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Sugata Marjit, Shrimoyee Ganguly, Rajat Acharyya

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Minimum Wage, Trade and Unemployment in General Equilibrium

Abstract

The path breaking work of Card and Krueger (1993), showing higher minimum wage can increase employment turned the age-old conventional wisdom on its head. This paper demonstrates that this apparently paradoxical result is perfectly plausible in a competitive general equilibrium production structure of a small open economy with a non-traded good, without taking any recourse to monopsony, spatial heterogeneity, heterogeneity of consumers etc. the usual theoretical drivers behind the result. Following Jones and Marjit (1992) we build up a simple general equilibrium model with complementary relationship in production and we show that higher minimum wage can raise aggregate employment. Expansion in the non-traded sector following a wage hike may be consistent with the overall expansion of the export sector in a multi good framework, an unlikely outcome in a conventional two good models which cannot accommodate complementary relationship in production.

JEL-Codes: J200, J300, F110, F160.

Keywords: minimum wage, employment, non-traded good.

*Sugata Marjit**
Indian Institute of Foreign Trade
India – Kolkata-700107
marjit@gmail.com

Shrimoyee Ganguly
Department of Economics
Jadavpur University / Kolkata / India
sssganguly@gmail.com

Rajat Acharyya
Department of Economics
Jadavpur University / Kolkata / India
rajat.acharyya@gmail.com

*corresponding author

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This paper is my humble tribute to Ron Jones- my teacher, mentor and long time collaborator. We are indebted to an anonymous referee for helpful comments. The usual disclaimer applies.

1. Introduction

Higher minimum wage can increase employment – a hypothesis that became talk of the town due to the remarkable contribution of Card and Krueger (1993) which showed the positive impact of a minimum wage law on the employment of New Jersey. This opened doors to a large body of literature, mostly empirical, critically looking at the age old neo-classical wisdom of a standard competitive market which predicts that higher minimum wage will reduce employment. Usually the conventional wisdom carries through the standard 2x2 or 2x3 models (Jones [1965], Jones [1971]). The purpose of this paper is to show how a simple general equilibrium structure in line with Jones and Marjit (1992) can get the Card and Krueger (1993) type result for a small economy with a non traded good without making special assumptions regarding demand, market structure etc. Thus the surprising result does not seem to be so surprising if one cares to look beyond the conventional neo-classical models.

Over the last few decades, an enormous body of research, primarily empirical, has been devoted to the employment effects of minimum wage with renewed interest after the seemingly paradoxical finding of Card and Krueger (1993) turned the age-old conventional wisdom of minimum wage hikes destroying jobs on its head. Surveying more than 400 fast-food restaurants in New Jersey and eastern Pennsylvania, they found no evidence that employment growth was slower in New Jersey following the minimum wage increase from $4.25 to $5.05 per hour in the year 1992. They concluded that despite the increase in wages, full-time-equivalent employment increased in New Jersey relative to Pennsylvania.

Such out of the box and paradoxical result compelled both critics and supporters of the minimum wage laws to rethink, giving space to possibilities of an employment improvement from a hike in wages. Since then, studies on this issue became more rigorous in terms of empirical analyses and theoretical explanations had new dimensions. Several empirical papers by Neumarck and Wascher (2008), Dube et.al (2010), Addison, Blackburn and Cotti (2009), Dube, Lester, and Reich (2016), Gittings and Schmutte (2015) etc. have discussed various empirical issues related to the problem.

For the Chinese economy, using a difference-in-difference methodology, Wang and Gunderson (2012) have found that overall minimum wages in China do have an adverse employment effect. Using a similar difference-in-difference approach, Cengiz et. al. (2019) found that the overall number of low-wage jobs in the United States remained essentially unchanged over the five years following the wage increase. What one can infer from these empirical studies is that varied research designs and their take on spatial heterogeneity account

for contrasting results about the effect of minimum wage on employment, as was pointed out in Neumark, Salas and Wascher (2013).

Whatever be the varied outcome of the empirical findings, with the exception of Brecher and Gross (2018) considering heterogeneous consumers, there is not much theoretical work in the literature in terms of competitive models, which looks rigorously at the production and demand effects of a minimum wage hike in a multi sector general equilibrium setting with standard assumptions. Among other theoretical studies, positive employment effects were derived by Ahn and Arcidiacono (2004) by developing a two-sided search model with endogenous labor supply and labor demand. As the number of matches increase with number of searching workers, an increase in the minimum wage induces more workers who were previously not searching, to participate in the labor market thus raising their chances of finding employment. Developing a heterogeneous firm model under perfect competition in a Heckscher-Ohlin setting, Bai et. al. (2018) showed that a binding minimum wage raises product prices of capacity constrained firms, encouraging substitution away from labour, and thus creating unemployment. They found robust evidence in support of their theoretical predictions using Chinese firm level production data. What the above studies provoke us to ponder upon is that the declining employment result can be just an artefact of the single sector simplified model as also pointed out by Fields (1986). Nonetheless, typically the one sector lesson on the classic negative relationship between minimum wage and employment has been taken as a building block and contradictions are treated as new results. Though there have been discussions on monopsony models [Aaronson and French (2007)] as one such alternative, but again all were in terms of partial equilibrium framework.

Such a partial equilibrium result readily carries through to the standard 2x2 structure of Jones (1965) or 2x3 structure of Jones (1971) for a small open economy with the addition of a fixed wage. Hence, the result seems to be robust to conventional general equilibrium modeling. In the 2x2 structure, with given commodity prices, it is hard to avoid complete specialization as capital moves to the sector promising higher real return. Then it boils down to a partial equilibrium world and a hike in minimum wage will reduce employment due to usual reasons. For the specific factor case, returns to specific factors are different. Each sector produces a separate good and partial equilibrium result is a natural outcome. Thus for a small open economy producing two goods, one cannot escape the conventional wisdom even if one goes from a partial equilibrium to a general equilibrium setting. Next consider a 2x2 structure with one of the two goods being a labor intensive non-traded good. As the minimum wage hike raises national income, the demand for import is likely to rise. To maintain trade balance, production of the export good must increase in the new equilibrium to match the rise in import. This can happen if the non-traded sector contracts. This will cause aggregate employment to fall via the output

magnification effect. Since there is redistributive effect as wage earners gain and capitalists lose, consumption bias with heterogeneous consumers can get the reverse result. Alternatively, an excess demand for non-traded good or a trade surplus must arise at the given level of employment. This would become plausible once we allow for more than one export good. The export sector as a whole then can expand along with the non traded good even if one of the export sectors contracts. Such a complementary relation is available from Jones and Marjit (1992) which generalized the small but elegant example of Gruen and Corden (1970) in a multi good framework. We use that mechanism to prove positive effect on employment following higher minimum wage.

The unique point of this paper is that it speaks about the necessity of a non-traded sector for the result. Hence, it leads to a theoretical question whether we can get the positive impact of a minimum wage hike when non traded component of the economy is insignificant. Note here that even Card and Kruegar in their original work had focussed on a local food market to come up with their unconventional results. If New Jersey and Pennsylvania had both exported the food to the rest of the world with prices determined externally, then they would have suffered in the short run from lowered employment levels. Existence of non-traded goods in the production set is not at odds with reality. Examples of such goods and services include hair cut, nursing, major components of construction, freshly prepared food and the like. Even most of the goods that are imported are actually value added locally before these are made available for consumption, rendering most of those goods for final consumption as non-traded as well, as was pointed out by Sanyal and Jones (1982).

The rest of the paper proceeds as follows. In section 2 we present the basic model and the main result. Section 3 discusses some extensions and robustness of the result and Section 4 concludes.

3. Minimum Wages and Employment: Formal Analysis

3.1 Analytical structure and basic assumptions

Consider a small open economy producing three-goods: two export goods – X and Z – and a non-traded good N. Export good X and the non-traded good N are produced under constant returns to scale technology by sectorally mobile physical capital (K) and unskilled labour (L), whereas the export good Z is produced by skilled labour (S) specific to this sector and capital.[1] While flexibility in the rate of return to capital (r) and skilled wage (w_S) along with competitive market forces ensure full utilization of capital stock and skilled labour, fixed minimum money wage for unskilled workers ($\vec{w}$) leads to an initial equilibrium with some unskilled workers remaining unemployed. To begin with, we assume that the economy consumes the non-traded good and another good Y, which is entirely imported, but not the two export goods. Moreover,

we assume that constant proportions of income are being spent on N and Y. Later we will consider domestic consumption of goods X and Z as well, and a more generalised demand structure for all these goods as a robustness check for our result.

Perfect competition in the three sectors yields the following price-average cost conditions:

$$P_Z^W = a_{SZ} w_S + a_{KZ} r \quad (1)$$

$$P_X^W = a_{LX} \overline{w} + a_{KX} r \quad (2)$$

$$P_N = a_{LN} \overline{w} + a_{KN} r \quad (3)$$

where, P_X^W and P_Z^W are the world prices exogenously given to this small open economy; P_N is the price of the non-traded good and a_{ij} ($i = L, K; j = X, N$) and a_{hZ} ($h = S, K$) denote the per unit requirement of input-i (input-h) in production of good-j (good Z), which depend on the relevant factor price ratios:

$$\begin{aligned} a_{ij} &= a_{ij}(\overline{w}/r), i = L, K, j = X, N \\ a_{hZ} &= a_{hZ}(w_S/r), h = S, K \end{aligned} \quad (4)$$

Full employment conditions for skilled labour and capital, and aggregate employment of unskilled labour are given as,

$$\overline{S} = a_{SZ} Z \quad (5)$$

$$\overline{K} = a_{KX} X + a_{KN} N + a_{KZ} Z \quad (6)$$

$$L_e = a_{LX} X + a_{LN} N \quad (7)$$

The market for non traded good must clear domestically which requires,

$$D_N\left(\frac{P_N}{P_Y^W}, y\right) = N \quad (8)$$

where, P_Y^W is the given world price of the imported good Y and y is the aggregate real income of our small open economy, measured in terms of good X.

Note that, by Walras' Law, the equilibrium in the non-traded market implies that trade is balanced. This is evident from the economy's budget constraint which, after normalizing the given world prices of the traded goods X, Y and Z, to unity, is given as:

$$D_Y + P_N D_N = P_N N + X + Z \quad (9)$$

Given our assumption of the two export goods not consumed domestically, the left hand side in (9) is the aggregate consumption expenditure. The right hand side gives us aggregate value of production and (produced) income of the economy, y. Thus, by the market clearing condition (9) for the non-traded good, trade is balanced:

$$D_Y = X + Z \quad (10)$$

We close our model by specifying the demand structure. We assume a Cobb Douglas aggregate utility function as a special case of homothetic taste. Such a utility function yields demand functions such that constant proportions of national income are spent on these goods. Let α and $1-\alpha$ be these proportions spent on imports and the non traded good respectively. Hence,

$$D_Y = \alpha(P_N N + X + Z)\,,\; D_N = \frac{(1-\alpha)(P_N N + X + Z)}{P_N} \qquad (11)$$

Equation system (1) – (8) describes our small open economy, with thirteen independent equations determining the thirteen variables – r, w_S, P_N, six a_{ij}s, X, N, Z and L_e.

Note that given the initial state of technology, the world price of the export good and money wage rigidity, the rate of return to capital (r) is solely and uniquely determined by the zero-profit condition for good X (see (2)). Once r gets determined from (2), it pegs the price of the non-traded good, that is, price of the non-traded good becomes *cost determined* and is invariant with respect to changes in demand for the non-traded good (see the zero profit condition (3)). On the other hand, for any set of factor prices and corresponding choices of input coefficients, the output of the skill based export good Z is determined by the availability of skilled labour (see (5)). Given such an output level of good Z and corresponding demand for capital outputs of the non-traded good and the export good X and the level of aggregate employment are determined simultaneously by (6), (7) and (8).

To illustrate such simultaneous determination, note that both the supply of the non-traded good and the demand for it varies with the aggregate level of employment. For any given set of parameters and corresponding production of good Z, an increase in aggregate employment raises the production and supply of the non-traded good (and reduces that of the export good X) if it is labour intensive relative to the export good X by the standard output magnification effect a la Jones (1965). This relationship is shown in Figure 1 by the positively sloped curve *NN*. On the other hand, as shown in the appendix, an increase in aggregate employment, ceteris paribus, raises aggregate output and produced income of the economy, raising output of N at the cost of output of X.[2] By (11), for any given P_N, the demand for non-traded good thus rises proportionately. This relationship is shown by the positively sloped curve $D_N D_N$. Note that by stability with Marshallian adjustment process, this curve must be flatter than the *NN* curve. At point E_0, the market for the non-traded good clears resulting in the equilibrium aggregate employment as L_e^0.

3.2 Employment effect of a hike in the minimum wage

Consider now a rise in minimum wage. This affects aggregate employment in three major ways. First, the hike in the minimum wage lowers the rate of return to capital for any given world price of the export good X, thereby raising the wage-rental ratio:

$$\hat{r} = -\frac{\theta_{LX}}{\theta_{KX}}\hat{\bar{w}} \qquad (12)$$

where, θ_{LX} and θ_{KX} are respectively share of labour and capital in average cost of producing good X; and "hat" over a variable denotes its proportional change. This, through the standard factor substitution effect, lowers the aggregate employment since producers everywhere would use relatively more capital intensive techniques than before the wage hike.

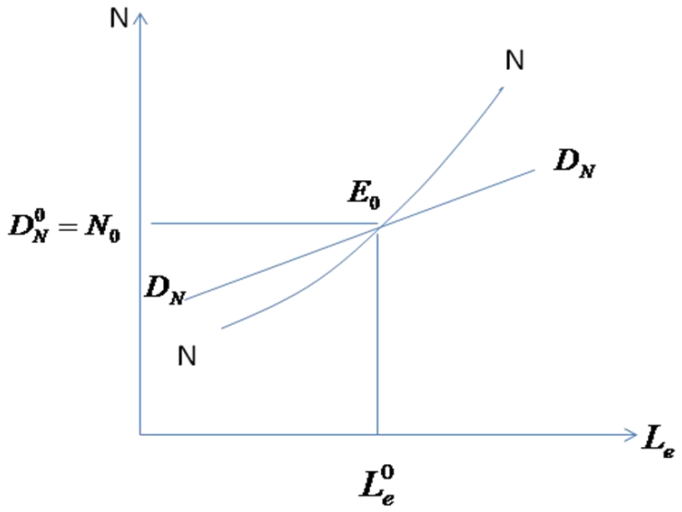

Figure 1: Aggregate Employment Determination

The second effect comes through a change in demand for the non-traded good. As the wage hike lowers the rate of return to capital, so it will raise (lower) the cost-determined price of the non-traded good if the non-traded good is relatively labour (capital) intensive:

$$\hat{P}_N = -\frac{|\theta|}{\theta_{KX}}\hat{\bar{w}} \qquad (13)$$

where, $|\theta| = \theta_{LX}\theta_{KN} - \theta_{KX}\theta_{LN} < 0$ if N is labour intensive relative to X and positive otherwise. Thus, if N is relatively labour intensive, increase in its price following the hike in the minimum wage will lower its demand and consequently its output. The aggregate employment should fall on this account,

The third effect on aggregate employment comes from the changes in the demand for non-traded good due to change in national income. Ceteris paribus, the hike in the minimum wage raises the national income and thus demand for imports. To finance the additional import bill and thus maintain trade balance, the aggregate value of exports, X + Z, must rise. The volume (and value) of skill-based exports Z increase unambiguously since the hike in minimum wage raises the skilled wage, which through substitution of skilled labour by capital per unit of output of good Z enables the producers to expand its scale of production:

$$\hat{Z} = \sigma_Z \frac{\theta_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}} \hat{\bar{w}} \qquad (14)$$

where, σ_Z is the factor substitution elasticity.

Now, at the initial production and exports of X, if this increase in the exports of Z is larger than the increased import bill, then a trade surplus develops, which in turn means an excess demand for the non-traded good and hence corresponding increase in the output of the non-traded good. However, since additional production of export good Z withdraws some capital from (X, N) nugget, increase in the output of non-traded good in response to such excess demand for it would necessitate a fall in the production of export good X. This would in turn lower the aggregate value of exports. Therefore, for non-traded output to rise in the final analysis, a sufficiently large expansion in the production and exports of the skill-based good Z (in the sense defined below) -- large enough to cause non-traded production to expand even with a contraction in the production of the other export good X and the two adverse effects on employment coming from increase in the price of the non-traded good and substitution of unskilled labour by capital in (X, N) nugget discussed above -- is required. As shown in the appendix, such a large expansion is ensured by a high value of the factor substitution elasticity:

$$\sigma_Z > \frac{\left[(1-\theta_N)\frac{|\theta||\lambda|}{\theta_{KX}} + \{2\theta_X + (\lambda_{LX}+\lambda_{KX})\theta_Z\}A\right]\theta_{SZ}\theta_{KX}}{\varphi\lambda_{KZ}\theta_{LX}} \equiv \tilde{\sigma}_Z \qquad (15)$$

where, $A = \frac{\lambda_{KX}\sigma_X\theta_{LX} + \lambda_{KN}\sigma_N\theta_{LN}}{\theta_{KX}} > 0$ and $\varphi \equiv \theta_Z(\lambda_{LN} - 2\lambda_{LX}) - \theta_X$.

For this condition to be satisfied the necessary condition is that $\varphi > 0$ and this, in turn, requires - a) $\lambda_{LN} > 2\lambda_{LX}$, b) $\theta_Z > \frac{\theta_X}{\lambda_{LN} - 2\lambda_{LX}}$.

Note that, despite a fall in employment due to contraction of the output of X, aggregate employment would rise through the increase in the production of the non-traded good as long as

it is unskilled-labour intensive relative to the export good X. Thus, this factor intensity condition is a necessary condition for employment expansion.

The adjustments following the wage hike is shown in Figure 2. At the initial equilibrium employment, the NN curve must shift upward as higher minimum wage leads to substitution of labour by capital generating a surplus which needs to be absorbed by a rise in N. This is a standard general equilibrium output effect. Note that in (15) low value of A (in the numerator) which helps our result is caused by low elasticities of factor substitution in X and N. Higher minimum wage in that case will release less labour, N will expand less and hence shift of NN will be smaller.

The demand line responds to the change in trade balance and the total X+Z. The rise in Z depends on the decline in r (the first component of the numerator in (15) and the elasticity of substitution of Z (the LHS in (15)). At the same L_e, there will be an excess demand for N if there is a rise in $X+Z$ and that exceeds the rise in Y, that is, there is a trade surplus. Hence the shift in demand for N will have to exceed the shift in NN, leading to a rise in L_e, as depicted in Figure-2. Thus, if demand shift is less than supply in Figure-2 it will imply a resultant trade deficit even with a rise in Z. Thus X needs to be pushed up to retain the trade balance condition leading to a drop in N and in L_e. Note that any change in L_e does not affect Z which is solely determined by the drop in r which remains unaffected by L_e. The rise in Z has to be adequate to allow the demand relationship in Figure-2 to shift up more than NN at initial L_e. Then employment will rise following a wage hike.

One should also note that the size of S i.e. available stock of skilled labour is important for the result. The expansion in Z depends on S as well as elasticity of factor substitution. For the same drop in per unit skilled labour due to a rise in w_S, greater will be the output of Z larger is the stock of S. Hence, higher minimum wage will raise employment of unskilled labour greater is the amount of skilled labour. Stronger will be the impact due to complementary effect.

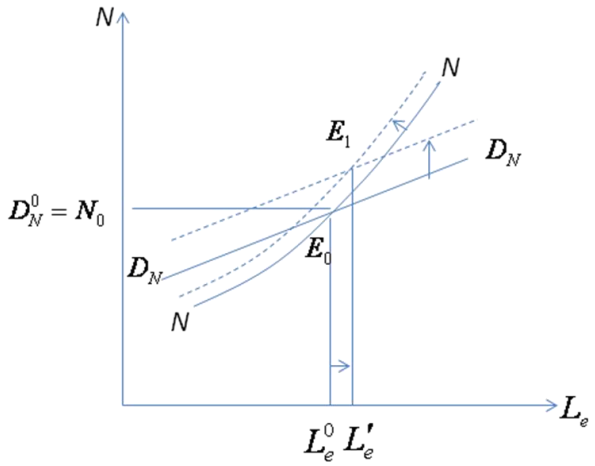

Figure 2: Hike in Minimum Wage and Employment

4. Robustness Check

4.1 A General Demand Structure

In this section we recast our analysis with more general demand pattern to demonstrate that our results derived in the previous section is not contingent upon the assumption that, first, the two export goods are not consumed domestically, and second, homothetic tastes (or constant fractions of income being spent on the goods consumed). Let D_j , j = Z, X, Y, N, denote the demand for the j-th good. Setting the *given* world prices of Z, X and Y to unity, the demand function for the j-th good would depend on the price of the non-traded good and the real income y as defined earlier:

$$D_j = D_j(P_N, y) \tag{17}$$

Thus, proportional change in the demand for j-th good is given as,

$$\hat{D}_i = \varepsilon_i \hat{P}_N + \eta_i \hat{y} \tag{18}$$

where, ε_i is the cross-price elasticity capturing substitution in consumption of good j by good N when P_N changes, and η_i is the income elasticity of good j. Using the change in real income $\hat{y} = \theta_X \hat{X} + \theta_Z \hat{Z} + \theta_N \hat{N} + \theta_N \hat{P}_N$ above boils down to,

$$\hat{D}_N = -(\varepsilon_N - \eta_N \theta_N)\hat{P}_N + \eta_N(\theta_X \hat{X} + \theta_Z \hat{Z} + \theta_N \hat{N}) \tag{19}$$

For any given minimum wage, P_N and Z do not change, so, the demand for non-traded good rises with an increase in aggregate employment as before and the $D_N D_N$ curve in Figure 1 is still positively sloped. The equilibrium aggregate employment along with the non-traded output is determined in the same fashion as before. However, these equilibrium values will vary with the income elasticity of the non-traded good, and thus the effect of a minimum wage hike on

aggregate employment will now be slightly different. To fix ideas, suppose, the demand for non-traded good is unitary income elastic, $\eta_N = 1$ like the homothetic taste discussed above. Then, the income effect of a minimum wage hike operating via changes in aggregate value of production, for any given price of the non-traded good would be the same as before. But, now the "net" effect of an increase in P_N brought about by the minimum wage hike (given that N is relatively unskilled labour intensive) may be smaller than before if the non-traded good is price inelastic in demand. To see this, note two things. First, only if the non-traded god is unitary price elastic as well as unitary income elastic, would the effect of the price increase on the demand for non-traded good and correspondingly on aggregate employment following the minimum wage hike would be the same. But, for price inelastic demand, the price effect would be smaller. Second, since the price effect (captured through ε_N) is composed of substitution effect and income effect (the latter captured through η_N), so $\varepsilon_N - \theta_N \eta_N$ would always be positive. At the same time, this net effect would be smaller than the case of homothetic tastes. This, a smaller value of factor substitution elasticity than $\tilde{\sigma}_Z$ defined in (15) would raise the employment expansion. Furthermore, this critical value of factor substitution elasticity would vary inversely with the value of the price elasticity of demand for the non traded good. The crux of the matter, however is that an employment expansion following a hike in the minimum wage is plausible under reasonable conditions even for a generalized demand structure.

4.2 Minimum Wage in Z

One may suggest that Z not being affected by the minimum wage is a critical assumption. Note that even when unskilled labour is used in Z, the possibility of w_S rising is still there since *r* will decline. One has to guarantee that w_S rises and the substitution effect between the unskilled and the skilled is weak relative to that between the skilled labour and capital. One can still get, however restricted, condition that will allow X+Z to rise.

5. Concluding Remarks

In this paper we have shown that in the context of a diversified export basket of a small open economy, an increase in the minimum wage can raise aggregate employment of unskilled workers if domestically produced set of goods contain non-traded goods, and such increase in minimum wage generates a trade surplus at initial level of employment. This result provides a theoretical support for the finding of Card and Kruger. The role of the non-traded good arises because of the demand augmenting effect that the minimum wage hike generates. There will be adverse supply effects of the wage hike, and thus employment expansion necessitates a larger demand effect. This in turn is contingent upon whether the minimum wage hike generates a trade surplus at initial level of employment, because the import demand rises too due to the minimum

wage hike and corresponding rise in income. However, since the economy has a diversified export basket, so trade surplus can arise even when the export good that uses unskilled labour falls as a consequence of the minimum wage hike, with the other skill-based export good expanding more than proportionately. In such a case, the minimum wage hike induced increase in the demand for non-traded good will be larger than its adverse supply effect and overall the aggregate employment will expand. The underlying condition for this employment expansion is that the factor substitution elasticity in production of the skill-based good is sufficiently large.

Appendix

Case of Fixed proportions of income spent on N and Y

Total differentiation of the demand for non-traded good which is specified in the text by equation (11), we obtain,

$$\hat{D}_N = -(1-\theta_N)\hat{P}_N + \theta_N\hat{N} + \theta_X\hat{X} + \theta_Z\hat{Z} \qquad \text{(A.1)}$$

where, θ_j, j = N, X, Z, is the share of good-j in (produced) income.

Now, from the aggregate employment equation we obtain,

$$\hat{L}_e = \lambda_{LX}\hat{X} + \lambda_{LN}\hat{N} + \lambda_{LX}\hat{a}_{LX} + \lambda_{LN}\hat{a}_{LN} \qquad \text{(A.2)}$$

Rewriting changes in input coefficients as:

$\hat{a}_{Lj} = \hat{a}_{Lj} - (\theta_{Lj}\hat{a}_{Lj} + \theta_{Kj}\hat{a}_{Kj}) = -\theta_{Kj}(\hat{a}_{Kj} - \hat{a}_{Lj}) = -\sigma_j\theta_{Kj}(\hat{\overline{w}} - \hat{r})$, j = X, N and substituting $\hat{r} = -\frac{\theta_{LX}}{\theta_{KX}}\hat{\overline{w}}$ in (A.2) yields,

$$\hat{L}_e = \lambda_{LX}\hat{X} + \lambda_{LN}\hat{N} - A\hat{\overline{w}} \qquad \text{(A.3)}$$

where, $A = \frac{(\lambda_{LX}\sigma_X\theta_{KX} + \lambda_{LN}\sigma_N\theta_{KN})}{\theta_{KX}} > 0$.

On the other hand, given $\hat{Z} = -\hat{a}_{sz}$ by the skilled-labour constraint, from total differentiation of the full employment condition for capital we obtain,

$$0 = \lambda_{KX}\hat{X} + \lambda_{KN}\hat{N} + (\lambda_{KX}\hat{a}_{KX} + \lambda_{KN}\hat{a}_{KN}) + \lambda_{KZ}(\hat{a}_{KZ} - \hat{a}_{SZ})$$

Finally, using factor substitution elasticity in Z sector $\hat{a}_{KZ} - \hat{a}_{SZ} = -\sigma_Z\theta_{KZ}(\hat{w}_S - \hat{r})$, $\hat{w}_S = -\frac{\theta_{KZ}}{\theta_{SZ}}\hat{r} = \frac{\theta_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}}\hat{\overline{w}}$, and $\hat{a}_{Kj} = \hat{a}_{Kj} - (\theta_{Lj}\hat{a}_{Lj} + \theta_{Kj}\hat{a}_{Kj}) = \theta_{Lj}(\hat{a}_{Kj} - \hat{a}_{Lj}) = \sigma_j\theta_{Lj}(\hat{\overline{w}} - \hat{r})$, the above expression can be rewritten as,

$$\lambda_{KN}\hat{N} + \lambda_{KX}\hat{X} = -B\hat{\bar{w}} \qquad \text{(A.4)}$$

where, $B = \left[\sigma_Z \frac{\lambda_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}} + \frac{(\sigma_X \lambda_{KX}\theta_{LX} + \sigma_N \lambda_{KN}\theta_{LN})}{\theta_{KX}} \right] > 0.$

(A.3) and (A.4) constitute the system of equations that will solve for $\hat{X}$ and $\hat{N}$ in terms of change in aggregate employment and change in the minimum wage:

$$\hat{X} = \frac{1}{|\lambda|}\left[\lambda_{KN}\hat{L}_e + (\lambda_{KN}A + \lambda_{LN}B)\hat{\bar{w}}\right] \qquad \text{(A.5)}$$

$$\hat{N} = \frac{1}{|\lambda|}\left[-\lambda_{KX}\hat{L}_e - (\lambda_{KX}A + \lambda_{LX}B)\hat{\bar{w}}\right] \qquad \text{(A.6)}$$

(A.5) and (A.6) reflect the standard trade off between output of X and N due to capital scarcity. If N is relatively labour intensive ($|\lambda| < 0$), then for any given minimum wage, an increase in aggregate employment will raise the output of N and lower that of X. (A.6) is in fact the equation of the *NN*-curve in Figure 1. On the other hand, from the following it is easy to check that despite such trade off between X and N, the aggregate value of production and produced income regardless of the factor intensity ranking. Note that, the price of the non-traded good and the production of Z increases only when the minimum wage changes. So for any given minimum wage, the aggregate value of production equals,

$$\hat{y}\big|_{\bar{w}} = \theta_N\hat{N} + \theta_X\hat{X} = \frac{1}{|\lambda|}[\theta_X\lambda_{KN} - \theta_N\lambda_{KX}] \qquad \text{(A.7)}$$

Now, $\theta_X\lambda_{KN} - \theta_N\lambda_{KX} > 0$ if $\frac{\theta_X}{\theta_N} > \frac{\lambda_{KX}}{\lambda_{KN}} \Rightarrow \frac{X}{P_N N} > \frac{a_{KX}X}{a_{KN}N} \Rightarrow \frac{ra_{KN}}{P_N} > ra_{KX} \Rightarrow \theta_{KN} > \theta_{KX}$ since $P_X^W = 1$

This condition is satisfied if N is relatively capital intensive, i.e., $|\lambda| > 0$. But $\theta_{KN} > \theta_{KX}$ if N is relatively labour intensive, i.e. $|\lambda| < 0$. In either case, $\hat{y} > 0$. Hence, at initial minimum wage (and corresponding P_N and output of Z), an increase in aggregate employment raises the aggregate value of production and income. Substitution of (A.5), (A.6), $\hat{Z} = \sigma_Z \frac{\lambda_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}}\hat{\bar{w}}$ and $\hat{P}_N = -\frac{|\theta|}{\theta_{KX}}\hat{\bar{w}}$ in (A.1) gives the equation of the D_ND_N curve in Figure 1 as:

$$\hat{D}_N = \frac{1}{|\lambda|}[\theta_X \lambda_{KN} - \theta_N \lambda_{KX}]\hat{L}_e + \left[(1-\theta_N)\frac{|\theta|}{\theta_{KX}} + \theta_Z \sigma_Z \frac{\lambda_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}} + \frac{1}{|\lambda|}[\theta_X \lambda_{KN} - \theta_N \lambda_{KX}]A + \frac{1}{|\lambda|}[\theta_X \lambda_{LN} - \theta_N \lambda_{LX}]B\right]\hat{\bar{w}} \qquad \text{(A.8)}$$

Thus, for any given minimum wage, demand for non-traded good rises with the aggregate employment which is reflected by the positively sloped $D_N D_N$ curve in Figure 1.

(A.6) and (A.8) constitute a pair of equations that solve for output of equilibrium non-traded output level and aggregate employment level, which the non-traded markets clears and thus trade is balanced. That is, from $\hat{D}_N = \hat{N}$ we get,

$$\hat{L}_e = \left\{\frac{\left[-(1-\theta_N)\frac{|\theta||\lambda|}{\theta_{KX}} - \theta_Z \sigma_Z \frac{\lambda_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}}|\lambda|\right]}{[\theta_X \lambda_{KN} + (1-\theta_N)\lambda_{KX}]} - A - \frac{[\theta_X \lambda_{LN} + (1-\theta_N)\lambda_{LX}]}{[\theta_X \lambda_{KN} + (1-\theta_N)\lambda_{KX}]}B\right\}\hat{\bar{w}} \qquad \text{(A.9)}$$

Now, it is easy to check that,

$$\theta_X \lambda_{KN} + (1-\theta_N)\lambda_{KX} = \theta_X \lambda_{KN} + (1-\theta_N)(1-\lambda_{KN}) = (\theta_X + \theta_N - 1)\lambda_{KN} + 1 - \theta_N$$
$$= -\theta_Z \lambda_{KN} + \theta_X + \theta_Z = \theta_Z \lambda_{KX} + \theta_X \text{ and } \theta_X \lambda_{LN} + (1-\theta_N)\lambda_{LX} = \theta_Z \lambda_{LX} + \theta_X$$

Substitution of these expressions, $|\lambda| = \lambda_{LX} - \lambda_{LN}$ and $B \equiv A + \sigma_Z \frac{\lambda_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}}$, (A.9) becomes:

$$\Rightarrow \hat{L}_e = \left[\frac{\sigma_Z\left(\theta_Z(\lambda_{LN} - 2\lambda_{LX}) - \theta_X\right)\frac{\lambda_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}} - (1-\theta_N)\frac{|\theta||\lambda|}{\theta_{KX}} - \{2\theta_X + (\lambda_{LX} + \lambda_{KX})\theta_Z\}A}{\theta_X + \lambda_{LX}\theta_Z}\right]\hat{\bar{w}}$$

Thus, given A>0, $|\lambda| < 0$ and $|\theta| < 0$, aggregate employment increases following a minimum wage hike if, $\left(\theta_Z(\lambda_{LN} - 2\lambda_{LX}) - \theta_X\right) > 0$ and $\sigma_Z > \frac{(1-\theta_N)\frac{|\theta||\lambda|}{\theta_{KX}} + \{2\theta_X + (\lambda_{LX} + \lambda_{KX})\theta_Z\}A}{\varphi\frac{\lambda_{KZ}\theta_{LX}}{\theta_{SZ}\theta_{KX}}}$

Finally note that $\varphi > 0$ if -- a) $\lambda_{LN} > 2\lambda_{LX}$, b) $\theta_Z > \theta_X / \lambda_{LN} - 2\lambda_{LX}$.

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Endnotes:

[1] This production structure and trade pattern follows Jones and Marjit (1992). A similar production structure with a quality differentiated skill-based export good has been developed in Acharyya and Jones (2001).

[2] As can be verified from the appendix, rise in aggregate output, despite the trade off between X and N due to capital scarcity, is independent of the factor intensity condition.