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Abstract

There are several empirical studies of ex post analysis of citations in academia. There is no ex ante analysis of citations. I consider a game-theoretic model of a contest between scholars on the basis of two widely-used measures of citations (i.e., the h-index and total citation count) and the newly-developed Euclidean index (Perry and Reny, American Economic Review, 2016). I find equilibria in which there are more and better-quality papers in the total citations contest than in the h-index contest. When the marginal cost of effort is constant, the scholars are indifferent between the number of papers and the quality of papers in the total citations contest but prefer quality of papers in the Euclidean contest although the total number of citations is the same in both contests. In some cases, the total citations contest yields the same quality of papers but more papers than the Euclidean contest, a result which holds when the marginal cost of effort is increasing but is not possible when the marginal cost of effort is constant. Consistent with previous empirical results, I find that as the cost of writing a paper increases, the h-index is inferior to the total citations index in both the quality and quantity of papers. This result is driven by how the cost of effort constrains the number of papers that a scholar can write and how the number of papers, in turn, constrains how the h-index counts citations.

JEL-Codes: D720.

Keywords: citation count, contests, Euclidean index, h-index, integer programming game.

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1. Introduction

A contest is a game in which players compete over a prize or set of prizes by making irreversible outlays or expenditures. This paper studies citations in academic contests.

Citations are used by government agencies, foundations, universities and departmental committees, and academic associations to determine funding, promotions, academic awards, teaching loads, etc (Ellison, 2013; Hamermesh, 2018; Perry and Reny, 2016). Ellison (2013), using economists at the top 50 US departments, found that a variant of the h-index of citations is strongly correlated with labor market outcomes. Using data on economists at 88 US economics departments, Hamermesh and Pfann (2012) found that an economist's total number of citations is a significant determinant of his/her salary. Perry and Reny (2016) tested how well the rankings of economists at the top 50 US universities on the basis of two citations indices (i.e., h-index and the Euclidean index) matched the US National Research Council's (NRC) rankings of the departments in which they were employed.

This paper studies a contest in which two scholars compete on the basis of their scholarly citations. The contestants have the option of writing more than one paper and thus the contest has multiple dimensions of efforts as in Amegashie (2002, 2019), Epstein and Hefeker (2003), Rai and Sarin (2009) and Arbatskaya and Mialon (2010).

I compare the incentive effects of two widely-used measures of citations (i.e., the total citation count and the h-index) and the Euclidean index. The h-index was proposed in Hirsch (2005). A scholar's h-index is the maximum number, h, of his/her papers that each have at least h citations (Hirsch, 2005). The Euclidean index is the Euclidean length of a scholar's citation list. The Euclidean index was axiomatically derived in a recent paper (Perry and Reny, 2016) while the h-index and the total citations index were axiomatized in Woeginger (2008) and

Marchant (2009).[1] Palacios-Huerta and Volij (2014) present a brief but interesting survey of various methods for ranking scholars and journals, which includes the three methods studied in this paper.

While the h-index is very popular and intuitively appealing, it has serious shortcomings that are very well known (e.g., Barnes, 2017; Egghe 2006; Lehmann et al, 2006; Perry and Reny, 2016; Waltman and van Eck, 2012; Yong, 2014). Ellison (2013) found that it was rather a *modification* of the h-index (i.e., the $h(a,b)$-index defined above) that performed better than the total citations index. Anderson (2017) and Ng (2017) have pointed out some shortcomings of the Euclidean index. A common critique of the total citation count is that it does not take the quality of journals into account and may be influenced a few highly-cited papers.

Perry and Reny (2016, p. 2724) note that their "... Euclidean index is intended to judge an individual's record *as it stands*. It is not intended as means to predict an individual's record at some future date." This point is applicable to any citations index. Therefore, to get a sense of the incentive effect of a citations index, one may have to resort to the kind of formal game-theoretic analysis in this paper. The approach in this paper is consistent with Perry and Reny's (2016, p. 2724) point that "If a prediction is important to the decision at hand (as in tenure decisions, for example) then separate methods must first be used to obtain a predicted citation list to which our index can then be applied." The predicted citation list in this paper is the Nash equilibrium of an academic contest that uses the Euclidean index as one of three citation indices. The results show that such *ex ante* analysis could clarify and confirm some results of *ex post* analysis of citations. It also leads to some different results.

[1]Ellison (2013) defines a variant of the h-index, $h(a,b)$-index which is the largest number h such that a researcher has at least h papers with ah^b citations each, where a and b are positive numbers. He estimates a and b to fit the data on the rankings of economists at the top 50 US universities. Kosmulski (2006) used $a = 1$ and $b = 2$ in his analysis.

I find that the nature of the marginal cost of effort matters. For example, when the marginal cost of effort is constant, the scholars are indifferent between the number of papers and the quality of papers in the total citations contest but prefer quality of papers in the Euclidean contest although the total number of citations is the *same* both contests. When the marginal cost is increasing, this result still holds in the Euclidean contest but is reversed in the total citations contest in the sense that the scholars prefer number of papers to the quality of papers. In some cases, the total citations index dominates the Euclidean index in the sense that the total citations contest yields the *same* quality of papers but *more* papers than the Euclidean contest. This result is not possible when the marginal cost of effort is constant. When there are differences in the qualities of journals, the scholars may strictly prefer quality of papers to the number of papers in the total citations contest, even if the marginal cost of effort is constant.

I also find that as the cost of writing a paper increases, the *h*-index is inferior to the total citations index in both the quality and quantity of papers. This result is driven by how the cost of effort constrains the number of papers that a scholar can write and how the number of papers, in turn, constrains how the *h*-index counts citations. This result is consistent with the empirical results in Ellison (2013), Hirsch (2007), Kosmulski (2006), and Yong (2014).

The paper is organized as follows: the next section presents a game-theoretic model and analysis of an academic contest. Section 3 discusses the results. Section 4 considers an extension in which the players can write more papers and another in which there are differences in the qualities of journals. Section 5 concludes the paper.

2. An academic contest

Consider an academic contest with two risk-neutral and identical scholars, A and B, who can write a maximum of $N = 2$ papers, 1 and 2. If player A invests an effort of x_k in paper k, it costs him cx_k^α and the papers gets x_k citations, where $\alpha > 0$, $c > 0$, and $k = 1,2$. If player B invests an effort of y_k in paper k, it costs him cy_k^α and the papers gets y_k citations, where $k = 1,2$. I normalize the value of winning the contest to 1 and assume that a zero effort gives a non-positive payoff.

Given cx_k^α , it is obvious that the cost of writing a more-cited paper is higher than the cost of writing a less-cited paper. I do not make any distinction between the quality or impact factor of journals and assume that cited papers differ only in their number of citations but otherwise have common characteristics.[2] I shall later relax this assumption.

Winning the contest is based on citations and I consider three criteria: total citation count (hereafter total citations); the h-index; and the Euclidean index in Perry and Reny (2016). Under total citations, the players' winning probabilities are:

$$P_A^T = \frac{X}{X+Y} \text{ and } P_B^T = \frac{Y}{X+Y}, \tag{1}$$

where $X \equiv x_1 + x_2$ and $Y \equiv y_1 + y_2$.

Similarly, when the Euclidean index is used, the players' winning probabilities are:

$$P_A^E = \frac{\sqrt{\sum_{k=1}^2 x_k^2}}{\sqrt{\sum_{k=1}^2 x_k^2}+\sqrt{\sum_{k=1}^2 y_k^2}} \text{ and } P_B^E = \frac{\sqrt{\sum_{k=1}^2 y_k^2}}{\sqrt{\sum_{k=1}^2 x_k^2}+\sqrt{\sum_{k=1}^2 y_k^2}}, \tag{2}$$

[2]Perry and Reny (2016) made the same assumption but later relaxed it by considering differences in average citations in different fields. Haley (2019a, 2019b) incorporates differences in journal quality in the index developed in Perry and Reny (2016). For a discussion of a different but related set of issues, see Chambers and Miller (2014).

where $\sqrt{\sum_{k=1}^{2} x_k^2}$ and $\sqrt{\sum_{k=1}^{2} y_k^2}$ are the Euclidean citation indices for players A and B respectively (Perry and Reny, 2016).[3] Finally, for the h-index contest the winning probabilities are

$P_A^h = \frac{h_A}{h_A+h_B}$ and $P_B^h = \frac{h_B}{h_A+h_B}$,

where h_j is the h-index of player $j = A, B$.

Note that a player with a higher citation index does not necessarily win the contest. Thus, citations alone do not determine success in the contest. This is consistent with their point that "… no single index number is intended to be sufficient for making decisions about funding, promotion, etc. It is but one tool among many for such purposes."[4]

I note that the number of citations per paper, x_k and y_k, must be a non-negative integer (hereafter integer), $k = 1,2$.[5] Thus, the game is an *integer programming game* (e.g., Koppe, Ryan, and Queyranne, 2011).

2.1 Constant marginal cost of effort

Suppose the marginal cost of effort is constant. Then α = 1, so the total cost of effort to player A of generating x_k citations for paper k is cx_k and the corresponding cost for player B is cy_k, k = 1,2.

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[3]The total citation count and the *h-index* are more popular than the Euclidean index. RePEc ranks journals using all three indices.

[4]See footnote 1 in Perry and Reny (2016). This is also consistent with Palacious-Huerta and Volij (2004, p. 964) who opined that "Citation analysis, however sophisticated it may be, cannot be a substitute for critical reading and expert judgment." See also Hamermesh (2018).

[5]The total number of citations and the *h*-index are integers but the Euclidean index need not be an integer. If the total number of citations and the *h*-index are divided by number of authors, the resulting index may not be an integer.

It is obvious that if c is too high, no papers will be written. I restrict the analysis to cases in which at least a paper is written in each of the three contests.

I begin with the h-index contest. Recall that a scholar's h-index is the maximum number, h, of his/her papers that each have at least h citations (Hirsch, 2005). Noting that effort is costly, a scholar can write a maximum of *two papers*, and given how the h-index links a scholar's number of citations to his number of papers[6], it follows that a players' effort per paper in the h-contest is restricted to the set $\{0,1,2\}$.

Consider a symmetric equilibrium in the h-index contest in which each contestant invests a total effort of 2 to each paper to generate a citation of 2 and thus an h-index of 2: $x_1^h = y_1^h = x_2^h = y_2^h = 2$. This gives an expected payoff:

$$\Pi_A^h = \Pi_B^h = \frac{1}{2} - c(2+2) \geq 0, \quad (3)$$

if $c \in \left(0, \frac{1}{8}\right]$. Suppose player A deviates to $x_1^h = 1$ and $y_1^h = 0$. Then his h-index is 1 and player B's h-index is 2. Then player A's payoff is $\frac{1}{1+2} - c \leq \frac{1}{2} - 4c$ if $c \leq \frac{1}{18} \in \left(0, \frac{1}{8}\right]$. Therefore, if $c \in \left(0, \frac{1}{18}\right]$, then the Nash equilibrium is $x_1^h = y_1^h = x_2^h = y_2^h = 2$ which gives an h-index of 2.

The Nash equilibrium is $x_1^h = y_1^h = 0$ and $x_2^h = y_2^h = 1$, which holds if and only if $c \in \left[\frac{1}{18}, \frac{1}{2}\right]$, where $c \leq \frac{1}{2}$ ensures that the players have a non-negative expected payoff and $c \geq \frac{1}{18}$ ensures that no player has a profitable deviation to writing 2 papers or no paper.[7] This equilibrium gives an h-index of 1 for each player.

[6]For example, a scholar with two papers each of which has 100 citations has an h-index of 2. I shall return to this issue in section 4.1.

[7]Or it is $x_1^h = y_1^h = 1$ and $x_2^h = y_2^h = 0$. In either case, they write only one paper.

Now consider the Euclidian index contest (hereafter Euclidean contest). The payoffs of players A and B are:

$$\Pi_A^E = \frac{\sqrt{\sum_{k=1}^{2} x_k^2}}{\sqrt{\sum_{k=1}^{2} x_k^2} + \sqrt{\sum_{k=1}^{2} y_k^2}} - c(x_1 + x_2), \tag{4}$$

and

$$\Pi_B^E = \frac{\sqrt{\sum_{k=1}^{2} y_k^2}}{\sqrt{\sum_{k=1}^{2} x_k^2} + \sqrt{\sum_{k=1}^{2} y_k^2}} - c(y_1 + y_2). \tag{5}$$

Consider an equilibrium in the Euclidean contest with efforts x_1^E and x_2^E for player A and y_1^E and y_2^E for player B. Note that P_A^E is strictly increasing in $\sqrt{\sum_{k=1}^{2} x_k^2}$ and P_B^E is strictly increasing in $\sqrt{\sum_{k=1}^{2} y_k^2}$. Also, for any given total effort $x_1^E + x_2^E$, player A maximizes $\sqrt{\sum_{k=1}^{2} x_k^2}$ by allocating all of this total effort to only one paper. This is also true for player B. Thus, without any loss of generality, we can restrict the analysis to a symmetric equilibrium with $x_1^E = y_1^E = 0$ and $x_2^E = y_2^E > 0$. Then (4) and (5) become:

$$\widehat{\Pi}_A^E = \frac{x_2}{x_2 + y_2} - c x_2, \tag{4a}$$

and

$$\widehat{\Pi}_B^E = \frac{y_2}{x_2 + y_2} - c y_2. \tag{5a}$$

For now, assume that x_k and y_k need not be integers. Then the unique Nash equilibrium is:

$$x_1^E = y_1^E = 0 \text{ and } x_2^E = y_2^E = \frac{1}{4c}. \tag{6}$$

Note that if $c \in \left\{\frac{1}{16}, \frac{1}{12}, \frac{1}{8}\right\}$, then $x_2^E = y_2^E = \frac{1}{4c} \in \{2, 3, 4\}$. These values of c belong to $c \in \left[\frac{1}{18}, \frac{1}{2}\right]$, the condition required for the equilibrium in the h-index contest to give an h-index of 1. When c is such that $\frac{1}{4c}$ is not an integer and $c \in \left[\frac{1}{18}, \frac{1}{2}\right]$, the symmetric equilibrium number of citations in the Euclidean contest is at least equal to 1. For example, when $c = 0.06 \in \left[\frac{1}{18}, \frac{1}{2}\right]$ and so $\frac{1}{4c} = 4.166$, the symmetric equilibrium number of citations in the Euclidean contest is 4.[8] When $c = 0.50$ and so $\frac{1}{4c} = 0.50$, the symmetric equilibrium number of citations in the Euclidean contests is 1.[9] Given that $c = 0.50$ is the largest marginal cost in the set $\left[\frac{1}{18}, \frac{1}{2}\right]$ and the equilibrium effort is non-increasing in c, it follows that if $c \in \left[\frac{1}{18}, \frac{1}{2}\right]$, the Euclidean contest induces the scholars to write better quality papers than the paper in the h-index contest. When $c = \frac{1}{18} = 0.055$, the symmetric equilibrium number of citations in Euclidean contest is 5. Given that the symmetric equilibrium effort is non-increasing in c, it follows that if $c \in \left(0, \frac{1}{18}\right]$, the symmetric equilibrium number of citations (per the sole paper) in the Euclidean contest is at least equal to 5. The corresponding equilibrium of citations per paper is only 2 in the h-index contest. This gives the following proposition:

Proposition 1: *Suppose the marginal cost of effort is constant and is sufficiently low. Then more papers are written in the h-index contest than in the Euclidean contest. If the marginal cost of effort is within an intermediate range, each contestant writes a single paper in the h-index contest and Euclidean contest. But in either case, the quality of the paper in the Euclidean*

[8]Given $c = 0.06$, the equilibrium effort of 4 gives a payoff of $0.5 - 4c = 0.26$ in the symmetric equilibrium of the Euclidean contest. To consider a range of deviations, I plotted $\Pi^d = \frac{4+m}{(4+m)+4} - 0.06(4+m)$ on the domains $m \in [-4, -1]$ and $m \in [1, \infty]$. This gave $\Pi^d < 0.26$ in all cases. Given $c = 0.06$, the highest integer effort that gives a non-negative payoff in a symmetric "equilibrium" is 8. Other than 4, there is no equilibrium with an integer effort that is 8 or less.

[9]Given $c = 0.50$, the equilibrium effort of 1 gives a payoff of $0.5 - c = 0$ in the symmetric equilibrium of the Euclidean contest. A deviation by a player to an effort of 2 gives a payoff of $\frac{2}{2+1} - 0.5(2) < 0.$ Zero effort gives a zero payoff. A deviation to an effort of 3 or higher gives a negative payoff.

contest is higher (significantly so in some cases) than the quality of the paper in the h-index contest.

Now consider the total citations contest. The payoffs of players A and B are:

$$\Pi_A^T = \frac{X}{X+Y} - cX, \tag{7}$$

and

$$\Pi_B^T = \frac{Y}{X+Y} - cY. \tag{8}$$

In this game, it is known that the individual efforts in the Nash equilibrium are not unique but the aggregate effort is unique and is given by (see, for example, Nitzan, 1991; Baik, 1993):

$$X_A^T \equiv x_1^T + x_2^T = y_1^T + y_2^T \equiv Y_B^T = \frac{1}{4c}. \tag{9}$$

This is also the equilibrium total number of citations for each player. There are multiple equilibria in the total citations contest.

Note that the equilibrium in (9) also exists in the Euclidean contest. But, unlike the Euclidean contest, the equilibrium in the total citations contest is *not* unique. For example, when $c = \frac{1}{18} = 0.055$, the symmetric equilibrium number of total citations is 5 in the Euclidean contest and total citations contest. But in the Euclidean contest, only one paper is written with 5 citations while in the total citations contest, two papers which have a total of 5 citations may be written. Based on the preceding analysis we know that, regardless of the fact that the number of citations per paper must be an integer, the following proposition holds:

Proposition 2: *Suppose the marginal cost of effort is constant. The total number of citations is the same in the Euclidean and total citations contests. However, in the total citations contest, the contestants are indifferent between the number of papers and the quality of papers. But in the Euclidean contest, the contestants strongly prefer quality of papers to the number of papers.*

2.2 Increasing marginal cost of effort

Now suppose $\alpha = 2$, so that the cost of effort is quadratic (i.e., increasing marginal costs). For the h-index contest, each contestant invests an effort of $x_1^h = y_1^h = x_2^h = y_2^h = 2$ and gets an h-index of 2 if:

$$\frac{1}{2} - c(2^2 + 2^2) = \frac{1}{2} - 8c \geq 0, \qquad (10)$$

and

$$\frac{1}{2} - 8c \geq \frac{1}{3} - c. \qquad (11)$$

The condition in (10) implies that the equilibrium payoff is non-negative and the condition in (11) is the requirement that there is no profitable deviation to an h-index of 1. Both conditions hold and thus the equilibrium h-index = 2 if $0 < c \leq \frac{1}{42}$. It can easily be shown the equilibrium h-index = 1 if $\frac{1}{42} \leq c \leq 0.5$. There are multiple equilibria if $c = \frac{1}{42}$.

Notice that for a *given* aggregate effort and a quadratic cost function, a contestant reduces his cost of total effort by spreading it over the two papers in the least unequal way while maintaining his probability of winning in the total citations contest. In particular, whenever possible, he allocates the *same* effort to each paper. If this not possible because of the integer requirement, then the optimal difference in effort between the two papers is 1. Thus, in the total citations contest, there is now a unique symmetric equilibrium for individual effort per paper.[10]

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[10]This is akin to the team contests in Esteban and Ray (2001) and Nitzan and Ueda (2009) where the two papers in this model are analogous to two players in a team. Note, however, that if the marginal cost of effort is constant but there is diminishing marginal returns from effort to citations, there will also be unique equilibrium for effort per paper.

Because of increasing marginal cost of effort, it is no longer the case that a player in the Euclidean contest will necessarily allocate all of his effort to a single paper. But if c is sufficiently small, then each contestant will allocate all of his effort to a single paper or will not write two papers (see appendix A).[11]

An analytical result in the Euclidean contest is challenging. Using the results in the continuous citations case as a guide (see Table 3), I present some numerical results in Table 1 below:

Table 1: *Equilibrium citations with quadratic cost of effort,* cx_k^2 and cy_k^2, $k = 1,2$ *and* $N = 2$ papers.

c	**Total Citations Contest** $\{x_1^T = y_1^T\}, \{x_2^T = y_2^T\}$	**Euclidean Contest** $\{x_1^E = y_1^E\}, \{x_2^E = y_2^E\}$	***h*-index contest** $\{x_1^h = y_1^h\}, \{x_2^h = y_2^h\}$
0.01	3,2	4,0	2,2
0.02	2,2	{2,0},{3,0}*	2,2
0.03	2,1	2,0	1,0
0.04	1,1	2,0	1,0
0.05	1,1	2,0	1,0
0.06	1,1	1,0	1,0
0.07	1,1	1,0	1,0
0.08	1,1	1,0	1,0
0.09	1,1	1,0	1,0
0.10	1,1	1,0	1,0

*Multiple equilibria

As expected in each contest, the total effort (number of citations) is non-increasing in c. The following propositions hold:

Proposition 3: *Suppose the marginal cost of effort is increasing and* c *is sufficiently small. In the total citations contest, the contestants prefer number of publications to quality of publications in*

[11]This, of course, does *not* necessarily imply that if c is sufficiently high, an equilibrium in the Euclidean contest will have two papers.

the sense that whenever their optimal total effort generates at least two citations, they write two papers instead of one paper. But in the Euclidean contest, the contestants prefer quality of publications to number of publications. Neither the quality or quantity of papers in the h*-index contest is higher than in the total citations contest.*

Proposition 4: *Suppose the marginal cost of effort is increasing and* c *is sufficiently small. In the Euclidean contest, the contestants write only one paper which is more cited than each paper in the h-index contest. If* c *is sufficiently small, the contestants in the h-index contest write two papers and get an h-index of 2 while if* c *is sufficiently big, they write no more than one paper but none of their papers has more citations than the single paper in the Euclidean contest.*

3. Discussion of results

Table 1 shows that, for increasing marginal cost of effort, it is possible to have more papers and better-quality papers in the total citations contest than in the h-index contest. This is consistent with Ellison (2013, p. 77-78) who, in his empirical analysis, found that, for economists "... using Hirsch's index instead of a citation count would ... be taking a substantial step backward." In the case of constant marginal cost, the possible superiority of the total citations index over the h-index can also be seen by combining the results in propositions 1 and 2.

Because the h-index de-emphasizes a scholar's most-cited papers, Ellison (2013) correctly argues that it has some similarities with the total citations index. In fact, Hirsch (2005) and Yong (2014) found that the h-index is directly proportional to the square root of the total number of citations. Yet, we find that the h-index and total citations have different incentive effects.

The result that in the Euclidean contest, the contestants only write one paper is an obvious consequence of the axiom of *depth relevance* in Perry and Reny (2016), one of the five axioms that underpins the Euclidean index. The axiom of depth relevance requires that "… it should *not* be the case that for any fixed number of citations, the index is maximized by spreading them as thinly as possible across as many publications as possible." (Perry and Reny, 2016, p. 2726). This encourages quality over quantity in the sense that a smaller number of highly cited papers is preferred to a bigger number of rarely cited papers. Unlike the Euclidean index, the total citations index does not satisfy the axiom of depth relevance. Yet, as stated in proposition 1, when the marginal cost of effort is constant there is an equilibrium in the total citations contest that has the *same* quantity and quality of papers as the equilibrium in the Euclidean contest. In fact, when the marginal cost of effort is increasing, Table 1 shows that there are cases in which the total citations index dominates the Euclidean index in the sense that the total citations contest yields the *same* quality of papers but *more* papers than the Euclidean contest.[12] This result is not possible when the marginal cost of effort is constant.

Some of the dominance of the total citation count over the Euclidean index is not driven by the small number of papers that the players can write. Suppose we increase the number of papers that the players can write. They will still write only one paper in the Euclidean contest with the same number of citations (e.g., 1 citation) as was the case with a maximum of two papers. Given the parameters used, there cannot be an equilibrium in the total citations contest with only one paper because when the players could write only two papers, an equilibrium with only one paper did not exist. Therefore, when they can write more than two papers, a one-paper equilibrium cannot exist in the total citations contest because the profitable deviations that were

[12]This occurs when $c \in \{0.02, 0.03, 0.06, 0.07, 0.08, 0.09, 0.10\}$ in Table 1.

available when they could only write two papers are still available. If there is an equilibrium with more than two papers in the total citations contest, the integer requirement will imply that each paper has at least 1 citation. Therefore, some of the dominance of the total citation index will hold even if they can write more papers. Of course, the claim is not that the total citation count always dominates the Euclidean index.

The preceding discussion and the propositions show that the behavior of the marginal cost of effort matters for the allocation of effort across papers and this allocation depends on the citation index used. To give one more example, note that when marginal cost is constant and c = 0.06, the single paper written in Euclidean contest has 4 citations but when marginal cost is increasing and c = 0.06, the single paper written in the Euclidean contest has only 1 citation.

4. Extensions

4.1 More than two papers

Suppose the contestants can write a maximum of $N \geq 2$ papers. In the case of a constant marginal cost, this extension is not challenging in the total citations and Euclidean contests. The players will write only one paper in the Euclidean contest and in the total citations contest, there is a unique equilibrium for the aggregate number of citations but its allocation across the N papers is not unique. In fact, proposition 2 will still hold. But in the h-index contest, this requires more work. Because of how the definition of the h-index is tied to the number of papers, the number of papers is an integer, and effort is costly, it follows that the effort and citations in the contest must be integers.

Suppose there are two contestants who can write N papers and the marginal cost of effort is constant. Given that effort is costly, it is not optimal to garner more than h citations on each of

h or less papers in the h-index contest. Consider a candidate symmetric Nash equilibrium with an h-index of $h \geq 1$. Then each candidate invests an effort of h in each of h papers. The total effort is h^2. Each player has an equilibrium payoff of $0.5 - ch^2$. Consider a deviation to an effort of $h + m$ for each of $h + m$ papers giving an h-index of $h + m$. Then this deviation gives a payoff of $\frac{h+m}{h+m+h} - c(h + m)^2$. Then the candidate equilibrium is indeed a Nash equilibrium if:

$$0.5 - ch^2 \geq \frac{h + m}{h + m + h} - c(h + m)^2.$$

This simplifies to:

$$H(m) \equiv 0.5 + c(m^2 + 2hm) - \frac{h + m}{2h + m} \geq 0.$$

Because the maximum number of papers is N and an h-index of zero gives a zero payoff, we require that $h + m \leq N$ and $h + m \geq 1$. These two inequalities give $-(h - 1) \leq m \leq N - h$. For a given $c > 0$, $0.5 - ch^2 \geq 0$ implies $h \leq \sqrt{0.5/c}$. Then, for a given $c > 0$ and $N \geq 2$, there is a symmetric Nash equilibrium with an h-index of h if $1 \leq h \leq \sqrt{0.5/c}$ and $H(m) \geq 0$ on the domain $m \in [-(h - 1), (N - h)]$, where m, h, and N are integers.[13]

Recall that we found that for constant marginal cost of effort, if $N = 2$ and $c \in \left[\frac{1}{18}, \frac{1}{2}\right]$, the Nash equilibrium is $x_1^h = y_1^h = 0$ and $x_2^h = y_2^h = 1$, which gives an h-index of 1. Because the players could write two papers but chose to write only one paper, it turns out that allowing more than two papers does not change the equilibrium for $c \in \left[\frac{1}{18}, \frac{1}{2}\right]$. The equilibrium will still

[13]Recall that I rule out equilibria in which c is so high that an effort of zero and thus an h-index of zero is optimal.

be an h-index of 1 except for $c = \frac{1}{18}$ where there is multiple equilibria. Therefore, for $N \geq 3$, I will consider $c \in \left(0, \frac{1}{18}\right)$. I present the results in Table 2 for $4 \leq N < \infty$.

Table 2: *Equilibrium citations with linear cost of effort and* $4 \leq N < \infty$.

c	**Total Citations Contest: Total citations**	**Euclidean Contest: Citations for a single paper**	**h-index contest: Values of h-index**
0.010	25	25	4
0.020	12,13*	12,13*	2,3*
0.030	8	8	2
0.040	6	6	2
0.050	5	5	2
0.052	5	5	2
0.054	5	5	2
0.055	5	5	2

*Multiple equilibria

When $c = 0.010$ and the players could only write a maximum of two papers, the equilibrium h-index was 2. But when the players could write at least four papers, Table 2 shows that the equilibrium h-index was 4. Being able to write even 100 papers had no effect on the equilibrium h-index of 4. And as the cost of writing a paper increased, the equilibrium h-index was 2 regardless of whether the players could write only two papers or more than two papers. Being able to write more papers does not change proposition 1.

The h-index was developed to discourage or penalize authors who have few papers that have many citations. However, in the model, increasing the number of papers, N, that the players can write did not necessarily induce them to write more papers because the cost of writing papers can be a constraint. Thus, relative to the other two indices, increasing N did not lead to highly-cited papers in the h-index contest because the number of papers written constrains how the h-index counts citations. An h-index of h puts zero weight on each of a scholar's papers that has less than h citations and, for each paper that has at least h citations, it puts zero weight on all citations in excess of h.[14] To strike a balance between quality and quantity, the h-index ends up wasting citations. In a recent and comprehensive survey about the h-index, Barnes (2017, p. 492) observed that:

"... a researcher's h-index cannot exceed his or her total number of publications, no matter how many citations any individual paper receives. As a result, the effective limit on many researcher's h-indexes is their number of published papers, not the citations these papers attract."

Given that the cost of effort constrains the number of papers written, it follows from the preceding point that in disciplines that write few papers --- perhaps because of cost constraints, lags in the peer-review process, demanding reviewers, etc -- the h-index will be less aligned to incentives and reward structures (Ellison, 2013). For example, Ellison (2013, p. 67) observed that "... the h index is unappealing when applied to economics. Economists write fewer papers than do physicists, and individual papers get many citations." In economics, Ellison (2013) found that variants of the h-index that put more weight on smaller numbers of highly-cited papers are more consistent with labor market outcomes than is the original h-index. In the same vein, Kosmulski (2006) argued that the h-index is probably appropriate in fields like mathematics or astronomy where the typical number of citations per paper is relatively low.[15] In physics, a field with relatively high number of papers and citations per author, Hirsch (2005, 2007) found that the h-index was a good predictor of scientific achievement.[16]

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[14]It is not surprising that the h-index is inelastic with respect to the number of citations. For example, as mentioned above, Hirsch (2005) and Yong (2014) found that a scholar's h-index was directly proportional to the square root of his total number of citations.

[15]But Yong (2014) observed that the h-index is not a good predictor of winners of the Fields Medal in mathematics.

[16]But in a subfield of physics, Lehmann et al. (2006) concluded that the h-index was inferior to the mean number of citations per paper as indicator of scientific quality.

In my model, how difficult it is to write papers is captured by the cost function for effort. Thus, in light of the preceding discussion and reiterating the fact that the number of papers constrains how the h-index counts citations we find, in table 1, that as the cost of writing a paper increases, the h-index is inferior to the total citations index in both the quality and quantity of papers.[17] And in table 2, it is not necessarily superior to the total citations index or the Euclidean index.

4.2 Two papers but different qualities of journals

In this section, I briefly consider differences in journal quality. Towards this end, suppose $N = 2$ and there are two journals, 1 and 2, with journal 1 being a high-quality journal and journal 2 being a lower-quality journal. Assume that if player A invests an effort of x_k in publishing in journal k (i.e., paper k) it costs him $c_k x_k^\alpha$ and the paper gets $r_k x_k$ citations, where $r_1 \geq r_2 > 0$ and $c_1 \geq c_2 > 0$. Therefore, publishing in the high-quality journal (high impact factor) requires more effort but garners more citations. A different but equivalent approach is to assume that a citation in the high-quality journal is equivalent to $r_1/r_2 \geq 1$ citations in the low-quality journal. This quality-adjusted citation of paper 1 relative to paper 2 is akin to the approach in Amegashie (2019)[18], Bodenhorn (2003), Conley et al. (2013),[19] and Kenny and Studley (1995). In this case, articles in high-quality journals need not garner more citations than articles in low-quality journals but citations in high-quality journals are given a bigger weight. [20]

[17]As mentioned in section 1, a number of authors have pointed out the shortcomings of the h-index.
[18]Amegashie (2019) is different from the current paper in several respects: (a) it has two prizes and two contests, (b) it does not consider the Euclidean index and h-index, (c) it is applicable to contests other than citations contests, and (d) it is not an *integer programming game* and so does not require that efforts be integers.
[19]Based on journal quality indices in Kalaitzidakis et al. (2003), Conley et al. (2013) define an *American Economic Review* (AER) equivalent publication as the number of publications in a journal that is equivalent to one publication in the AER. See Hamermesh (2018) and Oswald (2007) for a discussion of the limitations of this approach.
[20]See Palacious-Huerta and Volij (2004) for an analysis and discussion of the measurement of scholarly influence including weighting citations and Serrano (2004) for a critique of their work.

In what follows, I follow the first interpretation by assuming that a publication in a high-quality journal garners more citations than a publication in a lower-quality journal. Without loss of generality, let $r_2 = 1$ and $r_1 \equiv r > 1$. Then total citations for player A is $rx_1 + x_2$ and for player B, it is $ry_1 + y_2$. Similarly, the Euclidean citation index for players A and B are $\sqrt{r^2x_1^2 + x_2^2}$ and $\sqrt{r^2y_1^2 + y_2^2}$ respectively.

Suppose that $c_1 = c_2 > 0$ and the marginal cost of effort is constant. Then given $r > 1$, the equilibrium in the total citations contest and the Euclidean contest will have the players writing only one paper and that will be in journal 1. Generally, in the total citations contest, the players publish only one paper in journal 1 if $c_1 < rc_2$ and are indifferent between publishing in journal 1 or journal 2 if $c_1 = rc_2$. To see, this note that the players' expected payoffs are:

$$\Omega_A^T = \frac{rx_1+x_2}{rx_1+x_2+ry_1+y_2} - c_1x_1 - c_2x_2. \tag{12}$$

and

$$\Omega_B^T = \frac{ry_1+y_2}{rx_1+x_2+ry_1+y_2} - c_1y_1 - c_2y_2. \tag{13}$$

For player A, the first-order conditions are:

$$\frac{\partial\Omega_A^T}{\partial x_1} = \frac{r}{\theta} - \frac{r(rx_1+x_2)}{\theta^2} - c_1 \leq 0, \tag{14}$$

and

$$\frac{\partial\Omega_A^T}{\partial x_2} = \frac{1}{\theta} - \frac{(rx_1+x_2)}{\theta^2} - c_2 \leq 0, \tag{15}$$

where $\theta \equiv rx_1 + x_2 + ry_1 + y_2$. By symmetry, the conditions for player B are analogous.

Rewrite (14) as

$$\frac{1}{\theta} - \frac{(rx_1+x_2)}{\theta^2} - \frac{c_1}{r} \leq 0. \tag{14a}$$

Suppose $\frac{c_1}{r} < \frac{c_2}{1}$. Then the cost per benefit of publishing in journal 1 is bigger than the cost per benefit of publishing in journal 2. Consider an equilibrium in which $\frac{1}{\theta} - \frac{(rx_1+x_2)}{\theta^2} - \frac{c_1}{r} = 0$. Then, given $\frac{c_1}{r} < c_2$, the inequality in (15) gives $\frac{\partial \Omega_A^T}{\partial x_2} = \frac{1}{\theta} - \frac{(rx_1+x_2)}{\theta^2} - c_2 < 0$ for all $x_2 \geq 0$. Impose symmetry (i.e., $x_1 = y_1$ and $x_2 = y_2 = 0$) and solve $\frac{1}{\theta} - \frac{(rx_1+x_2)}{\theta^2} - \frac{c_1}{r} = 0$ to get the Nash equilibrium:

$$\hat{x}_1^T = \hat{y}_1^T = \frac{1}{4c_1} \text{ and } \hat{x}_2^T = \hat{y}_2^T = 0. \tag{16}$$

The equilibrium expected payoff is $\frac{1}{2} - c_1 \frac{1}{4c_1} = \frac{1}{4}$. The equilibrium number of citations is $\frac{r}{4c_1}$. Clearly, there are values of r and c_1 that give a positive integer value of $\frac{r}{4c_1}$. This requires $r \geq 4c_1$. Even if $\frac{r}{4c_1}$ is not an integer, it is easy to show that the players will only publish in journal 1.

Taking journal quality into account is similar to the axiom of *depth relevance* in Perry and Reny (2016). Therefore, it is not surprising that, for certain parameters (i.e., $c_1 < rc_2$), contestants in the total citations contest strictly prefer to write only one highly cited paper even though the marginal cost of effort is constant. Yet Perry and Reny (2016) concluded that a total citation count does not satisfy the axiom of *depth relevance*.[21] On the basis of how citations are used *ex post* (i.e., after papers have been published), Perry and Reny (2016) are right. But the analysis here shows that if a publication in a high-quality journal is cited more than a publication in a lower-quality journal, then the *ex ante* (incentive) effect of using total citation count will, at least in some cases, induce authors to put more weight on quality than quantity.

[21]See Perry and Reny (2016, p. 2726).

Publishing in a high-quality journal comes with professional prestige and rewards over and above what one gets on the basis of only total citations. In Economics, Roberto Serrano of Brown University has humorously dubbed this phenomenon, *Top5itis*. According to Serrano (2018), "Top5itis is a disease that currently affects the economics discipline. It refers to the obsession of the profession of academic economists with the so-called "top5 journals." Heckman and Moktan (2018) refer to this obsession as the "tyranny of the top five".[22] Whether this obsession is detrimental to the Economics profession is not my focus. I am interested in its incentive effect.

Suppose, as in Amegashie (2019), that in addition to the total citations contest, there is also a quality contest based on only publication in the high-quality journal. This contest has a prize, $V > 0$. To capture this, rewrite the payoffs in (12) and (13) as

$$\widehat{\Omega}_A^T = \frac{rx_1+x_2}{rx_1+x_2+ry_1+y_2} + \frac{rx_1}{rx_1+ry_1}V - c_1x_1 - c_2x_2. \quad (17)$$

and

$$\widehat{\Omega}_B^T = \frac{ry_1+y_2}{rx_1+x_2+ry_1+y_2} + \frac{ry_1}{rx_1+ry_1}V - c_1y_1 - c_2y_2. \quad (18)$$

Thus, publishing in the high-quality journal counts in both the total citations contest and the quality contest.[23] Then if $\frac{c_1}{r} < \frac{c_2}{1}$, there exists the Nash equilibrium (see Amegashie, 2019):

$$\tilde{x}_1^T = \tilde{y}_1^T = \frac{V+1}{4c_1} \text{ and } \tilde{x}_2^T = \tilde{y}_2^T = 0. \quad (19)$$

The equilibrium effort is, not surprisingly, increasing in V. I assume that the equilibrium number of citations, $\frac{r(V+1)}{4c_1}$ is an integer. More importantly, I note that by introducing a realistic feature

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[22]See also Attema et al. (2014).

[23]Based on several empirical studies, Hamermesh (2018) reports that total citations have an independent impact on salaries of scholars while Moore et al. (2001), for example, found that the quality of journals affects a scholar's monetary returns.

into a total citations contest, we can construct an equilibrium in which the contestants prefer the quality of papers to the number of papers.

While the assumption that articles in high-quality journals, *on average*, garner more citations than articles in lower-quality journals is not in doubt, there is a debate about whether articles in high-quality journals, merely because they appear in such journals, should be deemed as good or influential articles.[24] On the basis of work by Oswald (2007), Stern (2013), and his own work, Hamermesh (2018, p. 141) observed that:

"The main reason that a few economics journals are ranked more highly than all others is that the very few papers in these journals generate immensely more citations than other papers published in those journals or elsewhere. A very few outliers determine our perceptions of journal quality, perceptions that ignore the tremendous heterogeneity of articles across journals."[25]

Based on a similar observation in other fields (e.g., Seglen, 1997; Verma, 2015), the *San Francisco Declaration on Research Assessmen*t recommends that journal-based metrics such as journal impact factors should *not* be used "... as a surrogate measure of the quality of individual research articles, to assess an individual scientist's contributions, or in hiring, promotion, or funding decisions."[26] But to the extent, as noted by Hamermesh (2018, p. 141), that "[a] very few outliers determine our perceptions of journal quality", the incentive effects of total citation count identified here is plausible so long as we implicitly or explicitly weight citations in journals as in Amegashie (2019) and Conley et al. (2013).

[24]There is evidence that editors in various fields coerce authors to cite articles from their journals (Wilhite and Fong, 2012). Althouse et al. (2009) examine the factors that drive inflation in impact factors.

[25]See also Starbuck (2005) for the case of non-economics journals. Axarloglou and Theoharakis (2003) study some factors that influence economists' perceptions of economics journals.

[26]On December 16, 2012, a group of editors and publishers of scholarly journals (including *Proceedings of The National Academy Of Sciences*) met during the Annual Meeting of The American Society for Cell Biology in San Francisco, California. The group developed a set of recommendations which now known as the *San Francisco Declaration on Research Assessment*: https://www.andrewoswald.com/docs/SanFranciscoDeclarationFINAL.pdf

If journal impact factors (journal citations) are not desirable for comparing scholars, then citations per paper is a desirable option. Even so comparing scholars in different subfields of the same discipline or different fields (discipline) may require adjusting citations in a subfield or field by dividing a paper's citations by the average number of citations in its subfield or field (Radicchi et al., 2008; Ellison, 2013; Perry and Reny, 2016).

In the h-index contest with differences in journal quality, it is easy to construct an equilibrium in which the h-index is 2 for some parameters and it is 1 for other parameters. No further insights emerge beyond what was obtained in the case of $r = 1$, so I omit the proof.

5. Conclusion

This paper has taken the first step of studying the incentive effects of citations in a game-theoretic model. The h-index, in general, had a weak incentive effect especially when the cost of writing papers is high. This is because the number of papers constrains how the h-index counts citations and so when the cost of writing a paper is high, this constrains the number of papers written which, in turns, reduces a scholar's h-index.

The analysis can be extended in various directions. One could consider a model in which the relationship between effort and citations is explicitly modelled as a stochastic relationship; an index which is a combination of indices as in Hirsch (2007); research by teams; a different contest success function, and many other extensions. The paper models the incentives of authors in a contest. This methodology may also be used to study the incentives of journal editors and librarians.

The paper complements the axiomatic and empirical analysis of citations indices. It is hoped that the approach herein will be extended in subsequent work.

Appendix A

Proof that if the cost of effort is quadratic and c is sufficiently small, a contestant in the Euclidean contest will not write two papers.

Consider an equilibrium in which player *B*'s Euclidean index is $E_B \equiv \sqrt{y_1^2 + y_2^2} > 0$ and player *A*'s total effort is $X \equiv x_1 + x_2$ for two papers, where $x_1 > 0; x_2 > 0$. Then player *A* has a profitable deviation from this equilibrium by re-allocating $X \equiv x_1 + x_2$ to only one paper if:

$$\frac{\sqrt{(x_1+x_2)^2}}{\sqrt{(x_1+x_2)^2+E_B}} - c(x_1 + x_2)^2 > \frac{\sqrt{x_1^2+x_2^2}}{\sqrt{x_1^2+x_2^2+E_B}} - c(x_1^2 + x_2^2). \tag{A1}$$

The inequality in (A1) simplifies to:

$$\frac{\sqrt{x_1^2+x_2^2+2x_1x_2}}{\sqrt{x_1^2+x_2^2+2x_1x_2+E_B}} > \frac{\sqrt{x_1^2+x_2^2}}{\sqrt{x_1^2+x_2^2+E_B}} + 2cx_1x_2. \tag{A2}$$

Noting that $\frac{\sqrt{x_1^2+x_2^2+2x_1x_2}}{\sqrt{x_1^2+x_2^2+2x_1x_2+E_B}} > \frac{\sqrt{x_1^2+x_2^2}}{\sqrt{x_1^2+x_2^2+E_B}}$, it follows that (A2) holds if c is sufficiently small.

Thus if c is sufficiently small, an equilibrium in the Euclidean contest will have each player writing only one paper. **QED**

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Not for publication

Table 3: *Equilibrium citations with quadratic cost of effort,* cx_k^2 and cy_k^2, *assuming that the number of citations per paper need not be an integer,* $k = 1,2$.

c	**Total Citations Contest** $x_1^T = y_1^T = x_2^T = y_2^T$	**Euclidean Contest** $x_1^E = y_1^E = 0; x_2^E = y_2^E > 0$
0.01	2.500	3.535
0.02	1.768	2.500
0.03	1.443	2.401
0.04	1.250	1.768
0.05	1.118	1.581
0.06	1.021	1.443
0.07	0.945	1.336
0.08	0.883	1.250
0.09	0.833	1.179
0.10	0.790	1.118

Note: The equilibria were obtained by using the mathematics software, Maple V. Symmetry was not imposed. But, as expected, the equilibria were symmetric.