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Pricing the Pharmaceuticals when the Ability to Pay Differs: Taking Vertical Equity Seriously.

Abstract

A non-trivial fraction of people cannot afford to buy pharmaceutical products at unregulated market prices. Therefore, the paper analyzes the public insurance of the pharmaceutical products in terms of price controls and the socially optimal third-degree price discrimination. It characterizes first the Ramsey pricing rule in the absence of insurance and in the case where the producer price has to cover the R&D sunk cost of the firm. Subsequently, conditions for a welfare increasing departure from Ramsey pricing are stated in terms of price regulation and insurance coverage. The resulting outcome is second best. Unlike the earlier views expressed, increased consumption of pharmaceutical products is shown to be welfare increasing in the second best world. As the optimal means-tested insurance, two alternative criteria for vertical equity are examined in the spirit of the Rawlsian view. In the first scheme, the regulator chooses a higher insurance coverage for individuals with their income below a threshold. In the second scheme, the society imputes a social value to low-income patients in terms of the value-added they produce after the treatment. Under both schemes, the threshold is determined endogenously.

JEL-Codes: L100, L500.

Keywords: pharmaceutical products, price regulation, public health insurance, third-degree price discrimination, equity criterion.

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1 Introduction

The ability to pay for the pharmaceuticals varies among people. A non-trivial fraction of people cannot afford to buy pharmaceutical products at unregulated market prices. Therefore, the conflict between efficiency and equity has to be resolved in the optimal pricing. The pharmaceutical products are created through expensive R&D programs committing the pharmaceutical firms to rather high expenditures. Those expenses should subsequently be covered through prices, which, however, may turn out to be too high to be socially acceptable.

Apart from the efficiency considerations, policy-makers typically emphasize equitable access to services due to the fact that in many countries, if not in most, low-income people are not able to buy the medication they need. Indeed, the health policy concerning the medical industry often expressed in the official documents states that "the purpose of the medical policy is to provide to citizens high-quality and cost-efficient health program at reasonable prices....". Moreover, the PPRI Report 2018 provides information of currently existing pharmaceutical pricing and reimbursement policies in the 47 PPRI member countries. It turned out that 42 PPRI network member countries have mechanisms in place to set medicine prices at the ex-factory (or sometimes wholesale) price level, mostly targeting reimbursable medicines or prescription-only medicines. 46 PPRI network member countries have one or more reimbursement lists for outpatient medicines in place, and in 31 PPRI countries the reimbursement lists relate to both outpatient and inpatient sectors. In addition, hospital pharmaceutical formularies are managed at the level of hospitals in most PPRI countries. At least 43 countries charge co-payments for outpatient reimbursable medicines (frequently percentage co-payments, but also a prescription fee and/or a deductible). All these 43 countries apply exemptions from or reductions of co-payments for vulnerable and other defined population groups (Vogler et al., 2019).

The previous work based on efficient price regulation of pharmaceutical products and health insurance has produced a number of contributions. The basic idea has been cast in terms of the optimal product taxation in a one person or many person economy with Ramsey's (1927) idea of equal percentage reductions in (compensated) demands for all commodities (Diamond, 1975). Based on such foundations, Besley (1988) explored the trade-off between risk sharing and the incentives to consume medical care inherent in reimbursement insurance. Therefore, the price elasticity of demand appears to play a key role (Ringel et al.). Earlier, Feldstein (1973) had expressed concerns of the welfare cost of excess health insurance induced by the adverse incentives of the consumption of health care. The interaction of pricing and insurance coverage in the pharmaceutical market was addressed by Barros and Martinez-Giralt (2008) who considered the normative allocation of $R\&D$ costs across different markets served by a pharmaceutical firm. They showed that a higher insurance coverage calls for higher prices not only because of a lower demand elasticity but also due to a larger moral hazard effect in the consumption of the pharmaceuticals. The equilibrium pricing rule appeared to deviate from the standard Ramsey pricing rule: for equal demand elasticities, and given the distortion cost of funds, a country with a higher coverage rate will have a higher price of pharmaceuticals as well.

Gaynor et al. (2000) also focussed on the excessive consumption of the medical products caused by the insurance, that is, the moral hazard. In a related area, Grassi and Ma (2011, 2012) studied the provision of public supply of health care services but with non-price rationing when the income levels of people are different. When the rationing is based on wealth information (as is the case in the USA), the optimal policy in their analysis must implement a price reduction in the private market. If also the cost is observed, the optimal rationing turned out to be based on cost-effectiveness (as in most European countries and Canada).

In the current paper and in contrast to the existing work in the area, the question is raised how to introduce means-tested subsidies to low income citizens as part of the optimal regulation and yet to maintain the incentives of a pharmaceutical company in investing in the $R\&D$. To fix the ideas of the paper, a market for a pharmaceutical product with one firm having innovated a new product is considered. The firm is the sole producer of the product, say through the patent protection. The cost of innovation is sunk at the time the product is sold in the market, and it makes the average cost for the firm decreasing. Four policies are analyzed: Ramsey pricing without insurance, price regulation with an insurance, and two means-tested price and insurance policies. Throughout the analysis, we allow the consumer population to be heterogeneous in terms of the ability to pay (income) and analyze questions related to the access to the pharmaceutical care. Initially, the equity issues are ignored. As the equality between the marginal cost of production and the marginal revenue does not represent a feasible starting point for the price regulation, the Ramsey pricing rule is a natural candidate to be studied in the absence of insurance coverage. Subsequently, conditions for a welfare increasing departure from Ramsey pricing in terms of price regulation and optimal insurance coverage are derived taking the social cost of public funds into account. The resulting outcome is second best in general. The results provide an insight as to why both the price regulation and the social insurance are desirable.

Subsequently, the paper deals with the fact that a non-trivial fraction of patients cannot afford to buy pharmaceutical products even at regulated and subsidized market prices and asks if a means-tested insurance coverage is welfare improving. It thus arrives at the socially optimal third-degree out-of-pocket price discrimination. In addition, our paper also suggests how the health of people with low ability to pay can be valued in the social cost-benefit analysis. Such an analysis complements that of Grassi and Ma (2011,2012) who analyzed efficient non-price rationing schemes. Moreover, while Gaynor et al. (2000) worked with the case of a private insurance market for health care the focus in the current papers is instead in the public health insurance.

We find that the moral hazard in terms of increased consumption of pharmaceutical products is welfare increasing. Without the health insurance, the prices would be excessively high as the firm's R&D costs have to be recovered. It is the insurance coverage which provides the access to the desired consumption. Yet, the second-best equilibrium with public insurance also has some undesirable properties: the low-income people are left without the medication they need. As the optimal means-tested insurance, two alternative equity criteria are examined in the spirit of the Rawlsian view. In the first scheme, the regulator chooses a higher insurance coverage for individuals with their income below a threshold. In the second scheme, the society imputes a social value to low-income patients in terms of the value-added they produce after the treatment. Under both schemes, the income threshold between low-income and high-income patients is determined endogenously. For those with higher income, the insurance is based on the efficiency criterion.

Our findings indicate that in the Rawlsian world with equity based on maximizing the aggregate consumer surplus and conditional on the better access to medication by low-income people by means-tested insurance coverage, the consumption of the pharmaceutical and also the consumer surplus is split equally between the low- and high-income patients. It is also shown that the optimal means-tested price-insurance policy yields a strictly higher welfare than the optimal price-insurance policy with no equity concern.

In the model where the health care is viewed as an investment, we first solve for the optimal means-tested price-insurance policy. We then obtain a sharp result that if the quality of the drug is high, an increase in the social cost of public funds increases the number of people entitled to free medication. In addition, we show that the optimal means-tested price-insurance policy produces a higher welfare than the optimal price-insurance policy with no equity concern if the value of the recovered health of low-income people exceeds the social marginal cost of producing the medication.

Before presenting the model and the analysis of it, we comment on the potential information problems as follows. First, although the regulator is uninformed about the individual incomes of the patients in Sections $2-5$, it knows the income distribution. This is all it needs to know in the Ramsey problem and in the optimal price-insurance policy analysis. Second, in Sections 6 and 7, the regulator uses the means-tested approach to classify the patients into the low-income and high-income people and is assumed to have access to income information that is needed to construct optimal policies.

2 Model

We consider a market for a new pharmaceutical product. There is a single monopoly producer holding a patent to sell the product. The size of the consumer population is normalized to one. The fraction γ of the consumers is ill and in need of the pharmaceutical treatment, which is assumed to cure patients and help to recover their ability to work. We assume throughout the article that $\gamma = 1$.

Patients consuming the pharmaceutical have $v(s)$ as their utility of health. The parameter $s > 0$ measures the quality of the pharmaceutical product[1]. The better the quality of the product is, the better off a consumer is but at a decreasing rate, that is $v'(s) > 0$ and $v''(s) < 0$[2]. Patients not consuming the pharmaceutical have $v(0)$ as their utility of health.

2.1 Ability to pay, consumer surplus and demand

Each patient consumes regular (or consumption) goods and at most one unit of the medication. Patients are heterogeneous in their ability to pay for the pharmaceutical. We introduce a randomly distributed income variable w, assumed to follow the $U[0, 1]$ distribution. The income variable measures the disposable income and is adjusted for the patients' tax payments to the government.

We first show how willigness to pay for the pharmaceutical product, denoted θ, is determined by the patient's ability to pay and the quality of the pharmaceutical using the approach developed in Grassi and Ma (2011, 2012). Let the variables P and p denote the price of the (composite of the) consumption goods and the producer price of the pharmaceutical product, respectively, and let $x \geq 0$ denote the amount of the consumption goods. The budget constraint of a patient with income w can then be written as $w = Px + (1 - r)py$, where the binary variable $y = 1, 0$ describes whether the patient consumes the pharmaceutical or not, and the variable r stands for the insurance coverage. For simplicity, we adopt the normalization $P = 1$. Assuming separable utility and a minor generalization to Grassi and Ma (2011, 2012), the patient obtains indirect utility

$$u(w - (1 - r)p) + v(s) \tag{1}$$

by consuming the pharmaceutical product. The first part of the utility function $u(x)$ measures patient's utility obtained from the consumption goods. We assume that utility function $u(x)$ is strictly increasing and concave in the consumption of x.

If the pharmaceutical product is not consumed, the patient's indirect utility is

$$u(w) + v(0). \tag{2}$$

The willingness to pay for the pharmaceutical product θ for the patient with income w is now determined by the indifference condition

$$u(w - \theta) + v(s) = u(w) + v(0). \tag{3}$$

Intuitively, the health benefit due to the consumption of the pharmaceutical equals the utility sacrifice in terms of foregone consumption of the regular good, that is

$$v(s) - v(0) = u(w) - u(w - \theta). \tag{4}$$

The condition (3) defines implicitly patient's willingess to pay as a function of income and the quality of the pharmaceutical. The willingness to pay is increasing in income and in the quality of

[1]Alternatively, the parameter s can measure the health benefit that patients obtain from the consumption of the pharmaceutical.

[2]In the current paper, the quality s is exogenous. We, however, acknowledge the possibility that the price regulation may have implications for the quality of the pharmaceutical.

the pharmaceutical, because the implicit differentiation of condition (3) yields

$$\frac{\partial\theta}{\partial w} = \frac{-[u'(w) - u'(w-\theta)]}{u'(w-\theta)} > 0$$

and

$$\frac{\partial\theta}{\partial s} = \frac{v'(s)}{u'(w-\theta)} > 0.$$

Given the consumer price $p^c = (1-r)p$, patients with willingness to pay satisfying $\theta \geq p^c$ buy the pharmaceutical product while those with $\theta < p^c$ abstain from buying. The former group creates the demand for the pharmaceutical product. For the latter group, the sacrifice in terms of the foregone consumption of the regular goods is too high. We can thus define the low-income patients in our model as those with the ability to pay (hence also the willingness to pay) so low that that they abstain from buying the pharmaceutical. With the consumer price p^c, the consumer surplus for buying patients is given by $CS = \theta - p^c$. Patients with positive consumer surplus have $v(s)$ as their utility from health. Those patients who abstain from consuming the pharmaceutical will not be cured from the illness and have $v(0)$ as their utility from health.

In the above analysis, the willingness to pay $\theta(w,s)$ has been regarded as an endogenous variable determined by income and the quality of the pharmaceutical. A natural specification is given by

$$\theta(w,s) = w\eta(s), \tag{5}$$

where the function $\eta(s)$ is called the quality weight and is assumed to be an increasing and concave function of the quality. This parametrisation is consistent with the general approach presented above. Adopting the log-linearized Cobb-Douglas specification $u(x)+v(s) = \alpha\ln(x)+(1-\alpha)\ln(1+s)$, $0 < \alpha < 1$, the indifference condition (3) can be rewritten as

$$\alpha\ln(w-\theta) + (1-\alpha)\ln(1+s) = \alpha\ln(w). \tag{6}$$

Solving the above condition with respect to θ, we obtain

$$\theta(w,s) = w\left(1 - (1+s)^{\frac{\alpha-1}{\alpha}}\right), \tag{7}$$

where $\eta(s) = 1 - (1+s)^{\frac{\alpha-1}{\alpha}}$. It is straightforward to show that $\eta(0) = 0$, $\eta'(s) > 0$ and $\eta''(s) < 0$.

In what follows, we assume conditions allowing us to adopt the parametrisation (5). Then a patient with income w obtains a surplus

$$CS_w = w\eta - (1-r)p \tag{8}$$

if she consumes pharmaceutical, and zero surplus otherwise. For the indifferent (marginal) patient with income w_m, the equality $w_m\eta - (1-r)p = 0$ holds true. This equation can be solved with respect to the income of the indifferent consumer:

$$w_m = \frac{(1-r)p}{\eta}. \tag{9}$$

Those patients with incomes lower than w_m (low-income patients) do not buy the pharmaceutical while those patients with incomes higher than w_m (high-income patients) do buy.

Given the producer price and the insurance coverage, the demand for the pharmaceutical product is the number of buying patients:

$$q(p,r) = 1 - w_m = 1 - \frac{(1-r)p}{\eta}. \tag{10}$$

Thus, the inverse market demand function is given as

$$p = \frac{\eta}{1-r}(1-q). \tag{11}$$

For the intuition, the demand function is consistent with the idea that the product is a normal good with a positive income elasticity. The consumers are ordered on the declining (linear) demand function in regard to their ability to pay. We also note that an increase in the insurance coverage moves the inverse demand function right.

2.2 Producer

The profit of the pharmaceutical firm is given as follows

$$\pi = (p-c)q(p,r) - F, \tag{12}$$

where $c > 0$ is the marginal cost of production and $F > 0$ is a fixed (sunk) cost from $R\&D$ activities prior to the launch of the pharmaceutical product[3]. We assume throughout the following analysis that the quality weight of the pharmaceutical exceeds the marginal cost of production

Assumption 1 $c < \eta$.

Assumption 1 guarantees the existence of an active market for pharmaceuticals. If Assumption 1 would not hold true, there would be no patients whose willingness to pay for the pharmaceutical exceeds the marginal cost of producing the pharmaceutical. This implies that there would be no possibilities for market exchange.

In addition, to ensure a positive monopoly profit, we assume

Assumption 2 $0 < F < \frac{(\eta-c)^2}{4\eta}$.

Assumption 2 ensures that there are prices on the demand curve which exceed the average cost of production. Assumption 2 implies that there exist regulatory policies satisfying the firm's participation constraint

$$\pi \geq 0. \tag{13}$$

2.3 Regulator

The regulator is benevolent and chooses producer price and insurance coverage to maximize the social welfare. It is defined as the sum of the consumer surplus and the firm's profit subtracted by the cost of financing health insurance

$$W = CS + \pi - (1+\lambda)T. \tag{14}$$

In (14), T is the tax revenue raised to finance the health insurance. We assume that each € or $ raised through taxes to finance the pharmaceutical consumption costs $(1+\lambda)$ for the society, where $\lambda \geq 0$ measures the social cost of public funds. The regulator maximizes the social welfare subject to the budget constraint

$$T \geq rpq(p,r). \tag{15}$$

The right-hand side of the inequality (15) measures the insurance expenditure due to consumption of the pharmaceutical.

[3]Although that is not made explicit in the following analysis, both marginal and fixed costs can depend on the quality of the drug.

2.4 Timing

We will examine a strategic game between the regulator and the producer of the pharmaceutical. The sequence of moves is as follows. The regulator first chooses the producer price p and the insurance coverage r, after which the firm either accepts or rejects the proposal. If the firm accepts the proposal, patients decide whether or not to consume the pharmaceutical and the firm produces the amount of the pharmaceutical demanded by the patients.[4]

3 Setting up the problem

Since all patients with income higher than $[p(1-r)]/\eta$ consume the pharmaceutical, the consumer surplus arising from the consumption of pharmaceuticals is given as

$$CS(p,r) = \int_{\frac{p(1-r)}{\eta}}^{1} (w\eta - (1-r)\,p)\,dw. \tag{16}$$

Given the demand for the pharmaceutical (10), the firm's profit is defined by the equation (12). The aggregate insurance expenditure amounts to

$$IE(p,r) = rp \int_{\frac{p(1-r)}{\eta}}^{1} dw = rpq(p,r). \tag{17}$$

Since the value of the social welfare function (14) decreases as the tax revenue T increases, the regulator is not willing to collect more tax revenue than the amount of the aggregate insurance expenditure. This implies that the budget constraint (15) must be binding at any solution of the regulator's problem. The social welfare function can then be restated as follows:

$$W = CS(p,r) + \pi(p,r) - (1+\lambda)IE(p,r). \tag{18}$$

The regulator chooses the price-insurance policy (p,r) which maximizes the value of the social welfare (18) subject to the profit constraint

$$\pi(p,r) \geq 0 \tag{19}$$

and feasibility constraints

$$p \geq 0 \tag{20}$$

and

$$0 \leq r \leq 1. \tag{21}$$

In sections $5-7$, we will first analyse a relaxed problem in which the social welfare is maximized without the feasibility constraints. After solving the problem we then check whether the obtained solution satisfies the constraints (20) and (21). This approach has become a standard analytical tool in the principal-agent literature (see e.g. Martimort and Laffont, 2002).

An efficient benchmark to the regulator's problem is the first-best price and quantity which maximize the social welfare not influenced by the insurance coverage:

[4] The regulator acts as a Stackelberg leader relative to the producer and the consumers.

$$W_f = CS(p,0) + \pi(p,0). \tag{22}$$

In the first-best solution, the price equals the marginal cost, $p_f = c$. The amount of pharmaceuticals consumed at the first-best solution is $q(c,0) = 1 - (c/\eta)$. The corresponding value of the social welfare is

$$\bar{W}_f = CS(p_f,0) + \pi(p_f,0) = \frac{(\eta-c)^2}{2\eta} - F. \tag{23}$$

It is also understood that the regulator cannot implement the marginal-cost pricing schemes because they would yield the profit $-F$ that the firm is not willing to accept.

4 Ramsey price

Understanding that the marginal-cost pricing cannot be implemented, we first consider the pricing that maximizes welfare and satisfies the firm's profit constraint as the benchmark case. Furthermore, and to leave the analysis of the optimal social insurance to the subsequent sections, we assume in this section that the regulator does not subsidize the patients' pharmaceutical expenditures through social insurance and selects $r = 0$. Under such policy, the consumption of the pharmaceutical has no effect on public expenditures and there is no need to fund social insurance through taxation.

The problem of the regulator can be defined as finding the price of the pharmaceutical which maximizes the social welfare

$$W = CS(p,0) + \pi(p,0) \tag{24}$$

subject to the profit constraint

$$\pi(p,0) \geq 0. \tag{25}$$

The solution of the above problem defines the Ramsey-Boiteux price (e.g Armstrong and Sappington, 2007). With L denoting the value of the Lagrangean function, the necessary condition of the regulator's problem can be defined as follows

$$\begin{aligned}\frac{\partial L}{\partial p} &= \frac{\partial CS(p,0)}{\partial p} + (1+\mu)\frac{\partial \pi(p,0)}{\partial p} \\ &= -\left(1-\frac{p}{\eta}\right) + (1+\mu)\left(\left(1-\frac{p}{\eta}\right) - \frac{(p-c)}{\eta}\right) = 0,\end{aligned} \tag{26}$$

where μ is the positive-valued Lagrange multiplier of the profit constraint. In addition to the condition (26), the profit constraint and the complementary slackness conditions require that $-\pi(p,0) \leq 0$, $\mu \geq 0$ and $-\mu\pi(p,0) = 0$.

The first-order condition (26) can be rewritten in the form that is well-known in the literature of price regulation (e.g. Armstrong and Sappington, 2007):

$$\frac{p-c}{p} = \frac{\mu}{1+\mu}\frac{1}{|\epsilon|}, \tag{27}$$

where the price elasticity of the demand is $\epsilon = -p/(\eta - p)$. When evaluated at the Ramsey price, the price-cost margin is inversely related to the price elasticity of the demand for the pharmaceutical.

Straightforward computation shows that the social welfare (Eq. 24) is decreasing in the price of the pharmaceutical for all prices higher than the marginal cost[5]. Therefore, the regulator wants to reduce the price of the pharmaceutical until the excess profit of the pharmaceutical firm is exhausted. This implies that the firm must earn zero profit in the solution of the regulator's problem.

The first-order condition (26) for the price-cost margin can be solved together with the zero-profit condition

$$F - (p - c)\left(1 - \frac{p}{\eta}\right) = 0 \tag{28}$$

to have[6] the Ramsey price p_R:

$$p_R = \frac{1}{2}\left(\eta + c - \sqrt{(\eta - c)^2 - 4\eta F}\right) < \frac{1}{2}(\eta + c) \equiv p_M. \tag{29}$$

where $(1/2)(\eta + c) = \arg\max_p (p - c)(1/\eta)(\eta - p) - F$ is the monopoly price. Assumption 2 ensures that the Ramsey price is well-defined. The value of the Lagrange multiplier at the regulator's solution is strictly positive:

$$\mu_R = \frac{p_R - c}{\eta - 2p_R + c} > 0, \tag{30}$$

because the marginal monopoly profit $\eta - 2p_R + c$ is strictly positive when evaluated at the Ramsey price p_R.

Intuitively, the Ramsey price is sufficiently high so as to make the firm break even but it is lower than the monopoly price. The Ramsey price is related not only to the marginal or the fixed R&D costs but also to the demand elasticity.

The firm earns zero profit at the Ramsey solution, which implies that the maximum social welfare equals the equilibrium value of the consumer surplus. Therefore, the maximum social welfare is given as

$$W_R = CS(p_R, 0) = \frac{1}{2\eta}(\eta - p_R)^2 = \frac{1}{8\eta}\left(\eta - c + \sqrt{(\eta - c)^2 - 4\eta F}\right)^2. \tag{31}$$

We note from the Ramsey price that it even if it eliminates excess profits, it forcefully limits the number of people who are able to buy the pharmaceutical product when in need.

5 Second-best efficient price and insurance policy

We then introduce a social health insurance and ask whether adding a distortionary policy instrument to the regulator's strategy has potential to improve social welfare. Intuitively, the health insurance can improve patients' welfare by lowering the out-of-pocket price that patients pay for the pharmaceutical. However, the obvious social cost of the social health insurance is that it increases insurance expenditures financed through taxation. To examine whether social benefits exceed social costs, we first derive the optimal price-insurance policy and thereafter assess the welfare properties of it.

[5] The first derivative of the social welfare with respect to price is $-(p - c)\frac{1}{\eta}$ and the statement follows from this.

[6] The system of equations (28) and (27) has two solutions $r_1 = (p_1, \mu_1)$ and $r_2 = (p_2, \mu_2)$. The first (second, respectively) solution corresponds to the lower (higher) root of the zero profit condition. The value of social welfare is strictly decreasing at all price levels exceeding marginal cost. Since the prices in the feasible set (ie. prices which satisfy the profit constraint) all exceed the marginal cost, the lower root r_1 is the solution of the regulator's problem.

The regulator's policy problem is to choose price and insurance coverage (p, r) that maximize social welfare (18) subject to the profit constraint (19) and feasibility constraints (20) and (21). The solution of the regulator's problem is characterized in Proposition 1 below. A general feature of the solution is that $\mu = \lambda$ (Proof of Proposition 1). To explain intuitively this result, we note that the multiplier μ measures the marginal social benefit of relaxing the profit constraint of the pharmaceutical firm through tax funding while λ is the social marginal cost of the tax funding. It is part of the optimal solution that the social marginal benefit of relaxing firm's profit constraint equals the social marginal cost of tax funding.

Proposition 1. *If* $\lambda > 0$ *and*

$$\frac{(\eta - c)^2 \lambda (1 + \lambda)}{\eta (1 + 2\lambda)^2} < F, \tag{32}$$

the optimal price-insurance policy is

$$\tilde{p} = c + \frac{\eta F (1 + 2\lambda)}{(\eta - c)(1 + \lambda)} \tag{33}$$

and

$$\tilde{r} = \frac{\eta F (1 + 2\lambda)^2 - (\eta - c)^2 \lambda (1 + \lambda)}{(1 + 2\lambda) \left[\eta F (1 + 2\lambda) + c(\eta - c)(1 + \lambda) \right]}. \tag{34}$$

Proof. See Appendix.

The optimal price-insurance policy is designed so that it yields the zero profit to the firm. The producer price exceeds the marginal cost in order to cover the fixed $R\&D$ cost. It is also worth noting that the optimal price is increasing (and concave) in the social cost of public funds.

The condition (32) guarantees that $\tilde{r} > 0$ and the optimal policy is an interior solution. If the condition is not satisfied, the necessary conditions of the regulator's policy problem (Proof of Proposition 1) support the Ramsey solution that was examined in the previous section. Furthermore, it can be shown that an increase in the fixed cost is associated with an increase in the optimal insurance coverage and $\partial \tilde{r} / \partial F > 0$. Intuitively, these observations suggest that the regulator is more likely to introduce a greater insurance coverage, the higher the fixed cost is. The insurance coverage allows the regulator to increase consumer surplus by reducing the out-of-pocket payment. Were the health insurance not available, an increase in the fixed cost would increase the price of the pharmaceutical, decrease the demand for the pharmaceutical and consumer surplus.

The out-of-pocket price that the patients pay under the optimal price-insurance policy is

$$\tilde{p}(1 - \tilde{r}) = c + \frac{(\eta - c)\lambda}{(1 + 2\lambda)}. \tag{35}$$

When taxation is distortionary and $\lambda > 0$, the consumer price exceeds the marginal cost of producing the pharmaceutical. From this it also follows that the demand for the pharmaceuticals is below the first-best level and

$$q(\tilde{p}, \tilde{r}) = \frac{(\eta - c)(1 + \lambda)}{\eta (1 + 2\lambda)} < \frac{\eta - c}{\eta} = q(c, 0). \tag{36}$$

In addition, one can prove that that the patient's out-of-pocket price (35) under the optimal price-insurance policy is lower than the corresponding consumer price in the Ramsey solution (29), if the condition for the interior solution (32) holds true[7]. Provided that the demand for

[7] The proof is available from the authors

the pharmaceutical (10) decreases as the out-of-pocket price increases, such a decrease in the out-of-pocket price also increases the consumption of the pharmaceutical from that occurring in the Ramsey solution.

We then conduct the welfare analysis by analysing the consumer surplus, the insurance expenditure and the social welfare at the optimal price-insurance policy. Table 1 below displays these measures together with the corresponding measures in the first-best and Ramsey solutions. The consumer surplus associated with the optimal price-insurance policy is lower than the consumer surplus in the first-best solution with marginal cost pricing and no insurance coverage due to the positive marginal cost of taxation. On the contrary, the consumer surplus under the optimal price-insurance policy is higher than the consumer surplus in the Ramsey solution, if the condition for the interior solution (32) holds true[8]. The underlying reason for this is that the out-of-pocket price (35) is lower than the Ramsey price (29).

Under the condition for the interiorior solution (32), the insurance expenditure under the optimal price-insurance policy is positive. Furthermore, we note that, in the case of distortionary taxation, the expenditure is less than the fixed cost. On the other hand, when $\lambda \to 0$, the insurance expenditure approaches the fixed cost. The intuition behind this relationship between the optimal insurance expenditure and the social cost of public funds is as follows: the higher (lower) λ is the less (more) willing the regulator is to use taxation as a means to finance the fixed cost of producing the pharmaceuticals.

Table 1 Consumer surplus, profit, insurance expenditure and social welfare

	First-best	Ramsey	Price-insurance policy
CS	$\frac{(\eta-c)^2}{2\eta}$	$\frac{1}{8\eta}\left(\eta-c+\sqrt{(\eta-c)^2-4\eta F}\right)^2$	$\frac{(\eta-c)^2}{2\eta}\left(\frac{1+\lambda}{1+2\lambda}\right)^2$
π	0	0	0
IE	n.a.	n.a.	$F-\frac{(\eta-c)^2\lambda(1+\lambda)}{\eta(1+2\lambda)^2}$
W	$\frac{(\eta-c)^2}{2\eta}-F$	$\frac{1}{8\eta}\left(\eta-c+\sqrt{(\eta-c)^2-4\eta F}\right)^2$	$\frac{(\eta-c)^2}{2\eta}\frac{(1+\lambda)^2}{(1+2\lambda)}-F(1+\lambda)$

For the purpose of Proposition 2, we denote the social welfare under optimal price-insurance policy as follows:

$$\tilde{W}=\frac{(\eta-c)^2}{2\eta}\frac{(1+\lambda)^2}{(1+2\lambda)}-F(1+\lambda). \tag{37}$$

A comparison of the social welfare in the first-best solution and under the optimal price-insurance policy does to directly reveal that the first-best social welfare exceeds the social welfare under the optimal price-insurance policy (Table 1). However, Proposition 2 below demonstrates that the first-best welfare indeed exceeds the social welfare in the optimal price-insurance policy.

Comparing the social welfare under the optimal price-insurance policy (37) with the social welfare obtained from the Ramsey solution (31) leads to a striking observation. The social insurance improves the welfare because the resulting gain in consumer surplus exceeds the corresponding increase in the public expenditures (Proposition 2). This result is an illustration of the general theory of second best (Lipsey and Lancaster, 1956) where the introduction of a distortive policy instrument improves the welfare in an inefficient market.

The underlying reason for the finding that the health insurance is welfare improving is the fact that the optimal insurance in our model is combined with regulated producer prices. It is well-known in health economics that if the health insurance leads to higher prices of medical products (Pauly 1968; Feldstein 1973), the introduction of health insurance is detrimental to welfare. In the context of our model, insurance decreases out-of-pocket price, increases the demand of the

[8]The proof is available from the authors.

pharmaceutical but is associated with a lower producer price (due to economies of scale) than before the introduction of insurance, leaving space for a possible welfare improvement (Gaynor et al., 2000).

Proposition 2. *The welfare ranking between the first-best solution, the Ramsey solution and the optimal price-insurance policy is the following:*

$$\bar{W}_f > \tilde{W} > W_R. \tag{38}$$

Proof. See Appendix.

Intuitively, the Ramsey solution produces a smaller welfare than the optimal price-insurance policy, because a great many people are not able to acquire the drug at Ramsey prices. The optimal policy $(\tilde{p}, \tilde{r})$, however, does not reach the efficient solution because of the social cost of public funds.

6 Means-tested price-insurance policy

The previous analysis on the optimal price-insurance policy demonstrated how the introduction of health insurance can improve efficiency of the pharmaceutical market in comparison with the case where no health insurance is available. From the equity point of view, however, the optimal price-insurance policy has a serious limitation. Patients in the cohort of lowest incomes cannot afford to buy the pharmaceutical even in the presence of the health insurance. The number of such low-income patients is $1 - q(\tilde{p}, \tilde{r}) > 0$. Health is not like any other product, and equity considerations suggest that patients with low ability to pay should also have an access to the pharmaceutical treatment.

In this section, we examine an approach that adjusts the price-insurance policy to cope with vertical equity. In the welfare economics, the idea of equity has been introduced in terms of the Rawlsian welfare criterion. Based on Rawls (1999), it is typically expressed as the maximin rule of the social choice[9]. Accordingly, the policy should aim at considering the utility of the individual who is worst off. In this section, the implications of the Rawlsian equity principle are examined in terms of a means-tested insurance policy implemented in the form of a third-degree price discrimination. In particular, we examine an optimal insurance policy that offers a higher insurance coverage for low-income patients not able to purchase the pharmaceutical at the out-of-pocket price paid by high-income patients. The advantage of the suggested approach here is that it combines a solution for equity with an efficient insurance for those in higher income classes.

We analyze a model where people with high ability to pay and people with low ability to pay are entitled to different coverage rates, say $r_h \leq r_l$, where subscripts h and l refer to high-ability to pay (high-income) and low-ability to pay (low-income) patients, respectively. In particular, in what follows we will focus on the price-insurance mechanism (p, r_l, r_h) with the feature $r_h \leq r_l$. Under this mechanism, the regulator offers the price p for the firm and selects the parameters of insurance coverage for high-income and low-income patients so that the out-of-pocket price of low-income patients is lower than that of the high-income patients. Since the income variable is a continuous variable, we define low-income patients as the patient group not able to purchase pharmaceutical at price p and insurance coverage r_h. This implies that the groups of low- and high-income patients are determined endogenously on the basis of the policy parameters (p, r_l, r_h). The question is raised in particular, where to draw the demarcation lines between those who should have access to medication with price-insurance contracts (p, r_h) and (p, r_l).

[9]The Rawlsian view has been widely discussed in welfare economics. For a recent analysis, one can refer to Stark, Jakubek, and Falniowski (2014), for example.

We assume in what follows that the regulator has full information on patients' income and hence is able to fully identify low- and high-income patient groups and offer them different price-insurance coverage packages. If the regulator would not have full information on patients' income and offered two price-insurance contracts (p, r_l) and (p, r_h), all consumers in the market would prefer the contract offered to low income patients because of a higher insurance coverage. Hence, the optimal contract that we will derive next is not incentive compatible if incomplete information concerning patients' income exists between the regulator and patients. To make the contract implementable, we assume that the regulator is fully informed about the patients' income.

Given the price-insurance mechanisms (p, r_l, r_h), the aggregate consumer surplus is given as follows:

$$CS(p, r_l, r_h) = \int_{\frac{p(1-r_l)}{\eta}}^{\frac{p(1-r_h)}{\eta}} (w\eta - (1-r_l)p)\, dw + \int_{\frac{p(1-r_h)}{\eta}}^{1} (w\eta - (1-r_h)\, p)\, dw. \tag{39}$$

Under this mechanism, the aggregate demand is the sum of the demands of the buying high- and low-income patients:

$$\begin{aligned} q(p, r_l, r_h) &= q_l(p, r_l, r_h) + q_h(p, r_l, r_h) \\ &= \frac{p(1-r_h)}{\eta} - \frac{p(1-r_l)}{\eta} + 1 - \frac{p(1-r_h)}{\eta} = 1 - \frac{p(1-r_l)}{\eta}, \end{aligned} \tag{40}$$

and the profit of the firm is given as follows:

$$\pi(p, r_l, r_h) = (p-c)q(p, r_1, r_h) - F. \tag{41}$$

The total insurance expenditure consists of the insurance reimbursements paid to the high- and low-income patients

$$IE(p, r_l, r_h) = r_l p \left(\frac{p(1-r_h)}{\eta} - \frac{p(1-r_l)}{\eta} \right) + r_h p \left(1 - \frac{p(1-r_h)}{\eta} \right). \tag{42}$$

The regulator's problem is to choose the price and insurance policy (p, r_h, r_l) that maximizes social welfare (18) subject to the profit constraint (19), the constraint on insurance coverage rates $r_h \leq r_l$, and the feasibility constraints $p \geq 0$ and $0 \leq r_t \leq 1$ for $t = l, h$. The consumer surplus, profit and insurance expenditures in the current problem are defined in expressions (39), (41) and (42). The following proposition characterizes the optimal means-tested price-insurance mechanism.

Proposition 3. *If* $\lambda > 0$ *and*

$$\frac{(\eta - c)^2 2(1+\lambda)(1+2\lambda)}{\eta(2+3\lambda)^2} < F, \tag{43}$$

the optimal means-tested price-insurance policy in the interior solution is

$$\hat{p} = c + \frac{\eta F\,(2+3\lambda)}{(\eta - c)\,2\,(1+\lambda)} \tag{44}$$

and

$$\hat{r}_l = \frac{\eta F(2+3\lambda)^2 - (\eta - c)^2\, 2\lambda\,(1+\lambda)}{(2+3\lambda)\,(\eta F(2+3\lambda) + c(\eta - c)2(1+\lambda))} \tag{45}$$

$$\hat{r}_h = \frac{\eta F(2+3\lambda)^2 - (\eta - c)^2\, 2\,(1+\lambda)\,(1+2\lambda)}{(2+3\lambda)\,(\eta F(2+3\lambda) + c(\eta - c)2(1+\lambda))}. \tag{46}$$

Proof. See Appendix.

It follows from the fact that $(1+\lambda)(1+2\lambda) > (1+\lambda)\lambda$ that $r_h < r_l$ and that the insurance coverage of the low-income group exceeds that of the high-income group in the optimal means-tested solution. In addition, a direct comparison of the optimal prices $\hat{p}$ and $\tilde{p}$ demonstrates that $\hat{p} < \tilde{p}$ and that optimal price in the means-tested policy is lower than in the optimal policy in absence of means-testing (Section 5). Therefore, the introduction of the means-testing on the insurance policy also has implications on the price of the pharmaceutical.

The above results will become explicit when evaluating the welfare properties of the optimal means-tested policy. The out-of-pocket payment of the high-income patients is

$$\hat{p}(1 - \hat{r}_h) = \frac{\eta(1+2\lambda) + c(1+\lambda)}{2+3\lambda} \tag{47}$$

and that of the low-income patients is

$$\hat{p}(1 - \hat{r}_l) = \frac{\eta\lambda + 2c(1+\lambda)}{2+3\lambda}. \tag{48}$$

Because $\hat{r}_l > \hat{r}_h$, buying low-income patients pay less for pharmaceuticals out of pocket than buying high-income patients. Straightforward computation shows that the out-of-pocket price of high-income (low-income) patients is higher than the monopoly price $p^m = (\eta + c)/2$ (marginal cost). More strikingly, the optimal out-of-pocket payments ensure equal access to pharmaceutical treatment and low- and high-income patients consume the same amount of the pharmaceutical

$$q_l(\hat{p}, \hat{r}_h, \hat{r}_l) = q_h(\hat{p}, \hat{r}_h, \hat{r}_h) = \frac{(\eta - c)\,(1+\lambda)}{\eta(2+3\lambda)} \equiv x(\hat{p}, \hat{r}_h, \hat{r}_l). \tag{49}$$

The total consumption of pharmaceuticals is then $q(\hat{p}, \hat{r}_h, \hat{r}_l) = 2x(\hat{p}, \hat{r}_h, \hat{r}_l)$. Equal division of the market shows up also in the consumer surplus,

$$CS_l(\hat{p}, \hat{r}_h, \hat{r}_l) = CS_h(\hat{p}, \hat{r}_h, \hat{r}_l) = \frac{(\eta - c)^2\,(1+\lambda)^2}{2\eta(2+3\lambda)^2} \equiv S^c(\hat{p}, \hat{r}_h, \hat{r}_l), \tag{50}$$

The aggregate surplus is $CS(\hat{p}, \hat{r}_h, \hat{r}_l) = 2S^c(\hat{p}, \hat{r}_h, \hat{r}_l)$. We state these findings as follows:

Proposition 4. *Under the Rawlsian principle of equity based on maximizing the aggregate consumer surplus and conditional on the better access to medication by low-income people by means-tested insurance coverage, the final consumption of the pharmaceutical and also the consumer surplus is split equally between the low- and high-income patients.*

The result is sharp and it provides a yardstick when alternative equity principles are considered. Hence, and somewhat strikingly, although the patients with low ability to pay obtain the pharmaceutical at the lower out-of-pocket price, their surplus at the optimal solution is no higher than the surplus of the patients with high ability to pay.

When evaluated at the optimal solution, the aggregate insurance expenditure amounts to

$$IE(\hat{p}, \hat{r}_h, \hat{r}_l) = F - \frac{(\eta - c)^2(1 + 4\lambda + 3\lambda^2)}{\eta(2 + 3\lambda)^2}. \tag{51}$$

By Proposition 4 the high- and low-income consumers consume the same amount of the pharmaceutical. In addition, since $\hat{r}_l > \hat{r}_h$ and the optimal insurance coverage of low-income patients is higher than that of high-income patients, insurance expenditures paid to subsidize the consumption of low-income group is higher than the corresponding expenditures of the high-income group:

$$\hat{r}_1 \hat{p} q_h(\hat{p}, \hat{r}_h, \hat{r}_l) > \hat{r}_h \hat{p} q_l(\hat{p}, \hat{r}_h, \hat{r}_l). \tag{52}$$

Similarly as in Section 5, the pharmaceutical firms earns zero profit (Proof of Proposition 5). Then, the social welfare is given as follows:

$$\hat{W} = CS(\hat{p}, \hat{r}_h, \hat{r}_l) - (1 + \lambda) IE(\hat{p}, \hat{r}_h, \hat{r}_l) = \frac{(\eta - c)^2 (1 + \lambda)^2}{\eta(2 + 3\lambda)} - (1 + \lambda)F. \tag{53}$$

We then compare the welfare obtained from the policy paying explicit attention for equity with the welfare obtained from the optimal price-insurance policy with no concern for the low-income patients (Section 5).

Proposition 5. *Suppose that $\lambda > 0$. Then the optimal means-tested price-insurance policy $(\hat{p}, \hat{r}_l, \hat{r}_h)$ yields a strictly higher welfare than the optimal price-insurance policy $(\tilde{p}, \tilde{r})$ and $\hat{W} > \tilde{W}$.*

Proof. That $\hat{W} > \tilde{W}$ follows directly from the fact $2 + 3\lambda < 2(1 + 2\lambda)$. ‖

Stated verbally, under the Rawlsian criterion, the social welfare exceeds the social welfare under an optimal price-insurance policy with a uniform coverage rate (Section 5). Third-degree out-of-pocket price discrimination, or means-tested insurance benefits, further improve the social welfare by increasing the consumption of the low-income patients. At the same time, it also increases insurance expenditures but at a rate that is less than the increase of consumer surplus due to increased consumption of the pharmaceutical product.

7 Towards the value of life: health as an investment

The consumer surplus, based on the subjective valuation of pharmaceutical by those with a sufficiently high ability to pay, appears as an appropriate measure of welfare for those with high ability to pay. An alternative social criterion is also feasible when valuing the welfare of the low-income patients. Taking care of everyone may be regarded as a social norm and value as such. In order to study the implications of such a view, we denote the social value of a low-income patient (not being able to consume the pharmaceutical at optimal price-insurance policy $(\tilde{p}, \tilde{r})$) by $v > 0$.[10] A natural interpretation is that by providing the medication, the society can recover, say, the ability to work of these people. Then v can be taken to measure the social value of low-income patients in terms of the value-added they produce reflected in their market wage[11]. Such an approach points to the interpretation that the society can regard the expenditure on the state of health of the patients with a low ability to pay as an investment. The measure of value-added, v, can be viewed as an opportunity cost for the society in the case where the poor do not have access to medication. The criterion suggested in this section represents a complementary argument for that in Section 6 to deal with the Rawlsian principle.

Suppose that, instead of means-tested insurance coverage examined in Section 6, the regulator implements a policy (p^v, r^v) equivalent to that studied in Section 5 and provides full insurance[12] to low-income patients who are not able to purchase the drug at out-of-pocket price $p^v(1-r^v)$. As there are $1-q(p^v, r^v)$ such low-income patients, we introduce a term $v\,(1-q(p^v, r^v))$ into the social welfare. The appropriate policy target now is the maximization of the sum of the welfare of the self-paying patients and the social value of low-income patients whose access to the consumption of the pharmaceutical is financed through the full social health insurance (Arrow 1963; Pauly 1968). The coverage for the paying customers and the price of the pharmaceutical remain the optimizing variables of the regulator. We assume that both the consumer surplus of the high-income patients and the social value of low-income patients $v\,(1-q(p^v, r^v))$ are measured in monetary units. The aggregate social value of pharmaceutical consumption is then

$$CS(p^v, r^v) = v \int_0^{\frac{p^v(1-r^v)}{\eta}} dw + \int_{\frac{p^v(1-r^v)}{\eta}}^{1} (w\eta - (1-r^v)p^v)dw. \tag{54}$$

Given the policy (p^v, r_l, r_h), where $r_h = r^v$ and $r_l = 1$, all patients consume the pharmaceutical, because low-income patients have access to free medication. The profit of the firm is given as

$$\pi(p^v, r^v) = p - c - F, \tag{55}$$

and the insurance expenditure can be computed as the sum of insurance expenditures for low-income and high-income patients

$$IE(p^v, r^v) = p^v \left(\frac{p^v(1-r^v)}{\eta}\right) + r^v p^v \left(1 - \frac{p^v(1-r^v)}{\eta}\right) \tag{56}$$

As above, the regulator chooses the price and insurance policy (p^v, r^v) that maximizes social welfare (18) subject to the profit constraint (19) and the feasibility constraints $p^v \geq 0$ and $0 \leq r^v \leq 1$. The consumer surplus, profit and insurance expenditures in this problem are defined in

[10]If the medication is absolutely necessary for the survival of the patient, v can alternatively be regarded as the value of human life. The issue of the value of life has been long discussed in economics. Health care programs are but one of the many public policy initiatives that have mortality reductions as their primary goal. The proper social cost-benefit analysis requires an estimate of the value the society places on a life saved. Evaluating the economic value of a statistical life is now part of the generally accepted economic methodology and a large literature has developed to estimate it (Mrozek and Taylor (2001), Viscusi (2003), Brannon (2004,2005). Economists often estimate the value of life thus by looking at the risks that people are voluntarily willing to take, or how much they must be paid for taking them, see also Mankiw (2012). Richard Thaler, the 20th Nobel prize winner in Economics reports that the value of saving life, employed in the US cost-benefit analysis is about 7 million US$ (Thaler 2015).

[11]We take that v is the average productivity. Introducing the whole distribution is possible but would only add an unnecessary complexity in the model.

[12]We assume full insurance for low-income patients and examine under which conditions such a (restricted) policy is welfare improving relative to the optimal price-insurance policy (Section 5). The assumption on full insurance for low-income patients can be relaxed following the principles presented in Section 6.

expressions (54), (55) and (56). Proposition 6, below, displays the optimal means-tested price-insurance policy with a value-of-life criterion.

Proposition 6. *If* $\lambda > 0$ *and*

$$\frac{v+\eta\lambda}{1+2\lambda} < c+F, \tag{57}$$

the optimal means-tested price-insurance policy with a value-of-life criterion is given by

$$\hat{p}^v = c+F \tag{58}$$

and

$$\hat{r}^v = 1 - \frac{v+\eta\lambda}{(1+2\lambda)(c+F)}. \tag{59}$$

Proof. See Appendix.

Optimal means-tested policy under the value-of-life criterion and free medication to low-income patients creates a conflict of interest between the low-income and high-income patients groups. The conflict of interest between the high-income and low-income patients depends on by how much the society values the recovered health of the low-income patients. In particular, if the social preferences justify the regulator imputing an increased value of recovered health to patients with low ability to pay, the optimal coverage of those with high ability to pay is accordingly reduced, that is $\partial\hat{r}^v/\partial v < 0$.

This conflict of interest appears similarly in the analysis of out-of-pocket payments and demands of the low- and high-income patients. Firstly, the out-of-pocket payment of the high-income patients

$$\hat{p}^v(1-\hat{r}^v) = \frac{v+\eta\lambda}{1+2\lambda} \tag{60}$$

is strictly increasing in the social value of recovered health v of low-income patients. On the other hand, full insurance for low-income group implies free medication for patients with low ability to pay. The high-income patients' demand for the pharmaceutical

$$q(\hat{p}^v, \hat{r}^v) = \frac{\eta(1+\lambda)-v}{\eta(1+2\lambda)} \tag{61}$$

is decreasing in the of value of recovered health of low-income patients, while the demand for the pharmaceutical in the low-income group

$$1 - q(\hat{p}^v, \hat{r}^v) = \frac{v+\eta\lambda}{\eta(1+2\lambda)}. \tag{62}$$

is increasing in the social value of recovered health.

Quality of the drug and the social cost of public funds have striking implications on the demands and insurance expenditures of the low- and high-income patients. An increase in the quality of the drug raises the out-of-pocket price of the high-income patients and increases (decreases) the number of people classified as high-income (low-income) people. Secondly, consider the case when the social cost of public funds increases. Interesting enough, it is optimal to expand the share of population which is entitled to free medication, i.e $\partial[1-q(\hat{p}^v,\hat{r}^v)]/\partial\lambda > 0$, if $\eta > 2v$, and the quality of the drug is sufficiently high. The intuition behind this result is that though the share of population classified as low-income people increases the public expenditures, the high-income people have access to a more limited public insurance. We state this as follows:

Proposition 7. *If the quality of the drug is high i.e.* $\eta > 2v$, *an increase in the social cost of public funds increases the number of people entitled to free medication.*

Proof. The result follows by differentiating Eq. (62) with respect to λ. ‖

Results concerning consumer surpluses, profit, insurance expenditures and social welfare in the means-tested solution with value-of-life criterion are displayed in Table 2. Subscripts l and h in the consumer surpluses and insurance expenditures refer to the measures associated with low- and high-income patients.

Table 2 Consumer surpluses, profit, insurance expenditures and social welfare

		Means-testing solution with the value-of-life criterion
CS		$\frac{\eta^2(1+\lambda)^2-2v\eta(1-2\lambda^2)+v^2(3+4\lambda)}{2\eta(1+2\lambda)^2}$
	CS_l	$\frac{v(v+\eta\lambda)}{\eta(1+2\lambda)}$
	CS_h	$\frac{[\eta(1+\lambda)-v]^2}{2\eta(1+2\lambda)^2}$
π		0
IE		$F+c-\frac{\eta^2\lambda(1+\lambda)+v(\eta-v)}{\eta(1+2\lambda)^2}$
	IE_l	$\frac{(c+F)(v+\eta\lambda)}{\eta(1+2\lambda)}$
	IE_h	$\frac{[(c+F)(1+2\lambda)-(v+\eta\lambda)][\eta(1+\lambda)-v]}{\eta(1+\lambda)^2}$
W		$\frac{\eta^2(1+\lambda)^2+v(v+2\eta\lambda)}{2\eta(1+2\lambda)}-(1+\lambda)(c+F)$

It is worth noting that the consumer surplus of the high-income patients is monotonically decreasing in the social value of the pharmaceutical consumption of the low-income patients, when $v \leq \eta(1+\lambda)$ (Table 2) and the social value of the recovered health of low-income patients is sufficiently small. The same condition also ensures the positive consumption of the high-income group (61). Along the same lines, the insurance expenditures paid for the consumption of the high-income patients is positive under the assumption $(c+F)(1+2\lambda) > v+\eta\lambda$ ensuring the interior solution (Proposition 6), if social value of recovered health of low-income patients satisfies the condition $v \leq \eta(1+\lambda)$ (Table 2). Insurance expenditures paid for the consumption of the low-income patients are always positive-valued.

For the purposes of Proposition 8, we denote the social welfare under the means-tested solution with value-of-life criterion as follows

$$\hat{W}^v = \frac{\eta^2(1+\lambda)^2 + v(v+2\eta\lambda)}{2\eta(1+2\lambda)} - (1+\lambda)(c+F). \quad (63)$$

The following proposition derives the conditions for the social welfare $\hat{W}^v$ to exceed the social welfare in the situation with a uniform insurance coverage and no equity concern for low-income patients (Section 5).

Proposition 8. *Suppose that* $\lambda > 0$. *Then the optimal means-tested price-insurance policy* $(\hat{p}^v, \hat{r}_1^v, 1)$ *with the value-of-life criterion produces a higher welfare than the optimal price-insurance policy* $(\tilde{p}, \tilde{r})$ *with no equity concern, that is* $\hat{W}^v \geq \tilde{W}$, *if* $v \geq c(1+\lambda)$, *namely, if the value of the recovered health of low-income people exceeds the social marginal cost of producing the medication.*

Proof. See Appendix.

8 Final remarks

Previous work on the optimal price regulation of pharmaceutical products and the health insurance has produced a number of contributions. Our paper has extended the previous work in three ways. First, we have considered a market where the ability to pay differs in the patient population. Second, we have endogenized the insurance coverage and derived the optimal price regulation together with the insurance coverage. Third, we have examined various solutions that improve the access of low-income patients to the pharmaceutical treatment. While most of the earlier papers have abstracted from different abilities to pay for the pharmaceutical products, we have extended our analysis to the case of the socially optimal third-degree out-of-pocket price discrimination through means-tested insurance coverages. We have also derived an equilibrium where people with a low ability to pay have access to the full coverage while those with a high ability to pay have partial coverage. This turned out to be well motivated: the ability to pay of the high-income people is turned into willingness to pay while this cannot hold for the low-income people. Our results are informative in guiding decision-makers regulating the prices and the reimbursement of new pharmaceuticals.

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9 Appendix

Proof of Proposition 1. The regulator's problem is to find the price-insurance pair (p, r) which maximizes the social welfare

$$W = CS(p,r) + \pi(p,r) - (1+\lambda)IE(p,r) \tag{64}$$

subject to the profit constraint

$$-\pi(p,r) \leq 0 \tag{65}$$

and the feasibility constraints

$$-p \leq 0 \tag{66}$$

$$0 \leq r \leq 1. \tag{67}$$

The above problem is called as the original problem, OP. In what follows, we analyze the solutions of the original problem without the feasibility constraints. Such a problem will be called the relaxed problem, RP. This approach to finding the solution to the regulator's problem through the relaxed problem rests on the intuition that, if solutions of the relaxed problem also satisfy the feasibility constraints, then they must solve the original problem.

Let $(\tilde{p}, \tilde{r})$ denote the price-insurance policy solving the relaxed problem and μ the Lagrange multiplier of the profit constraint. The Lagrangian function of the relaxed problem can be written as

$$L = CS(p,r) + (1+\mu)\,\pi(p,r) - (1+\lambda)IE(p,r). \tag{68}$$

The solution of the relaxed problem must satisfy the first-order conditions:

$$\frac{\partial L}{\partial p} = -(1-r)\left[1 - \frac{p(1-r)}{\eta}\right] + (1+\mu)\left[1 - \frac{2p(1-r)}{\eta} + \frac{(1-r)c}{\eta}\right] - (1+\lambda)\,r\left[1 - \frac{2p(1-r)}{\eta}\right] = 0 \tag{69}$$

$$\frac{\partial L}{\partial r} = p\left[1 - \frac{p(1-r)}{\eta}\right] + (1+\mu)(p-c)\frac{p}{\eta} - (1+\lambda)\,p\left[1 - \frac{p(1-r)}{\eta} + \frac{pr}{\eta}\right] = 0. \tag{70}$$

Moreover, the solution must satisfy the profit constraint and the complementary slackness conditions $-\pi(p,r) \leq 0$, $\mu \geq 0$ and

$$\mu\left[F - (p-c)\left(1 - \frac{p(1-r)}{\eta}\right)\right] = 0. \tag{71}$$

Lemma 1.1 If $(\tilde{p}, \tilde{r}, \tilde{\mu})$ solves the relaxed problem, then $\tilde{\mu} = \lambda$.

Proof. Contrary to the claim, suppose that $\mu \neq \lambda$ in the solution of the relaxed problem. Then the first-order conditions (69) and (70) have two solutions. The first solution is $\hat{p} = 0$ and $\hat{r} = [\eta\mu + c(1+\mu)]/[\eta\lambda + c(1+\mu)]$, and the second solution is $\check{p} = [\eta(1+\lambda) - c(1+\mu)]/[\lambda - \mu]$ and $\check{r} = [(\eta - c)(1+\mu)]/[\eta(1+\lambda) - c(1+\mu)]$. When evaluated at these two solutions, the profit of the firm is $-\pi(\hat{p}, \hat{r}) = c + F$ and $-\pi(\check{p}, \check{r}) = F$, respectively. Therefore, the solutions of the first-order conditions (69) and (70) never satisfy the profit constraint. This implies that, if $\mu \neq \lambda$, there is no price-insurance pair which would satisfy the necessary conditions of the relaxed problem. For solutions to exist, we must therefore have $\mu = \lambda$. ‖

Lemma 1.2 If $(\tilde{p}, \tilde{r}, \tilde{\mu})$ solves the relaxed problem, then any pair $(\tilde{p}, \tilde{r})$ satisfying

$$p = \frac{\eta\lambda + c(1+\lambda)}{(1-r)(1+2\lambda)} \tag{72}$$

satisfies both first-order conditions (69) and (70).

Proof. Suppose that $(\tilde{p}, \tilde{r}, \tilde{\mu})$ solves the relaxed problem. Then, the first-order condition (69) holds true for any pair (p, r) for which

$$p = \frac{\eta\tilde{\mu} + c(1+\tilde{\mu}) - r\left[\eta\lambda + c(1+\tilde{\mu})\right]}{(1-r)\left[1 + 2\tilde{\mu} - r(1+2\lambda)\right]} \tag{73}$$

and the first-order condition (70) is satisfied for any pair (p, r) for which

$$p = \frac{\eta\lambda + c(1+\tilde{\mu})}{1 + \tilde{\mu} + \lambda - r(1+2\lambda)} \text{ or } p = 0. \tag{74}$$

The solution $p = 0$ can be ruled out because it does not satisfy the profit constraint. By Lemma 1, the solution of the relaxed problem must satisfy $\tilde{\mu} = \lambda$. Evaluating the right-hand sides of the equations (73) and (74) at $\tilde{\mu} = \lambda$ yields the equation (72). ‖

Let us then characterize the solution of the problem. By Lemma 1.1 and the assumption $\lambda > 0$, we must have $\tilde{\mu} = \lambda > 0$. Then it follows from the complementary slackness conditions that the zero profit condition $\pi(p, r) = 0$ must hold true at the solution of the regulator's problem. Solving the first-order conditions (69) and (70) together with the zero profit condition yields the price and insurance policy and the value of the Lagrange multiplier:

$$\tilde{p} = c + \frac{\eta F(1+2\lambda)}{(\eta - c)(1+\lambda)} \tag{75}$$

$$\tilde{r} = \frac{\eta F(1+2\lambda)^2 - (\eta - c)^2\lambda(1+\lambda)}{(1+2\lambda)\left[\eta F\left(1+2\lambda\right) + c(\eta - c)\left(1+\lambda\right)\right]} \tag{76}$$

$$\tilde{\mu} = \lambda. \tag{77}$$

When evaluated at the point $(\tilde{p}, \tilde{r}, \tilde{\mu})$, the determinant of the bordered Hessian matrix is

$$\left|\bar{H}\right| = \frac{\left(c(\eta - c)(1+\lambda) + \eta F\left(1+2\lambda\right)\right)^2}{\eta^3(1+2\lambda)} > 0, \tag{78}$$

which proves that the optimal policy is a local maximum.

Let us then check that the solution of the relaxed problem satisfies the feasibility conditions. It is straightforward to establish that the optimal insurance policy satisfies the condition $\tilde{r} < 1$. In addition, it holds true that $\tilde{r} > 0$ if the fixed cost satisfies the conditions (32). And since the optimal price $\tilde{p}$ is strictly positive, the solution satisfies the feasibility conditions of the original problem.‖

Proof of Proposition 2. We first prove that $\tilde{W} \geq W_R$. Define the welfare difference

$$DW(F) \equiv \tilde{W} - W_R = \tag{79}$$

$$\frac{(\eta - c)^2}{2\eta}\frac{(1+\lambda)^2}{1+2\lambda} - F(1+\lambda) - \frac{1}{8\eta}\left(\eta - c + \sqrt{(\eta-c)^2 - 4\eta F}\right)^2. \tag{80}$$

The first partial derivative of the welfare difference with respect to the fixed cost F is given as

$$\frac{\partial DW}{\partial F} = -(1+\lambda) + \frac{\eta - c + \sqrt{(\eta-c)^2 - 4\eta F}}{2\sqrt{(\eta-c)^2 - 4\eta F}}, \tag{81}$$

and the second partial derivative is

$$\frac{\partial^2 DW}{(\partial F)^2} = \frac{\eta(\eta - c)}{\left(\sqrt{(\eta-c)^2 - 4\eta F}\right)^3} > 0. \tag{82}$$

Hence, the welfare difference is a strictly convex function of the fixed cost F. The strict convexity of the function $DW(F)$ implies that the unconstrained minimum of the welfare difference with respect to the fixed cost (if one exists) must be unique. Solving the first-order condition $\partial DW/\partial F = 0$ with respect to F yields the minimum point

$$F_1 = \frac{(\eta - c)^2}{\eta}\frac{\lambda(1+\lambda)}{(1+2\lambda)^2} \geq 0, \tag{83}$$

which corresponds to the infimum of the interval of the fixed cost in the interior solution. This implies that $DW(F) > DW(F_1)$ for all values of the fixed cost, which satisfy the condition (32). When evaluated at the minimum point the value of the welfare difference is zero:

$$\begin{aligned} DW(F_1) &= \frac{(\eta - c)^2}{2\eta}\frac{(1+\lambda)^2}{1+2\lambda} - F_1(1+\lambda) - \frac{1}{8\eta}\left(\eta - c + \sqrt{(\eta-c)^2 - 4\eta F_1}\right)^2 \\ &= \frac{(\eta - c)^2}{2\eta}\left(\frac{1+\lambda}{1+2\lambda}\right)^2(1+2\lambda) - \frac{(\eta - c)^2}{2\eta}\left(\frac{1+\lambda}{1+2\lambda}\right)^2 2\lambda - \frac{(\eta - c)^2}{2\eta}\left(\frac{1+\lambda}{1+2\lambda}\right)^2 \\ &= 0 \end{aligned}$$

Hence $DW(F) > DW(F_1) = 0$ and $\tilde{W} > W_R$ for all solutions.

Secondly, we have $\bar{W}_f \geq \tilde{W}$ when

$$\frac{(\eta - c)^2}{2\eta} - F \geq \frac{(\eta - c)^2}{2\eta}\frac{(1+\lambda)^2}{(1+2\lambda)} - F(1+\lambda), \tag{84}$$

which implies that

$$\frac{(\eta - c)^2}{2\eta}\frac{\lambda}{1+2\lambda} \leq F. \tag{85}$$

But now

$$\frac{(\eta - c)^2}{2\eta}\frac{\lambda}{1+2\lambda} = \frac{(\eta - c)^2}{2\eta}\frac{\lambda(1+2\lambda)}{(1+2\lambda)^2} < \frac{(\eta - c)^2}{2\eta}\frac{2\lambda(1+\lambda)}{(1+2\lambda)^2}, \tag{86}$$

where the last expression corresponds to the infimum of the set of fixed costs inducing the optimal price-insurance solution to be interior. Hence, the condition (85) is satisfied as a strict inequality in the interior solution (with condition (32)), when $\lambda > 0$. ‖

Proof of Proposition 3. Suppose that $\lambda > 0$. We will concentrate on solving the relaxed problem. The Lagrangian function of the relaxed problem is given as follows

$$L = CS(p, r_h, r_l) + (1+\mu)\,\pi(p, r_h, r_h) - (1+\lambda)\,IE(p, r_h, r_l) - \kappa\,(r_h - r_l)\,, \tag{87}$$

where the consumer surplus, the firm's profit and the insurance expenditure are defined in (39), (41) and (42) and κ is the multiplier of the constraint $r_h \leq r_l$. The solution of the relaxed problem must satisfy the first-order conditions:

$$\begin{aligned}
\frac{\partial L}{\partial p} &= \frac{p\,(r_l - r_h)^2}{\eta} - (1-r_h)\left[1 - \frac{p\,(1-r_h)}{\eta}\right] \\
&+ (1+\mu)\left[1 - \frac{2p(1-r_l)}{\eta} + \frac{(1-r_l)c}{\eta}\right] \\
- (1+\lambda)&\left[r_l\left(\frac{2p(r_l - r_h)}{\eta}\right) + r_h\left(1 - \frac{2p(1-r_h)}{\eta}\right)\right] = 0
\end{aligned} \tag{88}$$

$$\begin{aligned}
\frac{\partial L}{\partial r_h} &= \frac{-p^2(r_l - r_h)}{\eta} + p\left(1 - \frac{p(1-r_h)}{\eta}\right) \\
-(1+\lambda)&\left[\frac{-p^2(r_l - r_h)}{\eta} + p\left(1 - \frac{p(1-r_h)}{\eta}\right)\right] - \kappa = 0
\end{aligned} \tag{89}$$

$$\begin{aligned}
\frac{\partial L}{\partial r_l} &= \frac{p^2(r_l - r_h)}{\eta} + (1+\mu)(p-c)\frac{p}{\eta} \\
-(1+\lambda)&\left(\frac{p^2(r_l - r_h)}{\eta} + \frac{p^2 r_l}{\eta}\right) + \kappa = 0
\end{aligned} \tag{90}$$

Moreover, the solution must satisfy the profit constraint and its complementary slackness conditions $-\pi(p, r_h, p_l) \leq 0$, $\mu \geq 0$ and

$$\mu\left[F - (p-c)\left(1 - \frac{p(1-r_l)}{\eta}\right)\right] = 0. \tag{91}$$

and the means-testing constraint $r_h \leq r_l$ and its complementary slackness conditions $r_h - r_l \leq 0, \kappa \geq 0$ and

$$\kappa(r_h - r_l) = 0. \tag{92}$$

From the perspective of the ensuing analysis it is important to note that effective means-testing, ie. $r_h < r_l$, occurs in the solution of the regulator's problem only if $\kappa = 0$. If this is not the case and $\kappa > 0$ then by the condition (92) we must have $r_h = r_l$ in the optimal solution. Both low- and high-income patients receive the same insurance reimbursement and means-testing does not take place. Furthermore, it is straightforward to show that the necessary conditions of the problem simplify to the same as those in the optimal price-insurance policy examined in Section 5. Therefore, the following analysis concentrates on the means-testing solution in which $\kappa = 0$.

Lemma 3.1 If $(\hat{p}, \hat{r}_h, \hat{r}_l, \hat{\mu})$ solves the relaxed problem, then $\hat{\mu} = \lambda$.

Proof. Contrary to the claim, suppose that $\mu \neq \lambda$ in the solution of the relaxed problem. Then the first-order conditions (88), (89) and (90) have two solutions $(\hat{p}, \hat{r})$ and $(\check{p}, \check{r})$. In the first solution $\hat{p} = 0$ and insurance coverage rates must satisfy the condition (multiple solutions)

$$\hat{r}_h = \frac{1}{\lambda}\left[\mu + (1+\mu)\frac{c}{\eta}(1 - \hat{r}_l)\right].$$

In the second solution $\check{p} = [\eta(1+\lambda) - c(1+\mu)]/[\lambda - \mu]$ and $\check{r}_h = \check{r}_l = [(\eta - c)(1+\mu)]/[\eta(1+\lambda) - c(1+\mu)]$. When evaluated at these two solutions, the profit of the firm is $-\pi(\hat{p}, \hat{r}) = c + F$ and $-\pi(\check{p}, \check{r}) = F$, respectively. Therefore, the solutions of the first-order conditions (88), (89) and (90) never satisfy the profit constraint. This implies that, if $\mu \neq \lambda$, there are no price and insurance policies that would satisfy the necessary conditions of the relaxed problem. For solutions to exist, we must have $\mu = \lambda$. ‖

Let us then derive the solution of the regulator's problem. By Lemma 3.1 and by assumption $\lambda > 0$, we must have $\hat{\mu} = \lambda > 0$. Complementary slackness conditions for the profit constraint then imply that $\pi(\hat{p}, \hat{r}) = 0$. Solving first-order conditions (88), (89) and (90) together with the zero-profit condition yields the means-tested price and insurance policy and the value of the Lagrange multiplier:

$$\hat{p} = c + \frac{\eta F(2+3\lambda)}{(\eta - c)2(1+\lambda)} \tag{93}$$

$$\hat{r}_l = \frac{\eta F(2+3\lambda)^2 - (\eta - c)^2 2\lambda(1+\lambda)}{(2+3\lambda)(\eta F(2+3\lambda) + c(\eta - c)2(1+\lambda))} \tag{94}$$

$$\hat{r}_h = \frac{\eta F(2+3\lambda)^2 - (\eta - c)^2 2(1+\lambda)(1+2\lambda)}{(2+3\lambda)(\eta F(2+3\lambda) + c(\eta - c)2(1+\lambda))} \tag{95}$$

$$\hat{\mu} = \lambda. \tag{96}$$

Lemma 3.2 If $\eta > c > 0$, the solution $(\hat{p}, \hat{r})$ is a local maximum.

Proof. To check that the above solution is a local maximum, first note that the relevant bordered Hessian is a 4×4 matrix with the profit constraint binding. When evaluated at the solution of the problem, the determinants of the last two (ie. $n - k = 3 - 1 = 2$) leading principal minors of the bordered Hessian are

$$\left|\bar{H}_4\right| = \frac{-\lambda\left[2c(\eta - c)(1+\lambda) + F\eta(2+3\lambda)\right]^4}{4(\eta - c)^2\eta^4(1+\lambda)^2(2+3\lambda)} < 0, \tag{97}$$

and

$$\left|\bar{H}_3\right| = \frac{A(F)}{2(\eta - c)^2\eta^3(1+\lambda)^2(2+3\lambda)^2}, \tag{98}$$

where $A(F) = B + CF + DF^2$ is a quadratic function in the fixed cost. The expressions for B, C and D are given as follows:

$$B = (1+\lambda)^4(1+2\lambda)\left[8c^2(c^4 + 6c^2\eta^2 + \eta^4) - 32c^3\eta(c^2 + \eta^2)\right] \tag{99}$$

$$\begin{aligned} C = {} & 4F\eta c(1+\lambda)^2(2+3\lambda)[-c^3(1+\lambda)(2+5\lambda) + \eta^3(2+\lambda)(1+2\lambda) \\ & -\eta^2 c(6+\lambda(17+9\lambda)) + \eta c^2(6+\lambda(19+12\lambda))] \end{aligned} \tag{100}$$

$$\begin{aligned} D = {} & \eta^2(2+3\lambda)^2[c^2(1+\lambda)^2(2+7\lambda) - 2c\eta(1+\lambda)(2+\lambda(5+\lambda)) \\ & +\eta^2(2+\lambda(7+4\lambda(2+\lambda)))] \end{aligned} \tag{101}$$

To show that proposed solution is a local maximum point, we need to show that $\left|\bar{H}_3\right| > 0$. To do this it suffices to show that $A(F) > 0$ for all relevant values. We do this in two steps.

Step 1. First, we first show that $A(F)$ is a strictly convex function of the fixed cost by showing that $D > 0$ for all relevant values. Define

$$D_p(\eta) = \frac{D}{\eta^2(2+3\lambda)^2} = c^2(1+\lambda)^2(2+7\lambda) - 2c\eta(1+\lambda)(2+\lambda(5+\lambda)) + \eta^2(2+\lambda(7+4\lambda(2+\lambda))) \tag{102}$$

Since $\eta^2(2+3\lambda)^2 > 0$, to prove that $D > 0$ it suffices to demonstrate that $D_p(\eta) > 0$ for all relevant values of η. The expression $D_p(\eta)$ is a strictly convex function in η with unique minimum point η_m, which can be found by solving $D_p'(\eta) = 0$ with respect to η. When evaluated at η_m the value of $D_p(\eta)$ is

$$D_p(\eta_m) = \frac{c^2\lambda(1+\lambda)^2(2+3\lambda)^3}{2+\lambda(7+4\lambda(2+\lambda))} > 0 \tag{103}$$

where strict inequality holds true by the assumptions $c > 0$ and $\lambda > 0$. This implies that $D_p(\eta) \geq D_p(\eta_m) > 0$ for all η and hence also for parameter values $\eta > c$.

Step 2. By the first step, the expression $A(F)$ has a unique minimum point with respect to F, denoted as F_m, which can be found by solving the $A'(F) = 0$ with respect to F. When evaluated at the minimum point, the value of the function $A(F)$ is

$$A(F_m) = \frac{4c^2(\eta-c)^2\lambda(1+\lambda)^4(2+3\lambda)(c(1+\lambda)+\eta(1+2\lambda))^2}{c^2(1+\lambda)^2(2+7\lambda) - 2c\eta(1+\lambda)(2+\lambda(5+\lambda)) + \eta^2(2+\lambda(7+4\lambda(2+\lambda)))} \tag{104}$$

Step 1 above showed that the denominator of $A(F_m)$ is strictly positive for all relevant parameter values. Similarly, the numerator of the $A(F_m)$ is strictly positive by the assumptions $\eta > c > 0$ and $\lambda > 0$. Therefore $|\bar{H}_3| > 0$ under the assumptions $\eta > c > 0$ and $\lambda > 0$. ‖

The solution of the relaxed problem satisfies the feasibility constraint $p \geq 0$, and the condition (43) ensures that $\hat{r}_h > 0$. That $\hat{r}_l > 0$ then follows from the fact $\hat{r}_h < \hat{r}_l$. Furthermore, straightforward computation shows that $\hat{r}_t < 1$ for both $t = l, h$. ‖

Proof of Proposition 6. Suppose that $\lambda > 0$. The Lagrangian function of the relaxed problem is

$$L = CS(p^v, r^v) + (1+\mu)\pi(p^v, r^v) - (1+\lambda)IE(p^v, r^v), \tag{105}$$

where the consumer surplus, profit and insurance expenditures are defined in expressions (54), (55) and (56). The solution of the relaxed problem must satisfy the first-order conditions

$$\frac{\partial L}{\partial p^v} = \frac{v(1-r^v)}{\eta} - (1-r_1)\left[1 - \frac{p^v(1-r^v)}{\eta}\right] + (1+\mu) - (1+\lambda)\left[r^v + \frac{2p^v(1-r^v)^2}{\eta}\right] = 0 \tag{106}$$

$$\frac{\partial L}{\partial r^v} = \frac{-vp^v}{\eta} + p^v\left[1 - \frac{p^v(1-r^v)}{\eta}\right] - (1+\lambda)\,p^v\left[1 - \frac{2p^v(1-r^v)}{\eta}\right] = 0. \tag{107}$$

together with the profit constraint and complementary slackness conditions $-\pi(p^v, r^v) \leq 0$, $\mu \geq 0$ and

$$\mu\,(F + c - p^v) = 0. \tag{108}$$

Lemma 1. The solution of the relaxed problem satisfies $\mu^v = \lambda$.

Proof. Contrary to the claim suppose that $\mu^v \neq \lambda$ in the solution of the relaxed problem. Then the first-order conditions (106) and (107) hold true simultaneously only when $\hat{p}^v = 0$ and $\hat{r}^v = [\eta\mu^v + v]/[\eta\lambda + v]$. At this point the firm's profit is $-\pi(\hat{p}^v, \hat{r}^v) = c + F > 0$. Therefore, we have no solution, which would satisfy the necessary conditions of the problem, if $\mu^v \neq \lambda$. For a solution to exist, we must have $\mu^v = \lambda$. ‖

Lemma 2 If $(\hat{p}^v, \hat{r}^v, \hat{\mu}^v)$ solves the relaxed problem, then any pair $(\hat{p}^v, \hat{r}^v)$ satisfying

$$p^v = \frac{\eta\lambda + v}{(1-r^v)(1+2\lambda)} \tag{109}$$

satisfies both first-order conditions conditions (106) and (107).

Proof. Suppose that $(\hat{p}^v, \hat{r}^v, \hat{\mu})$ solves the relaxed problem. Then the first-order condition (106) holds true for any pair (p^v, r^v) satisfying

$$p^v = \frac{\eta(\hat{\mu} - r^v\lambda) + v(1 - r^v)}{(1 - r^v)^2\,(1 + 2\lambda)} \tag{110}$$

and the first-order condition (107) is satisfied for any pair (p^v, r^v) for which

$$p^v = \frac{\eta\lambda + v}{(1 - r^v)(1 + 2\lambda)} \text{ or } p^v = 0. \tag{111}$$

The case $p^v = 0$ can be ruled out because that solution never satisfies the profit constraint. By Lemma 1, the solution of the relaxed problem must satisfy $\hat{\mu} = \lambda$. Evaluating the right-hand side of the equation (110) at $\hat{\mu} = \lambda$ yields the equation (109). ‖

Let us then assume that $\lambda > 0$. Then $\hat{\mu} = \lambda > 0$, and the optimal price is $\hat{p}^v = c + F$ by the zero-profit condition. This solution together with the condition (109) yields the optimal insurance:

$$\hat{r}^v = 1 - \frac{\eta\lambda + v}{(1 + 2\lambda)(c + F)}. \tag{112}$$

The optimal insurance coverage is strictly positive, if $c + F > (\eta\lambda + v)/(1 + 2\lambda)$. At the solution, the determinant of the bordered Hessian matrix is

$$\left|\bar{H}\right| = \frac{(c + F)^2(1 + 2\lambda)}{\eta} > 0, \tag{113}$$

which shows that the solution is a local maximum. The solution also satisfies the feasibility constraints.‖

Proof of Proposition 8. Now $\tilde{W} \leq \hat{W}^v$ if

$$\frac{(\eta - c)^2}{2\eta}\frac{(1 + \lambda)^2}{(1 + 2\lambda)} - F\,(1 + \lambda) \leq \frac{\eta^2(1 + \lambda)^2 + v(v + 2\eta\lambda)}{2\eta(1 + 2\lambda)} - (1 + \lambda)(c + F) \tag{114}$$

which, after some straightforward computation, simplifies to the inequality

$$v^2 + 2\lambda\eta v - c(1 + \lambda)[c(1 + \lambda) + 2\lambda\eta] \geq 0. \tag{115}$$

The above inequality holds true if $v \geq c(1 + \lambda)$ or if $v \leq -(c(1 + \lambda) + 2\eta\lambda)$. Since negative values of the parameter v are not feasible, we have $\tilde{W} \leq \hat{W}^v$, if $v \geq c(1 + \lambda)$. ‖