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Uncovering Gatsby Curves

Abstract

Empirical findings suggest a positive correlation between inequality and social immobility, a phenomenon coined the Gatsby curve. However, complete explanations of the phenomenon have not yet been proposed. This paper answers two questions: What are Gatsby curves? When do they exist? We build a theoretical environment in which parental investment and education improve the economic prospects of children. Gatsbian economies and Gatsby curves are formally defined, and we characterize the conditions under which they will arise. We show that an economy may go from being Gatsbian to non-Gatsbian. Finally, we show that the better network of relations of those with high-paying jobs may also generate a Gatsbian economy.

JEL-Codes: D310, H520, J310, J620.

Keywords: intergenerational mobility, income inequality, education.

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1 Introduction

In this paper, we provide answers to two important questions: What are Gatsby curves? When do they exist?

Recent empirical work suggests a positive correlation between income inequality and social immobility, a phenomenon coined the "Gatsby curve" by Krueger (2012). Figure 1, taken from Corak (2013), depicts a positive correlation between income inequality and social immobility for 22 countries. Jantti et al. (2006) also find evidence of a Gatsby-style correlation for a small panel of anglo-saxon and Scandinavian countries.

A Gatsby curve has also been observed within countries such as Canada or the United States (Conolly et al., 2018, 2019). However, it is not universal. Fan et al. (2015) show evidence that China has a negative correlation between inter-generational social immobility and cross-sectional inequality among parents. Chetty et al. (2014) also cast doubt on the positive slope of the Gatsby curve for the United States, pointing out to measurements issues.

So far, the literature on Gatsby curves has been almost entirely empirical and has been largely agnostic on the mechanisms which may lead to their existence. Rightly so, most authors have been careful not to present the positive correlation between income inequality and social immobility as a causal one. Thus, despite the high quality of the empirical work that has brought to light the Gatsby curve relationship, a complete and satisfactory theoretical explanation of the phenomenon yet remains to be proposed. This led Benabou (2017) to write: "Thus, after lagging behind theory for some time, measurement has now moved substantially ahead, and a renewed effort at closing the gap is well overdue."

Our paper is part of a literature attempting to fill this gap. It contribues to a theory of Gatsby curves in six ways.

First, we draw from the Becker & Tomes (1979) framework and build a theory in which

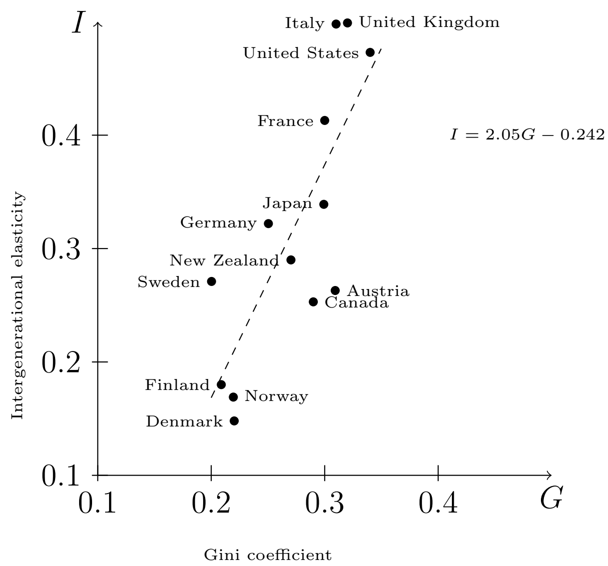


Figure 1: The Great Gatsby Curve (Corak, 2013).

parental investment is a core link between inequality and immobility. Parents either earn a low- or high- income and they invest in their children. More parental investment increases the likelihood that a child will earn a high-income when he becomes an adult. Thus, our theory encompasses mobility as the probability that a child earns an income different from that of his parent. It further defines inequality using the proportions of high- and low-income individuals. Our theory is rich and tractable, providing closed form expressions for inequality, immobility, as well as a complete characterization of the steady state transition matrix.

Second, we analyze the impact of an increase in education spending, or that of a widening of the income premium for high-paying jobs, so that we can formally characterize the conditions under which an economy may experience a simultaneous increase in inequality and immobility after such changes. An economy is then said to be x-Gatsbian when a change in x leads to a simultaneous increase (or a simultaneous decrease) in inequality and immobility. This is equivalent to moving along the positively-sloped portion of the x-expansion path in the inequality-immobility space. We characterize the conditions under which the economy is x-Gatsbian so that a Gatsby curve arises.

Third, we show that an economy may transition from being Gatsbian to non-Gatsbian (and *vice versa*). In other words, the x-expansion path in the inequality-immobility space may have both positively and negatively-sloped portions.

This last point is related to a key phenomenon that has been ignored in the applied theoretical literature on Gatsby curves, which is our fourth contribution: we show that inequality is a non-monotonic function of the proportion of low-income individuals in the economy. Starting from a situation in which everybody is low-income, reducing the proportion of low-income individuals at first increases inequality, but when the proportion of low-income individuals reaches a threshold, inequality will start — and keep on — decreasing with the proportion of low-income individuals.

Fifth, we show that, because of the non-monotonic behaviour of the Gini coefficient, changes usually viewed as inequality-increasing may instead reduce inequality. For example, as in the empirical findings in Barany & Siegel (2018), we show that an increase in the high-income premium may reduce inequality. This happens, after an increase in the premium, when the change in the proportion of low income individuals outweighs the change in the premium itself.

Sixth and lastly, as is suggested by the empirical literature, we show that the better network of relations of those with a high-paying jobs can generate a Gatsbian economy.

The remainder of the paper is structured as follows. In Section 2 we offer a brief survey of the relevant literature. We then turn to the presentation of our model of parental investment in Section 3. The characterization of the stationary state of the economy, of the measures of income inequality and of social immobility are in Section 4. We then turn to defining Gatsbian economies and Gatsby curves in Section 5. We provide a complete characterization of the conditions under which Gatsby curves arise. We then examine the situations in which a Gatsbian economy transitions to being non-Gatsbian (and *vice versa*) in Section 6. In Section 7, we analyze an alternative version of the model in which networks of relations may generate Gatsby curves. Concluding remarks follow.

2 Literature

To our knowledge, three theoretical papers have made key contributions to our understanding of the relationship between inequality and immobility: Solon (2004), Durlauf & Seshadri (2017), and Becker et al. (2018). In all of them, and as in the Becker & Tomes (1979) seminal paper, parents transmit their own advantage or disadvantage to their children. Thus, parents determine how mobile children are and how unequal the future distribution of income will be.

To be precise, in all those papers, an advantage in the form of a larger stock of human capital can be passed on to the next generation. In Solon (2004) and in Becker et al. (2018), the transmission occurs because parents invest in the human capital of their children. As is shown in these papers, when parents care for their children, wealthier parents — equivalent to those with a larger stock of human capital — invest more in their children than poorer parents. Thus, children of wealthier parents end up with a larger stock of human capital and that makes them wealthier. This is the approach we adopt. Alternatively, in Durlauf & Seshadri (2017)[1] and in Becker et al. (2018)[2], a child inherits a level of human capital that is larger when that of his parent is larger. Because a larger stock of human capital makes an individual wealthier, children of wealthy individuals end up being wealthier themselves.

All the aforementioned papers also include public education as an important ingredient. For example, Solon (2004) obtains that income inequality and social immobility can be accentuated by education policies that favour the children of richer families, as compared to

[1]Durlauf & Seshadri (2017) also introduce segregation and voting as mechanisms generating inequality and immobility.

[2]Thus, in Becker et al. (2018), the transmission of advantages is achieved through both parental investment and inheritance by the child of the traits of his parents.

the universal provision of education which would presumably be consumed by children of all backgrounds. As for Becker et al. (2018), they show that the degree of complementarity of public education with parental investment plays a key role in determining whether mobility is improved or reduced by in increase in public education expenditures.

We introduce public education in our analysis and we also obtain that the degree of complementarity between parental investment and public education plays an important role. As we will show, an increase in public education may generate a Gatsby curve, but that is not guaranteed and that will depend on the degree of complementarity.

While all these papers provide interesting answers as to the mechanisms that could be at play behind Gatsby curves, they nevertheless leave aside a number of important issues that we address in our analysis. Among them is the very nature of a Gatsby curve and that of the conditions under which they may be observed.

Also, our theory allows us to explore considerations that have been ignored so far in the literature. For example, all the above papers model income as a continuous variable and they do not characterize a continuous equivalent of a transition matrix describing the proportion of individuals transiting from one state to the other. Our approach, where income takes discrete values, allows for a complete characterization of the transition matrix. This is important because two identical stationary distributions of incomes may be associated with different patterns of inter-generational mobility. We can therefore push further the analysis of Gatsby curves and fully characterize our measures of inequality and of immobility.

Hence, we can find that inequality is a non-monotonic function of the proportion of low-income individuals in the economy, and that because of a composition effect, inequality may increase or decrease after an increase in the dispersion of incomes.[3] Our model also makes clear that a Gatsby curve is a positively-sloped expansion path in the inequality-immobility

[3]In the existing literature, an increase in the dispersion of income necessarily leads to an increase in income inequality because the composition of the population is fixed.

space. Further, our modelling allows for the full characterization of the conditions under which a Gatsby curve will be observed as well as that under which an economy will transition from being Gatsbian to non-Gatsbian

In the empirical literature, parental investment in children and education policies are often cited as potential explanations for the Gatsby curve (Duncan & Murnane, 2011; Black & Devereux, 2011). For example, Guryan et al. (2008) find a strong relationship between the level of education of parents, their incomes and the time they spend with their children.[4] This is consistent with theoretical works that model parental investments in children and public education as distinct, but complementary inputs into a child's human capital (Kaganovich & Zilcha, 1999; Benabou, 2017). Child-rearing practices may also play a role: because poorer parents are limited in their ability to mold their children's behaviour through pecuniary incentives, they may rely more on non-pecuniary incentives such as corporal punishment, which is harmful to children's development (Weinberg, 2001). Family resources and connections are also identified as a potential channel by both Corak (2013) and Becker & Tomes (1979). In particular, there appears to be a correlation between a father and his son's job (Corak & Piraino, 2011; Corak, 2013).

A number of papers have also explored the impact of public education on inequality and mobility.[5] For example, Corak (2013) observes that higher returns to college education are associated with lower social mobility. Moreover, his data shows that wealthier families engage in more enrichment expenditures, such as "books, computers, high-quality child care, summer camps, private schooling, and other things that promote the capabilities of their children" (p.15). Mazumder (2015) uses cross-country data and find correlations between social immobility and the level of inequality in test scores measuring literacy and numeracy skills in children. Finally, Rauh (2017) obtains that an increase in public education expenditures may increase inequality.

[4]We do not attempt here to survey this vast literature. Rather, we simply want to illustrate that the connection between parental investment and inequality and immobility has received attention from empiricists.

[5]Again, we do not attempt to make justice to this important literature.

Thus, the incorporation of parental investment and public education in models like ours, trying to explain Gatsby curves, is justified on empirical grounds.

3 Model

Each period t, there is a proportion Z^L of parents with a low-income w^L and a remaining proportion $Z^H = 1 - Z^L$ with a high-income w^H. Henceforth, we refer to each type of parent as a w^i-parent with $i \in \{L, H\}$. We denote by $\Delta w \equiv w^H - w^L$ the income premium of w^H-parents and the average income as $\bar{w} \equiv Z^L w^L + Z^H w^H = w^H - Z^L \Delta w$.

At any period t, each adult has a single child. During childhood, which lasts for one period, a w^i-parent invests ϕ^i in his child. This parental investment may capture the time spent with one's child, such as helping with homework, or the transmission of values and cultural traits (e.g. values of effort and hard work). The cost of parental investment is $c(w^i)\phi^i$, where $c(w^i)$, sometimes shortened as c^i, is decreasing in w^i. This implies that w^H-parents face a lower marginal cost than w^L-parents. We later consider the case in which all parents face the same marginal cost of investment, but with w^H-parents having a better network of relations (see section 7).

All children at period t become parents at period $t+1$. The probability that the child of a w^i-parent becomes a w^H-parent, which is captured by the function $p(\theta^i)$, depends on his employability, or human capital, which we denote θ^i. Employability depends on both parental investment and an exogenous, general provision of public education $e \geq 0$:

$$\theta^i = \lambda \phi^i e + (1 - \lambda)(\phi^i + e) \geq 0. \tag{1}$$

As a shorthand, we sometimes use $p^i \equiv p(\theta^i)$. The function $p(\cdot)$ is strictly increasing, strictly concave and satisfies $p(0) \geq 0$.

It should be noted that the properties of the probability function $p(\theta)$, most importantly its local curvature, capture the impact of education and parental investment on labor market outcomes. In the analysis that follows, we use this simple generic reduced-form function. However, it should be recognized that this generic probability function could be interpreted in several meaningful ways. For example, it could be microfounded by adapting the matching framework developed in Boadway & Cuff (2014). Other interpretations, reflecting the institutional features of the education system or that of the labor market could also be put forward.

Also note that the degree of complementarity between parental investments and public education is captured by the parameter $\lambda \in [0, 1]$. When $\lambda = 0$, public education is a perfect substitute to parental investment. Increasing λ reduces the degree of substitutability between education and parental investment.

3.1 The Parent's Problem

Parents value their net income (income minus parental investment) and the expected income of their children. The extent of parental altruism is denoted by $\beta \in]0, 1]$. They thus solve:

$$\max_{\phi} \; w^i - c(w^i)\phi + \beta[p(\theta^i)w^H + (1 - p(\theta^i))w^L]. \tag{2}$$

An equivalent formulation of the expected income is $p(\theta^i)\Delta w + w^L$, which makes explicit that (2) is concave in ϕ.

Our analysis focuses on interior solutions to the parental investment problem: $\phi \leq w^i$. Thus, we assume that the marginal cost of parental investment quickly becomes larger than its marginal benefit, ensuring an interior solution.

Note that if we did not make that assumption, optimal investment could be larger than current revenue ($\phi > w^i$). However, we could then ensure $\phi \leq w^i$ by assuming that capital markets are imperfect so that parents cannot borrow now to invest in exchange for a future reimbursement by their child.[6]

Henceforth, we use subscripts to denote partial derivatives ($f_x \equiv \partial f/\partial x$, $f_{xy} \equiv \partial^2 f/\partial x \partial y$). The first-order condition to (2) is:

$$-c^i + \beta p^i_\theta \theta_\phi \Delta w = 0, \tag{3}$$

where $\theta_\phi = \lambda e + (1-\lambda) \geq 0$ is independent of i. Since $p^i_{\theta\theta} < 0$, the second-order condition is satisfied and (2) yields a maximum. Denote by $\phi^{i*} \equiv \phi^*(e, w^i, \Delta w)$ the solution to (2). Similarly, let $\theta^{i*} \equiv \theta(\phi^{i*})$ and $p^{i*} = p(\theta^{i*})$. Reorganizing (3) yields some intuition:

$$\frac{1}{\beta \Delta w \theta_\phi} c^i = p^{i*}_\theta \tag{4}$$

On the left-hand side, only c^i is type dependent. Thus, differences in marginal probabilities are driven by differences in marginal investment costs. Since the difference favours w^H-parents ($c^H < c^L$), their parental investment is higher ($\phi^{H*} > \phi^{L*}$). Because, $p(\cdot)$ is increasing, it follows that $p^{H*} > p^{L*}$. [7] Using (4) for both types of parents yields:

$$\frac{1}{\beta \Delta w \theta_\phi} \left(c^H - c^L\right) = \left(p^{H*}_\theta - p^{L*}_\theta\right) < 0. \tag{5}$$

Since $c^H < c^L$, (5) implies that the marginal probability of landing in a w^H job for the child of a w^L-parent is higher than for one of a w^H-parent ($p^{H*}_\theta > p^{L*}_\theta$).

[6] Becker et al. (2018) provide an interesting analysis of the impact of imperfect capital markets on parental investment.

[7] This result obtains because of the simplifying assumptions that the utility of children and parents, given in (2), is linear in consumption and because the marginal cost of parental investment is lower for w^H-parents. It should be noted that an equivalent result would obtain with a strictly concave utility of consumption and a constant marginal cost of parental investment c independent of i.

3.2 Comparative Statics

Using (3), one can find the comparative statics of parental investment ϕ^{i*} with the provision of education e:

$$\phi_e^{*i} = -\frac{1}{\theta_\phi}\left(\theta_e^{*i} - \frac{\lambda}{-p_{\theta\theta}^{*i}/p_\theta^{*i}\theta_\phi}\right). \tag{6}$$

When the term in parentheses on the right-hand side is negative, education increases parental investment. This obtains if $-p_{\theta\theta}^{*i}/p_\theta^{*i}$ is small enough and satisfies:

$$\frac{-p_{\theta\theta}^{*i}}{p_\theta^{*i}} < \frac{\theta_{\phi e}}{\theta_\phi \theta_e^{*i}} = \frac{\lambda}{(\lambda e + (1-\lambda))(\lambda\phi^{*i} + (1-\lambda))}. \tag{7}$$

Thus, a greater provision of public education must have enough impact in terms of accrued probabilities to generate an increase in parental investment. Otherwise, if $-p_{\theta\theta}^{*i}/p_\theta^{*i}$ is large (i.e. such that (7) is not satisfied), then education saturates the probability of obtaining w^H which leads to lower parental investment. A noteworthy case is that in which public education is a perfect substitute for parental investment: $\lambda = 0$. In this extreme case, (7) cannot be satisfied and $\phi_e^{*i} = -1$: Public education then completely crowds out parental investment and θ^{*i} is constant.[8] It immediately follows that $\phi_e^{*i} < 0$ for λ sufficiently close to 0.

We now turn to the comparative statics of ϕ^{i*} with respect to w^i. Changes in income may affect the behaviour of a parent through two channels. First, it may change the marginal cost of parental investment $c(w^i)$ when the w^i-parent's own income varies. Second, it changes the benefits of all parental investments by affecting Δw. Because our analysis focuses on stationary states, we analyze changes in income that apply simultaneously to both parents

[8] An equivalent result is obtained in Becker et al. (2018).

and children. The impact of a change in w^L on ϕ^{i*} is as follows:

$$\phi^{L*}_{w^L} = -\frac{p^{L*}_{\theta}}{-p^{L*}_{\theta\theta}}\frac{1}{\theta_\phi}\left(\frac{c^L_{w^L}}{c^L} + \frac{1}{\Delta w}\right) \lesseqgtr 0, \tag{8a}$$

$$\phi^{H*}_{w^L} = -\frac{p^{H*}_{\theta}}{-p^{H*}_{\theta\theta}}\frac{1}{\theta_\phi}\frac{1}{\Delta w} < 0. \tag{8b}$$

Parental investment by w^L-parents is driven by the two aforementioned channels, which are illustrated within parentheses in (8a). First, increasing w^L reduces the marginal cost of investment $c(w^L)$ and leads to a larger investment. It also erodes the expected net return of the investment by reducing Δw. Absent of assumptions on c^L, the net effect is ambiguous for a w^L-parent. However, if we assume that $c(w^i)$ is strictly concave in w^i, then we can sign (8a) and $\phi^{L*}_{w^L} > 0$.[9] On the other hand, the sign of (8b) is unambiguously negative: the only effect of increasing w^L on w^H-parents is to reduce the expected return of their investment.

The effect of w^H on ϕ^{i*} is given by:

$$\phi^{L*}_{w^H} = \frac{p^{L*}_{\theta}}{-p^{L*}_{\theta\theta}}\frac{1}{\theta_\phi}\frac{1}{\Delta w} > 0, \tag{9a}$$

$$\phi^{H*}_{w^H} = \frac{p^{H*}_{\theta}}{-p^{H*}_{\theta\theta}}\frac{1}{\theta_\phi}\left(\frac{c^H_{w^H}}{c^H} - \frac{1}{\Delta w}\right) > 0. \tag{9b}$$

Increasing w^H affects positively the return of parental investment for both types of parents. For a w^H-parent, the marginal cost of investing is also reduced, which reinforces the first effect. The derivatives for both w^L- and w^H- parents are positive.

3.3 Individual Mobility and Immobility

For each type of parent, we have characterized the endogenous probability that their child will obtain a given income. We now use them to construct a typology of mobility and

[9] The expression $c^L + c^L_{w^L}\Delta w > 0$ is a first-degree Taylor expansion of $c(w)$ around w^L, evaluated at w^H. When c^i is strictly concave, the expansion is greater than $c(w^H)$, which implies the result.

immobility at the individual level.

Definition 1 (Types of individual mobility and immobility)**.**
(i) **Upward Mobility** $p(\theta^{L*})$**:** *The probability that the child of a* w^L*-parent earns* w^H*.*
(ii) **Downward Mobility** $1 - p(\theta^{H*})$**:** *The probability that the child of a* w^H*-parent earns* w^L*.*
(iii) **Sticky Ceilings** $p(\theta^{H*})$**:** *The probability that the child of a* w^H*-parent earns* w^H*.*
(iv) **Sticky Floors** $1 - p(\theta^{L*})$**:** *The probability that the child of a* w^L*-parent earns* w^L*.*

The model predicts that changes in the provision of education, e, or in incomes w^L and w^H, will have the following effects on individual mobility and immobility:

$$p_e^{L*} = \frac{(p_\theta^{L*})^2}{-p_{\theta\theta}^{L*}} \frac{\theta_{e\phi}}{\theta_\phi} \geq 0, \; p_{w^L}^{L*} = -\frac{(p_\theta^{L*})^2}{-p_{\theta\theta}^{L*}} \left(\frac{c_{w^L}}{c^L} + \frac{1}{\Delta w} \right) \lesseqgtr 0, \; p_{w^H}^{L*} = \frac{(p_\theta^{L*})^2}{-p_{\theta\theta}^{L*}} \frac{1}{\Delta w} > 0, \quad \text{(10a)}$$

$$p_e^{H*} = \frac{(p_\theta^{H*})^2}{-p_{\theta\theta}^{H*}} \frac{\theta_{e\phi}}{\theta_\phi} \geq 0, \; p_{w^L}^{H*} = -\frac{(p_\theta^{H*})^2}{-p_{\theta\theta}^{H*}} \frac{1}{\Delta w} < 0, \;\; p_{w^H}^{H*} = \frac{(p_\theta^{H*})^2}{-p_{\theta\theta}^{H*}} \left(\frac{1}{\Delta w} - \frac{c_{w^H}^H}{c^H} \right) > 0. \quad \text{(10b)}$$

Thus, when education and parental investment are not perfect substitutes ($\lambda \neq 0$), more spending in education increases both upward mobility p^{L*} and sticky ceilings p^{H*}. If they are perfect substitutes ($\lambda = 0$), then both upward mobility and sticky ceilings are unaffected by e because an increase in education results in a reduction in parental investment by an equivalent amount, leaving θ^{i*} unchanged for both types of parents.

With regard to income levels, the effect of increasing w^L on upward mobility is ambiguous unless $c(\cdot)$ is strictly concave. However, the effect of w^H on upward mobility is unambiguously positive. Finally, sticky ceilings are increasing in w^H and decreasing in w^L.

Note that in (10a) and (10b), marginal probabilities are expressed to clearly emphasize how the determinant of the inverted hessian of (2) (i.e.: $-(p_\theta^{L*})^2/p_{\theta\theta}^{L*} = p_\theta^{i*}(-p_{\theta\theta}^{i*})^{-1}p_\theta^{i*}$) affects their magnitude. Since it is instrumental in deriving the conditions under which a Gatsby curve may emerge, we later refer to and interpret it as weighted marginal probabilities, that is the product of the marginal probability and its local measure of concavity.

For later use, note that when weighted marginal probabilities increase with θ, an increase in θ (following, for example, an increase in education) may increase sticky ceilings $p(\theta^{H*})$ more than upward mobility $p(\theta^{L*})$. In such a case, there is a prejudice against children coming from lower-income families.

4 Stationary State, Inequality, and Immobility

In this section, we characterize the stationary state induced by the transition probabilities. We then provide a closed form for the Gini coefficient and characterize the manner in which it changes with exogenous variables. Likewise, we define social immobility and establish the conditions under which it increases or decreases with exogenous changes.

4.1 Stationary State

Since parental investments ϕ^i are endogenous, the transition probabilities p^i are also endogenous for each individual. In turn, they generate the equilibrium proportions of w^L- and w^H-parents. Characterizing these equilibrium proportions is a prerequisite to providing measurements of either income inequality or social immobility.

To emphasize dynamics, denote by $Z^{H,t}$ and $Z^{L,t}$ the proportions of w^L- and w^H-parents observed at time t. Their dynamics is described by the following Markov chain which contains, on its right-hand side, the two by two transition matrix of our economy:

$$\begin{bmatrix} Z^{H,t+1} \\ Z^{L,t+1} \end{bmatrix} = \begin{bmatrix} p(\theta^{H*}) & p(\theta^{L*}) \\ 1 - p(\theta^{H*}) & 1 - p(\theta^{L*}) \end{bmatrix} \begin{bmatrix} Z^{H,t} \\ Z^{L,t} \end{bmatrix}. \tag{11}$$

We show in the Appendix that since $p(\theta)$ is strictly increasing and strictly concave, (11) has a unique stationary state. It occurs when the proportions no longer change over time, a condition that boils down to a single defining equation:

$$Z^{L*} = 1 - p(\theta^{H*})Z^{H*} - p(\theta^{L*})Z^{L*}. \tag{12}$$

Using $Z^{L*} = 1 - Z^{H*}$, the stationary state proportions are then given by:

$$Z^{L*} = \frac{1 - p(\theta^{H*})}{p(\theta^{L*}) + 1 - p(\theta^{H*})}, \qquad Z^{H*} = \frac{p(\theta^{L*})}{p(\theta^{L*}) + 1 - p(\theta^{H*})}. \tag{13}$$

Figure 2 illustrates how upward and downward mobility define the net flow of persons transiting from one group to the other. As such, they are sufficient statistics to characterize the stationary state. The higher upward mobility p^{L*} is relative to downward mobility $(1 - p^{H*})$, the larger Z^{H*} must be to maintain stationarity.

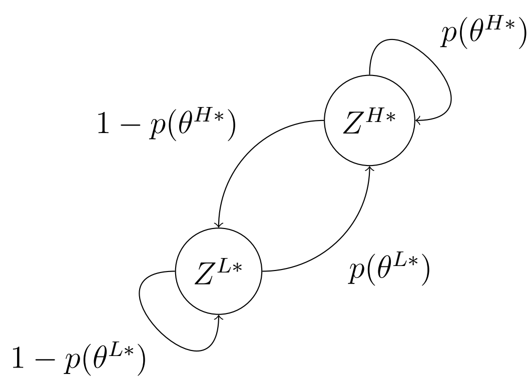


Figure 2: Transition Probabilities and the Stationary State

Since $Z^{L*} = 1 - Z^{H*}$, all comparative statics results for Z^{H*} take the opposite sign of that for Z^{L*}. We therefore provide results only for Z^{L*}. From (12), for any variable

$x \in \{e, w^H, w^L\}$, we obtain:

$$Z_x^{L*} = -p_x^{H*}\frac{p^{L*}}{(p^{L*}+1-p^{H*})^2} - p_x^{L*}\frac{1-p^{H*}}{(p^{L*}+1-p^{H*})^2}. \tag{14}$$

Using (10a) and (10b), it is easily obtained that $Z_e^{L*} < 0$ and $Z_{w^H}^{L*} < 0$. Thus, the stationary state proportion of w^L-parents decreases with education e and with the high-income level w^H. However, the sign of $Z_{w^L}^{L*}$ is ambiguous unless $c(\cdot)$ is assumed to be strictly concave, in which case $Z_{w^L}^{L*} < 0$.

4.2 Income Inequality

We use the standard Gini coefficient to account for income inequality in the economy. In the environment we developed, the following closed-form expression can be found:[10]

$$G = Z^L(1-Z^L)\frac{\Delta w}{w^H - Z^L\Delta w}. \tag{15}$$

A fundamental point which has gone unnoticed in the literature is that the Gini coefficient is a non-monotonic function of the proportion Z^L. It can be shown that G is a strictly concave function of Z^L which attains a global maximum at a unique critical value $\hat{Z}^L$. To identify $\hat{Z}^L$, one maximizes G with respect to Z^L and obtains that the sole solution within the unit interval is:

$$\arg\max_{Z^L} G \equiv \hat{Z}^L = \frac{\sqrt{w^H}}{\sqrt{w^H}+\sqrt{w^L}} \in]1/2, 1[. \tag{16}$$

A depiction of G as a function of Z^L is presented in Figure 3. It illustrates that inequality increases with Z^L when $Z^L < \hat{Z}^L$. Conversely, inequality decreases with Z^L when $Z^L > \hat{Z}^L$. For our purpose, the following definition will be useful:

[10] Calculations for this section are provided in the Appendix.

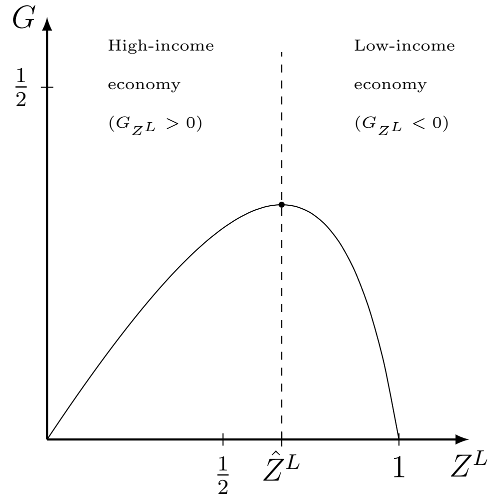


Figure 3: Inequality as a Function of Z^L: G is Maximized at $\hat{Z}^L$

Definition 2 (High-income and low-income economies)**.**
An economy is a **low-income economy** *when* $Z^L > \hat{Z}^L$.
An economy is a **high-income economy** *when* $Z^L < \hat{Z}^L$.

In what follows, we use this high- and low-income terminology to remind us of where Z^L stands relative to $\hat{Z} > 1/2$. Of course, it should not be interpreted *stricto sensu*. In particular, a low-income economy could be richer (in the sense of a higher GDP) than some other high-income economy.

As shown in Figure 3, the Gini coefficient is increasing with Z^L in a high-income economy while, in a low-income one, it decreases with Z^L. Thus, assessing the full effect of an exogenous change on the Gini coefficient requires knowing the impact on Z^L, as expressed in (14), and identifying the type of economy — high- or low-income — in which the change occurs.

We can now use (13) and (16) to characterize the Gini coefficient, G^*, at the stationary state proportion (Z^{L*}). Substituting the definition of Z^{L*} given by (13) and that of $\hat{Z}^L$ given by (16) in $Z^{L*} < \hat{Z}^L$, it is possible to show that a high-income economy occurs when the following inequality holds:

$$\sqrt{\frac{w^H}{w^L}}p^{L*} + p^{H*} > 1. \tag{17}$$

Equation (17) identifies the condition under which inequality is increasing in Z^L. Intuitively, when upward mobility and sticky ceilings are large enough to satisfy that inequality, the economy consists of a proportion of w^H-parents that is large, and of a small proportion of w^L-parents. We are then in a high-income economy, for which a marginal increase in Z^L generates a proportionally smaller decrease of the share of income earned by w^H-parents. Because $w^H > w^L$ and $p(\cdot) \in [0, 1]$ is strictly increasing, there exist values of θ^{H*} and θ^{L*} for which (17) is satisfied. As discussed in section 5, that may be the case if the provision of education e is large enough.

Conversely, when (17) does not hold, we have a low-income economy in which inequality decreases with Z^{L*}.[11] In such a case, upward mobility and sticky ceilings are not large enough to maintain a sufficient proportion of w^H-parents in the economy. Hence, a marginal increase in Z^{L*} reduces inequality by decreasing the share of high-income individuals. Inequality decreases because the population becomes more homogenous (and poorer on average at the same time).

4.3 Social Immobility

Our previous analysis has shown that the stationary state proportions Z^{L*} and Z^{H*} depend on the two by two transition matrix contained in (11). We use the transition probabilities it contains to define social immobility. To keep the definition as simple as possible, we use

[11] When the left-hand side of (17) equals one, inequality does not change with Z^L.

the trace of the transition matrix minus one:

$$I^* \equiv p^{H*} - p^{L*}. \tag{18}$$

There is no unanimity among economists regarding the measurement of social immobility. As discussed by Dardanoni (1993), different functions mapping transition probabilities in a social immobility index, such as the trace of the transition matrix, its determinant or its second eigenvalue, often yield contradictory results. However, a key feature of our environment is that these three definitions yield an identical measure, up to adding a constant (see Appendix). We thus avoid any controversy on the index selection.

Using (18) and the relevant marginal probabilities in (10a) and (10b), changes in the parameters of the model affect I^* as follows:

$$\frac{dI^*}{de} = \frac{\lambda}{\lambda e + (1-\lambda)} \left[\frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} - \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}} \right], \tag{19a}$$

$$\frac{dI^*}{dw^H} = \frac{1}{\Delta w} \left[\frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} - \frac{(p_\theta^{L*})^2}{-p_{\theta\theta}^{L*}} \right] + \frac{1}{-p_{\theta\theta}^{H*} c^H} [(p_\theta^{H*})^2 - c_{w^H}^H], \tag{19b}$$

$$\frac{dI^*}{dw^L} = -\frac{1}{\Delta w} \left[\frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} - \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}} \right] - \frac{1}{-p_{\theta\theta}^{L*} c^L} [(p_\theta^{L*})^2 - c_{w^L}^L]. \tag{19c}$$

When these expressions are positive, social immobility increases. The difference in weighted marginal probabilities, which appear within square brackets in (19a)—(19c), is crucial to determine their sign. Also, note that (19a) shows that the magnitude of the change in social immobility decreases with the level of education. Indeed, the term $\lambda/(\lambda e + 1 - \lambda)$ implies that the higher the level of education, the smaller the change in social immobility (provided $\lambda \neq 0$).

5 Gatsbian Economies

In the literature, a Gatsby curve is a graph depicting a positive relationship between income inequality and social immobility. We use this relationship as the starting point of our formal definition of Gatsby curves and characterize the circumstances in which they occur. An economy will be said to be Gatsbian when social immobility and income inequality move in tandem, meaning that both increase (or decrease) after a change in an exogenous variable. As the last sentence suggests, the relationship is not a causal one. When an exogenous parameter (e.g. income levels, education) changes, the economy moves towards a new stationary state at which inequality G^* and immobility I^* take some new values. Therefore, one would observe what the literature has called a Gatsby curve when G^* and I^* move in the same direction following a change in a parameter $x \in \{e, w^H, w^L\}$. We formally define this relationship below:

Definition 3 (Gatsbian economy)**.** *For $x \in \{e, w^H, w^L\}$, an x**-expansion path** is the curve of stationary state points $(G^*(x), I^*(x))$ that is induced by the value of x. We say that an economy is **Gatsbian** with respect to a variable x, or is equivalently **x-Gatsbian** if, at its stationary state, the economy is on the positively-sloped segment of the x-expansion path:*

$$x\text{-}Gatsbian \Leftrightarrow \frac{dI^*(x)/dx}{dG^*(x)/dx} > 0. \tag{20}$$

Otherwise, the economy is non-Gatsbian.

Definition 4 (Gatsby curve)**.** *A* **Gatsby curve** *is a positively-sloped segment of an expansion path in the (G, I) space.*

Figure 4 illustrates a portion of a positively-sloped x-expansion path with inequality and immobility moving in the same direction after increasing x_1 to x_2 and, finally, to x_3. On this section of the x-expansion path, the economy is $x-$Gatsbian.

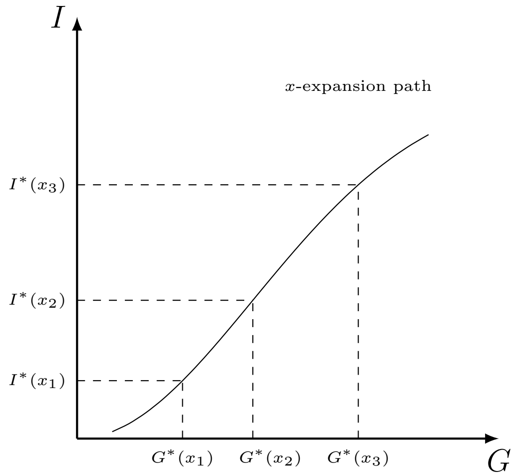


Figure 4: A positively-sloped x-expansion path is a Gatsby curve

From this explanation, it should be clear that using the signs of the total derivatives of immobility and inequality are useful to characterize the four cases under which an economy is x-Gatsbian. Table 1 classifies the possible cases, with two of them entailing a Gatsbian economy. As they bear no interest, we omit the cases in which total derivatives equal zero.

Table 1: Classification of Gatsbian and non-Gatsbian economies

		Inequality	
		$dG^*/dx > 0$	$dG^*/dx < 0$
Immobility	$dI^*/dx > 0$	*x-Gatsbian*	*Non-Gatsbian*
	$dI^*/dx < 0$	*Non-Gatsbian*	*x-Gatsbian*

Intuitively, the top-left case of Gatsbian economies is that in which a change in x moves the economy up along a Gatsby curve. The bottom-right case is that in which the economy moves down along a Gatsby curve after a change in x. As will become clear below, the key difference between the two cases is the position of the economy with respect to $\hat{Z}^L$ and the structure of the probability function $p(\cdot)$.

Using the impact of each variable $x \in \{e, w^L, w^H\}$ on inequality G^* and immobility I^*, we can now identify the conditions under which a Gatsbian economy and a Gatsby curve will obtain.

5.1 Education and Gatsbian Economies

It is well documented that in the last decades, the level of schooling around the world significantly increased (Global Change Data Lab, 2019). Could it then be that increases in schooling led to a simultaneous increase in inequality and immobility? As we show below, the answer is "yes, in certain circumstances". Thus, we show that schooling improvements may make an economy move up along a positively-sloped expansion path in the (G, I) space, i.e. a Gatsby curve. At the same time, we can also show that under different circumstances, increases in schooling lead to a simultaneous decrease in inequality and immobility, i.e. to an economy moving down along a Gatsby curve. Finally, we also show below that there are other circumstances in which the expansion path generated by increases in education is not positively sloped so there is simply no Gatsby curve.

We begin our analysis by characterizing the conditions under which an economy is e-Gatsbian.

Proposition 1 (e-Gatsbian)**.** *Assume education and parents' investment are not perfect substitutes ($\lambda \neq 0$). An economy is e-Gatsbian if and only if one of the following two situations occurs.*

a. A high-income economy ($Z^{L*} < \hat{Z}^L$) in which weighted marginal upward mobility is greater than weighted marginal sticky ceilings:

$$\lambda \neq 0, \qquad \sqrt{\frac{w^H}{w^L}} p(\theta^{L*}) + p(\theta^{H*}) > 1, \qquad \textit{and} \qquad \frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} < \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}}. \tag{21}$$

In this case, the economy moves down along a Gatsby curve.

b. A low-income economy ($Z^{L*} > \hat{Z}^L$) in which weighted marginal upward mobility is smaller than weighted marginal sticky ceilings:

$$\lambda \neq 0, \qquad \sqrt{\frac{w^H}{w^L}}p(\theta^{L*}) + p(\theta^{H*}) < 1 \qquad \textit{and} \qquad \frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} > \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}}. \tag{22}$$

In this alternative case, the economy moves up along a Gatsby curve.

Proof. First, equation (19a) tells us that a necessary condition for the change in immobility to be different from zero is some complementarity between education and the parents' investment ($\lambda > 0$).

Second, from (15), we have $dG^*/de = G^*_{Z^L} Z_e^{L*}$. Since $Z_e^{L*} < 0$, dG^*/de depends solely on the sign of $G^*_{Z^L}$, we have:

$$\frac{dG^*}{de} < 0 \;\Leftrightarrow\; \sqrt{\frac{w^H}{w^L}}p(\theta^{L*}) + p(\theta^{H*}) > 1, \quad \frac{dG^*}{de} > 0 \;\Leftrightarrow\; \sqrt{\frac{w^H}{w^L}}p(\theta^{L*}) + p(\theta^{H*}) < 1. \tag{23}$$

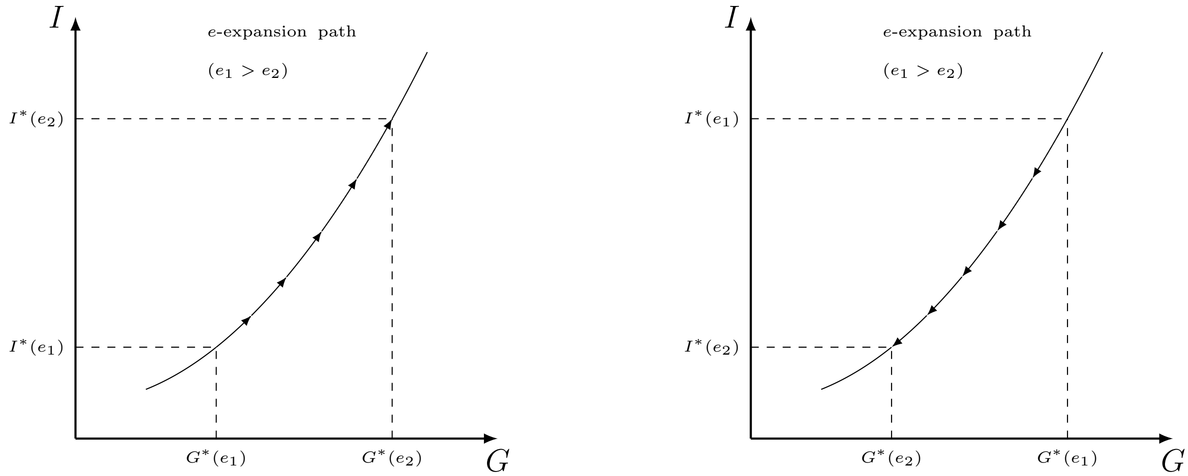
Finally, the sign of (19a) is tied to the difference in weighted marginal probabilities:

$$\frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} < \frac{\left(p_\theta^{L*}\right)}{-p_{\theta\theta}^{L*}} \quad \Leftrightarrow \quad \frac{dI^*}{de} < 0, \quad \frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} > \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}} \quad \Leftrightarrow \quad \frac{dI^*}{de} > 0, \tag{24}$$

Combining the relevant cases completes the proof. □

The two cases contained in Proposition 1 convey useful intuition. First note that in both the high-income and the low-income economy cases, increasing e has a positive effect on the probabilities p^{L*} (upward mobility) and p^{H*} (sticky ceilings). These increases in the probabilities in turn lead to a reduction in Z^L in both types of economies.

Now, in a high-income economy, a reduction in Z^L leads to a reduction in inequality (i.e.



(a) Low-income economy

(b) High-income economy

Figure 5: e-expansion paths in low- and high-income e-Gatsbian economies

a decrease in the Gini coefficient). But for the economy to be $e-$Gatsbian, the reduction in G must be accompanied with a reduction in immobility, which requires that the reduction in Z^L results mainly – though not exclusively – from the upward mobility channel. Thus, a high-income economy is $e-$Gatsbian when the marginal benefit of education accrues more intensively to low-income children through an increase in upward mobility, than it does to the high-income ones through the reinforcement of sticky ceilings. Figure 5b depicts the movement of an economy down the positively-sloped e-expansion path of an e-Gatsbian high-income economy.

On the other hand, a reduction in Z^L increases the Gini coefficient in a low-income economy. For immobility to increase at the same time, the reduction in Z^L must have been achieved mainly — though not exclusively — through a reinforcement of sticky ceilings. Thus, a low-income economy is $e-$Gatsbian when the children of high-income parents reap more of the marginal benefit of education than those of low-income ones. Depicted in Panel (a) of Figure 5a is the movement of an economy up the positively-sloped e-expansion path of an e-Gatsbian low-income economy.

5.2 High-Paying Jobs and Gatsbian Economies

Gatsby curves may have arisen because of the increase in the return to high-paying jobs that has been observed in the last decades. For example, Acemoglu & Autor (2011) show that the growth rate of the real composition-adjusted wages of workers increased more for those with more education. Table 2 reports some of their results. For example, between 1963 and 2008, men with 0-11 years of education experienced a decline of 5.1% in their real wages, while men with 18+ years of education saw theirs increase by 60.1%.

Table 2: Changes in real, composition-adjusted log weekly wages for full-time, full-year workers in the United States — 1963 - 2008

Education (Years)	Men	Women
0 – 11	-5.1	13.0
12	6.2	25.4
13 – 15	12.4	33.0
16 – 17	33.4	41.4
18 +	60.1	54.4

Source: Acemoglu & Autor (2011), Table 1.

In the same vein, Goos et al. (2009) and Barany & Siegel (2018) show evidence of job polarization, respectively, in Europe and in the United States. Bourdabat et al. (2010) also show that the real wage gap between high school and university graduates has steadily increased in Canada between 1980 and 2005.

In fact, some have argued that the increase in the return to high-paying jobs, combined with the stagnation of that of lower-paid jobs, is the cause for the increased inequality observed in the USA in the last decades.

We thus now turn our attention to the impact of an increase in w^H (with w^L unchanged) on inequality and immobility.

Proposition 2 (w^H-Gatsbian)**.** *An economy may be w^H-Gatsbian in the following situa-*

tions:

a. A high-income economy ($Z^{L*} < \hat{Z}^L$), in which case a necessary but non-sufficient set of conditions is that weighted marginal sticky ceilings be smaller than weighted marginal upward mobility:

$$\sqrt{\frac{w^H}{w^L}}p(\theta^{L*}) + p(\theta^{H*}) > 1, \qquad \text{and} \qquad \frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} < \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}}. \tag{25}$$

In such a case, the economy moves down a Gatsby curve.

b. A low-income economy ($Z^{L*} > \hat{Z}^L$) and a sufficient but non-necessary condition is that weighted marginal sticky ceilings be larger than weighted marginal upward mobility:

$$\sqrt{\frac{w^H}{w^L}}p(\theta^{L*}) + p(\theta^{H*}) < 1, \qquad \text{and} \qquad \frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} > \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}}. \tag{26}$$

In this alternative case, the economy moves up a Gatsby curve.

Proof. It can be seen from (15) that high-income w^H impacts G directly and indirectly:

$$\frac{dG^*}{dw^H} = G^*_{w^H} + G^*_{Z^L} Z^{L*}_{w^H} \tag{27}$$

Using (15) again, it is easily obtained that $G^*_{w^H} = w^L Z^{L*}(1 - Z^{L*})/(w^H - \Delta w Z^{L*})^2 > 0$. From (14), we also know that $Z^{L*}_{w^H} < 0$. Thus, as was the case with education, the sign of dG^*/dw^H depends on the sign of $G^*_{Z^L}$.

Consider now Part 1 of the proposition. A necessary but non-sufficient condition for $dG^*/dw^H < 0$ is that $G^*_{Z^L} > 0$, which amounts to condition (17) being met, i.e. that the economy be a high-income economy with $Z^{L*} < \hat{Z}^L$.

Then, using equation (19b), a necessary but non-sufficient condition for $dI^*/dw^H < 0$ is that the difference in weighted marginal probabilities be negative, i.e. $-\left(p_\theta^{H*}\right)^2/p_{\theta\theta}^{H*} +$

$\left(p_{\theta}^{L*}\right)^2/p_{\theta\theta}^{L*} < 0$.

Combined, these two statements amount to Part 1 of the proposition.

Part 2 of the proposition is obtained by using the logical inverse of the conditions on (27) and (19b). Here, however, $G_{Z^L}^* < 0$ (low-income economy) is sufficient to ensure that $dG^*/dw^H > 0$ and $-\left(p_{\theta}^{H*}\right)^2/p_{\theta\theta}^{H*} + \left(p_{\theta}^{L*}\right)^2/p_{\theta\theta}^{L*} > 0$ is sufficient to ensure that $dI^*/dw^H > 0$. □

Proposition 2 establishes the necessary conditions under which a high-income economy is w^H-Gatsbian, and the sufficient conditions under which a low-income economy is w^H-Gatsbian. From above, we know that an increase in w^H translates into an increase in both sticky ceilings and upward mobility, and that those in turn lead to a reduction in Z^L. In a high-income economy, this reduces inequality and a necessary condition for immobility to also decrease (so that the economy is Gatsbian) is that the reduction in Z^L be mainly achieved through the upward mobility channel. In a low-income economy, the reduction in Z^L induces an increase in inequality, and immobility will also increase (so that the economy is Gatsbian) if the reduction in Z^L is mainly accomplished through the reinforcement of sticky ceilings.

Clearly, these conditions for a w^H-Gatsbian economy (stated in Proposition 2) are identical to those of Proposition 1, except for the fact that those of Proposition 2 are either non-necessary or non-sufficient. This equivalence leads us to Corollary 1.

Corollary 1 (Relation between e-Gatsbian and w^H-Gatsbian)**.**
If a high-income economy is w^H-Gatsbian, then it is also e-Gatsbian.
If a low-income economy is e-Gatsbian, then it is also w^H-Gatsbian.

The core difference between an increase in high-income w^H and one in education e is that the former affects both the expected income of children and the parents' capacity to invest, while the latter only affects the expected income of children. The impact on the expected income of children, common to an increase in w^H or e, corresponds to the positive change in upward mobility and sticky ceilings. Regarding the accrued maneuver that an increase in w^H gives to w^H-parents to invest in their child, it impacts positively on sticky ceilings.

In a low-income economy, both effects push in the same direction. It is thus easier to meet the existence conditions. Hence, the conditions making the economy e-gatsbian are sufficient to make it w^H-Gatsbian. In a high-income economy, the two effects push in opposite directions. Thus, the conditions making the economy e-gatsbian become necessary, but are not sufficient to guarantee it is w^H-Gatsbian.

Note that the existence of a positive relationship between education and income is an empirical regularity. It can then be envisioned that increases in education could generate Gatsbian economies directly (as in Proposition 1) and indirectly through education-generated increases in w^H (as in Proposition 2).

5.3 Low-Income Jobs and Gatsbian Economies

As was discussed above, in some parts of the world, the increase in the real income of high-paying jobs has occurred simultaneously with a reduction of that for low-paying jobs (Table 2). The latter phenomenon may have generated some Gatsby curves. Thus, our analysis would not be complete without a characterization of the conditions under which an economy is w^L-Gatsbian.

In what follows, we further assume that the cost function $c(w)$ is strictly concave in w. This assumption – unnecessary until now – is sufficient to sign two useful expressions: $p^{L*}_{w^L}$ and $Z^{L*}_{w^L}$, which allows to expand the structure of propositions 1 and 2 to w^L:

Proposition 3 (w^L-Gatsbian). *Assume that $c(w)$ is strictly concave. Then, an economy is w^L-Gatsbian in the following situations:*

a. A high-income economy ($Z^{L*} < \hat{Z}^L$), in which case a necessary but non-sufficient condition is that weighted marginal sticky ceilings be smaller than weighted marginal upward mobility:

$$\sqrt{\frac{w^H}{w^L}}p(\theta^{L*}) + p(\theta^{H*}) > 1, \qquad \textit{and} \qquad \frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} < \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}}. \tag{28}$$

In such a case, the economy could move up along a Gatsby curve.

b. A low-income economy ($Z^{L*} > \hat{Z}^L$), in which case a sufficient but non-necessary conditions is that weighted marginal sticky ceilings be larger than weighted marginal upward mobility:

$$\sqrt{\frac{w^H}{w^L}}p(\theta^{L*}) + p(\theta^{H*}) < 1, \qquad \textit{and} \qquad \frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} > \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}}. \tag{29}$$

In such a case, the economy moves down along a Gatsby curve.

Proof. Using (15), it is possible to obtain:

$$\frac{dG^*}{dw^L} = G^*_{Z^L} Z^{L*}_{w^L} - \underbrace{\left[\frac{1}{\Delta w} + \frac{Z^{L*}}{w^H - \Delta w Z^L}\right] G^*}_{+}. \tag{30}$$

Under the assumption of the strict concavity of $c(\cdot)$, $Z^{L*}_{w^L} > 0$. Thus, a sufficient condition for $dG^*/dw^L < 0$ is that $G^*_{Z^L} < 0$, which is equivalent to $\sqrt{w^H/w^L}p(\theta^{L*}) + p(\theta^{H*}) < 1$.

Further, the negativity of the bracketed term in (19c) is sufficient to guarantee that $dI^*/dw^L < 0$.

Combining the two statement proves Part 2 of the Proposition. Part 1 of the Proposition is the logical inverse of Part 2. □

It is now possible to obtain the following corollary.

Corollary 2 (Relation between e-Gatsbian and w^L-Gatsbian). *Assume that $c(w)$ is strictly concave.*

If a high-income economy is w^L-Gatsbian, moving up along a Gatsby curve, then it is also e-Gatsbian, but moving down along a Gatsby curve.

If a low-income economy is e-Gatsbian, moving up along a Gatsby curve, then it is also w^L-Gatsbian, but moving down along a Gatsby curve.

Thus, if an economy witnesses an increase in education and in the return to low-income jobs, then it may simultaneously be e-Gatsbian and w^L-Gatsbian. However, as is made clear in Corollary 2, education e and low-income w^L would then push inequality and immobility in opposite directions.

6 From Gatsbian to Non-Gatsbian (and Vice Versa)

Is an e-Gatsbian low-income economy, in which the level of schooling improves, doomed to forever move up along a Gatsby curve? The short answer is no. In this section, we begin by showing that an economy can be e-Gatsbian below a certain level of education, and then become non-Gatsbian when the level of education crosses a threshold. We then present two examples to illustrate the transition.

The transitions can be understood intuitively. Suppose we start from an e-Gatsbian economy where everybody is low-income. Then, replacing a low-income individual by a high-income one increases inequality. As the level of education keeps increasing, the proportion Z^{L*} is reduced. Initially, this increases inequality and immobility. Recall that in an e-Gatsbian low-income economy, weighted marginal sticky ceilings are larger than weighted marginal upward mobility, so that when we replace a low-income individuals by an high-income one, the marginal increase is larger for immobility than for mobility. But, as we show below, continued increases in education above some threshold may end up displacing a large enough proportion of the population into w^H-parents for the economy to become a high-income one. Then, one possibility is that immobility keeps on rising with improvements in education. But as inequality now falls when the proportion of low-income parents is reduced, the economy may become non-Gatsbian.

We focus on the characteristics $p(\cdot), c(\cdot)$ and e (and assume $\lambda \neq 0$) to show the existence of transition paths.

Proposition 4. *Assume $\lambda \neq 0$. There exists classes of economies $(p(\cdot), c(\cdot))$ with a critical level of education $\bar{e}$ such that the economies are e-Gatsbian only if $e < \bar{e}$. Conversely, there exists classes of economies $(\tilde{p}(\cdot), \tilde{c}(\cdot))$ with a critical $\tilde{e}$ that are e-Gatsbian only if $e > \tilde{e}$.*

Proof. Let $\lambda = 1$ and consider the two following probability classes:

$$p(\theta) = \theta^\sigma, \quad \sigma \in]0,1[, \qquad\qquad \tilde{p}(\theta) = 1 - (1-\theta)^2. \tag{31}$$

For each class, the corresponding difference in weighted marginal probabilities is given by:

$$\frac{\left(p_\theta^{H*}\right)^2}{-p_{\theta\theta}^{H*}} - \frac{\left(p_\theta^{L*}\right)^2}{-p_{\theta\theta}^{L*}} = \frac{\sigma}{1-\sigma}\left[p(\theta^{H*}) - p(\theta^{L*})\right] > 0, \tag{32}$$

$$\frac{\left(\tilde{p}_\theta^{H*}\right)^2}{-\tilde{p}_{\theta\theta}^{H*}} - \frac{\left(\tilde{p}_\theta^{L*}\right)^2}{-\tilde{p}_{\theta\theta}^{L*}} = -2\left[\tilde{p}(\theta^{H*}) - \tilde{p}(\theta^{L*})\right] < 0. \tag{33}$$

Hence, an increase in education can only lead to an increase in social immobility with the first class while it can only lead to a decrease in social immobility with the second class.

Now, note that with $\lambda = 1$, $\theta = \phi e$, which means that when $e = 0$, $\theta^{H*} = \theta^{L*} = 0$. Consider the following functions, corresponding to the above two classes:

$$f(e) \equiv \sqrt{\frac{w^H}{w^L} p(\theta^{H*}(e)) + p(\theta^{L*}(e))} \qquad \tilde{f}(e) \equiv \sqrt{\frac{w^H}{w^L} \tilde{p}(\theta^{H*}(e)) + \tilde{p}(\theta^{L*}(e))} \tag{34}$$

Both functions are continuous and strictly increasing in e. Further, note that $f(0) = \tilde{f}(0) = 0 < 1$. Also, $f(e) \xrightarrow{e\to\infty} \sqrt{w^H/w^L} + 1 > 1$ and $\tilde{f}(e) \xrightarrow{e\to\infty} \sqrt{w^H/w^L} + 1 > 1$. From the mean value theorem, it follows that there exists levels of education $\bar{e}, \tilde{e}$ such that $f(\bar{e}) = \tilde{f}(\tilde{e}) = 1$. This means that inequality (17) may be satisfied or not, depending on whether e is greater or smaller than $\bar{e}$ (respectively e greater or smaller than $\tilde{e}$).

Given the above, for the first class of probability $p(\cdot)$, the economy is e-Gatsbian only when $e < \bar{e}$. As for the second class, the economy is e-Gatsbian only when $e > \tilde{e}$. □

We now go through two examples using the probability classes of the proof of Proposition 4. The parameters specifications are presented in Table 3 and all related equations are in Appendix.

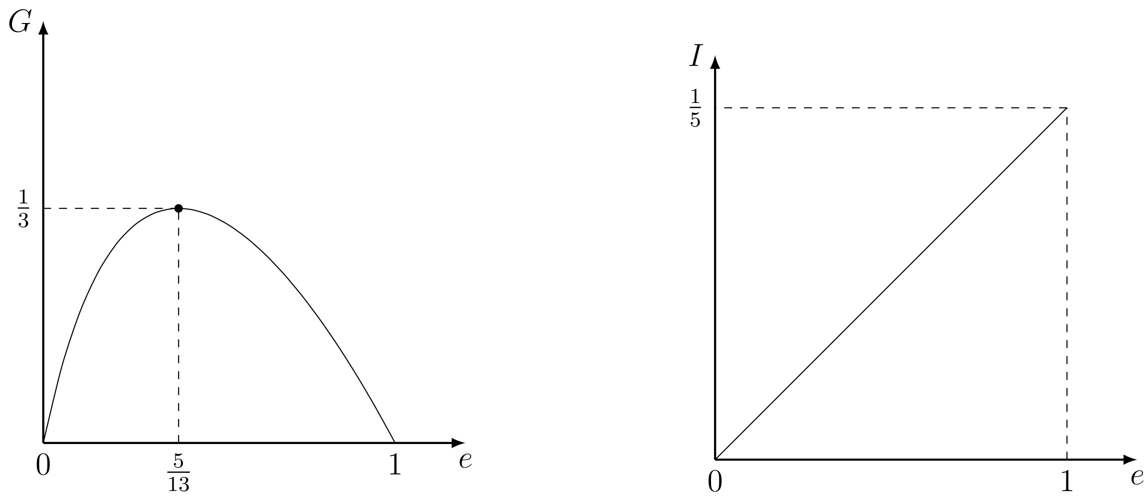
Table 3: Specific Instances of Examples

	From e-Gatsbian to non-Gatsbian	From non-Gatsbian to e-Gatsbian
$p(\theta)$	$\sqrt{\theta}$	$1-(1-\theta)^2$
$c(w)$	$2-\frac{w}{8}$	$6-\frac{w}{2}$
w^H	4	4
w^L, β, λ	1	1
e	$e \in [0,1]$	$e \in \left[\frac{11}{12}, \infty\right[$

Note: The range for e is chosen so as to avoid corner solutions.

The results for $p(\theta) = \sqrt{\theta}$ are presented in Figure 6. Figure 6a depicts the Gini coefficient of the economy when education increases. Starting from $e = 0$, we note that the initial increases in education translate into an increase in inequality. However, when e reaches a level of $\bar{e} = 5/13$, inequality reaches a maximum ($G = 1/3$) and then declines with further increases in education. As for 6b, it depicts immobility as a function of education. When the probability function $p(\theta) = \theta^\rho, \rho \in (0,1)$ is chosen, social immobility becomes a power function of education ($I \propto [\lambda e + 1 - \lambda]^{\frac{\rho}{1-\rho}}$) and, in the selected case where $\rho = 1/2$, the power function degenerates to the affine case.

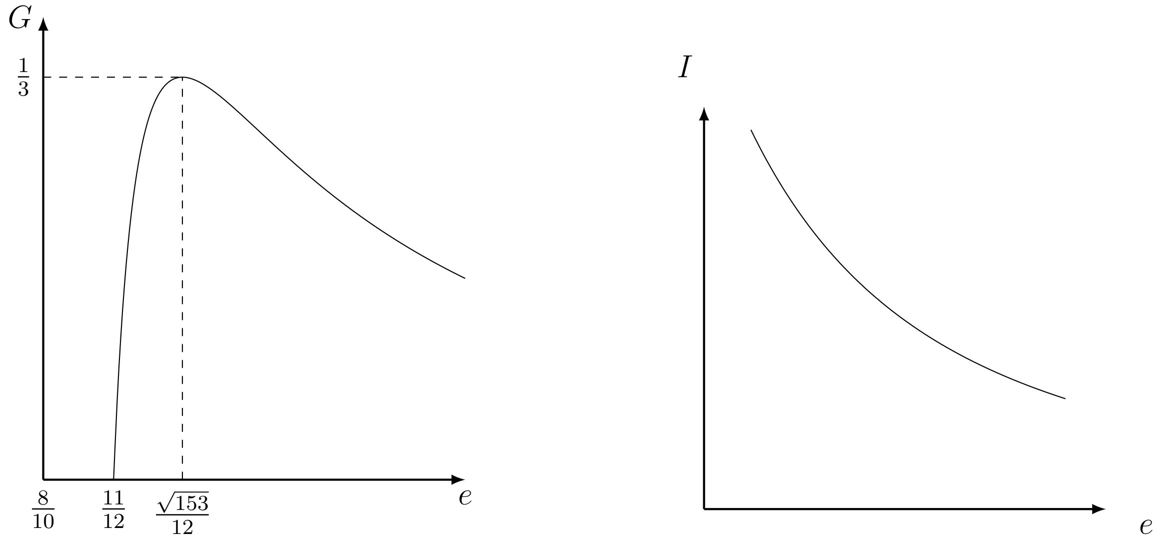
The interest here is in the combination of the two panels. In Figure 8a one can immediately note that for $e \in [0, 5/13[$, both inequality and immobility rise with education so that this segment of the e-expansion path is positively sloped and the economy is e-Gatsbian. However, when $e > 5/13$, inequality falls while immobility keeps on rising, so that portion of the e-expansion path is negatively sloped and the economy is no longer Gatsbian. When education increases steadily, as has been observed around the world in the last decades, it is possible for an economy to transition from a positively-sloped segment of its e-expansion path to a negatively-sloped one. In other words, the economy may transition from being e-Gatsbian to non-Gatsbian.



(a) Gini Coefficient as a Function of e (b) Immobility as a Function of e

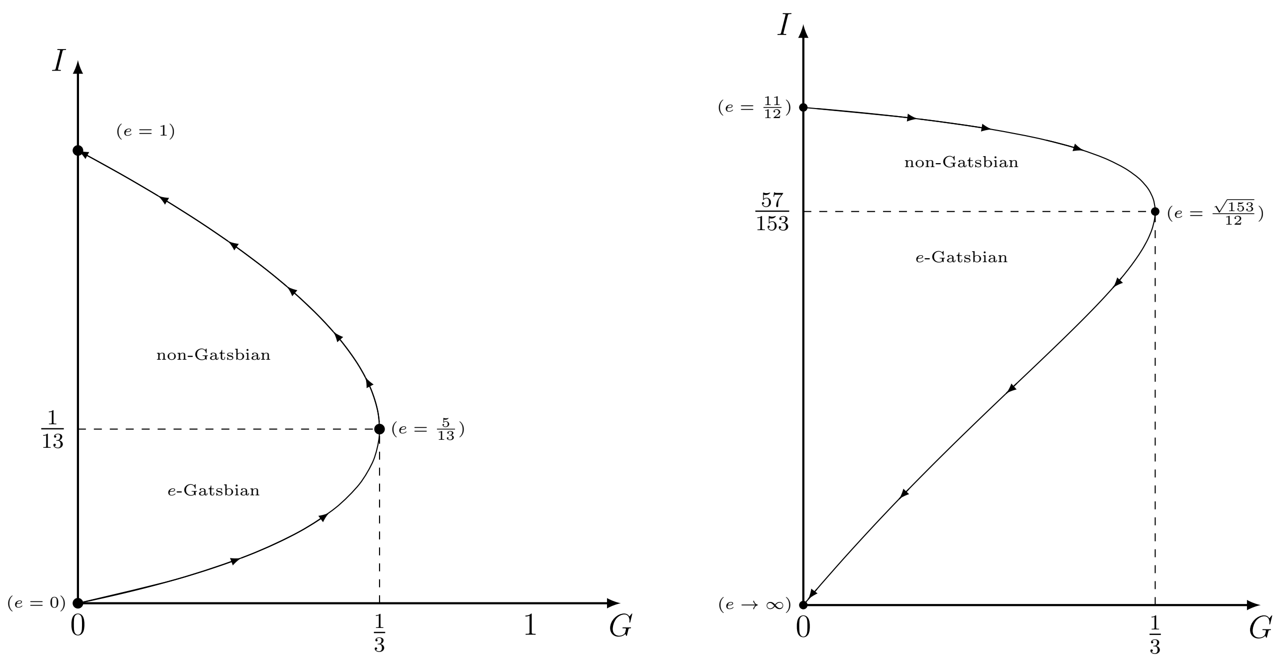
Figure 6: $p(\theta) = \sqrt{\theta}$.

We report the results of the second example ($\tilde{p}(\theta) = 1 - (1 - \theta)^2$) in Figure 7. Figure 7a depicts the Gini coefficient of the economy when education increases. Starting from $e = 11/12$, we note that the first increases in education translate into an increase in inequality. However, when e reaches a level of $\tilde{e} = \sqrt{153}/12$, inequality reaches a maximum ($G = 1/3$) and then declines with further increases in education. The mechanism at work here is the same as that in Figure 6.



(a) Gini Coefficient as a function of e (b) Immobility as a function of e

Figure 7: $\tilde{p}(\theta) = 1 - (1-\theta)^2$.



(a) $p(\theta) = \sqrt{\theta}$. (b) $\tilde{p}(\theta) = 1 - (1-\theta)^2$.

Figure 8: e-expansion Paths for Selected Examples

However, Figure 7b shows that immobility decreases with education. The curve depicted is inversely quadratic with e ($I \propto e^{-2}$), which happens with the cumulative probability function $\tilde{p}(\theta) = 1 - (1-\theta)^2$.

Combining the two panels yields Figure 8b. For $e \in [11/12, \sqrt{153}/12[$, the e-expansion path is negatively sloped, but for $e \in]\sqrt{153}/12, \infty[$, the path is positively sloped and the economy is e-Gatsbian. Thus, in this example, the economy transitions from being non-Gatsbian to being e-Gatsbian.

7 Alternative Model: Networks

In our core theory, w^H-parents invest more in their children than w^L-parents. It is so because the *marginal cost* of parental investment is lower for w^H-parents than for w^L-parents.

We now consider an alternative specification of our model in which the *return* of parental investment is larger for w^H-parents than for w^L-parents. This would be because, as has been argued in the literature, high-income parents tend to have a better network of relations (with high-income employers) than low-income parents. For example, Corak & Piraino (2011) document the fact that children tend to occupy jobs with an employer that has employed their parents. As a consequence, the likelihood that children will land in a high-income job is higher when their parents themselves earn a high-income.

Hence, we first shut down the marginal cost channel by assuming that the cost of parental investment for a w^i-parent is simply $c\phi$, where $c > 0$ is a parameter independent of w^i. Thus, all parents face the same cost of investing.

But now, reflecting the presence of networks in the job market, we assume that the probability that a child with employability θ will land in high-income job w^H is strictly increasing in the income of his parents w^i. Specifically, we assume that $p(\theta, w^i) = n(w^i)r(\theta)$, where $n_{w^i} > 0$, $r_\theta > 0$, and $r_{\theta\theta} < 0$. Thus, the return of parental investment is larger for w^H- than for w^L-parents.

Then, a w^i-parent's problem is:

$$\max_{\phi}\ w^i - c\phi + \beta[n(w^i)r(\theta)\Delta w + w^L], \tag{35}$$

where θ is given by equation (1). The first-order condition of problem (35) is as follows:

$$-c + \beta n^i r_\theta \theta_\phi \Delta w = 0. \tag{36}$$

where both r_θ and θ_ϕ are independent of w^i. From there, it is easily obtained that all the qualitative results of the previous sections follow.[12] For example, it can be shown that $\phi^{H*} > \phi^{L*}$ so that $\theta^{H*} > \theta^{L*}$ and $p^{H*} > p^{L*}$. Further, the comparative static results presented in subsection 3.2 all continue to hold with minor adaptations. Finally, and this is the important point, the conditions under which a Gatsbian economy arise remain the same (again with minor adaptations). Thus, all the results obtained in previous sections carry in this alternative formulation.

It follows that the source of the larger investment of w^H-parents in their child — whether a lower marginal cost or a larger return — makes no substantial difference in the generation of Gastby curves.

8 Conclusion

We have explained what Gatsby curves are and in what conditions they may arise.

Our analysis builds on a rich yet tractable Becker & Tomes (1979) environment in which parental investment and education both affect children's expected job market outcome. While high-income parents can invest in their children's employability at a lower marginal cost, children from low-income family can nonetheless land in high-income jobs, albeit at a

[12]The complete analysis is available upon request.

lower probability than those from high-income families.

We are able to derive closed-form expressions for inequality and immobility. We analyze the impact of changes in the economic environment on the stationary state of the economy and we assess their impact on inequality and immobility. In the inequality-immobility space, the locus of stationary states generated by those changes is simply an expansion path. When this expansion path is positively sloped, it is a Gatsby curve.

In the empirical literature, it is well documented that education (Global Change Data Lab, 2019) and the income level associated with better jobs (Acemoglu & Autor, 2011; Goos et al., 2009; Barany & Siegel, 2018; Bourdabat et al., 2010) have both significantly increased in the last decades in several countries. We show that Gatsby curves may have been generated by such increases. At the same time, we also show that circumstances exist in which Gatsby curves will not be generated by an increase in education or in the return to high-paying jobs, which could explain that in some countries, Gatsby curves have not been observed. We also show that an economy may transition from being Gatsbian to non-Gatsbian. Finally, we provide an alternative framework showing that our results can be extended to a social network environment, in which children of high-income families can benefit from the networks of their parents.

Are Gastby curves a good or a bad thing? Do they result from optimal policies? At a time where policymakers are showing keen interest in the Gatsby curve phenomenon, we must observe that these important questions have not yet received satisfactory answers in the literature. As it turns out, the framework developed in this paper naturally lends itself to a normative analysis of Gatsby curves and of education and taxation policies. It could therefore be used to provide answers to the above questions. However, prior to undertaking this task, one has to develop a proper measure of social welfare when both inequality and immobility may be affected by policies.[13]. These are promising avenues for future research.

[13] See Gottschalk & Spolaore (2002) for an interesting analysis of the key issues and difficulties that arise in this context.

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A Mathematical Appendix

A.1 Mixing Conditions For Transitionary Dynamics

A pair of eigenvalue and eigenvector (λ, v) satisfies the following equation:

$$\underbrace{\begin{bmatrix} \lambda - p(\theta^{*H}) & p(\theta^{*L}) \\ 1 - p(\theta^{*H}) & \lambda - (1 - p(\theta^{*L})) \end{bmatrix}}_{=\lambda I - P} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0, \tag{37}$$

which implies that the determinant of the matrix $\lambda I - P$ must be zero. This is true if

$$0 = \lambda^2 - (p(\theta^{*H}) + 1 - p(\theta^{*L}))\lambda - p(\theta^{*L})(1 - p(\theta^{*H})), \tag{38}$$

which is a quadratic equation in λ. Its first root is $\lambda = 1$ and its second root is

$$\lambda = p(\theta^{*H}) - p(\theta^{*L}) \in [0, 1]. \tag{39}$$

The transition matrix has a unique stationary state provided this second eigenvalue is smaller than one ($|\lambda| < 1$). This condition is always satisfied because $p(\theta)$ is strictly increasing and strictly concave, and $p(\theta^{*L}) < p(\theta^{*H}) < 1$.

Furthermore, it should be noted that the determinant of P is equal to:

$$p(\theta^{*H})(1 - p(\theta^{*L})) - p(\theta^{*L})(1 - p(\theta^{*H})) = p(\theta^{*H}) - p(\theta^{*L}), \tag{40}$$

which equals the second eigenvalue and the trace of the matrix (up to adding one).

A.2 Expression for the Gini Coefficient

The Lorenz curve links the cumulative proportion of the population with the cumulative proportion of total income that it earns. In our model, it is obtained using the fact that a proportion Z^L of w^L-parents earn $Z^L w^L/\bar{w}$ of the total income in the economy, while a proportion $(1 - Z^L)$ of w^H-parents earns a proportion $(1 - Z^L)w^H/\bar{w}$. Figure 9 below shows the Lorenz curve in the economy (black line). The blue-shaded area is the area between the Lorenz curve and the 45-degree line (dashed line). Algebraically, the shaded area of Figure

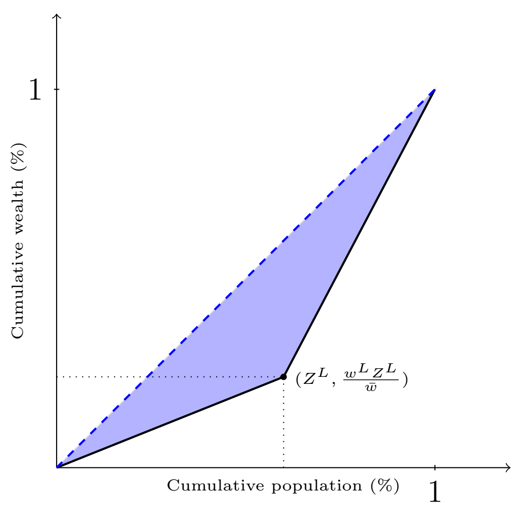

Figure 9: The Lorenz curve. The Gini coefficient equals twice the blue area.

9 is given by:

$$\frac{1}{2}Z^L(1 - Z^L)\left(\frac{\Delta w}{w^H - \Delta w Z^L}\right) \qquad \Rightarrow \qquad G = Z^L(1 - Z^L)\left(\frac{\Delta w}{w^H - \Delta w Z^L}\right), \tag{41}$$

since the Gini coefficient is exactly twice this surface.

A.3 Derivation of $\hat{Z}^L$

The first order condition of (41) with respect to Z^L is given by:

$$\frac{\Delta w}{w^H - \Delta w Z^L} - \frac{1}{1 - Z^L} + \frac{1}{Z^L} = 0, \qquad \Rightarrow \qquad w^H - 2w^H \hat{Z}^L + \Delta w (\hat{Z}^L)^2 = 0. \tag{42}$$

The second equation of (42) is a quadratic expression in $\hat{Z}^L$. Its two solutions are:

$$\hat{Z}^L = \frac{\sqrt{w^H}}{\sqrt{w^H} + \sqrt{w^L}} \in]1/2, 1[, \qquad \hat{Z}^L = \frac{\sqrt{w^H}}{\sqrt{w^H} - \sqrt{w^L}} > 1. \tag{43a}$$

Only the first solution of (43a) is smaller than 1.

A.4 Solutions to the Examples from Section 6

The two examples that are presented in section 6 have the following explicit (interior) solutions.

$$p(\theta^{H*}) = e, \qquad Z^{L*} = \frac{5(1-e)}{5-e}, \qquad I^* = \frac{1}{5}e, \tag{44}$$

$$p(\theta^{L*}) = \frac{4}{5}e, \qquad G(e) = \frac{4(1-e)}{5-e}\frac{15e}{5+11e}, \qquad \arg\max_e G(e) = \frac{5}{13}, \tag{45}$$

$$G(I) = \frac{4(1-5I)}{5(1-I)}\frac{15I}{1+11I}, \qquad \arg\max_I G(I) = \frac{1}{13}. \tag{46}$$

$$p(\theta^{H*}) = 1 - \frac{4}{9}\frac{1}{e^2}, \qquad Z^{L*} = \frac{64}{144e^2 - 57}, \qquad I^* = \frac{57}{144}\frac{1}{e^2}, \tag{47}$$

$$p(\theta^{L*}) = 1 - \frac{121}{144}\frac{1}{e^2}, \qquad G(e) = \frac{64}{144e^2 - 57}\frac{3(144e^2 - 121)}{576e^2 - 420}, \qquad \arg\max_e G(e) = \frac{\sqrt{153}}{12} \tag{48}$$

$$G(I) = \frac{64I}{57(1-I)}\frac{3(57 - 121I)}{228 - 420I}, \qquad \arg\max_I G(I) = \frac{57}{153}. \tag{49}$$