CESifo WORKING PAPERS

8096
2020

February 2020

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
www.cesifo-group.org/wp

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CESifo *Working Paper No. 8096*

Bribing in Team Contests

Abstract

We study bribing in a sequential team contest with multiple pairwise battles. We allow for asymmetries in winning prizes and marginal costs of effort; and we characterize the conditions under which (i) a player in a team is offered a bribe by the owner of the other team and (ii) she accepts the bribe. We show that these conditions depend on the ratios of players' winning prizes and marginal costs of effort: the team owner chooses to bribe the player with the most favorable winning prize to marginal cost of effort ratio, and offer a bribe that leaves her indifferent between accepting (and exerting zero effort) and not accepting (and exerting her optimal effort). In some cases, the competition between players and the negative consequences of one player receiving a bribe on the team performance can drag down equilibrium bribe to zero. We also study the impact of changes in winning prizes and marginal costs of effort on equilibrium bribing behavior.

JEL-Codes: C720, D730, D740.

Keywords: bribing, contest games, pairwise battles, team contests.

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1 Introduction

Bribing can be defined as giving someone money or something else of value in order to make her act in one's favor. Practically, all definitions incorporate phrases such as 'often illegally' or 'something dishonest'. Despite the illegality or immorality attached to the act, bribing in real life is ubiquitous. According to an OECD report (see OECD, 2014), bribes amount to 10% of the total value created by all commercial transactions and 34.5% of firm profits are spent on bribing. The World Bank estimates the amount of bribe exchanging hands in a year to be around 1 trillion dollars (see Kaufmann, 2005). Giving/accepting money to rig professional sport competitions is also very widespread. Transparency International, in its Global Corruption Report: Sport, describes match-fixing as a fully acknowledged, real threat to the integrity of sport and as such it is one of the six major topics in the report (see Sweeney, 2016). Some most well-known examples of bribing/match-fixing are Olympique de Marseille match-fixing scandal in 1993, Italian Football match-fixing scandal in 2011–2012, and Spanish Football League match-fixing scandal in 2019.

In this paper, we theoretically study bribing in a sequential team contest with multiple pairwise battles. In our model, there are two teams with equal number of players. To keep the model tractable yet sufficiently rich to derive interesting results, we fix the number of players in each team to three. Hence, the contest is made up of three pairwise battles. In each pairwise battle, the paired players simultaneously exert costly efforts, and a Tullock contest success function (see Tullock, 1980) determines who wins the battle. The team that wins at least two battles wins the contest, and the team members collect positive winning prizes while the members of the losing team receive zero. We allow individual winning prizes and marginal costs of effort to be heterogeneous across players. The pairs of players in each battle as well as the sequence of those battles are exogenously given at the beginning of the game.[1] At this point we make three modeling assumptions: (i) only one of the team owners can offer a bribe, (ii) she can offer a bribe to only one player in the other team, and (iii) the bribed player does not exert any effort.[2] The team owner that can offer a bribe chooses whom to bribe

[1]Obviously, the pairing can be modeled as an equilibrium phenomenon—especially when there are asymmetries across players. However, that is a different research question that should be handled in a separate paper.

[2]We discuss in detail how restrictive these assumptions are in the Conclusion section.

and by which amount, with the objective of maximizing her own expected payoff. The other team owner does not have a decision to make. Finally, all players are expected payoff maximizers.

We analytically solve for the subgame perfect Nash equilibrium of this sequential game. Our main result shows which player is offered a bribe in equilibrium, by which amount, and whether she accepts it or not. In particular, we show that the answers to all of these questions are closely connected to the ratios of individual winning prizes and marginal costs of effort. More precisely, the team owner chooses to bribe the player with the most favorable winning prize to marginal cost of effort ratio (after taking into account the same ratios for the players paired with them), and offer a bribe that leaves her indifferent between accepting (and exerting zero effort) and not accepting (and exerting her optimal effort). Our results show that even a zero-bribe can be accepted by a player under some circumstances. Furthermore, we conduct comparative static analyses on winning prize and marginal cost parameters in some illustrative examples to see the impact of a change in these parameters on equilibrium bribing behavior.

To the best of our knowledge, this is the first paper in the contest theory literature to incorporate bribing into team contests. In fact, bribing (or, cash transfers or side payments) is a very understudied topic even in individual contest games.[3] Preston and Syzmanski (2003) formulate a simple model of match-fixing, which takes into account the probability of getting caught and the uncertainty regarding the type of players (i.e., a moral type who does not accept bribes or immoral type who accepts bribes). The authors describe the conditions under which bribe exists in equilibrium. Schoonbeek (2009) studies a two-stage Tullock rent-seeking contest. In the first stage, the existing players in the game can choose to bribe a potential entrant to persuade her not to enter. In the second stage, the actual contest takes place. The author characterizes the conditions under which the potential entrant is bribed and hence stays out. Kimbrough and Sheremata (2013) study a two-stage game of (potential) conflict and show—both theoretically and experimentally—that with binding commitments, side payments can lead to

Let us be brief here and note that relaxing those assumptions does not bring significantly new insights, but it considerably complicates the analysis.

[3]Bribing is one type of a destructive/unethical action in contests. There are others such as sabotage, doping, and spying. For those, we refer the reader to Baik and Shogren (1995), Konrad (2000), Preston and Szymanski (2003), Amegashie (2012), Chowdhury and Gürtler (2015), and Doğan, Keskin, and Sağlam (2019) among others.

large efficiency gains through avoided conflicts. Finally, Esö and Schummer (2004), Rachmilevitch (2013), and Rachmilevitch (2015) all study bribing in the context of auctions, which can be considered as a contest-like interaction. As we mention above, neither these papers nor the others in the literature consider bribing in a team contest.

The organization of the paper is as follows. In Section 2, we introduce the baseline model of team contest with multiple pairwise battles without bribing. In Section 3, we introduce bribing into the baseline model, present an equilibrium analysis, and report the corresponding results. In Section 4, we conclude following a discussion on our modeling assumptions, some caveats, and possible future research questions.

2 The Baseline Model

We present the baseline model without bribing here and derive some results that will later be utilized in the model with bribing. There are two teams, denoted respectively by a and b, with equal number of players. They compete in a *sequential* team contest with multiple pairwise battles. Each player in team a is matched with a player in team b, and each pair of players compete in a *component battle* indexed by $t \in \mathbb{N}$. A player in team $i \in \{a, b\}$ is indexed by $i(t)$ if she is assigned to battle t. This ordering is exogenously given and does not change throughout the contest. Player $i(t)$ wins the component battle t with probability

$$p_{i(t)} = \frac{e_{i(t)}}{e_{a(t)} + e_{b(t)}} \tag{1}$$

where $e_{i(t)}$ denotes the battle effort exerted by $i(t)$ (see Tullock, 1980). Each player $i(t)$ is informed about the outcomes of the previous component battles $t' < t$ before exerting her effort in battle t. A team wins the contest if its members accumulate more victories than the members of the other team do.

The six-player version (i.e., teams composed of three players) of a team contest with multiple pairwise battles can be illustrated as in Figure 1.[4] The index of the component battle, which takes place at node (x, y), where x and y denote the number of victories team a and b collected until that point, respectively, is given by: $x + y + 1$.

Player $i(t)$ receives a prize of $V_{i(t)} > 0$ in case team $i \in \{a, b\}$ wins the contest. The members of the losing team receive 0. Each player $i(t)$ has

[4] We omit effort choices in this figure for the sake of expositional simplicity.

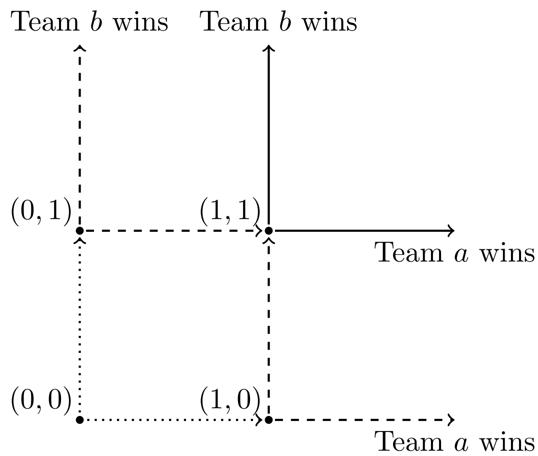

Figure 1: 3-vs-3 multiple pairwise battles

a constant marginal cost of effort: $c_{i(t)} > 0$. Apart from the asymmetries in winning prizes we allow here, our baseline model can be thought of as a special case of the model introduced by Fu, Lu, and Pan (2015).

In the following proposition, we derive the expected payoffs and the ratio of winning probabilities in any two-player contest with the standard Tullock contest success function, (1), and linear cost functions. In the corollary that follows, we then show that the same results are valid for each multiple pairwise battle of the team contest described above.

Proposition 1. *Consider a two-player contest with the standard Tullock contest success function and linear cost functions with coefficients c_1 and c_2. Assume that a victory by player 1 yields the payoff vector $(\mathcal{V}_{11}, \mathcal{V}_{21})$ and a victory by player 2 yields the payoff vector $(\mathcal{V}_{12}, \mathcal{V}_{22})$ such that $\mathcal{V}_{11} \geq \mathcal{V}_{12}$ and $\mathcal{V}_{22} \geq \mathcal{V}_{21}$. Denoting the winning probabilities of players 1 and 2 in the equilibrium by p_1 and p_2, respectively, we have*

$$\frac{p_1}{p_2} = \frac{c_2(\mathcal{V}_{11} - \mathcal{V}_{12})}{c_1(\mathcal{V}_{22} - \mathcal{V}_{21})}.$$

Also the expected payoff vector at the equilibrium is

$$\left(p_1^2\mathcal{V}_{11} + (1 - p_1^2)\mathcal{V}_{12}\ \ ,\ \ p_2^2\mathcal{V}_{22} + (1 - p_2^2)\mathcal{V}_{21}\right).$$

Proof. See Appendix A. $\square$

Now, consider the team contest with multiple pairwise battles as described above. Given that backward induction would yield a unique subgame perfect Nash equilibrium, the next result follows from Proposition 1.

Corollary 1. *In a team contest with multiple pairwise battles, consider a generic component battle between players $a(t)$ and $b(t)$. Let p_i be the probability of team i winning the contest after player $a(t)$ wins the component battle; and let q_i be the same probability after player $b(t)$ wins. Denoting the winning probabilities of players $a(t)$ and $b(t)$ in the equilibrium by $p_{a(t)}$ and $p_{b(t)}$, respectively, we have*

$$\frac{p_{a(t)}}{p_{b(t)}} = \frac{V_{a(t)}/c_{a(t)}}{V_{b(t)}/c_{b(t)}}$$

Also the expected payoff vector at the equilibrium is

$$\left((q_a + p_{a(t)}^2(p_a - q_a))V_{a(t)} \ \ , \ \ (p_b + p_{b(t)}^2(q_b - p_b))V_{b(t)}\right).$$

Proof. Using a notation consistent with Proposition 1, in this generic case, we write $V_{11} = p_a V_{a(t)}$, $V_{12} = q_a V_{a(t)}$, $V_{21} = (1 - p_a)V_{b(t)}$, and $V_{22} = (1 - q_a)V_{b(t)}$. By Proposition 1, we have

$$\frac{p_{a(t)}}{p_{b(t)}} = \frac{c_{b(t)}(p_a - q_a)V_{a(t)}}{c_{a(t)}((1 - q_a) - (1 - p_a))V_{b(t)}} = \frac{V_{a(t)}/c_{a(t)}}{V_{b(t)}/c_{b(t)}}$$

Utilizing Proposition 1, we can similarly write the expected payoff vector. □

Corollary 1 reveals that the winning probability of a player at any particular component battle is independent of the outcomes of past component battles. This result is similar, in essence, to the *history independence* observation made by Fu et al. (2015).

Utilizing the result above, we can evaluate the equilibrium winning probabilities for both teams at each node. From this point onward, for the sake of simplicity, we assume that there are three players in each team,[5] so that the victory threshold is two, and we set

$$p_{i(t)} = \frac{V_{i(t)}/c_{i(t)}}{V_{a(t)}/c_{a(t)} + V_{b(t)}/c_{b(t)}} \tag{2}$$

[5]We expect our results to be valid for teams of any finite size. We discuss the costs and benefits of relaxing this assumption in Section 4.

for any $i \in \{a, b\}$ and $t = 1, 2, 3$. Furthermore, for notational simplicity, we set $p_t \equiv p_{b(t)}$, which obviously implies that $p_{a(t)} = 1 - p_t$.

The winning probabilities at node $(1, 1)$ can now be written as $(1-p_3, p_3)$. At node $(1, 0)$, the winning probabilities are

$$(1 - p_2) \cdot (1, 0) + p_2 \cdot (1 - p_3, p_3) = (1 - p_2p_3\ ,\ p_2p_3).$$

Similarly, at node $(0, 1)$, the winning probabilities are

$$(1 - p_2) \cdot (1 - p_3, p_3) + p_2 \cdot (0, 1) = (1 - p_2 - p_3 + p_2p_3\ ,\ p_2 + p_3 - p_2p_3).$$

Finally, at node $(0, 0)$, the winning probabilities are

$$\begin{aligned}&(1 - p_1) \cdot (1 - p_2p_3\ ,\ p_2p_3) + p_1 \cdot (1 - p_2 - p_3 + p_2p_3\ ,\ p_2 + p_3 - p_2p_3)\\ &= (1 - p_1p_2 - p_1p_3 - p_2p_3 + 2p_1p_2p_3\ ,\ p_1p_2 + p_1p_3 + p_2p_3 - 2p_1p_2p_3).\end{aligned}$$

The following proposition states that the order of pairwise battles do not matter as far as players' expected payoffs are concerned. This result is similar to the *sequence independence* observation in Fu et al. (2015).

Proposition 2. *The expected payoff of player $b(t)$ from the team contest with multiple pairwise battles is independent of the order of battles.*

Proof. Note that team b's probability of winning the contest is given by $p_1p_2 + p_1p_3 + p_2p_3 - 2p_1p_2p_3$. Note also that if player $b(t)$ is assumed to win her own component battle, setting $p_t = 1$ yields the updated probability of winning for team b; and if she is assumed to lose that battle, setting $p_t = 1$ yields the updated probability of winning for team b. We can now write the expected payoffs of each team b member.

If player $b(1)$ wins, then her team wins the contest with probability $p_2 + p_3 - p_2p_3$; but if player $b(1)$ loses, then her team wins the contest with probability p_2p_3. Then, by Corollary 1, the expected payoff of player $b(1)$ at node $(0, 0)$ is

$$(p_2p_3 + p_1^2(p_2 + p_3 - 2p_2p_3))V_1.$$

From the perspective of player $b(2)$, notice that node $(0, 1)$ is reached with a probability of p_1, whereas the same probability is $1 - p_1$ for node $(1, 0)$. If player $b(2)$ wins at node $(0, 1)$, then her team definitely wins the contest; but in case player $b(2)$ loses at node $(0, 1)$, then her team wins the contest with probability p_3. The expected payoff of player $b(2)$ at node $(0, 1)$

is $(p_3 + p_2^2(1 - p_3))V_2$ and at node $(1, 0)$ is $(0 + p_2^2(p_3 - 0))V_2$. Accordingly, the overall expected payoff of player $b(2)$ can be written as

$$
\begin{aligned}
&(p_1(p_3 + p_2^2(1 - p_3)) + (1 - p_1)p_2^2 p_3)V_2 \\
&= (p_1 p_3 + p_2^2(p_1 + p_3 - 2p_1 p_3))V_2
\end{aligned}
$$

As for player $b(3)$, notice that the game may end after the first two component battles, so that she gets a payoff of V_3 with a probability of $p_1 p_2$ and a payoff of 0 with a probability of $(1 - p_1)(1 - p_2)$ without even playing in her component battle. Also, from her perspective, node $(1, 1)$ is reached with a probability of $p_1(1-p_2)+p_2(1-p_1)$; and at this node, player $b(3)$ has an expected payoff of $(0+p_3^2(1-0))V_3 = p_3^2 V_3$. Thus, her expected payoff as seen from node $(0, 0)$ can be written as

$$
\begin{aligned}
&p_1 p_2 V_3 + (p_1(1 - p_2) + p_2(1 - p_1))p_3^2 V_3 \\
&= (p_1 p_2 + p_3^2(p_1 + p_2 - 2p_1 p_2))V_3.
\end{aligned}
$$

The symmetrically-written expected payoffs imply the result. $\square$

3 Bribing in Team Contests

In this section, we introduce *bribing* into the baseline model and analyze its impact on the equilibrium strategies and winning probabilities. We allow, without loss of generality, the owner of team a to offer bribe to any player in team b; and we assume that only one player from team b can be bribed. The owner of team a receives a prize of V in case her team wins. In order to increase her expected earnings, by increasing her team's winning probability, she offers B_t amount of money to some player $b(t)$ in exchange for $b(t)$ not to exert any effort in the component battle t. Bribing occurs if the team owner becomes better off without making the bribed player worse off.

In this paper, we concentrate on a model in which the team owner decides whom to bribe before the contest starts. This means that player $b(t)$ is bribed before battle 1 takes place.[6]

We first recall that $p_t \equiv p_{b(t)}$ for every $t \in \{1, 2, 3\}$, and for the sake of notational simplicity, we further set $V_t \equiv V_{b(t)}$.

[6] For an analysis of an alternative model where we allow the owner of team a to offer bribe after battle 1 or battle 2 takes place, the interested reader is referred to Appendix B. The main intuition presented in this paper continues to follow in the alternative model.

3.1 Equilibrium Analysis

Our first result here is implied by Proposition 2, which states that the order of battles does not influence the expected payoffs.

Corollary 2. *In the team contest with multiple pairwise battles where the owner of one team has an option to bribe a single player from the opposing team, the respective conditions for a player to be indifferent between accepting and not accepting a bribe can be written symmetrically.*

Proof. Follows from Proposition 2. □

Below we present the conditions for player $b(1)$ to accept a bribe and provide similar conditions for the other players. Proposition 3 states an intermediate result where the player to be offered a bribe is fixed. We will later relax this assumption in proving our main result.

Proposition 3. *In the team contest with multiple pairwise battles, suppose that only player $b(1)$ can be bribed by the owner of team a. In case she accepts a bribe and forfeits from her component battle, her expected payoff would be $p_2p_3V_1$. Thus, the minimum acceptable bribe for player $b(1)$ is*

$$\mathcal{B}_1 = p_1^2(p_2 + p_3 - 2p_2p_3)V_1.$$

Symmetric expressions can be written for players $b(2)$ and $b(3)$ as follows:

$$\mathcal{B}_2 = p_2^2(p_1 + p_3 - 2p_1p_3)V_2 \quad \textit{and} \quad \mathcal{B}_3 = p_3^2(p_1 + p_2 - 2p_1p_2)V_3.$$

Proof. If player $b(1)$ forfeits from her component battle, the winning probability of her team would be p_2p_3. This yields an expected payoff of $p_2p_3V_1$ to the player. Given that her expected payoff from the baseline model with no bribing is $(p_2p_3 + p_1^2(p_2 + p_3 - 2p_2p_3))V_1$, the minimum acceptable bribe for player $b(1)$ can be calculated as the difference between the two expected payoffs. Symmetric expressions follow for the other players. □

In the following, we lay the groundwork for our main result, presented in Proposition 4, that characterizes the equilibrium conditions for bribing.

Notice that, by the assumption of common knowledge of rationality, a player can anticipate that her teammates may accept a bribe in case the player herself is not bribed. Accordingly, without loss of generality, we now evaluate the expected payoffs of players $b(2)$ and $b(3)$, in case it is known or anticipated that player $b(1)$ is bribed. Recall that when there is no bribe, the expected payoff of player $b(2)$ is $(p_1p_3 + p_2^2(p_1 + p_3 - 2p_1p_3))V_2$. The case of player $b(1)$ taking a bribe would yield the same expected payoff as in the case when $p_1 = 0$. Therefore, the expected payoff of player $b(2)$ would be $p_2^2p_3V_2$ and the expected payoff of player $b(3)$ would be $p_2p_3^2V_3$. Generally, if it is known or anticipated that player $b(t)$ accepts a bribe of B_t, then her expected payoff is

$$\frac{p_1p_2p_3}{p_t}V_t + B_t,$$

while the expected payoff of a non-bribed player $b(t')$ where $t \neq t'$ is

$$\frac{p_1p_2p_3}{p_t}p_{t'}V_{t'}.$$

Without loss of generality, suppose that the owner of team a decided to bribe player $b(1)$. We now evaluate whether player $b(2)$ would start accepting a bribe less than $\mathcal{B}_2$ once she anticipates that player $b(1)$ would be bribed if not her.[7] Given that player $b(1)$ is bribed, player $b(2)$ has an expected payoff of $p_2^2p_3V_2$; however, if player $b(2)$ accepts a bribe of B_2, then she would end up with an expected payoff of $p_1p_3V_2 + B_2$. Thus, B_2 should satisfy the following condition:

$$B_2 \geq p_3(p_2^2 - p_1)V_2 \equiv \mathcal{B}_2^1, \tag{3}$$

if player $b(2)$ is to accept a bribe in case she anticipates that her outside option is the case where player $b(1)$ is bribed. Notice that $\mathcal{B}_2^1 < \mathcal{B}_2$, so that player $b(2)$ indeed has a lower minimum acceptable bribe in such a case. Notice also that $\mathcal{B}_2^1$ may turn out to be non-positive, indicating that player $b(2)$ would be willing to forfeit from her component battle in return for a zero bribe.[8] We label this phenomenon as *the curse of rationality* since she would have been better off if she was naive and as such did not reason about the other players' behavior.

Utilizing our observations above, we now define the *expected total loss matrix* for the owner of team a, which we will later employ in proving our main result. To do that, for any $t \in \{1, 2, 3\}$, let

$$\mathbf{W}_{tt} = \frac{p_1p_2p_3}{p_t}V + \mathcal{B}_t;$$

[7] Similar arguments are valid for player $b(3)$ as well.

[8] This is true under the restriction that a negative bribe is not allowed.

and for any $t, t' \in \{1,2,3\}$, let

$$\mathbf{W}_{tt'} = \frac{p_1 p_2 p_3}{p_t} V + \max\{0, \mathcal{B}_t^{t'}\}$$

such that $\mathcal{B}_t^{t'} = p_{t''}(p_t^2 - p_{t'})V_t$ where $b(t'')$ is the remaining player. Here, the expression $\mathbf{W}_{tt}$ represents the amount of total loss suffered by the owner of team a in case player $b(t)$ is bribed under the condition that no other player would be bribed if not player $b(t)$. The first part is the expected payoff under the probable case that team a loses the contest, and the second part is the minimum bribe amount player $b(t)$ is willing to accept. The expression $\mathbf{W}_{tt'}$ represents the amount of total loss suffered by the owner of team a in case player $b(t)$ is bribed under the condition that player $b(t')$ would be bribed if not player $b(t)$. The first part is the expected payoff under the probable case that team a loses the contest, and the second part is the minimum bribe amount player $b(t)$ is willing to accept. Surely, the second part considers the possibility that $\mathcal{B}_t^{t'} < 0$. Now, we define the expected total loss matrix for the owner of team a:

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} & \mathbf{W}_{13} \\ \mathbf{W}_{21} & \mathbf{W}_{22} & \mathbf{W}_{23} \\ \mathbf{W}_{31} & \mathbf{W}_{32} & \mathbf{W}_{33} \end{bmatrix}.$$

In the equilibrium, the team owner would choose to bribe the player through which she would maximize her own expected payoff, or to put it differently, would minimize her expected total loss summarized in $\mathbf{W}$. In the following proposition, we characterize all conditions under which player $b(1)$ would be bribed by the owner of team a. We then argue that symmetric conditions would follow for the other players in team b.

Proposition 4. *In the team contest with multiple pairwise battles where the owner of team a has an option to bribe a single player from team b, the team owner intends to bribe player $b(1)$ under the following conditions:*

a positive bribe of $\mathcal{B}_1$ would be accepted, if

1. $\boldsymbol{W}_{11} < \min\{\boldsymbol{W}_{22}, \boldsymbol{W}_{33}\}$,
   $\boldsymbol{W}_{11} < \min\{\boldsymbol{W}_{21}, \boldsymbol{W}_{31}\}$;

and for every $j,k \in \{2,3\}$ with $j \neq k$: a bribe of $\max\{0, \mathcal{B}_1^j\}$ would be accepted, if

2. $\boldsymbol{W}_{11} < \min\{\boldsymbol{W}_{jj}, \boldsymbol{W}_{kk}\}$,
   $\boldsymbol{W}_{j1} < \min\{\boldsymbol{W}_{11}, \boldsymbol{W}_{k1}\}$,
   $\boldsymbol{W}_{1j} < \min\{\boldsymbol{W}_{j1}, \boldsymbol{W}_{kj}\}$; *or*

3. $\boldsymbol{W}_{11} < \min\{\boldsymbol{W}_{jj}, \boldsymbol{W}_{kk}\}$,
   $\boldsymbol{W}_{k1} < \min\{\boldsymbol{W}_{11}, \boldsymbol{W}_{j1}\}$,
   $\boldsymbol{W}_{jk} < \min\{\boldsymbol{W}_{1k}, \boldsymbol{W}_{k1}\}$,
   $\boldsymbol{W}_{1j} < \min\{\boldsymbol{W}_{jk}, \boldsymbol{W}_{kj}\}$; *or*

4. $\boldsymbol{W}_{jj} < \min\{\boldsymbol{W}_{11}, \boldsymbol{W}_{kk}\}$,
   $\boldsymbol{W}_{1j} < \min\{\boldsymbol{W}_{jj}, \boldsymbol{W}_{kj}\}$,
   $\boldsymbol{W}_{1j} < \min\{\boldsymbol{W}_{j1}, \boldsymbol{W}_{k1}\}$; *or*

5. $\boldsymbol{W}_{kk} < \min\{\boldsymbol{W}_{11}, \boldsymbol{W}_{jj}\}$,
   $\boldsymbol{W}_{jk} < \min\{\boldsymbol{W}_{1k}, \boldsymbol{W}_{kk}\}$,
   $\boldsymbol{W}_{1j} < \min\{\boldsymbol{W}_{jk}, \boldsymbol{W}_{kj}\}$,
   $\boldsymbol{W}_{1j} < \min\{\boldsymbol{W}_{j1}, \boldsymbol{W}_{k1}\}$; *or*

6. $\boldsymbol{W}_{kk} < \min\{\boldsymbol{W}_{11}, \boldsymbol{W}_{jj}\}$,
   $\boldsymbol{W}_{1k} < \min\{\boldsymbol{W}_{jk}, \boldsymbol{W}_{kk}\}$,
   $\boldsymbol{W}_{j1} < \min\{\boldsymbol{W}_{1k}, \boldsymbol{W}_{k1}\}$,
   $\boldsymbol{W}_{1j} < \min\{\boldsymbol{W}_{j1}, \boldsymbol{W}_{kj}\}$.

Furthermore, in case 1*, the team owner prefers giving the bribe if* $\boldsymbol{W}_{11}$ *is less than or equal to* $(p_1p_2 + p_2p_3 + p_1p_3 - 2p_1p_2p_3)V$*; whereas in the other cases, the same condition should be written with* $\boldsymbol{W}_{1j}$*.*

Proof. See Appendix A. □

The next result immediately follows.

Corollary 3. *In the team contest with multiple pairwise battles where the owner of team* a *has an option to bribe a single player from team* b*, the corresponding bribing conditions for players* $b(2)$ *and* $b(3)$ *can be written as in Proposition 4.*

Proof. Follows from Corollary 2 and Proposition 4. □

Below we make an observation to differentiate between the two case types presented in Proposition 4.

Remark 1. *Case 1 in Proposition 4 specifies a situation where player* $b(1)$ *is* so strong *that she is offered a positive bribe. Note that even when the other players would accept lower amounts of bribe (even, a zero-bribe) anticipating that player* $b(1)$ *would not exert any effort, the owner of team* a *does not deviate from her strategy of bribing* $b(1)$*. In all of the other cases, the team owner considers bribing player* $b(j)$ *at first, but then player* $b(1)$ *anticipates such a bribe and would be willing to lower her own acceptable bribe, which would make the team owner deviate to bribing player* $b(1)$*.*

It is worthwhile noting that there is no trivial expression with which we can ex-ante identify the player that will be offered a bribe and/or the amount of bribe. Instead we can, more generally, write that the answers to those questions depend on the comparison of the ratios of winning prize to marginal cost of effort (within each pair) across three pairs. For instance, a player with a high winning prize and low marginal cost of effort may not be offered a bribe if she is facing a *strong* opponent. Similarly, a player facing a *weak* opponent may not be offered a bribe if there is another player in her team who is relatively much stronger. As Proposition 4 reveals, in *intermediate* cases where no player is singled out as being *very strong*, the competition for receiving bribe drives the equilibrium bribe amount down, in some cases to zero. Note that in such cases, it is possible that a zero-bribe is accepted, since otherwise someone else could accept a bribe and spend zero effort, such that it would still be very likely that the team loses the contest; at least a zero-bribe saves the cost of effort for the player who accepts it. Section 3.2 presents some examples that reveal further insights.

3.2 Illustrative Examples

In this section, we present four different types of examples, varying in the level of heterogeneity and accompanied by graphical illustrations, to make our results more transparent.[9]

[9] In all graphs reported below, the color matrix is given by

$$\mathbf{C} = \begin{bmatrix} \text{Yellow} & \text{Green} & \text{Brown} \\ \text{Gray} & \text{Blue} & \text{Purple} \\ \text{Magenta} & \text{Orange} & \text{Red} \end{bmatrix},$$

which is matched with the entries of $\mathbf{W}$.

A common observation in the following two figures is that the three regions that specify the bribed player (i.e., the collection of yellow, green, and brown regions for player $b(1)$; the collection of gray, blue, and purple regions for player $b(2)$; and the collection of magenta, orange, and red regions for player $b(3)$) have a unique intersection point. That intersection point is surrounded by the regions related to cases (2)-(6) in Proposition 4, and as such the remaining regions that describe situations where *the bribed player is too strong* are completely separated.

In Figures 2 and 3, players in team a are assumed to be symmetric, with a winning prize of $V_{a(t)} = 10$ and a marginal cost of $c_{a(t)} = 2$.

Symmetric Costs, Possibly Asymmetric Winning Prizes: In Figure 2, the marginal costs of team b players are also equal to 2. The central point on the graph, which corresponds to the case when $V_b = (10, 10, 10)$, represents the case of complete symmetry. It is also the unique intersection point mentioned above.

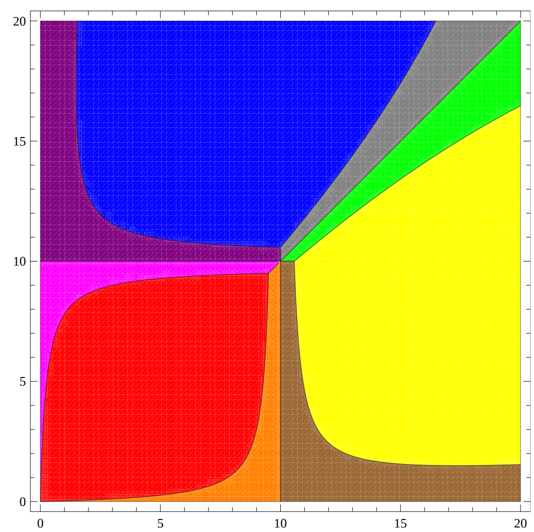

Figure 2: (Symmetric case) The regions for different types of bribing equilibrium when $V = 200$, $V_a = (10, 10, 10)$, $V_b = (\cdot, \cdot, 10)$, and $c_a = c_b = (2, 2, 2)$

Given that the central point is surrounded by six different regions, a deviation from the center leads to a dramatic change in the equilibrium type, depending on the direction of the deviation.

In case of a deviation towards the lower left, which indicates a decrease in both $V_{b(1)}$ and $V_{b(2)}$, player $b(3)$ would be bribed in the equilibrium. This is when player $b(3)$ is strong in terms of her $V_{b(t)}/c_{b(t)}$ ratio. In fact, as we move towards the origin, player $b(3)$ becomes too strong and guarantees a positive bribe as specified by the red region. In case of a deviation towards the upper right, which indicates an increase in both $V_{b(1)}$ and $V_{b(2)}$, it is either player $b(1)$ or $b(2)$ who is bribed, and which one is determined by the asymmetry between $V_{b(1)}$ and $V_{b(2)}$, favoring the player with the higher winning prize.

In case of a deviation towards the lower right, which indicates an increase in $V_{b(1)}$ and a decrease in $V_{b(2)}$, player $b(1)$ is bribed in the equilibrium; and in case of a deviation towards the upper left, which indicates an increase in $V_{b(2)}$ and a decrease in $V_{b(1)}$, player $b(2)$ is bribed in the equilibrium. If the increase in $V_{b(t)}$ for $t \in \{1,2\}$ is sufficiently high, then the equilibrium ends up in the yellow or blue regions showing the existence of too strong players.

Finally, one can comment on the effects of gradually changing one parameter value on the equilibrium type. For instance, in Figure 2, fixing a high level of V_2, say $V_2 = 15$, as V_1 increases, we move from $3 \to 2$, to 2, to $1 \to 2$, to $2 \to 1$, and to 1; whereas fixing a low level of V_2, say $V_2 = 5$, as V_1 increases, we move from $2 \to 3$, to 3, to $1 \to 3$, to $3 \to 1$, and to 1.[10]

Symmetric Winning Prizes, Possibly Asymmetric Costs: In Figure 3, the winning prizes of team b players are also equal to 10, but we now allow the marginal costs of players $b(1)$ and $b(2)$ to differ along the axes. The central point corresponds to the case of complete symmetry: $c_b = (2, 2, 2)$.

Similar to above, the deviation from the center leads to a dramatic change in the equilibrium type. Though, differently from above, player $b(3)$ is bribed in the upper right region where the marginal costs of the other players are high; whereas either player $b(1)$ or $b(2)$ would be bribed in the other parts of the graph, where at least one of them has a sufficiently low marginal cost. That is the reason why Figure 3 appears like a mirror image of Figure 2; locating the mirror in the center and ignoring the shapes of the region borders. This is intuitive, since a higher value of marginal cost creates an effect similar to that of a lower winning prize.

We can, again, comment on the effects of gradually changing one parameter value on the equilibrium type. In Figure 3, fixing a high level of c_2, say $c_2 = 3$, as c_1 increases, we move from 1, to $3 \to 1$, to $1 \to 3$, and to 3;

[10] Here, $k \to m$ indicates that player m is bribed, but player k would be bribed if not player m, whereas a single number represents the bribed player in case she is too strong.

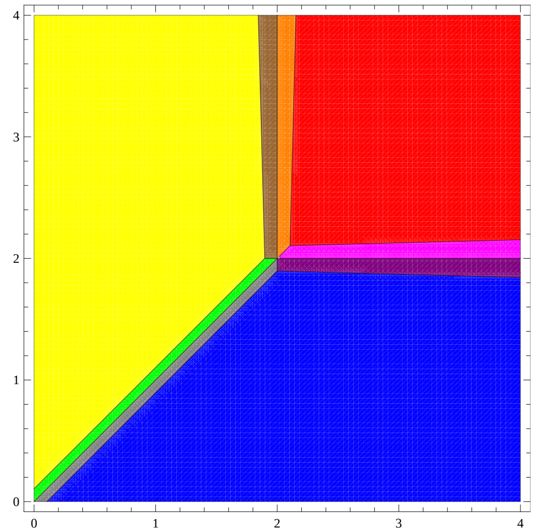

Figure 3: The regions for different types of bribing equilibrium when $V = 200$, $V_a = V_b = (10, 10, 10)$, $c_a = (2, 2, 2)$, and $c_b = (\cdot, \cdot, 2)$

whereas fixing a low level of c_2, say $c_2 = 1$, as c_1 increases, we move from 1, to $2 \to 1$, to $1 \to 2$, and to 2.

Asymmetric Winning Prizes and Possibly Asymmetric Costs: Figure 4 illustrates how the equilibrium types change depending on the winning prize and marginal cost of a fixed player: $b(1)$.

Although we intend to preserve the symmetric parameter values as in the previous two figures; here, for the sake of argument, we assume that $V_{b(2)} = 20$ and $V_{b(2)} = 10$, because in case those winning prizes are assumed to be equal, there would be regions where the owner of team a is indifferent between bribing players $b(2)$ and $b(3)$. In Figure 4, player $b(2)$ is always stronger than player $b(3)$, and accordingly, changing the parameter values for player $b(1)$ does not change the fact that bribing player $b(2)$ is preferred to bribing player $b(3)$. Apparently, such a partial asymmetry leads to a five-region graph.

We observe that under the 45-degree line, the owner of team a prefers to bribe player $b(1)$. This observation is related to the $V_{b(t)}/c_{b(t)}$ ratio for players $b(1)$ and $b(2)$. Simply, whichever player's ratio is greater than the other's, that player would be bribed in the equilibrium, given that they are

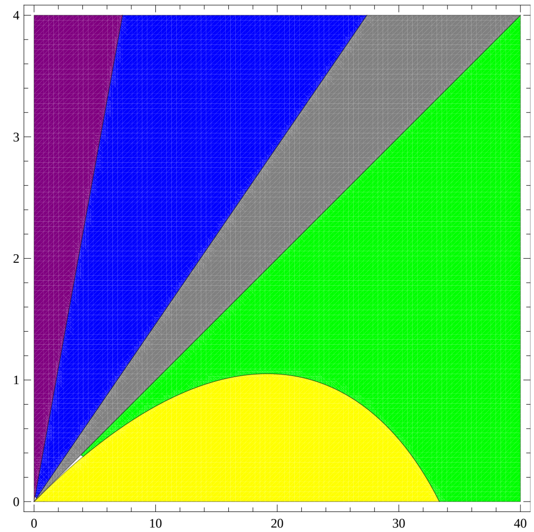

Figure 4: The regions for different types of bribing equilibrium when $V = 100$, $V_a = (10, 10, 10)$, $V_b = (\cdot, 20, 10)$, $c_a = (2, 2, 2)$, and $c_b = (\cdot, 2, 2)$

matched with symmetric players. Moreover, it seems that player $b(1)$ is too strong in regions where $c_{b(1)}$ is sufficiently low and $V_{b(1)}$ is sufficiently high; and interestingly, for sufficiently low values of $V_{b(1)}$, it is player $b(3)$ who is about to be bribed, but anticipating that outcome, player $b(2)$ accepts a lower amount of bribe such that the team owner deviates.

Asymmetric Winning Prizes and Costs: Finally, in Figure 5, we consider an asymmetric case. There are now two separate points at which four different regions intersect.
In this graph, player $b(3)$ is matched with a weak player in terms of the $V_{a(t)}/c_{a(t)}$ ratio. This makes player $b(3)$ relatively stronger, so that she is bribed for a large set of $(V_{b(1)}, V_{b(2)})$ values, even for some cases where the other players have relatively higher winning prizes: $(V_{b(1)}, V_{b(2)}) > (30, 40)$. As much higher values of $V_{b(1)}$ or $V_{b(2)}$ are realized, player $b(3)$'s advantage disappears and the team owner starts to bribe one of the other players.

Comparing players $b(1)$ and $b(2)$, we see that player $b(1)$ is strong as she has a lower marginal cost and player $b(2)$ is strong as she is matched with a weaker opponent in terms of the $V_{a(t)}/c_{a(t)}$ ratio. When both players have a winning prize of 60, player $b(1)$ is bribed, which indicates that player $b(1)$'s

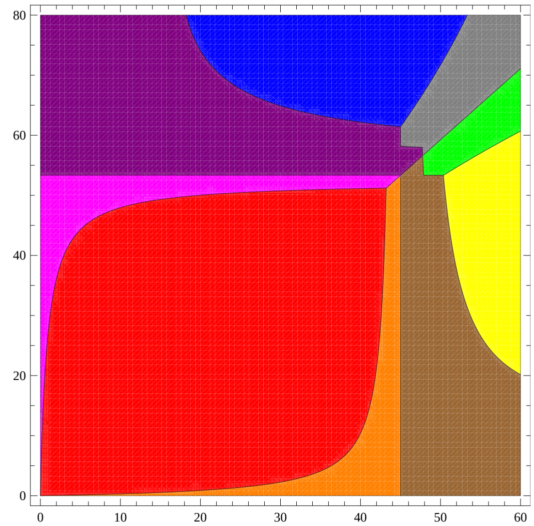

Figure 5: The regions for different types of bribing equilibrium when $V = 500$, $V_a = (30, 40, 20)$, $V_b = (\cdot, \cdot, 20)$, $c_a = (2, 3, 2)$, and $c_b = (3, 4, 2)$

advantage in marginal cost dominates player $b(2)$'s advantage in opponent strength. On the other hand, if one considers the upper right corner, which is when $(V_{b(1)}, V_{b(2)}) = (80, 60)$, where player $b(1)$'s advantage disappears due to a higher winning prize for player $b(2)$, one would see that player $b(2)$ is bribed. The interpretations and other observations are similar to those for Figure 2, given that both graphs allow for the the changes in $V_{b(1)}$ and $V_{b(2)}$.

4 Concluding Remarks

We presented the first theoretical analysis of bribing in team contests and analytically solved for the subgame perfect Nash equilibrium of the corresponding game. We allowed for heterogeneity at two levels (the winning prizes and marginal costs of players), since the presence and amount of bribing naturally depend on such heterogeneity.

Our results reveal an economically intuitive link between the choice of a player to be bribed (as well as the amount of bribe) and the ratios of the players' winning prizes and marginal costs of efforts. More precisely, the team owner who can offer a bribe takes into account the trade-off between the winning prize and marginal cost of effort for each player in the opponent team, and she chooses (also by taking into account the power balances in each pair) the one that maximizes her expected payoff. We also show that the *competition* between players and the negative externality that a bribe-receiving player imposes on her teammates can drive down equilibrium bribe amount, even to zero in some cases.

A few issues regarding our modeling assumptions are worthwhile discussing here. As we mentioned in the Introduction section, we kept the model simple for tractability, a clean and completely analytical equilibrium analysis, and transparent results. For instance, we allowed only the owner of team a to offer a bribe. A more general model could be considered in which a bribing strategy is available also to the owner of team b. As it is revealed in the equilibrium analysis, in such a model, similar bribing conditions could be derived for the owner of team b, but this would possibly not bring any significantly new insight. Obviously, allowing both team owners to offer bribes would introduce a meta-game (of bribing), and given that we have six cases for each player to be studied in the equilibrium analysis we conducted (see Proposition 4), we expect the analysis to be much more complicated since one would need to analyze too many cases in that more general model.

We assumed that the owner of team a can offer bribe to only one player in team b. Hence, another generalization could allow the owner of team a to bribe multiple players. However, given that each team has three players and that bribing two players would suffice for team a to guarantee winning the contest, the owner of team a would bribe two players if it is in her best interest to do so; and accordingly, generalizing the model in that dimension would not return new insights.

We assumed that the player who accepts a bribe does not exert any effort. Although this is a standard assumption in the literature, an alternative model may introduce the risk of getting caught by the owner of her team (for the player who accepted a bribe) with material consequences, in which case one may obtain a non-zero effort in equilibrium for the player who accepted a bribe (given that the probability of getting caught is a decreasing function of effort exerted). This more realistic feature would just add another layer of expectation in calculating the players' payoffs. If the probabilities of getting caught are symmetric for all players (after controlling for their winning prizes and marginal costs of effort), then our results would qualitatively not change. If there are asymmetries, however, then the equilibrium conditions we derived for the owner of team a regarding whom and how much to bribe would likely depend on the individual probabilities of getting caught in addition to winning prizes and marginal costs of effort. In that case, a lower probability of getting caught (compared to teammates) would be another factor that makes a player *strong*.

Future research may deal with questions related to (i) endogenous/strategic pairing of players in team contests where bribing is possible, (ii) optimal prize allocation within a team, or more generally, optimal contest design to prevent bribing, and (iii) the presence of honest players or whistleblowers.

Acknowledgments: *We would like to thank Azar Abizada and Wolfgang Leininger for useful discussions. We would also like to thank The Scientific and Technological Research Council of Turkey (TÜBİTAK) for the financial support under grant number 118K239. The usual disclaimers apply.*

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Appendix A

Proof of Proposition 1:
Let e_1 and e_2 denote the contest efforts exerted by players 1 and 2, respectively. Then the expected payoff of player 1 is

$$\frac{e_1}{e_1+e_2}\mathcal{V}_{11}+\frac{e_2}{e_1+e_2}\mathcal{V}_{12}-c_1e_1. \tag{4}$$

The first order condition with respect to e_1 is

$$\frac{e_2}{(e_1+e_2)^2}\mathcal{V}_{11}-\frac{e_2}{(e_1+e_2)^2}\mathcal{V}_{12}-c_1=0.$$

From here we obtain

$$\frac{e_2(\mathcal{V}_{11}-\mathcal{V}_{12})}{c_1}=(e_1+e_2)^2.$$

We can get a similar equation for player 2, so that

$$\frac{e_2(\mathcal{V}_{11}-\mathcal{V}_{12})}{c_1}=(e_1+e_2)^2=\frac{e_1(\mathcal{V}_{22}-\mathcal{V}_{21})}{c_2}.$$

Noting that the equilibrium is unique, we have

$$\frac{e_1}{e_2}=\frac{c_2(\mathcal{V}_{11}-\mathcal{V}_{12})}{c_1(\mathcal{V}_{22}-\mathcal{V}_{21})}.$$

Then

$$p_1=\frac{e_1}{e_1+e_2}=\frac{c_2(\mathcal{V}_{11}-\mathcal{V}_{12})}{c_1(\mathcal{V}_{22}-\mathcal{V}_{21})+c_2(\mathcal{V}_{11}-\mathcal{V}_{12})}$$

and

$$p_2=\frac{e_2}{e_1+e_2}=\frac{c_1(\mathcal{V}_{22}-\mathcal{V}_{21})}{c_1(\mathcal{V}_{22}-\mathcal{V}_{21})+c_2(\mathcal{V}_{11}-\mathcal{V}_{12})}.$$

By substituting these values into the first order conditions, we obtain

$$e_1=\frac{c_2(\mathcal{V}_{11}-\mathcal{V}_{12})^2(\mathcal{V}_{22}-\mathcal{V}_{21})}{(c_1(\mathcal{V}_{22}-\mathcal{V}_{21})+c_2(\mathcal{V}_{11}-\mathcal{V}_{12}))^2}$$

We can now see that $c_1e_1=p_1p_2(\mathcal{V}_{11}-\mathcal{V}_{12})$. Thus the expected payoff of player 1, given by equation (4), can be rewritten as

$$p_1\mathcal{V}_{11}+p_2\mathcal{V}_{12}-p_1p_2(\mathcal{V}_{11}-\mathcal{V}_{12}).$$

By substituting $p_2 = 1 - p_1$, we have

$$p_1\mathcal{V}_{11} + (1-p_1)\mathcal{V}_{12} - p_1(1-p_1)(\mathcal{V}_{11} - \mathcal{V}_{12}) = p_1^2\mathcal{V}_{11} + (1-p_1^2)\mathcal{V}_{12}$$

for the expected payoff of player 1. A similar result follows for player 2. $\square$

Proof of Proposition 4:
Without loss of generality, suppose that the owner of team a decided to bribe player $b(1)$. We know that player $b(2)$'s minimum acceptable bribe reduces from $\mathcal{B}_2$ to $\mathcal{B}_2^1$ after she anticipates that player $b(1)$ will be bribed.

There are three cases to consider: (i) $p_1 > p_2$; (ii) $p_2 > p_1 > p_2^2$; and (iii) $p_2^2 > p_1$. In case (i), there is a critical value $\overline{p}$ such that if $p_1 < \overline{p}$, then $\mathcal{B}_2^1$ is not sufficiently low such that deviating to offer a bribe of $\mathcal{B}_2^1$ to player $b(2)$ is not beneficial for the owner of team a. If $p_1 > \overline{p}$, however, then the team owner would deviate to bribe player $b(2)$. In case (ii), the team owner would deviate to offer a zero bribe to player $b(2)$, which is now acceptable since $\mathcal{B}_2^1 < 0$. Also in case (iii), the acceptable bribe is 0. However, there is now another critical value $\tilde{p}$. Accordingly, if $p_1 < \tilde{p}$, then it is as in case (ii). If not, this would mean that player $b(1)$ is too strong in the sense that she has a very high winning probability, such that deviating to bribe player $b(2)$ can never be beneficial although $b(1)$ is asking for a positive bribe whereas $b(2)$ would accept a zero bribe. Put differently, if the team owner will give up her right to bribe the other players by bribing player $b(2)$, then the team owner should be the one to be paid (which we do not allow in our model).

The analysis above considered a *first* deviation only. However, further deviations are possible following a similar reasoning. All information that would help us understand who would be the one to be bribed and by how much can be summarized by a 3×3 matrix. The same matrix is denoted by $\mathbf{W}$ above:

$$\begin{bmatrix} p_2p_3V + p_1^2(p_2+p_3-2p_2p_3)V_1 & p_2p_3V + \max\{0, p_3(p_1^2-p_2)V_1\} & p_2p_3V + \max\{0, p_2(p_1^2-p_3)V_1\} \\ p_1p_3V + \max\{0, p_3(p_2^2-p_1)V_2\} & p_1p_3V + p_2^2(p_1+p_3-2p_1p_3)V_2 & p_1p_3V + \max\{0, p_1(p_2^2-p_3)V_2\} \\ p_1p_2V + \max\{0, p_2(p_3^2-p_1)V_3\} & p_1p_2V + \max\{0, p_1(p_3^2-p_2)V_3\} & p_1p_2V + p_3^2(p_1+p_2-2p_1p_2)V_3 \end{bmatrix}$$

At first, the team owner aims to minimize over the diagonal. Whichever entry is the minimum, the corresponding player is bribed in the amount of the second part of that entry. Given that player $b(t)$ is bribed, the entries in the column t are compared. The team owner minimizes over this column. Suppose that the entry (t,t) is the minimum. Then the team owner chooses not to deviate from bribing player $b(t)$. And the process stops there. If otherwise, given that the entry (t',t) for $t \neq t'$ is the minimum, the team owner would deviate to bribe player $b(t')$ in the amount of the second part of the respective entry. Now, we can control for further deviations. The team owner compares the entry (t', t) with the non-diagonal entries in the column t'. Notice that this is a comparison between the cost in (t', t) where the team owner is currently at and the other cost levels the team owner can choose to incur. Once again, the team owner would choose the minimum and continue this process until there is no deviation.

The inequalities specified in the proposition statement keep track of such possible deviations. For instance, in case 1, since $\mathbf{W}_{11}$ is lower than other entries in Row 1 and Column 1, the owner of team a decides to bribe player $b(1)$ and does not deviate to bribing other players. Similarly, consider case 2 where $j = 2$ and $k = 3$. Once again, the owner of team a decides to bribe player $b(1)$, in the amount of $\mathcal{B}_1$, but now, since $\mathbf{W}_{21} < \mathbf{W}_{11}$ and $\mathbf{W}_{21} < \mathbf{W}_{31}$, the team owner would deviate to bribing player $b(2)$. Though, the team owner does not stop deviating: given that $\mathbf{W}_{12} < \mathbf{W}_{21}$, she would further deviate to bribing player $b(1)$. Notice that the bribe amount decreases to $\mathcal{B}_1^2$. Since $\mathbf{W}_{12}$ is lower than the other cost levels the team owner can choose to incur, there is no further deviation. Finally, consider case 5 where $j = 2$ and $k = 3$. Under these conditions, the owner of team a decides to bribe player $b(3)$ at first. When the other players anticipate such a bribe, they lower the amounts of bribe they are willing to accept, and the team owner deviates to bribing player $b(2)$, in the amount of $\mathcal{B}_2^3$. But then, since such an action can also be anticipated, the team owner further deviates to bribing player $b(1)$, in the amount of $\mathcal{B}_1^2$. The process stops here. The other cases follow similarly.

Finally, there is another condition to be checked. The minimum expected total loss found by the above-described procedure should be less than or equal to $(p_1p_2 + p_2p_3 + p_1p_3 - 2p_1p_2p_3)V$ in order for the owner of team a to prefer offering the respective amount to player $b(1)$. That expression is the expected loss of the team owner in case of no bribe, including team a's probability of losing the contest in parenthesis. If such an inequality is not satisfied, then the best bribing option for the team owner does not have a sufficiently high return, so that bribe does not occur. This completes the proof. $\square$

Appendix B

In this Appendix, we analyze an alternative model where we allow the owner of team a to offer bribe after battle 1 or battle 2 takes place, i.e., the team owner can intervene at any period t after observing the state. For this analysis, we first modify the original game tree provided in Figure 1, by extending an additional branch out of every node. These additional nodes are drawn in red as seen in Figure 6.

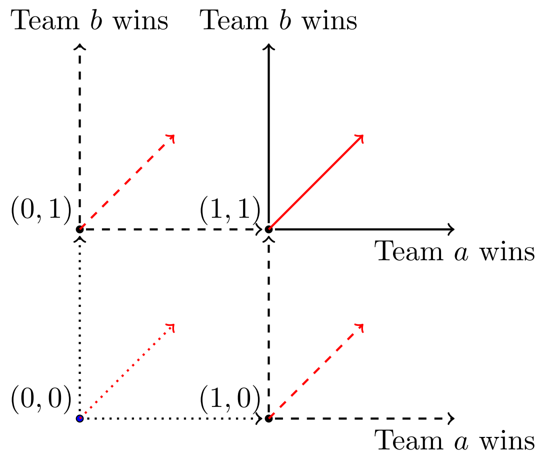

Figure 6: The alternative model of multiple pairwise battles with bribing

The red branch coming out of (x, y) directly leads to $(x, y+1)$, but rather with different future expected payoffs. Performing backward induction, we can also find that the vectors of winning probabilities attached to the red branch coming out of (1,1) and (1,0) are both (1,0); the vector of winning probabilities attached to the red branch coming out of (0,1) is $(1-p_3, p_3)$; and the vector of winning probabilities attached to the red branch coming out of (0,0) is $(1-p_2p_3, p_2p_3)$.

We now solve for a generic case summarized in Figure 7. In case of no bribe, if the generic battle t is won by $a(t)$, the game proceeds rightwards, and if otherwise, it proceeds upwards. If there occurs bribe, however, then the red line is followed. Each possible future state is represented by a triple. The first two components of a triple is the winning probabilities for team a and team b starting from that node, respectively; and the third component is the expected bribe to be given from that node onwards.

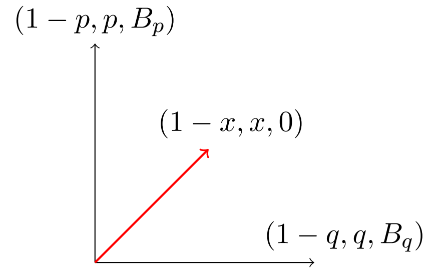

Figure 7: A generic case in the alternative model with bribing

Proposition 5. *Consider the battle at the generic node given by Figure 7.*

(i) Player $b(t)$ is bribed if

$$p_t \geq \frac{(x-q)V - B_q}{(p-q)V + B_p - B_q}. \tag{5}$$

(ii) The bribe is non-negative, i.e., $\mathcal{B}_t \geq 0$, if

$$p_t^2 \geq \frac{x-q}{p-q}. \tag{6}$$

(iii) If there occurs bribe, then the winning probabilities and the expected bribe at this node is

$$(1-x, x, \max\{0, \mathcal{B}_t\}).$$

In case there is no bribe, the same vector becomes

$$p_t(1-p, p, B_p) + (1-p_t)(1-q, q, B_q).$$

Proof. The expected payoff for player $b(t)$ is $(q + p_t^2(p-q))V_{b(t)}$ if there is no bribe in this generic case. On the other hand, if she takes a bribe of B, then her expected payoff would be $xV_{b(t)} + B$. Thus, if she is to accept a bribe, the following must hold:

$$(q + p_t^2(p-q))V_{b(t)} \leq xV_{b(t)} + B$$

$$(p_t^2(p-q) + q - x)V_{b(t)} \leq B$$

Let $\mathcal{B}_t = (p_t^2(p-q) + q - x)V_{b(t)}$ be the minimal acceptable bribe for this player. For the owner of team a, the expected payoff is

$$p_t((1-p)V - B_p) + (1-p_t)((1-q)V - B_q)$$

if there is no bribe in this generic case. On the other hand, if she gives a bribe of B in this case, then her expected payoff would be $(1-x)V - B$. Thus, if she is to offer bribe, the following must hold:

$$p_t((1-p)V - B_p) + (1-p_t)((1-q)V - B_q) \leq (1-x)V - B$$

$$B \leq p_t((p-q)V + B_p - B_q) - ((x-q)V - B_q)$$

Let $\bar{B} = p_t((p-q)V + B_p - B_q) - ((x-q)V - B_q)$ be the maximum bribe the team owner is willing to offer. Two conditions should be satisfied: $\mathcal{B}_t \leq \bar{B}$ and $0 \leq \bar{B}$. Thus, the bribing condition turns out to be

$$p_t((p-q)V + B_p - B_q) - ((x-q)V - B_q) \geq 0$$

which can also be written as

$$p_t \geq \frac{(x-q)V - B_q}{(p-q)V + B_p - B_q}.$$

Moreover, if $\mathcal{B}_t \leq 0$, then $b(t)$ would accept a bribe of 0; and otherwise, she would accept a bribe of $\mathcal{B}_t$. Thus the positivity condition is

$$\mathcal{B}_t = (p_t^2(p-q) + q - x)V_3 \geq 0$$

which reduces to

$$p_t^2 \geq \frac{x-q}{p-q}.$$

Finally, if there is no bribe at this generic node, then the winning probabilities and the expected bribe at this node are given by

$$p_t(1-p, p, B_p) + (1-p_t)(1-q, q, B_q).$$

On the other hand, if some bribe occurs, then the same vector would be $(1-x, x, \max\{0, \mathcal{B}_t\})$. □

The following proposition completes the equilibrium analysis for the alternative model of multiple pairwise battles with bribing.

Proposition 6. *Consider the alternative model of multiple pairwise battles with bribing given by Figure 6. In the equilibrium, depending on the model parameters, any player in team b can be bribed by the owner of team a. If the game arrives at node $(1,1)$ without any bribing, then player $b(3)$ is definitely* *bribed an amount of* $V_3 p_3^2$*. If the game arrives at node* $(1,0)$ *without any bribing, then player* $b(2)$ *would definitely accept a bribe of* 0*. If node* $(0,1)$ *is reached, however, there are three possibilities: (i) positive bribe:* $V_2(p_2^2 - p_3)$*; (ii) zero bribe; and (iii) no bribe. Finally, also at the starting node* $(0,0)$*, there are three possibilities; though, in case player* $b(1)$ *is bribed some positive amount, it is now either* $V_1 p_2 (p_1^2 - p_3)$ *or* $V_1 p_3 (p_1^2 - p_2)$*.*

Proof. The proof consists of characterizations of the equilibrium strategies for all players in team b as well as the owner of team a. Simply put, we perform backward induction and apply the observations in Proposition 5 to each node in the alternative model of multiple pairwise battles with bribing.

$\Rightarrow$ **At (1,1):** Since the game definitely ends after this node, we know that $p = 1$, $q = 0$, $x = 0$, and $B_p = B_q = 0$. The bribing condition is

$$p_3 \geq 0$$

i.e., the bribe is given for sure. We also find that $\mathcal{B}_3 = V_3 p_3^2$. Then, the respective equilibrium values are $(1, 0, V_3 p_3^2)$.

$\Rightarrow$ **At (1,0):** If the bribe will be given at node (1,1), team a wins the contest independent of who wins the battle at this node (without bribe). Accordingly, we know that $p = 0$, $q = 0$, $x = 0$, $B_p = V_3 p_3^2$, and $B_q = 0$. The bribing condition is now

$$p_2 \geq 0$$

i.e., the bribe is given for sure. We also find that $\mathcal{B}_2 = 0$. Then, the respective equilibrium values are $(1, 0, 0)$.

$\Rightarrow$ **At (0,1):** The contest is won by team b if player $b(2)$ wins at this node; and it proceeds to node (1,1) if player $a(2)$ wins at this node. Accordingly, we know that $p = 1$, $q = 0$, $x = p_3$, $B_p = 0$, and $B_q = V_3 p_3^2$. The bribing condition is

$$p_2 \geq \frac{p_3 V - V_3 p_3^2}{V - V_3 p_3^2}$$

and the bribe is positive only if

$$p_2^2 \geq p_3.$$

Thus we have three separate cases:

(i) If $p_2 \geq \frac{p_3 V - V_3 p_3^2}{V - V_3 p_3^2}$ and $p_2^2 \geq p_3$, then bribe occurs in the amount of $V_2(p_2^2 - p_3) > 0$. The respective equilibrium values are

$$(1 - p_3, p_3, V_2(p_2^2 - p_3)).$$

(ii) If $p_2 \geq \frac{p_3 V - V_3 p_3^2}{V - V_3 p_3^2}$ and $p_2^2 < p_3$, then bribe occurs in the amount of 0. The respective equilibrium values are

$$(1 - p_3, p_3, 0).$$

(iii) If $p_2 < \frac{p_3 V - V_3 p_3^2}{V - V_3 p_3^2}$, then there occurs no bribe. The respective equilibrium values are

$$(1 - p_2, p_2, (1 - p_2)p_3^2 V_3).$$

$\Rightarrow$ **At (0,0):** For any of the three cases (i), (ii), and (iii) analyzed above, we have $x = p_2 p_3$ and $(1 - q, q, B_q) = (0, 1, 0)$. On the other hand, as shown above, the triple $(1 - p, p, B_p)$ differs in each case.

(i) Suppose that $(1 - p, p, B_p) = (1 - p_3, p_3, V_2(p_2^2 - p_3))$. We know that bribe occurs if

$$p_1 \geq \frac{V p_2 p_3}{V p_3 + V_2(p_2^2 - p_3)},$$

and it would be positive only if $p_1^2 \geq p_2$. Thus ...

(i.a) if $p_1 \geq \frac{V p_2 p_3}{V p_3 + V_2(p_2^2 - p_3)}$ and $p_1^2 \geq p_2$, then bribe occurs in the amount of $V_1 p_3(p_1^2 - p_2) > 0$. The respective equilibrium values are

$$(1 - p_2 p_3, p_2 p_3, V_1 p_3(p_1^2 - p_2)).$$

(i.b) if $p_1 \geq \frac{V p_2 p_3}{V p_3 + V_2(p_2^2 - p_3)}$ and $p_1^2 < p_2$, then bribe occurs in the amount of 0. The respective equilibrium values are

$$(1 - p_2 p_3, p_2 p_3, 0).$$

(i.c) if $p_1 < \frac{V p_2 p_3}{V p_3 + V_2(p_2^2 - p_3)}$, then there occurs no bribe. The respective equilibrium values are

$$(1 - p_1 p_3, p_1 p_3, V_2 p_1(p_2^2 - p_3)).$$

(ii) Suppose that $(1-p, p, B_p) = (1-p_3, p_3, 0)$. We know that bribe occurs if

$$p_1 \geq \frac{Vp_2p_3}{Vp_3} = p_2,$$

and it would be positive only if $p_1^2 \geq p_2$. Thus ...

(ii.a) if $p_1 \geq p_2$ and $p_1^2 \geq p_2$, then bribe occurs in the amount of $V_1p_3(p_1^2 - p_2) > 0$. The respective equilibrium values are

$$(1 - p_2p_3, p_2p_3, V_1p_3(p_1^2 - p_2)).$$

(ii.b) if $p_1 \geq p_2$ and $p_1^2 < p_2$, then bribe occurs in the amount of 0. The respective equilibrium values are

$$(1 - p_2p_3, p_2p_3, 0).$$

(ii.c) if $p_1 < p_2$, then there occurs no bribe. The respective equilibrium values are

$$(1 - p_1p_3, p_1p_3, 0).$$

(iii) Suppose that $(1-p, p, B_p) = (1-p_2, p_2, (1-p_2)p_3^2V_3)$. We know that bribe occurs if

$$p_1 \geq \frac{Vp_2p_3}{Vp_2 + (1-p_2)p_3^2V_3},$$

and it would be positive only if $p_1^2 \geq p_3$. Thus ...

(iii.a) if $p_1 \geq \frac{Vp_2p_3}{Vp_2+(1-p_2)p_3^2V_3}$ and $p_1^2 \geq p_3$, then bribe occurs in the amount of $V_1p_2(p_1^2 - p_3) > 0$. The respective equilibrium values are

$$(1 - p_2p_3, p_2p_3, V_1p_2(p_1^2 - p_3)).$$

(iii.b) if $p_1 \geq \frac{Vp_2p_3}{Vp_2+(1-p_2)p_3^2V_3}$ and $p_1^2 < p_3$, then bribe occurs in the amount of 0. The respective equilibrium values are

$$(1 - p_2p_3, p_2p_3, 0).$$

(iii.c) if $p_1 < \frac{Vp_2p_3}{Vp_2+(1-p_2)p_3^2V_3}$, then there occurs no bribe. The respective equilibrium values are

$$(1 - p_1p_2, p_1p_2, p_1p_3^2(1-p_2)V_3).$$

The equilibrium analysis above verifies the results summarized in the proposition statement. This completes the proof. $\square$