CESifo WORKING PAPERS

7901
2019

Original Version: October 2019
This Version: December 2019

Male Investment in Schooling with Frictional Labour and Marriage Markets

Roberto Bonilla, Francis Kiraly

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
www.cesifo-group.org/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: www.CESifo-group.org/wp

CESifo *Working Paper No. 7901*
Category 4: Labour Markets

Male Investment in Schooling with Frictional Labour and Marriage Markets

Abstract

We present an equilibrium model with inter-linked frictional labour and marriage markets. Women's flow value of being single is treated as given, and it captures returns from employment. Single unemployed men conduct a so-called constrained sequential job search, and can choose to improve their labour market returns as well as their marriage prospects by undertaking a costly ex-ante investment in schooling. We establish the existence of market equilibria where a fraction of men get educated, and show that this fraction decreases if women's labour market returns increase. We also examine the robustness of such equilibria.

JEL-Codes: I260, D830, J120, J160, J310.

Keywords: returns to education, frictional markets, constrained search.

*Roberto Bonilla**
Newcastle University Business School
Newcastle University
UK – Newcastle upon Tyne, NE1 4SE
roberto.bonilla@ncl.ac.uk

Francis Kiraly
Newcastle University Business School
Newcastle University
UK – Newcastle upon Tyne, NE1 4SE
francis.kiraly@ncl.ac.uk

*corresponding author

1. Introduction

One of the most puzzling recent trends is that male schooling has recently lagged behind female education attainment. In an important study, Goldin et al.(2006) found that if women represented only 39% of U.S. undergraduates in 1960, within four decades they made up the majority of U.S. college students and of those graduating with a bachelor's degree. The trend is by no means limited to the U.S. The same study reported that, while school enrollment rates of women in 17 OECD countries were in the mid-80's below those of men, by 2002 women's college enrollment rates exceeded those of men in 15 of these countries.

In order to isolate the main mechanism we are interested in, we provide an equilibrium model of inter-linked frictional labour and marriage markets, with focus on men's choice of schooling investment. Education enhances not only the labour market returns of men (improved wage offers) but also, through this, their marriage prospects. We establish the existence and investigate the properties of a market equilibrium where some men choose to invest in education and women are selective about whom they marry. Crucially, we show that an increase in single women's labour market options (viewed as a proxy for their educational attainment) leads to a decrease in the equilibrium fraction of men who invest in schooling.

Our model is underpinned by the well known fact that couples tend to sort according to various traits (See Becker (1991)). In our paper, as all men are ex-ante homogeneous, the only relevant differentiating trait they bring to the marriage market is their wage, which in turn is affected by an ex-ante educational choice. Furthermore, as the study of equilibrium class formation is not one of the objectives of the present paper, the sorting aspect in our frictional marriage market essentially boils down to women having a reservation strategy. This setup requires the use of two important modelling devices.

First, with earned wages being the only distinguishing male trait in the marriage market, a single woman will only accept a single employed man if his wage is higher than a particular (endogenous) threshold wage. Consequently, a single unemployed man is involved in a so-called *constrained* sequential job search problem, whereby his marriage market prospects (marriageability) depend on his earnings, and therefore his labour market strategy is adjusted accordingly. That is, before being able to consider the question of male schooling investment, one has to characterise the optimal reservation wage policy of an unemployed single man, which is now a function of the threshold wage set by women. Second, in order to obtain such a meaningful female reservation wage in the frictional marriage market, we include as a *parameter* the flow utility of being single for a woman, which is meant to capture her labour market options and returns - possibly augmented through some ex-ante (not modelled here) educational investment. We assume that upon marriage women give up this flow value, there is no intra-marital bargaining, and the wage earned by the man becomes a public good for the marital partnership.

Some comments about the latter set of assumptions. First, in our model we consider direct selection into marriage based on the wage of a man. Grossbard-Shechtman and Neuman (2003) emphasise the importance of this "breadwinner" effect, while Ludwig and Brüderl (2018) provide recent empirical evidence of selection into marriage on wage levels and growth. Second, as our focus is on the effect of an increase in female returns/education on the fraction of men who invest in schooling, in this paper we choose to proxy the labour market returns of women with their flow utility outside marriage - just like in Blau et al. (2000). Of course this is a much more general parameter that could also be interpreted as a measure of women's (and, by comparison, men's) attitudes towards marriage, able to capture possible asymmetries in terms of how they perceive the gains from marriage. For example, the empirical results in Gould and Paserman (2003) suggest that men do not seem to care much about their partner's wage. In turn, Blundell et al. (2016) show that female attachment to the labour market weakens considerably after marriage, while Gould and Paserman (2003) among others provide evidence that women build this into their expectations and behaviour in the marriage market. Finally, the assumption that couples do not negotiate over the surplus from marriage simplifies the analysis: it does not qualitatively affect the nature of men's constrained job search decision, and it allows us to side-step the well-understood question of inefficiency in frictional markets generated by the hold-up problem.

Given this framework, our main result that an increase in female labour market returns (educational attainement) decreases the proportion of men who invest in education has the following intuitive explanation: In essence, we have a two-stage game in which first single unemployed men choose whether or not to invest in schooling, then this is followed by the interaction in the joint labour and marriage markets. In the latter subgame women and men simultaneously set their reservation wages, women taking as given the fraction of educated men. The partial search equilibrium in the inter-linked frictional markets determines the returns to education for men, both in terms of wages and marriage prospects. For any given cost of schooling, and with a binary education decision problem, a market equilibrium in which a fraction of men choose to undertake the schooling investment requires that the returns to education equals the cost of education. Crucially, it is the proportion of educated men that adjusts so that this market equilibrium condition holds. To see this, consider an increase in female labour market returns. Women become pickier in the marriage market and increase their reservation wage, but overall only the partial equilibrium of the joint frictional markets is affected directly. Importantly, with a fixed cost of schooling, the equilibrium returns from education for males needs to remain unchanged. Since a change in the fraction of educated men active in the frictional markets affects the female reservation wage in the same direction as a change in female labour market returns, it follows that if these returns increase, the fraction of educated men needs to decrease in order to restore the market equilibrium condition.

Our result is surprisingly robust and, apart from offering an explanation for the puzzling trend documented in Goldin et al. (2006) and others, it also seems to be in line with several other empirical findings. In terms of women's attitudes in the marriage market, Gould and Paserman (2003) conclude that women are pickier if female wages (their proxy for women's value of being single) are higher. Similarly, Blau et al. (2000) find that higher labour market returns for females has lead to lower marriage rates for women between ages 16-24 and 25-34. Finally, Oppenheimer (1988) and Oppenheimer and Lew (1995) argue that an improvement in labour market gains for women leads to them delaying the timing of marriage.

This paper is part of a research agenda whose main message is that many observed outcomes in labour markets (including human capital accumulation) can very well be the result of individuals' considerations and expectations in the marriage market - and vice-versa. As such, the present work builds on Bonilla and Kiraly (2013) and Bonilla et al. (2019), where the concept of constrained sequential job search was introduced and analysed in detail. For the specific question of the male-female schooling gap, our work complements the important contribution by Chiappori et al. (2009), who offer an explanation that also stresses the link between the marriage market and labour market.[1] However, their model is completely different from ours, as it investigates stable marriage assignments in a frictionless environment with transferable utility within couples, and their focus is on what determines women's educational choice, and when would it be likely to lag behind or overtake that of men's. In terms of models with a frictional labour market, Flinn and Mullins (2015) introduce endogenous productivity-enhancing schooling in a Pissarides-type general equilibrium model augmented with on-the-job search and potential wage renegotiations.

2. The Model

We consider steady state equilibria of an economy that consists of a continuum of risk neutral men and women, where all agents discount the future at rate r.

Men enter the economy unemployed, single and of type L. The distribution of wages faced by them is $F_L(.)$ with continuous support $[\underline{w}_L, \overline{w}_L]$. Men have a choice whether to enter the labour market immediately, or undertake an investment in education at a given cost c, same for all. An individual who undertakes the schooling investment becomes a type H man who now faces a

[1] See also Browning et al. (2014).

wage distribution $F_H(.)$ with support $[\underline{w}_H, \overline{w}_H]$. We assume that $F_H(.)$ first order stochastically dominates $F_L(.)$, so one can think of male types as representing educational attainment. Following the education decision, all men (H and L) become active in both the labour market and the marriage market. In the labour market, they look for wage offers using costless random sequential search, and job opportunities arrive at rate λ_0. If employed at wage w, a man receives the flow payoff w, and we assume there is no on-the-job search. While active in the labour market, single men also use costless sequential search in order to look for partners in the frictional marriage market. Marriage requires mutual acceptance, and we assume that divorce is not possible. For a man, marriage confers a flow payoff y which captures the non-economic utility of the partnership. Overall therefore, a married man employed at wage w has a flow payoff $w + y$.

Women enter the economy single, and let $x > 0$ denote the flow payoff of a single woman. This parameter is crucial for our investigation, as it captures a woman's options outside marriage. Here, we interpret this as her career opportunities, so an increase in x would mean higher labour market returns, possibly due to higher ex-ante schooling. Single women look for males using costless sequential search, but (as we will show) they are not interested in marrying unemployed men. Hence, for them the relevant wage distribution is that of wages earned by single type i men ($i = L, H$), denoted by $G_i(.)$. Once a marriage partnership is formed, a married woman simply enjoys a flow utility equal to her partner's wage.

Since for both sexes utilities are monotonic in wages, with sequential search, the optimal strategies for both women and men are characterised by the reservation property. Let R_i denote the reservation wage of unemployed type i men in the labour market. Similarly, let T_i denote the reservation wage of women in the marriage market, meaning an employed man of type i is marriageable only if his wage is no lower than T_i.

Everyone (irrespective of employment- and marital status) leaves the economy at rate δ. Let Γ denote the exogenous flow (measure) of new (unemployed and single) men who enter the economy at every instance, and let N_i denote the number of marriageable employed single men of type i. Similarly, let n denote the measure of single women; it is exogenous as we assume that a new single woman comes into the market every time a single woman gets married or exits the economy. Denote by λ_w^i the rate at which a single woman meets an eligible bachelor, and let λ_m denote the rate at which single men meet single women. We assume a quadratic matching function with parameter λ that measures the efficiency of the matching process. Then, we have $\lambda_w^i = \frac{\lambda(N_H+N_L)n}{n}\frac{N_i}{(N_H+N_L)} = \lambda N_i$, and $\lambda_m = \frac{\lambda(N_H+N_L)n}{(N_H+N_L)} = \lambda n$, where both N_i and λ_w^i are of course endogenous. Crucially, let τ denote the (endogenous) proportion of male entrants who decide to invest in schooling. We ask how does the steady state equilibrium fraction of educated men change with x.

3. Steady state and optimal search

3.1. Unemployed men, marriageable men and wages

Let u_i denote the number of unemployed men of type i. In steady state we require $\Gamma\tau_H = u_H[\delta + \lambda_0(1 - F_i(R_i))]$. That is, the inflow of unemployed men who choose to invest in schooling needs to equal the outflow of these educated unemployed, either into employment (at an acceptable wage) or full exit.

Consequently, in steady state we have:

$$u_H = \frac{\tau\Gamma}{\delta + \lambda_0\left[1 - F_H(R_H)\right]},$$

and

$$u_L = \frac{(1-\tau)\Gamma}{\delta + \lambda_0\left[1 - F_L(R_L)\right]}.$$

In order to obtain the number of single marriageable men of type i, we require

$$u_i\lambda_0[1 - F_i(T_i)] = N_i(\lambda n + \delta).$$

Then, using u_i as above, we obtain:

$$N_H = \frac{\tau\Gamma}{\delta + \lambda\left[1 - F_H(R_H)\right]}\frac{\lambda_0[1 - F_H(T_H)]}{\lambda n + \delta}$$

and

$$N_L = \frac{(1-\tau)\Gamma}{\delta + \lambda\left[1 - F_L(R_L)\right]}\frac{\lambda_0[1 - F_L(T_L)]}{\lambda n + \delta}.$$

Note the role of u_i in the determination of N_i. We will show that when the marriage market affects unemployed men's search behaviour, there are two possible outcomes:

(i) When $R_i < T_i$ the number of marriageable men increases with R_i. Given the exogenous wage distributions, if the reservation wages R_i increase, men of type i leave unemployment at a lower rate, so the steady state u_i increases. Then, since the rate at which men accept marriageable wages remains unchanged, this leads to an increase in N_i. Furthermore, N_i increases when the female reservation wage T_i decreases. (ii) When R_i is optimally set equal to T_i an increase in R_i results in a decrease in N_i.

Finally, the distribution of wages earned by marriageable men of type i is given by the steady state condition:

$$u_i\lambda_0[F_i(w) - F_i(T_i)] = G_i(w)N_i(\lambda n + \delta).$$

In the above, the number of marriageable men of type i with wages no higher than w is $G_i(w)N_i$, and they leave this stock if they get married or exit the economy altogether. The left-hand side captures the flow of unemployed men of type i who find and accept a job with a wage that confers marriageability but is no higher than w.

From here, also using the solution for N_i previously obtained, we have:

$$G_i(w) = \frac{F_i(w) - F_i(T_i)}{1 - F_i(T_i)}.$$

3.2. Optimal search: women

In this section, we derive the female reservation wages T_i. To do this, we first establish that women refuse to marry unemployed men of type i if the female reservation wage is high enough.[2] Consider a married and employed man of type i. Without the possibility of either on-the-job search or divorce, standard considerations give the discounted expected lifetime utility of such a man:

$$V_i^M(w) = \frac{w + y}{r + \delta}.$$

Although this utility is clearly independent of type (education), in the interest of clarity we will continue to use the subscripts whenever we refer to this value of employment.

Now consider a married but unemployed man of type i. As he is no longer active in the marriage market, his reservation wage is simply the standard pure labour market one:

$$\underline{R}_i = \frac{\lambda_0}{r + \delta} \int_{\underline{R}_i}^{\overline{w}_i} [1 - F_i(w)]\, dw \tag{1}$$

Note that $\underline{R}_H > \underline{R}_L$ because type H men have better job prospects in the labour market. Furthermore, as we will show later, $\underline{R}_i$ is in fact the lowest reservation wage for each type.[3]

In principle, women could of course marry unemployed men as well. Let us therefore examine the situation of a woman who is married to a type i jobless man. Her value function W_i^U is given by:

$$(r + \delta) W_i^U = \lambda_0 \int_{\underline{R}_i}^{\overline{w}_i} \left[W_i^M(w) - W_i^U \right] dF_i(w),$$

[2]This particular bit of analysis mirrors to some extent the one carried out in Bonilla et al. (2019), with the crucial difference that in this paper male types (here education) are endogenous.

[3]In Bonilla et al. (2019), $\underline{R}_i$ is the same for all types.

where $W_i^M(w) = w/(r+\delta)$ is the discounted lifetime utility of being married to a type i employed man who earns wage w. The above equation incorporates the fact that a married type i unemployed man has reservation wage $\underline{R}_i$.

For T_i to be a female reservation wage, it needs to satisfy the condition $W_i^U = T_i/(r+\delta)$. Given $W_i^M(w)$ above, we have:

$$T_i = \frac{\lambda_0}{r+\delta} \int_{\underline{R}_i}^{\overline{w}_i} [w - T_i]\, dF_i(w),$$

and the unique solution to this is $T_i = \underline{R}_i$. Now, if a woman's value of being single (denoted by W^S) increases, her reservation wage also increases. In contrast, W_i^U is independent of T_i. Hence, $W^S W_i^U$ if and only if $T_i \underline{R}_i$, and therefore we conclude that if $T_i > \underline{R}_i$, women will reject marriage to a type i unemployed man. Throughout, we work under the assumption that this is indeed the case.[4]

Next, we turn to the actual derivation of women's reservation wages T_i, emphasising that women cannot direct their search efforts and therefore contact with an H or an L man is completely random. Importantly, $W_H^M(w) = W_L^M(w) = w/(r+\delta)$: for a woman, the type of an employed man she is already married to is irrelevant. Using the definition of a reservation value we have $W^S = W_H^M(T_H) = W_L^M(T_L)$, which implies $T_H = T_L$. Consequently, from now on we drop the subscripts and use $T(= T_H = T_L)$ instead.

Recall that W^S denotes the value of being single for a woman. Standard derivations lead to the Bellman equation:

$$(\text{r+}\delta)W^S = x + \lambda N_H \int_T^{\overline{w}_H} \left[W_H^M(w) - W^S\right] dG_H(w) +$$

$$+ \lambda N_L \int_T^{\overline{w}_L} \left[W_L^M(w) - W^S\right] dG_L(w).$$

Making use of the solutions for N_i and $G_i(w)$ previously obtained, this becomes:

$$(\text{r+}\delta)W^S = x + \tfrac{\lambda\tau\Gamma\lambda_0}{[\delta+\lambda(1-F_H(R_H))](\lambda n+\delta)} \int_T^{\overline{w}_H} \left[W_H^M(w) - W^S\right] dF_H(w) +$$

$$+ \lambda\tau_L \tfrac{\lambda(1-\tau)\Gamma\lambda_0}{[\delta+\lambda(1-F_L(R_L))](\lambda n+\delta)} \int_T^{\overline{w}_L} \left[W_L^M(w) - W^S\right] dF_L(w).$$

[4]By doing so, we essentially eliminate the uninteresting equilibrium where the marriage market does not affect men's job search.

Finally, using $W^S = T/(r+\delta)$ and applying standard integration by parts, we obtain:

$$T = x + \frac{\lambda\tau\Gamma\lambda_0}{[\delta + \lambda(1 - F_H(R_H))](\lambda n + \delta)} \int_T^{\overline{w}_H} [1 - F_H(w)]dw + \tag{2}$$

$$+ \frac{\lambda(1-\tau)\Gamma\lambda_0}{(\delta + \lambda\left[1 - F_L(R_L)\right])(\lambda n + \delta)} \int_T^{\overline{w}_L} [1 - F_L(w)]dw.$$

At this point, we make three observations that are important for what follows:

1. Clearly, $\partial T/\partial x > 0$: as expected, women raise their reservation wage in the marriage market if their instantaneous utility from staying single increases.
2. $\partial T/\partial \tau > 0$: intuitively, a ceteris paribus increase in the fraction of educated men with better job prospects makes women pickier, since their marriage market prospects have also improved now.
3. $\partial T/\partial R_i > 0$: again, ceteris paribus, a higher reservation wage of type i men increases the number of marriageable men (see the discussion around N_i above), so women can afford to become choosier.

3.3. Optimal search: men

We are interested in equilibria in which the marriage market affects all men's decisions in the labour market. Nevertheless, it is instructive to consider the optimal job search behaviour of men under all possible circumstances. To that end, first recall that in any scenario where the marriage market does *not* influence labour market decisions, the male reservation wage is given by $\underline{R}_i$ obtained above.

When the marriage market *does* have an effect (through T) on male strategies, single unemployed men undertake a so-called *constrained* search, knowing that by accepting a particular wage (for life), they either become marriageable or loose the prospect of marriage forever. As a consequence, a man of type i searches from the wage offer distribution $F_i(.)$ and, for any given female reservation wage T which makes him acceptable for marriage, he uses a reservation wage *function* $R_i(T)$.

In what follows, we fully characterise the function $R_i(T)$. Although the derivation of the reservation wage function is the same for both types[5], the actual reservation wage functions will be different across types, essentially due

[5]For a much more detailed exposition of this, please consult Bonilla et al. (2019).

to the fact that men with different schooling choices face different wage distributions. The main insight is that this function is non-monotonic in the female reservation wage, and has a unique maximum, attained at $T = \widehat{T}_i$, where the latter is defined by:

$$\widehat{T}_i \equiv \frac{\lambda_0}{r+\delta}\left[\int_{\widehat{T}_i}^{\overline{w}_i}[1-F_i(w)]\,dw + \frac{\lambda n\left[1-F_i(\widehat{T}_i)\right]}{r+\delta+\lambda n}y\right]. \tag{3}$$

Clearly, for $y > 0$ and $F_i(\widehat{T}_i) < 1$, we have $\widehat{T}_i > \underline{R}_i$.

The formal reasoning is as follows. Overall, a man (of either type) can be in one of three states: unemployed and single, employed at wage w and single (S), or employed at wage w and married (M). Denote a type i man's value of being unemployed by U_i, and let $V_i^S(w)$ describe the value of being single and earning a wage w. Standard derivations lead to the Bellman equation for a type i unemployed man:

$$(r+\delta)U_i = \lambda_0 \int_{\underline{w}_i}^{\overline{w}_i} \max\left[V_i^S(w) - U_i, 0\right] dF_i(w).$$

Anticipating that $V_i^S(w)$ is not a continuous function (see below), we can define:

$$R_i(T) \equiv \min\left\{w : V_i^S(w) \geq U_i\right\}.$$

Since there is no divorce, the value of being married and earning a wage w is $V_i^M(w) = (w+y)/(r+\delta)$. Hence, for any T, we have:

$$V^S(w) = \left\{\begin{array}{lr} \frac{w}{r+\delta} & if \quad w < T \\ \frac{w}{r+\delta} + \frac{\lambda n}{(r+\delta+\lambda n)(r+\delta)}y & if \quad w \geq T \end{array}\right\}.$$

Please note that when $\lambda n = 0$ (i.e. no marriage market), we have $V_i^S(w) = w/(r+\delta)$ for all w and, from $U_i = V_i^S(R_i)$, standard manipulation yields $R_i = \underline{R}_i$. As stated before, this is the reservation wage that would be chosen by a hypothetical unemployed married man since, without divorce, this man is no longer involved in the marriage market.

The Proposition below presents the full characterisation of the male reservation wage function.

Proposition 1. *The reservation wage function $R_i(T)$ is continuous, piece-wise differentiable, and:*
(a) $R_i = \underline{R}_i$ for $T \leq \underline{R}_i$ and $T > \overline{w}_i$;
(b) $R_i = T$ for $T \in (\underline{R}_i, \widehat{T}_i]$;
(c) $R_i < T$ and decreasing for $T \in (\widehat{T}_i, \overline{w}_i]$.
Furthermore, $\widehat{T}_H > \widehat{T}_L$ and $\underline{R}_H > \underline{R}_L$.

Proof. See Appendix.

In essence, when the marriage market affects men's job search strategy, unemployed males can react in two ways. For relatively low values of female reservation wages, they choose to hold out for such wages and set R_i equal to T. At the critical $\widehat{T}_i$ the labour market related cost of holding out for it equals the gains from the marriage market. For even higher female reservation wages men gradually give up on trying to match T, so they only get married if they are lucky and land a high enough wage. This is because higher and higher female reservation wages make it less and less likely to encounter a marriageable wage, so the male reservation wage decreases.

Two further observations follow. First, men's value of unemployment U_i is not directly affected by x, so $\partial R_i(T)/\partial x = 0$ as women's flow utility of being single does not affect the male reservation wage functions. However, note that x will of course affect the equilibrium male reservation wages, through its direct effect on the female reservation wage function T. Second, the value of unemployment is not directly affected by τ either, so $\partial R_i/\partial \tau = 0$.

4. Equilibrium

In this section we investigate the existence and properties of a market equilibrium by first looking at the partial search equilibrium in the joint frictional markets, and then (using backward induction), pinning down the steady state fraction of educated men that is consistent with optimal schooling investment choices. Intuitively, as they face a binary decision, men will choose to invest in schooling as long as the returns from education - as captured here by the difference in the values of educated and uneducated single unemployed men, is higher than the cost of schooling. A mixed equilibrium with a fraction of educated men therefore requires $\Delta U(\equiv U_H - U_L) = c$, meaning all males are indifferent between investing or not in schooling.

The *exogenous* parameter x plays a key role in the determination of the partial equilibrium in the labour and marriage markets. In their behaviour in these inter-linked frictional markets agents also take τ as given. Crucially however, τ is the *only endogenous* variable that can adjust to ensure the equality of returns to education and cost of schooling.

The central results of our paper concern the nature of the interaction between these two variables. To get a flavour of the argument, consider a change in the flow utility of single women, x. This generates a change in $T(R_H, R_L)$ *only* - recall that the value of unemployment and reservation wages of men is not directly affected. However, the shift in $T(R_H, R_L)$ itself has an immediate impact on U_H and U_L, due to the change in the proportion of marriageable wages (different across male types). This leads to an adjustment in male reservation wages and with that, also a change in the returns to education. With comparative statics in mind recall that, just like for x, we have $\partial U_i/\partial\tau = 0$ so τ affects $T(R_H, R_L)$ only. Therefore, τ is indeed the only endogenous variable that can adjust in order to restore ΔU to its equilibrium level. Subsequent analysis of the adjustment of τ following a shock in x lead to further insights about the robustness of our equilibrium.

4.1. Partial search equilibrium

First, note that the number of steady-state educated single unemployed men (u_H) is essentially determined by the proportion of men who decide to invest in schooling, τ. Taking these two measures *as given*, a search equilibrium for the inter-linked frictional markets is the triplet $\{R_L^*, R_H^*, T^*\}$ together with steady state conditions, such that male reservation wages satisfy Proposition 1 and the female reservation wage satisfies (2). There are three types of potential equilibria: Type 1, characterised by $R_i^* < T^*$, $R_L^* < R_H^*$ and $\partial R_i/\partial T < 0$; Type 2, characterised by $R_H^* = T^*$, $R_L^* < T^*$ and $\partial R_L/\partial T < 0$; and Type 3, with $R_i^* = T^*$.

Proposition 2. *A partial search equilibrium exists and it is unique. In any such equilibrium $\partial T^*/\partial x > 0$ and $\partial T^*/\partial\tau > 0$.*

Proof. To show existence, note that $R_i(T)$ is continuous and non-monotonic in T, while $T(R_i)$ is continuous and increasing in R_i.

The second statement is proved by contradiction. Consider a potential Type 1 equilibrium. Let x increase, and assume a resulting new equilibrium with a lower T^*. Since $\partial R_i/\delta T < 0$ while the reservation wage of a particular male type is not directly affected by the reservation wage of the other type, this would necessarily involve higher R_i^*. From (2), a higher x, together with a higher R_i^* unambiguously results in a higher female reservation wage, which is a contradiction. Consider next a potential Type 3 equilibrium, and increase x. Imposing $R_i^* = T^*$ in (2) we have $\partial T/\partial x > 0$, and therefore a new equilibrium with a lower T^* would be a contradiction. Finally, consider a potential Type 2 equilibrium, and increase x. Given that $\partial R_L/\partial T < 0$ and R_H^* is optimally set equal to T^*, the above two arguments once again imply that a new equilibrium with a lower T^* would be a contradiction.

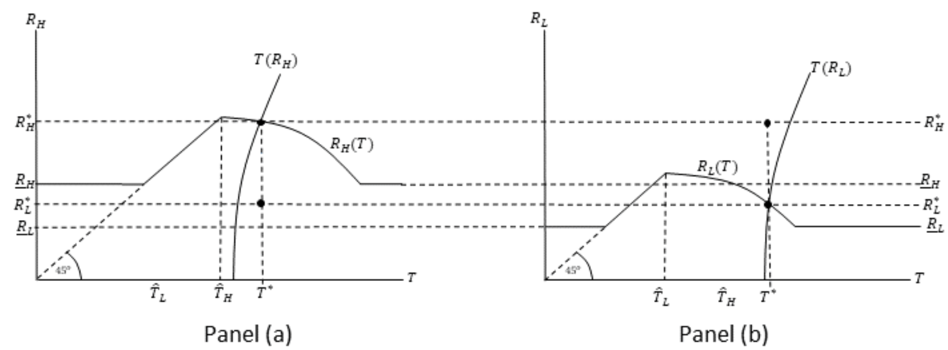
The reasoning which proves that $\partial T^*/\partial x > 0$ also implies that the female reservation wage function implicitly given in (2) crosses the 45 degree line from below, and hence the uniqueness of equilibrium follows. Finally, an increase in τ shifts the female reaction function in (2) to the right. **QED**

Although hidden, the role played by N_i in the results above is worth stressing. For the Type 1 equilibrium, an increase in R_i leads to an increase in N_i, which (ceteris paribus) makes women pickier. This, coupled with an increase in

x must result in a higher female reservation wage. For an equilibrium of Type 2 with $R_i^* = T^*$, a lower equilibrium female reservation wage would mean a higher N_i. But the combined effect of both a higher N_i and a higher x is that single women become pickier.

Panel (a) in Figure 1 captures a Type 1 partial search equilibrium for the joint labour and marriage markets, with the female reservation wage graphed against the reservation wage of educated men. Panel (b) captures the *same* partial search equilibrium, but this time with the female reservation wage graphed against the reservation wage of uneducated men.

Figure 1: Partial search equilibrium



Note that in Panel (b) the female reservation wage (graphed against R_L) is positioned more to the right compared to the case depicted in Panel (a), where it is graphed against R_H. This configuration always holds for $R_H > R_L$.

Finally, we are now also in the position to describe the range of the parameter x for which different types of equilibria obtain. To this end, define x_1, x_2, x_3, and x_4 such that $T^*(x_1) = \underline{R}_H$, $T^*(x_2) = \widehat{T}_L$, $T^*(x_3) = \widehat{T}_H$, and $T^*(x_3) = \overline{w}_L$. Then, for $x \in (x_3, x_4]$ a Type 1 equilibrium exists; for $x \in (x_2, x_3]$ a Type 2 equilibrium exists, while a Type 3 equilibrium exists for $x \in (x_1, x_2]$.

4.2. Market equilibrium with schooling

Our focus is on a mixed market equilibrium characterised by (i) a Type 1 partial search equilibrium in the joint frictional markets and (ii) a fraction of men who choose schooling. Men will choose to invest in schooling as long as $\Delta U \equiv U_H - U_L > c$, and hence a necessary condition for an interior equilibrium is that $\Delta U = c$, so all men are indifferent between paying or not for education. Because in a Type 1 partial search equilibrium $U_i = R_i/(r + \delta)$ the above condition for such a mixed market equilibrium amounts to:

$$\Delta R^* (\equiv R_H^* - R_L^*) = (r + \delta)c.$$

Clearly, this equality pins down the value of returns to education required for an equilibrium. Since $\partial R_i(T)/\partial T < 0$ for $T > \widehat{T}_H$, while $\partial R_L(T)/\partial R_H = \partial R_H(T)/\partial R_L = 0$, we can write $\Delta R = \Delta R(T)$. Therefore, the above equation also pins down the equilibrium value(s) of T, and with it the associated equilibrium values of R_H and R_L.[6].

Suppose for now that there are several equilibrium female reservation wages which satisfy the equality between returns and costs of education. Say there are k such values of T, so that $\Delta R(T_j) = (r + \delta)c$, with $j = 1, 2, ...k$. Then, the necessary and sufficient condition for a mixed market equilibrium requires that the female reservation wage that emerges from the partial search equilibrium coincides with one of the female reservation wages that ensures the equality of education returns and costs. That is, $T^* = T_j$ for some $j = 1, 2, ...k$. In turn, this equilibrium T^* pins down the equilibrium male reservation wages R_H^* and R_L^*, and thus the market equilibrium returns to education ΔR^*.

Although the proportion of men who choose to invest in schooling is endogenous in the overall market equilibrium, it acts as a *parameter* in the determination of T^* in the partial search equilibrium, so we have $T^*(x, \tau)$. That is, there needs to be a τ_j such that:

$$T^*(x, \tau_j) = T_j \tag{4}$$

Denoting by τ_j^* the equilibrium value of this endogenous variable, we want to know how does an increase in single women's flow utility affect the equilibrium fraction of men who invest in schooling. To carry out this comparative statics exercise, consider equilibrium condition (4), and an increase in x. Note that an increase in x affects the triplet $\{R_H^*, R_L^*, T^*\}$ through its direct effect on $T(R_i)$ *only*. Ceteris paribus, women are pickier in the marriage market. As T shifts, the male reservation wages of both types decrease, thereby changing the returns to education. But then the equilibrium condition (4) does not hold, and only a lower fraction of men who invest in education can ensure $T = T_j$. Our main result below formalises this argument:

Theorem 1. *Consider a mixed market equilibrium (MME) with $T^* = T_j$, characterised by:*

(i) $x \in (x_3, x_4]$,
(ii) R_i^* as in Proposition 1(c),
(iii) T^* given by (2),
(iv) $\tau_j^* \in (0, 1)$ solving (4).

Then, $\partial \tau_j^*/\partial x < 0$.

Proof. Condition (4) holds in an MME. Recall that x and τ are both parameters in the partial search equilibrium that determines T^* and hence

[6] If ΔR is monotonic in T (and this of course depends on the distribution functions F_i), the condition in fact pins down a *unique* equilibrium triplet.

ΔR^*. For (4) to hold after an increase in x, there must be a change in τ such that $dT^*(x,\tau) = 0$, so that in turn the returns to education remain unchanged ($d\Delta R^* = 0$). Total differentiation of T^* yields $dT^*(x,\tau) = \frac{\partial T^*}{\partial x}dx + \frac{\partial T^*}{\partial \tau}d\tau$. Since $\partial T^*/\partial x > 0$ (see Proposition 2), an increase in x leads to a higher T^*, so $\frac{\partial T^*}{\partial x}dx > 0$. Hence, $dT^*(x,\tau) = 0$ only if $\frac{\partial T^*}{\partial \tau}d\tau < 0$. As $\partial T^*/\partial \tau > 0$ (again, see Proposition 2), this in turn requires $\partial \tau/\partial x < 0$. **QED**

The result that the effect of an increase in x on the equilibrium proportion of men who choose education is negative follows because, while a change in x alters the actual returns to education through its effect on T, it clearly does not alter the cost of schooling, and hence neither does it affect the value of T that is consistent with the equilibrium. This, together with the fact that both x and τ affect T positively, delivers our main result.

Recall that the result in Theorem 1 applies to each of the possible mixed market equilibria, where $T^* = T_j$ and $\tau = \tau_j^*$. Assume we are in such an equilibrium. The way men actually change their schooling decision after a change in x determines the actual direction of adjustment of τ. In particular, note that an increase in T has a negative effect on all men: it leads to a decrease in the proportion of available marriageable wages, with this decrease being *different* across male types. Indeed, we have $\partial \Delta R/\partial T < 0$ if the negative effect of an increase in T is stronger for H than for L type men, that is:

$$\frac{\partial F_H(T)}{\partial T} > \frac{r + \delta + \lambda_0[1 - F_L(R_L)]}{r + \delta + \lambda_0[1 - F_H(R_H)]} \frac{\partial F_L(T)}{\partial T} \tag{5}$$

Intuitively, the returns to education diminish as the female reservation wage increases if the increase in the proportion of unmarriageable wages in the distribution $F_H(.)$ faced by educated men is high enough relative to that in the distribution faced by uneducated men $F_L(.)$, where "high enough" takes into account the fact that the female reservation wage affects employment probabilities through its effect on male reservation wages.

Given this, we obtain the following result.

Proposition 3. *Consider a mixed market equilibrium (MME) with $T^* = T_j$.*

(i) If inequality (5) *holds, an increase in x leads to either an MME with same T^* and a lower proportion of educated men, or a corner solution with no educated men.*

(ii) If the inequality (5) *holds in the opposite direction, an increase in x leads to either an MME with a higher T^* and a higher proportion of educated men, or a corner solution with no uneducated men.*

Proof. In the MME we have $\Delta R^*(T^*(x,\tau)) = (r+\delta)c$. An increase in x leads to an increase in T^*. When (5) holds we have $\partial \Delta R/\partial T < 0$, and therefore now $\Delta R(T) < (r+\delta)c$, so τ adjusts downwards (reversing T to T^*) until either the original equilibrium is restored or a corner solution emerges, with $\tau = 0$. When (5) holds in the opposite direction, $\partial \Delta R / \partial T > 0$, and hence $\Delta R(T) > (r + \delta)c$, so τ adjusts upwards, thus increasing T even further. If ΔR is monotonic, this process continues until $\tau = 1$; if ΔR is not monotonic, it is possible that the equilibrium condition $\Delta R^*(T^*(x, \tau)) = (r + \delta)c$ is met for a *different* $T^*(= T_{j+1})$. **QED**

We can now spell out in detail and interpret the chain of reactions that follow a positive shock in women's options outside marriage. Recall that, in our model, such a shock is meant to capture changes in female labour market returns that are either partly or entirely due to enhanced schooling investment on their part. The immediate effect of any such change is that women become pickier in the marriage market. In turn, the resulting increase in the female reservation wage has an adverse effect on the men active in the marriage market, as it leads to a decrease in the proportion of marriageable wages, for both educated and uneducated single men. Unemployed men adjust their labour market strategy, with all males reducing their reservation wages. If the increase in x harms (through the subsequent increase in T) the marriage market prospects of educated men relatively more than those of uneducated men, so the former reduce their reservation wage more than the latter, the respective changes in male reservation wages add up to a decrease in the returns to schooling. Now the returns to schooling are too low. Hence, the fraction of men who undertake investment in education decreases. At this point, the countervailing effect kicks in. With fewer educated men around, women become less picky, and the associated decrease in the female reservation wage continues until it reaches its old level. Only then will men be once again indifferent between acquiring or not education. Interestingly, this transition mechanism suggests that if women experience a positive shock in the labour market (higher returns, possibly due to increased educational attainment), this is entirely offset by a negative effect on the marriage market, where their prospects suffer as the pool of educated eligible men shrinks.

It is also possible to end up in a corner solution with no educated men. Nonetheless, women are now better off because the initial positive effect of an increase in their labour market returns is not fully eroded by the deterioration in their marriage market prospects.

In turn, if the increase in x harms the marriage market prospects of uneducated men relatively more than those of educated men, the respective changes in male reservation wages add up to an increase in the returns to schooling. This makes women even more picky, but men respond and the proportion of educated men increases further, in a virtuous cycle. Whether this leads to different type of equilibrium with no uneducated men, or stops at *another* mixed market equilibrium, the women are always better off.

Finally, what about the other two types of possible mixed market equilibria, characterised by Type 2 and Type 3 partial search equilibria? In terms of existence and uniqueness, the exact same arguments as before apply, suitably adjusted by substituting U_i for $R_i/(r+\delta)$. Furthermore, it can also be shown that $\partial F_H(T)/\partial T > \partial F_L(T)/\partial T$ is now a sufficient condition for a decrease in returns to education following an increase in female reservation wage.

5. Conclusion

In this paper we contribute to the discussion surrounding the fact that male schooling has recently lagged behind that of female education attainment. We show that in the steady state equilibrium of a model where a male educational decision is embedded in a framework of inter-linked frictional labour and marriage markets, the proportion of men who choose to undertake the (costly) schooling investment can decrease after an exogenous increase in the female labour market returns (viewed as a proxy for female educational attainment).

With education improving their prospects both in the labour and the marriage market (where marital selection occurs based on male wages), when searching for jobs men react to the expectations set by women. An increase in women's labour returns (education) increases their value of being single and hence their marriage market reservation wage, which in turn affects the reservation wages of men in the labour market. Crucially, due to the link between the male reservation wage and equilibrium values of unemployment, their returns to education are also affected. Only a decrease in the proportion of educated men can restore the tie-breaking equality between these returns and the cost of schooling, since a lower fraction of such men leads to a lower female reservation wage. Interestingly, although the constrained sequential job search results in a non-monotonic male reservation wage function, the above result is robust as it holds for any range of such wages.

Our analysis points towards a more general feature of models of interconnected frictional markets where (i) access to one market is conditional on the outcome of constrained sequential search in the other market, and (ii) the prospects in both markets are influenced by an ex-ante costly investment. In such models, the proportion of agents who undertake the investment is crucial, and is determined by the equality between the expected returns from this investment and its cost. If there is a change in an exogenous variable that affects these expected returns (through the change in the requirements to access the other market), the fraction of these agents is the only endogenous variable left to restore the market equilibrium condition.

References

[1] Becker, G., 1991. A Treatise on the Family. Harvard University Press, Cambridge, MA.

[2] Blau, F., Kahn, L., Waldfogel, J., 2000. Understanding young women's marriage decisions: the role of labor and marriage market conditions. Industrial & Labor Relations Review 53: 624-47.

[3] Blundell, R., Costa Dias, M., Meghir, C., Shaw, J., 2016. Female labour supply, human capital and welfare reform. Econometrica 84 (5): 1705-55.

[4] Bonilla, R., Kiraly, F., 2013. Marriage wage premium in a search equilibirum. Labour Economics 24: 107–15.

[5] Bonilla, R., Kiraly, F., Wildman, J., 2019. Beauty premium and marriage premium in search equilibrium: theory and empirical test. International Economic Review 60: 851-77.

[6] Browning, M., Chiappori, P-A., Weiss, Y., 2014. Economics of the Family. Cambridge University Press, Cambridge, UK.

[7] Chiappori, P-A., Iyigun, M., Weiss, Y., 2009. Investment in schooling and the marriage market. American Economic Review 99: 1689–1713.

[8] Flinn, C., Mullins, J., 2015. Labour market search and schooling investment. International Economic Review 56: 359-98.

[9] Goldin, C., Katz, LF., Kuziemko, I., 2006. The homecoming of American college women: the reversal of the college gender gap. Journal of Economic Perspectives 20: 133-56.

[10] Gould, ED., Paserman, MD., 2003. Waiting for Mr. Right: rising wage inequality and declining marriage rates. Journal of Urban Economics 53: 257-81.

[11] Grossbard-Shechtman, SA., Neuman, S., 2003. Marriage and work for pay. In: Grossbard-Shechtman, SA. (Ed.), Marriage and the Economy. Cambridge University Press. Cambridge, UK.

[12] Ludwig, V., Brüderl, J., 2018. Is there a male marital wage premium? New evidence from the United States. American Sociological Review 83 (4): 744-70.

[13] Oppenheimer, VK., 1988. A theory of marriage timing. The American Journal of Sociology 94: 563-91.

[14] Oppenheimer VK , Lew, V., 1995. American marriage formation in the 1980s: how important was women's economic independence? In: Oppenheim Mason, K., Jensen, AM. (Eds.), Gender and Family Change in Industrialized Countries. Clarendon Press, Oxford, UK.

APPENDIX - Proof of Proposition 1

First, consider $T \in (\widehat{T}_i, \overline{w}_i]$. Assume for a moment that $R_i(T) \leq T$. Then, using $V_i^S(R_i) = R_i/(r+\delta) = U_i$, the male reservation wage $R_i(T)$ is given by:

$$R_i(T) = \frac{\lambda_0}{r+\delta}\int_{R_i}^{\overline{w}_i} [1 - F_i(w)]\, dw + \frac{\lambda_0 \lambda n\, [1 - F_i(T)]}{(r+\delta)(r+\lambda n+\delta)} y.$$

From the above, $R_i(\widehat{T}_i) = \widehat{T}_i$, where $\widehat{T}_i$ as defined in (3). Call this reservation wage $\widehat{R}_i$. Also, from the above, when $T \geq \overline{w}_i$ we have $R_i(T) = \underline{R}_i$, since then $F_i(T) = 1$. It is easy to show that $\widehat{T}_i < \overline{w}_i$, and $R_i(T)$ is decreasing in T. Hence, $R_i(T) < T$ iff $T > \widehat{T}_i$. For $T \leq \widehat{T}_i$, the reservation function derived above does not survive as an optimal strategy. Consider then $T \in (\underline{R}_i, \widehat{T}_i]$. Unemployed men are still not marriageable, and the value of being a single unemployed with a reservation wage $R_i > T$ is given by:

$$(r+\delta)U_i = \frac{\lambda_0}{r+\delta}\int_{R_i}^{\overline{w}_i} [1 - F_i(w)] dw + \frac{\lambda_0 \lambda n}{(r+\delta)(r+\delta+\lambda n)} y.$$

On the other hand, when choosing T as the reservation wage, this value is given by:

$$(r+\delta)U_i = \frac{\lambda_0}{r+\delta}\int_{T}^{\overline{w}_i} [1 - F_i(w)] dw + \frac{\lambda_0 \lambda n}{(r+\delta)(r+\delta+\lambda n)} y.$$

For $R_i > T$, the latter is higher than the former. Intuitively, the only reason to increase R_i above $\underline{R}_i$ would be in order to become marriageable. But $R_i = T$ is already enough for that.

Next, consider $T \leq \underline{R}_i$. If men believe they are marriageable irrespective of their employment status, they choose $R_i = \underline{R}_i$ both when single and married. This is because $V_i^S(w) = \frac{w}{r+\delta} + \frac{\lambda n}{(r+\delta+\lambda n)(r+\delta)} y$ for all $w \geq \underline{R}_i$. For $T \leq \underline{R}_i$, they are indeed always marriageable. Now consider the case with $T > \overline{w}_i$. We have $1 - F_i(T) = 0$, and therefore a man of type i can never get married, as the highest available wage is $\overline{w}_i$. Men optimally set $R_i(T) = \underline{R}_i$.

Finally, $\widehat{T}_H > \widehat{T}_L$ and $\underline{R}_H > \underline{R}_L$ follow from the fact that $F_H(.)$ first order stochastically dominates $F_L(.)$. **QED**