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Platform Competition with Multi-Homing on Both Sides: Subsidize or Not?

Abstract

A major result in the study of two-sided platforms is the strategic interdependence between the two sides of the same platform, leading to the implication that a platform can maximize its total profits by subsidizing one of its sides. We show that this result largely depends on assuming that at least one side of the market single-homes. As technology makes joining multiple platforms easier, we increasingly observe that participants on both sides of two-sided platforms multi-home. The case of multi-homing on both sides is mostly ignored in the literature on competition between two-sided platforms. We help fill this gap by developing a model for platform competition in a differentiated setting (a Hoteling line), which is similar to other models in the literature but focuses on the case where at least some agents on each side multi-home. We show that when both sides in a platform market multi-home, the strategic interdependence between the two sides of the same platform will diminish or even disappear. Our analysis suggests that the common strategic advice to subsidize one side in order to maximize total profits may be limited or even incorrect when both sides multi-home, which is an important caveat given the increasing prevalence of multi-homing in platform markets.

JEL-Codes: O330, L110.

Keywords: multi-homing, platforms, two-sided platforms, network effects, platform subsidies.

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1 Introduction

Platforms have been at the center of the recent economics and business literatures on technology-based markets because of their increasing economic importance and their distinctive economic characteristics that can lead to certain results important both for theory and for management practice. A major such result in the two-sided platform literature is that there is interdependence between the two sides served by the same platform, meaning that lowering the price on one side can make the platform more competitive on the other side (without lowering its price there).

The policy implication is that under certain conditions, such as asymmetric network effects or different demand elasticities on each side, a platform may maximize its total profits by subsidizing one side (Rochet and Tirole 2003, Armstrong 2006). In a typical example, it may be optimal for a payment platform like PayPal to subsidize adoption by consumers in order to generate more merchant fees, or for a content distribution platform like Adobe to offer Acrobat Reader to consumers at a zero or even a negative price, in order to maximize its profits from the sale to content creators of the corresponding authoring tools.[1]

While this interdependence between the two sides of a platform has led to a significant management literature and practitioner advice promoting cross-subsidization as a competitive strategy in platform markets, we show in this paper that this interdependence depends on the assumption that at least one side of the platform single-homes, and is reduced or even disappears when both sides of the platform multi-home. This is an important finding because while multi-homing on both sides was uncommon in early platforms, such as operating systems, game consoles or optical disk players, as technology makes joining multiple platforms easier, participants in both sides of two-sided platforms frequently multi-home.

[1] Both the business and the economics literature recognize that subsidies are common in platform businesses (Rochet and Tirole 2006, Eisenmann, Parker and Van Alstyne 2006). A number of papers specifically study the role of subsidies in platofrm competition, under the name of *divide-and-conquer pricing* (Caillaud and Jullien 2003, Jullien 2011)

This is common, for instance, in platforms that can be joined on either side by simply downloading an app, such as ridesharing or food delivery. For platforms with multi-homing on both sides, the interdependence result and cross-subsidization policy implications thus need to be qualified or changed.[2]

The literature on competition between two-sided platforms, going back to Armstrong (2006) and Rochet and Tirole (2006), mostly ignores this case of multi-homing agents on both sides. Most analyses either consider single-homing by participants on both sides of a platform, or allow multi-homing by participants on one side while they impose single-homing on the other side. These models also typically assume full coverage on both sides, i.e. that all agents participate on at least one platform.

The usual argument for not considering multi-homing on both sides of the market is that if one side of the market fully multi-homes, there is no benefit to allowing the other side of the market to also multi-home as all possible pairs of agents could already connect with each other.[3] This argument relies on the assumption that all agents on the multi-homing side do multi-home, and that meeting the same agent for the second time on another platform does not bring any additional benefit. This assumption is limiting, however, if each side of the market only partially multi-homes, in which case multi-homing on both sides can generate new potential connections between the two sides of the market.

In this paper we develop a model for platform competition in a differentiated setting (a Hoteling line), which is similar to the standard models in the literature. However, we focus on equilibria where only some agents on each side multi-home. We show that in that case the strategic interdependence between the two sides of the same platform may be of lesser importance, or even not be present at all, in contrast to the models imposing single-homing

[2]Interestingly, these platforms frequently take strategic actions to discourage multi-homing, e.g., in the case of ridesharing loyalty rewards or nonlinear pricing such as upfront fees and lower marginal costs on the consumer side, and attempts to lock-in drivers, e.g., with vehicle financing or insurance programs that restrict participation in other platforms.

[3]See, for example, Armstrong (2006), p. 669.

on at least one side of the market. Thus when multi-homing is present on both sides of the market, the benefit of subsidizing one side is diminished or may not be present at all. This result is strongest when the market is fully covered and when connecting two agents that are already connected on a different platform does not create additional benefit.

Our results suggest that when both sides multi-home we need to be wary of overstating the importance of the interdependence between the two sides of a platform even when it does exist, or we risk to potentially offer inappropriate strategic advice.

2 Related Literature

While most of the literature on competition between two-sided platforms assumes single-homing on at least one side, some papers allow for multi-homing on both sides, typically in specialized settings. These papers differ from ours in that they address aspects of platform competition other than the effectiveness of subsidies, which is the focus of our paper.

An early example allowing multi-homing on both sides is Caillaud and Jullien (2003) who consider multi-homing in a matching setting, where multi-homing agents get additional chances at being matched, increasing the probability of successful matching. In another early paper, Doganoglu and Wright (2006) look at multihoming in a one-sided network market with two firms competing on a Hoteling line. They focus on the relation between multihoming and compatibility, exploring whether multihoming is a good substitute for compatibility, and specifically how the ability to multi-home affects prices, firm profits, and firms' incentives to make the two networks compatible. They find that multihoming increases prices, firm profits and social welfare, but it reduces firms' incentives to invest in compatibility. They illustrate how their setting and findings can be extended to two-sided networks; however, they only consider symmetric two-sided networks where firms charge identical prices on both sides, and thus do not address potential cross-subsidization between the two sides.

Choi (2010) and Choi, Jullien and LeFouili (2017) focus on the profitability and optimality of tying (i.e., bundling) the content provided to consumers, modeling competition between two platforms that provide content to spatially differentiated consumers; while the focus of their analysis is on tying, under certain parameters of their setting the resulting equilibria involve multi-homing on both sides of the market.

Ambrus, Calvano and Reisinger (2016) and Anderson, Foros and Kind (2016) analyze platforms in media markets and study the effect of multi-homing by advertisers and consumers on the provision of advertiser-supported content. Neither of these papers looks into the effectiveness of subsidies in maximizing profits. Athey, Calvano and Gans (2018) also model an advertising market, where two publishers connect consumers with advertisers that only value the first impression to a given consumer. In their model, the publishers can subsidize consumers by investing in the quality of free content in order to increase engagement; this subsidization can be profitable because more engaged consumers allow publishers to charge higher prices to advertisers. When both consumers and advertisers multi-home across publishers, the incentive to subsidize content quality is reduced as multi-homing increases, and disappears when all consumers multi-home. The settings of these papers differ from ours (and the standard model of two-sided platform competition along a Hotelling line) as there are no network effects or spatial differentiation.

Belleflamme and Peitz (2017) study how competition between two-sided platforms is shaped by the possibility of multi-homing. They do so by introducing multi-homing on one side into a model where both sides initially single-home. Previous analysis (e.g., Armstrong 2006) suggests that if users on one side can multi-home, platforms exert monopoly power on that side and compete on the single-homing side. Bellaflamme and Peltz find that the result can go either way, possibly benefiting rather than hurting the multi-homing side. They do not address multi-homing on both sides, but they recognize the importance of studying platform competition when both sides multi-home.

Jeitschko and Tremblay (2017) study entry and competition in a two-sided market where the homing decision is endogenized. Their setting differs from the canonical model of platform competition in that while one side (the “consumers”) is spatially differentiated in terms of its realized network benefit from joining a platform, the other side (the ”firms”) realize identical network benefits from either platform. They consider both a monopolist platform and a competitive setting with two platforms. They find a variety of equilibria, ranging from tipping where one platform dominates to competing platforms with a mix of multi-homing and single-homing on both sides of the market. They characterize their cases in terms of social surplus and intensity of competition in the case of more than one platform, and find that social surplus may be maximized under either monopoly or competition. They do not address the profitability of subsidies with or without multihoming in the settings with competing platforms.

Bryan and Gans (2018) model competition between two ride-sharing platforms that can reduce the expected wait time of riders by hiring a certain number of idling drivers. They study the impact of different multi-homing scenarios on market outcome, including the case where both riders and drivers multi-home. They find that in their ride-sharing setting it may be optimal for firms to pay drivers for idling, thus reducing consumer wait times, and the incentive to do so disappears when both sides of the market multi-home. Their model is specific to ride-sharing, with spatial differentiation only on the rider side, and the drivers' wage exogenously determined.

Liu et al (2019) offer an analysis of competition between two-sided platforms, in which buyers and sellers can multi-home, and platforms compete on transaction fees charged on both sides. They study outcomes with multi-homing on both sides, and find that the impact of increased platform competition depends on whether each side is allowed to multi-home. They assume that platform adoption is costless and consumers will join all platforms by default, and thus in their model there is no reason to subsidize platform adoption.

This emerging literature provides an increasing solid theoretical foundation for the importance of considering equilibria in platform competition where both sides multi-home. Given the centrality of the cross-subsidization result in the literature on platform competition, it is thus important to note that such cross-subsidization strategies are not profitable when both sides multi-home.

3 Model Set-Up and Benchmarks

We consider a setting with two types of potential participants (sides), X and Y, which are spatially differentiated and uniformly distributed; specifically $x \sim U[0,1]$ for side X and $y \sim U[0,Y]$ for side Y. We allow Y to be smaller, greater or equal to 1. There is two-sided Hoteling competition between the two platforms, A and B that are located at the ends of these segments, with A at 0 in both sides and B respectively at 1 and Y. The platforms charge p_i, $i = A, B$ on side X and r_i on side Y, and incur zero marginal cost in serving additional users.

A user located at x on side X (respectively y on Y) receives utility from joining platform $i = A, B$:

$$\begin{aligned} u(x; A) =& A_x + \alpha y_A - p_A - zx \\ u(x; B) =& B_x + \alpha(Y - y_B) - p_B - z(1 - x) \\ u(y; A) =& A_y + \beta x_A - r_A - qy \\ u(y; B) =& B_y + \beta(1 - x_B) - r_B - q(Y - y) \end{aligned} \tag{1}$$

where for platform A a mass of y_A agents participate on side Y and a mass x_A agents participate on side X, while for platform B a mass of $1 - x_B$ agents participate on side X and a mass of $Y - y_B$ agents participate on side Y; α and β is the "network effect" of the other side on side X and Y respectively; A_x, B_x and A_y, B_y are the stand-alone values users on side X and Y obtain from joining the respective platform; and z and q are the respective "transportation cost," i.e., the loss of utility due to preference mis-match or set-up costs. We assume that $qz > \alpha\beta$, i.e., that network effects are weaker than transportation costs, which is a typical assumption in models of competition with network effects on Hoteling line, as these models focus on the effects of differentiation. The utilities from multi-homing will be specified separately within each iteration of the model, since allowing for multihoming on one or both sides will effect these utilities.

In the rest of our analysis we assume that both platforms offer the same stand-alone (intrinsic) value $A_x = B_x = \sigma_x$ and $A_y = B_y = \sigma_y$, as this significantly streamlines the exposition of the benchmark cases. This assumption does not affect our qualitative results, and specifically the absence of subsidies when multi-homing on both sides. We present formulas for the general case in the online appendix.

For instance, an application of this model to gaming platforms (e.g., Xbox vs Playstation) would involve spatially differentiated preferences of consumers (e.g., based on user interface or previous experience with each platform) and game developers (e.g., based on each platform's development toolkits and the prior experience of application developers). If consumers are restricted to purchase one of the two systems then we would have single-homing on the consumer side. If developers can make their games available on one or both platforms, then we would be allowing multi-homing on the developer side, the actual outcome depending on the setting parameters and the resulting equilibria.

Similarly, an application to ride sharing could have the two platforms adopt different levels of idleness for their drivers as in Bryan and Gans (2018), which would affect the expected waiting time for consumers, with both drivers and consumers spatially differentiated based on their opportunity cost for idleness and waiting time respectively. In this case, single-homing agents would only consider one ride sharing platform as drivers or consumers, while multi-homing agents would consider both platforms and endogenously select to join one or both.

3.1 Single-homing Benchmark

We begin by analyzing as a benchmark the case with full coverage and single-homing imposed on both sides, as is typical in the literature on platform competition. In that case $x_A = x_B = \tilde{x}$, s.t. $u(\tilde{x}; A) = u(\tilde{x}; B)$ and similarly $y_A = y_B = \tilde{y}$. The platforms are setting their prices (p_A, r_A) and (p_B, r_B) to maximize their profits, $\Pi_A = p_A\tilde{x} + r_A\tilde{y}$ and $\Pi_B = p_B(1 - \tilde{x}) + r_B(Y - \tilde{y})$, resulting in the equilibrium familiar in the literature[4] with prices $p_A^S = p_B^S = z - Y\beta$, $r_A^S = r_B^S = qY - \alpha$, and allocation $\tilde{x}^S = \frac{1}{2}$, $\tilde{y}^S = \frac{Y}{2}$.

Platforms find it optimal to subsidize one side when $Y\beta > z$ or when $\alpha > qY$ (which is possible without violating $qz > \alpha\beta$).

Illustrative example. For instance, consider a setting with parameters $\alpha = 0.6$, $\beta = 1.5$, $z = 1.4$, $q = 1.6$, $\sigma_x = \sigma_y = 1.6$ and $Y = 1$, and single-homing imposed on both sides. At equilibrium the profit-maximizing prices are $p_A^S = p_B^S = -0.1$ and $r_A^S = r_B^S = 1$, yielding $\tilde{x}^S = \tilde{y}^S = \frac{1}{2}$ and $\Pi_A^S = \Pi_B^S = 0.45$. This example illustrates how subsidizing users on one side can be optimal in an environment where both sides single-home.

3.2 Benchmark with multi-homing on One Side Only

We now examine the second benchmark, with single-homing imposed on side X, multi-homing allowed on side Y, and full coverage of both sides. As before, $\tilde{x}$ is characterized by $u(\tilde{x}, A) = u(\tilde{x}, B)$. A user on side Y who multi-homes, i.e., joins both platforms, obtains utility $u(y; A\&B) = 2\sigma_x + \beta - r_A - r_B - qY = u(y; A) + u(y; B)$. It is preferable for user y to join both platforms when both $u(y; A) > 0$ and $u(y; B) > 0$. Therefore, all users $y < y_A$ join platform A, where y_A is characterized by $u(y_A; A) = 0$. All users $y > y_B$ join platform B, where y_B is characterized by $u(y_B; B) = 0$. Users $y \in (y_B, y_A)$ multi-home.

[4] For this equilibrium to hold, the utilities of the indifferent users on side X and Y need to be positive (to assure full coverage), which is the case if $2\sigma_x > 3z - Y(\alpha + 2\beta)$ and $2\sigma_y > 3qY - (2\alpha + \beta)$. Also platform profits must be nonnegative, which is the case as long as $3z - Y(\alpha + 2\beta) > 0$ and $3qY - (2\alpha + \beta) > 0$.

At equilibrium,[5] the platforms set profit-maximizing prices

$$p_A^M = p_B^M = \frac{4(qz - \alpha\beta) + \beta(\alpha - \beta) - 2\beta\sigma_y}{4q}$$

$$r_A^M = r_B^M = \frac{2\sigma_y - \alpha + \beta}{4}$$

For certain parameter values, these prices are negative for one side.

Illustrative example, continued. For instance, consider the parameter values in our illustrative example from Section 3.1 with single-homing imposed on side X, while side Y can multi-home. At equilibrium, the profit-maximizing prices are $p_A^M = p_B^M = -0.12$ and $r_A^M = r_B^M = 1.025$, yielding $\tilde{x} = \frac{1}{2}$, $y_A^M = 0.83$, $y_B^M = 0.17$ and $\Pi_A^M = \Pi_B^M = 0.79$. For these parameter values, when multi-homing is allowed on side Y, it is optimal to subsidize users on the single-homing side X even more and platform profits increase.

4 Allowing for multi-homing on both sides

We now allow for multi-homing on both sides of a platform. For instance, in the case of ridesharing platforms, a driver may participate in both Uber and Lyft, while a passenger may consider offerings from both Uber and Lyft in selecting a ride. Similarly, some consumers may own both Windows and MacOS computers, while developers often create applications for both operating systems.

Utility when multi-homing. We first characterize the utilities agents get when multi-homing, $u(x; A\&B)$ and $u(y; A\&B)$, in a situation when multi-homing occurs on both sides.

The market coverage of platform A is given by x_A, and $1 - x_B$ is the market coverage of platform B. Multi-homing on side X occurs when $x_A > x_B$. Multi-homing on both sides

[5] This equilibrium exists when $2\sigma_y + \alpha + \beta > 2qY$ and $2(\alpha + \beta)\sigma_y + 4q\sigma_x > 6(qz - \alpha\beta) - (\alpha - \beta)^2$.

occurs when $x_A > x_B$ and $y_A > y_B$. In such a case, a multi-homing agent from side X may meet certain agents from side Y on both platforms, as agents from side Y are multi-homing as well; and vice versa. Note that this issue is unique to the case of multi-homing on both sides. For example, when multi-homing is allowed only on one side, the multi-homing agent meets distinctive agents on the other side on each platform; thus, his multi-homing utility is simply equal to the sum of the utilities from joining each platform.

When meeting on both platforms, the agents may realize no additional benefit from the second meeting — there is "no double counting" of the network benefit. At the other extreme, the benefit received on each platform could be additive — there is "double counting" of the network benefit. In the intermediate case, meeting for the second time may yield partial additional network advantage — "partial double counting" of the network benefit. The same issue arises for the standalone (intrinsic) benefit of the two platforms under multi-homing.

For the base case of our analysis, we assume double counting of the stand-alone intrinsic values, but no double counting of the network effect from overlapping agents.[6] Even though X side users may meet some Y side users on both platforms, they only get the network benefit once. Thus joining both platforms when $y_A \geq y_B$ yields

$$u(x; A\&B) = 2\sigma_x + \alpha Y - p_A - p_B - z,$$

and similarly for $u(y; A\&B)$.

Decision to participate in both platforms. An agent multi-homes when multi-homing yields higher utility than joining only platform A, only platform B, or not joining either of the platforms. Utility of an agent joining A only is given by $u(x; A)$ as in (1). If A were the only platform in the market, uses $x < \bar{x}_A$ would prefer to join A while users $x > \bar{x}_A$ would

[6] Double counting the stand-alone values keeps the setting comparable to the benchmark case of multi-homing on one side. We later comment how double counting and partial double counting of network effects affects our results.

prefer not to join it, where $\bar{x}_A$ is characterized by $u(\bar{x}_A; A) = 0$, i.e.,

$$\bar{x}_A = \frac{\sigma_x + \alpha\, y_A - p_A}{z} \tag{2}$$

That is, $\bar{x}_A$ would be the market captured by platform A if it was the only platform (see Figure 1a).

Similarly, if B would be the only platform in the market, all users $x > \bar{x}_B$ would prefer to join B, while $x < \bar{x}_B$ would not join (see Figure 1b), where

$$\bar{x}_B = 1 - \frac{\sigma_x + \alpha\,(Y - y_B) - p_B}{z}\,.$$

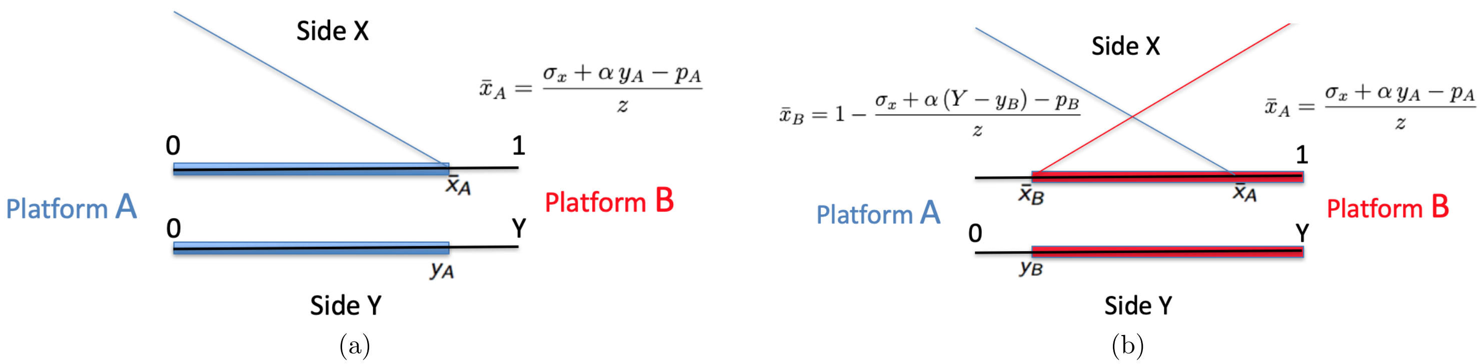

Figure 1: Market coverage if platform A or platform B would be the only platform in the market.

When $\bar{x}_A > \bar{x}_B$, there is potential for multi-homing on side X. Note, however, that if there is multi-homing on both sides, there will be fewer than $\bar{x}_A$ joining platform A. This is because marginal agents will consider joining A while they already participate in B. In such a case, user x's utility from joining A in addition to B is given by

$$u(x; A|B) = u(x; A\&B) - u(x; B)\,. \tag{3}$$

If there is multi-homing on side Y, this incremental utility $u(x; A|B)$ is smaller than $u(x; A)$. That means that some agents who might have joined A if no other platform was available, will not join A as the second platform. I.e., for some x, $u(x; A|B) < 0 < u(x; A)$. Thus, the actual market captured by platform A in the case of multi-homing on both sides is smaller than $\bar{x}_A$.

To characterize the size of the market captured by platform A in the case of multi-homing on both sides, we need to identify the agent who is indifferent between joining A in addition to B, and staying with B only. This agent, denoted by $\hat{x}_A$, is characterized by $u(\hat{x}_A; A|B) = 0$ which is equivalent to $u(\hat{x}_A; A\&B) = u(\hat{x}_A; B)$, i.e.,

$$\hat{x}_A = \frac{\sigma_x + \alpha\, y_B - p_A}{z} \tag{4}$$

In the case of multi-homing on both sides, platform A captures market of size $\hat{x}_A$ on side X. It is straightforward to note that since $y_A > y_B$, then $\hat{x}_A < \bar{x}_A$ (see Figure 2).

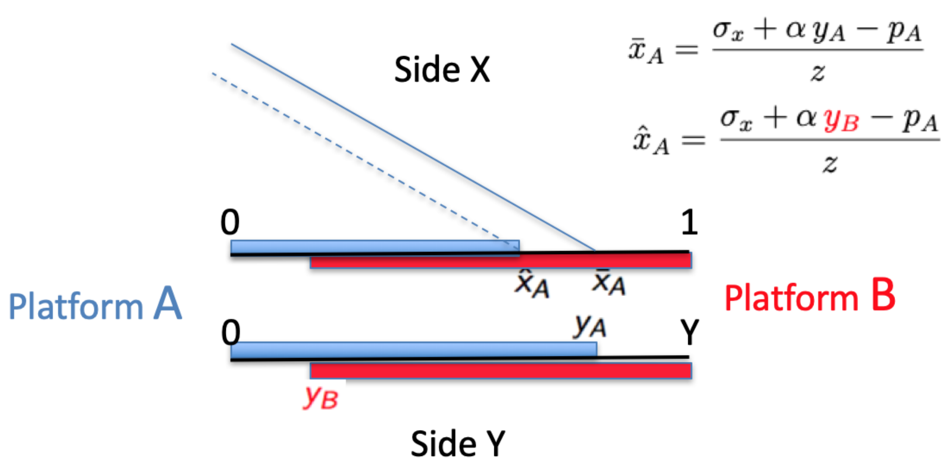

Figure 2: Market coverage of platform A when multi-homing on both sides occurs, $\hat{x}_A$

Notice the difference between formulas (2) and (4). The threshold $\bar{x}_A$ depends on the number of opposite-side agents available on the same platform, y_A. That is, it depends on pricing decision of the same platform A. But $\hat{x}_A$ depends on y_B, which depends on the pricing decision of *the other* platform.

Interestingly, platform A cannot make itself more appealing to agents on side X by increasing the number of Y agents it attracts. In fact, the attractiveness of platform A to agents on side X depends on side Y agents attracted by the other platform; this is because y_B represents the number of side Y agents that are *exclusive* to platform A. As platform B becomes more attractive to side Y agents, the number of such agents joining A exclusively decreases—even if A can increase its overall coverage of side Y. This lowers the attractiveness of joining A for the marginal side X agent, because the marginal side X agent is deciding whether to join A in addition to B, not whether to join either A or no platform at all; and the marginal agent on side X already has access to these side Y agents on platform B.

We can already see the intuition for our main result. To make sure it holds in full equilibrium, we next characterize the equilibrium.

Equilibria with partial multi-homing on both sides. We call *partial multi-homing* a situation where some agents on both sides multi-home, while others single-home. Partial multi-homing on both sides occurs at equilibrium when $0 < \hat{x}_B < \hat{x}_A < 1$ and $0 < \hat{y}_B < \hat{y}_A < 1$ (see Figure 3), where[7]

$$
\begin{aligned}
\hat{x}_A &= \frac{\sigma_x + \alpha \hat{y}_B - p_A}{z} \\
\hat{x}_B &= 1 - \frac{\sigma_x + \alpha (Y - \hat{y}_A) - p_B}{z} \\
\hat{y}_A &= \frac{\sigma_y + \beta \hat{x}_B - r_A}{q} \\
\hat{y}_B &= Y - \frac{\sigma_y + \beta (1 - \hat{x}_A) - r_B}{q}
\end{aligned}
\tag{5}
$$

[7]Formulas in (5) are obtained using the similar derivations as those leading to (4).

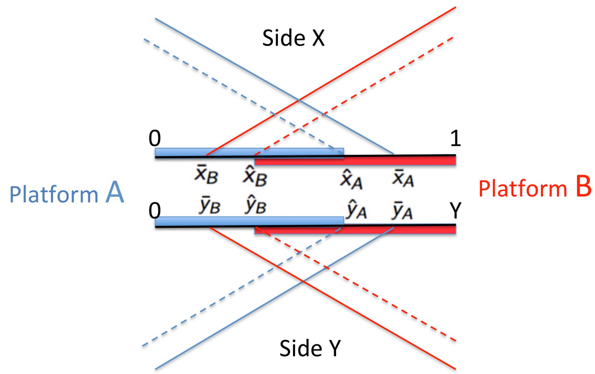

Figure 3: Participation decision with multi-homing on both sides

Lemma 1 *If both sides multi-home and there is no double counting of the network benefits from meeting the same other side agent on both platforms, there is no interdependence of prices on the two sides of the same platform when maximizing profit. I.e., the profit maximizing p_i^* does not depend on r_i^*.*

Proof. From (5) we can see that in equilibrium there is interaction between $\hat{x}_A$ and $\hat{y}_B$ (and therefore between p_A and r_B), but not between $\hat{x}_A$ and $\hat{y}_A$. That is, there is no strategic interaction between pricing on the two sides of the same platform. Formally, it follows from FOC's for profit maximization: r_i does not enter the FOC for maximizing profit of platform i with respect to p_i, and vice versa. ■

Interdependence of prices on the two sides of the same platform is the driver of subsidies in platform pricing; without it there is no incentive to subsidize one side, as according to Lemma 1 and (5) this will not affect the optimal price and quantity on the other side, and thus will not be profitable.

The pure strategy equilibrium with partial multi-homing on both sides is characterized by

$$
\begin{aligned}
\hat{x}_A^{MM} &= \frac{q[(2qz-\alpha\beta)(\sigma_x+\alpha Y)-\alpha z(\sigma_y+\beta)]}{(qz-\alpha\beta)(4qz-\alpha\beta)} \\
\hat{x}_B^{MM} &= \frac{(2qz-\alpha\beta)^2-(2qz-\alpha\beta)q(\sigma_x+Y\alpha)+\alpha qz\sigma_y}{(qz-\alpha\beta)(4qz-\alpha\beta)} \\
\hat{y}_A^{MM} &= \frac{z[(2qz-\alpha\beta)(\sigma_y+\beta)-\beta q(\sigma_x+Y\alpha)]}{(qz-\alpha\beta)(4qz-\alpha\beta)} \\
\hat{y}_B^{MM} &= \frac{Y(2qz-\alpha\beta)^2-(2qz-\alpha\beta)z(\sigma_y+\beta)+\beta qz\sigma_x}{(qz-\alpha\beta)(4qz-\alpha\beta)}
\end{aligned}
\tag{6}
$$

and

$$
\begin{aligned}
p_A^{MM} = p_B^{MM} &= \frac{(2qz-\alpha\beta)(\sigma_x+Y\alpha)-\alpha z(\sigma_y+\beta)}{4qz-\alpha\beta} \\
r_A^{MM} = r_B^{MM} &= \frac{(2qz-\alpha\beta)(\sigma_y+\beta)-\beta q(\sigma_x+Y\alpha)}{4qz-\alpha\beta}
\end{aligned}
$$

Proposition 1 *For range of parameters described by conditions (i)-(vi) below, there exists a pure strategy equilibrium with partial multi-homing on both sides.*

(i) $zq(4zq+\sigma_y\alpha) > q(Y\alpha+\sigma_x)(2qz-\alpha\beta)+\alpha\beta(4qz-\alpha\beta)$

(ii) $q(\sigma_x+Y\alpha)(2qz-\alpha\beta) < (qz-\alpha\beta)(4qz-\alpha\beta)+\alpha qz(\sigma_y+\beta)$

(iii) $zq(4zq+2\sigma_y\alpha+\alpha\beta) < 2q(2qz-\alpha\beta)(\sigma_x+Y\alpha)+\alpha\beta(4qz-\alpha\beta)$

(iv) $Y(2qz-\alpha\beta)^2+qz\beta\sigma_x > z(\sigma_y+\beta)(2qz-\alpha\beta)$

(v) $z(\sigma_y+\beta)(2qz-\alpha\beta) < (qz-\alpha\beta)(4qz-\alpha\beta)+\beta zq(\sigma_x+Y\alpha)$

(vi) $Y(2qz-\alpha\beta)^2+\beta zq(2\sigma_x+Y\alpha) < 2z(2qz-\alpha\beta)(\sigma_y+\beta)$

Proof. Suppose parameters satisfy conditions (i)-(vi). The region of parameters satisfying these conditions is non-empty as, for instance, the parameter values in our illustrating example from sections 3.1 and 3.2 satisfy all these conditions. As shown in the online appendix, these conditions guarantee an equilibrium with positive profits for the platforms and weakly positive utility for the marginal agents. Directly from conditions (i)-(iv) it follows that $\hat{x}_A^{MM}$, $\hat{x}_B^{MM}$, $\hat{y}_A^{MM}$ and $\hat{y}_B^{MM}$ calculated according to (6) are such that $0 < \hat{x}_B^{MM} < \hat{x}_A^{MM} < 1$ and $0 < \hat{y}_B^{MM} < \hat{y}_A^{MM} < 1$. Conditions $\hat{x}_B^{MM} < \hat{x}_A^{MM}$ and $\hat{y}_B^{MM} < \hat{y}_A^{MM}$ indicate multi-homing on both sides. And since all four thresholds are strictly between 0 and 1, the multi-homing is partial. ■

Proposition 2 *In an equilibrium with partial multi-homing on both sides, there are no subsidies, i.e.,* p_A^{MM}, r_A^{MM}, p_B^{MM} *and* r_B^{MM} *are strictly positive.*

Proof. Consider parameters for which conditions (i)-(vi) in Proposition 1 are satisfied. Since the conditions imply $0 < \hat{x}_B^{MM} < \hat{x}_A^{MM} < 1$ and $0 < \hat{y}_B^{MM} < \hat{y}_A^{MM} < 1$, they must also imply $\hat{x}_A^{MM} > 0$, $\hat{x}_B^{MM} < 1$, $\hat{y}_A^{MM} > 0$ and $\hat{y}_B^{MM} < 1$. Direct algebraic manipulations reveal that $\hat{x}_A^{MM} > 0 \iff p_A^{MM} > 0$, $\hat{y}_A^{MM} > 0 \iff r_A^{MM}$, $\hat{x}_B^{MM} < 1 \iff p_B^{MM} > 0$, and $\hat{y}_B^{MM} < 1 \iff r_B^{MM} > 0$. ■

Note that for other parameter values, different pure strategy equilibria are possible. For instance, equilibria may arise where the market on one or both sides is not fully covered, or where single-homing arises endogenously.

Illustrative example, continued. Once again considering the parameter values in the illustrative example from Sections 3.1 and 3.2, if multi-homing is allowed on both sides, users will chose to multi-home, as the profit maximizing prices are $p_A^{MM} = p_B^{MM} = 0.65$ and $r_A^{MM} = r_B^{MM} = 0.72$, yielding $\hat{x}_A^{MM} = 0.78$, $\hat{x}_B^{MM} = 0.22$, $\hat{y}_A^{MM} = 0.75$, $\hat{y}_B^{MM} = 0.25$ and $\Pi_A^{MM} = \Pi_B^{MM} = 1.06$. Compared to the previous settings with single-homing imposed on at least one side, prices are positive on both sides (i.e., there is no subsidy of one side).

5 Discussion and Extensions

Our analysis of platform competition with multi-homing on both sides assumes that participants that overlap on multiple platforms do not derive additional benefit from being able to meet on more than one platform, which we described as having no double counting of the corresponding network effects. Under this assumption, we showed that at equilibrium there is no interdependence between pricing decisions on the two sides by the same platform. This is a striking result when compared with the benchmarks of single-homing on at least one side, where such an interdependence has been a central result in the literature.

As we noted in Section 4, there are many possibilities for how the utility of these overlapping multi-homers is specified. In some markets the platforms may be differentiated enough to provide additional functionality; for instance seeing the same listing on AirBnB and HomeAway may offer incremental value to the prospective renter from accessing more photographs, property information, and additional reviews. Similarly, being exposed to the same prospective renter on both platforms may be valuable for a property, because of an additional chance to make an impression, or the ability to appeal when the consumer is planning business as well as vacation travel. This can be captured by extending our analysis to allow partial double counting of the network effects in the utility of multi-homers so that meeting the same agent on the second platform yields some incremental network benefit above the network benefit from meeting him on the first platform.

The utility of a multi-homing agent x on side X with $y_A - y_B$ agents multi-homing on side Y can thus be specified as

$$u(x; A\&B, \omega) = A_x + B_x + \alpha[Y + \omega(y_A - y_B)] - p_A - p_B - z\,,$$

where $\omega \in [0, 1]$ is the degree of double-counting the network effect. For $\omega = 0$, there is no double counting, as in our previous analysis. When $\omega = 1$, $u(x; A\&B) = u(x, A) + u(x, B)$, i.e., the X side agent gets fully additive benefit from meeting a Y agent on both platforms. That means the two platforms provide different benefits, and thus they are not competing with each other.

By accounting for ω in the analysis from Section 4, we get

$$\hat{x}_A(\omega) = \frac{\sigma_x + \alpha[\omega y_A + (1-\omega)y_B] - p_A}{z},$$

with similar results for side Y; ω thus determines the interdependence between the price platform A sets on side Y, and how attractive it is to agents on side X. If there is some incremental network benefit of meeting the same other-side agent on both networks (i.e., "double counting" the network effect from agents common to the two platforms), the strength of the interdependence between the prices charged on the two sides by the same platform is determined by the strength of this incremental benefit. For small values of ω this interdependence is correspondingly small, and the price set by platform B on side Y is much more important to determining $\hat{x}_A$, platform A's market coverage on the X side, than its own price on side Y, thus reducing A's incentives to subsidize Y.

In our main analysis, we have assumed a market that needs to be fully covered for multi-homing to occur. This assumption allowed us for clear presentation of our result. Spatially differentiated models of two-sided platforms in the literature sometimes add "hinterland" areas of consumers located beyond rather than between the platform locations, thus providing an alternative to positioning the platforms at the ends of the Hotelling segment—see for instance Hagiu and Halaburda (2014). These hinterlands are typically non-competitive in the sense that at equilibrium they are always served by the proximate platform; in our setting they would correspond to $x < 0$ and $x > 1$ on the X side, and $y < 0$ and $y > Y$ on the Y side. In the case of multi-homing on both sides, hinterlands that are not fully covered at equilibrium can reintroduce an interdependence between the prices a platform charges on the two sides, and thus possibly an incentive to subsidize one of these sides. This is because the subsidy could attract new participants from the platform's hinterland, if this hinterland is not fully covered, and these new participants would increase the platform's attractiveness to the other side. However, similarly to the case of partial double counting of network effects, this interdependence between the sides is weaker, as it affects only a small part of the market that the platform covers. And thus the potential subsidy would bring lower benefits to the platform, which significantly weakens the incentive to subsidize.

Our analysis of the setting with multi-homing on both sides established that while under platform competition the interdependence between the two sides plays a key role in environments with single-homing on at least one side, this interdependence is of lesser importance, and may disappear completely, when the platforms are competing in an environment where multi-homing on both sides is possible. In the absence of this interdependence, it is never optimal for the platforms to subsidize one side. This interdependence can be reintroduced when the benefit of meeting the same other-side participant on both platforms is at least partially additive, or when the market is not fully covered despite multi-homing. However, the presence of multi-homing on both sides makes this interdependence weaker, and therefore the benefits from subsidizing smaller or non-existing.

6 Conclusion

In this paper we analyze platform competition when agents multi-home on both sides. This is an increasingly important case as technology makes joining multiple platforms easier, and as a result participants on both sides of two-sided platforms increasingly multi-home. This case of multi-homing on both sides has been mostly ignored in the literature, including the work establishing the central result in platform competition for the pricing interdependence between the two sides of the market, which implies that it may be optimal for a platform to subsidize one side.

We develop a model for platform competition in a differentiated setting (a Hotelling line), which is similar to other models in the literature but focuses on the case where at least some agents on each side multi-home. Once we allow for multi-homing on both sides, it is important to specify what is the utility of the multi-homing users who meet multi-homing users on the other side — that is, they meet each other twice on the two platforms. Do they obtain the benefit of interaction twice, or only once?

In the base model, we analyze the case where participants meeting in both platforms obtain the benefit only once (no-double counting). It is reasonable to assume, for instance, that in online retailing there is little incremental value having a potential buyer see a seller's product listing on eBay, once that same listing has already been seen by that buyer on Amazon marketplace. For this specification, we show that:

– under certain conditions, equilibria exist with multi-homing on both sides

– when we have multi-homing on both sides, the interdependence between the two sides plays out differently than under single-homing; specifically, there is no interdependence between the two sides of the same platform

– optimal pricing for a platform on one side depends on the prices of the other platform only and thus it is never optimal to subsidize the other side (no divide-and-conquer strategy).

These results differ from most of the two-sided platform literature, where interdependence between the two sides served by the same platform is a major result leading to the implication that a platform will often maximize its total profits by subsidizing one side. Thus the common strategic advice to subsidize one side in order to maximize total profits may be limited or even incorrect when both sides multi-home, which can be significant given the increasing prevalence of multi-homing.

While we start with the assumption that meeting the same agent on both platforms brings no incremental benefit compared to meeting on one platform, we also discuss a more general formulation, where the agents can gain partial benefit from meeting each other the second time. In this case, the interdependence is present again. However, the degree of interdependence strongly depends on the size of the benefit from the second meeting. If the incremental benefit is small, the interdependence is also weak and pricing of the other platform is a much more important factor in determining a platform's optimal price than its own pricing on the other side. If, on the other hand, meeting again on the second platform is almost as valuable as meeting for the first time, the strong interdependence between the two sides of the same platform reappears, and the conventional platform pricing strategy advice may apply.

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Platform Competition with Multihoming on Both Sides: Subsidize or Not?

— ONLINE APPENDIX —

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In this appendix, we show that assuming $A_x = B_x$ and $A_y = B_y$ simplifies exposition of the benchmark cases without any loss to the our results.

1 General Model Set-up and Benchmarks

We consider a setting with two types of potential participants (sides), X and Y, which are spatially differentiated and uniformly distributed; specifically $x \sim U[0,1]$ for side X and $y \sim U[0,Y]$ for side Y. We allow Y to be smaller, greater or equal to 1. There is two-sided Hoteling competition between the two platforms, A and B that are located at the ends of these segments, with A at 0 in both sides and B respectively at 1 and Y. The platforms charge p_i, $i = A, B$ on side X and r_i on side Y, and incur zero marginal cost in serving additional users.

A user located at x on side X (respectively y on Y) receives utility from joining platform $i = A, B$:

$$\begin{aligned} u(x;A) =& A_x + \alpha y_A - p_A - zx \\ u(x;B) =& B_x + \alpha(Y - y_B) - p_B - z(1-x) \\ u(y;A) =& A_y + \beta x_A - r_A - qy \\ u(y;B) =& B_y + \beta(1 - x_B) - r_B - q(Y - y) \end{aligned} \tag{1}$$

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where for platform A a mass of y_A agents participate on side Y and a mass x_A agents participate on side X, while for platform B a mass of $1 - x_B$ agents participate on side X and a mass of $Y - y_B$ agents participate on side Y; α and β is the "network effect" of the other side on side X and Y respectively; A_x, B_x and A_y, B_y are the stand-alone values users on side X and Y obtain from joining the respective platform; and z and q are the respective "transportation cost," i.e., the loss of utility due to preference mis-match or set-up costs. We assume that $qz > \alpha\beta$, i.e., that network effects are weaker than transportation costs, which is a typical assumption in models of competition with network effects on Hoteling line, as these models focus on the effects of differentiation.

1.1 Single-homing benchmark

We begin by analyzing as a benchmark the case with full coverage and single-homing, as is typical in most of the literature on platform competition. Imposing single-homing, under full coverage, the platforms share the market according to $x_A = x_B = \tilde{x}$ s.t. $u(\tilde{x}; A) = u(\tilde{x}; B)$ and $y_A = y_B = \tilde{y}$ s.t., $u(\tilde{y}; A) = u(\tilde{y}; B)$; i.e.,

$$\tilde{x} = \frac{z + A_x - B_x + 2\alpha\tilde{y} - \alpha Y - p_A + p_B}{2z}$$

$$\tilde{y} = \frac{qY + A_y - B_y + 2\beta\tilde{x} - \beta - r_A + r_B}{2q}$$

First, solving for $\tilde{x}$ and $\tilde{y}$ for given prices yields

$$\tilde{x} = \frac{1}{2} + \frac{q(A_x - B_x) + \alpha(A_y - B_y) - q(p_A - p_B) - \alpha(r_A - r_B)}{2(zq - \alpha\beta)}$$

$$\tilde{y} = \frac{Y}{2} + \frac{\beta(A_x - B_x) + z(A_y - B_y) - \beta(p_A - p_B) - z(r_A - r_B)}{2(zq - \alpha\beta)}$$

Platform A is choosing p_A and r_A to maximize its profit, $\Pi_A = p_A\tilde{x} + r_A\tilde{y}$. The FOCs wrt p_A and r_A yield

$$zq - \alpha\beta + q(A_x - B_x) + \alpha(A_y - B_y) + qp_B + \alpha r_B - 2qp_A - r_A(\alpha + \beta) = 0$$

$$Y(zq - \alpha\beta) + \beta(A_x - B_x) + z(A_y - B_y) + \beta p_B + zr_B - (\alpha + \beta)p_A - 2zr_A = 0$$

Platform B is choosing p_B and r_B to maximize its profit, $\Pi_B = p_A(1 - \tilde{x}) + r_A(Y - \tilde{y})$. The

FOCs wrt p_B and r_B yield

$$zq - \alpha\beta - q(A_x - B_x) - \alpha(A_y - B_y) + qp_A + \alpha r_A - 2qp_B - r_B(\alpha + \beta) = 0$$

$$Y(zq - \alpha\beta) - \beta(A_x - B_x) - z(A_y - B_y) + \beta p_A + zr_A - (\alpha + \beta)p_B - 2zr_B = 0$$

Using these FOC to solve for the equilibrium, we obtain

$$\tilde{x}^S = \frac{1}{2} + \frac{3q(A_x - B_x) + (\alpha + 2\beta)(A_y - B_y)}{18(zq - \alpha\beta) - 4(\alpha - \beta)^2}$$

$$\tilde{y}^S = \frac{Y}{2} + \frac{(2\alpha + \beta)(A_x - B_x) + 3z(A_y - B_y)}{18(zq - \alpha\beta) - 4(\alpha - \beta)^2}$$

$$p_A^S = z - Y\beta + \frac{(A_x - B_x)(3qz - \beta(2\alpha + \beta)) + (A_y - B_y)z(\alpha - \beta)}{9(zq - \alpha\beta) - 2(\alpha - \beta)^2}$$

$$r_A^S = qY - \alpha + \frac{(A_y - B_y)(3qz - \alpha(\alpha + 2\beta) - q(A_x - Bx)(\alpha - \beta)}{9(zq - \alpha\beta) - 2(\alpha - \beta)^2}$$

$$p_B^S = z - Y\beta - \frac{(A_x - B_x)(3qz - \beta(2\alpha + \beta)) + (A_y - B_y)z(\alpha - \beta)}{9(zq - \alpha\beta) - 2(\alpha - \beta)^2}$$

$$r_B^S = qY - \alpha - \frac{(A_y - B_y)(3qz - \alpha(\alpha + 2\beta) - q(A_x - B_x)(\alpha - \beta)}{9(zq - \alpha\beta) - 2(\alpha - \beta)^2}$$

For this equilibrium to exist, we need to have the utilities of marginal users, $\tilde{x}^S$ and $\tilde{y}^S$ to be positive, and profits of the platforms Π_A^S and Π_B^S to be positive as well.
We get $u(\tilde{x}^*, A) = u(\tilde{x}^*, B) = \frac{1}{2}(A_x + B_x + Y(\alpha + 2\beta) - 3z)$ and $u(\tilde{y}^*, A) = u(\tilde{y}^*, B) = \frac{1}{2}(A_y + B_y + 2\alpha + \beta - 3qY)$. They are positive when

$$A_x + B_x > 3z - Y(\alpha + 2\beta) \qquad \text{and} \qquad A_y + B_y > 3qY - (2\alpha + \beta)$$

Note that the requirement that stand alone values are positive for the equilibrium is a general property of the Hoteling model, with or without network effects.

1.2 Multihoming on one side

We now examine the second benchmark, with single homing imposed on side X, multihoming allowed on side Y, and full coverage of both sides. As before, $\tilde{x}$ is characterized by $u(\tilde{x}, A) = u(\tilde{x}, B)$. A user on side Y who multihomes, i.e., joins both platforms, obtains utility $u(y;A\&B) = A_x + B_x + \beta - r_A - r_B - qY = u(y;A) + u(y;B)$. It is preferable for user y to join both platforms when $u(y;A) > 0$ and $u(y;B) > 0$. Therefore, all users $y < y_A$ join platform A, where y_A is characterized by $u(y_A;A) = 0$. All users $y > y_B$ join platform B, where y_B is characterized by $u(y_B;B) = 0$. Users $y \in (y_B, y_A)$ multihome.

Solving for $\tilde{x}$, y_A and y_B for given prices yields

$$\tilde{x} = \frac{1}{2} + \frac{q(A_x - B_x) + \alpha(A_y - B_y)}{2\,(zq - \alpha\beta)} - \frac{q(p_{AM} - p_{BM}) + \alpha(r_{AM} - r_{BM})}{2\,(zq - \alpha\beta)}$$

$$y_A = \frac{(A_x - B_x)q\beta + (A_y - B_y)\alpha\beta + (zq - \alpha\beta)(2A_y + \beta) - \beta q(p_{AM} - p_{BM}) - (2qz - \alpha\beta)r_{AM} + \alpha\beta r_{BM}}{2q(zq - \alpha\beta)}$$

$$y_B = Y - \frac{(B_x - A_x)q\beta + (B_y - A_y)\alpha\beta + (zq - \alpha\beta)(2B_y + \beta) + \beta q(p_{AM} - p_{BM}) + \alpha\beta r_{AM} - (2qz - \alpha\beta)r_{BM}}{2q(zq - \alpha\beta)}$$

Note that formula for $\tilde{x}$ is the same as in the case of single-homing on both sides.

In equilibrium, we obtain

$$\tilde{x}^M = \frac{1}{2} + \frac{2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{2(6(qz - \alpha\beta) - (\alpha - \beta)^2)}$$

$$y_A^M = \frac{2A_y + \alpha + \beta}{4q} + \frac{(\alpha - \beta)(2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{4q(6(qz - \alpha\beta) - (\alpha - \beta)^2}$$

$$y_B^M = Y - \frac{2B_y + \alpha + \beta}{4q} - \frac{(\alpha - \beta)(2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{4q(6(qz - \alpha\beta) - (\alpha - \beta)^2}$$

$$p_A^M = \frac{4(qz - \alpha\beta) + \beta(\alpha - \beta) - \beta(A_y + B_y)}{4q} + \frac{q(4(qz - \alpha\beta) + \beta(\alpha - \beta))(A_x - B_x) + (\alpha(qz - \alpha\beta) + qz(\alpha - \beta))(A_y - B_y)}{2q(6(qz - \alpha\beta) - (\alpha - \beta)^2)}$$

$$p_B^M = \frac{4(qz - \alpha\beta) + \beta(\alpha - \beta) - \beta(A_y + B_y)}{4q} - \frac{q(4(qz - \alpha\beta) + \beta(\alpha - \beta))(A_x - B_x) + (\alpha(qz - \alpha\beta) + qz(\alpha - \beta))(A_y - B_y)}{2q(6(qz - \alpha\beta) - (\alpha - \beta)^2)}$$

$$r_A^M = \frac{2A_y - \alpha + \beta}{4} - \frac{(\alpha - \beta)(2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{4(6(qz - \alpha\beta) - (\alpha - \beta)^2)}$$

$$r_B^M = \frac{2B_y - \alpha + \beta}{4} + \frac{(\alpha - \beta)(2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{4(6(qz - \alpha\beta) - (\alpha - \beta)^2)}$$

Such equilibrium exists if the platforms make positive profits and the marginal users obtain positive utility. Marginal users obtain positive utility when $A_y + B_y + \alpha + \beta > 2qY$ and $(\alpha + \beta)(A_y + B_y) + 2q(A_x + B_x) > 6(qz - \alpha\beta) - (\alpha - \beta)^2$.

2 Allowing for multihoming on both sides

We now allow for multihoming on both sides of a platform, i.e., $x_A > x_B$ and $y_A > y_B$. In such a case, multihoming agents on side X and Y obtain respective utility

$$u(x; A\&B) = A_x + B_x + \alpha Y - p_A - p_B - z$$
$$u(y; A\&B) = A_y + B_y + \beta - r_A - r_B - q\,.$$

An agent multihomes when multihoming yields higher utility than joining only platform A, only platform B, or not joining either of the platforms.

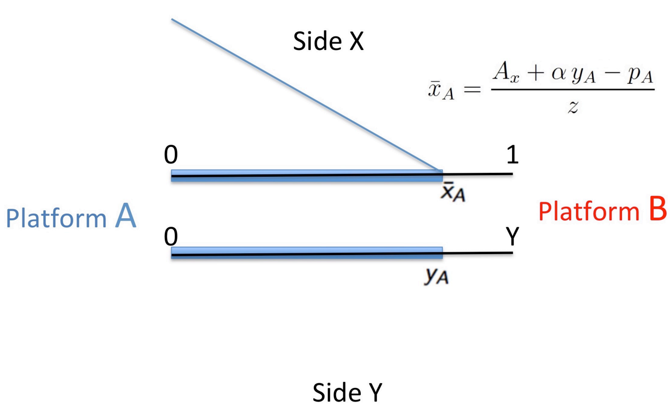
Utility of an agent joining A without having joined the other platform is given by $u(x; A)$ as in (1). Therefore, the agent indifferent between joining platform A only and not joining any platform at all, $\bar{x}_A$, is characterized by $u(\bar{x}_A; A) = 0$, i.e.,

$$\bar{x}_A = \frac{A_x + \alpha\, y_A - p_A}{z} \tag{2}$$

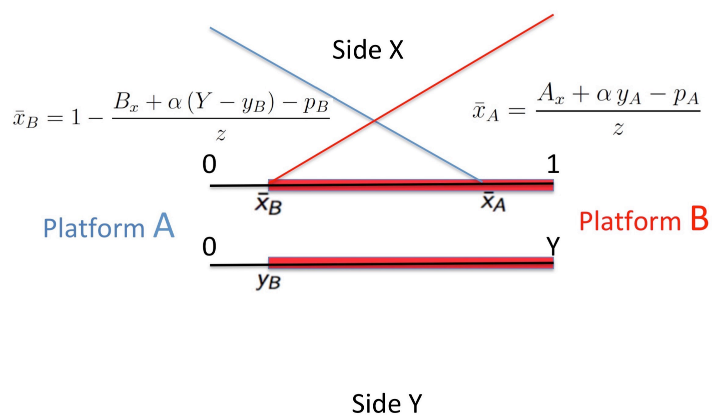
I.e., $\bar{x}_A$ would be the market captured by platform A if it was the only platform.

Similarly, given only the choice of platform B or no platform, all users $x > \bar{x}_B$ would prefer to join B, while $x < \bar{x}_B$ would not join, where

$$\bar{x}_B = 1 - \frac{B_x + \alpha\,(Y - y_B) - p_B}{z}$$



(a)



(b)

Figure 1

However, user x's utility from joining A in addition to B is given by

$$u(x; A|B) = u(x; A\&B) - u(x; B)\,. \tag{3}$$

If there is multihoming on side Y, this incremental utility $u(x; A|B)$ is smaller than $u(x; A)$. Thus, platform A's market coverage on side X is $\hat{x}_A$, characterized by $u(\hat{x}_A; A|B) = 0$ which is equivalent to $u(\hat{x}_A; A\&B) = u(\hat{x}_A; B)$. I.e.,

$$\hat{x}_A = \frac{A_x + \alpha\, y_B - p_A}{z} \tag{4}$$

Since $y_A > y_B$, then $\hat{x}_A < \bar{x}_A$.

Partial multihoming on both sides occurs in equilibrium when $0 < \hat{x}_B < \hat{x}_A < 1$ and $0 < \hat{y}_B < \hat{y}_A < 1$, where

$$\begin{aligned}
\hat{x}_A &= \frac{A_x + \alpha \hat{y}_B - p_A}{z} \\
\hat{x}_B &= 1 - \frac{B_x + \alpha(Y - \hat{y}_A) - p_B}{z} \\
\hat{y}_A &= \frac{A_y + \beta \hat{x}_B - r_A}{q} \\
\hat{y}_B &= Y - \frac{B_y + \beta(1 - \hat{x}_A) - r_B}{q}
\end{aligned} \tag{5}$$

After solving platforms' profit-maximizing problems, the pure strategy equilibrium with partial multihoming on both sides is characterized by

$$\begin{aligned}
\hat{x}_A^{MM} &= \frac{q[(2qz - \alpha\beta)(A_x + \alpha Y) - \alpha z(B_y + \beta)]}{(qz - \alpha\beta)(4qz - \alpha\beta)} \\
\hat{y}_A^{MM} &= \frac{z[(2qz - \alpha\beta)(A_y + \beta) - \beta q(B_x + Y\alpha)]}{(qz - \alpha\beta)(4qz - \alpha\beta)} \\
\hat{x}_B^{MM} &= \frac{(2qz - \alpha\beta)^2 - (2qz - \alpha\beta)q(B_x + Y\alpha) + \alpha qzA_y}{(qz - \alpha\beta)(4qz - \alpha\beta)} \\
\hat{y}_B^{MM} &= \frac{Y(2qz - \alpha\beta)^2 - (2qz - \alpha\beta)z(B_y + \beta) + \beta qzA_x}{(qz - \alpha\beta)(4qz - \alpha\beta)}
\end{aligned} \tag{6}$$

And

$$p_A^{MM} = \frac{(2qz - \alpha\beta)(A_x + Y\alpha) - \alpha z(B_y + \beta)}{4qz - \alpha\beta}$$
$$r_A^{MM} = \frac{(2qz - \alpha\beta)(A_y + \beta) - \beta q(B_x + Y\alpha)}{4qz - \alpha\beta}$$
$$p_B^{MM} = \frac{(2qz - \alpha\beta)(B_x + Y\alpha) - \alpha z(A_y + \beta)}{4qz - \alpha\beta}$$
$$r_B^{MM} = \frac{(2qz - \alpha\beta)(B_y + \beta) - \beta q(A_x + Y\alpha)}{4qz - \alpha\beta}$$

Proposition 1 *For range of parameters described by conditions (i)-(vi) below, there exists a pure strategy equilibrium with partial multihoming on both sides.*

(i) $zq(4zq + A_y\alpha) > q(Y\alpha + B_x)(2qz - \alpha\beta) + \alpha\beta(4qz - \alpha\beta)$

(ii) $q(A_x + Y\alpha)(2qz - \alpha\beta) < (qz - \alpha\beta)(4qz - \alpha\beta) + \alpha qz(B_y + \beta)$

(iii) $zq(4zq + A_y\alpha + B_y\alpha + \alpha\beta) < q(2qz - \alpha\beta)(A_x + B_x + 2Y\alpha) + \alpha\beta(4qz - \alpha\beta)$

(iv) $Y(2qz - \alpha\beta)^2 + qz\beta A_x > z(B_y + \beta)(2qz - \alpha\beta)$

(v) $z(A_y + \beta)(2qz - \alpha\beta) < (qz - \alpha\beta)(4qz - \alpha\beta) + \beta zq(B_x + Y\alpha)$

(vi) $Y(2qz - \alpha\beta)^2 + \beta zq(A_x + B_x + Y\alpha) < z(2qz - \alpha\beta)(A_y + B_y + 2\beta)$

Proof. Suppose parameters satisfy conditions (i)-(vi). It is straightforward to provide examples illustrating that the region of parameters satisfying these conditions is non-empty. Then $\hat{x}_A^{MM}$, $\hat{x}_B^{MM}$, $\hat{y}_A^{MM}$ and $\hat{y}_B^{MM}$ calculated according to (6) are such that $0 < \hat{x}_B^{MM} < \hat{x}_A^{MM} < 1$ and $0 < \hat{y}_B^{MM} < \hat{y}_A^{MM} < 1$. Conditions $\hat{x}_B^{MM} < \hat{x}_A^{MM}$ and $\hat{y}_B^{MM} < \hat{y}_A^{MM}$ indicate multihoming on both sides. And since all four thresholds are strictly between 0 and 1, the multihoming is partial. Moreover, multihoming brings positive utility on each side, i.e.,

$$\alpha z(A_y + B_y) + 2qz(A_x + B_x) > \alpha\beta(z - \alpha Y) + 4z(qz - \alpha\beta)$$

$$\beta q(A_x + B_x) + 2qz(A_y + B_y) > \alpha\beta(qY - \beta) + 4qY(qz - \alpha\beta)\,.$$

The final condition for existence of the equilibrium, platforms' positive profits, are assured by the result that the prices are non-negative. ■

Proposition 2 *In an equilibrium with partial multihoming on both sides, there are no subsidies, i.e.,* p_A^{MM}*,* r_A^{MM}*,* p_B^{MM} *and* r_B^{MM} *are strictly positive.*

Proof. Consider parameters for which conditions (i)-(vi) above are satisfied. Since the conditions imply $0 < \hat{x}_B^{MM} < \hat{x}_A^{MM} < 1$ and $0 < \hat{y}_B^{MM} < \hat{y}_A^{MM} < 1$, they must also imply $\hat{x}_A^{MM} > 0$, $\hat{x}_B^{MM} < 1$, $\hat{y}_A^{MM} > 0$ and $\hat{y}_B^{MM} < 1$. Direct algebraic manipulations reveal that $\hat{x}_A^{MM} > 0 \iff p_A^{MM} > 0$, $\hat{y}_A^{MM} > 0 \iff r_A^{MM}$, $\hat{x}_B^{MM} < 1 \iff p_B^{MM} > 0$, and $\hat{y}_B^{MM} < 1 \iff r_B^{MM} > 0$. ■

With these result, we achieve the goal of this appendix: showing that the assumption $A_x = B_x$ and $A_y = B_y$ simplifies exposition of the benchmark cases without any loss to the our results.