s_i F + s_j F - (1 - \phi)(n_i^* + v_j^*)] = \left(\theta^H\right)^{\sigma-1} \cdot \frac{1 - \phi}{\sigma f} s_j. \quad (\text{A.7})$$

where A is a normalization constant given by

$$A \equiv \frac{\beta}{\beta - \sigma + 1} \cdot \frac{\bar{\theta}^\beta - \bar{\theta}^{\sigma-1}}{\bar{\theta}^\beta - 1} > 0.$$

Hence, we have three equations, that is, (A.5)-(A.7) and four unknowns, n_j , v_i , n_i , v_j . As a result, there is a continuum of equilibria when $\omega = 1$.

Step 3. Consider a mixed equilibrium where the three types of firms coexist in region j . Then, all region- i firms are integrated when the productivity range of these firms is not “too” large, that is, $\pi_i^I(\theta) > \pi_i^V(\theta)$ and $\pi_i^I(\theta) > \pi_i^H(\theta)$ for all $\theta \in [1, \bar{\theta}]$ when $\bar{\theta}$ does not exceed some threshold.

Plugging (24) and (23) into (13)-(15) yields

$$\begin{aligned} \pi_i^I(\theta) - \pi_i^V(\theta) &= \frac{\theta^{\sigma-1} f}{\left(\theta_i^H\right)^{\sigma-1}} \left(\frac{1 - \phi\omega}{\omega - \phi} - \frac{\omega - \phi}{1 - \phi\omega} \right), \\ \pi_i^I(\theta) - \pi_i^H(\theta) &= f \cdot \left[1 - \left(\frac{\theta}{\theta_i^H} \right)^{\sigma-1} \cdot \frac{\omega - \phi}{1 - \phi\omega} \right]. \end{aligned}$$

First, we have $\pi_i^I(\theta) > \pi_i^V(\theta)$ for all θ because $1 - \phi\omega > \omega - \phi$. Second, since $\pi_i^I(\theta) - \pi_i^H(\theta)$ is decreasing in θ , $\pi_i^I(\theta) - \pi_i^H(\theta) > 0$ holds if

$$\pi_i^I(\bar{\theta}) - \pi_i^H(\bar{\theta}) > 0 \Leftrightarrow \bar{\theta} \leq K^{\frac{1}{\sigma-1}} \cdot \theta_i^H, \quad (\text{A.8})$$

where K is given by (25). In the worst case, $\theta_i^H \approx 1$ so that the desired inequality holds if $\bar{\theta} \leq K^{\frac{1}{\sigma-1}}$.

When firms are homogeneous ($\theta_i^H = 1$), (A.8) reduces to $\omega - \phi < 1 - \phi\omega$, which always holds. Q.E.D.

Appendix B

We first determine the candidate equilibrium values n_j^*, v_j^*, h_j^* when $n_i^* = s_i$ and, then, find the conditions for (20)–(22) to be positive. Finally, we show that $i = 1$ and $j = 2$.

Step 1. Substituting $v_i^* = 0$ into (12) and setting $A = 1$ leads to $\Delta_i^* = s_i + \omega s_j - (\omega - \phi)n_j^*$. Using (17) thus yields (20) for $j = 2$. Substituting $n_i^* = s_i$ and Δ_j^* into (12) yields (21) for $j = 2$. Substituting v_j^* and n_j^* into the condition $n_j + v_j + h_j = s_j$, we obtain (22) for $j = 2$.

Step 2. Set $S = s_j/s_i$. Using (20)–(22), the inequalities $n_j^* > 0$, $v_j^* > 0$ and $h_j^* > 0$ are, respectively, equivalent to the following conditions:

$$\sigma f > \frac{\omega - \phi}{1 + \omega S} \quad \sigma f > \frac{(1 - \phi\omega)S}{\phi + S} \quad \sigma f < \frac{(\omega - \phi)(1 - \phi\omega)(1 + S)}{(1 - \phi^2)(1 + \omega S)},$$

which amounts to (23) where

$$B_L \equiv \max \left\{ \frac{\omega - \phi}{1 + \omega S}, \frac{(1 - \phi\omega)S}{\phi + S} \right\} \quad \text{and} \quad B_R \equiv \frac{(\omega - \phi)(1 - \phi\omega)(1 + S)}{(1 - \phi^2)(1 + \omega S)}.$$

Step 3. Observe first that the inequality

$$\frac{\omega - \phi}{1 + \omega S} < \frac{(\omega - \phi)(1 - \phi\omega)(1 + S)}{(1 - \phi^2)(1 + \omega S)}$$

may be rewritten as follows:

$$S > \frac{\phi}{K}.$$

Furthermore, the inequality

$$\frac{(1 - \phi\omega)S}{\phi + S} < \frac{(\omega - \phi)(1 - \phi\omega)(1 + S)}{(1 - \phi^2)(1 + \omega S)}$$

is equivalent to

$$F(S) \equiv \phi K S^2 + (K - 1)S - \phi < 0. \quad (\text{B.1})$$

Let $\bar{S}$ be the positive root of $F(S) = 0$. Since $F(\phi/K) < 0$ and $F(1/K) > 0$, the condition (23) holds if and only if

$$\frac{\phi}{K} < S < \bar{S} < \frac{1}{K},$$

which implies $S < 1$. Therefore, it must be that $i = 1$ and $j = 2$.

Step 4. Since $1/(1 + \omega S)$ is decreasing in S while $S/(\phi + S)$ is increasing, the latter is smaller than the former if and only if this inequality holds when S takes on its lowest value, that is, $S = \phi/K$. Therefore, (23) and (24) are necessary and sufficient for Proposition 2 to hold. Q.E.D.

Appendix C

The proof involves several steps. First, we show that the solutions to the first-order conditions for, say, region i cannot be all positive and determine the optimal values of n_i , v_i and h_i under the assumption that the solutions to the first-order conditions for region j are strictly positive (Steps 1 and 2). Then, we determine the necessary and sufficient conditions for region j 's solutions to be strictly positive (Step 3), while Step 4 shows that the obtained solutions maximize the total welfare W .

The first letter in the subscript of a variable stands for the firm's HQ location while the second letter denotes the supplied market. We use the constraint $h_j = s_j - n_j - v_j > 0$ to replace h_j in U_i and W .

Step 1. Assume that the optimal solution is such that all three variables are strictly positive in region j . Differentiating (28) yields the following system of equations:

$$\begin{aligned}\frac{\partial W}{\partial n_j} &= s_j \frac{\partial U_j}{\partial n_j} + s_i \frac{\partial U_i}{\partial n_j} - C_j^I + C_j^H = 0, \\ \frac{\partial W}{\partial v_j} &= s_j \frac{\partial U_j}{\partial v_j} + s_i \frac{\partial U_i}{\partial v_j} - C_j^V + C_j^H = 0,\end{aligned}\tag{C.1}$$

and

$$\begin{aligned}\frac{\partial W}{\partial x_{jj}^I} &= s_j \frac{\partial U_j}{\partial x_{jj}^I} - n_j \frac{\partial C_j^I}{\partial x_{jj}^I} = 0 \Leftrightarrow x_{jj}^I = \left(\frac{1}{c\Omega_j} \right)^\sigma, \\ \frac{\partial W}{\partial x_{jj}^V} &= s_j \frac{\partial U_j}{\partial x_{jj}^V} - v_j \frac{\partial C_j^V}{\partial x_{jj}^V} = 0 \Leftrightarrow x_{jj}^V = \left(\frac{1}{\tau\gamma c\Omega_j} \right)^\sigma, \\ \frac{\partial W}{\partial x_{jj}^H} &= s_j \frac{\partial U_j}{\partial x_{jj}^H} - h_j \frac{\partial C_j^H}{\partial x_{jj}^H} = 0 \Leftrightarrow x_{jj}^H = \left(\frac{1}{c\Omega_j} \right)^\sigma, \\ \frac{\partial W}{\partial x_{ij}^I} &= s_j \frac{\partial U_j}{\partial x_{ij}^I} - n_i \frac{\partial C_j^I}{\partial x_{ij}^I} = 0 \Leftrightarrow x_{ij}^I = \left(\frac{1}{\tau c\Omega_j} \right)^\sigma, \\ \frac{\partial W}{\partial x_{ij}^V} &= s_j \frac{\partial U_j}{\partial x_{ij}^V} - v_i \frac{\partial C_j^V}{\partial x_{ij}^V} = 0 \Leftrightarrow x_{ij}^V = \left(\frac{1}{\gamma c\Omega_j} \right)^\sigma, \\ \frac{\partial W}{\partial x_{ij}^H} &= s_j \frac{\partial U_j}{\partial x_{ij}^H} - h_i \frac{\partial C_j^H}{\partial x_{ij}^H} = 0 \Leftrightarrow x_{ij}^H = \left(\frac{1}{\gamma c\Omega_j} \right)^\sigma,\end{aligned}\tag{C.2}$$

where

$$\Omega_j \equiv n_j(x_{jj}^I)^{\frac{\sigma-1}{\sigma}} + v_j(x_{jj}^V)^{\frac{\sigma-1}{\sigma}} + h_j(x_{jj}^H)^{\frac{\sigma-1}{\sigma}} + n_i(x_{ij}^I)^{\frac{\sigma-1}{\sigma}} + v_i(x_{ij}^V)^{\frac{\sigma-1}{\sigma}} + h_i(x_{ij}^H)^{\frac{\sigma-1}{\sigma}}.$$

Substituting (C.2) into Ω_j , we obtain

$$\Omega_j^\sigma = \left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \Lambda_j,$$

where

$$\Lambda_j \equiv s_j + \omega s_i - (\omega - \phi)n_i - (1 - \phi\omega)v_j\tag{C.3}$$

Furthermore, plugging (C.2) into the cost functions, we obtain:

$$C_j^I = f + \frac{s_j}{\Lambda_j} + \frac{s_i\phi}{\Lambda_i},\tag{C.4}$$

$$C_j^V = f + \frac{s_j\phi\omega}{\Lambda_j} + \frac{s_i\omega}{\Lambda_i},\tag{C.5}$$

$$C_j^H = 2f + \frac{s_j}{\Lambda_j} + \frac{s_i\omega}{\Lambda_i}.\tag{C.6}$$

Differentiating U_j and U_i with respect to n_j and v_j and plugging (C.2) in the resulting expressions, we obtain the following system of 4 equations:

$$\frac{\partial U_j}{\partial n_j} = \frac{\partial U_i}{\partial v_j} = 0, \quad (C.7)$$

$$\frac{\partial U_i}{\partial n_j} = \frac{\sigma}{\sigma-1}(\phi-\omega)\frac{1}{\Lambda_i} < 0, \quad (C.8)$$

$$\frac{\partial U_j}{\partial v_j} = \frac{\sigma}{\sigma-1}(\phi\omega-1)\frac{1}{\Lambda_j} < 0. \quad (C.9)$$

Substituting (C.4)–(C.6) and (C.7)–(C.9) into (C.1) and solving for Λ_i and Λ_j yields the following expressions:

$$\Lambda_j = \frac{s_j(1-\phi\omega)}{(\sigma-1)f} \quad \Lambda_i = \frac{s_i(\omega-\phi)}{(\sigma-1)f}, \quad (C.10)$$

which must hold at any interior optimal solution.

Step 2. Differentiating W with respect to n_i , using (C.4), (C.6), (C.7) and (C.8) in terms of i instead of j , and plugging (C.10) in the resulting expression yields:

$$\frac{\partial W}{\partial n_i} = \frac{(1-\omega)(1+\phi)}{1-\phi\omega}f > 0. \quad (C.11)$$

Therefore, the optimal solution cannot be interior. Moreover, it follows from (C.11) that $n_i^o = s_i$, hence $v_i^o = h_i^o = 0$, always maximize W when region $j \neq i$ accommodates the three types of firms at the optimum.

Step 3. We now show when the first-order conditions for region j yield a strictly positive solution when $n_i^o = s_i$ and $v_i^o = h_i^o = 0$. Setting $n_i = s_i$ and $v_i = h_i = 0$ into Λ_i and Λ_j defined in (C.3) yields the following two expressions:

$$\Lambda_j = s_j + \phi s_i - (1-\phi\omega)v_j \quad \Lambda_i = s_i + \omega s_j - (\omega-\phi)n_j. \quad (C.12)$$

Equalizing (C.10) and (C.12) leads to two equations in n_j and v_j , which have a unique solution given by (30) and (31). As for (32), it is given by $h_j^o = s_j - n_j^o - v_j^o$. These *three solutions* are positive if and only if the following conditions hold:

$$(\sigma-1)f > \frac{(\omega-\phi)s_i}{s_i + \omega s_j} \quad (\sigma-1)f > \frac{s_j(1-\phi\omega)}{s_j + \phi s_i} \quad (\sigma-1)f < \frac{(\omega-\phi)(1-\phi\omega)}{(1-\phi^2)(s_i + \omega s_j)},$$

which are equivalent to (29). Given $n_i^o = s_i$ and $v_i^o = h_i^o = 0$, (30)–(32) are, therefore, positive and the unique solution to the first-order conditions $\partial W/\partial n_j = \partial W/\partial v_j = \partial W/\partial h_j = 0$. If (29) holds, it must be $n_i^o = s_i$ and $v_i^o = h_i^o = 0$ because the solutions to the first-order conditions for region j are strictly positive.

Step 4. We now check that (30) and (31) maximize $W(n_j, v_j, s_j - n_j - v_j, n_i^o, v_i^o, h_i^o)$. Substituting the cost functions (C.4)–(C.6) and the first-order conditions (C.7)–(C.9) into (C.1), we obtain the following two expressions:

$$\frac{\partial W}{\partial n_j} = f - \frac{s_i(\omega-\phi)}{\Lambda_i} \frac{1}{\sigma-1} \quad \frac{\partial W}{\partial v_j} = f - \frac{s_j(1-\phi\omega)}{\Lambda_j} \frac{1}{\sigma-1}.$$

Differentiating (C.12) yields:

$$\frac{\partial \Lambda_i}{\partial n_j} = -(\omega-\phi) \quad \frac{\partial \Lambda_j}{\partial v_j} = -(1-\phi\omega) \quad \frac{\partial \Lambda_i}{\partial v_j} = \frac{\partial \Lambda_j}{\partial n_j} = 0.$$

It is thus readily verified that the Hessian

$$\begin{pmatrix} \frac{\partial^2 W}{\partial n_j^2} & \frac{\partial^2 W}{\partial n_j \partial v_j} \\ \frac{\partial^2 W}{\partial v_j \partial n_j} & \frac{\partial^2 W}{\partial v_j^2} \end{pmatrix} = \begin{pmatrix} -\frac{s_i(\omega-\phi)^2}{\Lambda_i^2} \frac{1}{\sigma-1} & 0 \\ 0 & -\frac{s_j(1-\phi\omega)^2}{\Lambda_j^2} \frac{1}{\sigma-1} \end{pmatrix}$$

has the following characteristic equation:

$$\lambda^2 + \frac{1}{\sigma - 1} \left[\frac{s_i(\omega - \phi)^2}{\Lambda_i^2} + \frac{s_j(1 - \phi\omega)^2}{\Lambda_j^2} \right] \lambda + \left(\frac{1}{\sigma - 1} \right)^2 \frac{s_i(\omega - \phi)^2}{\Lambda_i^2} \frac{s_j(1 - \phi\omega)^2}{\Lambda_j^2} = 0,$$

which has two negative eigenvalues. Therefore, when (29) holds (30) and (31) maximize $W(n_j, v_j, h_j, n_i, v_i, h_i)$.

Step 5. Finally, for $B_L < B_R$, we know from Appendix B that S is smaller than 1. This implies that $i = 1$ and $j = 2$. Q.E.D.

Appendix D

In what follows, we focus on **I - IH**, **I - HV**, and **I - IV**, which are the neighboring configurations of the mixed equilibrium. The corresponding equilibrium masses of firms are as follows. Proofs are given in the Supplementary Material.

Case 1. **I - IH**

$$n_2^* = \frac{1}{1 + S} \cdot \left(\frac{1 + \omega S}{\omega - \phi} - \frac{1}{\sigma f} \right) \quad h_2^* = s_2 - n_2^*, \quad (\text{C.1})$$

when

$$\max \left\{ \frac{(\omega - \phi)S}{\phi + S}, \frac{\omega - \phi}{1 + \omega S} \right\} < \sigma f < \min \left\{ \frac{(1 - \phi\omega)S}{\phi + S}, \frac{\omega - \phi}{1 + \phi S} \right\}$$

and $S > \hat{S}$, where $\hat{S}$ is the positive root of $\omega K S^2 + (K - 1)S - \phi = 0$, hold.

Case 2. **I - HV**

$$h_2^* = \frac{1}{1 + S} \cdot \left(\frac{S}{\sigma f} - \phi \frac{1 + \omega S}{1 - \phi\omega} \right) \quad v_2^* = s_2 - h_2^*, \quad (\text{C.2})$$

when

$$\frac{(1 - \phi\omega)S}{\phi + S} < \sigma f < \min \left\{ \frac{\omega - \phi}{1 + \omega S}, \frac{(1 - \phi\omega)S}{\phi + \phi\omega S} \right\}$$

and $S < \hat{S}$ hold.

Case 3. **I - IV**

$$v_2^* = \frac{1}{\omega - \phi} \cdot \left[\frac{1}{(1 + S)^2} \cdot \frac{(1 - \phi^2)(1 - \omega)}{1 - \phi\omega} - \frac{1}{1 + S} \cdot \frac{(1 - \phi)(1 - \omega - 2\phi\omega)}{1 - \phi\omega} - \phi \right] \quad n_2^* = s_2 - v_2^*.$$

when

$$\frac{(\omega - \phi)(1 - \phi\omega)(1 + S)}{(1 - \phi^2)(1 + \omega S)} < \sigma f$$

and (24) hold.

Appendix E

We determine the conditions on S for $1 < \theta_2^V < \theta_2^H < \bar{\theta}$ to hold.

Step 1. $\theta_2^V < \theta_2^H$. This inequality holds if and only if the first term in the right-hand side of (52) is negative. Since this inequality must hold for any value of $\bar{\theta}$, it boils down to (B.1) when $\bar{\theta}$ becomes arbitrarily large. Therefore, we have $S < \bar{S}$.

Step 2. $\theta_2^V > 1$. Since $\theta_2^H > 1$ and the right-hand side of (52) decreases with θ_2^H , $\theta_2^V > 1$ holds if and only if the right-hand side of (52) is smaller than 1 at $\theta_2^V = 1$:

$$\frac{\phi KS^2 + (K-1)S - \phi}{(\omega - \phi)KS} \cdot \left[1 - (\bar{\theta})^{-(\beta - \sigma + 1)}\right] + S < 0. \quad (\text{D.1})$$

Since (D.1) must hold for any value of $\bar{\theta}$, it boils down to

$$G_2(S) \equiv \omega KS^2 + (K-1)S - \phi < 0 \quad (\text{D.2})$$

when $\bar{\theta}$ grows indefinitely. Denoting by $\underline{S}$ the positive root of $G_2(S) = 0$, (D.2) holds if and only if $S < \underline{S}$. It is readily verified that $\underline{S} < \bar{S}$. Thus, combining (48) and (D.2), we have $1 < \theta_2^V < \theta_2^H < \bar{\theta}$ if and only if $\phi/K < S < \underline{S}$. Note also that these inequalities imply $\theta_2^H > 1$. Q.E.D.

Appendix F

We determine the conditions on S for $1 < \theta_2^H < \theta_2^V < \bar{\theta}$ to hold.

Step 1. $\theta_2^I < \theta_2^V$. This inequality holds if and only if the first term in the right-hand side (54) is negative:

$$\frac{\omega KS^2 + (K-1)S - \phi}{(1 - \phi\omega)S} \left[1 - (\bar{\theta})^{-(\beta - \sigma + 1)}\right] + 1 > 0,$$

which reduces to

$$G_3(S) \equiv \omega KS^2 + (K - \phi\omega)S - \phi > 0 \quad (\text{E.1})$$

when $\bar{\theta}$ becomes arbitrarily large. The positive root of $G_3(S) = 0$ being given by $S = \phi/K$, (E.1) holds if and only if $S > \phi/K$.

Step 2. $\theta_2^I > 1$. This holds if and only if the right-hand side of (54) is smaller than S at $\theta_2^I = 1$:

$$-\frac{\omega KS^2 + (K-1)S - \phi}{(1 - \phi\omega)S} \left[1 - (\bar{\theta})^{-(\beta - \sigma + 1)}\right] < 0,$$

which is equivalent to

$$G_2(S) > 0. \quad (\text{E.2})$$

when $\bar{\theta}$ becomes arbitrarily large.

Observe that (E.2) is the opposite of (D.2) and holds if and only if $S > \underline{S}$. Summing up, we have $1 < \theta_2^I < \theta_2^V < \bar{\theta}$ if and only if $\underline{S} < S < \bar{S}$. Q.E.D.

Appendix G

First, we study the impact of transportation and commuting costs on the mass of **H**-firms. The left-hand side of (53) is an increasing function of θ_2^H and does not depend on both ϕ and ω . The impact of changes in ϕ and ω on the right-hand side of (53) is captured by the first term, which can be rewritten as follows:

$$\frac{(1 + \phi K)S + \phi + K}{(1 - \phi\omega)(1 + S)} \cdot \left[1 - (\bar{\theta})^{-(\beta - \sigma + 1)}\right] = \frac{1 - (\bar{\theta})^{-(\beta - \sigma + 1)}}{1 + S} \cdot \left[\left(\frac{1}{1 - \phi\omega} + \frac{\phi}{\omega - \phi}\right)S + \frac{\phi}{1 - \phi\omega} + \frac{1}{\omega - \phi}\right]$$

By differentiating this expression with respect to ϕ and ω , we obtain:

$$\left[\left(\frac{1}{1-\phi\omega} + \frac{\phi}{\omega-\phi} \right) S + \frac{\phi}{1-\phi\omega} + \frac{1}{\omega-\phi} \right]_{\phi}' = \left[\frac{\omega}{(1-\phi\omega)^2} + \frac{\omega}{(\omega-\phi)^2} \right] S + \frac{1}{(1-\phi\omega)^2} + \frac{1}{(\omega-\phi)^2} > 0,$$

$$\left[\left(\frac{1}{1-\phi\omega} + \frac{\phi}{\omega-\phi} \right) S + \frac{\phi}{1-\phi\omega} + \frac{1}{\omega-\phi} \right]_{\omega}' = \left[\frac{\phi}{(1-\phi\omega)^2} - \frac{\phi}{(\omega-\phi)^2} \right] S + \frac{\phi^2}{(1-\phi\omega)^2} - \frac{1}{(\omega-\phi)^2} < 0.$$

Therefore, θ_2^H increases with ϕ and decreases with ω , which implies that the mass of **H**-firms decreases (increases) when transportation costs (communication costs) fall.

Second, the left-hand side of (52) increases with ω , while the first term of the right-hand side

$$\frac{\phi K S^2 + (K-1)S - \phi}{(\omega-\phi)KS} \cdot \left[1 - (\bar{\theta})^{-(\beta-\sigma+1)} \right] = \frac{1 - (\bar{\theta})^{-(\beta-\sigma+1)}}{S} \cdot \left[\frac{\phi}{\omega-\phi} S^2 + \left(\frac{1}{\omega-\phi} - \frac{1}{1-\phi\omega} \right) S - \frac{\phi}{1-\phi\omega} \right]$$

decreases with ω :

$$-\frac{\phi}{(\omega-\phi)^2} S^2 - \left[\frac{1}{(\omega-\phi)^2} + \frac{\phi}{(1-\phi\omega)^2} \right] S - \frac{\phi^2}{(1-\phi\omega)^2} < 0.$$

Therefore, θ_2^V decreases with ω , which leads to fewer **I**-firms.

Third, the left-hand side of (52) decreases with ϕ , while the behavior of first term in the right-hand side of (52) is a priori undetermined:

$$\left[\frac{\phi}{\omega-\phi} S^2 + \left(\frac{1}{\omega-\phi} - \frac{1}{1-\phi\omega} \right) S - \frac{\phi}{1-\phi\omega} \right]_{\phi}' = \frac{\omega K^2 S^2 + (K^2 - \omega)S - 1}{(1-\phi\omega)^2}.$$

The right-hand side of this expression has a unique positive root smaller than 1. Since the range of regions' asymmetry we work with is $\phi/K < S < \bar{S} < 1/K$, the derivative is positive at $S = 1/K$:

$$\left. \frac{\omega K^2 S^2 + (K^2 - \omega)S - 1}{(1-\phi\omega)^2} \right|_{S=\frac{1}{K}} = \frac{\omega K^2 \frac{1}{K^2} + (K^2 - \omega) \frac{1}{K} - 1}{(1-\phi\omega)^2} = \frac{(K-1)(\frac{\omega}{K} + 1)}{(1-\phi\omega)^2} > 0.$$

When $S = \phi/K$, the derivative

$$\left. \frac{\omega K^2 S^2 + (K^2 - \omega)S - 1}{(1-\phi\omega)^2} \right|_{S=\frac{\phi}{K}} = \frac{\omega K^2 \frac{\phi^2}{K^2} + (K^2 - \omega) \frac{\phi}{K} - 1}{(1-\phi\omega)^2} = \frac{\left(\frac{\omega\phi}{K} + 1 \right) (\phi K - 1)}{(1-\phi\omega)^2}$$

is also positive if $\phi K - 1 > 0$, which is equivalent to

$$\omega < \bar{\omega} = \frac{2\phi}{1+\phi^2}.$$

In sum, θ_2^V decreases with ϕ for all admissible regions' degrees of asymmetry when communication costs are not too large, i.e., $\omega < \bar{\omega}$.

Last, the left-hand side of (54) decreases with ϕ . In the right-hand side, only the second term given by

$$-\frac{\omega K S^2 + (K-1)S - \phi}{(1-\phi\omega)S} \left[1 - (\bar{\theta})^{-(\beta-\sigma+1)} \right] = -\frac{1 - (\bar{\theta})^{-(\beta-\sigma+1)}}{S} \left[\frac{\omega}{\omega-\phi} S^2 + \left(\frac{1}{\omega-\phi} - \frac{1}{1-\phi\omega} \right) S - \frac{\phi}{1-\phi\omega} \right].$$

is affected by ϕ . By differentiating the above expression, we obtain:

$$-\left[\frac{\omega}{\omega-\phi} S^2 + \left(\frac{1}{\omega-\phi} - \frac{1}{1-\phi\omega} \right) S - \frac{\phi}{1-\phi\omega} \right]_{\phi}' = -\frac{\omega K^2 S^2 + (K^2 - \omega)S - 1}{(1-\phi\omega)^2},$$

which is negative when $S = 1/K$ and positive at $S = \phi/K$ if $\omega > \bar{\omega}$. Moreover, when $\omega = 1$ the derivative is positive for $S = \underline{S}$. Therefore, $\omega^* > \bar{\omega}$ exists such that for $\omega > \omega^*$, there is a threshold value S^* such that the derivative is positive for $S \in (S^*, 1/K)$. Hence, θ_2^I increases with ϕ .

Bibliographie

- Aarland, K., Davis, J.C., Henderson, J.V., Ono, Y., 2007. Spatial organization of firms: The decision to split production and administration. *RAND J. Econ.* 38, 480-494.
- Acosta, C., Lyngemark, D.H., 2019. The internal spatial organization of firms: Evidence from Denmark. University of Toronto, memo.
- Antràs, P., Rossi-Hansberg, E., 2009. Organizations and trade. *Annual Rev. Econ.* 1, 43-64.
- Antràs, P., Yeaple, S.R., 2014. Multinational firms and the structure of international trade. In: Gopinath, G., Helpman, E., Rogoff, E. (Eds.), *Handbook of International Economics*, Volume 4, Elsevier, Amsterdam, pp. 55-130.
- Baldwin, R.E., 2016. *The Great Convergence*. Harvard University Press, Cambridge (MA).
- Baldwin, R.E., Forslid, R., Martin, Ph., Ottaviano, G.I.P., Robert-Nicoud, F., 2003. *Economic Geography and Public Policy*. Princeton University Press, Princeton (NJ).
- Battiston, D., Blanes i Vidal, J., Kirchmaier, T., 2017. Is distance dead? Face-to-face communication and productivity in teams. CEPR Discussion Paper N°11924.
- Behrens, K., Picard, P.M., 2007. Welfare, home market effects, and horizontal foreign direct investment. *Canadian J. Econ.* 40, 1118-148.
- Bergstrand, J.H., Egger, P., Larch, M., 2013. Gravity *Redux*: Estimation of gravity-equation coefficients, elasticities of substitution, and general equilibrium comparative statics under asymmetric bilateral trade costs. *J. Int. Econ.* 89, 110-121.
- Charnoz, P., Lelarge, C., Trevien, C., 2018. Communication costs and the internal organization of multi-plant businesses: Evidence from the impact of the French high-speed rail. *Econ. J.* 128 (610), 949-994.
- Dhingra, S., Morrow, J., 2019. Monopolistic competition and optimum product diversity under firm heterogeneity. *J. Pol. Econ.* 127, 196-232.
- Duranton, G., Puga, D., 2005. From sectoral to functional urban specialisation. *J. Urban Econ.* 57, 343-370.
- Elberfeld, W., 2003. A note on technology choice, firm heterogeneity and welfare. *Int. J. Indus. Organization* 21, 593-605.
- Fink, C., Mattoo, A., Neagu, I.C., 2005. Assessing the impact of communication costs on international trade. *J. Int. Econ.* 67, 428-445.
- Fujita, M., Gokan, T., 2005. On the evolution of the spatial economy with multi-unit · multi-plant firms: The impact of IT development. *Port. Econ. J.* 4, 73-105.
- Fujita, M., Thisse, J.-F., 2006. Globalization and the evolution of the supply chain: Who gains and who loses? *Int. Econ. Rev.* 47, 811-836.

- Gallagher, R.M., 2013. Shipping costs, information costs, and the sources of industrial coagglomeration. *J. Reg. Sci.* 53, 304-331.
- Garicano, L., 2000. Hierarchies and the organization of knowledge in production. *J. Pol. Econ.* 108, 874-904.
- Giroud, X., 2013. Proximity and investment: Evidence from plant-level data. *Q. J. Econ.* 128, 861-915.
- Gumpert, A., 2018. The organization of knowledge in multinational firms. *J. Euro. Econ. Association* 16, 1929–1976.
- Head K. and Mayer T., The empirics of agglomeration and trade, In: Henderson J.V. and Thisse J.F., (Eds.), *Handbook of Regional and Urban Economics*, Volume 4, 2004, Elsevier; Amsterdam, 2609–2669.
- Helpman, E., 2006. Trade, FDI, and the organization of firms. *J. Econ. Lit.* XLIV, 589-630.
- Helpman, E., Melitz, M., Yeaple, S., 2004. Export versus FDI with heterogeneous firms. *Am. Econ. Rev.* 94, 300-316.
- Henderson, J. V., 1997. Medium size cities. *Reg. Sci. Urban Econ.* 27, 583-612.
- Henderson, J.V., Ono, Y., 2008. Where do manufacturing firms locate their headquarters? *J. Urban Econ* 63, 431-450.
- Hillberry, R., Hummels, D., 2008. Trade responses to geographic frictions: A decomposition using micro-data. *Eur. Econ. Rev.* 52: 527–550.
- Kalnins, A. Lafontaine, F., 2013. Too far away? The effect of distance to headquarters on business establishment performance. *Am. Econ. J.: Microeconomics* 5, 157-179.
- Krugman, P. Scale economies, product differentiation, and the pattern of trade, *Am. Econ. Rev.* 70(5), 1980, 950-959.
- Krugman, P., 1995. Growing world trade: Causes and consequences. *Brookings Papers on Economic Activity* 1, 327-342.
- Leamer, E.E., Storper, M., 2001. The economic geography of the Internet Age. *J. Int. Business Stud.* 32, 641-665.
- Markusen, J.R., 2002. *Multinational Firms and the Theory of International Trade*. MIT Press, Cambridge (MA).
- Okubo, T., Tomiura, E. 2016. Multi-plant operation and headquarters separation: Evidence from Japanese plant-level panel data. *Japan and the World Economy* 39, 12-22.
- Ota, M., Fujita, M., 1993. Communication technologies and spatial organization of multi-unit firms in metropolitan areas. *Reg. Sci. Urban Econ.* 23, 695-729.
- Ottaviano, G.I.P., Thisse, J.-F., 2004. Agglomeration and economic geography. In: Henderson, J.V. and Thisse, J.-F. (Eds.), *Handbook of Regional and Urban Economics*, Volume 4, North-Holland, Amsterdam, pp. 2563-2608.
- Redding, S., 2011. Economic geography: A review of the theoretical and empirical literature. In: Bernhofen, D. Falvey, R. Greenaway, D., Kreickemeie, U. (Eds.), *The Palgrave Handbook of International Trade*, Palgrave Macmillan, London, pp. 497-531.

Redding, S.J., Turner, M.A., 2015. Transportation costs and the spatial organization of economic activity. In: Duranton, G., Henderson, J.V., Strange, W.C. (Eds.), *Handbook of Regional and Urban Economics*, Volume 5, Elsevier, Amsterdam, pp. 247-348.

Robert-Nicoud, F., 2008. Offshoring of routine tasks and (de)industrialisation: Threat or opportunity—And for whom? *J.Urban Econ.* 63, 517-535.

Rossi-Hansberg, E., Sarte, P.-D., and Owens III., R., 2009. Firm fragmentation and urban patterns. *Int. Econ. Rev.* 50, 143-186.

Yeaple, S.R., 2003. The complex integration strategies of multinationals and cross country dependencies in the structure of foreign direct investment. *J. Int. Econ.* 60, 293-314.