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A Unified Model of International Business Cycles and Trade

Abstract

We present a unified dynamic framework to study the interconnections between international trade and business cycle models. We prove an aggregate equivalence between a competitive, representative firm model that has aggregate production externalities and dynamic trade models that feature monopolistic competition, endogenous entry, and heterogeneous firms. The production externalities in the representative firm model have to be introduced in the intermediate and final good sectors so that the model is isomorphic to dynamic trade models that embody love-of-variety and selection effects. In a quantitative exercise with multiple shocks, we show that to improve the fit of the dynamic trade models with the data, the most important ingredient is negative capital externality in the intermediate good sector. This presents a puzzle for the literature as standard dynamic trade models provide micro-foundations for positive capital externality.

JEL-Codes: F120, F410, F440, F320.

Keywords: international business cycles, dynamic trade models, heterogeneous firms, production externalities, monopolistic competition, export costs, entry costs.

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1 Introduction

The standard international business cycle model, the IRBC model (e.g., Backus, Kehoe and Kydland (1994) and Heathcote and Perri (2002)), has been used extensively to answer quantitative questions. While successful on many fronts, the model has difficulty matching some important second moments such as a higher international correlation of output compared to consumption, the positive cross-country correlations of investment and hours, the high volatility of the trade balance, and the low cyclicality of the real exchange rate.

The basic IRBC model features a representative firm and perfectly competitive product markets. One can alternatively consider environments used in the modern trade literature, as developed in Krugman (1980) and Melitz (2003), which were introduced in the international business cycle literature by Ghironi and Melitz (2005).[1] A natural question then is whether these alternative environments — primarily, monopolistic competition, endogenous entry, and heterogeneous firms — lead to a better fit with the data in terms of aggregate international moments. Even more importantly, how precisely do these alternate environments affect the transmission mechanisms in response to aggregate shocks, and how do they impact international business cycle dynamics?

We provide a unified model of international business cycles and trade that can address these questions, both theoretically and quantitatively. On the theoretical front, our main result establishes an isomorphism between an *IRBC model extended with production externalities* in particular sectors — our unified model — and generalized versions of *dynamic trade models* (i.e., dynamic Krugman and Melitz models). On the quantitative front, the theoretical results first enable us to precisely pin-point how trade features affect the transmission of aggregate shocks over the business cycle. Second, they allow us to flexibly explore how the fit with the data can be improved. We find that the most important ingredient is negative capital externality in the intermediate good sector.

Let us now explain in detail the key components of our models and the results. In the basic IRBC model, each country uses capital and labor in a Constant Returns to Scale technology to produce a unique and traded intermediate good. Intermediate goods originating from different countries are combined into a final good using a Constant Elasticity of Substitution technology. The final good is used for consumption and investment into capital. Our unified model extends the basic IRBC model by introducing external economies

[1] The Krugman model features monopolistically competitive homogeneous firms that produce differentiated varieties and pay sunk costs of entry into the economy. The Melitz model additionally features heterogeneity of firm productivities and fixed costs of exporting. An additional standard assumption for the Melitz model is that firm productivities are drawn from a Pareto distribution.

of scale in production of intermediate (in terms of both labor and capital) and final goods. In addition to the unified model, we also formulate *generalized* dynamic versions of the Krugman (1980) and Melitz (2003) models by extending their *standard* counterparts. These set-ups then allow us to prove that the *generalized* dynamic versions of the Krugman and Melitz models are isomorphic to the *unified* model in their aggregate predictions.

Aggregate externalities introduced in the unified model are key to establishing the isomorphism, and these externalities do arise even in *standard* versions of the Krugman and Melitz models. These standard versions, however, allow only a one-way mapping to the unified model. This is because the standard models imply tight relationships between technological parameters in the corresponding unified model: Cobb-Douglas share of capital in the intermediate good technology, elasticity of substitution between intermediate goods in the final good technology, and strengths of external economies of scale. In particular, all of these parameters of the unified model are determined by only one structural parameter in the Krugman model — the elasticity of substitution between varieties, and by two structural parameters in the Melitz model — the elasticity of substitution between varieties and the shape of Pareto distribution. The essence of our generalizations of the Krugman and Melitz models is in breaking the implied tight relationships between technological parameters in the corresponding unified model, which is needed to establish a two-way mapping (that is, an isomorphism).

One building block for our results is that the measures of firms in the Krugman and Melitz models play the role of capital in the unified model. Thus, even though labor is the only factor of production in both the Krugman and Melitz models, the corresponding unified model features an aggregate production function for intermediate goods that uses both capital and labor. The *total* capital's exponent in this function is determined by the love-of-variety effect in the Krugman model and the selection effect in the Melitz model.[2]

Next, under the usual assumption that in the Krugman and Melitz models technology of production of differentiated varieties is linear in labor, the aggregate production function for intermediate goods in the corresponding unified model is also linear in labor. Only a part of capital and a part of labor used in this function are internalized by the representative firm, while the remaining parts induce (positive) externalities. The inter-

[2]As shown in the trade literature, the love-of-variety effect is the source of externalities in the (static) Krugman model, while the selection effect is the source of externalities in the (static) Melitz model (see, e.g., Kucheryavyy *et al.* (2019)). These are different effects, which gets reflected in different structural parameters governing the total capital's exponent in the aggregate production technology for intermediate goods in our unified model. It is the elasticity of substitution across varieties in the standard Krugman model, while it is the Pareto shape parameter in the standard Melitz model. Our dynamic perspective helps to bring to light the fact that only a part of the externality induced by each of these effects works through the measure of firms, while the other part works through labor.

nalized part of capital is equal to the share of firms' revenues accrued as profits in both the Krugman and Melitz models. Similarly, the internalized part of labor is equal to the share of firms' revenues that is paid as wages to labor used in production of varieties.

Besides externalities in production of intermediate goods, the Krugman model does not generate other externalities. The Melitz model however, additionally generates a (positive) externality in production of the final good. This externality in the Melitz model arises due to the selection effect that works through the importer's total demand for varieties: greater demand for varieties increases the number of exporters entering the market, which lowers importer's price index due to love-of-variety. The representative producer of the final good does not internalize this entry effect on the price index.

Our generalization of the Krugman model introduces correction for the love-of-variety effect in the production function for the final good, a labor externality in the production function for varieties, and an externality in the production function for the final good. The generalization of the Melitz model introduces correction of the selection effect in fixed costs of serving markets and a labor externality in the production function for varieties. Thus, to achieve the isomorphism with the unified model, we target the sources of externalities directly (love-of-variety and selection) when possible, or introduce externalities precisely into the production functions, thereby freeing the tight relationships among externalities implied by the standard Krugman and Melitz models.

Given our theoretical results, we undertake a quantitative exercise. First, we show that standard dynamic Krugman and Melitz models do not resolve the key empirical puzzles related to cross-country correlations. Our theoretical result offers the explanation: standard formulations and calibrations of these models lead to relatively small, tightly restricted, and positive externalities. This then leads to transmission mechanisms and aggregate second moments that are very similar to the IRBC model, as the endogenous cyclical movements in productivity introduced by the externalities are minor.[3]

Second, we pinpoint what is needed to achieve a better fit with the data. We consider two types of shocks, intermediate good and final good productivity shocks, and show that an essential feature to improve fit with the data is a negative capital externality in the intermediate goods technology for both shocks.[4] This exercise is possible only because

[3]In addition to assessing international correlations, we also explore the fit with the data in terms of domestic correlations with output of key open economy variables, such exports, imports, real exchange rate, and the trade balance. We find again that, due to the small externalities implied by the standard Krugman and Melitz models, they lead to very similar moments as the IRBC model.

[4]The intermediate good productivity shock is a canonical shock in the IRBC literature, while the final good productivity shock is new. The final good productivity shock leads to basic domestic comovements that look the same as those caused by the intermediate good productivity shock. That is, the final good productivity shock leads to a comovement of within-country consumption, output, hours, and investment,

our unified model perspective allows us to isolate different types of shocks and, more importantly, to vary the strengths of externalities independently from each other and from other parameters of the model.[5]

To understand why negative capital externality in the intermediate goods technology helps improve fit with the data, it is helpful to first review the main empirical puzzles associated with the basic two-country IRBC model and their underlying source. In the model, the international correlation of consumption is counterfactually higher than that for output, while the international correlations of labor hours and investment are lower than in the data.[6] A common source behind all these anomalous cross-country correlations is the tendency in the IRBC model for a positive intermediate good shock in the home country to lead to a substantial rise in factor use at home, while inducing a cut in factor use abroad. A negative capital externality makes the transmission of both the intermediate and final good productivity shocks endogenously negative. This limits the persistent rise in factor use at home, while limiting the fall in factor use abroad and, thus, helps with improving international correlations.[7]

Let us first discuss in more detail the standard intermediate good productivity shock. With negative capital externality, from the perspective of individual firms, it is as if the aggregate intermediate good productivity shock is less persistent, but has the same initial impact. This is because, in future, due to higher capital accumulation from a positive shock, the productivity faced by the firms is lower than the exogenous shock. The less persistent productivity increase leads to less persistent increase in hours, investment, and output at home. This endogenous decrease in persistence of productivity at home also acts against the reallocation of factors away from the foreign country. Moreover, with consumption smoothing motives, in the face of a less persistent rise in income at home, consumption increases by less initially and more importantly, its dynamic response changes non-trivially due to change in the path of investment.

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thereby generating a domestic business cycle. It is well understood in the closed economy literature that in a real economy, like the one studied in this paper, such domestic comovement is hard to generate for shocks other than the intermediate good productivity shock. In an open economy, however, due to a relative price effect, the final good productivity shock can generate a domestic business cycle. It is worth mentioning in this context that our isomorphism results hold for any aggregate shock that drives the business cycle.

[5]Given the isomorphism (two-way mapping) between the unified model and the generalized dynamic Krugman and Melitz models, we can then also interpret the underlying source of negative capital externality from either model perspectives.

[6]In fact, labor hours and investments across countries often co-move negatively in the model while they co-move positively in the data. With respect to the other key moment — the high cross-country correlation in consumption — note that this anomaly is not entirely due to perfect risk-sharing. To make this last point clear and for completeness, we present results for all three common variants of international risk-sharing: complete financial markets, bond economy, and financial autarky.

[7]Negative labor or final good externalities, in contrast, do not help improve the fit uniformly.

Overall, these changes to the transmission of the shock help increase output, investment, and hours correlation across countries while decreasing consumption correlation. In addition, negative capital externality also leads to an endogenous positive correlation of home productivity with the foreign, as typically the foreign country would decumulate capital. This also leads, comparatively, to an increase in hours, output, and investment in the foreign country.

For the final good productivity shock, a similar mechanism holds. The final good productivity shock does not directly affect the intermediate good production function. Endogenously, however, intermediate good productivity declines as there is higher future capital accumulation in response to a positive final good shock. This negative effect on intermediate good productivity acts in an opposite direction to the positive effect of the final good productivity shock on the home country, thereby limiting the persistent rise in factors use at home and the cut in factors use abroad.

Finally, we estimate our unified model with the intermediate and final good productivity shocks, which are not exogenously imposed to be correlated across countries, by matching several second moments from the data.[8] In particular, we match not just cross-country correlations, but also volatility and cyclicality of both domestic and open economy variables. We show that negative capital externality is important to improve fit with the data based on this comprehensive quantitative exercise. We additionally find that the final good productivity shock drives the international business cycle.[9] These quantitative conclusions hold for either complete markets or the bond economy.

Our results on negative capital externality thus pose a new puzzle for the quantitative literature that studies the interactions of international trade and business cycles models. In particular, our results point towards two possible implications for future research. One is to take a stance that existing models feature a case of "missing negative capital externality," and thus, move to fully micro-found it in existing models. The other is to modify the core structure of the dynamic trade models beyond those often considered in the literature such that positive capital externality, as embedded and micro-founded in dynamic trade models, can help improve the fit on international moments. One possibility is to

[8]Two key components of our estimation exercise are that we do not impose exogenously correlated shocks across countries and that we use a trade elasticity that is positive. Heathcote and Perri (2014) show that the standard IRBC model with intermediate good productivity shocks, even under complete markets, can match several key international correlations if the exogenous shock correlation is calibrated to match cross-country output correlation, and if the trade elasticity is negative (such that the foreign and domestic intermediate goods are complements).

[9]While both the intermediate and final good productivity shocks lead to a domestic business cycle and have similar implications for several open economy variables, for some open economy variables, the final good productivity shock enables a better fit with the data. We explain this in more detail later.

explore if introducing nominal/real wage rigidities affects the quantitative conclusions.

Our paper is related to several strands of the literature. In particular, Ghironi and Melitz (2005) and Jaef and Lopez (2014), by extending the set-up in Melitz (2003) to a dynamic setting, develop models most similar to the standard dynamic trade models we present, and they assess how important international trade features are for real exchange rate and business cycle dynamics.[10]

Our result on isomorphism is related to a similar result in a static environment demonstrated in Kucheryavyy *et al.* (2019). Kucheryavyy *et al.* (2019) present a version of the competitive model with multiple manufacturing sectors that feature external economies of scale in production. They show that their model is isomorphic to generalized static versions of multi-industry Krugman and Melitz models. Here, we focus on dynamic versions of Krugman and Melitz models that have only one manufacturing sector and other "non-manufacturing" sectors: final aggregate, investment and consumption.

Extension of the isomorphism from static to dynamic environments is non-trivial as it adds several new features due to capital accumulation and endogenous trade deficits. Most important are the the split of externalities between labor and capital, which plays an important role both qualitatively and quantitatively, and the final good externality that appears in the Melitz model because of endogenous trade balance.[11] The general formulation of externalities that can be used in both dynamic and static contexts constitutes one of our theoretical contributions. We then use the general model for a quantitative evaluation of business cycle statistics and transmission mechanisms.

Our paper is also related to the closed economy literature. In the closed economy endogenous growth literature (e.g., Romer, 1986), growth is generated by increasing returns in production, where externalities in the production function are modeled with respect to the capital input. In our unified open economy model, production externalities exist with respect to both capital and labor. In a closed-economy business cycle analysis, Benhabib and Farmer (1994) introduced production externalities in the RBC model to generate the possibility of multiple, bounded equilibria. In a closed economy set-up as well, Bilbiie *et al.* (2012) discuss how firm dynamics and firm entry in a model with monopolistic competition and sunk cost of entry look similar to capital stock dynamics and investment

[10] Eaton *et al.* (2016) develop a dynamic version of the competitive Eaton-Kortum model in Eaton and Kortum (2002) with physical capital accumulation. For completeness, we also formulate a general version of the competitive model of Eaton-Kortum with capital accumulation, but in that case, the equivalence with the IRBC model is immediate as there are no externalities. Jaef and Lopez (2014) also features physical capital accumulation, in an extension of Ghironi and Melitz (2005).

[11] Even in the static context, we show clearly that, whether entry costs are paid in terms of labor (standard assumption in the trade literature) or final good (more in line with the business cycle literature's assumption for investment), has important implications for the formulation of externalities.

in the standard competitive business cycles model. Our general model provides a similar interpretation as well, while additionally showing formally how a competitive open economy set-up with different levels and types of production externalities is in fact isomorphic to various generalized versions of monopolistic competition trade models with firm heterogeneity.

2 Unified Model of Business Cycles and Trade

We present our unified model, a dynamic stochastic general equilibrium model with multiple countries and international trade. Time is discrete and the horizon is infinite. The world consists of N countries with countries indexed by n, i, and j.[12] Each country has four production sectors: intermediate, final aggregate, consumption, and investment. Intermediate goods are produced from capital and labor. Final aggregate is assembled from intermediate goods. Consumption good is produced directly from the final aggregate. Investment good is produced from the final aggregate and labor. All markets are perfectly competitive. Labor is perfectly mobile within a country. Technology of production of intermediate goods and final aggregates features external economies of scale. There are three aggregate exogenous shocks in the economy: productivity shocks in the intermediate, final aggregate, and investment sectors. Only intermediate goods can be traded. Trade is subject to iceberg trade costs. International financial markets structure is one of the three standard alternatives: financial autarky, bond economy, or complete markets.

We now describe the model in detail.

2.1 Intermediate Goods and International Trade

Output of a country-n's intermediate good producer that in period t employs $k_{X,nt}$ units of capital and $l_{X,nt}$ units of labor is given by $S_{X,nt} k_{X,nt}^{\alpha_{X,K}} l_{X,nt}^{\alpha_{X,L}}$, where $\alpha_{X,K} \geq 0$ and $\alpha_{X,L} \geq 0$ with $\alpha_{X,K} + \alpha_{X,L} = 1$, and

$$S_{X,nt} \equiv \Theta_{X,n} Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}} \tag{1}$$

is aggregate productivity. The aggregate productivity consists of two parts: exogenous productivity, $\Theta_{X,n} Z_{X,nt}$, and endogenous productivity, $K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}$. The term $Z_{X,nt}$ in the exogenous productivity part is an aggregate productivity shock, while the term $\Theta_{X,n}$ is a normalization constant that is introduced to later show isomorphisms between the cur-

[12] In all our quantitative exercises we focus on the case of $N = 2$ as is standard in the business cycles literature. In formulating the theoretical framework with an arbitrary number of countries, we follow the modern quantitative trade literature.

rent setup and dynamic versions of trade models. The endogenous productivity part captures external economies of scale in production of intermediates, and it is taken by firms as given. The terms $K_{X,nt}$ and $L_{X,nt}$ are the total amounts of country n's capital and labor used in production of intermediates. Parameters $\psi_{X,K}$ and $\psi_{X,L}$ drive the strength of external economies of scale. Perfect competition in production of intermediates implies that the total output of intermediates in country n in period t is given by

$$X_{nt} = S_{X,nt} K_{X,nt}^{\alpha_{X,K}} L_{X,nt}^{\alpha_{X,L}}.$$

Let $P_{X,nt}$ denote the price of country n's intermediate good in period t, and W_{nt} and R_{nt} denote the wage and capital rental rate in country n in period t. Due to perfect competition,

$$K_{X,nt} = \alpha_{X,K} \frac{P_{X,nt} X_{nt}}{R_{nt}} \quad \text{and} \quad L_{X,nt} = \alpha_{X,L} \frac{P_{X,nt} X_{nt}}{W_{nt}}.$$

Moreover,

$$P_{X,nt} = \frac{R_{nt}^{\alpha_{X,K}} W_{nt}^{\alpha_{X,L}}}{\widetilde{\Theta}_{X,n} Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}}, \tag{2}$$

where $\widetilde{\Theta}_{X,n} \equiv \alpha_{X,K}^{\alpha_{X,K}} \alpha_{X,L}^{\alpha_{X,L}} \Theta_{X,n}$.

Intermediate goods are the only traded goods, and trade in these goods is costly. Trade costs are of the iceberg nature: in order to deliver one unit of intermediate good to country n, country i needs to ship $\tau_{ni,t} \geq 1$ units of this good. To guarantee absence of arbitrage in the transportation of goods, we require that trade costs satisfy the triangle inequality: $\tau_{nj,t}\tau_{ji,t} \geq \tau_{ni,t}$ for any countries n, i, and j. This implies that the price of country i's intermediate good sold in country n is given by $P_{ni,t} \equiv \tau_{ni,t} P_{X,it}$.

2.2 Final Aggregates and Consumption Goods

Final aggregate is produced by combining intermediate goods imported from different counties. Let $X_{ni,t}$ denote the amount of intermediate good that country n buys from country i in period t. The total output of final aggregate in country n at time t, Y_{nt}, is given by

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left(\omega_{ni} X_{ni,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where $\omega_{ni} \geq 0$ are exogenous importer-exporter specific weights, $\sigma > 0$ is an Armington elasticity of substitution between intermediate goods produced in different countries, and

$$S_{Y,nt} \equiv \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y} \tag{3}$$

is aggregate productivity with $P_{Y,nt}$ being the price of the final aggregate.[13]

As in production of intermediates, productivity in production of the final aggregate has two parts: exogenous productivity, $\Theta_{Y,n} Z_{Y,nt}$, and endogenous productivity, $\left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}$ with ψ_Y driving the strength of external economies of scale in production of the final aggregate. The term $Z_{Y,nt}$ is an aggregate productivity shock.[14] We do not put any restrictions on its correlation with the shock $Z_{X,nt}$ in the intermediate goods sector. The term $\Theta_{Y,n}$ is a normalization constant introduced to later show isomorphisms between different models. The endogenous part of $S_{Y,nt}$ captures external economies of scale in production of the final aggregate, and it is taken by firms as given. $(P_{Y,nt} Y_{nt}) / W_{nt}$ is the number of country-n's workers that produce the same value as the value of the final aggregate.[15]

Perfect competition in production of the final aggregate implies that the price of the final aggregate, $P_{Y,nt}$, is given by

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N} \left(\tau_{ni,t} P_{X,it} / \omega_{ni} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}}{\Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}}, \tag{4}$$

and country n's share of expenditure on country i's intermediate good, denoted by $\lambda_{ni,t}$, is

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni} \right)^{1-\sigma}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj} \right)^{1-\sigma}}. \tag{5}$$

Final aggregate in country n is used directly as the consumption good in this country as well as in the production process of the investment good, which we describe next.

[13] Since labor is perfectly mobile within a country, there is only one wage per country.

[14] The final good productivity shock $Z_{Y,nt}$ is new to the IRBC model and plays an important role quantitatively.

[15] The particular form in which the externality in production of the final aggregate is introduced is chosen to later show isomorphism with the dynamic Melitz model. This term appears in the Melitz model because of the fixed costs of serving markets that are paid in terms of the destination country labor.

2.3 Investment Goods

Let I_{nt} denote the total output of the investment good in country n in period t, and $P_{I,nt}$ the price of this good. Investment good is produced from labor and the final aggregate with the production technology given by

$$I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \tag{6}$$

where $0 \leq \alpha_I \leq 1$. Here $L_{I,nt}$ and $Y_{I,nt}$ are the total amounts of labor and final aggregate used in production of the investment good, $Z_{I,nt}$ is an exogenous aggregate productivity shock, and $\Theta_{I,n}$ is a normalization constant introduced to later show isomorphisms between different models. We do not put any restrictions on correlation of $Z_{I,nt}$ with the shocks $Z_{X,nt}$ and $Z_{Y,nt}$ in the intermediate and final goods sectors.[16]

Perfect competition in production of the investment good implies

$$L_{I,nt} = \alpha_I \frac{P_{I,nt} I_{nt}}{W_{nt}}, \quad \text{and} \quad Y_{I,nt} = (1-\alpha_I) \frac{P_{I,nt} I_{nt}}{P_{Y,nt}}.$$

Moreover,

$$P_{I,nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}, \tag{7}$$

where $\widetilde{\Theta}_{I,n} \equiv \alpha_I^{\alpha_I} (1-\alpha_I)^{1-\alpha_I} \Theta_{I,n}$.

2.4 Households

Each country n has a representative household with the period-t utility function given by $U(C_{nt}, L_{nt})$, where C_{nt} and L_{nt} are the household's consumption and supply of labor in period t. The household chooses consumption, supply of labor, investment, and holdings of financial assets (if allowed) so as to maximize the expected lifetime utility, $E_t \sum_{s=0}^{\infty} \beta^s U(C_{n,t+s}, L_{n,t+s})$, subject to the budget constraint and the law of motion of capital, where $\beta \in (0,1)$ is the discount factor, and E_t denotes the expectation over the states of nature taken in period t. The law of motion of capital is given by

$$K_{n,t+1} = (1-\delta) K_{nt} + I_{nt},$$

[16] In the IRBC model, investment is made directly from the final good. This standard technology can be obtained from (6) by setting $\Theta_{I,n} = 1$, $Z_{I,nt} = 1$, and $\alpha_I = 0$. As we will see later, the technology for producing the investment good in the standard versions of dynamic Krugman and Melitz models corresponds to setting $\alpha_I = 1$ and having $\Theta_{I,n} Z_{I,nt} \neq 1$. As we show later in detail, these differing choices can have non-trivial implications for the cyclicality of net exports, and, therefore, we take a general approach.

where I_{nt} is investment in period t and $\delta \in [0,1]$ is the depreciation rate.

Depending on the international financial markets structure, households face different budget constraints. Throughout the paper, we consider three standard alternatives for international financial markets: complete markets, bond economy, and financial autarky. In the case of financial autarky the budget constraint is given by[17]

$$P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} = W_{nt}L_{nt} + R_{nt}K_{nt}.$$

In the case of the bond economy and complete markets the budget constraints can be written by adding the expenditure and income from financial assets. Since the budget constraints associated with each of these financial market structures are standard, to conserve space, we delegate their formal description to Appendix A.1. Also, in the same appendix we provide the first-order conditions associated with the household problem.

In the case of complete financial markets and bond economy, international trade in assets allows unbalanced trade in intermediate goods. For future use, we define country n's real trade balance TB_{nt} as the value of net exports of intermediate goods in terms of the final good, $\mathrm{TB}_{nt} \equiv (P_{X,nt}X_{nt} - P_{Y,nt}Y_{nt})/P_{Y,nt}$, and define country n's real current account CA_{nt} as the change in this country's net financial assets position in terms of the final good.[18]

2.5 Market Clearing Conditions

The labor market clearing condition is given by

$$W_{nt}L_{X,nt} + W_{nt}L_{I,nt} = W_{nt}L_{nt} + aP_{Y,nt} \cdot \mathrm{TB}_{nt}, \quad \text{for } n = 1, \ldots, N, \tag{8}$$

where a is a constant. When $a = 0$, we have a standard labor market clearing condition. The extra term $aP_{Y,nt} \cdot \mathrm{TB}_{nt}$ is introduced to show isomorphism with the dynamic Melitz model, for which $a > 0$, and for which this term appears only if trade is unbalanced.

The rest of the market clearing conditions for the economy are standard. Since capital is used only in production of intermediate goods, we have

$$K_{X,nt} = K_{nt}, \quad \text{for } n = 1, \ldots, N.$$

[17] Observe that, since the consumption good is directly produced from the final aggregate (and there are no shocks in the consumption goods sector), the price of the consumption good is equal to the price of the final aggregate, $P_{Y,nt}$.

[18] The formal definition of CA_{nt} is standard, but requires use of additional notation. Therefore, we leave it to Appendix A.1.

The final aggregate is used in consumption and production of the investment good

$$C_{nt} + Y_{I,nt} = Y_{nt} \quad \text{for } n = 1, \ldots, N.$$

Demand for intermediate goods is equal to supply

$$\sum_{n=1}^{N} \tau_{ni,t} X_{ni,t} = X_{it}, \quad \text{for } i = 1, \ldots, N.$$

In the case of the bond economy and complete markets we also have the sets of market clearing conditions for financial assets.

The full set of equilibrium conditions is provided in Appendix A.2.

3 Generalized Versions of Krugman and Melitz Models

We next present the key elements of generalized dynamic versions the Krugman and Melitz models.[19] The focus of this section is to present the elements of these models that differ from their standard expositions, as they appear in the literature. Thus, our presentation omits all the derivations, which are provided in Appendix B. Anticipating isomorphisms between the unified, Krugman, and Melitz models, we use the same notation for parameters and variables of these models that map into each other. To mark some of the parameters and variables as being specific to a particular model, we use superscripts "K" for the Krugman model and "M" for the Melitz model.

3.1 Generalized Dynamic Version of the Krugman Model

3.1.1 Production of Varieties, International Trade, and Final Aggregate

Each country i produces a unique set of varieties Ω_{it}, which is endogenously determined in every period t. Let M_{it} be the measure of this set. All varieties can be internationally traded. Let $p_{ni,t}(\nu)$ denote the price of variety $\nu \in \Omega_{it}$ produced by country i and sold in country n. Assuming iceberg trade costs and no arbitrage in international trade implies that $p_{ni,t}(\nu) = \tau_{ni,t} p_{ii,t}(\nu)$.

Countries use varieties to produce non-traded final aggregates. Technology of produc-

[19] In Appendix B.1 we additionally present a generalized version of the Eaton and Kortum (2002) model and show that it can be mapped into the unified model in a straightforward manner.

tion of the final aggregate in country n is given by the nested CES production function

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left[M_{it}^{\phi_{Y,M} - \frac{1}{\sigma^K - 1}} \left[\int_{\nu \in \Omega_{it}} \left(\omega_{ni} x_{ni,t} (\nu) \right)^{\frac{\sigma^K - 1}{\sigma^K}} d\nu \right]^{\frac{\sigma^K}{\sigma^K - 1}} \right]^{\frac{\eta^K - 1}{\eta^K}} \right]^{\frac{\eta^K}{\eta^K - 1}}, \tag{9}$$

where $x_{ni,t}(\nu)$ is the amount of variety $\nu \in \Omega_{it}$ that country n buys from country i in period t, $\omega_{ni} \geq 0$ are exogenous importer-exporter specific weights, and

$$S_{Y,nt} \equiv \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}.$$

All terms of $S_{Y,nt}$ have the same meaning as in the corresponding definition (3) in the unified model.

The nested CES structure of (9) implies that the elasticity of substitution between varieties produced in one country, given by σ^K, is different from the elasticity of substitution between varieties produced in different countries, given by η^K. We assume that $\sigma^K > 1$ and $\eta^K > 1$. The term $M_{it}^{\phi_{Y,M} - \frac{1}{\sigma^K - 1}}$ introduces correction for the love-of-variety effect, which is the only source of externalities in the standard Krugman model with CES preferences. As is discussed in Benassy (1996), parameter $\phi_{Y,M}$ governs the taste for variety in the Krugman model (the standard Krugman model implies that the strength of the taste for variety is $1/(\sigma^K - 1)$). At the same time, as we shall see later, in the unified model, parameter $\phi_{Y,M}$ governs the strength of economies of scale induced by capital in production of intermediate goods. Having this parameter is critical for showing the isomorphism with the unified model.

Assuming perfect competition in production of the final aggregate, we get the usual CES demand:

$$x_{ni,t}(\nu) = S_{Y,nt}^{\eta^K - 1} M_{it}^{(\sigma^K - 1)\left(\phi_{Y,M} - \frac{1}{\sigma^K - 1}\right)} \omega_{ni}^{\sigma^K - 1} \left(\frac{p_{ni,t}(\nu)}{P_{ni,t}} \right)^{-\sigma^K} \left(\frac{P_{ni,t}}{P_{Y,nt}} \right)^{-\eta^K} Y_{nt}, \tag{10}$$

$$P_{ni,t} = M_{it}^{-\left(\phi_{Y,M} - \frac{1}{\sigma^K - 1}\right)} \left[\int_{\nu \in \Omega_{it}} p_{ni,t}(\nu)^{1 - \sigma^K} d\nu \right]^{\frac{1}{1 - \sigma^K}}, \tag{11}$$

$$P_{Y,nt} = S_{Y,nt}^{-1} \left[\sum_{i=1}^{N} \left(P_{ni,t} / \omega_{ni} \right)^{1 - \eta^K} \right]^{\frac{1}{1 - \eta^K}}. \tag{12}$$

Production of variety $\nu \in \Omega_{nt}$ requires only labor and is given by

$$x_{nt}(\nu) = S^{\kappa}_{X,nt} l_{nt}(\nu), \tag{13}$$

where $l_{nt}(\nu)$ is the amount of labor used in production of variety ν, and $S^{\kappa}_{X,nt} \equiv \Theta_{X,n} Z_{X,nt} L^{\phi_{X,L}}_{X,nt}$ is the aggregate productivity in production of varieties.[20] The aggregate productivity $S^{\kappa}_{X,nt}$ consists of two parts: exogenous productivity, $\Theta_{X,n} Z_{X,nt}$, and endogenous productivity, $L^{\phi_{X,L}}_{X,nt}$. Here $\Theta_{X,n}$ is a normalization constant, $Z_{X,nt}$ is an exogenous shock, and $L_{X,nt}$ is the total amount of labor allocated to production of varieties in country n in period t. The endogenous part of the aggregate productivity is an additional source of external economies of scale (on top of the love-of-variety effect) and is taken by firms as given. Having this additional source of externality is critical for showing the full isomorphism with the unified model.

Producers of varieties ν are engaged in monopolistic competition. Hence, the price of variety $\nu \in \Omega_{it}$ is

$$p_{ni,t}(\nu) = \frac{\sigma^{\kappa}}{\sigma^{\kappa} - 1} \cdot \frac{\tau_{ni,t} W_{it}}{S^{\kappa}_{X,it}},$$

the bilateral price index is $P_{ni,t} = \tau_{ni,t} P_{X,it}$, where

$$P_{X,it} \equiv \frac{\sigma^{\kappa}}{\sigma^{\kappa} - 1} \cdot \frac{W_{it}}{\Theta_{X,i} Z_{X,it} M^{\phi_{Y,M}}_{it} L^{\phi_{X,L}}_{X,it}}, \tag{14}$$

and the share of expenditure of country n on country i's varieties is

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{1-\eta^{\kappa}}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj}\right)^{1-\eta^{\kappa}}}. \tag{15}$$

[20] In Appendix B.2 we consider a more general (nonlinear) technology $x_{nt}(\nu) = S^{\kappa}_{X,nt} l_{nt}(\nu)^{\gamma}$, with $\gamma > 0$. This generalization allows us to demonstrate clearly the difference between internal versus external economies of scale in labor in the Krugman model, without conceptually changing them. In particular, from the perspective of the unified model, larger γ increases the Cobb-Douglas share of labor as well as the strength of labor externality in production of intermediate goods, while decreasing the share of capital and the strength of capital externality. Therefore, on one front, internal scale economies are similar to external scale economies as they both affect labor externality. We in the main text, however, consider external economies in the generalized trade models for two reasons. First, introduction of parameter γ in an otherwise standard Krugman model still leaves the model restricted. We show in Appendix B.2 that having $\gamma \neq 1$ implies that $\psi_{X,K} = \frac{\alpha_{X,K}}{1-\alpha_{X,K}} \psi_{X,L}$. Thus, both $\psi_{X,K}$ and $\psi_{X,L}$ are restricted to have the same sign. This matters substantively because, as we will see in Section 5, in order to match the data, we need to decrease capital externality $\psi_{X,K}$ until it becomes sufficiently negative, while having positive and large enough labor externality $\psi_{X,L}$. Second, introducing nonlinear-technology in the Melitz model breaks the isomorphism of the Melitz model with the unified model (see Footnote 23). Therefore, we choose to have a more general and completely flexible way of adjusting capital and labor externalities that can be applied in various models.

The total expenditure of country n on country-i's varieties is given by $\mathcal{X}_{ni,t} = \lambda_{ni,t} P_{Y,nt} Y_{nt}$. Substituting expression for $P_{ni,t}$ into (12), we get

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N} \left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{1-\eta^K}\right]^{\frac{1}{1-\eta^K}}}{\Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y}}. \tag{16}$$

Here, $P_{X,it}$ can be interpreted as the price of the output of varieties in country i in period t.

Let $\mathcal{X}_{nt}$ denote the value of total output of varieties in country n in period t, and D_{nt} denote the average profit of country n's producers of varieties Ω_{nt}. We have

$$\mathcal{X}_{nt} = \frac{\sigma^K}{\sigma^K - 1} W_{nt} L_{X,nt} \quad \text{and} \quad D_{nt} = \frac{1}{\sigma^K} \cdot \frac{\mathcal{X}_{nt}}{M_{nt}}. \tag{17}$$

3.1.2 Entry and Exit of Producers of Varieties

In order to enter the economy, producer of a variety in country n in period t needs to pay sunk cost equal to $\frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}$, where $0 \leq \alpha_I \leq 1$, and $\widetilde{\Theta}_{I,n} Z_{I,nt}$ is an exogenous cost shifter. Paying this sunk cost involves hiring $L_{I,nt} = \alpha_I \frac{V_{nt}}{W_{nt}}$ units of labor and using $Y_{I,nt} = (1 - \alpha_I) \frac{V_{nt}}{P_{Y,nt}}$ units of the final aggregate, where V_{nt} is the value of a variety in country n in period t.[21]

In every period t, each country has an unbounded mass of prospective entrants (firms) into the production of varieties. Entry into the economy is free, and, therefore, the value of a variety is equal to the sunk cost of entry:

$$V_{nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}. \tag{18}$$

Timing is as follows. Firms entering in period t start producing in the next period. At the end of each period t, an exogenous fraction δ of the total mass of firms (i.e., a fraction δ of M_{nt}) exits. The probability of exit is the same for all firms regardless of their age. Since exit occurs at the end of a period, any firm that entered into the economy produces for at least one period. Let $M_{I,nt}$ denote the number of producers of varieties that enter into the country n's economy in period t. Given the described process of entry and exit of firms,

[21] In Appendix B.2 we derive the sunk cost by introducing an R&D sector and specifying an invention process for new varieties. Labor and final aggregate needed to pay the sunk cost of entry are interpreted as the production factors used in the R&D sector for the invention of varieties.

the law of motion of varieties is

$$M_{n,t+1} = (1-\delta)\, M_{nt} + M_{I,nt}. \tag{19}$$

All producers of varieties are owned by households. We turn next to their problem.

3.1.3 Households

Similarly to the unified model, households in country n maximize the expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \beta^t U(C_{nt}, L_{nt})$, by choosing consumption C_{nt}, supply of labor L_{nt}, the number of new varieties $M_{I,nt}$, and holdings of financial assets (if allowed). Constraints faced by the households are the budget constraint and the law of motion of varieties given by (19). The specification of the budget constraint depends on the financial markets structure. In the case of financial autarky the budget constraint is given by

$$P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = W_{nt} L_{nt} + D_{nt} M_{nt}.$$

The left-hand side of this expression contains household's expenditure in period t: the household spends its budget on consumption and entry of new firms. The right-hand side of this expression contains household's income in period t: it consists of labor income and profits of firms. In the case of the bond economy and complete markets the budget constraints can be written by adding the expenditure and income from financial assets in the same manner as it is done in the unified model in Appendix A.1.

3.1.4 Markets Clearing Conditions

All market clearing conditions are standard. Labor is used for production and invention of varieties, $L_{x,nt} + L_{I,nt} = L_{nt}$, demand for varieties is equal to supply, $\sum_{n=1}^{N} \mathcal{X}_{ni,t} = \mathcal{X}_{it}$, and the final aggregate is used for consumption and invention of varieties, $C_{nt} + Y_{I,nt} = Y_{nt}$. The complete set of equilibrium conditions for the generalized Krugman model is provided in Appendix B.2.

3.2 Generalized Dynamic Version of the Melitz Model

Production side of the Melitz model is similar to the production side of the Krugman model in using only labor in production of intermediate goods, featuring monopolistic competition, and having sunk costs of entry into the economy. Additional features of the Melitz model are heterogeneous firms with Pareto distribution of efficiencies of production and the requirement that firms pay fixed costs of serving markets.

3.2.1 Production of Varieties, International Trade, and Final Aggregate

In every period t, country i can produce any of the varieties from an endogenously determined set of varieties Ω_{it} with measure M_{it}. All varieties from the set Ω_{it} can be internationally traded, but not all of them are available in a particular country n. The subset of country-i's varieties available in country n is denoted by $\Omega_{ni,t}$ (with $\Omega_{ni,t} \subseteq \Omega_{it}$), and its measure is denoted by $M_{ni,t}$. Subsets of varieties $\Omega_{ni,t}$ are endogenously determined. Importantly, only a subset $\Omega_{ii,t}$ of the whole set of varieties Ω_{it} is available in the domestic market i, and, generally, some varieties from Ω_{it} are not available in any country. Moreover, in general it can happen that some varieties from Ω_{it} are available in country $n \neq i$, but not in country i.

In order to sell in the country-n's market, a country-i's producer of a variety has to pay two types of costs: the usual per-unit iceberg trade costs $\tau^{\mathrm{M}}_{ni,t}$ and fixed cost $\Phi_{ni,t} > 0$, which are paid in terms of country-n's labor. The fixed cost $\Phi_{ni,t}$ is an endogenous object. Its formal definition is introduced later.

As in the Krugman model, countries combine varieties to produce non-traded final aggregates using the nested CES technology,

$$Y_{nt} = \left[\sum_{i=1}^{N} \left[\int_{\nu \in \Omega_{ni,t}} \left(\omega_{ni} x_{ni,t}\left(\nu\right) \right)^{\frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}}} d\nu \right]^{\frac{\sigma^{\mathrm{M}}}{\sigma^{\mathrm{M}}-1} \cdot \frac{\eta^{\mathrm{M}}-1}{\eta^{\mathrm{M}}}} \right]^{\frac{\eta^{\mathrm{M}}}{\eta^{\mathrm{M}}-1}}. \tag{20}$$

Differently from the Krugman model, we do not add correction for the love-of-variety effect in (20) — the reasons for this are discussed below in Section 4.2.[22] Also, (20), differently from (9), does not have an exogenous shock and external economies of scale. The reason for this is that the structure of the Melitz model endogenously generates both the exogenous shock and externalities in production of the final aggregate — both of these components of production function come from the fixed costs of serving markets, which are introduced below.

Perfect competition in production of the final aggregate implies the usual expressions for the CES demand that are almost the same as the corresponding expressions (10)-(12) in the Krugman model, except that there is no term correcting for the love of variety, and in the definition of $P_{ni,t}$ integration is over $\Omega_{ni,t}$ instead of Ω_{it}. For future reference, we

[22] Also, in Appendix B.3 we introduce the correction for the love-of-variety effect and formally explore implications of this correction.

provide the definition of $P_{ni,t}$,

$$P_{ni,t} = \left[\int_{\nu \in \Omega_{ni,t}} p_{ni,t} (\nu)^{1-\sigma^{\mathrm{M}}} d\nu \right]^{\frac{1}{1-\sigma^{\mathrm{M}}}}. \tag{21}$$

Production technology of variety $\nu \in \Omega_{it}$ is given by

$$x_{it} (\nu) = S^{\mathrm{M}}_{\mathrm{X},it} z_i (\nu)\, l_{it} (\nu), \tag{22}$$

where $l_{it} (\nu)$ is the amount of labor used in production of ν, $z_i (\nu)$ is the efficiency of production of ν, and $S^{\mathrm{M}}_{\mathrm{X},it} \equiv \Theta^{\mathrm{M}}_{\mathrm{X},i} Z_{\mathrm{X},it} \left[L^{\mathrm{M}}_{\mathrm{X},it} \right]^{\phi_{\mathrm{X},L}}$ is the aggregate productivity in production of varieties, with $L^{\mathrm{M}}_{\mathrm{X},it}$ being the total amount of labor used in production of varieties in country i.[23] As in the Krugman model, $S^{\mathrm{M}}_{\mathrm{X},it}$ features external economies of scale and is taken by firms as given. Monopolistic competition in production of varieties implies that the price of variety $\nu \in \Omega_{ni,t}$ is given by $p_{ni,t} (\nu) = \dfrac{\sigma^{\mathrm{M}}}{\sigma^{\mathrm{M}} - 1} \cdot \dfrac{\tau^{\mathrm{M}}_{ni,t} W_{it}}{S^{\mathrm{M}}_{\mathrm{X},it} z_i (\nu)}$.

3.2.2 Entry and Exit of Producers of Varieties

This part of the Melitz model is almost the same as the corresponding part of the Krugman model with one important difference that, upon entry, producer of a new variety in country n gets an idiosyncratic draw of efficiency of production, $z_n (\nu)$, from the Pareto distribution given by its cumulative distribution function with shape θ^{M} and minimal efficiency $z_{\min,n}$,

$$G_n (z) \equiv \mathrm{Prob} \left[z_n (\nu) \leq z \right] = 1 - \left(\frac{z_{\min,n}}{z} \right)^{\theta^{\mathrm{M}}}.$$

As in the Krugman model, the expected value of entry (before drawing the efficiency of production) is denoted by V_{nt}. The sunk cost of entry is equal to $\dfrac{W^{\alpha_I}_{nt} P^{1-\alpha_I}_{\mathrm{Y},nt}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}$. Assuming that entry is free, the sunk cost of entry is equalized with the expected value of entry in equilibrium,

$$V_{nt} = \frac{W^{\alpha_I}_{nt} P^{1-\alpha_I}_{\mathrm{Y},nt}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}. \tag{23}$$

The number of producers of varieties entering into the country n's economy in period t

[23]Differently from the Krugman model, having a more general (nonlinear) technology $x_{it} (\nu) = S^{\mathrm{M}}_{\mathrm{X},it} z_i (\nu)\, l_{it} (\nu)^{\gamma}$, with $\gamma > 0$, breaks the isomorphism of the Melitz model with the unified model even in a simple case of just two countries. In particular, a combination of such nonlinear technology with fixed costs of entry into markets generates variable trade elasticity that is a complicated function of other variables of the model. Also see Footnote 20 for the Krugman model.

is denoted by $M_{I,nt}$. The law of motion of varieties is $M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}$. Since the probability of exit is the same for all varieties $\nu \in \Omega_{nt}$, the distribution of efficiencies of production of varieties $\nu \in \Omega_{nt}$ in any period t is given by $G_n(z)$.

Under the assumption that efficiencies of production of varieties are distributed Pareto, we can derive that the set of country-i's varieties available in country n is given by

$$\Omega_{ni,t} = \left\{ \nu \in \Omega_{it} \,\middle|\, z_i(\nu) \geq z^*_{ni,t} \right\},$$

where $z^*_{ni,t}$ is given by

$$\left(\frac{z_{\min,i}}{z^*_{ni,t}} \right)^{\theta^{\mathrm{M}}} = \frac{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}{\theta^{\mathrm{M}} \sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{W_{nt} \Phi_{ni,t} M_{it}},$$

with $\mathcal{X}_{ni,t}$ being the total value of varieties that country n buys from i in period t.

3.2.3 Fixed Costs of Serving Markets

At this point we need to introduce the formal definition of the fixed costs of serving market n by firms from market i, $\Phi_{ni,t}$. Let $L_{F,nt}$ be the total amount of country n's labor that is used to pay the fixed costs of serving its market. We posit that

$$\Phi_{ni,t} \equiv \left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}} - \phi_{F,M}} L_{F,nt}^{\vartheta - \phi_{F,L}} \right]^{\frac{1}{\vartheta}} F_{ni,t}, \tag{24}$$

where $F_{ni,t}$ is an exogenous part of the fixed costs, $\left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}} - \phi_{F,M}} L_{F,nt}^{\vartheta - \phi_{F,L}} \right]^{\frac{1}{\vartheta}}$ is an endogenous part of the fixed costs that is taken by firms as given, and $\vartheta \equiv \frac{1}{\sigma^{\mathrm{M}} - 1} - \frac{1}{\theta^{\mathrm{M}}}$. Under the assumption that $\theta^{\mathrm{M}} > \sigma^{\mathrm{M}} - 1$, we have that $\vartheta > 0$. The term $\left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}} - \phi_{F,M}} L_{F,nt}^{\vartheta - \phi_{F,L}} \right]^{\frac{1}{\vartheta}}$ corrects for the externalities that arise due to the selection effects.[24] Parameter $\phi_{F,M}$ governs the strength of capital externality in production of intermediate goods in the corresponding unified model, while parameter $\phi_{F,L}$ governs the strength of externality in production of the final aggregate in the corresponding unified model.

Under the assumption (24) on the form on fixed costs of serving markets, we can

[24] Selection effects are the changes in the decomposition of country i's firms serving country n in response to changes in market conditions in country n. A detailed explanation is in Section 4.2 later.

derive that the bilateral price index is $P_{ni,t} = \tau^{M}_{ni,t} P_{X,it}$, where

$$P_{X,it} = \frac{\sigma^{M}}{\sigma^{M}-1} \cdot \frac{W_{it}}{z_{\min,i} \Theta^{M}_{X,i} Z_{X,it} M_{it}^{\phi_{F,M}} \left[L^{M}_{X,it} \right]^{\phi_{X,L}}} \tag{25}$$

is interpreted as the price of the output of varieties in country i in period t. The price of the final aggregate is

$$P_{Y,nt} = \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}} \right)^{-\frac{1}{\sigma^{M}-1}+\phi_{F,L}} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^{M} W_{nt}} \right)^{-\phi_{F,L}} \left[\sum_{i=1}^{N} \left(F^{\vartheta}_{ni,t} \tau^{M}_{ni,t} P_{X,it} / \omega_{ni} \right)^{-\theta^{M}\xi} \right]^{-\frac{1}{\theta^{M}\xi}}, \tag{26}$$

where

$$\xi \equiv \frac{1}{\left(\frac{1}{\eta^{M}-1} - \frac{1}{\sigma^{M}-1} \right) \theta^{M} + 1}, \tag{27}$$

and the share of expenditure of country n on country i's varieties is

$$\lambda_{ni,t} = \frac{\left(F^{\vartheta}_{ni,t} \tau^{M}_{ni,t} P_{X,it} / \omega_{ni} \right)^{-\theta^{M}\xi}}{\sum_{l=1}^{N} \left(F^{\vartheta}_{nl,t} \tau^{M}_{nl,t} P_{X,lt} / \omega_{nl} \right)^{-\theta^{M}\xi}}. \tag{28}$$

The total expenditure of country n on country-i's varieties is given by $\mathcal{X}_{ni,t} = \lambda_{ni,t} P_{Y,nt} Y_{nt}$. The value of total output of varieties in country n, $\mathcal{X}_{nt}$, and total average profits of country n's producers of varieties, D_{nt}, are given by

$$\mathcal{X}_{nt} = \frac{\sigma^{M}}{\sigma^{M}-1} W_{nt} L^{M}_{X,nt} \quad \text{and} \quad D_{nt} = \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}} \cdot \frac{\mathcal{X}_{nt}}{M_{nt}}. \tag{29}$$

The total amount of country n's labor used to serve its market is $L_{F,nt} = \frac{\theta^{M}+1-\sigma^{M}}{\theta^{M}\sigma^{M}} \cdot \frac{P_{Y,nt} Y_{nt}}{W_{nt}}$, which, using $P_{Y,nt} Y_{nt} = \mathcal{X}_{nt} - P_{Y,nt} \cdot \mathrm{TB}_{nt}$ and the above expression for $\mathcal{X}_{nt}$, can also be written as

$$L_{F,nt} = \left(\frac{1}{\sigma^{M}-1} - \frac{1}{\theta^{M}} \right) L^{M}_{X,nt} - \frac{\theta^{M}+1-\sigma^{M}}{\theta^{M}\sigma^{M}} \cdot \frac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{W_{nt}}. \tag{30}$$

3.2.4 Household's Problem and Markets Clearing Conditions

The household's problem is identical to the one in the Krugman model. Labor market clearing condition is different from the corresponding condition in the Krugman model — it involves labor used for serving markets, $L_{F,nt}$,

$$L^{\text{M}}_{X,nt} + L_{F,nt} + L_{I,nt} = L_{nt}. \tag{31}$$

The other conditions are the same as in the Krugman model: $\sum_{n=1}^{N} \mathcal{X}_{ni,t} = \mathcal{X}_{it}$ and $C_{nt} + Y_{I,nt} = Y_{nt}$.

The complete set of equilibrium conditions for the generalized Melitz model is provided in Appendix B.3.

4 Theoretical Results

We now formulate our main theoretical result: isomorphisms between the unified model of Section 2 and the dynamic trade models of Section 3. In the rest of this section, for brevity, when there is no risk of confusion, we refer to the generalized dynamic international trade models of Section 3 simply as "the Krugman model" and "the Melitz model".

4.1 Formal Characterization

The key results in establishing the link between the unified model of Section 2 and the models of Section 3 are the following three lemmas.

Lemma 1. *By an appropriate relabeling of variables and parameters, the price of country n's output of varieties in the Krugman and Melitz models — given, correspondingly, by expressions* (14) *and* (25) *— can be written as the price of country n's intermediates in the unified model given by expression* (2)*.*

Proof. Consider the Krugman model first. Expression (14) for $P_{X,nt}$ can be written as

$$P_{X,nt} = \left(1 - \frac{1}{\sigma^{\text{K}}}\right)^{-1} \frac{D_{nt}^{\frac{1}{\sigma^{\text{K}}}} W_{nt}^{1-\frac{1}{\sigma^{\text{K}}}}}{\Theta_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M}} L_{X,nt}^{\phi_{X,L}} D_{nt}^{\frac{1}{\sigma^{\text{K}}}} W_{nt}^{-\frac{1}{\sigma^{\text{K}}}}},$$

while expressions (17) can be written as $W_{nt} = \left(1 - \frac{1}{\sigma^{\text{K}}}\right) \mathcal{X}_{nt} / L_{X,nt}$ and $D_{nt} = \frac{1}{\sigma^{\text{K}}} \mathcal{X}_{nt} / M_{nt}$. Substituting W_{nt} and D_{nt} into the denominator of expression for $P_{X,nt}$, we get

$$P_{X,nt} = \frac{D_{nt}^{\frac{1}{\sigma^{\text{K}}}} W_{nt}^{1-\frac{1}{\sigma^{\text{K}}}}}{\widetilde{\Theta}^{\text{K}}_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M}-\frac{1}{\sigma^{\text{K}}}} L_{X,nt}^{\phi_{X,L}+\frac{1}{\sigma^{\text{K}}}}}, \tag{32}$$

where $\widetilde{\Theta}^{\text{K}}_{X,n}$ is a constant.

Now consider the Melitz model. Using the same manipulations as above for the Krugman model, we get that in the Melitz model expression (25) for $P_{X,nt}$ can be written as

$$P_{X,nt} = \frac{D_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} W_{nt}^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}}}{\widetilde{\widetilde{\Theta}}_{X,n}^{M} Z_{X,nt} M_{nt}^{\phi_{F,M}-\frac{\sigma^M-1}{\sigma^M\theta^M}} \left[L_{X,nt}^{M}\right]^{\phi_{X,L}+\frac{\sigma^M-1}{\sigma^M\theta^M}}},$$

where $\widetilde{\widetilde{\Theta}}_{X,n}^{M}$ is a constant. Defining

$$L_{X,nt} \equiv \left(\frac{\sigma^M}{\sigma^M-1} - \frac{1}{\theta^M}\right) L_{X,nt}^{M}, \tag{33}$$

we get that $P_{X,nt}$ can be written as

$$P_{X,nt} = \frac{D_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} W_{nt}^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}}}{\widetilde{\Theta}_{X,n}^{M} Z_{X,nt} M_{nt}^{\phi_{F,M}-\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{\phi_{X,L}+\frac{\sigma^M-1}{\sigma^M\theta^M}}}, \tag{34}$$

where $\widetilde{\Theta}_{X,n}^{M}$ is a constant.

By examining expressions (32) and (34), we see that they become identical to expression (2) for price in the unified model, if we (i) relabel variables D_{nt} as R_{nt} and M_{nt} as $K_{X,nt}$; (ii) map exponents of all variables in (32) and (34) to the corresponding exponents in (2); and (iii) multiply the amount of labor used in production of varieties in the Melitz model, $L_{X,nt}^{M}$, by $\left(\frac{\sigma^M}{\sigma^M-1} - \frac{1}{\theta^M}\right)$ to map it to the amount of labor used in production of intermediates in the unified model, $L_{X,nt}$. □

Informally, the average firms' profit in country n and the measure of country n's varieties in the Krugman and Melitz models play the role of, correspondingly, return on capital in country n and the stock of country n's capital in the unified model. The adjustment to $L_{X,nt}^{M}$ in the Melitz model has to be done because in the Melitz model — differently from the other models — there is an extra use of labor to pay fixed costs of serving markets. The labor used to pay fixed costs of serving markets can be written as

$$L_{F,nt} = \left(\frac{1}{\sigma^M-1} - \frac{1}{\theta^M}\right) L_{X,nt}^{M} - \frac{\theta^M+1-\sigma^M}{\theta^M\sigma^M} \cdot \frac{P_{Y,nt}\cdot \mathrm{TB}_{nt}}{W_{nt}}. \tag{35}$$

The sum of $L_{X,nt}^{M}$ and the first term on the right-hand side of the above expression gives the right-hand side of (33). The second term on the right-hand side of the above expression is mapped into the additional term on the right-hand side of the labor market clearing condition (8) in the unified model with

$$a = \frac{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}{\theta^{\mathrm{M}} \sigma^{\mathrm{M}}}.$$

Mappings between exponents in expressions (34) and (2) are summarized in Table 1 and discussed later in this section.

In order to formulate the next lemma, we need to introduce an additional assumption for the Melitz model:

Assumption 1. ***(Melitz)*** *(i)* $(F_{ni,t}/F_{nn,t})^{\vartheta}\,\tau^{\mathrm{M}}_{ni,t} \geq 1$ *for all* n, i *and all* t*; and (ii)* $(F_{nl,t}F_{li,t})^{\vartheta}\,\tau^{\mathrm{M}}_{nl,t}\tau^{\mathrm{M}}_{li,t} \geq (F_{ni,t}F_{nn,t})^{\vartheta}\,\tau^{\mathrm{M}}_{ni,t}$ *for all* n, l, i *and all* t.

Observe that, since $\vartheta = \frac{1}{\sigma^{\mathrm{M}}-1} - \frac{1}{\theta^{\mathrm{M}}}$ and $\theta^{\mathrm{M}} > \sigma^{\mathrm{M}} - 1$, we have that $\vartheta > 0$. So the sufficient conditions to guarantee Assumption 1 are (i) $F_{ni,t} \geq F_{nn,t}$ for all n, i and all t; and (ii) $F_{nl,t}F_{li,t} \geq F_{ni,t}F_{nn,t}$ for all n, l, i and all t.

Lemma 2. *By an appropriate relabeling of variables and parameters, the price of country n's final aggregate in the Krugman model given — by expression (16) — can be written as the price of country n's final aggregate in the unified model given by expression (4). Moreover, under Assumption 1, the price of country n's final aggregate in the Melitz model — given by expression (26) — can also be written as the price of country n's final aggregate in the unified model.*

Proof. Comparing expression (16) for the Krugman model with expression (4) for the unified model, we see that they are almost identical. The only difference is in the exponents of the aggregators of the CES price indices.

One way to achieve a mapping between expression (26) for the Melitz model and (4) for the unified model is by making two redefinitions in the Melitz model. First, we can redefine iceberg trade cost as

$$\tau_{ni,t} \equiv \left(\frac{F_{ni,t}}{F_{nn,t}}\right)^{\vartheta} \tau^{\mathrm{M}}_{ni,t}.$$

Assumption 1 guarantees that $\tau_{ni,t}$ defined this way are, indeed, iceberg trade costs that satisfy the no-arbitrage condition. Second, we can write $F^{-\vartheta}_{nn,t} = \Theta^{\mathrm{M}}_{Y,n} Z_{Y,nt}$ and define

$$\Theta_{Y,n} \equiv \left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}\right)^{\frac{1}{\sigma^{\mathrm{M}}-1} - \phi_{F,L}} \left[\sigma^{\mathrm{M}}\right]^{-\phi_{F,L}} \Theta^{\mathrm{M}}_{Y,n}. \tag{36}$$

Then we get expression for $P_{Y,nt}$ in the Melitz model that is almost identical to (4). Again, the only difference is in the exponents of the aggregators of the CES price indices. □

Mappings between exponents in expressions (16) and (26) for the Krugman and Melitz models and expression (4) for the unified model are summarized in Table 1 and discussed later in this section.

Lemma 3. *By an appropriate relabeling of variables and parameters, the value of a variety before entry in the economy in the Melitz and Krugman models — given, correspondingly, by expressions* (18) *and* (23) *— can be written as the price of country n's investment good in the unified model given by expression* (7)*.*

Proof. To prove the statement for the lemma, all we need to do is to relabel the value of a variety before entry in the economy, V_{nt}, as the price of the investment good in the unified model, $P_{I,nt}$. After this relabeling, expressions (18) and (23) become identical to (7). □

Lemmas 1-3 lead to our main theoretical result formulated in the next proposition.

Proposition 1. *By an appropriate relabeling of variables and parameters in the Krugman and Melitz models, and by making an additional Assumption 1 for the Melitz model, we can write the equilibrium system of equations in both models in a form identical to the equilibrium system of equations in the unified model. Thus, these models are isomorphic to each other in their aggregate predictions.*

Proof. Appendix B. □

Proposition 1 says that, up to relabeling, the generalized versions of the Krugman and Melitz models are essentially the same. Moreover, under certain parameterizations, these models are identical to a standard IRBC model extended to allow for external economies of scale in production and iceberg trade costs, despite having very different micro-foundations.

4.2 Economic Explanation

We now provide economic explanation of and detailed intuition for our results in Section 4.1. We start by formally introducing the standard Krugman and Melitz models. To do so, we describe parameter restrictions that need to be imposed in the generalized Krugman and Melitz models to obtain the standard versions of these models. In order to keep track of these restrictions, it helps to refer to Table 1.

In order to obtain the standard Krugman model from its generalized version, we first need to set the elasticity of substitution between varieties produced in different countries equal to the elasticity of substitution between varieties produced in one country (i.e., assume that $\eta^{K} = \sigma^{K}$). Second, we need to remove the correction for the love-of-variety effect in the production technology for the final aggregate by imposing $\phi_{Y,M} = \frac{1}{\sigma^{K}-1}$. Third,

Model	$\alpha_{X,K}$	$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y	α_I	Trade elasticity
Standard Krugman	$\frac{1}{\sigma^{K}}$	$\frac{1}{\sigma^{K}-1}-\frac{1}{\sigma^{K}}$	$\frac{1}{\sigma^{K}}$	0	1	$\sigma^{K}-1$
Standard Melitz	$\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}$	$\frac{1}{\sigma^{M}\theta^{M}}$	$\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}$	$\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}$	1	θ^{M}
Generalized Krugman	$\frac{1}{\sigma^{K}}$	$\phi_{Y,M}-\frac{1}{\sigma^{K}}$	$\phi_{X,L}+\frac{1}{\sigma^{K}}$	ψ_Y	α_I	$\eta^{K}-1$
Generalized Melitz	$\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}$	$\phi_{F,M}-\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}$	$\phi_{X,L}+\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}$	$\phi_{F,L}$	α_I	$\theta^{M}\xi$

Notes: $\alpha_{X,K}$ is the capital share in production of intermediates in the unified model. $\psi_{X,K}$ and $\psi_{X,L}$ are the scale elasticities of capital and labor in production of intermediates in the unified model. ψ_Y is the scale elasticity of real output of the final aggregate in production of the final aggregate in the unified model. σ^K and σ^M are the elasticities of substitution between varieties in the Melitz and Krugman models. θ^M is the shape of Pareto distribution in the Melitz model. $\phi_{Y,M}$ is the correction for the love-of-variety effect in the generalized Krugman model. $\phi_{X,L}$ is the scale elasticity of labor in production of varieties in the generalized Krugman and Melitz models. $\phi_{F,M}$ and $\phi_{F,L}$ are the scale elasticities of total measure of varieties and total amount of labor in fixed costs of serving markets in the generalized Melitz model. α_I is the labor share in production of the investment good in the unified model as well as the labor share in the cost of entry into the economy in the Krugman and Melitz models. Trade elasticity in the unified model is given by the exponent of $\tau_{ni,t}$ in expression (5). η^K is the elasticity of substitution between varieties produced by different countries in the Krugman model. $\xi=\left[\left(\frac{1}{\eta^{M}-1}-\frac{1}{\sigma^{M}-1}\right)\theta^{M}+1\right]^{-1}$.

Table 1: Parameter mappings between models

we need to shut down external economies of scale in production of varieties by setting $\phi_{X,L}=0$. And, fourth, we need to shut down external economies of scale and exogenous shocks in production of the final aggregate by imposing $S_{Y,nt}=1$.

Similarly, in order to obtain the standard Melitz model from its generalized version, we first need to set $\eta^M=\sigma^M$. Second, we need to remove correction for the externalities that arise due to the selection effects. This involves imposing $\phi_{F,M}=\frac{1}{\theta^M}$ and $\phi_{F,L}=\vartheta$. Third, we need to shut down external economies of scale in production of varieties by setting $\phi_{X,L}=0$.

For the rest of this section, we focus on explaining our results on isomorphisms of Proposition 1. One way to understand these results is to consider them from three perspectives: (i) the difference between the standard Krugman and Melitz models that we described above; (ii) the difference between the generalized Krugman and Melitz models we develop versus their standard counterparts; (iii) the difference between dynamic and static environments. We take these three perspectives next.

4.2.1 Standard Krugman versus Melitz Model

The key difference between standard Krugman and Melitz models is that in the Krugman model, externalities arise due to the love-of-variety effect, while in the Melitz model, they arise due to the selection effect. To see this clearly, as in Costinot and Rodríguez-Clare (2014), consider expressions for the price index of goods available in country n that are produced in country i. In the generalized Krugman and Melitz models, these price indices were defined in (11) and (21). It can be shown that in the standard Krugman model

$$P_{ni,t} = \kappa^{K} \times \underbrace{\tau_{ni,t} c^{K}_{X,it}}_{\text{Intensive Margin}} \times \underbrace{M_{it}^{\frac{1}{1-\sigma^{K}}}}_{\text{Extensive Margin: Entry}}, \tag{37}$$

while in the standard Melitz model

$$P_{ni,t} = \kappa^{M} \times \underbrace{\tau^{M}_{ni,t} c^{M}_{X,it}}_{\text{Intensive Margin}} \times \underbrace{\left(\left[\frac{\mathcal{Y}_{nt}}{W_{nt} F_{ni,t}} \right]^{\frac{1}{1-\sigma^{M}}} \frac{\tau^{M}_{ni,t} c^{M}_{X,it}}{P_{Y,nt}} \right)^{\frac{\theta^{M}}{\sigma^{M}-1}-1}}_{\text{Extensive Margin: Selection}} \times \underbrace{M_{it}^{\frac{1}{1-\sigma^{M}}}}_{\text{Extensive Margin: Entry}}, \tag{38}$$

where $\mathcal{Y}_{nt} \equiv P_{Y,nt} Y_{nt}$ is the total expenditure in n, $c^{K}_{X,it} \equiv W_{it} \big/ S^{K}_{X,it}$ and $c^{M}_{X,it} \equiv W_{it} \big/ \left(z_{\min,i} S^{M}_{X,it} \right)$, and κ^{K} and κ^{M} are constants. Here, changes at the *intensive* margin are changes in the prices of goods that n buys from i, $p_{ni,t}(\nu)$; while changes at the *extensive* margin are changes in the number and/or decomposition of firms from i that sell their goods in n (i.e., changes in Ω_{it} in the Krugman model and $\Omega_{ni,t}$ in the Melitz model).[25]

Expressions (37) and (38) look almost identical to the corresponding expressions in the static environment and capture similar effects.[26] They show that in the Krugman model an increase in M_{it}, everything else equal, leads to a fall in the price index with elasticity

[25] Sometimes the trade literature uses the terms the "intensive" and "extensive" margin to refer to changes in bilateral trade flows ($\mathcal{X}_{ni,t}$ in the notation of the current paper) rather than price indices (e.g., Chaney, 2008). The price indices' perspective is more insightful for our purposes as it allows us to see clearly the sources of externalities and, moreover, our key results to showing isomorphisms across models are Lemmas 1 and 2 on mappings of prices.

[26] See e.g., equation (14) in Costinot and Rodríguez-Clare (2014). Briefly, in both the Krugman and Melitz models, the price index is affected at the *intensive* margin by variable costs of production, as captured by $\tau_{ni,t} c^{K}_{X,it}$ and $\tau^{M}_{ni,t} c^{M}_{X,it}$. Also, in both models, the price index is affected at the *extensive* margin by entry of firms, as captured by terms $M_{it}^{1/(1-\sigma^{K})}$ and $M_{it}^{1/(1-\sigma^{M})}$. That is, higher entry lowers the price index in both models. The Melitz model features an additional term that also affects the *extensive* margin by causing entry/exit of firms into exporting activities in response to changes in market conditions captured by changes in $\mathcal{Y}_{nt}$, $W_{nt} F_{ni,t}$, and/or $\tau^{M}_{ni,t} c^{M}_{X,it} \big/ P_{Y,nt}$. Finally, we note that while some studies (e.g., Chaney, 2008) are interested only in how variable and fixed costs of trade ($\tau_{ni,t}$, $\tau^{M}_{ni,t}$, and $F_{ni,t}$) impact the intensive and extensive margins, we are interested in understanding all sources of changes in these margins.

$1/\left(\sigma^{K}-1\right)$, which is the love-of-variety effect. In the Melitz model, however, selection effects are in operation, and we can distinguish between two selection effects.

The first selection effect in the Melitz model is triggered by changes in M_{it}: an increase in M_{it} leads to a fall in $P_{ni,t}$, implying a fall in $P_{Y,nt}$, which, in turn, leads to an increase in the relative cost of serving market n with goods from country i. This increase in the cost is reflected by the term $\tau^{M}_{ni,t}c^{M}_{X,it}\Big/P_{Y,nt}$ of (38), and it implies that a set of the least productive firms from i can no longer serve market n, and so they exit from n. The number of varieties dropped is such that the total amount of varieties left available in n, $M_{ni,t}$, is unchanged.[27] Since the remaining varieties have higher average efficiency relative to the previously available set of varieties, the price index $P_{ni,t}$ settles at a lower level with elasticity $1/\theta^{M}$ with respect to M_{it} (as opposed to elasticity $1/\left(\sigma^{K}-1\right)$ in the Krugman model).[28]

The second selection effect in the Melitz model is triggered by changes in $\mathcal{Y}_{nt}/(W_{nt}F_{ni,t})$ as given in (38): everything else equal, an increase in n's market size, $\mathcal{Y}_{nt}$, or a fall in the costs of entry into market n, $W_{nt}F_{ni,t}$, cause more exporters from i to enter market n. Since the total number of firms in i, M_{it}, is kept unchanged, these new exporters are the less productive firms from i that previously were unable to earn positive profits in n. Entry of these exporters results in a fall in $P_{ni,t}$. This, as in the first selection effect, implies a fall in $P_{Y,nt}$ and, thus, an increase in the relative cost of serving market n with goods from country i, which leads to exit of the least productive firms from i that have just entered into n. The price index $P_{ni,t}$ settles at a lower level with elasticity $\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}$.[29] Here, the term $\frac{1}{\sigma^{M}-1}$ reflects the love-of-variety effect (price index falls because of the entry of new varieties), while the term $\frac{1}{\theta^{M}}$ corrects for the fact that the new varieties have lower productivity than the average productivity of the original set of varieties.

The love-of-variety effect in the Krugman model and the first selection effect in the Melitz model are the sources of the externalities in production of intermediate goods. This is the reason why, from the perspective of the unified model, the sum of capital and labor externalities in production of intermediate goods (i.e., $\psi_{X,K}+\psi_{X,L}$) is $1/\left(\sigma^{K}-1\right)$ in the standard Krugman model and $1/\theta^{M}$ in the standard Melitz model (see the first two rows of Table 1).[30] The source of the externality in production of the final aggregate in the Melitz

[27] In Appendix B.3 we show that in the standard Melitz model $M_{ni,t}=\frac{\theta^{M}+1-\sigma^{M}}{\theta^{M}\sigma^{M}}\,\mathcal{X}_{ni,t}/(W_{nt}F_{ni,t})$. Thus, indeed, M_{it} does not directly affect $M_{ni,t}$.

[28] That the elasticity of $P_{ni,t}$ with respect to M_{it} is $1/\theta^{M}$ can be seen by taking both sides of (38) to the power $(1-\sigma^{M})$, summing over i, and solving for $P^{1/\theta^{M}}_{Y,nt}$. Then, $P^{1/\theta^{M}}_{Y,nt}$ is a sum of N terms, which are the bilateral price indices taken to the power θ^{M}. Each of these price indices negatively depends on $M^{1/\theta^{M}}_{it}$.

[29] Again, that the elasticity of $P_{ni,t}$ with respect to $\mathcal{Y}_{nt}/(W_{nt}F_{ni,t})$ is $\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}$ can be seen by taking both sides of (38) to the power $(1-\sigma^{M})$, summing over i, and solving for $P^{1/\theta^{M}}_{Y,nt}$. See Footnote 28.

[30] This provides explanation for total externality only, and we explain the split between capital and labor, which plays a critical role in our dynamic setting, in Subsection 4.2.3.

model is the second selection effect, which is why in the corresponding unified model $\psi_Y = \frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}$. All of these externalities arise because firms do not internalize their impact on the extensive margin. The isomorphisms with the unified model are achieved by collapsing the extensive margin effects into the intensive margin.

The discussion above of the first selection effect in the Melitz model makes it clear why in the generalized Melitz model we do not have a correction for the love-of-variety effect in production technology of the final aggregate given by (20). To show this even more clearly, in Appendix B.3 we introduce the correction for the love-of-variety effect in (20) to explore its implications for the isomorphism. As expected, this correction does not affect the elasticity $1/\theta^M$ with which price $P_{ni,t}$ falls as M_{it} increases. This correction impacts the trade elasticity and the size of externality in production of the final aggregate (in this externality, the first term of $\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}$ is replaced by the parameter governing the strength of the love of variety). Moreover, it implies particular restrictions on the combination of the trade elasticity and the size of the final aggregate externality that can be used in the unified model to map it to the Melitz model. Thus, because of these restrictions, the correction for the love-of-variety effect does not allow a isomorphism between the Melitz and unified models. At the same time, the isomorphism can be achieved by correcting the selection effects directly: by introducing externalities in the fixed costs of serving markets, $\Phi_{ni,t}$, which is what we choose to do in the generalized Melitz model that we discuss next. We elaborate more on this point at the end of Appendix B.3.

4.2.2 Generalized versus Standard Dynamic Krugman and Melitz Models

As one can see from Table 1, in the standard Krugman model, the elasticity of substitution between varieties governs four out of five key parameters of the corresponding unified model: the share of capital in production of intermediates, $\alpha_{X,K}$; strengths of economies of scale in production of intermediates, given by $\psi_{X,K}$ for capital and $\psi_{X,L}$ for labor; and trade elasticity, given by the (minus of) exponent of $\tau_{ni,t}$ in expression (5) for trade shares. Thus, the standard Krugman model implies tight links between key parameters of the corresponding unified model. The modeling assumptions of the generalized Krugman model of Section 3.1 allow us to break these tight links.

In the generalized Krugman model, trade elasticity is given by the (minus of) exponent of $\tau_{ni,t}$ in expression (15) for trade shares and is equal to $(\eta^K - 1)$. By assuming that $\eta^K \neq \sigma^K$, we break the link between σ^K and trade elasticity.[31] By introducing correction for the

[31] A combination of the nested CES production technology with the monopolistic competition environment is also used in Alessandria and Choi (2007), Jaef and Lopez (2014), Feenstra *et al.* (2018), and Kucheryavyy *et al.* (2019), among others.

love-of-variety effect in the generalized Krugman model — by assuming that $\phi_{Y,M} \neq \frac{1}{\sigma^K - 1}$ — we break the tight link between σ^K and the strength of economies of scale for capital. We can get any desired value of the parameter $\psi_{X,K}$ in the unified model by varying $\phi_{Y,M}$. The correction for the love-of-variety effect however, does not break the link between σ^K and the strength of economies of scale for labor. To break this last link, we directly introduce external economies of scale in the technology of production of varieties given by (13) — with the strength of these economies of scale given by the parameter $\phi_{X,L}$. With this generalization, we can get any desired level of the strength of economies of scale for labor in production of intermediates in the unified model.

Let us now turn to the Melitz model. Two parameters of the standard Melitz model — elasticity of substitution between varieties, σ^M, and the shape of Pareto distribution, θ^M, govern the five key parameters of the corresponding unified model: $\alpha_{X,K}$, $\psi_{X,K}$, $\psi_{X,L}$, ψ_Y, and trade elasticity. Thus, as is the case with the standard Krugman model, the standard Melitz model implies tight links between these key parameters of the corresponding unified model. Again, the modeling assumptions of the generalized Melitz model of Section 3.2 allow us to break these tight links.

In the generalized Melitz model, trade elasticity is given by the (minus of) exponent of $\tau_{ni,t}$ in expression (28) for trade shares and is equal to $\theta^M \xi$, while in the standard Melitz model trade elasticity is equal to θ^M. By assuming that $\eta^M \neq \sigma^M$, we break the link between θ^M and trade elasticity. By introducing correction for the first selection effect in the Melitz model — by assuming that $\phi_{F,M} \neq \frac{1}{\theta^M}$ — we break the tight link between θ^M and the strength of economies of scale for capital. We can get any desired value of parameter $\psi_{X,K}$ in the unified model by varying $\phi_{F,M}$.

Similar to the correction for the love-of-variety effect in the Krugman model correction for the first selection effect in the Melitz model, however, does not break the link between θ^M and σ^M on the one hand and the strength of economies of scale for labor on the other. To break this link, we directly introduce external economies of scale in the technology of production of varieties given by (22) — with the strength of these economies of scale given by the parameter $\phi_{X,L}$. With this generalization, we can get any desired level of the strength of economies of scale for labor in production of intermediates in the unified model. Finally, by introducing correction for the second selection effect in the Melitz model — by assuming that $\phi_{F,L} \neq \vartheta$ — we break the tight link between θ^M and σ^M on the one hand and the strength of externality in production of the final aggregate on the other. By varying $\phi_{F,L}$, we can get any value of parameter ψ_Y in the unified model.

4.2.3 Dynamic versus Static Environment

At this point the reader might wonder what is new in the dynamic environment relative to the static environment. We explain now how the dynamic environment in fact has non-trivial implications for the isomorphisms as it brings to light the split between capital and labor (and the corresponding capital and labor externalities) as well as the final good externality — features that are typically (implicitly) assumed away in static environments (as, e.g., in Kucheryavyy *et al.* (2019)).

Consider first the Krugman model. The proof of Lemma 1 shows that the split between W_{nt} and D_{nt} in $P_{x,nt}$ takes that form because labor gets $\left(1 - \frac{1}{\sigma^K}\right)$ share of total revenue, while the remaining $\frac{1}{\sigma^K}$ share of revenue are profits of firms. These shares are constant as firms charge constant markups over costs, due to CES preferences. Since labor is the only factor of production of varieties, and the technology of production is linear in labor, from the perspective of the unified model it is as if the representative firm uses only $\left(1 - \frac{1}{\sigma^K}\right)$ log-share of labor in its technology, while the remaining $\frac{1}{\sigma^K}$ log-share of labor induces a technological externality. Next, from the perspective of the unified model, the technology of production of intermediate goods uses capital to the power of $\frac{1}{\sigma^K-1}$ (which is the love-of-variety effect). Part of this capital is internalized by firms in terms of $\frac{1}{\sigma^K}$ share of revenue, while the remaining part — equal to $\left(\frac{1}{\sigma^K-1} - \frac{1}{\sigma^K}\right)$ — induces an externality.

Now consider the Melitz model. Similar to the Krugman model, the split between capital and labor in production technology for intermediate goods in the Melitz model arises because labor gets $\left(1 - \frac{\sigma^M-1}{\sigma^M\theta^M}\right)$ share of total revenue with the remaining $\frac{\sigma^M-1}{\sigma^M\theta^M}$ share accruing as profits of firms. Again, since the technology of production is linear in labor, from the perspective of the unified model, the remaining $\frac{\sigma^M-1}{\sigma^M\theta^M}$ log-share of labor induces a technological externality. The technology of production of intermediate goods uses capital to the power of $\frac{1}{\theta^M}$ (which is the selection effect). Out of this capital, the $\frac{\sigma^M-1}{\sigma^M\theta^M}$ log-share is internalized by firms, while the remaining part — equal to $\frac{1}{\theta^M} - \frac{\sigma^M-1}{\sigma^M\theta^M} = \frac{1}{\sigma^M\theta^M}$ —induces an externality.

The split between capital and labor in production of intermediate goods is absent from the corresponding results in the literature on isomorphisms in the static environment with free entry of firms, where costs of entry are assumed to be paid in terms of labor only (a typical assumption). To understand the reason, observe that (17) and (29) imply that total firms' profits in the dynamic Krugman and Melitz models are given by $D_{nt}M_{nt} = \frac{1}{\sigma^K-1}W_{nt}L_{x,nt}$ and $D_{nt}M_{nt} = \frac{1}{\theta^M}W_{nt}L^M_{x,nt}$, respectively. In the static versions of these models, expressions for profits are the same (the time index is irrelevant). In addition to these expressions, in both the static Krugman and Melitz models where the cost of entry per firm, $\Theta_{I,n}^{-1}$, is paid in terms of labor only, the total cost of entry is equal to $\Theta_{I,n}^{-1} W_{nt} M_{nt}$. Invoking the free entry condition and equating the total costs of entry with profits, we get standard results for static Krugman and Melitz models that the number of firms is proportional to the total labor used in production of varieties. Thus, in this case there is no split between capital and labor in production of intermediate goods.

If the costs of entry are paid in terms of the final good — as we assume in most of our quantitative, and all estimation, exercises in Section 5 — then even the static environment would feature a split between capital and labor. This is because, when the costs of entry are paid in terms of the final good, both in the Krugman and Melitz models, the total costs of entry are equal to $\Theta_{I,n}^{-1} P_{Y,nt} M_{nt}$, while the expressions for total profits, $D_{nt} M_{nt}$, are not affected. Therefore, in this case, $M_{nt} = \frac{1}{\sigma^{K}-1} \Theta_{I,n} \left(W_{nt}/P_{Y,nt} \right) L_{X,nt}$ in the Krugman model and $M_{nt} = \frac{1}{\theta^{M}} \Theta_{I,n} \left(W_{nt}/P_{Y,nt} \right) L_{X,nt}^{M}$ in the Melitz model, and so the relationship between the number of firms and labor is affected by the real wage $W_{nt}/P_{Y,nt}$.

In our dynamic environment, the number of firms is a state variable, while the costs of entry are paid only by the firms entering the economy in the current period, $M_{I,nt}$. Thus, at a given time period, there is no fixed relationship between the total number of firms in the economy and the total amount of labor used in production of varieties, even when entry costs are paid in terms of labor. In Section 5, we show that the capital and labor externalities are both qualitatively and quantitatively different when we assess implications for transmission of shocks. Thus this difference is not just a theoretical curiosity, but also has first-order implications for the substantive business cycle questions of our paper.

Another important element of our results is the final good externality in the Melitz model, which also plays a unique role in a dynamic environment. While this externality in principle can arise even in the static environment featuring endogenous (elastic) labor supply, it has a non-trivial behavior in the dynamic environment due to trade imbalances and, in the case of investment done in terms of labor, due to period-by-period firm entry decisions (or, equivalently, capital accumulation decisions). To understand these points, let us write the expression for the final good externality as

$$\frac{P_{Y,nt} Y_{nt}}{W_{nt}} = \frac{\theta^{M} \sigma^{M}}{\theta^{M} \sigma^{M} + 1 - \sigma^{M}} \left(L_{nt} - L_{I,nt} \right) - \frac{\theta^{M} \left(\sigma^{M} - 1 \right)}{\theta^{M} \sigma^{M} + 1 - \sigma^{M}} \cdot \frac{\mathrm{TB}_{nt} \cdot P_{Y,nt}}{W_{nt}}, \tag{39}$$

where we used $P_{Y,nt} Y_{nt} = \mathcal{X}_{nt} - P_{Y,nt} \cdot \mathrm{TB}_{nt}$, and additionally used (29) for $\mathcal{X}_{nt}$, (30) for $L_{F,nt}$, and the labor market clearing condition (31).

Consider then the steady state of the dynamic version of the Melitz model. In the steady state, $\mathrm{TB}_{nt} = 0$. If investment is done in terms of final good only, then $L_{I,nt} = 0$, and expression (39) implies that the final good externality is proportional to L_{nt}. Alternatively, if investment is done in terms of labor only, then in the steady state (assuming $\beta = 1$ and $\delta = 1$ for simplicity), $L_{I,nt} = \frac{1}{\theta^{M}} L^{M}_{X,nt}$, which, using (39), allows us to show that $P_{Y,nt} Y_{nt} / W_{nt} = \frac{\sigma^{M}}{\sigma^{M}-1} L^{M}_{X,nt}$. Thus, again, the final good externality is proportional to L_{nt}. Therefore, the final good externality would matter in the static Melitz model only if labor supply is endogenous. Otherwise the final good externality would be absorbed by a constant term in the production function for the final aggregate in the isomorphic static competitive model.

In the dynamic setting with borrowing and lending, $\text{TB}_{nt} \neq 0$ and, thus, the final good externality has a non-trivial behavior even if labor supply is inelastic and investment is done in terms of the final good only. If investment is done in terms of labor, then there is no fixed relationship between $L_{I,nt}$ and L_{nt}. Then, even under balanced trade (as in the case of financial autarky), and irrespective of whether labor supply is endogenous or inelastic, period-by-period firm entry decisions generate a potentially non-trivial behavior of the final good externality.

The upshot is that the precise formulations of the externalities depend on four factors: dynamic vs. static setting; endogenous vs. fixed labor supply; endogenous international borrowing/lending vs. balanced trade; and whether investment is done in terms of final good or labor. Our presentation on isomorphism in terms of various externalities is the most general as it nests all these various cases (as well as those that have appeared in the literature previously in static settings). Moreover, the general formulation we present, in particular the split between capital and labor externality, is also substantively important for our main quantitative questions, as we show next.

5 Quantitative Results

We now quantitatively assess the international business cycle implications of the dynamic trade models and show how the transmission mechanisms in response to shocks get altered compared to the standard IRBC model. We start by comparing the fit of the IRBC and standard dynamic Krugman and Melitz models. We then use the unified model of Section 2 to fully explore the ingredients needed to achieve better fit with the data, while explaining in detail the transmission mechanisms that change when we vary externalities. We then end with a quantitative exercise where we match a comprehensive set of domestic and international moments.

Common Parameters	$\beta = 0.99$, $\gamma = 2$, $\delta = 0.025$, $\mu = 0.34$, $\sigma = 2$, $\tau_{ni,t} = 5.67$, $\omega_{ni} = 0.5$, $\Theta_{X,n} = \Theta_{I,n} = 1$, $\Theta_{Y,n} = 2.069$, $b_{adj} = 0.0025$ Productivity process in the intermediate goods sector: $\begin{bmatrix} \log(Z_{X,1t}) \\ \log(Z_{X,2t}) \end{bmatrix} = \begin{bmatrix} \rho_{X,11} & 0 \\ 0 & \rho_{X,22} \end{bmatrix} \times \begin{bmatrix} \log(Z_{X,1,t-1}) \\ \log(Z_{X,2,t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon_{X,1t} \\ \varepsilon_{X,2t} \end{bmatrix}$, $\begin{bmatrix} \varepsilon_{X,1t} \\ \varepsilon_{X,2t} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{X,1} & 0 \\ 0 & \sigma^2_{X,2} \end{bmatrix} \right)$, with $\rho_{X,11} = \rho_{X,22} = 0.97$, $\sigma_{X,1} = \sigma_{X,2} = 0.0073$ Productivity process in the final goods sector, $Z_{Y,nt}$, has the same structure as $Z_{X,nt}$ with the same autocorrelation parameter values, $\rho_{Y,11} = \rho_{Y,22} = 0.97$, and normally distributed uncorrelated shocks with variances $\sigma_{Y,1} = \sigma_{Y,2} = 0.0073$
IRBC	$\alpha_{X,K} = 0.36$, $\psi_{X,K} = \psi_{X,L} = \psi_Y = 0$, $\alpha_I = 0$, $a = 0$, $Z_{I,nt} = Z_{Y,nt} = 1$
Krugman	$\alpha_{X,K} = \frac{1}{3.8} \approx 0.26$, $\psi_{X,K} = \frac{1}{3.8-1} - \frac{1}{3.8} \approx 0.094$, $\psi_{X,L} = \frac{1}{3.8} \approx 0.26$, $\psi_Y = 0$, $\alpha_I = 1$, $a = 0$, $Z_{I,nt} = Z_{X,nt}$, $Z_{Y,nt} = 1$
Melitz	$\alpha_{X,K} = \frac{3.8-1}{3.8*3.4} \approx 0.22$, $\psi_{X,K} = \frac{1}{3.8*3.4} \approx 0.077$, $\psi_{X,L} = \frac{3.8-1}{3.8*3.4} \approx 0.22$, $\psi_Y = \frac{1}{3.8-1} - \frac{1}{3.4} \approx 0.063$, $\alpha_I = 1$, $a = \frac{3.4+1-3.8}{3.8*3.4} \approx 0.046$, $Z_{I,nt} = Z_{X,nt}$, $Z_{Y,nt} = [Z_{X,nt}]^{\frac{1}{3.8-1} - \frac{1}{3.4}} \approx [Z_{X,nt}]^{0.063}$

Table 2: Standard calibrations of models.

5.1 Calibration

We focus on the world economy that consists of two symmetric countries for direct comparison with the business cycle literature. We consider the following preferences that are consistent with a balanced growth path and widely used in the literature (e.g., Heathcote and Perri (2002)):

$$U(C_{nt}, L_{nt}) = \frac{1}{1-\gamma} \left[C^{\mu}_{nt} (1 - L_{nt})^{1-\mu} \right]^{1-\gamma}.$$

We start with a calibration that we call "standard". It is summarized in Table 2. For this calibration we choose three sets of parameter values of the unified model that correspond to standard IRBC, Krugman, and Melitz models. We choose parameter values of the unified model corresponding to the Krugman and Melitz models so that in the Krugman and Melitz models, all generalizations are shut down except that we allow for the nested CES production technology of the final aggregate.[32] Formally, the implied parameterization for the Krugman model is $\phi_{Y,M} = \dfrac{1}{\sigma^{K} - 1}$ and $\phi_{X,L} = 0$, but allowing for $\eta^{K} \neq \sigma^{K}$. Similarly, the implied parameterization for the Melitz model is $\phi_{F,M} = \dfrac{1}{\theta^{M}}$, $\phi_{F,L} = \vartheta$, and $\phi_{X,L} = 0$, but allowing for $\eta^{M} \neq \sigma^{M}$.

We first choose a set of common parameter values for the three models. Most of these values are taken from the literature. Periods are interpreted as quarters. Values of parameters β, γ, δ, and μ are the same as in, for example, Heathcote and Perri (2002) and Ghironi and Melitz (2005). We follow the macro literature (as apposed to the international trade literature) and set the elasticity of substitution between intermediate goods in production of the final good to 2, i.e., we set $\sigma = 2$. This implies that the trade elasticity is equal to 1.[33] We choose the level of iceberg trade costs $\tau_{ni,t} = 5.67$ for $n \neq i$ to match the steady-state share of imports of intermediate goods of 0.15. Differently from Heathcote and Perri (2002), we do not have home bias in production of the final aggregate and set $\omega_{ni} = 0.5$ for all n and i.[34] Values of autocorrelations $\rho_{X,11}$ and $\rho_{X,22}$ of the productivity process in the intermediate goods sector, $Z_{X,nt}$, as well as volatilities of shocks $\sigma_{X,1}$ and $\sigma_{X,2}$ to $Z_{X,nt}$ are taken from Heathcote and Perri (2002).[35] The productivity process in the final goods sector, $Z_{Y,nt}$, has the same parameterization as $Z_{X,nt}$.

We set the normalization constants in the intermediate goods and investment sectors to 1, $\Theta_{X,n} = \Theta_{I,n} = 1$. In order to match the value of fixed costs of serving foreign mar-

[32] With a slight abuse of terminology compared to the previous Section, we will refer to these models as standard Krugman and Melitz models in this Section.

[33] See, for example, Hillberry and Hummels (2013) on the choice between "macro" versus "micro" trade elasticity. Later we do a sensitivity analysis with lower (than 1) and higher elasticities of substitution.

[34] In the case of two symmetric countries, the steady state prices of intermediate goods are the same across the two countries: $P_{X,1} = P_{X,2}$ (here we drop the time index t to emphasize that these are the steady state values of prices). Therefore, the steady state trade share — obtained from (5) by substituting steady state values of prices of intermediate goods — is simply

$$\lambda_{ni} = \frac{(\tau_{ni}/\omega_{ni})^{1-\sigma}}{(\tau_{n1}/\omega_{n1})^{1-\sigma} + (\tau_{n2}/\omega_{n2})^{1-\sigma}}.$$

With the same values of taste parameters ω_{ni} across countries, the steady state trade share depends only on iceberg trade costs and parameter σ. In this case, we can find that $\tau_{12} = \tau_{21} = \left(\lambda_{12}^{-1} - 1\right)^{1/(\sigma-1)}$.

[35] Differently from Heathcote and Perri (2002), we do not allow for spillovers in the process for $Z_{X,nt}$, and we do not allow for correlation of shocks to $Z_{X,nt}$. We do this to ensure that the dynamics are driven by endogenous propagation mechanisms in the models. We later show sensitivity results when we allow for spillovers and correlation of shocks.

kets in Ghironi and Melitz (2005) (which is discussed below), we set the normalization constant in the final aggregates sector to 2.069, $\Theta_{Y,n} = 2.069$. Finally, for the case of the bond economy, we choose a relatively low value of the bond holdings adjustment cost, $b_{adj} = 0.0025$.

The values of the remaining parameters are different between the IRBC, Krugman, and Melitz models. For the IRBC model, we set the same share of capital in production of intermediate goods as in Heathcote and Perri (2002), $\alpha_{X,K} = 0.36$, and require that investment is made in terms of the final good only (i.e., set $\alpha_I = 1$). The IRBC model does not have any externalities ($\psi_{X,K} = \psi_{X,L} = \psi_Y = 0$), it does not have productivity shocks in the investment and final aggregate sectors ($Z_{I,nt} = Z_{Y,nt} = 1$), and it does not have the additional term $a\text{TB}_{nt}$ in the labor market clearing condition ($a = 0$).

For the parameterization corresponding to the Krugman model, we use the value of $\sigma^{\text{K}} = 3.8$ from Bilbiie *et al.* (2012). This choice immediately implies values for all key parameters specific to the Krugman model: $\alpha_{X,K} = \frac{1}{\sigma^{\text{K}}} \approx 0.26$, $\psi_{X,K} = \frac{1}{\sigma^{\text{K}} - 1} - \frac{1}{\sigma^{\text{K}}} \approx 0.094$, and $\psi_{X,L} = \frac{1}{\sigma^{\text{K}}} \approx 0.26$ (see Table 1 for parameter mappings between the models). The standard Krugman model has neither externalities nor productivity shocks in production of the final aggregate ($\psi_Y = 0$ and $Z_{Y,nt} = 1$), and it does not have the additional term $a\text{TB}_{nt}$ in the labor market clearing condition ($a = 0$). Investment is made in terms of labor only ($\alpha_I = 0$). We follow Bilbiie *et al.* (2012) in setting the productivity shock in production of investment goods identical to the productivity shock in production of intermediate goods ($Z_{I,nt} = Z_{X,nt}$). The choice of the investment-sector normalization constant $\Theta_{I,n} = 1$ implies that the sunk entry cost into the economy in the Krugman model — given by $\widetilde{\Theta}_{I,n}^{-1}$ — is also equal to 1.[36] Finally, trade elasticity equal to 1 in the unified model implies that the elasticity of substitution between varieties from different countries in the Krugman model is equal to $\eta^{\text{K}} = 2$.[37]

Turning to the parameterization corresponding to the Melitz model, let us first consider fixed and variable costs of serving markets in the Melitz model. We assume that in the Melitz model $F_{12,t} = F_{11,t}$ and $F_{21,t} = F_{22,t}$ for all t. This implies that $\tau^{\text{M}}_{ni,t} = \tau_{ni,t} = 5.67$. Following Ghironi and Melitz (2005), we further assume that the fixed costs of serving markets in the Melitz model are subject to the same shock as the production technology of varieties. Formally, we assume that $F_{nn,t} = f_{nn}/Z_{X,nt}$, where f_{nn} is a time-independent

[36] Bilbiie *et al.* (2012) also have the value of the sunk costs of entry into the economy equal to 1. As Bilbiie *et al.* (2012) note, this value does not affect any impulse-responses under CES preferences.

[37] Recall that having an independent parameter η^{K} in the generalized Krugman model — that is, differently from the standard Krugman model, having $\eta^{\text{K}} \neq \sigma^{\text{K}}$ — allows us to vary trade elasticity independently from the parameters of the production function for intermediate goods (see Table 1).

constant (defined below). We proved the part of Lemma 2 concerning the Melitz model by defining $F_{nn,t}^{-\vartheta} = \Theta_{Y,n}^{M} Z_{Y,nt}$. This definition implies that $Z_{Y,nt} = [Z_{X,nt}]^{\vartheta}$ and $f_{nn} = [\Theta_{Y,n}^{M}]^{-\frac{1}{\vartheta}}$. Using mapping (36), we find that the fixed costs of serving markets are given by

$$f_{nn} = \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{\frac{\sigma^{M}-1}{\theta^{M}+1-\sigma^{M}}} \frac{1}{\sigma^{M}} [\Theta_{Y,n}]^{-\frac{1}{\vartheta}}. \tag{40}$$

Next, following Ghironi and Melitz (2005), we choose $\sigma^{M} = 3.8$ (which is also the same as σ^{K}) and $\theta^{M} = 3.4$. The choices of σ^{M} and θ^{M} imply that $\alpha_{X,K} = \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}} \approx 0.22$, $\psi_{X,K} = \frac{1}{\sigma^{M}\theta^{M}} \approx 0.077$, $\psi_{X,L} = \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}} \approx 0.22$, $\psi_{Y} = \frac{1}{\sigma^{M}-1} - \frac{1}{\theta^{M}} \approx 0.063$, and $Z_{Y,nt} \approx [Z_{X,nt}]^{0.063}$. Using expression (40) we get that the implied value of the fixed costs of serving markets in the Melitz model is $f_{nn} \approx 0.0084$, which is the same as the fixed cost of serving *foreign* markets in Ghironi and Melitz (2005). The labor market clearing condition now features the additional term $aP_{Y,nt} \cdot \mathrm{TB}_{nt}$ with $a = \frac{\theta^{M}+1-\sigma^{M}}{\sigma^{M}\theta^{M}} \approx 0.046$. As in the calibration corresponding to the Krugman model, $Z_{I,nt} = Z_{X,nt}$ and $\alpha_{I} = 1$. The implied sunk entry cost into the economy is equal to 1. Finally, the choice of $\sigma = 2$ in the unified model implies that in the Melitz model the elasticity of substitution between varieties from different countries is equal to[38]

$$\eta^{M} = 1 + \left(\frac{1}{\sigma-1} + \vartheta\right)^{-1} \approx 1.94.$$

5.2 Comparison Across IRBC and Standard Dynamic Trade Models

Moments, both domestic and international, across models under the calibration in Table 2 are presented in Tables 3 (for the intermediate good productivity shock) and Table 4 (for the final good productivity shock).[39] Column (1) provides data moments from Heathcote and Perri (2002). In Columns (2) and (5), we present results for the standard IRBC model and in Columns (3), (6), and (4), (7) for standard versions of the Krugman and Melitz models respectively. We note that for both shocks, the domestic moments show the presence of a business cycle, that is a positive co-movement of within-country output, consumption, investment, and labor. They both are thus natural candidates for a study

[38]Similarly to the generalized Krugman model, having $\eta^{M} \neq \sigma^{M}$ in the generalized Melitz model allows us to vary trade elasticity independently from the parameters of the production function for intermediate goods (see Table 1 and Footnote 37).

[39]The final good productivity shock is new to the IRBC literature, and we will explain the results for this shock in detail later when presenting transmission mechanisms. We also note that we consider quite a comprehensive set of moments, and we further add more moments in the estimation exercise.

of international business cycle moments.[40] We report results for the two most common financial market arrangements in the literature, complete markets and bond economy. To conserve space, we report the financial autarky case in Table 8 in Appendix D.1.

		Benchmark calibration						Investment final good			
		Complete			Bond			Complete		Bond	
	Data	IRBC	Krug	Mel	IRBC	Krug	Mel	Krug	Mel	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
International moments:											
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	0.58	−0.03	−0.10	−0.09	0.02	−0.02	−0.02	−0.09	−0.11	0.01	−0.03
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.47	0.45	0.45	0.11	0.19	0.21	0.41	0.37	0.15	0.15
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.39	−0.27	−0.26	−0.35	−0.19	−0.18	−0.41	−0.44	−0.34	−0.38
$\text{Corr}\left(L_1, L_2\right)$	0.42	−0.30	−0.45	−0.46	−0.04	−0.21	−0.23	−0.40	−0.43	−0.11	−0.19
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.49	0.58	0.61	−0.60	−0.01	0.19	−0.25	−0.20	−0.53	−0.50
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.32	0.36	0.85	0.88	0.13	0.64	0.77	0.60	0.60	0.32	0.35
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.81	0.93	0.25	0.25	0.96	0.78	0.76	0.86	0.84	0.94	0.93
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.13	0.61	0.64	0.67	0.50	0.52	0.60	0.68	0.69	0.60	0.64
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.21	0.16	0.17	0.27	0.08	0.08	0.19	0.21	0.24	0.25
Domestic moments:											
$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.86	0.94	0.93	0.94	0.98	0.98	0.98	0.95	0.95	0.98	0.98
$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.87	0.98	0.96	0.96	0.99	0.97	0.97	0.98	0.98	0.99	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.97	0.97	0.97	0.97	0.97	0.97	0.96	0.96	0.96	0.96
$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.81	0.37	0.45	0.47	0.43	0.52	0.54	0.42	0.43	0.48	0.49
$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.66	0.46	0.51	0.47	0.40	0.44	0.41	0.42	0.41	0.36	0.36
$\frac{\text{Std}\left(P_{I,1}I_1/P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	2.84	3.33	3.80	4.20	3.29	3.75	4.13	3.94	4.61	3.91	4.57

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. For Columns (8)-(11), parameterizations are $\alpha_I = 0$ and $Z_{I,nt} = 1$ and $Z_{Y,nt} = 1$ (i.e., there are no shocks to the investment and final good sectors).

Table 3: Moments from standard calibrations and formulations of models. Shock to the intermediate goods sector.

[40]Later in our estimation exercise, as well as in our discussion of transmission using impulse responses, we will discuss how the two shocks affect some international moments differently.

		Benchmark calibration						Investment final good			
		Complete			Bond			Complete		Bond	
	Data	IRBC	Krug	Mel	IRBC	Krug	Mel	Krug	Mel	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
International moments:											
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	0.58	−0.17	−0.06	−0.06	−0.15	−0.03	−0.03	−0.17	−0.20	−0.12	−0.17
$\text{Corr}\left(C_1, C_2\right)$	0.36	−0.10	0.10	0.11	−0.19	0.05	0.05	−0.03	−0.07	−0.14	−0.15
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.62	−0.13	−0.13	−0.60	−0.04	−0.04	−0.59	−0.61	−0.56	−0.59
$\text{Corr}\left(L_1, L_2\right)$	0.42	−0.22	−0.69	−0.70	−0.11	−0.44	−0.48	−0.27	−0.33	−0.08	−0.19
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.69	0.72	0.72	−0.69	0.71	0.71	−0.63	−0.63	−0.66	−0.66
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.32	−0.18	0.98	0.98	−0.18	0.99	0.99	0.11	0.12	0.00	0.01
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.81	0.92	0.97	0.97	0.92	0.99	0.99	0.93	0.93	0.93	0.93
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.13	0.74	0.73	0.73	0.73	0.72	0.72	0.75	0.75	0.73	0.74
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.53	0.08	0.08	0.54	0.04	0.05	0.44	0.45	0.49	0.49
Domestic moments:											
$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.86	0.98	1.00	1.00	0.98	1.00	1.00	0.98	0.98	0.99	0.99
$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.87	0.99	0.93	0.93	0.99	0.97	0.96	0.99	0.98	0.99	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.95	1.00	0.99	0.95	1.00	1.00	0.95	0.94	0.95	0.94
$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.81	0.43	0.83	0.84	0.46	0.87	0.87	0.48	0.49	0.52	0.53
$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.66	0.40	0.14	0.13	0.38	0.11	0.10	0.37	0.36	0.33	0.33
$\frac{\text{Std}\left(P_{I,1}I_1/P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	2.84	3.90	1.56	1.65	3.83	1.51	1.59	4.48	5.20	4.46	5.18

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. For Columns (8)-(11), parameterizations are $\alpha_I = 0$ and $Z_{I,nt} = 1$ (i.e., there is no shock to the investment sector).

Table 4: Moments from standard calibrations and formulations of models. Shock to the final goods sector.

Performance of Krugman vs. Melitz model. Tables 3 and 4 show that there is not much qualitative or quantitative difference between the Krugman and Melitz models, for either shocks and for either domestic and international moments. From the point of view of the unified model, the Melitz model has three different features relative to the Krugman model: external economies of scale, shocks in production of the final aggregate, and the additional term $a\text{TB}_{nt}$ in the labor market clearing condition. The standard calibration used for the Melitz model, however, implies that these features have a small impact quantitatively as the relevant parameters are small: $\psi_Y \approx 0.063$, $Z_{Y,nt} \approx [Z_{X,nt}]^{0.063}$, and $a \approx 0.046$. In the calibration for the Melitz model model, three parameters — $\alpha_{X,K}$, $\psi_{X,K}$, and $\psi_{X,L}$ — have values different from the calibration for the Krugman model. But again, this difference is small given our parameterization, which was tailored to be in line with the literature. Table 2 in fact shows clearly how the calibration implies small differences between the Krugman and Melitz models, using the perspective of our unified model.

Performance of Krugman and Melitz models vs. standard IRBC model. From the point of view of the standard IRBC model, the Krugman and Melitz models have several key modifications that could potentially have opposite or hard to understand effects on second moments and transmission of shocks. Most interesting among them are external economies of scale in production of intermediate and final aggregate goods. Before we focus on the role played by these externalities, however, we perform one exercise related to an important moment, the cyclicality of trade balance. Looking at Columns (2)-(3) in both Tables 3 and 4, we see that one striking difference between the IRBC model and the Krugman and Melitz models is the cyclicality of the trade balance. The correlation of trade balance with output is counterfactually positive for the Krugman and Melitz models.[41] This change is, however, only because standard trade models imply that the investment good (from the perspective of the unified model) is produced using domestic labor, while in the standard IRBC model the investment good is produced using the final aggregate.

To show that the difference in investment, indeed, is the reason for opposite cyclicality of the trade balance, we change the standard formulations of the Krugman and Melitz models. In both Tables 3 and 4, in Columns (8)-(9) (for complete markets) and in Columns (10)-(11) (for bond economy) we now model the investment good as being produced using the final good only (by setting $\alpha_I = 0$).[42] Then the trade balance is countercyclical and similar to the IRBC model. As expected, what noticeably changes now are the moments associated with investment that drive trade balance dynamics. The volatility of investment increases and the cross-country correlation of investment becomes more negative.

[41] While it is true that the cyclicality is negative for the Krugman model in the case of the bond economy and the shock to the intermediate goods sector (Table 3, Column (6)), still, the extent of countercyclicality is dramatically reduced compared to the IRBC model. So our point is valid even for the bond economy, although we mostly focus on complete markets in our discussion to save on space.

[42] Note that in the intermediate good shock case, the Melitz model endogenously introduces a shock to the final aggregate sector (see Table 2). We shut down this shock (by setting $Z_{Y,nt} = 1$) when we make investment in terms of the final good for easier comparison, and this does not affect any results here.

This also then changes the cyclicality of imports and exports.[43]

Having resolved the issue of cyclicality of net exports, comparing Columns (2), (8), and (9) of Tables 3 and 4 (for complete markets) and comparing Columns (5), (10), and (11) of Tables 3 and 4 (for bond economy), we see that the Krugman and Melitz models perform no better than the standard IRBC model. *Broadly, for both shocks, the Krugman and Melitz models perform well and fail in the same cross-country moments as the standard IRBC model.*[44]

What is the main reason for the similar performance of the IRBC model on the one hand and the Krugman and Melitz models on the other? Our result on isomorphism provides the answer: Even though the Krugman and Melitz models feature external economies of scale, their magnitudes implied by the parameterization from the literature (see Table 2) are not large enough to make a quantitative difference. Moreover, these standard parameterizations imply positive capital externalities, whereas, as we show in the next Section, we need negative capital externalities in order for the Krugman and Melitz models to achieve an improvement over the standard IRBC model.

5.3 Changing Production Externalities

From now on, we assume that investment is done in terms of the final good. We then use our unified model of Section 2 to explore if it is possible to achieve a better fit with the data. The unified model perspective is critical as we can vary each externality independently.[45] This allows for a clean inspection of the transmission mechanisms in the models. We do comparative statics for all the three externalities: capital and labor input in the intermediate goods production technology as well as the externality in the final good production technology. We focus, however, mostly on the role of capital externality as it turns out to be most crucial quantitatively. We discuss other externalities in more detail in Section 6.3.3.

[43] Thus overall, it is not the case that simply adding investment to an international business cycles model ensures a countercyclical trade balance by countervailing the consumption smoothing intuition in models without capital accumulation. It is critically important how the investment good is produced. Consider the well-understood case of the intermediate good productivity shock. If investment good is produced with labor input only, while investment certainly increases with a positive productivity shock, investment is less volatile, and it does not render net exports countercyclical for then, the rise in imports is much more muted. This is because now imports follow consumption closely (as investment good production does not use the foreign intermediate good), which is smoothed over time due to standard consumption smoothing incentives. This plays a key role in making net exports procyclical.

[44] This outcome of similarity between IRBC and a Melitz-type dynamic model for the case of the intermediate good productivity shock was also reported in a numerical analysis by Jaef and Lopez (2014).

[45] Given the isomorphism (two-way mapping) between the unified model and the generalized dynamic Krugman and Melitz models that we established in Section 4.1, we can in principle also interpret this exercise of flexibly varying externalities from either model perspectives.

5.3.1 Role of Negative Capital Externality

Table 5 shows that an essential feature to improve fit with the data is negative capital externality in intermediate goods production. In this table, we provide moments from the model without any externality, as well as with a positive and a negative capital externality, for both the intermediate good and the final good productivity shocks. For concreteness, we focus our discussions here on the complete international financial markets case, while still presenting the bond economy results for completeness in Table 5 .

As a starting point, we note that the main cross-country empirical puzzles from the perspective of the IRBC model are associated with co-movement across countries in output, consumption, hours, and investment. That is, as is clear from Tables 3 and 4, in the IRBC model, the co-movement of consumption is counterfactually higher than that of GDP. Moreover, while in the data labor hours and investment co-move positively, in standard models they co-move negatively. Additionally, the canonical IRBC model with the intermediate good productivity shocks leads to a more procyclical real exchange rate and a less volatile trade balance compared to the data.

Table 5 then shows that negative capital externality helps bring the model closer to the data on these important international moments. This is seen from comparing Column (3) with (1) for the intermediate good productivity shock, and Column (6) with (4) for the final good productivity shock. For both shocks, compared to the standard IRBC model with no externalities, negative capital externality leads to higher cross-country output, investment, and labor correlations and a lower consumption correlation. Moreover, it leads to a more volatile trade balance. Let us discuss the mechanisms behind this result for each shock.

Intermediate good productivity shock We now provide an economic interpretation for the results in Columns (1)-(3) of Table 5 by analyzing the transmission mechanisms using impulse-response functions, in which a 1% exogenous technology shock in the intermediate goods sector hits the home country. Figure 1 shows the results under complete markets, where we vary only the externality in capital input, $\psi_{X,K}$.

To set the stage, let us discuss quickly the basic transmission mechanism under no externality, that is, the basic IRBC model. When a positive intermediate good productivity shock, which is persistent but mean-reverting, hits home, the substitution effect of increased wage dominates the income effect, and the household supplies more labor. Moreover, given increased productivity, there is an increase in investment at home. With higher income currently and in the future, consumption also increases, and it is smoothed over time as the usual permanent income hypothesis intuition applies. The flip side of

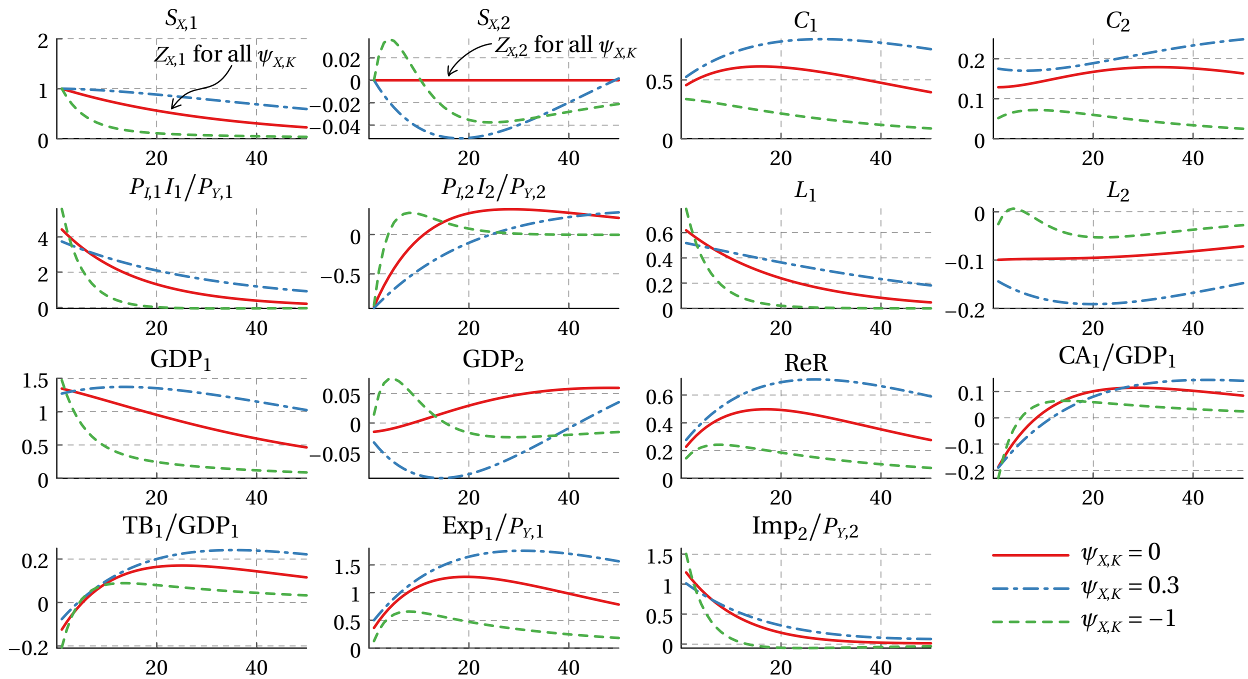
	Complete markets						Bond economy					
	Int. good shock			Final good shock			Int. good shock			Final good shock		
$\psi_{X,K} =$	0	0.3	-1	0	0.3	-1	0	0.3	-1	0	0.3	-1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
International moments:												
$\text{Corr}\,(\text{GDP}_1, \text{GDP}_2)$	-0.03	-0.07	0.08	-0.17	-0.21	-0.08	0.02	0.02	0.09	-0.15	-0.14	-0.09
$\text{Corr}\,(C_1, C_2)$	0.47	0.55	0.34	-0.10	0.03	-0.33	0.11	0.06	0.15	-0.19	-0.17	-0.20
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	-0.39	-0.47	-0.26	-0.62	-0.68	-0.53	-0.35	-0.36	-0.25	-0.60	-0.60	-0.53
$\text{Corr}\,(L_1, L_2)$	-0.30	-0.52	0.00	-0.22	-0.42	0.01	-0.04	-0.02	0.08	-0.11	-0.09	-0.05
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	-0.49	-0.40	-0.57	-0.69	-0.66	-0.73	-0.60	-0.60	-0.60	-0.69	-0.66	-0.73
$\text{Corr}\,(\text{Exp}_1, \text{GDP}_1)$	0.36	0.52	0.17	-0.18	-0.06	-0.29	0.13	0.17	0.12	-0.18	-0.05	-0.25
$\text{Corr}\,(\text{Imp}_1, \text{GDP}_1)$	0.93	0.90	0.96	0.92	0.91	0.94	0.96	0.96	0.97	0.92	0.91	0.94
$\text{Corr}\,(\text{ReR}, \text{GDP}_1)$	0.61	0.67	0.46	0.74	0.77	0.58	0.50	0.54	0.42	0.73	0.75	0.59
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.21	0.15	0.28	0.53	0.46	0.60	0.27	0.24	0.30	0.54	0.45	0.57
Domestic moments:												
$\text{Corr}\,(C_1, \text{GDP}_1)$	0.94	0.93	0.92	0.98	0.99	0.91	0.98	0.99	0.93	0.98	1.00	0.92
$\text{Corr}\,(L_1, \text{GDP}_1)$	0.98	0.97	0.99	0.99	0.99	0.98	0.99	0.99	0.99	0.99	0.99	0.99
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.97	0.96	0.97	0.95	0.94	0.96	0.97	0.97	0.97	0.95	0.95	0.97
$\frac{\text{Std}\,(C_1)}{\text{Std}\,(\text{GDP}_1)}$	0.37	0.44	0.25	0.43	0.50	0.33	0.43	0.56	0.28	0.46	0.58	0.30
$\frac{\text{Std}\,(L_1)}{\text{Std}\,(\text{GDP}_1)}$	0.46	0.42	0.53	0.40	0.35	0.49	0.40	0.31	0.52	0.38	0.29	0.51
$\frac{\text{Std}\,(P_{I,1}I_1/P_{Y,1})}{\text{Std}\,(\text{GDP}_1)}$	3.33	3.01	3.81	3.90	3.57	4.40	3.29	2.86	3.80	3.83	3.30	4.38

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 5: Capital externalities in the unified model.

consumption smoothing is the large response of investment. These positive domestic correlations lead to a standard business cycle at home. Next, given the increased productivity at home, the home country finances increased investment by running a current account deficit.

For the foreign country, as consumption increases due to risk-sharing, the wealth effect leads to a decrease in labor supply. Moreover, the foreign country cuts down on investment as it is optimal to concentrate production in the more productive home country. Over time, the foreign country runs a current account surplus, using the saving to rebuild the depleted capital stock. These transmission mechanisms are behind the failure of the model to match the cross-country correlations in output, investment, and hours.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediate good sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$). All cases are for the complete markets economy. The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{X,K} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{X,K}$.

Figure 1: Impulse-response functions for $Z_{X,1}$. Capital externalities in the intermediate goods sector in the unified model. Complete markets.

Let us now turn to the explanation of why negative capital externality helps move the model closer to the data in terms of these international correlations. The key to answering this question is the observation that, *in the presence of negative capital externalities in production of intermediate goods, individual firms perceive the aggregate country-specific shock as being less persistent with the same initial impact*. This is because, in future, due to positive capital accumulation, the productivity increase faced by the firms — which is measured by $S_{X,1}$ — is lower than the exogenous productivity shock — which is measured by $Z_{X,1}$. Given this insight, we can now analyze what happens to labor supply at home when the productivity increase has the same initial size but is more transient (compared to the no-externality case). Under a more transient shock, the substitution effect of wage increase is even stronger than the income effect (again, compared to the no-externality case). This implies that households supply more labor today, as is clear in Figure 1. Given the same initial capital stock, a larger initial supply of labor leads to a larger initial response of output at home.

Next, while the initial effect on income is higher, in future, as the productivity process is more transient, income will be lower than in the model without externalities. This lack of a persistent rise in home output helps mitigate the failure of the IRBC model to generate output co-movement across countries. Moreover, due to the desire to smooth consumption over time, consumption rises by less than in the no-externality case. The smaller rise of consumption at home, as well as altered path of consumption due to change in investment, help reduce the correlation in consumption across countries. Also, the smaller rise in consumption at home implies that home investment increases more on impact.[46] But this does not worsen international correlation in investment. Over time, both investment and labor at home follow the less transient path of productivity, leading to higher cross-country correlations in investment and labor. Overall, endogenous productivity being less persistent than the productivity shock at home decreases the extent of productivity differences across countries that plagues the IRBC model and improves cross-country correlations in both factors and output.[47]

To understand the dynamic responses of foreign variables in Figure 1, observe that while the country-specific productivity shocks are uncorrelated in our experiments, negative capital externality leads to an endogenous positive correlation in the productivity faced by the two countries. In particular, from the foreign country's perspective, starting from the next period, there is a positive effect on productivity, as typically there would be negative investment in the foreign country following a positive home productivity shock. This positive effect on productivity faced by the foreign country then leads to increased labor hours and increased investment for very standard reasons. The resulting increase in output in the foreign country helps further with increasing output co-movement across countries. Moreover, note how foreign output and labor supply completely track the dy-

[46] The initial impact effect of investment at home is higher even though the productivity faced by firms is less persistent over time. Unless the trade elasticity is very high, such that the domestic and foreign goods are very substitutable, this result holds. With a very high trade elasticity, under complete markets, the initial impact on home investment and trade balance can be lower with negative capital externality. The higher initial impact on hours however, is independent of trade elasticity.

[47] To make this even more clear, in Table 15 in Appendix D.3 we show in an estimation exercise that the canonical IRBC model requires a very low persistence in productivity shocks to fit the data on international correlations.

namics of $S_{X,2}$, as is common in RBC models, but which would have been very difficult to interpret without the perspective of the unified model that leads us to follow the dynamics of $S_{X,2}$ for intuition, instead of $Z_{X,2}$.[48]

Having discussed international correlations, we close this subsection with the discussion of the fit in terms of two other important open economy variables: the cyclicality of the real exchange rate and volatility of the trade balance. First, as negative capital externalities lead to a larger initial increase in home investment, there is a sharper response of trade balance. This then helps with increasing the volatility of the trade balance.[49] Second, the change in the dynamic path of output that we discussed above reduces the cyclicality of the real exchange rate with output. Finally, negative capital externality also helps decrease the cyclicality of exports, which is affected by the change in path of investment in the foreign country.[50]

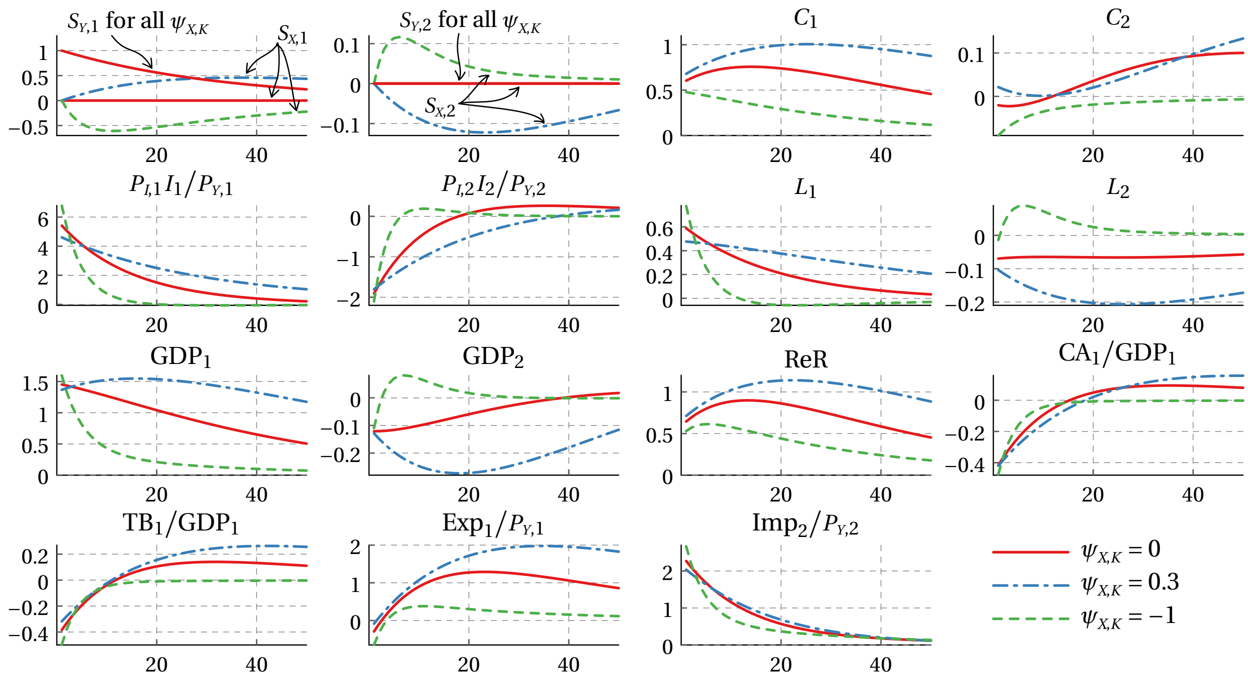
Final good productivity shock We now consider the productivity shock in the final good sector, instead of the intermediate good sector. For economic explanations, we again turn to an analysis of impulse response functions in which a 1% exogenous productivity shock in the final goods sector hits the home country. Figure 2 shows the results under complete markets where we vary only the externality in capital input, $\psi_{X,K}$.

As this shock is new to the IRBC model, the transmission even under no externality requires an explanation. The first result to note is that, as we emphasized before, when the final good productivity shock hits, domestic output, labor, investment, and consumption all co-move, thereby, generating a home business cycle. In RBC models, such a business cycle pattern is notoriously difficult to generate for shocks other than the intermediate good productivity shock. What explains the new result here?

[48] As we noted earlier, with negative capital externality and high trade elasticity, home investment does not increase on impact following a less persistent productivity process. In this case, the margin affected most is the cut in foreign investment, which gets reduced more significantly compared to the case of no externality. Thus, even with high trade elasticity, negative capital externality plays a similar role: following the same shock, negative capital externality makes productivity differences across countries less severe and helps generate higher international correlations in output and factors.

[49] Over time, as is standard, trade balance switches to positive with investment increasing in the foreign country as it rebuilds its capital stock.

[50] One exception here is that negative capital externalities lead to more procyclical imports, which makes the fit worse with the data as the IRBC model already leads to imports that are more procyclical than the data. The reason imports become more procyclical is that the behavior of imports closely follows that of investment (as can be clearly seen in Figure 1). This is because with consumption smoothed over time, investment response is comparatively larger. As the investment good is produced with the final aggregate good, which uses the foreign intermediate good, the behavior of imports closely mirrors that of investment, with only the magnitude being smaller as determined by the import share. Given our explanation above on how investment and output increase more sharply initially with negative capital externalities, imports follow a similar pattern, thereby increasing its pro-cyclicality.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the final good sector in country 1, $Z_{Y,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$). All cases are for the complete markets economy.

Figure 2: Impulse-response functions for $Z_{Y,1}$. Capital externalities in the intermediate goods sector in the unified model. Complete markets.

Following a final good shock at home, productivity of the final good increases, which leads to a decrease in the relative price of the home final good. Naturally, this leads to an increase in both consumption and investment, as they both are produced using the final good. Importantly, labor supply also increases, even though the intermediate good production function, where labor is used, has not experienced a positive shock. Usually, in a closed economy model, one would expect the positive wealth effect due to increased consumption to lead to a decrease in labor supply, with real wages not getting affected much. The situation in the open economy environment here is different, because the price of consumption/final good and the price of the home good are not the same. To understand why this matters, let us look at the optimal labor supply condition of the household, given by

$$-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}} = \frac{W_{nt}}{P_{X,nt}} \cdot \frac{P_{X,nt}}{P_{Y,nt}}.$$

For intuition, we can hold $\frac{W_{nt}}{P_{X,nt}}$ constant, as the intermediate good productivity shock, which affects the marginal product of labor, is not the one being considered here. When the final good productivity shock hits, the first-order effect is an increase in the relative price $\frac{P_{X,nt}}{P_{Y,nt}}$, driven by the fall in the final good price. Then the household finds it optimal to supply more labor, and we can get both consumption and hours at home to increase.

Let us now consider the international transmissions of this final good productivity shock under no externality. The home country finances increased investment and consumption by running a current account deficit. Moreover, as we mentioned above, the real exchange rate depreciates as the final good produced by the home country is cheaper with increased productivity. The depreciation of the real exchange rate is high enough that it requires foreign consumption to fall to ensure that relative consumption is equated with the real exchange rate. This happens even though the financial markets are complete, which is a unique aspect of the final good shock. This explains why in Table 5, for the case of no externality, there is a negative cross-country correlation in consumption. Next, like with the intermediate good productivity shock, the foreign country cuts down on its labor supply and investment, which leads to a negative cross-country correlation in hours and investment, with the wealth effect on labor supply muting a bit the negative co-movement in labor across countries.

Finally, because of the large effect on relative prices, two key moments get affected more compared to the intermediate good productivity shock. First, the trade balance response is stronger, as a decline in relative price leads to an increased trade balance response. And, second, exports are much less cyclical. These two features will play an important role in the estimation exercise.

Overall, while the final good productivity shock also leads to a domestic business cycle like the intermediate good productivity shock, for the IRBC model, Table 4 shows that international correlations are still hard to match. Thus, while consumption correlation across countries is significantly reduced, it is still higher than output correlation, and labor and investment do not co-move positively. At the same time, as shown in Columns (4)-(6) of Table 4, negative capital externality helps move the model closer to the data on international correlations for the final good shock, similarly to the intermediate good shock. That is, negative capital externality increases cross-country output, investment, and hours correlations and decreases consumption correlation.[51]

[51] In fact, for our illustrative parameterization of negative capital externality, consumption correlation

For international correlations, a mechanism similar to the intermediate good productivity shock holds. As shown in Figure 2, while the final good productivity shock does not directly affect intermediate good productivity, with negative externality, endogenously, intermediate good productivity declines as there is typically higher capital accumulation in response to this shock. Again, on impact, there is no effect on intermediate good productivity, but dynamically, there is a negative effect. This negative endogenous effect on intermediate good productivity then negates the positive effects of the final good productivity shock. That is, it is as if now there are two aggregate productivity shocks in the model, one off-setting the other. The home country's overall increase in productivity is thus muted over time, thereby again leading to less persistent effects on home variables, combined with a larger effect on hours, investment, and output initially, as shown in Figure 2. These less persistent effects at home act against the cut in factors of production in the foreign country, improving international correlations overall and moving them more in line with the data.

Overall, a comparison of Figure 1 with 2 shows that for these two different shocks the changes in transmission due to negative capital externality are quite close. Like with the intermediate good productivity shock case with negative externality, here as well, the foreign country's endogenous productivity increases, which further helps with comovement. Finally, the fit is also improved in terms of generating a less cyclical real exchange rate as well as a more volatile trade balance thanks to the same underlying mechanisms as in the case with intermediate good productivity shock.

At the end, we note that our main claims above also generally hold for the bond economy, as shown in Table 5.[52] We relegate impulse responses for this case to Appendix E, which look quite similar to the complete markets case discussed in detail above.

5.3.2 Role of Negative Labor and Final Good Externalities

We now briefly discuss the role of negative labor and final good externalities, relegating a detailed discussion to Section 6.3.3. In a dynamic model, labor and capital externality play very different roles in affecting the transmission of shocks. The effect of final good externality, in turn, is distinct from labor externality, as they operate through completely different channels. Most importantly, while negative capital externality in production helps with moving the model closer to the data in terms of various international moments

across countries is less than output correlation.

[52]One exception is that for the intermediate good productivity shock, negative capital externality does not decrease cross-country consumption correlation. However, even in this case, cross-country consumption correlation becomes closer to output correlation.

	Interm. good shock				Final good shock			
	$\psi_{X,L}$		ψ_Y		$\psi_{X,L}$		ψ_Y	
	0.7	−1	0.2	−1	0.7	−1	0.2	−1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
International moments:								
Corr $(\text{GDP}_1, \text{GDP}_2)$	−0.17	0.10	−0.31	0.12	−0.21	−0.18	−0.50	0.21
Corr (C_1, C_2)	0.25	0.62	0.19	0.74	−0.06	−0.19	−0.34	0.19
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	−0.48	−0.31	−0.70	0.01	−0.57	−0.69	−0.83	−0.20
Corr (L_1, L_2)	−0.35	−0.25	−0.52	−0.30	−0.28	−0.16	−0.59	0.24
Corr $\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.55	−0.45	−0.66	0.62	−0.68	−0.71	−0.79	−0.56
Corr $(\text{Exp}_1, \text{GDP}_1)$	0.20	0.47	−0.21	0.94	−0.09	−0.29	−0.53	0.42
Corr $(\text{Imp}_1, \text{GDP}_1)$	0.92	0.93	0.91	0.49	0.94	0.90	0.92	0.95
Corr $(\text{ReR}, \text{GDP}_1)$	0.64	0.58	0.68	0.44	0.72	0.75	0.79	0.62
Std $\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.46	0.12	0.56	0.08	0.76	0.45	1.03	0.18
Domestic moments:								
Corr (C_1, GDP_1)	0.94	0.94	0.94	0.91	0.97	0.99	0.97	0.99
Corr (L_1, GDP_1)	0.99	0.98	0.99	0.97	0.99	0.99	0.99	0.99
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.97	0.96	0.95	0.99	0.96	0.94	0.95	0.97
$\frac{\text{Std}(C_1)}{\text{Std}(\text{GDP}_1)}$	0.32	0.41	0.34	0.39	0.36	0.49	0.38	0.49
$\frac{\text{Std}(L_1)}{\text{Std}(\text{GDP}_1)}$	0.49	0.44	0.48	0.46	0.46	0.36	0.44	0.36
$\frac{\text{Std}(P_{I,1}I_1/P_{Y,1})}{\text{Std}(\text{GDP}_1)}$	3.54	3.17	4.04	2.63	3.83	4.09	4.64	3.09

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 6: Labor and final good externalities in the unified model. Compete markets.

as, shown above, negative labor and final good externalities do not uniformly do so.

We present these results in Table 6 for both the intermediate and final good productivity shocks, focusing on complete markets. We see that for the intermediate good productivity shock, negative labor and final good externalities make trade balance less volatile and less countercyclical, while also increasing consumption correlation across countries.

The same holds the final good productivity shock with final good externality. For labor externality and the final good productivity shock, trade balance does not get less countercyclical and consumption correlation does not increase, while the investment correlation decreases and the real exchange rate becomes more procyclical. Overall, compared to negative capital externality in Table 5 , the fit does not improve uniformly for these two other externalities for this shock as well.

We discuss the detailed transmission mechanisms for the two shocks as we vary labor and final good externalities in Section 6.3.3 and present them in Figures 5-8 in Appendix E. Here we just briefly note the key economic differences compared to negative capital externality.

Consider for concreteness the intermediate good productivity shock. The main difference from negative capital externality is that with negative labor externality, while the productivity process faced by the home country firms is less transient than the shock in future, as typically there would be an increase in labor hours, the initial impact also shifts down. This is because unlike the capital stock, which is pre-determined today, labor hours respond positively today as well. This then looks like a productivity process for the home country that has shifted downwards at every point in time, and the response of other variables then follows the path of productivity.

For negative final good externality, the channel is different. This externality does not affect at all the path of productivity in the intermediate goods sector, unlike labor externality. Instead, it only endogenously affects the productivity in the final aggregate sector. Note that this externality acts in terms of $\left(P_{Y,nt}Y_{nt}\right)/W_{nt}$, and thus we need to understand the effects on home and foreign country GDP.[53] When this externality is negative, since output increases at home with a productivity increase in the intermediate good sector, it means that productivity in the final aggregate sector at home endogenously decreases. Then the macroeconomic transmission that follows is as if there were a negative final good productivity shock.

5.4 Estimation Exercise

Motivated by our findings above on how varying production externalities affects important international moments, we now undertake a more formal moment-matching exercise to show carefully the need for negative capital externality to improve fit with the data. In particular, for both the complete markets and the bond economy cases, we now match a comprehensive list of moments, a larger set than the one presented above, while estimating the parameters governing the shock processes and all three externalities. Our

[53] As we pointed out before, for the particular case of financial autarky, this externality term is proportional to total hours.

Moment	Data	Compl.	Bond	Moment	Data	Compl.	Bond
$\text{Corr}(\text{GDP}_1, \text{GDP}_2)$	0.58	0.50	0.43	$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.33	0.33
$\text{Corr}(C_1, C_2)$	0.36	0.37	0.38	$\text{Corr}(C_1, \text{GDP}_1)$	0.86	0.96	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	0.22	0.18	$\text{Corr}(L_1, \text{GDP}_1)$	0.87	1.00	1.00
$\text{Corr}(L_1, L_2)$	0.42	0.50	0.43	$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.99	0.99
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.50	−0.46	$\text{Std}(\text{GDP}_1)$	1.67	1.85	1.93
$\text{Corr}(\text{Exp}_1, \text{GDP}_1)$	0.32	0.45	0.46	$\frac{\text{Std}(C_1)}{\text{Std}(\text{GDP}_1)}$	0.81	0.15	0.15
$\text{Corr}(\text{Imp}_1, \text{GDP}_1)$	0.81	0.99	0.94	$\frac{\text{Std}(L_1)}{\text{Std}(\text{GDP}_1)}$	0.66	0.59	0.59
$\text{Corr}(\text{ReR}, \text{GDP}_1)$	0.13	0.15	0.14	$\frac{\text{Std}(P_{I,1}I_1/P_{Y,1})}{\text{Std}(\text{GDP}_1)}$	2.84	3.86	3.83

Parameter estimates:

	$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y	σ_X	σ_Y	ρ_X	ρ_Y
Complete	−2.70	0.91	−0.06	0.000	0.002	0.00	0.99
Bond	−4.00	0.90	−0.20	0.000	0.004	0.00	0.90

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$, $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 7: Results of the estimation of the unified model for $\sigma = 2$.

criterion for model fit is the equally-weighted mean squared error.[54]

Table 7 below reports data moments, model moments, and the parameter estimates under the best fit. From Table 7, the main estimate of interest is −2.70 (complete markets) and −4.00 (bond economy) for $\psi_{X,K}$. This highlights the key quantitative role of negative capital externality in accounting for international business cycle moments.

In addition, we see that the shock driving the business cycle now is the final good productivity shock, for both the complete markets and the bond economy cases.[55] In particular, there is no need for the intermediate good productivity shock. While our discus-

[54]Note that we still do not use any data moments based on autocorrelations directly. This is deliberate as we want to emphasize that persistence of shocks can be identified from cross-country correlations, which is at the heart of our mechanism. In the grid search, we construct a 7-dimensional grid for parameters $\psi_{X,K}$, $\psi_{X,L}$, ψ_Y, σ_X, σ_Y, ρ_X, and ρ_Y, and compute moments in each point of the grid. We then calculate the loss function $\mathcal{L} = \sqrt{\sum_{m=1}^{M} \left([\text{Mom}^{\text{model}}]_m / [\text{Mom}^{\text{data}}]_m - 1 \right)^2}$, where $\left[\text{Mom}^{\text{model}}\right]_m$ and $\left[\text{Mom}^{\text{data}}\right]_m$ are moments calculated in the model and data, and find the point on the grid with the lowest value of $\mathcal{L}$. See Appendix C for more details.

[55]The estimates and fit are generally quite similar for the complete markets and the bond economy. The negative capital externality and persistence of the shock is lower for complete markets.

sion in the previous section highlighted how the two shocks lead to very similar domestic and international business cycle dynamics, the final good productivity shock is estimated to be dominant here as it helps to reduce export cyclicality and increase trade balance volatility in line with the data, for reasons discussed in Section 5.3.1.

In terms of fit, a moment that is hard to match is the relative volatility of consumption, which is lower than the data. While even the model without externality, that is, the canonical IRBC model, also had this issue (as shown in Table 3), here it gets worse as negative capital externality improves fit on several dimensions simultaneously by endogenously reducing the persistence of the shock. This then means that consumption is very smooth, for standard consumption smoothing reasons.

6 Discussion, Extensions, and Sensitivity

We now present a more detailed discussion of various models and connection to the literature, as well as a series of extensions and sensitivity analyses.

6.1 Discussion of Theoretical Models and Literature

The unified model described in Section 2 is a generalization of the standard IRBC model studied in the previous literature. For example, Heathcote and Perri (2002)'s model can be obtained as a special case of the unified model by shutting down externalities, requiring that capital investment uses the final aggregate only (i.e., it does not use labor), leaving exogenous shocks only in production of intermediate goods, and dropping the additional term $aP_{Y,nt} \cdot \mathrm{TB}_{nt}$ in the labor market clearing condition. Formally, this requires setting $\psi_{X,K} = \psi_{X,L} = \psi_Y = 0$, $\alpha_I = 0$, $Z_{Y,nt} = Z_{I,nt} = 1$, $\Theta_{X,n} = \Theta_{Y,n} = \Theta_{I,n} = 1$, and $a = 0$. We further need to remove iceberg trade costs (i.e., set $\tau_{ni,t} = 1$) in order to obtain exactly the environment considered by Heathcote and Perri (2002), which features home bias in production of the final aggregate.

There are no direct analogs in the existing literature of the generalized Melitz model of Section 3.2. There are two important differences of the generalized Melitz model with the dynamic versions of the Melitz model described in Ghironi and Melitz (2005) and Jaef and Lopez (2014). First, fixed costs of serving markets in the generalized Melitz model are paid in terms of the destination-country labor, while in the existing dynamic Melitz models the fixed costs are paid in terms of the source-country labor. Second, there are non-zero fixed costs of serving domestic markets in the generalized Melitz model, while in the existing dynamic Melitz models there are no fixed costs of serving domestic markets. The presence of such costs in the generalized Melitz model creates the situation when in every period there are some firms that neither produce nor exit. These firms have too low efficiency of production to overcome fixed costs of serving markets, but had high enough efficiency of production to enter the economy at some point. In the existing dynamic Melitz models all firms that enter the economy produce for at least the domestic market. Quantitatively, the effects of these different assumptions are small in a model with two symmetric countries, which is traditionally the focus of the international business cycles literature (as in our paper). The benefit of the assumptions about fixed costs of serving markets made in the generalized Melitz model of Section 3.2 is that these assumptions allow us to establish isomorphism with the unified model.[56]

If we shut down external economies of scale in production of varieties and in the fixed costs of serving markets (by setting $\phi_{X,L} = 0$, $\phi_{F,M} = \frac{1}{\theta^{M}}$, and $\phi_{F,L} = \vartheta$), and if we require that the sunk costs of entry into the economy are paid in terms of labor only (by setting $\alpha_{I} = 1$), then the only essential differences between the generalized Melitz model of Section 3.2 and the model presented in Ghironi and Melitz (2005) will be the differences in the assumptions about fixed costs of serving markets described in the previous paragraph. In Jaef and Lopez (2014), production technology for intermediate varieties uses capital together with labor, and so the model features capital accumulation in addition to entry and exit of producers of varieties.[57]

6.2 Quantitative Comparison Across Models

We consider several extensions and sensitivity exercises below related to one of our main findings: dynamic trade extensions of the IRBC model do not change business cycle moments significantly. Our theoretical results suggest that as the dynamic trade extensions of the IRBC model introduce externalities that are small in magnitude, one might expect them to behave similarly over the business cycle generally. It could nevertheless still be possible that these small externalities have a larger effect under alternate model environments on the household side, such as risk sharing arrangements across countries, and under different parameterizations, such as that of trade elasticity or of parameters

[56]The generalized Melitz model can be considered as an extension to a dynamic environment of the static version of the Melitz model described in Kucheryavyy *et al.* (2019), who make the same assumptions about fixed costs of serving markets as in the current paper. These assumptions allow Kucheryavyy *et al.* (2019) to establish isomorphism between a static multi-industry version of the Melitz model and a static multi-industry version of the Eaton-Kortum model with external economies of scale.

[57]The environment in Alessandria and Choi (2007) does not have period-by-period firm entry decisions, which is an important feature of our environment. Instead, the environment in Alessandria and Choi (2007) features sunk costs of entry into exporting markets, which create exporters hysteresis — the feature absent from our generalized Melitz model.

governing the shock process. We address these issues next.

6.2.1 Financial Autarky

To conserve on space, we presented results for complete markets and the bond economy in Section 5.2 when we compared various models with the data. Heathcote and Perri (2002) show that compared to complete markets or the bond economy, the IRBC model under financial autarky leads to international correlations closer to the data. Financial autarky, by construction, however, cannot account for trade balance dynamics and the differential cyclicality of exports and imports. Still, for completeness, in Table 8 in Appendix D.1, we show results under financial autarky for both productivity shocks, with the rest of the parameterization as given in Table 2. First, while under financial autarky, the IRBC model does lead to more positive international correlations in output, investment, and hours, they are still lower than the data and additionally, consumption correlation is still higher than output.[58] More importantly, and what constitutes our main point, is that the IRBC model and the Krugman and Melitz models lead to very similar moments for both shocks, even under financial autarky.

6.2.2 Different Trade Elasticity

We presented our results for a baseline calibration of trade elasticity of 1, that is, $\sigma = 2$. One relevant extension to consider is whether our comparison across models is affected by this parameterization. This is pertinent because the international trade literature estimates/calibrates a much higher trade elasticity. In Table 9 in Appendix D.1, we show results for both a higher ($\sigma = 6$) and a lower ($\sigma = 0.9$) trade elasticity than our baseline calibration.[59] We focus on complete markets for concreteness and the rest of the parameterization is as given in Table 2. The main takeaway from Table 9 that we want

[58] The fact that international correlations in investment and labor are lower here than in Heathcote and Perri (2002) for the IRBC model with intermediate good productivity shock is the different calibration of the elasticity of substitution between intermediate goods in production of the final good (we use $\sigma = 2$ while Heathcote and Perri (2002) use $\sigma = 0.90$). When domestic and foreign goods are complements, as with $\sigma = 0.90$, the international transmission of shocks changes non-trivially in the IRBC model irrespective of whether one considers financial autarky or complete markets. We show this in more detail next as well.

[59] When we change trade elasticity, we also change iceberg trade costs $\tau_{ni,t}$ for $n \neq i$ to match the steady-state share of imports of intermediate goods of 0.15. For $\sigma = 6$ this implies $\tau_{12} = \tau_{21} \approx 1.415$, while for $\sigma = 0.9$ this implies $\tau_{12} = \tau_{21} \approx 2.93 \times 10^{-8}$ (see Footnote 34). Observe that in the case of $\sigma = 0.9$ we have negative iceberg trade costs, which are very small in absolute values. This fact is concealed if we match the share of imports by parameter $\omega_{ni}^{\frac{\sigma-1}{\sigma}}$ (as in Heathcote and Perri (2002)) instead of $\tau_{ni,t}$. In this case, we have $\tau_{ni,t} = 1$ for all n and i, and $\omega_{12}^{\frac{\sigma-1}{\sigma}} = \omega_{21}^{\frac{\sigma-1}{\sigma}} = \left(\lambda_{12}^{-1} - 1\right)^{-1/\sigma}$ and $\omega_{11} = \omega_{22} = 1$. For $\sigma = 0.9$ this implies $\omega_{12}^{\frac{\sigma-1}{\sigma}} = \omega_{21}^{\frac{\sigma-1}{\sigma}} \approx 0.146$.

to emphasize is that, for both shocks, the differences across the three models (the IRBC, dynamic Krugman, and dynamic Melitz) for the key moments are minor, regardless of trade elasticity.[60]

6.2.3 Spillovers and Correlated Shocks

Our baseline results were for the case of uncorrelated productivity shocks across countries that do not spillover exogenously. This specification is our preferred one as it separates endogenous transmission mechanisms from exogenously imposed shock correlations clearly, and is also the one we use in the estimation exercise. Nevertheless, as a specification with spillovers and correlated shocks is also common in the quantitative literature, e.g., as estimated in Heathcote and Perri (2002), we adopt such a calibration next. With this alternate calibration of shock processes, Tables 10 and 11 in Appendix D.1 show analogous results to those in Tables 3 and 4. For the IRBC models, while the output comovement across countries improves as expected, there still is a mismatch with the data, as there is even higher correlation in consumption compared to output, and low, even negative, hours and investment correlations across countries. Moreover, again, the IRBC models and the Krugman and Melitz models do not show major differences.

6.2.4 IRBC with Investment Using Labor

We emphasized before how the cyclicality of trade balance depends critically on whether the investment good is produced using labor or the final good. We demonstrated that in the context of the dynamic Krugman and Melitz models. For completeness here, we do the same for the IRBC model. In Table 12 in Appendix D.1, we change the canonical IRBC model with the intermediate good productivity shock such that investment is done in terms of home labor only (by setting $\alpha_I = 1$). For completeness, we show two cases under that specification. First, one in which there is no shock in the investment technology ($Z_{I,nt} = 1$), which leads to the most direct comparison with the IRBC model. Second, one in which the intermediate good productivity shock also perturbs the investment production function ($Z_{I,nt} = Z_{x,nt}$), which is what is implied by the dynamic Krugman and Melitz models. It is clear there that the trade balance is now pro-cyclical, unlike in Table 3.

[60] For a higher trade elasticity, $\sigma = 6$, as is well-known, the fit of the IRBC model itself worsens significantly as international correlations become much weaker. That is, generally, with the elasticity of substitution across the domestic and foreign goods increasing, the cross-country correlations of output, investment, and labor decrease, while that of consumption increases. The key reason is that when a productivity shock hits the home economy, as trade elasticity is higher now, it is optimal for the foreign country to cut its labor supply and investment by even more. Thus, higher trade elasticity worsens further, the relative productivity differences across countries, and makes trade balance more volatile. Higher trade elasticity, as expected, helps on making the real exchange rate less pro-cyclical.

6.3 Changing Production Externalities

We next consider several extensions and sensitivity exercises below related to our main point on improving the fit of the unified model: negative capital externality plays an important role, both qualitatively and quantitatively.

6.3.1 Capital Externality Under Other Risk-Sharing Arrangements

To conserve on space, we showed comparative statics results for capital externality under complete markets and the bond economy cases only above. Here, we additionally discuss results for financial autarky, which we present in Table 13 in Appendix D.2. Overall, our main point, that negative capital externality helps improve the fit by decreasing consumption correlation and increasing output, investment, and hours correlation across countries, continues to be valid even under financial autarky.

Figures 3-4 in Appendix E next present the impulse responses for both the bond economy and financial autarky cases. These figures show the transmission mechanisms that underlie the second moments in Table 5 for the bond economy and in Table 13 for financial autarky. We note that for financial autarky, while the mechanisms are still related to how negative capital externality endogenously reduces the persistence of the shocks, the transmission is different for the foreign country. For the intermediate good productivity shock case, this can be seen by comparing Figure 4 with the baseline complete market variant in Figure 1. The difference arises because under financial autarky, in response to a positive intermediate good productivity shock at home that leads to a real exchange rate depreciation, the resulting positive wealth effect for the foreign country leads to an increase in investment, hours, and output. Thus, when there is negative capital externality, it endogenously reduces productivity in the foreign country, leading to responses of variables that are very similar to those in the home country. Thus, investment, hours, and output increase more on impact, but decay faster, even in the foreign country.

6.3.2 Capital Externality Under Correlated Shocks and Spillovers

We now discuss results for correlated shocks that spillover across countries, as estimated in Heathcote and Perri (2002). Under this calibration, Table 14 in Appendix D.2 presents results on varying capital externality for both the complete markets and bond economy cases. The same point that we have emphasized before applies: negative capital externality helps bring the model closer to the data.

6.3.3 Labor and Final Good Externalities

In the discussion in Section 5.3, we focused mostly on comparative statics related to capital externality, as they are the most important quantitatively in our estimation exercise. Moreover, as was clear from Table 6, even qualitatively, while negative capital externality helps move the model closer to the data, negative labor and final good externalities do not uniformly do so. We now discuss the transmission mechanisms for these two externalities in detail here, emphasizing the differences from capital externality.

We start with the intermediate good productivity shock. From Table 6, we see that negative labor externality increases consumption correlation across countries, while making trade balance less volatile and also less countercyclical. Figure 5 in Appendix E shows the impulse responses to this shock under complete markets where we vary only the externality in labor input, $\psi_{X,L}$. The key to understanding the transmission is that with negative labor externality, while the productivity process faced by the home country is also less transient, as typically there would be an increase in labor hours in future, the initial impact also shifts down. This is because, unlike the capital stock which is pre-determined, labor hours respond positively today as well. This then looks like a productivity process for the home country that has shifted downwards at every point in time, as can be seen from the path of $S_{x,1}$. Then, unlike the case of negative capital externality, the home household does not increase hours initially, which in turn means that the initial increase in investment and output also does not happen. The effect of negative labor externality is thus not as strong as that of negative capital externality in moving the co-movement of hours and investment towards positive.

Given lower GDP both on impact and in future, consumption smoothing implies that consumption drops uniformly at home with negative externality, compared to the case of no externality. This lower response of investment and consumption means that, unlike the case of negative capital externality, net exports does not become more volatile or more countercyclical.

Furthermore, as typically there would be a negative response of foreign labor hours in response to this shock, there is an endogenous correlation of home and foreign productivities.[61] This helps, at least qualitatively, with generating a less negative response of foreign investment and hours. For consumption response in the foreign country, the effects are less clear overall, because of the combination of perfect risk-sharing and the different response of hours at home when labor externality is negative compared to when capital externality is negative. Overall, consumption in the foreign country does not change its

[61] The dynamic positive correlation of productivity across countries that occurs with negative capital externality however, does not happen, as is clear in Figure 5.

dynamic response and in fact, changes in a non-monotonic way across various levels of labor externality, as there is relatively less difference in its investment and output paths. This contributes to an increase in cross-country consumption correlation.

Next, we vary the final good externality. We see in Table 6 that for the intermediate good productivity shock, negative final good externality increases consumption correlation across countries while making trade balance both less volatile and less countercyclical. We show detailed transmission mechanisms in Figure 7 in Appendix E, where we vary only the externality in the final good aggregator technology, ψ_Y. While the overall patterns appear similar to labor externality , the channel is however, different. The reason is that this externality does not affect at all the path of productivity in the intermediate goods sector, unlike labor externality. Instead, this externality only affects endogenously the productivity in the final good sector, as can be seen in Figure 7 from the path of $S_{Y,1}$.

To understand the direction of effects, note that the final good externality acts in terms of $(P_{Y,nt} Y_{nt}) / W_{nt}$, the number of country-$n$'s workers that produce the same value as the value of the final aggregate. Thus, we need to understand the effects of home and foreign country GDP.[62] When this externality is negative, since output increases at home with a productivity increase in the intermediate good sector, it means that productivity in the final good sector at home endogenously decreases. This effect holds both on impact and dynamically. Then the transmission that follows is as if there were a negative final good productivity shock.[63] Thus, it drives both consumption and investment at home down, compared to the case of no externality. This lower demand for the aggregate final good translates to lower production of the home intermediate good and lower home labor supply, given the low import share. Like with negative labor externality, this lower effect on investment plays an important role in making net exports less countercyclical (in this example, trade balance is positive under negative externality), and perhaps more importantly, less volatile.

In the foreign country, again unlike labor externality, there is no impact on productivity in the intermediate good sector, and the effect is only on the final good productivity. In particular, as foreign GDP increases, it endogenously has a negative effect on final good productivity. Critically, however, this negative productivity effect is much stronger for the home country compared to the foreign country. Thus, relative to no externality, this is still a positive effect for the foreign country compared to the home country. As a result, there is an increase in foreign hours and investment, compared to the no externality

[62] For the particular case of financial autarky only, as we pointed out before, this externality term is proportional to total hours.

[63] It would thus be the inverse of the transmission we described in Figure 2 for the shock.

case.[64]

Now we move to the final good productivity shock. Table 6 shows that for the final good productivity shock as well, unlike capital externality, labor and final good externalities do not uniformly help improve the fit of the model. Again, the main difference is in terms of consumption correlation across countries and the volatility of trade balance.

Figures 11-12 in Appendix E, show the transmission mechanism underlying these results. The main economic insights are very similar to the intermediate good productivity shock. Note that with this shock, the effects of externality happen through endogenous changes in intermediate good productivity, as was the case with capital externality. Thus for instance here, with negative labor externality, intermediate good productivity has an endogenous negative effect at home, both on impact and over time. The overall effects are a parallel shift downward in hours, investment, and output at home, as was the case in Figure 5 for the intermediate good productivity shock. For the foreign country, as was the case with the intermediate good productivity shock, the effect is positive on intermediate good productivity, which helps decrease the negative effect on hours and investment. The final good externality for this shock looks similar to the intermediate good productivity shock as well.

6.4 Estimation

We finally consider extensions and sensitivity exercises regarding our estimation exercise, which additionally also help provide more explanation for our baseline results.

6.4.1 Untargeted Moments Under Best Fit

Table 16 in Appendix D.3 contains results in terms of some untargeted moments for the estimated model. Our estimated model underpredicts the volatility of exports and imports, as well as that of the real exchange rate. While the volatility of these variables is still higher than what would be obtained in the baseline IRBC model, future work can address mechanisms to further improve fit along these dimensions.

6.4.2 Best Fit for IRBC

Throughout the paper, we have emphasized the key role played by negative capital externality in improving the fit of the model with the data. The channel we have highlighted is

[64]For completeness, we also provide results for the bond economy in Figures 6 and 8 in Appendix E. As can be seen from these figures, the transmission is similar to the compete market case we have just described.

that negative capital externality endogenously decreases the persistence of productivity shocks hitting the economy, with the impact effect unchanged. To show that this channel is in fact in operation in all international business cycle models, we estimate the canonical IRBC model with the intermediate good productivity shock, to match the same set of moments as in our baseline exercise. The results are in Table 15 in Appendix D.3, which show that the estimated persistence of the shock is much lower than the values often calibrated in the literature ($\rho_X = 0.37$ for complete markets and $\rho_X = 0.43$ for the bond economy).

7 Conclusion

We present a unified framework to fully study the interconnections between international trade and business cycle models. We prove an aggregate equivalence between a competitive, representative firm open economy model that has production externalities and dynamic trade models that feature monopolistic competition, heterogeneous firms, and costs of entry and exporting. Such isomorphism holds even though the dynamic trade models have very different micro foundations from the competitive, representative firm model.

Our theoretical results shed light on why the business cycle implications of the IRBC and the standard dynamic trade models that appear in the literature are similar: the implied externalities are small, positive, and tightly restricted across factors. In a quantitative exercise with multiple shocks, we show that to resolve some well known empirical puzzles in the international business cycle literature, the most important ingredient is negative capital externality.

In future work, we plan to extend the analysis in some key directions. It would be of interest to study optimal trade policy, in order to provide a unified treatment in a dynamic context of normative issues that have been explored in various modern international trade models. It would also be worthwhile to consider models with frictions that can endogenously generate negative capital externality in the aggregate.

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A Unified Model

A.1 Households Budget Constraints and First-Order Conditions

A.1.1 Financial Autarky

In the case of financial autarky, there is no international trade in financial assets. Households in country n face the following flow budget constraint

$$P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} = W_{nt}L_{nt} + R_{nt}K_{nt}.$$

First-order conditions for the household's optimization problem are given by

$$P_{I,nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[R_{n,t+1} + (1-\delta) P_{I,n,t+1}\right] \right\}, \tag{41}$$

$$-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \tag{42}$$

where $U_1(\cdot,\cdot)$ and $U_2(\cdot,\cdot)$ are derivatives of the utility function with respect to consumption and labor, correspondingly. Here (41) is the standard Euler equation, while (42) is the standard labor supply equation.

A.1.2 Bond Economy

We consider a bond economy where each country issues a non-state-contingent bond denominated in its consumption units. Holdings of country i's bond by country n are denoted by $B_{ni,t}$. The household's flow budget constraint is given by

$$P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} + \sum_{i=1}^{N} P_{Y,it}\left(B_{ni,t} + \frac{b_{adj}}{2}B_{ni,t}^2\right)$$
$$= W_{nt}L_{nt} + R_{nt}K_{nt} + \sum_{i=1}^{N} P_{Y,it}\left(1 + r_{i,t-1}\right)B_{ni,t-1} + T_{nt}^B,$$

where $r_{i,t-1}$ is period-t return on country-i's bond, and $T_{nt}^B \equiv \frac{b_{adj}}{2}\sum_{i=1}^N P_{Y,it}B_{ni,t}^2$ is the bond fee rebate, taken as given by the household. Here b_{adj} is the adjustment cost of bond holdings, which is introduced to ensure stationarity. First-order conditions are given by conditions (41) and (42), plus an additional set of Euler equations:

$$P_{Y,it}\frac{U_1\left(C_{nt},L_{nt}\right)}{P_{Y,nt}}\left(1+b_{adj}B_{ni,t}\right)=\beta E_t\left\{\frac{U_1\left(C_{n,t+1},L_{n,t+1}\right)}{P_{Y,n,t+1}}P_{Y,i,t+1}\left(1+r_{it}\right)\right\},$$

for $i=1,\ldots,N$.

International trade in bonds allows unbalanced trade in intermediate goods. Define country n's real trade balance TB_{nt} as the value of net exports of intermediate goods in terms of the final good:

$$\mathrm{TB}_{nt}\equiv\left(P_{X,nt}X_{nt}-P_{Y,nt}Y_{nt}\right)/P_{Y,nt},$$

and define country n's real current account CA_{nt} as the change in this country's net financial assets position in terms of the final good:[65]

$$\mathrm{CA}_{nt}\equiv\sum_{i=1}^{N}\frac{P_{Y,it}}{P_{Y,nt}}\left(B_{ni,t}-B_{ni,t-1}\right).$$

A.1.3 Complete Financial Markets

To introduce the household's budget constraint in the case of complete markets, we employ notation for the states of nature in period t, denoted by s_t, and history of states in period t, denoted by s^t. In each state with history s^t, countries trade a complete set of state-contingent nominal bonds denominated in the numeraire currency. Let $\mathcal{B}_{n,t+1}\left(s^t,s_{t+1}\right)$ denote the amount of the nominal bond with return in state s_{t+1} that country n acquires in the state with history s^t. Assuming that there are no costs of trading currency or securities between countries, we can denote by $P_{B,t}\left(s^t,s_{t+1}\right)$ the international price of this bond in the state with history s^t. Country n's budget constraint is given by

$$\begin{aligned}P_{Y,nt}\left(s^t\right)C_{nt}\left(s^t\right)+P_{I,nt}\left(s^t\right)I_{nt}\left(s^t\right)+\mathcal{A}_{nt}\left(s^t\right)\\=W_{nt}\left(s^t\right)L_{nt}\left(s^t\right)+R_{nt}\left(s^t\right)K_{nt}\left(s^t\right)+\mathcal{B}_{nt}\left(s^t\right),\end{aligned}$$

[65]Using markets clearing conditions (described later), it can be shown that trade balance and current account can also be written as

$$\mathrm{TB}_{nt}=\left(W_{nt}L_{nt}+R_{nt}K_{nt}-P_{Y,nt}C_{nt}-P_{I,nt}I_{nt}\right)/P_{Y,nt},\text{ and }\mathrm{CA}_{nt}=\mathrm{TB}_{nt}+\sum_{i=1}^{N}r_{i,t-1}P_{Y,it}B_{ni,t-1}/P_{Y,nt}.$$

where

$$\mathcal{A}_{nt}\left(s^t\right) \equiv \sum_{s_{t+1}} P_{B,t}\left(s^t, s_{t+1}\right) \mathcal{B}_{n,t+1}\left(s^t, s_{t+1}\right)$$

is country n's net foreign assets position in period t. First-order conditions in the case of complete markets are given by conditions (41) and (42) (with the state-dependent notation added to them), plus an additional set of conditions:

$$\begin{aligned}
P_{B,t}\left(s^t, s_{t+1}\right) &= \beta \frac{\pi_{t+1}\left(s^{t+1}\right)}{\pi_t\left(s^t\right)} \cdot \frac{P_{Y,nt}\left(s^t\right)}{P_{Y,n,t+1}\left(s^{t+1}\right)} \cdot \frac{U_1\left(C_{n,t+1}\left(s^{t+1}\right), L_{n,t+1}\left(s^{t+1}\right)\right)}{U_1\left(C_{nt}\left(s^t\right), L_{nt}\left(s^t\right)\right)}, \\
Q_{ni,t}\left(s^t\right) &= \kappa_{ni} \frac{U_1\left(C_{nt}\left(s^t\right), L_{nt}\left(s^t\right)\right)}{U_1\left(C_{it}\left(s^t\right), L_{it}\left(s^t\right)\right)}, \quad \text{for each } i,
\end{aligned} \tag{43}$$

where $\pi_t\left(s^t\right)$ is the probability of history s^t occurring in period t,

$$Q_{ni,t}\left(s^t\right) \equiv \frac{P_{Y,nt}\left(s^t\right)}{P_{Y,it}\left(s^t\right)}$$

is the real exchange rate, and $\kappa_{ni} \equiv \left(\frac{U_1\left(C_{n0}\left(s^0\right), L_{n0}\left(s^0\right)\right) / P_{Y,n0}\left(s^0\right)}{U_1\left(C_{i0}\left(s^0\right), L_{i0}\left(s^0\right)\right) / P_{Y,i0}\left(s^0\right)}\right)^{-1}$.

In what follows, we drop the state-dependent notation for brevity. Condition (43) is the standard Backus-Smith condition that says that the real exchange co-moves with the ratio of marginal utilities. As in the case of the bond economy, trade balance is defined as net exports of intermediate goods, and current account is defined as the change in net foreign assets position,

$$\begin{aligned}
\mathrm{TB}_{nt} &= \left(P_{X,nt} X_{nt} - P_{Y,nt} Y_{nt}\right) / P_{Y,nt} , \\
\mathrm{CA}_{nt} &= \left(\mathcal{A}_{nt} - \mathcal{A}_{n,t-1}\right) / P_{Y,nt} .
\end{aligned}$$

A.2 Equilibrium Conditions

Equilibrium conditions of the unified model are given by:

$$
\begin{aligned}
&P_{I,nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[R_{n,t+1} + (1-\delta) P_{I,n,t+1}\right] \right\}, \\
&- \frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
&K_{n,t+1} = (1-\delta) K_{nt} + I_{nt}, \\
&X_{nt} = \left(\Theta_{X,n} Z_{X,nt} K_{nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}\right) K_{nt}^{\alpha_{X,K}} L_{X,nt}^{\alpha_{X,L}}, \\
&Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\omega_{ni} \frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \\
&I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
&W_{nt} L_{X,nt} + W_{nt} L_{I,nt} = W_{nt} L_{nt} + a P_{Y,nt} \cdot \mathrm{TB}_{nt}, \\
&C_{nt} + Y_{I,nt} = Y_{nt}, \\
&\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it}, \\
&\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{1-\sigma}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj}\right)^{1-\sigma}}, \\
&K_{nt} = \alpha_{X,K} \frac{P_{X,nt} X_{nt}}{R_{nt}}, \\
&L_{X,nt} = \alpha_{X,L} \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
&L_{I,nt} = \alpha_I \frac{P_{I,nt} I_{nt}}{W_{nt}}, \\
&Y_{I,nt} = (1-\alpha_I) \frac{P_{I,nt} I_{nt}}{P_{Y,nt}}.
\end{aligned}
$$

The household's budget constraint in the case of financial autarky is given by

$$
P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} = W_{nt} L_{nt} + R_{nt} K_{nt},
$$

in the case of the bond economy it is given by

$$
P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} + \sum_{i=1}^{N} P_{Y,it} B_{ni,t} = W_{nt} L_{nt} + R_{nt} K_{nt} + \sum_{i=1}^{N} P_{Y,it} \left(1 + r_{i,t-1}\right) B_{ni,t-1},
$$

and in the case of complete markets it is given by

$$
P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} + \mathcal{A}_{nt} = W_{nt} L_{nt} + R_{nt} K_{nt} + \mathcal{B}_{nt},
$$

with

$$\mathcal{A}_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \mathcal{B}_{n,t+1} \right\}.$$

Additional conditions in the case of the bond economy are

$$P_{Y,it}\left(1 + b_{adj} B_{ni,t}\right) = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} P_{Y,i,t+1}\left(1 + r_{it}\right) \right\},$$

$$\text{for } i = 1, \ldots, N,$$

$$\sum_{n=1}^{N} B_{ni,t} = 0,$$

while in the case of complete markets they are

$$\frac{P_{Y,it}}{P_{Y,jt}} = \kappa_{ij} \frac{U_1\left(C_{it}, L_{it}\right)}{U_1\left(C_{jt}, L_{jt}\right)}, \quad \text{for each } i \text{ and } j,$$

$$\sum_{i=1}^{N} \mathcal{A}_{it} = 0,$$

where

$$\kappa_{ij} \equiv \left(\frac{U_1\left(C_{i0}, L_{i0}\right) / P_{Y,i0}}{U_1\left(C_{j0}, L_{j0}\right) / P_{Y,j0}} \right)^{-1}.$$

is found in the steady state.

A.3 Steady State

Given L_n, Y_n, R_n, W_n, $P_{X,n}$, $P_{I,n}$, $P_{Y,n}$, we can find the rest of the variables using the following conditions:

$$
\begin{aligned}
\lambda_{ni} &= \frac{\left(\tau_{ni} P_{X,i}/\omega_{ni}\right)^{1-\sigma}}{\sum_{j=1}^{N}\left(\tau_{nj} P_{X,j}/\omega_{nj}\right)^{1-\sigma}},\\
X_i &= \frac{1}{P_{X,i}} \sum_{n=1}^{N} \lambda_{ni} P_{Y,n} Y_n,\\
K_n &= \alpha_{X,K} \frac{P_{X,n} X_n}{R_n},\\
L_{X,n} &= \alpha_{X,L} \frac{P_{X,n} X_n}{W_n},\\
I_n &= \delta K_n,\\
C_n &= \left(W_n L_n + R_n K_n - P_{I,n} I_n\right)/P_{Y,n},\\
L_{I,n} &= \alpha_I \frac{P_{I,n} I_n}{W_n},\\
Y_{I,n} &= \left(1-\alpha_I\right) \frac{P_{I,n} I_n}{P_{Y,n}}.
\end{aligned}
$$

Conditions that determine L_n, Y_n, R_n, W_n, $P_{X,n}$, $P_{I,n}$, $P_{Y,n}$, are:

$$
\begin{aligned}
& L_n - L_{X,n} - L_{I,n} = 0,\\
& Y_n - C_n - Y_{I,n} = 0,\\
& R_n - \left(\frac{1}{\beta} - 1 + \delta\right) P_{I,n} = 0,\\
& -\frac{U_2\left(C_n, L_n\right)}{U_1\left(C_n, L_n\right)} - \frac{W_n}{P_{Y,n}} = 0,\\
& X_n - \Theta_{X,n} Z_{X,n} K_n^{\alpha_{X,K}+\psi_{X,K}} L_{X,n}^{\alpha_{X,L}+\psi_{X,L}} = 0,\\
& I_n - \Theta_{I,n} Z_{I,n} L_{I,n}^{\alpha_I} Y_{I,n}^{1-\alpha_I} = 0,\\
& Y_n - \Theta_{Y,n} \left(\frac{P_{Y,n} Y_n}{W_n}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\omega_{ni} \frac{\lambda_{ni} P_{Y,n} Y_n}{\tau_{ni} P_{X,i}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = 0.
\end{aligned}
$$

B Generalized Dynamic Versions of the Standard Trade Models

B.1 Generalized Dynamic Version of the Eaton-Kortum Model

Household's problem is identical to the one in the unified model. Moreover, as in the unified model, the production side consists of intermediate, final, consumption, and investment goods. All markets are perfectly competitive. The intermediate goods sector here is different from the intermediate goods sector in the unified model — it consists of a continuum of varieties indexed by $\nu \in [0,1]$. Any country has a technology to produce any of the varieties $\nu \in [0,1]$. The production technology of variety ν in country n in period t is given by

$$x_{nt}(\nu) = S_{X,nt} z_n(\nu) k_{X,nt}(\nu)^{\alpha_{X,K}} l_{X,nt}(\nu)^{\alpha_{X,L}},$$

where $k_{X,nt}(\nu)$ and $l_{X,nt}(\nu)$ are capital and labor used in production of variety ν, $z_n(\nu)$ is the efficiency of production of variety ν, and $S_{X,nt} \equiv \Theta_{X,n} Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}$ is aggregate productivity. All terms of $S_{X,nt}$ have similar meanings as the corresponding terms of the aggregate productivity in the intermediate goods sector in the unified model given by expression (1). In particular, $K_{X,nt}$ and $L_{X,nt}$ denote total amounts of capital and labor used in production of all varieties in country n in period t.[66] As in the unified model, aggregate productivity $S_{X,nt}$ captures external economies of scale in the production of varieties and is taken by firms as given.

Profit maximization problem of producer of variety ν implies

$$R_{nt} k_{X,nt}(\nu) = \alpha_{X,K} p_{nn,t}(\nu) x_{nt}(\nu), \tag{44}$$

$$W_{nt} l_{X,nt}(\nu) = \alpha_{X,L} p_{nn,t}(\nu) x_{nt}(\nu). \tag{45}$$

And the cost of production is

$$p_{nn,t}(\nu) = \alpha_{X,K}^{-\alpha_{X,K}} \alpha_{X,L}^{-\alpha_{X,L}} \frac{R_{nt}^{\alpha_{X,K}} W_{nt}^{\alpha_{X,L}}}{S_{X,nt} z_n(\nu)}.$$

In equilibrium,

$$K_{X,nt} = \int_0^1 k_{X,nt}(\nu)\, d\nu \quad \text{and} \quad L_{X,nt} = \int_0^1 l_{X,nt}(\nu)\, d\nu.$$

Denote the value of total output of varieties by $\mathcal{X}_{nt}$:

$$\mathcal{X}_{nt} \equiv \int_0^1 p_{nn,t}(\nu) x_{nt}(\nu)\, d\nu.$$

[66] This production technology generalizes the production technology used in Kucheryavyy *et al.* (2019) by introducing capital in addition to labor as a factor of production and adding capital externality in addition to labor externality. This generalization is a natural extension of the static environment of Kucheryavyy *et al.* (2019) with no capital to the dynamic environment of the current paper with capital accumulation.

Integrating conditions (44)-(45) over ν, we get

$$R_{nt} = \alpha_{X,K} \frac{\mathcal{X}_{nt}}{K_{X,nt}} \quad \text{and} \quad W_{nt} = \alpha_{X,L} \frac{\mathcal{X}_{nt}}{L_{X,nt}}.$$

Varieties are traded. Trade is costly and is subject to iceberg trade costs $\tau_{ni,t}$. Varieties are combined into the non-tradeable final aggregate:

$$Y_{nt} = S^{EK}_{Y,nt} \left[\int_0^1 \left[\sum_{i=1}^{N} \omega_{ni} x_{ni,t}(\nu) \right]^{\frac{\sigma^{EK}-1}{\sigma^{EK}}} d\nu \right]^{\frac{\sigma^{EK}}{\sigma^{EK}-1}},$$

where $x_{ni,t}(\nu)$ is the amount of variety ν that country n buys from country i in period t, $\omega_{ni} \geq 0$ are exogenous importer-exporter specific weights, and, similarly to the unified model,

$$S^{EK}_{Y,nt} \equiv \Theta^{EK}_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}$$

is aggregate productivity. All terms of $S^{EK}_{Y,nt}$ have similar meanings as the corresponding terms of the aggregate productivity in the final goods sector in the unified model given by expression (3). Production function for Y_{nt} implies that varieties produced by different countries are perfect substitutes in production of the final aggregate. Hence, producers of the final aggregate in country n buy each variety ν from the cheapest source (taking into account taste parameters ω_{ni}).

Let $p_{ni,t}(\nu) = \tau_{ni,t} p_{ii,t}(\nu)$ be the price in country n of variety ν produced in country i. Let $\Omega_{ni,t} \subseteq [0,1]$ be the (endogenously determined) set of varieties that country n buys from i. We can write

$$Y_{nt} = S^{EK}_{Y,nt} \left[\sum_{i=1}^{N} \int_{\nu \in \Omega_{ni,t}} \left(\omega_{ni} x_{ni,t}(\nu) \right)^{\frac{\sigma^{EK}-1}{\sigma^{EK}}} d\nu \right]^{\frac{\sigma^{EK}}{\sigma^{EK}-1}}.$$

Demand for individual varieties $\nu \in \Omega_{ni,t}$ is given by

$$x_{ni,t}(\nu) = \left[S^{EK}_{Y,nt} \right]^{\sigma^{EK}-1} \omega_{ni}^{\sigma^{EK}-1} \left(\frac{p_{ni,t}(\nu)}{P_{Y,nt}} \right)^{-\sigma^{EK}} Y_{nt},$$

with the price index

$$P_{Y,nt} = \left[S^{EK}_{Y,nt} \right]^{-1} \left[\sum_{i=1}^{N} \left(P_{ni,t} / \omega_{ni} \right)^{1-\sigma^{EK}} \right]^{\frac{1}{1-\sigma^{EK}}},$$

where

$$P_{ni,t} \equiv \left[\int_{\Omega_{ni,t}} p_{ni,t} \left(\nu \right)^{1-\sigma^{\text{EK}}} d\nu \right]^{\frac{1}{1-\sigma^{\text{EK}}}}.$$

Assume that efficiencies $z_n(\nu)$ are drawn from the Fréchet distribution given by its cumulative distribution function

$$\text{Prob}\left[z_{nt} \left(\nu \right) \leq z \right] = e^{-z^{-\theta^{\text{EK}}}}.$$

We can derive that

$$P_{ni,t}^{1-\sigma^{\text{EK}}} = \Gamma\left(\frac{\theta^{\text{EK}} + 1 - \sigma^{\text{EK}}}{\theta^{\text{EK}}} \right) \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni} \right)^{-\theta^{\text{EK}}}}{\left[\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{ni} \right)^{-\theta^{\text{EK}}} \right]^{\frac{\theta^{\text{EK}}+1-\sigma^{\text{EK}}}{\theta^{\text{EK}}}}},$$

where

$$P_{X,it} \equiv \frac{R_{it}^{\alpha_{X,K}} W_{it}^{\alpha_{X,L}}}{\widetilde{\Theta}_{X,i} Z_{X,it} K_{X,it}^{\psi_{X,K}} L_{X,it}^{\psi_{X,L}}},$$

with $\widetilde{\Theta}_{X,i} \equiv \alpha_{X,K}^{\alpha_{X,K}} \alpha_{X,L}^{\alpha_{X,L}} \Theta_{X,i}$. Therefore

$$P_{Y,nt}^{1-\sigma^{\text{EK}}} = \Gamma\left(\frac{\theta^{\text{EK}} + 1 - \sigma^{\text{EK}}}{\theta^{\text{EK}}} \right) \left[S_{Y,nt}^{\text{EK}} \right]^{\sigma^{\text{EK}}-1} \sum_{i=1}^{N} \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni} \right)^{-\theta^{\text{EK}}}}{\left[\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj} \right)^{-\theta^{\text{EK}}} \right]^{\frac{\theta^{\text{EK}}+1-\sigma^{\text{EK}}}{\theta^{\text{EK}}}}},$$

which gives

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N} \left(\tau_{ni,t} P_{X,it} / \omega_{ni} \right)^{-\theta^{\text{EK}}} \right]^{-\frac{1}{\theta^{\text{EK}}}}}{\Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}},$$

with $\Theta_{Y,n} \equiv \Gamma\left(\frac{\theta^{\text{EK}} + 1 - \sigma^{\text{EK}}}{\theta^{\text{EK}}} \right)^{\frac{1}{\sigma^{\text{EK}}-1}} \Theta_{Y,n}^{\text{EK}}$, where $\Gamma(\cdot)$ is the gamma-function. The expenditure share of country n on varieties produced in country i is similar to the corresponding expression (5) in the unified model and is given by

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni} \right)^{-\theta^{\text{EK}}}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj} \right)^{-\theta^{\text{EK}}}}.$$

The final aggregate is used for consumption and investment. As in the unfed model, the consumption good is directly produced from the final good, and so the price of the consumption good in country n is $P_{Y,nt}$. The technology of production of the investment good is also assumed to be the same as in the unified model, i.e., it assumed to be given by expression (6). Hence, the price of the investment good is the same as in the unified model and is given by

$$P_{I,nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}} \tag{46}$$

Since the household's problem is identical to the one in the unified model of Section 2, it yields the same set of equilibrium conditions.

Denote $X_{nt} \equiv \mathcal{X}_{nt}/P_{X,nt}$. After some manipulations, the set of equilibrium conditions that are common across all financial market structures can be written as

$$\begin{aligned}
&P_{I,nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[R_{n,t+1} + (1-\delta) P_{I,n,t+1}\right] \right\}, \\
&-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
&K_{n,t+1} = (1-\delta) K_{nt} + I_{nt}, \\
&X_{nt} = \left(\Theta_{X,n} Z_{X,nt} K_{nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}\right) K_{nt}^{\alpha_{X,K}} L_{X,nt}^{\alpha_{X,L}}, \\
&Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^{EK}}{\theta^{EK}+1}}\right]^{\frac{\theta^{EK}+1}{\theta^{EK}}}, \\
&I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
&L_{X,nt} + L_{I,nt} = L_{nt}, \\
&C_{nt} + Y_{I,nt} = Y_{nt}, \\
&\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it}, \\
&\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^{EK}}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt}/\omega_{nj}\right)^{-\theta^{EK}}}, \\
&K_{nt} = \alpha_{X,K} \frac{P_{X,nt} X_{nt}}{R_{nt}}, \\
&L_{X,nt} = \alpha_{X,L} \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
&L_{I,nt} = \alpha_I \frac{P_{I,nt} I_{nt}}{W_{nt}}, \\
&Y_{I,nt} = (1-\alpha_I) \frac{P_{I,nt} I_{nt}}{P_{Y,nt}}, \\
&C_{nt} + I_{nt} = Y_{nt}.
\end{aligned}$$

Conditions, that are specific to different financial market structures, are identical to the ones in the unified model. Comparing the above equilibrium system with the equilibrium system in the unified model, we see that these systems are identical. Thus, the generalized version of the Eaton-Kortum model is isomorphic to the unified model.

B.2 Generalized Dynamic Version of the Krugman Model

Production of Varieties, International Trade, and Final Aggregate. Here we modify the generalized Krugman model in one more way: we consider a nonlinear technology of production of varieties. Namely, we assume that production of variety $\nu \in \Omega_{nt}$ is given by

$$x_{nt}(\nu) = S^{\kappa}_{X,nt} l_{nt}(\nu)^{\gamma}, \quad \gamma > 0.$$

If $\gamma = 1$, then we get the linear technology studied in the main text. For $0 < \gamma < 1$ the technology features decreasing returns to scale, while for $\gamma > 1$ the technology features increasing returns to scale. While here we allow for $\gamma \neq 1$, we keep the assumption that $\sigma^{\kappa} > 1$.

The profit maximization problem of producer of variety $\nu \in \Omega_{it}$ is given by

$$\max_{p_{ii,t}(\nu), x_{ni,t}(\nu), l_{it}(\nu)} \sum_{n=1}^{N} p_{ii,t}(\nu) \tau_{ni,t} x_{ni,t}(\nu) - W_{it} l_{it}(\nu)$$

s.t.

$$x_{ni,t}(\nu) = S^{\eta^{\kappa}-1}_{Y,nt} M_{it}^{(\sigma^{\kappa}-1)\left(\phi_{Y,M} - \frac{1}{\sigma^{\kappa}-1}\right)} \omega_{ni}^{1-\sigma^{\kappa}} \tau_{ni,t}^{-\sigma^{\kappa}} p_{ii,t}(\nu)^{-\sigma^{\kappa}} P_{ni,t}^{\sigma^{\kappa}-\eta^{\kappa}} P^{\eta^{\kappa}}_{Y,nt} Y_{nt},$$

$$\sum_{n=1}^{N} \tau_{ni,t} x_{ni,t}(\nu) = S^{\kappa}_{X,it} l_{it}(\nu)^{\gamma}.$$

Denote by

$$q_{it}(\nu) \equiv \sum_{n=1}^{N} \tau_{ni,t} x_{ni,t}(\nu),$$

and

$$B_{it} \equiv \left[\sum_{n=1}^{N} S^{\eta^{\kappa}-1}_{Y,nt} M_{it}^{(\sigma^{\kappa}-1)\left(\phi_{Y,M} - \frac{1}{\sigma^{\kappa}-1}\right)} \omega_{ni}^{1-\sigma^{\kappa}} \tau_{ni,t}^{1-\sigma^{\kappa}} P_{ni,t}^{\sigma^{\kappa}-\eta^{\kappa}} P^{\eta^{\kappa}}_{Y,nt} Y_{nt} \right]^{\frac{1}{\sigma^{\kappa}}}.$$

Then $p_{ii,t}(\nu) = B_{it} q_{it}(\nu)^{-\frac{1}{\sigma^{\kappa}}}$, and we can write the above maximization problem as

$$\max_{q_{it}(\nu)} \pi_{it}(q_{it}(\nu)),$$

where

$$\pi_{it}\left(q_{it}\left(\nu\right)\right) \equiv B_{it} q_{it}\left(\nu\right)^{1-\frac{1}{\sigma^{\kappa}}} - W_{it}\left[S^{\kappa}_{x,it}\right]^{-\frac{1}{\gamma}} q_{it}\left(\nu\right)^{\frac{1}{\gamma}}.$$

Observe that if $1-\frac{1}{\sigma^{\kappa}} > \frac{1}{\gamma}$, which is equivalent to $\gamma > \frac{\sigma^{\kappa}}{\sigma^{\kappa}-1}$, then $\lim_{q_{it}(\nu)\to+\infty} \pi_{it}\left(q_{it}\left(\nu\right)\right) = +\infty$. This is the case when economies of scale in production of varieties are too large, and so monopolists would want to produce infinite amounts of varieties and earn infinite profits. Thus, there is no equilibrium in this case.

In the case with $\gamma = \frac{\sigma^{\kappa}}{\sigma^{\kappa}-1}$ we also do not have an equilibrium. Indeed, if in this case $B_{it} < W_{it}\left[S^{\kappa}_{x,it}\right]^{-\frac{1}{\gamma}}$, then monopolists would not produce anything. If $B_{it} > W_{it}\left[S^{\kappa}_{x,it}\right]^{-\frac{1}{\gamma}}$, then monopolists would want to produce an infinite amount of each variety. Finally, if $B_{it} = W_{it}\left[S^{\kappa}_{x,it}\right]^{-\frac{1}{\gamma}}$, then each monopolist earns zero profits, and would not be able to cover the fixed costs of entry into the economy.

Thus, the only possibility of an equilibrium is the case with $0 < \gamma < \frac{\sigma^{\kappa}}{\sigma^{\kappa}-1}$. For the rest of this section we assume that γ satisfies this restriction.

We have

$$\begin{aligned}\pi'_{it}\left(q_{it}\left(\nu\right)\right) &= \left(1-\frac{1}{\sigma^{\kappa}}\right) B_{it} q_{it}\left(\nu\right)^{-\frac{1}{\sigma^{\kappa}}} - \frac{1}{\gamma} W_{it}\left[S^{\kappa}_{x,it}\right]^{-\frac{1}{\gamma}} q_{it}\left(\nu\right)^{\frac{1}{\gamma}-1} \\ &= \frac{1}{\gamma} W_{it}\left[S^{\kappa}_{x,it}\right]^{-\frac{1}{\gamma}} q_{it}\left(\nu\right)^{-\frac{1}{\sigma^{\kappa}}}\left(\left[q^{*}_{it}\right]^{\frac{1}{\sigma^{\kappa}}+\frac{1}{\gamma}-1} - q_{it}\left(\nu\right)^{\frac{1}{\sigma^{\kappa}}+\frac{1}{\gamma}-1}\right),\end{aligned}$$

where

$$q^{*}_{it} \equiv \left[\frac{\gamma\left(\sigma^{\kappa}-1\right)}{\sigma^{\kappa}} \cdot \frac{B_{it}\left[S^{\kappa}_{x,it}\right]^{\frac{1}{\gamma}}}{W_{it}}\right]^{\frac{1}{\frac{1}{\sigma^{\kappa}}+\frac{1}{\gamma}-1}}.$$

From here we see that, since $\frac{1}{\sigma^{\kappa}}+\frac{1}{\gamma}-1 > 0$, we have that $\pi'_{it}\left(q_{it}\left(\nu\right)\right) > 0$ for $q_{it}\left(\nu\right) < q^{*}_{it}$, $\pi'_{it}\left(q_{it}\left(\nu\right)\right) < 0$ for $q^{*}_{it} > q_{it}\left(\nu\right)$, and $\pi'_{it}\left(q_{it}\left(\nu\right)\right) = 0$ for $q_{it}\left(\nu\right) = q^{*}_{it}$. Thus, q^{*}_{it} is the global maximum of $\pi_{it}\left(\cdot\right)$. The associated labor allocation is given by $l^{*}_{it} = \left[q^{*}_{it}\big/S^{\kappa}_{x,it}\right]^{\frac{1}{\gamma}}$, and price is

$$p^{*}_{ii,t} = \left[\frac{\gamma\left(\sigma^{\kappa}-1\right)}{\sigma^{\kappa}} \cdot \frac{\left[S^{\kappa}_{x,it}\right]^{\frac{1}{\gamma}}}{W_{it}}\right]^{\frac{1}{\sigma^{\kappa}-\frac{\sigma^{\kappa}}{\gamma}-1}} B_{it}^{\frac{\sigma^{\kappa}-\frac{\sigma^{\kappa}}{\gamma}}{\sigma^{\kappa}-\frac{\sigma^{\kappa}}{\gamma}-1}},$$

Denote by $L_{x,it} = M_{it} l^{*}_{it}$ the total amount of labor used in production of varieties. Then, using $p^{*}_{ii,t}\left[q^{*}_{it}\right]^{\frac{1}{\sigma^{\kappa}}} = B_{it}$ and $q^{*}_{it} = S^{\kappa}_{x,it}\left[l^{*}_{it}\right]^{\gamma} = S^{\kappa}_{x,it}\left[L_{x,it}/M_{it}\right]^{\gamma}$, the monopolist's price can be written as

$$p^*_{ii,t} = \frac{\sigma^\kappa}{\gamma\left(\sigma^\kappa - 1\right)} \cdot \frac{W_{it}}{S^\kappa_{X,it} M^{1-\gamma}_{it} L^{\gamma-1}_{X,it}}.$$

This allows us to find the price index

$$\begin{aligned}
P_{ni,t} &= M_{it}^{-\left(\phi_{Y,M} - \frac{1}{\sigma^\kappa - 1}\right)} \left[\int_{\nu\in\Omega_{it}} p_{ni,t}\left(\nu\right)^{1-\sigma^\kappa} d\nu\right]^{\frac{1}{1-\sigma^\kappa}} \\
&= M_{it}^{-\left(\phi_{Y,M} - \frac{1}{\sigma^\kappa - 1}\right)} \left[M_{it}\left[\frac{\sigma^\kappa}{\gamma\left(\sigma^\kappa - 1\right)} \cdot \frac{\tau_{ni,t} W_{it}}{S^\kappa_{X,it} M^{1-\gamma}_{it} L^{\gamma-1}_{X,it}}\right]^{1-\sigma^\kappa}\right]^{\frac{1}{1-\sigma^\kappa}} \\
&= \tau_{ni,t} P_{X,it},
\end{aligned}$$

where

$$P_{X,it} \equiv \frac{\sigma^\kappa}{\gamma\left(\sigma^\kappa - 1\right)} \cdot \frac{W_{it}}{\Theta_{X,i} Z_{X,it} M^{\phi_{Y,M}+1-\gamma}_{it} L^{\phi_{X,L}+\gamma-1}_{X,it}}.$$

From here we can find total demand of country n for country i's varieties:

$$\begin{aligned}
\mathcal{X}_{ni,t} &= \int_{\nu\in\Omega_{it}} \tau_{ni,t} p_{ii,t}\left(\nu\right) x_{ni,t}\left(\nu\right) d\nu \\
&= S^{\eta^\kappa - 1}_{Y,nt} \left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\sigma^\kappa} P^{\sigma^\kappa - \eta^\kappa}_{ni,t} P^{\eta^\kappa}_{Y,nt} Y_{nt} \\
&= \lambda_{ni,t} P_{Y,nt} Y_{nt},
\end{aligned}$$

where

$$\lambda_{ni,t} \equiv \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\eta^\kappa}}{\sum_{j=1}^N \left(\tau_{nj,t} P_{X,jt}/\omega_{nj}\right)^{1-\eta^\kappa}}$$

is the expenditure share.

Next, multiplying both sides of $q^*_{it} = S^\kappa_{X,it}\left[L_{X,it}/M_{it}\right]^\gamma$ on price $p^*_{ii,t}$, gives

$$p^*_{ii,t} q^*_{it} = \frac{\sigma^\kappa}{\gamma\left(\sigma^\kappa - 1\right)} \cdot \frac{W_{it} L_{X,it}}{M_{it}},$$

which implies that the total value of output of all varieties in country i, $\mathcal{X}_{it} = M_{it} p^*_{ii,t} q^*_{it}$, is

$$\mathcal{X}_{it} = \frac{\sigma^\kappa}{\gamma\left(\sigma^\kappa - 1\right)} W_{it} L_{X,it}.$$

Let $D_{it} \equiv \pi_{it}\left(q^*_{it}\right)$ be the average profit of country i's producers of varieties Ω_{it}. It is given by

$$D_{it} = \frac{\mathcal{X}_{it} - W_{it} L_{X,it}}{M_{it}} = \left(1 - \frac{\gamma\left(\sigma^{\kappa} - 1\right)}{\sigma^{\kappa}}\right) \cdot \frac{\mathcal{X}_{it}}{M_{it}}.$$

Invention of Varieties, Entry and Exit of Producers of Varieties. Varieties are invented in the R&D sector. The invention process uses labor and final aggregate. Specifically, a combination of l_I units of labor and y_I units of the final aggregate results in $\Theta_{I,n} Z_{I,nt} l_I^{\alpha_I} y_I^{1-\alpha_I}$ new varieties, where $0 \leq \alpha_I \leq 1$, and $\Theta_{I,n} Z_{I,nt}$ is an exogenous productivity in the R&D sector. Assuming perfect competition in the R&D sector and letting V_{nt} be the value of an invented variety, we get that invention of one variety requires $\alpha_I \frac{V_{nt}}{W_{nt}}$ units of labor and $\left(1 - \alpha_I\right) \frac{V_{nt}}{P_{Y,nt}}$ units of the final aggregate. Perfect competition also implies that $V_{nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}$, where $\widetilde{\Theta}_{I,n} \equiv \alpha_I^{\alpha_I} \left(1 - \alpha_I\right)^{1-\alpha_I} \Theta_{I,n}$.

In every period t each country has an unbounded mass of prospective entrants (firms) into the production of varieties. All varieties invented in a particular country in period t are sold to these prospective entrants in the same period. A producer of a variety enters into the economy by buying this variety from the R&D sector. Entry into the economy is free, and so any entrant pays for the variety its value V_{nt}.

Let $M_{I,nt}$ denote the number of varieties that are invented in country n in period t (which is also the number of firms that enter into the economy). The total amount of labor and final aggregate used in the R&D sector are, respectively,

$$L_{I,nt} = \alpha_I \frac{V_{nt} M_{I,nt}}{W_{nt}}, \quad \text{and} \quad Y_{I,nt} = \left(1 - \alpha_I\right) \frac{V_{nt} M_{I,nt}}{P_{Y,nt}}.$$

From here we also get that

$$M_{I,nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}.$$

Households. Here we describe only financial autarky. Derivations for bond economy and complete markets can be done in a similar way. The problem of country n's households is

$$\max_{C_{nt}, L_{nt}, M_{I,nt}, M_{n,t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_{nt}, L_{nt}\right)$$

s.t.

$$P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = W_{nt} L_{nt} + D_{nt} M_{nt},$$

$$M_{n,t+1} = \left(1 - \delta\right) M_{nt} + M_{I,nt}.$$

First-order conditions for this problem imply:

$$V_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[D_{n,t+1} + (1-\delta) V_{n,t+1}\right] \right\},$$
$$-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}.$$

Equilibrium System of Equations Let us manipulate the expression for $P_{X,nt}$ to bring it to a form isomorphic to the price of the intermediate good in the unified model. We have

$$\begin{aligned} P_{X,nt} &= \frac{\sigma^{K}}{\gamma\left(\sigma^{K}-1\right)} \cdot \frac{W_{it}}{\Theta_{X,i} Z_{X,it} M_{it}^{\phi_{Y,M}+1-\gamma} L_{X,it}^{\phi_{X,L}+\gamma-1}} \\ &= \frac{\sigma^{K}}{\gamma\left(\sigma^{K}-1\right)} \cdot \frac{D_{nt}^{1-\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}} W_{nt}^{\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}}}{\Theta_{X,it} Z_{X,nt} M_{it}^{\phi_{Y,M}+1-\gamma} L_{X,it}^{\phi_{X,L}+\gamma-1} D_{nt}^{1-\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}} W_{nt}^{\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}-1}}. \end{aligned}$$

Using the facts that $D_{nt} = \left(1 - \frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}\right) \cdot \frac{\mathcal{X}_{it}}{M_{it}}$ and $W_{nt} = \frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}} \cdot \frac{\mathcal{X}_{nt}}{L_{X,nt}}$, we get

$$P_{X,nt} = \frac{D_{nt}^{1-\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}} W_{nt}^{\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}}}{\widetilde{\Theta}_{X,n}^{K} Z_{X,nt} M_{it}^{\phi_{Y,M}-\frac{\gamma}{\sigma^{K}}} L_{X,it}^{\phi_{X,L}+\frac{\gamma}{\sigma^{K}}}},$$

where $\widetilde{\Theta}_{X,n}^{K} \equiv \left[1 - \frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}\right]^{1-\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}} \left[\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}\right]^{\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}} \Theta_{X,n}$. Let $X_{nt} \equiv \mathcal{X}_{nt}/P_{X,nt}$ be the real output of varieties. By substituting the expressions for D_{nt} and W_{nt} into the above expression for $P_{X,nt}$, we get

$$X_{nt} = \left(\Theta_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M}-\frac{\gamma}{\sigma^{K}}} L_{X,nt}^{\phi_{X,L}+\frac{\gamma}{\sigma^{K}}}\right) M_{nt}^{1-\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}} L_{X,nt}^{\frac{\gamma\left(\sigma^{K}-1\right)}{\sigma^{K}}}.$$

Next, we have

$$\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}} = S_{Y,nt}^{\eta^{K}-1} \left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\eta^{K}} P_{Y,nt}^{\eta^{K}} Y_{nt},$$

which gives

$$\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{\eta^{K}} = S_{Y,nt}^{\eta^{K}-1} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{-1} P_{Y,nt}^{\eta^{K}} Y_{nt}.$$

Taking both sides to the power of $\frac{1-\eta^{\kappa}}{\eta^{\kappa}}$, we get

$$(\tau_{ni,t} P_{X,it}/\omega_{ni})^{1-\eta^{\kappa}} = S_{Y,nt}^{(\eta^{\kappa}-1)\frac{1-\eta^{\kappa}}{\eta^{\kappa}}} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\eta^{\kappa}-1}{\eta^{\kappa}}} P_{Y,nt}^{1-\eta^{\kappa}} Y_{nt}^{\frac{1-\eta^{\kappa}}{\eta^{\kappa}}}.$$

Summing over i and using the fact that

$$P_{Y,nt}^{1-\eta^{\kappa}} = S_{Y,nt}^{-(1-\eta^{\kappa})} \sum_{i=1}^{N} P_{ni,t}^{1-\eta^{\kappa}} = S_{Y,nt}^{-(1-\eta^{\kappa})} \sum_{i=1}^{N} (\tau_{ni,t} P_{X,it}/\omega_{ni})^{1-\eta^{\kappa}},$$

we get

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\eta^{\kappa}-1}{\eta^{\kappa}}}\right]^{\frac{\eta^{\kappa}}{\eta^{\kappa}-1}}.$$

Combining all expressions and definitions, we get the equilibrium system in isomorphic form:

$$
\begin{aligned}
&V_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[D_{n,t+1} + (1-\delta) V_{n,t+1}\right] \right\}, \\
&-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
&M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}, \\
&X_{nt} = \left(\Theta_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M} - \frac{\gamma}{\sigma^K}} L_{X,nt}^{\phi_{X,L} + \frac{\gamma}{\sigma^K}}\right) M_{nt}^{1-\frac{\gamma(\sigma^K-1)}{\sigma^K}} L_{X,nt}^{\frac{\gamma(\sigma^K-1)}{\sigma^K}}, \\
&Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^N \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it} / \omega_{ni}}\right)^{\frac{\eta^K-1}{\eta^K}}\right]^{\frac{\eta^K}{\eta^K-1}}, \\
&M_{I,nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
&L_{X,nt} + L_{I,nt} = L_{nt}, \\
&C_{nt} + Y_{I,nt} = Y_{nt}, \\
&\sum_{n=1}^N \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it}, \\
&\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it}\right)^{1-\eta^K}}{\sum_{l=1}^N \left(\tau_{nl,t} P_{X,lt}\right)^{1-\eta^K}}, \\
&M_{nt} = \left(1 - \frac{\gamma\left(\sigma^K - 1\right)}{\sigma^K}\right) \cdot \frac{P_{X,nt} X_{nt}}{D_{nt}}, \\
&L_{X,nt} = \frac{\gamma\left(\sigma^K - 1\right)}{\sigma^K} \cdot \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
&L_{I,nt} = \alpha_I \frac{V_{nt} M_{I,nt}}{W_{nt}}, \\
&Y_{I,nt} = \left(1 - \alpha_I\right) \frac{V_{nt} M_{I,nt}}{P_{Y,nt}}, \\
&P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = W_{nt} L_{nt} + D_{nt} M_{nt}.
\end{aligned}
$$

The Role of Nonlinear Production Function in the Krugman Model From the point of view of the unified model, having free parameter γ in an otherwise standard Krugman model allows us to have more flexibility in the production function for intermediate goods. Indeed, with $\gamma \neq 1$, we have $X_{nt} = S_{X,nt} M_{nt}^{1-\frac{\gamma(\sigma^K-1)}{\sigma^K}} L_{X,nt}^{\frac{\gamma(\sigma^K-1)}{\sigma^K}}$ with $S_{X,nt} = \Theta_{X,n} Z_{X,nt} M_{nt}^{\frac{1}{\sigma^K-1} - \frac{\gamma}{\sigma^K}} L_{X,nt}^{\frac{\gamma}{\sigma^K}}$, and we can now use two fundamental parameters of the Krugman model — γ and σ^K — to control three parameters of the unified model — $\alpha_{X,K}$, $\psi_{X,K}$, and $\psi_{X,L}$. In order to understand whether introduction of γ helps with bringing us closer to the estimates for $\psi_{X,K}$ and $\psi_{X,L}$, use $\alpha_{X,K} = 1 - \frac{\gamma(\sigma^K-1)}{\sigma^K}$ and $\psi_{X,L} = \frac{\gamma}{\sigma^K}$ to find that $\gamma = \psi_{X,L} + 1 - \alpha_{X,K}$

and $\sigma^{K} = \frac{\psi_{X,L}+1-\alpha_{X,K}}{\psi_{X,L}}$. This then implies that $\psi_{X,K} = \frac{\alpha_{X,K}}{1-\alpha_{X,K}}\psi_{X,L}$. From here we see that, given a calibrated value for $\alpha_{X,K}$, having γ in the Krugman model does not help us with matching our estimates of $\psi_{X,K} = -2.70$ and $\psi_{X,L} = 0.91$ for complete markets, and $\psi_{X,K} = -4.00$ and $\psi_{X,L} = 0.90$ for the bond economy.

B.3 Generalized Version of the Melitz Model

In order to show what role the love-of-variety effect plays in the Melitz model, let us introduce correction for this effect in the technology of production of final aggregate. Assume that the final aggregate technology is given by

$$Y_{nt} = \left[\sum_{i=1}^{N} \left[M_{ni,t}^{\phi_{Y,M}-\frac{1}{\sigma^{M}-1}} \left[\int_{\nu\in\Omega_{ni,t}} \left(\omega_{ni} x_{ni,t}\left(\nu\right)\right)^{\frac{\sigma^{M}-1}{\sigma^{M}}} d\nu \right]^{\frac{\sigma^{M}}{\sigma^{M}-1}} \right]^{\frac{\eta^{M}-1}{\eta^{M}}} \right]^{\frac{\eta^{M}}{\eta^{M}-1}},$$

where $M_{ni,t}^{\phi_{Y,M}-\frac{1}{\sigma^{M}-1}}$ is the correction term for the love-of-variety effect with the strength of the effect given by parameter $\phi_{Y,M}$. Denote, for convenience, $\widetilde{\phi}_{Y,M} \equiv \phi_{Y,M} - \frac{1}{\sigma^{M}-1}$. Demand for individual varieties is given by

$$\begin{aligned}
x_{ni,t}\left(\nu\right) &= M_{ni,t}^{(\sigma^{M}-1)\widetilde{\phi}_{Y,M}} \omega_{ni}^{\sigma^{M}-1} \left(\frac{p_{ni,t}\left(\nu\right)}{P_{ni,t}}\right)^{-\sigma^{M}} \left(\frac{P_{ni,t}}{P_{Y,nt}}\right)^{-\eta^{M}} Y_{nt}. \\
P_{ni,t} &= M_{ni,t}^{-\widetilde{\phi}_{Y,M}} \left[\int_{\nu\in\Omega_{ni,t}} p_{ni,t}\left(\nu\right)^{1-\sigma^{M}} d\nu \right]^{\frac{1}{1-\sigma^{M}}}, \\
P_{Y,nt} &= \left[\sum_{i=1}^{N} \left(P_{ni,t}/\omega_{ni}\right)^{1-\eta^{M}} \right]^{1-\eta^{M}}.
\end{aligned}$$

The profit that producer of variety $\nu \in \Omega_{it}$ can earn in market n is given by

$$\begin{aligned}
D_{ni,t}\left(\nu\right) &= \frac{1}{\sigma^{M}} p_{ni,t}\left(\nu\right) x_{ni,t}\left(\nu\right) - W_{nt}\Phi_{ni,t} \\
&= \frac{1}{\sigma^{M}} M_{ni,t}^{(\sigma^{M}-1)\widetilde{\phi}_{Y,M}} \left(\frac{\sigma^{M}}{\sigma^{M}-1} \cdot \frac{\tau_{ni,t}^{M} W_{it}}{\omega_{ni} S_{X,it}^{M} z_{i}\left(\nu\right)} \right)^{1-\sigma^{M}} P_{ni,t}^{\sigma^{M}-\eta^{M}} P_{Y,nt}^{\eta^{M}} Y_{nt} - W_{nt}\Phi_{ni,t}.
\end{aligned}$$

As long as $D_{ni,t}\left(\nu\right) \geq 0$, variety $\nu \in \Omega_{it}$ will be sold in country n. Condition $D_{ni,t}\left(\nu\right) = 0$ gives the cutoff efficiency $z_{ni,t}^{*}$ such that only producers with $z_{i}\left(\nu\right) \geq z_{ni,t}^{*}$ serve market n.

After some algebra, we get

$$\frac{z_{ni,t}^{*}}{z_{\min,i}}=M_{ni,t}^{-\widetilde{\phi}_{Y,M}}\left(\frac{\sigma^{M}}{\sigma^{M}-1}\cdot\frac{\tau_{ni,t}^{M}W_{it}}{\omega_{ni}S_{X,it}^{M}z_{\min,i}}\right)\left(\frac{P_{Y,nt}Y_{nt}}{\sigma^{M}W_{nt}\Phi_{ni,t}}\right)^{\frac{1}{1-\sigma^{M}}}P_{ni,t}^{-\frac{\sigma^{M}-\eta^{M}}{\sigma^{M}-1}}P_{Y,nt}^{-\frac{\eta^{M}-1}{\sigma^{M}-1}}.$$

With Pareto distribution of efficiencies of production, we have that

$$M_{ni,t}=M_{it}\int_{z_{ni,t}^{*}}^{\infty}dG_{i}\left(z\right)=M_{it}\left(1-G_{i}\left(z_{ni,t}^{*}\right)\right)=M_{it}\left(\frac{z_{ni,t}^{*}}{z_{\min,i}}\right)^{-\theta^{M}}.$$

This gives

$$\left(\frac{z_{ni,t}^{*}}{z_{\min,i}}\right)^{1-\widetilde{\phi}_{Y,M}\theta^{M}}=M_{it}^{-\widetilde{\phi}_{Y,M}}\left(\frac{\sigma^{M}}{\sigma^{M}-1}\cdot\frac{\tau_{ni,t}^{M}W_{it}}{\omega_{ni}S_{X,it}^{M}z_{\min,i}}\right)\left(\frac{P_{Y,nt}Y_{nt}}{\sigma^{M}W_{nt}\Phi_{ni,t}}\right)^{\frac{1}{1-\sigma^{M}}}P_{ni,t}^{-\frac{\sigma^{M}-\eta^{M}}{\sigma^{M}-1}}P_{Y,nt}^{-\frac{\eta^{M}-1}{\sigma^{M}-1}}. \tag{47}$$

Next, let us find the bilateral price indices. We have

$$\begin{aligned}
P_{ni,t}^{1-\sigma^{M}}&=M_{ni,t}^{(\sigma^{M}-1)\phi_{Y,M}-1}M_{it}\int_{z_{ni,t}^{*}}^{\infty}\left(\frac{\sigma^{M}}{\sigma^{M}-1}\cdot\frac{\tau_{ni,t}^{M}W_{it}}{S_{X,it}^{M}z}\right)^{1-\sigma^{M}}dG_{i}\left(z\right)\\
&=\theta^{M}z_{\min,i}^{\theta^{M}}\left(\frac{\sigma^{M}}{\sigma^{M}-1}\cdot\frac{\tau_{ni,t}^{M}W_{it}}{S_{X,it}^{M}}\right)^{1-\sigma^{M}}M_{ni,t}^{(\sigma^{M}-1)\phi_{Y,M}-1}M_{it}\int_{z_{ni,t}^{*}}^{\infty}z^{\sigma^{M}-\theta^{M}-2}dz\\
&=\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\left(\frac{\sigma^{M}}{\sigma^{M}-1}\cdot\frac{\tau_{ni,t}^{M}W_{it}}{z_{\min,i}S_{X,it}^{M}}\right)^{1-\sigma^{M}}M_{ni,t}^{(\sigma^{M}-1)\phi_{Y,M}-1}M_{it}\left(\frac{z_{ni,t}^{*}}{z_{\min,i}}\right)^{\sigma^{M}-\theta^{M}-1}\\
&=\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\left(\frac{\sigma^{M}}{\sigma^{M}-1}\cdot\frac{\tau_{ni,t}^{M}W_{it}}{z_{\min,i}S_{X,it}^{M}}\right)^{1-\sigma^{M}}M_{it}^{(\sigma^{M}-1)\phi_{Y,M}}\left(\frac{z_{ni,t}^{*}}{z_{\min,i}}\right)^{(\sigma^{M}-1)\left(1-\theta^{M}\phi_{Y,M}\right)}.
\end{aligned} \tag{48}$$

In order to ensure that the right-hand side of this expression is finite, we need to make the technical assumption that $\theta^{M}>\sigma^{M}-1$.

Without risk of confusion, let us redefine constant ϑ in definition (24) of $\Phi_{ni,t}$ to be $\vartheta\equiv\phi_{Y,M}-\frac{1}{\theta^{M}}$. Without correction for the love-of-variety effect (i.e., when $\phi_{Y,M}=1/\left(\sigma^{M}-1\right)$), we have the same definition of ϑ as in the main text. Substituting the expression (47) for the cutoff threshold into (48) and using the definition of $\Phi_{ni,t}$, we get:

$$P_{ni,t}^{1-\sigma^{M}}=\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\left(\tau_{ni,t}^{M}P_{X,it}\right)^{-\frac{\theta^{M}}{1-\widetilde{\phi}_{Y,M}\theta^{M}}}\left(\frac{P_{Y,nt}Y_{nt}}{\sigma^{M}L_{F,nt}^{\frac{\vartheta-\phi_{F,L}}{\vartheta}}W_{nt}F_{ni,t}}P_{ni,t}^{\sigma^{M}-\eta^{M}}P_{Y,nt}^{\eta^{M}-1}\right)^{\frac{\vartheta\theta^{M}}{1-\widetilde{\phi}_{Y,M}\theta^{M}}},$$

where

$$P_{X,it} \equiv \frac{\sigma^M}{\sigma^M - 1} \cdot \frac{W_{it}}{z_{\min,i} S^M_{X,it} M_{it}^{\phi_{F,M}}}.$$

Solving for $P_{ni,t}$, we get

$$P_{ni,t}^{1-\eta^M} = \left(\frac{\theta^M}{\theta^M + 1 - \sigma^M}\right)^{\left(1-\widetilde{\phi}_{Y,M}\theta^M\right)\xi} \left(\tau^M_{ni,t} P_{X,it}\right)^{-\theta^M \xi} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^M L_{F,nt}^{\frac{\vartheta - \phi_{F,L}}{\vartheta}} W_{nt} F_{ni,t}} P_{Y,nt}^{\eta^M - 1}\right)^{\vartheta \theta^M \xi},$$

where

$$\xi \equiv \frac{1}{\left(\frac{1}{\eta^M - 1} - \phi_{Y,M}\right)\theta^M + 1}.$$

This allows us to find expression for the price index,

$$P_{Y,nt} = \left(\frac{\theta^M}{\theta^M + 1 - \sigma^M}\right)^{-\left(\frac{1}{\theta^M} - \widetilde{\phi}_{Y,M}\right)} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^M W_{nt}}\right)^{-\vartheta} L_{F,nt}^{\vartheta - \phi_{F,L}} \left[\sum_{i=1}^{N} \left(F_{ni,t}^{\vartheta} \tau^M_{ni,t} P_{X,it} / \omega_{ni}\right)^{-\theta^M \xi}\right]^{-\frac{1}{\theta^M \xi}}.$$

Next, bilateral trade flows are given by:

$$\begin{aligned} \mathcal{X}_{ni,t} &= M_{it} \int_{\Omega_{ni,t}} p_{ni,t}(\nu)\, x_{ni,t}(\nu)\, d\nu \\ &= \left(\frac{P_{ni,t}/\omega_{ni}}{P_{Y,nt}}\right)^{1-\eta^M} P_{Y,nt} Y_{nt}. \end{aligned}$$

Substituting expressions for price indices, we get

$$\mathcal{X}_{ni,t} = \lambda_{ni,t} P_{Y,nt} Y_{nt},$$

where

$$\lambda_{ni,t} = \frac{\left(\Phi^{\vartheta}_{ni,t} \tau^M_{ni,t} P_{X,it} / \omega_{ni}\right)^{-\theta^M \xi}}{\sum_{l=1}^{N} \left(\Phi^{\vartheta}_{nl,t} \tau^M_{nl,t} P_{X,lt} / \omega_{nl}\right)^{-\theta^M \xi}}.$$

Let us now find profits. For this, we need to have the expression for $z^*_{ni,t}$. After some algebra, we get

$$\left(\frac{z^*_{ni,t}}{z_{\min,i}}\right)^{1-\widetilde{\phi}_{Y,M}\theta^M} = \frac{\tau^M_{ni,t} P_{X,it}/\omega_{ni}}{P_{ni,t}} M_{it}^{\frac{\phi_{Y,M} - \phi_{F,M}}{\vartheta}\left(\frac{1}{\sigma^M - 1} - \vartheta\right)} \left(\frac{\mathcal{X}_{ni,t}}{\sigma^M W_{nt} F_{ni,t}}\right)^{\frac{1}{1-\sigma^M}} L_{F,nt}^{\frac{\vartheta - \phi_{F,L}}{\vartheta(\sigma^M - 1)}}.$$

Next, we have

$$\frac{\tau^{\mathrm{M}}_{ni,t} P_{X,it}/\omega_{ni}}{P_{ni,t}} = \left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}\right)^{\frac{1-\widetilde{\phi}_{Y,M}\theta^{\mathrm{M}}}{\theta^{\mathrm{M}}}} \left(\frac{P_{ni,t}}{P_{Y,nt}}\right)^{(1-\eta^{\mathrm{M}})\vartheta} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^{\mathrm{M}} W_{nt} F_{ni,t}}\right)^{\vartheta} L_{F,nt}^{-(\vartheta-\phi_{F,L})}$$
$$= \left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}\right)^{\frac{1-\widetilde{\phi}_{Y,M}\theta^{\mathrm{M}}}{\theta^{\mathrm{M}}}} \left(\frac{\mathcal{X}_{ni,t}}{\sigma^{\mathrm{M}} W_{nt} F_{ni,t}}\right)^{\vartheta} L_{F,nt}^{-(\vartheta-\phi_{F,L})},$$

which allows us to find

$$\left(\frac{z^*_{ni,t}}{z_{\min,i}}\right)^{\theta^{\mathrm{M}}} = \left[\frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{W_{nt}F_{ni,t}}\right]^{-1} M_{it}^{\frac{\phi_{Y,M}-\phi_{F,M}}{\vartheta}} L_{F,nt}^{\frac{\vartheta-\phi_{F,L}}{\vartheta}},$$

and

$$M_{ni,t} = \left(\frac{z^*_{ni,t}}{z_{\min,i}}\right)^{-\theta^{\mathrm{M}}} M_{it} = \left(\frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{W_{nt}F_{ni,t}}\right) \left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}}-\phi_{F,M}} L_{F,nt}^{\vartheta-\phi_{F,L}}\right]^{-\frac{1}{\vartheta}}.$$

Observe that in the standard Melitz model $\phi_{F,M} = \frac{1}{\theta^{\mathrm{M}}}$ and $\phi_{F,L} = \vartheta$, and so $M_{ni,t} = \frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \mathcal{X}_{ni,t}/(W_{nt}F_{ni,t})$. Thus, M_{it} does not directly affect $M_{ni,t}$ (see Section 4.2 and footnote 27).

To get average profits of country i from exports to n, we need to calculate the following expression:

$$D_{ni,t} = \frac{1}{\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{M_{it}} - W_{nt} \left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}}-\phi_{F,M}} L_{F,nt}^{\vartheta-\phi_{F,L}}\right]^{\frac{1}{\vartheta}} F_{ni,t} \frac{M_{ni,t}}{M_{it}}$$
$$= \frac{1}{\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{M_{it}} - \frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{M_{it}}$$
$$= \frac{\sigma^{\mathrm{M}}-1}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{M_{it}}.$$

Hence, total average profits of country i are

$$D_{it} = \sum_{n=1}^{N} D_{ni,t} = \frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{it}}{M_{it}},$$

where $\mathcal{X}_{it}$ is total output of intermediates in country i. We can find that, as in the Krugman model,

$$\mathcal{X}_{it} = \frac{\sigma^{\mathrm{M}}}{\sigma^{\mathrm{M}}-1} W_{it} L^{\mathrm{M}}_{X,it}.$$

The amount of country n's labor that country i uses to serve country n's market is

$$L_{F,ni,t} = \left[M_{it}^{\frac{1}{\theta^M} - \phi_{F,M}} L_{F,nt}^{\vartheta - \phi_{F,L}} \right]^{\frac{1}{\vartheta}} F_{ni,t} M_{ni,t} = \frac{\theta^M + 1 - \sigma^M}{\theta^M \sigma^M} \cdot \frac{\mathcal{X}_{ni,t}}{W_{nt}}.$$

Hence, the total amount of country n's labor used to serve its market is

$$\begin{aligned} L_{F,nt} &= \sum_{i=1}^{N} L_{F,ni,t} = \frac{\theta^M + 1 - \sigma^M}{\theta^M \sigma^M} \cdot \sum_{i=1}^{N} \frac{\mathcal{X}_{ni,t}}{W_{nt}} \\ &= \frac{\theta^M + 1 - \sigma^M}{\theta^M \sigma^M} \cdot \frac{P_{Y,nt} Y_{nt}}{W_{nt}}. \end{aligned}$$

This allows us to write

$$P_{Y,nt} = \left(\frac{\theta^M}{\theta^M + 1 - \sigma^M} \right)^{-\frac{1}{\sigma^M - 1}} L_{F,nt}^{-\phi_{F,L}} \left[\sum_{i=1}^{N} \left(F_{ni,t}^{\vartheta} \tau_{ni,t}^{M} P_{X,it} / \omega_{ni} \right)^{-\theta^M \xi} \right]^{-\frac{1}{\theta^M \xi}}.$$

or

$$P_{Y,nt} = \left(\frac{\theta^M}{\theta^M + 1 - \sigma^M} \right)^{-\frac{1}{\sigma^M - 1} + \phi_{F,L}} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^M W_{nt}} \right)^{-\phi_{F,L}} \left[\sum_{i=1}^{N} \left(F_{ni,t}^{\vartheta} \tau_{ni,t}^{M} P_{X,it} / \omega_{ni} \right)^{-\theta^M \xi} \right]^{-\frac{1}{\theta^M \xi}}.$$

Also, we can write

$$L_{F,nt} = \frac{\theta^M + 1 - \sigma^M}{\theta^M \sigma^M} \cdot \frac{\mathcal{X}_{nt}}{W_{nt}} \cdot \frac{P_{Y,nt} Y_{nt}}{\mathcal{X}_{nt}} = \left(\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M} \right) \frac{P_{Y,nt} Y_{nt}}{\mathcal{X}_{nt}} L_{X,nt}^{M}.$$

Equilibrium System of Equations In order to write the equilibrium system in the isomorphic form, we need to do transformations of some of the equilibrium conditions. Define trade deficit as the real value of net exports of varieties in terms of the final good,

$$\mathrm{TB}_{nt} \equiv (\mathcal{X}_{nt} - P_{Y,nt} Y_{nt}) / P_{Y,nt}.$$

We can write

$$\begin{aligned} L_{F,nt} &= \left(\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M} \right) \frac{P_{Y,nt} Y_{nt}}{\mathcal{X}_{nt}} L_{X,nt}^{M} = \left(\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M} \right) \frac{\mathcal{X}_{nt} - P_{Y,nt} \cdot \mathrm{TB}_{nt}}{\mathcal{X}_{nt}} L_{X,nt}^{M} \\ &= \left(\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M} \right) L_{X,nt}^{M} - \left(\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M} \right) \frac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{\mathcal{X}_{nt}} L_{X,nt}^{M}. \end{aligned}$$

Using expression $\mathcal{X}_{nt} = \dfrac{\sigma^{M}}{\sigma^{M}-1} W_{nt} L^{M}_{X,nt}$, we can write

$$\left(\frac{1}{\sigma^{M}-1} - \frac{1}{\theta^{M}}\right) \frac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{\mathcal{X}_{nt}} L^{M}_{X,nt} = \frac{\theta^{M} + 1 - \sigma^{M}}{\theta^{M}\sigma^{M}} \cdot \frac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{W_{nt}}.$$

Define

$$L_{X,nt} \equiv L^{M}_{X,nt} + \left(\frac{1}{\sigma^{M}-1} - \frac{1}{\theta^{M}}\right) L^{M}_{X,nt} = \left(\frac{\sigma^{M}}{\sigma^{M}-1} - \frac{1}{\theta^{M}}\right) L^{M}_{X,nt}.$$

With this definition the labor market clearing condition can be written as

$$L_{X,nt} + L_{I,nt} = L_{nt} + \frac{\theta^{M} + 1 - \sigma^{M}}{\theta^{M}\sigma^{M}} \cdot \frac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{W_{nt}}.$$

Next, rewrite condition for $\mathcal{X}_{nt}$,

$$\mathcal{X}_{nt} = \frac{\sigma^{M}}{\sigma^{M}-1} W_{nt} L^{M}_{X,nt} = \frac{1}{1 - \dfrac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} \cdot W_{nt} L_{X,nt}.$$

Manipulate the expression for $P_{X,nt}$,

$$\begin{aligned} P_{X,nt} &= \frac{\sigma^{M}}{\sigma^{M}-1} \cdot \frac{W_{nt}}{z_{\min,n} \Theta^{M}_{X,n} Z_{X,nt} M_{nt}^{\phi_{F,M}} \left[L^{M}_{X,nt}\right]^{\phi_{X,L}}} \\ &= \frac{\sigma^{M}}{\sigma^{M}-1} \cdot \left(\frac{\sigma^{M}}{\sigma^{M}-1} - \frac{1}{\theta^{M}}\right)^{\phi_{X,L}} \frac{D_{nt}^{\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} W_{nt}^{1-\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}}}{z_{\min,n} \Theta^{M}_{X,n} Z_{X,nt} M_{nt}^{\phi_{F,M}} L_{X,nt}^{\phi_{X,L}} D_{nt}^{\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} W_{nt}^{-\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}}}. \end{aligned}$$

Using the facts that $D_{nt} = \dfrac{\sigma^{M}-1}{\sigma^{M}\theta^{M}} \cdot \dfrac{\mathcal{X}_{nt}}{M_{nt}}$ and $W_{nt} = \left(1 - \dfrac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}\right) \dfrac{\mathcal{X}_{nt}}{L_{X,nt}}$, we get

$$P_{X,nt} = \frac{D_{nt}^{\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} W_{nt}^{1-\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}}}{\widetilde{\Theta}^{M}_{X,n} Z_{X,nt} M_{nt}^{\phi_{F,M} - \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} L_{X,nt}^{\phi_{X,L} + \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}}},$$

where

$$\widetilde{\Theta}^{M}_{X,n} \equiv \left(\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}\right)^{\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} \left(1 - \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}\right)^{1-\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} \left(\frac{\sigma^{M}}{\sigma^{M}-1} - \frac{1}{\theta^{M}}\right)^{-1-\phi_{X,L}} \Theta^{M}_{X,n} z_{\min,n}.$$

Let $X_{nt} \equiv \mathcal{X}_{nt}/P_{X,nt}$ be the real output of varieties. By substituting expressions for D_{nt} and W_{nt} into the above expression for $P_{X,nt}$ we get

$$X_{nt} = \left(\Theta_{X,n} M_{nt}^{\phi_{F,M}-\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{\phi_{X,L}+\frac{\sigma^M-1}{\sigma^M\theta^M}}\right) M_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}},$$

where

$$\Theta_{X,n} \equiv \left(\frac{\sigma^M}{\sigma^M-1} - \frac{1}{\theta^M}\right)^{-1-\phi_{X,L}} \Theta_{X,n}^M z_{\min,n}.$$

Next, we have

$$\sum_{i=1}^N \left(F_{ni,t}^{\vartheta}\tau_{ni,t}^M P_{X,it}/\omega_{ni}\right)^{-\theta^M\xi} = \left(\frac{\theta^M}{\theta^M+1-\sigma^M}\right)^{-\left(\frac{1}{\sigma^M-1}-\phi_{F,L}\right)\theta^M\xi} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^M W_{nt}}\right)^{-\phi_{F,L}\theta^M\xi} P_{Y,nt}^{-\theta^M\xi},$$

and so

$$\lambda_{ni,t} = \left(\frac{\theta^M}{\theta^M+1-\sigma^M}\right)^{\left(\frac{1}{\sigma^M-1}-\phi_{F,L}\right)\theta^M\xi} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^M W_{nt}}\right)^{\phi_{F,L}\theta^M\xi} \left(F_{ni,t}^{\vartheta}\tau_{ni,t}^M P_{X,it}/\omega_{ni}\right)^{-\theta^M\xi} P_{Y,nt}^{\theta^M\xi},$$

which gives

$$\frac{\lambda_{ni,t}P_{Y,nt}Y_{nt}}{F_{ni,t}^{\vartheta}\tau_{ni,t}^M P_{X,it}/\omega_{ni}} = \left(\frac{\theta^M}{\theta^M+1-\sigma^M}\right)^{\left(\frac{1}{\sigma^M-1}-\phi_{F,L}\right)\theta^M\xi} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^M W_{nt}}\right)^{\phi_{F,L}\theta^M\xi} \times \left(F_{ni,t}^{\vartheta}\tau_{ni,t}^M P_{X,it}/\omega_{ni}\right)^{-(1+\theta^M\xi)} P_{Y,nt}^{1+\theta^M\xi} Y_{nt}.$$

Taking both sides to the power of $\frac{\theta^M\xi}{1+\theta^M\xi}$, we get

$$\left(\frac{\lambda_{ni,t}P_{Y,nt}Y_{nt}}{F_{ni,t}^{\vartheta}\tau_{ni,t}^M P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^M\xi}{1+\theta^M\xi}} = \left[\left(\frac{\theta^M}{\theta^M+1-\sigma^M}\right)^{\left(\frac{1}{\sigma^M-1}-\phi_{F,L}\right)\theta^M\xi} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^M W_{nt}}\right)^{\phi_{F,L}\theta^M\xi} Y_{nt}\right]^{\frac{\theta^M\xi}{1+\theta^M\xi}} \times \left(F_{ni,t}^{\vartheta}\tau_{ni,t}^M P_{X,it}/\omega_{ni}\right)^{-\theta^M\xi} P_{Y,nt}^{\theta^M\xi}.$$

Summing over i and doing some algebra, we get

$$Y_{nt} = \left(\frac{\theta^M}{\theta^M+1-\sigma^M}\right)^{\frac{1}{\sigma^M-1}-\phi_{F,L}} [\sigma^M]^{-\phi_{F,L}} \left(\frac{P_{Y,nt}Y_{nt}}{W_{nt}}\right)^{\phi_{F,L}} \left[\sum_{i=1}^N \left(\frac{\lambda_{ni,t}P_{Y,nt}Y_{nt}}{F_{ni,t}^{\vartheta}\tau_{ni,t}^M P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^M\xi}{1+\theta^M\xi}}\right]^{\frac{1+\theta^M\xi}{\theta^M\xi}}.$$

Let us redefine iceberg trade costs as

$$\tau_{ni,t} \equiv \left(\frac{F_{ni,t}}{F_{nn,t}}\right)^{\vartheta} \tau^{\mathrm{M}}_{ni,t}.$$

Under Assumption 1, the redefined iceberg trade costs $\tau_{ni,t}$ satisfy $\tau_{ni,t} \geq 1$ for all n, i, and t, and they also satisfy the triangle inequality.[67] Using the definition of $\tau_{ni,t}$, we can write the expression for the final aggregate as

$$Y_{nt} = \left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}\right)^{\frac{1}{\sigma^{\mathrm{M}}-1} - \phi_{F,L}} \left[\sigma^{\mathrm{M}}\right]^{-\phi_{F,L}} F_{nn,t}^{-\vartheta} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^{\mathrm{M}}\xi}{1+\theta^{\mathrm{M}}\xi}}\right]^{\frac{1+\theta^{\mathrm{M}}\xi}{\theta^{\mathrm{M}}\xi}}.$$

Let us write $F_{nn,t}^{-\vartheta} = \Theta^{\mathrm{M}}_{Y,n} Z_{Y,nt}$, where $Z_{Y,nt}$ is supposed to be the same exogenous shock as in the unified model. Define

$$\Theta_{Y,n} \equiv \left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}\right)^{\frac{1}{\sigma^{\mathrm{M}}-1} - \phi_{F,L}} \left[\sigma^{\mathrm{M}}\right]^{-\phi_{F,L}} \Theta^{\mathrm{M}}_{Y,n}.$$

Then we can write

$$Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^{\mathrm{M}}\xi}{1+\theta^{\mathrm{M}}\xi}}\right]^{\frac{1+\theta^{\mathrm{M}}\xi}{\theta^{\mathrm{M}}\xi}}.$$

Combining all expressions and definitions, we get the equilibrium system in isomor-

[67] In Assumption 1 we use the definition of ϑ from the main text, i.e., we use $\vartheta \equiv \frac{1}{\sigma^{\mathrm{M}}-1} - \frac{1}{\theta^{\mathrm{M}}}$. Formally speaking, for the purposes of the current appendix we need to modify Assumption 1 and use the definition $\vartheta \equiv \phi_{Y,M} - \frac{1}{\theta^{\mathrm{M}}}$. This slight abuse of notation should not create confusion.

phic form (for the case of financial autarky):

$$
\begin{aligned}
V_{nt} &= \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[D_{n,t+1} + (1-\delta) V_{n,t+1}\right] \right\}, \\
-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} &= \frac{W_{nt}}{P_{Y,nt}}, \\
M_{n,t+1} &= (1-\delta) M_{nt} + M_{I,nt}, \\
X_{nt} &= \left(\Theta_{X,n} M_{nt}^{\phi_{F,M} - \frac{\sigma^M - 1}{\sigma^M \theta^M}} L_{X,nt}^{\phi_{X,L} + \frac{\sigma^M - 1}{\sigma^M \theta^M}} \right) M_{nt}^{\frac{\sigma^M - 1}{\sigma^M \theta^M}} L_{X,nt}^{1 - \frac{\sigma^M - 1}{\sigma^M \theta^M}}, \\
Y_{nt} &= \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it} / \omega_{ni}} \right)^{\frac{\theta^M \xi}{1+\theta^M \xi}} \right]^{\frac{1+\theta^M \xi}{\theta^M \xi}}, \\
M_{I,nt} &= \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
L_{X,nt} + L_{I,nt} &= L_{nt} + \frac{\theta^M + 1 - \sigma^M}{\theta^M \sigma^M} \cdot \frac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{W_{nt}}, \\
C_{nt} + Y_{I,nt} &= Y_{nt}, \\
\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} &= P_{X,it} X_{it}, \\
\lambda_{ni,t} &= \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{-\theta^M \xi}}{\sum_{l=1}^{N} \left(\tau_{nl,t} P_{X,lt} / \omega_{nl}\right)^{-\theta^M \xi}}, \\
M_{nt} &= \frac{\sigma^M - 1}{\sigma^M \theta^M} \cdot \frac{P_{X,nt} X_{nt}}{D_{nt}}, \\
L_{X,nt} &= \left(1 - \frac{\sigma^M - 1}{\sigma^M \theta^M}\right) \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
L_{I,nt} &= \alpha_I \frac{V_{nt} M_{I,nt}}{W_{nt}}, \\
Y_{I,nt} &= (1-\alpha_I) \frac{V_{nt} M_{I,nt}}{P_{Y,nt}}, \\
P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} &= D_{nt} M_{nt} + W_{nt} L_{nt}.
\end{aligned}
$$

Love of Variety in the Melitz Model Let us discuss the role that the strength of the love-of-variety effect — given by parameter $\phi_{Y,M}$ — plays in the generalized Melitz model. The love-of-variety effect impacts the above system in two places. First, it impacts the trade elasticity, which is given by the exponent of $\tilde{\tau}_{ni,t}$ in the expression for trade shares $\lambda_{ni,t}$ and is equal to $\theta^M \xi$ with

$$\xi = \frac{1}{\left(\frac{1}{\eta^M - 1} - \phi_{Y,M}\right)\theta^M + 1}. \tag{49}$$

Second, if we remove labor externality in the fixed costs of serving markets by assuming that $\phi_{F,L} = \vartheta$, then the strength of economies of scale in production of the final aggregate will be given by $-\vartheta$ with $\vartheta = \phi_{Y,M} - \frac{1}{\theta^M}$. These changes are due to the fact that correction for the love of variety changes the impact of the extensive margin through selection on the bilateral price index. Formally, this correction changes the exponent of the middle term in expression (38).

Importantly, not all combinations of the trade elasticity and the strength of economies of scale in production of the final aggregate in the unified model can be mapped into a valid trade elasticity $\theta^M \xi$ in the generalized Melitz model, if we keep parameter restriction that $\phi_{F,L} = \vartheta$. For example, the value $\psi_Y = \frac{1}{\eta^M - 1}$ can be used in the unified model, but not in the corresponding Melitz model. Indeed, having $\psi_Y = \frac{1}{\eta^M - 1}$ in the unified model implies that in the corresponding generalized Melitz model we need to have $\vartheta = -\psi_Y = -\frac{1}{\eta^M - 1}$ and $\phi_{Y,M} = -\vartheta + \frac{1}{\theta^M} = \frac{1}{\eta^M - 1} + \frac{1}{\theta^M}$. But this, in turn, implies that the denominator in expression (49) for ξ is zero. In other words, having $\psi_Y = \frac{1}{\eta^M - 1}$ in the unified model implies a non-valid value for ξ in the corresponding generalized Melitz model.

If we relax parameter restriction that $\phi_{F,L} = \vartheta$, and, thus, allow for labor externalities in the fixed costs of serving markets, then the only place where parameter $\phi_{Y,M}$ impacts the equilibrium system in the generalized Melitz model is the trade elasticity. Then any combination of trade elasticity and the strength of economies of scale in production of the final aggregate in the unified model can be mapped into the corresponding parameters in the generalized Melitz model. Thus, we can have isomorphism. However, in this case, the trade elasticity in the generalized Melitz model is governed by two free parameters: η^M and $\phi_{Y,M}$. So, one of these parameters is redundant for the purposes of isomorphism. It makes more economic sense to adjust parameter η^M — elasticity of substitution between varieties produced in different countries — rather than $\phi_{Y,M}$ to change the trade elasticity. Hence, parameter $\phi_{Y,M}$ is not needed in this case. This is why we choose to not to have correction for the love-of-variety in the generalized Melitz model in the main text, i.e., in the main text we have $\phi_{Y,M} = \frac{1}{\sigma^M - 1}$, $\vartheta \equiv \frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}$, and

$$\xi = \frac{1}{\left(\frac{1}{\eta^M - 1} - \frac{1}{\sigma^M - 1}\right)\theta^M + 1}.$$

C Grid Search

In the grid search, we consider the following values for our parameters: $\psi_{X,K} \in [-10, 10]$ with step 0.1, $\psi_{X,L} \in [-5, 5]$ with step 0.1, $\psi_Y \in \{-0.35, -0.3, -0.2, -0.1, 0, 0.1\}$, $\sigma_X \in [0, 0.01]$ with step 0.001, $\sigma_X \in [0, 0.008]$ with step 0.001,
$\rho_X, \rho_Y \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.92, 0.94, 0.96, 0.98, 0.99\}$. For some of these parameter combinations the steady state does not exists, or the Blanchard-Kahn conditions are not satisfied, and thus moments cannot be calculated. For each point on the grid where we can compute moments, we calculate the loss function

$$\mathcal{L} = \sqrt{\sum_{m=1}^{M} \left(\left[\text{Mom}^{\text{model}}\right]_m / \left[\text{Mom}^{\text{data}}\right]_m - 1 \right)^2},$$

where $\left[\text{Mom}^{\text{model}}\right]_m$ and $\left[\text{Mom}^{\text{data}}\right]_m$ are moments calculated in the model and data. With then find the point on the grid with the minimal value of $\mathcal{L}$.

For the case of complete markets, after finding the point with the minimal value of $\mathcal{L}$, we additionally consider a finer grid around the set of points with the value of $\mathcal{L}$ within 1% of the the minimal value of $\mathcal{L}$. In the finer grid, we use step 0.01 for $\psi_{X,K}$, $\psi_{X,L}$, and ψ_Y, and step 0.01 for ρ_X and ρ_Y, while we leave the same grid for σ_X and σ_Y. We calculate the moments on this finer grid and the associated loss functions. We then, again, find the set of points with the value of $\mathcal{L}$ within 1% of the the minimal value of $\mathcal{L}$ on this finer grid. We repeat this procedure until such set of points stops changing. For the result for complete markets in Table 7, we report the result for the point with the minimal value of $\mathcal{L}$ in this set.

D Additional Tables with Moments

We tabulate below model moments for various extensions and sensitivity analysis.

D.1 Comparison Across Models

		Benchmark calibration						Investment final good			
		Int. good shock			Final good shock			Int. good shock		Final good shock	
	Data	IRBC	Krug	Mel	IRBC	Krug	Mel	Krug	Mel	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
International moments:											
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	0.58	0.15	0.06	0.06	0.03	−0.01	0.00	0.16	0.16	0.05	0.05
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.17	0.12	0.11	0.05	0.00	0.00	0.18	0.18	0.07	0.06
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	0.14	0.01	0.01	0.02	−0.01	−0.01	0.15	0.15	0.03	0.04
$\text{Corr}\left(L_1, L_2\right)$	0.42	0.13	−0.01	0.00	0.01	−0.02	−0.02	0.14	0.15	0.02	0.03
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49										
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.32	0.89	0.86	0.88	1.00	1.00	1.00	0.90	0.90	0.99	0.99
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.81	0.89	0.86	0.88	1.00	1.00	1.00	0.90	0.90	0.99	0.99
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.13	0.65	0.67	0.67	0.70	0.71	0.71	0.65	0.65	0.69	0.69
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45										
Domestic moments:											
$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.86	0.98	0.98	0.98	0.98	1.00	1.00	0.98	0.98	0.98	0.98
$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.87	0.99	0.97	0.96	0.99	0.98	0.98	0.98	0.98	0.98	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	1.00	0.98	0.98	1.00	1.00	1.00	0.99	0.99	0.99	0.99
$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.81	0.45	0.56	0.58	0.45	0.91	0.91	0.51	0.53	0.51	0.53
$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.66	0.39	0.41	0.37	0.39	0.08	0.08	0.34	0.33	0.34	0.33
$\frac{\text{Std}\left(P_{I,1}I_1/P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	2.84	2.64	3.50	3.86	2.64	1.51	1.59	3.18	3.66	3.18	3.66

Notes: See notes to Table 3 for the intermediate good shock and notes to Table 4 for the final good shock.

Table 8: Moments from standard calibrations and formulations of models. Financial autarky.

	$\sigma = 0.9$						$\sigma = 6$					
	Int. good shock			Final good shock			Int. good shock			Final good shock		
	IRBC	Krug	Mel $Z_{Y,n} = 1$	IRBC	Krug	Mel $Z_{Y,n} = 1$	IRBC	Krug	Mel $Z_{Y,n} = 1$	IRBC	Krug	Mel $Z_{Y,n} = 1$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
International moments:												
Corr $(\text{GDP}_1, \text{GDP}_2)$	0.25	0.31	0.29	−0.13	−0.08	−0.10	−0.27	−0.49	−0.54	−0.22	−0.34	−0.43
Corr (C_1, C_2)	0.21	0.29	0.28	−0.11	−0.04	−0.06	0.69	0.50	0.37	−0.13	−0.04	−0.16
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	−0.06	−0.01	−0.04	−0.40	−0.37	−0.39	−0.77	−0.84	−0.87	−0.86	−0.86	−0.89
Corr (L_1, L_2)	0.28	0.33	0.27	−0.16	−0.12	−0.17	−0.61	−0.78	−0.81	−0.26	−0.51	−0.60
Corr $\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.60	−0.57	−0.58	−0.73	−0.71	−0.71	−0.48	−0.29	−0.27	−0.60	−0.49	−0.51
Corr $(\text{Exp}_1, \text{GDP}_1)$	0.58	0.70	0.70	0.41	0.57	0.59	−0.25	−0.08	−0.09	−0.44	−0.28	−0.32
Corr $(\text{Imp}_1, \text{GDP}_1)$	0.99	0.99	0.99	1.00	1.00	1.00	0.68	0.47	0.43	0.72	0.65	0.66
Corr $(\text{ReR}, \text{GDP}_1)$	0.58	0.56	0.57	0.71	0.70	0.71	0.05	0.40	0.68	0.78	0.82	0.84
Std $\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.19	0.18	0.18	0.22	0.22	0.22	0.77	0.90	1.13	1.52	1.42	1.60
Domestic moments:												
Corr (C_1, GDP_1)	0.98	0.98	0.98	0.98	0.98	0.98	0.83	0.84	0.87	0.99	0.98	0.98
Corr (L_1, GDP_1)	0.99	0.98	0.98	0.99	0.98	0.98	0.98	0.98	0.98	0.99	0.99	0.99
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.98	0.98	0.97	0.98	0.98	0.97	0.89	0.88	0.86	0.84	0.83	0.82
$\frac{\text{Std}(C_1)}{\text{Std}(\text{GDP}_1)}$	0.46	0.52	0.53	0.45	0.51	0.52	0.30	0.30	0.31	0.43	0.43	0.44
$\frac{\text{Std}(L_1)}{\text{Std}(\text{GDP}_1)}$	0.39	0.34	0.33	0.40	0.35	0.34	0.53	0.52	0.51	0.40	0.40	0.39
$\frac{\text{Std}(P_{I,1}I_1/P_{Y,1})}{\text{Std}(\text{GDP}_1)}$	3.03	3.64	4.22	3.15	3.81	4.42	4.67	5.60	6.84	6.14	6.77	8.15

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 9: Robustness with respect to different values of σ. Complete markets.

		Benchmark calibration						Investment final good			
		Complete			Bond			Complete		Bond	
	Data	IRBC	Krug	Mel	IRBC	Krug	Mel	Krug	Mel	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
International moments:											
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	0.58	0.14	0.03	0.06	0.17	0.09	0.10	−0.01	−0.04	0.05	0.00
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.79	0.72	0.71	0.65	0.60	0.61	0.69	0.65	0.57	0.54
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.48	−0.43	−0.44	−0.45	−0.39	−0.40	−0.60	−0.65	−0.57	−0.63
$\text{Corr}\left(L_1, L_2\right)$	0.42	−0.51	−0.65	−0.66	−0.38	−0.55	−0.57	−0.66	−0.68	−0.55	−0.60
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.49	0.48	0.51	−0.55	0.12	0.26	−0.37	−0.34	−0.51	−0.49
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.32	0.38	0.86	0.88	0.27	0.75	0.82	0.50	0.48	0.35	0.35
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.81	0.95	0.50	0.50	0.96	0.76	0.74	0.90	0.88	0.94	0.93
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.13	0.53	0.56	0.59	0.46	0.48	0.54	0.63	0.65	0.57	0.61
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.21	0.12	0.14	0.24	0.08	0.08	0.20	0.22	0.23	0.24
Domestic moments:											
$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.86	0.91	0.91	0.92	0.95	0.95	0.95	0.91	0.92	0.95	0.95
$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.87	0.94	0.91	0.91	0.95	0.92	0.91	0.93	0.93	0.94	0.94
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.93	0.95	0.94	0.93	0.95	0.94	0.93	0.92	0.92	0.91
$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.81	0.50	0.55	0.57	0.53	0.58	0.60	0.52	0.52	0.56	0.56
$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.66	0.40	0.47	0.43	0.36	0.42	0.39	0.38	0.38	0.34	0.34
$\frac{\text{Std}\left(P_{I,1}I_1/P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	2.84	3.11	3.58	3.94	3.07	3.53	3.88	3.83	4.55	3.80	4.51

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. For Columns (8)-(11), parameterizations are $\alpha_I = 0$ and $Z_{I,nt} = 1$ and $Z_{Y,nt} = 1$ (i.e., there are no shocks to the investment and final good sectors).

Table 10: Correlated shocks across countries and spillovers with an otherwise standard calibration. Shock to the intermediate goods sector.

	Data	Benchmark calibration						Investment final good			
		Complete			Bond			Complete		Bond	
	Data	IRBC	Krug	Mel	IRBC	Krug	Mel	Krug	Mel	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
International moments:											
$\text{Corr}\,(\text{GDP}_1, \text{GDP}_2)$	0.58	−0.05	0.16	0.17	−0.04	0.18	0.19	−0.14	−0.20	−0.11	−0.18
$\text{Corr}\,(C_1, C_2)$	0.36	0.45	0.41	0.41	0.40	0.38	0.38	0.38	0.31	0.31	0.25
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.73	−0.29	−0.34	−0.72	−0.24	−0.29	−0.77	−0.80	−0.76	−0.79
$\text{Corr}\,(L_1, L_2)$	0.42	−0.52	−0.98	−0.98	−0.48	−0.97	−0.97	−0.65	−0.69	−0.58	−0.64
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.66	0.63	0.63	−0.66	0.62	0.62	−0.64	−0.65	−0.65	−0.67
$\text{Corr}\,(\text{Exp}_1, \text{GDP}_1)$	0.32	−0.19	0.99	0.99	−0.18	0.99	0.99	−0.05	−0.06	−0.10	−0.11
$\text{Corr}\,(\text{Imp}_1, \text{GDP}_1)$	0.81	0.91	0.97	0.97	0.91	0.99	0.98	0.92	0.92	0.92	0.92
$\text{Corr}\,(\text{ReR}, \text{GDP}_1)$	0.13	0.68	0.65	0.64	0.67	0.64	0.64	0.72	0.73	0.71	0.72
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.56	0.06	0.07	0.55	0.04	0.04	0.49	0.51	0.52	0.54
Domestic moments:											
$\text{Corr}\,(C_1, \text{GDP}_1)$	0.86	0.95	0.99	0.99	0.96	0.99	0.99	0.95	0.95	0.97	0.96
$\text{Corr}\,(L_1, \text{GDP}_1)$	0.87	0.96	0.72	0.71	0.96	0.73	0.72	0.95	0.95	0.95	0.95
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.91	0.97	0.96	0.91	0.97	0.97	0.90	0.90	0.90	0.90
$\frac{\text{Std}\,(C_1)}{\text{Std}\,(\text{GDP}_1)}$	0.81	0.51	0.89	0.89	0.52	0.91	0.91	0.54	0.54	0.56	0.56
$\frac{\text{Std}\,(L_1)}{\text{Std}\,(\text{GDP}_1)}$	0.66	0.37	0.13	0.12	0.36	0.11	0.10	0.35	0.35	0.33	0.33
$\frac{\text{Std}\,(P_{I,1}I_1/P_{Y,1})}{\text{Std}\,(\text{GDP}_1)}$	2.84	3.95	1.42	1.50	3.88	1.39	1.46	4.69	5.53	4.67	5.50

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. For Columns (8)-(11), parameterizations are $\alpha_I = 0$ and $Z_{I,nt} = 1$ (i.e., there is no shock to the investment sector).

Table 11: Correlated shocks across countries and spillovers with an otherwise standard calibration. Shock to the final goods sector.

Moment	Data	Compl. markets		Bond economy		Fin. autarky	
		IRBC $Z_{I,n}=1$	IRBC $Z_{I,n}=Z_{X,n}$	IRBC $Z_{I,n}=1$	IRBC $Z_{I,n}=Z_{X,n}$	IRBC $Z_{I,n}=1$	IRBC $Z_{I,n}=Z_{X,n}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
International moments:							
$\text{Corr}(\text{GDP}_1, \text{GDP}_2)$	0.58	0.07	−0.06	0.13	0.00	0.11	0.06
$\text{Corr}(C_1, C_2)$	0.36	0.58	0.47	0.36	0.17	0.11	0.12
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	0.12	−0.19	0.34	−0.11	0.11	0.02
$\text{Corr}(L_1, L_2)$	0.42	−0.84	−0.38	−0.56	−0.16	0.10	0.00
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	0.68	0.63	0.66	0.07		
$\text{Corr}(\text{Exp}_1, \text{GDP}_1)$	0.32	0.93	0.91	0.98	0.77	0.89	0.86
$\text{Corr}(\text{Imp}_1, \text{GDP}_1)$	0.81	0.10	0.31	0.44	0.81	0.89	0.86
$\text{Corr}(\text{ReR}, \text{GDP}_1)$	0.13	0.68	0.67	0.66	0.62	0.67	0.67
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.23	0.14	0.14	0.05		
Domestic moments:							
$\text{Corr}(C_1, \text{GDP}_1)$	0.86	0.96	0.95	0.99	0.98	1.00	0.98
$\text{Corr}(L_1, \text{GDP}_1)$	0.87	0.86	0.97	0.93	0.98	0.99	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	1.00	0.99	0.99	0.99	1.00	0.99
$\frac{\text{Std}(C_1)}{\text{Std}(\text{GDP}_1)}$	0.81	0.74	0.44	0.82	0.51	0.90	0.54
$\frac{\text{Std}(L_1)}{\text{Std}(\text{GDP}_1)}$	0.66	0.26	0.55	0.16	0.49	0.10	0.46
$\frac{\text{Std}(P_{I,1}I_1/P_{Y,1})}{\text{Std}(\text{GDP}_1)}$	2.84	1.37	3.02	1.29	2.97	1.41	2.85

Notes: See notes to Table 3.

Table 12: Standard IRBC model with investment in terms of labor.

D.2 Role of Externalities

	Int. good shock			Final good shock		
$\psi_{X,K} =$	0	0.3	-1	0	0.3	-1
	(1)	(2)	(3)	(4)	(5)	(6)
International moments:						
Corr $(\text{GDP}_1, \text{GDP}_2)$	0.15	0.13	0.16	0.03	0.01	0.05
Corr (C_1, C_2)	0.17	0.18	0.13	0.05	0.06	0.01
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.14	0.09	0.17	0.02	-0.03	0.06
Corr (L_1, L_2)	0.13	0.06	0.18	0.01	-0.06	0.06
Corr $\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$						
Corr $(\text{Exp}_1, \text{GDP}_1)$	0.89	0.89	0.90	1.00	1.00	0.99
Corr $(\text{Imp}_1, \text{GDP}_1)$	0.89	0.89	0.90	1.00	1.00	0.99
Corr $(\text{ReR}, \text{GDP}_1)$	0.65	0.66	0.65	0.70	0.70	0.68
Std $\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$						
Domestic moments:						
Corr (C_1, GDP_1)	0.98	0.99	0.93	0.98	0.99	0.93
Corr (L_1, GDP_1)	0.99	0.99	0.99	0.99	0.99	0.99
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	1.00	1.00	1.00	1.00	1.00	1.00
$\frac{\text{Std}(C_1)}{\text{Std}(\text{GDP}_1)}$	0.45	0.56	0.29	0.45	0.56	0.29
$\frac{\text{Std}(L_1)}{\text{Std}(\text{GDP}_1)}$	0.39	0.31	0.51	0.39	0.31	0.51
$\frac{\text{Std}(P_{I,1}I_1/P_{Y,1})}{\text{Std}(\text{GDP}_1)}$	2.64	2.30	3.14	2.64	2.31	3.14

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 13: Capital externalities. Financial autarky.

	Complete markets						Bond economy					
	Int. good shock			Final good shock			Int. good shock			Final good shock		
$\psi_{X,K} =$	0	0.3	−1	0	0.3	−1	0	0.3	−1	0	0.3	−1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
International moments:												
Corr $(\text{GDP}_1, \text{GDP}_2)$	0.14	0.09	0.28	−0.05	−0.10	0.09	0.17	0.15	0.29	−0.04	−0.05	0.08
Corr (C_1, C_2)	0.79	0.81	0.72	0.45	0.51	0.26	0.65	0.60	0.65	0.40	0.39	0.37
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	−0.48	−0.63	−0.17	−0.73	−0.82	−0.52	−0.45	−0.56	−0.16	−0.72	−0.77	−0.52
Corr (L_1, L_2)	−0.51	−0.80	0.07	−0.52	−0.79	−0.02	−0.38	−0.63	0.11	−0.48	−0.70	−0.06
Corr $\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.44	−0.52	−0.66	−0.63	−0.67	−0.55	−0.55	−0.53	−0.66	−0.63	−0.67
Corr $(\text{Exp}_1, \text{GDP}_1)$	0.38	0.49	0.31	−0.19	−0.13	−0.22	0.27	0.29	0.29	−0.18	−0.08	−0.19
Corr $(\text{Imp}_1, \text{GDP}_1)$	0.95	0.93	0.97	0.91	0.89	0.92	0.96	0.97	0.97	0.91	0.90	0.93
Corr $(\text{ReR}, \text{GDP}_1)$	0.53	0.59	0.38	0.68	0.73	0.50	0.46	0.51	0.36	0.67	0.71	0.50
Std $\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.21	0.16	0.25	0.56	0.50	0.60	0.24	0.21	0.26	0.55	0.46	0.57
Domestic moments:												
Corr (C_1, GDP_1)	0.91	0.90	0.91	0.95	0.94	0.92	0.95	0.97	0.92	0.96	0.97	0.92
Corr (L_1, GDP_1)	0.94	0.87	0.98	0.96	0.91	0.98	0.95	0.91	0.98	0.96	0.93	0.98
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.93	0.91	0.96	0.91	0.89	0.94	0.93	0.92	0.96	0.91	0.89	0.94
$\frac{\text{Std}(C_1)}{\text{Std}(\text{GDP}_1)}$	0.50	0.60	0.33	0.51	0.60	0.35	0.53	0.66	0.34	0.52	0.64	0.34
$\frac{\text{Std}(L_1)}{\text{Std}(\text{GDP}_1)}$	0.40	0.37	0.49	0.37	0.33	0.48	0.36	0.28	0.49	0.36	0.28	0.49
$\frac{\text{Std}(P_{I,1}I_1/P_{Y,1})}{\text{Std}(\text{GDP}_1)}$	3.11	2.83	3.58	3.95	3.67	4.35	3.07	2.69	3.57	3.88	3.41	4.33

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_nL_n + R_nK_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 14: Correlated shocks across countries and spillovers with an otherwise standard calibration. Capital externalities.

D.3 Additional Estimation Results

Moment	Data	Compl.	Bond	Moment	Data	Compl.	Bond
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	0.58	0.02	0.03	$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.53	0.53
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.10	0.06	$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.86	0.94	0.93
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	-0.32	-0.32	$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.87	1.00	1.00
$\text{Corr}\left(L_1, L_2\right)$	0.42	0.01	0.02	$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.98	0.98
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	-0.49	-0.67	-0.67	$\text{Std}\left(\text{GDP}_1\right)$	1.67	1.99	2.01
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.32	-0.05	-0.05	$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.81	0.18	0.18
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.81	0.98	0.98	$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.66	0.58	0.58
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.13	0.35	0.36	$\frac{\text{Std}\left(P_{I,1}I_1/P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	2.84	4.20	4.18

Parameter estimates:

	$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y	σ_X	σ_Y	ρ_X	ρ_Y
Complete	0.00	0.00	0.00	0.013	0.000	0.37	0.00
Bond	0.00	0.00	0.00	0.013	0.000	0.43	0.00

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$, $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

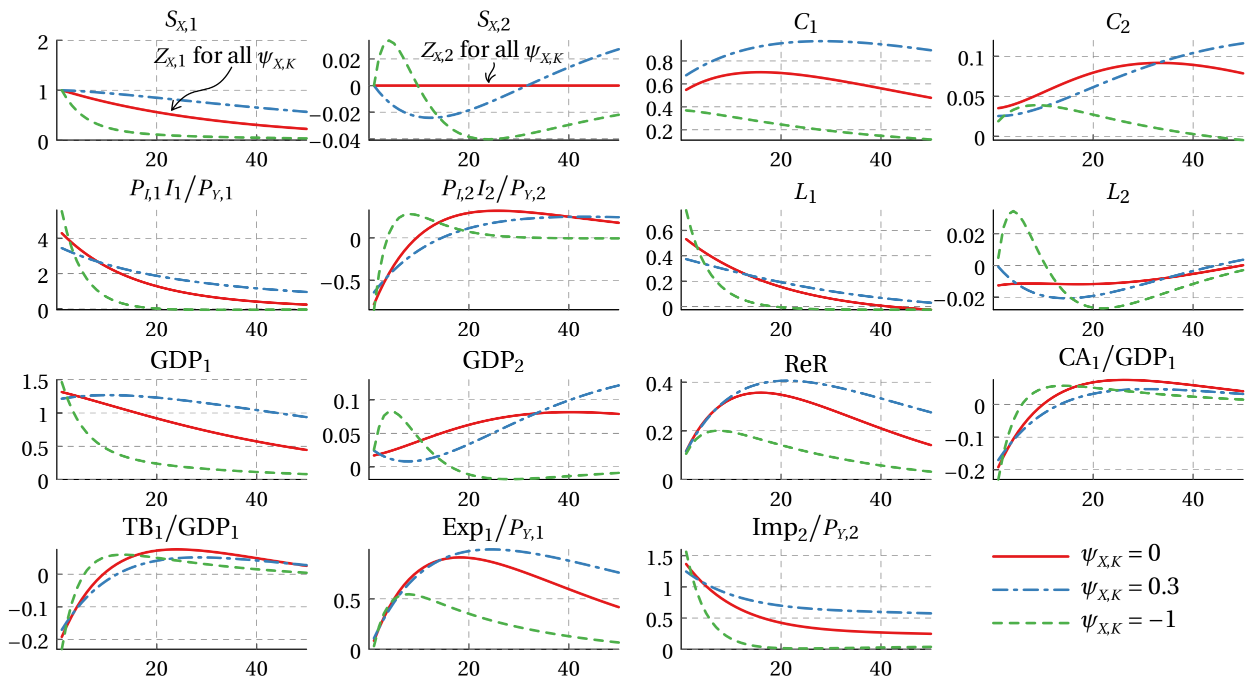
Table 15: Results of the estimation for $\sigma = 2$: IRBC economy.

Moment	Data	Compl.	Bond
Std (Exp_1)	3.94	1.92	1.93
Std (Imp_1)	5.42	1.97	2.03
$\frac{\text{Std (ReR)}}{\text{Std}(\text{GDP}_1)}$	2.23	0.23	0.24

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21} X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12} X_{12}/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

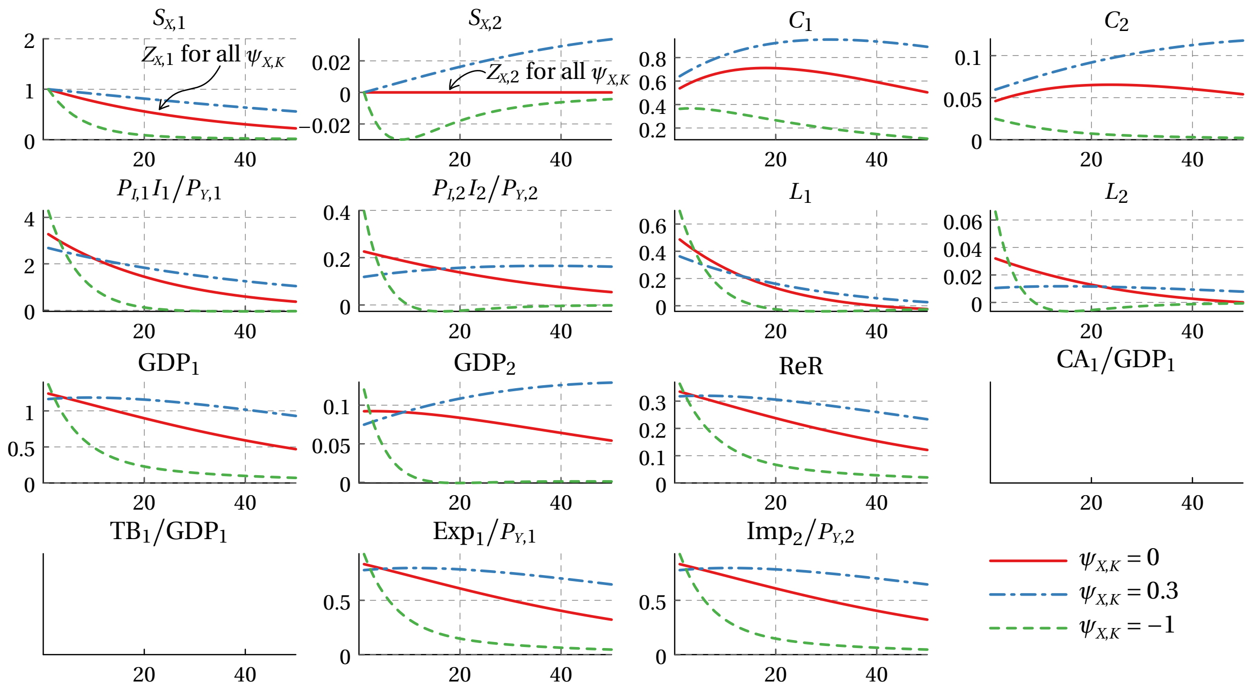
Table 16: Results of the estimation for $\sigma = 2$: untargeted moments.

E Additional Impulse-Response Functions



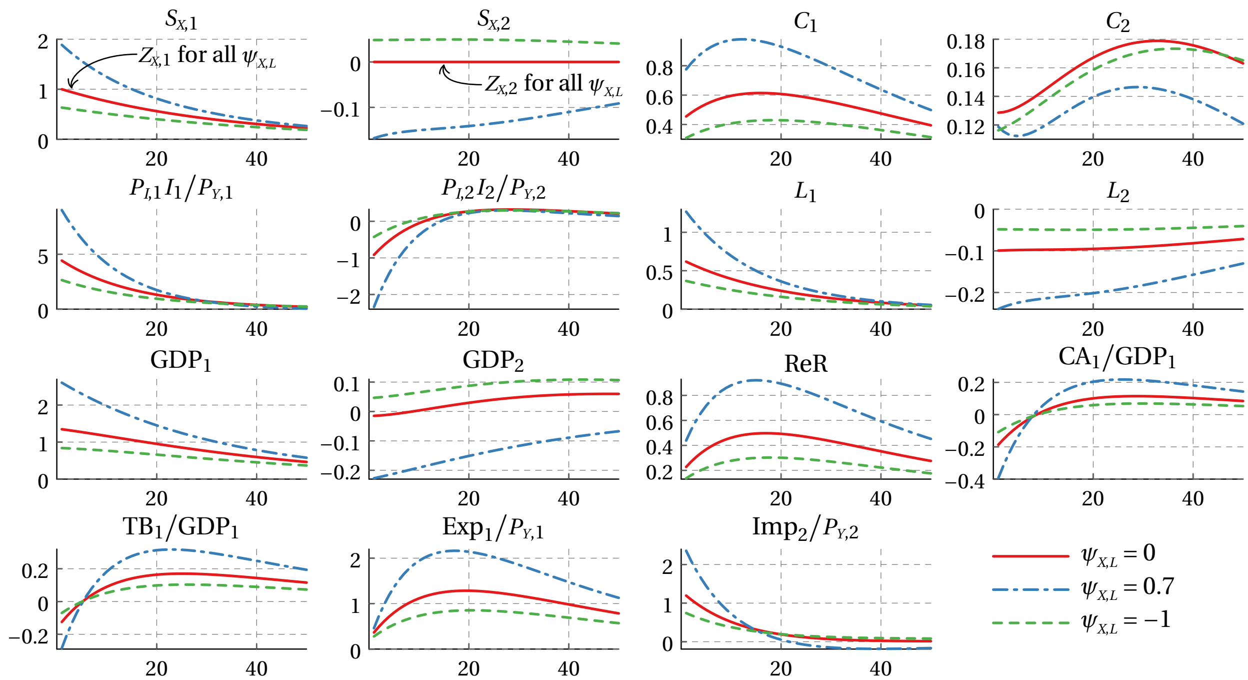
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$). All cases are for the bond economy. The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{X,K} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{X,K}$.

Figure 3: Impulse-response functions for $Z_{X,1}$. Capital externalities in the intermediate goods sector. Bond economy.



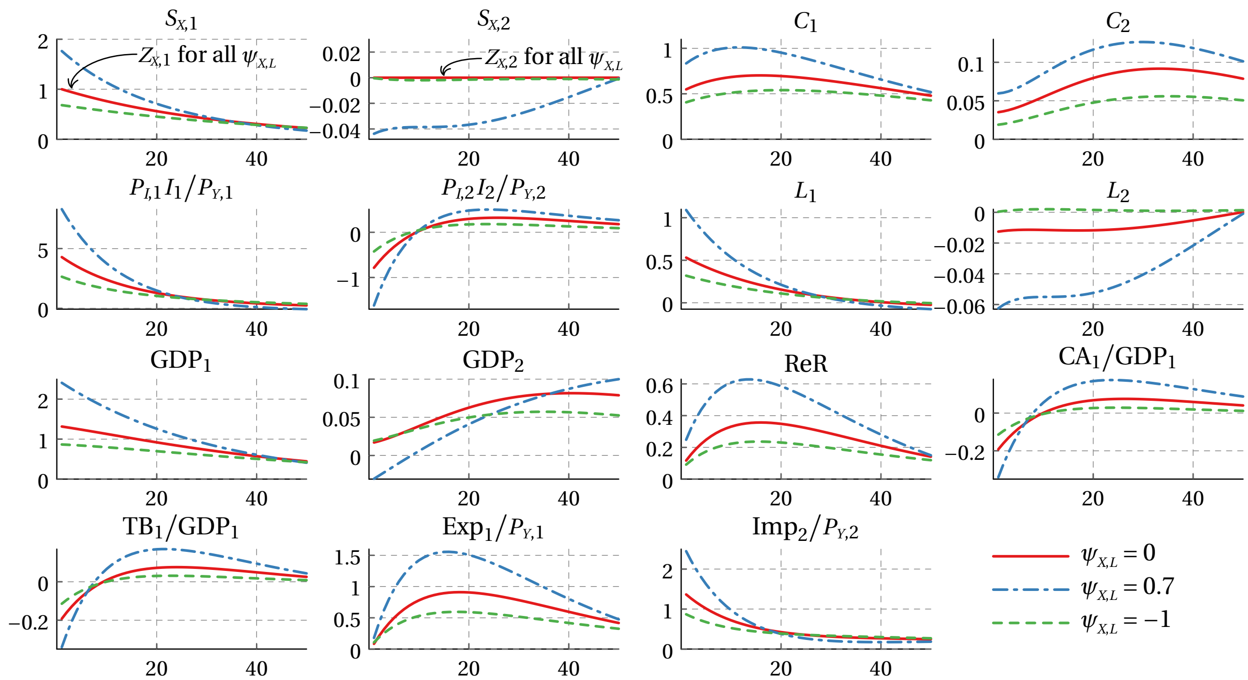
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$). All cases are for the financial autarky. The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{X,K} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{X,K}$.

Figure 4: Impulse-response functions for $Z_{X,1}$. Capital externalities in the intermediate goods sector. Financial autarky.



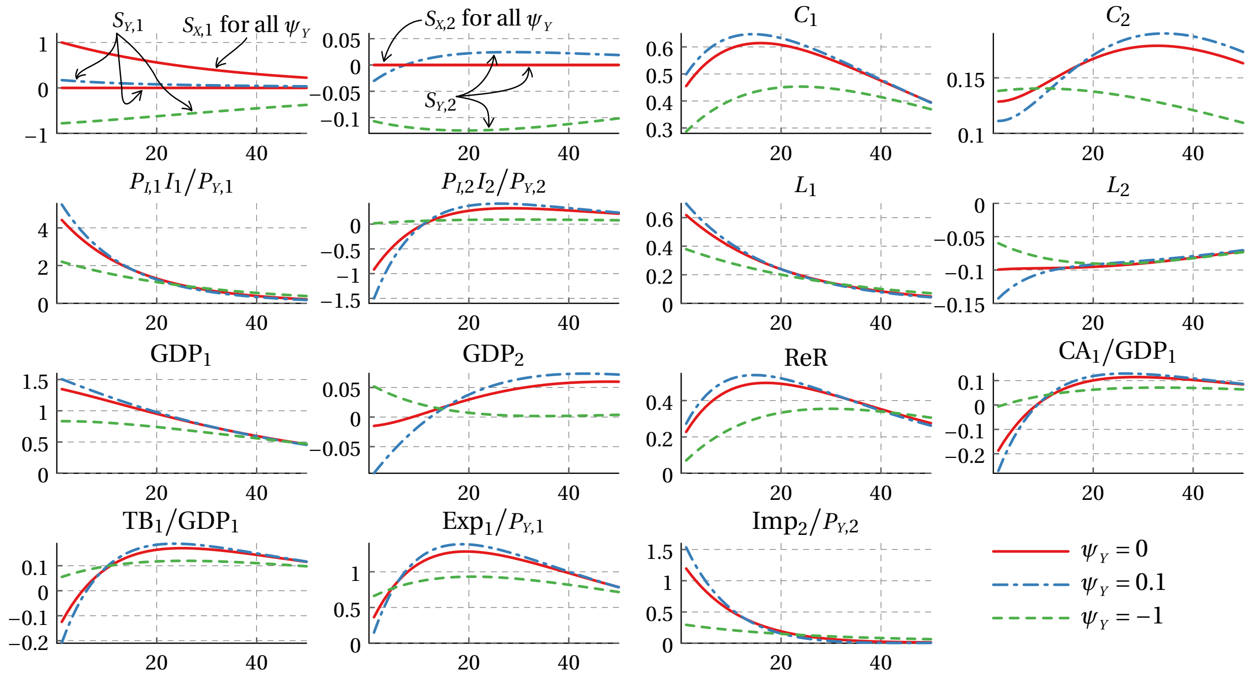
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,L} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,L} = 0.7$ and $\psi_{X,L} = -1$ differ from the case with $\psi_{X,L} = 0$ only in having labor externality in the production of intermediates (with the corresponding value for $\psi_{X,L}$). All cases are for the complete markets economy. The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{X,L} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{X,L}$.

Figure 5: Impulse-response functions for $Z_{X,1}$. Labor externalities in the intermediate goods sector. Complete markets.



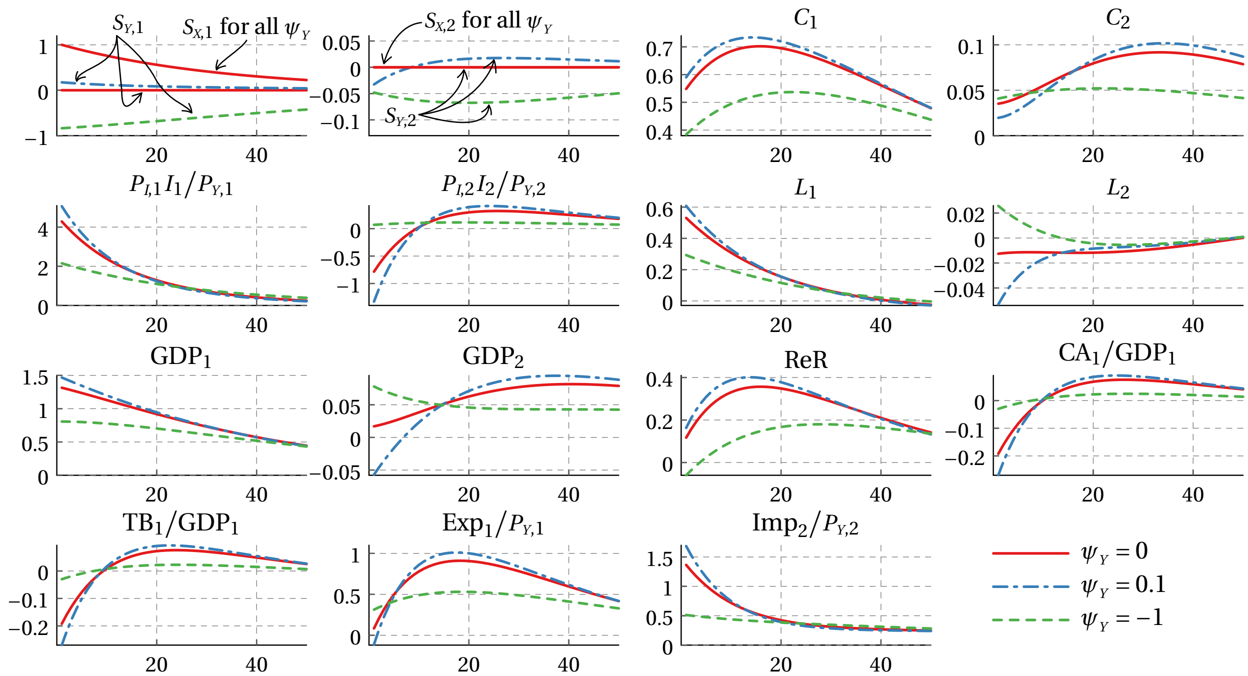
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,L} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,L} = 0.7$ and $\psi_{X,L} = -1$ differ from the case with $\psi_{X,L} = 0$ only in having labor externality in the production of intermediates (with the corresponding value for $\psi_{X,L}$). All cases are for the bond economy. The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{X,L} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{X,L}$.

Figure 6: Impulse-response functions for $Z_{X,1}$. Labor externalities in the intermediate goods sector. Bond economy.



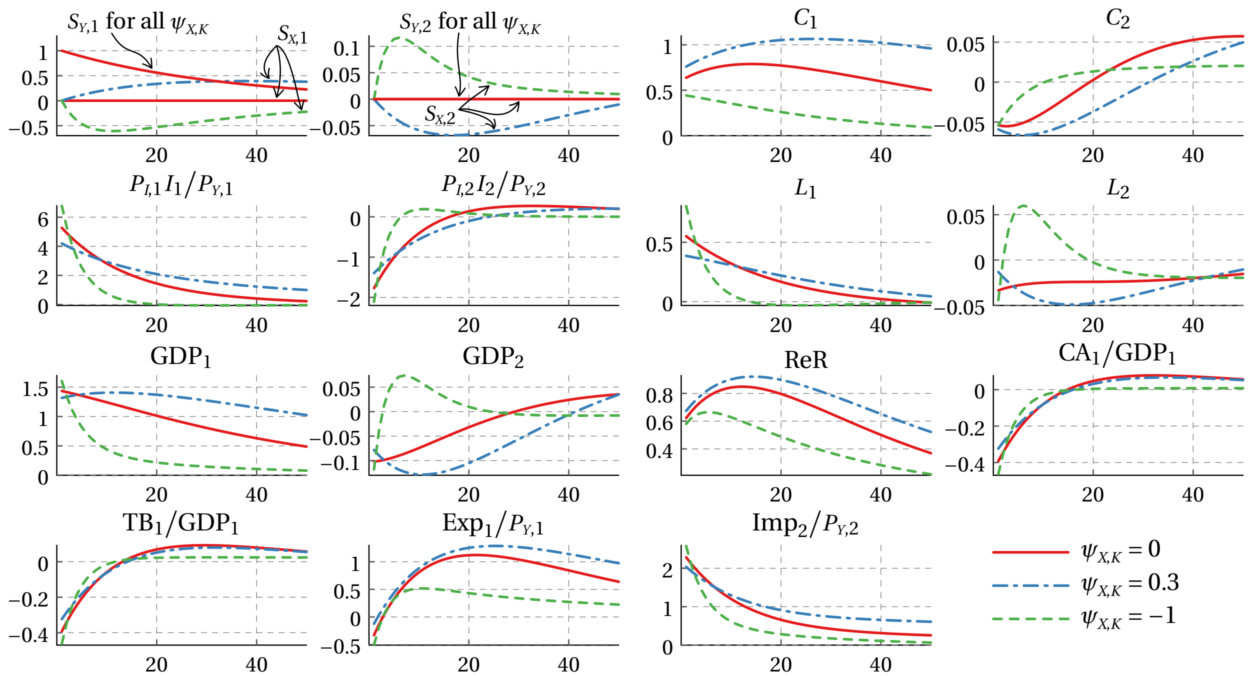
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_Y = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_Y = 0.1$ and $\psi_Y = -1$ differ from the case with $\psi_Y = 0$ only in having externality in production of the final aggregates (with the corresponding value for ψ_Y). All cases are for the complete markets economy.

Figure 7: Impulse-response functions for $Z_{X,1}$. Externality in the final aggregates sector. Complete markets.



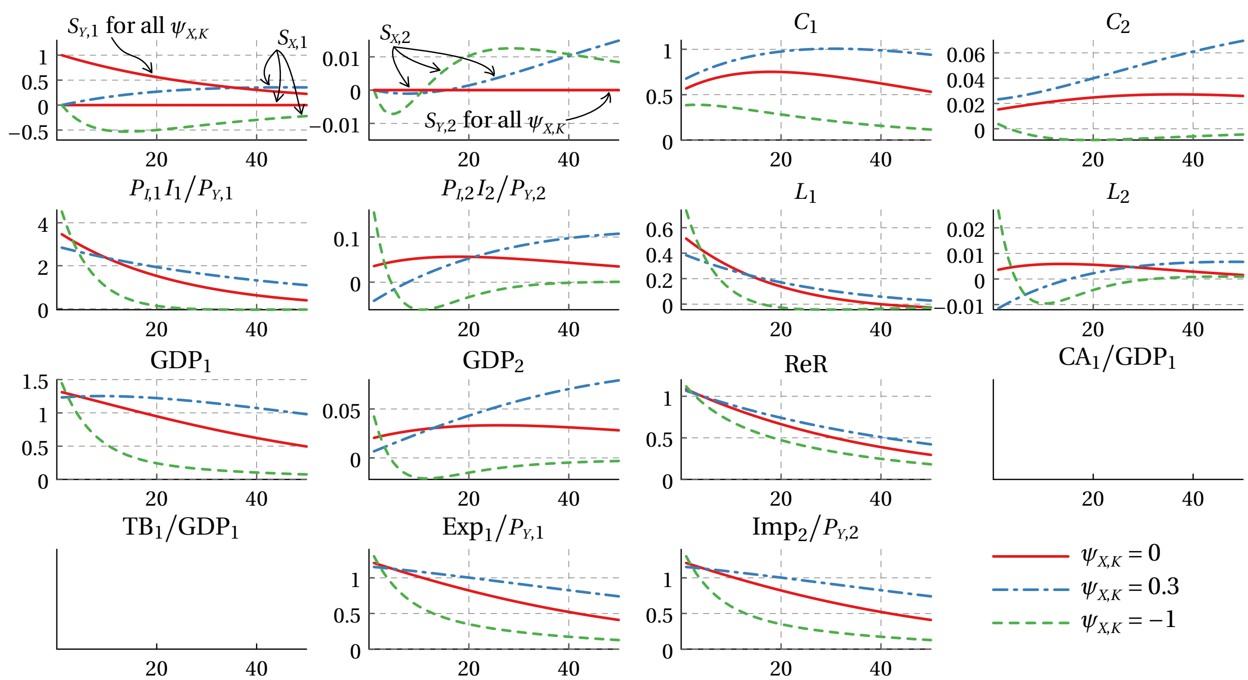
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_Y = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_Y = 0.1$ and $\psi_Y = -1$ differ from the case with $\psi_Y = 0$ only in having externality in production of the final aggregates (with the corresponding value for ψ_Y). All cases are for the bond economy.

Figure 8: Impulse-response functions for $Z_{X,1}$. Externality in the final aggregates sector. Bond economy.



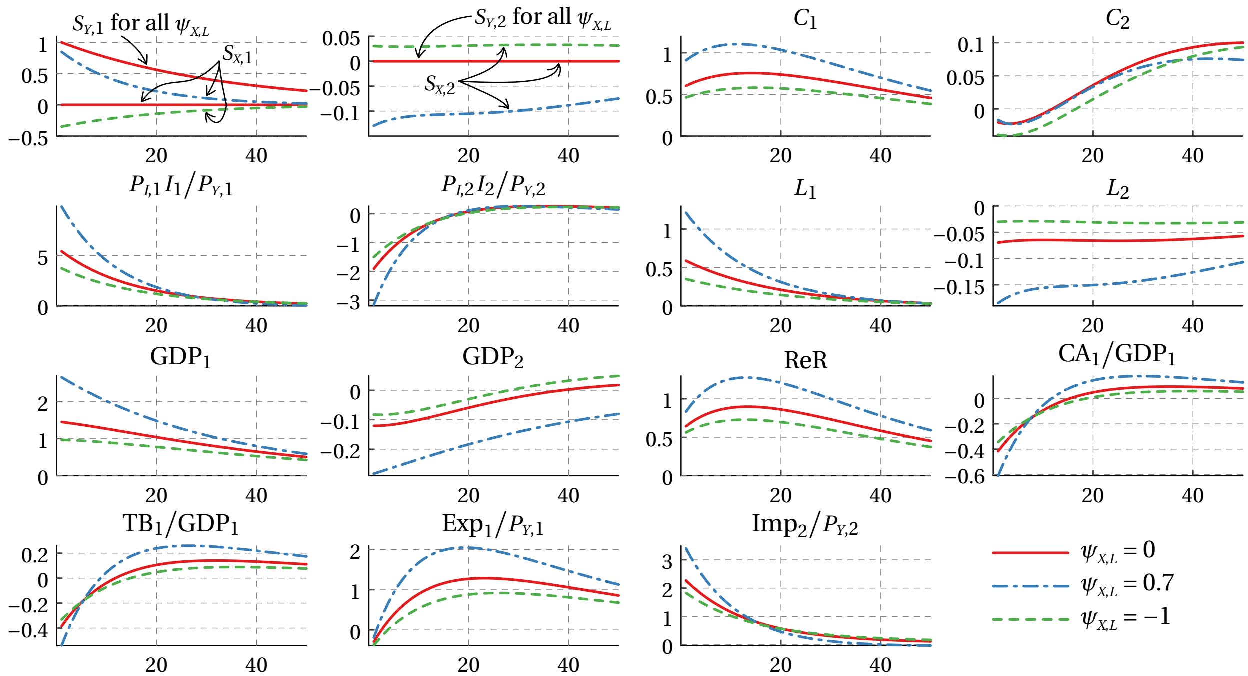
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the final good sector in country 1, $Z_{Y,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$). All cases are for the bond economy.

Figure 9: Impulse-response functions for $Z_{Y,1}$. Capital externalities in the intermediate goods sector. Bond economy.



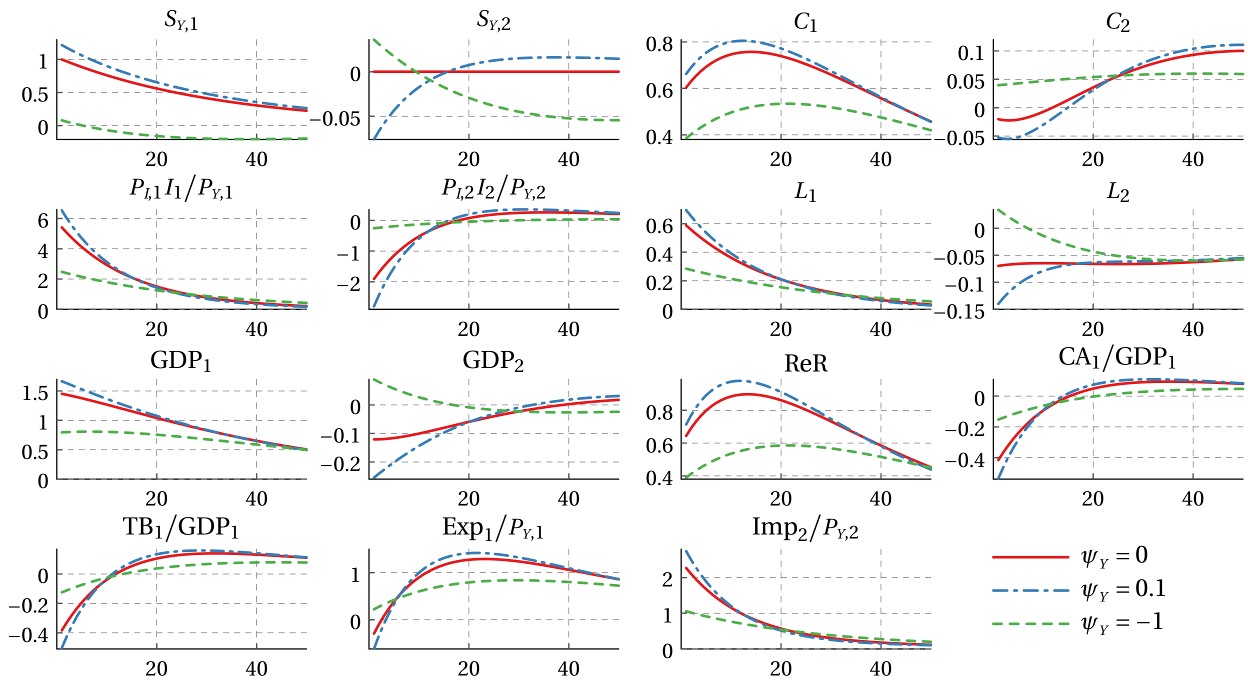
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the final good sector in country 1, $Z_{Y,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$). All cases are for financial autarky.

Figure 10: Impulse-response functions for $Z_{Y,1}$. Capital externalities in the intermediate goods sector. Financial autarky.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the final good sector in country 1, $Z_{Y,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,L} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,L} = 0.7$ and $\psi_{X,L} = -1$ differ from the case with $\psi_{X,L} = 0$ only in having labor externality in the production of intermediates (with the corresponding value for $\psi_{X,L}$). All cases are for the complete markets economy.

Figure 11: Impulse-response functions for $Z_{Y,1}$. Labor externalities in the intermediate goods sector. Complete markets.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the final good sector in country 1, $Z_{Y,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_Y = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_Y = 0.1$ and $\psi_Y = -1$ differ from the case with $\psi_Y = 0$ only in having externality in production of the final aggregates (with the corresponding value for ψ_Y). All cases are for complete markets. $S_{X,1}$ and $S_{X,2}$ do not respond to shock and, thus, are not depicted.

Figure 12: Impulse-response functions for $Z_{Y,1}$. Externality in the final aggregates sector. Complete markets.