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Abstract

A large literature suggests that the expected equity risk premium is countercyclical. Using a variety of different measures for this risk premium, we document that it also exhibits growth asymmetry, i.e. the risk premium rises sharply in recessions and declines much more gradually during the following recoveries. We show that a model with recursive preferences, in which agents cannot perfectly observe the state of current productivity, can generate the observed asymmetry in the risk premium. Key for this result are endogenous fluctuations in uncertainty which induce procyclical variations in agent's nowcast accuracy. In addition to matching moments of the risk premium, the model is also successful in generating the growth asymmetry in macroeconomic aggregates observed in the data, and in matching the cyclical relation between quantities and the risk premium.

JEL-Codes: E200, E300, G100.

Keywords: risk premium, business cycles, Bayesian learning, asymmetry, uncertainty, nowcasting.

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1 Introduction

The most recent U.S. investment and housing booms that ended abruptly in 2001 and 2007 have been associated with highly optimistic beliefs about profitability. In the former case, beliefs about profitability were linked to information technology and in the latter to house price gains. Both booms were associated with times of low uncertainty and saw long hikes in stock markets. Adjustments of beliefs about profitability resulted in sharp recessions, heightened uncertainty, and strong corrections in stock markets (see e.g. Beaudry and Portier (2006) and Shiller (2007)). In this situation, investors were less willing to bear financial risk and, for a given level of stock market risk, they required a higher compensation to hold stocks instead of a risk free short-term asset. Indeed, this equity risk premium increased sharply at the brink of both recessions, very much in contrast to the slow and gradual decline that could be observed during the preceding booms. This growth asymmetry shows in positively skewed distributions of risk premium growth — skewness is 0.64 and 1.00, respectively over these two business cycles.[1]

There is a large body of work in the finance literature on risk premia, which for example provides substantial empirical evidence that the equity risk premium varies over time and is countercyclical.[2] A recent and growing literature jointly studies both the behavior of risk premia and macroeconomic dynamics. We contribute to this body of work which has not considered the important asymmetric feature of risk premia. We first document the degree of asymmetry in the data and then develop a structural model with endogenous countercyclical variations in uncertainty that is consistent with this feature in the data. To the best of our knowledge this is the first paper that tackles this issue.

We start by computing statistics on growth asymmetry for a variety of expected equity risk premium measures that have been found relevant in the literature. We document for the post-WWII U.S. economy that growth rates of all risk premium measures exhibit positive skewness. This growth asymmetry is not only a salient feature over the entire sample, but it is

[1] These skewness statistics over the two business cycles are significant with p-values of 0.065 and 0.017, respectively. The positively skewed distribution implies that positive changes in risk premia are more extreme than negative changes.

[2] See e.g. Fama and French (1989), Ferson and Harvey (1991), and Bekaert and Harvey (1995).

also present across subsamples — which we specify to include two consecutive business cycles as defined by the NBER's Business Cycle Dating Committee — and for different investment horizons of risk premia. In particular, we construct risk premium measures using models based on the historical mean of realized stock market returns in excess of Treasury bond yields, and models based on predictive time series regressions of equity returns on selected fundamentals. We further employ direct risk premium measures based on responses of Chief Financial Officers recorded in the Duke CFO Global Business Outlook Survey. The broad support for growth asymmetry is remarkable given the substantial diversity in assumptions underlying the various employed risk premium measures.

We then build a structural model that is consistent with the empirically observed growth asymmetry in the risk premium. The core of the model is a standard real business cycle model with capital adjustment costs and preferences of the Epstein and Zin (1989) type. This setup allows disentangling relative risk aversion and the elasticity of intertemporal substitution and gives rise to a risk premium. Our empirical measures for expected risk premia incorporate information about future returns, so that their value can differ from realized ex-post risk premia. In the model, deviations from fundamentals are possible, because agents need to form nowcasts about the state of total factor productivity (TFP). Nowcasting is required as the otherwise standard Cobb-Douglas type production function includes an additive noise term and neither TFP nor the noise term can be observed separately at the time decisions about production inputs are made.[3]

The key mechanism to generate growth asymmetry in the risk premium are endogenous changes in the degree of uncertainty about the state of productivity, which in turn induce procyclical variations in agent's nowcast precision. TFP follows a two-state Markov process and agents employ a Bayesian learning technology to form a nowcast about the state of current productivity. Intensified use of production inputs amplifies the signal on the state of productivity relative to the noise and results in endogenous procyclical variations of the signal-to-noise ratio. When production inputs are high, agent's nowcasts are rela-

[3]This setup is consistent with the nowcasting process of statistical agencies documented in Faust et al. (2005). They describe the preliminary nowcast to be the sum of the final GDP announcement and an additive noise term.

tively accurate as uncertainty about the state of productivity is low. The end of a boom, implied by a change from the high to the low productivity state, can be nowcasted with high accuracy and the risk premium will increase sharply. The reduced use of production inputs during the following recession leads to a lower signal-to-noise ratio and a situation of heightened uncertainty about the state of productivity. For this reason, agent's nowcasting accuracy increases only slowly during the following recovery. The associated gradual decline in uncertainty comes along with a slow and gradual decline in the risk premium.

The model is calibrated to match nowcast precision in the Survey of Professional Forecasters (SPF). Overall, it is successful in generating the observed growth asymmetry in the risk premium and in matching the risk premium's relation with macroeconomic activity in the data. The model captures the empirically observed positive skewness in the risk premium, while a framework without the endogenous variation in nowcast accuracy does not imply a skewness substantially different from zero. The model's ability to generate growth asymmetry rests on the procyclicality of nowcast precision and the associated variations in uncertainty. This mechanism finds strong support in the data. Firstly, the median absolute nowcast error for real GDP growth from the SPF varies countercyclically and is notably heightened when the economy contracts. We also find this absolute median nowcast error is particularly high at times when the risk premium rises strongly. Secondly, we employ the dispersion in nowcasts for GDP growth from the SPF as a proxy for uncertainty to provide further corroborative evidence for the model mechanism. We report uncertainty varies countercyclically — in line with evidence in the literature, see e.g. Bloom (2014) — and is notably heightened at times when the economy contracts and when risk premia are high. Endogenous procyclical variations in agent's nowcasting precision are crucial also for the model's ability to generate the well known stylized business cycle fact of negatively skewed growth rates in macroeconomic aggregates — i.e. expansions in economic activity are long and gradual while recessions are sharp and short. In addition to the skewness statistics, it is notable that our model is also successful in matching the countercyclical movements of risk premia observed in the data.

Our paper is related to several strands of the literature. There is a large body of work

in the empirical finance literature on risk premia. Yet, existing theoretical work in finance has mostly been confined to endowment economies that do not consider feedback between time-varying risk premia and macroeconomic aggregates.[4] On the other hand, most standard macroeconomic models do not include a meaningful role for the risk premium. Our work links to a growing literature that jointly studies the behavior of macroeconomic aggregates and risk premia in bond or equity markets (e.g. Jermann (1998) and Kaltenbrunner and Lochstoer (2010)). Gilchrist and Zakrajšek (2012) empirically document a close link between increases in the excess bond premium and a deterioration of macroeconomic conditions. Gourio (2013) develops a macroeconomic framework driven by variations in disaster risk that reproduces key features of corporate bond risk premia — such as their countercyclicality — and studies their implications for business cycles. Campbell et al. (2019) show how macroeconomic dynamics drive risk premia in bond and equity markets and Corradi et al. (2013) find that the level and volatility of fluctuations in the stock market are largely explained by business cycle factors. Bekaert et al. (2009) highlight the role of uncertainty for the countercyclical volatility of asset returns. We contribute to this literature by explaining the growth asymmetry in risk premia and macroeconomic aggregates.

Our paper is also related to work that highlights the importance of beliefs about current and future TFP for fluctuations in macroeconomic and financial aggregates (e.g. Beaudry and Portier (2006), Barsky and Sims (2011), Cascaldi-Garcia and Vukotic (2019)). Görtz et al. (2019) document a close link between changes in expectations about future TFP, stock prices and risk premia. Risk premia incorporate expectations about future stock market returns and as such, they can differ from ex-post realizations. In our model, this can be the case as agents need to form nowcasts to learn about the current state of productivity. Milani (2011) highlights the relevance of expectations and learning for output fluctuations. He relaxes the rational expectations assumption to allow for agent's learning in a New Keynesian framework and estimates the model using forecast data from the SPF. Enders et al. (2017) compute GDP nowcast errors based on the SPF and show that these are sizable

[4] Jermann (1998) and Lettau and Uhlig (2000) stress that many asset pricing models which are successful in endowment economies do not generalize well to production economies.

and play a non-negligible role, accounting for up to 15% of output fluctuations.

While the above literature typically does not consider asymmetries, in our framework agents have to solve a signal extraction problem with time varying parameters to explain growth asymmetries in the data. In this respect, our work links closely to a literature that considers an asymmetric speed of learning and time variation in uncertainty (e.g. Veldkamp (2005), Boldrin and Levine (2001), Fajgelbaum et al. (2017)). Our mechanism for endogenous variations in the signal-to-noise ratio is closely related to Van Nieuwerburgh and Veldkamp (2006) and Ordoñez (2013) who employ it to explain steepness asymmetry in macroeconomic aggregates observed at business cycle frequencies. A similar mechanism is used in Saijo (2017). In this paper, agents learn about the efficiency of investments in an environment where uncertainty varies endogenously and has adverse effects on economic activity. Common across this literature is that endogenous variations in uncertainty imply state dependencies in the strength of agents' responses to shocks. While also our model relies on such a type of mechanism, the studies above use it to explain empirical facts related to macroeconomic aggregates. We add to this literature by studying asymmetries in risk premia.

The remainder of the paper is structured as follows. Section 2 provides an overview about the data. In Section 3 we provide details on the estimation of risk premium measures and document their growth asymmetry. Section 4 describes the model and Section 5 the calibration and computational details. Section 6 discusses the model mechanism that gives rise to asymmetries and results from simulations. Section 7 concludes.

2 Data

We construct measures for U.S. risk premia over a horizon from 1957Q3 to 2019Q2. For comparability with the existing literature, we follow the common practice and use the S&P 500 as a measure for equity prices and treasury yields for the risk-free rate (see e.g. Graham and Harvey (2007)). Quarterly time series for the S&P 500 index are from Robert Shiller's website. Since it has become standard in the empirical literature to estimate risk premia with a horizon of one year and shorter, we consider investment horizons of one and four

quarters.[5] Consistent with the respective horizon, we either use the 3-Month Treasury Bill rate (TB3MS) or the 1-Year Treasury Constant Maturity rate (DGS1) as measures for the risk-free rate which are obtained from the Board of Governors of the Federal Reserve System.

For the fundamentals in the regression based method to estimate risk premia, we use the cyclically adjusted price-earning Ratio (CAPE) available from Robert Shiller's website. As an alternative fundamental, we compute the cyclically adjusted price-dividend ratio (CAPD) based on data from the same source. Consistent with Shiller's cyclical adjustment to the price earnings-ratio, we compute the CAPD as the current real price of equity divided by the average of dividends over the previous ten years. The real price of equity is defined as the S&P 500 index deflated with the CPI.

The U.S. Bureau of Economic Analysis provides time series for real gross domestic product (GDPC1), real gross private domestic investment (GPDIC1), and real personal consumption expenditures (PCECC96). These series are quarterly, seasonally adjusted, and in billions of chained 2012 Dollars. Hours worked by all persons in the non-farm business sector (HOANBS) is available from the US Bureau of Labor Statistics. This source also provides a time series of civilian non-institutional population (CNP16OV) used to express the above macroeconomic aggregates in per-capita terms.

3 Empirical Evidence on Risk Premia

In this section, we estimate risk premia using a variety of models that have been found relevant in the literature. We then document that all measures for risk premia exhibit growth asymmetry.

The equity risk premium is the compensation required to make agents indifferent at the margin between investing in a risky market portfolio and a risk-free bond. Formally, the equity risk premium at time t over investment horizon k, $ERP_{t,t+k}$, is defined as the difference between the expected return on equity, $R^e_{t,t+k}$, and the risk-free rate, $R^f_{t,t+k}$, over

[5] See for example Goyal and Welch (2008), Lettau and Ludvigson (2001a), Lettau and Ludvigson (2001b).

horizon k,

$$E_t[ERP_{t,t+k}] = E_t[R^e_{t,t+k}] - R^f_{t,t+k}. \tag{1}$$

The term $R^f_{t,t+k}$, as it is risk-free, is known at time t, while the future expected performance of the stock market is not. Investors can only observe with certainty the past returns of the stock market up to time t, and can use the information available to form expectations.

To compute the risk premium in equation (1), a variety of methods have been suggested in the literature. Duarte and Rosa (2015) provide an extensive overview about the most widely used models and classify these in five categories. We will estimate risk premia based on four models for investment horizon $k = 4$ and three models for investment horizon $k = 1$ which, according to Duarte and Rosa (2015)'s classification, are part of three of these categories. The first category comprises models based on the historical mean of realized equity premia, the second includes models that employ time series regressions and the third is based on survey data. Models in these three categories have the advantage that they rely on a minimum of assumptions, and importantly, allow us to compute long time series for risk premia. In addition, Goyal and Welch (2008) and Campbell and Thompson (2008) show that models based on the historical mean of realized equity premia and based on time series regressions are hard to improve upon in terms of out-of-sample predictability. The other two methods classified by Duarte and Rosa (2015) are undoubtedly very useful in other circumstances, but have substantial drawbacks for our purposes.[6] We now provide a brief overview over the models we employ to compute risk premia.

3.1 Historical Mean of Realized Returns

This method is the most straightforward of all approaches to compute the future risk premium from time t to $t + k$. Following Goyal and Welch (2008), it is simply the historical

[6]Models based on cross-sectional regressions (see e.g. Adrian et al. (2013)) impose tight restrictions on the estimation of risk premia and results are heavily dependent on the portfolios, state variables and risk factors used (Harvey et al. (2016)). While models in our three considered categories use information in real time where investors don't have information sets that include future realizations, this method uses full-sample regression estimates which is particularly problematic in our context with a focus on asymmetries. Risk premium estimates based on dividend discount models (see e.g. Damodaran (2019)) require additional strong assumptions, for example on the computation of future expected dividends and a discount rate for these dividends.

mean of realized stock market returns in excess of the risk-free rate over H periods preceding time t. This can be formalized as

$$ERP_{t,t+k} = \frac{1}{H}\sum_{h=0}^{H}(R^e_{t-k-h,t-h} - R^f_{t-k-h,t-h}).$$

We specify $H = t - k$ as in Goyal and Welch (2008) who use systematically all the available historical data since the beginning of the sample.

The validity of this method relies on the assumption about consistent behavior between past and future. This means the mean of excess returns should either be constant or very slow moving to avoid a systematic bias in the estimates. We verify that there is no trend in realized excess stock market returns using the augmented Dickey-Fuller test (for details see Appendix A.1).

3.2 Time series regressions

This method is based on the idea to utilize the relationship between time series of economic variables and stock market returns to predict future equity returns from a linear regression. One can then subtract the contemporaneous risk-free rate to recover an estimate of the risk premium, as in Fama and French (1988), Fama and French (2002) and Campbell and Thompson (2008). We estimate the following predictive regression

$$R^e_{t,t+k} = \alpha + \beta \cdot fundamental_t + \varepsilon_t, \tag{2}$$

where $fundamental_t$ represents a variable that theory and practice have found likely to drive future excess stock returns. This method links as directly as possible to equation (1) by computing the equity risk premium

$$E_t[ERP_{t,t+k}] = \hat{\alpha} + \hat{\beta} \cdot fundamental_t - R^f_{t,t+k}, \tag{3}$$

based on the estimates $\hat{\alpha}$ and $\hat{\beta}$ for α and β. Generally the literature relies on a single fundamental in this regression, as using several variables at once has been found to reduce model's out-of-sample accuracy. The fundamental used is typically a valuation ratio such as the price-dividend ratio or the price-earning ratio. These valuation ratios are known to be negatively correlated with future stock returns since the works of Rozeff (1984), Campbell and Shiller (1988a), and Campbell and Shiller (1988b).

We compute risk premia from two different models based on the above time series regressions and follow the detailed methodology in Campbell and Thompson (2008). The models differ in the variable used as fundamental, where we either employ Shiller's cyclically adjusted price-earning ratio (CAPE) or the cyclically adjusted price-dividend ratio (CAPD). For each quarter t in our sample, we estimate parameters in equation (2) based on a sample up to time $t-1$. The risk premium is then constructed according to equation (3) using an out-of-sample forecast. To estimate α and β we use a sample that begins 20 years prior to 1957Q3.[7] We further implement the two restrictions suggested by Campbell and Thompson (2008), i.e. $\hat{\beta}$ must have the sign predicted by theory, otherwise it is replaced by zero, and the predicted risk premium must be positive, otherwise the historical mean is used as a predictor instead. Out-of-sample forecasts are produced for each quarter t from 1957Q3 to 2019Q2.

3.3 Survey based risk premium measures

The third method we consider to derive a measure for the risk premium is based on survey data. The Duke CFO Global Business Outlook Survey is the longest ongoing survey about the expected equity return (conducted quarterly since 2000Q2) in the United States.[8] Graham and Harvey (2018) then recover the 10-year ahead expected risk premium by subtracting the known risk-free Treasury bond annual yield to the median forecast of future

[7] Our results are robust also to using a sample beginning in 1881Q1, when both fundamentals are first available.

[8] Every quarter, on average about 350 Chief Financial Officers from a sample of representative US firms respond to the following question: "The current annual yield on a 10-year Treasury bond is x%. Please complete the following: Over the next 10 years, I expect the average annual S&P 500 return will be: ...%". Here x% is replaced by the the actual yield on a 10-year Treasury bond at the time of the survey. A corresponding question is asked for a one-year investment horizon.

S&P 500 annual returns. Since 2004Q1, the survey also includes a question on the expected return of the S&P 500 over the next year. We use the responses to this question to compute, analogously to Graham and Harvey (2018), the expected risk premium for an investment horizon of one year. Responses and questions based upon which we could construct risk premia with an investment horizon of one quarter are not available in this survey.

3.4 Asymmetries in Risk Premia

In this section, we show skewness statistics for the growth rate of the risk premium measures described above. In particular, we report results based on a model that relies on the historical mean of realized returns, results based on two time series regression models (using either the cyclically adjusted price-dividend or the price-earning ratio as fundamental), and results based on survey evidence. Table 1 summarizes results based on each of the four models for an investment horizon of one year ($k = 4$). All four risk premium measures exhibit growth asymmetry which manifests in positively skewed distributions. Over the entire sample (1957Q3 - 2019Q2) the growth in the risk premium based on the historical average method and the two time series regression models has a skewness of 2.55, 0.15 and 0.14, respectively. The positive skewness implies that the risk premium exhibits growth asymmetry: it declines gradually and rises much more sharply. This result is robust also when considering parts of our sample. Table 1 shows skewness statistics for subsamples designed to cover two business cycles from peak to peak as defined by the NBER's Business Cycle Dating Committee. It is evident that the vast majority of risk premia also exhibit a positive skewness over these subsamples. The risk premium measure based on survey evidence covers a much shorter sample, starting in 2004Q1. Nonetheless, its use is appealing to confirm our results since this measure is based on a very different methodology. Over the available sample, the survey based measure exhibits a skewness of 0.80 and hence also provides evidence for steepness asymmetry in the risk premium.

Next, we discuss skewness statistics at an investment horizon of one quarter ($k = 1$) for the three risk premium measures based on the historical average and time series regressions.[9]

[9]The Duke CFO Global Business Outlook Survey does not include a question that corresponds to the

These are summarized in Table 2, where we again provide statistics over the entire sample as well as subsamples. Also results at the one quarter investment horizon document positive skewness over the entire sample and the majority of subsamples. While qualitatively consistent, quantitatively the degree of skewness varies considerably across measures. The risk premium based on the historical average method implies a skewness of 0.12 while the measures based on time series exhibit skewness of 0.25 and 1.12. These quantitative differences are not surprising — and consistent with findings for first and second moments in the literature (see e.g. Duarte and Rosa (2015)) — in light of the substantial diversity in assumptions and the underlying methodologies to derive the risk premium measures. Given this, it is striking that all considered measures feature positive skewness. Overall, this section provides broad evidence that the risk premium — measured in a variety of ways and at different investment horizons — exhibits growth asymmetries: declines are long and gradual and rises are sharp and short.

4 The model

The core of our model is a representative agent real business cycle (RBC) model which is extended with two key mechanisms. Firstly, households have recursive preferences of the Epstein and Zin (1989) type. It is well known that standard RBC models with Arrow-Pratt preferences and a reasonable degree of relative risk aversion (RRA) fail to account for the existence of risk premia. This is due to the fact that the intertemporal elasticity of substitution (EIS) and the RRA are reciprocal of each other. A small EIS of the magnitude necessary to justify meaningful risk premia necessarily leads to an excessively large RRA. Recursive preferences separate the RRA and the EIS. Secondly, agents cannot directly observe productivity. Instead, they receive a noisy signal about previous period's productivity and use a Bayesian learning technology to form nowcasts. Agent's varying speed of learning over the business cycle is the key to match empirically observed asymmetries in risk premia

one quarter investment horizon. Based on this survey, Graham and Harvey (2018) provide a risk premium measure for a 10 year investment horizon though. Skewness for growth in this measure is 0.151 over a 2000Q2-2019Q2 sample.

Table 1: Skewness statistics for growth in different measures of risk premia based on an investment horizon of one year

	Time series (fundamental = CAPE)	Time series (fundamental = CAPD)
1957Q3 - 2019Q2	0.15 (0.324)	0.14 (0.367)
1957Q3 - 1969Q4	1.64 (0.000)	1.99 (0.000)
1969Q4 - 1980Q1	0.08 (0.804)	0.65 (0.080)
1980Q1 - 1990Q3	0.39 (0.251)	0.13 (0.704)
1990Q3 - 2007Q4	-2.41 (0.000)	-1.34 (0.000)
2007Q4 - 2019Q2	1.85 (0.000)	2.51 (0.000)
	Historical average	
1957Q3 - 2019Q2	2.55 (0.000)	
1957Q3 - 1969Q4	-1.98 (0.000)	
1969Q4 - 1980Q1	1.89 (0.000)	
1980Q1 - 1990Q3	-0.01 (0.968)	
1990Q3 - 2007Q4	1.20 (0.000)	
2007Q4 - 2019Q2	3.12 (0.000)	
	Survey	
2004Q1 - 2019Q2	0.80 (0.012)	
2007Q4 - 2019Q2	0.71 (0.042)	

Notes. 1957Q3-2019Q2 is the full sample for the historical mean and time series methods. Smaller sub-samples are constructed such as to cover two peak-to-peak cycles each as defined by the NBER's Business Cycle Dating Committee, with the exception of the last sub-sample that covers the time from the most recent peak. Survey results are available only from 2004Q1 to 2019Q2. "Historical average", refers to the expectations obtained using the historical average method. "Time series (fundamental = CAPE)" and "Time series (fundamental = CAPD)" refer to the expectations obtained using the time series regression method, using the CAPE and CAPD ratios respectively as fundamentals. "Survey" refers to a risk premium measure based on the Duke CFO Global Business Outlook Survey. Skewness statistics are calculated from the first difference of the logarithm of the risk premium. P-values, in parenthesis, are based on D'Agostino, Belanger and D'Agostino (1990) and Royston (1991) test statistics.

Table 2: Skewness statistics for growth in different measures of risk premia based on an investment horizon of one quarter

	Time series (fundamental = CAPE)	Time series (fundamental = CAPD)
1957Q3 - 2019Q2	0.25 (0.104)	1.12 (0.000)
1957Q3 - 1969Q4	0.07 (0.821)	0.05 (0.886)
1969Q4 - 1980Q1	0.23 (0.495)	1.62 (0.000)
1980Q1 - 1990Q3	0.68 (0.053)	0.27 (0.426)
1990Q3 - 2007Q4	2.16 (0.000)	1.63 (0.000)
2007Q4 - 2019Q2	2.88 (0.000)	3.10 (0.000)
	Historical average	
1957Q3 - 2019Q2	0.12 (0.042)	
1957Q3 - 1969Q4	-0.17 (0.596)	
1969Q4 - 1980Q1	0.72 (0.043)	
1980Q1 - 1990Q3	0.26 (0.430)	
1990Q3 - 2007Q4	0.32 (0.237)	
2007Q4 - 2019Q2	-0.14 (0.661)	

Notes. 1957Q3-2019Q2 is the full sample for the historical mean and time series methods. Smaller sub-samples are constructed such as to cover two peak-to-peak cycles each as defined by the NBER's Business Cycle Dating Committee, with the exception of the last sub-sample that covers the time from the most recent peak. "Historical average", refers to the expectations obtained using the historical average method. "Time series (fundamental = CAPE)" and "Time series (fundamental = CAPD)" refer to the expectations obtained using the time series regression method, using the CAPE and CAPD ratios respectively as fundamentals. Survey results are not available for this horizon. Skewness statistics are calculated from the first difference of the logarithm of the risk premium. P-values, in parenthesis, are based on D'Agostino, Belanger and D'Agostino (1990) and Royston (1991) test statistics.

and macroeconomic variables.

4.1 Production and technology

The economy comprises of a continuum of perfectly competitive identical firms with unit mass. Firms use the following Cobb-Douglas production function to produce output, y_t,

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha} + \nu_t, \qquad 0 < \alpha < 1, \tag{4}$$

by employing capital, k_t, and labour, l_t. Output further depends on a productivity shock, A_t, and an additive noise shock, ν_t. This production function is based on Van Nieuwerburgh and Veldkamp (2006) and is consistent with the notion in Faust et al. (2005) who characterize the preliminary GDP announcement of statistical agencies as the sum of a final GDP announcement and a noise term. The productivity shock takes the form of a Markov process with two states, high and low $A_t = \{A_t^H, A_t^L\} \; \forall \; t$, and a standard deviation σ_A. The Markov chain is ergodic and has a symmetric transition matrix, Π, to ensure any asymmetry in the resulting model dynamics is endogenous. The noise shock is independent and identically normal distributed with zero mean and standard deviation σ_ν.

The assumptions about agent's information set are such that — even though they know the underlying shock processes — they cannot separately observe the productivity and noise shock. Further, agents make decisions about production inputs before they know the level of output since both shocks are realized only at the end of each period. To make an informed decision about production inputs, agents use a Bayesian learning technology to infer the level of current period's productivity based on their noisy observation of output in the previous period.[10]

Both, firms as well as households have the same belief about current productivity since all agents have the same information set and have access to the same Bayesian updating

[10] These timing assumptions are consistent with nowcasting in public policy institutions. Bok et al. (2017) document that the New York Fed Staff Nowcast for GDP on the last quarter is only observable at about the beginning of the next quarter. They also describe that nowcasts and forecasts are based on surveys and limited number of reporting units, i.e. they filter.

technology. In the following sections, we will discuss optimal decision making of firms and households, given their beliefs about productivity, and show how these agents employ the Bayesian learning technology to update their beliefs.

4.2 Firms

Firms enter the period with knowledge about their capital stock. They use Bayesian updating, to be described in detail below, to form a belief about productivity at the beginning of the period. Given this information, firms decide about labor demand and investment, where the latter determines next period's capital stock. Firms own the capital stock, rather than rent it from households, but issue shares and pay out dividends.

At the beginning of the period, after firms have formed a belief about productivity, they expect cash flow, $\tilde{f}_t$, to be

$$\tilde{f}_t = \tilde{A}_t k_t^{\alpha} l_t^{d^{1-\alpha}} + \tilde{\nu}_t - w_t l_t^d - i_t,$$

where $\tilde{A}_t$ denotes the beliefs about productivity and $\tilde{\nu}_t$ the belief about the noise. Following the discussion in the section above, agent's expectation about the noise, $\tilde{\nu}_t$, is zero. w_t denotes the real wage, l_t^d stands for labor demand, and i_t for investment. In general, notation $\tilde{x}_t$ indicates agent's belief about a particular variable. This belief is formed at the beginning of the current period, t, given the information set at the beginning of the current period, $\mathcal{I}_t$, such that $\tilde{x}_t = \mathbb{E}_t[x_t \mid \mathcal{I}_t]$. Then, x_t denotes the realization of this variable at the end of period t.

Firms have to respect their investment financing constraint

$$i_t = \tilde{y}_t - w_t l_t^d - \tilde{d}_t s_t^s + p_t(s_{t+1}^s - s_t^s), \tag{5}$$

where the difference between s_{t+1}^s and s_t^s represents the supplied number of shares to be traded at price p_t between firms and households. Expected dividends, $\tilde{d}_t$, communicated to

the households at the beginning of the period, are given by

$$\tilde{d}_t = \frac{\tilde{y}_t - w_t l_t^d - i_t + p_t(s_{t+1}^s - s_t^s)}{s_t^s}. \tag{6}$$

Actual dividends are paid out at the end of the period and will absorb the effects of incorrect beliefs and balance out the investment financing constraint. Note that realized cash flow,

$$f_t = A_t k_t^\alpha l_t^{d^{1-\alpha}} + \nu_t - w_t l_t^d - i_t,$$

will differ from expected cash flow most of the time as they include realized productivity as well as the realization of the noise term.

The law of motion for capital is

$$k_{t+1} = \left[(1-\delta) + \Phi\left(\frac{i_t}{k_t}\right)\right] k_t, \tag{7}$$

where δ is the depreciation rate and the capital adjustment cost function $\Phi\left(\frac{i_t}{k_t}\right)$ is positive and concave. The concavity implies that large changes in the investment ratio are more expensive than gradual adjustments. As in Hayashi (1982) and Jermann (1998), the adjustment cost has the functional form

$$\Phi\left(\frac{i_t}{k_t}\right) = \frac{a_1}{1-\chi}\left(\frac{i_t}{k_t}\right)^{1-\chi} + a_2, \qquad \chi > 1,$$

where χ is the elasticity of the investment ratio with respect to Tobin's q and parameters a_1 and a_2 ensure costs are zero in the steady state. The use of these capital adjustment costs allows us to derive the expression for the return on equity shown as shown in Appendix B.2.

Firms maximize their value, which is equivalent to the sum of discounted expected cash flow

$$\max_{l_t^d, i_t, k_{t+1}} \mathbb{E}_t \left[\sum_{j=0}^{+\infty} m_{t,t+j} \left(A_{t+j} k_{t+j}^\alpha l^{d}{}_{t+j}^{1-\alpha} - w_{t+j} l_{t+j}^d - i_{t+j} \right) \middle| \mathcal{I}_t \right], \tag{8}$$

where $m_{t,t+j}$ is the household's discount factor to be specified in the next section. We maxi-

mize equation (8) with respect to i_t, k_{t+1} and l_t^d subject to equation (7) and the constraints $i_t \geq 0$, $k_t \geq 0$ to obtain the first order conditions

$$q_t = \frac{1}{\Phi'(i_t/k_t)}, \tag{9}$$

$$q_t = \mathbb{E}_t \left\{ m_{t,t+1} \left[A_{t+1} l_{t+1}^{d\,1-\alpha} \alpha k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right] \middle| \mathcal{I}_t \right\}, \tag{10}$$

$$w_t = (1-\alpha) \tilde{A}_t l_t^{d-\alpha} k_t^{\alpha}, \tag{11}$$

where q_t denotes the Lagrange multiplier and can be interpreted as Tobin's q. Equation (9) determines the real price of investment and equation (10) determines optimal investment. The labor supply function (11) states that the real wage is equal to the expected marginal productivity of labour, since the actual marginal productivity is unobservable.

4.3 Households

There is a continuum of identical households with unit mass. At the beginning of each period, households decide how much labor to supply and how many shares to buy. Based on their expected cash flow, firms also inform households on the amount of dividends, $\tilde{d}_t$, they expect to pay. When firms observe their realized cash flow at the end of the period, they pay dividends, d_t, which may differ from the expected dividends. At this point, households update their views about their income which they subsequently use for consumption, c_t. In other words, consumption expenditures absorb any unexpected realizations due to incorrect beliefs to satisfy the households' budget at the end of the period. At the beginning of the period, the households' expected budget constraint is

$$\tilde{c}_t + p_t(s_{t+1}^d - s_t^d) = w_t l_t^s + \tilde{d}_t s_t^d, \tag{12}$$

where $\tilde{c}_t$ is the expected consumption level, labor supply is l_t^s, and the difference between s_{t+1}^d and s_t^d represents the demand for the number of new shares.

Households have preferences as in Epstein and Zin (1989) so that recursive utility is a

CES aggregate of their period utility function and a certainty equivalent for next period utility,

$$U_t = \left[(1-\beta) u_t(\tilde{c}_t, l_t^s)^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \mid \mathcal{I}_t \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{13}$$

where $\psi > 1$ is the elasticity of inter-temporal substitution, $\gamma \in [0, +\infty) \setminus \{1\}$ is the relative risk aversion, and $\beta \in (0,1)$ is the discount factor. Period utility takes the form

$$u_t(\tilde{c}_t, l_t^s) = \tilde{c}_t^{\kappa} (1 - l_t^s)^{1-\kappa}, \tag{14}$$

with $\kappa \in (0,1)$ which controls labor supply.

Households maximize equation (13) subject to (12) and to the interiority conditions $\tilde{c}_t \geq 0$, $c_t \geq 0$ and $0 \leq l_t^s \leq 1$. We obtain the following labor supply function from the household's maximization problem (details are provided in Appendix B.1)

$$\tilde{c}_t = \frac{\kappa}{1-\kappa} (1 - l_t^s) w_t, \tag{15}$$

which provides an intratemporal link between labor supply, the real wage and the beginning of period belief about consumption. Combining the first order condition with respect to s_{t+1}^d and the Envelope Theorem for s_t^d (shown in Appendix B.1) we obtain the Lucas equation

$$1 = \mathbb{E}_t \left[m_{t,t+1} \frac{d_{t+1} + p_{t+1}}{p_t} \,\middle|\, \mathcal{I}_t \right], \tag{16}$$

where

$$m_{t,t+1} = \beta \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \mid \mathcal{I}_t \right]} \right)^{1-\frac{1}{\theta}} \left(\frac{c_{t+1}}{\tilde{c}_t} \right)^{\frac{\kappa(1-\gamma)}{\theta} - 1} \left(\frac{1 - l_{t+1}^s}{1 - l_t^s} \right)^{\frac{(1-\gamma)(1-\kappa)}{\theta}}, \tag{17}$$

is the stochastic discount factor, using $\theta := (1-\gamma)/(1-\frac{1}{\psi})$. The risk-free rate between period t and $t+1$ is thus defined as

$$R_{t,t+1}^f = \frac{1}{\mathbb{E}_t[m_{t+1,t} \mid \mathcal{I}_t]}, \tag{18}$$

and the expected return on equity between period t and $t+1$ is

$$\mathbb{E}_t[R^e_{t,t+1} \mid \mathcal{I}_t] = \mathbb{E}_t\left[\frac{d_{t+1}+p_{t+1}}{p_t} \,\middle|\, \mathcal{I}_t\right]. \tag{19}$$

Then the expected risk premium is given by

$$\mathbb{E}_t[ERP_{t,t+1} \mid \mathcal{I}_t] = \mathbb{E}_t[R^e_{t,t+1} \mid \mathcal{I}_t] - R^f_{t,t+1}.$$

Note that, as for example in Heer and Maußner (2012), we do not explicitly take into account equation (5) in the maximization programme of the representative firm. This is because irrespective of the choice of labour and investment, it is always possible to find a combination of dividends and number of shares that satisfies equation (5). Since we also do not impose a specific dividend policy, we cannot directly compute the return on equity based on dividends and share prices. However, as shown in Appendix B.2, we can recover the expected return on equity to be

$$\mathbb{E}_t\left[\frac{d_{t+1}+p_{t+1}}{p_t} \,\middle|\, \mathcal{I}_t\right] = \mathbb{E}_t\left[\frac{q_{t+1}k_{t+2}+y_{t+1}-w_{t+1}l_{t+1}-i_{t+1}}{q_t k_{t+1}} \,\middle|\, \mathcal{I}_t\right]. \tag{20}$$

Variables on the right hand side of this equation can be recovered using household's and firm's programmes, given expectations about future productivity. We will discuss in the next section how these expectations about productivity can be formed. Using the right hand side of equation (20) to compute the expected return on equity has the advantage that it limits the state space of the dynamic programming problem and thereby keeps our computational problem tractable.

4.4 Bayesian learning

We now turn to a description of the Bayesian learning mechanism which agents use to form a belief about current technology, $\tilde{A}_t$. Information set $\mathcal{I}_t$ contains all information

available to the agents at the beginning of period t

$$\mathcal{I}_t := \{y^{t-1}, c^{t-1}, d^{t-1}, p^t, w^t, l^{d^t}, l^{st}, i^t, k^t, s^{d^{t+1}}, s^{st+1}\},$$

where x^t denotes the history of variable x up to time t. The technology and the noise shocks are never individually observed, but agents have information about their underlying processes. This includes the transition matrix, Π, which consists of the probabilities of a state change as detailed in Section 4.1.

Agents use the following Bayesian filter to forecast A_t given information set $\mathcal{I}_t$

$$P(A_{t-1} = A^H \mid \mathcal{I}_t) = \frac{\phi(y_{t-1} \mid A^H, \mathcal{I}_{t-1}) P(A_{t-1} = A^H \mid \mathcal{I}_{t-1})}{\phi(y_{t-1} \mid A^H, \mathcal{I}_{t-1}) P(A_{t-1} = A^H \mid \mathcal{I}_{t-1}) + \phi(y_{t-1} \mid A^L, \mathcal{I}_{t-1}) P(A_{t-1} = A^L \mid \mathcal{I}_{t-1})}, \quad (21)$$

$$[P(A_t = A^H \mid \mathcal{I}_t), P(A_t = A^L \mid \mathcal{I}_t)] = [P(A_{t-1} = A^H \mid \mathcal{I}_t), P(A_{t-1} = A^L \mid \mathcal{I}_t)]\mathbf{\Pi}. \quad (22)$$

This filter comprises a Bayesian updating formula, equation (21), and an adjustment for the possibility of a state change, equation (22), where ϕ is the normal probability density function. In equation (21) Bayes' law gives the posterior probability at time t for productivity to be in a high state in the previous period. The reciprocal posterior probability for a low state, $P(A_{t-1} = A^L | \mathcal{I}_t)$, is obtained analogously. Then, agents adjust for the possibility of a state change from period $t-1$ to t using equation (22) by multiplying the vector of posterior probabilities with the transition matrix, to obtain a prior belief about the current state of productivity. Agents can subsequently form a belief about the productivity level in the current period by multiplying the vector of priors with the vector of productivity states

$$\tilde{A}_t = [P(A_t = A^H \mid \mathcal{I}_t), P(A_t = A^L \mid \mathcal{I}_t)][A^H, A^L]'. \quad (23)$$

Note that for agents to compute the risk-free rate and the return on equity they need to form expectations about several variables in period $t+1$. To do so, they need to estimate the probability that productivity will be in the high or low state in $t+1$, given their beliefs

about the current state of productivity, $P(A_t = A^H \mid \mathcal{I}_t)$ and $P(A_t = A^L \mid \mathcal{I}_t)$. Then they multiply the vector of prior probabilities with the transition matrix,

$$[P(A_{t+1} = A^H \mid \mathcal{I}_t), P(A_{t+1} = A^L \mid \mathcal{I}_t)] = [P(A_t = A^H \mid \mathcal{I}_t), P(A_t = A^L \mid \mathcal{I}_t)]\mathbf{\Pi}, \qquad (24)$$

so that agents employ part of the learning technology analogously to the case described above.

4.5 Equilibrium and social planner problem

Equilibrium. At the end of each period the equilibrium in the decentralized economy presented above is a sequence of quantities $\{c_t, l_t^s, l_t^d, i_t, d_t, k_t, s_t^d, s_t^d\}_{t=0}^{\infty}$ and prices $\{w_t, p_t\}_{t=0}^{\infty}$, given k_0, s_0, and A_0, such that the problem of firms is solved, the problem of households is solved, the markets for goods, labour and firm's shares clear

$$y_t = i_t + c_t, \qquad l_t^s = l_t^d = l_t, \qquad s_t^s = s_t^d = s_t.$$

Social Planner Problem. The decentralized economy has a social planner analogue which can be solved in a recursive fashion. At the beginning of each period the planner maximizes the utility of the representative household, equation (13), subject to the capital accumulation constraint (7), the aggregate resource constraint $\tilde{y}_t = i_t + \tilde{c}_t$ (which is the combination of the households' budget constraint (12) and the firms' investment financing constraint (5)), and the interiority conditions $c_t \geq 0$, $\tilde{c}_t \geq 0$, $0 \leq l_t \leq 1$, $k_t \geq 0$ and $i_t \geq 0$.

The benevolent planner enters the period with knowledge about two state variables: the capital stock, k_t, and a belief about current period's productivity, $\tilde{A}_t$. The belief is established by using the Bayesian updating mechanism in equations (21)-(23). Given these state variables, the planner chooses hours worked, l_t, and investment, i_t, which then implies beliefs for the levels of output, $\tilde{y}_t$, and consumption, $\tilde{c}_t$. The planner uses this information together with the technology (24) to derive the risk-free rate, the expected return on equity and subsequently the risk premium. Then, the actual productivity shock A_t and the noise

ν_t are realized, but not observed separately. The realization of these shocks implies that the planner can observe the actual level of output, y_t, which will typically differ from the belief about output, $\tilde{y}_t$. Subsequently, actual consumption, c_t is realized as a residual.

Formally, the planner solves the following Bellman equation, where V denotes the value function:

$$V(k_t, \tilde{A}_t) = \max_{l_t, i_t, k_{t+1}} \left[(1-\beta)(\tilde{c}_t^{\kappa}(1-l_t)^{1-\kappa})^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t V^{1-\gamma}(k_{t+1}, \tilde{A}_{t+1} \mid \mathcal{I}_t))^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}$$

$$\text{s.t. } k_{t+1} = \left(1 - \delta + \Phi\left(\frac{i_t}{k_t}\right)\right) k_t,$$

$$\tilde{c}_t = \tilde{y}_t - i_t,$$

$$i_t \geq 0, \quad k_t \geq 0, \quad \tilde{c}_t \geq 0, \quad c_t \geq 0, \quad 0 \leq l_t \leq 1, \quad \text{and } k_0,\ A_0 \text{ given},$$

where

$$\Phi\left(\frac{i_t}{k_t}\right) = \frac{b_1}{1-\kappa}\left(\frac{i_t}{k_t}\right)^{1-\kappa} + b_2 \qquad \text{and} \qquad \tilde{y}_t = \tilde{A}_t k_t^{\alpha} l_t^{1-\alpha} + \tilde{\nu},$$

and the updating rules (21)-(24) are taken as given.

The social planner equilibrium is achievable in the decentralized economy since the planner uses information that is available to all agents at no cost, the constraints and first order conditions of the planner are consistent with those of the agents, technology is convex and the preferences are insatiable.[11]

[11] It is important to note that the formulation with a social planning economy rules out agent's active experimentation. In our setup there is no feedback between actions and beliefs and learning is passive. This is a common assumption in the literature, see e.g. Van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013) and Saijo (2017). Active learning would invalidate the Welfare Theorems in the social planning economy and hence there would be no decentralized counterpart to the planner's equilibrium. The passive learning is reflected in the planner's recursive problem above: the state variables, including beliefs about productivity, are determined before optimal production decisions are made, after which subsequently beliefs are updated again. This process can be repeated until beliefs about productivity coincide with its actual realization.

5 Calibration and computation

5.1 Calibration

Table 3 summarizes the parameter values used to calibrate the model. Consistent with the empirical sections above the model is calibrated at quarterly frequency. Several values are standard in the literature. We calibrate the share of capital in production, $\alpha = 0.36$, the discount factor, $\beta = 0.98$, and the capital depreciation rate, $\delta = 0.025$ (see e.g. Kydland and Prescott (1982)). We set the steady state labour supply to 1/3 which then implies $\kappa = 0.37$ for equation (15) to hold in steady state. The capital adjustment cost parameter is set to $\chi = 4$, consistent with the value in Jermann (1998). The two parameters related to the adjustment costs, a_1 and a_2, ensure zero capital adjustment costs in steady state and can be expressed as functions of other parameters (derivations of their functional forms are shown in Appendix B.3).

Given these parameters, we calibrate the elasticity of inter-temporal substitution, ψ, to be 0.01, so that the mean risk premium in the model matches the equivalent moment in the data. This calibration is also consistent with the empirical estimates in Yogo (2004) and Gomes and Paz (2011) for the elasticity of inter-temporal substitution. We use the risk premium based on the historical average measure as a benchmark to calibrate our model as it relies on a minimum of assumptions while at the same time we observe a long time series. We consider this measure at an investment horizon of one quarter, which is consistent with the setup of our model. While the model is calibrated to match the level of the mean risk premium in the data (0.063 vs. 0.069), it is reassuring that given the above parameters the model also delivers levels for the expected return on equity (0.103 vs. 0.088) and the risk-free rate (0.042 vs. 0.045) that are comparable to their data equivalents.[12] Based on Caldara et al. (2012), we calibrate the degree of relative risk aversion, γ, to be 5 as our benchmark, which is also in line with the value used in Gourio (2012). Empirical evidence on the degree of relative risk aversion is scarce. For robustness we verify $\gamma \in \{1, 10\}$ which

[12] The difference between statistics for the expected return on equity and the risk-free rate do not exactly match the ones provided for the risk premium. The reason is that the risk premium is computed for every period before the average is taken across all simulations.

does not significantly alter our results.[13]

The model's learning technology relies on three parameters that require calibrating. We set the states of the two-state Markov chain to be $A^H = 1 + 0.032$ and $A^L = 1 - 0.032$ so that the standard deviation of the technology process, σ_A, is consistent with the findings in Cooley and Prescott (1995) and Fernald (2019) based on estimates of Solow residuals. Note that the distance between the two states matters for the volatility of the process, but since we evaluate deviations from the steady state, the absolute level of the technology process is not important. Let p_{ij} denote the probability for a change from state $i = \{H, L\}$ to state $j = \{H, L\}$, then the ergodicity of the Markov chain implies $p_{ij} \in (0, 1)$ and $p_{iH} + p_{iL} = 1$. In combination with the symmetry assumption on the transition matrix this implies $p_{LH} = p_{HL}$ and $p_{HH} = p_{LL} = (1 - p_{LH})$. Hence, the autocorrelation for technology can be pinned down by the probability of a state change, p_{LH}, which we set to 0.05. This implies an auto-correlation for productivity of 0.95, which is consistent with the estimate in Cooley and Prescott (1995), and gives an autocorrelation for output in the model (0.93) that is in line with the corresponding statistic in the data (0.84). Finally, we calibrate the standard deviation of the noise shock to be $\sigma_\nu = 0.01$, so that our model matches the negative correlation between the median absolute nowcast error for GDP growth and real GDP growth in the data.[14] The variance of the noise shock affects the signal-to-noise ratio and thereby determines the speed of learning. If the volatility of the noise shock is too large, it becomes impossible to extract any information from the signal received. If the volatility of the noise shock is too small, it becomes straightforward to infer real productivity and learning is trivial. Our value for σ_ν is between these extreme cases so that learning is neither impossible nor trivial.

[13] These results are available upon request.

[14] The nowcast data is from the Survey of Professional Forecasters (SPF). The SPF provides quarterly nowcasts over a horizon 1968Q4-2019Q2. The nowcasts are on GNP growth, up to 1991Q4, and GDP growth, from 1992Q1. Throughout the paper we compute nowcast errors using the corresponding series for GNP and GDP growth from the Bureau of Economic Analysis.

Table 3: Calibrated Parameters

Description	Parameter	Value
Income share of capital	α	0.36
Discount factor	β	0.98
Depreciation rate of capital	δ	0.025
Probability of state change in transition matrix	p_{LH}	0.05
Standard deviation of productivity shock	σ_A	0.032
Standard deviation of noise shock	σ_ν	0.01
Relative risk aversion	γ	5
Elasticity of inter-temporal substitution	ψ	0.01
Capital adjustment cost parameter	χ	4
Period utility parameter	κ	0.37

5.2 Computational details

We solve the model using Value Function Iteration. Epstein and Zin (1989) show that a version of the contraction mapping theorem still holds with recursive preferences. The algorithm requires the choice of two grids, for hours and for capital. We use 1000 grid points for capital and 500 grid points for hours. The upper and lower bounds of the grids are equal to 125% and 75% of the respective steady state values of the variables. These values ensure that the choices of the representative agent are not constrained by the boundaries, while maintaining a high grid density for precision of the solution. During simulations we do not visit the grid points at the boundaries of the state space. Consumption and the belief about consumption do not require a specific grid, as their values can be recovered using the grids for capital and labour. We use the policy functions to simulate 500 time series of 248 quarters after 50 periods are discarded. This is consistent with the length of the time horizon in the empirical sections above.

6 Results

6.1 Nowcasting, uncertainty and asymmetry

We have documented in Section 3.4 that there is substantial growth asymmetry across a variety of measures for the risk premium. In this section, we show that our model can resemble this asymmetry due to endogenous variations in agent's nowcasting precision about productivity.[15] The key for this mechanism is the formulation for output (4), which consists of the product of TFP and the function of production inputs, as well as the additive noise term. Agents employ output realized at the end of the previous period in the Bayesian learning technology to infer the current state of productivity. When production inputs are low, agents learn slowly about productivity because the noise variance is relatively large in comparison to the variance of the signal. A recession is hence a time of high uncertainty and low nowcast accuracy. During a recovery, intensified use of production inputs amplifies changes in technology so that the variance of the signal increases. Given our assumption of a constant noise variance this implies a rising signal-to-noise ratio during a recovery. This decline in uncertainty raises nowcast precision so that agent's speed of learning increases with output and the risk premium declines gradually. At the peak, a situation of low uncertainty and high output, a decline in the state of productivity can be observed relatively precisely. The result is a strong negative adjustment in production inputs, an increase in uncertainty, and a sharp rise in the risk premium. Hence, procyclical fluctuations in the signal-to-noise ratio lead to endogenous variations in nowcasting accuracy which generate asymmetries in the risk premium and the other macroeconomic aggregates.

Important for the functionality of the learning mechanism is a procyclical signal-to-noise ratio resulting from endogenous variations in nowcast accuracy and the degree of uncertainty.[16] Empirical evidence supports this model mechanism. We employ the median

[15] Enders et al. (2017) find that productivity shocks have a statistically and economically significant impact on nowcast errors and report evidence for Granger causality. They also investigate potential links between a variety of other non-technology shocks and nowcast errors, but cannot find significant effects of such shocks on nowcast accuracy.

[16] In Section 4.1 we assumed the variance of the noise to be constant. This assumption has been made for simplicity to keep the computational problem tractable. In principle, we can relax this assumption so that the noise variance can even rise when the use of production inputs increases. As long as it rises at a rate

Table 4: Nowcast accuracy, economic contractions and surges in risk premia

	Negative GDP growth dummy	GDP growth rate	Positive risk premium growth dummy
$\hat{\alpha}$	1.436 (0.000)	2.160 (0.000)	0.753 (0.034)
$\hat{\beta}$	2.537 (0.000)	-0.177 (0.000)	1.638 (0.000)
Adjusted R^2	0.23	0.09	0.02

Results of the time series regression $y_t = \alpha + \beta x_t + \varepsilon$ where y_t is the median absolute nowcast error for real GDP growth from the Survey of Professional Forecasters (SPF), and x_t is either the quarter-on-quarter growth rate of real GDP, or a dummy variable equal to one when the quarter-on-quarter growth of real GDP is negative, or a dummy variable equal to one when the quarter-on-quarter growth rate of risk premium exceeds 2%. The sample is limited to 1968Q4-2019Q2 by the availability of the SPF. P-values are reported in parenthesis.

absolute nowcast error for GDP growth from the Survey of Professional Forecasters (SPF) as a measure for nowcast accuracy and hence the speed of learning. Table 4 shows results from a regression of this median absolute nowcast error on either GDP growth or a dummy indicating a contraction of the economy. We find a positive relationship between GDP growth and nowcast accuracy and we document that nowcast errors are particularly large during contractions. Our results based on nowcasting accuracy are consistent with findings in the literature on procyclical forecast precision, see e.g. evidence in Jaimovich and Rebelo (2009) based on the Livingston Survey. Table 4 also provides evidence on the relationship between nowcast accuracy and growth of expected risk premia. We regress the median absolute nowcast error for GDP growth on a dummy for large increases in risk premia — indicating a quarter-on-quarter growth rate of expected risk premia of at least to 2%.[17] The result of this regression indicates that surges in risk premia coincide with times of a slow speed of learning.

Next, we turn to regressions where we employ the dispersion of nowcasts for GDP growth from the SPF as dependent variable. Dispersion is defined as the difference between the 75th and the 25th percentile of the projections for quarter-on-quarter growth. Disagreement of

less than $k_t^{\alpha} l_t^{1-\alpha}$, this still guarantees a procyclical signal-to-noise ratio.

[17] This is a conservative classification for a period to exhibit a large increase in the risk premium. The dummy is one in 71 out of a total of 203 quarters. Results in Tables 4 and 5 are robust also when we apply a tighter threshold that includes surges in risk premia above about 4%. This implies the dummy is unity in 25 periods which is the same number of quarters covered by the dummy indicating a contraction in GDP.

Table 5: Uncertainty, economic contractions and surges in risk premia

	Negative GDP growth dummy	GDP growth rate	Positive risk premium growth dummy
$\hat{\alpha}$	1.267 (0.000)	1.570 (0.000)	1.352 (0.000)
$\hat{\beta}$	1.139 (0.000)	-0.069 (0.000)	0.369 (0.034)
Adjusted R^2	0.19	0.05	0.02

Results of the time series regression $y_t = \alpha + \beta x_t + \varepsilon$ where y_t is the dispersion of individual nowcasts for real GDP growth from the Survey of Professional Forecasters (SPF), and x_t is either the quarter-on-quarter growth rate of real GDP, or a dummy variable equal to one when the quarter-on-quarter growth of real GDP is negative, or a dummy variable equal to one when the quarter-on-quarter growth rate of risk premium exceeds 2%. The dispersion of nowcasts is measured as the difference between the 75th percentile and the 25th percentile of the nowcasts for quarter-on-quarter GDP growth nowcasts, expressed in annualized percentage points. The sample is limited to 1968Q4-2019Q2 by the availability of the SPF. P-values are reported in parenthesis.

private sector expectations, as reported in the SPF, are a widely used proxy for uncertainty.[18] Considering GDP growth as independent variable, Table 5 reveals a negative link between output growth and uncertainty. A regression with a dummy — indicating times when the economy contracts — as independent variable corroborates this finding, reporting a significant positive relationship between contractions and nowcast dispersion. Our results on the adverse link between uncertainty and economic activity are consistent with findings in the literature (see e.g. Bachmann et al. (2013), Bloom (2014)). Using the dummy for strong surges in risk premia as independent variable shows that risk premia are heightened at times of high uncertainty. Investors tend to be more uncertain about the current and future state of the economy during economic contractions which requires compensation through higher risk premia. Our results are consistent with evidence in Corradi et al. (2013) who report the volatility of risk premia to be strongly countercyclicyal and with Baker et al. (2012) who use firm level data to document that uncertainty raises stock price volatility.

The evidence from Tables 4 and 5 corroborates the model assumption of a procyclical signal-to-noise ratio. It further provides empirical support for the key elements to generate growth asymmetry in risk premia, as it implies a link between uncertainty, strong risk premium growth, the state of the business cycle and variations in the speed of learning.

[18] See e.g. Bachmann et al. (2013).

6.2 Asymmetries in the model

We now evaluate the model's ability to resemble the risk premium's growth asymmetry observed in the data. We report moments for the risk premium based on the historical average method at the one-quarter investment horizon as this measure has been employed to calibrate the model. Table 6 reports a selection of moments for the risk premium in the data (Panel A) and implied by the model (Panel B). Second moments are computed based on the cyclical components of HP(1600) filtered series. The appropriate transformation to detect growth asymmetry, as shown in Sichel (1993), is by computing the skewness from log first-differences. The model matches the countercyclicality of the risk premium and the autocorrelation observed in the data rather well. Also the risk premium's volatility relative to output volatility is reasonably close to the statistic reported in the data. Most notable however is that the model is able to generate positive skewness in the risk premium (0.156) that comes close to the one observed in the data (0.122). The positive skewness implies that increases in the risk premium are larger than decreases. Together with the observed negative correlation with output, this is consistent with the mechanism outlined in Section 6.1 above: the risk premium declines gradually during a recovery and increases sharply when a recession occurs.

It is interesting to contrast this result with statistics based on a model without the learning mechanism. The difference to the baseline model is that the state of productivity is revealed at the beginning of the period. The corresponding moments are shown in Table 6, Panel C. While the baseline model can generate the empirically observed positive skewness in the risk premium rather well, the model without the learning mechanism fails to generate this asymmetry. Skewness in this model is not substantially different from zero; in fact it is slightly negative (-0.054). Concerning the risk premium statistics, this is the main difference to the baseline model with learning. The model without learning nevertheless implies a countercyclical risk premium, correctly ranks the risk premium to be more volatile than output, and generates a positive autocorrelation, albeit the latter is somewhat weaker than in the data.

We also report in Table 6 moments for macroeconomic aggregates. Overall, our baseline model (as well as the model without learning) matches the corresponding volatilities and correlations in the data reasonably well. Consumption is less volatile and investment more volatile than output.[19] A well known issue of real business cycle models is the low volatility of hours worked which is larger than the volatility of output in the data.[20] Our baseline model is able to replicate the negative skewness of macroeconomic aggregates observed in the data. The growth asymmetry in macroeconomic aggregates implies that increases are long and gradual and declines are short and sharp. This is a well documented feature of business cycles. It is consistent with the evidence in Van Nieuwerburgh and Veldkamp (2006), Görtz and Tsoukalas (2013) and Ordoñez (2013) who employ signal extraction processes similar to ours to generate asymmetries in macroeconomic aggregates. Görtz and Tsoukalas (2013) report asymmetry in these variables is a salient feature of business cycles across G7 countries and Ordoñez (2013) show it is stronger even in countries with less developed financial sectors. Given the above discussion, it is not surprising that the model without learning fails also to generate the growth asymmetry in macroeconomic aggregates. Skewness of output and consumption is close to zero. Investment and hours are only slightly negatively skewed, of about the size of one standard error, and by far not as much as in the data. It is hence apparent that the learning mechanism is crucial to align the model outcomes with the empirically observed asymmetry in the risk premium and the macroeconomic aggregates.

7 Conclusion

The expected risk premium on equity is the expected excess return above the risk-free rate that investors require as compensation for the higher uncertainty associated with risky

[19] To make learning non-trivial the variance of the noise shock needs to be large enough to disguise the true technology state. This however implies an unrealistically low autocorrelation and high volatility of output. We follow Van Nieuwerburgh and Veldkamp (2006) to resolve this conflict between learning and output volatility and report all moments for the model's output based on a filtered series given public information available at the end or the period, i.e. the persistent component of end-of-period output $\hat{y}_t = \mathbb{E}_t[A_t \mid \mathcal{I}_{t+1}]k_t^{\alpha} l_t^{1-\alpha}$. They show that $\hat{y}$ can be interpreted as revised data which is typically collected by data agencies who would like to report $y_t - \nu_t$ but don't observe the noise. For the national income accounts to balance also consumption must be filtered so that $\hat{y}_t = \hat{c}_t - i_t$.

[20] A remedy discussed in the literature can e.g. be to use the indivisible labor approach of Hansen (1985).

Table 6: Key moments of the risk premium and macroeconomic aggregates

	Relative std deviation	Correlation with output	1st order auto-cor.	Skewness
Panel A: U.S. data				
Risk premium	2.177	-0.486	0.772	0.122
Output	1.000	1.000	0.836	-0.523
Investment	4.373	0.898	0.817	-0.697
Hours	1.226	0.854	0.909	-0.965
Consumption	0.795	0.872	0.862	-0.672
Panel B: Baseline model (with learning)				
Risk premium	2.972 (0.028)	-0.511 (0.006)	0.640 (0.007)	0.156 (0.028)
Output	1.000 (0.000)	1.000 (0.000)	0.932 (0.003)	-0.591 (0.058)
Investment	2.038 (0.007)	0.735 (0.005)	0.890 (0.004)	-0.461 (0.063)
Hours	0.209 (0.001)	0.696 (0.006)	0.859 (0.004)	-0.453 (0.067)
Consumption	0.944 (0.004)	0.911 (0.001)	0.825 (0.007)	-0.137 (0.040)
Panel C: Model without learning				
Risk premium	3.676 (0.057)	-0.456 (0.006)	0.185 (0.007)	-0.054 (0.018)
Output	1.000 (0.000)	1.000 (0.000)	0.921 (0.004)	-0.029 (0.036)
Investment	1.992 (0.005)	0.983 (0.001)	0.908 (0.004)	-0.059 (0.049)
Hours	0.205 (0.001)	0.933 (0.003)	0.883 (0.000)	-0.071 (0.054)
Consumption	0.725 (0.002)	0.989 (0.000)	0.923 (0.003)	0.042 (0.046)

Values reported in parentheses are standard errors. The sample in panel A is 1957Q3 - 2019Q2. Statistics shown for the risk premium in Panel A are based on the historical average measure with one quarter investment horizon. The models in panels B and C are simulated 500 times over 298 periods after which the first 50 periods are discarded. Second moments are calculated based on percentage deviations from HP(1600) filter trend. Skewness is calculated from log first-differenced series.

assets. We estimate a variety of measures for the expected risk premium on equity for the post-WWII U.S. economy based on models that have been found relevant in the literature. We document these measures exhibit growth asymmetry in the sense that increases in the risk premium are sharp and short while declines are more gradual and long. We show this positive skewness is a salient feature of risk premium growth at different investment horizons and over different subsamples. A real business cycle model with Epstein-Zin preferences is consistent with this fact in the data. We demonstrate that the key mechanism to generate growth asymmetry in risk premia are procyclical variations in nowcast accuracy due to endogenous changes in the degree of uncertainty about productivity. This mechanism finds support in the data using measures for uncertainty and nowcast precision from the Survey of Professional Forecasters. In addition to matching the growth asymmetry in risk premia, the model is also successful in generating the empirically observed countercyclicality of risk premia and the negative skewness in macroeconomic aggregates.

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Appendix

A Additional evidence on the estimation on risk premia

A.1 ADF test results for realized risk premia

Tables 7 and 8 show that ADF tests overwhelmingly reject the null hypothesis of a unit root in realized ex-post risk premia at the yearly or quarterly horizon. These are computed as the ex-post difference between the stock market return and the risk-free rate. This test statistic validates the use of the historical mean method to compute the risk premium using the historical mean method.

Table 7: Yearly risk premia

Test statistic	1% critical value	5% critical value	10% critical value
-5.247	-3.461	-2.880	-2.570
MacKinnon approximate p-value = 0.000			

Table 8: Quarterly risk premia

Test statistic	1% critical value	5% critical value	10% critical value
-13.401	-3.461	-2.880	-2.570
MacKinnon approximate p-value = 0.000			

B Model derivations

B.1 Household's optimality conditions

The recursive structure of the utility function immediately implies the Bellman equation

$$F(s_t^d, \tilde{A}_t) = \max_{s_{t+1}^d, l_t^s, \tilde{c}_t} W\left(u(\tilde{c}_t, l_t^s), \mu_t\right),$$

where

$$W\left(u(\tilde{c}_t, l_t^s), \mu_t\right) = \left[(1-\beta)u(\tilde{c}_t, l_t^s)^{\frac{1-\gamma}{\theta}} + \beta\mu_t^{\frac{1-\gamma}{\theta}}\right]^{\frac{\theta}{1-\gamma}},$$

with

$$\mu_t = \left(\mathbb{E}_t\left[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t\right]\right)^{\frac{1}{1-\gamma}}$$

and $\theta := \frac{1-\gamma}{1-\frac{1}{\psi}}$, subject to

$$\tilde{c}_t = w_t l_t^s + \tilde{d}_t s_t^d - p_t(s_{t+1}^d - s_t^d).$$

To ease notation in the following algebraic derivations, we use the simplified notations u_t to denote the period utility function $u(\tilde{c}_t, l_t)$, W_t to denote the CES aggregation $W\left(u(\tilde{c}_t, l_t^s), \mu_t\right)$, and F_t to denote the value function $F(s_t^d, \tilde{A}_t)$.

B.1.1 Derivation of the Lucas equation and the SDF

The first order condition with respect to s_{t+1} yields

$$\frac{\partial W_t}{\partial u_t}\frac{\partial u_t}{\partial \tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial s_{t+1}^d} + \frac{\partial W_t}{\partial \mu_t}\frac{\partial \mu_t}{\partial \mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]}\mathbb{E}_t\left[\frac{\partial F_{t+1}^{1-\gamma}}{\partial F_{t+1}}\frac{\partial F_{t+1}}{\partial s_{t+1}^d} \,\middle|\, \mathcal{I}_t\right] = 0.$$

The Envelope theorem for s_t yields

$$\frac{\partial F_t}{\partial s_t^d} = \frac{\partial W_t}{\partial u_t}\frac{\partial u_t}{\partial \tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial s_t^d}.$$

Combining both conditions, we obtain

$$\frac{\partial W_t}{\partial u_t}\frac{\partial u_t}{\partial \tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial s_{t+1}^d} + \frac{\partial W_t}{\partial \mu_t}\frac{\partial \mu_t}{\partial \mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]}\mathbb{E}_t\left[\frac{\partial F_{t+1}^{1-\gamma}}{\partial F_{t+1}}\frac{\partial W_{t+1}}{\partial u_{t+1}}\frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial s_{t+1}^d} \,\middle|\, \mathcal{I}_t\right] = 0. \tag{B.1}$$

We can then recover

$$\frac{\partial W_t}{\partial u_t} = F_t^{1-\frac{1-\gamma}{\theta}}(1-\beta)u_t^{\frac{1-\gamma}{\theta}-1},$$

$$\frac{\partial W_t}{\partial \mu_t} = \beta F_t^{1-\frac{1-\gamma}{\theta}}\left(\mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]\right)^{\frac{1}{\theta}-\frac{1}{1-\gamma}},$$

$$\frac{\partial \mu_t}{\partial \mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]} = \frac{1}{1-\gamma}\mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]^{\frac{1}{1-\gamma}-1},$$

$$\frac{\partial \tilde{c}_{t+1}}{\partial s^d_{t+1}} = \mathbb{E}_t[d_{t+1} + p_{t+1} \mid \mathcal{I}_t],$$

and

$$\frac{\partial \tilde{c}_t}{\partial s^d_{t+1}} = -p_t.$$

Plugging the last 5 equations into equation (B.1) yields after re-arranging

$$0 = \mathbb{E}_t \left[m_{t+1,t} \frac{d_{t+1} + p_{t+1}}{p_t} - 1 \,\middle|\, \mathcal{I}_t \right],$$

where

$$m_{t+1,t} = \beta \left(\frac{F^{1-\gamma}_{t+1}}{\mathbb{E}_t \left[F^{1-\gamma}_{t+1} \mid \mathcal{I}_t \right]} \right)^{1-\frac{1}{\theta}} \left(\frac{u_{t+1}}{u_t} \right)^{\frac{1-\gamma}{\theta}-1} \frac{\frac{\partial u_{t+1}}{\partial c_{t+1}}}{\frac{\partial u_t}{\partial \tilde{c}_t}}.$$

These are the Lucas equation (16) and the stochastic discount factor (17).

B.1.2 Optimal labour supply

The first order condition with respect to l_t yields

$$\frac{\partial W_t}{\partial u_t} \left[\frac{\partial u_t}{\partial \tilde{c}_t} w_t + \frac{\partial u_t}{\partial l^s_t} \right] = 0. \tag{B.2}$$

We can then recover

$$\frac{\partial u_t}{\partial \tilde{c}_t} = \kappa \tilde{c}_t^{\kappa-1} (1 - l^s_t)^{1-\kappa}$$

and

$$\frac{\partial u_t}{\partial l^s_t} = -\tilde{c}_t^{\kappa} (1-\kappa)(1 - l^s_t)^{-\kappa}.$$

Plugging the last 2 equations into equation (B.2) yields after re-arranging

$$\tilde{c}_t = \frac{\kappa}{(1-\kappa)} (1 - l^s_t) w_t,$$

which is the labor supply function (15).

B.2 Derivation of the return on equity

Firm's expected cash flow at the beginning of the period is defined as

$$\tilde{f}_t = \tilde{d}_t s_t - p_t(s_{t+1} - s_t) = \tilde{y}_t - w_t l_t - i_t.$$

From equation (11), it holds that $w_t l_t = (1-\alpha)\tilde{y}_t$. Hence we can simplify the above equation for expected cash flow to become

$$\tilde{f}_t = \alpha \tilde{y}_t - i_t.$$

Using equations (10) and (7), and due to the specific capital adjustment costs we apply, we can write

$$\begin{aligned}
q_t k_{t+1} =& \mathbb{E}_t \left\{ m_{t,t+1} \left[A_{t+1} l_{t+1}^{1-\alpha} \alpha k_{t+1}^{\alpha} - i_{t+1} + q_{t+1} \left(1 - \delta + \Phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) k_{t+1} \right] \middle| \mathcal{I}_t \right\} \\
\Leftrightarrow q_t k_{t+1} =& \mathbb{E}_t \left[m_{t,t+1} (\alpha y_{t+1} - i_{t+1} + q_{t+1} k_{t+2}) \mid \mathcal{I}_t \right] \\
\Leftrightarrow q_t k_{t+1} =& \mathbb{E}_t \left[m_{t,t+1} (f_{t+1} + q_{t+1} k_{t+2}) \mid \mathcal{I}_t \right].
\end{aligned}$$

Iterating forward, we obtain

$$q_t k_{t+1} = \mathbb{E}_t \left[\sum_{i=1}^{+\infty} m_{t,t+i} f_{t+i} \middle| \mathcal{I}_t \right], \tag{B.3}$$

assuming that $\lim_{i \to +\infty} \mathbb{E}_t [m_{t,t+i} q_{t+i} k_{t+i+1} \mid \mathcal{I}_t] = 0$. Following Altug and Labadie (2008), the value of a firm on the stock market is equal its present value of future discounted cash flow. This allows us to rewrite equation (B.3) as

$$q_t k_{t+1} = p_t s_{t+1}.$$

Finally, using the above expressions, we can derive a formulation for the return on equity which depends on variables that have been pinned down uniquely in firm's and household's

maximization problems

$$\begin{aligned}\mathbb{E}_t\left[\frac{d_{t+1}+p_{t+1}}{p_t}\,\middle|\,\mathcal{I}_t\right] =&\mathbb{E}_t\left[\frac{s_{t+1}d_{t+1}-p_{t+1}(s_{t+2}-s_{t+1})+s_{t+2}p_{t+1}}{s_{t+1}p_t}\,\middle|\,\mathcal{I}_t\right]\\ =&\mathbb{E}_t\left[\frac{f_{t+1}+k_{t+2}q_{t+1}}{k_{t+1}q_t}\,\middle|\,\mathcal{I}_t\right]\\ =&\mathbb{E}_t\left[\frac{q_{t+1}k_{t+2}+y_{t+1}-w_{t+1}l^d_{t+1}-i_{t+1}}{q_tk_{t+1}}\,\middle|\,\mathcal{I}_t\right],\end{aligned}$$

which is equation (20) in the main body.

B.3 Derivation of functional forms for parameters a_1 and a_2

The parameters a_1 and a_2 are calibrated to ensure that adjustment costs are zero in steady state, so that steady state investment and Tobin's q are $i = \delta k$ and $q = 1$. From equations (7) and (9), we can see that the latter is satisfied if

$$\Phi(\delta) = \delta \text{ and } \Phi^{'}(\delta) = 1.$$

Given the functional form of Φ, this implies

$$\frac{a_1}{1-\chi}\delta^{1-\chi} + a_2 = \delta \text{ and } a_1\delta^{-\chi} = 1,$$

from where we deduce

$$a_1 = \delta^{\chi} \text{ and } a_2 = -\frac{\delta\chi}{1-\chi}.$$