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Rotemberg and Imperfect Common Knowledge: A Solution Algorithm

Abstract

This paper develops an algorithm that enables to solve macroeconomic models with Rotemberg pricing and imperfect common knowledge. Under the concept of imperfect common knowledge, Rotemberg pricing requires the solution algorithm to take prices explicitly into account. The state space includes the hierarchy of average higher-order expectations as well as the aggregate price level. In addition to determining the usual policy functions of output, inflation, and the nominal interest rate, the algorithm has to search for the policy function of the aggregate price and for the policy function of the firm-specific price.

JEL-Codes: C630, D820, E310.

Keywords: Rotemberg pricing, dispersed information, heterogenous beliefs, Kalman filter, higher-order expectations.

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1 Introduction

This is the first paper that presents how to solve macroeconomic models with Rotemberg pricing and imperfect common knowledge. The presented algorithm deals with dynamic beauty contests in which firms suffer from a nominal rigidity (Rotemberg, 1982) as well as from an information rigidity (Woodford, 2002). In contrast to the usual assumption, firms in this class of models are not fully informed. They instead receive noisy information about the underlying economic fundamentals. Through firm-specific signals, each firm obtains a different piece of information, which the other firms do not observe. The private nature of the received signals creates information dispersion among the firms.

The vast majority of macroeconomic models still assumes the existence of full-information rational expectations—a concept pioneered by Muth (1961). An increasing number of empirical studies nevertheless disproves this assumption (Coibion, Gorodnichenko and Kamdar, 2018). Survey evidence on the expectations-formation process points to substantial departures from full-information rational expectations. Data rejects the null hypothesis of full-information rational expectations not because of irrationality but because of information rigidities (Coibion and Gorodnichenko, 2015). A promising modeling technique that could reconcile survey data with macroeconomic theory is the concept of imperfect common knowledge. In models that feature imperfect common knowledge, economic agents act rationally but are not fully informed. Pieces of information disperse throughout the economy. Such information dispersion causes agents to disagree about the future economic development. Imperfect common knowledge is therefore able to reproduce the well-documented fact of forecast disagreement (Coibion and Gorodnichenko, 2012; Andrade et al., 2016).

In the class of models that this paper analyzes, strategic complementarity represents a crucial mechanism. Each firm wants to set its price close to competitors' prices. The resulting relative price should be neither too low nor too high. Because firms are dispersedly informed, they have to nowcast the prices of their competitors. Strategic complementarity together with dispersed information prompts firms to form higher-order expectations. Firms start forecasting the forecasts of others (Townsend, 1983). In usual macroeconomic models, where firms have identical information sets and hence identical forecasts, higher-order expectations do not emerge.

Nimark (2008) and Melosi (2017) describe how to solve models of imperfect common knowledge under the pricing assumption of Calvo (1983). They derive the Calvo version of the imperfect-common-knowledge Phillips curve. They are then able to express and solve models of imperfect common knowledge with Calvo pricing in three endogenous variables: output, inflation, and the nominal interest rate. Such a solution approach is however infeasible under the alternative pricing assumption of Rotemberg (1982). As Šauer (2016) shows, inflation in the Rotemberg version of the imperfect-common-knowledge Phillips curve depends on higher-order expectations of future relative prices, which cannot be rewritten in terms of output, inflation, and the nominal interest rate. This means that one has to explicitly consider prices in the solution algorithm if one wants to solve models with Rotemberg pricing and imperfect common knowledge.

The algorithm presented here builds on the method of undetermined coefficients. The unknown coefficients of the policy functions are determined by inserting the conjectured solution into the corresponding equilibrium conditions. The dynamics of higher-order expectations are pinned down by the Kalman filter, which firms utilize in the expectations-formation process. The state space is formed by the hierarchy of average higher-order expectations and additionally by the aggregate price level. Besides determining the behavior of output, inflation, and the nominal interest rate, the solution algorithm searches for the policy function of the aggregate price as well as for the policy function of the firm-specific price.

My paper contributes to the relatively new strand of literature that intends to generate more knowledge about the expectations-formation process of firms. Recently, Coibion et al. (2018*b*) have called for more intensive research on firms' expectations. They argue that a better understanding of what firms expect would enable central banks to conduct active management of inflation expectations. If central banks understood the exact mechanism of expectations formation, they would know how to affect beliefs and thus decisions of price-setting firms. It would consequently become easier for monetary policy to reach its goals. Before the active management of inflation expectations can enrich the toolbox of central banks, economists have to, among others, study various models that departure from the assumption of full-information rational expectations. This paper shows how to solve one class of such models, which have the potential to offer insights into how firms form their expectations and how firms' expectations influence the economy.

As recent research demonstrates, higher-order expectations—the keystone of the imperfect-common-knowledge framework—can resolve a variety of puzzles that arise in models with full-information rational expectations. Under common knowledge, which prevails in models with full-information rational expectations, announcements about future policy or future fundamentals immediately trigger a non-negligible response of the model economy. The sharp and immediate reaction is typically perceived as unrealistic because distant future should be discounted accordingly. Higher-order expectations can fix this issue by giving rise to endogenous myopia, which attenuates general-equilibrium effects of announcements that convey information about the future. The forward-guidance puzzle can hence be solved by allowing for higher-order expectations (Angeletos and Lian, 2018). Moreover, the sluggishness of higher-order expectations can help to explain the high persistence of the real exchange rate, which usual full-information models struggle to replicate (Candian, 2019). For a long time, higher-order expectations have been a theoretical concept that has not been measured in a real-world setting. Coibion et al. (2018*a*) represents the first study that reports data on higher-order expectations of a macroeconomic variable; it analyzes first-order as well as second-order inflation expectations of firm managers.

The rest of the paper is structured as follows. Section 2 presents a small new-Keynesian model in which firms encounter Rotemberg pricing and imperfect common knowledge. I develop an algorithm that enables to solve such a model in Section 3. I then apply the solution algorithm and carry out several simulation exercises in Section 4. Finally, Section 5 concludes.

2 The Model

The paper aims to find a way how to solve macroeconomic models with Rotemberg pricing and imperfect common knowledge. Because the aim of the paper is very practical, I use a concrete model to demonstrate the solution algorithm. I present the algorithm by a model that closely relates to Melosi (2017) and Nimark (2008). Of course, my model differs from Melosi (2017) and Nimark (2008) in the underlying pricing assumption; I assume the Rotemberg pricing instead of the Calvo pricing.

I discuss the model in the following subsections. It consists of a representative household, bundler, monopolistically-competitive firms, and a central bank. As usual in the imperfect-common-knowledge literature, the representative household, the bundler, and the central bank are perfectly informed. In quarter t, they know all variables of quarter t and also know all variables of previous quarters. The monopolistically-competitive firms are, in contrast, imperfectly informed. The firms know only a few variables, which they learn with a certain degree of imprecision. The imprecision results from idiosyncratic noise, which generates disagreement among the firms over the past, present, and future realizations of variables. In other words, the firms suffer from the imperfect common knowledge.

Additionally, the concept of the common knowledge of rationality is imposed on the model. Everybody in the model knows that all agents behave rationally in line with their information sets; every agent knows the parameters of the model and understands how the economy operates.

2.1 The Representative Household

As in other papers that study information frictions (e.g., Nimark, 2008; Paciello and Wiederholt, 2014; Melosi, 2017), the representative household is perfectly informed. The household maximizes expected utility with respect to a budget constraint.

$$\max_{C_t, N_t, B_t} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{C_\tau^{1-\gamma}}{1-\gamma} - \kappa \frac{N_\tau^{1+\varphi}}{1+\varphi} \right) \exp\left(d_\tau\right)$$

$$\text{s.t.}$$

$$P_\tau C_\tau + B_\tau = W_\tau N_\tau + R_{\tau-1} B_{\tau-1} + \Gamma_\tau$$

The operator E_t represents expectations that are based on perfect information. The operator captures what the household expects from the perspective of quarter t knowing all variables of quarter t and knowing all variables of previous quarters. The household derives utility from consumption C_t and experiences disutility from supplying labor N_t. The consumption costs P_t. The household can invest in nominal bonds B_t, which yield gross nominal interest rate R_t. Work brings nominal hourly wage W_t. The household also receives nominal profits Γ_t from the monopolistically-competitive firms. The demand shock d_t takes the form of a Gaussian AR(1) process.

$$d_t = \rho_d d_{t-1} + \sigma_d \epsilon_t^d$$

$$\epsilon_t^d \sim \mathcal{N}\left(0, 1\right)$$

2.2 The Bundler

The bundler, who is perfectly informed, maximizes its profit. Using a Dixit–Stiglitz aggregator, the bundler combines differentiated goods $Y_t(j)$ to form the aggregate output Y_t. The bundler takes the aggregate price P_t and the prices of the differentiated goods $P_t(j)$ as given.

$$\max_{Y_t, Y_t(j) \forall j \in [0;1]} P_t Y_t - \int_0^1 P_t(j) Y_t(j) \, \mathrm{d}j$$

$$\text{s.t.}$$

$$Y_t = \left[\int_0^1 \left(Y_t(j)\right)^{\frac{\nu-1}{\nu}} \, \mathrm{d}j \right]^{\frac{\nu}{\nu-1}}$$

2.3 Monopolistically-Competitive Firms

A continuum of firms $[0;1]$ engages in monopolistic competition. A firm $j \in [0;1]$ produces a differentiated good $Y_t(j)$ by a production function that is linear in labor $N_t(j)$.

$$Y_t(j) = \exp\left(a_t(j)\right) N_t(j)$$

The firm-specific productivity $a_t(j)$ consists of two components: aggregate productivity a_t and idiosyncratic Gaussian shock $\eta_t^a(j)$.

$$a_t(j) = a_t + \tilde{\sigma}_a \eta_t^a(j)$$
$$\eta_t^a(j) \sim \mathcal{N}(0,1)$$

The aggregate productivity a_t follows a Gaussian AR(1) process.

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_t^a$$
$$\epsilon_t^a \sim \mathcal{N}(0,1)$$

The monopolistically-competitive firms suffer from imperfect information. The firms have limited information sets that contain private information. Specifically, the information set of the firm j takes the form

$$\mathcal{I}_t^j = \left\{P_{\tau-1}, R_\tau, P_\tau(j), a_\tau(j), d_\tau(j) | \tau = t, t-1, t-2, \ldots\right\}.$$

In quarter t, the firm j knows its own price $P_t(j)$, its own productivity $a_t(j)$, and the economy-wide nominal interest rate R_t. The firm j learns the aggregate price P_{t-1} with a lag of one quarter. Additionally, it receives an idiosyncratic signal $d_t(j)$ about the demand shock d_t.

$$d_t(j) = d_t + \tilde{\sigma}_d \eta_t^d(j)$$
$$\eta_t^d(j) \sim \mathcal{N}(0,1)$$

Based on the information set $\mathcal{I}_t^j$, the firm j forms expectations E_t^j.

The firms maximize expected profits by setting prices a la Rotemberg (1982), knowing that the demand for their products depends on their respective relative prices $Z_t(j) = P_t(j)/P_t$.

$$\max_{P_t(j)} E_t^j \sum_{\tau=t}^{\infty} \Lambda_{\tau|t} \left[P_\tau(j) Y_\tau(j) - P_\tau MC_\tau(j) Y_\tau(j) - P_\tau \frac{\Xi}{2} \left(\frac{P_\tau(j)}{P_{\tau-1}(j)} - \Pi \right)^2 Y_\tau \right]$$

s.t.

$$Y_\tau(j) = (Z_\tau(j))^{-\nu} Y_\tau$$

The price-adjustment costs have the usual quadratic form, which is scaled by the aggregate output Y_τ. When the firm-specific inflation $\Pi_\tau(j) = P_\tau(j)/P_{\tau-1}(j)$ keeps up with the trend inflation Π, no price-adjustment costs arise. The firm j discounts the nominal profits by the factor

$$\Lambda_{\tau|t} = \beta^{\tau-t} \frac{C_\tau^{-\gamma} \exp(d_\tau) P_t}{C_t^{-\gamma} \exp(d_t) P_\tau}$$

and faces real marginal costs

$$MC_\tau(j) = \frac{1}{\exp(a_\tau(j))} \frac{W_\tau}{P_\tau}.$$

2.4 The Central Bank

Central banks pay close attention to expectations of private agents (e.g., Bernanke, 2007). They monitor market-based as well as survey-based measures of expectations. Information from surveys of firms, consumers, and professional forecasters represents a key input into the decision-making process of every central bank. Accordingly, monetary policy in the model is conducted by a simple rule in which the central bank reacts to private-sector expectations:

$$R_t = R \left(\int_0^1 E_t^j \frac{\Pi_{t+1}}{\Pi} \, \mathrm{d}j \right)^{\phi_\pi} \left(\int_0^1 E_t^j \frac{Y_{t+1}}{Y_{t+1}^*} \, \mathrm{d}j \right)^{\phi_y} \exp(m_t).$$

The central bank responds to firms' expectations of future inflation Π_{t+1} ($\Pi_{t+1} = P_{t+1}/P_t$) and to firms' expectations of future output gap Y_{t+1}/Y_{t+1}^*. The variable Y_{t+1}^* denotes the natural output, which I define as the output that would arise under flexible prices and perfect information. The monetary shock m_t follows a Gaussian AR(1) process.

$$m_t = \rho_m m_{t-1} + \sigma_m \epsilon_t^m$$
$$\epsilon_t^m \sim \mathcal{N}(0, 1)$$

2.5 Market Clearing

Labor supply equals labor demand:

$$N_t = \int_0^1 N_t(j)\,\mathrm{d}j.$$

The bonds are in zero net supply:

$$B_t = 0.$$

The aggregate output is split between consumption and price-adjustment costs:

$$Y_t = C_t + \int_0^1 \frac{\Xi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi\right)^2 Y_t\,\mathrm{d}j.$$

2.6 The Linearized Model

The linearized Euler equation of the household leads to a dynamic IS curve

$$y_t = E_t y_{t+1} - \frac{1}{\gamma}\left(r_t - E_t\pi_{t+1}\right) + \frac{1-\rho_d}{\gamma} d_t, \tag{1}$$

where $y_t = \left(Y_t - Y\right)/Y$, $r_t = R_t - R$, and $\pi_{t+1} = \Pi_{t+1} - \Pi$.

The first-order condition of the firm j with respect to its price $P_t(j)$ specifies the behavior of the firm-specific inflation $\pi_t(j) = \Pi_t(j) - \Pi$:

$$\pi_t(j) = \beta E_t^j \pi_{t+1}(j) + \frac{\nu - 1}{\Xi\Pi} E_t^j mc_t(j) - \frac{\nu - 1}{\Xi\Pi} E_t^j z_t(j). \tag{2}$$

The firm-specific inflation depends on three variables: the firm's forecast of its own inflation $E_t^j \pi_{t+1}(j)$, the firm's nowcast of its own marginal costs $E_t^j mc_t(j)$ ($mc_t(j) = (MC_t(j) - MC(j))/MC(j)$), and the firm's nowcast of its own relative price $E_t^j z_t(j)$ ($z_t(j) = (Z_t(j) - Z(j))/Z(j)$). The marginal costs can be expressed in terms of aggregate output, aggregate productivity, and firm-specific productivity:

$$mc_t(j) = \left(\gamma + \varphi\right) y_t - \varphi a_t - a_t(j).$$

The linearized interest-rate rule of the central bank reads:

$$r_t = \phi_\pi \int_0^1 E_t^j \pi_{t+1}\,\mathrm{d}j + \phi_y \int_0^1 E_t^j\left(y_{t+1} - y_{t+1}^*\right)\mathrm{d}j + m_t. \tag{3}$$

I can express the natural output $y_{t+1}^* = \left(Y_{t+1}^* - Y\right)/Y$, which would materialize in the absence of nominal and information rigidities, solely in terms of the aggregate productivity:

$$y_t^* = \frac{1+\varphi}{\gamma+\varphi} a_t.$$

The Dixit–Stiglitz aggregator of the bundler implies that the aggregate log price equals the sum of firm-specific log prices:

$$p_t = \int_0^1 p_t(j)\,\mathrm{d}j. \tag{4}$$

The difference between the present and the past aggregate log price defines inflation:

$$\pi_t = p_t - p_{t-1} - (\Pi - 1)\,. \tag{5}$$

3 The Solution Algorithm

The method of undetermined coefficients (i.a., Uhlig, 2001; Christiano, 2002) constitutes the core of the algorithm. I conjecture a solution; I insert the solution into equilibrium conditions and receive updates for the undetermined coefficients. The Kalman filter delivers updates for the hierarchy of higher-order expectations. The equations of the linearized model from Section 2.6 deliver updates for the policy functions. In what follows, vectors and matrices are denoted by bold symbols to ensure readability.

3.1 State Variables

I summarize the state variables of the model in vector $\boldsymbol{\xi_t}$, which consists of $2 + 3(k+1)$ elements:

$$\boldsymbol{\xi_t} = \begin{bmatrix} \Pi - 1 & p_{t-1} & \boldsymbol{X_{t|t}^{(0:k)\prime}} \end{bmatrix}'.$$

Similarly to the setup with the Calvo pricing (Nimark, 2008; Melosi, 2017), the higher-order expectations of the shocks $\boldsymbol{X_{t|t}^{(0:k)}}$ enter the state vector $\boldsymbol{\xi_t}$. The hierarchy of average higher-order expectations takes the form

$$\boldsymbol{X_{t|t}^{(0:k)}} = \begin{bmatrix} a_t & m_t & d_t & a_{t|t}^{(1)} & m_{t|t}^{(1)} & d_{t|t}^{(1)} & \dots & a_{t|t}^{(k)} & m_{t|t}^{(k)} & d_{t|t}^{(k)} \end{bmatrix}'.$$

The average higher-order expectations of the shocks are defined in the following way:

$$
\begin{aligned}
a_{t|t}^{(1)} &= \int_0^1 E_t^j a_t \,\mathrm{d}j \\
a_{t|t}^{(l)} &= \int_0^1 E_t^j a_{t|t}^{(l-1)} \,\mathrm{d}j \quad \forall l = 2, 3, \ldots \\
m_{t|t}^{(1)} &= \int_0^1 E_t^j m_t \,\mathrm{d}j \\
m_{t|t}^{(l)} &= \int_0^1 E_t^j m_{t|t}^{(l-1)} \,\mathrm{d}j \quad \forall l = 2, 3, \ldots \\
d_{t|t}^{(1)} &= \int_0^1 E_t^j d_t \,\mathrm{d}j \\
d_{t|t}^{(l)} &= \int_0^1 E_t^j d_{t|t}^{(l-1)} \,\mathrm{d}j \quad \forall l = 2, 3, \ldots
\end{aligned}
$$

The hierarchy of higher-order expectations has to be included in the state vector because of the strategic complementarity that is inherent in the price setting of every firm. An optimal firm-specific price hinges on what a specific firm thinks of the aggregate price, which is defined as the average firm-specific price. Every firm-specific price additionally reflects what a specific firm thinks of the realized aggregate shocks. When a firm j forms during the price setting the nowcast $E_t^j p_t = \int_0^1 E_t^j p_t(i) \,\mathrm{d}i$, it implicitly nowcasts what other firms think of the realized shocks (for instance: $E_t^j(E_t^i a_t)$). Due to the strategic complementarity, the other firms are interested in the price that the firm j plans to choose. Therefore, they nowcast what the firm j thinks of their nowcasts (for instance: $E_t^i(E_t^j(E_t^i a_t))$). These new nowcasts are again nowcasted by the firm j. The process of nowcasting each other continues and leads to an infinite regress problem. This phenomenon is known as forecasting the forecasts of others (Townsend, 1983).

An exact solution of the model would require to track an infinite hierarchy of higher-order expectations: $k \to \infty$. But in practice the solution of the model has to be based on objects of finite length. The models of imperfect common knowledge are hence solved up to the order k, where $k \in \mathbb{N}$ is sufficiently large to mimic infinity. Restricting the hierarchy of higher-order expectations to a finite order k still results into an accurate solution because gradually with increasing order the average expectations lose importance in the policy functions (Nimark, 2017).

Under Calvo, it is possible to combine equilibrium conditions and express the model in three endogenous variables: output, inflation, and the interest rate. The hierarchy $\boldsymbol{X}_{\boldsymbol{t|t}}^{\boldsymbol{(0:k)}}$ is then enough to fully describe the state of the Calvo model. However, this approach is impossible under the Rotemberg pricing. There is no way how to substitute for the aggregate price p_t in the equilibrium conditions and to express the Rotemberg model solely in output, inflation, and the interest rate. The Rotemberg solution algorithm has to take into account the aggregate price. Because prices are sticky, today's aggregate price p_t depends on the past aggregate price p_{t-1}. The past aggregate price p_{t-1} hence represents an additional state variable that I have to consider in the state vector $\boldsymbol{\xi_t}$. The element $\Pi - 1$ in the state vector allows for non-zero trend inflation.

I conjecture the state follows a VAR(1) process:

$$\boldsymbol{\xi_t} = \boldsymbol{M}\boldsymbol{\xi_{t-1}} + \boldsymbol{N}\boldsymbol{\epsilon_t}.$$

The vector $\boldsymbol{\epsilon_t}$ contains the innovations of the shock processes a_t, m_t, and d_t:

$$\boldsymbol{\epsilon_t} = \begin{bmatrix} \epsilon_t^a & \epsilon_t^m & \epsilon_t^d \end{bmatrix}'.$$

The matrix $\boldsymbol{M}$, which describes the persistence of the state, can be partitioned as follows:

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 1 & 1 & & & \boldsymbol{H_p} & \\ 0 & 0 & \rho_a & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & 0 & \rho_m & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & 0 & 0 & \rho_d & \boldsymbol{0_{1\times 3k}} \\ \boldsymbol{0_{3k\times 1}} & \boldsymbol{0_{3k\times 1}} & & & \boldsymbol{K} & \end{bmatrix}.$$

The first row of the matrix $\boldsymbol{M}$ transmits the net trend inflation $\Pi - 1$ from the past state vector $\boldsymbol{\xi_{t-1}}$ into today's state vector $\boldsymbol{\xi_t}$. The second row of $\boldsymbol{M}$ captures the dynamics of the aggregate price p_{t-1}. The aggregate price keeps up with the trend inflation, has a unit root, and reacts to the hierarchy of higher-order expectations. The third, fourth, and fifth row describe the persistence of the actual shock processes a_t, m_t, and d_t. The remaining $3k$ rows capture the history dependence of the average expectations $\boldsymbol{X_{t|t}^{(1:k)}}$. The matrix $\boldsymbol{N}$ can be split into:

$$\boldsymbol{N} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sigma_a & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_d \\ & \boldsymbol{F} & \end{bmatrix}.$$

The first and the second row of $\boldsymbol{N}$ contain only zeros because today's innovations $\boldsymbol{\epsilon_t}$ affect neither the trend inflation nor the past aggregate price p_{t-1}. The third, fourth, and fifth row scale down the innovations for the actual shock processes a_t, m_t, and d_t. The remaining $3k$ rows describe how the innovations $\boldsymbol{\epsilon_t}$ impact the average expectations $\boldsymbol{X^{(1:k)}_{t|t}}$.

For a later step, it is useful to find a simple expression for the average nowcast of the state vector $\boldsymbol{\xi_t}$. I can express the average nowcast of the state by the truncation matrix $\boldsymbol{T_\xi}$:

$$\int_0^1 E_t^j \boldsymbol{\xi_t} \, \mathrm{d}j = \boldsymbol{T_\xi} \boldsymbol{\xi_t},$$

$$\boldsymbol{T_\xi} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 1 & 0 & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ & & \boldsymbol{0_{3k\times 5}} & & & \boldsymbol{I_{3k\times 3k}} \\ & & \boldsymbol{0_{3\times[2+3(k+1)]}} & & & \end{bmatrix},$$

where $\boldsymbol{I_{3k\times 3k}}$ stands for an identity matrix. Similarly, I can define a truncation matrix for the hierarchy of average higher-order expectations:

$$\int_0^1 E_t^j \boldsymbol{X^{(0:k)}_{t|t}} \, \mathrm{d}j = \boldsymbol{T_X} \boldsymbol{X^{(0:k)}_{t|t}},$$

$$\boldsymbol{T_X} = \begin{bmatrix} \boldsymbol{0_{3k\times 3}} & \boldsymbol{I_{3k\times 3k}} \\ \multicolumn{2}{c}{\boldsymbol{0_{3\times 3(k+1)}}} \end{bmatrix}.$$

3.2 Jump Variables

I place the jump variables of the model into vector $\boldsymbol{s_t}$:

$$\boldsymbol{s_t} = \begin{bmatrix} y_t & \pi_t & r_t \end{bmatrix}'.$$

The vector consists of output y_t, inflation π_t, and the nominal interest rate r_t. The jump variables $\boldsymbol{s_t}$ are determined by the state $\boldsymbol{\xi_t}$:

$$\boldsymbol{s_t} = \boldsymbol{v} \boldsymbol{\xi_t}.$$

I can partition the policy matrix $\boldsymbol{v}$ in the following way:

$$\boldsymbol{v} = \begin{bmatrix} 0 & 0 & \boldsymbol{H_y} \\ 0 & 0 & \boldsymbol{H_\pi} \\ 0 & 0 & \boldsymbol{H_r} \end{bmatrix}.$$

The jump variables, which represent deviations from the steady state, exhibit stationarity and zero mean. Therefore, the jump variables do not depend on the trend inflation $\Pi - 1$ and the past aggregate price p_{t-1}. The first two columns of the policy matrix $\boldsymbol{v}$ are accordingly zero vectors. Solely the hierarchy of higher-order expectations determines the jump variables. The vectors $\boldsymbol{H_y}$, $\boldsymbol{H_\pi}$, and $\boldsymbol{H_r}$ formalize the dependence of the jump variables y_t, π_t, and r_t on the hierarchy $\boldsymbol{X_{t|t}^{(0:k)}}$.

3.3 The Formation of Expectations

The monopolistically-competitive firms, which suffer from dispersed information, form their expectations by the constant-gain Kalman filter (Hamilton, 1994). The observation equation of a monopolistically-competitive firm $j \in [0;1]$ reads in the general form:

$$\boldsymbol{Z_t(j)} = \boldsymbol{D\xi_t} + \boldsymbol{Q\eta_t(j)}.$$

In accordance with the information set $\mathcal{I}_t^j$, the vector of observables $\boldsymbol{Z_t(j)}$ consists of the past aggregate price, the firm-specific productivity, the idiosyncratic signal about the demand shock, and the nominal interest rate:

$$\boldsymbol{Z_t(j)} = \begin{bmatrix} p_{t-1} & a_t(j) & d_t(j) & r_t \end{bmatrix}'.$$

The firm-specific price $p_t(j)$, which is part of the information set $\mathcal{I}_t^j$, does not need to be included in the vector of observables $\boldsymbol{Z_t(j)}$. In the filtering problem of the firm j, the price $p_t(j)$ represents a redundant signal. The firm j chooses the price $p_t(j)$ as an optimal response to the information that the other signals reveal. Therefore, the price $p_t(j)$ can only reveal information that is already contained in the signals p_{t-1}, $a_t(j)$, $d_t(j)$, and r_t. The matrix $\boldsymbol{D}$ expresses the observables in terms of the state vector:

$$\boldsymbol{D} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & 1 & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & 0 & 0 & 1 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & & & \boldsymbol{H_r} & \end{bmatrix}.$$

The vector $\boldsymbol{\eta_t(j)}$ stacks the idiosyncratic shocks of the model:

$$\boldsymbol{\eta_t(j)} = \begin{bmatrix} \eta_t^a(j) & \eta_t^d(j) \end{bmatrix}'.$$

For the signals $a_t(j)$ and $d_t(j)$, the idiosyncratic shocks have to be scaled down by the matrix $\boldsymbol{Q}$:

$$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 \\ \tilde{\sigma}_a & 0 \\ 0 & \tilde{\sigma}_d \\ 0 & 0 \end{bmatrix}.$$

The constant-gain Kalman filter implies that the firm j nowcasts the state of the economy $\boldsymbol{\xi_t}$ by the following device:

$$E_t^j \boldsymbol{\xi_t} = E_{t-1}^j \boldsymbol{\xi_t} + \boldsymbol{P D' E}^{-1} \left(\boldsymbol{Z_t(j)} - E_{t-1}^j \boldsymbol{Z_t(j)} \right). \tag{6}$$

When the firm j forms its nowcast $E_t^j \boldsymbol{\xi_t}$, it updates its past forecast $E_{t-1}^j \boldsymbol{\xi_t}$ by the newly-incoming information $\boldsymbol{Z_t(j)} - E_{t-1}^j \boldsymbol{Z_t(j)}$. The symbol $\boldsymbol{P}$ denotes the mean-squared-error matrix of the state. The matrix $\boldsymbol{P}$ solves the fixed-point problem

$$\boldsymbol{P} = \boldsymbol{M} \left[\boldsymbol{P} - \boldsymbol{P D'} \left(\boldsymbol{D P D'} + \boldsymbol{Q Q'} \right)^{-1} \boldsymbol{D P} \right] \boldsymbol{M'} + \boldsymbol{N N'}. \tag{7}$$

The symbol $\boldsymbol{E}$ stands for the mean-squared-error matrix of the observables:

$$\boldsymbol{E} = \boldsymbol{D P D'} + \boldsymbol{Q Q'}. \tag{8}$$

The firm j forecasts future observables and future state variables as follows:

$$E_{t-1}^j \boldsymbol{Z_t(j)} = \boldsymbol{D} E_{t-1}^j \boldsymbol{\xi_t},$$
$$E_{t-1}^j \boldsymbol{\xi_t} = \boldsymbol{M} E_{t-1}^j \boldsymbol{\xi_{t-1}}.$$

I use these forecasting relations to rewrite the nowcasting device (6) into:

$$\begin{aligned} E_t^j \boldsymbol{\xi_t} = {} & \left(\boldsymbol{M} - \boldsymbol{P D' E}^{-1} \boldsymbol{D M} \right) E_{t-1}^j \boldsymbol{\xi_{t-1}} + \boldsymbol{P D' E}^{-1} \boldsymbol{D M} \boldsymbol{\xi_{t-1}} \\ & + \boldsymbol{P D' E}^{-1} \boldsymbol{D N} \boldsymbol{\epsilon_t} + \boldsymbol{P D' E}^{-1} \boldsymbol{Q} \boldsymbol{\eta_t(j)}. \end{aligned} \tag{9}$$

If I integrate over (9) with respect to the firms $j \in [0;1]$, I obtain an expression for the average nowcast of the state:

$$\begin{aligned} \int_0^1 E_t^j \boldsymbol{\xi_t} \, \mathrm{d}j = {} & \left(\boldsymbol{M} - \boldsymbol{P D' E}^{-1} \boldsymbol{D M} \right) \int_0^1 E_{t-1}^j \boldsymbol{\xi_{t-1}} \, \mathrm{d}j + \boldsymbol{P D' E}^{-1} \boldsymbol{D M} \boldsymbol{\xi_{t-1}} \\ & + \boldsymbol{P D' E}^{-1} \boldsymbol{D N} \boldsymbol{\epsilon_t}. \end{aligned} \tag{10}$$

I can express the average nowcasts and the actual state vector in (10) more explicitly:

$$\begin{bmatrix} \Pi - 1 \\ p_{t-1} \\ \boldsymbol{X}_{t|t}^{(1:k)} \\ \boldsymbol{X}_{t|t}^{((k+1):(k+1))} \end{bmatrix} = \left(\boldsymbol{M} - \boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{M}\right) \begin{bmatrix} \Pi - 1 \\ p_{t-2} \\ \boldsymbol{X}_{t-1|t-1}^{(1:k)} \\ \boldsymbol{X}_{t-1|t-1}^{((k+1):(k+1))} \end{bmatrix} + \boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{M} \begin{bmatrix} \Pi - 1 \\ p_{t-2} \\ \boldsymbol{X}_{t-1|t-1}^{(0:0)} \\ \boldsymbol{X}_{t-1|t-1}^{(1:k)} \end{bmatrix} + \boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{N}\boldsymbol{\epsilon}_t. \tag{11}$$

Because the model is solved up to the finite order k, I can crop the rows and columns that correspond to the redundant order $k+1$. After cropping the unnecessary rows and columns in (11), I learn the dynamics of the average expectations $\boldsymbol{X}_{t|t}^{(1:k)}$:

$$\boldsymbol{X}_{t|t}^{(1:k)} = \left[\left(\boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{M}\right)_{(3:(3k+2),3:5)} \left(\left(\boldsymbol{M}-\boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{M}\right)_{(3:(3k+2),3:(3k+2))} + \left(\boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{M}\right)_{(3:(3k+2),6:(3k+5))}\right) \right] \times \begin{bmatrix} \boldsymbol{X}_{t-1|t-1}^{(0:0)} \\ \boldsymbol{X}_{t-1|t-1}^{(1:k)} \end{bmatrix} + \left(\boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{N}\right)_{(3:(3k+2),1:3)} \boldsymbol{\epsilon}_t. \tag{12}$$

In Section 3.1, I conjecture that the hierarchy of higher-order expectations $\boldsymbol{X}_{t|t}^{(1:k)}$ behaves according to:

$$\boldsymbol{X}_{t|t}^{(1:k)} = \boldsymbol{K}\boldsymbol{X}_{t-1|t-1}^{(0:k)} + \boldsymbol{F}\boldsymbol{\epsilon}_t.$$

I determine the coefficients of the state $\boldsymbol{K}$ and $\boldsymbol{F}$ by comparing the conjectured behavior of the hierarchy $\boldsymbol{X}_{t|t}^{(1:k)}$ to (12):

$$\boldsymbol{K} = \left[\left(\boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{M}\right)_{(3:(3k+2),3:5)} \left(\left(\boldsymbol{M}-\boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{M}\right)_{(3:(3k+2),3:(3k+2))} + \left(\boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{M}\right)_{(3:(3k+2),6:(3k+5))}\right) \right], \tag{13}$$

$$\boldsymbol{F} = \left(\boldsymbol{P}\boldsymbol{D}'\boldsymbol{E}^{-1}\boldsymbol{D}\boldsymbol{N}\right)_{(3:(3k+2),1:3)}. \tag{14}$$

3.4 Policy Functions

To determine the policy function of output y_t, I insert the conjectured solution into the dynamic IS curve (1). The policy vector $\boldsymbol{H_y}$ is determined by:

$$\boldsymbol{H_y} = \boldsymbol{H_y} \begin{bmatrix} \rho_a & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & \rho_m & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & \rho_d & \boldsymbol{0_{1\times 3k}} \\ & & \boldsymbol{K} & \end{bmatrix} - \frac{1}{\gamma}\boldsymbol{H_r} + \frac{1}{\gamma}\boldsymbol{H_\pi} \begin{bmatrix} \rho_a & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & \rho_m & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & \rho_d & \boldsymbol{0_{1\times 3k}} \\ & & \boldsymbol{K} & \end{bmatrix} + \frac{1-\rho_d}{\gamma}\boldsymbol{1_3^X}, \quad (15)$$

where $\boldsymbol{1_3^X}$ denotes a row unit vector which has the same length as the hierarchy $\boldsymbol{X_{t|t}^{(0:k)}}$ and whose third element equals one.

The interest-rate rule of the central bank concretizes the undetermined policy vector of the nominal interest rate r_t. If I express the interest-rate rule (3) in terms of the conjectured solution, I receive the following condition for the policy vector $\boldsymbol{H_r}$:

$$\begin{aligned} \boldsymbol{H_r} = \phi_\pi \boldsymbol{H_\pi} & \begin{bmatrix} \rho_a & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & \rho_m & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & \rho_d & \boldsymbol{0_{1\times 3k}} \\ & & \boldsymbol{K} & \end{bmatrix} \boldsymbol{T_X} + \phi_y \boldsymbol{H_y} \begin{bmatrix} \rho_a & 0 & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & \rho_m & 0 & \boldsymbol{0_{1\times 3k}} \\ 0 & 0 & \rho_d & \boldsymbol{0_{1\times 3k}} \\ & & \boldsymbol{K} & \end{bmatrix} \boldsymbol{T_X} \\ & - \phi_y \frac{1+\varphi}{\gamma+\varphi}\rho_a \boldsymbol{1_4^X} + \boldsymbol{1_2^X}. \end{aligned} \quad (16)$$

The symbols $\boldsymbol{1_2^X}$ and $\boldsymbol{1_4^X}$ represent row unit vectors that have the same length as the hierarchy $\boldsymbol{X_{t|t}^{(0:k)}}$; their second and fourth element respectively equal one.

Papers that impose noisy information and Calvo pricing on the model economy always derive the imperfect-common-knowledge Phillips curve (Nimark, 2008; Melosi, 2017). The papers do so for a straightforward reason. By a series of substitutions, the derivation of the Phillips curve eliminates prices from the conditions that characterize the economy. The models with Calvo pricing and imperfect common knowledge can then be expressed in three endogenous variables: aggregate output, aggregate inflation, and the nominal interest rate. Under the assumption of Calvo pricing, this solution strategy succeeds because inflation in the Phillips curve depends on higher-order expectations of future inflation and on higher-order expectations of today's marginal costs. The marginal costs can be rewritten in terms of shocks and aggregate output.

However, the solution strategy applied by models with Calvo pricing fails under the competing Rotemberg price setting. Under the assumption of Rotemberg pricing, inflation in the imperfect-common-knowledge Phillips curve additionally depends on higher-order expectations of future relative prices (Šauer, 2016). Relative prices cannot be reformulated in terms of aggregate output, aggregate inflation, and the nominal interest rate. Every substitute for a firm-specific relative price contains an endogenous firm-specific variable. Therefore, the solution algorithm under Rotemberg pricing has to explicitly consider prices, in addition to aggregate output, aggregate inflation, and the nominal interest rate.

Because I have to consider prices in my Rotemberg algorithm, I can refrain from working with the imperfect-common-knowledge Phillips curve. Instead, it is now more convenient to focus directly on the first-order condition from which the Phillips curve originates. I can rewrite the first-order condition (2), which specifies the optimal price of the monopolistically-competitive firm j, into:

$$\begin{aligned} p_t(j) = {} & \frac{(1-\beta)\,\Xi\Pi\,(\Pi-1)}{(1+\beta)\,\Xi\Pi+\nu-1} + \frac{\Xi\Pi}{(1+\beta)\,\Xi\Pi+\nu-1} p_{t-1}(j) \\ & - \frac{\nu-1}{(1+\beta)\,\Xi\Pi+\nu-1} a_t(j) - \frac{\nu-1}{(1+\beta)\,\Xi\Pi+\nu-1} \varphi E_t^j a_t \\ & + \frac{\nu-1}{(1+\beta)\,\Xi\Pi+\nu-1} (\gamma+\varphi)\, E_t^j y_t + \frac{\nu-1}{(1+\beta)\,\Xi\Pi+\nu-1} E_t^j p_t \\ & + \frac{\beta\Xi\Pi}{(1+\beta)\,\Xi\Pi+\nu-1} E_t^j p_{t+1}(j). \end{aligned} \tag{17}$$

Next, I conjecture the policy function of the firm j, which formalizes how the firm chooses its price. The policy function translates the state variables of the firm into the choice variable $p_t(j)$:

$$p_t(j) = \iota_p p_{t-1}(j) - \iota_a a_t(j) + \boldsymbol{\iota_\xi} E_t^j \boldsymbol{\xi_t}. \tag{18}$$

The state space of the firm consists of three dimensions. Due to the price-adjustment costs, the firm has to take into account its past price $p_{t-1}(j)$. The firm furthermore decides on the basis of its productivity $a_t(j)$. Finally, the chosen price reflects firm's beliefs about the state of the economy $E_t^j \boldsymbol{\xi_t}$. If I insert the conjectured policy function (18) into the rewritten first-order condition (17), I learn the undetermined coefficients ι_p, ι_a, and $\boldsymbol{\iota_\xi}$.

$$\iota_p = \frac{(1+\beta)\,\Xi\Pi+\nu-1-\sqrt{\left[(1+\beta)\,\Xi\Pi+\nu-1\right]^2-4\beta\Xi^2\Pi^2}}{2\beta\Xi\Pi} \tag{19}$$

$$\iota_a = \frac{\nu-1}{\left[1+\beta\,(1-\iota_p)\right]\Xi\Pi+\nu-1} \tag{20}$$

$$\begin{aligned} \boldsymbol{\iota_\xi} = {} & \left\{ \frac{(1-\beta)\,\Xi\Pi}{\nu-1} \iota_a \mathbf{1_1^\xi} - \left(\varphi + \frac{\beta\Xi\Pi\rho_a\iota_a}{\nu-1}\right) \iota_a \mathbf{1_3^\xi} + (\gamma+\varphi)\,\iota_a \begin{bmatrix} 0 & 0 & \boldsymbol{H_y} \end{bmatrix} \right. \\ & \left. + \iota_a \begin{bmatrix} 1 & 1 & \boldsymbol{H_p} \end{bmatrix} \right\} \left(\boldsymbol{I}_{[\mathbf{2+3(k+1)}]\times[\mathbf{2+3(k+1)}]} - \frac{\beta\Xi\Pi}{\nu-1} \iota_a \boldsymbol{M} \right)^{-1} \end{aligned} \tag{21}$$

The symbols $\mathbf{1}_{\mathbf{1}}^{\boldsymbol{\xi}}$ and $\mathbf{1}_{\mathbf{3}}^{\boldsymbol{\xi}}$ denote row unit vectors that have the same length as the state vector $\boldsymbol{\xi_t}$ and whose first and third element respectively equal one.

By knowing how the firm-specific price $p_t(j)$ behaves, I can determine the behavior of the aggregate price p_t. After I replace prices in the definition of the aggregate price level (4) by the corresponding policy functions, I get a condition for the policy vector $\boldsymbol{H_p}$:

$$\boldsymbol{H_p} = -\iota_a \mathbf{1}_{\mathbf{1}}^{\boldsymbol{X}} + (\boldsymbol{\iota_\xi})_{\mathbf{(1:1,3:(3}\boldsymbol{k}\mathbf{+5))}} \boldsymbol{T_X}, \tag{22}$$

where $\mathbf{1}_{\mathbf{1}}^{\boldsymbol{X}}$ stands for a row unit vector that has the same length as the hierarchy $\boldsymbol{X}_{\boldsymbol{t|t}}^{\mathbf{(0:}\boldsymbol{k}\mathbf{)}}$ and whose first element equals one. From the definition of the inflation rate (5), it follows that the policy vector of inflation is identical to the policy vector of the aggregate price:

$$\boldsymbol{H_\pi} = \boldsymbol{H_p}. \tag{23}$$

3.5 The Summary of the Algorithm

Algorithm 1 summarizes the solution procedure in eight steps. Calibrated parameters straightforwardly deliver the solution coefficients ι_p and ι_a. All other coefficients require an iterative scheme. One has to update $\boldsymbol{\iota_\xi}$, $\boldsymbol{H_p}$, $\boldsymbol{H_\pi}$, $\boldsymbol{H_r}$, $\boldsymbol{H_y}$, $\boldsymbol{K}$, and $\boldsymbol{F}$ by the determining conditions from Section 3.3 and 3.4 till convergence is reached.

Algorithm 1 Solution Algorithm for Models with Rotemberg Pricing and Imperfect Common Knowledge

Step 1: compute ι_p by (19); compute ι_a by (20)
Step 2: initialize $\boldsymbol{\iota}_{\boldsymbol{\xi}}^0$, $\boldsymbol{H}_{\boldsymbol{p}}^0$, $\boldsymbol{H}_{\boldsymbol{\pi}}^0$, $\boldsymbol{H}_{\boldsymbol{r}}^0$, $\boldsymbol{H}_{\boldsymbol{y}}^0$, $\boldsymbol{K}^0$, $\boldsymbol{F}^0$
Step 3: update $\boldsymbol{\iota}_{\boldsymbol{\xi}}^{l+1}$ by (21); update $\boldsymbol{H}_{\boldsymbol{p}}^{l+1}$ by (22); update $\boldsymbol{H}_{\boldsymbol{\pi}}^{l+1}$ by (23); update $\boldsymbol{H}_{\boldsymbol{r}}^{l+1}$ by (16); update $\boldsymbol{H}_{\boldsymbol{y}}^{l+1}$ by (15)
Step 4: obtain $\boldsymbol{P}^{l+1}$ by solving the fixed-point problem (7); update $\boldsymbol{E}^{l+1}$ by (8)
Step 5: update $\boldsymbol{K}^{l+1}$ by (13); update $\boldsymbol{F}^{l+1}$ by (14)
Step 6: compute $\delta^{l+1} = \max\left\{ \left\|\boldsymbol{\iota}_{\boldsymbol{\xi}}^{l+1} - \boldsymbol{\iota}_{\boldsymbol{\xi}}^{l}\right\|_{\max}, \left\|\boldsymbol{H}_{\boldsymbol{p}}^{l+1} - \boldsymbol{H}_{\boldsymbol{p}}^{l}\right\|_{\max}, \left\|\boldsymbol{H}_{\boldsymbol{\pi}}^{l+1} - \boldsymbol{H}_{\boldsymbol{\pi}}^{l}\right\|_{\max}, \left\|\boldsymbol{H}_{\boldsymbol{r}}^{l+1} - \boldsymbol{H}_{\boldsymbol{r}}^{l}\right\|_{\max}, \left\|\boldsymbol{H}_{\boldsymbol{y}}^{l+1} - \boldsymbol{H}_{\boldsymbol{y}}^{l}\right\|_{\max}, \left\|\boldsymbol{K}^{l+1} - \boldsymbol{K}^{l}\right\|_{\max}, \left\|\boldsymbol{F}^{l+1} - \boldsymbol{F}^{l}\right\|_{\max} \right\}$
Step 7: if $\delta^{l+1} < \sqrt{\text{machine } \varepsilon}$, stop; else repeat steps 3–6
Step 8: construct $\boldsymbol{M}$, $\boldsymbol{N}$, $\boldsymbol{v}$

4 Simulations

I apply the new algorithm to receive the solution of the dynamic beauty contest in which firms suffer from imperfect common knowledge and face the nominal rigidity a la Rotemberg. I carry out several simulations, which offer insights into the workings of this model class. The underlying calibration of the simulation exercises is summarized in Table 1. Because the time periods in the model represent quarters, I calibrate the discount factor β to 0.995. Through the calibration, the felicity function of the representative household becomes logarithmic in consumption ($\gamma = 1$) and linear in labor ($\varphi = 0$). I set the price elasticity ν to six and the parameter of price-adjustment costs Ξ to twenty. The central bank targets inflation of two percent ($\Pi = 1.005$) and reacts more strongly to inflation expectations than to output-gap expectations ($\phi_\pi = 1.5$, $\phi_y = 0.1$). By all shock processes, I calibrate the persistence to 0.5. The shocks of demand (σ_d, $\tilde{\sigma}_d$) and productivity (σ_a, $\tilde{\sigma}_a$) exhibit standard deviations of one percent. Similarly, the interest-rate shock has an annualized standard deviation of one percentage point ($\sigma_m = 0.0025$). The highest order of expectations k that the simulations consider is the order one hundred.

Group	Symbol	Description	Value
Household	β	discount factor	0.995
	γ	relative risk aversion	1
	φ	inverse of Frisch elasticity	0
Firms	ν	price elasticity	6
	Ξ	price-adjustment costs	20
Central Bank	Π	gross trend inflation	1.005
	ϕ_π	reaction to inflation expectations	1.5
	ϕ_y	reaction to output-gap expectations	0.1
Shocks	ρ_a	persistence of aggregate-productivity process	0.5
	ρ_m	persistence of monetary process	0.5
	ρ_d	persistence of demand process	0.5
	σ_a	s.d. of aggregate-productivity innovation	0.01
	σ_m	s.d. of monetary innovation	0.0025
	σ_d	s.d. of demand innovation	0.01
	$\tilde{\sigma}_a$	s.d. of idiosyncratic-productivity innovation	0.01
	$\tilde{\sigma}_d$	s.d. of demand noise	0.01
Infinite Regress	k	highest order of expectations	100

Table 1: Calibration

4.1 Impulse Responses

Figure 1 depicts impulse responses to a positive aggregate-productivity shock. After the shock hits the economy in quarter 1, the higher aggregate productivity a_t elevates output y_t. On average, firms start experiencing higher firm-specific productivities $a_t(j)$, which limit price increases. Because firms decide to keep their prices low, realized inflation, along with inflation expectations, lies below the trend. The central bank responds to the weaker inflation expectations by cutting the interest rate r_t. Firms then observe the confounding signal of a lower interest rate, which they partly misinterpret. They think that additional factors, not just the higher aggregate productivity, lead the central bank to the observed interest-rate cut. On impact, firms incorrectly believe that the central bank reacts to a negative demand shock and carries out monetary expansion. In the model, the signal of the nominal interest rate generates the Fed information effect—a phenomenon described by Romer and Romer (2000) and Nakamura and Steinsson (2018). Private agents try to gain additional information on the underlying economic fundamentals from the communication and policy actions of the Federal Reserve. However, agents can easily misunderstand the extra piece of information.

In Figure 2, I show how the model economy adjusts to a positive interest-rate shock. The nominal interest rate r_t rises in response to the shock. The monetary tightening induces the representative household to consume less. Weaker consumption results into lower output y_t. After firms observe the interest-rate hike in quarter 1, they immediately nowcast a positive interest-rate shock. However, the nowcasted interest-rate shock is smaller than the actually materialized one. Firms believe the interest-rate shock is not the only explanation for the rise in the nominal interest rate. From the viewpoint of firms, a positive demand shock d_t and a negative aggregate-productivity shock a_t represent additional reasons for the interest-rate hike. Due to this misperception of shocks, firms let their prices grow roughly at the pace of trend inflation. Inflation in quarter 1 hence lies in the neighborhood of trend inflation. In quarter 2, firms learn the aggregate price of quarter 1. The richer information set forces firms to revise their beliefs. Firms realize that the interest-rate shock has to be the main driver behind the higher interest rate. Because firms now believe that a restrictive monetary policy is the main explanation for the observed economic data, they decide to keep their prices relatively low. Inflation consequently lies below the trend for several quarters.

Impulse responses to a positive demand shock are presented in Figure 3. The representative household increases its consumption in reaction to the positive demand shock. The increased consumption spending raises output y_t. From firm-specific signals about the demand shock, firms get the opportunity to detect that a positive demand shock spreads throughout the economy. The belief in a positive demand shock leads firms to increase their prices by more than trend inflation. Furthermore, firms revise their inflation expectations

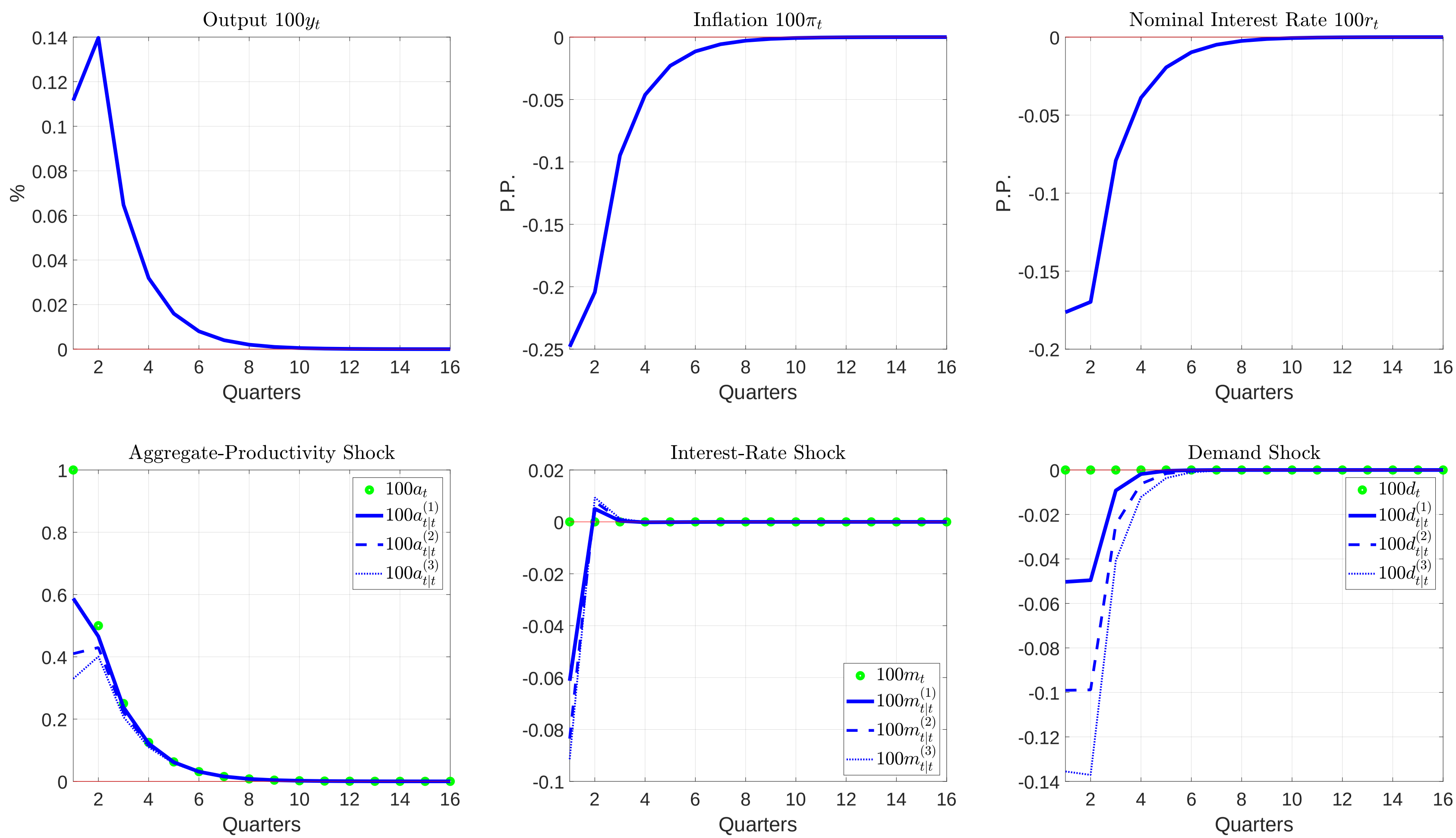

Figure 1: Impulse Responses to a Positive Aggregate-Productivity Shock ($\epsilon^a_1 = 1$)

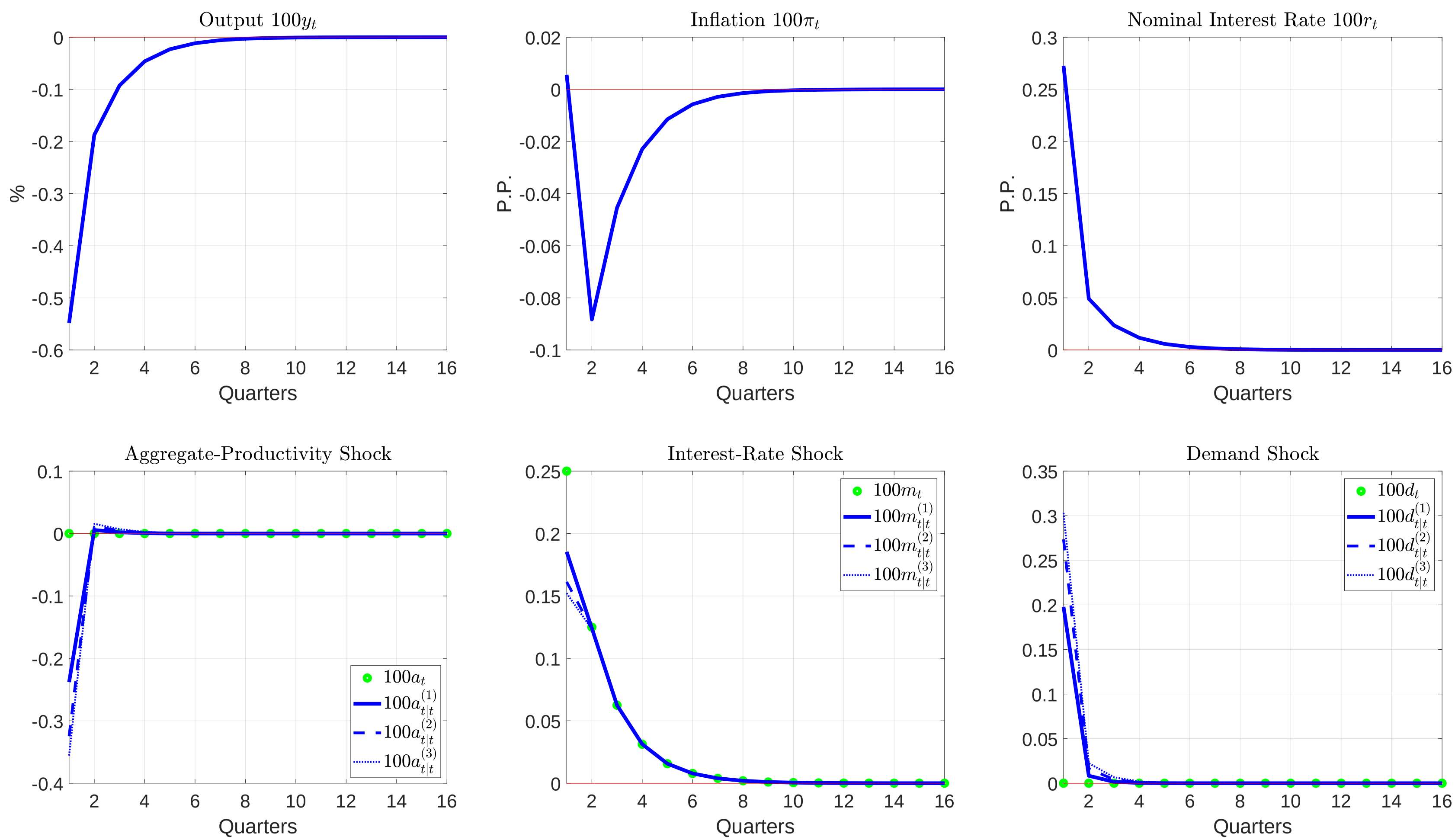

Figure 2: Impulse Responses to a Positive Interest-Rate Shock ($\epsilon^m_1 = 1$)

upward. The central bank responds to the higher inflation expectations by raising the nominal interest rate r_t. The signal of the higher interest rate supports the firms' view that a positive demand shock currently affects the economy. But in addition, the interest-rate signal misleads firms to believing that the economy is also hit by a negative aggregate-productivity shock a_t and a positive interest-rate shock m_t.

4.2 The Decomposition of Inflation

Under the assumption of dispersed information, nominal rigidities imply Phillips curves that substantially differ from the usual New-Keynesian form, which arises under full-information rational expectations. The firms' price-setting condition (17) can be rewritten into the following imperfect-common-knowledge Phillips curve (Šauer, 2016):

$$\begin{aligned}\pi_t = &\frac{(\nu-1)\Pi}{(1+\beta)\Xi\Pi^2+\nu-1}\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1} mc_{t|t}^{(l)}\\ &+\frac{\beta\Xi\Pi^2}{(1+\beta)\Xi\Pi^2+\nu-1}\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1} \pi_{t+1|t}^{(l)}\\ &+\frac{\beta\Xi\Pi^3}{(1+\beta)\Xi\Pi^2+\nu-1}\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1} z_{t+1|t}^{(l)}.\end{aligned}$$

The Rotemberg version of the imperfect-common-knowledge Phillips curve consists of three components. The first term of the Phillips curve—the marginal-costs component—captures higher-order expectations of today's marginal costs:

$$\begin{aligned}mc_{t|t}^{(1)} &= \int_0^1 E_t^j mc_t(j)\,\mathrm{d}j,\\ mc_{t|t}^{(l)} &= \int_0^1 E_t^j mc_{t|t}^{(l-1)}\,\mathrm{d}j \quad \forall l=2,3,\dots k.\end{aligned}$$

The second term of the Phillips curve—the inflation-expectations component—depends on higher-order expectations of future inflation:

$$\begin{aligned}\pi_{t+1|t}^{(1)} &= \int_0^1 E_t^j \pi_{t+1}\,\mathrm{d}j,\\ \pi_{t+1|t}^{(l)} &= \int_0^1 E_t^j \pi_{t+1|t}^{(l-1)}\,\mathrm{d}j \quad \forall l=2,3,\dots k.\end{aligned}$$

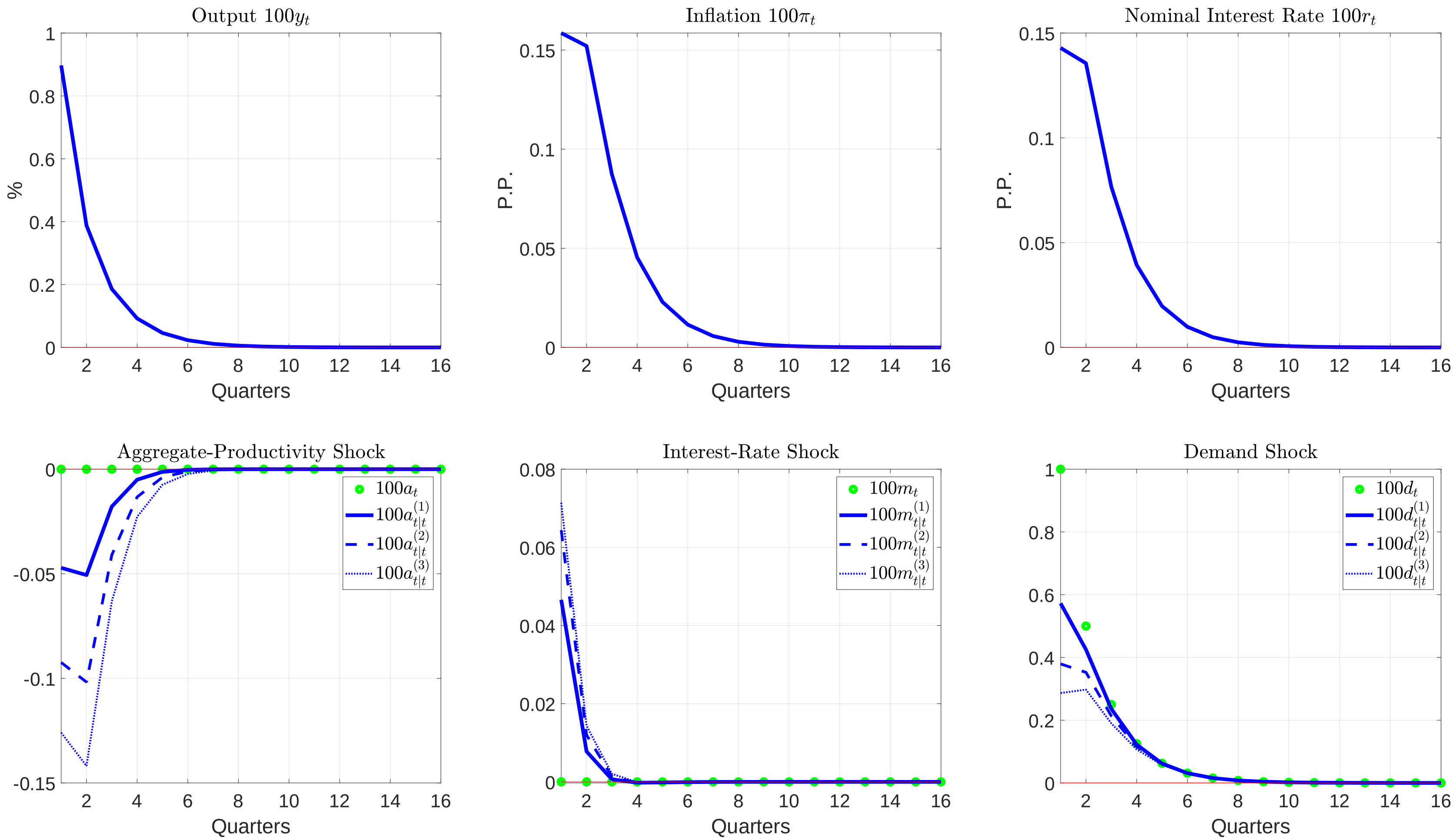

Figure 3: Impulse Responses to a Positive Demand Shock ($\epsilon^d_1 = 1$)

The third term—the relative-prices component—contains higher-order expectations of future relative prices:

$$z^{(1)}_{t+1|t} = \int_0^1 E^j_t z_{t+1}(j)\,\mathrm{d}j,$$
$$z^{(l)}_{t+1|t} = \int_0^1 E^j_t z^{(l-1)}_{t+1|t}\,\mathrm{d}j \quad \forall l = 2, 3, \ldots k.$$

The three components of the Phillips curve can be expressed in terms of the matrices that characterize the solution of the model (Appendix A). After I obtain the solution of the model, I quantify each component of the imperfect-common-knowledge Phillips curve. In Figure 4, I simulate one hundred quarters of inflation and decompose it according to the logic of the Phillips curve. The figure makes clear that the main contributions come from the marginal-costs component and the inflation-expectations component. In contrast, the relative-prices component contributes minimally to inflation.

I repeat the simulation exercise of Figure 4 ten thousand times. For each simulated sample of one hundred quarters, I compute average absolute contributions of the three Phillips-curve components. Figure 5 plots the resulting distributions of the average absolute contributions. In absolute terms, the marginal-costs component and the inflation-expectations component contribute each on average by around 0.15 percentage points to inflation $100\pi_t$. By comparison, the average absolute contribution of the relative-prices component lies just by around 0.04 percentage points.

5 Conclusion

The paper presented a solution algorithm for a class of macroeconomic models that abandon the usual assumption of full-information rational expectations. The algorithm is designed for models in which firms cope with Rotemberg pricing and imperfect common knowledge. Under the concept of imperfect common knowledge, Rotemberg pricing forces the solution algorithm to take prices explicitly into account. The state vector therefore contains not only the hierarchy of average higher-order expectations but also the aggregate price. The algorithm has to explicitly determine the policy function of the aggregate price as well as the policy function of the firm-specific price.

References

Andrade, Philippe, Richard K. Crump, Stefano Eusepi, and Emanuel Moench.

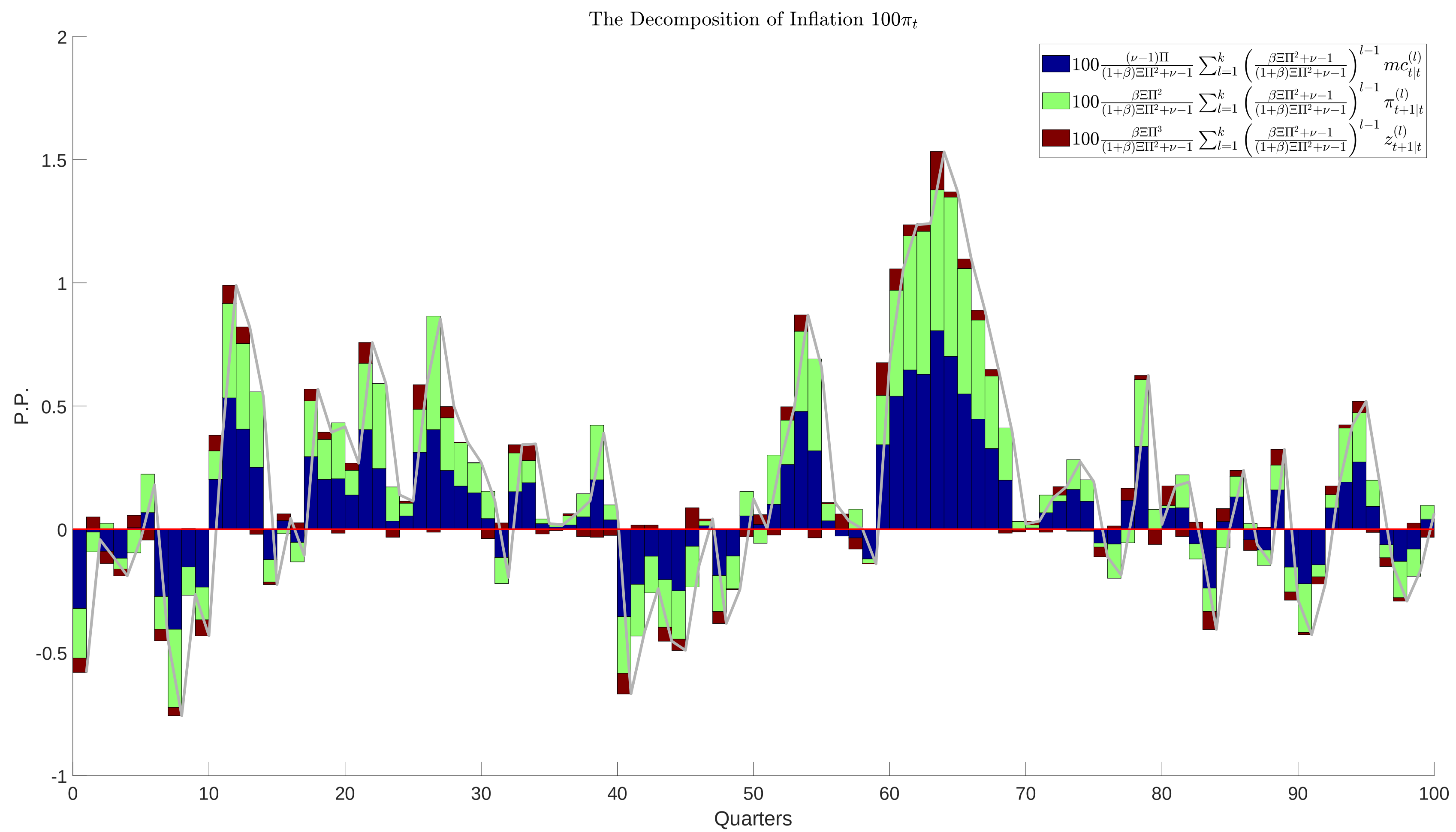

Figure 4: Simulated Inflation $100\pi_t$. Inflation (gray line) is decomposed according to the logic of the imperfect-common-knowledge Phillips curve into three components: the marginal-costs component (blue bars), the inflation-expectations component (green bars), and the relative-prices component (brown bars).

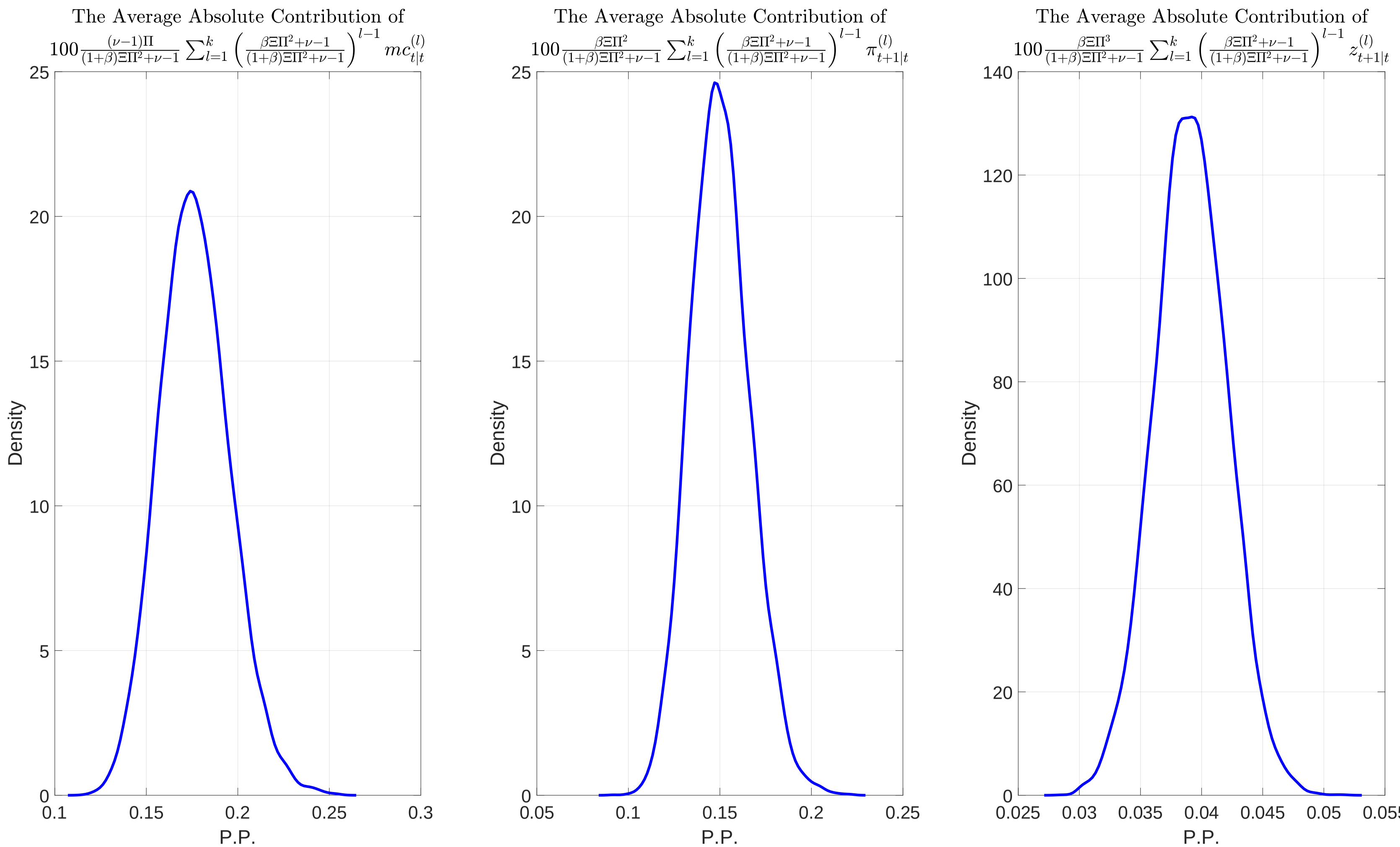

Figure 5: Average Absolute Contributions of Phillips-Curve Components. The first subplot depicts the distribution of the average absolute contribution of the marginal-costs component; the second subplot depicts the distribution of the average absolute contribution of the inflation-expectations component; the third subplot depicts the distribution of the average absolute contribution of the relative-prices component. The distributions are based on 10,000 simulated samples; each sample consists of 100 quarters.

2016. “Fundamental disagreement.” *Journal of Monetary Economics*, 83: 106–128.

Angeletos, George-Marios, and Chen Lian. 2018. “Forward Guidance without Common Knowledge.” *American Economic Review*, 108(9): 2477–2512.

Bernanke, Ben S. 2007. “Inflation Expectations and Inflation Forecasting.” *Speech at the Monetary Economics Workshop of the NBER Summer Institute*, `https://www.federalreserve.gov/newsevents/speech/Bernanke20070710a.htm`.

Calvo, Guillermo A. 1983. “Staggered prices in a utility-maximizing framework.” *Journal of Monetary Economics*, 12(3): 383–398.

Candian, Giacomo. 2019. “Information frictions and real exchange rate dynamics.” *Journal of International Economics*, 116: 189–205.

Christiano, Lawrence J. 2002. “Solving Dynamic Equilibrium Models by a Method of Undetermined Coefficients.” *Computational Economics*, 20(1-2): 21–55.

Coibion, Olivier, and Yuriy Gorodnichenko. 2012. “What Can Survey Forecasts Tell Us about Information Rigidities?” *Journal of Political Economy*, 120(1): 116–159.

Coibion, Olivier, and Yuriy Gorodnichenko. 2015. “Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts.” *American Economic Review*, 105(8): 2644–2678.

Coibion, Olivier, Yuriy Gorodnichenko, and Rupal Kamdar. 2018. “The Formation of Expectations, Inflation, and the Phillips Curve.” *Journal of Economic Literature*, 56(4): 1447–1491.

Coibion, Olivier, Yuriy Gorodnichenko, Saten Kumar, and Jane Ryngaert. 2018*a*. “Do You Know That I Know That You Know...? Higher-Order Beliefs in Survey Data.” National Bureau of Economic Research, Inc NBER Working Papers 24987.

Coibion, Olivier, Yuriy Gorodnichenko, Saten Kumar, and Mathieu Pedemonte. 2018*b*. “Inflation Expectations as a Policy Tool?” National Bureau of Economic Research, Inc NBER Working Papers 24788.

Hamilton, James D. 1994. “Time Series Analysis.” Chapter 13, 372–408. Princeton University Press.

Melosi, Leonardo. 2017. “Signalling Effects of Monetary Policy.” *Review of Economic Studies*, 84(2): 853–884.

Muth, John F. 1961. "Rational Expectations and the Theory of Price Movements." *Econometrica*, 29(3): 315–335.

Nakamura, Emi, and Jón Steinsson. 2018. "High-Frequency Identification of Monetary Non-Neutrality: The Information Effect." *The Quarterly Journal of Economics*, 133(3): 1283–1330.

Nimark, Kristoffer. 2008. "Dynamic pricing and imperfect common knowledge." *Journal of Monetary Economics*, 55(2): 365–382.

Nimark, Kristoffer P. 2017. "Dynamic Higher Order Expectations." `http://www.kris-nimark.net/pdf/papers/DHOEv3.pdf`.

Paciello, Luigi, and Mirko Wiederholt. 2014. "Exogenous Information, Endogenous Information, and Optimal Monetary Policy." *Review of Economic Studies*, 81(1): 356–388.

Romer, Christina D., and David H. Romer. 2000. "Federal Reserve Information and the Behavior of Interest Rates." *American Economic Review*, 90(3): 429–457.

Rotemberg, Julio J. 1982. "Monopolistic Price Adjustment and Aggregate Output." *Review of Economic Studies*, 49(4): 517–531.

Šauer, Radek. 2016. "The imperfect-common-knowledge Phillips curve: Calvo vs Rotemberg." *Economics Letters*, 148: 45–47.

Townsend, Robert M. 1983. "Forecasting the Forecasts of Others." *Journal of Political Economy*, 91(4): 546–588.

Uhlig, Harald. 2001. "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily." In *Computational Methods for the Study of Dynamic Economies.* , ed. Ramon Marimon and Andrew Scott, 30–61. Oxford University Press.

Woodford, Michael. 2002. "Imperfect Common Knowledge and the Effects of Monetary Policy." In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps.* , ed. Philippe Aghion, Roman Frydman, Joseph Stiglitz and Michael Woodford, 25–58. Princeton University Press.

A Components of the Phillips Curve

Marginal-Costs Component:

$$\frac{(\nu-1)\Pi}{(1+\beta)\Xi\Pi^2+\nu-1}\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1} mc_{t|t}^{(l)} =$$
$$= \frac{(\nu-1)\Pi}{(1+\beta)\Xi\Pi^2+\nu-1}\left[(\gamma+\varphi)\boldsymbol{H_y T_X}-\varphi\mathbf{1_4^X}-\mathbf{1_1^X}\right]\left[\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1}(\boldsymbol{T_X})^{l-1}\right]\boldsymbol{X_{t|t}^{(0:k)}}.$$

Inflation-Expectations Component:

$$\frac{\beta\Xi\Pi^2}{(1+\beta)\Xi\Pi^2+\nu-1}\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1}\pi_{t+1|t}^{(l)} =$$
$$= \frac{\beta\Xi\Pi^2}{(1+\beta)\Xi\Pi^2+\nu-1}\boldsymbol{H_\pi}\begin{bmatrix}\rho_a & 0 & 0 & \mathbf{0_{1\times 3k}}\\ 0 & \rho_m & 0 & \mathbf{0_{1\times 3k}}\\ 0 & 0 & \rho_d & \mathbf{0_{1\times 3k}}\\ & & \boldsymbol{K} & \end{bmatrix}\boldsymbol{T_X}\left[\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1}(\boldsymbol{T_X})^{l-1}\right]\boldsymbol{X_{t|t}^{(0:k)}}.$$

Relative-Prices Component:

$$\frac{\beta\Xi\Pi^3}{(1+\beta)\Xi\Pi^2+\nu-1}\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1} z_{t+1|t}^{(l)} =$$
$$= \frac{\beta\Xi\Pi^3}{(1+\beta)\Xi\Pi^2+\nu-1}\left(\iota_p\begin{bmatrix}1 & 1 & \boldsymbol{H_p}\end{bmatrix}-\iota_a\rho_a\mathbf{1_6^\xi}+\boldsymbol{\iota_\xi M T_\xi}-\begin{bmatrix}1 & 1 & \boldsymbol{H_p}\end{bmatrix}\boldsymbol{M T_\xi}\right)$$
$$\times\left[\sum_{l=1}^{k}\left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\Xi\Pi^2+\nu-1}\right)^{l-1}(\boldsymbol{T_\xi})^{l-1}\right]\boldsymbol{\xi_t}.$$