CESifo WORKING PAPERS

8109
2020

February 2020

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
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CESifo *Working Paper No. 8109*

Race Meets Bargaining in Product Development

Abstract

We introduce a model of product development in a firm. Our model describes the process as a multi-stage contest (i.e., race) with an endogenous length (with one stage or two stages) between two workers. We model the payments to workers from the new product using the normatively appealing Nash bargaining solution (see Nash, 1950). In our model the disagreement payoffs endogenously depend on the contest outcome. More precisely, a bargaining advantage is given to the leading worker in the product development contest. We analytically characterize subgame perfect equilibrium effort levels of workers and describe the conditions under which a full-edged final (as opposed to, say, a prototype) product is developed. Our comparative static analyses reveal economically intuitive insights. Finally, we provide an answer to the firm's problem of optimal incentive provision (considering both collective and individual incentives).

JEL-Codes: C720, C780, D860, O310, O320.

Keywords: product development, contests, Nash bargaining solution, optimal contracts, subgame perfect Nash equilibrium, race.

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1 Introduction

Most new product development or product innovation activities in companies or research centers are undertaken by teams rather than single agents (see Levi and Slem, 1995; Sethi, 2000; Sethi et al., 2001; Akgün and Lynn, 2002). It could be argued that the team, as a whole, has a collective interest yet individuals may have competing interests regarding who among them achieves a breakthrough or contributes the most (see Natter et al., 2001; Beersma et al. 2003; Kistruck et al. 2016). Hence, balancing collective and individual incentives in these team environments is a challenge for the firms (see Sarin and Mahajan, 2001; Chang et al., 2007; Garbers and Konradt, 2014; Hutchison-Krupat and Chao, 2014; Lazaric and Raybaut, 2014; Nyberg et al., 2018). Naturally, two important questions arise: (i) to what extent do these incentives influence workers' behavior? and (ii) what are the optimal (collective and individual) incentives that take into account the fairness aspect? To answer such questions and alike, it is of interest to theoretically study these environments with rich models that encompass both competitive and cooperative aspects of the interaction.

In this paper, we model product development by two workers in a firm.[1] We describe this process as a multi-stage contest (i.e., *race*) between the workers assigned to the corresponding product development project. Our model incorporates the payments to workers as an outcome of a cooperative bargaining problem. The *fairness* aspect is introduced by utilizing the normatively appealing Nash bargaining solution (see Nash, 1950).[2] Moreover, the disagreement point in the bargaining problem we study is not exogenously given but rather depends on workers' performances in the race. More precisely, the better-performing worker receives an advantageous bargaining position, a la *Lockean desert*. Finally, in contrast to the *industry standard* in the literature on multi-stage contest games, our product development race does not have an exogenously fixed length. Instead, two workers need to unanimously reach an agreement on the length of the race. A direct consequence is that they may end up building only a prototype or a full-fledged final product depending on their productivity levels, the values of the prototype and the final product, the incentives provided by the firm, and the team-decision rule. We also study optimal incentive

[1] Throughout the paper, we use the words *agents* and *workers* interchangeably.

[2] Fairness and the Nash bargaining solution are almost synonymously used in empirical and applied work due to the appealing normative properties of this solution. We follow that tradition here and argue that Nash bargaining solution guarantees that the division of contest prizes is done in a *fair* fashion.

provision by the firm, which would influence workers' decisions regarding the race length.[3]

We first solve for the subgame perfect equilibrium effort levels, expected payoffs, and *stop or continue* decisions of agents. The equilibrium analyses point toward two critical values for the difference between the returns from the first stage and second stage contests. Given that the full-fledged final product is more valuable than the prototype, if the difference between their values is sufficiently large, then both agents decide to continue to develop the final product; if the difference is sufficiently small, then both agents decide to stop at the end of the first stage contest and hence only develop the prototype, and finally if the difference is neither too large nor too small, one agent decides to continue whereas the other decides to stop. In the last case, whether they continue or not depends on the particular team-decision rule (i.e., "both should agree to stop" or "both should agree to continue") employed. This last case emphasizes the importance of preference aggregation or decision-making process in teams.

Second, we conduct comparative static analyses on model parameters. Our analyses, again, offer economically intuitive results: (i) an increase in the difference between the values of the final product and the prototype enlarges the set of parameter values for which the final product is developed; (ii) an increase in the bargaining power given to the winner of the first (or, the second) stage contest enlarges the set of parameter values for which the winner/loser in the first stage contest prefers to stop/continue; (iii) an increase in the bargaining power given to the winner of the first stage contest for sharing the prototype information enlarges the set of parameter values for which the loser/winner in the first stage contest prefers to stop/continue; and finally (iv) an increase in own marginal cost of effort shrinks the set of parameter values for which the agent prefers to continue after the first stage contest, whereas an increase in the other agent's marginal cost of effort leads to an opposite effect.

Finally, we study the optimal incentive provision problem of the firm. The equilibrium analysis is conducted for all possible values of contest prizes and bargaining powers given to agents. In this part, we consider a firm management optimally choosing these incentives to maximize its own revenue. We first show that there exists a set of parameter values for which the firm prefers agents to stop after the first stage contest and hence only develop the prototype. In this case, the firm sets the winning prize for the first stage contest equal to the disagreement point of the winning agent

[3] Since we explain all details of the model in Section 2, we intentionally refrain from doing it here to avoid repetition.

and also sets the respective losing prize to zero. We derive the optimal value of the winning prize as a function of marginal costs of effort. Second, there also exist parameter values for which the firm prefers agents to continue to the second stage contest, thereby developing the final product. Our analysis shows that the firm should set the bargaining advantage given for sharing the prototype information to its minimum possible level and set the value of the prize for the second stage contest equal to the sum of the two disagreement points (i.e., assigned for the contest victory and for information sharing).

The organization of the paper is as follows. In Section 2, we introduce the model of product development in a firm as a race with endogenous length. In Section 3, we present our equilibrium analysis, comparative static analyses, and results on optimal incentive provision. In Section 4, we discuss our modeling assumptions and possible future research questions, and we conclude.

2 The Model

Consider two agents, denoted by $i \in \{1, 2\}$, working in the same firm and competing in a multi-stage innovation contest. There are potentially two stages. The first stage can be thought of as developing a prototype or an unfinished product, which still has some market value. The second stage can be thought of as developing a full-fledged final product, which naturally has a higher market value than the unfinished one. Below, we describe the sequence of events in these two stages in detail.

In the first stage, agents compete in a one-shot contest game such that the winner gains an advantageous position. One possible interpretation is that the winner of the first stage contest achieved a breakthrough that leads to a prototype for a new product. In this contest, each agent chooses an effort level, denoted by x for agent 1 and y for agent 2, and the winner is determined by a standard Tullock contest success function (see Tullock, 1980):

$$p_1 = \frac{x}{x+y} \qquad \text{and} \qquad p_2 = \frac{y}{x+y}.$$

As usual, contest efforts are assumed to be costly and irreversible: $C_i(a) = c_i a$ for each $i \in \{1, 2\}$ and $a \in [0, \infty)$.[4] The total compensation for workers' efforts in the development of the prototype

[4] Note that once agents exert efforts in a product development contest, then the corresponding product is developed

is $V_p \geq 0$, and it will be allocated by the firm using the Nash bargaining solution (see Nash, 1950). Noting that a higher disagreement payoff is a source of bargaining power in the Nash bargaining solution, in our model the winner of the first stage contest is given a more advantageous disagreement point, $d_r \geq 0$, whereas the loser's disagreement point is normalized to 0.

The game does not necessarily end after the first stage: agents have an option to continue working on the project to develop the final version of the product. In other words, our innovation contest model is one with an *endogenous length*. Each agent individually decides whether to stop or continue, and the *unanimity rule* determines whether they will or not. In Section 3, we consider both cases for this rule separately: "both agents should agree to stop, otherwise they will continue" or "both agents should agree to continue, otherwise they will stop".

If agents jointly decide to continue, they forfeit their right of earning V_p and proceed to the second stage in which there is another one-shot contest game. If that happens, it is assumed that all information regarding the prototype built by the winner of the first stage contest is shared with the other agent. In the second stage, agents collect a total earning of $V_f \geq V_p$ (again, to be allocated by the firm), capturing the fact that the full-fledged final product has a value greater than the prototype's. The contest structure and the sharing rule are assumed to be the same. Similarly, the winner in the second stage obtains an advantageous disagreement point, $d_r \geq 0$, whereas the loser's disagreement point is normalized to 0. However, now the winner of the first stage contest is additionally given an advantage in return for the prototype information shared, $d_s \in [0, d_r]$.

In line with the Nash's axiomatic model of bargaining, we assume that $d_r \leq V_p$ and $d_r + d_s \leq V_f$. These imply that the possible disagreement points are elements of the bargaining set. Note that the formal definition of the Nash bargaining solution will be provided in Section 3.1 below.

Our innovation contest model formulated above can be classified as a *race* (see Figure 1)[5] (see Harris and Vickers, 1985, 1987; Klumpp and Polborn, 2006; Konrad and Kovenock, 2009; Doğan et al., 2018), but with a number of important differences: (i) We present an outside option at the end of the first stage: agents can choose to leave the game at nodes (0,1) and (1,0) in the figure. This leads to an endogenously and strategically determined winning threshold in a

for sure. In reality, whether a product is developed or not may be a stochastic function of team's efforts. Since such a stochastic process is not the focus of this paper, here we assume a deterministic process for the sake of simplicity.

[5]This is a standard illustration for a race model in the contest theory literature. A node represents the number of battle victories each agent has, for instance, at node (m, n), agent 1 has m victories and agent 2 has n victories. Two battle victories collected by an agent would move the game to a terminal node in which the agent wins the race.

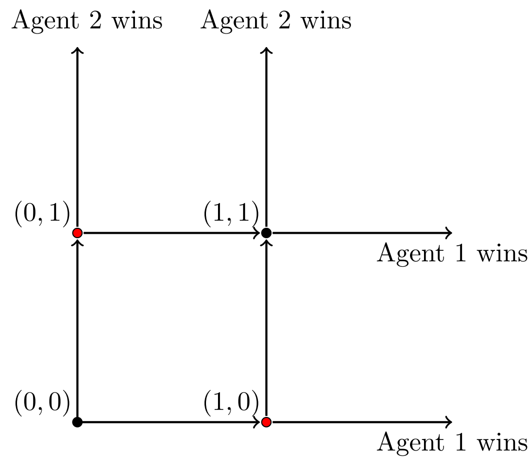

Figure 1: Product Development Race

race. (ii) We incorporate state-dependent winning prizes, which are determined in a cooperative bargaining model that takes the contest outcome as an input—something not observed in earlier work on contest theory. (iii) With the current structure, it is as if we assume that there is a pre-specified jump from nodes (0,1) and (1,0) to node (1,1) in the figure. An interpretation is that the organizational environment makes the contest winner in the first stage share all the information regarding the successfully-built prototype with the losing side. This is relevant in our setting, since agents work in the same firm and it is in firm management's interest to make any such information common knowledge among agents because spending further resources for an already existing prototype would be rather inefficient (see Foss et al., 2010; Hu and Randel, 2014).

3 The Results

This section is divided in to three subsections. In the first part, we present the equilibrium analyis of the extensive form game formulated in Section 2. In the second part, we conduct comparative static analyses on $V_f - V_p$, d_r, d_s, c_1, and c_2. Finally, in the third part, we investigate optimal incentives (from the perspective of the firm, which aims to maximize its net earnings) that must be provided to workers.

3.1 Equilibrium Analysis

The extensive form game presented in Section 2 is a finite-horizon game with complete information. Therefore, we can use backward induction to find the subgame perfect Nash equilibrium of the game. Along those lines, we first analyze a generic Nash bargaining problem. Suppose that two agents with linear utility functions share a pie of size $V > 0$ with disagreement points $d_1, d_2 \geq 0$ satisfying $d_1 + d_2 \leq V$. The standard Nash bargaining solution (see Nash, 1950), which is given by

$$\arg \max_{a_1 \in [0,V]} \; (a_1 - d_1)((V - a_1) - d_2),$$

yields

$$a_1^* = \frac{V + d_1 - d_2}{2} \qquad \text{and} \qquad a_2^* = \frac{V + d_2 - d_1}{2}.$$

Now, utilizing this generic solution, we apply backward induction below. Consider node $(1, 1)$, which can only be reached if agents decided to continue developing the product. Without loss of generality, assume that it was agent 1 who managed to develop the prototype in the first stage. This means that the race proceeded from node $(1, 0)$ to node $(1, 1)$. Recall that this brings an additional term to the disagreement point of agent 1, who then aims to maximize

$$\frac{x_{1,0}}{x_{1,0} + y_{1,0}} \frac{V_f + d_r + d_s}{2} + \frac{y_{1,0}}{x_{1,0} + y_{1,0}} \frac{V_f - d_r + d_s}{2} - c_1 x_{1,0}$$

where $x_{1,0}$ and $y_{1,0}$ denote the respective efforts exerted by the agents. Similarly, agent 2 aims to maximize

$$\frac{x_{1,0}}{x_{1,0} + y_{1,0}} \frac{V_f - d_r - d_s}{2} + \frac{y_{1,0}}{x_{1,0} + y_{1,0}} \frac{V_f + d_r - d_s}{2} - c_2 y_{1,0}$$

After taking the first order conditions, the equilibrium analysis yields

$$x_{1,0}^* = \frac{c_2 d_r}{(c_1 + c_2)^2} \qquad \text{and} \qquad y_{1,0}^* = \frac{c_1 d_r}{(c_1 + c_2)^2}$$

with the equilibrium expected payoffs of

$$\begin{aligned} EU_{1,0}^{c,1} &= \frac{V_f - d_r + d_s}{2} + \frac{{c_2}^2 d_r}{(c_1+c_2)^2} \\ EU_{1,0}^{c,2} &= \frac{V_f - d_r - d_s}{2} + \frac{{c_1}^2 d_r}{(c_1+c_2)^2} \end{aligned} \tag{1}$$

On the other hand, if agents decided to stop at $(1,0)$, so that the game does not proceed to node $(1,1)$, the agents immediately collect their payoffs:

$$U_{1,0}^{s,1} = \frac{V_p + d_r}{2} \qquad \text{and} \qquad U_{1,0}^{s,2} = \frac{V_p - d_r}{2}. \tag{2}$$

Accordingly, we can write that agent 1 prefers to continue if

$$V_f - V_p > 2d_r - d_s - \frac{2{c_2}^2 d_r}{(c_1+c_2)^2},$$

whereas agent 2 prefers to continue if

$$V_f - V_p > d_s - \frac{2{c_1}^2 d_r}{(c_1+c_2)^2}.$$

It is then easy to see that for sufficiently high values of $V_f - V_p$, both agents prefer to continue; for sufficiently low values of $V_f - V_p$, both agents prefer to stop; and for intermediate values of $V_f - V_p$, either agent 1 or agent 2 prefers to continue, and in such a case, how the unanimity rule is applied would be critical in determining the outcome.

Notice that in the case where it was agent 2 who managed to develop the prototype in the first stage, so that the race follows the path of nodes: $(0,0) \to (0,1) \to (1,1)$, the equilibrium strategies and expected payoffs for both agents can symmetrically be written. Accordingly, we can write that agent 1 prefers to continue if

$$V_f - V_p > d_s - \frac{2{c_2}^2 d_r}{(c_1+c_2)^2}.$$

whereas agent 2 prefers to continue if

$$V_f - V_p > 2d_r - d_s - \frac{2{c_1}^2 d_r}{(c_1+c_2)^2},$$

This completes the analysis of the second stage contest. For the one-shot contest in the first stage, backward induction dictates that we should consider four possibilities: (i) agents would continue no matter who wins, (ii) agents would continue if agent 1 wins, but stop if agent 2 wins, (iii) agents would stop if agent 1 wins, but continue if agent 2 wins, and (iv) agents would stop no matter who wins.

We analyze the equilibrium strategies case by case. In case (i), agents anticipate that they will jointly decide to proceed to the second stage independent of who wins the first stage. Then, at node $(0,0)$, agent 1 aims to maximize

$$\frac{x_{0,0}^{cc}}{x_{0,0}^{cc}+y_{0,0}^{cc}}EU_{1,0}^{c,1}+\frac{y_{0,0}^{cc}}{x_{0,0}^{cc}+y_{0,0}^{cc}}EU_{0,1}^{c,1}-c_1x_{0,0}^{cc}$$

where $x_{0,0}^{cc}$ and $y_{0,0}^{cc}$ denote the respective efforts exerted by the agents in this particular case. Similarly, agent 2 aims to maximize

$$\frac{x_{0,0}^{cc}}{x_{0,0}^{cc}+y_{0,0}^{cc}}EU_{1,0}^{c,2}+\frac{y_{0,0}^{cc}}{x_{0,0}^{cc}+y_{0,0}^{cc}}EU_{0,1}^{c,2}-c_2y_{0,0}^{cc}$$

The equilibrium analysis yields

$$x_{0,0}^{cc*}=\frac{c_2d_s}{(c_1+c_2)^2}\qquad\text{and}\qquad y_{0,0}^{cc*}=\frac{c_1d_s}{(c_1+c_2)^2}$$

with the equilibrium expected payoffs of

$$\begin{aligned}EU_{0,0}^{cc,1}&=\frac{V_f-d_r-d_s}{2}+\frac{{c_2}^2(d_r+d_s)}{(c_1+c_2)^2}\\EU_{0,0}^{cc,2}&=\frac{V_f-d_r-d_s}{2}+\frac{{c_1}^2(d_r+d_s)}{(c_1+c_2)^2}\end{aligned}\tag{3}$$

In case (ii), agents anticipate that they will proceed to the second stage in case agent 1 wins the first stage and they will stop in case agent 2 wins the first stage. At node $(0,0)$, agent 1 aims to maximize

$$\frac{x_{0,0}^{cs}}{x_{0,0}^{cs}+y_{0,0}^{cs}}EU_{1,0}^{c,1}+\frac{y_{0,0}^{cs}}{x_{0,0}^{cs}+y_{0,0}^{cs}}EU_{0,1}^{s,1}-c_1x_{0,0}^{cs}$$

where $x_{0,0}^{cs}$ and $y_{0,0}^{cs}$ denote the respective efforts exerted by the agents in this particular case.

Similarly, agent 2 aims to maximize

$$\frac{x_{0,0}^{cs}}{x_{0,0}^{cs} + y_{0,0}^{cs}} EU_{1,0}^{c,2} + \frac{y_{0,0}^{cs}}{x_{0,0}^{cs} + y_{0,0}^{cs}} EU_{0,1}^{s,2} - c_2 y_{0,0}^{cs}$$

The equilibrium analysis yields

$$x_{0,0}^{cs*} = \frac{c_2 \left((c_1+c_2)^2(V_f - V_p - d_s) - (2c_2^2 + 4c_1c_2)d_r\right)\left((c_1+c_2)^2(V_f - V_p + d_s) - 2c_2^2 d_r\right)^2}{2(c_1+c_2)^2\left((c_1+c_2)(c_2^2(V_f - V_p + 2d_r + d_s) - c_1^2(V_f - V_p - d_s)) + 2c_1c_2(c_2 d_s + c_1(2d_r + d_s)\right)^2}$$
$$y_{0,0}^{cs*} = \frac{c_1 \left((c_1+c_2)^2(V_f - V_p - d_s) - (2c_2^2 + 4c_1c_2)d_r\right)^2\left((c_1+c_2)^2(V_f - V_p + d_s) - 2c_2^2 d_r\right)}{2(c_1+c_2)^2\left((c_1+c_2)(c_2^2(V_f - V_p + 2d_r + d_s) - c_1^2(V_f - V_p - d_s)) + 2c_1c_2(c_2 d_s + c_1(2d_r + d_s)\right)^2}$$

The equilibrium expected payoffs can be calculated by plugging these equilibrium strategies into the expected payoff functions above.

Noting that case (iii) is symmetric to case (ii), here we omit the respective equilibrium analysis. Finally, in case (iv), agents anticipate that they will jointly decide to stop after developing the prototype independent of who wins the first stage. Then, at node $(0,0)$, agent 1 aims to maximize

$$\frac{x_{0,0}^{ss}}{x_{0,0}^{ss} + y_{0,0}^{ss}} U_{1,0}^{s,1} + \frac{y_{0,0}^{ss}}{x_{0,0}^{ss} + y_{0,0}^{ss}} U_{0,1}^{s,1} - c_1 x_{0,0}^{ss}$$

where $x_{0,0}^{ss}$ and $y_{0,0}^{ss}$ denote the respective efforts exerted by the agents in this particular case. Similarly, agent 2 aims to maximize

$$\frac{x_{0,0}^{ss}}{x_{0,0}^{s} + y_{0,0}^{ss}} U_{1,0}^{s,2} + \frac{y_{0,0}^{ss}}{x_{0,0}^{s} + y_{0,0}^{ss}} U_{0,1}^{s,2} - c_2 y_{0,0}^{ss}$$

The equilibrium analysis yields

$$x_{0,0}^{ss*} = \frac{c_2 d_r}{(c_1+c_2)^2} \qquad \text{and} \qquad y_{0,0}^{ss*} = \frac{c_1 d_r}{(c_1+c_2)^2}$$

with the equilibrium expected payoffs of

$$EU_{0,0}^{ss,1} = \frac{V_p - d_r}{2} + \frac{c_2^2 d_r}{(c_1+c_2)^2} \quad \text{and} \quad EU_{0,0}^{ss,2} = \frac{V_p - d_r}{2} + \frac{c_1^2 d_r}{(c_1+c_2)^2} \tag{4}$$

3.2 Comparative Statics

Here we conduct comparative static analyses on various model parameters.

a) Changes in $V_f - V_p$: As it can be seen above in equilibrium analysis, the difference between the total earnings in two contests directly enters into the inequalities that determine whether agents would want to continue to stage 2 or not. It is clear that an increase in $V_f - V_p$ enlarges the parameter space where the second stage contest is reached and the final product is developed in equilibrium. If $V_f - V_p$ decreases from such a high value under which both agents prefer to continue, then agent $i \in \{1, 2\}$ prefers to stop in case she wins in the first stage before she stops in case of a loss; however, if $V_f - V_p$ decreases to a sufficiently low level, then both agents would want to stop at the end of stage 1 no matter who wins the first stage contest. This is economically intuitive, because a higher difference between the total earnings would make it more worthwhile for both agents to continue.

b) Changes in d_r or d_s: Observing the same inequalities reported for the stop or continue decision, one can see that in the inequality for the contest winner, the sign of the coefficient for d_r is positive and the sign of d_s is negative; whereas the situation is the converse in the inequality for the contest loser. Therefore, an increase in d_r or a decrease in d_s enlarges the set of parameter values under which the winner in the first stage prefers to stop before stage 2 and the loser in the first stage prefers to continue to stage 2. This is economically intuitive, because such changes in these parameters would increase the importance of winning the first stage contest in case of stopping and decrease the importance of the same in case of continuing to stage 2. As a result, it would be better for the contest winner to stop and for the contest loser to continue.

c) Changes in c_1 or c_2: Taking the derivatives of the right-hand-side of each inequality reported for the stop or continue decision with respect to the cost parameters, one can see that in the inequality for agent $i \in \{1, 2\}$, an increase in c_i leads to an increase while an increase in c_{-i} leads to a decrease in the expression. Accordingly, as c_i increases, the set of parameters under which agent i prefers to continue to stage 2 would get smaller. The same is true when c_{-i} decreases. This is economically intuitive, because a higher marginal cost parameter for an agent results in a lower expected return form the contest, so that the agent would opt out of the contest if possible.

3.3 Optimal Incentive Provision

In this section, we study an optimal design problem from the perspective of the firm management who employs the two competing agents in product development. Assume now that contest efforts are productive such that the first stage efforts either influence the revenue generated by the prototype (if the agents stop after stage 1) or the technology to be used in the final version production (if the agents continue to stage 2), whereas the second stage efforts influence the revenue generated by the final version. All revenue is completely collected by the firm management. By specifying three items in the agents' contracts, namely the (V_f, V_p) pair, the (d_r, d_s) pair, and the unanimity rule, the firm aims to maximize its net earnings, calculated as the total revenue collected minus the total winning prize offered to its workers.

In the following analysis, we consider a Cobb-Douglas type revenue function for the firm. In particular, we assume that the revenue generated by the prototype is $A\sqrt{e_w^1}$ where $A > 0$ is the technology level and e_w^1 is the effort exerted by the winner in the first stage. Also, we assume that the revenue generated by the final version is $A_j\sqrt{e_w^2}$ for any $j \in \{0, +\}$ where $A_j > 0$ is the technology level and e_w^2 is the effort exerted by the winner in the second stage. In this context, $A_+ > 0$ represents a positive technology and $A_0 = 0$ represents a zero technology such that the positive technology will be achieved if a given effort threshold is reached in the first stage.

Before starting the optimal design analysis, notice that the firm simply cares about the stop or continue decision, hence cases (ii) and (iii) above, where the two agents would continue if one agent wins in the first stage but they would stop if the other agent wins, can only be sub-optimal. Accordingly, we concentrate on cases (i) and (iv) above. The firm maximizes its revenue in two cases separately: (**a**) maximization after stage 1 (corresponding to case iv) and (**b**) maximization after stage 2 (corresponding to case i).

Our analysis below shows that there exist sets of parameter values under which the firm management prefers each case: (**a**) stopping after stage 1 or (**b**) continuing to stage 2, after achieving a positive technology level in stage 2.[6] The respective conditions will be revealed at the end of the analysis.

Case (a): Given our result in the previous subsection, we know that the winning probability for

[6] Note that staying below the effort threshold and ending up with a zero technology for the final product can never be optimal for the firm management.

agent $i \in \{1,2\}$ is $c_i/(c_1+c_2)$ in the equilibrium. The objective function can be written as

$$\max_{V_p \geq d_r} A\left(\frac{c_2}{c_1+c_2}\sqrt{x_{0,0}^{s*}} + \frac{c_1}{c_1+c_2}\sqrt{y_{0,0}^{s*}}\right) - V_p$$

$$\max_{V_p \geq d_r} A\left(\frac{c_1\sqrt{c_1 d_r} + c_2\sqrt{c_2 d_r}}{(c_1+c_2)^2}\right) - V_p$$

The solution to this problem is trivial: $V_p^* = d_r$. This indicates that the firm chooses an extreme value for the winner's disagreement point, so that it would be as if the Nash solution is not used in the equilibrium. But then one can further maximize over d_r:

$$\max_{d_r \geq 0} \frac{A(c_1\sqrt{c_1} + c_2\sqrt{c_2})}{(c_1+c_2)^2}\sqrt{d_r} - d_r \tag{5}$$

Taking the derivative with respect to d_r and setting it equal to zero yields

$$\frac{A(c_1\sqrt{c_1} + c_2\sqrt{c_2})}{(c_1+c_2)^2}\frac{1}{2\sqrt{d_1}} = 1$$

$$d_r^{s*} = \left[\frac{A(c_1\sqrt{c_1} + c_2\sqrt{c_2})}{2(c_1+c_2)^2}\right]^2 \tag{6}$$

To sum up, if the firm prefers the agents to stop after stage 1, $V_p^* = d_r^{s*}$, which is given by (6), is the optimal winning prize that should be offered to the winner in the first stage, while the loser receives nothing.

Case (b): As mentioned earlier, in the second stage, there exists a threshold level for total exerted effort, such that if it is reached, the firm achieves a positive technology of production, $A_+ > 0$; but if not, it ends up with a zero technology level, $A_0 = 0$. Given that the equilibrium effort for each agent i is given by

$$\frac{c_{-i} d_s}{(c_1+c_2)^2},$$

so that the total equilibrium effort is $d_s/(c_1+c_2)$, let $\delta_{crit} > 0$ be the respective critical level for d_s, which returns the total effort threshold in the equilibrium.

Then, the objective function can be written as

$$\max_{V_f \geq d_r + d_s} A_j \left[\frac{c_2}{c_1+c_2} \left(\frac{c_2}{c_1+c_2} \sqrt{x^*_{1,0}} + \frac{c_1}{c_1+c_2} \sqrt{y^*_{1,0}} \right) + \frac{c_1}{c_1+c_2} \left(\frac{c_2}{c_1+c_2} \sqrt{x^*_{0,1}} + \frac{c_1}{c_1+c_2} \sqrt{y^*_{0,1}} \right) \right] - V_f$$

for any $j \in \{0, +\}$. Since, by symmetry, the terms in the parentheses are equal to each other, the objective function reduces to

$$\max_{V_f \geq d_r + d_s} A_j \left(\frac{c_2}{c_1+c_2} \sqrt{x^*_{1,0}} + \frac{c_1}{c_1+c_2} \sqrt{y^*_{1,0}} \right) - V_f$$

$$\max_{V_f \geq d_r + d_s} A_j \left(\frac{c_1\sqrt{c_1 d_r} + c_2\sqrt{c_2 d_r}}{(c_1+c_2)^2} \right) - V_f$$

This is very similar to the problem in the first stage. We know that the solution is trivial: $V_f^* = d_r + d_s$. But then one can further maximize over d_r and d_s:

$$\max_{d_r \geq 0,\ d_s \geq 0} \frac{A_j(c_1\sqrt{c_1} + c_2\sqrt{c_2})}{(c_1+c_2)^2} \sqrt{d_r} - d_r - d_s \tag{7}$$

This immediately implies that d_s should take its minimum value. Given that zero technology level can never be optimal, since it would always return a zero revenue in the second stage, we can conclude that the firm prefers $d_s^* = \delta_{crit}$, aiming to achieve a technology level of A_+. Then, taking the derivative with respect to d_r and setting it equal to zero yields

$$\frac{A_+(c_1\sqrt{c_1} + c_2\sqrt{c_2})}{(c_1+c_2)^2} \frac{1}{2\sqrt{d_1}} = 1$$

$$d_r^{c*} = \left[\frac{A_+(c_1\sqrt{c_1} + c_2\sqrt{c_2})}{2(c_1+c_2)^2} \right]^2 \tag{8}$$

The optimization analysis above singles out the optimal selections of d_r and d_s values for each possible case: (**a**) and (**b**). The final step is to compare the earnings those optimal selections yield in order to identify which case would be preferred by the firm. Note that the optimal selections in a specific case should be *consistent* in the sense that the equilibrium strategies must lead to that particular case. For instance, if the firm prefers to produce the final version with a positive technology level, then it should be able to incentivize agents towards that direction. If this cannot be achieved under the restriction that the firm made the optimal selections of d_r and d_s in case (**b**), then further analysis would be necessary, which would possibly return a corner solution for d_r.

Now, let $\mathcal{E}_s^*(A)$ be the maximum level of earnings the firm collects after stage 1, which can be calculated by implementing d_r^{s*} in (6) into (5) and setting $V_p = d_r^{s*}$. Similarly, let $\mathcal{E}_c^*(A_+)$ be the maximum level of earnings the firm collects after stage 2 in case of high technology level, which is equivalent to the case when d_r^{c*} in (8), $d_s^* = \delta_{crit}$, and $A_j = A_+$ are implemented into (7) and (V_p, V_f) is set to $(d_r^{c*}, d_r^{c*} + \delta_{crit})$.

To control for consistency, we make the following observations. If it turns out that $\mathcal{E}_s^*(A) > \mathcal{E}_c^*(A_+)$, then the firm would choose the values $V_p = V_f = d_r^{s*}$, $d_r = d_r^{s*}$ and $d_s = 0$. If the firm further sets the unanimity rule in such a way that "both agents should agree to continue, otherwise they will stop", one can see that agents would stop, because the winner in the first stage prefers stopping before moving onto the second stage. On the other hand, if $\mathcal{E}_c^*(A_+) > \mathcal{E}_s^*(A)$, then the firm would choose the values $V_p = d_r^{c*}$, $V_f = d_r^{c*} + \delta_{crit}$, $d_r = d_r^{c*}$ and $d_s = \delta_{crit}$. If the firm further sets the unanimity rule in such a way that "both agents should agree to stop, otherwise they will continue", one can see that agents would continue, since the loser in the first stage prefers moving onto the second stage. This results in a high technology level. With these arguments, we have shown that all optimal selections found above are consistent with the respective cases. This completes the analysis of optimal design problem for the firm.

4 Conclusion

We formulate a model of product development where (i) two agents in a firm compete to receive a more advantageous share of earnings from a new product, (ii) the competition between agents is modeled as a race with an endogenous length (with one stage or two stages), and (iii) the agents' contest payoffs are determined using the Nash bargaining solution with endogenously-determined disagreement points depending on the race outcome. Agents may choose not to proceed to the second stage, which would lead to a semi-developed product, or a prototype, that has a lower value than the full-fledged final product. We solve for the subgame perfect Nash equilibrium of the corresponding sequential game, describe the conditions under which the final product is developed, conduct comparative static analyses on model parameters, and solve the firm management's optimal contract problem.

Our paper contributes to literature on applied industrial organization (more precisely, product development and optimal contract design) and the literature on contest theory. Our contribution to the applied literature is the introduction of a rich yet tractable model of product innovation, which allows us to study (i) cooperation and competition in product development teams, (ii) fair and optimal incentives (both collective and individual) to be given to workers, (iii) knowledge sharing in teams, and (iv) influence of different team decision rules on product development. Our theoretical innovation is threefold: to the best of our knowledge, ours is the first model to study a race with (i) an endogenous length, (ii) state-dependent prizes, and (iii) a cooperative bargaining game embedded in it (i.e., prize determination).

A few words about our modeling assumptions is in order. We model product development process and the corresponding race between agents with two-stages (representing a prototype and a final product). We could have had more than two stages. However, that would have complicated the analysis yet not have brought significantly new insights (on top of what the current model offers). A similar argument is valid regarding a possible extension to a multi-agent setup. Another possible extension would about the form of the effort cost functions. We expect our qualitative results to hold under a more general form of cost functions (e.g., increasing and convex). Finally, in line with Nash (1950), the source of bargaining power comes from disagreement points in our model. Another way of introducing bargaining power is by using the asymmetric Nash bargaining solution, which maximizes $(x - x_d)^{\alpha}(y - y_d)^{\beta}$. The relative values of α and β can be thought of as a measure of power asymmetry. We believe that this way of incorporating bargaining power would likely lead to a less tractable model without bringing significantly new insights.

Finally, we believe that our model can be used in studying further questions related to product development teams. For instance, future research may model the demand side in greater detail by introducing (heterogenous) consumer preferences for the prototoype and for the final product, and explicitly solving for their utility maximization problems. Another potentially fruitful venue could be modeling duopolistic (or, more generally, oligopolistic) competition using the framework we introduced here. Also, the optimal formation of product development teams can be studied to answer questions such as "Should teams be formed with similar or different agents?".

Acknowledgments

Emin Karagözoğlu and Çağrı Sağlam would like to thank the Scientific and Technological Research Council of Turkey (TÜBİTAK) for financial support under grant number 118K239. The usual disclaimers apply.

References

[1] Akgün AE, Lynn GS (2005) Antecedents and consequences of team stability on new product development performance, *Journal of Engineering and Technology Management*, **19**, 263–286.

[2] Beersma B, Hollenbeck JR, Humphrey SE, Moon H, Conlon DE, Ilgen DR (2003) Cooperation, competition, and team performance: toward a contingency approach, *Academy of Management Journal*, **46**, 572–590.

[3] Chang TJ, Yeh SP, Yeh I (2007), The effects of joint reward system in new product development, *International Journal of Manpower*, **28**, 276–297.

[4] Doğan S, Karagözoğlu E, Keskin K, Sağlam Ç (2018) Multi-player race, *Journal of Economic Behavior and Organization*, **149**, 123–136.

[5] Foss NJ, Husted K, Michailova S (2010) Governing knowledge sharing in organizations: Levels of analysis, governance mechanisms, and research directions, *Journal of Management Studies*, **47**, 455–482.

[6] Garbers Y, Konradt U (2014) The effect of financial incentives on performance: a quantitative review of individual and team-based financial incentives, *Journal of Occupational and Organizational Psychology*, **87**, 102–137.

[7] Harris C, Vickers J (1985) Perfect equilibrium in a model of a race, *Review of Economic Studies*, **52**, 193–209.

[8] Harris C, Vickers J (1987) Racing with uncertainty, *Review of Economic Studies*, **54**, 1–21.

[9] Hu L, Randel AE (2014) Knowledge sharing in teams: social capital, extrinsic incentives, and team innovation, *Group and Organization Management*, **39**, 213–243.

[10] Hutchison-Krupat J, Chao RO (2014) Tolerance for failure and incentives for collaborative innovation, *Production and Operations Management*, **23**, 1265–1285.

[11] Kistruck GM, Lount RB, Smith BR, Bergman BJ, Moss TW (2016) Cooperation vs. competition: alternative goal structures for motivating groups in a resource scarce environment, *Academy of Management Journal*, **59**, 1174–1198.

[12] Klumpp T, Polborn MK (2006) Primaries and the New Hampshire effect, *Journal of Public Economics*, **90**, 1073–1114.

[13] Konrad K, Kovenock D (2009) Multi-battle contests, *Games and Economic Behavior*, **66**, 256–274.

[14] Lazaric N, Raybaut A (2014) Do incentive systems spur work motivation of inventors in high tech firms? A group-based perspective, *Journal of Evolutionary Economics*, **24**, 135–157.

[15] Levi D, Slem C (1995) Team work in research and development organizations: The characteristics of successful teams, *International Journal of Industrial Ergonomics*, **16**, 29–42.

[16] Nash JF (1950) The bargaining problem, *Econometrica*, **18**, 155–162.

[17] Natter M, Mild A, Feurstein M, Dorffner G, Taudes A (2001) The effect of incentive schemes and organizational arrangements on the new product development process, *Management Science*, **47**, 1029–1045.

[18] Nyberg MA, Maltarich MA, Abdulsalam DD, Essman SM, Cragun O (2018) Collective pay for performance: a cross-disciplinary review and meta-analysis, *Journal of Management*, **44**, 2433–2472.

[19] Sarin S, Mahajan M (2001) The effect of reward structures on the performance of cross-functional product development teams, *Journal of Marketing*, **65**, 35–53.

[20] Sethi R (2000) New product quality and product development teams, *Journal of Marketing*, **64**, 1–14.

[21] Sethi R, Smith DC, Park CW (2001) Cross-functional product development teams, creativity, and the innovativeness of new consumer products, *Journal of Marketing Research*, **38**, 73–85.

[22] Tullock G (1980) Efficient rent seeking, *Toward a Theory of the Rent-Seeking Society*, Eds: Buchanan JM and Tollison RD, Texas, Texas AM University Press.