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Working Time Accounts and Turnover

Abstract

Working time account is an organization tool that allows firms to smooth their demand for hours employed. Descriptive literature suggests that working time accounts are likely to reduce layoffs and inhibit increases in unemployment during recessions. In a model of optimal labour demand I show that working time account does not necessarily guarantee less layoffs at the firm level. These may be reduced or increased depending on whether the firm meets economic downturn with surplus or deficit of hours and on how productive the firm is. In expected terms, however, working time account reduces net job destruction at almost any level of firm's productivity. Model predictions are consistent with dynamics of aggregate turnover in Germany during the Great Recession.

JEL-Codes: J230, J630.

Keywords: labour demand, working hours, working time account, turnover, layoff, Great Recession, Germany.

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1 Introduction

European unemployment has increased dramatically during the recession that followed the global financial crises of 2007-2008 (the *Great Recession*). Though a number of major OECD countries have reported soaring unemployment rates, notably the US where unemployment rate has increased by unprecedented 5.5 percentage points reaching 9.9% at its peak (OECD, 2018), the unemployment rate in Germany has shown nearly no changes.[1] Comparing Germany and the US in particular, although both countries have experienced a sharp decline in real GDP and a substantial reduction in person-hours worked, there were two important differences. First, while layoffs were characteristic for the US, in Germany instead there was a large decrease in hours worked per person with little job loss. Second, composition of sectors affected by the crises and patterns of sectoral post-crises recovery differed widely. In Germany the crises hit the exporting branch of the manufacturing (as measured by the drop in the value added). In the US, to the contrary, housing market, construction, retail services and financial services have suffered most. Germany has recovered faster than the US.[2]

In a landmark descriptive study Burda and Hunt (2011) look into multiplicity of factors that could help explain the surprisingly weak reaction of the German unemployment to the crises. Among others, they put forward a flexible working hours scheme called *working time account*.[3] Working time account is essentially a bookkeeping tool used by firms to track under- and overtime work. Firms that operate working time accounts for their personnel may, for instance, let employees work overtime but do not need to pay for this overtime work. Instead overtime work is written onto an account as a "debt" of the firm to its employee, such that at some point in future the employee may work less, running down

[1]In fact German unemployment rate has continued to fall, losing 0.5 percentage points in the first quarter of the recession. It did not change in the second quarter and started to go up only thereafter, picking 0.7 percentage points during the next two quarters. With the entire recession lasting one year, the economy entered recession with the unemployment rate of 7.7% and left recession with the unemployment rate of 7.9%. Once the recession was over unemployment rate started falling again (see OECD, 2018).

[2]For excellent descriptions of the US and German labour markets during the Great Recession see Elsby et al. (2010) and Burda and Hunt (2011), respectively.

[3]Möller (2010) and Rinne and Zimmermann (2013) also mention the potential of working time accounts.

overtime hours accumulated on her account. Hourly wage rate as well as per period pay stay constant regardless of whether the employee currently has surplus or deficit on her working time account. There are defined limits on the accumulated surplus and deficit of hours. By the end of the pre-specified time interval, called compensation period, the account must be balanced, i.e. both the firm's debt to the workers and workers' debt to the firm, measured in hours, should be equal to zero.[4]

Shortly before the financial crises almost 45% of all German employees were in possession of a working time account (Zapf, 2012), which made 41% of civil employment. Pre-crises years show a distinct pattern of changes in the balances of working time accounts at German establishments. While years 2005-2007 saw a gradual increase in surplus of hours, year 2008 has been marked with their unusually sharp fall (Zapf, 2012). Such dynamics has led the literature to suggest the mechanism through which working time accounts could have contributed to inhibiting the increase in German unemployment during the Great Recession. In particular, Burda and Hunt (2011) argue that by building up surpluses of hours worked in good times and running them down in bad times firms avoided firing workers immediately. A worker will not be fired unless she is compensated for the unpaid overtime hours worked previously. This compensation takes a form of working for a while at reduced hours with no change in worker's salary, consistent with the stylized fact of falling hours worked per person in Germany during the Great Recession. Since the crises in Germany was rather a consequence of a drop in demand for German export goods at the world's market, the nature of the negative shock to the economy was temporary. By running down the surplus first, working time accounts postponed job destruction and gave many jobs sufficient time to survive until world's demand started showing signs of recovery. With increasing pace of recovery slashing jobs has become increasingly unnecessary. This lack of job destruction has found reflection in the absence of increase of the unemployment rate.[5]

[4]Options for monetary compensation of overtime work may exist and details in configuration of working time accounts may vary. See Herzog-Stein and Zapf (2014) for excellent review of the organization of working time accounts in Germany.

[5]Recent empirical studies of Bellmann and Hübler (2015) and Balleer et al. (2017), however, question the strength of working time accounts' contribution to harnessing unemployment in Germany in 2008-2009.

In the present paper I first show that theoretical relationship between working time accounts and job destruction, which I equivalently call turnover unless indicated otherwise, is more general than the one described by Burda and Hunt (2011). I demonstrate that working time accounts do not always restrain turnover at the firm level when a negative demand shock hits the goods market. Sometimes turnover can be amplified. The ultimate impact of working time accounts on turnover depends on two factors: (i) on profitability of a firm, and (ii) on whether a firm has surplus or deficit on its working time accounts balance while facing demand downturn. I find that at relatively high-productive firms working time accounts lead to *lower* turnover if a firm has surplus of hours and *higher* turnover if a firm has deficit of hours prior to an adverse demand shock. At relatively low-productive firms converse is true: working time accounts lead to *higher* turnover if there is surplus of hours and *lower* turnover if there is deficit of hours in face of a negative demand shock. In all the cases above turnover is always compared to that of an identical firm without working time accounts.

Intuition for the possible harmful impact of working time accounts on high-productive firms that face negative demand shock with deficit of hours can be illustrated by looking at the combined influence of direct revenues realized during the demand downturn and reserves accumulated by the firm before the downturn. With deficit of hours prior to the shock less reserves will be accumulated comparatively to an identical firm without working time account. Once negative demand shock realizes, revenue loss may be too large to be compensated by available reserves even despite the increase in hours required to balance the account. As a result, jobs at a firm with working time account can be destroyed by a weaker shock than jobs at an identical firm without the account, where hours worked always remained constant. Intuition for the harmful effect of working time accounts on low-productive firms that face negative demand shock with surplus of hours is similar. Low productivity reduces profit and leads to low accumulation of reserves regardless of the use of working time account. Despite accumulated reserves will still be higher than at an identical firm without working time account due to surplus of hours, the necessity to reduce hours in order to balance the account once negative demand shock realizes may lead to a revenue loss too large to be compensated by reserves available. As a result jobs at a firm with working time accounts can again become more vulnerable.

While I show that the use of working time account leads to a variety of turnover outcomes, all of these outcomes are necessarily firm- and state-specific. Therefore, having demonstrated that working time accounts may increase turnover for *some* firms in *certain* states of demand at the goods market, I ask whether the same should be expected in aggregate terms. To answer this question I set up a numerical analysis in which I compute the expected turnover over the entire range of firm productivity distribution. Expectations are taken with respect to present and future demand fluctuations at the goods market. The result of this numerical analysis shows strong turnover-reducing aggregate effect of working time accounts, in line with Burda and Hunt (2011).

All my results are obtained in a purpose-built dynamic model of labour demand by a firm that operates a working time account and faces uncertainty about future state of the goods market. The firm is a local monopolist who chooses working hours subject to existing working time account regulations. There is no borrowing but the firm is allowed to build its own reserve fund. In the basic version of the model labour adjustment is permitted only along the intensive margin. In the generalized version of the model I add size adjustment via costly hiring/firing. The suggested framework is rather stylized as it purposefully omits many omnipresent features of the labour market and disregards equilibrium effects. Nevertheless, even in this stylized environment I can already generate ambiguity of the impact of working time accounts on turnover at the firm level.

With this, the present paper makes three important contributions to the existing literature. First, and to the best of my knowledge, this is the only paper to date that theoretically formalizes working time account and explains its mechanics, filling the obvious gap. Second, it discovers that the use of working time accounts may lead to excess job destruction, which is a completely new insight. Third, it shows that despite multiplicity of turnover outcomes at the firm level working time account reduces expected negative turnover. The latter is particularly important as it provides a micro-founded theoretical underpinning to the already existing hypothesis of Burda and Hunt (2011).

The paper is organized as follows. Section 2 presents a basic two-period model of a firm with a working time account where labour adjustment is permitted only along the intensive margin. Section 3 analyzes realized and expected turnover in this model. Section 4 introduces adjustment along the extensive margin and reviews the results for turnover in this generalized setting. Section 5 concludes and sets directions for future research.

2 Basic model of working time account

2.1 Market structure and characteristics of a firm

- Output and demand at the goods market

A firm is equipped with production technology $Y_t = Anh_t$, where A is the productivity of the firm, n is the firm size and h_t are the *actual* hours worked per worker. All workers are identical. For simplicity, in the basic model I assume that n is exogenously given. Endogenous hires/layoffs are considered in Section 4. Let m_t denote the demand for produced good. The demand function is specified as in Bentolila and Bertola (1990) suggesting that the firm is a local monopolist. With this, the reduced-form demand function becomes

$$m_t = z_t p_t^{1/(\epsilon-1)}, \quad \epsilon \in (0,1)\,, \tag{1}$$

where p_t is the price of a good and ϵ is the inverted price mark-up that reflects the monopoly power of the firm. Similarly to Bentolila and Bertola (1990), scale parameter z_t in this demand function is subject to stochastic fluctuations at the goods market. I suggest that z_t is a realization of a random variable Z_t, where $Z_t \sim F(z_t)$ and $F(z)$ is stationary. Stochastic fluctuations of z_t will constitute the only source of uncertainty influencing the optimal choice of hours employed by the firm in this model.

As in Bentolila and Bertola (1990) I assume that the firm produces a non-storable good i.e. output needs to equal demand at the goods market, $m_t = Anh_t$.

- Working hours and working time accounts

Supply of hours is perfectly elastic. I make an important distinction between *actual* hours and *contracted* hours employed by the firm. Despite a worker has actually worked h_t for her firm, the firm does not pay the worker on the basis of h_t. Wage bill of the firm is calculated on the basis of a *contracted* amount of hours $\bar{h}$ instead, where $\bar{h}$ does not change over time. At any given t it need not be that $h_t = \bar{h}$. Consequently, there may exist either surplus or deficit of actual hours worked relative to contracted hours. Surplus will be viewed as credit from worker to firm and deficit will be viewed as credit from firm to worker. In addition at any given t there are limits on actual hours worked such that a person cannot work more than $h^{\max}$ and less than $h_{\min}$, i.e. $h_{\min} \leq h_t \leq h^{\max}$.

At any t the surplus/deficit of hours worked is written onto a *working time account*. Let b_t denote the balance of the working time account. In addition let $b^{\max}$ stand for the upper limit of surplus accumulation, $b^{\max} > 0$, and let $b_{\min}$ stand for the lower limit of deficit accumulation, $b_{\min} < 0$. At the moment of opening the working time account, which I normalize to zero, the balance of the account is necessarily zero, $b_0 = 0$. For all the dates to follow the balance of the working time account may take any value between $b_{\min}$ and $b^{\max}$. However, at the end of each compensation period it must hold that the account is balanced, such that the total amount of actual hours worked is equal to the total amount of contracted hours within each compensation period. Equivalently, at the end of each compensation period all credit from worker to firm must be compensated by the firm as well as all credit from firm to worker must be compensated by the worker. Denoting the length of the compensation period by τ I therefore require that $b_{j\tau} = 0$, where $j = 1, 2, ...$[6]

Maintaining that time is discrete, the above argument implies the following law of motion for the balance of the working time account

$$b_t = b_{t-1} + (h_t - \bar{h}), \tag{2}$$

where $b_{\min} \leq b_t \leq b^{\max}$, $b_{j\tau} = 0$ with $j = 0, 1, 2, ...$ and $t = 1, 2, ...$

[6] According to Herzog-Stein and Zapf (2014) in 2007 in Germany average limit of surplus (deficit) accumulation was equal to $+103$ (-63) hours and average duration of compensation period was 38 weeks.

- Profit function and borrowing constraints

Profit function of a firm reads

$$\pi_t(h_t) = z_t^{1-\epsilon} [Anh_t]^{\epsilon} - wn\bar{h}. \tag{3}$$

where w is an exogenous hourly wage rate (see Online Appendix). The firm needs to pay its wage bill $wn\bar{h}$ at any t. If at some t wage bill cannot be paid, the firm shrinks in size leaving some of its workers unemployed till $t+1$. As a result, there arises demand for credit when in a given period t firm's revenue becomes insufficient to pay workers their contracted wage. This occurs, for instance, when a negative shock hits the goods market. Consistent with the credit crunch during the Great Recession, I do not allow firms to finance labour costs through borrowing at the financial market. However a firm is allowed to accumulate own reserves by retaining past undistributed profits. Reserves can be held in a form of a riskless asset with an interest rate r.

- Adjustment margin and other modeling choices

Several justifications for the above modeling choices are in place at this point.

First, intensive margin in this model is represented exclusively by the working time account. This implicitly omits variable-hours contracts, paid overtime work and other similar contractual agreements. There are two equally important reasons for that. One reason is that I would like to study the working time account as a single tool, abstracting from interactions with other variable-hours instruments. Another reason is that, although variable-hours contracts are not completely unheard of, the use of working time accounts in Germany at the establishment level is directly overseen by establishment's works council that traditionally promotes fixed-hours contracts. Moreover, without consent of works council it is impossible to change the arrangement of the contract (Herzog-Stein and Zapf, 2014). Works councils in Germany are particularly strong at large firms in the industrial sector, whereas exactly these firms took the heaviest toll of the Great Recession in Germany (see e.g. Möller, 2010).

Second, the model is partial in that it abstracts from any equilibrium effects through wages. Partiality in terms of the absence of equilibrium effects can be justified by the actual length of the compensation period that is typically less than a year. In presence of industry-level collective agreements wages are not expected to be adjusted faster than accounts are compensated (faster than once a year at the very least), which supports exogeneity of w.

2.2 Optimal choice of hours

- Time horizon and uncertainty

In the basic model I assume that a firm lives only for two periods (i.e. $t = 1{,}2$) and the compensation period for a working time account is equal to two model periods (i.e. $\tau = 2$). The rationale for setting the lifetime of a firm equal to the length of the compensation period is that working time account must be balanced at the end of each compensation period, so the optimal policy over a longer lifetime can be represented as a sequence of optimal policies each formed *within* a single compensation period.

I assume that demand level at the goods market reveals itself at the beginning of each model period. A firm chooses hours at the beginning of the first period. Thus it observes z_1 but still needs to form expectations about the value of z_2. These expectations are formed at $t = 1$ with respect to $F(z)$.

- Objective function and constraints

Consider the first period. As the firm observes z_1, the wage bill of the firm active at the market will always be paid in the first period, i.e. $\pi_1(h_1) \geq 0$ always holds. By the end of the first period the firm possesses $(1 + r)\,\pi_1(h_1)$ in terms of accumulated reserves.

Consider the second period. If the realized value of z_2 in the second period is small enough, such that $\pi_2(h_2)$ becomes negative, part of the wage bill in the second period will be paid out from accumulated reserves. If the realized value of z_2 is too small, such that the necessity to pay the wage bill in the second period consumes all reserves available, the firm starts laying off workers. Any claims of laid off workers, including claims on accumulated hours, are not honoured. Thus $(1+r)\,\pi_1(h_1)$ provides the lower bound for the loss in the second period while maintaining the size of the firm constant and defines the limit of liability of the firm towards its employees. Limited liability is important because it plays a role of transmission mechanism mapping demand fluctuations into fluctuations of turnover. Its presence makes turnover possible.[7]

Let $\beta \equiv 1/(1+r)$ denote the period discount factor. Then the problem of the firm writes

$$V = \max_{\{h_1,h_2\}} \{\pi_1(h_1) + \beta E_1(\pi_2(h_2))\}, \tag{4}$$

subject to (i) the law of motion for the balance of the working time account (2) with $b_2 = 0$, (ii): $h_{\min} \leq h_1, h_2 \leq h^{\max}$, (iii): $\pi_1(h_1) \geq 0$. In this problem E_1 is the expectation at $t = 1$.

- Optimal solution

Differentiating (4) with respect to hours, the optimal solution for both periods is

$$h_1^* = \frac{2}{\frac{1}{z_1}\left[\beta E_1\left(z_2^{1-\epsilon}\right)\right]^{1/(1-\epsilon)} + 1}\bar{h}, \tag{5a}$$

$$h_2^* = \frac{2}{1 + z_1\left[\beta E_1\left(z_2^{1-\epsilon}\right)\right]^{1/(\epsilon-1)}}\bar{h}, \tag{5b}$$

when constraints (ii)-(iii) are not binding (see Online Appendix). Appendix A.1 shows a general solution with kinks once any of (ii)-(iii) becomes binding. Figure A.1 in this appendix presents a typical hours profile in both periods.

From (5) two facts are evident. First, the demand for hours in both periods is influenced by uncertainty about the demand level at the goods market in the second pe-

[7] Current formulation suggests that the firm completely shifts the layoff risk associated with the use of working time account onto its employees. Alternative formulation with layoff risk shared between the firm and its employees requires introduction of works council as a worker representative at the firm level. While such extension is not unthinkable, in Germany at the very least, works councils do not have power to stop layoffs. Although they must be consulted at each instance of layoff, the most they can do is express objection (*Betriebsverfassungsgesetz, § 102 Mitbestimmung bei Kündigungen, Absatz 3*). This objection increases the chances of a worker to win the case in the court, but the ultimate decision is always by the court. The worker remains formally employed till the end of the litigation (*Betriebsverfassungsgesetz, § 102 Mitbestimmung bei Kündigungen, Absatz 5*). Works council does not represent workers in court.

riod. Second, there exists a unique demand level at the goods market in the first period, $z_1' : z_1' = \left[\beta E_1\left(z_2^{1-\epsilon}\right)\right]^{1/(1-\epsilon)}$, at which optimal hours are equal to contracted hours $\bar{h}$. Optimal hours in (5) have two more interesting analytical properties summarized in the following proposition (see Online Appendix for the proof).

Proposition 1 *Optimal hours in the first(second) period are an increasing(decreasing) function of the current demand at the goods market, z_1, and a decreasing(increasing) function of the expected demand at the goods market, $E_1(z_2)$.* □

These properties are particularly insightful if placed in the context of expansion/recession. If one associates the higher than average demand level at the goods market with an expansion and lower than average demand with a recession, then with values of z_1 sufficiently higher than $E_1(z_2)$ a firm will employ more hours in the expansion and with values of z_1 sufficiently lower than $E_1(z_2)$ a firm will employ less hours in the recession. Consequently, the optimal solution displays coherence with the observation that German firms have accumulated high surpluses on their working time accounts during the expansion and were running down these surpluses during the recession, as noted by Burda and Hunt (2011).

3 Turnover in the basic model

3.1 Realized turnover

- Two firms

Consider a firm with working time account and optimal demand for hours as described in Section 2. Consider next a completely identical firm that does not have working time account for some exogenous reason. The only difference between the two firms is that actual hours at the latter are restricted to be equal to contracted hours, i.e. $h_1 = h_2 = \bar{h}$. I seek to answer whether and under which conditions adverse demand shocks at the goods market can lead to higher job destruction at a firm with working time account, once compared to an identical firm without working time account.

For the entire Section 3.1 I will interchangeably use terms "layoff", "fire" and "job destruction" to refer to a forced reduction of the firm size due to exogenous demand shock. This size reduction is not the optimal choice of the firm in any way.[8]

- Layoffs with and without working time account

Consider a threshold level of the realized demand parameter in the second period at which a firm is just able to cover wage cost in the second period. Let z_2^* denote this threshold level for a firm with working time account and let and $\bar{z}_2$ denote this threshold level for a firm without working time account. Then for any realization of z_2 such that $z_2 < z_2^*$ ($z_2 < \bar{z}_2$) demand downturn at the goods market leads a firm with(without) working time account to start laying off workers. The respective layoff threshold levels are

$$z_2^* = \left(\frac{wn\bar{h} - (1+r)\,\pi_1\,(h_1^*)}{[Anh_2^*]^\epsilon} \right)^{1/(1-\epsilon)}, \tag{6a}$$

$$\bar{z}_2 = \left(\frac{wn\bar{h} - (1+r)\,\pi_1\,(\bar{h})}{\left[An\bar{h}\right]^\epsilon} \right)^{1/(1-\epsilon)}. \tag{6b}$$

(see Online Appendix). Both of them unambiguously increase in wage rate and decrease in productivity, i.e. the higher is the wage rate (the lower is the productivity) the weaker shock is needed to make the firm reduce its size. Most importantly, z_2^* and $\bar{z}_2$ are *not* equal to each other. The intriguing question therefore is: Is it always true that $z_2^* < \bar{z}_2$? If this is the case, then for intermediate realizations of the demand parameter z_2, namely for $z_2^* < z_2 < \bar{z}_2$, all workers at a firm with working time account will survive the demand downturn whereas some workers at an identical firm without working time account will be laid off. Consequently, working time account will contribute to reducing turnover.

It turns out that $z_2^* < \bar{z}_2$ may not always hold. Whether it does will ultimately depend on the relationship between the realized demand conditions at the goods market in the first period and the expected demand conditions at the goods market in the second period, which determines when the firm will have surplus and deficit at the working time account.

[8] One way of rationalizing this is to assume that no worker is allowed supply hours at a rate below w, so once an adverse shock hits, employment of a certain fraction of workers must be discontinued.

Proposition 2 establishes the result (see Online Appendix for the proof).

Proposition 2 *When productivity of a firm is high enough relative to its hourly wage and/or size, working time account: (i) reduces turnover if a firm meets demand downturn with surplus of actual hours employed; (ii) increases turnover if a firm meets demand downturn with deficit of actual hours employed.* □

Necessary condition for Proposition 2 to hold is given in the proof, though it is not particularly intuitive. However, if a firm is productive enough to ensure that revenues from the first period cover the present value of wage cost of both periods in absence of working time account, i.e. whenever $z_1^{1-\epsilon}\left[An\bar{h}\right]^{\epsilon} \geq (1+\beta)\, wn\bar{h}$, it is sufficient for Proposition 2 to apply. This sufficient condition is likewise derived in the proof.

Proposition 2 highlights one of the key messages of the paper. It shows that general dependence between working time account and turnover is ambiguous, i.e. working time account can reduce as well as increase turnover. While turnover-reducing effect has already been conjectured in the literature to date, possibility of turnover-enhancing effect has not yet been explored. For this reason emergence of a turnover-enhancing effect in the above theoretical framework deserves particular attention.

Two points are remarkable. First, in my model the firm commits to its working hours policy only within each compensation period. Therefore, to meet the demand downturn with deficit it must be that: $^{a)}$ when the firm was choosing its working hours policy demand conditions at the goods market were already poor (low z_1), which motivated the choice of initially running into deficit, and $^{b)}$ contrary to expectations, aggregate demand conditions did not improve thereafter (low z_2), i.e. production continued to be too low to repay wage credit given by workers to the firm. Thus the harmful effect of the working time account for high-productive firms materializes in *protracted recessions*, where recovery of demand at the goods market takes longer than initially expected.

Second, the very relationship between productivity on the one hand and wage rate and firm size on the other hand also plays a role. While for relatively high-productive firms surplus of hours on working time account insures against higher turnover, it turns out that for relatively low-productive firms the result is completely opposite. This follows from the proof of Proposition 2 and is summarized by the corollary below.

Corollary to Proposition 2 *When productivity of a firm is low enough relative to its hourly wage and/or size, working time account: (i) increases turnover if a firm meets demand downturn with surplus of actual hours employed; (ii) reduces turnover if a firm meets demand downturn with deficit of actual hours employed.* □

Necessary condition is given in the Proof of Proposition 2, though for Corollary to apply, it is sufficient that the firm operates at the break even point or at least epsilon-above it in absence of a working time account (see Online Appendix). Thus, under normal circumstances, it takes a weaker negative demand shock at the goods market to destroy jobs at a low-productive firm with working time account vis-a-vis the firm of the same productivity without the account; at least at the lower end of the productivity distribution.[9]

- How working time account influences turnover?

Working time account influences turnover through the two channels: intertemporal shifting of hours and accumulation/decumulation of reserves.

Consider first the situation in which a firm with working time account meets downturn with surplus of hours on the account and compare this firm to an identical firm without working time account. If a firm with working time account meets downturn with surplus, its profit in the first period is higher and its profit in the second period is lower than respective profits of a firm without working time account, due to intertemporal shifting of hours. Higher profit in the first period means larger reserve accumulated for the second period than at a firm without working time account. Thus, facing downturn in the second period, a firm with working time account has lower direct revenue in the second period but larger reserve

[9]It is worth acknowledging that in the present analysis Proposition 2 and corollary to this proposition focus on layoff thresholds, whereas more generally, turnover is a combination of the probability of a layoff, indicated by the threshold, and the size of layoff conditional on layoff occurring. Unfortunately analytical treatment of *both* the threshold and layoff size for shocks below this threshold is infeasible. This analytical intractability is partly compensated by the numerical analysis of Section 3.2, where I consider expected turnover across the entire range of the productivity distribution.

available for the second period than an identical firm without working time account. It will be able to withstand a stronger demand downturn without firing workers only if larger reserve outweighs the higher loss due to reduced hours. The higher is the productivity of a firm relative to its wage cost and size, the higher is the share of reserve in the total wage fund available to cover wage bill in the second period. Since a firm with working time account has formed a larger reserve, the result of Proposition 2 applies and this firm withstands stronger shock than the identical firm without the account. Once productivity of a firm relative to wage cost and size gets lower, the lower gets the share of reserve in the total wage fund and hence the more important become hours in the second period. So by Corollary 1 workers at the firm with working time account get fired after a weaker shock than at an identical firm without the account.

Consider now the situation in which a firm with working time account meets downturn with deficit of hours on the account. The mechanics is just the opposite. Once meeting downturn with deficit, the profit in the first period is lower and the profit in the second period is higher than respective profits of an identical firm without working time account, as implied by intertemporal shifting of hours. Lower profit in the first period means smaller reserve accumulated for the second period than at a firm without working time account. Thus, facing downturn in the second period, a firm with working time account has higher direct revenue in the second period but smaller reserve available for the second period than an identical firm without working time account. To be able to withstand a stronger demand downturn without layoffs, the loss incurred in the second period, despite being lower than at an identical firm without working time account, should still not be too large so that it could be covered by the relatively smaller accumulated reserve. The higher is the productivity of a firm relative to wage cost and size, the higher is the share of reserve in the total wage fund. Since a firm with working time account has formed a smaller reserve, according to Proposition 2 it takes a weaker shock to destroy a job at this firm comparatively to an identical firm without working time account. Once productivity gets sufficiently low relative to wage cost and size, direct effect of higher hours acquires more importance than reserve accumulation. So Corollary 1 applies and a firm with working time account withstands a stronger demand downturn without layoffs.

3.2 Expected turnover

- Measurement

The analysis so far was implicitly conditional on a given constellation of realized demand parameters z_1 and z_2. It has been shown that there exist realizations of z_1 and z_2 under which a firm with working time account will experience lower/higher turnover than an identical firm without the account. Does working time account also lead to a lower/higher *expected* turnover? To answer this question consider a firm with working time account and define φ^* as a fraction of workers that have to be laid off in the wake of the adverse demand shock in the second period, $\varphi^* \in (0, 1]$. As $\pi_2 (h_2^*)$ cannot drop below $-(1+r)\, \pi_1 (h_1^*)$, for any given realization of z_2, $z_2 \in (0, z_2^*)$, the fraction $\varphi^* (z_2)$ of workers to be laid off must equalize the loss in the second period on the one hand and accumulated reserves on the other hand,

$$\pi_2 \left(h_2^*, [1-\varphi^* (z_2)]\, n\right) = -(1+r)\, \pi_1 (h_1^*, n) . \tag{7}$$

Since $\pi_1 (h_1^*, n)$ directly depends on z_1, the productivity, wage rate, firm size and contracted hours through (3), the fraction φ^* does so too. To reduce dimensionality of the analysis I normalize (7) by the period wage cost defining $\widetilde{A} \equiv \frac{[An\bar{h}]^{\epsilon}}{wnh}$, such that the fraction of workers to be laid off ultimately becomes a function of demand realizations in both periods given the normalized productivity, $\varphi^*(z_1, z_2; \widetilde{A})$. Considering an identical firm without working time account, the fraction of workers to be laid off in the second period, $\bar{\varphi}(z_1, z_2; \widetilde{A})$, is defined by repeating the same steps.

Let z_1^* denote the realization of demand in the first period at which the firm with working time account breaks even and let $\bar{z}_1$ denote the realization of demand in the first period at which an identical firm without working time account breaks even. Then the expected turnover at both firms is measured by

$$E(\varphi^*; \widetilde{A}) = \int_{z_1^*}^{\infty} \left[\int_0^{z_2^*} \varphi^*(z_1, z_2; \widetilde{A}) f(z_2)\, dz_2 \right] f(z_1)\, dz_1, \quad (8a)$$

$$E(\bar{\varphi}; \widetilde{A}) = \int_{\bar{z}_1}^{\infty} \left[\int_0^{\bar{z}_2} \bar{\varphi}(z_1, z_2; \widetilde{A}) f(z_2)\, dz_2 \right] f(z_1)\, dz_1\,, \quad (8b)$$

respectively. Since the probability distribution of $\widetilde{A}$ is very difficult to establish in practice, in what follows I will consider expected turnover as a function of a given $\widetilde{A}$ on the entire range of the distribution of normalized productivity.

- Numerical analysis

Due to intractability of (8a) and (8b) I cannot show analytically under which circumstances one measure exceeds the other. Therefore for the rest of Section 3.2 I resort to numerical analysis. Parameter choice and robustness are discussed in Appendix A.2.

Before computing both measures it is instructive to see how the fraction of layoffs at both firms depends on demand fluctuations. Figure 1 illustrates. The column dimension of this figure shows dependence on a given value of the normalized productivity: low (left column, $\widetilde{A} = 0.97$) and high (right column, $\widetilde{A} = 1.00$).

First row of Figure 1 plots the ratio of layoff thresholds in the second period (6a)-(6b), defined as $\bar{z}_2/z_2^*$. The left panel in this row shows that for small enough realizations of z_1 layoff threshold of the firm with working time account is larger than that of the firm without the account ($\bar{z}_2/z_2^* < 1$). In other words there exists a set of realizations of z_2 within which the firm with working time account will start laying off workers while the firm without the account will not, as discussed in Section 3.1. The right panel demonstrates that for higher productivity this ceases to be the case. At both panels the leftmost value of $\bar{z}_2/z_2^*$ corresponds to the lowest realization of demand in the first period at which both firms (with and without the account) are active at the market in the first period.

Second row shows the fraction of workers that need to be fired in the second period as a function of z_2 for selected values of z_1.[10] It demonstrates that worse demand conditions in

[10] These are roots of (7) and roots of a similar equation for a firm without working time account.

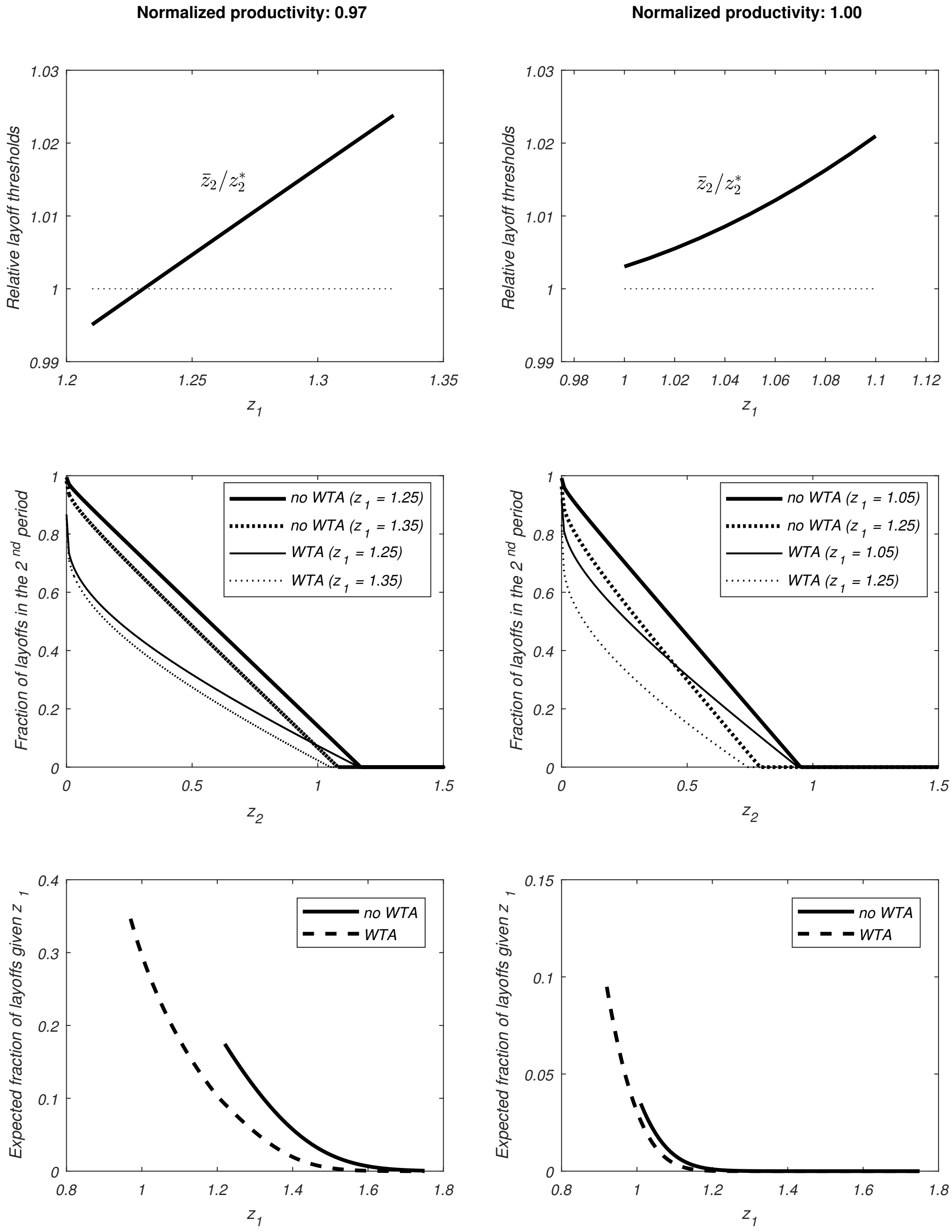

Figure 1: Job destruction with and without working time account (WTA)

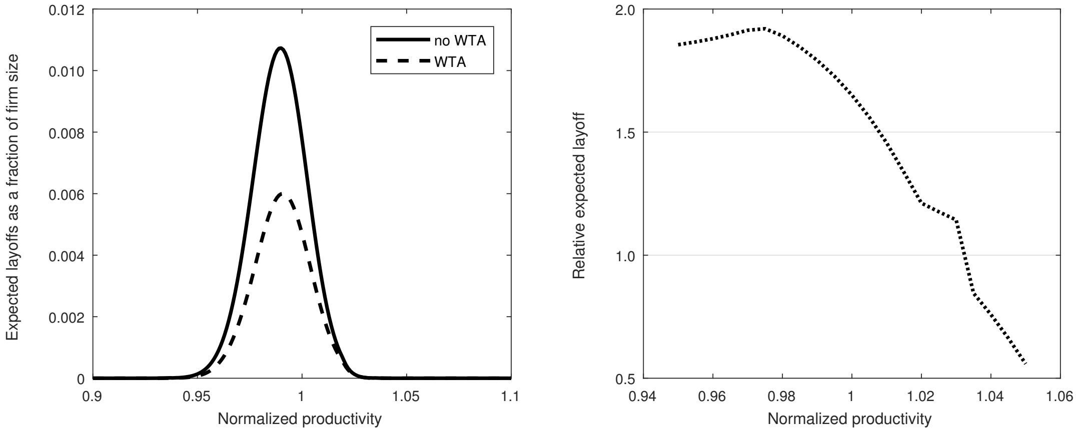

Figure 2: Expected layoff with and without working time account (WTA)

the first period (solid lines) lead to higher fraction of layoffs than better demand conditions in the first period (dotted line of the same thickness) and that this holds for both choices of $\widetilde{A}$. Furthermore, for the chosen values of z_1 a firm without working time account (thin lines) will fire less workers than an identical firm without the account (thick lines), again regardless of productivity.

Third row shows the expected fraction of workers that need to be fired in the second period as a function of z_1 , where expectation is over the distribution of shocks in the second period.[11] The leftmost value of the solid(dashed) line corresponds to the lowest value of z_1 at which a firm without(with) working time account can be active at the market in the first period. This row shows that for the set of z_1 at which *both* firms with and without the account are active, the firm with the account will fire less workers in expected terms (the dashed line is below the solid line) and both firms will fire less at higher $\widetilde{A}$.

Finally, left panel of Figure 2 shows for any value of the normalized productivity $\widetilde{A}$ the expected fraction of workers that will be fired at both firms. These are expressions (8a)-(8b) computed for all z_1 at which both firms are active in the first period to facilitate a well-defined comparison.[12] Left panel of Figure 2 demonstrates that expected turnover at a firm without working time account is larger than at an identical firm with working time account for all levels of normalized productivity where the size of turnover is non-negligible.

[11]These are inner integrals in (8a) and (8b).

[12]Lower bound of integration of the outer integral in both (8a)-(8b) is replaced by $\max\{z_1^*, \bar{z}_1\}$.

At the highest, working time account reduces expected turnover by 0.48% of the firm size n. Right panel of Figure 2 plots the expected turnover at a firm without working time account relative to the expected turnover at a firm with the account. It shows that at very high levels of normalized productivity working time account will increase expected turnover (the ratio becomes less then one). However expected fractions of workers to be fired at such firms are negligibly small to make any difference once aggregation over the distribution of $\widetilde{A}$ is made (at most 0.0012% and 0.0010% of the firm size at the firm with and without working time account, respectively).

Concluding, although there exist demand conditions at the goods market under which working time account will enhance turnover, in expected terms the use of working time account will have a clear turnover-reducing effect everywhere, except at a extremely high end of the distribution of normalized productivity, where turnover itself is negligible in size.

3.3 Discussion

- Germany during the Great Recession: Policy experience and implications

Despite its relative simplicity the model provides an able framework to mirror the pattern of turnover in Germany during the Great Recession as suggested in the earlier empirical literature, as well as consider possible alternatives in light of some recent evidence. As for the main suggested pattern, four observations in the literature appear relevant. First, Burda and Hunt (2011) argue that German firms have met the Great Recession with high surpluses on their working time accounts. Second, Möller (2010) states that the recession has primarily hit German exporting firms in manufacturing, which are regarded by Möller (2010) as "strong firms in economically strong regions". Third, Dustmann et al. (2014) emphasize low unit labour costs throughout the entire period of interest. Fourth, the recession itself was lasting just one year, in contrast to much longer recovery in many OECD countries. These four observations fit well into the prediction of Proposition 2, which states that positive surpluses, high productivity relative to wage cost and no protraction of the recession make working time accounts reduce turnover.

As for some recent evidence, two new studies question the extent of positive influence of working time accounts on turnover. Bellmann and Hübler (2015) find that working time accounts reduce profits. Balleer at al. (2017) do not find changes in firing/hiring behavior of firms due to working time accounts. While weakening the conjecture of Burda and Hunt (2011), these new evidence can still be well accommodated in the predictions of my model. Notably, the model suggests that harmful effect of working time accounts materializes after the accounts start reducing profit, whereas the latter is what Bellmann and Hübler (2015) document.

While all the empirical work to date tells that influence of working time accounts on turnover in Germany during the Great Recession was either positive or neutral at the very least, these conclusions are derived from just one recession episode in just one country. In theory I put forward that this need not hold universally because the influence of working time accounts depends on the very nature of the shock. Two important situations in which presence of accounts can be more destructive than their absence are when recessions take much longer than expected and when recessions primarily hit low-productive firms.[13] More generally, there will always be high-productive firms and low-productive firms caught at a wrong foot during any recession. So, for the total effect of working time accounts what will ultimately matter is the aggregation of all positive/negative firm-specific effects across the entire economy. I perform such an aggregation numerically to find out that the expected effect of working time accounts on reducing job destruction is very positive. Thus the ultimate conclusion of the analysis in the basic model with working time account is that despite there may have been firms at which working time accounts have contributed to higher turnover, the overall effect was an unambiguous reduction of job loss due to use of working time accounts.

Thinking of mitigation of undesired effect of working time accounts for those particular firms affected, another tool from the German institutional palette, the so-called short-time work, can help if combined with introduction of the accounts. Short-time work supports the

[13] An example of such a setting would be Spain where shock to construction industry during the Great Recession was permanent rather than temporary.

firm by temporarily paying salaries to employees using public funds. Thereby it postpones layoffs and hedges from protraction of the recession, weakening possible harmful effect of working time accounts. Indeed Möller (2010) documents the pattern in which firms that were eventually applying for access to short-time work would first run down surpluses on their working time accounts, if operating ones. Balleer et al. (2016) demonstrate, though, that it is important to distinguish the automatic stabilization effect of short-time work from a discretionary intervention. They find that while discretionary intervention did not help during the Great Recession mere presence of short-time work in capacity of automatic stabilizer has significantly contributed to restraining increase in unemployment. Combining my results with those of Balleer et al. (2016), policy advise would go for introduction both working time accounts and short-time work simultaneously and permanently.

Lastly, although the model predicts an unambiguous reduction in expected turnover, due to its partial nature the model cannot quantify the contribution of working time accounts to gross labour reallocation. Literature suggests that low job destruction during the Great Recession in Germany is at least partly due to structural reforms that took place in 2003-2005 (see e.g. Bauer and King, 2018; Hartung et al., 2018; Carrillo-Tudela et al., 2019), so the impact of working time accounts needs to be evaluated conditional on these reforms. Furthermore, the strength of working time accounts' impact is constrained by accumulation limits for surplus/deficit (Herzog-Stein and Zapf, 2014).

- Working time accounts in other countries

Legislative base for regulating working time accounts is in place in all the member states of the European Union that acceded prior to eastern enlargement, as well as in some other states of the European Economic Area, e.g. in Norway (EU, 2010). Yet, actual use of working time accounts is seen only in the Nordic countries, Germany and Austria. According to EU (2010), in Norway 32% of male and 27% of female employees accumulate either hours or days; 18% of Danish and 16% of Finnish employees accumulate days; in Sweden hours or days accumulation is reported as "common" among employees. Such geographical localization is explained by high degree of social partnership and the managerial culture in those countries.

In particular, managerial culture supports worker representation not only through unions but also through works councils. Works councils deal with firm-specific aspects of worker-firm relations at the level of establishment or of a department of an establishment complementing a more aggregated industry-level representation through unions. Enforcement of working time accounts at the firm level is a legislative prerogative of works councils.

Through the looking glass of the numerical analysis of Section 3.2 any of these countries is different from Germany only in terms of markup, limits of hours accumulation, duration of balancing period and variance of the shocks at the goods market. Since moderate variation in these parameters does not overturn the main prediction, expected effect of working time accounts in these countries should most likely be positive too.

4 Extended model with two margins of adjustment

In this section I enable firms to adjust their size through costly hiring and firing. I then review predictions of the basic model in presence of the intensive and extensive margins of adjustment.

4.1 Model structure and solution

- Adjustment margins and timing

I assume that a firm chooses its hours and size simultaneously, at the beginning of the first period. I do not allow the firm to revise its hiring/firing decision at the beginning of the second period, when uncertainty about demand at the goods market is revealed, in order to avoid situations in which the firm will fire workers while still making positive profits.[14] Nevertheless, just as in the basic model, the firm may still fire workers in the second period for exogenous reason, namely when realized demand at the goods market in the second period

[14] Alternatively, I can allow the firm to fire at an additional cost that acts as a penalty for early termination of the two-period working hours contract signed at the beginning of the first period. Large enough penalty will rule out potential downward size adjustment while still retaining the possibility for an upward size adjustment. While not impossible, this introduces a substantial modeling complication, as the firm will be managing working time accounts with different balancing dates.

is too low to sustain the initially chosen size.[15] Thus in the extended model there is turnover in both periods: in the first period there is hiring/firing induced by profit maximization; in the second period there is potentially only firing induced by an adverse demand shock. Since there is a possibility of hiring, turnover in the extended model ceases to be equivalent to job destruction, as was the case in the basic model.

- Objective function and constraints

I introduce firm size adjustment similarly to Bertola and Bentolila (1990). Let n_h denote the number of hired workers and n_f denote the number fired workers.[16] Let H denote hiring cost per worker and F denote firing cost per worker. As workers are homogeneous the firm can either hire or fire. Let $\mathbb{I}_{n_h>0}$ denote the indicator function that takes value 1 if the firm hires and $\mathbb{I}_{n_f>0}$ denote the indicator function that takes value 1 if the firm fires. Mutual exclusivity of hiring and firing is reflected by $\mathbb{I}_{n_h>0}\mathbb{I}_{n_f>0} = 0$. With this, the profit function of the firm writes

$$\begin{aligned}\pi_t\left(h_t, n_h, n_f\right) = z_t^{1-\epsilon}\left[A\left(n + \mathbb{I}_{n_h>0}n_h - \mathbb{I}_{n_f>0}n_f\right)h_t\right]^{\epsilon} \\ - w\left(n + \mathbb{I}_{n_h>0}n_h - \mathbb{I}_{n_f>0}n_f\right)\bar{h} - \left(\mathbb{I}_{n_h>0}n_hH + \mathbb{I}_{n_f>0}n_fF\right)\mathbb{I}_{t=1}, \quad (9)\end{aligned}$$

where $t = 1, 2$ and $\mathbb{I}_{t=1}$ is an indicator function that takes value one in the first period, reflecting the fact that size adjustment decision is made in the first period and that this decision is influenced by uncertainty about future demand at the goods market.

With equation (9) in place of (3) the problem of the firm becomes

$$V = \max_{\{h_1,h_2,n_h,n_f\}}\{\pi_1\left(h_1, n_h, n_f\right) + \beta E_1\left(\pi_2\left(h_2, n_h, n_f\right)\right)\}, \quad (10)$$

subject to (i) the law of motion for the balance of working time account (2) with $b_2 = 0$, (ii): $\pi_1\left(h_1, n_h, n_f\right) \geq 0$, (iii): $h_{\min} \leq h_1, h_2 \leq h^{\max}$, (iv) $\mathbb{I}_{n_h>0}\mathbb{I}_{n_f>0} = 0$.

[15]Similarly to the basic model, terms “layoff”, “fire” and “job destruction” in the second period of the model mean forced reduction of the firm size that is not related to optimizing behavior of the firm.

[16]For analytical convenience I maintain that n_h and n_f are continuous variables.

- Optimal solution

Optimal choice of hours in both periods remains identical to the one already reported in (5a)-(5b). Independence of optimal hours and optimal size adjustment decisions obtains because the firm does not internalize layoff risk, shifting it entirely onto workers. Optimal amount of workers hired (fired) is given by n_h^* (n_f^*), respectively

$$n_h^* = [\epsilon A^\epsilon]^{1/(1-\epsilon)} \left(\frac{z_1^{1-\epsilon}[h_1^*]^\epsilon + \beta E_1 \left(z_2^{1-\epsilon}\right) [h_2^*]^\epsilon}{(1+\beta)\, w\bar{h} + H} \right)^{1/(1-\epsilon)} - n, \tag{11a}$$

$$n_f^* = n - [\epsilon A^\epsilon]^{1/(1-\epsilon)} \left(\frac{z_1^{1-\epsilon}[h_1^*]^\epsilon + \beta E_1 \left(z_2^{1-\epsilon}\right) [h_2^*]^\epsilon}{(1+\beta)\, w\bar{h} - F} \right)^{1/(1-\epsilon)}, \tag{11b}$$

(see Online Appendix for all derivations). Remarkable about adjustment along the extensive margin in (11a)-(11b) is that a firm can hire/fire regardless of $h_1^* \gtreqless h_2^*$, i.e. no matter if working time account in the first period has surplus or deficit of hours. This tells that extensive and intensive margin can be substitutes or complements.

4.2 Turnover with two margins of adjustment

- Working time account and realized turnover

To see whether ambiguous influence of working time account on turnover also obtains in the model with extensive margin first I need to consider an identical firm that offers the same two-period wage contract but does not have working time account for some exogenous reason. The problem of such a firm is a special case of (10) with $h_1 = h_2 = \bar{h}$ subject to only two constraints: (ii) and (iv). When such a firm hires or fires, optimal amount of workers hired (fired) is given by $\bar{n}_h$ ($\bar{n}_f$), respectively

$$\bar{n}_h = \left(\epsilon \left[A\bar{h}\right]^\epsilon \frac{z_1^{1-\epsilon} + \beta E_1 \left(z_2^{1-\epsilon}\right)}{(1+\beta)\, w\bar{h} + H} \right)^{1/(1-\epsilon)} - n, \tag{12a}$$

$$\bar{n}_f = n - \left(\epsilon \left[A\bar{h}\right]^\epsilon \frac{z_1^{1-\epsilon} + \beta E_1 \left(z_2^{1-\epsilon}\right)}{(1+\beta)\, w\bar{h} - F} \right)^{1/(1-\epsilon)}, \tag{12b}$$

(see Online Appendix). Comparison of hiring/firing decisions of the firms with and without working time accounts uncovers an interesting picture of turnover in the first period, summarized in Proposition 3 (see Online Appendix for the proof).

Proposition 3 (Turnover in the first period) *A firm with working time account always fires less and hires more workers than an identical firm without working time account.*

When a firm with working time account fires, an identical firm without working time account will necessarily fire. When a firm with working time account hires, an identical firm without working time account may either hire or fire. □

The first statement of Proposition 3 implies that regardless the initial size n, a firm with working time account will be larger than an identical firm without working time account once the size adjustment decision is made. This result is a manifestation of the firm's use of working time account to hoard labour by shifting some of its labour adjustment to the costless intensive margin in view of costly adjustment along the extensive margin. The second statement of Proposition 3 tells that working time account matters not only quantitatively, but also qualitatively as it opens the scope for hiring in the situations where an identical firm without the account would actually shrink.

Implications for turnover in the first period are non-trivial because firms with and without working time account do not necessarily use identical size adjustment strategies. Furthermore, the very definition of turnover will matter. For instance, if turnover is defined as the absolute change in firm size relative to the initial firm size (e.g. Boeri, 1996, equation 1), working time account will increase turnover if both firms hire, reduce turnover if both firms fire and have an ambiguous effect if a firm with working time account hires whereas a firm without the account fires its employees. If one considers gross job creation and gross job destruction rates as relevant measures of turnover (Davis and Haltiwanger, 1992)[17], working time account unambiguously increases gross job creation and unambiguously reduces gross job destruction.

[17] POS_{st} and NEG_{st}, respectively, reduced to a single establishment within a single sector; see Davis and Haltiwanger (1992), page 828.

Differences in size adjustment decisions of a firm with and without working time accounts in the first period shape the patterns of potential job destruction in the second period. For convenience of exposition let Σ denote the set of all possible size adjustment strategies, $\Sigma \equiv \{\text{'}hire\text{'}, \text{'}fire\text{'}\}$. Let $\sigma \in \Sigma$ be a strategy chosen by a firm with working time account, let $\sigma' \in \Sigma$ be a strategy chosen by an identical firm without working time account and let (σ, σ') be a pair of such strategies. Let S denote the set of all possible pairs of size adjustment strategies, $S = \{(\text{'}fire\text{'},\text{'}fire\text{'}), (\text{'}hire\text{'},\text{'}hire\text{'}), (\text{'}hire\text{'},\text{'}fire\text{'})\}$, as per Proposition 3. The effect of working time account on turnover in the second period is summarized by the following proposition (see Online Appendix for the proof).

Proposition 4 (Turnover in the second period) *When labour adjustment strategies are ('fire','fire') and productivity of a firm is high enough relative to its hourly wage and/or initial size, working time account: (i) reduces turnover if a firm meets demand downturn with surplus of actual hours employed; (ii) increases turnover if a firm meets demand downturn with deficit of actual hours employed.*

When labour adjustment strategies are ('hire','hire'), regardless of productivity, hourly wage and initial size working time account: (i) reduces turnover if a firm meets demand downturn with surplus of actual hours employed; (ii) increases turnover if a firm meets demand downturn with deficit of actual hours employed.

When labour adjustment strategies are ('hire','fire'), working time account: (i) reduces turnover if a firm meets demand downturn with surplus of actual hours employed, regardless of productivity, hourly wage and initial size; (ii) increases turnover if a firm meets demand downturn with deficit of actual hours employed and productivity of a firm is high enough relative to its hourly wage and/or initial size. □

Proposition 4 reiterates ambiguity of the effect of working time account on turnover in the second period. Compared to the basic model, turnover in protracted recessions becomes more common, as for certain combinations of hiring strategies it obtains regardless of productivity, wage and initial firm size. In the basic model, to the contrary, this was the case only for relatively productive firms. Mechanism of influence remains as described in Section 3.1.

Summing up, Propositions 3 and 4 tell that even after allowing for adjustment along the extensive margin there will still exist states of demand at the goods market in which working time account will increase job destruction in the second period. Whether this effect will be offset by turnover in the first period (either through reduced job destruction or through increased hiring) and in expected terms is the question for the subsequent numerical analysis.

- Working time account and expected turnover

Similar to the analysis of Section 3.2 I consider a firm with working time account and define φ^* as a fraction of workers to be laid off in the wake of an adverse demand shock in the second period. For any given realization of z_2 this fraction $\varphi^*(z_2)$ solves,[18]

$$\pi_2\left(h_2^*, n_h^*, n_f^* | \varphi^*(z_2)\right) = -(1+r)\,\pi_1\left(h_1^*, n_h^*, n_f^*\right). \tag{13}$$

Since (11a)-(11b) directly depend on hiring and firing costs in addition to the rest of the parameters of the model, dimensionality of the problem increases. I normalize (13) by the period total wage cost $wn\bar{h}$ and define a per-worker hiring(firing) cost as a share of worker's period wage bill, $\widetilde{H} \equiv \frac{H}{w\bar{h}}$ and $\widetilde{F} \equiv \frac{F}{w\bar{h}}$, respectively. The fraction of workers to be laid off ultimately becomes the function of demand realizations in both periods given the normalized productivity, hiring and firing cost: $\varphi^*(z_1, z_2; \widetilde{A}, \widetilde{H}, \widetilde{F})$.

The size of realized layoffs in the second period is given by $\varphi^*(z_1, z_2)(n + \mathbb{I}_{n_h>0} n_h^* - \mathbb{I}_{n_f>0} n_f^*)$. Since there is turnover in both periods, I consider net job destruction,

$$\delta^*(z_1, z_2; \widetilde{A}, \widetilde{H}, \widetilde{F}) = \frac{1}{n}\left(\mathbb{I}_{n_f>0} n_f^* - \mathbb{I}_{n_h>0} n_h^* + \varphi^*(z_1, z_2)(n + \mathbb{I}_{n_h>0} n_h^* - \mathbb{I}_{n_f>0} n_f^*)\right), \tag{14}$$

which is the total size of layoffs in both periods net of possible hiring in the first period, normalized by the initial size of the firm. Next, I restrict my attention only to shrinking firms, i.e. to realizations of demand at which net job destruction is greater than zero. I do so

[18] $\pi_2\left(h_2^*, n_h^*, n_f^* | \varphi^*(z_2)\right)$ in equation (13), is a simplified notation for $\pi_2\left(h_2^*, n_h^*, n_f^* | \varphi^*(z_2)\right) = z_2^{1-\epsilon}\left[A\left[1-\varphi^*(z_2)\right](n + \mathbb{I}_{n_h>0} n_h^* - \mathbb{I}_{n_f>0} n_f^*) h_2^*\right]^{\epsilon} - w\left[1-\varphi^*(z_2)\right](n + \mathbb{I}_{n_h>0} n_h^* - \mathbb{I}_{n_f>0} n_f^*)\bar{h}$.

to focus on the job-destruction-reducing effect of the working time account. Consequently, for any given combination of $\widetilde{A}$, $\widetilde{H}$ and $\widetilde{F}$, I construct the measure of an *expected net turnover* over both periods at a firm that shrinks

$$E(\delta^*; \widetilde{A}, \widetilde{H}, \widetilde{F}) = \int_{z_1^*}^{\infty} \left[\int_0^{z_2^*} \max\left\{0, \delta^*(z_1, z_2; \widetilde{A}, \widetilde{H}, \widetilde{F})\right\} f\left(z_2\right) dz_2 \right] f\left(z_1\right) dz_1, \qquad (15)$$

where z_1^* is the threshold demand parameter at which the firm becomes active at the market in the first period and z_2^* is the threshold demand parameter for the firm to start laying off workers in the second period.

Repeating the same steps, the *expected net turnover* at an identical shrinking firm without working time account is

$$E(\bar{\delta}; \widetilde{A}, \widetilde{H}, \widetilde{F}) = \int_{\bar{z}_1}^{\infty} \left[\int_0^{\bar{z}_2} \max\left\{0, \bar{\delta}(z_1, z_2; \widetilde{A}, \widetilde{H}, \widetilde{F})\right\} f\left(z_2\right) dz_2 \right] f\left(z_1\right) dz_1, \qquad (16)$$

where and $\bar{z}_1$ and $\bar{z}_2$ are the corresponding entry and layoff thresholds.

As in the basic model, I am interested to see whether $E(\bar{\delta}; \widetilde{A}, \widetilde{H}, \widetilde{F})$ exceeds $E(\delta^*; \widetilde{A}, \widetilde{H}, \widetilde{F})$. Since both expressions condition on a three-dimensional set of parameters, graphical expression is not straightforward. Figure 3 shows the difference $E(\bar{\delta}; \widetilde{A}, \widetilde{H}, \widetilde{F}) - E(\delta^*; \widetilde{A}, \widetilde{H}, \widetilde{F})$ computed for a sequence of the values of normalized productivity $\widetilde{A}$ at arbitrary combinations of hiring and firing costs, given that both firms are active at the market in the first period.[19] This figure generalizes the right panel of Figure 2: for any value of $\widetilde{A}$ it shows a plane in a three-dimensional space instead of a single point.[20] Figure 3 shows that working time account will always be associated with lower expected net turnover (positive difference), except for the very high levels of $\widetilde{A}$. However, as in the basic model, the amount of job destruction at these high levels turns out to be negligible (below 0.0010% of the initial firm size) to have any effect. Finally, and again similar to the basic model, the highest reduction of net expected turnover due to working time account makes 0.47% of the initial firm size.

[19]The latter means that lower bound of integration in (15)-(16) is replaced by $\max\{z_1^{*\cdot}, \bar{z}_1\}$. Choice of $\widetilde{H}$ and $\widetilde{F}$ is discussed in Appendix A.2.

[20]Note however that here I use the difference in expected turnover rather than the ratio of expected turnover to circumvent the problem of division by small numbers at too low levels of $\widetilde{F}$.

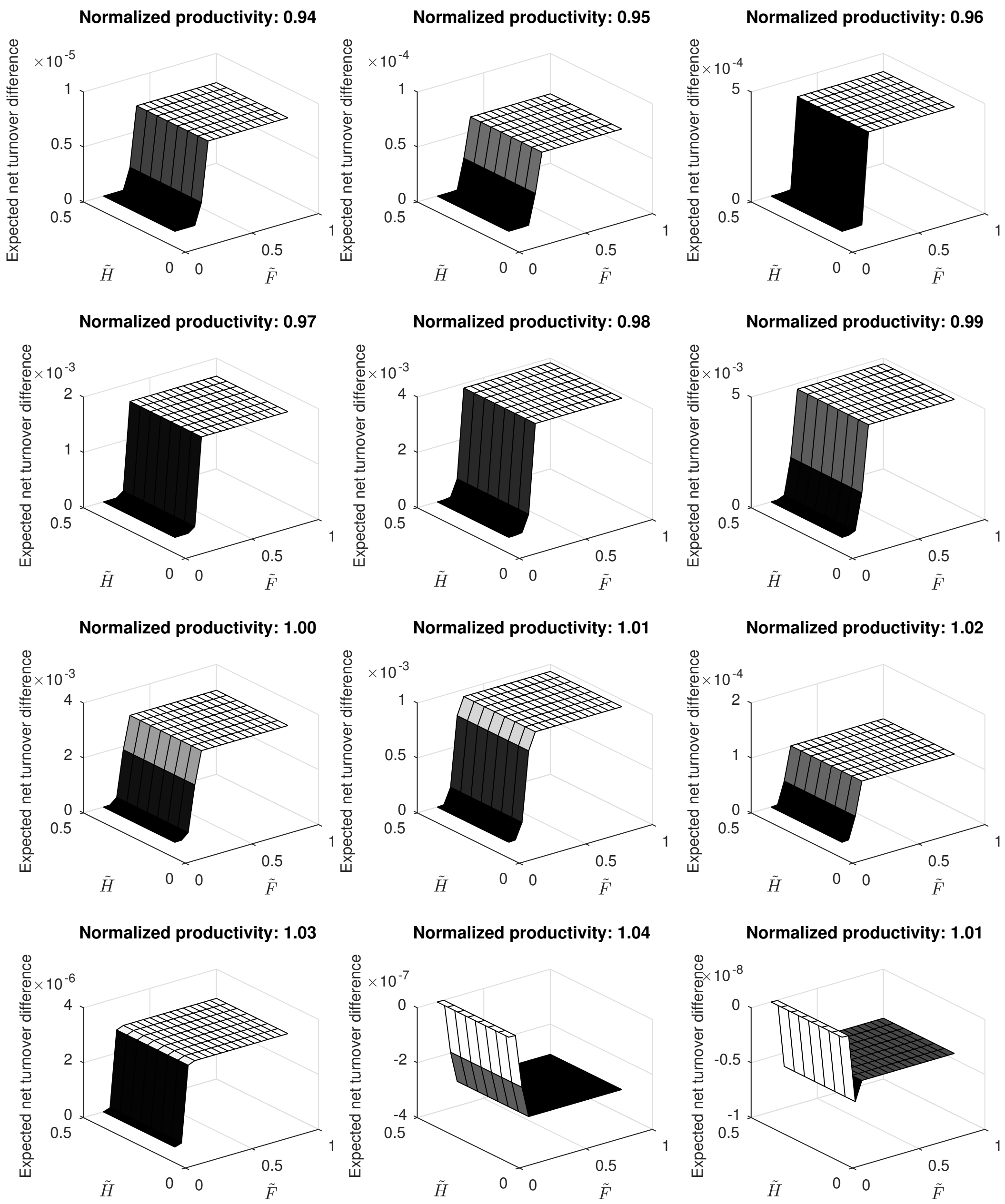

Figure 3: Reduction of expected net turnover due to working time account

Concluding, the message delivered by the model with adjustment along the extensive and intensive margins is identical to the one already generated by the model with intensive margin only. The entire discussion of Section 3.3 remains therefore unchanged.

5 Concluding remarks

In this paper I construct a simple yet powerful model of labour demand by a local monopolist who operates a working time account. Labour adjustment decisions are made in face of uncertain demand conditions at the goods market and under constraints imposed by working time accounts regulation. Firms do not have access to credit, but can form own reserves. Motivated by the hypothesis of Burda and Hunt (2011) on the performance of working time accounts in Germany during the Great Recession, I use this model to investigate the relationship between working time accounts and realized turnover, measured by layoffs.

Contrary to initial expectations, I find that firms with working time accounts need not necessarily have lower turnover than firms without such accounts. Working time account may increase turnover when a high-productive firm has deficit of actual hours worked and expects improvement of demand at the goods market in future. It may also increase turnover when a low-productive firm has surplus of actual hours worked and expects deterioration of demand at the goods market in future. In both situations a firm without working time account will be able to withstand stronger demand downturns than an identical firm with the account without slashing jobs. At the same time the model also encompasses the pattern suggested by Burda and Hunt (2011). I show that when a high-productive firm has surplus on its working time account and expects demand downturn at the goods market in future it will be able to withstand a stronger negative demand shock than a firm without working time account, provided that the recession that follows is not protracted.

In view of the finding that for certain firms and under certain demand conditions working time account will increase realized turnover I consider the measure of expected turnover, where turnover is measured either by layoffs or by total layoffs net of earlier hiring, depending on the version of the model. Expectations are taken with respect to present and future demand fluctuations at the goods market. I find that for any level of (normalized) firm productivity at which turnover is not negligible, working time account unambiguously reduces expected turnover at firms that shrink.

Overall the paper provides a strong case for a positive contribution of working time accounts to maintaining aggregate job stability. Aggregate job stability increases as labour hoarding outweighs moral hazard of increasing short-term profits on expense of longer-term employment, both induced by the use of working time account. In a simple world of exogenous wages this implies positive welfare gains.

Although the conclusion is quite optimistic, the analysis of the paper is not without limitations. The most serious limitation is the one-sided nature of the theoretical framework, which inevitably makes the analysis stylized. Endogenizing wages, for example, may see some results changing as the use of working time account is likely to reduce wages (due to lower risk of separation, substitution for overtime work etc.). I see the introduction of the worker side together with endogenous wage setting as promising direction for future work. Furthermore, since theoretical modeling of working time accounts is still new, alternative ways of formalizing the account as a tool in a choice set of a firm are worth exploring.

Appendix

A.1 Optimal choice of hours

To complete characterization of optimal hours consider the solution where any/all of (ii): $h_{\min} \leq h_1, h_2 \leq h^{\max}$, (iii): $\pi_1(h_1) \geq 0$, is/are binding. First, optimal hours in the first period may not be lower than $\tilde{h} = \frac{1}{An}\left(z_1^{\epsilon-1}[wn\bar{h}]\right)^{1/\epsilon}$, where $\tilde{h}$ solves $\pi_1(\tilde{h}) = 0$. Second, optimal hours in the first period may not be lower than $h_{\min}$ and may not be higher than $h^{\max}$. Combining the two, optimal solution for h_1^* can be expressed as

$$h_1^* = \max\left\{\min\left\{h^{\max}, \frac{2}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}+1}\bar{h}\right\}, \max\left\{h_{\min}, \tilde{h}\right\}\right\}, \tag{A.1}$$

with $h_2^* = 2\bar{h} - h_1^*$. Figure A.1 illustrates the typical pattern of optimal hours.

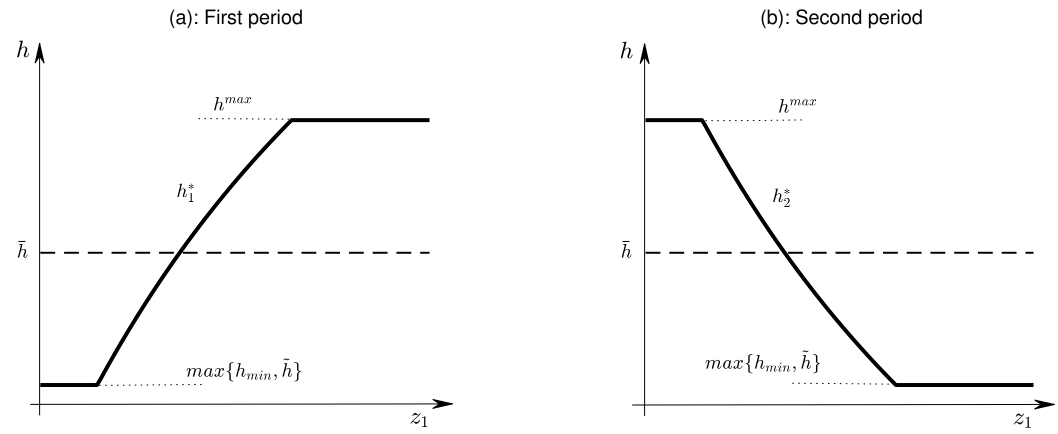

Figure A.1: Optimal hours in the basic model

A.2 Numerical analysis

- Parameters

Numerical exercises of Sections 3.2 and 4.2 share the same set of parameters. Parameters are chosen to be consistent with German labour market shortly before the Great Recession. All their values are summarized in Table A.1.

I let one model period last six months. First, this corresponds to the time window within which the economy may technically enter recession (two consecutive quarters). Second, the length of the compensation period in manufacturing frequently lasts up to one year (Herzog-Stein and Zapf, 2014). The average annual long-term interest rate was 3.8% in 2006-2009 (OECD, 2018). This implies a period (six-months) interest rate $r = 1.88\%$ and period (six month) discount factor $\beta = 0.9815$. Inverted mark-up ϵ is set to $\epsilon = 1/1.19$ which is informed by the estimated price mark-up of 19% in German manufacturing (Christopoulou and Vermeulen, 2012). I assume that uncertainly at the goods market follows the unit-mean lognormal distribution, $Z_t \sim \mathcal{LN}(-\frac{\sigma^2}{2}, \sigma)$. The choice of σ equal to 0.10 implies the 95% confidence interval of $(0.82, 1.08)$ for the draws of z_t, such that the goods market would typically not contract by more than 18% and not expand by more than 8% in any period. I set $h^{\max} = 1.15 \times \bar{h}$ and, symmetrically, $h_{\min} = 0.85 \times \bar{h}$. Finally, in the extended model I let $\widetilde{F}$ vary from 15% to 100% of the worker's period wage bill, which corresponds to severance

parameter	value	comments
r	1.88%	period interest rate
β	0.9815	period discount factor
ϵ^{-1}	19%	price mark-up
$h^{\max}/\bar{h}$	1.15	period limit of hours accumulation
$h^{\min}/\bar{h}$	0.85	period limit of hours decumulation
σ	0.10	variance parameter for demand fluctuations

Table A.1: Parameter values

payments of 1 to 6 monthly wages. $\widetilde{H}$ varies from 15% to 50% of the period wage bill.

Qualitative and quantitative results of Sections 3.2 and 4.2 are remarkably robust to variation of all parameter values within a reasonable range. Furthermore, replacing $\mathcal{LN}(-\frac{\sigma^2}{2}, \sigma)$ by a continuous uniform distribution defined on $(0.9, 1.1)$ leads to similar results.

- Further quantitative aspects

Quantitative part of the paper relies on aggregate statistics, which inevitably makes the analysis quite stylized. I have to resort to aggregate statistics because of the lack of longitudinal data on working time accounts balances at the establishment level. To the best of my knowledge the only data source containing hours balances for German firms is the Works Council Survey of the Institute of Economic and Social Research (WSI) used by Herzog-Stein and Zapf (2014). Yet, even in these data the balances are available only in the special survey conducted in July-September 2009, still during the recession, and in the subsequent extension of a regular survey undertaken in January-April 2010, immediately after the recovery. The span of less than one year and the particular timing of surveys make these data hardly useful for informing and quantifying more sophisticated theoretical models.

A more commonly used longitudinal data on firm dynamics in Germany, and an easier accessible data too, is the IAB Establishment Panel of the Institute for Employment Research (IAB). Albeit it contains an indicator variable on whether a firm operates working time accounts, it does not report the total balance of these accounts. Absence of numeric information about the balance size is a serious impediment.

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Online Appendix

I. Basic model

I.a Profit function

Consider the demand function given in (1). Solving (1) for price, $p_t = z_t^{1-\epsilon} m_t^{\epsilon-1}$. Inserting $m_t = Anh_t$ for output, revenue $p_t m_t$ becomes

$$\begin{aligned} p_t m_t &= z_t^{1-\epsilon} m_t^{\epsilon-1} m_t \\ &= z_t^{1-\epsilon} m_t^{\epsilon} = z_t^{1-\epsilon} \left[Anh_t\right]^{\epsilon}. \end{aligned}$$

Since wage cost is given by $wn\bar{h}$ profit function writes

$$\pi_t\left(h_t\right) = z_t^{1-\epsilon}\left[Anh_t\right]^{\epsilon} - wn\bar{h}.$$

I.b Optimal solution

First, (2), $b_0 = 0$ and $b_2 = 0$ imply that $h_2 = 2\bar{h} - h_1$. Inserting this into (4) and differentiating w.r.t. h_1 f.o.c. for h_1 writes $\pi_1'\left(h_1\right) - \beta E_1\left(\pi_2'\left(2\bar{h} - h_1\right)\right) \stackrel{!}{=} 0$. This leads to

$$\begin{aligned} \epsilon z_1^{1-\epsilon}\left(An\right)^{\epsilon}\left[h_1\right]^{\epsilon-1} &= \beta E_1\left(\epsilon z_2^{1-\epsilon}\left(An\right)^{\epsilon}\left[2\bar{h} - h_1\right]^{\epsilon-1}\right) \\ z_1^{1-\epsilon} h_1^{\epsilon-1} &= \left[2\bar{h} - h_1\right]^{\epsilon-1} \beta E_1(z_2^{1-\epsilon}) \\ z_1^{(1-\epsilon)/(\epsilon-1)} h_1 &= \left[2\bar{h} - h_1\right]\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)} \\ z_1^{(1-\epsilon)/(\epsilon-1)} h_1 &= 2\bar{h}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)} - h_1\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}, \end{aligned}$$

such that

$$\begin{aligned} h_1 &= \frac{2\bar{h}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}}{z_1^{(1-\epsilon)/(\epsilon-1)} + \left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}} = \frac{2\bar{h}}{\left[\beta E_1(z_2^{1-\epsilon})\right]^{-1/(\epsilon-1)} z_1^{(1-\epsilon)/(\epsilon-1)} + 1} \\ &= \frac{2\bar{h}}{\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}\left[\frac{1}{z_1}\right]^{(1-\epsilon)/(1-\epsilon)} + 1} = \frac{2\bar{h}}{\left[\beta \frac{1}{z_1^{1-\epsilon}} E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} + 1}, \end{aligned}$$

and finally

$$h_1 = \frac{2}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} + 1}\bar{h}.$$

Solution for h_2 follows from

$$\begin{aligned} h_2 &= 2\bar{h} - h_1 = 2\bar{h}\left(1 - \frac{1}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} + 1}\right) \\ &= \frac{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} + 1} 2\bar{h} = \frac{2}{1 + z_1\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}}\bar{h}. \end{aligned}$$

I.c Proof of Proposition 1

Proof. Consider (5a). Differentiating h_1^* w.r.t. z_1,

$$\begin{aligned}\frac{\partial h_1^*}{\partial z_1} &= \frac{\partial}{\partial z_1}\left(\frac{2}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}+1}\bar{h}\right) \\ &= \frac{2\bar{h}}{\left(\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}+1\right)^2}\frac{1}{z_1^2}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} > 0.\end{aligned}$$

Defining $E \equiv E_1({z_2}^{1-\epsilon})$ and differentiating h_1^* w.r.t. E,

$$\begin{aligned}\frac{\partial h_1^*}{\partial E} &= \frac{\partial}{\partial E}\left(\frac{2}{\frac{1}{z_1}\left[\beta E\right]^{1/(1-\epsilon)}+1}\bar{h}\right) \\ &= -\frac{2\bar{h}}{(1-\epsilon)\,z_1\left(\frac{1}{z_1}\left[\beta E\right]^{1/(1-\epsilon)}+1\right)^2}\beta^{1/(1-\epsilon)}E^{\epsilon/(1-\epsilon)} < 0.\end{aligned}$$

Since h_1^* decreases in $E_1({z_2}^{1-\epsilon})$ and ${z_2}^{1-\epsilon}$ is a monotone increasing transformation of z_2, h_1^* decreases in $E_1(z_2)$.

As $h_2^* = 2\bar{h} - h_1^*$, the above means that h_2^* decreases in z_1 and increases in $E_1(z_2)$. ■

I.d Layoff thresholds

For a firm with working time account z_2^* is defined to equalize wage cost in the second period to the revenue in the second period plus reserves accumulated before the second period. Thus z_2^* solves

$$\begin{aligned}(1+r)\,\pi_1\left(h_1^*\right) + \left[z_2^*\right]^{1-\epsilon}\left[Anh_2^*\right]^{\epsilon} - wn\bar{h} &= 0 \\ \left[z_2^*\right]^{1-\epsilon}\left[Anh_2^*\right]^{\epsilon} &= wn\bar{h} - (1+r)\,\pi_1\left(h_1^*\right) \\ z_2^* &= \left(\frac{wn\bar{h} - (1+r)\,\pi_1\left(h_1^*\right)}{\left[Anh_2^*\right]^{\epsilon}}\right)^{1/(1-\epsilon)}.\end{aligned}$$

For a firm without working time account $\bar{z}_2$ is defined in the same way. With $\bar{h}$ replacing h_1^* and h_2^* it becomes

$$\bar{z}_2 = \left(\frac{wn\bar{h} - (1+r)\,\pi_1\left(\bar{h}\right)}{\left[An\bar{h}\right]^{\epsilon}}\right)^{1/(1-\epsilon)}.$$

I.e Proof of Proposition 2

Proof. Assume that $z_2^* < \bar{z}_2$ holds. Inserting (6a) and (6b),

$$\left(\frac{wn\bar{h} - (1+r)\,\pi_1\left(h_1^*\right)}{\left[Anh_2^*\right]^{\epsilon}}\right)^{1/(1-\epsilon)} < \left(\frac{wn\bar{h} - (1+r)\,\pi_1\left(\bar{h}\right)}{\left[An\bar{h}\right]^{\epsilon}}\right)^{1/(1-\epsilon)}$$

After some algebra and rearrangement (see Supplementary Derivations) this inequality can be expressed as

$$\left\{\left[\bar{h}\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}\right\}-\frac{z_1^{1-\epsilon}\left[An\bar{h}\right]^{\epsilon}}{(1+\beta)\,wn\bar{h}}\left\{\left[h_1^*\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}\right\}<0. \tag{OA.1}$$

Consider statement (i) of the proposition. Surplus at the working time account in the first period means that $h_1^* > \bar{h} > h_2^*$, implying that $[h_1^*]^{\epsilon} > \left[\bar{h}\right]^{\epsilon} > [h_2^*]^{\epsilon}$ and $[h_1^*]^{\epsilon} - [h_2^*]^{\epsilon} > \left[\bar{h}\right]^{\epsilon} - [h_2^*]^{\epsilon} > 0$. Consequently, (OA.1) implies that any $\frac{z_1^{1-\epsilon}\left[An\bar{h}\right]^{\epsilon}}{(1+\beta)wn\bar{h}} \geq 1$ is sufficient for $z_2^* < \bar{z}_2$ to hold. Rearranging (OA.1), $z_2^* < \bar{z}_2$ holds as long as

$$(1+\beta)\,\frac{\bar{h}^{1-\epsilon}}{z_1^{1-\epsilon}}\frac{\left[\bar{h}\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}<\frac{A^{\epsilon}}{wn^{1-\epsilon}}. \tag{OA.2}$$

Consider statement (ii) of the proposition. Deficit at the working time account in the first period means that $h_1^* < \bar{h} < h_2^*$, implying that $[h_1^*]^{\epsilon} < \left[\bar{h}\right]^{\epsilon} < [h_2^*]^{\epsilon}$ and $[h_1^*]^{\epsilon} - [h_2^*]^{\epsilon} < \left[\bar{h}\right]^{\epsilon} - [h_2^*]^{\epsilon} < 0$. Consequently, (OA.1) implies that any $\frac{z_1^{1-\epsilon}\left[An\bar{h}\right]^{\epsilon}}{(1+\beta)wn\bar{h}} \geq 1$ is sufficient for $z_2^* < \bar{z}_2$ *not* to hold. Rearranging (OA.1) again, $z_2^* < \bar{z}_2$ will be violated as long as (OA.2) holds.

Inequality (OA.2) provides the necessary condition for this Proposition to hold. From the above also follows that $\frac{z_1^{1-\epsilon}\left[An\bar{h}\right]^{\epsilon}}{(1+\beta)wn\bar{h}} \geq 1$, more conveniently: $z_1^{1-\epsilon}\left[An\bar{h}\right]^{\epsilon} \geq (1+\beta)\,wn\bar{h}$, is the sufficient condition. ■

I.f Sufficient condition for Corollary to Proposition 2

Consider statement (i) of the corollary. Since surplus at the working time account in the first period implies $[h_1^*]^{\epsilon} - [h_2^*]^{\epsilon} > \left[\bar{h}\right]^{\epsilon} - [h_2^*]^{\epsilon} > 0$, it has to be that $\frac{z_1^{1-\epsilon}\left[An\bar{h}\right]^{\epsilon}}{(1+\beta)wn\bar{h}}$ is sufficiently small for $z_2^* < \bar{z}_2$ *not* to hold. For any given β, the lowest value is the break-even point in the first period $z_1^{1-\epsilon}\left[An\bar{h}\right]^{\epsilon} = wn\bar{h}$. For a firm at a break even point, if

$$\frac{\left[\bar{h}\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}<\frac{1}{1+\beta}. \tag{OA.3}$$

is violated then $z_2^* < \bar{z}_2$ does not hold. For $\bar{h} < h_1^* < 2\bar{h}$ I show that the l.h.s. of (OA.3) is an increasing function in h_1^* (see Supplementary Derivations). Letting $h_1^* \to \bar{h}$ and applying L'Hôpital's rule,

$$\begin{aligned}
\lim_{h_1^*\to\bar{h}}\frac{\left[\bar{h}\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}} &= \frac{\lim\limits_{h_1^*\to\bar{h}}\frac{\partial}{\partial h_1^*}\left(\left[\bar{h}\right]^{\epsilon}-\left[2\bar{h}-h_1^*\right]^{\epsilon}\right)}{\lim\limits_{h_1^*\to\bar{h}}\frac{\partial}{\partial h_1^*}\left(\left[h_1^*\right]^{\epsilon}-\left[2\bar{h}-h_1^*\right]^{\epsilon}\right)} \\
&= \frac{\lim\limits_{h_1^*\to\bar{h}}\left(\epsilon\left[2\bar{h}-h_1^*\right]^{\epsilon-1}\right)}{\lim\limits_{h_1^*\to\bar{h}}\left(\epsilon\left[h_1^*\right]^{\epsilon-1}+\epsilon\left[2\bar{h}-h_1^*\right]^{\epsilon-1}\right)} = \frac{\epsilon\left[\bar{h}\right]^{\epsilon-1}}{\epsilon\left[\bar{h}\right]^{\epsilon-1}+\epsilon\left[\bar{h}\right]^{\epsilon-1}} = \frac{1}{2}.
\end{aligned}$$

Letting $h_1^* \to 2\bar{h}$, $\lim\limits_{h_1^*\to\bar{h}}\frac{\left[\bar{h}\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}} = \frac{1}{2^{\epsilon}}$. Since the r.h.s. of (OA.3) is a constant arbitrary close to $1/2$, for typical values of ϵ and β the inequality in (OA.3) ceases to hold. Sufficient condition for statement (ii) of Corollary 1 is established in the same way.

II. Extended model with two margins of adjustment

II.a Optimal Solution

- A firm with working time account

Consider f.o.c. for the extensive margin. $\partial V / \partial n_h \overset{!}{=} 0$ implies

$$
\begin{aligned}
\epsilon z_1^{1-\epsilon}\left[A h_1\right]^\epsilon\left[n+n_h\right]^{\epsilon-1}-w \bar{h}-H+\beta\left\{\epsilon E_1\left(z_2^{1-\epsilon}\right)\left[A h_2\right]^\epsilon\left[n+n_h\right]^{\epsilon-1}-w \bar{h}\right\} &= 0 \\
\epsilon A^\epsilon\left[n+n_h\right]^{\epsilon-1}\left\{z_1^{1-\epsilon} h_1^\epsilon+\beta E_1\left(z_2^{1-\epsilon}\right) h_2^\epsilon\right\} &= (1+\beta) w \bar{h}+H \\
\left[n+n_h\right]^{1-\epsilon} &= \epsilon A^\epsilon \frac{z_1^{1-\epsilon} h_1^\epsilon+\beta E_1\left(z_2^{1-\epsilon}\right) h_2^\epsilon}{(1+\beta) w \bar{h}+H} \\
n_h &= \left[\epsilon A^\epsilon\right]^{1/(1-\epsilon)}\left(\frac{z_1^{1-\epsilon} h_1^\epsilon+\beta E_1\left(z_2^{1-\epsilon}\right) h_2^\epsilon}{(1+\beta) w \bar{h}+H}\right)^{1/(1-\epsilon)}-n, \qquad \text{(OA.4)}
\end{aligned}
$$

and $\partial V / \partial n_f \overset{!}{=} 0$ implies

$$
\begin{aligned}
-\epsilon z_1^{1-\epsilon}\left[A h_1\right]^\epsilon\left[n-n_f\right]^{\epsilon-1}+w \bar{h}-F+\beta\left\{-\epsilon E_1\left(z_2^{1-\epsilon}\right)\left[A h_2\right]^\epsilon\left[n-n_f\right]^{\epsilon-1}+w \bar{h}\right\} &= 0 \\
(1+\beta) w \bar{h}-F &= \epsilon A^\epsilon\left[n-n_f\right]^{\epsilon-1}\left\{z_1^{1-\epsilon} h_1^\epsilon+\beta E_1\left(z_2^{1-\epsilon}\right) h_2^\epsilon\right\} \\
\left[n-n_f\right]^{1-\epsilon} &= \epsilon A^\epsilon \frac{z_1^{1-\epsilon} h_1^\epsilon+\beta E_1\left(z_2^{1-\epsilon}\right) h_2^\epsilon}{(1+\beta) w \bar{h}-F} \\
n_f &= n-\left[\epsilon A^\epsilon\right]^{1/(1-\epsilon)}\left(\frac{z_1^{1-\epsilon} h_1^\epsilon+\beta E_1\left(z_2^{1-\epsilon}\right) h_2^\epsilon}{(1+\beta) w \bar{h}-F}\right)^{1/(1-\epsilon)}. \qquad \text{(OA.5)}
\end{aligned}
$$

Consider f.o.c. for the intensive margin. As before (2), $b_0 = 0$ and $b_2 = 0$ imply that $h_2 = 2\bar{h} - h_1$. Inserting this into (10), $\partial V / \partial h_1 \overset{!}{=} 0$ implies

$$
\begin{aligned}
\epsilon z_1^{1-\epsilon}\left[A\left(n+\mathbb{I}_{n_h>0} n_h-\mathbb{I}_{n_f>0} n_f\right)\right]^\epsilon h_1^{\epsilon-1}-\beta \epsilon E_1\left(z_2^{1-\epsilon}\right)\left[A\left(n+\mathbb{I}_{n_h>0} n_h-\mathbb{I}_{n_f>0} n_f\right)\right]^\epsilon\left[2 \bar{h}-h_1\right]^{\epsilon-1} &= 0 \\
\epsilon\left[A\left(n+\mathbb{I}_{n_h>0} n_h-\mathbb{I}_{n_f>0} n_f\right)\right]^\epsilon z_1^{1-\epsilon} h_1^{\epsilon-1} &= \epsilon\left[A\left(n+\mathbb{I}_{n_h>0} n_h-\mathbb{I}_{n_f>0} n_f\right)\right]^\epsilon \beta E_1\left(z_2^{1-\epsilon}\right)\left[2 \bar{h}-h_1\right]^{\epsilon-1} \\
z_1^{1-\epsilon} h_1^{\epsilon-1} &= \beta E_1\left(z_2^{1-\epsilon}\right)\left[2 \bar{h}-h_1\right]^{\epsilon-1}
\end{aligned}
$$

at which point it becomes identical to f.o.c. for h_1 in the basic model (see Section I.b of this Online Appendix). Thus, the solution for optimal hours is

$$
h_1=\frac{2}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}+1} \bar{h}, \quad \text{and} \quad h_2=\frac{2}{1+z_1\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}} \bar{h}.
$$

- A firm without working time account

F.o.c. for the extensive margin are identical to (OA.4) and (OA.5) except that $\bar{h}$ replaces h_1 and h_2.

$$
n_h=\left[\epsilon A^\epsilon\right]^{1/(1-\epsilon)}\left(\frac{z_1^{1-\epsilon}\left[\bar{h}\right]^\epsilon+\beta E_1\left(z_2^{1-\epsilon}\right)\left[\bar{h}\right]^\epsilon}{(1+\beta) w \bar{h}+H}\right)^{1/(1-\epsilon)}-n,
$$

$$n_f = n - [\epsilon A^{\epsilon}]^{1/(1-\epsilon)} \left(\frac{z_1^{1-\epsilon}\left[\bar{h}\right]^{\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right)\left[\bar{h}\right]^{\epsilon}}{(1+\beta)\, w\bar{h} - F} \right)^{1/(1-\epsilon)}.$$

Rearranging,

$$\begin{aligned} n_h &= [\epsilon A^{\epsilon}]^{1/(1-\epsilon)} \left(\frac{z_1^{1-\epsilon}\left[\bar{h}\right]^{\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right)\left[\bar{h}\right]^{\epsilon}}{(1+\beta)\, w\bar{h} + H} \right)^{1/(1-\epsilon)} - n \\ &= [\epsilon A^{\epsilon}]^{1/(1-\epsilon)} \left(\left[\bar{h}\right]^{\epsilon} \frac{z_1^{1-\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\, w\bar{h} + H} \right)^{1/(1-\epsilon)} - n, \end{aligned}$$

so

$$n_h = \left(\epsilon \left[A\bar{h}\right]^{\epsilon} \frac{z_1^{1-\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\, w\bar{h} + H} \right)^{1/(1-\epsilon)} - n, \tag{OA.6}$$

and similarly

$$n_f = n - \left(\epsilon \left[A\bar{h}\right]^{\epsilon} \frac{z_1^{1-\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\, w\bar{h} - F} \right)^{1/(1-\epsilon)}. \tag{OA.7}$$

II.b Proof of Proposition 3

Proof. I.(i): Consider the statement $n_h^* > \bar{n}_h$. Inserting from (OA.4) and (OA.6),

$$\begin{aligned} [\epsilon A^{\epsilon}]^{1/(1-\epsilon)} \left(\frac{z_1^{1-\epsilon} h_1^{\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right) h_2^{\epsilon}}{(1+\beta)\, w\bar{h} + H} \right)^{1/(1-\epsilon)} &> \left(\epsilon \left[A\bar{h}\right]^{\epsilon} \frac{z_1^{1-\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\, w\bar{h} + H} \right)^{1/(1-\epsilon)} \\ z_1^{1-\epsilon}[h_1]^{\epsilon} + [h_2]^{\epsilon} \beta E_1\left(z_2^{1-\epsilon}\right) &> z_1^{1-\epsilon}\left[\bar{h}\right]^{\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right)\left[\bar{h}\right]^{\epsilon} \end{aligned}$$

After some algebra (see Supplementary Derivations) this inequality becomes

$$2 > \left[\frac{h_1}{\bar{h}}\right]^{1-\epsilon} + \left[2 - \frac{h_1}{\bar{h}}\right]^{1-\epsilon}. \tag{OA.8}$$

Define $x \equiv \frac{h_1}{\bar{h}}$ and consider $f(x) = [x]^{1-\epsilon} + [2-x]^{1-\epsilon}$. Its first and second derivatives are

$$\begin{aligned} f'(x) &= (1-\epsilon)[x]^{-\epsilon} - (1-\epsilon)[2-x]^{-\epsilon} \\ f''(x) &= \left((1-\epsilon)[x]^{-\epsilon} - (1-\epsilon)[2-x]^{-\epsilon}\right)' \\ &= -\epsilon(1-\epsilon)[x]^{-\epsilon-1} - \epsilon(1-\epsilon)[2-x]^{-\epsilon-1} < 0 \end{aligned}$$

i.e. $f(x)$ is a concave function. It attains maximum at $x=1$, where $f(1)=2$.[1] Thus, unless $h_1 = \bar{h}$ the inequality (OA.8) always holds, establishing that $n_h^* > \bar{n}_h$.

I.(ii): Consider the statement $\bar{n}_f > n_f^*$. Inserting from (OA.5) and (OA.7),

$$n - \left(\epsilon \left[A\bar{h}\right]^{\epsilon} \frac{z_1^{1-\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\, w\bar{h} - F} \right)^{1/(1-\epsilon)} > n - [\epsilon A^{\epsilon}]^{1/(1-\epsilon)} \left(\frac{z_1^{1-\epsilon} h_1^{\epsilon} + \beta E_1\left(z_2^{1-\epsilon}\right) h_2^{\epsilon}}{(1+\beta)\, w\bar{h} - F} \right)^{1/(1-\epsilon)}$$

[1] $f'(x) = 0$: $(1-\epsilon)[x]^{-\epsilon} - (1-\epsilon)[2-x]^{-\epsilon} = 0 \quad \Leftrightarrow \quad [x]^{-\epsilon} = [2-x]^{-\epsilon} \quad \Leftrightarrow \quad x = 2-x \quad \Leftrightarrow \quad x = 1.$

$$\left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)}\left(\frac{z_1^{1-\epsilon}h_1^{\epsilon}+\beta E_1\left(z_2^{1-\epsilon}\right)h_2^{\epsilon}}{(1+\beta)\,w\bar{h}-F}\right)^{1/(1-\epsilon)} > \left(\epsilon\left[A\bar{h}\right]^{\epsilon}\frac{z_1^{1-\epsilon}+\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\,w\bar{h}-F}\right)^{1/(1-\epsilon)}$$
$$z_1^{1-\epsilon}h_1^{\epsilon}+\beta E_1\left(z_2^{1-\epsilon}\right)h_2^{\epsilon} > z_1^{1-\epsilon}\left[\bar{h}\right]^{\epsilon}+\beta E_1\left(z_2^{1-\epsilon}\right)\left[\bar{h}\right]^{\epsilon}$$

at which point it becomes identical to the above. Thus $\bar{n}_f > n_f^*$.

II.(i): Let a firm with working time account fire, i.e. it is true that $n_f^* > 0$. Using (OA.5), $n_f^* > 0$ implies

$$n^{1-\epsilon} > \epsilon A^{\epsilon}\frac{\left[h_1^*\right]^{\epsilon}z_1^{1-\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\,w\bar{h}-F}.$$

Suppose that an identical firm without working time account will hire instead, i.e. $\bar{n}_h > 0$. Using (OA.6), $\bar{n}_h > 0$ implies

$$\epsilon A^{\epsilon}\frac{\left[\bar{h}\right]^{\epsilon}z_1^{1-\epsilon}+\left[\bar{h}\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\,w\bar{h}+H} > n^{1-\epsilon}.$$

Both inequalities suggest that

$$\epsilon A^{\epsilon}\frac{\left[\bar{h}\right]^{\epsilon}z_1^{1-\epsilon}+\left[\bar{h}\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\,w\bar{h}+H} > \epsilon A^{\epsilon}\frac{\left[h_1^*\right]^{\epsilon}z_1^{1-\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)\,w\bar{h}-F}$$
$$\frac{\left[\bar{h}\right]^{\epsilon}z_1^{1-\epsilon}+\left[\bar{h}\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{\left[h_1^*\right]^{\epsilon}z_1^{1-\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)} > \frac{(1+\beta)\,w\bar{h}+H}{(1+\beta)\,w\bar{h}-F}.$$

Considering the r.h.s. and invoking $\frac{1}{z_1^{1-\epsilon}}\left[\beta E_1(z_2^{1-\epsilon})\right]=\left(\frac{h_2}{h_1}\right)^{1-\epsilon}$, one obtains

$$\begin{aligned}
\frac{\left[\bar{h}\right]^{\epsilon}z_1^{1-\epsilon}+\left[\bar{h}\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{\left[h_1^*\right]^{\epsilon}z_1^{1-\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)} &= \frac{\left[\bar{h}\right]^{\epsilon}+\left[\bar{h}\right]^{\epsilon}\left(\frac{h_2^*}{h_1^*}\right)^{1-\epsilon}}{\left[h_1^*\right]^{\epsilon}+\left[h_2^*\right]^{\epsilon}\left(\frac{h_2^*}{h_1^*}\right)^{1-\epsilon}} = \frac{\left[\bar{h}\right]^{\epsilon}\left(h_1^*\right)^{\epsilon-1}\left[\left(h_1^*\right)^{1-\epsilon}+\left(h_2^*\right)^{1-\epsilon}\right]}{\left(h_1^*\right)^{\epsilon-1}\left[h_1^*+h_2^*\right]}\\
&= \frac{\left[\bar{h}\right]^{\epsilon}\left[\left(h_1^*\right)^{1-\epsilon}+\left(h_2^*\right)^{1-\epsilon}\right]}{2\bar{h}} = \frac{1}{2}\left(\frac{h_1^*}{\bar{h}}\right)^{1-\epsilon}+\frac{1}{2}\left(\frac{h_2^*}{\bar{h}}\right)^{1-\epsilon}\\
&< \left(\frac{1}{2}\frac{h_1^*}{\bar{h}}+\frac{1}{2}\frac{h_2^*}{\bar{h}}\right)^{1-\epsilon} = \left(\frac{h_1^*+h_2^*}{2\bar{h}}\right)^{1-\epsilon} = 1
\end{aligned}$$

where the last line is due to Jensen's inequality for an increasing concave function, with $h_1^*, h_2^* \neq \bar{h}$. Thus

$$1 > \frac{\left[\bar{h}\right]^{\epsilon}z_1^{1-\epsilon}+\left[\bar{h}\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{\left[h_1^*\right]^{\epsilon}z_1^{1-\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)} > \frac{(1+\beta)\,w\bar{h}+H}{(1+\beta)\,w\bar{h}-F},$$

which is a contradiction since $H, F > 0$. So when a firm with working time account fire, an identical firm without working time account will only fire too.

II.(ii): Let a firm with working time account hire, i.e. it is true that $n_h^* > 0$. Using (OA.4),

$n_h^* > 0$ implies

$$\epsilon A^{\epsilon} \frac{[h_1^*]^{\epsilon} z_1^{1-\epsilon} + [h_2^*]^{\epsilon} \beta E_1 \left(z_2^{1-\epsilon}\right)}{(1+\beta) w\bar{h} + H} > n^{1-\epsilon}.$$

Suppose that an identical firm without working time account will fire instead, i.e. $\bar{n}_f > 0$. Using (OA.7), $\bar{n}_f > 0$ implies

$$n^{1-\epsilon} > \epsilon A^{\epsilon} \frac{\left[\bar{h}\right]^{\epsilon} z_1^{1-\epsilon} + \left[\bar{h}\right]^{\epsilon} \beta E_1 \left(z_2^{1-\epsilon}\right)}{(1+\beta) w\bar{h} - F}.$$

Both inequalities suggest that

$$\begin{aligned}
\epsilon A^{\epsilon} \frac{[h_1^*]^{\epsilon} z_1^{1-\epsilon} + [h_2^*]^{\epsilon} \beta E_1 \left(z_2^{1-\epsilon}\right)}{(1+\beta) w\bar{h} + H} &> \epsilon A^{\epsilon} \frac{\left[\bar{h}\right]^{\epsilon} z_1^{1-\epsilon} + \left[\bar{h}\right]^{\epsilon} \beta E_1 \left(z_2^{1-\epsilon}\right)}{(1+\beta) w\bar{h} - F} \\
1 &> \frac{\left[\bar{h}\right]^{\epsilon} z_1^{1-\epsilon} + \left[\bar{h}\right]^{\epsilon} \beta E_1 \left(z_2^{1-\epsilon}\right)}{[h_1^*]^{\epsilon} z_1^{1-\epsilon} + [h_2^*]^{\epsilon} \beta E_1 \left(z_2^{1-\epsilon}\right)} \frac{(1+\beta) w\bar{h} + H}{(1+\beta) w\bar{h} - F} \\
1 &> \underbrace{\left[\frac{1}{2}\left(\frac{h_1^*}{\bar{h}}\right)^{1-\epsilon} + \frac{1}{2}\left(\frac{h_2^*}{\bar{h}}\right)^{1-\epsilon}\right]}_{<1} \underbrace{\frac{(1+\beta) w\bar{h} + H}{(1+\beta) w\bar{h} - F}}_{>1}
\end{aligned}$$

as already shown above in (i). Thus, for H and F small enough the inequality may hold implying that while a firm with working time account hires an identical firm without the account may fire. For H and F large enough both firms hire. ■

II.c Proof of Proposition 4

Proof. (i) Let labour adjustment strategies be ('$fire$','$fire$'). Break-even thresholds similar to those in (6a)-(6b) for this pair of labour adjustment strategies are given by

$$\begin{aligned}
z_{2,f}^* &= \left(\frac{w(n-n_f^*)\bar{h} - (1+r)\,\pi_{1,f}(h_1^*, n_f^*)}{[A(n-n_f^*)h_2^*]^{\epsilon}}\right)^{1/(1-\epsilon)}, \\
\bar{z}_{2,f} &= \left(\frac{w\,(n-\bar{n}_f)\,\bar{h} - (1+r)\,\pi_{1,f}\left(\bar{h}, \bar{n}_f\right)}{\left[A\,(n-\bar{n}_f)\,\bar{h}\right]^{\epsilon}}\right)^{1/(1-\epsilon)},
\end{aligned}$$

where $\pi_{1,f}$ stands for (9) when $\mathbb{I}_{n_f>0} = 1$ and $\mathbb{I}_{n_h>0} = 0$. One seeks to establish conditions under which $z_{2,f}^* < \bar{z}_{2,f}$. Inserting for $z_{2,f}^*$ and $\bar{z}_{2,f}$ this inequality implies

$$\frac{w(n-n_f^*)\bar{h} - (1+r)\,\pi_{1,f}(h_1^*, n_f^*)}{[A(n-n_f^*)h_2^*]^{\epsilon}} < \frac{w\,(n-\bar{n}_f)\,\bar{h} - (1+r)\,\pi_{1,f}\left(\bar{h}, \bar{n}_f\right)}{\left[A\,(n-\bar{n}_f)\,\bar{h}\right]^{\epsilon}}.$$

After some algebra (see Supplementary Derivations) the above inequality becomes

$$\left[1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right] A^{\epsilon/(1-\epsilon)} > \frac{nF}{(1-\epsilon)\,z_1} \left[\frac{(1+\beta)\,w\bar{h} - F}{2\bar{h}\epsilon}\right]^{\frac{\epsilon}{1-\epsilon}} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}\right].$$

- Surplus of hours in the first period ($h_1^* > h_2^*$) means

$$A^{\epsilon/(1-\epsilon)} > \frac{nF}{(1-\epsilon)\, z_1} \left[\frac{(1+\beta)\, w\bar{h} - F}{2\bar{h}\epsilon}\right]^{\frac{\epsilon}{1-\epsilon}} \frac{1 - \left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}}{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}},$$

so $z_{2,f}^* < \bar{z}_{2,f}$ for A sufficiently large relative to w and/or n. Thus working time account reduces turnover if a firm meets downturn with surplus.

- Deficit of hours in the first period ($h_1^* < h_2^*$) means

$$A^{\epsilon/(1-\epsilon)} < \frac{nF}{(1-\epsilon)\, z_1} \left[\frac{(1+\beta)\, w\bar{h} - F}{2\bar{h}\epsilon}\right]^{\frac{\epsilon}{1-\epsilon}} \frac{1 - \left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}}{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}},$$

so $z_{2,f}^* < \bar{z}_{2,f}$ for A sufficiently small relative to w and/or n. Consequently, for A sufficiently large relative to w and/or n working time account increases turnover if a firm meets downturn with deficit.

(ii) Let labour adjustment strategies be (‘*hire*’,‘*hire*’). Break-even thresholds for this pair of labour adjustment strategies are given by

$$\begin{aligned} z_{2,h}^* &= \left(\frac{w\,(n+n_h^*)\,\bar{h} - (1+r)\,\pi_{1,h}\,(h_1^*, n_h^*)}{\left[A\,(n+n_h^*)\,h_2^*\right]^{\epsilon}}\right)^{1/(1-\epsilon)}, \\ \bar{z}_{2,h} &= \left(\frac{w\,(n+\bar{n}_h)\,\bar{h} - (1+r)\,\pi_{1,h}\,(\bar{h}, \bar{n}_h)}{\left[A\,(n+\bar{n}_h)\,\bar{h}\right]^{\epsilon}}\right)^{1/(1-\epsilon)}, \end{aligned}$$

where $\pi_{1,h}$ stands for (9) when $\mathbb{I}_{n_f>0} = 0$ and $\mathbb{I}_{n_h>0} = 1$. One seeks to establish conditions under which $z_{2,h}^* < \bar{z}_{2,h}$. Inserting for $z_{2,h}^*$ and $\bar{z}_{2,h}$ this inequality implies

$$\frac{w\,(n+n_h^*)\,\bar{h} - (1+r)\,\pi_{1,h}\,(h_1^*, n_h^*)}{\left[A\,(n+n_h^*)\,h_2^*\right]^{\epsilon}} < \frac{w\,(n+\bar{n}_h)\,\bar{h} - (1+r)\,\pi_{1,h}\,(\bar{h}, \bar{n}_h)}{\left[A\,(n+\bar{n}_h)\,\bar{h}\right]^{\epsilon}}.$$

After some algebra (see Supplementary Derivations) the above inequality becomes

$$\left[1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right] A^{\epsilon/(1-\epsilon)} > \frac{nH}{(1-\epsilon)\, z_1} \left(\frac{(1+\beta)\, w\bar{h} + H}{2\bar{h}\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} \left[\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)} - 1\right].$$

- Surplus of hours in the first period ($h_1^* > h_2^*$) means that $z_{2,h}^* < \bar{z}_{2,h}$ always holds because the r.h.s. of the above inequality becomes negative while the l.h.s. remains positive. Thus working time account always reduces turnover if a firm meets downturn with surplus.

- Deficit of hours in the first period ($h_1^* < h_2^*$) means that $z_{2,h}^* < \bar{z}_{2,h}$ never holds because the l.h.s. of the above inequality becomes negative while the r.h.s. remains positive. Thus working time account always increases turnover if a firm meets downturn with deficit.

(iii) Let labour adjustment strategies be (‘*hire*’,‘*fire*’). One seeks to establish conditions under which $z_{2,h}^* < \bar{z}_{2,f}$, where both break-even thresholds are given in (i) and (ii) of this proof. Inserting for $z_{2,h}^*$ and $\bar{z}_{2,f}$ this inequality implies

$$\frac{w\left(n+n_h^*\right)\bar{h}-(1+r)\,\pi_{1,h}\left(h_1^*,n_h^*\right)}{\left[A\left(n+n_h^*\right)h_2^*\right]^{\epsilon}} < \frac{w\left(n-\bar{n}_f\right)\bar{h}-(1+r)\,\pi_{1,f}\left(\bar{h},\bar{n}_f\right)}{\left[A\left(n-\bar{n}_f\right)\bar{h}\right]^{\epsilon}}.$$

After some algebra (see Supplementary Derivations) the above inequality becomes

$$\left[1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right]A^{\epsilon/(1-\epsilon)} > -\frac{n}{z_1\,(1-\epsilon)\left(2\bar{h}\epsilon\right)^{\epsilon/(1-\epsilon)}}$$
$$\times\left[H\left[(1+\beta)\,w\bar{h}+H\right]^{\epsilon/(1-\epsilon)}+F\left[(1+\beta)\,w\bar{h}-F\right]^{\epsilon/(1-\epsilon)}\left(\frac{2}{\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}+1}\right)^{\epsilon/(1-\epsilon)}\right].$$

- Surplus of hours in the first period ($h_1^* > h_2^*$) means that $z_{2,h}^* < \bar{z}_{2,f}$ always holds because the r.h.s. of the above inequality remains negative while the l.h.s. becomes positive. Thus working time account always reduces turnover if a firm meets downturn with surplus.

- Deficit of hours in the first period ($h_1^* < h_2^*$) means

$$A^{\epsilon/(1-\epsilon)} < \frac{n}{z_1\,(1-\epsilon)\left(2\bar{h}\epsilon\right)^{\epsilon/(1-\epsilon)}}$$
$$\times\frac{H\left[(1+\beta)\,w\bar{h}+H\right]^{\epsilon/(1-\epsilon)}+F\left[(1+\beta)\,w\bar{h}-F\right]^{\epsilon/(1-\epsilon)}\left(\frac{2}{\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}+1}\right)^{\epsilon/(1-\epsilon)}}{\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}-1},$$

  so $z_{2,h}^* < \bar{z}_{2,f}$ for A sufficiently small relative to w and/or n. Consequently, for A sufficiently large relative to w and/or n working time account increases turnover if a firm meets downturn with deficit.

■

Supplementary Derivations

1. Basic model

1.1 Derivations for: " I.e Proof of Proposition 2"

Consider

$$\begin{gathered}
\left(\frac{wn\bar{h}-(1+r)\,\pi_1\left(h_1^*\right)}{\left[Anh_2^*\right]^\epsilon}\right)^{1/(1-\epsilon)}<\left(\frac{wn\bar{h}-(1+r)\,\pi_1\left(\bar{h}\right)}{\left[An\bar{h}\right]^\epsilon}\right)^{1/(1-\epsilon)} \\
\left[wn\bar{h}-(1+r)\,\pi_1\left(h_1^*\right)\right]\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon<wn\bar{h}-(1+r)\,\pi_1\left(\bar{h}\right) \\
\left[\frac{1}{1+r}wn\bar{h}-\pi_1\left(h_1^*\right)\right]\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon<\left[\frac{1}{1+r}wn\bar{h}-\pi_1\left(\bar{h}\right)\right]
\end{gathered}$$

With $\beta\equiv 1/(1+r)$,

$$\begin{aligned}
\left[\beta wn\bar{h}-\left\{z_1^{1-\epsilon}\left[Anh_1^*\right]^\epsilon-wn\bar{h}\right\}\right]\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon &< \beta wn\bar{h}-\left\{z_1^{1-\epsilon}\left[An\bar{h}\right]^\epsilon-wn\bar{h}\right\} \\
(1+\beta)\,wn\bar{h}\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon-z_1^{1-\epsilon}\left[Anh_1^*\right]^\epsilon\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon &< (1+\beta)\,wn\bar{h}-z_1^{1-\epsilon}\left[An\bar{h}\right]^\epsilon \\
(1+\beta)\,wn\bar{h}\left\{\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon-1\right\} &< z_1^{1-\epsilon}\left[An\bar{h}\right]^\epsilon\left\{\left[\frac{h_1^*}{h_2^*}\right]^\epsilon-1\right\} \\
\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon-1 &< \frac{z_1^{1-\epsilon}\left[An\bar{h}\right]^\epsilon}{(1+\beta)\,wn\bar{h}}\left\{\left[\frac{h_1^*}{h_2^*}\right]^\epsilon-1\right\} \\
\left[\bar{h}\right]^\epsilon-\left[h_2^*\right]^\epsilon &< \frac{z_1^{1-\epsilon}\left[An\bar{h}\right]^\epsilon}{(1+\beta)\,wn\bar{h}}\left\{\left[h_1^*\right]^\epsilon-\left[h_2^*\right]^\epsilon\right\}
\end{aligned}$$

and finally

$$\left\{\left[\bar{h}\right]^\epsilon-\left[h_2^*\right]^\epsilon\right\}-\frac{z_1^{1-\epsilon}\left[An\bar{h}\right]^\epsilon}{(1+\beta)\,wn\bar{h}}\left\{\left[h_1^*\right]^\epsilon-\left[h_2^*\right]^\epsilon\right\}<0.$$

1.2 Derivations for: "I.f Sufficient condition for Corollary to Proposition 2"

Consider $\frac{\partial}{\partial h_1^*}\left(\frac{\left[\bar{h}\right]^\epsilon-\left[h_2^*\right]^\epsilon}{\left[h_1^*\right]^\epsilon-\left[h_2^*\right]^\epsilon}\right)=\frac{\partial}{\partial h_1^*}\left(\frac{\left[\bar{h}\right]^\epsilon-\left[2\bar{h}-h_1^*\right]^\epsilon}{\left[h_1^*\right]^\epsilon-\left[2\bar{h}-h_1^*\right]^\epsilon}\right)$ and replace h_1^* by y to make notation more compact. Then $\frac{\partial}{\partial y}\left(\frac{\bar{h}^\epsilon-\left[2\bar{h}-y\right]^\epsilon}{y^\epsilon-\left[2\bar{h}-y\right]^\epsilon}\right)>0$ means

$$\begin{gathered}
\frac{\epsilon\left[2\bar{h}-y\right]^{\epsilon-1}\left(y^\epsilon-\left[2\bar{h}-y\right]^\epsilon\right)-\left(\bar{h}^\epsilon-\left[2\bar{h}-y\right]^\epsilon\right)\left[\epsilon y^{\epsilon-1}+\epsilon\left[2\bar{h}-y\right]^{\epsilon-1}\right]}{\left[y^\epsilon-\left[2\bar{h}-y\right]^\epsilon\right]^2}>0 \\
\left[2\bar{h}-y\right]^{\epsilon-1}\left(y^\epsilon-\left[2\bar{h}-y\right]^\epsilon\right)>\left(\bar{h}^\epsilon-\left[2\bar{h}-y\right]^\epsilon\right)\left[y^{\epsilon-1}+\left[2\bar{h}-y\right]^{\epsilon-1}\right] \\
\left[2\bar{h}-y\right]^{\epsilon-1}\overset{h_1^*>h_2^*}{>}\frac{\bar{h}^\epsilon-\left[2\bar{h}-y\right]^\epsilon}{y^\epsilon-\left[2\bar{h}-y\right]^\epsilon}\left[y^{\epsilon-1}+\left[2\bar{h}-y\right]^{\epsilon-1}\right]
\end{gathered}$$

$$
\begin{aligned}
\left[2\bar{h}-y\right]^{\epsilon-1}-\frac{\bar{h}^{\epsilon}-\left[2\bar{h}-y\right]^{\epsilon}}{y^{\epsilon}-\left[2\bar{h}-y\right]^{\epsilon}}\left[2\bar{h}-y\right]^{\epsilon-1} &> \frac{\bar{h}^{\epsilon}-\left[2\bar{h}-y\right]^{\epsilon}}{y^{\epsilon}-\left[2\bar{h}-y\right]^{\epsilon}}y^{\epsilon-1} \\
\left[2\bar{h}-y\right]^{\epsilon-1}\frac{y^{\epsilon}-\bar{h}^{\epsilon}}{y^{\epsilon}-\left[2\bar{h}-y\right]^{\epsilon}} &> \frac{\bar{h}^{\epsilon}-\left[2\bar{h}-y\right]^{\epsilon}}{y^{\epsilon}-\left[2\bar{h}-y\right]^{\epsilon}}y^{\epsilon-1} \\
\left[2\bar{h}-y\right]^{\epsilon-1}\left(y^{\epsilon}-\bar{h}^{\epsilon}\right) &\overset{h_1^*>h_2^*}{>} \left(\bar{h}^{\epsilon}-\left[2\bar{h}-y\right]^{\epsilon}\right)y^{\epsilon-1} \\
\left[2\bar{h}-y\right]^{\epsilon-1}\left(y^{\epsilon}-\bar{h}^{\epsilon}\right)+\left[2\bar{h}-y\right]^{\epsilon}y^{\epsilon-1} &> \bar{h}^{\epsilon}y^{\epsilon-1} \\
\left[2\bar{h}-y\right]^{\epsilon-1}\left\{y^{\epsilon}-\bar{h}^{\epsilon}+\left[2\bar{h}-y\right]y^{\epsilon-1}\right\} &> \bar{h}^{\epsilon}y^{\epsilon-1} \\
\left[2\bar{h}-y\right]^{\epsilon-1}\left\{2\bar{h}y^{\epsilon-1}-\bar{h}^{\epsilon}\right\} &> \bar{h}^{\epsilon}y^{\epsilon-1} \\
\left[2\bar{h}-y\right]^{\epsilon-1}\frac{2\bar{h}y^{\epsilon-1}-\bar{h}^{\epsilon}}{\bar{h}^{\epsilon}y^{\epsilon-1}} &> 1 \\
2\bar{h}^{1-\epsilon}-y^{1-\epsilon} &> \left[2\bar{h}-y\right]^{1-\epsilon} \\
\bar{h}^{1-\epsilon} &> \frac{\left[2\bar{h}-y\right]^{1-\epsilon}+y^{1-\epsilon}}{2}.
\end{aligned}
$$

Inserting back h_1^* for y, and h_2^* for $2\bar{h}-y$

$$
\bar{h}^{1-\epsilon} > \frac{[h_1^*]^{1-\epsilon}+[h_2^*]^{1-\epsilon}}{2}.
$$

Since $\bar{h}=(h_1^*+h_2^*)/2$, ultimately

$$
\left(\frac{h_1^*+h_2^*}{2}\right)^{1-\epsilon} > \frac{[h_1^*]^{1-\epsilon}+[h_2^*]^{1-\epsilon}}{2},
$$

which holds by Jensen's inequality for concave function.

2. Extended model with two margins of adjustment

2.1 Derivations for "II.b Proof of Proposition 3"

I.(i): Consider

$$
\begin{aligned}
z_1^{1-\epsilon}[h_1]^{\epsilon}+[h_2]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right) &> z_1^{1-\epsilon}\left[\bar{h}\right]^{\epsilon}+\beta E_1\left(z_2^{1-\epsilon}\right)\left[\bar{h}\right]^{\epsilon} \\
[h_1]^{\epsilon}+[h_2]^{\epsilon}\frac{\beta E_1\left(z_2^{1-\epsilon}\right)}{z_1^{1-\epsilon}} &> \left[\bar{h}\right]^{\epsilon}+\left[\bar{h}\right]^{\epsilon}\frac{\beta E_1\left(z_2^{1-\epsilon}\right)}{z_1^{1-\epsilon}} \\
[h_1]^{\epsilon}+[h_2]^{\epsilon}\left[\frac{h_2}{h_1}\right]^{1-\epsilon} &> \left[\bar{h}\right]^{\epsilon}+\left[\bar{h}\right]^{\epsilon}\left[\frac{h_2}{h_1}\right]^{1-\epsilon} \\
[h_1]^{1-\epsilon}[h_1]^{\epsilon}+[h_2]^{\epsilon}[h_2]^{1-\epsilon} &> [h_1]^{1-\epsilon}\left[\bar{h}\right]^{\epsilon}+\left[\bar{h}\right]^{\epsilon}[h_2]^{1-\epsilon} \\
h_1+h_2 &> [h_1]^{1-\epsilon}\left[\bar{h}\right]^{\epsilon}+\left[\bar{h}\right]^{\epsilon}[h_2]^{1-\epsilon} \\
2\bar{h} &> [h_1]^{1-\epsilon}\left[\bar{h}\right]^{\epsilon}+\left[\bar{h}\right]^{\epsilon}[h_2]^{1-\epsilon} \\
2 &> [h_1]^{1-\epsilon}\left[\bar{h}\right]^{\epsilon-1}+\left[\bar{h}\right]^{\epsilon-1}[h_2]^{1-\epsilon} \\
2 &> \left[\frac{h_1}{\bar{h}}\right]^{1-\epsilon}+\left[\frac{h_2}{\bar{h}}\right]^{1-\epsilon}
\end{aligned}
$$

and finally

$$2 > \left[\frac{h_1}{\bar{h}}\right]^{1-\epsilon} + \left[2 - \frac{h_1}{\bar{h}}\right]^{1-\epsilon}.$$

2.2 Derivations for "II.c Proof of Proposition 4"

- Labour adjustment strategies are ('$fire$','$fire$')

$$\begin{aligned}
\frac{w(n-n_f^*)\bar{h} - (1+r)\,\pi_{1,f}(h_1^*, n_f^*)}{[A(n-n_f^*)h_2^*]^\epsilon} &< \frac{w\,(n-\bar{n}_f)\,\bar{h} - (1+r)\,\pi_{1,f}\left(\bar{h}, \bar{n}_f\right)}{\left[A\,(n-\bar{n}_f)\,\bar{h}\right]^\epsilon} \\
w(n-n_f^*)\bar{h} - (1+r)\,\pi_{1,f}(h_1^*, n_f^*) &< \left(\frac{n-n_f^*}{n-\bar{n}_f}\right)^\epsilon \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon \left[w\,(n-\bar{n}_f)\,\bar{h} - (1+r)\,\pi_{1,f}\left(\bar{h}, \bar{n}_f\right)\right]
\end{aligned}$$

$$\begin{aligned}
& w(n-n_f^*)\bar{h} - w\,(n-\bar{n}_f)\,\bar{h}\left(\frac{n-n_f^*}{n-\bar{n}_f}\right)^\epsilon \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon \\
& \qquad < (1+r)\left\{\pi_{1,f}(h_1^*, n_f^*) - \pi_{1,f}\left(\bar{h}, \bar{n}_f\right)\left(\frac{n-n_f^*}{n-\bar{n}_f}\right)^\epsilon \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon\right\}
\end{aligned}$$

$$\begin{aligned}
& w\bar{h}(n-n_f^*)\left[1 - \left(\frac{n-\bar{n}_f}{n-n_f^*}\right)^{1-\epsilon} \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon\right] \\
& \qquad < (1+r)\left\{\left[z_1^{1-\epsilon}\left[A(n-n_f^*)h_1^*\right]^\epsilon - w(n-n_f^*)\bar{h} - n_f^*F\right]\right. \\
& \qquad\qquad \left. - \left[z_1^{1-\epsilon}\left[A(n-\bar{n}_f)\bar{h}\right]^\epsilon - w(n-\bar{n}_f)\bar{h} - \bar{n}_f F\right]\left(\frac{n-n_f^*}{n-\bar{n}_f}\right)^\epsilon \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon\right\}
\end{aligned}$$

$$\begin{aligned}
& w\bar{h}\left[1 - \left(\frac{n-\bar{n}_f}{n-n_f^*}\right)^{1-\epsilon} \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon\right] \\
& \qquad < (1+r)\left\{\left[z_1^{1-\epsilon}\left[Ah_1^*\right]^\epsilon (n-n_f^*)^{\epsilon-1} - w\bar{h} + F - \frac{nF}{(n-n_f^*)}\right]\right. \\
& \qquad\qquad \left. - \left[z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon \left[n-\bar{n}_f\right]^{\epsilon-1} - w\bar{h} + F - \frac{nF}{(n-\bar{n}_f)}\right]\left(\frac{n-\bar{n}_f}{n-n_f^*}\right)^{1-\epsilon} \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon\right\}
\end{aligned}$$

$$\begin{aligned}
& \left[(1+\beta)\,w\bar{h} - F\right]\left[1 - \left(\frac{n-\bar{n}_f}{n-n_f^*}\right)^{1-\epsilon} \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon\right] \\
& \qquad < z_1^{1-\epsilon}\left[Ah_1^*\right]^\epsilon (n-n_f^*)^{\epsilon-1}\left[1 - \left(\frac{h_2^*}{h_1^*}\right)^\epsilon\right] - \frac{nF}{(n-n_f^*)}\left[1 - \left(\frac{n-\bar{n}_f}{n-n_f^*}\right)^{-\epsilon} \left(\frac{h_2^*}{\bar{h}}\right)^\epsilon\right].
\end{aligned}$$

For $(n-\bar{n}_f)/(n-n_f^*)$ one can show that

$$\frac{n-\bar{n}_f}{n-n_f^*} = \frac{\left(\epsilon A^\epsilon \frac{\left[\bar{h}\right]^\epsilon z_1^{1-\epsilon} + \left[\bar{h}\right]^\epsilon \beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)w\bar{h}-F}\right)^{1/(1-\epsilon)}}{\left(\epsilon A^\epsilon \frac{\left[h_1^*\right]^\epsilon z_1^{1-\epsilon} + \left[h_2^*\right]^\epsilon \beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)w\bar{h}-F}\right)^{1/(1-\epsilon)}} = \left(\frac{\left[\bar{h}\right]^\epsilon z_1^{1-\epsilon} + \left[\bar{h}\right]^\epsilon \beta E_1\left(z_2^{1-\epsilon}\right)}{\left[h_1^*\right]^\epsilon z_1^{1-\epsilon} + \left[h_2^*\right]^\epsilon \beta E_1\left(z_2^{1-\epsilon}\right)}\right)^{1/(1-\epsilon)}$$

$$= \left(\left(\frac{\bar{h}}{h_2^*}\right)^{\epsilon} \frac{1 + \frac{1}{z_1^{1-\epsilon}} \beta E_1 \left(z_2^{1-\epsilon}\right)}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} + \frac{1}{z_1^{1-\epsilon}} \beta E_1 \left(z_2^{1-\epsilon}\right)} \right)^{1/(1-\epsilon)},$$

which leads to

$$\text{(a)} \quad : \qquad 1 - \left(\frac{n - \bar{n}_f}{n - n_f^*}\right)^{1-\epsilon} \left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon} = 1 - \frac{1 + \frac{1}{z_1^{1-\epsilon}} \beta E_1 \left(z_2^{1-\epsilon}\right)}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} + \frac{1}{z_1^{1-\epsilon}} \beta E_1 \left(z_2^{1-\epsilon}\right)} = \frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} - 1}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} + \frac{1}{z_1^{1-\epsilon}} \beta E_1 \left(z_2^{1-\epsilon}\right)}$$

$$= \quad \frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} - 1}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} + \left(\frac{h_2^*}{h_1^*}\right)^{1-\epsilon}} = \frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} \left[1 - \left(\frac{h_1^*}{h_2^*}\right)^{-\epsilon}\right]}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} \left[1 + \left(\frac{h_2^*}{h_1^*}\right)\right]} = \frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \left(\frac{h_2^*}{h_1^*}\right)}.$$

$$\text{(b)} \quad : \qquad 1 - \left(\frac{n - \bar{n}_f}{n - n_f^*}\right)^{-\epsilon} \left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon} = 1 - \left(\frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} + \frac{1}{z_1^{1-\epsilon}} \beta E_1 \left(z_2^{1-\epsilon}\right)}{1 + \frac{1}{z_1^{1-\epsilon}} \beta E_1 \left(z_2^{1-\epsilon}\right)} \frac{h_2^*}{\bar{h}} \right)^{\epsilon/(1-\epsilon)}$$

$$= \quad 1 - \left(\frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon} + \left(\frac{h_2^*}{h_1^*}\right)^{-\epsilon} \left(\frac{h_2^*}{h_1^*}\right)}{1 + \left(\frac{h_2^*}{h_1^*}\right) \left(\frac{h_2^*}{h_1^*}\right)^{-\epsilon}} \frac{h_2^*}{\bar{h}} \right)^{\epsilon/(1-\epsilon)} = 1 - \left(\frac{1 + \left(\frac{h_2^*}{h_1^*}\right)}{\left(\frac{h_1^*}{h_2^*}\right)^{-\epsilon} + \left(\frac{h_2^*}{h_1^*}\right)} \frac{h_2^*}{\bar{h}} \right)^{\epsilon/(1-\epsilon)}$$

$$= \quad 1 - \left(\frac{2 \frac{h_2^*}{h_1^*}}{\left(\frac{h_1^*}{h_2^*}\right)^{-\epsilon} + \left(\frac{h_2^*}{h_1^*}\right)} \right)^{\epsilon/(1-\epsilon)} = 1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)}.$$

Inserting this back

$$\left[(1+\beta) w \bar{h} - F\right] \left[\frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \left(\frac{h_2^*}{h_1^*}\right)} \right]$$

$$< z_1^{1-\epsilon} \left[A h_1^*\right]^{\epsilon} (n - n_f^*)^{\epsilon - 1} \left[1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right] - \frac{nF}{(n - n_f^*)} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)} \right]$$

$$\left[(1+\beta) w \bar{h} - F\right] \left[\frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \left(\frac{h_2^*}{h_1^*}\right)} \right] < \frac{z_1^{1-\epsilon} \left[A h_1^*\right]^{\epsilon} \left[(1+\beta) w \bar{h} - F\right]}{\epsilon A^{\epsilon} \left\{ \left[h_1^*\right]^{\epsilon} z_1^{1-\epsilon} + \left[h_2^*\right]^{\epsilon} \beta E_1 \left(z_2^{1-\epsilon}\right) \right\}} \left[1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right]$$

$$- \frac{nF}{(n - n_f^*)} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)} \right]$$

$$\left[(1+\beta) w \bar{h} - F\right] \left[\frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \left(\frac{h_2^*}{h_1^*}\right)} \right] < \frac{(1+\beta) w \bar{h} - F}{\epsilon \left[1 + \left(\frac{h_2^*}{h_1^*}\right)\right]} \left[1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right] - \frac{nF}{(n - n_f^*)} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}} \right]$$

$$\left[(1+\beta)\,w\bar{h}-F\right]\left[\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1+\left(\frac{h_2^*}{h_1^*}\right)}\right]\frac{\epsilon-1}{\epsilon}<-\frac{nF}{\left(n-n_f^*\right)}\left[1-\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}\right]$$

$$\left[(1+\beta)\,w\bar{h}-F\right]\left[\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1+\left(\frac{h_2^*}{h_1^*}\right)}\right]\frac{\epsilon-1}{\epsilon}\left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)}$$

$$<-\frac{nF\left[(1+\beta)\,w\bar{h}-F\right]^{1/(1-\epsilon)}}{\left(\left[h_1^*\right]^{\epsilon}z_1^{1-\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)\right)^{1/(1-\epsilon)}}\left[1-\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}\right]$$

$$\left[(1+\beta)\,w\bar{h}-F\right]\left[\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1+\left(\frac{h_2^*}{h_1^*}\right)}\right]\frac{1-\epsilon}{\epsilon}\left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)}$$

$$>\frac{nF\left[(1+\beta)\,w\bar{h}-F\right]^{1/(1-\epsilon)}}{\left[z_1^{1-\epsilon}\left[h_1^*\right]^{\epsilon}\left(1+\frac{h_2^*}{h_1^*}\right)\right]^{1/(1-\epsilon)}}\left[1-\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}\right]$$

$$\left[(1+\beta)\,w\bar{h}-F\right]\frac{\left[1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right]}{\frac{1}{h_1^*}\left(2\bar{h}\right)}\frac{1-\epsilon}{\epsilon}\left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)}$$

$$>\frac{nF\left[(1+\beta)\,w\bar{h}-F\right]^{1/(1-\epsilon)}}{\frac{z_1}{h_1^*}\left(2\bar{h}\right)^{1/(1-\epsilon)}}\left[1-\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}\right]$$

and finally

$$\left[1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right]A^{\epsilon/(1-\epsilon)}>\frac{nF}{(1-\epsilon)\,z_1}\left[\frac{(1+\beta)\,w\bar{h}-F}{2\bar{h}\epsilon}\right]^{\frac{\epsilon}{1-\epsilon}}\left[1-\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}\right].$$

- Labour adjustment strategies are (‘*hire*’,‘*hire*’)

$$\frac{w\left(n+n_h^*\right)\bar{h}-(1+r)\,\pi_{1,h}\left(h_1^*,n_h^*\right)}{\left[A\left(n+n_h^*\right)h_2^*\right]^{\epsilon}} < \frac{w\left(n+\bar{n}_h\right)\bar{h}-(1+r)\,\pi_{1,h}\left(\bar{h},\bar{n}_h\right)}{\left[A\left(n+\bar{n}_h\right)\bar{h}\right]^{\epsilon}}$$

$$\left[w\left(n+n_h^*\right)\bar{h}-(1+r)\,\pi_{1,h}\left(h_1^*,n_h^*\right)\right] < \left(\frac{n+n_h^*}{n+\bar{n}_h}\right)^{\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\left[w\left(n+\bar{n}_h\right)\bar{h}-(1+r)\,\pi_{1,h}\left(\bar{h},\bar{n}_h\right)\right]$$

$$w\bar{h}\left(n+n_h^*\right)\left[1-\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right]$$

$$<(1+r)\left\{\left[z_1^{1-\epsilon}\left[A\left(n+n_h^*\right)h_1^*\right]^{\epsilon}-w\left(n+n_h^*\right)\bar{h}-n_h^*H\right]\right.$$

$$\left.-\left[z_1^{1-\epsilon}\left[A\left(n+\bar{n}_h\right)\bar{h}\right]^{\epsilon}-w\left(n+\bar{n}_h\right)\bar{h}-\bar{n}_hH\right]\left(\frac{n+n_h^*}{n+\bar{n}_h}\right)^{\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right\}$$

$$\frac{w\bar{h}}{1+r}\left[1-\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right]$$
$$<\left[z_1^{1-\epsilon}\frac{\left[A\left(n+n_h^*\right)h_1^*\right]^{\epsilon}}{\left(n+n_h^*\right)}-w\bar{h}-H+\frac{nH}{\left(n+n_h^*\right)}\right]$$
$$-\left[z_1^{1-\epsilon}\frac{\left[A\left(n+\bar{n}_h\right)\bar{h}\right]^{\epsilon}}{\left(n+\bar{n}_h\right)}-w\bar{h}-H+\frac{nH}{\left(n+\bar{n}_h\right)}\right]\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}$$

$$\left[(1+\beta)\,w\bar{h}+H\right]\left[1-\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right]$$
$$<\left[z_1^{1-\epsilon}\frac{\left[A\left(n+n_h^*\right)h_1^*\right]^{\epsilon}}{\left(n+n_h^*\right)}+\frac{nH}{\left(n+n_h^*\right)}\right]-\left[z_1^{1-\epsilon}\frac{\left[A\left(n+\bar{n}_h\right)\bar{h}\right]^{\epsilon}}{\left(n+\bar{n}_h\right)}+\frac{nH}{\left(n+\bar{n}_h\right)}\right]\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}$$

$$\left[(1+\beta)\,w\bar{h}+H\right]\left[1-\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right]$$
$$<z_1^{1-\epsilon}\left[Ah_1^*\right]^{\epsilon}\left(n+n_h^*\right)^{\epsilon-1}\left[1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right]+\frac{nH}{\left(n+n_h^*\right)}\left[1-\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right].$$

For $(n+\bar{n}_h)/(n+n_h^*)$ one can show that

$$\frac{n+\bar{n}_h}{n+n_h^*}=\frac{\left(\epsilon A^{\epsilon}\frac{\left[\bar{h}\right]^{\epsilon}z_1^{1-\epsilon}+\left[\bar{h}\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)w\bar{h}+H}\right)^{1/(1-\epsilon)}}{\left(\epsilon A^{\epsilon}\frac{z_1^{1-\epsilon}\left[h_1^*\right]^{\epsilon}+\left[h_1^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)w\bar{h}+H}\right)^{1/(1-\epsilon)}}=\left(\left(\frac{\bar{h}}{h_2^*}\right)^{\epsilon}\left(\frac{1+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}\right)\right)^{1/(1-\epsilon)},$$

which leads to

$$\text{(a)}\quad:\quad 1-\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}=1-\left(\frac{\bar{h}}{h_2^*}\right)^{\epsilon}\left(\frac{1+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}\right)\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}$$
$$=\frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}-1}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}=\frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}-1}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}+\left(\frac{h_2^*}{h_1^*}\right)^{1-\epsilon}}=\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1+\left(\frac{h_2^*}{h_1^*}\right)}.$$

$$\text{(b)}\quad:\quad 1-\left(\frac{n+\bar{n}_h}{n+n_h^*}\right)^{-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}=1-\left(\frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}{1+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}\right)^{\epsilon/(1-\epsilon)}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon/(1-\epsilon)}$$
$$=1-\left(\frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}{1+\frac{1}{z_1^{1-\epsilon}}\beta E_1\left(z_2^{1-\epsilon}\right)}\frac{h_2^*}{\bar{h}}\right)^{\epsilon/(1-\epsilon)}=1-\left(\frac{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}+\left(\frac{h_2^*}{h_1^*}\right)^{1-\epsilon}}{1+\left(\frac{h_2^*}{h_1^*}\right)^{1-\epsilon}}\frac{h_2^*}{\bar{h}}\right)^{\epsilon/(1-\epsilon)}$$
$$=1-\left(\frac{\left(\frac{h_2^*}{h_1^*}\right)^{-\epsilon}\left[1+\left(\frac{h_2^*}{h_1^*}\right)\right]}{\left(\frac{h_2^*}{h_1^*}\right)^{-\epsilon}\left[\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}+\left(\frac{h_2^*}{h_1^*}\right)\right]}\frac{h_2^*}{\bar{h}}\right)^{\epsilon/(1-\epsilon)}=1-\left(\frac{1+\left(\frac{h_2^*}{h_1^*}\right)}{\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}+\left(\frac{h_2^*}{h_1^*}\right)}\frac{h_2^*}{\bar{h}}\right)^{\epsilon/(1-\epsilon)}$$

$$= 1 - \left(\frac{2 \frac{h_2^*}{h_1^*}}{\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon} + \left(\frac{h_2^*}{h_1^*}\right)} \right)^{\epsilon/(1-\epsilon)} = 1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)} .$$

Inserting this back

$$\left[(1+\beta) w\bar{h} + H\right] \frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \left(\frac{h_2^*}{h_1^*}\right)}$$

$$< z_1^{1-\epsilon} \left[A h_1^*\right]^{\epsilon} (n + n_h^*)^{\epsilon - 1} \left[1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right] + \frac{nH}{(n + n_h^*)} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)}\right]$$

$$\left[(1+\beta) w\bar{h} + H\right] \frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \left(\frac{h_2^*}{h_1^*}\right)}$$

$$< \frac{(1+\beta) w\bar{h} + H}{\epsilon} \frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{\left[1 + \left[\frac{h_2^*}{h_1^*}\right]^{\epsilon} \frac{1}{z_1^{1-\epsilon}} \beta E_1\left(z_2^{1-\epsilon}\right)\right]} + \frac{nH}{(n + n_h^*)} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)}\right]$$

$$\left[(1+\beta) w\bar{h} + H\right] \frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \frac{h_2^*}{h_1^*}}$$

$$< \frac{(1+\beta) w\bar{h} + H}{\epsilon} \frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \frac{h_2^*}{h_1^*}} + \frac{nH}{(n + n_h^*)} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)}\right]$$

$$\frac{\epsilon - 1}{\epsilon} \left[(1+\beta) w\bar{h} + H\right] \frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \frac{h_2^*}{h_1^*}} \left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)}$$

$$< \frac{nH}{\left(\frac{z_1^{1-\epsilon}\left[h_1^*\right]^{\epsilon} + \left[h_2^*\right]^{\epsilon} \beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta) w\bar{h} + H}\right)^{1/(1-\epsilon)}} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)}\right]$$

$$\frac{\epsilon - 1}{\epsilon} \left[(1+\beta) w\bar{h} + H\right] \frac{1 - \left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1 + \frac{h_2^*}{h_1^*}} \left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)}$$

$$< \frac{nH \left[(1+\beta) w\bar{h} + H\right]^{1/(1-\epsilon)}}{\left(z_1^{1-\epsilon} \left[h_1^*\right]^{\epsilon} + \left[h_2^*\right]^{\epsilon} \beta E_1\left(z_2^{1-\epsilon}\right)\right)^{1/(1-\epsilon)}} \left[1 - \left(\frac{2}{1 + \left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}} \right)^{\epsilon/(1-\epsilon)}\right]$$

$$\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1+\frac{h_2^*}{h_1^*}}\left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)} > \frac{\epsilon}{1-\epsilon}\frac{nH\left[(1+\beta)\,w\bar{h}+H\right]^{\frac{\epsilon}{1-\epsilon}}}{\left(z_1^{1-\epsilon}\left[h_1^*\right]^{\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)\right)^{1/(1-\epsilon)}}\left[\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}-1\right]$$

$$\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1+\frac{h_2^*}{h_1^*}}\left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)} > \frac{\epsilon}{1-\epsilon}\frac{nH\left[(1+\beta)\,w\bar{h}+H\right]^{\frac{\epsilon}{1-\epsilon}}}{\left[z_1^{1-\epsilon}\left[h_1^*\right]^{\epsilon}\left(1+\frac{h_2^*}{h_1^*}\right)\right]^{1/(1-\epsilon)}}\left[\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}-1\right]$$

$$\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{2\bar{h}}h_1^*\left[\epsilon A^{\epsilon}\right]^{1/(1-\epsilon)} > \frac{\epsilon}{1-\epsilon}h_1^*\frac{nH\left[(1+\beta)\,w\bar{h}+H\right]^{\frac{\epsilon}{1-\epsilon}}}{z_1\left(2\bar{h}\right)^{1/(1-\epsilon)}}\left[\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}-1\right]$$

and finally

$$\left[1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right]A^{\epsilon/(1-\epsilon)} > \frac{nH}{(1-\epsilon)\,z_1}\left(\frac{(1+\beta)\,w\bar{h}+H}{2\bar{h}\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}}\left[\left(\frac{2}{1+\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}}\right)^{\epsilon/(1-\epsilon)}-1\right].$$

- Labour adjustment strategies are (‘*hire*’,‘*fire*’)

$$\frac{w\left(n+n_h^*\right)\bar{h}-(1+r)\,\pi_{1,h}\left(h_1^*,n_h^*\right)}{\left[A\left(n+n_h^*\right)h_2^*\right]^{\epsilon}} < \frac{w\left(n-\bar{n}_f\right)\bar{h}-(1+r)\,\pi_{1,f}\left(\bar{h},\bar{n}_f\right)}{\left[A\left(n-\bar{n}_f\right)\bar{h}\right]^{\epsilon}}$$

$$w\left(n+n_h^*\right)\bar{h}-(1+r)\,\pi_{1,h}\left(h_1^*,n_h^*\right) < \left(\frac{n+n_h^*}{n-\bar{n}_f}\right)^{\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\left[w\left(n-\bar{n}_f\right)\bar{h}-(1+r)\,\pi_{1,f}\left(\bar{h},\bar{n}_f\right)\right]$$

$$\begin{aligned}&w\left(n+n_h^*\right)\bar{h}-w\left(n-\bar{n}_f\right)\bar{h}\left(\frac{n+n_h^*}{n-\bar{n}_f}\right)^{\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\\&\qquad<(1+r)\left\{\pi_{1,h}\left(h_1^*,n_h^*\right)-\pi_{1,f}\left(\bar{h},\bar{n}_f\right)\left(\frac{n+n_h^*}{n-\bar{n}_f}\right)^{\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right\}\end{aligned}$$

$$\begin{aligned}&w\bar{h}\left(n+n_h^*\right)\left[1-\left(\frac{n-\bar{n}_f}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right]\\&\qquad<(1+r)\left\{\left[z_1^{1-\epsilon}\left[A\left(n+n_h^*\right)h_1^*\right]^{\epsilon}-w\left(n+n_h^*\right)\bar{h}-n_h^*H\right]\right.\\&\qquad\qquad\left.-\left[z_1^{1-\epsilon}\left[A(n-\bar{n}_f)\bar{h}\right]^{\epsilon}-w(n-\bar{n}_f)\bar{h}-\bar{n}_fF\right]\left(\frac{n+n_h^*}{n-\bar{n}_f}\right)^{\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right\}\end{aligned}$$

$$\begin{aligned}&(1+\beta)\,w\bar{h}\left[1-\left(\frac{n-\bar{n}_f}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right]\\&\qquad<\left[z_1^{1-\epsilon}\frac{\left[A\left(n+n_h^*\right)h_1^*\right]^{\epsilon}}{\left(n+n_h^*\right)}-H+\frac{nH}{n+n_h^*}\right]\\&\qquad\qquad-\left[z_1^{1-\epsilon}\frac{\left[A(n-\bar{n}_f)\bar{h}\right]^{\epsilon}}{\left(n-\bar{n}_f\right)}+F-\frac{nF}{n-\bar{n}_f}\right]\left(\frac{n-\bar{n}_f}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\end{aligned}$$

$$
\begin{aligned}
&\left[(1+\beta)\,w\bar{h}+H\right]-\left[(1+\beta)\,w\bar{h}-F\right]\left(\frac{n-\bar{n}_f}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\\
&\quad< z_1^{1-\epsilon}\left[Ah_1^*\right]^{\epsilon}\left[n+n_h^*\right]^{\epsilon-1}\left[1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right]+\frac{n}{n+n_h^*}\left[H+F\left(\frac{n-\bar{n}_f}{n+n_h^*}\right)^{-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}\right]
\end{aligned}
$$

For $(n-\bar{n}_f)/(n+n_h^*)$ one can show that

$$
\begin{aligned}
\frac{n-\bar{n}_f}{n+n_h^*} &= \frac{\left(\epsilon A^{\epsilon}\frac{\left[\bar{h}\right]^{\epsilon}z_1^{1-\epsilon}+\left[\bar{h}\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)w\bar{h}-F}\right)^{1/(1-\epsilon)}}{\left(\epsilon A^{\epsilon}\frac{\left[h_1^*\right]^{\epsilon}z_1^{1-\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{(1+\beta)w\bar{h}+H}\right)^{1/(1-\epsilon)}}\\
&= \left(\frac{\left[\bar{h}\right]^{\epsilon}z_1^{1-\epsilon}+\left[\bar{h}\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}{\left[h_1^*\right]^{\epsilon}z_1^{1-\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)}\right)^{1/(1-\epsilon)}\left(\frac{(1+\beta)\,w\bar{h}+H}{(1+\beta)\,w\bar{h}-F}\right)^{1/(1-\epsilon)}\\
&= \left(\left(\frac{\bar{h}}{h_2^*}\right)^{\epsilon}\frac{1+\left(\frac{h_2^*}{h_1^*}\right)^{1-\epsilon}}{\left(\frac{h_1^*}{h_2^*}\right)^{\epsilon}+\left(\frac{h_2^*}{h_1^*}\right)^{1-\epsilon}}\frac{(1+\beta)\,w\bar{h}+H}{(1+\beta)\,w\bar{h}-F}\right)^{1/(1-\epsilon)}\\
&= \left(\left(\frac{\bar{h}}{h_2^*}\right)^{\epsilon}\left[\frac{\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}+\frac{h_2^*}{h_1^*}}{1+\frac{h_2^*}{h_1^*}}\right]\frac{(1+\beta)\,w\bar{h}+H}{(1+\beta)\,w\bar{h}-F}\right)^{1/(1-\epsilon)}
\end{aligned}
$$

which leads to

$$
\text{(a)}\quad:\qquad \left(\frac{n-\bar{n}_f}{n+n_h^*}\right)^{1-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}=\left[\frac{\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}+\frac{h_2^*}{h_1^*}}{1+\frac{h_2^*}{h_1^*}}\right]\frac{(1+\beta)\,w\bar{h}+H}{(1+\beta)\,w\bar{h}-F}
$$

$$
\begin{aligned}
\text{(b)}\quad:\qquad &\left(\frac{n-\bar{n}_f}{n+n_h^*}\right)^{-\epsilon}\left(\frac{h_2^*}{\bar{h}}\right)^{\epsilon}=\left(\left[\frac{\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}+\frac{h_2^*}{h_1^*}}{1+\frac{h_2^*}{h_1^*}}\right]\frac{(1+\beta)\,w\bar{h}+H}{(1+\beta)\,w\bar{h}-F}\right)^{-\epsilon/(1-\epsilon)}\left(\frac{h_2^*}{\bar{h}}\right)^{\frac{\epsilon}{1-\epsilon}}\\
&= \left(\frac{\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon-1}+1}{\frac{2\bar{h}}{h_2^*}}\frac{(1+\beta)\,w\bar{h}+H}{(1+\beta)\,w\bar{h}-F}\right)^{-\epsilon/(1-\epsilon)}\left(\frac{h_2^*}{\bar{h}}\right)^{\frac{\epsilon}{1-\epsilon}}=\left(\frac{2}{\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}+1}\frac{(1+\beta)\,w\bar{h}-F}{(1+\beta)\,w\bar{h}+H}\right)^{\epsilon/(1-\epsilon)}.
\end{aligned}
$$

Inserting this back

$$
\begin{aligned}
&\frac{\epsilon-1}{\epsilon}\left[(1+\beta)\,w\bar{h}+H\right]\left[\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1+\frac{h_2^*}{h_1^*}}\right]\\
&< n\left(\frac{\left[(1+\beta)\,w\bar{h}+H\right]}{\epsilon A^{\epsilon}\left[z_1^{1-\epsilon}\left[h_1^*\right]^{\epsilon}+\left[h_2^*\right]^{\epsilon}\beta E_1\left(z_2^{1-\epsilon}\right)\right]}\right)^{1/(1-\epsilon)}\left[H+F\left(\frac{2}{\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}+1}\frac{(1+\beta)\,w\bar{h}-F}{(1+\beta)\,w\bar{h}+H}\right)^{\epsilon/(1-\epsilon)}\right]
\end{aligned}
$$

$$\frac{\epsilon-1}{\epsilon}\left[(1+\beta)\,w\bar{h}+H\right]\left[\frac{1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}}{1+\frac{h_2^*}{h_1^*}}\right]A^{\epsilon/(1-\epsilon)}$$

$$< \frac{h_1^*}{z_1}\frac{n\left[(1+\beta)\,w\bar{h}+H\right]^{1/(1-\epsilon)}}{\left\{\epsilon 2\bar{h}\right\}^{1/(1-\epsilon)}}\left[H+F\left(\frac{2}{\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}+1}\frac{(1+\beta)\,w\bar{h}-F}{(1+\beta)\,w\bar{h}+H}\right)^{\epsilon/(1-\epsilon)}\right]$$

and finally

$$\left[1-\left(\frac{h_2^*}{h_1^*}\right)^{\epsilon}\right]A^{\epsilon/(1-\epsilon)} > -\frac{n}{z_1\,(1-\epsilon)\left(2\bar{h}\epsilon\right)^{\epsilon/(1-\epsilon)}}$$

$$\times\left[H\left[(1+\beta)\,w\bar{h}+H\right]^{\epsilon/(1-\epsilon)}+F\left[(1+\beta)\,w\bar{h}-F\right]^{\epsilon/(1-\epsilon)}\left(\frac{2}{\left(\frac{h_1^*}{h_2^*}\right)^{1-\epsilon}+1}\right)^{\epsilon/(1-\epsilon)}\right].$$